

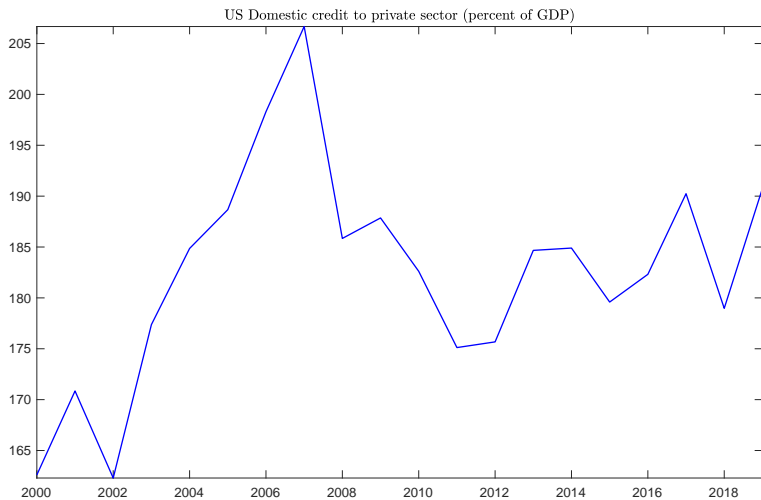
The Effects of Balance Sheet Constraints on Non Financial Firms

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Background

- Several shortcomings of standard New Keynesian model.
 - It assumes that the interest rate satisfies an Euler equation with the consumption of a single, representative household.
 - Evidence against that Euler equation is strong (Hall (JPE1978), Hansen-Singleton (ECMA1982), Canzoneri-Cumby-Diba (JME2007))
- Here, discuss Buera-Moll (AEJ-Macro2015) model of heterogeneous households and firms.
 - Shows how a model with heterogeneous households breaks Euler equation.
 - Shows how deleveraging can lead to many of the things observed in the Financial Crisis and Great Recession.
 - fall in output, investment, consumption, TFP, real interest rate.
- ‘Toy’ model that can be solved analytically, great for intuition.
- Earlier, similar models: Kahn-Thomas (JPE2013), Liu-Wang-Zha (ECMA2013).



Source: [World Bank](#)

Outline

- Hand-to-mouth workers
- Entrepreneurs (where all the action is)
- Aggregates: Loan Market, GDP, TFP, Consumption, Capital, Consumption
- Equilibrium
 - Computation.
 - Parameter values.
 - The dynamic effects of deleveraging.

Hand-to-mouth Workers

- Hand-to-mouth workers maximize

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t^W)^{1-\sigma}}{1-\sigma} - \frac{1}{1+\chi} L_t^{1+\chi} \right]$$

subject to:

$$C_t^W \leq w_t L_t.$$

- Solution:

$$L_t^{\frac{\chi+\sigma}{1-\sigma}} = w_t, \tag{1}$$

and labor supply is upward-sloping for $0 < \sigma < 1$.

Entrepreneurs

- i^{th} entrepreneur would like to maximize utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_{i,t}), \quad u(c) = \log c.$$

- i^{th} entrepreneur can do one of two things in t :
 - use time t resources plus debt, $d_{i,t} \geq 0$, to invest in capital and run a production technology in period $t+1$.
 - will do this if i 's technology is sufficiently productive.
 - use time t resources to make loans, $d_{i,t} < 0$, to financial markets.
 - will do this if i 's technology is unproductive.

Rate of Return on Entrepreneurial Investment

- i^{th} entrepreneur can invest $x_{i,t}$ and increase its capital in $t + 1$:

$$k_{i,t+1} = (1 - \delta) k_{i,t} + x_{i,t}, \delta \in (0, 1)$$

- In $t + 1$ entrepreneur can use $k_{i,t+1}$ to produce output:

$$y_{i,t+1} = (z_{i,t+1} k_{i,t+1})^\alpha l_{i,t+1}^{1-\alpha}, \alpha \in (0, 1),$$

where $l_{i,t+1}$ ~ amount of labor hired in $t + 1$ for wage, w_{t+1} .

- Technology shock, $z_{i,t+1}$, observed at time t , and
 - independent and identically distributed:
 - across i for a given t ,
 - across t for given i .
 - Density of z , $\psi(z)$; CDF of z , $\Psi(z)$.

