Online Appendix

Expectations-Driven Liquidity Traps:
Implications for Monetary and Fiscal Policy

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A Sunspot equilibrium in the baseline model without fiscal stabilization policy

A.1 Proof of Proposition 1

To prove Proposition 1 on the necessary and sufficient conditions for existence of the sunspot equilibrium, it is useful to proceed in four steps. Each step is associated with an auxiliary proposition.

Let

$$A := -\beta\lambda(1-p_H), \tag{A.1}$$

$$B := \kappa^2 + \lambda(1-\beta p_H), \tag{A.2}$$

$$C := \frac{(1-p_L)}{\sigma\kappa}(1-\beta p_L + \beta(1-p_H)) - p_L, \tag{A.3}$$

$$D := -\frac{(1-p_L)}{\sigma\kappa}(1-\beta p_L + \beta(1-p_H)) - (1-p_L) = -1-C, \tag{A.4}$$

and

$$E := AD - BC. \tag{A.5}$$

Proposition A.1 *There exists a vector* $\{y_H, \pi_H, i_H, y_L, \pi_L, i_L\}$ *that solves the system of linear equations (8)–(13).*

Proof: Rearranging the system of equations (8)–(13) and eliminating y_H and y_L, we obtain two unknowns for π_H and π_L in two equations

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \pi_L \\ \pi_H \end{bmatrix} = \begin{bmatrix} \kappa^2\pi^* \\ r^n \end{bmatrix}. \tag{A.6}$$

For what follows, it is useful to show that $E = 0$ is generically inconsistent with existence of the sunspot equilibrium. Since $B > 0$, we can always write $\pi_H = \kappa^2/B\pi^* - A/B\pi_L$. Plugging this into $C\pi_L + D\pi_H = r^n$ and multiplying both sides by B, we get $D\kappa^2\pi^* - E\pi_L = Br^n$. Since the right-hand side of this equation is strictly positive, $E = 0$ is inconsistent with the existence of the sunspot equilibrium for generic π^*.

Hence, we can invert the matrix on the left-hand-side of (A.6)

$$\begin{bmatrix} \pi_L \\ \pi_H \end{bmatrix} = \frac{1}{AD - BC} \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \begin{bmatrix} \kappa^2 \pi^* \\ r^n \end{bmatrix}. \tag{A.7}$$

Thus,

$$\pi_H = -\frac{C\kappa^2}{E}\pi^* + \frac{A}{E}r^n \tag{A.8}$$

and

$$\pi_L = \frac{D\kappa^2}{E}\pi^* - \frac{B}{E}r^n. \tag{A.9}$$

From the Phillips curves in both states, we obtain

$$y_H = \frac{\kappa\left(\beta(1-p_H) - (1-\beta)C\right)}{E}\pi^* + \frac{\beta\kappa(1-p_H)}{E}r^n \tag{A.10}$$

and

$$y_L = \frac{\kappa\left(\beta p_L - 1 - (1-\beta)C\right)}{E}\pi^* - \frac{(1-\beta p_L)\kappa^2 + (1-\beta)(1-\beta p_L + \beta(1-p_H))\lambda}{\kappa E}r^n. \tag{A.11}$$

Proposition A.2 *Suppose equations (8)–(13) are satisfied. Then* $\lambda y_L + (\kappa\pi_L - \pi^*) < 0$ *if and only if (i)* $E > 0$ *and* $\pi^* > -\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2}r^n$ *or (ii)* $E < 0$ *and* $\pi^* < -\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2}r^n$.

Proof: Using (A.9) and (A.11), we have

$$\lambda y_L + \kappa(\pi_L - \pi^*) = -\frac{\kappa^2 + \lambda\left(1 - \beta p_L + \beta(1-p_H)\right)}{E}\kappa\left(\pi^* + \frac{\kappa^2 + \lambda(1-\beta)}{\kappa^2}r^n\right). \tag{A.12}$$

Notice that $(\kappa^2 + \lambda\left(1 - \beta p_L + \beta(1-p_H)\right))\kappa > 0$, and $\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2}r^n > 0$. Thus, if $E > 0$ and $\pi^* > -\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2}r^n$, then $\lambda y_L + \kappa(\pi_L - \pi^*) < 0$. Similarly, if $E < 0$ and $\pi^* < -\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2}r^n$, then $\lambda y_L + \kappa(\pi_L - \pi^*) < 0$.

Proposition A.3 *Suppose equations (8)–(13) are satisfied,* $E > 0$ *and* $\pi^* > -\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2}r^n$. *Then* $i_H > 0$ *if and only if* $p_L - (1-p_H) - \frac{1-p_L+1-p_H}{\kappa\sigma}\left(1 - \beta p_L + \beta(1-p_H)\right) > 0$.

Proof: i_H is given by

$$\begin{aligned} i_H =& \frac{1-p_H}{\sigma}\left(y_L - y_H\right) + p_H\pi_H + (1-p_H)\pi_L + r^n \\ =& \frac{\left(p_L - (1-p_H) - \frac{1-p_L+1-p_H}{\kappa\sigma}\left(1 - \beta p_L + \beta(1-p_H)\right)\right)\kappa^2}{E}\left(\pi^* + \frac{\kappa^2 + \lambda(1-\beta)}{\kappa^2}r^n\right), \end{aligned} \tag{A.13}$$

where in the second row we made use of (A.8)–(A.11).

Proposition A.4 *Suppose equations (8)–(13) are satisfied,* $E < 0$ *and* $\pi^* < -\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2} r^n$*. Then* $i_H < 0$.

Proof: First, substitute equations (A.1), (A.2), and (A.4) into equation (A.5) to obtain

$$E = \beta\lambda(1-p_H) - \left(\kappa^2 + \lambda(1-\beta)\right) C. \tag{A.14}$$

Hence, $E < 0$ implies $C > 0$.

Corollary A.1 $C < 0$ *implies* $E > 0$.

Next, note that

$$p_L - (1-p_H) - \frac{1-p_L+1-p_H}{\kappa\sigma}\left(1-\beta p_L + \beta(1-p_H)\right) = -C - (1-p_H)\frac{1-\beta p_L + \beta(1-p_H) + \kappa\sigma}{\kappa\sigma}.$$

Hence, $C > 0$ implies $p_L - (1-p_H) - \frac{1-p_L+1-p_H}{\kappa\sigma}\left(1-\beta p_L + \beta(1-p_H)\right) < 0$.

Corollary A.2 $p_L - (1-p_H) - \frac{1-p_L+1-p_H}{\kappa\sigma}\left(1-\beta p_L + \beta(1-p_H)\right) > 0$ *implies* $C < 0$.

From equation (A.13), it follows that $p_L - (1-p_H) - \frac{1-p_L+1-p_H}{\kappa\sigma}\left(1-\beta p_L + \beta(1-p_H)\right) < 0$, $E < 0$ and $\pi^* < -\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2} r^n$ imply $i_H < 0$.

We are now ready to proof Proposition 1. For notational convenience, define

$$\Omega(p_L, p_H, \kappa, \sigma, \beta) \equiv p_L - (1-p_H) - \frac{1-p_L+1-p_H}{\kappa\sigma}\left(1-\beta p_L + \beta(1-p_H)\right). \tag{A.15}$$

Proof of "if" part: Suppose that $\Omega(\cdot) > 0$ and $\pi^* > -\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2} r^n$. According to Proposition A.1 there exists a vector $\{y_H, \pi_H, i_H, y_L, \pi_L, i_L\}$ that solves equations (8)–(13). According to Corollary A.2, $\Omega(\cdot) > 0$ implies $C < 0$. According to Corollary A.1, $C < 0$ implies $E > 0$. According to Proposition A.2, $E > 0$ and $\pi^* > -\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2} r^n$ imply $\lambda y_L + \kappa(\pi_L - \pi^*) < 0$. According to Proposition A.3, given $E > 0$ and $\pi^* > -\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2} r^n$, $\Omega(\cdot) > 0$ implies $i_H > 0$.

Proof of "only if" part: Suppose that the vector $\{y_H, \pi_H, i_H, y_L, \pi_L, i_L\}$ solves (8)–(13), and satisfies $\lambda y_L + \kappa(\pi_L - \pi^*) < 0$ and $i_H > 0$. According to Proposition A.2, $\lambda y_L + \kappa(\pi_L - \pi^*) < 0$ implies that either (i) $E > 0$ and $\pi^* > -\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2} r^n$ or (ii) $E < 0$ and $\pi^* < -\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2} r^n$. According to Proposition A.4, (ii) is inconsistent with $i_H > 0$. Hence, $E > 0$ and $\pi^* > -\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2} r^n$. According to Proposition A.3, given $E > 0$ and $\pi^* > -\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2} r^n$, $i_H > 0$ implies $\Omega(\cdot) > 0$.

A.2 Proof of Proposition 2

The allocations and prices in the sunspot equilibrium are given by

$$\pi_L = -\frac{(C+1)\kappa^2}{E}\pi^* - \frac{\kappa^2 + \lambda(1-\beta p_H)}{E} r^n \tag{A.16}$$

$$y_L = \frac{\kappa\left(\beta p_L - 1 - (1-\beta)C\right)}{E}\pi^* - \frac{(1-\beta p_L)\kappa^2 + (1-\beta)(1-\beta p_L + \beta(1-p_H))\lambda}{\kappa E} r^n \tag{A.17}$$

$$\pi_H = -\frac{C\kappa^2}{E}\pi^* - \frac{\beta\lambda(1-p_H)}{E} r^n \tag{A.18}$$

$$y_H = \frac{\kappa\left(\beta(1-p_H) - (1-\beta)C\right)}{E}\pi^* + \frac{\beta\kappa(1-p_H)}{E} r^n \tag{A.19}$$

Assuming $\pi^* = 0$ and $\lambda > 0$, it holds

$$\begin{aligned}
\pi_L &= -\frac{\kappa^2 + \lambda(1-\beta p_H)}{E} r^n < 0 \\
y_L &= -\frac{(1-\beta p_L)\kappa^2 + (1-\beta)(1-\beta p_L + \beta(1-p_H))\lambda}{\kappa E} r^n < 0 \\
\pi_H &= -\frac{\beta\lambda(1-p_H)}{E} r^n \leq 0 \\
y_H &= \frac{\beta\kappa(1-p_H)}{E} r^n \geq 0
\end{aligned}$$

When $p_H < 1$, $\pi_H < 0$ and $y_H > 0$.

A.3 Proof of Proposition 3

Keeping in mind that $-1 < C < 0$ in the sunspot equilibrium, it holds,

$$\frac{\partial \pi_L}{\partial \pi^*} = -\frac{C+1}{E}\kappa^2 < 0 \tag{A.20}$$

$$\frac{\partial y_L}{\partial \pi^*} = -\frac{\beta(1-p_L) + (1-\beta)(C+1)}{E}\kappa < 0, \tag{A.21}$$

and

$$\frac{\partial \pi_H}{\partial \pi^*} = -\frac{C}{E}\kappa^2 > 0 \tag{A.22}$$

$$\frac{\partial y_H}{\partial \pi^*} = \frac{\beta(1-p_H) - (1-\beta)C}{E}\kappa > 0. \tag{A.23}$$

A.4 Proof of Lemma 1

If π^0 exists, it holds $-\frac{C\kappa^2}{E}\pi^0 - \frac{\beta\lambda(1-p_H)}{E}r^n = 0$. Solving for π^0, one obtains

$$\pi^0 = -\frac{\beta\lambda(1-p_H)}{C\kappa^2} r^n, \tag{A.24}$$

where $C < 0$, and hence $\pi^0 > 0$.

A.5 Proof of Proposition 4

Note first that

$$\mathrm{E}V = -\frac{1}{1-\beta}\frac{1}{2}\left[\frac{1-p_L}{1-p_L+1-p_H}\left(\pi_H^2+\bar{\lambda}y_H^2\right)+\frac{1-p_H}{1-p_L+1-p_H}\left(\pi_L^2+\bar{\lambda}y_L^2\right)\right], \tag{A.25}$$

where V is defined in equation (3).

Assuming $\lambda = \bar{\lambda}$, the partial derivative of $\mathrm{E}V$ with respect to π^* is

$$\begin{aligned}\frac{\partial \mathrm{E}V}{\partial \pi^*} = -&\frac{1}{(1-\beta)(1-p_L+1-p_H)E^2}\Bigg\{\Big[\left(\kappa^2+\bar{\lambda}(1-\beta)^2\right)\left((1-p_H)(C+1)^2+(1-p_L)C^2\right)\\ &+\bar{\lambda}\beta(1-p_H)(1-p_L)(1-\beta p_L+1-\beta p_H)\Big]\kappa^2\pi^* + \Big[\bar{\lambda}\left(\kappa^2+\bar{\lambda}(1-\beta)\right)(1-\beta p_L+\beta(1-p_H))\\ &(\beta(1-p_L)+(1-\beta)(C+1))+\left(\kappa^2+\bar{\lambda}(1-\beta+\beta^2(1-p_L+1-p_H))\right)\kappa^2(C+1)\\ &-(\beta\kappa)^2\bar{\lambda}(1-p_L)\Big](1-p_H)r^n\Bigg\}.\end{aligned}$$

Note that all terms in the square brackets which are multiplied by π^* are positive. In the square brackets which are multiplied by r^n all terms are positive except for the last one, $-(\beta\kappa)^2\bar{\lambda}(1-p_L) < 0$.

