Online Appendix

Optimal Public Debt with Life Cycle Motives

William B. Peterman
Federal Reserve Board

Erick Sager
Federal Reserve Board

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A Construction of the Balanced Growth Path

This appendix provides a formal construction of the Balanced Growth Path for the set of economies described in Section 2. We construct the Balanced Growth Path in multiple parts. In Appendix A.1, we construct the Balanced Growth Path for aggregates. In Appendix A.2, we construct the Balanced Growth Path for individual agents' allocations. Finally, in Appendix A.3, we put these elements together to characterize the balanced growth path for the distribution and the aggregation of households.

A.1 Aggregate Conditions

Balanced Growth Path: A Balanced Growth Path (BGP) is a sequence

$$\{C_t, A_t, Y_t, K_t, L_t, B_t, G_t, Tr_t\}_{t=0}^{\infty}$$

such that (i) for all $t = 0, 1, \ldots$ $C_t, A_t, Y_t, K_t, B_t, G_t, Tr_t$ grow at a constant rate g_y,

$$\frac{Y_{t+1}}{Y_t} = \frac{C_{t+1}}{C_t} = \frac{A_{t+1}}{A_t} = \frac{K_{t+1}}{K_t} = \frac{B_{t+1}}{B_t} = \frac{G_{t+1}}{G_t} = \frac{Tr_{t+1}}{Tr_t} = 1 + g_y$$

(ii) per capita variables all grow at the same constant rate g_w:

$$\frac{Y_{t+1}/N_{t+1}}{Y_t/N_t} = \frac{C_{t+1}/N_{t+1}}{C_t/N_t} = \frac{A_{t+1}/N_{t+1}}{A_t/N_t} = \frac{K_{t+1}/N_{t+1}}{K_t/N_t} = \frac{B_{t+1}/N_{t+1}}{B_t/N_t} = \frac{G_{t+1}/N_{t+1}}{G_t/N_t} = \frac{Tr_{t+1}/N_{t+1}}{Tr_t/N_t} = 1 + g_w$$

and (iii) effective labor per capita is constant:

$$\frac{L_{t+1}}{N_{t+1}} = \frac{L_t}{N_t} = \frac{L_0}{N_0}$$

Denote time 0 variables without a time subscript, for example $L \equiv L_0$.

Growth Rates: Let all growth derive from TFP $g_z > 0$ and population $g_n > 0$ growth. Then on a balanced growth path we assume:

$$Z_t = (1 + g_z)^t Z$$

$$N_t = (1 + g_n)^t N$$

where Z and N are steady state values. Then, from part (iii) of the definition, growth in labor is:

$$\frac{L_{t+1}}{L_t} = \frac{L_{t+1}/N_{t+1}}{L_t/((1+g_n)N_t)} = 1 + g_n$$

In steady state $Y = ZK^{\alpha}L^{1-\alpha}$. Let output growth be given by $g_y > 0$. Therefore the production function gives:

$$Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha} \quad \Longrightarrow \quad (1+g_y) = (1+g_z)^{\frac{1}{1-\alpha}}(1+g_n)$$

Lastly, from parts (ii) and (iii) of the Balanced Growth Path definition, we can solve for the growth of per capita variables:

$$\frac{Y_{t+1}/N_{t+1}}{Y_t/N_t} = \frac{Z_{t+1}}{Z_t}\left(\frac{K_{t+1}/N_{t+1}}{K_t/N_t}\right)^{\alpha}\left(\frac{L_{t+1}/N_{t+1}}{L_t/N_t}\right)^{1-\alpha} \Longrightarrow (1+g_w) = (1+g_z)^{\frac{1}{1-\alpha}}$$

Prices: From Euler's theorem we know:

$$Y_t = \alpha Y_t + (1-\alpha)Y_t = (r_t + \delta)K_t + w_t L_t$$

Accordingly, the wage and interest rate depend on the capital-labor ratio. Growth in the capital-labor ratio is:

$$\frac{K_{t+1}/L_{t+1}}{K_t/L_t} = (1+g_z)^{\frac{1}{1-\alpha}} = 1 + g_w$$

Therefore, the growth rate for the wage is:

$$\frac{w_{t+1}}{w_t} = \frac{Z_{t+1}}{Z_t} \cdot \left(\frac{K_{t+1}/L_{t+1}}{K_t/L_t}\right)^{\alpha} = 1 + g_w$$

and the growth rate for the interest rate is:

$$\frac{r_{t+1}+\delta}{r_t+\delta} = \frac{Z_{t+1}}{Z_t} \cdot \left(\frac{K_{t+1}/L_{t+1}}{K_t/L_t}\right)^{\alpha-1} = 1$$

Therefore wages grow while interest rates do not.

Equilibrium Conditions: The detrended *asset market clearing condition* is:

$$K_t = A_t + B_t \quad \Longrightarrow \quad K = A + B$$

The detrended *resource constraint* is:

$$C_t + K_{t+1} + G_t + Tr_t = Y_t + (1-\delta)K_t \quad \Longrightarrow \quad C + (g_y + \delta)K + G + Tr = Y$$

and the detrended *government budget constraint* is:

$$G_t + Tr_t + B_{t+1} - B_t = R_t + rB_t \quad \Longrightarrow \quad G + Tr = R + (r - g_y)B$$

A.2 Individual Conditions

Preferences: We assume that labor disutility and utility over bequests have a time-dependent component. Specifically, we assume labor disutility grows at the same rate as the utility over consumption, such that $v_{t+1}(h,d) = (1+g_w)^{1-\sigma} v_t(h,d)$. Therefore, total utility over consumption, hours and retirement status is:

$$u(c_t) - v_t(h_t, d_t) = \left[(1+g_w)^{1-\sigma}\right]^t (u(c) - v(h,d)) \, .$$

Similarly, assume that the non-homothetic component of the utility over bequests grows at the same rate as the utility over consumption, such that

$$\phi(a_{t+1}) = \left[(1+g_w)^{1-\sigma}\right]^{t+1} \phi(a') \; .$$

Bequests: Because bequest inflows $b_j(\kappa)$ are generated from the households' savings from the previous period, bequests grow at the same rate as savings.

Social Security: In order for the AIME to grow at the same rate as the wage, we assume a cost of living adjustment (COLA) on Social Security taxes and payments. For social security taxes, the cap on eligible income grows at the rate of wage growth, $\bar{m}_t = (1+g_w)^t \bar{m}$. Furthermore, base payment bend points $b^{ss}_{i,t} = (1+g_w)^t b^{ss}_i$ and base payment values $\tau_{r,i,t} = (1+g_w)^t \tau_{r,i}$ for $i = 1,2,3$.

Medical Expenses and Policy: We assume that medical expenses are indexed to wage growth and so $\mu_t = (1+g_w)^t \mu$ at all ages. We also assume that the consumption floor achieved by medical transfers, $\underline{c}_t$, also receives a COLA so that it grows at the same rate as the wage and therefore $\underline{c}_t = (1+g_w)^t \underline{c}$. By construction, this implies that individuals' medical transfers also grow at the same rate as the wage, $Tr_t = (1+g_w)^t Tr$.

Tax Function: On a Balanced Growth Path, (c_t, a'_{t+1}, a_t) and $\tilde{y}_t$ must all grow at the same rate as the wage. Furthermore, the tax function must grow at the same rate as the wage. Recalling the tax function, $\Upsilon_t(\tilde{y}_t)$, τ_2 must grow at the same rate as $\tilde{y}_t^{-\tau_1}$. Rewrite as:

$$\Upsilon_t(\tilde{y}_t) = \tau_0 \left((1+g_w)^t \tilde{y} - \left([(1+g_w)^t]^{-\tau_1} \tilde{y}^{-\tau_1} + [(1+g_w)^t]^{-\tau_1} \tau_2 \right)^{-\frac{1}{\tau_1}} \right) = (1+g_w)^t \, \Upsilon(\tilde{y})$$

Individual Budget Constraint: Let the function $T(h, a, \varepsilon, m, d)$ contain income taxes, social security taxes or payments, medical expenses or transfers, and bequest inflows that a household faces. A household's time t budget constraint is:

$$c_t + a'_{t+1} \leq w_t \varepsilon_t h_t + (1+r_t) a_t - T_t(h_t, a_t, \varepsilon_t, m_t, d_t)$$

$$c + (1+g_w) a' \leq w \varepsilon h + (1+r) a - T(h, a, \varepsilon, m, d)$$

where $\{c, a', a, h, w, r, \varepsilon\}$ are stationary variables. Given that the tax function $\Upsilon(\tilde{y})$ grows at rate g_w, so will the transfer function $T(h, a, \varepsilon)$ in the infinitely lived agent model. Furthermore, given that the Social Security program $\{\bar{m}, b_i^{ss}, \tau_{r,i}\}$, medical system $\{\underline{c}, Tr, \mu\}$ and bequest inflows $\{b(\kappa)\}$ grow at rate g_w, so will the transfer $T(h, a, \varepsilon, m, d)$ function in the life cycle model.