Rate of Return on Entrepreneurial Investment

- i^{th} entrepreneur's time $t + 1$ profits:

$$\begin{aligned}\max_{l_{i,t+1}} \left[(z_{i,t+1} k_{i,t+1})^\alpha l_{i,t+1}^{1-\alpha} - w_{t+1} l_{i,t+1} \right] \\ = \pi_{t+1} z_{i,t+1} k_{i,t+1}\end{aligned}$$

$$\pi_{t+1} \equiv \alpha \left(\frac{1-\alpha}{w_{t+1}} \right)^{\frac{1-\alpha}{\alpha}}.$$

- Rate of return on one unit of investment in t :

$$\pi_{t+1} z_{i,t+1} + 1 - \delta.$$

The Decision to Invest or Lend

- The i^{th} entrepreneur can make a one period loan at t , and earn $1 + r_{t+1}$ at $t + 1$.
- Let \bar{z}_{t+1} denote value of $z_{i,t+1}$ such that return on investment same as return on making a loan:

$$\pi_{t+1}\bar{z}_{t+1} + 1 - \delta = 1 + r_{t+1}.$$

- If $z_{i,t+1} > \bar{z}_{t+1}$,
 - borrow as much as possible, subject to collateral constraint:

$$d_{i,t+1} \leq \theta_t k_{i,t+1}, \quad \theta_t \in [0, 1],$$

and invest as much as possible in capital.

- In this case borrow:

$$d_{i,t+1} = \theta_t k_{i,t+1}.$$

- If $z_{i,t+1} < \bar{z}_{t+1}$, then set $k_{i,t+1} = 0$ and make loans, $d_{i,t} < 0$.

Entrepreneur's Problem

- At t , maximize utility,

$$E_t \sum_{j=0}^{\infty} \beta^j u(c_{i,t+j})$$

subject to:

- given $k_{i,t}$ and $d_{i,t}$
- borrowing constraint
- budget constraint:

$$c_{i,t} + \overbrace{k_{i,t+1} - (1 - \delta) k_{i,t}}^{\text{investment, } x_{it}} \leq \overbrace{y_{i,t} - w_t l_{i,t}}^{\text{if entrepreneur invested in } t-1} + \underbrace{\pi_t z_{i,t} k_{i,t}}_{\text{increase in debt, net of financial obligations}} + \overbrace{d_{i,t+1} - (1 + r_t) d_{i,t}}$$

- Alternative representation of budget constraint:

$$c_{i,t} + \underbrace{k_{i,t+1} - d_{i,t+1}}_{\equiv a_{i,t+1}, \text{ 'net worth' }} \leq \underbrace{[\pi_t z_{i,t} + 1 - \delta] k_{i,t} - (1 + r_t) d_{i,t}}_{\equiv m_{i,t}, \text{ 'cash on hand' }}$$

Entrepreneur's Problem

- At t , maximize utility,

$$E_t \sum_{j=0}^{\infty} \beta^j u(c_{i,t+j}),$$

$u(c) = \log(c)$, subject to:

- given $k_{i,t}$ and $d_{i,t}$
- borrowing constraint
- budget constraint:

$$c_{i,t} + a_{i,t+1} \leq m_{i,t}$$

where,

$$a_{i,t+1} = k_{i,t+1} - d_{i,t+1}, \quad m_{i,t} = [\pi_t z_{i,t} + 1 - \delta] k_{i,t} - (1 + r_t) d_{i,t}$$

- Optimal choice of next period's net worth:

$$a_{i,t+1} = \beta m_{i,t}, \quad c_{i,t} = (1 - \beta) m_{i,t}.$$

Entrepreneur's Problem

- For $z_{i,t+1} \geq \bar{z}_{t+1}$, max debt and capital:

$$d_{i,t+1} = \theta_t k_{i,t+1} = \theta_t (d_{i,t+1} + a_{i,t+1})$$
$$\rightarrow d_{i,t+1} = \frac{\theta_t}{1 - \theta_t} a_{i,t+1}, \quad k_{i,t+1} = \frac{1}{1 - \theta_t} a_{i,t+1}$$

– Example:

- if $\theta_t = \frac{2}{3}$, then leverage = $1 / (1 - \theta_t) = 3$.
 - if net worth, $a_{i,t+1} = 100$, then $k_{i,t+1} = 300$ and $d_{i,t+1} = 200$.
- For $z_{i,t+1} < \bar{z}_{t+1}$, $k_{i,t+1} = 0$ and $d_{i,t+1} < 0$ (i.e., lend)
 - upper bound on lending: $d_{i,t+1} = -m_{i,t}$, all cash on hand.
 - won't go to upper bound with log utility.

