ONLINE-APPENDIX
NOT FOR PUBLICATION

Evaluating Deliberative Competence:
A Simple Method with an Application to Financial Choice

Sandro Ambuehl, B. Douglas Bernheim, Annamaria Lusardi

Table of Contents

A Proofs of Propositions 1-3

A.1 Proof of Proposition 1

In proving this proposition, we write the evaluation standard as $u \in V, U$, rather than as V, so that we can invoke this result for both V and U in the proof of Proposition 2.

Step 1: Proof of equation (4).

For $u \in \{U, V\}$, we can rewrite equation (4) as follows:

$$\lim_{\alpha \to 0} \left(\frac{l_M^u \left(r^u(y_c^\alpha), y_s^\alpha \right)}{u_m(0, y_0) \left| r^u\left(y_c^\alpha\right) - r^u\left(y_s^\alpha\right) \right|} \right) = 1. \tag{10}$$

The numerator and denominator both converge to zero as $\alpha \to 0$, so we apply L'Hôpital's rule to the terms $\frac{l_M^u\left(r^u(y_c^\alpha), y_s^\alpha\right)}{u_m(0, y_0)}$ and $\left| r^u\left(y_c^\alpha\right) - r^u\left(y_s^\alpha\right) \right|$.

Suppose first that $\left.\frac{\partial r^u(y_s^\alpha)}{\partial \alpha}\right|_{\alpha=0} - \left.\frac{\partial r^u(y_c^\alpha)}{\partial \alpha}\right|_{\alpha=0} > 0$, so that $r^u\left(y_s^\alpha\right) > r^u\left(y_c^\alpha\right)$ for small α. In that case we can replace $\left| r^u\left(y_c^\alpha\right) - r^u\left(y_s^\alpha\right) \right|$ in the denominator with $r^u\left(y_s^\alpha\right) - r^u\left(y_c^\alpha\right)$.

By definition,

$$l_M^u \left(r^u(y_c^\alpha), y_s^\alpha \right) = u\left(-r^u\left(y_c^\alpha\right), y_s^\alpha\right) - u\left(0, y_0\right).$$

It follows that

$$\frac{d}{d\alpha} \left(\frac{l_M^u\left(r^u(y_c^\alpha), y_s^\alpha\right)}{u_m(0, y_0)} \right) = -\frac{u_m\left(-r^u\left(y_c^\alpha\right), y_s^\alpha\right)}{u_m(0, y_0)} \frac{\partial r^u\left(y_c^\alpha\right)}{\partial \alpha} + \frac{u_y\left(-r^u\left(y_c^\alpha\right), y_s^\alpha\right)\left(y_s - y_0\right)}{u_m(0, y_0)}$$

(Note that the gradient of u with respect to y, denoted u_y, is a row vector and $(y_s - y_0)$ is a column vector). Using the fact that

$$\frac{d r^u\left(y_s^\alpha\right)}{d\alpha} = \frac{u_y\left(-r^u\left(y_s^\alpha\right), y_s^\alpha\right)\left(y_s - y_0\right)}{u_m\left(-r^u\left(y_s^\alpha\right), y_s^\alpha\right)},$$

we have

$$\frac{d}{d\alpha} \left(\frac{l_M^u\left(r^u(y_c^\alpha), y_s^\alpha\right)}{u_m(0, y_0)} \right) = -\frac{u_m\left(-r^u\left(y_c^\alpha\right), y_s^\alpha\right)}{u_m\left(0, y_0\right)} \frac{\partial r^u\left(y_c^\alpha\right)}{\partial \alpha}$$

$$+\frac{u_y\left(-r^u\left(y_c^\alpha\right), y_s^\alpha\right)\left(y_s - y_0\right)}{u_y\left(-r^u\left(y_s^\alpha\right), y_s^\alpha\right)\left(y_s - y_0\right)} \frac{u_m\left(-r^u\left(y_s^\alpha\right), y_s^\alpha\right)}{u_m(0, y_0)} \frac{\partial r^u\left(y_s^\alpha\right)}{\partial \alpha}$$

Taking limits, we have

$$\lim_{\alpha \to 0} \frac{d}{d\alpha} \left(\frac{l_M^u\left(r^u(y_c^\alpha), y_s^\alpha)\right)}{u_m(0, y_0)} \right) = \left.\frac{\partial r^u\left(y_s^\alpha\right)}{\partial \alpha}\right|_{\alpha=0} - \left.\frac{\partial r^u\left(y_c^\alpha\right)}{\partial \alpha}\right|_{\alpha=0}$$

The desired conclusion then follows directly from l'Hôpital's rule.

An analogous argument for the case of $\left.\frac{\partial r^u(y_s^\alpha)}{\partial \alpha}\right|_{\alpha=0} - \left.\frac{\partial r^u(y_c^\alpha)}{\partial \alpha}\right|_{\alpha=0} < 0$, which implies $r^u\left(y_s^\alpha\right) > r^u\left(y_c^\alpha\right)$ for small α, completes the proof of equation (10).

Step 2: Proof of equation (5).

For $u \in \{U, V\}$, we can rewrite equation (5) as follows:

$$\lim_{\alpha\to 0}\left(\frac{\alpha l_W^u\left(r^u(y_c^\alpha), y_s^\alpha, H^\alpha\right)}{\frac{1}{2}\left(r^u\left(y_c^\alpha\right) - r^u\left(y_s^\alpha\right)\right)^2 hu_m(0, y_0)}\right) = 1. \tag{11}$$

The numerator and denominator both converge to zero as $\alpha \to 0$, so we apply L'Hôpital's rule to the terms $\frac{\alpha l_W^u\left(r^u(y_c^\alpha), y_s^\alpha, H^\alpha\right)}{hu_m(0,y_0)}$ and $\frac{1}{2}\left(r^u\left(y_c^\alpha\right) - r^u\left(y_s^\alpha\right)\right)^2$. As we show below, the derivatives of both terms also converge to zero as $\alpha \to 0$, so two applications of the rule are required.

We begin with $\frac{1}{2}\left(r^u\left(y_c^\alpha\right) - r^u\left(y_s^\alpha\right)\right)^2$. We have:

$$\frac{d}{d\alpha}\frac{1}{2}\left(r^u\left(y_c^\alpha\right) - r^u\left(y_s^\alpha\right)\right)^2 = \left(r^u\left(y_c^\alpha\right) - r^u\left(y_s^\alpha\right)\right)\left(\frac{dr^u\left(y_c^\alpha\right)}{d\alpha} - \frac{dr^u\left(y_s^\alpha\right)}{d\alpha}\right),$$

Given our assumptions concerning bounds on the derivatives of the utility functions, $\frac{dr^u\left(y_f^\alpha\right)}{d\alpha}$ is bounded. Consequently, this expression converges to zero as $\alpha \to 0$ (because both valuations converge to zero). So we consider the second derivative:

$$\frac{d^2}{d\alpha^2}\frac{1}{2}\left(r^u\left(y_c^\alpha\right) - r^u\left(y_s^\alpha\right)\right)^2 = \left(\frac{dr^u\left(y_c^\alpha\right)}{d\alpha} - \frac{dr^u\left(y_s^\alpha\right)}{d\alpha}\right)^2 + \left(r^u\left(y_s^\alpha\right) - r^u\left(y_c^\alpha\right)\right)\left(\frac{d^2r^u\left(y_c^\alpha\right)}{d\alpha^2} - \frac{d^2r^u\left(y_s^\alpha\right)}{d\alpha^2}\right)$$

It is straightforward to verify that $\frac{d^2r^u\left(y_f^\alpha\right)}{d\alpha^2}$ is bounded under our assumptions. It follows that

$$\lim_{\alpha\to 0}\frac{1}{2}\frac{d^2}{d\alpha^2}\left[\left(r^u\left(y_s^\alpha\right)\right)^2 - \left(r^u\left(y_c^\alpha\right)\right)^2\right] = \left(\lim_{\alpha\to 0}\frac{dr^u\left(y_s^\alpha\right)}{d\alpha} - \lim_{\alpha\to 0}\frac{dr^u\left(y_c^\alpha\right)}{d\alpha}\right)^2, \tag{12}$$

which is non-zero by assumption.

Next consider $\frac{\alpha l_W^u\left(r^u(y_c^\alpha), y_s^\alpha, H^\alpha\right)}{hu_m(0,y_0)}$. Suppose first that $\left.\frac{dr^u(y_s^\alpha)}{d\alpha}\right|_{\alpha=0} - \left.\frac{\partial r^u(y_c^\alpha)}{\partial \alpha}\right|_{\alpha=0} > 0$, so that $r^u\left(y_s^\alpha\right) > r^u\left(y_c^\alpha\right)$ for small α. By definition,

$$\frac{\alpha}{h}l_W^u\left(r^u(y_c^\alpha), y_s^\alpha, H^\alpha\right) = \int_{r^u(y_c^\alpha)}^{r^u(y_s^\alpha)}\left[u\left(-p, y_s^\alpha\right) - u\left(0, y_0\right)\right]dp$$

Noting that the integrand is zero at the upper limit of integration, it follows that

$$\frac{d}{d\alpha}\left[\frac{\alpha}{h}l_W^u\left(r^u(y_c^\alpha), y_s^\alpha, H^\alpha\right)\right] = -\left[u\left(-r^u\left(y_c^\alpha\right), y_s^\alpha\right) - u\left(0, y_0\right)\right]\frac{dr^u\left(y_c^\alpha\right)}{d\alpha} + \int_{r^u(y_c^\alpha)}^{r^u(y_s^\alpha)}\left[u_y\left(-p, y_s^\alpha\right)\left(y_s - y_0\right)\right]dp$$

Given our assumptions concerning bounds on the derivatives of the utility functions, $\frac{dr^u(y_c^\alpha)}{d\alpha}$ and $u_y\left(-p, y_s^\alpha\right)(y_s - y_0)$ are bounded. Consequently, both terms of this expression converge to zero as $\alpha \to 0$. We therefore take the second derivative:

$$\begin{aligned}\frac{d^2}{d\alpha^2}\left[\frac{\alpha}{h} l_W^u\left(r^u(y_c^\alpha), y_s^\alpha, H^\alpha\right)\right] &= -\left[u\left(-r^u\left(y_c^\alpha\right), y_s^\alpha\right) - u\left(0, y_0\right)\right]\frac{d^2 r^u(y_c^\alpha)}{d\alpha^2} + u_m\left(-r^u\left(y_c^\alpha\right), y_s^\alpha\right)\left(\frac{dr^u(y_c^\alpha)}{d\alpha}\right)^2 \\ &\quad -2u_y\left(-r^u\left(y_c^\alpha\right), y_s^\alpha\right)\left(y_s - y_0\right)\frac{dr^u(y_c^\alpha)}{d\alpha} + u_y\left(-r^u\left(y_s^\alpha\right), y_s^\alpha\right)(y_s - y_0)\frac{dr^u(y_s^\alpha)}{d\alpha} \\ &\quad + \int_{r^u(y_c^\alpha)}^{r^u(y_s^\alpha)} \frac{d}{d\alpha}\left[u_y\left(-p, y_s^\alpha\right)(y_s - y_0)\right] dp\end{aligned}$$

Recall that

$$\frac{dr^u\left(y_s^\alpha\right)}{d\alpha} = \frac{u_y\left(-r^u\left(y_s^\alpha\right), y_s^\alpha\right)\left(y_s - y_0\right)}{u_m\left(-r^u\left(y_s^\alpha\right), y_s^\alpha\right)},$$

We use this expression to substitute for $u_y\left(-r^u\left(y_s^\alpha\right), y_s^\alpha\right)\left(y_s - y_0\right)$ in the fourth term. We also multiply the third term by $\frac{dr^u(y_s^\alpha)}{d\alpha}\frac{u_m(-r^u(y_s^\alpha), y_s^\alpha)}{u_y(-r^u(y_s^\alpha), y_s^\alpha)(y_s - y_0)} = 1$. Then we take limits as $\alpha \to 0$. The boundedness of $\frac{d^2 r^u\left(y_f^\alpha\right)}{d\alpha^2}$ (noted above) guarantees that the first term vanishes. The boundedness of the first and second derivatives of the utility functions guarantees that the last term vanishes. We thus obtain:

$$\begin{aligned}\lim_{\alpha\to 0}\frac{d^2}{d\alpha^2}\left[\frac{\alpha}{h} l_W^u\left(r^u(y_c^\alpha), y_s^\alpha, H^\alpha\right)\right] &= u_m\left(0, y_0\right)\Bigg[\left(\lim_{\alpha\to 0}\frac{dr^u\left(y_c^\alpha\right)}{d\alpha}\right)^2 - 2\left(\lim_{\alpha\to 0}\frac{dr^u\left(y_s^\alpha\right)}{d\alpha}\right)\left(\lim_{\alpha\to 0}\frac{dr^u\left(y_c^\alpha\right)}{d\alpha}\right) \\ &\qquad + \left(\lim_{\alpha\to 0}\frac{dr^u\left(y_s^\alpha\right)}{d\alpha}\right)^2\Bigg] \\ &= u_m\left(0, y_0\right)\left[\lim_{\alpha\to 0}\frac{dr^u\left(y_c^\alpha\right)}{d\alpha} - \lim_{\alpha\to 0}\frac{dr^u\left(y_s^\alpha\right)}{d\alpha}\right]^2 \qquad (13)\end{aligned}$$

Equation (11) follows immediately from equations (12) and (13).