A.3 Aggregation

Distributions: For j-th cohort at time t, the measure over $(a, \varepsilon, m, d_{-1})$ is given by:

$$\begin{aligned}
\lambda_{j,t}(a_t, \varepsilon, m_t, d_{-1}) &= \lambda_{j,t-1}\left(\frac{a_t}{1+g_w}, \varepsilon, \frac{m_t}{1+g_w}, d_{-1} \right)(1+g_n) \\
&= \lambda_{j,t-i}\left(\frac{a_t}{(1+g_w)^i}, \varepsilon, \frac{m_t}{(1+g_w)^i}, d_{-1} \right)(1+g_n)^i \quad \forall\, i \leq t \\
&= \lambda_j(a, \varepsilon, m, d_{-1}) N_{t-j+1}.
\end{aligned}$$

Therefore, $\lambda_j(a, \varepsilon, m, d_{-1})$ is a stationary distribution over age j households that integrates to one.

Aggregation: Aggregate consumption in the life cycle model is constructed as follows.

Define the relative size of cohorts as $\omega_1 = 1$ and:

$$\omega_{j+1} = \frac{N_{t-j}}{N_t} \cdot \prod_{i=1}^{j} \psi_i = (1+g_n)^{-j} \prod_{i=1}^{j} \psi_i = \frac{\psi_j \omega_j}{1+g_n} \quad \forall\, j = 1, \ldots, J-1$$

Let $C_{j,t}$ be aggregate consumption per age-j household, which is derived from the age-j household's allocation:

$$C_{j,t} = \int (1+g_w)^t c_j(a, \varepsilon, m, d_{-1}) \mathbf{d}\lambda_j = (1+g_w)^t \int c_j(a, \varepsilon, m, d_{-1}) \mathbf{d}\lambda_j = (1+g_w)^t C_j$$

where C_j is the stationary aggregate consumption per age-j household. Accordingly, aggregate consumption is:

$$\begin{aligned} C_t &= N_t \Big(C_{1,t} + \psi_1 (1+g_n)^{-1} C_{2,t} + \cdots + \left(\prod_{i=1}^{J-1} \psi_i \right) (1+g_n)^{-(J-1)} C_{J,t} \Big) \\ &= (1+g_w)^t N_t \sum_{j=1}^{J} \omega_j C_j \\ &= (1+g_y)^t C \end{aligned}$$

where C is the stationary level of aggregate consumption and where we have normalized $N = 1$.

We can similarly construct the remaining aggregates $\{A, K, Y, B, G\}$ on the balanced growth path. Notably, however, labor per capita does not grow. Aggregate labor per capita is constructed as:

$$L_t = N_t \sum_{j=1}^{J} \omega_j L_j \quad \Longrightarrow \quad L = \frac{L_t}{N_t} = \sum_{j=1}^{J} \omega_j \int d_j(a, \varepsilon, m, d_{-1}) \varepsilon h_j(a, \varepsilon, m, d_{-1}) \mathbf{d}\lambda_j$$

which is the sum over ages of aggregate labor per age-j household.

B Infinitely Lived Agent Model

In this section, we describe the economic environments for the Infinitely Lived Agent Model from Section 4.3. We then detail any additional calibration procedures the infinitely lived agent model requires relative to the Life Cycle model. In general, we keep as many parameters in common as possible and, where that is not possible, allow parameters to vary across the models while matching the same moments.

Model Overview: The infinitely lived agent model differs from the life cycle model in three ways. First, agents in the infinitely lived agent model lifetimes are infinite ($J \to \infty$) and, therefore, agents neither experience mortality risk ($\psi_j = 0$ for all $j \geq 1$) nor give bequests ($b_j(\kappa) = 0$ for all κ and $j \geq 1$). Second, labor productivity no longer has an age-dependent component ($\theta_j = \bar{\theta}$ for all $j \geq 1$). Lastly, there is no retirement ($\underline{J}_{ret} \to \infty$ such that $d_j = 1$ for all $j \geq 1$), there is no Social Security program ($\tau_{ss} = 0$ and $b_{ss}(m) = 0$ for all x), and there are no medical expenditures ($\mu_j(\kappa) = 0$ for all κ and $j \geq 1$).

Accordingly, we study a stationary recursive competitive equilibrium along a balanced growth path in which the initial endowment of wealth and labor productivity shocks no longer affects individual decisions and the distribution over wealth and labor productivity is time invariant.

Balanced Growth Path: In order to isolate the effects on optimal policy due to fundamental differences in the life cycle and infinitely lived agent models, and not due to differences in balanced growth path constructs, we want sources of output growth (e.g. TFP and population growth) to be consistent across models. Thus, we incorporate population growth into the infinitely lived agent model. To be consistent with the life cycle model, we construct a balanced growth path in which the infinitely lived agent model's income and wealth distributions grow homothetically. Our representation of this growth concept is consistent with a dynastic model in which population growth arises from agents producing offspring and valuing the utility of their offspring.

To elaborate in more detail, two additional assumptions admit a balanced growth path with population growth. First, agents exogenously reproduce at rate g_n and next period's offspring are identical to each other. Second, the parent values each offspring identically, and furthermore values each offspring as much as they value their self. Formally, if the parent has continuation value $\beta\mathbb{E}[v(a', \varepsilon')]$, then the parent values all its offspring with total value of $g_n\beta\mathbb{E}[v(a', \varepsilon')]$.

These two assumptions imply two features. First, each offspring is identical to its parent. That is, if the parent's state vector is (a', ε') next period, then so is each offspring's state vector. As a result, the value function of each offspring upon birth is $v(a', \varepsilon')$. Second, since the parent values each offspring equal to its own continuation value, it is optimal for the parent to save save $(1 + g_n)a'$ in total. The portion $g_n a'$ is transfered to offspring, and the portion a' is kept for next period.

On the balanced growth path of the Infinitely Lived Agent Model, the stationary dynamic program is:

$$v(a,\varepsilon) = \max_{c,a',h} [u(c) - v(h)] + [\beta(1+g_w)^{1-\sigma}](1+g_n)\sum_{\varepsilon'} \pi(\varepsilon'|\varepsilon)v(a',\varepsilon')$$

$$\text{s.t.} \qquad c + (1+g_n)(1+g_w)a' \leq we(\varepsilon)h + (1+r)a - \Upsilon(y(h,a,\varepsilon))$$

where $a' \geq \underline{a}$ and $y(h,a,\varepsilon) \equiv we(\varepsilon)h + ra$. The distribution evolves according to:

$$\lambda_{t+1}(a_{t+1},\varepsilon_{t+1}) = \sum_{\varepsilon_t} \pi(\varepsilon_{t+1}|\varepsilon_t) \int_A \mathbb{1}\left[a'_{t+1}(a_t,\varepsilon_t) = a_{t+1}\right] \lambda_t(a_t,\varepsilon_t)da_t$$

The stationary distribution $\lambda(a,\varepsilon)$ has measure 1 over $\mathcal{A} \times \mathcal{E}$ but the mass of agents grows at rate g_n:

$$\begin{aligned}
\lambda_t(a_t,\varepsilon) &= \lambda_{t-1}\left(\frac{a_t}{1+g_w},\varepsilon\right)(1+g_n) \\
&= \lambda_{t-s}\left(\frac{a_t}{(1+g_w)^s},\varepsilon\right)(1+g_n)^s \quad \forall\, s \leq t \\
&= \lambda(a,\varepsilon)N_t
\end{aligned}$$

To construct aggregate consumption, wealth, savings and labor, multiply individual allocations by the size of the population (N_t) and sum using the stationary distribution λ. For example, aggregate consumption is:

$$C_t = N_t \int (1+g_w)^t c(a,\varepsilon)d\lambda = (1+g)^t \int c(a,\varepsilon)d\lambda = (1+g)^t C$$

We can similarly construct the remaining aggregates $\{A,K,Y,B,G\}$ on the balanced growth path. Notably, however, aggregate labor per capita does not grow:

$$\frac{L_t}{N_t} = \int \varepsilon h(a,\varepsilon)d\lambda$$

where again $N_0 = 1$ by normalization.