The first-order necessary condition for the welfare-maximizing inflation target is $\frac{\partial \mathrm{E}V}{\partial \pi^*} = 0$. Solving for π^*, one obtains

$$\pi^{**} = -\frac{1-p_H}{\kappa^2}\frac{\bar{\lambda}\left(\kappa^2+\bar{\lambda}(1-\beta)\right)(1-\beta p_L+\beta(1-p_H))(\beta(1-p_L)+(1-\beta)(C+1))+\left(\kappa^2+\bar{\lambda}(1-\beta+\beta^2(1-p_L+1-p_H))\right)\kappa^2(C+1)-(\beta\kappa)^2\bar{\lambda}(1-p_L)}{\left(\kappa^2+\bar{\lambda}(1-\beta)^2\right)\left((1-p_H)(C+1)^2+(1-p_L)C^2\right)+\bar{\lambda}\beta(1-p_H)(1-p_L)(1-\beta p_L+1-\beta p_H)}r^n$$

Note that $\pi^{**} > -\frac{\kappa^2+\bar{\lambda}(1-\beta)}{\kappa^2}r^n$ whenever existence condition (22) is satisfied. Specifically, $\pi^{**} > -\frac{\kappa^2+\bar{\lambda}(1-\beta)}{\kappa^2}r^n$ if and only if

$$\begin{aligned}&\left(\kappa^2+\bar{\lambda}(1-\beta)\right)\left\{(\kappa^2+\bar{\lambda}(1-\beta)^2)C\left[(1-p_L+1-p_H)C+1-p_H\right]\right\}\\ &>\left[\left(\kappa^2+\bar{\lambda}(1-\beta)\right)\bar{\lambda}\beta(1-\beta)(1-p_H)+(\beta\kappa)^2\bar{\lambda}(1-p_H)\right]\left[(1-p_L+1-p_H)C+1-p_H\right],\end{aligned}$$

where $(1-p_L+1-p_H)C+1-p_H = -(1-p_L)\Omega(p_L,p_H,\kappa,\sigma,\beta) < 0$. Hence, the left-hand side of the inequality is positive and the right-hand side is negative, so that the inequality is satisfied.

Next, we show that $\pi^{**} < \pi^0$. This requires

$$-\frac{\beta\bar{\lambda}}{C} > -\frac{\bar{\lambda}\left(\kappa^2+\bar{\lambda}(1-\beta)\right)(1-\beta p_L+\beta(1-p_H))\left(\beta(1-p_L)+(1-\beta)(C+1)\right)+\left(\kappa^2+\bar{\lambda}(1-\beta+\beta^2(1-p_L+1-p_H))\right)\kappa^2(C+1)-(\beta\kappa)^2\bar{\lambda}(1-p_L)}{\left(\kappa^2+\bar{\lambda}(1-\beta)^2\right)\left((1-p_H)(C+1)^2+(1-p_L)C^2\right)+\bar{\lambda}\beta(1-p_H)(1-p_L)(1-\beta p_L+1-\beta p_H)},$$

which can be rewritten as

$$\begin{aligned}&\beta\bar{\lambda}\kappa^2(1-p_L)(1-\beta)C^2+\beta\bar{\lambda}^2(1-\beta)^2(1-p_L)C^2+\beta\bar{\lambda}\left(\kappa^2+\bar{\lambda}(1-\beta)^2\right)(1-p_H)(C+1)^2+(\beta\bar{\lambda})^2(1-p_L)(1-p_H)(1-\beta p_L+1-\beta p_H)\\&>(\beta\kappa)^2\bar{\lambda}(1-p_L)(1-p_H)C+\kappa^2\left(\kappa^2+\bar{\lambda}(1-\beta p_H)\right)C+\left[\kappa^2(1-\beta p_L)+\bar{\lambda}(1-\beta)(1-\beta p_L+1-\beta p_H)\right]\left[\beta(1-p_L)+(1-\beta)(C+1)\right]\bar{\lambda}C.\end{aligned}$$

Note that all terms on the left-hand side of the inequality sign are strictly positive and all terms on the right-hand side are strictly negative. This completes the proof.

A.6 Proof of Proposition 6

Suppose $\pi^* = 0$ and $p_H < 1$. It holds

$$\begin{aligned}\frac{\partial\pi_L}{\lambda} &= \frac{\beta\kappa^2(1-p_H)(1-p_L)}{E^2}\frac{\kappa\sigma+(1-\beta p_L+\beta(1-p_H))}{\kappa\sigma}r^n > 0\\ \frac{\partial y_L}{\partial\lambda} &= \frac{\beta\kappa(1-p_H)(1-p_L)}{E^2}\frac{\kappa\sigma+(1-\beta)(1-\beta p_L+\beta(1-p_H))}{\kappa\sigma}r^n > 0\\ \frac{\partial\pi_H}{\partial\lambda} &= -\frac{\beta\kappa^2(1-p_H)}{E^2}\left[\Omega(p_L,p_H,\kappa,\sigma,\beta)+(1-p_H)\frac{\kappa\sigma+(1-\beta p_L+\beta(1-p_H))}{\kappa\sigma}\right]r^n < 0\\ \frac{\partial y_H}{\partial\lambda} &= -\frac{\beta\kappa(1-p_H)}{E^2}\left[(1-\beta)\Omega(p_L,p_H,\kappa,\sigma,\beta)+(1-p_H)\frac{\kappa\sigma+(1-\beta)(1-\beta p_L+\beta(1-p_H))}{\kappa\sigma}\right]r^n < 0\end{aligned}$$

A.7 Proof of Proposition 7

Note first that

$$\mathrm{E}V = -\frac{1}{1-\beta}\frac{1}{2}\left[\frac{1-p_L}{1-p_L+1-p_H}\left(\pi_H^2+\bar{\lambda}y_H^2\right)+\frac{1-p_H}{1-p_L+1-p_H}\left(\pi_L^2+\bar{\lambda}y_L^2\right)\right], \tag{A.26}$$

where V is defined in equation (3).

Assuming $\pi^* = 0$, the partial derivative of $\mathrm{E}V$ with respect to λ is

$$
\begin{aligned}
\frac{\partial \mathrm{EV}}{\partial \lambda} = & \frac{\beta\left((1-p_H)r^n\right)^2}{(1-\beta)(1-p_L+1-p_H)E^3}\Bigg\{\Big[\beta\kappa^2(1-p_L)C+\kappa^2(1-\beta p_H)(C+1) \\
& +\bar{\lambda}(1-\beta)(1-\beta p_L+\beta(1-p_H))\left((1-\beta)(C+1)+\beta(1-p_L)\right)\Big]\lambda \\
& +\beta\kappa^2\left[(1-p_L)(1-p_H)\beta\bar{\lambda}-(1-p_L)(1-\beta)C\bar{\lambda}\right]+\kappa^4(C+1) \\
& +\bar{\lambda}(1-\beta p_L)\kappa^2\left((1-\beta)(C+1)+\beta(1-p_L)\right)\Bigg\}.
\end{aligned}
$$

Note that since $(C+1) > 0$ and $C < 0$, all terms in curly brackets are positive except for the very first one, $\beta\kappa^2(1-p_L)C < 0$. Also note that since in the sunspot equilibrium $E > 0$, the term in front of the curly brackets is positive for any $\lambda \geq 0$. Since the only negative term in curly brackets is multiplied by λ, $\frac{\partial \mathrm{EV}}{\partial \lambda}_{|\lambda=0} > 0$, and therefore $\lambda^* > 0$.

Furthermore, if

$$\kappa^2\beta(1-p_L)C+\kappa^2(1-\beta p_H)(C+1)+\bar{\lambda}(1-\beta)\left(1-\beta p_L+\beta(1-p_H)\right)\left((C+1)(1-\beta)+\beta(1-p_L)\right) \geq 0,$$

then $\frac{\partial \mathrm{EV}}{\partial \lambda} > 0$ for all $\lambda \geq 0$. Hence, in this case no interior solution for λ^* exists and $\lambda^* = \infty$.

If instead

$$\kappa^2\beta(1-p_L)C+\kappa^2(1-\beta p_H)(C+1)+\bar{\lambda}(1-\beta)\left(1-\beta p_L+\beta(1-p_H)\right)\left((C+1)(1-\beta)+\beta(1-p_L)\right) < 0,$$

then

$$\lambda^* = -\frac{\beta\kappa^2\left[(1-p_L)(1-p_H)\beta\bar{\lambda}-(1-p_L)(1-\beta)C\bar{\lambda}\right]+\kappa^4(C+1)+\bar{\lambda}(1-\beta p_L)\kappa^2\left((1-\beta)(C+1)+\beta(1-p_L)\right)}{\kappa^2\beta(1-p_L)C+\kappa^2(1-\beta p_H)(C+1)+\bar{\lambda}(1-\beta)\left(1-\beta p_L+\beta(1-p_H)\right)\left((C+1)(1-\beta)+\beta(1-p_L)\right)}$$

In this case, $\lambda^* > \bar{\lambda}$ if

$$(\beta\kappa)^2(1-p_L)\bar{\lambda}\underbrace{(C+1-p_H)}_{<0}+\kappa^2\left(\kappa^2+(1-\beta p_H)\bar{\lambda}\right)(C+1)+\left(\kappa^2(1-\beta p_L)+(1-\beta)\bar{\lambda}(1-\beta p_L+\beta(1-p_H))\right)\left(\beta(1-p_L)+(1-\beta)(C+1)\right)\bar{\lambda} > 0$$

A.8 Proof of Proposition 8

Let $X_{S|\lambda=\bar{\lambda},\pi^*=\hat{\pi}^*}$ denote the outcome of variable $X \in \{\pi, y\}$ in state $S \in \{H, L\}$ of the sunspot equilibrium when $\lambda = \bar{\lambda}$ and $\pi^* = \hat{\pi}^*$, and $X_{S|\lambda=\hat{\lambda},\pi^*=0}$ when $\lambda = \hat{\lambda}$ and $\pi^* = 0$. We need to show that $X_{S|\lambda=\bar{\lambda},\pi^*=\hat{\pi}^*} = X_{S|\lambda=\hat{\lambda},\pi^*=0}$ for all $X \times S$ and any $\hat{\lambda} \geq 0$.

High-state inflation:

$$\begin{aligned}
\pi_{H|\lambda=\bar{\lambda},\pi^*=\hat{\pi}^*} =& -\frac{C\kappa^2}{[\beta\bar{\lambda}(1-p_H)-(\kappa^2+\bar{\lambda}(1-\beta))C]}\frac{\beta(1-p_H)(\bar{\lambda}-\hat{\lambda})}{[\beta\hat{\lambda}(1-p_H)-(\kappa^2+\hat{\lambda}(1-\beta))C]}r^n \\
& -\frac{\beta\bar{\lambda}(1-p_H)}{\beta\bar{\lambda}(1-p_H)-(\kappa^2+\bar{\lambda}(1-\beta))C}r^n \\
=& -\frac{\beta\hat{\lambda}(1-p_H)}{\beta\hat{\lambda}(1-p_H)-(\kappa^2+\hat{\lambda}(1-\beta))C}r^n \\
=& \pi_{H|\lambda=\hat{\lambda},\pi^*=0}
\end{aligned}$$

High-state output:

$$\begin{aligned}
y_{H|\lambda=\bar{\lambda},\pi^*=\hat{\pi}^*} =& \frac{\kappa\left(\beta(1-p_H)-(1-\beta)C\right)}{[\beta\bar{\lambda}(1-p_H)-(\kappa^2+\bar{\lambda}(1-\beta))C]}\frac{\beta(1-p_H)(\bar{\lambda}-\hat{\lambda})}{[\beta\hat{\lambda}(1-p_H)-(\kappa^2+\hat{\lambda}(1-\beta))C]}r^n \\
& +\frac{\beta\kappa(1-p_H)}{\beta\bar{\lambda}(1-p_H)-(\kappa^2+\bar{\lambda}(1-\beta))C}r^n \\
=& \frac{\beta\kappa(1-p_H)}{\beta\hat{\lambda}(1-p_H)-(\kappa^2+\hat{\lambda}(1-\beta))C}r^n \\
=& y_{H|\lambda=\hat{\lambda},\pi^*=0}
\end{aligned}$$

Low-state inflation:

$$\begin{aligned}
\pi_{L|\lambda=\bar{\lambda},\pi^*=\hat{\pi}^*} =& \frac{D\kappa^2}{[\beta\bar{\lambda}(1-p_H)-(\kappa^2+\bar{\lambda}(1-\beta))C]}\frac{\beta(1-p_H)(\bar{\lambda}-\hat{\lambda})}{[\beta\hat{\lambda}(1-p_H)-(\kappa^2+\hat{\lambda}(1-\beta))C]}r^n \\
& -\frac{\kappa^2+\bar{\lambda}(1-\beta p_H)}{\beta\bar{\lambda}(1-p_H)-(\kappa^2+\bar{\lambda}(1-\beta))C}r^n \\
=& -\frac{\kappa^2+\hat{\lambda}(1-\beta p_H)}{\beta\hat{\lambda}(1-p_H)-(\kappa^2+\hat{\lambda}(1-\beta))C}r^n \\
=& \pi_{L|\lambda=\hat{\lambda},\pi^*=0}
\end{aligned}$$

Low-state output:

$$\begin{aligned}
y_{L|\lambda=\bar{\lambda},\pi^*=\hat{\pi}^*} =& \frac{\kappa(\beta p_L-1-(1-\beta)C)}{[\beta\bar{\lambda}(1-p_H)-(\kappa^2+\bar{\lambda}(1-\beta))C]}\frac{\beta(1-p_H)(\bar{\lambda}-\hat{\lambda})}{[\beta\hat{\lambda}(1-p_H)-(\kappa^2+\hat{\lambda}(1-\beta))C]}r^n \\
& -\frac{\kappa^2(1-\beta p_L)+\bar{\lambda}(1-\beta)\left(1-\beta p_L+\beta(1-p_H)\right)}{\beta\bar{\lambda}(1-p_H)-(\kappa^2+\bar{\lambda}(1-\beta))C}r^n \\
=& -\frac{\kappa^2(1-\beta p_L)+\hat{\lambda}(1-\beta)\left(1-\beta p_L+\beta(1-p_H)\right)}{\beta\hat{\lambda}(1-p_H)-(\kappa^2+\hat{\lambda}(1-\beta))C}r^n \\
=& y_{L|\lambda=\hat{\lambda},\pi^*=0}
\end{aligned}$$

A.9 Numerical example

This subsection provides a numerical example of the sunspot equilibrium in the model without fiscal policy. One period is assumed to correspond to one quarter, and the parameterisation follows Table 1.

