Simple Model

- Consider a large group of entrepreneurs, each of whom has net worth, *N*.
- Each entrepreneur has access to a project with rate of return,

$$(1+R^k)\omega$$

- Here, ω is a unit mean, idiosyncratic shock experienced by the individual entrepreneur after the project has been started,

$$\int_0^\infty \omega dF(\omega) = 1$$

- *F* is lognormal cumulative distribution function

- Entrepreneur receives a contract from a bank, which specifies a rate of interest, *Z*, and a loan amount, *B*.
  - If entrepreneur cannot pay the interest rate, Z, the bank pays a monitoring cost and takes everything.

- Total assets acquired by the entrepreneur:

$$\overbrace{A}^{\text{total assets}} = \overbrace{N}^{\text{net worth}} + \overbrace{B}^{\text{loans}}$$

- Entrepreneur who experiences sufficiently bad luck, $\omega \leq \bar{\omega}$, looses everything.

- Cutoff, $\bar{\omega}$

$$\overbrace{(1+R^k)\bar{\omega}}^{\text{gross rate of return experience by entrepreneur with 'luck', } \bar{\omega}} \times \overbrace{A}^{\text{total assets}}$$

$$= \overbrace{ZB}^{\text{interest and principle owed by the entrepreneur}}$$

$$(1+R^k)\bar{\omega}A = ZB \rightarrow$$

$$\bar{\omega} = \frac{Z}{(1+R^k)} \frac{\frac{B}{N}}{\frac{A}{N}} = \frac{Z}{(1+R^k)} \frac{\overbrace{\frac{A}{N}}^{\text{leverage} = L} - 1}{\frac{A}{N}} = \frac{Z}{(1+R^k)} \frac{L-1}{L}$$

- Cutoff higher with:
  - higher leverage, *L*
  - higher $Z/(1+R^k)$

- Expected return to entrepreneur, over opportunity cost of funds:

Expected payoff for entrepreneur

$$\frac{\int_{\bar{\omega}}^{\infty}[(1+R^{k})\omega A - ZB]\,dF(\omega)}{N(1+R)}$$

For lower values of ω , entrepreneur receives nothing 'limited liability'.

opportunity cost of funds

- Rewriting entrepreneur's rate of return:

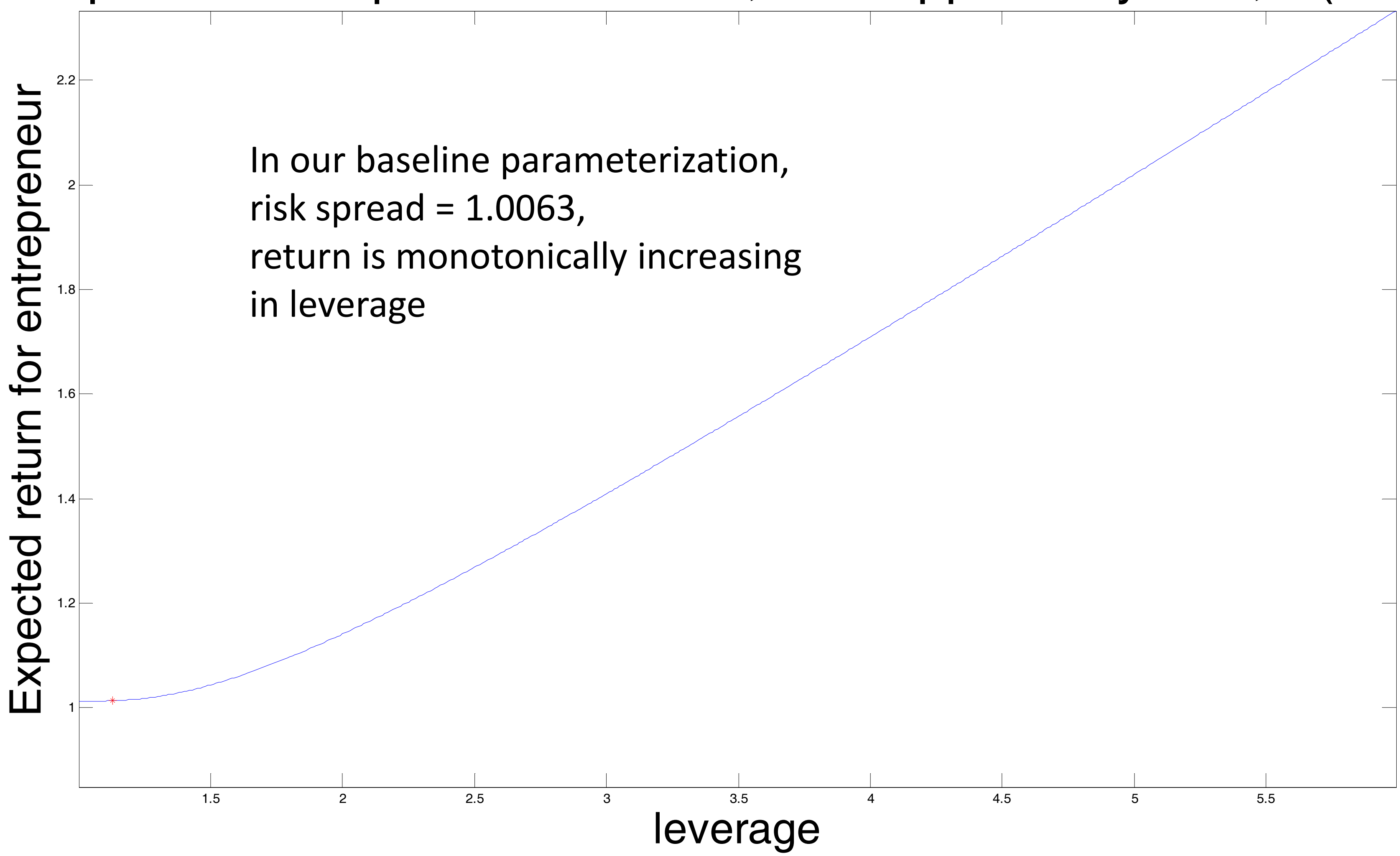
$$\frac{\int_{\bar{\omega}}^{\infty}[(1+R^k)\omega A - ZB]dF(\omega)}{N(1+R)} = \frac{\int_{\bar{\omega}}^{\infty}[(1+R^k)\omega A - (1+R^k)\bar{\omega}A]dF(\omega)}{N(1+R)}$$