Definition (Equilibrium): Given a government policy (G,B,B',R), a *stationary recursive competitive equilibrium* is (i) an allocation for consumers described by policy functions (c,a',h) and consumer value function V, (ii) an allocation for the representative firm (K,L), (iii) prices (w,r), and (v) a distribution over agents' state vector λ that satisfy:

(a) Given prices and policies, $V(a,\varepsilon)$ solves the following Bellman equation:

$$V(a,\varepsilon) = \max_{c,a',h} \ \left[u(c) - v(h)\right] + \beta(1+g_w)^{1-\sigma}(1+g_n)\sum_{\varepsilon'}\pi(\varepsilon'|\varepsilon)V(a',\varepsilon') \tag{B1}$$

$$\begin{aligned} \text{s.t.} \quad c + (1+g_w)(1+g_n)a' &\leq we(\varepsilon)h + (1+r)a - \Upsilon(y(h,a,\varepsilon)) \\ a' &\geq \underline{a} \end{aligned}$$

with associated policy functions $c(a,\varepsilon)$, $a'(a,\varepsilon)$ and $h(a,\varepsilon)$.

(b) Given prices (w,r), the representative firm's allocation minimizes cost.

(c) Government policies satisfy budget balance:

$$G + (1+g_y)B' - B = rB + R$$

where aggregate income tax revenue is given by:

$$R \equiv \int \Upsilon\big(y(h(a,\varepsilon),a,\varepsilon)\big)\ \mathbf{d}\lambda(a,\varepsilon) \tag{B2}$$

(d) Given policies and allocations, prices clear asset and labor markets:

$$K - B = \int a\ \mathbf{d}\lambda(a,\varepsilon) \tag{B3}$$

$$L = \int e(\varepsilon)h(a,\varepsilon)\ \mathbf{d}\lambda(a,\varepsilon) \tag{B4}$$

and the allocation satisfies the resource constraint (guaranteed by Walras' Law):

$$\int c(a,\varepsilon)\mathbf{d}\lambda(a,\varepsilon) + (1+g_y)K' + G = ZF(K,L) + (1-\delta)K \tag{B5}$$

(e) Given consumer policy functions, the distribution over wealth and productivity shocks is given recursively from the law of motion $T^* : \mathbf{M} \to \mathbf{M}$ such that T^* is given by:

$$\lambda'(\mathcal{A}\times\mathcal{E}) = \int_{A\times E} Q_j\left((a,\varepsilon),\mathcal{A}\times\mathcal{E}\right)\mathbf{d}\lambda$$

where $\mathcal{S} \equiv \mathcal{A}\times\mathcal{E} \subset \mathbf{S}$, and $Q : \mathbf{S}\times\mathcal{B}(\mathbf{S}) \to [0,1]$ is a transition function on $(\mathbf{S},\mathcal{B}(\mathbf{S}))$ that gives the probability that an agent with current state $\mathbf{s} \equiv (a,\varepsilon)$

Table A1: Target Moments, Data and Infinitely Lived Agent Model

Moments	**Data**	**Model**
Capital-to-Output Ratio	2.900	2.903
Hours Worked	0.205	0.206
Top 40% Wealth Share	0.946	0.946
Borrowers' Debt-to-Wealth	-0.048	-0.048
Top 20% Labor Income Share	0.635	0.636

transits to the set $\mathcal{S} \subset \mathbf{S}$ in the next period. The transition function is given by:

$$Q\left((a, \varepsilon), \mathcal{S}\right) = \begin{Bmatrix} \pi(\mathcal{E}|\varepsilon) & \text{if } a'(\mathbf{s}) \in \mathcal{A}, \\ 0 & \text{otherwise} \end{Bmatrix}$$

(f) Aggregate capital, governmental debt, prices and the distribution over consumers are stationary, such that $K' = K$, $B' = B$, $w' = w$, $r' = r$, and $\lambda' = \lambda$.

Calibration: The infinitely lived agent model does not have an age-dependent wage profile. For comparability across models, we replace the age-dependent wage profile with the population-weighted average of θ_j's, such that $\bar{\theta} = \sum_{j=1}^{J_{ret}} (\omega_j / \sum_{j=1}^{J_{ret}} \omega_j)\theta_j \approx 2.47$.[36]

With respect to preferences, in the absence of a retirement decision or a bequest motive, we set $\chi_2 = \chi_a = \chi_b = 0$. Furthermore, the infinitely lived agent model does not have age-variation in household size.

For labor productivity shocks, we set the probability of realizing the superstar shock at 1.8%, the persistence of the superstar shock at 95% and the value of the shock at 12.55. This calibration implies, that for both the life cycle and infinitely lived agent models, the value of the superstar shock so that the bottom 60% of the population holds 5.4% of total wealth or, equivalently, the top 40% of the population holds 94.6% of total wealth.

[36]When calibrating the stochastic process for idiosyncratic productivity shocks, we use the same process in the both the life cycle and infinitely lived agent models. Using the same underlying process will imply that cross-sectional wealth inequality will be different across the two models. One reason is that the life cycle model will have additional cross-sectional inequality due to the humped shaped savings profiles, which induces the accumulation, stationary, and deaccumulation phases. We view these difference in inequality as a fundamental difference between the two models and, therefore, choose not to specially alter the infinitely lived agent model to match a higher level of cross-sectional inequality.

Table A2: Infinitely Lived Agent Model, Aggregates and Prices

	Baseline	**Counterfactual**	**% Difference**
Public Savings/Output	**-0.67**	**1.68**	**235%**
Interest Rate	4.1%	1.1%	-2.9%
Wage	1.17	1.36	16.3%
Income Tax, τ_0	25.8%	29.7%	3.9%
Consumption	1.86	1.84	-1.1%
Hours	0.206	0.215	4.5%
Private Savings	12.1	8.7	-28.2%
Pre-Tax Total Income	2.66	2.67	0.3%
Output	3.38	4.03	19.2%
Productive Capital	9.81	15.45	57.4%
Labor	1.86	1.89	1.9%
Standard Deviations			
PV Asset Income	8.0	1.8	-77.6%
PV Labor Income	25.2	30.0	18.9%
PV Total Income	31.8	31.6	-0.8%

Lastly, we recalibrate the parameters (β, χ_1) to the same targets as in the life cycle model and choose τ_2 to balance the government's budget, obtaining $\beta = 0.929$, $\chi_1 = 62.1$ and $\tau_2 = 2.2$. Since the infinitely lived agent model does not contain a retirement decision, we use average total hours worked instead of calibrating the model to average hours worked before the normal retirement age. Remaining parameters are held constant across the two models.[37]

Table A1 shows that the infinitely lived agent model fits the set of targeted moments closely. Table A2 provides the model's baseline and optimal equilibrium outcomes.

[37] We choose total factor productivity to be $Z = 1$, which implies that the baseline level of the wage is equal across the infinitely lived agent model and the life cycle model. Alternatively, we could have chosen Z to equalize output across models but allow for a different wage. We found that under this alternative calibration that the optimal public savings policy was very similar, 27 percent of output instead of 36 percent of output.