■

A.2 Proof of Proposition 2

Step 1. We claim that there exists a constant K such that, for all y_c and y_s,

$$\lim_{\alpha\to 0}\left(\frac{r^U\left(y_c^\alpha\right) - r^U\left(y_s^\alpha\right)}{r^V\left(y_c^\alpha\right) - r^V\left(y_s^\alpha\right)}\right) = K. \qquad (14)$$

Because the numerator and denominator both converge to zero, we apply L'Hopital's rule:

$$\lim_{\alpha\to 0}\left(\frac{r^U\left(y_c^\alpha\right)-r^U\left(y_s^\alpha\right)}{r^V\left(y_c^\alpha\right)-r^V\left(y_s^\alpha\right)}\right)=\frac{\lim_{\alpha\to 0}\frac{dr^U\left(y_c^\alpha\right)}{d\alpha}-\lim_{\alpha\to 0}\frac{dr^U\left(y_s^\alpha\right)}{d\alpha}}{\lim_{\alpha\to 0}\frac{dr^V\left(y_c^\alpha\right)}{d\alpha}-\lim_{\alpha\to 0}\frac{dr^V\left(y_s^\alpha\right)}{d\alpha}}$$

Recall that $r^u(y)$ is defined by the following equation:

$$u\left(-r^u(y),\varphi(y)\right)=u\left(0,\varphi\left(y_0\right)\right)$$

Differentiating implicitly with respect to α and evaluating at y_f^α , we obtain:

$$\frac{dr^u\left(y_f^\alpha\right)}{d\alpha}=\frac{u_\varphi\left(-r^u\left(y_f^\alpha\right),\varphi\left(y_f^\alpha\right)\right)}{u_m\left(-r^u\left(y_f^\alpha\right),\varphi\left(y_f^\alpha\right)\right)}\left(\nabla\varphi\left(y_f^\alpha\right)\cdot\left(y_f-y_0\right)\right)$$

Using the fact that $\lim_{\alpha\to 0} r^u\left(y_f^\alpha\right)=0$, it follows that

$$\lim_{\alpha\to 0}\frac{\partial r^u\left(y_f^\alpha\right)}{\partial\alpha}=\frac{u_\varphi\left(0,\varphi\left(y_0\right)\right)}{u_m\left(0,\varphi\left(y_0\right)\right)}\left(\nabla\varphi\left(y_0\right)\cdot\left(y_f-y_0\right)\right)$$

Therefore,

$$\lim_{\alpha\to 0}\frac{\partial r^u\left(y_c^\alpha\right)}{\partial\alpha}-\lim_{\alpha\to 0}\frac{\partial r^u\left(y_s^\alpha\right)}{\partial\alpha}=\frac{u_\varphi\left(0,\varphi\left(y_0\right)\right)}{u_m\left(0,\varphi\left(y_0\right)\right)}\left(\nabla\varphi\left(y_0\right)\cdot\left(y_c-y_s\right)\right)$$

Accordingly, we have

$$\lim_{\alpha\to 0}\left(\frac{r^U\left(y_c^\alpha\right)-r^U\left(y_s^\alpha\right)}{r^V\left(y_c^\alpha\right)-r^V\left(y_s^\alpha\right)}\right)=\frac{U_\varphi\left(0,\varphi\left(y_0\right)\right)V_m\left(0,\varphi\left(y_0\right)\right)}{U_m\left(0,\varphi\left(y_0\right)\right)V_\varphi\left(0,\varphi\left(y_0\right)\right)}\equiv K>0$$

As claimed, K does not depend on y_s or y_c (even though the limits of the numerator and denominator do).

Step 2. Proof of equations (6) and (7).

First consider $e=M$. We have

$$\lim_{\alpha\to 0}\left(\frac{L_M^U\left(r^U(y_c^\alpha),y_s^\alpha\right)}{d_M\left(r^V\left(y_c^\alpha\right),r^V\left(y_s^\alpha\right)\right)}\right)=\lim_{\alpha\to 0}\left(\frac{L_M^U\left(r^U(y_c^\alpha),y_s^\alpha\right)}{d_M\left(r^U\left(y_c^\alpha\right),r^U\left(y_s^\alpha\right)\right)}\right)\lim_{\alpha\to 0}\frac{d_M\left(r_c^U\left(y_c^\alpha\right),r_s^U\left(y_s^\alpha\right)\right)}{d_M\left(r_c^V\left(y_c^\alpha\right),r_s^V\left(y_s^\alpha\right)\right)}$$

From Proposition 1, we know that the first term after the equals sign is unity. For the second term, we have

$$\lim_{\alpha\to 0}\frac{d_M\left(r_c^U\left(y_c^\alpha\right),r_s^U\left(y_s^\alpha\right)\right)}{d_M\left(r_c^V\left(y_c^\alpha\right),r_s^V\left(y_s^\alpha\right)\right)}=\lim_{\alpha\to 0}\left|\frac{r_c^U\left(y_c^\alpha\right)-r_s^U\left(y_s^\alpha\right)}{r_c^V\left(y_c^\alpha\right)-r_s^V\left(y_s^\alpha\right)}\right|=\left|\lim_{\alpha\to 0}\frac{r_c^U\left(y_c^\alpha\right)-r_s^U\left(y_s^\alpha\right)}{r_c^V\left(y_c^\alpha\right)-r_s^V\left(y_s^\alpha\right)}\right|=K$$

Consequently, equation (6) holds for $K_M=K$.

Now consider $e = W$. We have

$$\lim_{\alpha\to 0}\left(\frac{L_W^U\left(r^U(y_c^\alpha), y_s^\alpha, H^\alpha\right)}{d_W\left(r^V\left(y_c^\alpha\right), r^V\left(y_s^\alpha\right)\right)}\right) = \lim_{\alpha\to 0}\left(\frac{L_W^U\left(r^U(y_c^\alpha), y_s^\alpha, H^\alpha\right)}{d_W\left(r^U\left(y_c^\alpha\right), r^U\left(y_s^\alpha\right)\right)}\right)\lim_{\alpha\to 0}\frac{d_W\left(r_c^U\left(y_c^\alpha\right), r_s^U\left(y_s^\alpha\right)\right)}{d_W\left(r_c^V\left(y_c^\alpha\right), r_s^V\left(y_s^\alpha\right)\right)}$$

From Proposition 1, we know that the first term after the equals sign is h. For the second term, we have

$$\lim_{\alpha\to 0}\frac{d_W\left(r_c^U\left(y_c^\alpha\right), r_s^U\left(y_s^\alpha\right)\right)}{d_W\left(r_c^V\left(y_c^\alpha\right), r_s^V\left(y_s^\alpha\right)\right)} = \lim_{\alpha\to 0}\left(\frac{r_c^U\left(y_c^\alpha\right) - r_s^U\left(y_s^\alpha\right)}{r_c^V\left(y_c^\alpha\right) - r_s^V\left(y_s^\alpha\right)}\right)^2 = \left(\lim_{\alpha\to 0}\frac{r_c^U\left(y_c^\alpha\right) - r_s^U\left(y_s^\alpha\right)}{r_c^V\left(y_c^\alpha\right) - r_s^V\left(y_s^\alpha\right)}\right)^2 = K^2$$

Consequently, equation (7) holds for $K_W = hK^2$.

■

A.3 Proof of Proposition 3

Define

$$\kappa(y_s) \equiv \left(\frac{U_\varphi\left(0, \varphi\left(y_0\right)\right)\nabla\varphi\left(y_0\right)\cdot\left(y_s - y_0\right)}{U_m\left(0, \varphi\left(y_0\right)\right)}\right)$$

First consider $e = M$. Proceeding as in the proof of Proposition 2, we have

$$\lim_{\alpha\to 0}\left(\frac{L_M^U\left(r^U(y_{c\theta}^\alpha), y_s^\alpha\right)}{d_M\left(r^V\left(y_{c\theta}^\alpha, \theta\right), r^V\left(y_s^\alpha, \theta\right)\right)}\right) = \frac{U_\varphi\left(0, \varphi\left(y_0\right)\right)V_m\left(0, \varphi\left(y_0\right), \theta\right)}{U_m\left(0, \varphi\left(y_0\right)\right)V_\varphi\left(0, \varphi\left(y_0\right), \theta\right)}$$

Furthermore,

$$\lim_{\alpha\to 0}\left(\frac{\alpha\kappa(y_s)}{r^V(y_s^\alpha, \theta)}\right) = \lim_{\alpha\to 0}\left(\frac{r^V(y_s^\alpha, \theta)}{\alpha}\right)^{-1}\kappa(y_s) = \left(\left.\frac{dr^V(y_s^\alpha, \theta)}{d\alpha}\right|_{\alpha=0}\right)^{-1}\kappa(y_s)$$

$$= \left(\frac{V_m\left(0, \varphi\left(y_0\right), \theta\right)}{V_\varphi\left(0, \varphi\left(y_0\right), \theta\right)\nabla\varphi\left(y_0\right)\cdot\left(y_s - y_0\right)}\right)\kappa(y_s) = \frac{U_\varphi\left(0, \varphi\left(y_0\right)\right)V_m\left(0, \varphi\left(y_0\right), \theta\right)}{U_m\left(0, \varphi\left(y_0\right)\right)V_\varphi\left(0, \varphi\left(y_0\right), \theta\right)}$$

Equation (8) is thereby verified.

Now consider $e = W$. Proceeding as in the proof of Proposition 2, we have

$$\lim_{\alpha\to 0}\left(\frac{L_W^U\left(r^U(y_{c\theta}^\alpha), y_s^\alpha, H^\alpha\right)}{d_W\left(r^V\left(y_{c\theta}^\alpha, \theta\right), r^V\left(y_s^\alpha, \theta\right)\right)}\right) = h\left(\frac{U_\varphi\left(0, \varphi\left(y_0\right)\right)V_m\left(0, \varphi\left(y_0\right), \theta\right)}{U_m\left(0, \varphi\left(y_0\right)\right)V_\varphi\left(0, \varphi\left(y_0\right), \theta\right)}\right)^2$$

Furthermore,

$$\lim_{\alpha\to 0}h\left(\frac{\alpha\kappa(y_s)}{r^V(y_s^\alpha, \theta)}\right)^2 = \lim_{\alpha\to 0}h\left(\frac{r^V(y_s^\alpha, \theta)}{\alpha}\right)^{-2}\left(\kappa(y_s)\right)^2 = \left(\left.\frac{dr^V(y_s^\alpha, \theta)}{d\alpha}\right|_{\alpha=0}\right)^{-2}\left(\kappa(y_s)\right)^2$$

$$= h\left(\frac{V_m\left(0,\varphi\left(y_0\right),\theta\right)\kappa(y_s)}{V_\varphi\left(0,\varphi\left(y_0\right),\theta\right)\nabla\varphi\left(y_0\right)\cdot\left(y_s-y_0\right)}\right)^2 = h\left(\frac{U_\varphi\left(0,\varphi\left(y_0\right)\right)V_m\left(0,\varphi\left(y_0\right),\theta\right)}{U_m\left(0,\varphi\left(y_0\right)\right)V_\varphi\left(0,\varphi\left(y_0\right),\theta\right)}\right)^2$$

Equation (9) is thereby verified.

Finally, note that when y is a scalar, we have $\kappa\left(y_s\right) = \left(y_s - y_0\right)\kappa^*$, where

$$\kappa^* \equiv \left(\frac{U_\varphi\left(0,\varphi\left(y_0\right)\right)\varphi'\left(y_0\right)}{U_m\left(0,\varphi\left(y_0\right)\right)}\right).$$

■

B Additional theoretical results

For the approximations in the main test, we held the the magnitude of the misunderstanding fixed and let the consequences of the instrument shrink. Another possibility is to hold the scale of the instrument constant and let the magnitude of the misunderstanding shrink. As we noted in the text, the latter approach may provide better approximations for large instruments (which may be particularly useful in settings where risk preferences play a central role), but worse approximations for large misunderstandings. This section develops this alternative approach.

Formally, we define $y_c^\alpha = \alpha y_c + (1-\alpha)y_s$, and study the relationship between the welfare loss and the valuations $r^V\left(y_c^\alpha\right)$ and $r^V(y_s)$ as α shrinks to zero. Because the scale of the instrument remains fixed as we vary α, we take the distribution of prices, H, to be fixed as well. We also assume that the density, $h(p)$, is bounded and differentiable with a bounded derivative. This alternative formulation requires a slight modification of the welfarist criterion. As before, we convert l_e^u to money-metric utility (for $e \in \{M, W\}$), but in this case we divide by $u_m\left(-r^u(y_s), y_s\right)$.

We now state and discuss counterparts for the three propositions in the main text. Proofs follow.

Proposition 4. *For all y_c satisfying $\left.\frac{dr^V(y_c^\alpha)}{d\alpha}\right|_{\alpha=0} \neq 0$, we have*

$$\lim_{\alpha\to 0}\left(\frac{L_M^V\left(r^V(y_c^\alpha), y_s\right)}{d_M\left(r^V(y_c^\alpha), r^V(y_s)\right)}\right) = 1 \tag{15}$$

and

$$\lim_{\alpha\to 0}\left(\frac{L_W^V\left(r^V(y_c^\alpha), y_s, H\right)}{d_W\left(r^V(y_c^\alpha), r^V(y_s)\right)}\right) = h\left(r^V(y_s)\right) \tag{16}$$

Two distinctions between Propositions 1 and 4 merit discussion. First, the constant of proportionality in equation (16) involves $h\left(r^V(y_s)\right)$ rather than the scalar h. In either case, to aggregate over instruments, one must assume that the pertinent density is uncorrelated with measured deliberative competence. Second, for Proposition 4, the money-metric scaling factor $u_m\left(-r^u(y_s), y_s\right)$ also depends on the instrument through y_s. Aggregation over instruments therefore requires either that the differences in the marginal utility of income are small (e.g., because curvature is modest over the relevant range), or that measured deliberative competence is uncorrelated with simply framed valuations across instruments.

