

# **Business Cycle Anatomy**

## Online Appendix

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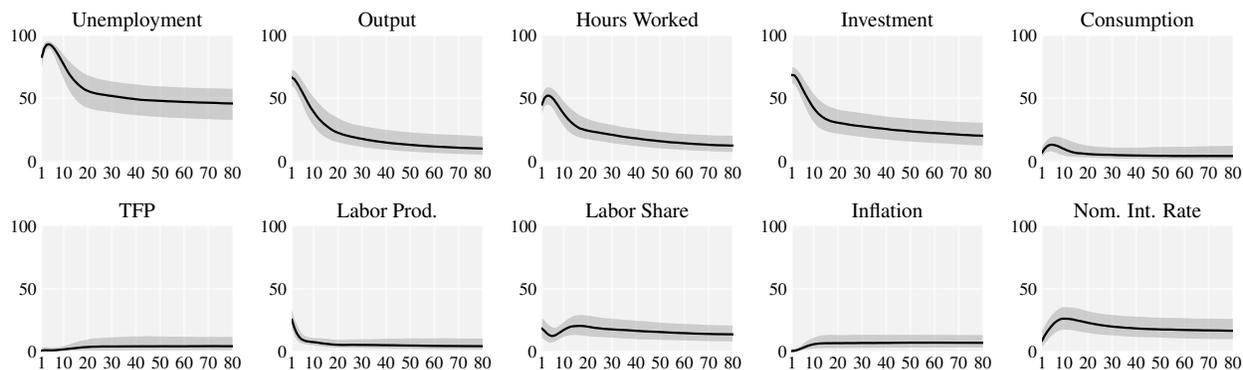
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## **D Variance Contributions in the Time Domain**

Figure 12 complements Table 1 in the main text by reporting the contribution of the identified MBC shock to the FEV of the variables at different horizons. To avoid any confusion, let us emphasize that the shock is still identified in the frequency domain, by targeting the volatility of unemployment over the band of the business-cycle frequencies. The time domain is used only in the calculation of variance contributions.

The picture that emerges is fully consistent with that painted in the main text: the identified shock explain the bulk of the short-run variation in the key macroeconomic quantities, and has a negligible footprint to TFP and inflation at all horizons. The only subtlety worth noting here is that “short run” in the time domain maps to a horizon of about 4 to 8 quarters. This is evident not only in the FEV contributions reported here but also in the IRFs shown in the main text, which pick within the first few quarters. And it anticipates the choice of the horizon targeted in a variant, time-domain identification considered in Online Appendix E.

Figure 12: Variance Contributions at Different Horizons



*Note:* Variance contributions of the MBC shock in the time domain. Horizontal axis: time horizon in quarters. Shaded area : 68% HPDI.

## E Business Cycles in the Frequency vs Time Domain

In this appendix we explore how our method, which identifies the MBC shock in the frequency domain, maps to the time domain. In the first part, we use a simple model to illustrate why targeting the 6-32 quarters range in the frequency domain (FD) is not tautologically the same as targeting the 6-32 quarters horizon range in the time domain (TD). In that model, the shock that targets 6-32 quarters range in the FD is instead best proxied by the shock that targets 4 quarters in the TD. The second part completes the picture by showing how these properties characterizes the actual data as well.

### E.1 Time Domain vs Frequency Domain: A Simple Theoretical Example

In this section we use a 3 equation-3 shock model as a laboratory for investigating the relation between MBC shock identification in the frequency and time domain. The properties of the primitive shocks are chosen in a way that the model gives rise to an MBC shock that replicates our identified empirical MBC shock in the frequency domain (maximizes contribution to volatility of certain variables over the band of 6-32 quarters). We then derive two shocks in the time domain: One by targeting FEVs at 4 quarters; and another by targeting FEVs over the horizons 6-32 quarters. We then compare the properties of the FD MBC shock to those derived in the time domain. The objective of this subsection is to establish that targeting FEV at 4 quarters in the time domain gives rise to the same object as targeting volatility in the band of 6-32 quarters produces; while targeting FEVs over the 6-32 quarter horizons produces a distinctly different object (something that exerts relatively little influence in the short run but has more important effects in the medium term ).

Let us consider a model featuring 3 shocks ( $x_{i,t}$ ,  $i=1,2,3$ )

$$\begin{aligned}x_{1,t} &= \varphi_1 x_{1,t-1} + \varphi_2 x_{1,t-2} + \varepsilon_{1,t} \\x_{2,t} &= \rho x_{2,t-1} + \varepsilon_{2,t} \\ \Delta x_{3,t} &= \rho \Delta x_{3,t-1} + \varepsilon_{3,t}\end{aligned}$$

where, without loss of generality,  $\varepsilon_t \equiv (\varepsilon_{1,t}, \varepsilon_{2,t}, \varepsilon_{3,t}) \rightsquigarrow \mathcal{N}(0, I)$ . In the sequel, we set  $\varphi_1 = 1.55$  and  $\varphi_2 = 0.6$  such that the AR(2) model displays persistence and generates a hump in the response to a  $\varepsilon_{1,t}$  shock in period 4. The persistence of the  $x_{2,t}$  shock is set to  $\rho = 0.25$  such that the shock is stationary and displays low persistence. Finally, we used  $\rho = 0.8$ , such that the diffusion is slow but the bulk of it has taken place in quarter 32.

Endogenous variables ( $y_{i,t}$ ,  $i=1,2,3$ ) are then determined by

$$y_{i,t} = x_{1,t} + a_i x_{2,t} + b_i x_{3,t}$$

The coefficients  $a_i$  and  $b_i$  are determined such that the contribution of the various shocks to the volatility of  $y_i$  are as reported in Table 13.

Table 13: Variance contribution of “structural” shocks (6-32 Quarters)

	$y_1$	$y_2$	$y_3$
$x_1$	75.00	60.00	10.00
$x_2$	15.00	10.00	80.00
$x_3$	10.00	30.00	10.00

Thus,  $x_2$  is the MBC shock,  $y_1$  and  $y_2$  correspond to macroeconomic quantities such as output and employment and  $y_3$  could be a variable such as inflation.

Table 14: Variance Contribution of Identified Shocks

	Targeting $y_1$			Targeting $y_2$			Targeting $y_3$		
	$y_1$	$y_2$	$y_3$	$y_1$	$y_2$	$y_3$	$y_1$	$y_2$	$y_3$
FD	80.47	73.12	13.80	74.30	78.54	16.36	16.25	11.96	81.47
TD 4	76.70	74.05	18.07	65.88	75.16	17.25	17.01	11.88	75.90
TD 6-32	37.87	59.31	15.21	20.13	42.86	12.61	16.17	38.10	11.09

Table 15: Structural Decomposition of Identified Shocks

	Targeting $y_1$			Targeting $y_2$			Targeting $y_3$		
	$x_1$	$x_2$	$x_3$	$x_1$	$x_2$	$x_3$	$x_1$	$x_2$	$x_3$
FD	98.98	0.05	0.97	95.14	0.00	4.86	0.69	98.12	1.19
TD 4	96.41	1.18	2.41	90.59	0.67	8.74	9.75	89.11	1.15
TD 6-32	64.45	0.07	35.48	23.82	0.01	76.17	12.88	0.47	86.65

Figure 13: IRF to Identified Shocks

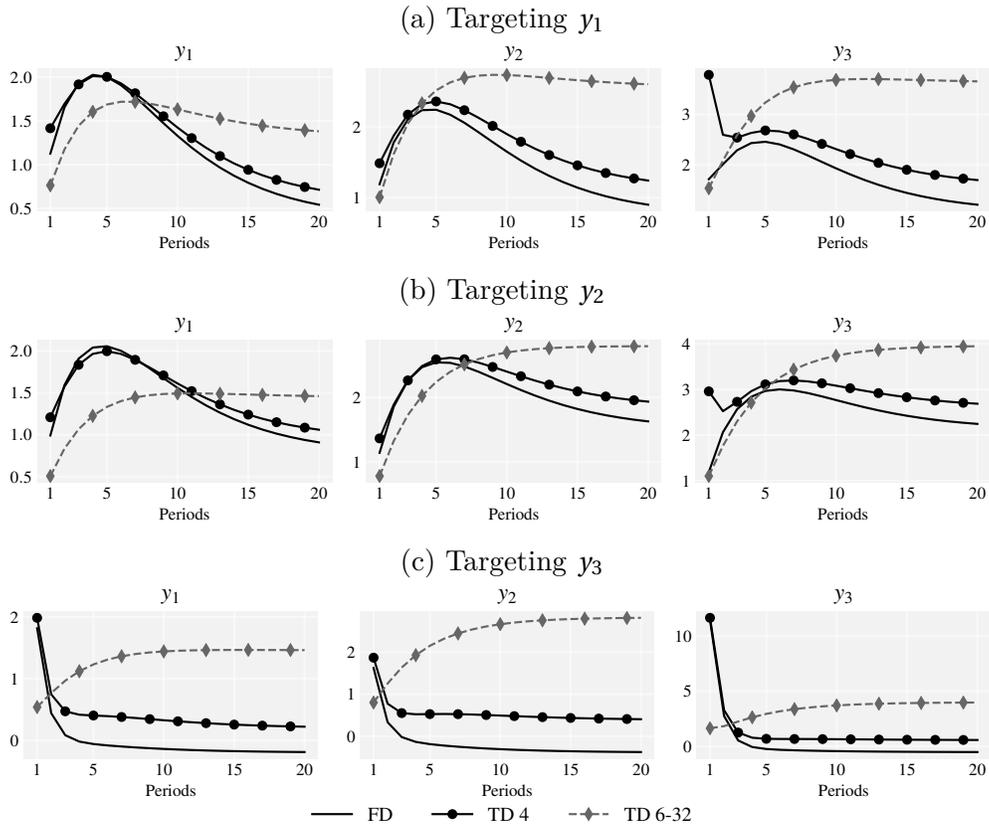
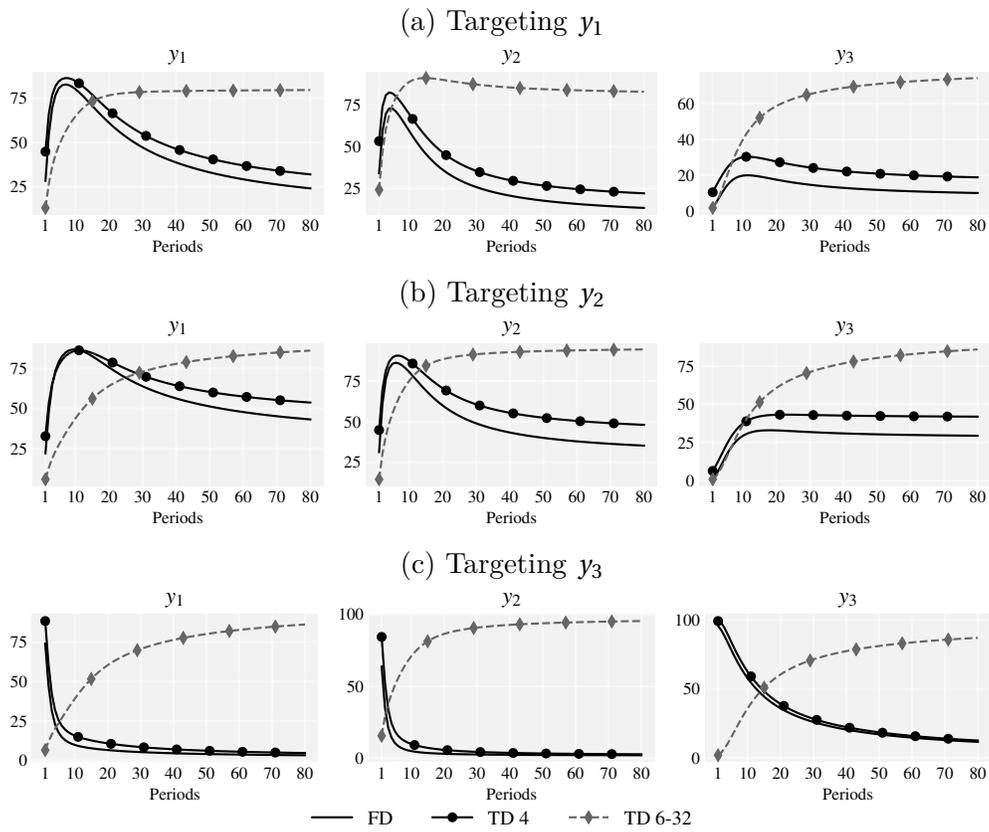


Figure 14: FEV of Identified Shocks



## E.2 Time Domain vs Frequency Domain: Empirical Counterpart

We now illustrate how the lessons of the preceding controlled experiment apply to the actual data, too. Figure 15 and Table 16 compare the properties of our MBC shock, which is identified by target the 6-32 quarter band in the frequency domain (FD), to two time-domain (TD) alternatives, the shock that targets the 6-32 quarter horizon range and the shock that targets the 4 quarter horizon. Clearly, the picture seen in the data is the same as that seen in our controlled experiment.

Figure 1 in the main text and Figure 12 in Online Appendix D paint a complementary picture in terms of the TD properties of our FD-identified MBC shock: its IRFs and FEV contributions peak within 1 to 4 quarters. Together, these results clarify the following point: in the actual data, as in the preceding controlled experiment, targeting the business-cycle frequencies in the frequency domain is essentially the same as targeting a horizon of about a year in the time domain.

Figure 15: Frequency-Domain vs Time-Domain Identification (IRFs)

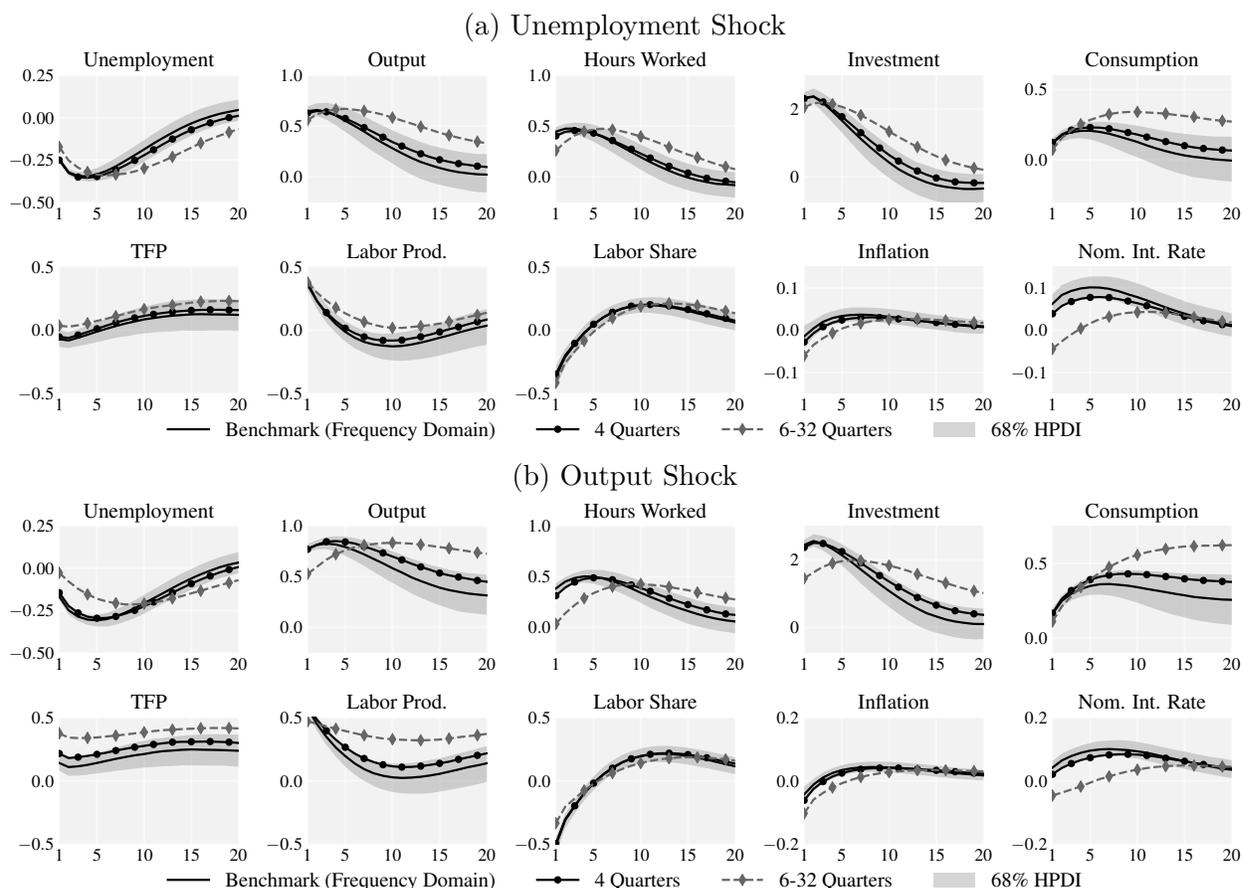


Figure 16: Frequency-Domain vs Time-Domain Identification (FEV)