Aggregates: Demand for Loans

- The total amount of cash on hand for all entrepreneurs, M_t , is

$$M_t = \int_i m_{i,t} di.$$

- Total demand for loans:
 - Since the $z_{i,t+1}$'s are distributed randomly to entrepreneurs, the cash in hand of the $[1 - \Psi(\bar{z}_{t+1})]$ investing entrepreneurs is:

$$[1 - \Psi(\bar{z}_{t+1})] M_t.$$

- Each of these entrepreneurs borrows $d_{i,t+1} = \theta_t / (1 - \theta_t) \beta m_{i,t}$, so total borrowing by investing entrepreneurs is

$$\beta \frac{\theta_t}{1 - \theta_t} [1 - \Psi(\bar{z}_{t+1})] M_t.$$

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$$\beta \frac{\theta_t}{1 - \theta_t} [1 - \Psi(\bar{z}_{t+1})] M_t.$$

Aggregates: Supply of Loans and Loan Market Clearing

- Total supply of loans:
 - Since the $z_{i,t+1}$'s are distributed randomly to entrepreneurs, the cash in hand of the $\Psi(\bar{z}_{t+1})$ non-investing entrepreneurs is:

$$\Psi(\bar{z}_{t+1}) M_t.$$

- Each of these entrepreneurs lends $-d_{i,t+1} = \beta m_{it}$, so total borrowing by investing entrepreneurs is

$$\beta \Psi(\bar{z}_{t+1}) M_t.$$

- Loan market clearing implies:

$$\beta \frac{\theta_t}{1 - \theta_t} [1 - \Psi(\bar{z}_{t+1})] M_t = \beta \Psi(\bar{z}_{t+1}) M_t,$$

or,

$$\Psi(\bar{z}_{t+1}) = \theta_t. \quad (2)$$

Aggregates: Gross Domestic Product

- The i^{th} firm's production function is:

same for each i , because all face same w_t

$$y_{it} = (z_{i,t}k_{i,t})^\alpha l_{i,t}^{1-\alpha} = \overbrace{\left(\frac{z_{i,t}k_{i,t}}{l_{i,t}}\right)^\alpha} l_{i,t}.$$

- Ratios equal ratio of sums:

$$\frac{z_{i,t}k_{i,t}}{l_{i,t}} = \frac{\int_i z_{i,t}k_{i,t}di}{\int_i l_{i,t}di} = \frac{\int_i z_{i,t}k_{i,t}di}{L_t}.$$

- GDP

$$\begin{aligned} Y_t &= \int_i y_{i,t}di = \left(\frac{\int_i z_{i,t}k_{i,t}di}{L_t}\right)^\alpha \int_i l_{i,t}di \\ &= \left(\frac{\int_i z_{i,t}k_{i,t}di}{L_t}\right)^\alpha L_t \end{aligned}$$

Aggregates: GDP, TFP and wage

- With some algebra, can establish:

$$Y_t = \left(\underbrace{E[z|z > \bar{z}_t]}_{=E[z|z > \bar{z}_t]} \times K_t \right)^\alpha L_t^{1-\alpha} = Z_t K_t^\alpha L_t^{1-\alpha}, \quad (3)$$

$$Z_t \equiv (E[z|z > \bar{z}_t])^\alpha. \quad (4)$$

- Simple intuition:
 - Aggregate output, Y_t , a function of aggregate capital and labor, and (endogenous) TFP, Z_t .
 - Z_t average TFP of firms in operation.
- Aggregate wage:

$$w_t = (1 - \alpha) \frac{Y_t}{L_t}. \quad (5)$$

Aggregates: Consumption

- Integrating over entrepreneurs' budget constraints:

$$\begin{aligned} & \int_i [c_{i,t} + k_{i,t+1} - d_{i,t+1}] di \\ &= \int_i [y_{i,t} - w_t l_{i,t} + (1 - \delta) k_{i,t} - (1 + r_t) d_{i,t}] di \end{aligned}$$

- Using loan market clearing, $\int_i d_{i,t} di = 0$:

$$C_t^E + K_{t+1} - (1 - \delta) K_t = Y_t - \underbrace{\overbrace{w_t \int_i l_{i,t} di}^{=C_t^W}}_{(1 - \alpha) Y_t},$$

where

$$C_t^E = \int_i c_{i,t} di$$

- Then,

$$C_t^E + K_{t+1} - (1 - \delta) K_t = \alpha Y_t. \quad (6)$$

Aggregates: Capital Accumulation

- Entrepreneur decision rule:

$$\begin{aligned}a_{i,t+1} &\equiv k_{i,t+1} - d_{i,t+1} \\ &= \beta [y_{i,t} - w_t l_{i,t} + (1 - \delta) k_{i,t} - (1 + r_t) d_{i,t}]\end{aligned}$$

- Integrating over all entrepreneurs (using $\int_i d_{i,t} di = 0$):

$$K_{t+1} = \beta [\alpha Y_t + (1 - \delta) K_t] \quad (7)$$

- Note: K_{t+1} is not a direct function of θ_t .
 - If θ_t falls, then borrowing drops by investing entrepreneurs, driving down r_{t+1} .
 - Lower r_{t+1} encourages unproductive entrepreneurs who previously were lending, to switch to borrowing and buying more capital.
 - The positive and negative effects on capital purchases cancel, which is why K_{t+1} is not a function of θ_t .