Proposition 5. *For each $e \in \{W, M\}$, there exists a strictly positive function $K_e(y_s)$, such that for all y_c satisfying $\left.\frac{dr^V(y_c^\alpha)}{d\alpha}\right|_{\alpha=0} \neq 0$, we have*

$$\lim_{\alpha\to 0}\left(\frac{L_M^U\left(r^U\left(y_c^\alpha\right), y_s\right)}{d_M\left(r^V\left(y_c^\alpha\right), r^V\left(y_s\right)\right)}\right) = K_M\left(y_s\right) \tag{17}$$

$$\lim_{\alpha\to 0}\left(\frac{L_W^U\left(r^U\left(y_c^\alpha\right), y_s, H\right)}{d_W\left(r^V\left(y_c^\alpha\right), r^V\left(y_s\right)\right)}\right) = K_W\left(y_s\right) \tag{18}$$

An important difference between Propositions 2 and 5 is that, for the latter, the constants of proportionality depend on the instrument (even with no differences in the density term across instruments for the welfarist criterion). Once again, this feature introduces some qualifications with respect to aggregating over instruments for the alternative approach.

Our next proposition concerns potential policy-induced confounds. It identifies an adjustment to measured deliberative competence required to restore comparability across policies when they potentially impact simply framed valuations. It involves a function $\rho^V\left(y_s, \theta\right)$, defined as follows. Let $y_s^\beta = \beta y_s + (1-\beta)y_0$, where β is a scalar. Then $\rho^V\left(y_s, \theta\right) = \left.\frac{dr^V(y_s^\beta, \theta)}{d\beta}\right|_{\beta=1}$. In words, $\rho^V\left(y_s, \theta\right)$ is the amount by which the simply framed valuation changes as we scale up the instrument's consequences. Critically, the value of this term is easily inferred from the type of data we collected in our experiment.

Proposition 6. *For each $e \in \{W, M\}$, there exists a strictly positive function $\kappa_e(y_s)$ such that, for all $y_{c\theta}$ satisfying $\left.\frac{dr^V(y_c^\alpha, \theta)}{d\alpha}\right|_{\alpha=0} \neq 0$ and policies θ, we have*

$$\lim_{\alpha\to 0}\left(\frac{L_M^U\left(r^U(y_c^\alpha), y_s\right)}{d_M^V\left(r^V\left(y_{c\theta}^\alpha, \theta\right), r^V\left(y_s, \theta\right)\right)}\right) = \frac{\kappa_M(y_s)}{\rho^V\left(y_s, \theta\right)} \tag{19}$$

and

$$\lim_{\alpha\to 0}\left(\frac{L_W^U\left(r^U(y_c^\alpha), y_s, H\right)}{d_W^V\left(r^V\left(y_{c\theta}^\alpha, \theta\right), r^V\left(y_s, \theta\right)\right)}\right) = \frac{\kappa_W(y_s)}{\left[\rho^V\left(y_s, \theta\right)\right]^2} \tag{20}$$

According to this proposition, we can address the confound simply by rescaling our measures of deliberative competence. In particular, once we divide $d_M^V\left(r^V\left(y_{c\theta}, \theta\right), r^V\left(y_s, \theta\right)\right)$ by $\rho^V\left(y_s, \theta\right)$, and $d_W^V\left(r^V\left(y_{c\theta}, \theta\right), r^V\left(y_s, \theta\right)\right)$ by $\left[\rho^V\left(y_s, \theta\right)\right]^2$, the main implications of Proposition 5 follow.

Section D.6 uses Proposition 6 to correct for policy-induced framing effects and shows that our empirical results are qualitatively unchanged.

B.1 Proof of Proposition 4

In proving this proposition, we write the evaluation standard as $u \in \{V, U\}$, rather than as V, so that we can invoke this result for both V and U in the proof of Proposition 5.

The proof involves two steps.

Step 1: Proof of equation (15).

Suppose first that $\left.\frac{dr^u(y_c^\alpha)}{d\alpha}\right|_{\alpha=0} < 0$. Hence, for all sufficiently small α we have that $r^u\left(y_s\right) > r^u\left(y_c^\alpha\right)$ and that $r^u\left(y_c^\alpha\right)$ is strictly increasing. In this case the denominator of the expression in equation (15) is $r^u\left(y_s\right) - r^u\left(y_c^\alpha\right)$. Because $L_M^u\left(r^u(y_c^\alpha), y_s\right)$ (the numerator) and $r^u\left(y_s\right) - r^u\left(y_c^\alpha\right)$ both converge to zero as $\alpha \to 0$, we apply l'Hopital's rule.

For the numerator, the facts that u is strictly increasing in its first component, and that $r^u\left(y_c^\alpha\right)$ is strictly increasing, imply

$$L_M^u\left(r^u(y_c^\alpha), y_s\right) = \max_{p\in[r^u(y_c^\alpha), r^u(y_s)]}\left[\frac{u\left(-p, y_s\right) - u\left(0, y_0\right)}{u_m(-r^u(y_s), y_s)}\right] = \frac{u\left(-r^u\left(y_c^\alpha\right), y_s\right) - u\left(0, y_0\right)}{u_m(-r^u(y_s), y_s)}$$

It follows that

$$\frac{dL_M^u\left(r^u(y_c^\alpha), y_s\right)}{d\alpha} = -\frac{u_m\left(-r^u\left(y_c^\alpha\right), y_s\right)}{u_m(-r^u(y_s), y_s)}\frac{dr^u\left(y_c^\alpha\right)}{d\alpha},$$

and accordingly that

$$\lim_{\alpha\to 0}\frac{dL_M^u\left(r^u(y_c^\alpha), y_s\right)}{d\alpha} = -\lim_{\alpha\to 0}\frac{dr^u\left(y_c^\alpha\right)}{d\alpha}$$

For the denominator, the derivative is $-\frac{dr^u(y_c^\alpha)}{d\alpha}$, so the limit is the same as for the numerator. Equation (15) follows (in this case) immediately from L'Hopital's rule.

Now suppose $\left.\frac{dr^u(y_c^\alpha)}{d\alpha}\right|_{\alpha=0} > 0$, so that $r^u\left(y_s\right) > r^u\left(y_c^\alpha\right)$ for small α, in which case the denominator of the expression in equation (15) is $r^u\left(y_c^\alpha\right) - r^u\left(y_s\right)$. In this case, for the numerator, we have

$$L_M^u\left(r^u(y_c^\alpha), y_s\right) = \max_{p\in[r^u(y_s), r^u(y_c^\alpha)]}\left[\frac{u\left(0, y_0\right) - u\left(-p, y_s\right)}{u_m(-r^u(y_s), y_s)}\right] = \frac{u\left(0, y_0\right) - u\left(-r^u\left(y_c^\alpha\right), y_s\right)}{u_m(-r^u(y_s), y_s)}$$

The remainder of the argument is the same, except that the signs of the numerator and denominator are both switched, so the limiting ratio of derivatives is again unity.

Step 2: Proof of equation (16).

Suppose first $\left.\frac{dr^u(y_c^\alpha)}{d\alpha}\right|_{\alpha=0} < 0$, so that $r^u\left(y_s\right) > r^u\left(y_c^\alpha\right)$ for small α. Because $L_W^u\left(r^u(y_c^\alpha), y_s, H\right)$ (the numerator of the expression in equation (16)) and $d_W\left(r^u\left(y_c^\alpha\right), r^u\left(y_s\right)\right)$ (the denominator) both converge to zero as $\alpha \to 0$, we apply l'Hopital's rule.

Consider the numerator, $L_W^u\left(r^u(y_c^\alpha), y_s, H\right)$. By definition,

$$L_W^u\left(r^u(y_c^\alpha), y_s, H\right) = \frac{1}{u_m(-r^u(y_s), y_s)}\int_{r^u(y_c^\alpha)}^{r^u(y_s)}\left[u\left(-p, y_s\right) - u\left(0, y_0\right)\right]h(p)dp$$

It follows that

$$\frac{d}{d\alpha}L_W^u\left(r^u(y_c^\alpha), y_s, H\right) = -\left[\frac{u\left(-r^u\left(y_c^\alpha\right), y_s\right) - u\left(0, y_0\right)}{u_m(-r^u(y_s), y_s)}\right]\frac{dr^u\left(y_c^\alpha\right)}{d\alpha}h\left(r^u\left(y_c^\alpha\right)\right)$$

Given our assumptions concerning bounds on the density and derivatives of the utility functions, $\frac{dr^u(y_c^\alpha)}{d\alpha}$, $\frac{1}{u_m(-r^u(y_s),y_s)}$, and $h\left(r^u\left(y_c^\alpha\right)\right)$ are all bounded, so this expression converges to zero as $\alpha \to 0$. Anticipating the same property for the denominator, we take the second derivative:

$$\frac{d^2}{d\alpha^2} L_W^u\left(r^u(y_c^\alpha), y_s, H\right) = -\left[\frac{u\left(-r^u\left(y_c^\alpha\right), y_s\right) - u\left(0, y_0\right)}{u_m(-r^u(y_s), y_s)}\right]\left[\frac{d^2 r^u\left(y_c^\alpha\right)}{d\alpha^2} h\left(r^u\left(y_c^\alpha\right)\right) + \frac{dr^u\left(y_c^\alpha\right)}{d\alpha}\frac{dh\left(r^u\left(y_c^\alpha\right)\right)}{d\alpha}\right]$$

$$+\frac{u_m\left(-r^u\left(y_c^\alpha\right), y_s\right)}{u_m(-r^u(y_s), y_s)}\left(\frac{dr^u\left(y_c^\alpha\right)}{d\alpha}\right)^2 h\left(r^u\left(y_c^\alpha\right)\right)$$

As before, our assumptions imply that $\frac{dr^u(y_c^\alpha)}{d\alpha}$, $\frac{1}{u_m(-r^u(y_s),y_s)}$, and $h\left(r^u\left(y_c^\alpha\right)\right)$ are bounded. Boundedness of the derivative of the density then implies that $\frac{dh(r^u(y_c^\alpha))}{d\alpha}$ is bounded. It is also straightforward to verify that $\frac{d^2 r^u(y_c^\alpha)}{d\alpha^2}$ is bounded under our assumptions. It follows that, as $\alpha \to 0$, the first term converges to zero. Because the ratio in the second term converges to unity, we have:

$$\lim_{\alpha\to 0}\frac{d^2}{d\alpha^2} L_W^u\left(r^u(y_c^\alpha), y_s, H\right) = h\left(r^u\left(y_s\right)\right)\lim_{\alpha\to 0}\left(\frac{dr^u\left(y_c^\alpha\right)}{d\alpha}\right)^2$$

Now consider the denominator, $\frac{1}{2}\left(r^u\left(y_s\right) - r^u\left(y_c^\alpha\right)\right)^2$. Taking the derivative yields

$$\frac{d}{d\alpha}\left[\frac{1}{2}\left(r^u\left(y_s\right) - r^u\left(y_c^\alpha\right)\right)^2\right] = -\left(r^u\left(y_s\right) - r^u\left(y_c^\alpha\right)\right)\frac{dr^u\left(y_c^\alpha\right)}{d\alpha}$$

Because $\frac{dr^u(y_c^\alpha)}{d\alpha}$ is bounded, this expression converges to 0 as $\alpha \to 0$, as claimed above. Taking the second derivative yields

$$\frac{d^2}{d\alpha^2}\left[\frac{1}{2}\left(r^u\left(y_s\right) - r^u\left(y_c^\alpha\right)\right)^2\right] = -\left(r^u\left(y_s\right) - r^u\left(y_c^\alpha\right)\right)\frac{d^2 r^u\left(y_c^\alpha\right)}{d\alpha^2} + \left(\frac{dr^u\left(y_c^\alpha\right)}{d\alpha}\right)^2$$

As $\alpha \to 0$, the first term converges to zero (given the boundedness of the second derivative noted above), so we have

$$\lim_{\alpha\to 0}\frac{d^2}{d\alpha^2}\left[\frac{1}{2}\left(r^u\left(y_s\right) - r^u\left(y_c^\alpha\right)\right)^2\right] = \lim_{\alpha\to 0}\left(\frac{dr^u\left(y_c^\alpha\right)}{d\alpha}\right)^2$$

Thus, the ratio of the limit of second derivatives for the numerator and the denominator is $h\left(r^u\left(y_s\right)\right)$. Equation (16) follows (in this case) immediately from L'Hopital's rule.

Now suppose $\left.\frac{dr^u(y_c^\alpha)}{d\alpha}\right|_{\alpha=0} > 0$, so that $r^u\left(y_s\right) > r^u\left(y_c^\alpha\right)$ for small α. The same analysis of the denominator applies. For the numerator, the limits of the integral change places, as do the two terms in the integrand. As a result, the first and second derivatives are unchanged. Consequently, the same arguments deliver equation (16) in this case as well.

■

B.2 Proof of Proposition 5

The proof involves two steps.