Figure A.1 plots the region of existence for the sunspot equilibrium in the (p_H, p_L) space (black area), and the region of existence for the fundamental equilibrium in the (p_H^f, p_L^f) space (gray area).[52]

Figure A.1: Existence regions for sunspot equilibrium and fundamental equilibrium

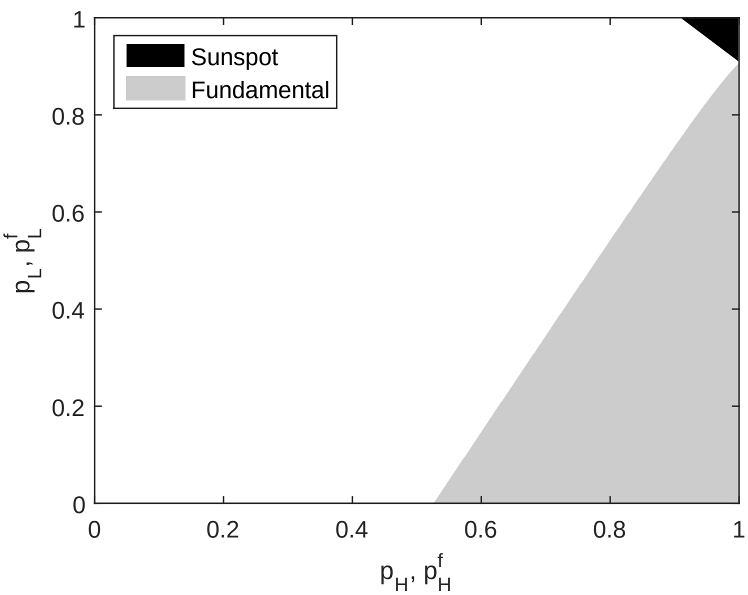


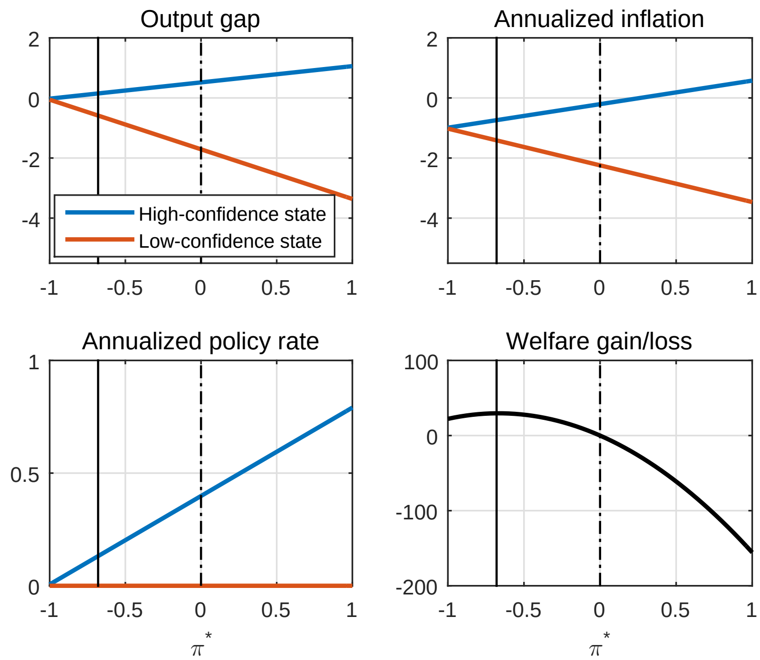
Figure A.2 shows how allocations and welfare in the sunspot equilibrium depend on the central bank's inflation target π^*. We set $p_L = 0.9375$ and $p_H = 0.98$. In this particular example, the optimal inflation target is negative.

B The no-sunspot equilibrium in the baseline model without fiscal stabilization policy: Case where condition (23) is violated

When condition (23) is violated, i.e. when the inflation target π^* is sufficiently negative for a given value of λ, the sunspot equilibrium fails to exist. In this section, we characterize the remaining no-sunspot equilibrium.

[52] In case of the fundamental equilibrium, the condition for equilibrium existence depends on the value of the natural real rate in the low-fundamental state, r_L^n. The region of existence is shrinking in the absolute value of r_L^n.

Figure A.2: Allocations and welfare as a function of π^*



Note: Dash-dotted vertical lines indicate the case where the central bank has the same objective function as society as a whole, i.e. $\pi^* = 0$. Solid vertical lines indicate the welfare-maximizing inflation target. The welfare gain/loss is expressed relative to the welfare level achieved when the inflation target is zero (in percent).

B.1 Allocation and prices

We first show that the no-sunspot equilibrium features a binding lower bound on nominal interest rates, deflation, and a negative output gap.

Given that the sunspot shock does not affect agents' decisions in no-sunspot equilibria, we can abstract from the confidence states. The equilibrium conditions read:

$$\pi = \kappa y + \beta \pi \tag{B.1}$$

$$i = \pi + r^n \tag{B.2}$$

$$0 = i[\kappa(\pi - \pi^*) + \lambda y] \tag{B.3}$$

where $\kappa(\pi - \pi^*) + \lambda y = 0$ if $i > 0$ and $\kappa(\pi - \pi^*) + \lambda y < 0$ if $i = 0$.

Suppose, first, that $i > 0$. Solving the system of equations under this assumption, we obtain

$$i = r^n + \frac{\kappa^2}{\kappa^2 + \lambda(1-\beta)} \pi^*. \tag{B.4}$$

When condition (23) does not hold, equation (B.4) implies $i \leq 0$, which contradicts the initial assumption that $i > 0$.

Hence, in the no-sunspot equilibrium, we must have $i = 0$. Solving the system of equations under this assumption, we obtain

$$\pi = -r^n < 0 \tag{B.5}$$

$$y = -\frac{1-\beta}{\kappa} r^n < 0. \tag{B.6}$$

Note that this no-sunspot equilibrium is unique, and that the equilibrium values of output and inflation do not depend on the value of the inflation target.

B.2 Proof of Proposition 5

To show that unconditional welfare $\mathrm{E}V_t$ in the no-sunspot equilibrium with an inflation target low enough such that condition (23) fails to hold is lower than in the sunspot equilibrium with an optimized inflation target, we proceed as follows.

First, note that the policy functions for inflation and the output gap associated with the sunspot equilibrium are continuous functions of the inflation target π^* on the domain that includes $\underline{\pi} \equiv -\frac{\kappa^2+\lambda(1-\beta)}{\kappa^2} r^n$ as a boundary. Second, for $\pi^* \to \underline{\pi}$, the function values of the policy functions associated with the sunspot equilibrium converge to the function values of the policy functions associated with the no-sunspot equilibrium when $\pi^* \leq \underline{\pi}$. That is

$$\lim_{\pi^* \to \underline{\pi}} \pi_L(\pi^*) = -r^n \tag{B.7}$$

$$\lim_{\pi^* \to \underline{\pi}} y_L(\pi^*) = -\frac{1-\beta}{\kappa} r^n \tag{B.8}$$

$$\lim_{\pi^* \to \underline{\pi}} \pi_H(\pi^*) = -r^n \tag{B.9}$$

$$\lim_{\pi^* \to \underline{\pi}} y_H(\pi^*) = -\frac{1-\beta}{\kappa} r^n, \tag{B.10}$$

where $\pi_L(\pi^*), y_L(\pi^*), \pi_H(\pi^*)$ and $y_H(\pi^*)$ are defined in equations (A.16) – (A.19).

Finally, note that when deriving the welfare-maximizing inflation target in Section A.5 to proof Proposition 4, we do not restrict the domain for π^* to exclude $\underline{\pi}$. Given that we show in Section A.5 that the welfare-maximizing inflation target is strictly larger than $\underline{\pi}$ as long as (22)—the other condition for existence of the sunspot equilibrium—is satisfied, society prefers being in the sunspot equilibrium with an optimized inflation target over being in the no-sunspot equilibrium with an inflation target $\pi^* \leq \underline{\pi}$.

C Policy problem in the baseline model with fiscal stabilization policy

At the beginning of time, society delegates monetary and fiscal policy to a discretionary policymaker. The objective function of the policymaker is given by

$$V_t^{MF} = -\frac{1}{2}\mathrm{E}_t \sum_{j=0}^{\infty} \beta^j \left(\pi_{t+j}^2 + \bar{\lambda} x_{t+j}^2 + \lambda_g g_{t+j}^2\right), \tag{C.1}$$

where for $\lambda_g = \bar{\lambda}_g$, the policymaker's objective function coincides with society's objective function.

The optimization problem of a generic policymaker acting under discretion is as follows. Each period t, she chooses the inflation rate, the modified output gap, government spending, and the nominal interest rate to maximize its objective function (C.1) subject to the behavioral constraints of the private sector and the lower bound constraint, with the policy functions at time $t+1$ taken as given. Since the model features no endogenous state variable, the policymaker solves a sequence of static optimization problems

$$\max_{\pi_t, x_t, g_t, i_t} -\frac{1}{2}\left(\pi_t^2 + \bar{\lambda} x_t^2 + \lambda_g g_t^2\right) \tag{C.2}$$

subject to

$$\pi_t = \kappa x_t + \beta \mathrm{E}_t \pi_{t+1} \tag{C.3}$$

$$x_t = \mathrm{E}_t x_{t+1} + (1-\Gamma)(g_t - \mathrm{g}_{t+1}) - \sigma\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^n\right) \tag{C.4}$$

$$i_t \geq 0 \tag{C.5}$$

The consolidated first order conditions are

$$(\kappa \pi_t + \bar{\lambda} x_t) i_t = 0 \tag{C.6}$$

$$\kappa \pi_t + \bar{\lambda} x_t \leq 0 \tag{C.7}$$

$$i_t \geq 0 \tag{C.8}$$

$$\lambda_g g_t + (1-\Gamma)(\kappa \pi_t + \bar{\lambda} x_t) = 0 \tag{C.9}$$

together with the private sector behavioral constraints.

D Sunspot equilibrium in the baseline model with fiscal stabilization policy

D.1 Proof of Proposition 9

To proof Proposition 9 on the necessary and sufficient condition for existence of the sunspot equilibrium, it is useful to proceed in three steps. Each step is associated with an auxiliary proposition.

Let

$$\tilde{C} := \lambda_g C + \left(\kappa^2 + \bar{\lambda}(1-\beta p_L)\right) \frac{(1-\Gamma)^2}{\kappa\sigma}(1-p_L), \tag{D.1}$$

$$\tilde{D} := \lambda_g D - \beta\bar{\lambda}\frac{(1-\Gamma)^2}{\kappa\sigma}(1-p_L)^2, \tag{D.2}$$

and

$$\begin{aligned}\tilde{E} :=& A\tilde{D} - B\tilde{C} \\ =& \lambda_g E - \frac{(1-\Gamma)^2(1-p_L)}{\kappa\sigma}\left(\kappa^2 + \bar{\lambda}(1-\beta)\right)\left[\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))\right],\end{aligned} \tag{D.3}$$

where A, B, C, D and E are defined in (A.1)–(A.5).

Proposition D.1 *There exists a vector* $\{x_H, \pi_H, i_H, g_H, x_L, \pi_L, i_L, g_L\}$ *that solves the system of linear equations (33)–(40).*

Proof: Rearranging the system of equations (33)–(40) and eliminating x_H, i_H, g_H, x_L, i_L and g_L, we obtain two unknowns for π_H and π_L in two equations

$$\begin{bmatrix} A & B \\ \tilde{C} & \tilde{D} \end{bmatrix} \begin{bmatrix} \pi_L \\ \pi_H \end{bmatrix} = \begin{bmatrix} 0 \\ \lambda_g r^n \end{bmatrix}. \tag{D.4}$$

For what follows, it is useful to show that $\tilde{E} = 0$ is inconsistent with existence of the sunspot equilibrium. Since $B > 0$, we can always write $\pi_H = -A/B\pi_L$. Plugging this into $\tilde{C}\pi_L + \tilde{D}\pi_H = \lambda_g r^n$ and multiplying both sides by B, we get $-\tilde{E}\pi_L = B\lambda_g r^n$. Since the right-hand side of this equation is strictly positive for $\lambda_g > 0$, $\tilde{E} = 0$ is inconsistent with the existence of the sunspot equilibrium. Hence, we can invert the matrix on the left-hand-side of (D.4)

$$\begin{bmatrix} \pi_L \\ \pi_H \end{bmatrix} = \frac{1}{A\tilde{D} - B\tilde{C}} \begin{bmatrix} \tilde{D} & -B \\ -\tilde{C} & A \end{bmatrix} \begin{bmatrix} 0 \\ \lambda_g r^n \end{bmatrix}. \tag{D.5}$$

Thus,

$$\pi_H = \frac{A}{\tilde{E}}\lambda_g r^n \tag{D.6}$$

and

$$\pi_L = \frac{-B}{\tilde{E}} \lambda_g r^n. \tag{D.7}$$

From the Phillips curves in both states, we obtain

$$x_H = \frac{\beta\kappa(1-p_H)}{\tilde{E}} \lambda_g r^n \tag{D.8}$$

and

$$x_L = -\frac{(1-\beta p_L)\kappa^2 + (1-\beta)(1-\beta p_L + \beta(1-p_H))\bar{\lambda}}{\kappa\tilde{E}} \lambda_g r^n. \tag{D.9}$$

Using the target criterion for fiscal policy in the low-confidence state (39), we obtain

$$g_L = \frac{(1-\Gamma)\left(\kappa^2 + \bar{\lambda}(1-\beta)\right)\left(\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))\right)}{\kappa\tilde{E}} r^n. \tag{D.10}$$

Using the consumption Euler equation in the high-confidence state (33), we obtain

$$\begin{aligned} i_H = &\left[1 - \frac{1-p_H}{\tilde{E}}\left(\lambda_g\left(\kappa^2 + \bar{\lambda} + (\kappa^2 + \bar{\lambda}(1-\beta))\frac{1-\beta p_L + \beta(1-p_H)}{\kappa\sigma}\right)\right.\right. \\ &\left.\left. + \frac{(1-\Gamma)^2}{\kappa\sigma}(\kappa^2 + \bar{\lambda}(1-\beta))\left(\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))\right)\right)\right] r^n. \end{aligned} \tag{D.11}$$

Finally, from equations (35) and (40), we have $g_H = 0$, and $i_L = 0$.

Proposition D.2 *Suppose equations (33)–(40) are satisfied. Then* $\bar{\lambda}x_L + \kappa\pi_L < 0$ *if and only if* $\tilde{E} > 0$.

Proof: Using (D.7) and (D.9), we have

$$\bar{\lambda}x_L + \kappa\pi_L = -\frac{\left(\kappa^2 + \bar{\lambda}(1-\beta)\right)\left(\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))\right)}{\kappa\tilde{E}} \lambda_g r^n \tag{D.12}$$

Notice that $\lambda_g r^n > 0$ and $\left(\kappa^2 + \bar{\lambda}(1-\beta)\right)\left(\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))\right) > 0$. Thus, if $\bar{\lambda}x_L + \kappa\pi_L < 0$, then $\tilde{E} > 0$. Similarly, if $\tilde{E} > 0$, then $\bar{\lambda}x_L + \kappa\pi_L < 0$.