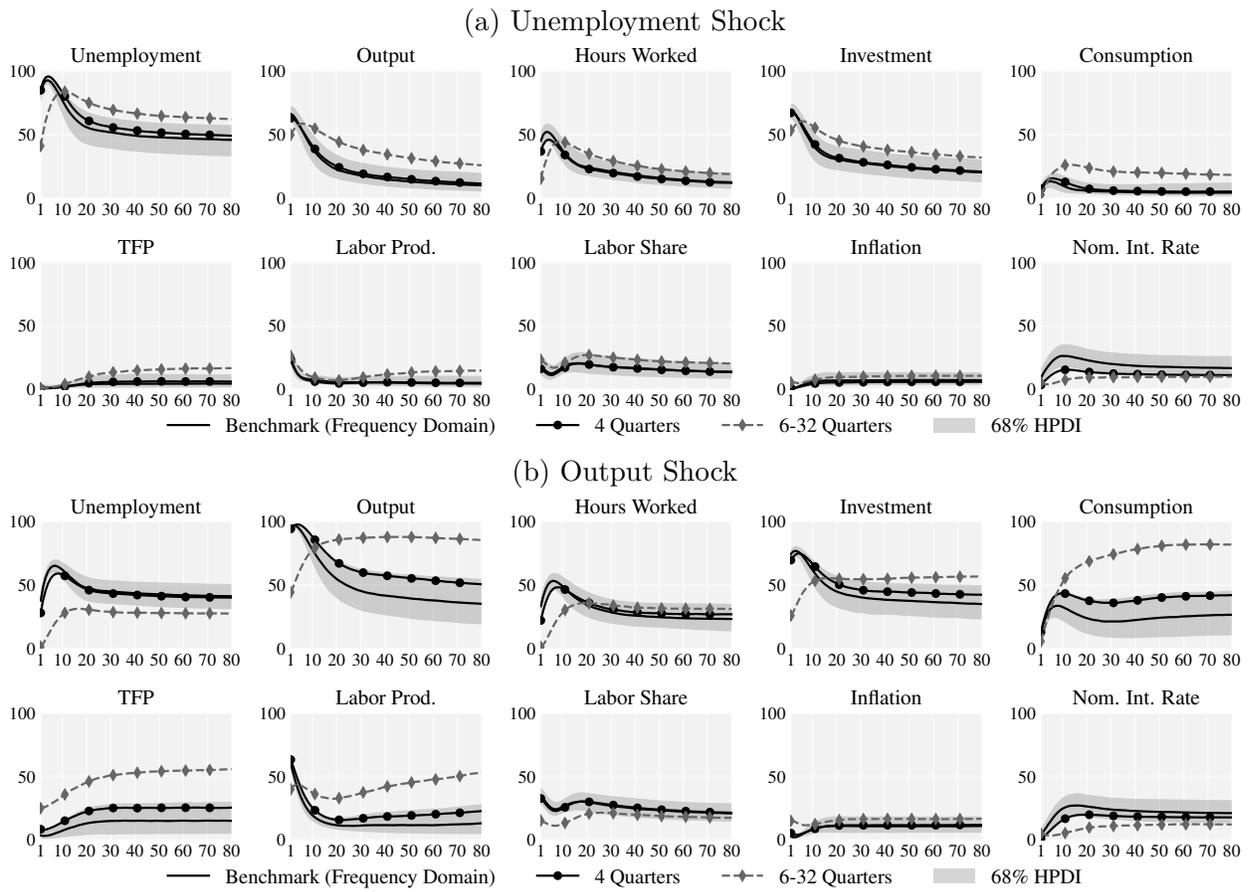


Table 16: Frequency-Domain vs Time-Domain Identification (Variance Contributions)

	$u$	$Y$	$h$	$I$	$C$	TFP	$Y/h$	$Wh/Y$	$\pi$	$R$
<i>Unemployment Shock</i>										
Benchmark	73.7	57.8	46.9	61.1	20.0	5.7	23.6	26.9	6.8	21.8
	[66.7,79.8]	[50.5,65.1]	[39.6,53.9]	[54.7,67.9]	[13.7,27.0]	[2.6,10.8]	[17.2,31.0]	[18.5,35.6]	[3.3,12.0]	[14.6,30.9]
4 Qrts	71.0	53.0	40.2	57.1	19.2	5.2	19.6	24.6	5.6	12.8
	[62.7,77.6]	[46.9,59.2]	[33.3,47.9]	[51.3,63.4]	[13.3,25.5]	[2.2,9.6]	[14.4,25.2]	[18.3,30.4]	[2.8,9.8]	[7.5,19.1]
6-32 Qrts	55.9	50.0	35.6	51.8	22.9	3.8	20.9	31.3	11.1	11.0
	[37.2,68.0]	[34.0,62.0]	[23.6,47.6]	[33.4,64.5]	[15.6,30.9]	[1.4,8.4]	[11.9,30.7]	[19.9,42.9]	[5.4,19.8]	[5.1,21.8]
<i>Output Shock</i>										
Benchmark	55.6	79.8	44.0	66.5	32.6	4.1	41.0	40.5	10.6	16.8
	[49.6,61.7]	[72.9,86.2]	[36.7,51.3]	[61.0,72.6]	[26.0,39.2]	[1.7,8.4]	[35.3,47.2]	[33.7,46.8]	[6.1,16.1]	[10.3,25.0]
4 Qrts	46.8	77.1	36.3	61.6	34.5	6.7	40.6	38.0	11.9	10.2
	[40.5,53.7]	[69.2,84.1]	[29.3,44.1]	[54.3,68.1]	[27.7,40.9]	[3.3,11.9]	[34.6,46.6]	[31.9,44.1]	[7.5,17.6]	[5.7,16.3]
6-32 Qrts	18.4	43.9	18.6	30.0	30.0	18.9	30.2	18.7	20.2	9.0
	[10.6,28.5]	[29.3,59.7]	[12.0,26.3]	[18.9,42.8]	[22.2,39.2]	[9.3,31.2]	[20.3,40.9]	[8.8,30.8]	[12.2,30.3]	[3.5,16.6]

*Note:* The two parts of the table correspond to different targeted variables, unemployment or GDP. In each part, the first row correspond to our benchmark, frequency-domain identification of the shock, while the other rows correspond to time-domain identification. In particular, three cases are reported, depending on whether the shock is constructed by maximizing its contribution to the FEV of the respective variable at horizons of 4 quarters and 6 to 32 quarters. The columns report the contributions of the thus-identified shocks to the business-cycle volatilities of all the variables. 68% HPDI into brackets.

Table 17: Frequency-Domain vs Time-Domain Identification (Long-run Variance Contributions)

	$u$	$Y$	$h$	$I$	$C$	TFP	$Y/h$	$Wh/Y$	$\pi$	$R$
<i>Unemployment Shock</i>										
Benchmark	21.6	4.9	5.3	5.0	4.3	4.4	4.2	3.6	5.4	8.6
	[9.2,38.6]	[0.7,16.5]	[1.4,15.2]	[0.9,17.1]	[0.5,15.7]	[0.6,15.4]	[0.5,14.8]	[0.9,11.9]	[1.6,13.9]	[3.0,19.3]
4 Qrts	30.1	5.9	5.3	6.2	5.3	5.7	5.3	4.6	5.3	6.1
	[16.2,45.1]	[0.9,16.4]	[1.5,14.1]	[1.2,17.1]	[0.7,15.4]	[0.8,16.4]	[0.7,15.5]	[1.2,12.0]	[1.5,13.3]	[1.9,14.7]
6-32 Qrts	58.8	16.1	11.8	16.9	15.2	16.9	16.3	14.3	9.7	10.0
	[34.7,76.3]	[3.4,35.9]	[3.2,29.5]	[4.3,37.4]	[3.0,34.6]	[3.5,36.7]	[3.5,35.3]	[3.9,32.7]	[2.9,23.5]	[2.8,27.9]
<i>Output Shock</i>										
Benchmark	25.7	28.3	16.1	25.6	27.1	25.7	26.7	18.7	10.6	11.6
	[12.8,41.1]	[9.5,49.8]	[3.9,35.6]	[7.6,48.0]	[9.0,48.3]	[8.0,48.3]	[8.3,48.6]	[4.8,38.8]	[4.0,21.6]	[4.5,24.2]
4 Qrts	30.7	41.0	19.3	38.1	40.1	40.0	40.8	30.8	12.7	12.0
	[17.3,46.7]	[21.1,60.4]	[4.8,43.1]	[17.6,58.7]	[20.9,59.6]	[20.0,59.4]	[20.5,59.7]	[12.7,51.0]	[4.9,24.4]	[5.2,26.1]
6-32 Qrts	32.8	74.4	25.0	67.8	74.5	75.7	77.4	64.2	18.9	22.0
	[15.6,54.2]	[44.2,90.8]	[5.6,60.4]	[34.2,88.9]	[44.6,91.1]	[49.4,89.9]	[51.4,91.3]	[38.7,81.8]	[7.0,38.1]	[7.8,47.0]

*Note:* The two parts of the table correspond to different targeted variables, unemployment or GDP. In each part, the first row correspond to our benchmark, frequency-domain identification of the shock, while the other rows correspond to time-domain identification. In particular, three cases are reported, depending on whether the shock is constructed by maximizing its contribution to the FEV of the respective variable at horizons of 4 quarters and 6 to 32 quarters. The columns report the contributions of the thus-identified shocks to the business-cycle volatilities of all the variables. 68% HPDI into brackets

## F Long Run PCA

Table 7 in Section III.A reported the first principal component over the business-cycle frequencies (the band corresponding to 6–32 quarters). For completeness, Table 18 here reports the corresponding object over the long-run frequencies (the band corresponding to 80– $\infty$  quarters). The picture that emerges corroborates the existence of a single unit-root force driving almost the entirety of the long-run fluctuations in TFP and the key macroeconomic quantities.

Table 18: First Principal Component, Long Term, 1955-2017

	$u$	$Y$	$h$	$I$	$C$	$TFP$	$Y/h$	$wh/Y$	$\pi$	$R$
Raw data	10.4	99.9	64.9	98.1	99.7	98.3	98.8	73.9	6.2	7.0
VAR-Based	6.7	97.8	31.9	82.3	92.3	70.9	69.0	4.7	4.9	4.2
Normalized data	10.4	99.2	62.6	95.6	99.8	96.7	98.8	78.3	9.8	10.3
VAR Normalized	13.9	92.0	29.6	74.0	88.3	75.8	76.9	13.5	12.1	15.5

## G Robustness of Empirical Findings

In Section III.C of the main text, we established the robustness of the empirical properties of the shock that targets unemployment across eleven specifications. In Subsection G.1 of this online appendix, we first show that the same robustness property characterizes the other shocks that form our anatomy. In Subsections G.2 and G.3, we expand on some additional findings from the two extended VARs that show up as rows 9 and 10 in these tables. Then, in Subsections G.4 and G.5, we fill in a few details regarding the VECM specifications and measurement of the relative prices of investment. Finally, in Subsections G.6 and G.7, we examine the robustness of our results to the definition of inflation and the addition of the unemployment gap.

### G.1 Beyond the unemployment shock: other elements of the anatomy

Table 8 in the main text reported the variance contributions of the shock that targets unemployment across eleven specifications. Table 19 through Table 23 here repeat the exercise of a select subset of the other elements comprising our anatomy: the shocks that target GDP, hours, investment, and inflation. Although omitted here for the shake of saving space, the same robustness property is also present in terms of IRFs.

Table 19: The MBC Shock, Targeting Unemployment, Variance Contributions (6-32 Quarters)

		$u$	$Y$	$h$	$I$	$C$	TFP	$Y/h$	$Wh/Y$	$\pi$	$R$
[1]	Benchmark	73.7 [66.7,79.8]	57.8 [50.5,65.1]	46.9 [39.6,53.9]	61.1 [54.7,67.9]	20.0 [13.7,27.0]	5.7 [2.6,10.8]	23.6 [17.2,31.0]	26.9 [18.5,35.6]	6.8 [3.3,12.0]	21.8 [14.6,30.9]
[2]	4 Lags	74.4 [67.1,80.6]	57.7 [49.7,64.8]	48.8 [41.7,55.8]	62.0 [54.7,69.0]	20.8 [13.9,28.3]	6.3 [2.7,11.3]	23.0 [16.6,29.7]	27.2 [19.4,36.2]	6.7 [3.1,12.1]	23.9 [15.7,33.1]
[3]	VECM(1)	62.5 [56.4,68.9]	52.4 [44.6,59.3]	48.2 [40.9,54.8]	54.6 [47.8,60.7]	36.6 [26.9,47.1]	13.1 [6.3,23.4]	23.6 [16.5,31.5]	29.9 [19.4,39.6]	8.8 [3.9,18.1]	28.1 [15.2,43.3]
[4]	VECM(2)	65.1 [58.7,72.0]	54.7 [47.7,62.3]	49.8 [43.5,56.2]	54.4 [48.3,61.8]	43.7 [32.1,54.5]	14.4 [7.3,23.2]	19.0 [12.6,26.3]	26.4 [18.3,35.5]	12.2 [6.1,20.4]	21.2 [11.7,33.8]
[5]	1948-2017	78.2 [72.4,84.1]	64.8 [58.6,70.9]	49.2 [43.4,55.9]	63.5 [57.0,69.7]	19.9 [13.7,26.7]	6.2 [2.5,11.2]	26.3 [19.8,33.3]	29.1 [21.0,37.0]	5.7 [2.3,10.2]	17.6 [10.5,24.7]
[6]	1960-2007	69.1 [61.9,75.4]	60.4 [51.6,67.5]	49.8 [42.4,56.7]	62.7 [54.8,69.8]	24.5 [16.6,34.9]	5.3 [2.2,11.3]	26.6 [18.6,36.1]	29.7 [19.4,41.4]	12.4 [6.1,20.3]	26.7 [16.6,38.7]
[7]	pre-Volcker	73.6 [63.6,82.3]	56.1 [46.1,64.8]	42.6 [32.2,52.5]	60.7 [51.6,69.2]	22.1 [12.6,33.5]	7.1 [2.7,15.2]	29.4 [20.4,40.8]	27.5 [17.2,40.1]	17.4 [9.5,27.9]	27.2 [16.6,40.0]
[8]	post-Volcker	73.4 [64.8,79.7]	50.4 [41.8,58.2]	50.4 [41.7,58.8]	58.3 [50.1,65.7]	20.0 [12.3,29.0]	7.5 [3.1,14.3]	18.8 [12.3,27.5]	22.6 [14.0,33.7]	4.8 [1.9,10.6]	15.8 [7.9,25.2]
[9]	Extended	59.5 [53.8,65.5]	50.5 [43.1,58.2]	45.8 [39.4,51.3]	53.2 [44.7,59.7]	22.1 [15.1,31.3]	5.0 [2.0,10.3]	27.1 [19.5,34.7]	27.9 [15.0,44.3]	11.9 [6.3,20.7]	29.0 [17.4,42.5]
[10]	Financial	68.5 [62.1,75.1]	57.9 [50.0,64.3]	46.6 [39.5,54.1]	59.9 [52.6,66.7]	26.0 [17.9,34.6]	7.1 [2.9,13.2]	27.3 [19.2,36.1]	27.2 [18.3,37.7]	8.6 [3.9,14.8]	25.9 [17.1,36.3]
[11]	Chained C&I	81.7 [75.8,86.7]	58.5 [52.1,64.5]	46.1 [39.3,52.1]	61.1 [55.1,67.1]	17.1 [11.7,23.4]	4.2 [1.6,8.2]	19.8 [14.0,25.9]	19.6 [12.7,26.4]	5.6 [2.5,10.2]	23.0 [16.3,30.3]

Note: 68% HPDI into brackets.