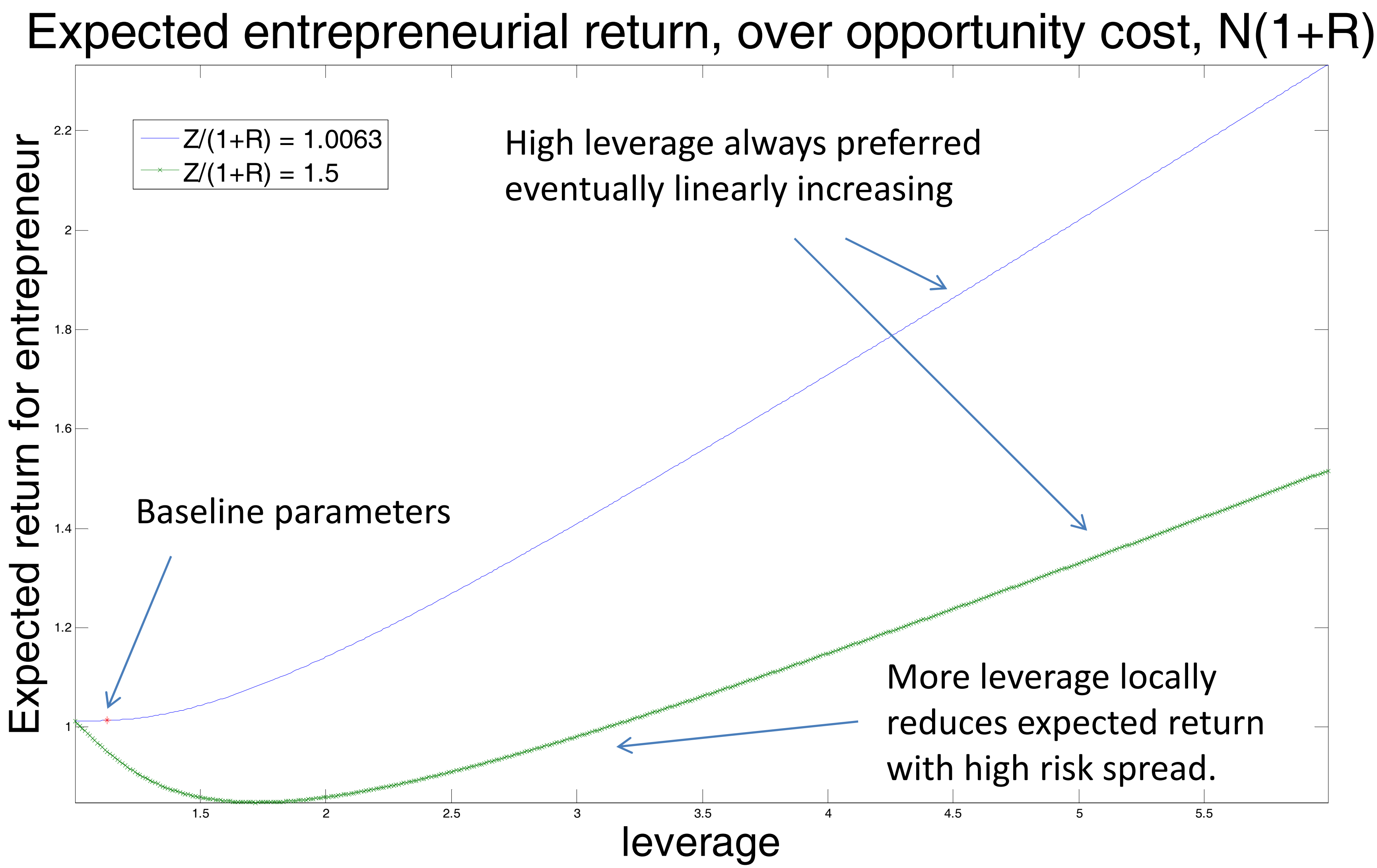
$$= \int_{\bar{\omega}}^{\infty}[\omega - \bar{\omega}]dF(\omega)\left(\frac{1+R^k}{1+R}\right)L$$

$$\bar{\omega} = \frac{Z}{(1+R^k)}\frac{L-1}{L} \rightarrow_{L\rightarrow\infty} \frac{Z}{(1+R^k)}$$

- Entrepreneur's return unbounded above
  - Risk neutral entrepreneur would always want to borrow an infinite amount (infinite leverage).

Expected entrepreneurial return, over opportunity cost, N(1+R)




Expected entrepreneurial return, over opportunity cost, N(1+R)
Z/(1+R) = 1.0063
Z/(1+R) = 1.5
High leverage always preferred
eventually linearly increasing
Baseline parameters
More leverage locally
reduces expected return
with high risk spread.
Expected return for entrepreneur
leverage
2.2
2
1.8
1.6
1.4
1.2
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5