Step 1: We claim that there exists a constant $K(y_s)$ such that, for all y_c and y_s,

$$\lim_{\alpha\to 0}\left(\frac{r^U\left(y_c^\alpha\right)-r^U\left(y_s\right)}{r^V\left(y_c^\alpha\right)-r^V\left(y_s\right)}\right)=K(y_s). \tag{21}$$

Because the numerator and denominator of the preceding expression both converge to zero, we apply L'Hopital's rule:

$$\lim_{\alpha\to 0}\left(\frac{r^U\left(y_c^\alpha\right)-r^U\left(y_s\right)}{r^V\left(y_c^\alpha\right)-r^V\left(y_s\right)}\right)=\frac{\lim_{\alpha\to 0}\frac{dr^U\left(y_c^\alpha\right)}{d\alpha}}{\lim_{\alpha\to 0}\frac{dr^V\left(y_c^\alpha\right)}{d\alpha}}$$

Recall that $r^V(y)$ is defined by the following equation:

$$V\left(-r^V(y),\varphi(y)\right)=V\left(0,\varphi\left(y_0\right)\right)$$

Evaluating $r^V(y)$ at y_c^α and differentiating implicitly with respect to α, we obtain:

$$\frac{dr^V\left(y_c^\alpha\right)}{d\alpha}=\frac{V_\varphi\left(-r^V\left(y_c^\alpha\right),\varphi\left(y_c^\alpha\right)\right)}{V_m\left(-r^V\left(y_c^\alpha\right),\varphi\left(y_c^\alpha\right)\right)}\left(\nabla\varphi\left(y_c^\alpha\right)\cdot\left(y_c-y_s\right)\right)$$

Using the fact that $\lim_{\alpha\to 0} r^V\left(y_c^\alpha\right)=r^V(y_s)$, it follows that

$$\lim_{\alpha\to 0}\frac{dr^V\left(y_c^\alpha\right)}{d\alpha}=\frac{V_\varphi\left(-r^V(y_s),\varphi\left(y_0\right)\right)}{V_m\left(-r^V(y_s),\varphi\left(y_0\right)\right)}\left(\nabla\varphi\left(y_0\right)\cdot\left(y_c-y_s\right)\right)$$

Repeating these calculations for U, we obtain

$$\lim_{\alpha\to 0}\frac{dr^U\left(y_c^\alpha\right)}{d\alpha}=\frac{U_\varphi\left(-r^U(y_s),\varphi\left(y_0\right)\right)}{U_m\left(-r^U(y_s),\varphi\left(y_0\right)\right)}\left(\nabla\varphi\left(y_0\right)\cdot\left(y_c-y_s\right)\right)$$

Accordingly, we have

$$\lim_{\alpha\to 0}\left(\frac{r^U\left(y_c^\alpha\right)-r^U\left(y_s\right)}{r^V\left(y_c^\alpha\right)-r^V\left(y_s\right)}\right)=\frac{U_\varphi\left(-r^U(y_s),\varphi\left(y_s\right)\right)V_m\left(-r^V(y_s),\varphi\left(y_s\right)\right)}{U_m\left(-r^U(y_s),\varphi\left(y_s\right)\right)V_\varphi\left(-r^V(y_s),\varphi\left(y_s\right)\right)}\equiv K(y_s)>0$$

As claimed, K does not depend on y_c (even though the limits of the derivatives of the numerator and denominator with respect to α in equation (21) do).

Step 2: Proof of equations (17) and (18).

Consider $e = M$. We have

$$\lim_{\alpha\to 0}\left(\frac{L_M^U\left(r^U(y_c^\alpha), y_s\right)}{d_M\left(r^V\left(y_c^\alpha\right), r^V\left(y_s\right)\right)}\right) = \lim_{\alpha\to 0}\left(\frac{L_M^U\left(r^U(y_c^\alpha), y_s\right)}{d_M\left(r^U\left(y_c^\alpha\right), r^U\left(y_s\right)\right)}\right)\lim_{\alpha\to 0}\left(\frac{d_M\left(r^U\left(y_c^\alpha\right), r^U\left(y_s\right)\right)}{d_M\left(r^V\left(y_c^\alpha\right), r^V\left(y_s\right)\right)}\right)$$

From Proposition 1 (which holds for U as well as V), we know that the first limit after the equals sign converges to unity. For the second term, we have

$$\lim_{\alpha\to 0}\frac{\left|r^U\left(y_c^\alpha\right) - r^U\left(y_s\right)\right|}{\left|r^V\left(y_c^\alpha\right) - r^V\left(y_s\right)\right|} = \lim_{\alpha\to 0}\left|\frac{r^U\left(y_c^\alpha\right) - r^U\left(y_s\right)}{r^V\left(y_c^\alpha\right) - r^V\left(y_s\right)}\right| = \left|\lim_{\alpha\to 0}\frac{r^U\left(y_c^\alpha\right) - r^U\left(y_s\right)}{r^V\left(y_c^\alpha\right) - r^V\left(y_s\right)}\right| = K(y_s)$$

Therefore, equation (17) holds for $K_M(y_s) = K(y_s)$.

Now consider $e = W$. We have

$$\lim_{\alpha\to 0}\left(\frac{L_W^U\left(r^U(y_c^\alpha), y_s, H\right)}{d_W\left(r^V\left(y_c^\alpha\right), r^V\left(y_s\right)\right)}\right) = \lim_{\alpha\to 0}\left(\frac{L_W^U\left(r^U(y_c^\alpha), y_s, H\right)}{d_W\left(r^U\left(y_c^\alpha\right), r^U\left(y_s\right)\right)}\right)\lim_{\alpha\to 0}\left(\frac{d_W\left(r^U\left(y_c^\alpha\right), r^U\left(y_s\right)\right)}{d_W\left(r^V\left(y_c^\alpha\right), r^V\left(y_s\right)\right)}\right)$$

From Proposition 1, we know that the limit after the equals sign converges to $h\left(r^U(y_s)\right)$. For the second term, we have

$$\lim_{\alpha\to 0}\frac{\left(r^U\left(y_c^\alpha\right) - r^U\left(y_s\right)\right)^2}{\left(r^V\left(y_c^\alpha\right) - r^V\left(y_s\right)\right)^2} = \lim_{\alpha\to 0}\left(\frac{r^U\left(y_c^\alpha\right) - r^U\left(y_s\right)}{r^V\left(y_c^\alpha\right) - r^V\left(y_s\right)}\right)^2 = \left(\lim_{\alpha\to 0}\frac{r^U\left(y_c^\alpha\right) - r^U\left(y_s\right)}{r^V\left(y_c^\alpha\right) - r^V\left(y_s\right)}\right)^2 = \left[K\left(y_s\right)\right]^2$$

Therefore, equation (18) holds for $K_W\left(y_s\right) = h\left(r^U(y_s)\right)\left[K\left(y_s\right)\right]^2$.

■

B.3 Proof of Proposition 6

Proceeding as in the proof of Proposition 5, Step 1, we see that, for any given θ,

$$\lim_{\alpha\to 0}\left(\frac{r^U\left(y_{c\theta}^\alpha\right) - r^U\left(y_s\right)}{r^V\left(y_{c\theta}^\alpha, \theta\right) - r^V\left(y_s, \theta\right)}\right) = \frac{U_\varphi\left(-r^U(y_s), \varphi\left(y_s\right)\right)V_m\left(-r^V(y_s,\theta), \varphi\left(y_s\right), \theta\right)}{U_m\left(-r^U(y_s), \varphi\left(y_s\right)\right)V_\varphi\left(-r^V(y_s,\theta), \varphi\left(y_s\right), \theta\right)} \tag{22}$$

Moreover, it is straightforward to check that

$$\rho^V\left(y_s, \theta\right) = \left.\frac{\partial r^V\left(y_s^\beta, \theta\right)}{\partial\beta}\right|_{\beta=1} = \frac{V_\varphi\left(-r^V(y_s,\theta), \varphi\left(y_s\right), \theta\right)}{V_m\left(-r^V(y_s,\theta), \varphi\left(y_s\right), \theta\right)}\left(\nabla\varphi\left(y_s\right)\cdot\left(y_s - y_0\right)\right) \tag{23}$$

Defining $\kappa(y_s) \equiv \frac{U_\varphi(-r^U(y_s),\varphi(y_s))}{U_m(-r^U(y_s),\varphi(y_s))}\nabla\varphi\left(y_s\right)\cdot\left(y_s - y_0\right)$, equations (22) and (23) then imply

$$\lim_{\alpha\to 0}\left(\frac{r^U\left(y_{c\theta}^\alpha\right) - r^U\left(y_s\right)}{r^V\left(y_{c\theta}^\alpha, \theta\right) - r^V\left(y_s, \theta\right)}\right) = \frac{\kappa(y_s)}{\rho^V\left(y_s, \theta\right)} \tag{24}$$

Consider $e = M$. Analogously to Step 2 of Proposition 5, we have

$$\lim_{\alpha\to 0}\left(\frac{L_M^U\left(r^U(y_{c\theta}^{\alpha}), y_s\right)}{d_M\left(r^V\left(y_{c\theta}^{\alpha},\theta\right), r^V\left(y_s,\theta\right)\right)}\right) = \lim_{\alpha\to 0}\left(\frac{L_M^U\left(r^U(y_{c\theta}^{\alpha}), y_s\right)}{d_M\left(r^U\left(y_{c\theta}^{\alpha}\right), r^U\left(y_s\right)\right)}\right)\lim_{\alpha\to 0}\left(\frac{d_M\left(r^U\left(y_{c\theta}^{\alpha}\right), r^U\left(y_s\right)\right)}{d_M\left(r^V\left(y_{c\theta}^{\alpha},\theta\right), r^V\left(y_s,\theta\right)\right)}\right) \tag{25}$$

Proposition 4, as applied to U, is unchanged. Therefore, the first term on the right equals 1. Using the same arguments as in Step 2 of Proposition 5, equation (19) follows directly from equations (24) and (25), with $\kappa_M(y_s) = \kappa(y_s)$.

Now consider $e = W$. Analogously to Step 2 of Proposition 5, we have

$$\lim_{\alpha\to 0}\left(\frac{L_W^U\left(r^U(y_{c\theta}^{\alpha}), y_s, H\right)}{d_W\left(r^V\left(y_{c\theta}^{\alpha},\theta\right), r^V\left(y_s,\theta\right)\right)}\right) = \lim_{\alpha\to 0}\left(\frac{L_W^U\left(r^U(y_{c\theta}^{\alpha}), y_s, H\right)}{d_W\left(r^U\left(y_{c\theta}^{\alpha}\right), r^U\left(y_s\right)\right)}\right)\lim_{\alpha\to 0}\left(\frac{d_W\left(r^U\left(y_{c\theta}^{\alpha}\right), r^U\left(y_s\right)\right)}{d_W\left(r^V\left(y_{c\theta}^{\alpha},\theta\right), r^V\left(y_s,\theta\right)\right)}\right) \tag{26}$$

Again, Proposition 4, as applied to U, is unchanged. Therefore, the first term on the right equals $h\left(r^U\left(y_s\right)\right)$. Using the same arguments as in Step 2 of Proposition 5, equation (20) follows directly from equations (24) and (26), with $\kappa_W(y_s) = \left[\kappa(y_s)\right]^2 h\left(r^U(y_s)\right)$.

∎

C Experiment details

In this section we detail the implementation of the experiment. Screenshots of the instructions and the experimental interface are in Appendix E.

Amazon Mechanical Turk Workers log on to AMT through an interface that displays a list of *Human Intelligence Tasks* (HITs), each with a title, an estimated duration, and an estimated remuneration rate. Other HITs include taking surveys, categorizing images, writing product descriptions, and identifying performers on music recordings.

To ensure that subjects were *technically* able to view the videos, we told them at the outset of the study that access to youtube.com was required. We also asked them to reproduce the last word spoken in the welcome video, and the last word of the title slide of whichever treatment video they viewed. Subjects who were not able to complete these tasks correctly were not allowed to continue with the study. The videos were embedded in the survey so that subjects could not find the other treatment videos used in this study.

We ensured that each subject participated in our study only once using the unique identifying numbers assigned by AMT.[1] A subject can only receive payment for participation in the study if she correctly provides this information, and hence has no incentive for misrepresentation.

Initial Financial Literacy Before participating in the main stages of the experiment, subjects completed the unincentivized financial literacy test in Table A1. This test of financial literacy originated with Lusardi and Mitchell (2017) and van Rooij et al. (2011), and has been used in many other studies (Lusardi and Mitchell, 2014).

[1] Nonetheless, one subject managed to participate in our study twice. Both times, this subject exhibited multiple switching points, and hence is excluded from all analyses.

Attention to the Video Before subjects watched the treatment video, we informed them that, with 25% probability, their earnings would be entirely determined by their performance on a test,[2] and that 'to be able to answer the questions in the test, you need to both understand and know the contents of the video.' We also explained that the video could help them make better decisions both during the experiment and in real life, inasmuch as it was made by 'internationally recognized academic experts on financial decision making.' Finally, we disabled the *continue* button for the duration of the video.

Iterated Multiple Price List Each line of each price list was a binary choice between the future reward and a specified dollar amount to be received no more than two days after completion of the experiment. For the first price list, the immediate payment varied from \$0 to \$20 in increments of \$2. For the second price list, it varied from $\$x$ to $\$(x+1.8)$ in increments of \$0.20, where $x+2$ is the smallest amount chosen over the future reward in the first list. (See appendix E for screenshots of the computer interface.) If a subjects' payment was determined according to a price list, the randomization over lines proceeded as follows. A line was randomly selected from the first price list. If that line did not correspond to x (defined above), it was implemented. Otherwise, a random line from the second price list was selected, and the decision for that line was implemented. With this procedure, truthful revelation of preferences is optimal.

Questionnaire Questions concerning decision strategies employed the following wording. Use of the rule of 72 in complexly framed problems: "Sometimes in this experiment, you were given a choice such as 'We will invest \$10 in an account with 1% interest per week. Interest is compounded weekly. We will pay you the proceeds in 72 days.' When deciding about this choice, did you use the rule of 72?"[3] Use of the rule of 72 in simply framed problems: "Sometimes in this experiment, you were given a choice such as 'We will pay you \$20 in 36 days.' When deciding about such a choice, did you use the rule of 72?" In both cases, subjects answered either "Yes", "No", or "I don't know the rule of 72." Number of problems for which the future reward was calculated explicitly: "In total, you were given 10 rounds in which one of the options was something like 'we will invest \$... in an account with ...% interest per day. Interest is compounded daily. We will pay you the proceeds in... days.' Out of these 10 rounds, how many times did you explicitly calculate the money amount that this investment would yield within the specified time?" Subjects responded by selecting an integer between 0 and 10. Use of external help on the test: "When you completed the test about the video on financial investing, did you use external resources (such as other websites, books, etc.) to find the right answers?" Subjects answered either "Yes" or "No."

We also asked subjects how much attention they had paid to their choices, how much attention they had paid to the video, whether they had any suggestions about the study, and whether they had experienced any technical difficulties. The overwhelming majority of subjects reported the highest level of attention in answer to both questions—a finding we interpret with caution.

[2] Hastings et al. (2013) criticize most existing studies that use such test scores as outcome measures on the grounds that the tests are unincentivized. One of the few exceptions is Levy and Tasoff (2016).

[3] The survey question incorrectly described the interest rate as pertaining to a week rather than a day. We believe the meaning of the question was nevertheless clear despite this typo.

Table A1: Financial Literacy questionnaire.