Table 20: The MBC Shock, Targeting Output, Variance Contributions (6-32 Quarters)

		$u$	$Y$	$h$	$I$	$C$	TFP	$Y/h$	$Wh/Y$	$\pi$	$R$
[1]	Benchmark	55.6	79.8	44.0	66.5	32.6	4.1	41.0	40.5	10.6	16.8
		[49.6,61.7]	[72.9,86.2]	[36.7,51.3]	[61.0,72.6]	[26.0,39.2]	[1.7,8.4]	[35.3,47.2]	[33.7,46.8]	[6.1,16.1]	[10.3,25.0]
[2]	4 Lags	56.2	79.2	44.3	67.3	32.9	5.1	40.1	40.4	10.9	16.5
		[49.2,62.8]	[71.9,86.3]	[36.8,51.6]	[60.6,73.0]	[25.8,40.9]	[2.3,9.8]	[34.3,46.2]	[33.6,46.9]	[6.2,17.1]	[10.4,25.2]
[3]	VECM(1)	51.7	64.8	43.6	55.9	46.4	8.1	32.5	41.1	9.8	19.3
		[44.3,58.7]	[57.4,71.4]	[36.0,50.9]	[49.3,62.3]	[37.1,56.4]	[4.5,14.0]	[25.8,39.9]	[32.1,49.2]	[4.9,17.1]	[9.8,34.2]
[4]	VECM(2)	52.8	68.2	43.8	56.2	54.7	8.1	32.0	33.9	9.6	16.9
		[46.0,60.0]	[61.3,75.8]	[36.2,51.0]	[49.6,63.5]	[44.1,64.4]	[4.4,14.1]	[25.8,38.6]	[26.5,41.1]	[4.8,17.3]	[9.2,27.4]
[5]	1948-2017	61.8	85.7	52.3	70.5	34.3	3.2	44.1	40.8	5.8	14.7
		[56.2,67.0]	[80.1,91.0]	[46.2,58.2]	[65.3,75.3]	[27.7,41.5]	[1.3,6.2]	[37.8,49.6]	[34.4,47.2]	[2.8,9.9]	[9.0,22.1]
[6]	1960-2007	55.1	78.7	47.7	69.7	36.3	7.7	43.8	42.8	14.8	20.4
		[48.1,61.2]	[70.9,85.4]	[40.1,55.0]	[62.5,75.6]	[28.1,44.8]	[3.2,14.5]	[36.9,50.8]	[34.7,50.7]	[7.8,22.3]	[11.2,30.2]
[7]	pre-Volcker	60.0	71.2	45.3	62.1	38.1	5.6	44.2	42.8	19.5	23.3
		[50.5,68.2]	[61.5,79.6]	[34.8,55.4]	[52.3,70.8]	[27.3,50.5]	[2.1,13.1]	[35.4,53.8]	[31.1,53.5]	[10.9,29.9]	[13.2,35.9]
[8]	post-Volcker	46.4	77.5	40.5	66.4	35.7	7.3	26.3	26.6	3.7	17.8
		[37.4,54.0]	[68.7,84.5]	[31.9,48.9]	[58.1,72.8]	[24.8,46.0]	[3.1,14.3]	[19.7,33.5]	[19.0,35.7]	[1.4,8.6]	[9.2,29.2]
[9]	Extended	47.7	65.4	40.3	56.6	31.4	4.7	40.5	42.8	11.2	17.6
		[41.7,53.7]	[58.4,72.1]	[33.5,46.5]	[50.5,62.7]	[23.8,39.4]	[2.1,9.5]	[34.7,46.9]	[33.1,51.3]	[6.4,17.9]	[10.4,27.9]
[10]	Financial	54.0	75.1	43.3	62.4	35.8	5.5	41.3	38.5	11.6	20.1
		[47.4,60.4]	[68.4,82.1]	[36.0,51.3]	[56.0,68.5]	[27.7,43.2]	[2.9,9.7]	[34.7,47.7]	[30.9,46.2]	[6.8,17.9]	[12.2,29.2]
[11]	Chained C&I	57.5	85.5	43.1	69.6	33.2	2.8	39.2	30.9	8.6	17.9
		[51.2,62.8]	[80.3,90.1]	[35.9,49.8]	[64.7,74.0]	[25.3,40.5]	[1.4,5.4]	[33.0,44.8]	[24.3,37.6]	[4.5,13.7]	[11.6,25.5]

Note: 68% HPDI into brackets.

Table 21: The MBC Shock, Targeting Hours Worked, Variance Contributions (6-32 Quarters)

		$u$	$Y$	$h$	$l$	$C$	TFP	$Y/h$	$Wh/Y$	$\pi$	$R$
[1]	Benchmark	49.0	46.5	70.0	46.7	21.7	11.5	22.0	19.4	7.0	22.4
		[41.8,56.3]	[38.0,55.3]	[63.0,76.7]	[37.3,55.6]	[15.6,28.5]	[6.4,18.2]	[15.4,29.4]	[11.1,29.5]	[3.5,12.5]	[14.6,31.5]
[2]	4 Lags	51.1	46.1	69.8	45.4	22.7	10.9	19.5	17.9	6.7	23.9
		[43.6,58.0]	[37.4,55.0]	[62.5,76.4]	[36.3,54.4]	[16.0,30.4]	[5.5,17.3]	[13.1,26.5]	[10.4,27.0]	[3.0,11.8]	[16.2,33.4]
[3]	VECM(1)	52.4	48.3	56.9	50.1	34.5	21.8	21.9	23.1	10.9	35.4
		[45.0,58.9]	[39.3,56.7]	[52.4,62.1]	[42.2,57.5]	[23.6,46.8]	[9.7,34.5]	[14.6,30.0]	[11.7,39.2]	[4.4,22.2]	[18.0,52.1]
[4]	VECM(2)	53.8	49.1	58.8	49.8	38.5	21.3	17.3	20.6	11.6	26.9
		[46.4,61.6]	[40.1,58.2]	[53.8,64.6]	[42.5,57.2]	[24.2,52.9]	[11.3,32.4]	[10.9,24.9]	[12.3,32.7]	[5.5,19.3]	[13.9,45.8]
[5]	1948-2017	51.6	57.0	75.8	56.3	23.4	8.9	24.2	25.1	8.6	16.6
		[45.9,57.5]	[49.9,63.3]	[70.0,81.8]	[49.1,62.8]	[17.1,30.2]	[4.4,15.1]	[17.7,31.1]	[16.7,33.9]	[4.2,13.7]	[10.4,23.6]
[6]	1960-2007	48.7	50.9	69.9	49.7	26.0	6.1	23.1	22.2	10.5	26.3
		[41.0,56.0]	[41.0,59.3]	[62.0,76.9]	[40.1,58.3]	[18.1,35.2]	[2.4,12.4]	[15.6,32.6]	[13.2,33.9]	[5.3,18.2]	[16.7,36.9]
[7]	pre-Volcker	44.5	46.5	68.7	49.0	22.9	20.7	27.3	20.4	17.3	24.8
		[33.3,54.8]	[36.2,57.2]	[59.1,77.2]	[37.7,60.0]	[14.0,34.7]	[9.7,32.4]	[17.2,38.8]	[10.9,34.8]	[10.4,28.0]	[14.2,38.7]
[8]	post-Volcker	50.0	44.3	71.8	44.5	19.2	6.4	16.0	14.4	3.8	13.9
		[41.0,58.4]	[35.3,52.6]	[62.8,79.5]	[34.4,53.5]	[12.0,28.7]	[2.5,13.6]	[9.7,24.4]	[7.7,23.9]	[1.4,9.3]	[6.7,23.6]
[9]	Extended	42.8	41.1	61.1	42.7	23.7	10.4	21.8	14.1	12.7	27.3
		[36.3,49.4]	[33.8,48.4]	[55.3,67.3]	[34.9,50.2]	[17.7,31.1]	[5.2,17.3]	[15.2,30.1]	[6.7,25.0]	[7.0,20.9]	[17.0,38.8]
[10]	Financial	50.2	49.2	63.8	49.3	27.0	11.6	25.5	21.9	8.8	26.2
		[42.4,58.1]	[39.5,58.2]	[57.6,70.0]	[39.0,59.4]	[19.2,36.7]	[5.6,19.1]	[16.7,35.3]	[12.4,34.7]	[4.1,15.2]	[17.4,36.5]
[11]	Chained C&I	48.1	46.0	79.5	45.6	20.0	11.8	19.9	13.5	5.5	20.2
		[41.7,54.2]	[38.9,52.9]	[73.3,84.7]	[38.5,52.3]	[13.8,26.2]	[6.3,18.1]	[13.4,26.5]	[7.7,20.3]	[2.5,10.5]	[13.6,27.6]

Note: 68% HPDI into brackets.

Table 22: The MBC Shock, Targeting Investment, Variance Contributions (6-32 Quarters)

		$u$	$Y$	$h$	$I$	$C$	TFP	$Y/h$	$Wh/Y$	$\pi$	$R$
[1]	Benchmark	58.2	66.2	44.4	80.1	18.8	3.7	33.9	36.5	7.4	20.6
		[52.3,64.0]	[60.2,72.3]	[36.8,51.9]	[73.5,86.6]	[12.5,26.3]	[1.3,7.9]	[27.7,40.2]	[29.5,43.4]	[3.6,12.5]	[13.3,29.1]
[2]	4 Lags	59.6	66.7	43.1	79.8	20.2	4.9	31.7	36.4	7.2	20.9
		[52.8,65.7]	[59.9,72.6]	[36.1,50.6]	[72.3,86.4]	[13.7,28.5]	[1.9,9.4]	[25.7,38.4]	[29.1,43.5]	[3.4,12.7]	[13.6,29.1]
[3]	VECM(1)	54.4	57.1	45.5	63.0	36.0	9.2	28.1	37.6	9.1	26.3
		[47.8,61.1]	[49.9,63.9]	[38.2,52.6]	[56.5,69.3]	[26.2,47.3]	[4.4,17.8]	[20.6,35.5]	[27.9,46.5]	[4.0,17.5]	[15.0,41.4]
[4]	VECM(2)	56.3	60.2	46.8	63.6	42.3	10.0	26.0	33.3	10.5	22.8
		[49.8,63.0]	[53.1,67.4]	[39.8,53.7]	[57.1,70.5]	[29.8,54.9]	[4.8,17.5]	[19.6,32.7]	[25.0,41.5]	[5.1,18.5]	[12.5,36.3]
[5]	1948-2017	61.2	71.5	53.0	84.8	21.3	3.0	36.8	36.5	8.2	18.6
		[55.8,66.6]	[66.5,76.3]	[46.6,59.1]	[79.0,89.7]	[14.4,29.2]	[1.1,6.3]	[30.8,43.1]	[30.1,43.2]	[4.0,13.4]	[11.9,26.1]
[6]	1960-2007	55.8	67.9	44.9	80.1	23.9	6.9	35.6	38.5	12.2	24.6
		[49.1,62.3]	[61.1,74.2]	[36.6,52.4]	[71.8,86.9]	[16.0,33.3]	[2.6,13.3]	[28.6,43.1]	[30.3,46.7]	[6.2,20.1]	[15.4,34.6]
[7]	pre-Volcker	62.6	60.3	47.7	72.3	24.3	7.4	36.1	32.4	17.8	28.9
		[52.9,70.3]	[50.1,68.8]	[37.4,57.4]	[62.4,81.3]	[13.6,37.3]	[2.7,15.8]	[26.5,46.2]	[21.9,44.1]	[10.3,28.6]	[18.6,42.5]
[8]	post-Volcker	50.7	62.6	39.9	82.1	22.1	5.3	19.4	25.1	3.7	18.1
		[42.6,58.4]	[54.5,69.3]	[30.8,48.3]	[73.0,89.2]	[13.5,31.3]	[2.1,11.1]	[13.4,26.4]	[17.4,33.3]	[1.5,8.6]	[10.3,28.0]
[9]	Extended	49.7	56.7	42.7	65.3	20.4	4.0	35.0	41.6	10.7	21.3
		[43.6,55.5]	[50.4,62.8]	[35.8,48.8]	[58.6,72.0]	[13.4,28.3]	[1.7,8.2]	[29.3,41.1]	[31.4,50.7]	[5.7,17.6]	[13.6,31.3]
[10]	Financial	57.0	63.4	44.9	74.1	23.7	5.1	34.9	35.2	8.6	24.0
		[50.4,62.9]	[57.3,69.3]	[37.4,52.6]	[66.9,80.6]	[15.6,32.7]	[2.3,10.1]	[28.1,41.8]	[27.8,43.4]	[3.9,15.0]	[15.6,34.0]
[11]	Chained C&I	58.9	68.6	42.0	86.1	18.6	2.6	30.6	27.5	6.5	22.3
		[53.1,64.6]	[63.8,73.8]	[35.0,48.7]	[80.5,90.5]	[12.6,25.3]	[1.0,5.3]	[25.3,37.0]	[21.2,33.9]	[2.9,11.4]	[15.3,29.5]

Note: 68% HPDI into brackets.

Table 23: The Inflation Shock, Variance Contributions (6-32 Quarters)

		$u$	$Y$	$h$	$I$	$C$	TFP	$Y/h$	$Wh/Y$	$\pi$	$R$
[1]	Benchmark	4.2	8.0	3.4	3.0	15.6	3.7	7.8	1.9	83.3	8.0
		[1.8,8.6]	[4.0,13.1]	[1.3,6.8]	[1.1,6.3]	[9.8,22.3]	[2.0,7.1]	[4.2,12.4]	[0.7,4.4]	[76.2,88.3]	[3.2,14.2]
[2]	4 Lags	5.1	9.3	4.1	3.5	16.1	3.7	9.6	2.5	82.4	7.7
		[2.3,9.4]	[5.1,14.4]	[1.6,8.0]	[1.3,7.0]	[10.7,21.9]	[1.9,6.8]	[5.5,15.3]	[0.8,5.5]	[76.1,87.6]	[3.0,13.7]
[3]	VECM(1)	10.3	13.1	10.0	8.5	19.9	9.9	14.2	5.5	86.2	16.3
		[4.7,17.4]	[6.8,20.7]	[4.6,17.3]	[3.5,15.2]	[12.0,29.2]	[5.9,15.7]	[8.2,21.6]	[2.2,11.1]	[79.4,91.0]	[9.1,25.5]
[4]	VECM(2)	4.6	2.3	5.0	3.5	1.8	11.2	3.2	4.2	85.8	5.3
		[1.4,9.6]	[0.6,6.0]	[1.9,9.6]	[1.1,8.2]	[0.4,5.7]	[6.5,17.2]	[1.2,6.4]	[2.0,7.5]	[79.0,91.2]	[2.8,9.9]
[5]	1948-2017	2.7	2.6	4.6	6.1	12.0	7.1	6.6	2.1	86.2	6.5
		[1.0,6.2]	[0.9,5.4]	[2.0,8.4]	[3.5,10.0]	[6.9,18.2]	[3.3,12.8]	[3.6,10.8]	[0.6,4.9]	[81.1,90.4]	[2.6,12.3]
[6]	1960-2007	9.9	8.8	7.2	5.2	19.1	3.8	8.5	4.1	76.6	11.4
		[4.8,16.9]	[3.8,16.1]	[2.8,13.7]	[1.7,10.9]	[12.0,27.0]	[1.8,7.3]	[3.9,15.0]	[1.5,9.3]	[68.6,83.2]	[5.3,19.6]
[7]	pre-Volcker	10.3	14.2	6.5	10.9	21.2	14.4	17.4	9.0	66.3	9.4
		[3.5,22.3]	[6.4,27.0]	[2.0,17.1]	[3.9,22.7]	[12.3,31.5]	[5.8,26.3]	[9.0,28.8]	[3.1,20.9]	[55.1,76.5]	[3.2,22.9]
[8]	post-Volcker	7.5	9.3	7.4	5.5	14.6	2.5	8.0	2.7	87.4	24.1
		[3.0,14.3]	[4.6,16.4]	[3.1,13.9]	[2.2,10.9]	[8.0,23.6]	[0.9,6.0]	[3.2,14.9]	[1.0,6.4]	[81.0,92.3]	[13.7,34.5]
[9]	Extended	8.4	9.6	7.2	5.3	14.1	5.5	11.7	3.7	75.3	13.7
		[3.9,14.9]	[5.1,15.8]	[3.3,13.4]	[2.1,10.7]	[8.4,21.3]	[2.8,9.4]	[6.3,17.7]	[1.3,8.5]	[67.9,81.8]	[7.2,22.2]
[10]	Financial	4.8	8.3	3.9	3.5	14.3	4.1	8.2	2.2	80.8	8.2
		[1.9,9.4]	[4.1,13.6]	[1.4,7.9]	[1.2,7.5]	[8.3,20.4]	[2.0,7.4]	[4.3,13.0]	[0.9,4.9]	[73.2,86.8]	[3.5,15.6]
[11]	Chained C&I	2.1	4.9	1.5	2.1	6.5	3.2	6.3	1.7	80.5	7.0
		[0.6,4.7]	[1.9,9.0]	[0.5,3.8]	[0.8,4.7]	[3.0,11.3]	[1.4,6.4]	[3.2,10.6]	[0.6,4.1]	[73.7,86.2]	[2.7,14.0]

Note: 68% HPDI into brackets.