C Welfare Decomposition

This appendix constructs the welfare decomposition in Section 4.1 of the main text:

$$(1+\Delta_{CEV}) = (1+\Delta_{level})(1+\Delta_{age})(1+\Delta_{distr}).$$

To construct these three components (the levels effect, the age effect and the distribution effect), we must construct a composite of consumption, hours, retirement and bequest effects, as follows:

$$\begin{aligned}
(1+\Delta_{level}) &\equiv (1+\Delta_{C_{level}}) \cdot (1+\Delta_{H_{level}}) \cdot (1+\Delta_{R_{level}}) \cdot (1+\Delta_{B_{level}}) \\
(1+\Delta_{age}) &\equiv (1+\Delta_{C_{age}}) \cdot (1+\Delta_{H_{age}}) \cdot (1+\Delta_{R_{age}}) \cdot (1+\Delta_{B_{age}}) \\
(1+\Delta_{distr}) &\equiv (1+\Delta_{C_{distr}}) \cdot (1+\Delta_{H_{distr}}) \cdot (1+\Delta_{R_{distr}}) \cdot (1+\Delta_{B_{distr}})
\end{aligned}$$

These terms are explicitly defined in equation (C1) through equation (C13) below.

In the remainder of the section, first we define notation and construct the CEV in the context of the model in Appendix C.1. Then, in order to construct the level, age and distribution effects, we define and decompose the consumption, hours and bequest welfare effects in Appendix C.2. Finally, in Appendix C.3, we formally describe the partial equilibrium decomposition of the CEVs.

C.1 Preliminaries

Social Welfare Function: Consider two economies, $i \in \{1,2\}$. Define ex ante welfare in economy $i \in \{1,2\}$ derived from consumption, hours, retirement and bequest allocations $\{c_j^i(\mathbf{s}), h_j^i(\mathbf{s}), d_j^i(\mathbf{s}), a_{j+1}^i(\mathbf{s})\}_{j=1}^J$ over states $\mathbf{s} \equiv (a, \varepsilon, m, d_{-1})$ distributed with $\lambda_j^i(\mathbf{s})$ as:

$$S^i = U(c^i) - V(h^i, d^i) + V^b(a^i)$$

where for notational compactness we define,

$$V(h^i, d^i) \equiv V^h(h^i) + V^d(d^i)$$

and where the ex ante utility over allocations is given by,

$$U(c^i) \equiv \int \mathbb{E}_0 \left[\sum_{j=1}^{J} \beta^{j-1} \left(\prod_{i=1}^{j-1} \psi_i \right) u\left(c_j^i\right) \right] \mathbf{d}\lambda_1^i$$

$$V^h(h^i) \equiv \int \mathbb{E}_0 \left[\sum_{j=1}^{J} \beta^{j-1} \left(\prod_{i=1}^{j-1} \psi_i \right) v\left(h_j^i\right) \right] \mathbf{d}\lambda_1^i$$

$$V^d(d^i) \equiv \int \mathbb{E}_0 \left[\sum_{j=1}^{J} \beta^{j-1} \left(\prod_{i=1}^{j-1} \psi_i \right) \; \chi_2 \; d_j^i \right] \mathbf{d}\lambda_1^i$$

$$V^b(a^i) \equiv \int \mathbb{E}_0 \left[\sum_{j=1}^{J} \beta^{j-1} \left(\prod_{i=1}^{j-1} \psi_i \right) \beta(1-\psi_j) \; \phi\left(a_{j+1}^i\right) \right] \mathbf{d}\lambda_1^i.$$

Aggregations: For consumption, hours, retirement decisions and bequests (assets) we define aggregates for the total population and for each age-cohort. Aggregate consumption in economy i is computed as

$$C^i = \sum_{j=1}^{J} \omega_j \int_{\mathbf{S}} c_j^i(\mathbf{s}) \mathbf{d}\lambda_j^i(\mathbf{s}),$$

while the total consumption for agents of age j that inhabit economy i is computed as

$$C_j^i = \int_{\mathbf{S}} c_j^i(\mathbf{s}) \mathbf{d}\lambda_j^i(\mathbf{s}).$$

We similarly define aggregate hours, retirement and asset bequests, (H^i, I^i, A^i), as well as total hours, retirement and asset bequests by age, $\{H_j^i, I_j^i, A_{j+1}^i\}_{j=1}^J$.

Furthermore, we compute the ex ante utility of consumption from an allocation that consists of consuming C^i in each period of life,

$$U(C^i) \equiv \int \mathbb{E}_0 \left[\sum_{j=1}^{J} \beta^{j-1} \left(\prod_{i=1}^{j-1} \psi_i \right) u\left(C^i\right) \right] \mathbf{d}\lambda_1^i.$$

and we compute the ex ante utility of consumption from an allocation in economy i that consists of consuming the age-cohort aggregates at each age, $\{C_j^i\}_{j=1}^J$,

$$U(C_j^i) \equiv \int \mathbb{E}_0 \left[\sum_{j=1}^{J} \beta^{j-1} \left(\prod_{i=1}^{j-1} \psi_i \right) u\left(C_j^i\right) \right] \mathbf{d}\lambda_1^i.$$

We similarly compute the utility from consuming aggregates of hours, retirement decisions and asset bequests, denoted by $V^h(H^i)$, $V^d(I^i)$ and $V^b(A^i)$ respectively. Finally, we similarly compute the utility from consuming aggregates of hours, retirement decisions and asset bequests *by age*, denoted by $V^h(H^i_j)$, $V^d(I^i_j)$ and $V^b(A^i_{j+1})$ respectively.

Consumption Equivalent Variation: Denote the Consumption Equivalent Variation (CEV) by Δ_{CEV}, which is defined as the percent of expected lifetime consumption that an agent inhabiting economy $i = 1$ would pay *ex ante* in order to inhabit economy $i = 2$:

$$S((1+\Delta_{CEV})c^1, h^1, d^1, a^1) = S(c^2, h^2, d^2, a^2),$$

which, according to the preferences laid out in Appendix C.1, can be expressed as,

$$(1+\Delta_{CEV})^{1-\sigma}U(c^1) - V(h^1, d^1) + V^b(a^1) = U(c^2) - V(h^2, d^2) + V^b(a^2) \tag{C1}$$

or rewritten,

$$1+\Delta_{CEV} = \left[\frac{U(c^2)}{U(c^1)} - \left(\frac{V^h(h^2)}{V^h(h^1)} - 1\right)\frac{V^h(h^1)}{U(c^1)} - \left(\frac{V^d(d^2)}{V^d(d^1)} - 1\right)\frac{V^d(d^1)}{U(c^1)} + \left(\frac{V^b(a^2)}{V^b(a^1)} - 1\right)\frac{V^b(a^1)}{U(c^1)}\right]^{\frac{1}{1-\sigma}}.$$

C.2 CEV Decomposition

In order to decompose the CEV into a level effect, age and distribution effect we (i) construct the allocation $(\hat{c}^1, \hat{h}^1, \hat{d}^1, \hat{a}^1)$ as a perturbation of the allocation (c^1, h^1, d^1, a^1) such that $\hat{c}^1 = \delta_C c^1$, $\hat{h}^1 = \delta_H h^1$,$\hat{d}^1 = \delta_R d^1$ and $\hat{a}^1 = \delta_B a^1$, and (ii) construct the allocation $(\tilde{c}^1, \tilde{h}^1, \tilde{d}^1, \tilde{a}^1)$ as a perturbation of the allocation (c^1, h^1, d^1, a^1) such that $\tilde{c}^1 = \delta_c c^1$, $\tilde{h}^1 = \delta_h h^1$, $\tilde{d}^1 = \delta_r d^1$ and $\tilde{a}^1 = \delta_b a^1$.

Perturbations: Next, we define these perturbations.