Proposition D.3 *Suppose equations (33)–(40) are satisfied and* $\tilde{E} > 0$. *Then* $i_H > 0$ *if and only if* $\lambda_g\Omega(p_L, p_H, \kappa, \sigma, \beta) - (1-\Gamma)^2\frac{1-p_L+1-p_H}{\kappa\sigma}\left[\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))\right] > 0$, *where* $\Omega(\cdot)$ *is defined in (A.15).*

Proof: First, notice that i_H is given by

$$
\begin{aligned}
i_H =& \frac{1-p_H}{\sigma}\left(x_L - x_H + (1-\Gamma)(g_H - g_L)\right) + p_H\pi_H + (1-p_H)\pi_L + r^n \\
=& \frac{\left(\kappa^2 + \bar{\lambda}(1-\beta)\right) r^n}{\tilde{E}}\left[\lambda_g \Omega(p_L, p_H, \kappa, \sigma, \beta) - (1-\Gamma)^2 \frac{1-p_L+1-p_L}{\kappa\sigma}\left(\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))\right)\right],
\end{aligned}
\tag{D.13}
$$

where in the second row we made use of (D.6)–(D.10). Notice also that $\frac{\left(\kappa^2+\bar{\lambda}(1-\beta)\right)r^n}{\tilde{E}} > 0$. Thus, if $\lambda_g\Omega(p_L, p_H, \kappa, \sigma, \beta) - (1-\Gamma)^2 \frac{1-p_L+1-p_H}{\kappa\sigma}\left[\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))\right] > 0$ then $i_H > 0$. Similarly, if $i_H > 0$ then $\lambda_g\Omega(p_L, p_H, \kappa, \sigma, \beta) - (1-\Gamma)^2 \frac{1-p_L+1-p_H}{\kappa\sigma}\left[\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))\right] > 0$.

We are now ready to proof Proposition 9. For notational convenience, define

$$
\tilde{\Omega}(p_L, p_H, \kappa, \sigma, \beta, \Gamma, \lambda_g) = \lambda_g\Omega(p_L, p_H, \kappa, \sigma, \beta) - (1-\Gamma)^2 \frac{1-p_L+1-p_H}{\kappa\sigma}\left[\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))\right]
\tag{D.14}
$$

Proof of "if" part: Suppose that $\tilde{\Omega}(\cdot) > 0$. According to Proposition D.1 there exists a vector $\{x_H, \pi_H, i_H, g_H, x_L, \pi_L, i_L, g_L\}$ that solves equations (33)–(40). Notice that

$$
\begin{aligned}
(\kappa^2 + \bar{\lambda}(1-\beta))\tilde{\Omega}(\cdot) =& \tilde{E} - (1-p_H)\Bigg[\lambda_g\left(\kappa^2 + \bar{\lambda} + (\kappa^2 + \bar{\lambda}(1-\beta))\frac{1-\beta p_L + \beta(1-p_H)}{\kappa\sigma}\right) \\
&+ \frac{(1-\Gamma)^2}{\kappa\sigma}(\kappa^2 + \lambda(1-\beta))\left(\kappa^2 + \lambda(1-\beta p_L + \beta(1-p_H))\right)\Bigg].
\end{aligned}
$$

Hence, $\tilde{\Omega}(\cdot) > 0$ implies $\tilde{E} > 0$. According to Proposition D.2, $\tilde{E} > 0$ implies $\bar{\lambda}x_L + \kappa\pi_L < 0$. According to Proposition D.3, given $\tilde{E} > 0$, $\tilde{\Omega}(\cdot) > 0$ implies $i_H > 0$.

Proof of "only if" part: Suppose that the vector $\{x_H, \pi_H, i_H, g_H, x_L, \pi_L, i_L, g_L\}$ solves (33)–(40), and satisfies $\bar{\lambda}x_L + \kappa\pi_L < 0$ and $i_H > 0$. According to Proposition D.2, $\bar{\lambda}x_L + \kappa\pi_L < 0$ implies $\tilde{E} > 0$. According to Proposition D.3, $\tilde{E} > 0$ and $i_H > 0$ imply $\tilde{\Omega}(\cdot) > 0$.

D.2 Proof of Proposition 10

In the sunspot equilibrium, allocations and prices are given by

$$\pi_L = -\frac{\kappa^2 + \bar{\lambda}(1-\beta p_H)}{\tilde{E}}\lambda_g r^n < 0 \tag{D.15}$$

$$x_L = -\frac{(1-\beta p_L)\kappa^2 + (1-\beta)(1-\beta p_L + \beta(1-p_H))\bar{\lambda}}{\kappa\tilde{E}}\lambda_g r^n < 0 \tag{D.16}$$

$$g_L = \frac{(1-\Gamma)\left(\kappa^2 + \bar{\lambda}(1-\beta)\right)\left(\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))\right)}{\kappa\tilde{E}} r^n > 0 \tag{D.17}$$

$$\pi_H = -\frac{\beta\bar{\lambda}(1-p_H)}{\tilde{E}}\lambda_g r^n \leq 0 \tag{D.18}$$

$$x_H = \frac{\beta\kappa(1-p_H)}{\tilde{E}}\lambda_g r^n \geq 0 \tag{D.19}$$

$$g_H = 0, \tag{D.20}$$

where $\tilde{E} > 0$ is defined in equation (D.3). When $p_H < 0$, $\pi_H < 0$ and $x_H > 0$.

D.3 Proof of Proposition 11

In the sunspot equilibrium, it holds

$$\begin{aligned}
\frac{\partial \pi_L}{\partial \lambda_g} &= \frac{(\kappa^2 + \bar{\lambda}(1-\beta p_H))(1-\Gamma)^2(\kappa\sigma)^{-1}(1-p_L)(\kappa^2 + \bar{\lambda}(1-\beta))\left[\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))\right]}{\tilde{E}^2} r^n > 0 \\
\frac{\partial x_L}{\partial \lambda_g} &= \left[\kappa^2(1-\beta p_L) + \bar{\lambda}(1-\beta)(1-\beta p_L + \beta(1-p_H))\right] \\
&\times \frac{(1-\Gamma)^2(\kappa\sigma)^{-1}(1-p_L)(\kappa^2 + \bar{\lambda}(1-\beta))\left[\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))\right]}{\kappa\tilde{E}^2} r^n > 0 \\
\frac{\partial g_L}{\lambda_g} &= -\frac{(1-\Gamma)\left(\kappa^2 + \bar{\lambda}(1-\beta)\right)\left(\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))\right)}{\kappa\tilde{E}^2} E r^n < 0
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial \pi_H}{\partial \lambda_g} &= \frac{\beta\bar{\lambda}(1-p_H)(1-\Gamma)^2(\kappa\sigma)^{-1}(1-p_L)(\kappa^2 + \bar{\lambda}(1-\beta))\left[\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))\right]}{\tilde{E}^2} r^n \geq 0 \\
\frac{\partial x_H}{\partial \lambda_g} &= -\frac{\beta\kappa(1-p_H)(1-\Gamma)^2(\kappa\sigma)^{-1}(1-p_L)(\kappa^2 + \bar{\lambda}(1-\beta))\left[\kappa^2 + \bar{\lambda}(1-\beta p_L + \beta(1-p_H))\right]}{\tilde{E}^2} r^n \leq 0.
\end{aligned}$$

When $p_H < 1$, $\frac{\partial \pi_H}{\partial \lambda_g} > 0$ and $\frac{\partial x_H}{\partial \lambda_g} < 0$.

D.4 Comparison with an exogenous increase in government spending

In our analysis of fiscal stabilization policy, government spending is an endogenous variable set by an optimizing policymaker. A more common approach in the literature on fiscal policy in expectations-driven liquidity traps is to treat the fiscal policy instrument as an exogenous variable (e.g. Mertens and Ravn, 2014; Bilbiie, 2018). We therefore provide a brief comparison of these two approaches.

Suppose that government spending follows an exogenous process that is perfectly correlated with the sunspot shock, i.e. $g_t = g_L$ if $\xi_t = \xi_L$ and $g_t = g_H$ if $\xi_t = \xi_H$, where $g_L > g_H = 0$. For this case, the definition of the sunspot equilibrium has to be slightly modified.

Definition 5 *The sunspot equilibrium in the model with the sunspot shock and* **exogenous** *fiscal policy is given by a vector $\{x_H, \pi_H, i_H, x_L, \pi_L, i_L\}$ that solves the system of linear equations (33), (34), (36), (37), (38), (40), and satisfies the inequality constraints (41) and (42).*

Assuming that the high-confidence state is absorbing ($p_H = 1$), the low-confidence-state AD and AS curves in the model with exogenous fiscal stabilization policy are given by

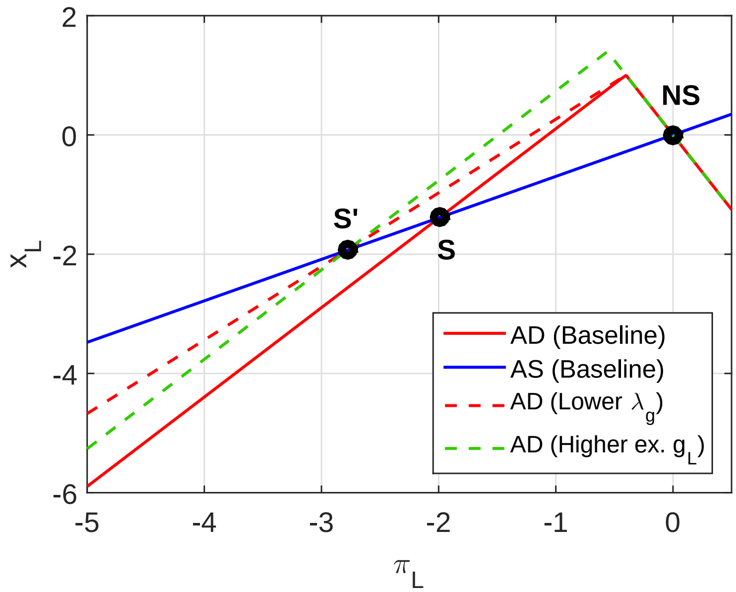
$$\textbf{AD-sunspot g-ex:} \quad x_L = \min\left[\left(\frac{\sigma}{1-p_L}r^n + (1-\Gamma)g_L\right) + \frac{\sigma p_L}{1-p_L}\pi_L, -\frac{\kappa}{\bar{\lambda}}\pi_L\right] \tag{D.21}$$

$$\textbf{AS-sunspot g-ex:} \quad x_L = \frac{1-\beta p_L}{\kappa}\pi_L \tag{D.22}$$

Figure D.1 compares the effects of a reduction in λ_g—which in equilibrium results in an increase in g_L—on the AD-AS curves in the model with endogenous fiscal stabilization policy to those of an increase in g_L in the model with exogenous fiscal policy interventions. For the baseline, it is assumed that $\lambda_g = \infty$ in the model with endogenous fiscal policy and $g_L = 0$ in the model with exogenous fiscal policy. Hence, in the baseline, the low-state AD curve is the same whether fiscal policy is endogenous or exogenous. The sunspot equilibrium outcomes for inflation and the output gap in the baseline are represented by the intersection of the AD curve (red solid line) with the AS curve (blue solid line), marked by point S. When considering an increase in low-state government spending in the model with exogenous fiscal policy, we calibrate the stimulus to be of the same size as the equilibrium increase in government spending that occurs in the model with endogenous fiscal policy in response to the reduction in λ_g.

In the model with endogenous fiscal stabilization policy a change in λ_g affects the slope of the AD curve to the left of the kink. A reduction in λ_g makes the AD curve flatter (red dashed line). In the model with exogenous fiscal policy interventions, a change in low-state government spending instead affects the intercept term in the AD curve and results in a level shift to the left of the kink. An increase in low-state government spending shifts the AD curve upwards (green dashed line). While the sunspot equilibria in the two models are observationally equivalent by construction (see point S'), the two AD curves are not observationally equivalent. Since an exogenous increase in low-state government spending does not affect the slope of the AD curve, a policy intervention of this type is in general unsuited to eliminate the sunspot equilibrium.

Figure D.1: Low-confidence state AD-AS curves: Endogenous vs exogenous fiscal policy

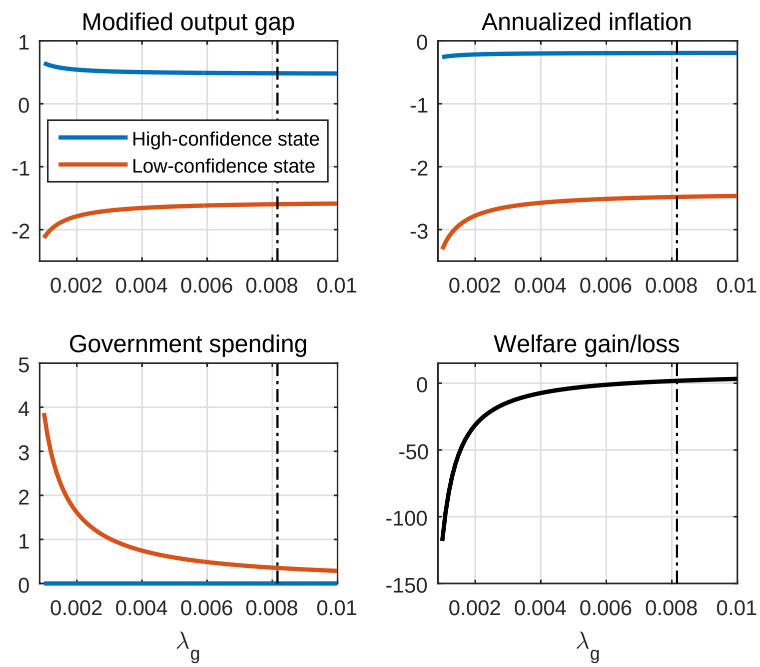


Note: Solid lines: $\lambda_g = \infty$ (fiscal policy endogenous), $g_L = 0$ (fiscal policy exogenous); red dashed line: $\lambda_g = \bar{\lambda}_g/10$ (fiscal policy endogenous); green dashed line: $g_L = 4$ (fiscal policy exogenous). Inflation is expressed in annualized terms.