## G.2 Stock Prices, Relative Price of Investment, and Utilization

Here, we describe additional properties of the specification in row 9 (“Extended”) of Tables 8 and 20-23. Recall that this specification contains three additional variables: stock prices ( $SP$ ); the relative price of investment ( $P_i/P_c$ ); and capital utilization ( $z$ ). Our measure of stock prices is in real terms, is the same as that used by Beaudry and Portier, and is taken from Robert Shiller’s website ([http://www.econ.yale.edu/~shiller/data/ie\\_data.xls](http://www.econ.yale.edu/~shiller/data/ie_data.xls)). The relative price of investment is the ratio of the price of Gross Private Domestic Investment and Durables to the price of Non Durables and Services; its computation is detailed in Online Appendix G.5. Finally the capacity utilization rate variable corresponds to the Capacity Utilization in Manufacturing (SIC), CUMFNS in the Federal Reserve Economic Database.

The inclusion of stock prices and the relative price of investment is motivated by works that uses these variable in the identification of, respectively news shocks and investment-specific technology shocks. The inclusion of capacity utilization, on the other hand, helps shed light on why labor productivity moves with the MBC shock while TFP does not. Last but not least, the inclusion of all three variables at once helps illustrate the robustness of our main findings to the addition of more information—a point already made in Tables 8 and 20-23.

Here, Tables 24-25 and Figure 17 complete the picture by reporting the contribution of the MBC shock to the short-run and long-run volatility of the aforementioned three variables, as well as the properties of the shock that targets the business-cycle volatility of stock prices.<sup>1</sup> The most noteworthy new findings are the following.

First, the disconnect between the business cycle and technology applies to both TFP and investment-specific technology, as measured by the relative price of investment. For instance, the MBC shock explains 5% of the volatility of either of these variables at either the business-cycle or the long-run frequencies.

Second, the shock that targets Stock Prices accounts for 22 to 25% of the business-cycle volatility in unemployment, output and investment, and 15 to 22% of the long-run volatility in TFP, output and investment. In this regard, the fluctuations in stock prices appear to be disconnected from current technology and to contain non-trivial statistical information about both the business cycle and the long-term prospects of the economy. The extent to which these patterns reflect the presence of a news shock is explored further in Appendix C.

Finally, the shock that targets utilization at the business-cycle frequencies is similar to the MBC shock in terms of both variance contributions and IRFs (Figure 17). This helps understand why labor productivity increases in response to the MBC shock, while TFP does not move.

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<sup>1</sup>The shocks that target business cycle volatility in TFP and the relative price of investment lack novelty as they contribute negligibly to the volatility of the macroeconomic quantities.

Table 24: Extended VAR, Business Cycle Variance Contributions

	$u$	$Y$	$h$	$I$	$C$	$z$		
MBC Shock	59.5	50.5	45.8	53.2	22.1	51.7		
	[53.8,65.5]	[43.1,58.2]	[39.4,51.3]	[44.7,59.7]	[15.1,31.3]	[45.6,57.7]		
SP Shock	24.4	23.3	16.2	21.8	25.0	18.4		
	[18.3,31.0]	[17.1,29.4]	[10.5,22.5]	[16.1,28.4]	[18.5,31.5]	[12.8,24.7]		
	TFP	$Y/h$	$P_i/P_c$	$SP$	$wh/Y$	$\pi$	$R$	
MBC Shock	5.0	27.1	4.7	11.5	27.9	11.9	29.0	
	[2.0,10.3]	[19.5,34.7]	[1.7,9.9]	[5.3,23.2]	[15.0,44.3]	[6.3,20.7]	[17.4,42.5]	
SP Shock	4.5	10.9	3.6	82.0	11.4	9.0	5.3	
	[2.5,7.7]	[6.2,16.4]	[1.3,7.9]	[76.0,87.8]	[6.3,17.6]	[4.5,14.7]	[2.3,10.5]	

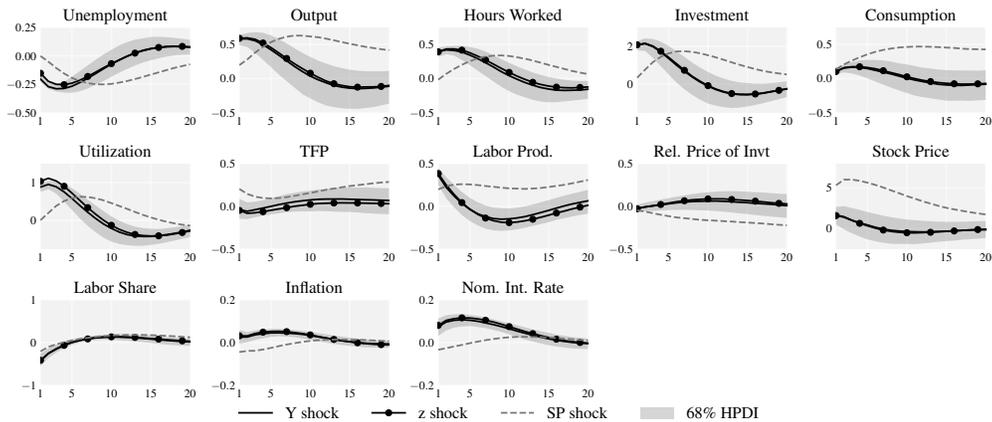
Note: The rows correspond to the shocks targeting business-cycle variation in unemployment (MBC shock) and Stock Prices (SP shock), respectively. The columns correspond to the 13 variables in the VAR. These are the 10 variables from our baseline specification, and also capacity utilization  $z$ , the Relative Price of Investment  $P_i/P_c$  and stock prices  $SP$ . 68% HPDI into brackets.

Table 25: Extended VAR, Long-Run Variance Contributions (80- $\infty$  Quarters)

	$u$	$Y$	$h$	$I$	$C$	$z$		
MBC Shock	9.1	3.8	3.8	4.5	3.6	6.2		
	[2.6,24.1]	[0.5,16.9]	[1.1,10.9]	[0.9,16.9]	[0.5,16.8]	[2.2,14.5]		
SP Shock	29.5	15.8	9.8	15.4	16.1	17.8		
	[15.0,46.5]	[2.5,38.0]	[2.7,27.6]	[3.6,35.8]	[2.6,38.5]	[7.8,32.3]		
	TFP	$Y/h$	$P_i/P_c$	$SP$	$wh/Y$	$\pi$	$R$	
MBC Shock	4.0	3.9	3.9	5.0	4.0	6.7	10.1	
	[0.6,17.3]	[0.5,18.1]	[0.5,17.5]	[1.1,16.1]	[0.8,14.4]	[2.2,15.6]	[3.3,23.1]	
SP Shock	22.9	22.8	27.5	34.5	25.1	8.0	13.9	
	[6.2,44.3]	[6.0,45.0]	[10.8,47.6]	[18.3,52.4]	[11.6,43.1]	[2.7,21.5]	[3.5,31.6]	

Note: The rows correspond to the shocks targeting business-cycle frequencies variation in unemployment (MBC shock) and Stock Prices (SP shock) respectively. The columns correspond to the 13 variables in the VAR. These are the 10 variables from our baseline specification, plus capacity utilization ( $z$ ), the Relative Price of Investment ( $P_i/P_c$ ) and stock prices ( $SP$ ). 68% HPDI into brackets.

Figure 17: Extended VAR, IRFs



### G.3 Financial Variables

Here we provide additional information on the VAR that adds the credit spread ( $CS$ ) and appears as row 10 (“Financial”) of Tables 8 and 20-23. We also consider a more comprehensive specification, called “Financial-Full,” that contains three additional financial variables at the expense of a shorter sample period. The additional variables are the slope of the term structure ( $TS$ ), the level of credit to non-financial firms ( $Cr$ ), and the net worth of such firms ( $WS$ ).

Our measurement of all these variables follows Christiano et al. (2014). The credit spread ( $CS$ ) is the difference between the interest rate on BAA-rated corporate bonds and the 10 year US government bond rate. The slope of the term structure ( $TS$ ) is the difference between the 10-year constant maturity US government bond yield and the Federal Funds rate. The level of credit ( $Cr$ ) is taken from the Flow of Funds of the US Federal Reserve Board. Finally, net worth ( $WS$ ) is measured by the Dow Jones Wilshire 5000 index.<sup>2</sup> Because this index only starts in 1971 and the measure of credit is only available until 2014, the VAR that contains all four financial variables (“Financial-Full”) is estimated for the period running from 1971Q1 to 2014Q4. By contrast, the VAR that contains only the credit spread (“Financial”, or row 10 of the aforementioned tables) spans the entire 1955Q1-2017Q4 period.

For the purposes of the model evaluation done in Section V, we have also considered a third specification, which is obtained by restricting the second specification to 1985Q1-2010Q4. This is the period used in the original estimation of the model in Christiano et al. (2014). We refer to this specification as “Financial-CMR.”

Figure 18 reports the IRFs of the various facets of the MBC shock obtained from these three specifications. Although there are some differences,<sup>3</sup> the main picture remains the same: the reduced-form shocks obtained by targeting unemployment, hours, output, investment and consumption are highly interchangeable.

Perhaps more interestingly, we can now detect the empirical footprint of the MBC shock on the new, financial variables. In particular, we see that the credit spread spikes on impact, while output and the other key macroeconomic quantities respond with a delay, in a hump-shaped manner. From this perspective, the credit spread leads the business cycle. As discussed in Section V, this property, which is presumably informative about the real-financial nexus, is unfortunately not captured by the model of Christiano et al. (2014).<sup>4</sup>

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<sup>2</sup>Note that the measure of net worth is a stock-market valuation, which differs from that used in the previous subsection (SP500) because the present specification aims at replicating the data used in CMR, while the previous one followed Beaudry and Portier. In any case, it makes little difference which one of these two measures is used as their business-cycle behavior is nearly identical.

<sup>3</sup>Most notably, consumption appears to more closely connected to the MBC shock in the third specification.

<sup>4</sup>Although we have omitted it here, we have also looked at the shock that targets the credit spread itself. This shock is similar to the MBC shock in terms of IRFs (comovements), although less so with regard to variance contributions. Importantly, this shock, too, gives rise to pattern mentioned above, with the credit spread itself moving before the key macroeconomic quantities.

Figure 18: Comparing Business-Cycle Factors

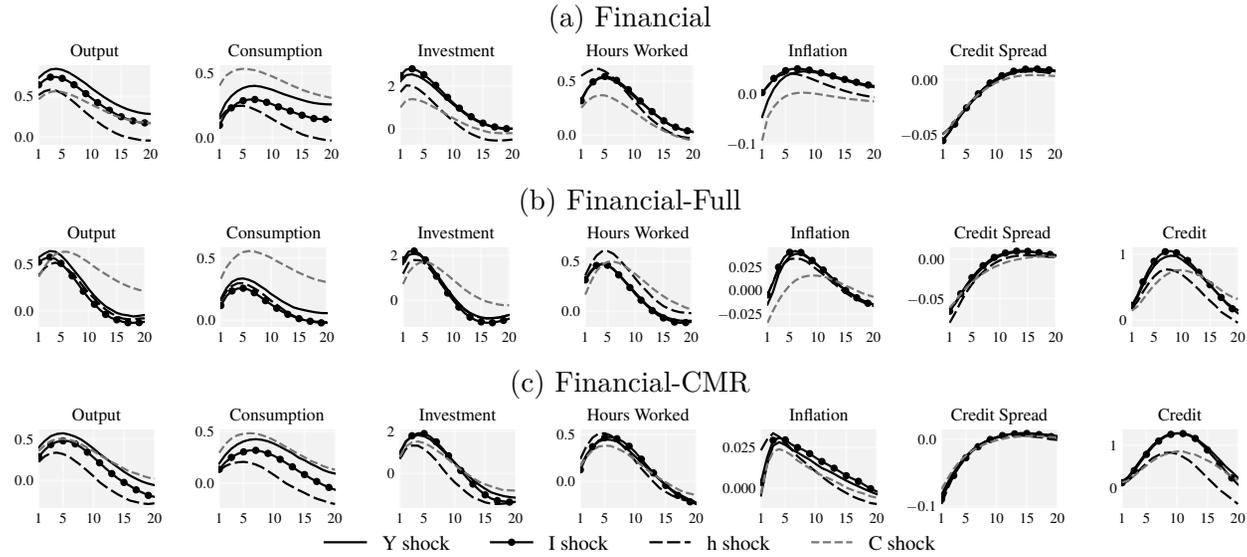


Table 26: Financial VARs, Short-Run Contributions of MBC Shock

	$u$	$Y$	$h$	$I$	$C$	$\pi$	$CS$	$Cr$
Financial	68.5	57.9	46.6	59.9	26.0	8.6	42.5	
	[62.1,75.1]	[50.0,64.3]	[39.5,54.1]	[52.6,66.7]	[17.9,34.6]	[3.9,14.8]	[30.5,55.4]	
Financial-Full	60.8	51.7	52.9	55.0	33.1	13.3	48.6	40.9
	[54.8,67.5]	[43.8,59.5]	[44.7,61.4]	[47.2,62.9]	[21.9,45.8]	[6.0,24.9]	[27.7,62.6]	[28.3,52.3]
Financial-CMR	65.7	52.9	59.9	56.3	37.1	15.9	57.2	47.3
	[56.4,74.4]	[40.5,64.3]	[49.5,69.9]	[45.0,67.9]	[21.8,51.8]	[6.8,31.7]	[35.1,71.2]	[28.7,62.0]

*Note:* The rows correspond to the shocks targeting business-cycle frequencies variation in unemployment (MBC shock) for the various financial VARs described in the text.  $CS$  denotes the Credit Spread,  $Cr$  the measure of credit. 68% HPDI into brackets.

## G.4 Description of VECMs

We now fill in the details of the VECMs reported in rows 3 and 4 of Tables 8 and 20-23. Both of these VECMs are nested in the following form:

$$\Delta X_t = \Gamma_0 \Theta X_{t-1} + \sum_{i=1}^p \Gamma_i \Delta X_{t-i} + v_t$$

where  $\Theta$  is the matrix of co-integration coefficients and  $\Gamma_0$  is the matrix of loadings of these co-integration relationships. The difference between the two VECMs is the specification of the number of unit roots and the co-integration relations.

In  $VECM_1$ , we assume that the real quantities ( $Y, C, I, APL$ ) and  $TFP$  share a single stochastic trend, while the remaining variables are assumed to be stationary. The co-integrating relationship is of the type  $x_t = \alpha_x + \beta_x TFP_t$  for each variable  $x \in \{Y, C, I, APL\}$ .

In  $VECM_2$ , the real quantities ( $Y, C, I, APL$ ) and  $TFP$  share one stochastic trend; the nominal variables,  $\pi$  and  $R$ , share another stochastic trend; and the remaining variables (the unemployment, hours, and the labor share) are stationary. The co-integration relationships are of the type  $x_t = \alpha_x + \beta_x TFP_t$  for  $x \in \{Y, C, I, APL\}$  and  $R_t = \delta + \gamma \pi_t$ .

We have also considered a third specification that allows the number of stochastic trends and the co-integration relationships to be determined completely a-theoretically, by means of the standard maximum eigenvalue and trace tests proposed by Johansen and Juselius (1990). Relative to the aforementioned two specifications, this “unrestricted” VECM marginally reinforces the disconnect between the short run and the long run;<sup>5</sup> but it also produces six (!) unit roots, which makes little sense from the perspective of theory.

## G.5 Measuring the Relative Price of Investment

We now describe the measure of the relative price of investment that is used in one of our robustness exercises, the one appearing as row 9 (“Extended”) of Tables 8 and 20-23.

Let  $P_t^x$  denote the chained price index of aggregate  $x$  at time  $t$ , and similarly  $Q_t^x$  the quantity of aggregate  $x$  at time  $t$ , where  $x$  can denote either gross domestic private investment (GPDI), durable consumption (D), non durable consumption (ND) or services (S). The change in investment (I=GPDI+D) price, is then given by

$$\Delta P_t^I = \sqrt{\Delta P_t^I(Q_{t-1}^I) \Delta P_t^I(Q_t^I)} - 1$$

where

$$\Delta P_t^I(Q_{t-1}^I) = \frac{P_t^{GPDI} Q_{t-1}^{GPDI} + P_t^D Q_{t-1}^D}{P_{t-1}^{GPDI} Q_{t-1}^{GPDI} + P_{t-1}^D Q_{t-1}^D} \quad \text{and} \quad \Delta P_t^I(Q_t^I) = \frac{P_t^{GPDI} Q_t^{GPDI} + P_t^D Q_t^D}{P_{t-1}^{GPDI} Q_t^{GPDI} + P_{t-1}^D Q_t^D}$$

Similarly, we define the change in the consumption (C=ND+S) price as

$$\Delta P_t^C = \sqrt{\Delta P_t^C(Q_{t-1}^C) \Delta P_t^C(Q_t^C)} - 1$$

---

<sup>5</sup>In particular, the unemployment shock accounts 10% of the long-run volatility in output and TFP, compared to 14% in  $VECM_1$  or  $VECM_2$ .

where

$$\Delta P_t^C(Q_{t-1}^C) = \frac{P_t^{\text{ND}} Q_{t-1}^{\text{ND}} + P_t^{\text{S}} Q_{t-1}^{\text{S}}}{P_{t-1}^{\text{ND}} Q_{t-1}^{\text{ND}} + P_{t-1}^{\text{S}} Q_{t-1}^{\text{S}}} \text{ and } \Delta P_t^C(Q_t^C) = \frac{P_t^{\text{ND}} Q_t^{\text{ND}} + P_t^{\text{S}} Q_t^{\text{S}}}{P_{t-1}^{\text{ND}} Q_t^{\text{ND}} + P_{t-1}^{\text{S}} Q_t^{\text{S}}}$$

Let us denote by  $Q_t$  the relative price of investment as  $Q_t = P_t^I/P_t^C$ , then  $Q_t$  satisfied

$$Q_t = (1 + \Delta P_t^I - \Delta P_t^C) Q_{t-1}$$

## G.6 Varying The Definition of Inflation

In this section, we vary the definition of inflation. We first repeat our benchmark exercise that relies on the GDP deflator and then complement it with exercises where the inflation measure is built using, respectively, the Consumer Price Index (All items, All Urban Consumers) (CPI), the Consumer Price Index (All Items Less Food and Energy, All Urban Consumers) (CORE) and the Producer Price Index for All Commodities (PPI). Our main results are found to be robust to the exact definition of inflation, in particular the disconnect between the evolution of the core business cycle variables and inflation, and the disconnect between inflation and the labor share. Table 27 revisits the variance contribution of the unemployment, inflation and labor share shocks to the variables of the VAR.

