Why Do People Give? Testing Pure and Impure Altruism*

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Online Appendix A:
Monotonically decreasing balanced-budget and unfunded crowd-out.

In this appendix we derive three results. First, we derive sufficient conditions for balanced-budget crowd-out ($\frac{\mathrm{d}g_i^*}{\mathrm{d}G_{-i}}|_{dw_i=-dG_{-i}}$) to be monotonically decreasing in giving-by-others. Second, we show that unfunded crowd-out ($\frac{\mathrm{d}g_i^*}{\mathrm{d}G_{-i}}|_{dw_i=0}$) is monotonically decreasing with Cobb-Douglas impure altruism. Third, we present necessary and sufficient conditions for separable impure altruism utility functions to have monotonically decreasing unfunded crowd-out.

Following Andreoni (1989) an impure altruist's preferences are given by $U(x_i, G, g_i)$, where x_i denotes private consumption, G the charity's output, and g_i i's gift to the charity. $G = \sum_{i=1}^{n} g_i$ is the sum of the charitable gifts, and $G_{-i} = \sum_{j \neq i} g_j$ the amount given-by-others. Normalizing prizes i's budget constraint is: $x_i + g_i \leq w_i$, where w_i denotes own income. Adding G_{-i} to both sides the budget constraint can be re-written as: $x_i + G \leq w_i + G_{-i}$. Setting $g_i = G - G_i$ the resulting first-order condition equals:

$$-U_x(x_i, G, G - G_{-i}) + U_G(x_i, G, G - G_{-i}) + U_{g_i}(x_i, G, G - G_{-i}) = 0. \tag{A.1}$$

The Engel curve for the public good derived from the first-order condition is a function of two arguments, social income ($Z_i = w_i + G_{-i}$) and giving-by-others (G_{-i})

$$G^* = q(w_i + G_{-i}, G_{-i}) \tag{A.2}$$

Thus in the impure altruism model the response to a change in own income $\mathrm{d}G^*/\mathrm{d}w_i \triangleq q_1$ does not equal that seen for a change in giving-by-others $\mathrm{d}G^*/\mathrm{d}G_{-i} \triangleq q_1 + q_2$. q_2 is the difference between the two effects = $\mathrm{d}G^*/\mathrm{d}G_{-i} - \mathrm{d}G^*/\mathrm{d}w_i$.

Equation (A.2) implies:

$$g_i^* = -G_{-i} + q(w_i + G_{-i}, G_{-i})$$

and

$$\mathrm{d}g_i^* = -\mathrm{d}G_{-i} + q_1[\mathrm{d}w_i + \mathrm{d}G_{-i}] \quad + \ q_2\, \mathrm{d}G_{-i}$$

Thus balanced-budget crowd-out equals $\frac{\mathrm{d}g_i^*}{\mathrm{d}G_{-i}}|_{dw_i=-dG_{-i}} = -1 + q_2$. A one dollar decrease in own income accompanied by a one dollar increase in the giving-by-others increases *i's* preferred provision of the public good by the amount q_2.

If $q_1 > 0$, $q_2 > 0$ and $q_1 + q_2 < 1$, then at the margin both altruism and warm-glow influence giving. The model reduces to the pure altruism model if $q_1 > 0$ and $q_2 = 0$, and it reduces to a model of pure warm-glow model if *i's* preferred level of the public good increases dollar-for-dollar with the unfunded amount provided by others: $dG^*/dG_{-i} = q_1 + q_2 = 1$; hence, if individuals are motivated at the margin by warm-glow only (no altruism), crowd-out in response to an unfunded increase in G_{-i} is $\frac{dg_i^*}{dG_{-i}}|_{dw_i=0} = -1 + q_1 + q_2 = 0$.

Examining the impure altruism model Ribar and Wilhelm (2002) show that although impure altruists are predicted to respond to changes in giving by others, unfunded crowd-out $\frac{dg_i^*}{dG_{-i}}|_{dw_i=0} = -1 + q_1 + q_2 < 0$, this prediction does not hold in the limit. Rather, they show that under weak conditions on preferences (concave utility and strictly operative warm-glow at all levels of G) as giving-by-others $G_{-i} \rightarrow \infty \Rightarrow q_1 + q_2 \rightarrow 1$. Hence, $\frac{dg_i^*}{dG_{-i}}|_{dw_i=0} \rightarrow 0$ as $G_{-i} \rightarrow \infty$. That is, the impure altruism model converges to a model where, at the margin, giving is motived only by pure warm-glow.[1]

To obtain a comparative static from the impure altruism model that is testable in an experiment, we need to secure that the associated shift in the marginal preferences from impure altruism to warm-glow is monotonic. We begin by showing sufficient conditions on preferences to secure that balanced-budget crowd-out is monotonically decreasing.