D.5 Numerical example

This subsection provides a numerical example of how allocations and welfare depend on the relative weight that the policymaker's objective function puts on government spending stabilization λ_g. The parameterisation follows Table 1 except that we account for a non-zero steady-state government spending to output ratio of 0.2, which implies that the inverse of the elasticity of the marginal utility of private consumption with respect to output σ becomes 0.4. The inverse of the elasticity of the marginal utility of public consumption with respect to output ν is set to 0.1, as in Section 5. This implies $\bar{\lambda}_g = 0.0082$. In addition, $p_L = 0.9375$ and $p_H = 0.98$.

Figure D.2: Allocations and welfare as a function of λ_g



Note: Dash-dotted vertical lines indicate the case where the policymaker has the same objective function as society as a whole, i.e. $\lambda_g = \bar{\lambda}_g$. The welfare gain/loss is expressed relative to the welfare level achieved when $\lambda_g = \bar{\lambda}_g$ (in percent).

E Fundamental equilibrium in the baseline model with fiscal stabilization policy

E.1 Existence of the fundamental equilibrium

Proposition 13 *The fundamental equilibrium in the model with government spending and a two-state natural real rate shock exists if and only if*

$$\begin{aligned}\tilde{E}^f <& (1-p_H^f)\frac{r_L^n}{r_H^n}\left[\lambda_g\left(\kappa^2+\bar{\lambda}+(\kappa^2+\bar{\lambda}(1-\beta))\frac{1-\beta p_L^f+\beta(1-p_H^f)}{\kappa\sigma}\right)\right.\\ &\left.+\frac{(1-\Gamma)^2}{\kappa\sigma}(\kappa^2+\bar{\lambda}(1-\beta))\left(\kappa^2+\bar{\lambda}(1-\beta p_L^f+\beta(1-p_H^f))\right)\right] \quad \text{(E.1)}\end{aligned}$$

where $\tilde{E}^f \equiv \lambda_g E^f - \frac{(1-\Gamma)^2(1-p_L^f)}{\kappa\sigma}\left(\kappa^2+\bar{\lambda}(1-\beta)\right)\left[\kappa^2+\bar{\lambda}(1-\beta p_L^f+\beta(1-p_H^f))\right]$.

To proof Proposition 13, we proceed again in three steps. Each step is associated with an auxiliary proposition.

Proposition E.1 *There exists a vector* $\{x_H, \pi_H, i_H, g_H, x_L, \pi_L, i_L, g_L\}$ *that solves the system of linear equations (35), (36), (39), (40), and (43)–(46).*

Proof: Let

$$A^f := -\beta\bar{\lambda}(1 - p_H^f), \tag{E.2}$$
$$B^f := \kappa^2 + \bar{\lambda}(1 - \beta p_H^f), \tag{E.3}$$
$$C^f := \frac{(1 - p_L^f)}{\sigma\kappa}(1 - \beta p_L^f + \beta(1 - p_H^f)) - p_L^f, \tag{E.4}$$
$$D^f := -\frac{(1 - p_L^f)}{\sigma\kappa}(1 - \beta p_L^f + \beta(1 - p_H^f)) - (1 - p_L^f) = -1 - C^f, \tag{E.5}$$

and

$$E^f := A^f D^f - B^f C^f. \tag{E.6}$$

Rearranging the system of equations and eliminating x_H, i_H, g_H, x_L, i_L and g_L, we obtain two unknowns for π_H and π_L in two equations

$$\begin{bmatrix} A^f & B^f \\ \tilde{C}^f & \tilde{D}^f \end{bmatrix} \begin{bmatrix} \pi_L \\ \pi_H \end{bmatrix} = \begin{bmatrix} 0 \\ \lambda_g r_L^n \end{bmatrix}, \tag{E.7}$$

where

$$\tilde{C}^f := \lambda_g C^f + \left(\kappa^2 + \bar{\lambda}(1 - \beta p_L^f)\right) \frac{(1 - \Gamma)^2}{\kappa\sigma}(1 - p_L^f), \tag{E.8}$$
$$\tilde{D}^f := \lambda_g D^f - \beta\bar{\lambda}\frac{(1 - \Gamma)^2}{\kappa\sigma}(1 - p_L^f)^2. \tag{E.9}$$

Define $\tilde{E}^f := A^f \tilde{D}^f - B^f \tilde{C}^f$. For what follows, it is useful to show that $\tilde{E}^f = 0$ is inconsistent with existence of the fundamental equilibrium. Since $B > 0$, we can always write $\pi_H = -A^f/B^f \pi_L$. Plugging this into $\tilde{C}^f \pi_L + \tilde{D}^f \pi_H = \lambda_g r_L^n$ and multiplying both sides by B^f, we get $-\tilde{E}^f \pi_L = B^f \lambda_g r_L^n$. Since the right-hand side of this equation is strictly negative for $\lambda_g > 0$, $\tilde{E}^f = 0$ is inconsistent with the existence of the fundamental equilibrium. Hence, we can invert the matrix on the left-hand-side of (E.7)

$$\begin{bmatrix} \pi_L \\ \pi_H \end{bmatrix} = \frac{1}{A^f \tilde{D}^f - B^f \tilde{C}^f} \begin{bmatrix} \tilde{D}^f & -B^f \\ -\tilde{C}^f & A^f \end{bmatrix} \begin{bmatrix} 0 \\ \lambda_g r_L^n \end{bmatrix}. \tag{E.10}$$

Thus,

$$\pi_H = \frac{A^f}{\tilde{E}^f}\lambda_g r_L^n \tag{E.11}$$

and

$$\pi_L = \frac{-B^f}{\tilde{E}^f}\lambda_g r_L^n. \tag{E.12}$$

From the Phillips curves in both states, we obtain

$$x_H = \frac{\beta\kappa(1-p_H^f)}{\tilde{E}^f}\lambda_g r_L^n \tag{E.13}$$

and

$$x_L = -\frac{(1-\beta p_L^f)\kappa^2 + (1-\beta)(1-\beta p_L^f + \beta(1-p_H^f))\bar{\lambda}}{\kappa\tilde{E}^f}\lambda_g r_L^n. \tag{E.14}$$

Using the target criterion for fiscal policy in the low-confidence state (39), we obtain

$$g_L = \frac{(1-\Gamma)\left(\kappa^2+\bar{\lambda}(1-\beta)\right)\left(\kappa^2+\bar{\lambda}(1-\beta p_L^f+\beta(1-p_H^f))\right)}{\kappa\tilde{E}^f}r_L^n. \tag{E.15}$$

Using the consumption Euler equation in the high-confidence state (43), we obtain

$$\begin{aligned} i_H =& r_H^n - \frac{1-p_H^f}{\tilde{E}^f}\left(\lambda_g\left(\kappa^2+\bar{\lambda}+(\kappa^2+\bar{\lambda}(1-\beta))\frac{1-\beta p_L^f+\beta(1-p_H^f)}{\kappa\sigma}\right)\right. \\ &\left. + \frac{(1-\Gamma)^2}{\kappa\sigma}(\kappa^2+\bar{\lambda}(1-\beta))\left(\kappa^2+\bar{\lambda}(1-\beta p_L^f+\beta(1-p_H^f))\right)\right)r_L^n. \end{aligned} \tag{E.16}$$

Finally, from equations (35) and (40), we have $g_H = 0$, and $i_L = 0$.

Proposition E.2 *Suppose equations (35), (36), (39), (40), and (43)–(46) are satisfied. Then* $\bar{\lambda}x_L + \kappa\pi_L < 0$ *if and only if* $\tilde{E}^f < 0$.

Proof: Using (E.12) and (E.14), we have

$$\bar{\lambda}x_L + \kappa\pi_L = -\frac{\left(\kappa^2+\bar{\lambda}(1-\beta)\right)\left(\kappa^2+\bar{\lambda}(1-\beta p_L^f+\beta(1-p_H^f))\right)}{\kappa\tilde{E}^f}\lambda_g r_L^n \tag{E.17}$$

Notice that $\lambda_g r_L^n < 0$ and $\left(\kappa^2+\bar{\lambda}(1-\beta)\right)\left(\kappa^2+\bar{\lambda}(1-\beta p_L^f+\beta(1-p_H^f))\right) > 0$. Thus, if $\bar{\lambda}x_L + \kappa\pi_L < 0$, then $\tilde{E}^f < 0$. Similarly, if $\tilde{E}^f < 0$, then $\bar{\lambda}x_L + \kappa\pi_L < 0$.

Proposition E.3 *Suppose equations (35), (36), (39), (40), and (43)–(46) are satisfied and* $\tilde{E}^f < 0$. *Then* $i_H > 0$ *if and only if* $\tilde{E}^f < \underline{\tilde{E}}^f$,
where $\underline{\tilde{E}}^f \equiv (1-p_H^f)\frac{r_L^n}{r_H^n}\left[\lambda_g\left(\kappa^2+\bar{\lambda}+(\kappa^2+\bar{\lambda}(1-\beta))\frac{1-\beta p_L^f+\beta(1-p_H^f)}{\kappa\sigma}\right)+\frac{(1-\Gamma)^2}{\kappa\sigma}(\kappa^2+\bar{\lambda}(1-\beta))\left(\kappa^2+\bar{\lambda}(1-\beta p_L^f+\beta(1-p_H^f))\right)\right.$

Proof: First, notice that i_H is given by

$$
\begin{aligned}
i_H =& \frac{1-p_H^f}{\sigma}\left(x_L - x_H + (1-\Gamma)(g_H - g_L)\right) + p_H^f \pi_H + (1-p_H^f)\pi_L + r_H^n \\
=& r_H^n - \frac{1-p_H^f}{\tilde{E}^f}\left[\lambda_g\left(\kappa^2 + \bar{\lambda} + (\kappa^2 + \bar{\lambda}(1-\beta))\frac{1-\beta p_L^f + \beta(1-p_H^f)}{\kappa\sigma}\right)\right. \\
&\left. + \frac{(1-\Gamma)^2}{\kappa\sigma}(\kappa^2 + \bar{\lambda}(1-\beta))\left(\kappa^2 + \bar{\lambda}(1-\beta p_L^f + \beta(1-p_H^f))\right)\right] r_L^n, \qquad \text{(E.18)}
\end{aligned}
$$

The term in square brackets is strictly positive, $r_H^n > 0$, $r_L^n < 0$ and $\tilde{E}^f < 0$. Thus, if $\tilde{E}^f < \underline{\tilde{E}}^f$ then $i_H > 0$. Similarly, if $i_H > 0$ then $\tilde{E}^f < \underline{\tilde{E}}^f$.

We are now ready to proof Proposition 13.

Proof of "if" part: Suppose that $\tilde{E}^f < \underline{\tilde{E}}^f$. According to Proposition E.1 there exists a vector $\{x_H, \pi_H, i_H, g_H, x_L, \pi_L, i_L, g_L\}$ that solves equations (35), (36), (39), (40), and (43)–(46). Notice that $\underline{\tilde{E}}^f < 0$. Hence, $\tilde{E}^f < \underline{\tilde{E}}^f$ implies $\tilde{E}^f < 0$. According to Proposition E.2, $\tilde{E}^f < 0$ implies $\bar{\lambda} x_L + \kappa \pi_L < 0$. According to Proposition E.3, given $\tilde{E}^f < 0$, $\tilde{E}^f < \underline{\tilde{E}}^f$ implies $i_H > 0$.

Proof of "only if" part: Suppose that the vector $\{x_H, \pi_H, i_H, g_H, x_L, \pi_L, i_L, g_L\}$ solves (35), (36), (39), (40), (43)–(46), and satisfies $\bar{\lambda} x_L + \kappa \pi_L < 0$ and $i_H > 0$. According to Proposition E.2, $\bar{\lambda} x_L + \kappa \pi_L < 0$ implies $\tilde{E}^f < 0$. According to Proposition E.3, $\tilde{E}^f < 0$ and $i_H > 0$ imply $\tilde{E}^f < \underline{\tilde{E}}^f$.

E.2 Allocations and prices

In the fundamental equilibrium, allocations and prices are given by:

$$
\begin{aligned}
\pi_L =& -\frac{\kappa^2 + \bar{\lambda}(1-\beta p_H^f)}{\tilde{E}^f}\lambda_g r_L^n < 0 && \text{(E.19)} \\
x_L =& -\frac{(1-\beta p_L^f)\kappa^2 + (1-\beta)(1-\beta p_L^f + \beta(1-p_H^f))\bar{\lambda}}{\kappa \tilde{E}^f}\lambda_g r_L^n < 0 && \text{(E.20)} \\
g_L =& \frac{(1-\Gamma)\left(\kappa^2 + \bar{\lambda}(1-\beta)\right)\left(\kappa^2 + \bar{\lambda}(1-\beta p_L^f + \beta(1-p_H^f))\right)}{\kappa \tilde{E}^f} r_L^n > 0 && \text{(E.21)} \\
\pi_H =& -\frac{\beta\bar{\lambda}(1-p_H^f)}{\tilde{E}^f}\lambda_g r_L^n \leq 0 && \text{(E.22)} \\
x_H =& \frac{\beta\kappa(1-p_H^f)}{\tilde{E}^f}\lambda_g r_L^n \geq 0 && \text{(E.23)} \\
g_H =& 0. && \text{(E.24)}
\end{aligned}
$$

When $p_H^f < 1$, $\pi_H < 0$ and $x_H > 0$.