Define the aggregate consumption level under allocation i as C^i, aggregate hours under allocation i as H^i, aggregate retirement under allocation i as I^i, and aggregate savings under allocation i as A^i. The scalars $(\delta_C, \delta_H, \delta_R, \delta_B)$ are chosen as equivalent variations of aggregate consumption, aggregate hours and aggregate wealth that satisfy

$$S(\delta_C C^1, H^1, I^1, A^1) = S(C^2, H^1, I^1, A^1)$$

$$S(C^2, \delta_H H^1, I^1, A^1) = S(C^2, H^2, I^1, A^1)$$

$$S(C^2, H^1, \delta_R I^1, A^1) = S(C^2, H^2, I^2, A^1)$$

$$S(C^2, H^2, I^2, \delta_B A^1) = S(C^2, H^2, I^2, A^2)$$

Similarly, define the aggregate consumption level of an age-j cohort under allocation i as C_j^i, aggregate hours of an age-j cohort under allocation i as H_j^i, aggregate retirement of an age-j cohort under allocation i as I_j^i, and aggregate savings of an age-j cohort under allocation i as A_{j+1}^i. The scalars $(\delta_c, \delta_h, \delta_r, \delta_b)$ are chosen as equivalent variations of life cycle consumption, hours and wealth that satisfy

$$S(\delta_c C_j^1, H_j^1, I_j^1, A_{j+1}^1) = S(C_j^2, H_j^1, I_j^1, A_{j+1}^1)$$

$$S(C_j^2, \delta_h H_j^1, I_j^1, A_{j+1}^1) = S(C_j^2, H_j^2, I_j^1, A_{j+1}^1)$$

$$S(C_j^2, H_j^1, \delta_r I_j^1, A_{j+1}^1) = S(C_j^2, H_j^2, I_j^2, A_{j+1}^1)$$

$$S(C_j^2, H_j^2, I_j^2, \delta_b A_{j+1}^1) = S(C_j^2, H_j^2, I_j^2, A_{j+1}^2)\ .$$

We use the properties of the social welfare function to explicitly derive expressions for these perturbations in Appendix C.2.1.

Welfare Decomposition: Given these constructed allocations $(\hat{c}_j, \hat{h}_j, \hat{d}_j, \hat{a}_j)$ and $(\tilde{c}_j, \tilde{h}_j, \tilde{d}_j, \tilde{a}_j)$ we can decompose the CEVs according to the change in consumption, hours, retirement and bequests into level, age and hours effects. Recall that these CEVs are used to construct the CEV decomposition into overall level, overall age and overall distribution effects.

Decompose the consumption CEV into the level effect, age effect and distribution effect according to the following conditions,

$$S((1+\Delta_{C_{level}})c^1, h^1, d^1, a^1) = S(\hat{c}^1, h^1, d^1, a^1)$$

$$S((1+\Delta_{C_{age}})\hat{c}^1, h^1, d^1, a^1) = S(\tilde{c}^1, h^1, d^1, a^1)$$

$$S((1+\Delta_{C_{distr}})\tilde{c}^1, h^1, d^1, a^1) = S(c^2, h^1, d^1, a^1)$$

decompose the hours decomposition according to,

$$S((1+\Delta_{H_{level}})c^2, h^1, d^1, a^1) = S(c^2, \hat{h}^1, d^1, a^1)$$

$$S((1+\Delta_{H_{age}})c^2, \hat{h}^1, d^1, a^1) = S(c^2, \tilde{h}^1, d^1, a^1)$$

$$S((1+\Delta_{H_{distr}})c^2, \tilde{h}^1, d^1, a^1) = S(c^2, h^2, d^2, a^1)$$

decompose the retirement decomposition according to,

$$S((1+\Delta_{R_{level}})c^2, h^1, d^1, a^1) = S(c^2, h^1, \hat{d}^1, a^1)$$
$$S((1+\Delta_{R_{age}})c^2, h^1, \hat{d}^1, a^1) = S(c^2, h^1, \tilde{d}^1, a^1)$$
$$S((1+\Delta_{R_{distr}})c^2, h^1, \tilde{d}^1, a^1) = S(c^2, h^2, d^2, a^1)$$

and the bequest decomposition according to,

$$S((1+\Delta_{B_{level}})c^2, h^2, d^2, a^1) = S(c^2, h^2, d^2, \hat{a}^1)$$
$$S((1+\Delta_{B_{age}})c^2, h^2, d^2, \hat{a}^1) = S(c^2, h^2, d^2, \tilde{a}^1)$$
$$S((1+\Delta_{B_{distr}})c^2, h^2, d^2, \tilde{a}^1) = S(c^2, h^2, d^2, a^2)$$

Given the level, age and distribution effects for the consumption, hours, retirement and bequest allocations, we construct the overall level, age and distribution effects as follows:

$$\begin{aligned}
(1+\Delta_{level}) &\equiv (1+\Delta_{C_{level}}) \cdot (1+\Delta_{H_{level}}) \cdot (1+\Delta_{R_{level}}) \cdot (1+\Delta_{B_{level}}) \\
(1+\Delta_{age}) &\equiv (1+\Delta_{C_{age}}) \cdot (1+\Delta_{H_{age}}) \cdot (1+\Delta_{R_{age}}) \cdot (1+\Delta_{B_{age}}) \\
(1+\Delta_{distr}) &\equiv (1+\Delta_{C_{distr}}) \cdot (1+\Delta_{H_{distr}}) \cdot (1+\Delta_{R_{distr}}) \cdot (1+\Delta_{B_{distr}})
\end{aligned}$$

C.2.1 Derivations

Finally, we explicitly derive the functional forms for the perturbations and the level, age and distribution effects of the consumption, hours, retirement and bequest CEVs.

Consumption Decomposition: The allocation $\hat{c}^1$ is as a CEV for the aggregate consumption allocations using the perturbation $\hat{c}^1 = \delta_C c^1$ such that δ_C is defined as,

$$S(\delta_C C^1, H^1, I^1, A^1) = S(C^2, H^1, I^1, A^1)$$
$$U(\delta_C C^1) = U(C^2)$$
$$\delta_C = \left(\frac{U(C^2)}{U(C^1)}\right)^{\frac{1}{1-\sigma}}$$

and for $\tilde{c}^1$ is as a CEV for the cohort aggregate consumption allocations using the perturbation $\tilde{c}^1 = \delta_c c^1$ such that δ_c is defined as,

$$S(\delta_c C_j^1, H^1, I^1, A^1) = S(C_j^2, H^1, I^1, A^1)$$

$$U(\delta_c C_j^1) = U(C_j^2)$$

$$\delta_c = \left(\frac{U(C_j^2)}{U(C_j^1)}\right)^{\frac{1}{1-\sigma}}$$

Put together, this gives consumption CEVs of

$$1 + \Delta_{C_{level}} = \left(\frac{\delta_C^{1-\sigma} U(c^1)}{U(c^1)}\right)^{\frac{1}{1-\sigma}} = \left(\frac{U(C^2)}{U(C^1)}\right)^{\frac{1}{1-\sigma}} \tag{C2}$$

$$1 + \Delta_{C_{age}} = \left(\frac{\delta_c^{1-\sigma} U(c^1)}{\delta_C^{1-\sigma} U(c^1)}\right)^{\frac{1}{1-\sigma}} = \frac{[U(C_j^2)/U(C_j^1)]^{\frac{1}{1-\sigma}}}{[U(C^2)/U(C^1)]^{\frac{1}{1-\sigma}}} \tag{C3}$$

$$1 + \Delta_{C_{distr}} = \left(\frac{U(c^2)}{\delta_c^{1-\sigma} U(c^1)}\right)^{\frac{1}{1-\sigma}} = \frac{[U(c^2)/U(c^1)]^{\frac{1}{1-\sigma}}}{[U(C_j^2)/U(C_j^1)]^{\frac{1}{1-\sigma}}} \tag{C4}$$

Accordingly, the consumption effect decomposition is verified as follows,

$$\begin{aligned}
(1 + \Delta_C) &= (1 + \Delta_{C_{level}}) \quad \cdot \quad (1 + \Delta_{C_{age}}) \quad \cdot \quad (1 + \Delta_{C_{distr}}) \\
&= (C^2/C^1) \quad \cdot \quad \frac{(U(C_j^2)/U(C_j^1))^{\frac{1}{1-\sigma}}}{C^2/C^1} \quad \cdot \quad \frac{(U(c^2)/U(c^1))^{\frac{1}{1-\sigma}}}{(U(C_j^2)/U(C_j^1))^{\frac{1}{1-\sigma}}} \\
&\overset{\checkmark}{=} \left(\frac{U(c^2)}{U(c^1)}\right)^{\frac{1}{1-\sigma}}
\end{aligned}$$