PROPOSITION 1. Consider a concave impurely altruistic utility function, with strictly operative warm-glow, and that satisfies the technical conditions described in footnote 22. Further, if utility is additively separable with positive third derivatives, then q_2 is monotonically increasing in G_{-i}.[2]

Proof: Differentiating the first-order condition (A.1) with respect to G_{-i} yields:

$$q_2 = (U_{gG} + U_{gg} - U_{gx}) / (U_{xx} + U_{gg} + U_{GG} - 2\, U_{Gx} - 2\, U_{gx} + 2\, U_{gG}) \tag{A.3}$$

which for additively separable utility functions reduces to:

$$q_2 = U_{gg} / (U_{xx} + U_{gg} + U_{GG}). \tag{A.4}$$

Differentiating the second derivatives with respect to G_{-i} yields:

$$\frac{dU_{xx}}{dG_{-i}} = U_{xxx} \frac{dx^*}{dG_{-i}} = U_{xxx}\,(1 - q_1 - q_2) > 0 \tag{A.5}$$

$$\frac{dU_{GG}}{dG_{-i}} = U_{GGG} \frac{dG^*}{dG_{-i}} = U_{GGG}\,(q_1 + q_2) > 0 \tag{A.6}$$

$$\frac{dU_{gg}}{dG_{-i}} = U_{ggg} \frac{dg^*}{dG_{-i}} = U_{ggg}\,(q_1 + q_2 - 1) < 0 \tag{A.7}$$

where the inequalities follow from the assumed positive third derivatives. Now differentiating (A.4) with respect to G_{-i}:

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[1] In addition there are several technical conditions: utility is twice continuously differentiable, has strictly positive first derivatives, U_G is finite for all $g_i > 0$, the second derivatives of $U(.,.,.)$ with respect to the two private goods x_i and g_i are finite for all levels of G, and $U_{xx} - 2U_{xg} + U_{gg}$ is bounded away from zero (again, for all levels of G). The assumption that warm-glow is operative also is needed to secure that the impure altruism model, in contrast to the pure altruism model, can predict individual giving in a large economy (Andreoni, 1989). As in Andreoni (1989) it is also assumed that the giving-by-others is addressing a need, through the charity, that itself remains constant.

[2] In the analysis of risk, a positive third derivative corresponds to prudence, which can be interpreted as the disutility of being faced with a specified risk decreasing as wealth gets higher (Eeckhoudt and Schlesinger, 2006).

$$\frac{dq_2}{dG_{-i}} = \frac{\frac{dU_{gg}}{dG_{-i}}(U_{xx} + U_{GG}) - U_{gg}\left(\frac{dU_{xx}}{dG_{-i}} + \frac{dU_{GG}}{dG_{-i}}\right)}{(U_{xx} + U_{GG} + U_{gg})^2}. \tag{A.8}$$

Concavity combined with the signs in (A.5)–(A.7) imply that $\frac{dq_2}{dG_{-i}}$ is positive.$\blacksquare$

Monotonically decreasing balanced-budget crowd-out secures a testable prediction of the impure altruism, because monotonicity implies balanced-budget crowd-out decreases when increasing G_{-i} between any two finite levels G_{-i}^{Low} and G_{-i}^{High}. Cobb-Douglas preferences meet the conditions in Proposition 1, thus securing decreasing balanced-budget crowd-out as shown in Figure 1.

For Cobb-Douglas preferences we can also show our second result. Namely that Cobb-Douglas preferences also have monotonically decreasing unfunded crowd-out ($\frac{dg_i^*}{dG_{-i}}|_{dw_i=0} \rightarrow 0$), and presents a set of conditions on preferences such that decreasing unfunded crowd-out is monotonic—hence the marginal motive for giving monotonically moves from impure altruism to warm-glow ($q_1 + q_2 \rightarrow 1$).