FL1. Suppose you had \$100 in a savings account and the interest rate was 2 percent per year. After 5 years, how much do you think you would have in the account if you left the money to grow?
More than \$102 (92.03%), Exactly \$102 (4.61%), Less than \$102 (1.99%), Do not know (1.37%)

FL2. Suppose you had \$100 in a savings account and the interest rate is 20 percent per year and you never withdraw money or interest payments. After 5 years, how much would you have on this account in total?
More than \$200 (75.84%), Exactly \$200 (19.68%), Less than \$200 (2.74%), Do not know (1.74%)

FL3. Imagine that the interest rate on your savings account was 1 percent per year and inflation was 2 percent per year. After 1 year, how much would you be able to buy with the money in this account?
More than today (7.35%), exactly the same (4.98%), less than today (84.43%), do not know (3.24%)

FL4. Assume a friend inherits \$10,000 today and his sibling inherits \$10,000 3 years from now. Who is richer because of the inheritance?
My friend (56.79%), his sibling (6.72%), they are equally rich (29.89%), do not know (6.60%)

FL5. Suppose that in the year 2015, your income has doubled and prices of all goods have doubled too. In 2015, how much will you be able to buy with your income?
More than today (4.23%), the same (90.16%), less than today (4.73%), do not know (0.87%)

Notes: Numbers in brackets indicate the percentage of subjects who chose a given answer. The fraction of correct answers to each question only differs slightly across experiments and treatments, see Table A1.

Test questions in Control interventions Panels A and B of Table A2 display the test questions about the Control intervention in Experiments A and B, respectively. The two experiments involve different sets of questions about their respective Control interventions because the Control interventions differ. We decided to use a different Control intervention for Experiment B because the Control intervention in Experiment A is largely descriptive, and hence is not well-suited to incorporating practice questions with individualized feedback.

Table A2: Test questions concerning the Control interventions.

Experiment A

Which of the following quotes is attributed to Benjamin Franklin?
Compound interest is the most powerful force in the universe; Youth is wasted on the young; Money makes money. And the money that money makes, makes money;

Which quote is attributed to the author Upton Sinclair?
Only liars manage always to be out of the market during bad times and in during good times; It is difficult to get a man to understand something when his salary depends upon his not understanding it; There are three classes of people who do not believe that markets work: the Cubans, the North Koreans, and active managers; Nobody knows more than the market.

What percentage of mutual funds tends to be outperformed by the market (S&P 500 Index) each year?
between 10 and 30% between 30 and 50% between 50 and 70% between 70 and 90%

What is an "indexing" investment strategy?
Buying index funds, which hold assets that have been indexed as particularly profitable by financial experts; Buying index funds, which hold stocks of companies that provide information about the stock market as a whole (stock market indices); Buying index funds, which hold the market portfolio; Buying index funds, which hold optimally diversified, custom tailored portfolios.

Professional investors as a whole are responsible for what percentage of stock market trading?
30%; 50%; 70%; 90%.

Experiment B

In order to limit your risk, you might invest in which of the following pairs of stocks?
Microsoft and Google; General Motors and Chrysler; Coca-Cola and Pepsi; General Motors and Microsoft; Facebook and Twitter.

We would expect the degree of relation between the returns of Coca-Cola stock and the returns of Pepsi stock to be closest to _____? [-1 means perfect negative relation and +1 means perfect positive relation]:
-0.7; -0.3; 0; 0.3; 0.7.

Considering a long time period (for example 10 or 20 years), which asset normally gives the highest return?
Savings accounts; Corporate bonds; Government bonds; T-Bills; Stocks.

Normally, which asset displays the highest fluctuations over time?
Savings accounts; Corporate bonds; Government bonds; T-Bills; Stocks.

A degree of relation of _____between two assets will NOT help reduce your risk.
1; 0.5; 0; -0.5; -1.

D Additional Data Analysis

D.1 Demographics

Table A1 presents detailed demographics of our subject pool by treatment, as well as their initial financial literacy.[4] Column 5 lists data for the representative US citizen. Demographic variables are taken from the 2010 US Census. Employment variables are for April 2014, and come from the Bureau of Labor Statistics. Financial literacy scores are from Lusardi (2011), and from the 2012 FED bulletin for stock holdings.[5] (Representative data on financial literacy only exist for questions FL1 and FL3.) For empty cells, no representative data are available. Column 6 reports, for each variable, the p-value of an F-test for differences across treatments. The number of significant differences is well within the range we would expect given the number of tests performed.

As reported in section I.B, our sample is poorer, better educated, and more likely to live in larger households than the average US citizen. While the incidence of full-time employment in our sample mirrors that of the general population, the fraction of respondents who classify themselves as employed part-time is double that of the general population. Our subjects are also disproportionately male and white, younger, slightly more urban, and more likely to have never been married than the representative US citizen.

D.2 Main results controlling for demographics

Table A2 presents our main results in a regression that includes data from both experiments and controls for demographics. Demographic controls consist of all variables listed in Table A1, except for the summary statistics "FL1-FL3 all correct" and "FL1-FL5 all correct". For brevity, we pool across the timeframes.

In each case we see that coefficient estimates are barely changed in comparison to the estimates in the main text, which do not control for demographic characteristics. We conclude that the differences between experiments A and B reflect differences in the interventions rather than differences in subject characteristics.

[4] These statistics only include subjects who did not exhibit multiple switching points in any of the price lists.

[5] J. Bricker, A. B. Kennickell K. B. Moore, and J. Sabelhaus, 2012, *Changes in U.S. Family Finances from 2007 to 2010: Evidence from the Survey of Consumer Finances*, Federal Reserve Bulletin, 98(2).

Table A1: Demographics and financial literacy.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Experiment A					Experiment B			
Treatment	Treat.	Cont.	Subst. only	Rhet. only	p-value	Treat.	Cont.	p-value	US
FL1	0.93	0.92	0.92	0.95	0.83	0.90	0.92	0.59	0.65
FL2	0.81	0.73	0.73	0.71	0.32	0.75	0.80	0.29	-
FL3	0.82	0.82	0.83	0.85	0.93	0.88	0.85	0.38	0.64
FL4	0.58	0.64	0.50	0.59	0.17	0.53	0.59	0.31	-
FL5	0.96	0.90	0.87	0.91	0.09*	0.91	0.88	0.30	-
FL1 - FL3 all correct	0.71	0.63	0.62	0.62	0.44	0.68	0.70	0.72	-
FL1 - FL5 all correct	0.47	0.45	0.34	0.40	0.20	0.40	0.42	0.60	-
Male	0.57	0.57	0.61	0.50	0.40	0.47	0.53	0.28	0.49
Age (median)	32	28	29	29	0.02**	36	34	0.14	37.2
Household Income (median)	45000	35000	45000	35000	0.19	57500	57500	1.00	53,046
Race									
African-American	0.08	0.06	0.08	0.04	0.68	0.05	0.10	0.10*	0.13
Asian	0.08	0.11	0.12	0.05	0.22	0.06	0.06	0.90	0.05
Caucasian	0.81	0.72	0.72	0.77	0.34	0.82	0.79	0.41	0.63
Hispanic	0.03	0.07	0.03	0.10	0.06*	0.04	0.04	0.88	0.17
Other	0.01	0.04	0.05	0.04	0.44	0.02	0.01	0.37	0.02
Education									
Less than high school	0.01	0.00	0.00	0.00	0.35	0.00	0.00	1.00	0.14
High school	0.13	0.12	0.15	0.14	0.92	0.14	0.13	0.71	0.31
Vocational / technical	0.08	0.08	0.08	0.03	0.29	0.07	0.06	0.56	0.09
Some college	0.35	0.37	0.33	0.44	0.34	0.31	0.34	0.67	0.19
College	0.39	0.37	0.38	0.34	0.90	0.37	0.37	0.97	0.18
Graduate degree	0.05	0.06	0.07	0.05	0.88	0.11	0.11	0.88	0.09
Employment									
Full time employed	0.50	0.50	0.48	0.43	0.70	0.61	0.68	0.16	0.48[a)]
Part time employed	0.21	0.23	0.26	0.27	0.72	0.18	0.16	0.60	0.11[b)]
Marital Status									
Married	0.28	0.30	0.32	0.29	0.94	0.46	0.47	0.80	0.27
Widowed	0.00	0.00	0.00	0.00	1.00	0.01	0.01	0.53	0.56
Divorced	0.07	0.05	0.04	0.04	0.80	0.04	0.06	0.37	0.06
Never married	0.64	0.65	0.64	0.64	1.00	0.49	0.46	0.69	0.10
Urban / Rural									
Urban and suburban	0.17	0.17	0.11	0.17	0.48	0.25	0.17	0.06*	0.81
Rural	0.83	0.83	0.89	0.83	0.48	0.75	0.83	0.06*	0.19
Household size									
1	0.18	0.13	0.11	0.19	0.26	0.10	0.13	0.33	0.22
2	0.22	0.24	0.25	0.24	0.95	0.27	0.23	0.42	0.36
3	0.14	0.19	0.17	0.22	0.46	0.18	0.22	0.35	0.17
4 or more	0.46	0.44	0.47	0.35	0.23	0.45	0.41	0.50	0.26
Owns stocks	0.16	0.23	0.20	0.23	0.54	0.43	0.39	0.44	0.15
N	106	109	128	112	-	169	179	-	-

[a)] Percentage of civilian noninstitutional population that is full-time employed.
[b)] Percentage of civilian noninstitutional population that is part-time employed.

Notes: The sample includes all subjects who completed the study and did not exhibit any multiple switching points. Column 5 presents the *p*-values of an *F*-test for joint equality of the coefficients listed in columns 1 - 4. Column 8 presents the *p*-value of a *t*-test for joint equality of the coefficients in columns 6 - 7. Column 9 lists comparison values for the representative US citizen whenever available. Figures for marital status and income exclude three and five individuals, respectively, who preferred to withhold this information. See text for data sources.

Table A2: Main results controlling for demographics in joint analysis of both experiments.

VARIABLES	(1) Test scores on module	(2)	(3) Valuation in frame	(4)	(5) Deliberative Competence	(6)
	Treatment	Control	Complex	Simple		
Correction for changes in valuations in simple frame					No	Yes
Difference to Control B						
Treatment B	1.644***	-0.350***	14.591***	6.723***	8.211***	13.149***
	(0.140)	(0.126)	(2.826)	(2.190)	(1.788)	(2.585)
Treatment A	1.528***	-0.823***	12.361***	-0.542	2.619	-0.750
	(0.163)	(0.131)	(3.365)	(2.778)	(2.103)	(4.189)
Control A	0.112	0.232	-0.959	-0.287	1.907	3.624
	(0.171)	(0.147)	(3.120)	(2.601)	(2.069)	(2.762)
Substance-Only	1.364***	-1.066***	3.805	0.778	3.290*	-0.418
	(0.157)	(0.136)	(3.168)	(2.687)	(1.903)	(5.053)
Rhetoric-Only	0.614***	-0.842***	18.108***	5.695**	6.087***	8.378***
	(0.181)	(0.140)	(3.559)	(2.775)	(1.912)	(2.950)
Demographic controls	Yes	Yes	Yes	Yes	Yes	Yes
p-value						
Control A = Treatment A	0.000	0.000	0.000	0.930	0.748	0.286
Observations	788	788	7,880	7,880	7,880	7,870
Subjects	788	788	788	788	788	787

Notes: Each column corresponds to a separate regression. Deliberative Competence measured as $d_M^{i,R,t} = -\left|r_c^{i,R,t} - r_s^{i,R,t}\right|$. Regressions exclude individuals who withheld information about one of the control variables. Column 6 omits the subject for whom $r_S^{i,R,t} = 0$ for all instruments. Standard errors in parentheses, clustered on the subject level. $***p < 0.01$,$***p < 0.05$,$*p < 0.1$.

D.3 Effects on individual test questions

We analyze the effect of the treatments on answers to individual test questions in table A3. The test questions differ by how closely they follow the material in the education intervention, and by how easily they are answered without knowledge of the rule of 72.

Q1 is the only question for which the answer was explicitly given in the education video (including in the Substance-Only treatment but not in the Rhetoric-Only treatment). The video also discussed an example that is similar, but not identical, to Q2.[6]

The remaining questions require more flexible thinking. Q3 and Q4 can easily be answered with the rule of 72. Knowledge of this rule, however, is not necessary to answer these questions correctly. Q3 can be answered by iteratively multiplying a starting value with 1.07, and counting the number of iterations required for the amount to increase to the desired value. Likewise, Q4 can be answered by calculating the factor by which an investment grows within 8 years at 9 percent interest (either iteratively, or using the compound interest formula), and then dividing 500 by this number. Q5 is a standard compound interest calculation, and parallels the calculations that need to be made in the complexly framed decision problems.

[6]The example is: "To double your money in 10 years, what rate of return do you need? The answer: 10 times $X = 72$, so $X = 7.2$ percent."

Table A3 displays the treatment effects on the success rates for each of these questions. Baseline rates of correct answers are highly similar across the two experiments. Moreover, in both experiments, the significant effect of the Full and Substance-Only treatments on the total score derive from questions Q1, Q2, and Q5. The fact that performance in Q5 increased in these treatments is reassuring, as it demonstrates that the increase in test scores is at least partly due to subjects' increased ability to analyze previously unseen problems properly. Moreover, while treatment effects are similar across the experiments for questions Q1 to Q4, the treatment effect on Q5 in Experiments B is more than double that in Experiment A, tentatively hinting at our finding that our intervention in Experiment B is more effective than that in Experiment A.

Table A3: Fraction of correct responses on individual questions in the test about the Treatment intervention.