E.3 Effects of a marginal change in λ_g

The partial derivatives of the policy functions with respect to λ_g are

$$\frac{\partial \pi_L}{\partial \lambda_g} = \frac{(\kappa^2 + \bar{\lambda}(1-\beta p_H^f))(1-\Gamma)^2(\kappa\sigma)^{-1}(1-p_L^f)(\kappa^2+\bar{\lambda}(1-\beta))\left[\kappa^2+\bar{\lambda}(1-\beta p_L^f+\beta(1-p_H^f))\right]}{(\tilde{E}^f)^2} r_L^n < 0$$

$$\frac{\partial x_L}{\partial \lambda_g} = \left[\kappa^2(1-\beta p_L^f) + \bar{\lambda}(1-\beta)(1-\beta p_L^f + \beta(1-p_H^f))\right]$$
$$\times \frac{(1-\Gamma)^2(\kappa\sigma)^{-1}(1-p_L^f)(\kappa^2+\bar{\lambda}(1-\beta))\left[\kappa^2+\bar{\lambda}(1-\beta p_L^f+\beta(1-p_H^f))\right]}{\kappa(\tilde{E}^f)^2} r_L^n < 0$$

$$\frac{\partial g_L}{\lambda_g} = -\frac{(1-\Gamma)\left(\kappa^2+\bar{\lambda}(1-\beta)\right)\left(\kappa^2+\bar{\lambda}(1-\beta p_L^f+\beta(1-p_H^f))\right)}{\kappa(\tilde{E}^f)^2} E^f r_L^n,$$

and

$$\frac{\partial \pi_H}{\partial \lambda_g} = \frac{\beta\bar{\lambda}(1-p_H^f)(1-\Gamma)^2(\kappa\sigma)^{-1}(1-p_L^f)(\kappa^2+\bar{\lambda}(1-\beta))\left[\kappa^2+\bar{\lambda}(1-\beta p_L^f+\beta(1-p_H^f))\right]}{(\tilde{E}^f)^2} r_L^n \leq 0$$

$$\frac{\partial x_H}{\partial \lambda_g} = -\frac{\beta\kappa(1-p_H^f)(1-\Gamma)^2(\kappa\sigma)^{-1}(1-p_L)(\kappa^2+\bar{\lambda}(1-\beta))\left[\kappa^2+\bar{\lambda}(1-\beta p_L^f+\beta(1-p_H^f))\right]}{(\tilde{E}^f)^2} r_L^n \geq 0.$$

When $p_H^f < 1$, $\frac{\partial \pi_H}{\partial \lambda_g} < 0$ and $\frac{\partial x_H}{\partial \lambda_g} > 0$.

F Extension: Model with fundamental and sunspot shocks

In the main body of the paper, we separately consider optimal policy design in the model with a sunspot shock alone, and contrast it with that in the model with a fundamental shock alone. Another way to understand the implication of a sunspot shock on optimal policy design is to examine the implication of introducing a sunspot shock to a model with a fundamental shock for optimal policy design. In this section, we extend our log-linearized baseline model to a model with both fundamental and sunspot shocks. Considering an equilibrium with both fundamental-driven and expectations-driven liquidity traps, we first analyze monetary policy design in the absence of fiscal stabilization policy, and then consider the design of fiscal stabilization policy.

F.1 Setup without fiscal stabilization policy

The model, society's objective function, and the central bank's objective function are identical to those described in the main text. The only difference is that the model features both a sunspot shock and a natural real rate shock.

The transition probabilities for the fundamental (natural real rate) shock are:

$$\text{Prob}\left(r_{t+1}^n = r_H^n | r_t^n = r_H^n\right) = p_H^f \tag{F.1}$$

$$\text{Prob}\left(r_{t+1}^n = r_L^n | r_t^n = r_L^n\right) = p_L^f, \tag{F.2}$$

and the transition probabilities for the sunspot shock are:

$$\text{Prob}\left(\xi_{t+1} = \xi_H | \xi_t = \xi_H\right) = p_H \tag{F.3}$$

$$\text{Prob}\left(\xi_{t+1} = \xi_L | \xi_t = \xi_L\right) = p_L \tag{F.4}$$

These two shocks are uncorrelated. Let x_{IJ} denote x in the I-state for the fundamental shock and the J-state for the sunspot shock.

The equilibrium with both shocks is defined as a vector $\{y_{HH}$, π_{HH}, i_{HH}, y_{LH}, π_{LH}, i_{LH}, y_{HL}, π_{HL}, i_{HL}, y_{LL}, π_{LL}, $i_{LL}\}$ that solves

High-confidence block:

$$\begin{aligned} y_{HH} = & \left[p_H^f p_H y_{HH} + (1-p_H^f) p_H y_{LH} + p_H^f (1-p_H) y_{HL} + (1-p_H^f)(1-p_H) y_{LL}\right] \\ & + \sigma \left[p_H^f p_H \pi_{HH} + (1-p_H^f) p_H \pi_{LH} + p_H^f (1-p_H) \pi_{HL} + (1-p_H^f)(1-p_H) \pi_{LL} - i_{HH} + r_H^n\right] \end{aligned} \tag{F.5}$$

$$\pi_{HH} = \kappa y_{HH} + \beta \left[p_H^f p_H \pi_{HH} + (1-p_H^f) p_H \pi_{LH} + p_H^f (1-p_H) \pi_{HL} + (1-p_H^f)(1-p_H) \pi_{LL}\right] \tag{F.6}$$

$$0 = \kappa(\pi_{HH} - \pi^*) + \lambda y_{HH} \tag{F.7}$$

$$i_{HH} > 0 \tag{F.8}$$

$$\begin{aligned} y_{LH} = & \left[(1-p_L^f) p_H y_{HH} + p_L^f p_H y_{LH} + (1-p_L^f)(1-p_H) y_{HL} + p_L^f (1-p_H) y_{LL}\right] \\ & + \sigma \left[(1-p_L^f) p_H \pi_{HH} + p_L^f p_H \pi_{LH} + (1-p_L^f)(1-p_H) \pi_{HL} + p_L^f (1-p_H) \pi_{LL} - i_{LH} + r_L^n\right] \end{aligned} \tag{F.9}$$

$$\pi_{LH} = \kappa y_{LH} + \beta \left[(1-p_L^f) p_H \pi_{HH} + p_L^f p_H \pi_{LH} + (1-p_L^f)(1-p_H) \pi_{HL} + p_L^f (1-p_H) \pi_{LL}\right] \tag{F.10}$$

$$i_{LH} = 0 \tag{F.11}$$

$$0 > \kappa(\pi_{LH} - \pi^*) + \lambda y_{LH} \tag{F.12}$$

Low-confidence block:

$$y_{HL} = \left[p_H^f(1-p_L)y_{HH} + (1-p_H^f)(1-p_L)y_{LH} + p_H^f p_L y_{HL} + (1-p_H^f)p_L y_{LL}\right] \\ + \sigma\left[p_H^f(1-p_L)\pi_{HH} + (1-p_H^f)(1-p_L)\pi_{LH} + p_H^f p_L \pi_{HL} + (1-p_H^f)p_L \pi_{LL} - i_{HL} + r_H^n\right] \tag{F.13}$$

$$\pi_{HL} = \kappa y_{HL} + \beta\left[p_H^f(1-p_L)\pi_{HH} + (1-p_H^f)(1-p_L)\pi_{LH} + p_H^f p_L \pi_{HL} + (1-p_H^f)p_L \pi_{LL}\right] \tag{F.14}$$

$$i_{HL} = 0 \tag{F.15}$$

$$0 > \kappa(\pi_{HL} - \pi^*) + \lambda y_{HL} \tag{F.16}$$

$$y_{LL} = \left[(1-p_L^f)(1-p_L)y_{HH} + p_L^f(1-p_L)y_{LH} + (1-p_L^f)p_L y_{HL} + p_L^f p_L y_{LL}\right] \\ + \sigma\left[(1-p_L^f)(1-p_L)\pi_{HH} + p_L^f(1-p_L)\pi_{LH} + (1-p_L^f)p_L \pi_{HL} + p_L^f p_L \pi_{LL} - i_{LL} + r_L^n\right] \tag{F.17}$$

$$\pi_{LL} = \kappa y_{LL} + \beta\left[(1-p_L^f)(1-p_L)\pi_{HH} + p_L^f(1-p_L)\pi_{LH} + (1-p_L^f)p_L \pi_{HL} + p_L^f p_L \pi_{LL}\right] \tag{F.18}$$

$$i_{LL} = 0 \tag{F.19}$$

$$0 > \kappa(\pi_{LL} - \pi^*) + \lambda y_{LL} \tag{F.20}$$

Once alloactions are computed, one can solve for V_{HH}, V_{LH}, V_{HL}, V_{LL} using,

$$V_{HH} = u_{HH} + \beta\left[p_H^f p_H V_{HH} + (1-p_H^f)p_H V_{LH} + p_H^f(1-p_H)V_{HL} + (1-p_H^f)(1-p_H)V_{LL}\right] \tag{F.21}$$

$$V_{LH} = u_{LH} + \beta\left[(1-p_L^f)p_H V_{HH} + p_L^f p_H V_{LH} + (1-p_L^f)(1-p_H)V_{HL} + p_L^f(1-p_H)V_{LL}\right] \tag{F.22}$$

$$V_{HL} = u_{HL} + \beta\left[p_H^f(1-p_L)V_{HH} + (1-p_H^f)(1-p_L)V_{LH} + p_H^f p_L V_{HL} + (1-p_H^f)p_L V_{LL}\right] \tag{F.23}$$

$$V_{LL} = u_{LL} + \beta\left[(1-p_L^f)(1-p_L)V_{HH} + p_L^f(1-p_L)V_{LH} + (1-p_L^f)p_L V_{HL} + p_L^f p_L V_{LL}\right] \tag{F.24}$$

Note that welfare is measured by the unconditional expectation of the value function. In the model with both shocks, we can show that welfare is given by

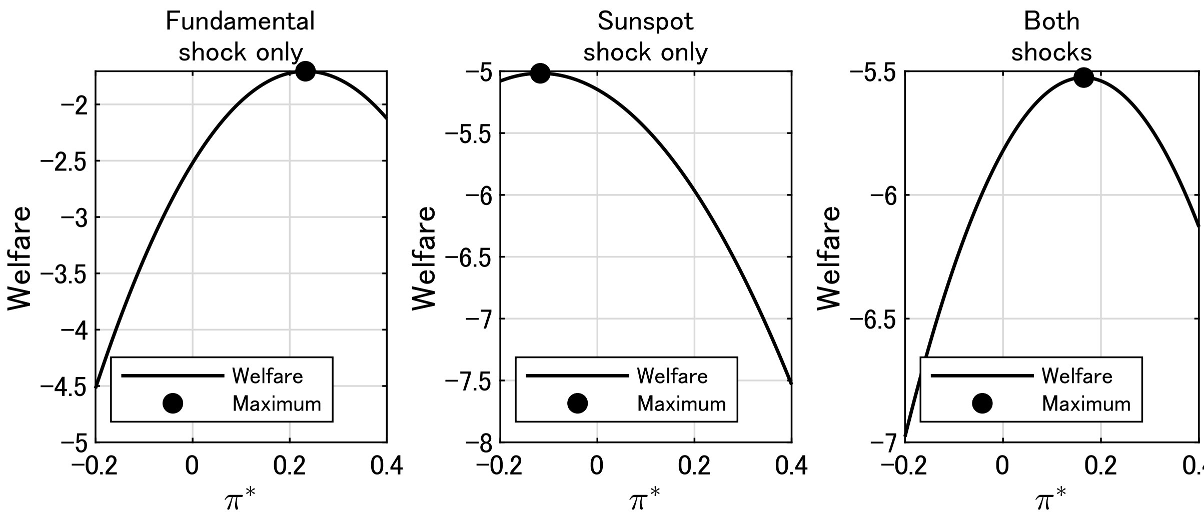
$$W = \frac{1}{1-\beta}\Big[\frac{1-p_L^f}{1-p_L^f+1-p_H^f}\frac{1-p_L}{1-p_L+1-p_H}u_{HH} + \frac{1-p_H^f}{1-p_L^f+1-p_H^f}\frac{1-p_L}{1-p_L+1-p_H}u_{LH} \\ + \frac{1-p_L^f}{1-p_L^f+1-p_H^f}\frac{1-p_H}{1-p_L+1-p_H}u_{HL} + \frac{1-p_H^f}{1-p_L^f+1-p_H^f}\frac{1-p_H}{1-p_L+1-p_H}u_{LL}\Big] \tag{F.25}$$

F.2 Monetary policy frameworks

F.2.1 A non-zero inflation target

Figure F.1 shows welfare as a function of the inflation target π^* in (i) the fundamental equilibrium of the model with the fundamental shock only (left panel), (ii) the sunspot equilibrium of the model with the sunspot shock only (middle panel), and (iii) the equilibrium defined above in the model with both fundamental and sunspot shocks (right panel). The parameterization is identical to the

Figure F.1: Optimal inflation target in the model with a fundamental shock only, the model with a sunspot shock only, and the model with both shocks

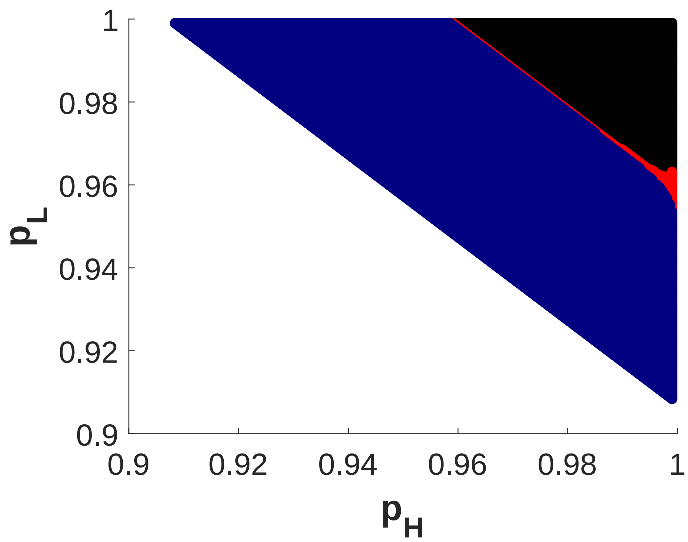


baseline parameterization used in the main body of the paper. For the transition probabilities, we use $p_H = 0.99$, $p_L = 0.95$, $p_H^f = 0.99$, and $p_L^f = 0.86$. Consistent with the results shown in Nakata and Schmidt (2019a), the optimal inflation target is positive in the model with a fundamental shock only. As shown in the main body of the paper, in the sunspot equilibrium of the model with the sunspot shock only, the optimal inflation target can be negative or positive, depending on parameter values. In the example considered here, the optimal inflation target is negative. The right panel shows that the introduction of the sunspot shock to the model with a fundamental shock lowers the optimal inflation target compared to the case with a fundamental shock only, but the optimal target remains strictly positive.