Aggregates: Consumption Euler Equation

- Interestingly, aggregate entrepreneurial consumption satisfies Euler equation:

$$\begin{aligned}\frac{C_{t+1}^E}{C_t^E} &= \frac{(1 - \beta) [\alpha Y_{t+1} + (1 - \delta) K_{t+1}]}{(1 - \beta) [\alpha Y_t + (1 - \delta) K_t]} \\ &= \beta \frac{\alpha Y_{t+1} + (1 - \delta) K_{t+1}}{K_{t+1}} \\ &= \beta \left[\alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta \right]\end{aligned}$$

- But,
 - does not hold for aggregate consumption, $C_t = C_t^W + C_t^E$.
 - does not hold relative to the interest rate.

Equilibrium

- Seven variables:

$$L_t, w_t, C_t^E, Y_t, K_{t+1}, \bar{Z}_t, Z_t.$$

- Seven equations: (1), (2), (3), (4), (5), (6), (7).
- Exogenous variables:

$$K_1, \theta_0, \theta_1, \theta_2, \dots, \theta_T$$

Equilibrium Computation

- Responses to exogenous variables:
 - For $t = 1, 2, \dots, T$, $\bar{z}_t = \Psi^{-1}(\theta_{t-1})$ using (2);
 $Z_t \equiv (E[z|z > \bar{z}_t])^\alpha$ using (4),
 - Using (1) and (5) for L_t and w_t ; (3) for Y_t ; (7) for K_{t+1} ; and (6) for C_t^E :

$$L_t = [(1 - \alpha) Z_t K_t^\alpha]^{\frac{1-\sigma}{\chi+\sigma+(1-\sigma)\alpha}}$$

$$w_t = L_t^{\frac{\chi+\sigma}{1-\sigma}}$$

$$Y_t = Z_t K_t^\alpha L_t^{1-\alpha}$$

$$K_{t+1} = \beta [\alpha Y_t + (1 - \delta) K_t]$$

$$C_t^E = (1 - \beta) [\alpha Y_t + (1 - \delta) K_t]$$

sequentially, for $t = 1, 2, 3, \dots, T$.

Equilibrium Computation

- Other variables: interest rate and profits for $t = 1, 2, \dots, T$:

$$\pi_t = \alpha \left(\frac{1 - \alpha}{w_t} \right)^{\frac{1-\alpha}{\alpha}}$$
$$1 + r_t = \pi_t \bar{z}_t + 1 - \delta$$

- Pareto distribution:

$$\psi(z) = \eta z^{-(\eta+1)}, \eta = 2.1739, 1 \leq z$$
$$\Psi(\bar{z}) = 1 - \bar{z}^{-\eta}, Ez = \frac{\eta}{\eta - 1} = 1.85.$$

Parameter Values and Steady State

- Other parameters:

$$\alpha = 0.36, \delta = 0.10, \beta = 0.97, \chi = 1, \sigma = 0.9.$$

- Steady state, with $\theta = \frac{2}{3}$:

$$Y = 3.45, K = 9.50, L = 1.04, C = 2.50,$$

$$w = 2.12, \bar{z} = 1.66, Z = 1.50,$$

$$Z^{\frac{1}{\alpha}} = E[z|z > \bar{z}] = 3.07, 1 + r = 0.97,$$

$$C^E/C = 0.12, C^W/C = 0.88,$$

after rounding.