Hours Decomposition: To construct allocations $(\hat{h}^1, \tilde{h}^1)$, we follow Floden (2001) and Conesa, Kitao, and Krueger (2009), who define the allocation $\hat{h}^1$ as a compensating differential for labor disutility for the aggregate hours allocations using the perturbation $\hat{h}^1 = \delta_H h^1$ such that δ_H is defined as,

$$S(C^2, \delta_H H^1, I^1, A^1) = S(C^2, H^2, I^1, A^1)$$

$$V^h(\delta_H H^1) = V^h(H^2)$$

$$\delta_H = \left(\frac{V^h(H^2)}{V^h(H^1)}\right)^{\frac{1}{1+\frac{1}{\gamma}}}$$

and for $\tilde{h}^1$ as a compensating differential for labor disutility for the cohort aggregate hours allocations using the perturbation $\hat{h}^1 = \delta_h h^1$ such that δ_h is defined as,

$$\begin{aligned}
S(C^2, \delta_h H_j^1, I_j^1, A^1) &= S(C^2, H_j^2, I_j^1, A^1) \\
V^h(\delta_h H_j^1) &= V^h(H_j^2) \\
\delta_h &= \left(\frac{V^h(H_j^2)}{V^h(H_j^1)}\right)^{\frac{1}{1+\frac{1}{\gamma}}}
\end{aligned}$$

Put together, this gives us hours CEVs of

$$\begin{aligned}
1 + \Delta_{H_{level}} &= \left(\frac{U(c^2) - \delta_H^{1+\frac{1}{\gamma}} V^h(h^1) + V^h(h^1)}{U(c^2)}\right)^{\frac{1}{1-\sigma}} \\
&= \left(1 + \left(1 - \frac{V^h(H^2)}{V^h(H^1)}\right) \frac{V^h(h^1)}{U(c^2)}\right)^{\frac{1}{1-\sigma}} \qquad \text{(C5)}
\end{aligned}$$

$$\begin{aligned}
1 + \Delta_{H_{age}} &= \left(\frac{U(c^2) - \delta_h^{1+\frac{1}{\gamma}} V^h(h^1) + \delta_H^{1+\frac{1}{\gamma}} V^h(h^1)}{U(c^2)}\right)^{\frac{1}{1-\sigma}} \\
&= \left(1 + \left(\frac{V^h(H^2)}{V^h(H^1)} - \frac{V^h(H_j^2)}{V^h(H_j^1)}\right) \frac{V^h(h^1)}{U(c^2)}\right)^{\frac{1}{1-\sigma}} \qquad \text{(C6)}
\end{aligned}$$

$$\begin{aligned}
1 + \Delta_{H_{distr}} &= \left(\frac{U(c^2) - V^h(h^2) + \delta_h^{1+\frac{1}{\gamma}} V^h(h^1)}{U(c^2)}\right)^{\frac{1}{1-\sigma}} \\
&= \left(1 + \left(\frac{V^h(H_j^2)}{V^h(H_j^1)} - \frac{V^h(h^2)}{V^h(h^1)}\right) \frac{V^h(h^1)}{U(c^2)}\right)^{\frac{1}{1-\sigma}} \qquad \text{(C7)}
\end{aligned}$$

The decomposition can be verified using a first order approximation of the $i = 2$ allocation around the $i = 1$ allocation and therefore a first order approximation of

$\Delta_{H_{level}}, \Delta_{H_{age}}, \Delta_{H_{distr}}$ around zero. Noting that $u'(c)c/u(c) = (1-\sigma)$ and $v'(h)h/v(h) = 1 + 1/\gamma$, the first order approximations yield the following expressions for the hours welfare decomposition:

$$\Delta_H \approx -\frac{1}{1-\sigma}\left(\frac{V^h(h^2)}{V^h(h^1)} - 1\right)\frac{V^h(h^1)}{U(c^2)}$$

$$\Delta_{H_{level}} \approx -\frac{1}{1-\sigma}\left(\delta_H^{1+\frac{1}{\gamma}} - 1\right)\frac{V^h(h^1)}{U(c^2)}$$

$$\Delta_{H_{age}} \approx -\frac{1}{1-\sigma}\left(\delta_h^{1+\frac{1}{\gamma}} - \delta_H^{1+\frac{1}{\gamma}}\right)\frac{V^h(h^1)}{U(c^2)}$$

$$\Delta_{H_{distr}} \approx -\frac{1}{1-\sigma}\left(\frac{V^h(h^2)}{V^h(h^1)} - \delta_h^{1+\frac{1}{\gamma}}\right)\frac{V^h(h^1)}{U(c^2)}$$

Since $\log(1+\Delta) \approx \Delta$,

$$\log(1+\Delta_H) = \log(1+\Delta_{H_{level}}) + \log(1+\Delta_{H_{age}}) + \log(1+\Delta_{H_{distr}})$$

implies

$$\Delta_H \approx \Delta_{H_{level}} + \Delta_{H_{age}} + \Delta_{H_{distr}}.$$

Retirement Decomposition: To construct allocations $(\hat{d}^1, \tilde{d}^1)$, define the allocation $\hat{d}^1$ as a compensating differential of disutility from working life for the aggregate retirement allocations using the perturbation $\hat{d}^1 = \delta_R d^1$ such that δ_R is defined as,

$$S(C^2, H^2, \delta_R I^1, A^1) = S(C^2, H^2, I^2, A^1)$$

$$V^d(\delta_R I^1) = V^d(I^2)$$

$$\delta_R = \frac{V^d(I^2)}{V^d(I^1)}$$

and for $\tilde{d}^1$ as a compensating differential of disutility from working life for the cohort aggregate retirement allocations using the perturbation $\hat{d}^1 = \delta_r d^1$ such that δ_d is defined as,

$$S(C^2, H_j^2, \delta_r I_j^1, A^1) = S(C^2, H_j^2, I_j^2, A^1)$$

$$V^d(\delta_r I_j^1) = V^d(I_j^2)$$

$$\delta_r = \frac{V^d(I_j^2)}{V^d(I_j^1)}$$

Put together, this gives us hours CEVs of

$$1 + \Delta_{R_{level}} = \left(\frac{U(c^2) - \delta_R V^d(d^1) + V^d(d^1)}{U(c^2)} \right)^{\frac{1}{1-\sigma}}$$

$$= \left(1 + \left(1 - \frac{V^d(I^2)}{V^d(I^1)} \right) \frac{V^d(d^1)}{U(c^2)} \right)^{\frac{1}{1-\sigma}} \tag{C8}$$

$$1 + \Delta_{R_{age}} = \left(\frac{U(c^2) - \delta_r V^d(d^1) + \delta_R V^d(d^1)}{U(c^2)} \right)^{\frac{1}{1-\sigma}}$$

$$= \left(1 + \left(\frac{V^d(I^2)}{V^d(I^1)} - \frac{V^d(I_j^2)}{V^d(I_j^1)} \right) \frac{V^d(d^1)}{U(c^2)} \right)^{\frac{1}{1-\sigma}} \tag{C9}$$

$$1 + \Delta_{R_{distr}} = \left(\frac{U(c^2) - V^d(d^2) + \delta_r V^d(d^1)}{U(c^2)} \right)^{\frac{1}{1-\sigma}}$$

$$= \left(1 + \left(\frac{V^d(I_j^2)}{V^d(I_j^1)} - \frac{V^d(d^2)}{V^d(d^1)} \right) \frac{V^d(d^1)}{U(c^2)} \right)^{\frac{1}{1-\sigma}} \tag{C10}$$

Similarly to the hours decomposition, the retirement decomposition can be readily verified by linearizing around the $i = 1$ economy's allocation.