Table 27: Inflation Index: Variance Contributions (6-32 Quarters)

	$u$	$Y$	$h$	$I$	$C$	TFP	$Y/h$	$Wh/Y$	$\pi$	$R$
<i>Unemployment Shock</i>										
GDP Deflator	73.7	57.8	46.9	61.1	20.0	5.7	23.6	26.9	6.8	21.8
	[66.7,79.8]	[50.5,65.1]	[39.6,53.9]	[54.7,67.9]	[13.7,27.0]	[2.6,10.8]	[17.2,31.0]	[18.5,35.6]	[3.3,12.0]	[14.6,30.9]
CPI	73.9	56.8	46.0	60.5	18.3	5.8	22.6	25.6	7.3	22.0
	[66.5,80.1]	[49.7,63.5]	[39.2,52.9]	[53.7,67.0]	[12.0,25.2]	[2.5,10.6]	[16.0,30.6]	[17.6,35.8]	[3.7,12.4]	[14.8,31.1]
CORE	73.4	59.6	48.4	61.5	24.1	5.8	24.3	31.1	9.2	22.6
	[66.9,79.2]	[52.5,66.1]	[41.6,55.1]	[54.9,68.3]	[17.7,32.2]	[2.4,10.9]	[18.0,32.2]	[22.4,40.6]	[4.8,15.4]	[14.4,31.8]
PPI	73.6	56.9	46.6	61.4	18.5	6.0	22.4	25.2	3.1	23.3
	[67.1,80.0]	[49.4,63.4]	[39.4,53.4]	[54.2,67.9]	[12.0,25.5]	[2.6,11.2]	[16.3,30.1]	[17.3,35.0]	[1.1,6.7]	[15.2,32.0]
<i>Inflation Shock</i>										
GDP Deflator	4.2	8.0	3.4	3.0	15.6	3.7	7.8	1.9	83.3	8.0
	[1.8,8.6]	[4.0,13.1]	[1.3,6.8]	[1.1,6.3]	[9.8,22.3]	[2.0,7.1]	[4.2,12.4]	[0.7,4.4]	[76.2,88.3]	[3.2,14.2]
CPI	5.1	7.3	3.7	3.3	8.5	2.2	6.6	1.5	79.7	11.5
	[2.5,9.2]	[4.0,12.1]	[1.5,7.3]	[1.4,6.8]	[4.9,12.9]	[0.8,5.2]	[3.2,10.8]	[0.5,4.0]	[72.7,85.1]	[6.0,18.0]
CORE	9.1	9.1	6.1	4.7	13.9	2.8	5.2	1.7	79.2	15.6
	[4.8,15.1]	[4.8,15.2]	[3.0,11.1]	[2.1,8.8]	[8.6,20.9]	[1.2,6.3]	[2.0,9.6]	[0.6,4.3]	[72.1,85.7]	[10.0,23.4]
PPI	3.6	3.4	4.5	3.1	8.2	5.3	4.6	1.3	92.5	9.4
	[1.6,6.5]	[1.7,6.1]	[2.0,7.8]	[1.5,5.9]	[5.3,12.2]	[2.1,10.4]	[2.1,8.1]	[0.4,3.1]	[88.7,95.3]	[5.1,14.6]
<i>Labor Share Shock</i>										
GDP Deflator	26.0	35.6	22.5	31.5	13.1	3.2	35.0	85.4	4.0	8.2
	[18.7,33.5]	[28.8,42.6]	[15.4,30.2]	[24.3,38.9]	[7.6,19.5]	[1.5,6.4]	[29.0,41.3]	[79.8,90.4]	[1.7,7.6]	[4.0,14.5]
CPI	25.1	35.5	21.9	31.1	12.8	3.6	34.8	86.6	3.4	9.3
	[16.8,33.0]	[28.1,42.7]	[14.8,30.1]	[23.2,38.9]	[7.6,19.3]	[1.7,7.0]	[28.2,41.2]	[80.6,90.9]	[1.4,6.8]	[4.4,16.0]
CORE	28.1	38.3	23.0	33.5	16.1	3.8	34.3	86.3	3.7	10.9
	[20.4,36.3]	[31.4,46.3]	[15.1,31.2]	[26.4,41.1]	[9.9,23.4]	[1.7,7.6]	[28.6,40.8]	[80.9,90.1]	[1.4,7.9]	[5.2,17.7]
PPI	24.1	35.4	21.0	30.4	12.0	3.5	34.9	86.6	1.4	8.8
	[16.5,32.9]	[28.5,43.0]	[14.0,29.1]	[22.8,38.3]	[6.7,18.2]	[1.6,6.6]	[28.7,41.1]	[81.0,90.9]	[0.5,3.2]	[4.1,15.1]

## G.7 Adding Output and Unemployment Gaps

We now consider two additional versions of the VAR where we add, respectively, the output gap (measured as the difference between the actual GDP and the potential GDP as reported in the Federal Reserve Database) and the unemployment gap (measured as the difference between the actual rate of unemployment and the NAIRU as reported in the Federal Reserve Database). We then compared our unemployment and output shocks to the shocks we recover when we target the gaps. Figure 19 and Table 28 report the IRFs and variance contributions of these various shocks. The results clearly indicate that the gap-shocks are very similar if not identical.

Figure 19: Impulse Response Functions

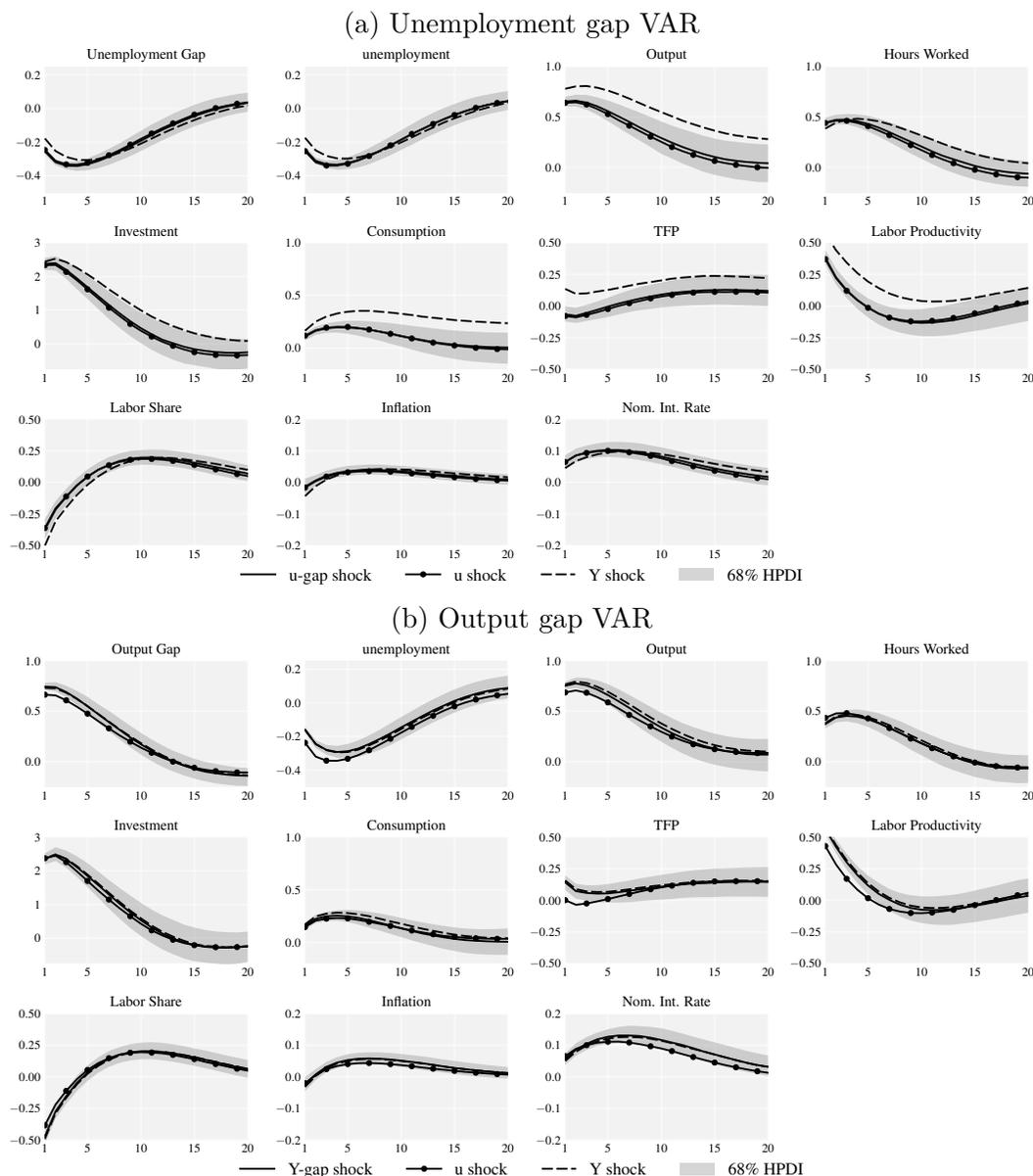


Table 28: Adding Gaps: Variance Contributions (6-32 Quarters)

	Gap	$u$	$Y$	$h$	$I$	$C$	TFP	$Y/h$	$Wh/Y$	$\pi$	$R$
Output Gap VAR: 1955Q1-2017Q4											
$u$	65.8	74.7	63.5	47.7	64.4	23.8	4.1	28.0	28.5	9.0	23.9
	[59.0,71.8]	[68.1,80.9]	[56.1,69.9]	[40.7,54.8]	[57.2,70.7]	[16.1,32.0]	[1.5,8.5]	[20.9,35.9]	[19.9,37.4]	[4.6,15.4]	[16.5,32.5]
$y$	82.9	60.6	82.8	48.1	69.5	33.7	3.4	45.9	39.1	14.2	26.3
	[77.0,87.4]	[54.3,66.1]	[76.2,88.0]	[40.8,55.5]	[62.9,75.1]	[25.5,41.9]	[1.6,6.5]	[40.1,52.4]	[31.2,45.7]	[8.3,22.2]	[18.5,34.3]
$y - gap$	85.5	60.3	80.0	46.3	69.6	30.1	3.2	45.1	37.0	13.8	28.6
	[79.5,90.3]	[54.0,65.7]	[73.6,85.0]	[38.7,54.4]	[62.8,75.1]	[22.1,38.5]	[1.5,6.2]	[39.0,51.4]	[29.8,44.4]	[8.0,21.9]	[20.7,36.2]
Unemployment Gap VAR: 1955Q1-2017Q4											
$u$	71.6	74.7	58.0	47.2	61.4	19.9	6.2	22.9	26.0	6.7	22.0
	[65.6,77.4]	[68.2,80.2]	[50.8,64.6]	[40.0,54.2]	[54.5,68.3]	[14.1,26.8]	[2.7,11.1]	[16.1,29.9]	[18.3,34.5]	[3.4,11.2]	[14.8,30.2]
$y$	57.1	55.8	80.3	43.8	66.9	34.2	4.0	40.0	39.2	10.4	16.7
	[51.6,63.1]	[49.7,62.0]	[73.4,86.1]	[36.2,50.5]	[61.1,72.7]	[27.0,40.9]	[1.8,7.6]	[34.7,45.6]	[32.3,46.0]	[6.2,15.9]	[10.4,25.0]
$u - gap$	74.0	72.3	59.5	47.6	62.8	18.7	5.9	24.4	27.3	7.2	22.1
	[68.0,80.0]	[66.1,77.9]	[52.2,66.1]	[40.4,54.9]	[55.9,69.6]	[12.8,25.6]	[2.6,10.6]	[17.9,31.5]	[19.6,35.8]	[3.7,12.1]	[14.5,30.1]

## H Bayesian vs Classical Approach

In this Appendix we first describe the details of the Minnesota prior we used to make Bayesian inference from our VARs. We then explore how the main results are robust to a “classical” alternative.

### H.1 Priors

We used the Minnesota prior, which incorporates the prior belief that the endogenous variables included in the VAR follow either a random walk process or a stationary AR(1) process. For a VAR( $p$ ) process of the form

$$X_t = C + \sum_{k=1}^p A^{(k)} X_{t-k} + u_t$$

where  $X_t = (x_{1t}, \dots, x_{Nt})$ , the Minnesota prior implies  $C = \mathbf{0}$ ,

$$A^{(1)} = \begin{pmatrix} a_{11} & 0 & \dots & 0 \\ 0 & a_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & a_{NN} \end{pmatrix} \quad \text{with} \quad a_{ii} = \begin{cases} 1 & \text{if Random walk} \\ \rho & \text{with } |\rho| < 1 \text{ if AR(1)} \end{cases}$$

and  $A^{(k)} = \mathbf{0}$  for all  $k = 2, \dots, p$ .

In our benchmark experiment, we left the possibility that all variables exhibit a random walk component. However, as a robustness check, we also investigated the case where hours worked, unemployment, the labor share, the inflation rate and the nominal interest rate are, in line with most standard theoretical models, described by stationary AR(1) processes with a persistence,  $\rho$ , lower than 1. We found that this is not playing a role for our main results (see Table 29). The Minnesota prior also assumes that the variance of the prior distribution for the coefficients  $a_{ij}$  is given by

$$\begin{cases} \left( \frac{\gamma_1}{k\gamma_3} \right)^2 & \text{if } i = j \\ \left( \frac{\sigma_i \gamma_1 \gamma_2}{\sigma_j k \gamma_3} \right)^2 & \text{if } i \neq j \end{cases}$$

and by  $(\sigma_i \gamma_4)^2$  for the constant.  $\sigma_i$  denotes the standard deviation of the residuals as estimated by a standard OLS regression and  $k$  is the lag. Finally the parameters  $\gamma_1$ ,  $\gamma_2$  and  $\gamma_4$  control for the tightness of the priors on the own lags, other variables lags and the constant term. The parameter  $\gamma_3$  controls the degree to which coefficients on lags higher than 1 are likely to be zero. We follow Canova (2007, p.380) and use  $\gamma_1 = 0.2$ ,  $\gamma_2 = 0.5$ ,  $\gamma_3 = 2$  and  $\gamma_4 = 10^5$  which implies a relatively loose prior on the VAR coefficients and an uninformed prior for the constant terms.. The posterior distribution is then computed relying on a Gibbs sampler (see Canova (2007), p. 361-366), performing 50,000 draws and only keeping the last 1,000 draws. We checked the robustness of our results to longer simulations.

## H.2 Robustness: Classical vs Bayesian

We now compare our baseline results to two alternatives. The one remains Bayesian but changes the Minnesota prior in the manner described above. The other uses classical inference.