For the Cobb-Douglas result i's voluntary contribution is given by

$$g_i^* = -G_{-i} + ½\,[(1-\beta)\,G_{-i} + (\alpha+\beta)\,Z_i + \{[(1-\beta)\,G_{-i} + (\alpha+\beta)\,Z_i\,]^2 - 4\,\alpha\,G_{-i}Z_i\,\}^{½}\,]\,. \tag{A.9}$$

Differentiating with respect to G_{-i} to get unfunded crowd-out $\frac{dg_i^*}{dG_{-i}}|_{dw_i=0} = -1 + q_1 + q_2$, yields:

$$q_1 + q_2 = \frac{1}{2}\left[1 + \alpha + \frac{N}{S^{1/2}}\right] \tag{A.10}$$

where:

$$N \equiv (1-\alpha)^2 G_{-i} + [(\beta-\alpha) + (\alpha+\beta)\alpha]w_i \tag{A.11}$$

$$S \equiv (1-\alpha)^2 G_{-i}^2 + 2[(\beta-\alpha) + (\alpha+\beta)\alpha]G_{-i}w_i + (\alpha+\beta)^2 w_i^2. \tag{A.12}$$

Differentiating (A.10) with respect to G_{-i} indicates that:

$$sign\left[\frac{d(q_1+q_2)}{dG_{-i}}\right] = sign\left[S\frac{dN}{dG_{-i}} - N(\tfrac{1}{2})\frac{dS}{dG_{-i}}\right] \tag{A.13}$$

Noting that $\frac{dN}{dG_{-i}} = (1-\alpha)^2$ and $\frac{dS}{dG_{-i}} = 2N$, the term in square brackets on the right-hand side of (A.13) reduces to $S(1-\alpha)^2 - N^2$, and (A.11) and (A.12) used to show:

$$\begin{aligned} S(1-\alpha)^2 - N^2 &= \{(\alpha+\beta)^2(1-\alpha)^2 - [(\beta-\alpha) + (\alpha+\beta)\alpha]^2\}w_i^2 \\ &= 4\alpha\beta(1-\alpha-\beta)w_i^2 \end{aligned} \tag{A.14}$$

If (and only if) $\alpha + \beta < 1$, $\alpha > 0$, and $\beta > 0$, the right-hand side of (A.14) is strictly positive, implying $\frac{d(q_1+q_2)}{dG_{-i}}$ is positive and $\frac{\mathrm{d}g_i^*}{\mathrm{d}G_{-i}}|_{dw_i=0}$ monotonically decreases as G_{-i} increases. █

PROPOSITION 2. Consider a concave impurely altruistic utility function, with strictly operative warm-glow, that satisfies the technical conditions described in footnote 22, and further is additively separable. $q_1 + q_2$ is monotonically increasing in G_{-i} if and only if $\frac{U_{GGG}}{U_{GG}^2} > \frac{U_{xxx} - U_{ggg}}{(U_{xx} + U_{gg})^2}$.

Proof: In fashion parallel to obtaining equation (A.3), partially differentiating the first-order condition (A.1) with respect to social income Z_i yields:

$$q_1 = (U_{xx} - U_{xG} - U_{xg}) / (U_{xx} + U_{gg} + U_{GG} - 2\,U_{Gx} - 2\,U_{gx} + 2\,U_{gG}) \quad \text{(A.15)}$$

which for additively separable utility functions reduces to:

$$q_1 = U_{xx} / (U_{xx} + U_{gg} + U_{GG}), \quad \text{(A.16)}$$

which adding to (A.4):

$$q_1 + q_2 = (U_{xx} + U_{gg}) / (U_{xx} + U_{gg} + U_{GG}), \quad \text{(A.17)}$$

Differentiating (A.17) with respect to G_{-i}:

$$\frac{d(q_1+q_2)}{dG_{-i}} = \frac{U_{GG}\left(\frac{dU_{xx}}{dG_{-i}} + \frac{dU_{gg}}{dG_{-i}}\right) - (U_{xx} + U_{gg})\frac{dU_{GG}}{dG_{-i}}}{(U_{xx} + U_{GG} + U_{gg})^2}.$$