Asset Bequest Decomposition: Finally we construct allocations $(\hat{a}^1, \tilde{a}^1)$ equivalently as a perturbation that preserves utility across changes in aggregate wealth and the life cycle profile of wealth. That is, we define the allocation $\hat{a}^1$ as a compensating differential for bequest utility for the aggregate wealth allocations using the perturbation $\hat{a}^1 = \delta_H h^1$ such that δ_A is defined as,

$$S(C^2, H^2, I^2, \delta_A A^1) = S(C^2, H^2, I^2, A^2)$$

$$V^b(\delta_A A^1) = V^b(A^2)$$

$$\delta_A = \left(\frac{V^b(A^2)}{V^b(A^1)} \right)^{\frac{1}{1-\sigma}} = \frac{A^2 + \chi_a}{A^1 + \chi_a}$$

and for $\tilde{a}^1$ as a compensating differential for bequest utility for the cohort aggregate wealth allocations using the perturbation $\hat{a}^1 = \delta_a a^1$ such that δ_a is defined as,

$$S(C_j^2, H_j^2, I_j^2, \delta_a A_{j+1}^1) = S(C_j^2, H_j^2, I_j^2, A_{j+1}^2)$$

$$V^b(\delta_a A_{j+1}^1) = V^b(A_{j+1}^2)$$

$$\delta_a = \left(\frac{V^b(A_{j+1}^2)}{V^b(A_{j+1}^1)}\right)^{\frac{1}{1-\sigma}}$$

Put together, this gives us bequest CEVs of

$$1 + \Delta_{B_{level}} = \left(\frac{U(c^2) - \delta_B^{1-\sigma} V^b(a^1) + V^b(a^1)}{U(c^2)}\right)^{\frac{1}{1-\sigma}}$$

$$= \left(1 + \left(1 - \frac{V^b(A^2)}{V^b(A^1)}\right) \frac{V^b(a^1)}{U(c^2)}\right)^{\frac{1}{1-\sigma}} \tag{C11}$$

$$1 + \Delta_{B_{age}} = \left(\frac{U(c^2) - \delta_b^{1-\sigma} V^b(a^1) + \delta_B^{1-\sigma} V^b(a^1)}{U(c^2)}\right)^{\frac{1}{1-\sigma}}$$

$$= \left(1 + \left(\frac{V^b(A^2)}{V^b(A^1)} - \frac{V^b(A_{j+1}^2)}{V^b(A_{j+1}^1)}\right) \frac{V^b(a^1)}{U(c^2)}\right)^{\frac{1}{1-\sigma}} \tag{C12}$$

$$1 + \Delta_{B_{distr}} = \left(\frac{U(c^2) - V^b(a^2) + \delta_b^{1-\sigma} V^b(a^1)}{U(c^2)}\right)^{\frac{1}{1-\sigma}}$$

$$= \left(1 + \left(\frac{V^b(A_{j+1}^2)}{V^b(A_{j+1}^1)} - \frac{V^b(a^2)}{V^b(a^1)}\right) \frac{V^b(a^1)}{U(c^2)}\right)^{\frac{1}{1-\sigma}} \tag{C13}$$

Similarly to the hours decomposition, the bequest decomposition can be readily verified by linearizing around the $i = 1$ economy's allocation.

C.3 Total Change in Social Welfare from Prices and Policies

This appendix details the construction of the partial equilibrium contributions to the CEV from Section 4.2 .

Denote the baseline public debt policy by B and the optimal public savings policy by B^*. Likewise, denote competitive equilibrium outcomes under the baseline debt policy $(r, w, \tau_0, \tau_{ss}, \bar{b})$ and under the optimal public savings policy as $(r^*, w^*, \tau_0^*, \tau_{ss}^*, \bar{b}^*)$. Accordingly, the social welfare function can be written as a function of equilibrium outcomes, $S(r, w, \tau_0, \tau_{ss}, \bar{b})$. Denote the change in social welfare from the baseline to the optimal policy as,

$$\mathbf{d}S(r, w, \tau_0, \tau_{ss}, \bar{b}) \equiv S(r^*, w^*, \tau_0^*, \tau_{ss}^*, \bar{b}^*) - S(r, w, \tau_0, \tau_{ss}, \bar{b})$$

Now we consider the following computation. First, we compute the total change in social welfare from a change in the optimal public savings policy from B to B^*. Next, we compute the partial equilibrium change in the social welfare function with respect to each of the model's equilibrium outcomes (interest rate r, wage w, income tax rate τ_0, Social Security payroll tax τ_{ss}, and the aggregate level of bequests denoted by $\bar{b}$). For explication, consider the interest rate r. Set the interest rate to the optimal value $r^* \equiv r(B^*)$ while holding $(w, \tau_0, \tau_{ss}, \bar{b})$ constant at their baseline values. Given outcomes $(r^*, w, \tau_0, \tau_{ss}, \bar{b})$, we recompute individual decision rules to obtain the following,

$$\Big\{ c_j(a, \varepsilon, m, d_{-1} | r^*, w, \tau_0, \tau_{ss}, \bar{b}), a_{j+1}(a, \varepsilon, m, d_{-1} | r^*, w, \tau_0, \tau_{ss}, \bar{b}),$$
$$h_j(a, \varepsilon, m, d_{-1} | r^*, w, \tau_0, \tau_{ss}, \bar{b}), d_j(a, \varepsilon, m, d_{-1} | r^*, w, \tau_0, \tau_{ss}, \bar{b}) \Big\}_{j=1}^{J}.$$

Given the partial equilibrium decision rules, we compute the implied social welfare as $S(r^*, w, \tau_0, \tau_{ss}, \bar{b})$. Denote the partial equilibrium change in social welfare function with respect to a change in the interest rate to r^* alone as,

$$\mathbf{d}_r S(r, w, \tau_0, \tau_{ss}, \bar{b}) \equiv S(r^*, w, \tau_0, \tau_{ss}, \bar{b}) - S(r, w, \tau_0, \tau_{ss}, \bar{b})$$

Similarly we compute the partial equilibrium changes in social welfare with respect to the wage w, income tax rate τ_0, payroll tax rate τ_{ss} and aggregate bequests $\bar{b}$ as,

$$\mathbf{d}_w S(r, w, \tau_0, \tau_{ss}, \bar{b}) \equiv S(r, w^*, \tau_0, \tau_{ss}, \bar{b}) - S(r, w, \tau_0, \tau_{ss}, \bar{b})$$
$$\mathbf{d}_{\tau_0} S(r, w, \tau_0, \tau_{ss}, \bar{b}) \equiv S(r, w, \tau_0^*, \tau_{ss}, \bar{b}) - S(r, w, \tau_0, \tau_{ss}, \bar{b})$$
$$\mathbf{d}_{\tau_{ss}} S(r, w, \tau_0, \tau_{ss}, \bar{b}) \equiv S(r, w, \tau_0, \tau_{ss}^*, \bar{b}) - S(r, w, \tau_0, \tau_{ss}, \bar{b})$$
$$\mathbf{d}_{\bar{b}} S(r, w, \tau_0, \tau_{ss}, \bar{b}) \equiv S(r, w, \tau_0, \tau_{ss}, \bar{b}^*) - S(r, w, \tau_0, \tau_{ss}, \bar{b})$$

The total change in welfare up to a first order approximation is,

$$S(r^*, w^*, \tau_0^*, \tau_{ss}^*, \bar{b}^*) - S(r, w, \tau_0, \tau_{ss}, \bar{b}) = \sum_{i \in \{r, w, \tau_0, \tau_{ss}, \bar{b}\}} \mathbf{d}_i S(r, w, \tau_0, \tau_{ss}, \bar{b}) + \tilde{\epsilon}_S$$

where $\tilde{\epsilon}_S$ is a residual.[38]

These partial changes in social welfare are then converted into CEVs. Additionally, we apply this decomposition to the level, age and distribution effects, as well as to the overall CEV. These CEV measures are reported in Table 3 of the main text.

D Data and Empirical Measures

In this appendix, we detail the construction of empirical measures that we feed into the model.

Demographics: To measure household survival rates from mortality tables, we account for demographic changes such as variation in the size of the household and mortality rates by age and sex, and we define household mortality as when either both members of a married household die or when the sole remaining adult of a household dies. Accordingly, we construct the household-level mortality rate as,

$$\psi_j \equiv \omega_j^{single} \left[\omega_j^{male} \psi_j^{male} + (1 - \omega_j^{male}) \psi_j^{female} \right] + (1 - \omega_j^{single}) \psi_j^{male} \psi_j^{female} \ ,$$

where ψ_j^{male} and ψ_j^{female} are survival probabilities for age j males and females respectively, and $\omega_j^{single} \omega_j^{male}$ is the fraction of single-adult households with an age-j male head of household. We obtain mortality rates by age and sex from Bell and Miller (2002) and derive $\{\psi_j\}_{j=1}^{J-1}$ by applying a quartic polynomial in age to the raw series.