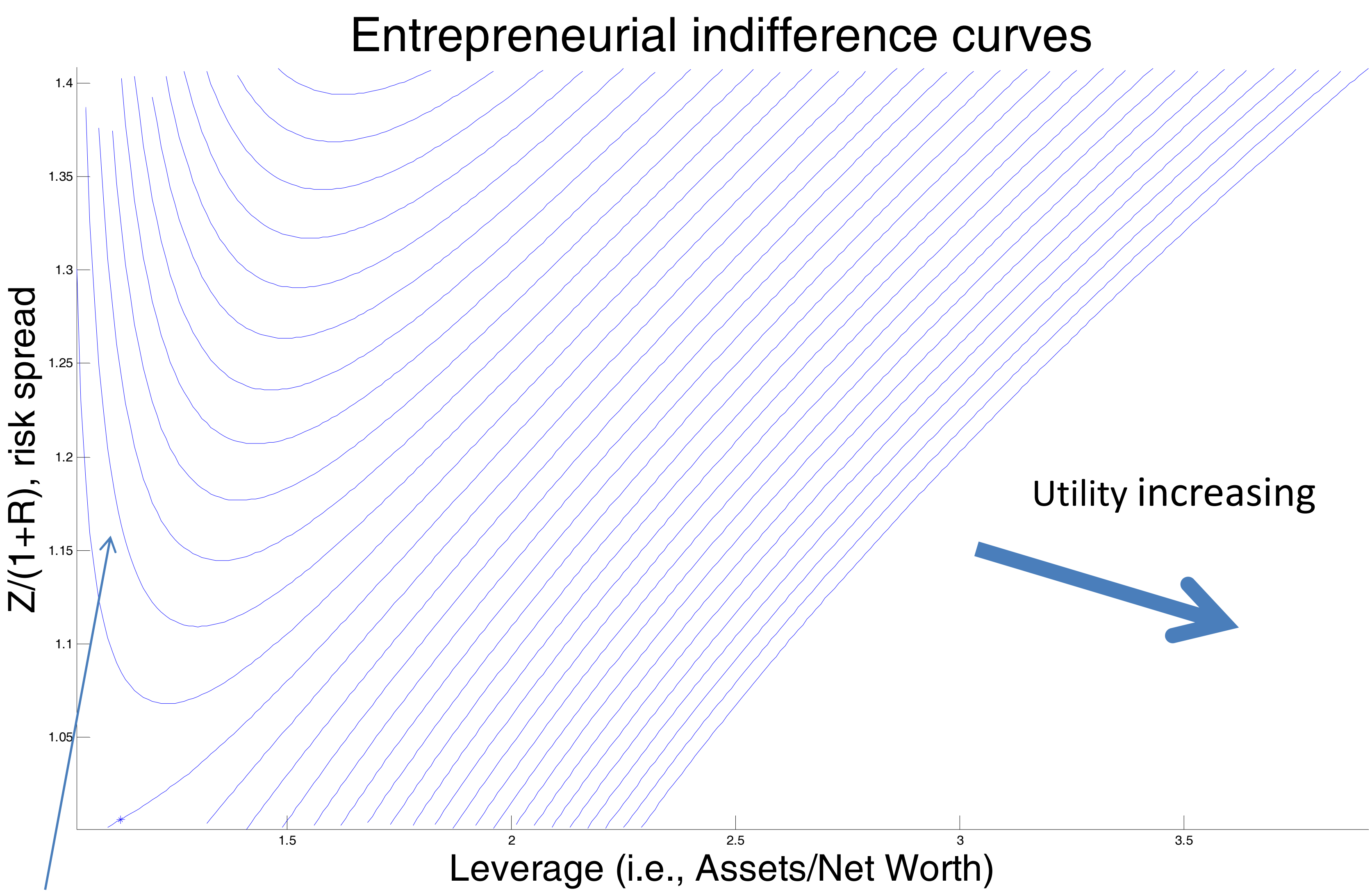
- If given a fixed interest rate, entrepreneur with risk neutral preferences would borrow an unbounded amount.

- Infinite leverage would break the bank, which would suffer excessive bankruptcy costs associated with bankrupt entrepreneurs.

- This is why a loan contract must specify *both* an interest rate, Z, and a loan amount, B.

- Need to represent preferences of entrepreneurs over Z and B.
  - Problem, local decrease in utility with more leverage makes entrepreneur indifference curves 'strange' ….