Using equations (A.5)–(A.7), the numerator of the right-hand side reduces to:

$$Numerator\left\{\frac{d(q_1 + q_2)}{dG_{-i}}\right\} = U_{GG}\,(U_{xxx} - U_{ggg})(1 - q_1 - q_2) - (U_{xx} + U_{gg})\,U_{GGG}(q_1 + q_2) \quad \text{(A.18)}$$

$$= \frac{1}{U_{xx} + U_{gg} + U_{GG}}\left[U_{GG}^2(U_{xxx} - U_{ggg}) - (U_{xx} + U_{gg})^2 U_{GGG}\right].$$

The (A.18) right-hand side term in square brackets is negative, implying $\frac{d(q_1+q_2)}{dG_{-i}}$ positive, if and only if $\frac{U_{GGG}}{U_{GG}^2} > \frac{U_{xxx} - U_{ggg}}{(U_{xx} + U_{gg})^2}$. █

Remark: Positive third derivatives and $U_{ggg} > U_{xxx}$ would satisfy the condition in Proposition 2 and therefore lead to $q_1 + q_2$ monotonically increasing in G_{-i}. Positive third derivatives ensure that the (negative) second derivatives monotonically move toward zero as G_{-i} increases. That combined with $U_{ggg} > U_{xxx}$

ensures that the second derivative with respect to giving moves toward zero faster than does the second derivative with respect to own consumption.

Online Appendix B: Instructions

Claim Check____________

Welcome

Thank you for agreeing to participate in our study on decision making. There are two parts of the study today. In the first part you are asked to make six decisions and in the second part you are asked to fill out a survey. When you have completed your decisions we will randomly select one of your six decisions for payment. Your total payment from the study will be the sum of the payment that results from your decision and $5 for showing up to the study. The entire study should take about an hour, and at the end you will be paid privately and in cash. A research foundation has provided the funds for this study.

We ask that you do not speak to each other or make comments, except to ask questions about the procedures of the study. We also ask that you do not discuss the procedures of the study with others outside this room.

Your Identity

Your identity is secret. You will never be asked to reveal it to anyone during the course of the study. Your name will never be associated with your decisions or with your answers on the survey. Neither the assistants nor the other participants will be able to link you to any of the decisions you make. In order to keep your decisions private, *please do not reveal your choices to any other participant.*

Claim Check

Attached to the top of this page is a yellow piece of paper with a number on it. This is your Claim Check. Each participant has a different number. We use claim checks to maintain secrecy about your decisions, payment, and identity. You will present your Claim Check to an assistant at the end of the study to receive your cash payment.

Please remove your claim check now, and put it in a safe place.

Decision Tasks

For the decision tasks you will be paired with a child in Southwestern Pennsylvania (Allegheny, Washington, Greene, and Fayette Counties). The child is between 1 and 12 years old, and the child's family home has suffered extensive fire damage. Most or all of the family's possessions have been lost. For each of your decisions you will be given an amount of money which you will be asked to allocate between the child and yourself. The money allocated towards the child will be spent on children's books. These books will be distributed to the child by the American Red Cross of Southwestern Pennsylvania, immediately after the child has been affected by a severe fire.

As soon as a fire is reported in Southwestern Pennsylvania, the American Red Cross is contacted and volunteers are dispatched to the site. They help the affected families find temporary shelter, provide them with clothing, a meal, and give them a comfort bag with essential toiletries. Each day an average of one family in Southwestern Pennsylvania experiences a severe fire. These families depend on the American Red Cross for emergency help to cope with the sudden loss of their home and belongings. Unfortunately the American Red Cross only has funds to provide these families with the bare essentials, and they do not provide any “comfort” items for the children of the affected families.

For the study today we have joined the American Red Cross of Southwestern PA to collect funds to buy books for the affected children. In each of the six decisions you will be given an amount of money which you are asked to allocate between the child you are paired with and yourself. In addition the foundation has agreed to donate a fixed amount of money towards the child independent of your allocation. Thus the total amount to be spent on the child is the sum of the foundation’s fixed donation and the allocation you make to the child. The amount of money that you can allocate between the child and you, as well as the foundation’s fixed donation to the child, will vary across the six decisions.

The American Red Cross will use the funds collected from your allocation and that of the foundation to purchase the child books. Each participant in this study is paired with a different child. If you choose not to allocate any funds to the child, then the money to be spent on the child will be limited to the research foundation’s fixed donation. Only you have the opportunity to allocate additional funds to the child. Neither the American Red Cross nor any other donors provide books to the child. Your decision alone determines how much will be spent on the child.