As shown in the main body of the text, the optimal inflation target can be positive in the sunspot equilibrium of the model with the sunspot shock alone when both p_H and p_L are sufficiently close to one. When the persistence of both confidence states is very high, adding the sunspot shock to the model with the fundamental shock increases the optimal inflation target (in the equilibrium defined above) compared to the value of the optimal target in the model with the fundamental shock only. Figure F.2 shows for which pairs of p_H and p_L the optimal inflation target is higher in the equilibrium with both shocks than in the fundamental equilibrium with the fundamental shock only.[53]

[53] Note that, while we vary the values of p_H and p_L, we keep the transition probabilities of the fundamental shock, as well as all other parameter values, unchanged.

Figure F.2: The effect of introducing a sunspot shock on the optimal inflation target



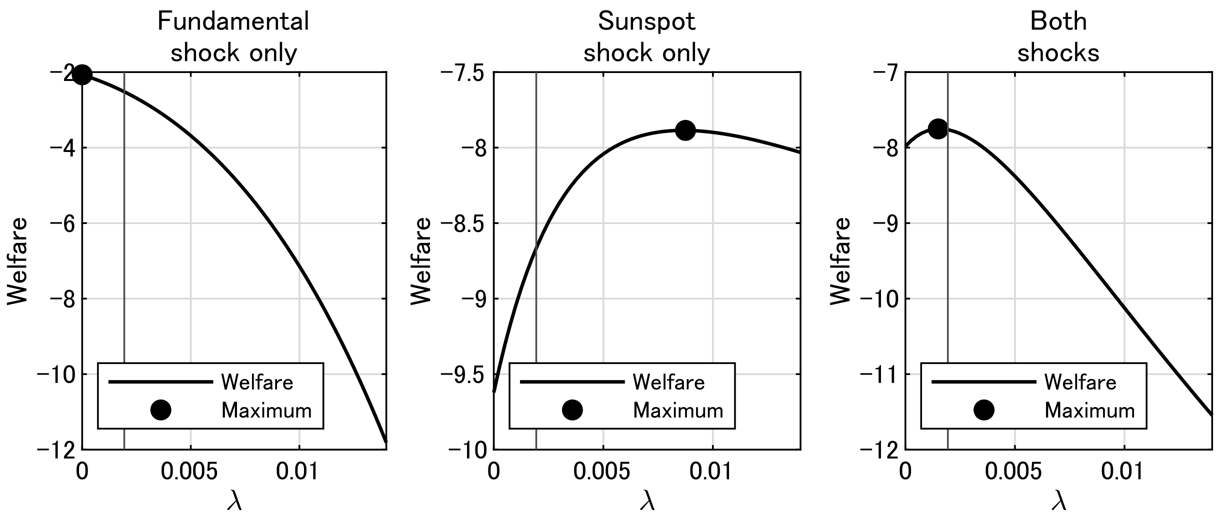
Note: The black area indicates pairs of p_H and p_L under which the optimal inflation target is higher in the model with both shocks than in the model with a fundamental shock only. The red area indicates pairs of p_H and p_L under which the optimal inflation target is the same across the model with both shocks and the model with a fundamental shock only. The blue area indicates pairs of p_H and p_L under which the optimal inflation target is lower in the model with both shocks than in the model with a fundamental shock only. We keep the transition probability matrix for the fundamental shock unchanged as we vary p_H and p_L.

F.2.2 Inflation conservatism

Figure F.3 shows welfare as a function of the relative weight on output gap stabilization in the policymaker's objective function λ in (i) the fundamental equilibrium of the model with the fundamental shock only (left panel), (ii) the sunspot equilibrium of the model with the sunspot shock only (middle panel), and (iii) the equilibrium defined above in the model with both fundamental and sunspot shocks (right panel). Consistent with the results in Nakata and Schmidt (2019a), the optimal weight on the output gap term is zero in the model with the fundamental shock only. For the discussion that follows it is useful to note that this is a corner solution and that, if there were no lower bound on λ imposed, the optimal λ would be negative.[54] Let us call the unconstrained optimal value of λ the shadow optimal weight. On the other hand, as shown in the main body of the paper and in the left panel of Figure F.3, the optimal value of λ is strictly above zero in the sunspot equilibrium of the model with the sunspot shock only. When we introduce the sunspot shock to the model with the fundamental shock, we would expect that the (shadow) optimal λ becomes an average of the (shadow) optimal λ in the model with a fundamental shock only and the optimal λ in the model with a sunspot shock only. This is indeed the case. As shown in the right panel of Figure F.3, for our baseline calibration, the optimal λ is strictly positive, but smaller

[54]We impose λ to be non-negative because a central bank objective function that values output volatility is unrealistic.

Figure F.3: Optimal inflation conservatism in the model with a fundamental shock only, the model with a sunspot shock only, and the model with both shocks



In each panel, the thin black vertical line indicates $\bar{\lambda}$, the society's weight on the output gap volatility term.

than in the sunspot equilibrium of the model with the sunspot shock only.

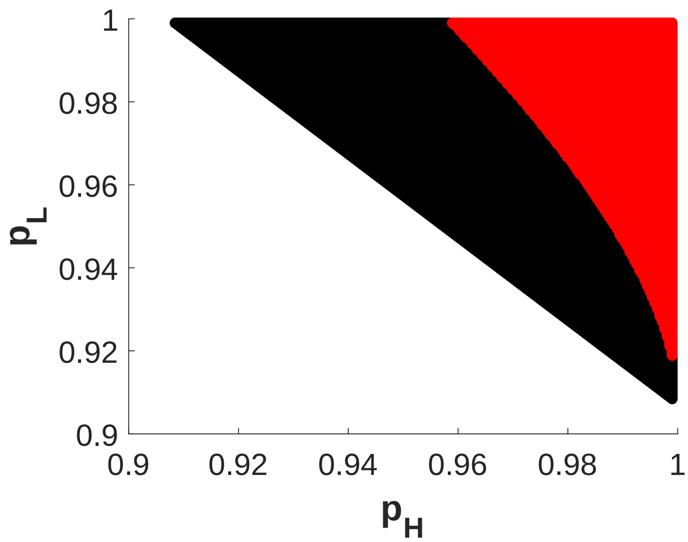
Figure F.4 shows that for a sufficiently high value of p_H—higher than in our baseline calibration—the optimal λ is zero in the model with both shocks, reflecting the fact that the shadow optimal λ is negative in this case.

F.3 Setup with fiscal stabilization policy

We now extend the analysis to include fiscal stabilization policy. The model, society's objective function, the central bank's objective function are the same as in the main text. The only difference is that the model features both a sunspot shock and a natural real rate shock. The structure of these two shocks is the same in the previous subsection.

The equilibrium with fiscal stabilization policy and occasional liquidity traps is defined as a vector $\{x_{HH}, \pi_{HH}, i_{HH}, g_{HH}, x_{LH}, \pi_{LH}, i_{LH}, g_{LH}, x_{HL}, \pi_{HL}, i_{HL}, g_{HL}, x_{LL}, \pi_{LL}, i_{LL}, g_{LL}\}$ that solves the following system of linear equations

Figure F.4: The effect of introducing a sunspot shock on the optimal degree of inflation conservatism



Note: The black area indicates pairs of p_H and p_L under which the optimal λ is higher in the model with both shocks than in the model with a fundamental shock only. The red area indicates pairs of p_H and p_L under which the optimal λ is the same across the model with both shocks and the model with a fundamental shock only. We keep the transition probability matrix for the fundamental shock unchanged as we vary p_H and p_L.

$$\begin{aligned}
x_{HH} = {} & (1-\Gamma)g_{HH} \\
& + p_H^f p_H \left[x_{HH} - (1-\Gamma)g_{HH}\right] + (1-p_H^f)p_H \left[x_{LH} - (1-\Gamma)g_{LH}\right] \\
& + p_H^f (1-p_H) \left[x_{HL} - (1-\Gamma)g_{HL}\right] + (1-p_H^f)(1-p_H) \left[x_{LL} - (1-\Gamma)g_{LL}\right] \\
& + \sigma \left[p_H^f p_H \pi_{HH} + (1-p_H^f)p_H \pi_{LH} + p_H^f(1-p_H)\pi_{HL} + (1-p_H^f)(1-p_H)\pi_{LL} - i_{HH} + r_H^n\right]
\end{aligned} \tag{F.26}$$

$$\pi_{HH} = \kappa x_{HH} + \beta \left[p_H^f p_H \pi_{HH} + (1-p_H^f)p_H \pi_{LH} + p_H^f(1-p_H)\pi_{HL} + (1-p_H^f)(1-p_H)\pi_{LL}\right] \tag{F.27}$$

$$\lambda_g g_{HH} = -(1-\Gamma)\left(\kappa \pi_{HH} + \bar{\lambda} x_{HH}\right) \tag{F.28}$$

$$0 = \kappa \pi_{HH} + \bar{\lambda} x_{HH} \tag{F.29}$$

$$i_{HH} > 0 \tag{F.30}$$

$$\begin{aligned}
x_{LH} = {} & (1-\Gamma)g_{LH} \\
& + (1-p_L^f)p_H\left[x_{HH}-(1-\Gamma)g_{HH}\right] + p_L^f p_H\left[x_{LH}-(1-\Gamma)g_{LH}\right] \\
& + (1-p_L^f)(1-p_H)\left[x_{HL}-(1-\Gamma)g_{HL}\right] + p_L^f(1-p_H)\left[x_{LL}-(1-\Gamma)g_{LL}\right] \\
& + \sigma\left[(1-p_L^f)p_H\pi_{HH} + p_L^f p_H\pi_{LH} + (1-p_L^f)(1-p_H)\pi_{HL} + p_L^f(1-p_H)\pi_{LL} - i_{LH} + r_L^n\right]
\end{aligned} \tag{F.31}$$

$$\pi_{LH} = \kappa x_{LH} + \beta\left[(1-p_L^f)p_H\pi_{HH} + p_L^f p_H\pi_{LH} + (1-p_L^f)(1-p_H)\pi_{HL} + p_L^f(1-p_H)\pi_{LL}\right] \tag{F.32}$$

$$\lambda_g g_{LH} = -(1-\Gamma)\left(\kappa\pi_{LH} + \bar{\lambda}x_{LH}\right) \tag{F.33}$$

$$0 > \kappa\pi_{LH} + \bar{\lambda}x_{LH} \tag{F.34}$$

$$i_{LH} = 0 \tag{F.35}$$

$$\begin{aligned}
x_{HL} = {} & (1-\Gamma)g_{HL} \\
& + p_H^f(1-p_L)\left[x_{HH}-(1-\Gamma)g_{HH}\right] + (1-p_H^f)(1-p_L)\left[x_{LH}-(1-\Gamma)g_{LH}\right] \\
& + p_H^f p_L\left[x_{HL}-(1-\Gamma)g_{HL}\right] + (1-p_H^f)p_L\left[x_{LL}-(1-\Gamma)g_{LL}\right] \\
& + \sigma\left[p_H^f(1-p_L)\pi_{HH} + (1-p_H^f)(1-p_L)\pi_{LH} + p_H^f p_L\pi_{HL} + (1-p_H^f)p_L\pi_{LL} - i_{HL} + r_H^n\right]
\end{aligned} \tag{F.36}$$

$$\pi_{HL} = \kappa x_{HL} + \beta\left[p_H^f(1-p_L)\pi_{HH} + (1-p_H^f)(1-p_L)\pi_{LH} + p_H^f p_L\pi_{HL} + (1-p_H^f)p_L\pi_{LL}\right] \tag{F.37}$$

$$\lambda_g g_{HL} = -(1-\Gamma)\left(\kappa\pi_{HL} + \bar{\lambda}x_{HL}\right) \tag{F.38}$$

$$0 > \kappa\pi_{HL} + \bar{\lambda}x_{HL} \tag{F.39}$$

$$i_{HL} = 0 \tag{F.40}$$

$$\begin{aligned}
x_{LL} = {} & (1-\Gamma)g_{LL} \\
& + (1-p_L^f)(1-p_L)\left[x_{HH}-(1-\Gamma)g_{HH}\right] + p_L^f(1-p_L)\left[x_{LH}-(1-\Gamma)g_{LH}\right] \\
& + (1-p_L^f)p_L\left[x_{HL}-(1-\Gamma)g_{HL}\right] + p_L^f p_L\left[x_{LL}-(1-\Gamma)g_{LL}\right] \\
& + \sigma\left[(1-p_L^f)(1-p_L)\pi_{HH} + p_L^f(1-p_L)\pi_{LH} + (1-p_L^f)p_L\pi_{HL} + p_L^f p_L\pi_{LL} - i_{LL} + r_L^n\right]
\end{aligned} \tag{F.41}$$

$$\pi_{LL} = \kappa x_{LL} + \beta\left[(1-p_L^f)(1-p_L)\pi_{HH} + p_L^f(1-p_L)\pi_{LH} + (1-p_L^f)p_L\pi_{HL} + p_L^f p_L\pi_{LL}\right] \tag{F.42}$$

$$\lambda_g g_{LL} = -(1-\Gamma)\left(\kappa\pi_{LL} + \bar{\lambda}x_{LL}\right) \tag{F.43}$$

$$0 > \kappa\pi_{LL} + \bar{\lambda}x_{LL} \tag{F.44}$$

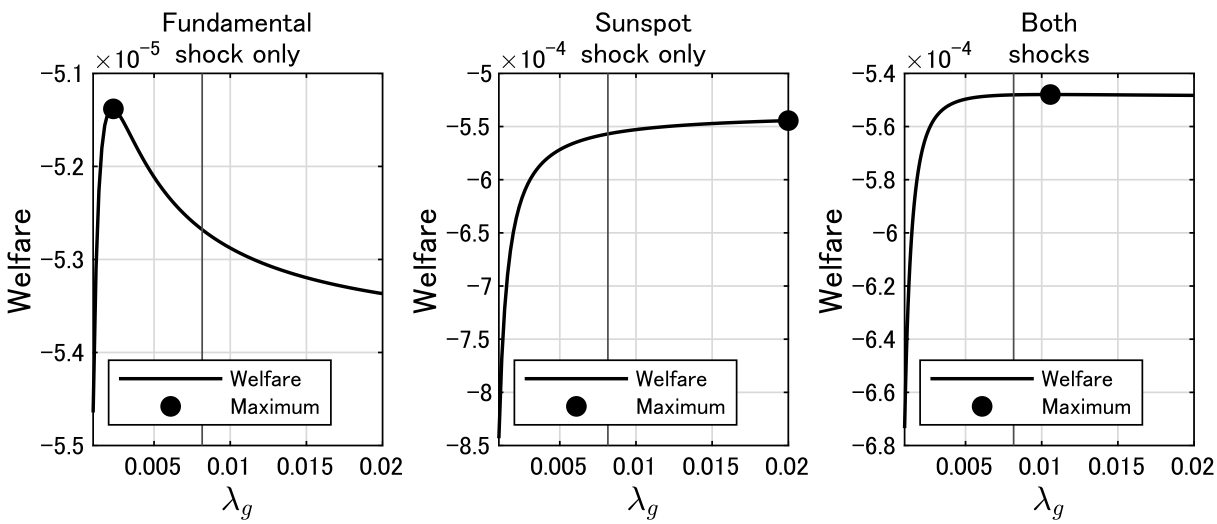
$$i_{LL} = 0 \tag{F.45}$$

Once allocations are computed, the value function and welfare can be computed in a manner similar to that described in the subsection on the model without fiscal stabilization policy.