Question	Q1	Q2	Q3	Q4	Q5
			Experiment A		
Treatment effects					
Treatment	0.566***	0.619***	0.062	0.021	0.174***
	(0.054)	(0.053)	(0.068)	(0.068)	(0.067)
Substance-Only	0.584***	0.592***	-0.037	0.023	0.109*
	(0.051)	(0.053)	(0.065)	(0.065)	(0.065)
Rhetoric-Only	0.072	0.191***	0.067	0.114*	0.050
	(0.065)	(0.061)	(0.067)	(0.067)	(0.067)
Level in Control	0.330***	0.220***	0.514***	0.422***	0.477***
	(0.045)	(0.040)	(0.048)	(0.047)	(0.048)
Observations	455	455	455	455	455
			Experiment B		
Treatment effect	0.559***	0.696***	0.045	-0.075	0.375***
	(0.042)	(0.038)	(0.054)	(0.052)	(0.048)
Level in Control	0.346***	0.168***	0.464***	0.436***	0.436***
	(0.036)	(0.028)	(0.037)	(0.037)	(0.037)
Observations	348	348	348	348	348

Notes: Standard errors in parentheses. $***p < 0.01$, $**p < 0.05$, $*p < 0.1$.

D.4 Self-reported behavior

The study ends with a brief non-incentivized questionnaire. We ask subjects whether they had used the rule of 72 in the complexly framed problems, and whether they had used it in the simply framed problems. We also elicit the number of complexly framed valuation tasks for which subjects explicitly calculated the future value of the investment, and ask whether they obtained help when taking the test on compound interest. The questionnaire also addresses a small number of additional issues.

Subjects in the Control condition report similar numbers of decisions for which they engaged in explicit calculations in each of the experiments (6.4 and 6.7 in Experiments A and B, respectively). The Treatment condition significantly increases that number, by 1.7 problems in Experiment A (column 1, $p < 0.01$) and by 2.6 problems in Experiment B (column 2, $p < 0.01$). The treatment effect on the fraction of subjects reporting to have used the rule of 72 in their decision making in complexly framed decisions does not differ substantially across experiments (57.9% and 60.5% in Experiments A and B, respectively). There is, however, a difference in levels. Only 12.8% of subjects in the Control condition of Experiment A report using the rule, whereas 31.8% of subjects in the Control condition of Experiment B do so.

As expected, the fraction of subjects reporting to have used the rule of 72 for simply framed problems is substantially smaller; averaging 9.2% and 22.3% in the Control conditions of Experiments A and B, respectively. In both experiments the Treatment condition increases the frequency of such reports, but does so almost twice as much in Experiment A (by 17.2 percentage points) than in Experiment B (by 9.6 percentage points). Finally, when asked about the use of external help with the test questions at the end of the experiment, we do not find treatment effects in either experiment, although the fraction of subjects reporting the use of such help exceeds 20% in Experiment A, whereas it is lower than 8% in Experiment B.

Unlike performance on test scores and directional behavioral changes, these self-reported behaviors suggest that the effects of the Treatment interventions differ across the experiments, though that interpretation is complicated by the fact that baseline levels differ across the experiments. Like the conventional measures, however, data on self-reported behavior suggest that the Treatment interventions are effective in either experiment.

Table A4: Self-reported behavior.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Self-report	Engages in explicit calculation		Uses of rule of 72 in complex frame		Uses of rule of 72 in simple frame		External help with test	
Experiment	A	B	A	B	A	B	A	B
Levels								
Control	6.404***	6.693***	0.128***	0.318***	0.092***	0.223***	0.220***	0.078***
	(0.377)	(0.277)	(0.032)	(0.035)	(0.028)	(0.031)	(0.040)	(0.020)
Treatment	8.142***	9.331***	0.708***	0.923***	0.264***	0.320***	0.208***	0.059***
	(0.342)	(0.209)	(0.044)	(0.021)	(0.043)	(0.036)	(0.040)	(0.018)
Difference	1.738***	2.639***	0.579***	0.605***	0.172***	0.096**	-0.013	-0.019
	(0.509)	(0.347)	(0.055)	(0.041)	(0.051)	(0.048)	(0.056)	(0.027)
Observations	215	348	215	348	215	348	215	348

Notes: Each column displays the coefficients of a separate OLS regression. Standard errors in parentheses. $^{***}p < 0.01$, $^{***}p < 0.05$, $^{*}p < 0.1$.

D.5 Analysis using the welfarist measure of Deliberative Competence

Here, we present our main empirical results on Deliberative Competence using the measure $d_W^{i,R,t} = -\left(r_c^{i,R,t} - r_s^{i,R,t}\right)^2$ which approximates the average rather than the maximal loss from characterization failure.

Table A5 replicates Table 7 with this alternative measure. Panel A shows that the intervention in Experiment A leaves $d_W^{i,R,t}$ nearly unchanged on average whereas the intervention in Experiment B leads to a substantial and statistically highly significant increase both in the pooled sample and separately within each timeframe.

Panel B applies Proposition 6 to correct for changes in valuations in the simple frame.[7] Again, we find that the intervention in Experiment A, if anything, harms subjects, whereas the intervention in Experiment B significantly increases their welfare. The relative magnitude of these effects differs from those in Table 7. Here, we find that the harm caused by the intervention in Experiment A is of a similar magnitude as the benefits caused by the intervention in Experiment B, whereas in Panel B of Table 7 the benefits of the intervention in Experiment B exceed the magnitude of the harm in Experiment A severalfold. One reason for this divergence is the stronger sensitivity of $d_W^{i,R,t}$ to large valuation differences, $r_C^{i,R,t} - r_S^{i,R,t}$.

[7] Proceeding as in Table 7, we account for noise in the elicitation of valuations in the simple frame. Specifically, we calculate the mean valuation for simply framed choices for each timeframe and use the square of the resulting average as the correction factor. Moreover, by our normalization, $y_{sI} - y_0 = 1$ for all instruments I.

Table A5: Deliberative Competence based on expected welfare loss.

A. Deliberative Competence: Welfarist Measure

	(1)	(2)	(3)	(4)	(5)	(6)
Delay in days	72 and 36	72 and 36	72	72	36	36
Experiment	A	B	A	B	A	B
Levels						
Control	-5.846***	-6.356***	-5.782***	-6.486***	-5.911***	-6.227***
	(0.616)	(0.535)	(0.688)	(0.568)	(0.623)	(0.549)
Treatment	-5.924***	-4.323***	-5.851***	-4.088***	-5.996***	-4.558***
	(0.810)	(0.453)	(0.793)	(0.448)	(0.982)	(0.498)
Substance-Only	-5.116***		-4.745***		-5.487***	
	(0.563)		(0.554)		(0.650)	
Rhetoric-Only	-4.574***		-4.764***		-4.384***	
	(0.585)		(0.626)		(0.611)	
p-value of difference to Control						
Treatment	0.939	0.004	0.947	0.001	0.941	0.025
Substance-Only	0.382		0.241		0.638	
Rhetoric-Only	0.135		0.274		0.081	
Observations	4,550	3,480	2,275	1,740	2,275	1,740
Subjects	455	348	455	348	455	348

B. Deliberative Competence: Welfarist Measure corrected for changes in simply framed valuations

	(1)	(2)	(3)	(4)	(5)	(6)
Delay in days	72 and 36	72 and 36	72	72	36	36
Experiment	A	B	A	B	A	B
Levels						
Control	-11.609***	-14.580***	-13.053***	-15.365***	-10.164***	-13.794***
	(1.371)	(1.686)	(2.065)	(1.809)	(1.127)	(2.176)
Treatment	-20.816***	-8.700***	-19.831***	-8.855***	-21.800***	-8.544***
	(5.086)	(1.257)	(4.690)	(1.253)	(6.815)	(1.415)
Substance-Only	-34.069**		-49.831*		-18.307***	
	(16.927)		(28.233)		(5.928)	
Rhetoric-Only	-12.097***		-14.309***		-9.886***	
	(2.256)		(2.782)		(2.112)	
p-value of difference to Control						
Treatment	0.081	0.005	0.187	0.003	0.093	0.044
Substance-Only	0.187		0.195		0.178	
Rhetoric-Only	0.853		0.717		0.907	
Observations	4,550	3,470	2,275	1,735	2,275	1,735
Subjects	455	347	455	347	455	347

Notes: Each column displays the coefficients of a separate OLS regression of the welfarist measure of Deliberative Competence, $d_W^{i,R,t} = -\left(r_c^{i,R,t} - r_s^{i,R,t}\right)^2$, on treatment indicators. Standard errors in parentheses, clustered by subject. The reason for the smaller number of observations in Panel B in Experiment B is one subject who consistently made choices consistent with a valuation of zero in the simple frame. As the correction consists in dividing by simply framed valuations, this subject is excluded from that analysis. $^{***}p < 0.01$,$^{***}p < 0.05$,$^{*}p < 0.1$.

Table A6 replicates Table 11 using using the welfarist measure of Deliberative Competence, $d_W^{i,R,t} = -\left(r_c^{i,R,t} - r_s^{i,R,t}\right)^2$. As in Table 11, we find that the intervention in Experiment A harms subjects in the lowest quartile of valuations in the simple frame ($p < 0.1$), and has beneficial effects for subjects in the highest quartile ($p < 0.1$). Also paralleling the result in Table 11, the intervention in Experiment B does not significantly harm subjects in any quartile, but has substantially positive effects for subjects in the second-highest ($p < 0.1$) and highest quartiles ($p < 0.01$). We conclude that our inferences regarding practice and feedback are not driven by an unintended relationship between welfare weights and biases that impact simply framed valuations, also based on the welfarist measure.

Table A6: Welfarist measure of Deliberative Competence by quartiles of simply framed valuations.

Experiment A

VARIABLE	Deliberative Competence			
	Quartile simply framed valuation			
	1	2	3	4
Levels				
Control	-3.068	-4.501	-6.615	-9.082
	(0.703)	(1.100)	(1.310)	(1.368)
Treatment	-7.803	-4.991	-5.140	-5.803
	(2.594)	(1.050)	(0.864)	(1.396)
Effect	-4.735*	-0.490	1.476	3.279*
	(2.688)	(1.521)	(1.569)	(1.954)
Observations	2,150			
Subjects	215			

Experiment B

VARIABLE	Deliberative Competence			
	Quartile simply framed valuation			
	1	2	3	4
Levels				
Control	-3.378	-5.964	-6.051	-9.967
	(0.669)	(0.955)	(0.849)	(1.407)
Treatment	-3.539	-5.016	-3.953	-4.773
	(0.916)	(0.874)	(0.751)	(1.037)
Effect	-0.161	0.947	2.098*	5.193***
	(1.134)	(1.295)	(1.133)	(1.748)
Observations	3,480			
Subjects	348			

Notes: Effect of treatments on the welfarist measure of Deliberative Competence, $d_W^{i,R,t} = -\left(r_c^{i,R,t} - r_s^{i,R,t}\right)^2$, by quartiles of simply framed valuations. Each panel presents the output of a single OLS regression. Standard errors in parentheses, clustered by subject. $***p < 0.01$,$***p < 0.05$,$*p < 0.1$, omitted for levels.

D.6 Deliberative Competence with correction for policy-induced confounds based on the approach of Appendix B

In this section, we provide estimates of Deliberative Competence corrected for policy-induced framing effects using the alternative approximation approach developed in Section B.

We estimate the correction factor ρ by regressing valuations in the simple frame on the future amount the subject can receive from purchasing the instrument. We run a separate OLS regression for each timeframe and for each treatment within each Experiment. The slope coefficient is a consistent estimate for $\rho^V\left(y_s, \theta\right)$; in each case, we divide the absolute valuation difference by this estimate.

Panel A of Table A7 applies this correction factor to replicate Table 7. Panel B performs parallel analysis using the welfarist measure d_W and the corresponding correction factor ρ^2. In each case we find large and statistically highly significant treatment effects for Experiment B but not for Experiment A.

Table A7: Deliberative Competence corrected for policy-induced confounds based on the approach of Appendix B

A. Deliberative Competence, d_M

	(1)	(2)	(3)	(4)	(5)	(6)
Delay in days	72 and 36	72 and 36	72	72	36	36
Experiment	A	B	A	B	A	B
Levels						
Control	-36.875***	-38.517***	-39.765***	-40.982***	-33.985***	-36.052***
	(2.472)	(2.036)	(2.880)	(2.255)	(2.304)	(1.978)
Treatment	-31.591***	-26.576***	-33.291***	-27.800***	-29.891***	-25.353***
	(2.402)	(1.684)	(2.609)	(1.821)	(2.476)	(1.699)
Substance-Only	-30.356***		-30.824***		-29.888***	
	(1.972)		(2.062)		(2.127)	
Rhetoric-Only	-27.375***		-29.278***		-25.471***	
	(1.941)		(2.115)		(1.977)	
p-value of difference to Control						
Treatment	0.126	0.000	0.096	0.000	0.227	0.000
Substance-Only	0.040		0.012		0.192	
Rhetoric-Only	0.003		0.004		0.005	
Observations	4,550	3,480	2,275	1,740	2,275	1,740
Subjects	455	348	455	348	455	348

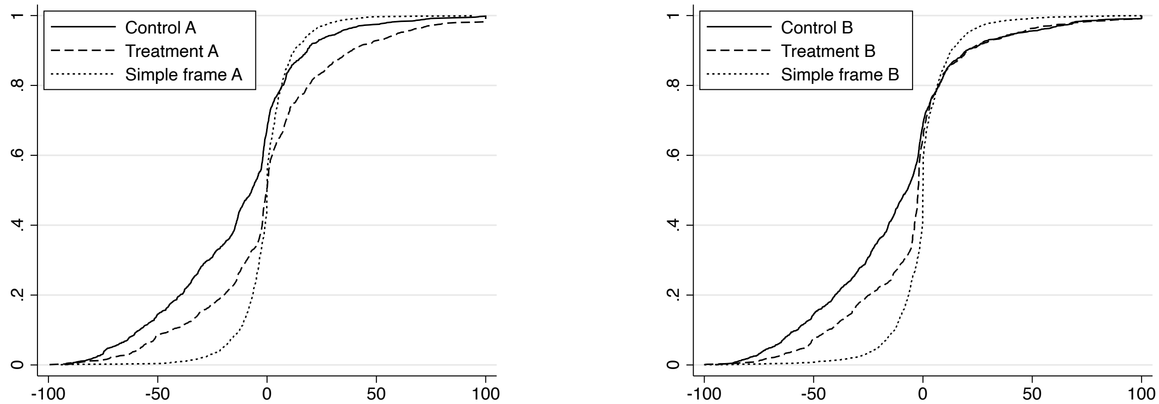
B. Deliberative Competence, d_W

	(1)	(2)	(3)	(4)	(5)	(6)
Delay in days	72 and 36	72 and 36	72	72	36	36
Experiment	A	B	A	B	A	B
Levels						
Control	-72.230***	-77.119***	-82.620***	-84.680***	-61.840***	-69.557***
	(5.363)	(4.666)	(6.715)	(5.487)	(4.695)	(4.263)
Treatment	-64.912***	-58.021***	-69.717***	-61.145***	-60.106***	-54.898***
	(5.925)	(4.223)	(6.712)	(4.658)	(5.982)	(4.224)
Substance-Only	-55.189***		-57.307***		-53.071***	
	(4.140)		(4.605)		(4.133)	
Rhetoric-Only	-60.167***		-66.526***		-53.808***	
	(5.291)		(6.112)		(4.959)	
p-value of difference to Control						
Treatment	0.360	0.003	0.175	0.001	0.820	0.015
Substance-Only	0.012		0.002		0.162	
Rhetoric-Only	0.110		0.077		0.240	
Observations	4,550	3,480	2,275	1,740	2,275	1,740
Subjects	455	348	455	348	455	348

Notes: Each column displays the coefficients of a separate OLS regression of Deliberative Competence, on treatment indicators. Deliberative Competence is corrected for the change in simply framed valuations using the approximation strategy developed in Section B. Panel A uses the maximal loss measure of Deliberative Competence, d_M; Panel B uses the welfarist measure d_W.