In explaining why the American Red Cross is seeking funds for books, their Emergency Preparedness Coordinator Sandi Wraith states “Children's needs are often overlooked in the immediate aftermath of a disaster because everyone is concerned primarily with putting the fire out, reaching safety, and finding shelter, food and clothing...just the basics of life. So many times, I've seen children just sitting on the curb with no one to talk to about what's happening...for this reason I've found trauma recovery experts in the community to work with us to train our volunteer responders in how to address children's needs at the scene of a disaster.......being able to give the children fun and distracting books will provide a great bridge for our volunteers to connect with kids and get them talking about what they've experienced.”

Once we are ready to proceed to the decisions, you will be given a decision folder and a calculator. The folder contains a decision task with six decisions on it, and an envelope. For each decision you will have to enter your preferred allocation. If you wish to receive a receipt from the American Red Cross for your allocation to the child, you will need to fill out the acknowledgment form. Note however that by doing so you will relinquish your anonymity. If you wish to remain anonymous, leave the acknowledgment form blank. When you have completed the decision form please place it in the envelope along with the acknowledgment form, instructions and the calculator.

When we have collected all the envelopes we will draw a number between 1 and 6 to determine which one of the decisions counts for payment. Since one decision is randomly selected for payment, you should be making your decision as if every decision counts.

Sample Decisions

Here is an example of the type of decision you will have to make. This is just an example to demonstrate how everything is calculated. The example is not meant to guide your decision in any way. On the actual decision sheets we want you to select the allocation that you like best.

Example: You have been given \$20 to allocate between the child and yourself. The research foundation's fixed donation towards the child is \$5. You must choose how much money to allocate towards the child and yourself.

You may choose to allocate nothing towards the child's books and \$20 to yourself. If this decision is selected for payment the foundation's fixed donation of \$5 is spent on the child and your payment from the decision will be \$20.

Alternatively you may choose to allocate \$20 towards the child and nothing to yourself. The money to be spent on the child's books will be \$20+\$5 = \$25, and your payment from the decision is \$0.

Finally, you may choose to allocate any amount between \$0 and \$20 to the child and the remainder to yourself. Suppose you choose to allocate \$8 towards the child and \$12 to yourself. If selected for payment the American Red Cross will receive \$8+\$5 = \$13 to spend on the child's books and your payment for the decision will be \$12.

Monitor Role

To verify that all the procedures of this study are followed we will select a participant to be the monitor of the study. If your Claim Check number is 8 you will be the monitor. The monitor will follow the assistants around to see that everything takes place as explained in these instructions. The monitor will receive a fixed payment for his or her time.

Once all decision forms have been collected all participants will be given a survey. While you are completing the survey the monitor will walk with two assistants to a separate room to oversee that the calculation of the funds for the child and you are performed as described in the instructions. Your payment will be placed along with a receipt in an envelope that has your claim check number on the face of it. The assistant will make out a check to the American Red Cross of Southwestern PA for the amount corresponding to the funds for the child determined by your allocation. One check will be made out for each child. This check as well as any relevant acknowledgment form will be placed in an addressed and stamped envelope to the American Red Cross. Once all the calculations have been completed an assistant will walk the monitor back to this room. A box of envelopes with your payments will be given to an assistant who has not seen your decision sheets. The monitor will then make a statement to you on the extent to which the instructions were followed as described in the instructions. Once you have completed your survey you may come to the front to collect your payment by showing your claim check. An assistant who has not seen your decision form will hand you the sealed envelope with your payment.

After the study is completed the monitor and an assistant will walk to the nearest mailbox (on Forbes next to the Hillman Library) where the monitor will drop the envelope in the mailbox. To prove that all procedures are followed the monitor will be asked to sign a certificate to that effect. This certificate will be posted outside 4916 Posvar Hall.

Upon receipt of the check and acknowledgment form the American Red Cross will send a letter affirming that the check has been used to buy books for the child according to the description above. This letter will be posted outside 4916 Posvar Hall.

If you are the monitor of this study please identify yourself by coming to the front of the room now.

If you have any questions about the procedures, please raise your hand now and one of us will come to your seat to answer your question.

Before we proceed to the decision task we want you to complete a brief quiz, to make sure you know how everything will be calculated.