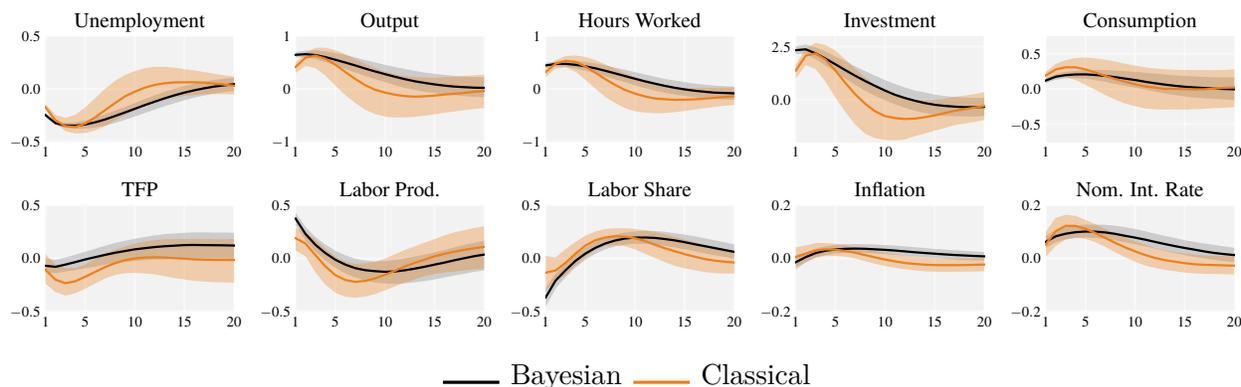
Table 29 and Figure 20 illustrate that the change in the method of inference does not alter the key properties of the MBC shock (defined as the shock that targets unemployment at the 6-32 quarters frequency band). In particular, the change in the prior has a completely negligible effect. And as we move from Bayesian to classical, only two small differences deserve mentioning.

First, the contribution of the MBC shock to the variability of some of the main macroeconomic quantities is somewhat reduced, while it increases for consumption. That is, the MBC shock loses a bit in terms of variance contribution but gains in terms of co-movement.

Second, the MBC shock now accounts for a larger (but still relatively small) share of the variance of TFP over the business cycle frequencies. Note, though, that this finding does not suggest a greater relevance of either RBC or TFP-news types of shocks. As can be seen in Figure 20, the response of TFP to the MBC shock is negative in the short run (while that of output and employment is positive). So the identified MBC shock does not seem to be related to the force that drives business cycles in the RBC model. Moreover, as can be seen in Table 29, the contribution of the MBC shock to long term TFP remains essentially zero, consistent with our baseline results and at odds with TFP-news being the main driver.

Focusing more explicitly on news shocks, we have also repeated the exercise of Appendix C, which extracts a news shock out of the two factors that drive the majority of TFP at all frequencies, using classical inference. As seen in Figures 22 and 23, the main lesson of that exercise, too, is unaffected: once enough information is used from the data (in the form of sufficiently large VARs), the identified news shock explains a small fraction of the business cycle, despite the fact that it now explains an even larger fraction of the long-run movements in TFP. And as seen in Figure 21, the empirical footprint of the identified news shocks in terms of IRFs is also unaffected.

Figure 20: Impulse Response Functions to the MBC Shock: Bayesian vs Classical Inference



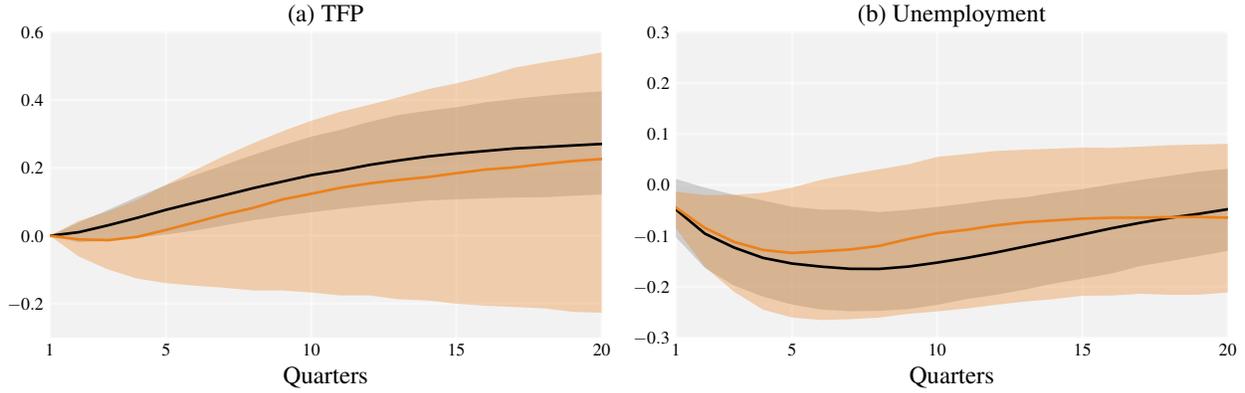
*Note:* Impulse Response Functions of all the variables in our VAR to the identified MBC shock. Horizontal axis: time horizon in quarters. Gray Shaded area : 68% Highest Posterior Density Interval. Red Shaded area : 68% Confidence Interval obtained from Kilian's (1996) bias corrected Bootstrap.

Table 29: Variance Contributions

	$u$	$Y$	$h$	$I$	$C$
<i>Short Run (6-32 quarters)</i>					
Bayesian	73.7	57.8	46.9	61.1	20.0
	[66.7,79.8]	[50.5,65.1]	[39.6,53.9]	[54.7,67.9]	[13.7,27.0]
Bayesian (AR)	74.4	58.2	47.0	61.9	19.6
	[67.8,80.5]	[50.6,65.4]	[40.2,53.7]	[54.5,68.6]	[13.4,26.0]
Classical	62.0	51.3	52.8	53.4	36.4
	[55.3,70.6]	[43.2,59.8]	[44.9,61.4]	[46.0,61.9]	[26.5,47.0]
<i>Long Run (80-∞ quarters)</i>					
Bayesian	21.6	4.9	5.3	5.0	4.3
	[9.2,38.6]	[0.7,16.5]	[1.4,15.2]	[0.9,17.1]	[0.5,15.7]
Bayesian (AR)	23.3	4.8	4.9	5.2	4.4
	[9.2,40.1]	[0.6,15.7]	[1.2,14.9]	[0.8,16.2]	[0.5,14.7]
Classical	8.9	4.6	5.9	5.1	4.7
	[2.1,24.2]	[0.6,17.8]	[1.1,17.8]	[0.8,17.7]	[0.6,17.5]
	TFP	$Y/h$	$wh/Y$	$\pi$	$R$
<i>Short Run (6-32 quarters)</i>					
Bayesian	5.7	23.6	26.9	6.8	21.8
	[2.6,10.8]	[17.2,31.0]	[18.5,35.6]	[3.3,12.0]	[14.6,30.9]
Bayesian (AR)	5.7	24.1	28.0	6.4	21.8
	[2.6,10.3]	[17.5,31.0]	[19.7,36.9]	[3.2,11.5]	[13.9,30.6]
Classical	19.5	26.9	27.4	14.9	39.5
	[10.0,29.9]	[18.6,36.5]	[17.1,39.6]	[6.7,27.6]	[22.5,56.9]
<i>Long Run (80-∞ quarters)</i>					
Bayesian	4.4	4.2	3.6	5.4	8.6
	[0.6,15.4]	[0.5,14.8]	[0.9,11.9]	[1.6,13.9]	[3.0,19.3]
Bayesian (AR)	4.2	3.9	3.4	5.4	8.2
	[0.5,14.2]	[0.5,14.0]	[0.7,10.3]	[1.6,13.1]	[2.4,19.0]
Classical	4.5	4.5	5.5	6.7	8.8
	[0.8,18.1]	[0.7,18.1]	[1.1,17.7]	[1.8,16.4]	[2.6,21.0]

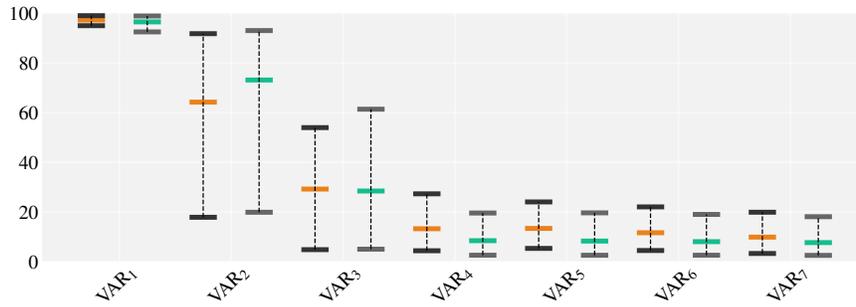
*Note:* Variance contributions of the MBC shock at two frequency bands. The first row (Short Run) corresponds to the range between 6 and 32 quarters, the second row (Long Run) to the range between 80 quarters and  $\infty$ . The shock is constructed by targeting unemployment over the 6-32 range. 68% HPDI into brackets in the Bayesian case. In the Classical exercise, we report the 68% confidence band obtained from Kilian's (1996) bias corrected Bootstrap. The Bayesian (AR) case corresponds to the situation where the priors assume that unemployment, hours worked, inflation, the interest rate, the labor share are stationary AR processes.

Figure 21: IRF of TFP and Unemployment to News Shock (Benchmark VAR)



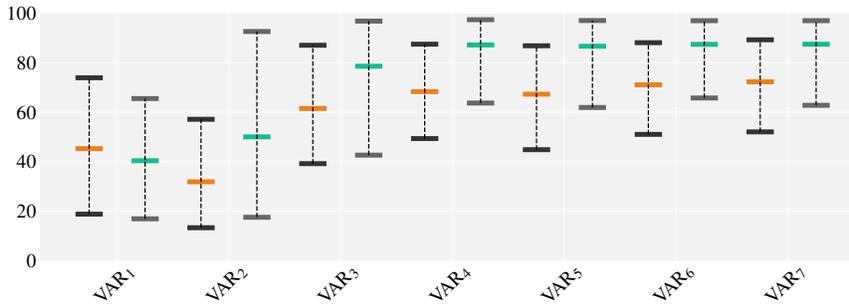
— Bayesian; — Classical;  
 Shaded area: Gray: 68% HPDI Red: 68% Confidence band from 1,000 bootstrap draw

Figure 22: Variance Contribution of News Shock to Unemployment



*Note:* Contribution of news shock to unemployment at business-cycle frequencies. Red (resp. Green) line gives median for the Bayesian (resp. Classical) case, upper and lower black lines give 68% HPDI.  $VAR_1 = \{u, TFP\}$ ,  $VAR_2 = VAR_1 \cup \{I\}$ ,  $VAR_3 = VAR_2 \cup \{Y, C, h\}$ ,  $VAR_4 =$  Baseline VAR,  $VAR_5 = VAR_4 \cup \{SP500\}$ ,  $VAR_6 = VAR_5 \cup \{\text{utilization}\}$ ,  $VAR_7 = VAR_6 \cup \{\text{credit spread}\}$ .

Figure 23: Long-Run Variance Contribution of News Shock to TFP



*Note:* Contribution of news shock to unemployment at business-cycle frequencies. Red (resp. Green) line gives median for the Bayesian (resp. Classical) case, upper and lower black lines give 68% HPDI.  $VAR_1 = \{u, TFP\}$ ,  $VAR_2 = VAR_1 \cup \{I\}$ ,  $VAR_3 = VAR_2 \cup \{Y, C, h\}$ ,  $VAR_4 =$  Baseline VAR,  $VAR_5 = VAR_4 \cup \{SP500\}$ ,  $VAR_6 = VAR_5 \cup \{\text{utilization}\}$ ,  $VAR_7 = VAR_6 \cup \{\text{credit spread}\}$ .

# I An AD-AS Example

In this appendix we conduct two “pedagogical” exercises motivated by the AD-AS example mentioned in Section IV. In the first, which is semi-structural in nature, we show that the narrative of offsetting demand and supply shocks does not work insofar as the supply shock is proxied by the productivity shock identified via our method. In the second exercise, which is fully structural, we show that this story is also inconsistent with a textbook New Keynesian model calibrated to the relevant elements of our anatomy.

## I.1 Proxying the AS shock with the TFP shock

Our first, semi-structural exercise is based on the following simple idea. If the MBC shock is a mixture of an inflationary demand shock and a disinflationary supply shock, and if the supply shock reflects movements in productivity, then the documented disconnect between the MBC shock and inflation should be weakened, and the role of the demand shock be revealed, if we control for the effect of productivity. This in turn can be done by purging from the data the reduced-form shock that targets TFP over the business-cycle frequencies.<sup>6</sup> We thus repeat our identification of the shocks that target unemployment, GDP, and inflation after this purging and ask whether this reduces the disconnect between the MBC shock and inflation.

As evident in Table 30 and Figure 24, the answer is clearly negative. Whether we look at original reduced-form shocks or the ones obtained after purging the effects of productivity, the aforementioned disconnect and indeed the shocks themselves remain almost unchanged.

Table 30: Variance Contributions

	<i>Unemployment Shock</i>			<i>Output Shock</i>			<i>Inflation Shock</i>		
	<i>u</i>	<i>Y</i>	$\pi$	<i>u</i>	<i>Y</i>	$\pi$	<i>u</i>	<i>Y</i>	$\pi$
Baseline	73.7	57.8	6.8	55.6	79.8	10.6	4.2	8.0	83.3
Purged	70.7	60.9	7.9	57.2	78.3	9.4	3.7	6.0	80.1

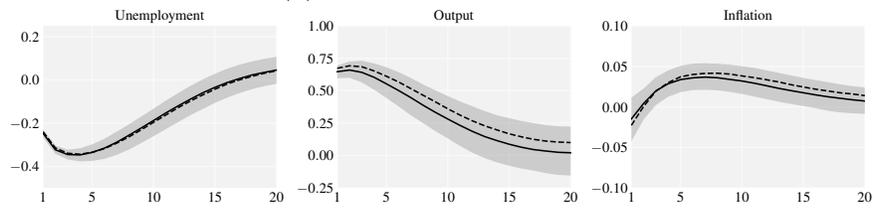
Furthermore, insofar as one accepts the interpretation of the MBC shock identified in the data as the AD shock in the theory, the challenge for the theory is twofold: not only does the MBC shock accounts for a small fraction of volatility in inflation, but it has such a small impact on inflation that the theory can make sense of only if the AS curve is extremely flat.

We illustrate this point in Figure 25. The solid black line shows the actual response of inflation to the MBC shock in the data. The dashed red line shows the response predicted by the New Keynesian Philips Curve, under a textbook calibration and with the real marginal cost proxied by

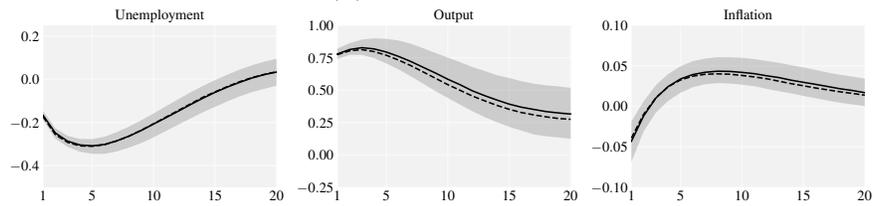
<sup>6</sup>We have obtained almost identical results with a variant specification that proxies the supply shock with the technology shock identified as in Galí (1999), as well as with one that purges both the short-run and the long-run TFP shocks identified via our method. These alternatives, however, seem less appropriate for the present purposes, because they amount to purging also the effects of news about future productivity, which in standard models maps to a demand rather than a supply shock.

Figure 24: Impulse Response Functions

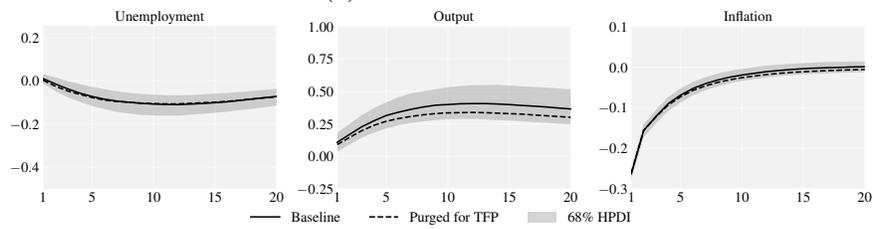
(a) Unemployment Shock



(b) Output Shock

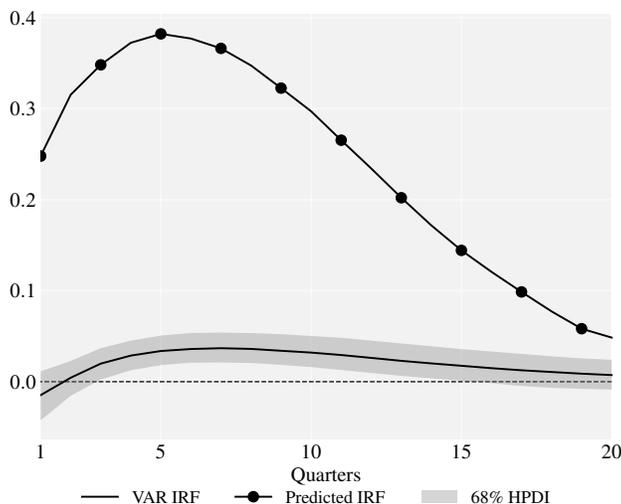


(c) Inflation Shock



the response of the labor share to the MBC shock.<sup>7</sup> The large gap between the two lines illustrates that, even after controlling for the possible sluggishness in the response of the real marginal cost due to wage rigidities, the predicted response of inflation is over 10 times larger than the actual one. Conversely, the Phillips curve has to be very flat for the theory to match the observed inflation response. A similar picture is painted in the next subsection, which takes a fully-structural approach to both the MBC shock and the shock that accounts for the volatility in inflation.