D.7 Valuation difference compared to noise in the simple frame

Figure A1: Replication of CDFs from Figure 3 with a measure of the distribution of noise in the simple frame superimposed.



Experiment A

Experiment B

Here we investigate the possibility that the measured overestimation of compound interest exhibited by a fraction of our subjects is solely attributable to elicitation noise. To test the hypothesis, we estimate the amount of noise in simply framed decisions. We then compare the frequency of excessive valuations we would observe based on that amount of noise alone to the frequency of excessive valuations we actually observe in the complex frame. Specifically, we calculate, for each subject and each timeframe $t \in \{36, 72\}$, the values $\Delta_1^t = r_S^{j,20,t} - r_S^{j,18,t}$, $\Delta_2^t = r_S^{j,18,t} - r_S^{j,20,t}$ $\Delta_3^t = r_S^{j,16,t} - r_S^{j,14,t}$, and $\Delta_4^t = r_S^{j,14,t} - r_S^{j,16,t}$ (recall that valuations $r_S^{i,R,t}$ involve the normalization of the future value to \$1). Figure A1 superimposes the CDF of Δ_k^t (pooled across $k \in \{1, .., 4\}$ and $t \in \{36, 72\}$) on the CDF of the valuation difference from Figure 3. If excessive valuations in the complex frame were solely attributable to elicitation noise, the CDF of Δ_k^t should coincide with the CDF of $r_C^{i,R,t}$ to the left of zero. By contrast, we find substantial differences between these curves in each experiment, and especially for the treatment in Experiment A. Accordingly, elicitation noise alone cannot explain the overestimation of compound interest in either condition of either experiment.

E Instructions

This is a research study run by the department of economics at Stanford University.

IMPORTANT

This study may take up to ONE AND A HALF HOURS to complete. Please start this study only if you do have that much time in a single session.

If you do not complete the study, or if the HIT times out on you, we will not be able to pay you. (The HIT is set to time out in 3 hours.)

You will earn $10 just for completing this study. In addition, you will receive up to $20, depending on the decisions you make in this study.

Do not start this study if you do not have access to youtube.com. Some browsers will block embedded videos. Please make sure your browser will display them, as you may otherwise not be able to complete this study.

Click here to start the study: https://stanforduniversity.qualtrics.com/SE/?SID=SV_0GPXNo1f9TX5YIR

Provide the survey code here:

WELCOME

This is a research study run by the department of economics at Stanford University.

IMPORTANT

This study may take up to ONE AND A HALF HOURS to complete. Please start this study only if you do have that much time in a single session.

If you do not complete the study, or if the HIT times out on you, we will not be able to pay you. (The HIT is set to time out in 3 hours.)

You will earn $10 just for completing this study. In addition, you will receive up to $20, depending on the decisions you make in this study.

Do not start this study if you do not have access to youtube.com. Some browsers will block embedded videos. Please make sure your browser will display them.

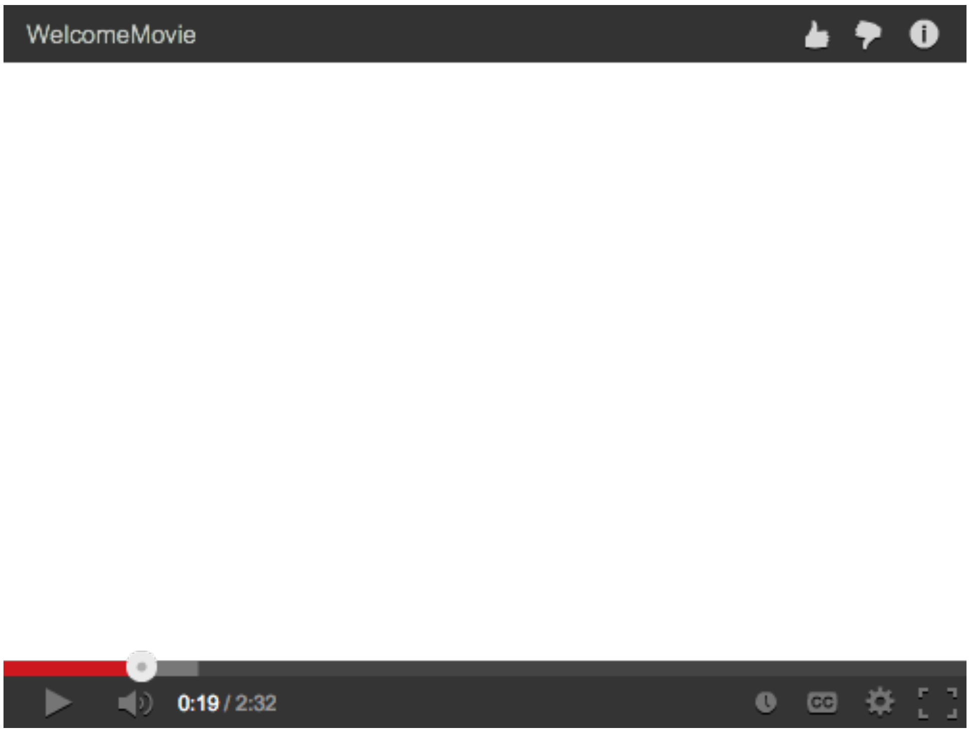
By clicking the button below, you consent to participating in this research study.

Questions, Concerns, or Complaints: If you have any questions, concerns or complaints about this research study, its procedures, risks and benefits, you should ask the Protocol Director, Sandro Ambuehl, sambuehl@stanford.edu

Independent contact: If you are not satisfied with how this study is being conducted, or if you have any concerns, complaints, or general questions about the research or your rights as a participant, please contact the Stanford Institutional Review Board (IRB) to speak to someone independent of the research team at (650)-723-2480 or toll free at 1-866-680-2906. You can also write to the Stanford IRB, Stanford University, Stanford, CA 94305-5401

>>

[Some browsers will ask you whether you want to display this content. Please click "display all content".]



[There should be a video here. If if does not load, please click [here](#)]

Links to researchers' personal homepages

[Professor B. Douglas Bernheim](#)

[Sandro Ambuehl](#)

To continue, please enter the LAST word that Doug Bernheim said in this video. A continue button will appear after the duration of the video.

>>

Before we start this study, we would like to ask you a few questions about yourself. Please answer these questions truthfully. Your answers will not affect your payment from this experiment.

What is your gender?

- male
- female

What is your age?

What is your ethinicity?

- African-American
- Asian
- Caucasian
- Hispanic
- Other

Please indicate the highest level of education you completed.

- Elementary School
- Middle School
- High School or equivalent
- Vocational/Technical School (2 year)
- Some College
- College Graduate (4 year)
- Master's Degree (MS)
- Doctoral Degree (PhD)
- Professional Degree (MD, JD, etc.)

What is your current marital status?

- Divorced
- Living with another
- Married
- Separated
- Single
- Widowed
- Prefer not to say

Please answer the following questions as well as you can. Your answers to these questions will not affect your payment from this study.

Imagine that the interest rate on your savings account was 1 percent per year and inflation was 2 percent per year. After 1 year, how much would you be able to buy with the money in this account?

- More than today
- Exactly the same
- Less than today
- Do not know

Suppose you had $100 in a savings account and the interest rate is 20 percent per year and you never withdraw money or interest payments. After 5 years, how much would you have on this account in total?

- More than $200
- Exactly $200
- Less than $200
- Do not know

Assume a friend inherits $10,000 today and his sibling inherits $10,000 3 years from now. Who is richer because of the inheritance?

- My friend
- His sibling
- They are equally rich
- Do not know

Suppose you had $100 in a savings account and the interest rate was 2 percent per year. After 5 years, how much do you think you would have in the account if you left the money to grow?

- More than $102
- Exactly $102
- Less than $102
- Do not know

Suppose that in the year 2015, your income has doubled and prices of all goods have doubled too. In 2015, how much will you be able to buy with your income?

- More than today
- The same
- Less than today
- Do not know

>>

You will now watch a

12-MINUTE VIDEO ABOUT FINANCIAL INVESTING.

Please follow this video carefully.

Please watch the ENTIRE video.

(a "continue" button will appear after 12 minutes.)

Doing so will be useful to you for three reasons:

1. *TEST with PAYMENT FOR CORRECT ANSWERS.*

Your earnings from this experiment may be entirely determined by a test on this video. **The final part of this experiment is a test about the contents of this video. There is a one in four chance that your earnings from this experiment are wholly determined by your performance in this test. The test has 10 questions. For each question you answer correctly, you will receive $1 within at most two days from today. For each question you answer incorrectly, you will receive $0. To be able to answer the questions in the test, you need to both *understand* and *know* the contents of the video. You may scroll back to watch parts of the video multiple times if you wish.**

2. *REMAINDER OF THIS STUDY.*

The video may help you with your decisions in the remainder of this experiment.
In each remaining part of this experiment, you will make financial investment decisions. There is a three in four chance that one of these decisions wholly determines your earnings from this experiment.

3. REAL LIFE

The video may help you with your decisions in real life.
This video was made by internationally recognized academic experts on financial decision making (Burton G. Malkiel, Charles D. Ellis, and B. Douglas Bernheim). This video may help you make financial decisions in your life in general.

>>

PLEASE FOLLOW THIS VIDEO CAREFULLY

PLEASE WATCH THE ENTIRE VIDEO

[Some browsers will ask you whether you want to display this content. Please click "**display all content**".]

[There should be a video here. If it does not load, please click here.]

To continue, enter the FOURTH word of the FIRST slide of this video. A continue button will appear after the duration of the video.

>>

PLEASE READ THESE INSTRUCTIONS CAREFULLY

The remainder of this experiment consists of 20 rounds of decision making.

Your payment may be determined entirely by ONE RANDOMLY CHOSEN decision you make in this part of the experiment.

This will happen with a three in four chance. Otherwise, your payment is determined by your performance in the test about the video you just watched.

Hence, you should make every decision as if it is the one that counts, because it might be!

>>

PLEASE READ THESE INSTRUCTIONS CAREFULLY

In each round, you will be presented with two lists. The first list will be like the following:

	you will get the specified dollar amount within two days from today	Option X
$20	○	○
$18	○	○
$16	○	○
$14	○	○
$12	○	○
$10	○	○
$8	○	○
$6	○	○
$4	○	○
$2	○	○
$0	○	○

Option X will vary from round to round. For instance, option X may be "get $15 in 8 weeks".

PLEASE READ THESE INSTRUCTIONS CAREFULLY

Our payment procedure is designed such that it is in your best interest to choose, on each line of each decision list, the option you genuinely prefer.

Here's why: You'll get exactly what you chose, for one randomly drawn decision.

Read this paragraph if you want to know more details.

Question: When will I be paid according to the first decision list, and when will I be paid according to the second decision list in a round?

Answer: Suppose you filled in the *first* decision list of a round as follows:

YOU WILL NOW MAKE YOUR DECISIONS

It is in your best interest to choose as you genuinely prefer. Please think about your choices carefully.

There are no right or wrong choices!

>>

Please choose, on each line, the option you genuinely prefer.

If you pick the option on the LEFT,

you will get the specified dollar amount within two days from today.

If you pick the option on the RIGHT,

we will invest $4.50 in an account with 2% interest per day. Interest is compounded daily. We will pay you the proceeds in 72 days.

You may switch from left to right at most once.

This is the
first
decision list for these options.

	you will get the specified dollar amount within two days from today	we will invest $4.50 in an account with 2% interest per day. Interest is compounded daily. We will pay you the proceeds in 72 days.
$20	○	○
$18	○	○
$16	○	○
$14	○	○
$12	○	○
$10	○	○
$8	○	○
$6	○	○
$4	○	○
$2	○	○
$0	○	○

>>

Please choose, on each line, the option you genuinely prefer.

If you pick the option on the LEFT,

you will get the specified dollar amount within two days from today.

If you pick the option on the RIGHT,

we will invest $4.50 in an account with 2% interest per day. Interest is compounded daily. We will pay you the proceeds in 72 days.

You may switch from left to right at most once.