Indifference Curves Over *Z* and *B* Problematic

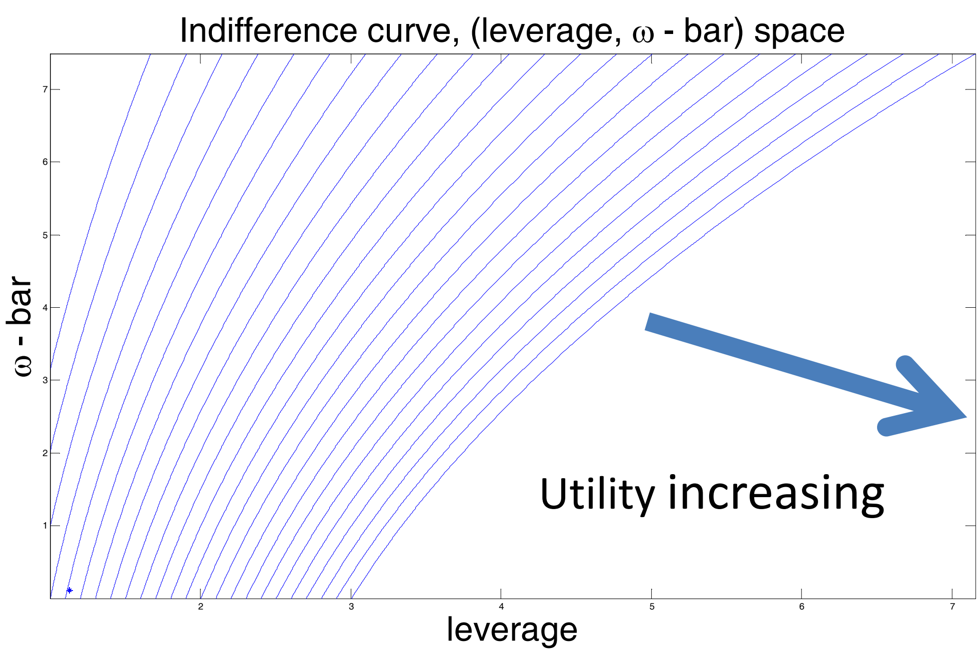


Downward-sloping indifference curves reflect local fall in net worth with rise in leverage when risk premium is high.

Solution to Technical Problem Posed by Result in Previous Slide

- Think of the loan contract in terms of the loan amount (or, leverage, (N+B)/N) and the cutoff, $\bar{\omega}$

$$\frac{\int_{\bar{\omega}}^{\infty}[(1+R^k)\omega A - ZB]dF(\omega)}{N(1+R)} = \int_{\bar{\omega}}^{\infty}[\omega - \bar{\omega}]dF(\omega)\left(\frac{1+R^k}{1+R}\right)L$$



$$L = \frac{A}{N} = \frac{N+B}{N}$$

The Bank

- Source of funds from households, at fixed rate, R

- Bank borrows B units of currency, lends proceeds to entrepreneurs.

- Provides entrepreneurs with standard debt contract, (Z,B)

Banks, cont'd

- Monitoring cost for bankrupt entprepreneur with $\omega < \bar{\omega}$

Bankruptcy cost parameter

$$\mu \omega dF(\omega)(1+R^k)A$$

- Bank zero profit condition

$$\overbrace{[1-F(\bar{\omega})]}^{\text{fraction of entrepreneurs with } \omega>\bar{\omega}} \quad \overbrace{ZB}^{\text{quantity paid by each entrepreneur with } \omega>\bar{\omega}}$$

$$+ \quad \overbrace{(1-\mu)\int_0^{\bar{\omega}} \omega dF(\omega)(1+R^k)A}^{\text{quantity recovered by bank from each bankrupt entrepreneur}}$$

$$= \quad \overbrace{(1+R)B}^{\text{amount owed to households by bank}}$$

Banks, cont'd

- Simplifying zero profit condition:

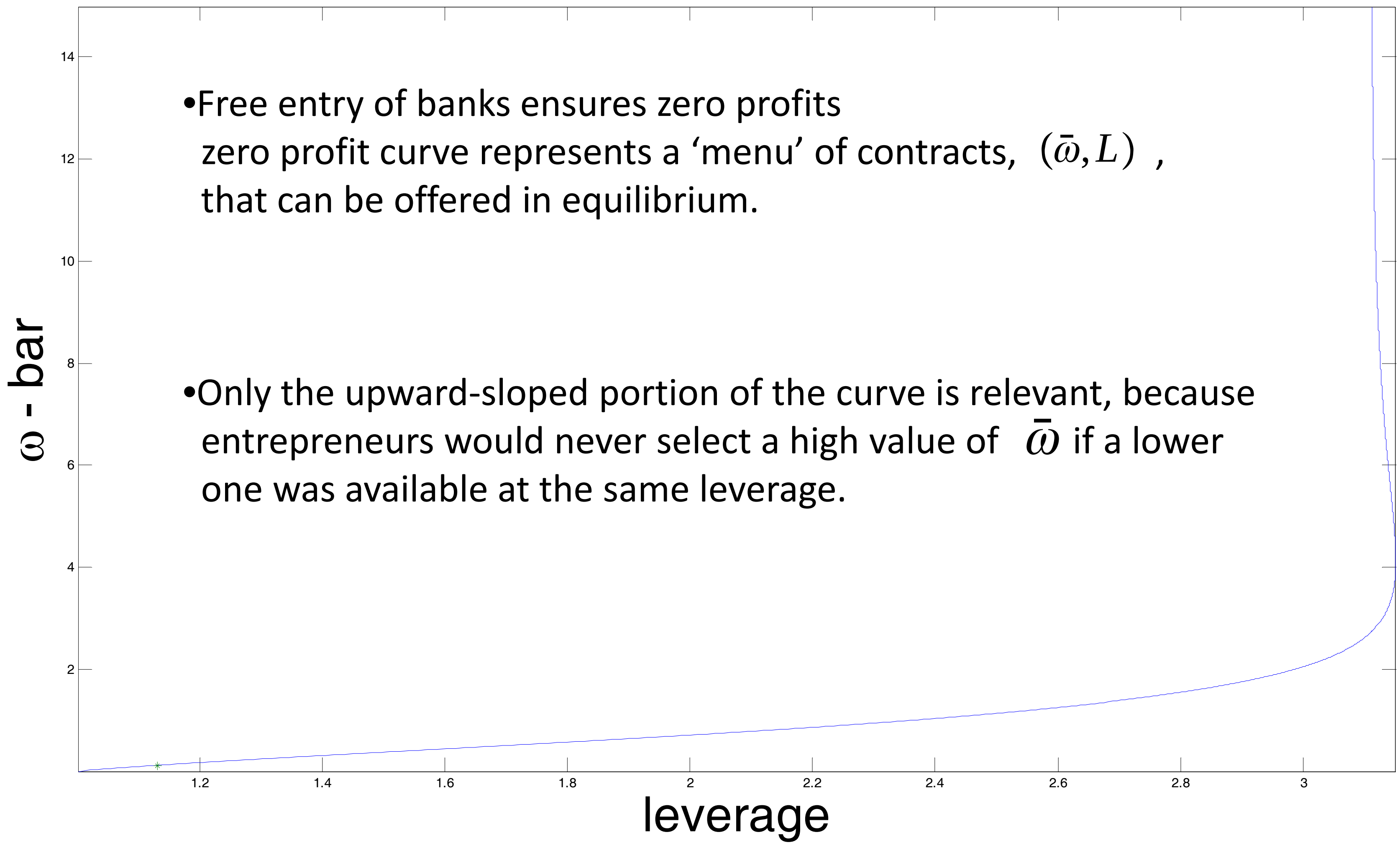
$$[1-F(\bar{\omega})]ZB + (1-\mu)\int_0^{\bar{\omega}} \omega dF(\omega)(1+R^k)A = (1+R)B$$