Preferences: We compute the adult equivalent scale at each age (of the head of household) to convert households of varying sizes into a standardized measure,

$$\tilde{n}_j \equiv \left[\omega_j^{single} \cdot 1 \right] + \left[(1 - \omega_j^{single}) \cdot 1.5 \right] + (1/3) n_j^c$$

where ω_j^{single} is the fraction of single-adult households with an age-j head of house-

[38]The residual contains all interaction terms (e.g., change in welfare from the baseline allocation to an allocation in which more than one price or policy changes at a time). The residual term can be expressed as the difference between the overall change in social welfare net of the sum of partial equilibrium changes in one price or policy at a time. In our computations we verify that the residual term is indeed small across various exercises.

hold, and n_j^c is the average number of children in a household with an age-j head of household. For each household in the 2007 Survey of Consumer Finances, we observe a head of household, a spouse (if there is one), and children (if there are any). We compute the share of single-adult households with an age-j head of household, denoted ω_j^{single}, and the share of married households with age-j head of household, given by $1-\omega_j^{single}$. We derive $\{n_j\}_{j=1}^J$ by applying a quartic polynomial in age to the measured profile of $\{\tilde{n}_j\}_{j=1}^J$.

Bequests: When constructing the bequest distribution, we normalize the distribution by aggregate labor income. We use the HRS-AHEAD dataset, using estate inheritances and excluding intra-household bequests by dropping observations in which estates were transferred to a spouse. We use the CPI to convert to \$2002 and normalize by the Social Security's labor income adjustment (the 2002 Average Wage Index, https://www.ssa.gov/oact/cola/awidevelop.html).

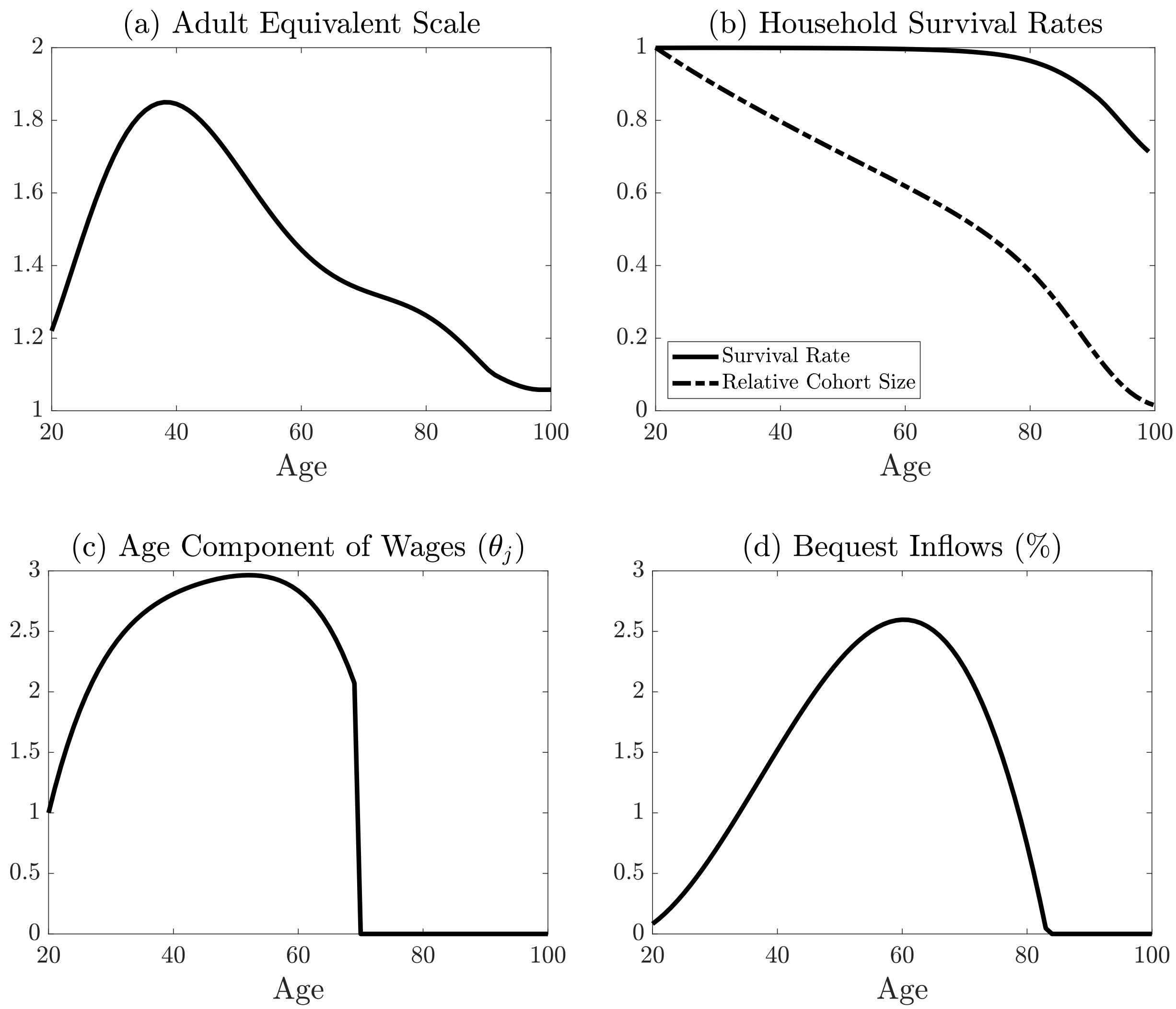
The total level of bequests to type-κ households is determined by the total amount of wealth held by type-κ households upon death. For each type, these bequests are allocated to living households to match shares of bequests received by age in the Survey of Consumer Finances, according to the function $b_j(\kappa)$. In particular, we construct the function $b_j(\kappa)$ from the relationship

$$(1+g_n)b_j(\kappa) = \omega_j^b \cdot \sum_{j=1}^{J} \omega_j(1-\psi_j) \int a_j'(a,\varepsilon,m,d_{-1})\mathbf{d}\lambda_j(a,\varepsilon,m,d_{-1}|\kappa)$$

which must hold for each household-type κ and age-j, and where $\{\omega_j^b\}_{j=1}^J$ are the shares of bequests received from the Survey of Consumer Finances, scaled by the total bequests by type-κ households in the model. To compute $\{\omega_j^b\}_{j=1}^J$, we take total household-level bequests by age from Feiveson and Sabelhaus (2018, 2019) and apply a quartic polynomial in age and normalize to convert to lifetime shares by age such that $\sum_{j=1}^J \omega_j^b = 1$.

Medical Expenditures: We compute medical expenditures at each age as the weighted average for single and married households, controlling for the household's composition of men and women. To do so, denote average medical expenditures for age-j men and women by μ_j^m and μ_j^f, respectively, and recall that ω_j^{single} and ω_j^{male} denote the share of single households and share of single households with a male head, respectively. Following DeNardi, French, and Jones (2010) and Kopecky and Koreshkova (2014), we compute out-of-pocket medical expenditures by sex and age (μ_j^m, μ_j^f) in

Figure A1: Household Size, Mortality, Wage, and Bequest Profiles



the HRS-AHEAD dataset by regressing individual expenditures on a quartic polynomial in age and include individual-specific fixed effects in order to alleviate measurement error and survivorship bias. Accordingly, the average medical expenditures for a household with head of age-j are the weighted average of single male, single female and married household expenditures,

$$\tilde{\mu}_j = \omega_j^{single}\left[\omega_j^{male}\mu_j^m + (1-\omega_j^{male})\mu_j^f\right] + (1-\omega_j^{single})\left(\frac{1}{2}\mu_j^m + \frac{1}{2}\mu_j^f\right) .$$

We derive μ_j by applying a quartic polynomial in age to $\{\tilde{\mu}_j\}_{j=\bar{J}_{ret}}^J$.