This is the
second
decision list for these options.

	you will get the specified dollar amount within two days from today	we will invest $4.50 in an account with 2% interest per day. Interest is compounded daily. We will pay you the proceeds in 72 days.
$ 9.8	○	○
$ 9.6	○	○
$ 9.4	○	○
$ 9.2	○	○
$ 9	○	○
$ 8.8	○	○
$ 8.6	○	○
$ 8.4	○	○
$ 8.2	○	○
$ 8	○	○

>>

TEST

You will now participate in a test about the video you have watched at the beginning of the experiment. The test has 10 questions.

There is a one in four chance that your earnings from this study are entirely determined by your performance in this test.

IF you are randomly chosen to be paid according to this test, THEN: For each question you answer correctly, you will earn $1. For each question you answer incorrectly, you will earn $0. You will be paid within at most two days from today.

>>

What is an "indexing" investment strategy?

- Buying index funds, which hold assets that have been indexed as particularly profitable by financial experts
- Buying index funds, which hold stocks of companies that provide information about the stock market as a whole (stock market indices)
- Buying index funds, which hold the market portfolio
- Buying index funds, which hold optimally diversified, custom tailored portfolios

Paul had invested his money into an account which paid 9% interest per year (interest is compounded yearly). After 8 years, he had $500. How big was the investment that Paul had made 8 years ago?

- $200
- $210
- $220
- $230
- $240
- $250
- $260
- $270
- $280
- $290
- $300
- $310
- $320
- $330
- $340
- $350
- $360
- $370
- $380
- $390
- $400

if the interest rate is 10% per year (interest is compounded yearly), how many years does it take until an investment doubles?

- 7 years
- 7.2 years
- 7.4 years
- 7.8 years
- 8 years

If an investment grows at 8 percent per year (interest is compounded yearly), by how much has it grown after 4 years?

- by 30%
- by 31%
- by 32%
- by 33%
- by 34%
- by 35%
- by 36%
- by 37%
- by 38%
- by 39%
- by 40%

Please answer the following questions truthfully. Your answers to these questions DO NOT AFFECT YOUR PAYMENT for this study.

How much attention did you pay to your choices?

- I paid quite a bit of attention for all of my choices.
- For some choices I paid attention, for others I didn't pay much attention
- I clicked through most of the choices without paying much attention.

At the beginning of the experiment, we asked you to watch a video about financial investing. Please indicate which of the following describes your situation best

- I watched the entire video, and paid close attention
- I watched the entire video, but sometimes didn't pay attention
- I skipped parts of the video, because I already knew the material
- I skipped parts of the video, because it was boring (but I did not already know the material)
- I did not watch the video.

Sometimes in this experiment, you were given a choice such as "We will invest $10 in an account with 1% interest per day. Interest is compounded weekly. We will pay you the proceeds in 72 days." When deciding about this choice, did you use the rule of 72?

- Yes
- No
- I don't know the rule of 72

Sometimes in this experiment, you were given a choice such as "We will pay you $20 in 36 days." When deciding about such a choice, did you use the rule of 72?

- Yes
- No
- I don't know the rule of 72

In total, you were given 10 rounds in which one of the options was something like "we will invest $... in an account with ...% interest per week. Interest is compounded weekly. We will pay you the proceeds in … days". Out of these 10 rounds, how many times did you explicitly calculate the money amount that this investment would yield within the specified time?

When you completed the test about the video on financial investing, did you use external resources (such as other websites, books, etc.) to find the right answers?

- Yes
- No

Do you have any suggestions for us about this experiment?

Did you experience any technical difficulties with this study?

>>

F Practice problems with personalized feedback

Education Intervention Part 1: `https://www.youtube.com/watch?v=EnFVLiM1dTs`

Part 1 Practice Question

If you invest $100 at 2% (compounded yearly), how much will be in your account after 36 years?

- ○ $102
- ○ $172
- ○ $200
- ○ $202
- ○ $300
- ○ $302
- ○ $400
- ○ $402

BACK NEXT

If the answer is correct in the first trial:

Great job! Watch the next part of the video to hone your skills even more.

BACK NEXT

If the answer is incorrect in the first trial:

Hmm, that's not quite right.

Please try again. The rule of 72 will help!

If you invest $100 at 2% (compounded yearly), how much will be in your account after 36 years?

- ○ $102
- ○ $172
- ○ $200
- ○ $202
- ○ $300
- ○ $302
- ○ $400
- ○ $402

BACK NEXT

If the answer is correct in the second time:

Nice! You got it this time!

Please watch the next part of the video to hone your skills even more!

BACK NEXT

If the answer is incorrect in the second trial:

Hmm, that's still not quite right.

Watch the next part of the video, so you see how you can get a good idea about how compound interest works.

BACK NEXT

PLEASE FOLLOW THIS VIDEO CAREFULLY

PLEASE WATCH THE ENTIRE VIDEO

The next button will appear automatically when the video ends (after time equal to the duration of the video passes.).

You can stop the video by clicking on it once and make it full screen by clicking on it twice.

If you reload the page, you will again need to wait for the next button. So please do not close your web browser or reload the page unless it is necessary.

[Some browsers will ask you whether you want to display this content. Please click "display all content".]

Education Intervention Part 2: https://youtu.be/3pjkVd0XMlk

Part 2 Practice Questions

Now you try:

$100 is invested at 9% for 32 years, compounded yearly. How much will be in the account after these 32 years?

- ○ $100
- ○ $200
- ○ $388
- ○ $400
- ○ $600
- ○ $800
- ○ $1200
- ○ $1600

BACK NEXT

If the answer is correct in the first trial:

This is correct!

Please click next to move on the next part of the video.

BACK NEXT

If the answer is incorrect in the first trial:

Subjects see one of the following explanations depending on their previous answer and they re-attempt the question.

If the subject selected answer $100:

You selected $100. That's not quite right.

You start out with $100. Then you get 9% interest each year! Hence after 32 years, you will have MORE than $100!

Please watch the video again to understand how much you will have.

BACK NEXT

If the subject selected answer \$200:

You selected \$200. That's not quite right.

You probably remembered from the example above that at 9%, an investment doubles in 8 years.

Thus, the \$100 double to \$200 after 8 years.

These \$200 then double to \$400 in the next 8 years. (That is, until year 16).

In the next 8 years, from year 16 to year 24, these \$400 double to \$800!

Then, in the next 8 years, from year 24 to year 32, it will double again.

Please give it another try.

BACK NEXT

If the subject selected answer $388:

You selected $388. That's not quite right.

You probably got this because you thought you'd get 32 times the interest of 9% on your $100, which is $9.

But, starting from the second year, you also get interest on the interest you earned!

Here's how: You do start out with $100. In the first year, you get 9% interest. That's $9. You start the second year with $109 in your account. Your interest in the second year is 9% of $109, which is MORE than $9. In fact, you'll get 9% of $109, which is $9.80.

Please watch the video again, so you'll understand how compound interest works.

BACK NEXT

If the subject selected answer $400:

You selected $400. That's not quite right.

In this question, the $100 are invested for 32 years, not just for 16 years, as in the example above.

Please give it another try.

BACK NEXT

If the subject selected answer \$600:

You selected \$600. That's not quite right.

You probably remembered from the example above that at 9%, an investment doubles in 8 years.

Thus, the \$100 double to \$200 after 8 years.

These \$200 then double to \$400 in the next 8 years. (That is, until year 16).

In the next 8 years, from year 16 to year 24, these \$400 double to \$800!

Then, in the next 8 years, from year 24 to year 32, it will double again.

Please give it another try.

BACK NEXT

If the subject selected answer $800:

You selected $800. That's not quite right.

You probably remembered from the example above that at 9%, an investment doubles in 8 years.

Thus, the $100 double to $200 after 8 years.

These $200 then double to $400 in the next 8 years. (That is, until year 16).

In the next 8 years, from year 16 to year 24, these $400 double to $800!

Then, in the next 8 years, from year 24 to year 32, it will double again.

Please give it another try.

BACK NEXT

If the subject selected answer \$1200:

You selected \$1200. That's not quite right.

You probably remembered from the example above that at 9%, an investment doubles in 8 years.

Thus, the \$100 double to \$200 after 8 years.

These \$200 then double to \$400 in the next 8 years. (That is, until year 16).

In the next 8 years, from year 16 to year 24, these \$400 double to \$800!

Then, in the next 8 years, from year 24 to year 32, it will double again.

Please give it another try.

BACK NEXT

Re-attempt the question:

Please try again:

$100 is invested at 9% for 32 years, compounded yearly. How much will be in the account after these 32 years?

- ◯ $100
- ◯ $200
- ◯ $388
- ◯ $400
- ◯ $600
- ◯ $800
- ◯ $1200
- ◯ $1600

BACK NEXT

If the answer is correct in the second time:

This is correct!

Please click next to move on the next part of the video.

BACK NEXT

If the answer is incorrect in the second trial:

Hmm, that's still not quite right.

But let's move to the next part of the video.

BACK NEXT

The next button will appear automatically when the video ends (after time equal to the duration of the video passes.).

You can stop the video by clicking on it once and make it full screen by clicking on it twice.

If you reload the page, you will again need to wait for the next button. So please do not close your web browser or reload the page unless it is necessary.

Education Intervention, part 3: `https://youtu.be/kjPYqcZNzPI`

Practice Questions at the end of the Intervention

Thanks for watching this video!

We'll now ask you to solve a bunch of problems on your own. We'll first walk you through in steps, and then it's up to you to find the right steps.

These questions are still a part of the education. They don't count for money, but you need to get them right so you can continue with the survey.

BACK NEXT

Question 1(a)

You invest $50 at 8%. Eventually, we want to know how much will be in your account after 27 years. But we'll get there in three easy steps.

1. How long does it take for the money to double at 8%?
2. How many times does it double in 27 years?
3. Hence, how much will be in the account after it doubles that many times?

So let's start with the first one of these.

How many years does it take for this investment to double?

BACK NEXT

If the answer to part (a) is incorrect in the first trial:

Your answer isn't quite correct. Remember: The rule of 72 says

percentage interest rate X number of years it takes for the investment to double = 72

Please try again: You invest $50 at 8%. How many years does it take for this investment to double?

BACK NEXT

If the answer to part (a) is incorrect in the second trial:

That's still not quite correct.

Here's how you can do it correctly:

The rule of 72 says that

percentage interest rate *times* the number of years it takes for the investment to double = 72

or, in mathematical notation,

$$\mathbf{X \times Y = 72}$$

In this problem, the percentage interest rate is 8%. Hence you just need to know: 8 times *what* equals 72?

That's how long it takes for the investment to double!

Enter your answer below.

BACK

NEXT

If the answer to part (a) is correct in the first trial or later trials:

Great, you've got it!

Question 1(b)

We're still looking at that $50 invested at 8%. As you've figured out, at 8%, the investment doubles in 9 years.

Remember the three steps?

1. How long does it take for the money to double at 8%?
2. How many times does it double in 27 years?
3. Hence, how much will be in the account after it doubles that many times?

We now tackle the second step:

How many times does this investment double over the course of 27 years?

- ○ once
- ○ twice
- ○ three times
- ○ four times
- ○ five times
- ○ six times
- ○ seven times
- ○ eight times
- ○ nine times
- ○ ten times

If the answer to part (b) is incorrect in the first trial:

Unfortunately, that's not quite right.

As you've figured out, the investment doubles in 9 years. It doubles in every 9 years over the course of 27 years!

Hence, 9 times *what* equals 27?

The answer to this question tells you how many times the investment doubles!

Please choose one of the answers below.

- ○ once
- ○ twice
- ○ three times
- ○ four times
- ○ five times
- ○ six times
- ○ seven times
- ○ eight times
- ○ nine times
- ○ ten times

BACK NEXT

If the answer to part (b) is correct in the first trial or later trials:

Nice job!

Now to the last one of the three steps.

1. How long does it take for the money to double at 8%?
2. How many times does it double in 27 years?
3. Hence, how much will be in the account after it doubles that many times?

You figured out that over the course of 27 years, your $50, invested at 8% double three times.

Hence, how much will be in your account after 27 years?

- ○ $50
- ○ $100
- ○ $150
- ○ $200
- ○ $250
- ○ $300
- ○ $350
- ○ $400
- ○ $450
- ○ $500
- ○ $600
- ○ $700
- ○ $800

If the answer to part (c) is incorrect in the first trial:

Oops, that is not quite right.

In the first 9 years, your investment doubles by $50 and is then worth $100. In the second 9 years, these entire $100 double again. So after the second 9 years (that is after 18 years), you have $200.

So, how much will you have after 27 years?

- ○ $50
- ○ $100
- ○ $150
- ○ $200
- ○ $250
- ○ $300
- ○ $350
- ○ $400
- ○ $450
- ○ $500
- ○ $600
- ○ $700
- ○ $800

BACK NEXT

If the answer to part (c) is correct in the first trial or later trials:

Awesome job!

Now it's up to you to go through the steps in the right order.

Let's try this example:

Question 2

You invest $100 at 6%. How much will be in your account after 24 years?

- ○ $100
- ○ $106
- ○ $148
- ○ $200
- ○ $288
- ○ $300
- ○ $306
- ○ $400
- ○ $406
- ○ $500
- ○ $506
- ○ $600
- ○ $606

If the answer to Question 2 is incorrect in the first trial:

Oops, that's not quite right. Remember the three steps for using the rule of 72:

1. How long does it take for the money to double?
2. How many times will it double over the years?
3. Hence, how much will be in the account after it doubles that many times?

Give it another shot:

Question 2

You invest $100 at 6%. How much will be in your account after 24 years?

- ◯ $100
- ◯ $106
- ◯ $148
- ◯ $200
- ◯ $288
- ◯ $300
- ◯ $306
- ◯ $400
- ◯ $406
- ◯ $500
- ◯ $506
- ◯ $600
- ◯ $606

If the answer to Question 2 is correct in the first trial or later trials:

Great! Thanks for paying attention to this education module. Before moving forward, we would like to ask you a question about the education module. Your answer to this question will not affect your payments from this study.

BACK NEXT