Figure 25: The MBC Shock and the NKPC



## I.2 A 2×2 New Keynesian model

We now turn to second, fully-structural exercise: we employ a two-shock, two-variable version of the New Keynesian model and ask what it takes for this model to account for the relevant elements of our anatomy.

In particular, we estimate both the shock processes and the main parameters of the model—those that govern the slopes of the AS and AD curves and the sluggishness of the inflation and output dynamics—by minimizing the distance between four empirical IRFs and their theoretical counterparts. These are the IRFs of output and inflation to the output shock and to the inflation shock, as identified by our method. We focus on these objects because the simple, textbook-style model considered here is meant to speak to the only dynamics of output and inflation.<sup>8</sup>

We then use the estimated model to answer two questions. First, what parameter values (for instance, the slope of the Phillips curve) does the model need in order to achieve maximum fit vis-a-vis our facts? And second, does the MBC shock identified via our method correspond to

<sup>7</sup>To construct this line, we proceed as follows. First, we take the New Keynesian Phillips Curve:  $\pi_t = \kappa x_t + \beta E_t[\pi_{t+1}]$ , where  $\beta \in (0, 1)$  is the discount factor,  $\kappa = (1 - \theta)(1 - \beta\theta)/\theta$ , and  $\theta$  is the Calvo parameter. Next, we set  $\theta = 2/3$  (prices are, on average, reset every 3 quarters) and  $\beta = 0.99$  (an annual discount rate of 4%). Finally, we feed  $x_t$  with the response of the labor share to the MBC shock.

<sup>8</sup>The empirical IRFs are obtained from our VAR by targeting the inflation rate or output (see Figure 2 for example). The theoretical IRFs are constructed in an analogous manner, treating the model as the DGP.

a single structural shock in the model or to a mixture of structural shocks, as suggested by the AD-AS example used in Section IV?

Like the textbook version of the New Keynesian model, the version considered here reduces to two equations in the  $(y, \pi)$  space, one representing aggregate demand (AD) and the other representing aggregate supply (AS). At the same time, our version mimics richer DSGE versions by allowing for a flat Philips curve, habit persistence and price indexation. These enhancements may lack empirical micro-foundations but are customarily used in the literature in order to improve the model's empirical performance.

Let us start with the textbook version of the New Keynesian model, which can be expressed by the following equations:

$$y_t = -\sigma(R_t - \mathbb{E}_t[\pi_{t+1}]) + \mathbb{E}_t[y_{t+1}] + \sigma \xi_t \quad (1)$$

$$\pi_t = \lambda mc_t + \beta \mathbb{E}_t[\pi_{t+1}] + \lambda \mu_t \quad (2)$$

$$mc_t = \kappa y_t - \frac{1+\nu}{\alpha} a_t + \zeta_t \quad (3)$$

$$R_t = \varphi \pi_t + \psi y_t + m_t \quad (4)$$

The interpretation is familiar: (1) is the Dynamic IS curve, (2) is the NKPC, (3) describes the real marginal cost as a function of output and productivity, and (4) specifies monetary policy. The notation is also standard:  $y_t$  is output,  $\pi_t$  is inflation,  $mc_t$  is the real marginal cost,  $R_t$  is the nominal interest rate,  $\mathbb{E}_t$  is the rational expectations operator,  $a_t$  is the productivity shock,  $\xi_t$  is the discount-rate shock,  $\mu_t$  is the markup shock,  $\zeta_t$  is the cost-push shock,  $m_t$  is the monetary-policy shock,  $\sigma > 0$  is the elasticity of intertemporal substitution,  $\beta \in (0, 1)$  is the steady-state discount factor,  $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$  is the slope of the NKPC with respect to the real marginal cost (and to the markup shock, too),  $\theta$  is the Calvo parameter (the probability of a firm's not being able to reset its price),  $\kappa \equiv \frac{1+\nu}{\alpha} + \frac{1-\sigma}{\sigma} > 0$  is the slope of the real marginal cost with respect to output,  $\nu \geq 0$  is the Frisch elasticity of labor supply,  $\alpha \in (0, 1]$  is the short-run elasticity of output with respect to labor, and  $\varphi > 1$  and  $\psi \geq 0$  parameterize the responsiveness of monetary policy to, respectively, inflation and output.

To simplify the exposition of the AD and AS curves below, we set  $\psi = 0$ .<sup>9</sup> For the reported experiments, we also interpret a period as a quarter and set  $\beta = .99$ ,  $\varphi = 2$ ,  $\alpha = 1$ , and  $\nu = 0$ .<sup>10</sup> More crucially, the parameters  $\lambda$  and  $\sigma$ , which govern the slopes of the two curves, and two additional parameters, which are introduced momentarily and which govern the endogenous persistence in the model, are left free to be estimated in one of the experiments.

Substituting (4) in (1) and (3) in (2), we can reduce the model to the following two equations

---

<sup>9</sup>Since the experiments conducted here do not utilize data on the interest rate, the effect of a positive  $\psi$  on the dynamics of output and inflation can be proxied by appropriately adjusted values for other model parameters. Accordingly, we have verified that our findings about the model's performance remain essentially unchanged if we let, for example,  $\psi = 0.5$ .

<sup>10</sup>The values of  $\beta$  and  $\varphi$  are standard, while those for  $\alpha$  and  $\nu$  help reduce the sensitivity of the real marginal cost to output (intuitively, a high value for  $\alpha$  mimics variable utilization and a low value for  $\nu$  mimics real wage rigidity), which in turn helps improve the empirical performance of the model (and makes our own job harder)

in output and inflation alone:

$$y_t = -\sigma\varphi\pi_t + \sigma\mathbb{E}_t[\pi_{t+1}] + \mathbb{E}_t[y_{t+1}] + u_t^d \quad (5)$$

$$\pi_t = \lambda\kappa y_t + \beta\mathbb{E}_t[\pi_{t+1}] - u_t^s \quad (6)$$

where  $u_t^d \equiv \sigma\xi_t - \sigma m_t$  and  $u_t^s \equiv \lambda\kappa a_t - \lambda\kappa\zeta_t - \lambda\mu_t$ . Condition (5) represents aggregate demand, AD, (6) represents aggregate supply, AS. Accordingly,  $u_t^d$  and  $u_t^s$  are the (composite) demand and supply shocks. We assume that these shocks follow independent  $AR(1)$  process and let  $(\sigma_d, \sigma_s)$  denote their standard deviations and  $(\rho_d, \rho_s)$  their autocorrelations.

This completes the description of the baseline version of the New Keynesian model, which is the building block for the enhanced, DSGE-like variant used here. This variant is obtained by including habit persistence in the Dynamic IS curve and by replacing the standard NKPC with the hybrid one. The modified equations are given by

$$\begin{aligned} y_t &= -\sigma\frac{1-h}{1+h}(\varphi\pi_t - \mathbb{E}_t\pi_{t+1}) + \frac{1}{1+h}\mathbb{E}_t y_{t+1} + \frac{h}{1+h}y_{t-1} + u_t^d \\ \pi_t &= \lambda\left(\kappa y_t + \frac{h}{\sigma(1-h)}(y_t - y_{t-1})\right) + \frac{\beta\theta}{\theta + \omega(1-\theta(1-\beta))}\mathbb{E}_t\pi_{t+1} + \frac{\omega}{\theta + \omega(1-\theta(1-\beta))}\pi_{t-1} - u_t^s \end{aligned}$$

for some  $h \in [0, 1)$  and  $\omega \in [0, 1)$ . These capture the inertia added to the aggregate demand and aggregate supply equations, respectively.<sup>11</sup> Finally,  $\lambda$  is allowed to take low enough values so as to accommodate a relatively weak positive co-movement between inflation and output in response to demand shocks.

Let  $\Theta \equiv (\sigma_d, \sigma_s, \rho_d, \rho_s; \lambda, \sigma, h, \omega)$  collect the parameters that regulate the shock processes and the internal propagation, namely the slopes of the AS and AD curves and the corresponding sources of sluggishness. We estimate  $\Theta$  by minimizing the distance between the IRFs of output and inflation to the output and inflation shocks identified in the data via our method and the corresponding objects in the model.

Table 31 reports the estimated parameter values. Table 32 reports the variance contributions of the model's two structural shocks. The most notable features are that  $\lambda$  is nearly zero, that the output fluctuations are dominated by a non-inflationary demand shock, and that the inflation fluctuations are dominated by a disinflationary supply shock. That is, confronted with the relevant elements of our anatomy, the model demands a very flat AS (or Philips) curve and specialized structural shocks, a picture consistent with that painted in Section II.<sup>12</sup>

<sup>11</sup>The standard interpretation of  $h$  is as the degree of habit persistence in consumption. But as there is no capital in the model,  $h$  represents all the adjustment frictions in aggregate demand. On the other hand,  $\omega$  corresponds to the fraction of irrational, backward-looking firms in Galí and Gertler (1999), or the degree of automatic past-price indexation in Christiano et al. (2005). These model enhancements lack solid empirical micro-foundations but are customarily used in the DSGE literature.

<sup>12</sup>Another interesting finding, which is though not particularly relevant for the present purposes, is that the estimation of the model based on our anatomy yields  $\omega = 0$ , that is, no past-price indexation or backward-looking element in the Philips curve. This appears to be driven by the absence of sluggishness in the response of inflation to the inflation shock and suggests that the "right" model is one that somehow allows for such sluggishness in the response of inflation to the main driver of the real quantities without however introducing such sluggishness in the overall inflation dynamics.

Table 31: Parameters

$\sigma_s$	$\sigma_d$	$\rho_s$	$\rho_d$	$h$	$\omega$	$\lambda$	$\sigma$
0.0792	0.0305	0.7022	0.9569	0.1915	0.0000	0.0004	0.2829

Table 32: Variance Contributions

	Output	Inflation
Supply Shock	8.23	98.76
Demand Shock	91.77	1.24

The purpose of this—pedagogical—exercise was to illustrate how the combination of our anatomy with a model can help discipline the AD-AS narrative offered in Section. The same strategy is applied to, and works well for, the three state-of-the-art DSGE models considered in Section V. Naturally, while all of these exercises support the interpretation of the empirical MBC shock as a non-inflationary demand shock, they cannot establish its universality.

## J Robustness of Model Evaluations

This appendix assesses the robustness of the lessons drawn in Section V regarding the evaluation of the JPT and ACD models under the lenses of our method.

### J.1 Running the Same VAR on Data and Models

In the main text, we evaluated the ability of JPT and ACD to account for the MBC shock in the data using the theoretical, asymptotic properties of the two models. We now explore the robustness of our findings to a Monte Carlo exercise that runs the same, small-size VAR on artificial data from each model and on the actual US data.

Because both models have a stochastic dimension smaller than that of our benchmark VAR, we also rerun our empirical specification on a restricted VAR featuring Output, Consumption, Investment, Hours worked, Fernald’s measure of Total Factor Productivity (corrected for utilization), the nominal interest rate and the inflation rate. As can be seen in the first two rows of Figure 26, this smaller VAR gives rise to the same picture as our baseline VAR: the shocks that target output, hours, investment and consumption are essentially indistinguishable from one another.

Because the smaller VAR run here has exactly the same stochastic dimension as the JPT model, it can be readily run on artificial data generated by that model. By contrast, the ACD model has one dimension less: being a flexible-price, no-monetary model, it makes no prediction about inflation (and nominal variables). To be able run the same VAR on artificial data from that model, we augment it with the simplest model of inflation we could think of: an exogenous AR(1) process.<sup>13</sup>

<sup>13</sup>We estimated this process using inflation data alone. This gave an estimate of 0.89 for the persistence parameter and 0.27% for the standard deviation of the innovation. All the other (real) parameters of the model were fixed at

Clearly, this add-on has no effect on the model’s predictions regarding any of the real variables. It only permits us to run the same VAR on the two models under consideration.

Each model is then simulated 1000 times to generate artificial time series for the aforementioned set of variables. Each artificial time series has the same length as in the data (192 quarters from 1960Q1 to 2007Q4). Note that, in order to avoid any dependence on initial conditions, we actually simulated 292 observations and discarded the first 100. Then, for each set of simulated data, we estimated the same VAR as in actual data and applied our methodology to extract the various VAR-based shocks, or “factors,” and build their IRFs.

The third and the fourth row of Figure 26 show the median of the so-obtained distribution of IRFs for the JPT and ACD models, respectively. The comparison of these rows to one another and with the first row (the data) corroborates the lesson obtained in the main text on the basis of the theoretical state-space representation of the two models: the factors in JPT are less interchangeable than their counterparts either in ACD or the data. The visual impression is corroborated by Table 33, which reports the metric discussed in the main text.

Table 33: Interchangeability of Factors, Simulated VARs

	$Y$	$C$	$I$	$h$	Average
Data (Baseline VAR, 1960-2007)	0.47	0.51	1.38	0.18	0.63
Data (Small VAR, 1960-2007)	0.47	0.50	1.76	0.29	0.76
JPT	1.76	1.39	3.99	0.94	2.02
ACD	0.42	0.47	1.34	0.25	0.62

*Note:* The metric is the same as that in Table 9. A number closer to zero indicates a larger degree of interchangeability.

## J.2 Re-estimating JPT/ACD

We now turn to the remaining two robustness exercises mentioned in Section V.

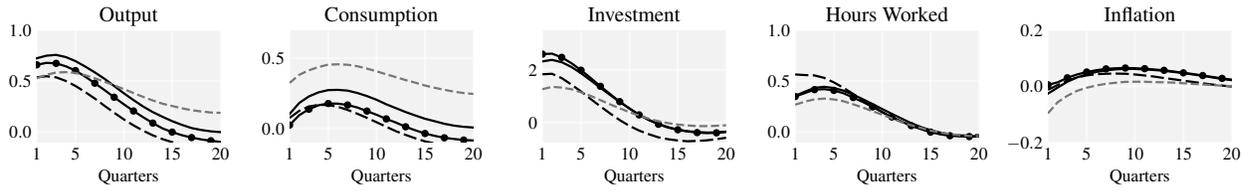
First, in order to offer a proper comparison between JPT and ACD, we re-estimated the JPT model the same frequency-domain Bayesian technique used to estimate ACD. More precisely, the model is estimated over the business-cycle band of frequencies (6-32 quarters), using the levels of all variables, and using the 1960-2007 data. This set of results is labeled *JPT - Freq. Domain* in the tables and figures that follow.

Second, we re-estimated both models using a minimum-distance estimation technique, with the parameters selected in order to minimize the distance between IRFs of output, consumption, investment and hours worked to the output, consumption, investment and hours worked factors over the horizon of 20 quarters (a set of 320 moments). Denoting by  $IRF_{j,h}^i$  (resp.  $\widehat{IRF}_{j,h}^i(\Theta)$ ) the response of variable  $j$  to factor  $i$  at horizon  $h$  found in the data (resp. in the model) and  $\sigma_{j,h}^i$  the

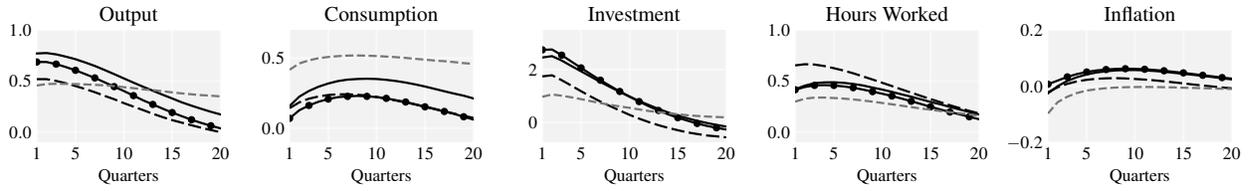
their values in the original article. Finally, the nominal interest rate was obtained directly from the Fisher equation, using the AR(1) process for inflation and the model’s prediction about the real rate.