$$[1-F(\bar{\omega})]\bar{\omega}(1+R^k)A + (1-\mu)\int_0^{\bar{\omega}} \omega dF(\omega)(1+R^k)A = (1+R)B$$

$$\begin{aligned}[1-F(\bar{\omega})]\bar{\omega} + (1-\mu)\int_0^{\bar{\omega}} \omega dF(\omega) &= \frac{1+R}{1+R^k}\frac{B/N}{A/N} \\ &= \frac{1+R}{1+R^k}\frac{L-1}{L}\end{aligned}$$

- Expressed naturally in terms of $(\bar{\omega}, L)$

Bank zero profit condition, in (leverage, ω - bar) space



- Free entry of banks ensures zero profits
  zero profit curve represents a 'menu' of contracts, $(\bar{\omega}, L)$,
  that can be offered in equilibrium.

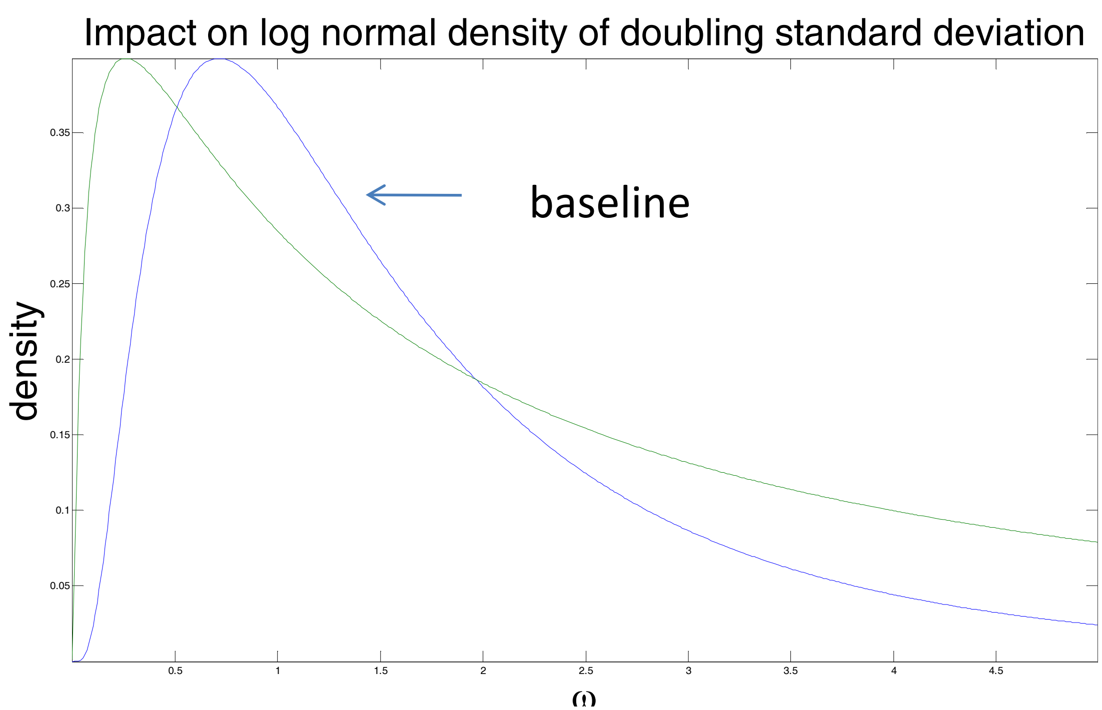
- Only the upward-sloped portion of the curve is relevant, because
  entrepreneurs would never select a high value of $\bar{\omega}$ if a lower
  one was available at the same leverage.

Effect of Increase in Risk, σ

- Keep

$$\int_0^\infty \omega dF(\omega) = 1$$

- But, double standard deviation of Normal underlying *F*.



Effect of Doubling in Risk, σ