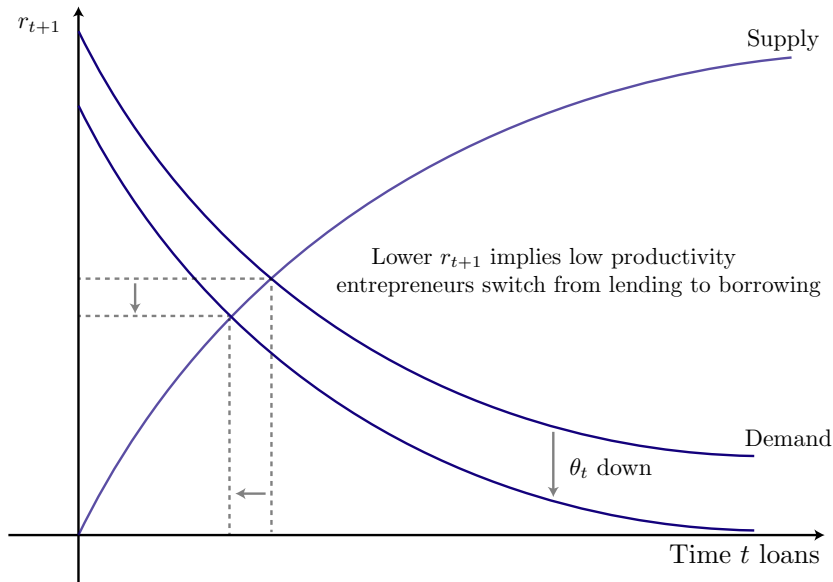
Tighter Lending Standards: θ_t down

- 'MIT shock'
 - economy in a steady state, $t = -\infty, \dots, 1, 2$, and expected to remain there.
 - In $t = 3$, θ_3 drops unexpectedly from 0.67 to 0.60, and gradually returns to its steady state level:
 - $\theta_3 = \theta \times 0.9$, $\theta_t = (1 - \rho)\theta + \rho\theta_{t-1}$, for $t = 4, 5, \dots$
 - $\rho = 0.8$.

Immediate Impact of Negative θ_t Shock

- Period $t = 3$ impact of shock:
 - Deleveraging associated with drop in θ_3 reduces demand for debt by each investing entrepreneur, driving down period $t = 3$ interest rate, r_4 .
 - Marginally productive firms which previously were lending, switch to borrowing and making low-return investments with the drop in r_4 .
 - No impact on total investment in period $t = 3$, as the cut-back by high productivity entrepreneurs is replaced by expanded investment by lower productivity entrepreneurs.
 - No impact on consumption, wages, etc., in period $t = 3$.

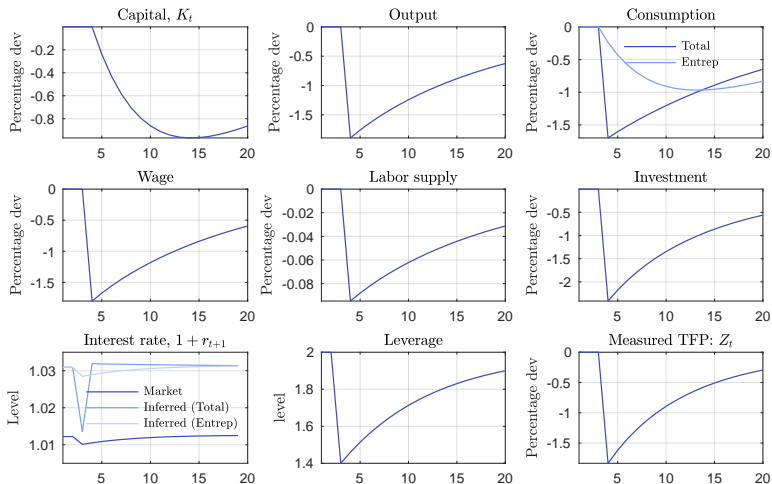
Immediate Impact of Negative θ_t Shock



Dynamic Effects of Drop in θ_t

- The cut in leverage by highly productive, but collateral-poor, firms is the trigger for the over 1.8 percent drop in TFP in period $t = 4$.
 - Until the drop in capital is more substantial, by say period $t = 20$, the drop in TFP is the main factor driving GDP down.
- Total consumption drops substantially, driven by the drop in income of hand-to-mouth workers, who consume $2/3$ of GDP.
 - Entrepreneurial consumption, directly related to GDP, also drops.
- Investment drops by over 2 percent.
- Employment drops by (a modest) 0.1 percent.

Response to Collateral Constraint Shock



Conclusion

- Buera-Moll model gives a flavor of the sort of analysis one can do with heterogeneous agent models with balance sheet constraints.
 - Illustrates the value of simple models for gaining intuition.
- Model provides an 'endogenous theory of TFP'.
 - Stems from poor allocation of resources due to frictions in financial market.
 - See also Song-Storesletten-Zilibotti (AER2011).
- Deleveraging shock gets a surprising number of things right, but
 - how important was deleveraging per se, for the crisis?
 - what is the 'deleveraging shock a stand-in for?'