Figure 26: The MBC Shock

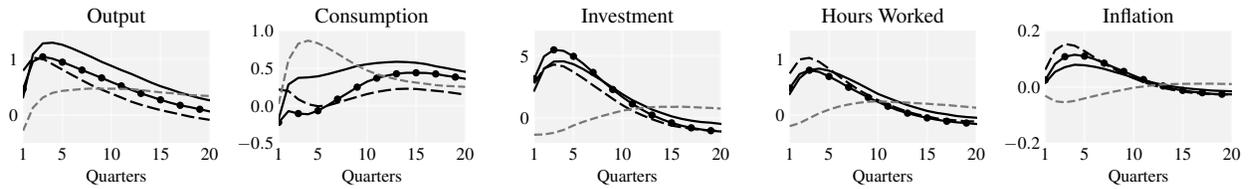
(a) Data (Baseline VAR, 1960-2007)



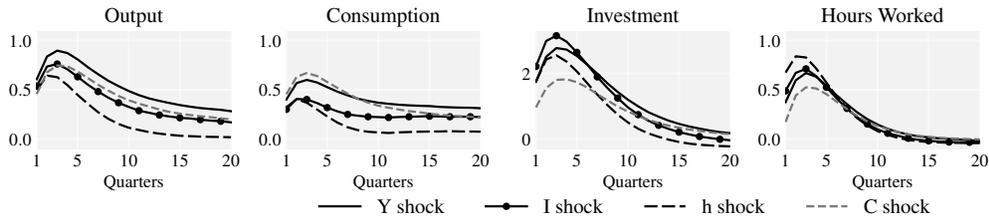
(a) Data (Small VAR, 1960-2007)



(b) JPT



(d) ACD



— Y shock    ● I shock    - - - h shock    - · - · C shock

variance of  $IRF_{j,h}^i$ , the vector of structural parameters  $\Theta$  is found by solving the problem

$$\min_{\Theta} \sum_{i=1}^4 \sum_{j=1}^4 \sum_{h=1}^{20} \frac{(\widehat{IRF}_{j,h}^i(\Theta) - IRF_{j,h}^i)^2}{\sigma_{j,h}^i}$$

Given our focus on the real IRFs, the parameters pertaining to the nominal part of JPT (Calvo probabilities, indexation parameters, parameters of nominal shocks) are not identified. We therefore set the values of these parameters to those estimated by JPT and re-estimated the parameters pertaining to the real side of the model (preferences, technology, adjustment costs, parameters of real shock processes). The relevant set of results is labeled *JPT - Matching Factors* and *ACD - Matching Factors*.

Figure 27 and Table 34, which extend Figure 6 and Table 9 from the main text, provide a comprehensive comparison of the dynamic properties of the two models under alternative specifications. The main findings are as follows. Re-estimating the JPT model in the frequency domain has a significant but still insufficient impact on the model’s ability to reproduce the interchangeability of factors in the data. Re-estimating it by targeting the factors helps the model even more, but it still falls short of that in the data. Re-estimating the ACD by targeting the factors does not upset its already good performance, but it overshoots in the direction of producing too much interchangeability. All in all, the metric of how different the factors are is systematically greater for JPT than ACD, irrespective of the estimation method.

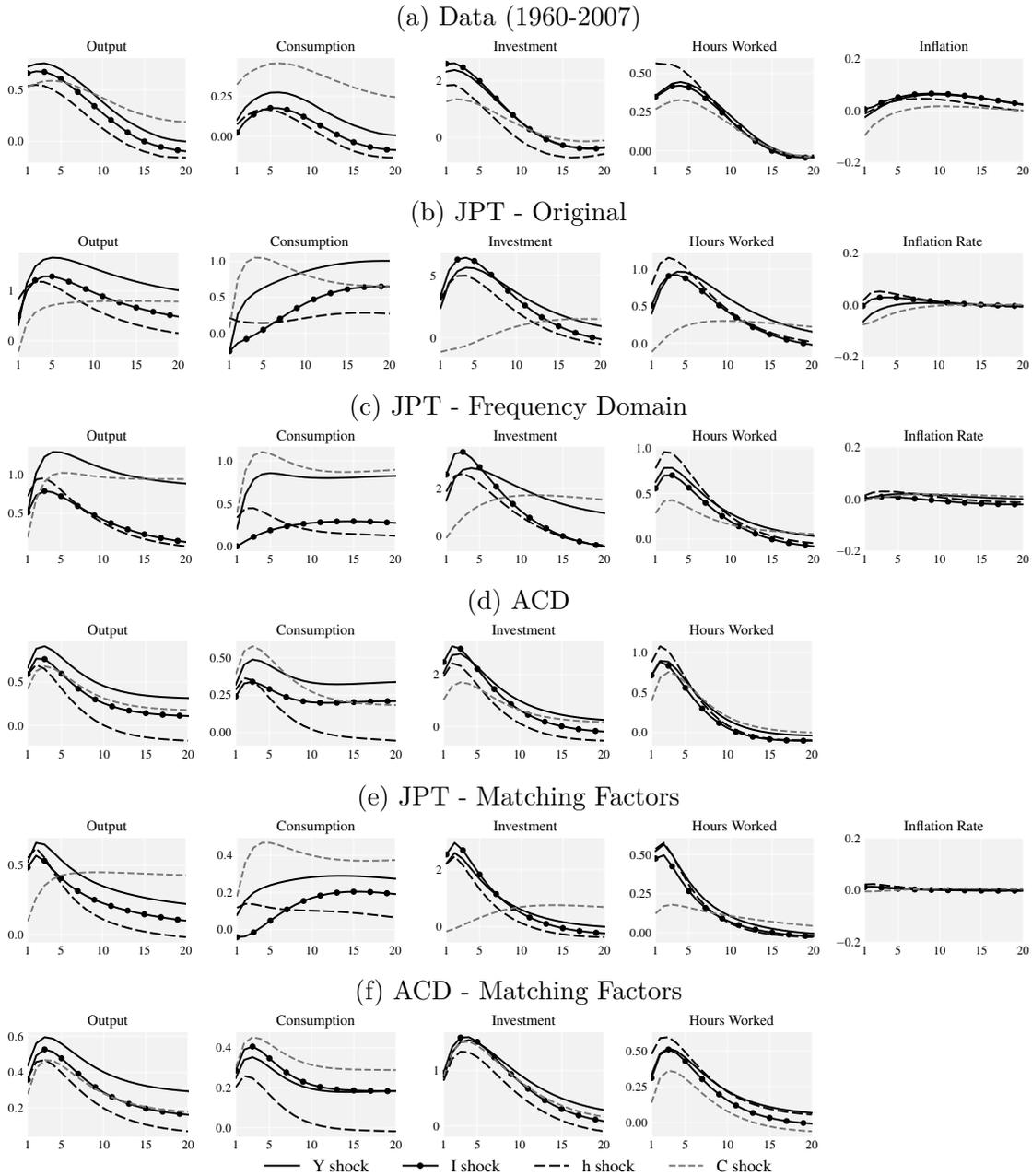
Table 34: Interchangeability of Factors

	<i>Y</i>	<i>C</i>	<i>I</i>	<i>h</i>	Average
Data (1960-2007)	0.47	0.51	1.38	0.18	0.63
JPT - Original	2.90	2.21	6.29	1.35	3.19
JPT - Freq. Domain	1.41	1.42	3.24	0.42	1.62
ACD	0.56	0.49	1.61	0.30	0.74
JPT - Matching Factors	0.56	0.51	2.26	0.27	0.90
ACD - Matching Factors	0.26	0.36	0.49	0.26	0.34

*Note:* The metric is the same as that in Table 9. A smaller number indicates greater interchangeability.

In conclusion, let us reiterate that the main goal of the application of our method to ACD and JPT is not to judge the superiority of one model over the other, but rather to illustrate the probing power of our method in the context of existing, medium-scale, DSGE models that have already been estimated and evaluated via other methods. This is best exemplified by the exercise conducted in the main text. The second robustness exercise in this appendix serves a complementary objective, namely to inform on whether is at all possible for these models to be replicate the propagation mechanism we observe in the data and, if so, what this requires in terms of their parameters. In short, the two exercises illustrate two different ways in which our anatomy of the data can inform theory.

Figure 27: Comparing Business-Cycle Factors



## K The Secondary Business Cycle Shock

For each of the five macroeconomic quantities,  $X \in \{u, Y, h, I, C\}$ , we now identify *two* shocks. The first shock is the one already reported in the main text: it is obtained by maximizing its contribution to the business-cycle volatility of that variable. The second shock is obtained by maximizing its contribution to the residual, business-cycle volatility of the targeted variable after filtering out the effect of the first shock. This procedure produces a collection of five new shocks, one for each of the macroeconomic quantities of interest.

Figure 28 reports the IRFs to these shocks and Table 35 their variance contributions. The IRFs are nearly the same, suggesting that these shocks, too, represent interchangeable facets of one shock—the “secondary” business cycle shock, or SBC for short.

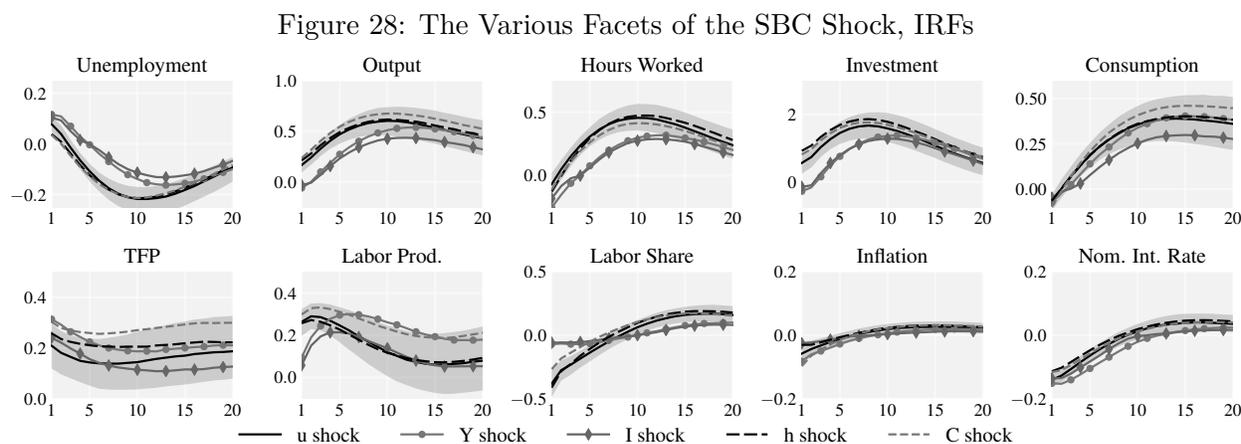


Figure 28 also reveals that the impact of the SBC shock on the economy builds up slowly over time, peaking after several quarters. By contrast, the impact of the MBC shock peaks within a year and fades shortly after. Furthermore, the SBC shock contains relatively more information about TFP and the prospects of the economy in the medium to long run. In this sense, whereas the MBC shock fits the profile of a quick-moving demand shock, the SBC shock fits the profile of a slow-moving supply shock. Both shocks, however, have a similar, zero to weakly positive, effect on inflation. Neither of them therefore fits easily in the traditional AD-AS framework.<sup>14</sup> In the main text, we focused on the MBC shock as the main probing tool of our anatomy and treated the SBC shock as part of the residual. While the SBC shock represents subsidiary rather than primary variation in the data, it can still serve as an additional or complementary validation tool in exercises like those conducted in Section V. For instance, consider Figure 29. This figure redoes Figure 6 for the SBC shock in place of the MBC shock. That is, it compares the various second largest shocks to the corresponding objects in JPT and ACD, the models considered in Section V. Clearly, JPT does rather poorly vis-a-vis the SBC shock, too. But now this failure is shared by ACD.

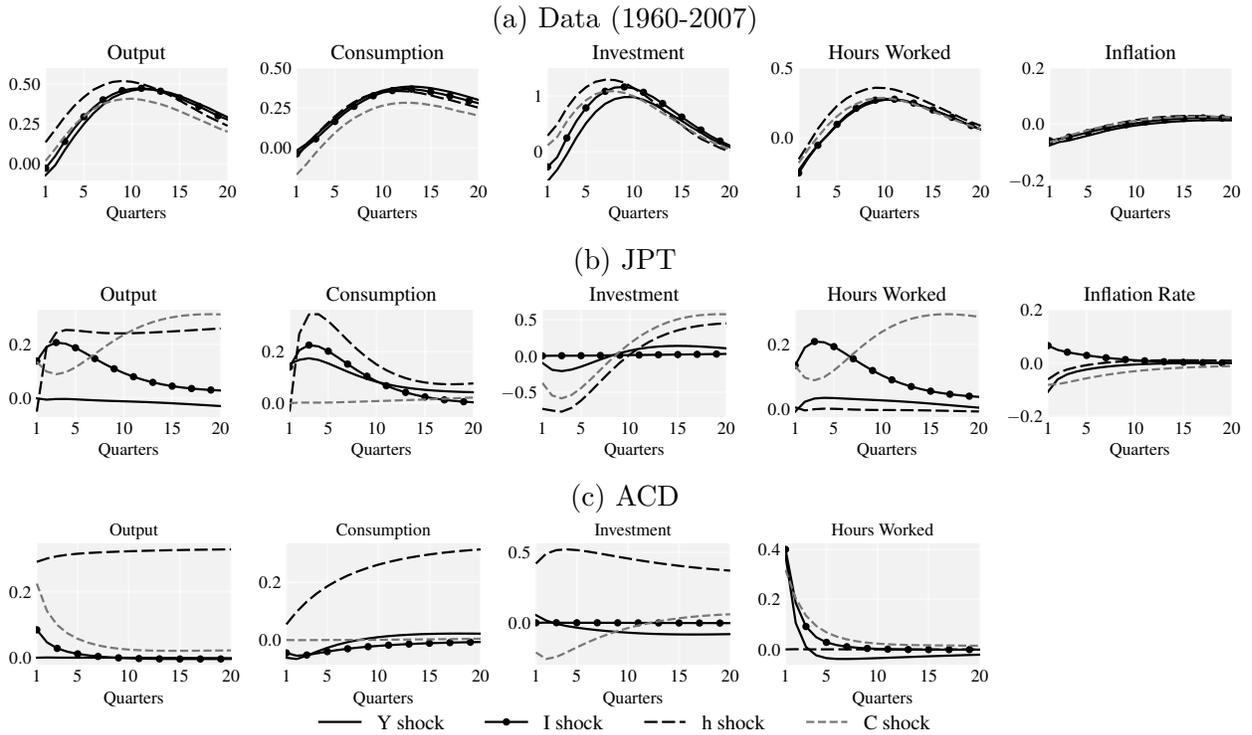
<sup>14</sup>Of course, any attempt to offer a structural interpretation of the MBC and SBC shocks, either jointly or in isolation, faces the basic challenge discussed in detail in Section IV that such objects could be different combinations of multiple theoretical shocks, none of which fits the profile of either of these empirical objects. The aforementioned interpretation is therefore possible but not necessary.

Table 35: Variance Contributions of Second Largest Shocks

Target	$u$	$Y$	$h$	$I$	$C$
Unemployment	24.7 [18.6,31.4]	22.2 [15.6,28.8]	24.5 [17.8,31.4]	20.5 [14.2,27.0]	22.0 [14.9,29.3]
Output	24.4 [18.0,30.8]	18.9 [12.8,25.5]	25.0 [17.5,32.7]	17.6 [11.7,23.9]	23.2 [16.3,30.7]
Hours Worked	21.5 [15.1,29.1]	23.2 [15.7,31.4]	28.3 [21.8,35.4]	25.2 [18.2,33.9]	22.1 [15.2,29.1]
Investment	24.5 [18.3,31.0]	18.0 [11.9,24.8]	26.8 [19.5,34.7]	18.6 [12.4,24.9]	22.2 [15.1,29.9]
Consumption	21.1 [14.4,28.9]	26.8 [19.0,36.1]	23.1 [16.3,30.3]	22.9 [15.1,32.5]	28.6 [21.9,35.1]
	TFP	$Y/h$	$Wh/Y$	$\pi$	$R$
Unemployment	6.3 [2.3,12.5]	20.3 [14.4,26.8]	26.7 [17.6,36.9]	11.5 [4.9,20.1]	38.1 [29.3,47.5]
Output	18.7 [9.2,29.8]	17.5 [11.4,24.1]	6.6 [2.3,13.9]	21.1 [11.0,33.4]	54.1 [43.7,62.4]
Hours Worked	10.3 [5.0,18.5]	18.0 [11.5,24.8]	30.1 [19.7,41.0]	8.2 [3.4,16.1]	28.9 [18.8,38.7]
Investment	17.9 [8.7,30.1]	16.5 [10.4,23.1]	9.1 [3.3,16.8]	12.0 [4.5,22.2]	51.7 [41.4,60.7]
Consumption	14.0 [6.5,23.5]	21.7 [14.7,29.3]	15.8 [7.0,26.8]	22.2 [13.5,31.9]	30.7 [20.6,41.4]

Note: 68% HPDI into brackets.

Figure 29: The SBC in the Data and the Models



On the one hand, these findings confirm the validation process of our empirical strategy. On the other hand, they serve as an additional warning that the postulated propagation mechanisms of state-of-the-art models, even of the most successful ones, remain crude representations of the propagation mechanisms that best characterize the data.

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