F.4 Fiscal policy design

Figure F.5 shows welfare as a function of the relative weight on government spending stabilization in the policymaker's objective function λ_g in (i) the fundamental equilibrium of the model with the fundamental shock only (left panel), (ii) the sunspot equilibrium of the model with the sunspot shock only (middle panel), and (iii) the equilibrium defined above in the model with both fundamental and sunspot shocks (right panel). The parameterization is identical to the baseline parameterization used in the main body of the paper, and the transition probabilities are $p_H = 0.99$, $p_L = 0.95$, $p^f_H = 0.99$, and $p^f_L = 0.86$, as in the case without fiscal stabilization policy. Consistent

Figure F.5: Optimal fiscal activism in the model with a fundamental shock only, the model with a sunspot shock only, and the model with both shocks

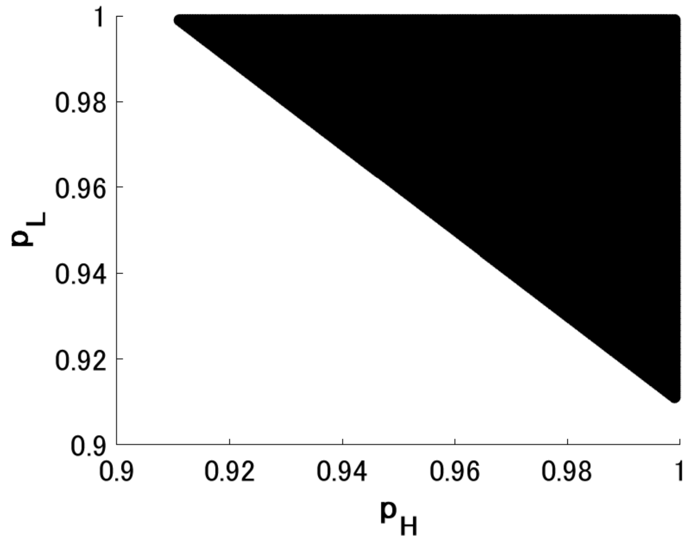


In each panel, the thin black vertical line indicates $\bar{\lambda}_g$, the society's weight on the government spending volatility term.

with the results in Schmidt (2017), in the model with the fundamental shock only, the optimal value of λ_g is lower than the weight on the government spending term in society's objective function $\bar{\lambda}_g$. As shown in the main body of the paper, it is optimal to put a very large value on the government spending stabilization term in the sunspot equilibrium of the model with the sunspot shock model—conditional on the existence of the sunspot equilibrium—to prevent the use of government spending as a stabilization tool. When we introduce a sunspot shock to the model with a fundamental shock, we would expect that the optimal λ_g is somewhere in between the two values from the two single-shock models. This is indeed the case. Furthermore, in our example the optimal λ_g in the model with both shocks is slightly larger than $\bar{\lambda}_g$.

Figure F.6 shows that the optimal λ_g is higher in the model with both shocks than in the model with a fundamental shock alone for any pairs of p_H and p_L.

Figure F.6: The effect of introducing a sunspot shock on the optimal degree of fiscal activism



Note: The black area indicates pairs of p_H and p_L under which the optimal λ_g is higher in the model with both shocks than in the model with a fundamental shock only. We keep the transition probability matrix for the fundamental shock unchanged as we vary p_H and p_L.

G Extension: A fully non-linear model

This section provides a generic description of the fully non-linear model that is used for the analyses presented in Section 6 of the paper and in Sections H and I. The description is generic in the sense that it allows for time variation in government spending and a fundamental shock. When considering the model variant without fiscal stabilization policy, government spending is assumed to be constant (at zero). When considering the model variant with a sunspot shock only, the discount factor shock is assumed to be constant (at one).

G.1 Private sector block of the model

We describe the generic model where government spending is non-zero and potentially time-varying.

Representative household. The representative household maximizes expected lifetime utility

$$V_0 = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\prod_{s=-1}^{t-1} \delta_s \right] \left(\frac{C_t^{1-\frac{1}{\sigma}} - 1}{1-\frac{1}{\sigma}} - \chi_Y \frac{H_t^{1+\eta}}{1+\eta} + \chi_G \frac{G_t^{1-\frac{1}{\nu}}}{1-\frac{1}{\nu}} \right), \tag{G.1}$$

subject to a sequence of budget constraints

$$P_t C_t + \mathrm{E}_t Q_{t,t+1} B_t \leq W_t H_t + B_{t-1} - P_t T_t + P_t D_t \tag{G.2}$$

and a no-Ponzi game condition. The household obtains utility from private consumption C_t and from the provision of public goods G_t and dislikes labor H_t. δ_t is a discount factor shock that alters the weight of felicity in period $t+1$ relative to felicity in period t. When abstracting from fundamental shocks, $\delta_t = 1$ for all t. The household has access to state-contingent, one-period, nominal assets B_t. She earns labor income $W_t H_t$, where W_t is the nominal wage rate, pays lump-sum taxes T_t and receives dividend payments from the intermediate-goods-producing firms D_t. The last two variables are expressed in real terms.

The first-order necessary conditions to the optimization problem are given by

$$R_t^{-1} = \beta\delta_t \mathrm{E}_t \frac{C_{t+1}^{-\frac{1}{\sigma}}}{C_t^{-\frac{1}{\sigma}}} \Pi_{t+1}^{-1} \tag{G.3}$$

$$w_t = \chi_Y H_t^{\eta} C_t^{\frac{1}{\sigma}}, \tag{G.4}$$

as well as the transversality condition

$$\lim_{T\to\infty} \mathrm{E}_t(Q_{t,T} B_T) = 0, \tag{G.5}$$

where $Q_{t,T} \equiv \beta^{T-t}\left[\prod_{s=t}^{T-1}\delta_s\right]\frac{C_T^{-\frac{1}{\sigma}}/P_T}{C_t^{-\frac{1}{\sigma}}/P_t}$ is the stochastic discount factor between periods t and $T \geq t$, $R_t^{-1} = \mathrm{E}_t Q_{t,t+1}$, $\Pi_t = P_t/P_{t-1}$ is the gross inflation rate between periods $t-1$ and t, and $w_t = W_t/P_t$ is the real wage rate.

Firms. The final consumption good is produced under perfect competition using the following technology

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}}, \tag{G.6}$$

where $\theta > 1$ and $Y_t(j)$ denotes the intermediate input j.

The market for intermediate goods exhibits monopolistic competition. Expenditure minimization by the producer of the final good results in the following demand for intermediate good j

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\theta} Y_t, \tag{G.7}$$

where $P_t(j)$ denotes the price charged by firm j and $P_t \equiv \left(\int_0^1 P_t(j)^{1-\theta} dj\right)^{\frac{1}{1-\theta}}$ represents the price for the final consumption good.

Intermediate goods are produced using labor

$$Y_t(j) = H_t(j). \tag{G.8}$$

The intermediate-goods-producing firms are owned by the representative household and face quadratic price adjustment costs. In period t, firm j chooses the price of good j, $P_t(j)$, to maximize expected discounted profits

$$\mathrm{E}_t \sum_{l=0}^{\infty} Q_{t,t+l} \left[Y_{t+l}(j) \left((1+\nu) P_{t+l}(j) - W_{t+l} \right) - \frac{\phi}{2} \left(\frac{P_{t+l}(j)}{P_{t+l-1}(j)} - (\Pi^*)^h \right)^2 P_{t+l}(C_{t+l} + G_{t+l}) \right] \tag{G.9}$$

subject to (G.7). The parameter ν denotes a constant production subsidy that eliminates the distortions arising from monopolistic competition, and G_t is government consumption. We allow for (partial) indexation of price changes to the central bank's inflation target Π^*, where $h \in [0,1]$. The first-order necessary condition for the optimization problem of firm j in period t is

$$\begin{aligned} &(1-\theta)(1+\nu)Y_t(j) + \theta w_t \frac{P_t}{P_t(j)} Y_t(j) - \phi \left(\frac{P_t(j)}{P_{t-1}(j)} - (\Pi^*)^h \right) \frac{P_t}{P_{t-1}(j)} (C_t + G_t) \\ &+ \beta \delta_t \mathrm{E}_t \left(\frac{C_{t+1}^{-\frac{1}{\sigma}}}{P_{t+1}} \frac{P_t}{C_t^{-\frac{1}{\sigma}}} \phi \left(\frac{P_{t+1}(j)}{P_t(j)} - (\Pi^*)^h \right) \frac{P_{t+1}(j) P_{t+1}}{P_t(j)^2} (C_{t+1} + G_{t+1}) \right) = 0. \end{aligned} \tag{G.10}$$

We assume that all firms are symmetric, $P_t(j) = P_t$ for all j. Hence, $Y_t(j) = Y_t$ for all j and $H_t = Y_t$, where $H_t = \int_0^1 H_t(j)dj$. Equation (G.10) can then be written as a New Keynesian Phillips curve

$$Y_t(\chi_Y Y_t^{\eta} C_t^{\frac{1}{\sigma}} - 1) = \frac{\phi}{\theta} \left[\left(\Pi_t - (\Pi^*)^h \right) \Pi_t (C_t + G_t) - \beta \delta_t \mathrm{E}_t \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\sigma}} \left(\Pi_{t+1} - (\Pi^*)^h \right) \Pi_{t+1} (C_{t+1} + G_{t+1}) \right], \tag{G.11}$$

where the real wage rate has been substituted out using the representative household's labor supply condition (G.4) and the production subsidy satisfies $1 + \nu = \frac{\theta}{\theta - 1}$.

Aggregate resource constraint. Total output is used for private consumption, for government spending and for price adjustments

$$Y_t = C_t + G_t + \frac{\phi}{2} \left(\Pi_t - (\Pi^*)^h \right)^2 (C_t + G_t). \tag{G.12}$$

G.2 The policy problem of the benevolent policymaker

Policy is Ricardian. Each period t, the discretionary policymaker chooses the gross inflation rate Π_t, output Y_t, private consumption C_t, government spending G_t, and the gross nominal interest rate R_t to maximize household welfare subject to the consumption Euler equation, the resource constraint, the Phillips curve and the lower bound constraint, with the policy functions at time $t+1$ taken as given. Since the model features no endogenous state variable, the policymaker solves a sequence of static optimization problems. Formally

$$\max_{\Pi_t, Y_t, C_t, G_t, R_t} \frac{C_t^{1-\frac{1}{\sigma}} - 1}{1-\frac{1}{\sigma}} - \chi_Y \frac{Y_t^{1+\eta}}{1+\eta} + \chi_G \frac{G_t^{1-\frac{1}{\nu}} - 1}{1-\frac{1}{\nu}}$$

subject to

$$\frac{C_t^{-\frac{1}{\sigma}}}{R_t} = \beta \delta_t \mathrm{E}_t \frac{C_{t+1}^{-\frac{1}{\sigma}}}{\Pi_{t+1}}$$

$$Y_t = (C_t + G_t)\left(1 + \frac{\phi}{2}\left(\Pi_t - (\Pi^*)^h\right)^2\right)$$

$$Y_t\left(\chi_Y Y_t^\eta C_t^{\frac{1}{\sigma}} - 1\right) = \frac{\phi}{\theta}\left(\left(\Pi_t - (\Pi^*)^h\right)\Pi_t(C_t + G_t) - \beta\delta_t \mathrm{E}_t\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\sigma}}\left(\Pi_{t+1} - (\Pi^*)^h\right)\Pi_{t+1}(C_{t+1} + G_{t+1})\right)$$

$$R_t \geq 1$$

The first order conditions are

$$C_t^{-\frac{1}{\sigma}} - \frac{1}{\sigma}\frac{C_t^{-\frac{1}{\sigma}-1}}{R_t}\lambda_t^{EE} - \left(1 + \frac{\phi}{2}\left(\Pi_t - (\Pi^*)^h\right)^2\right)\lambda_t^{RC} + \Big(\frac{\chi_Y}{\sigma}Y_t^{1+\eta}C_t^{\frac{1}{\sigma}-1} - \frac{\phi}{\theta}\left(\Pi_t - (\Pi^*)^h\right)\Pi_t$$

$$+ \frac{\beta\phi}{\theta\sigma}\delta_t \mathrm{E}_t\left(\frac{C_t}{C_{t+1}}\right)^{\frac{1}{\sigma}} C_t^{-1}\left(\Pi_{t+1} - (\Pi^*)^h\right)\Pi_{t+1}(C_{t+1} + G_{t+1})\Big)\lambda_t^{PC} = 0$$

$$\chi_Y Y_t^\eta - \lambda_t^{RC} - \left(\chi_Y(1+\eta)Y_t^\eta C_t^{\frac{1}{\sigma}} - 1\right)\lambda_t^{PC} = 0$$

$$\left(\Pi_t - (\Pi^*)^h\right)(C_t + G_t)\lambda_t^{RC} + \frac{1}{\theta}\left(2\Pi_t - (\Pi^*)^h\right)(C_t + G_t)\lambda_t^{PC} = 0$$

$$\frac{C_t^{-\frac{1}{\sigma}}}{R_t^2}\lambda_t^{EE} - \lambda_t^{LB} = 0$$

$$\chi_G G_t^{-\frac{1}{\nu}} - \left(1 + \frac{\phi}{2}\left(\Pi_t - (\Pi^*)^h\right)^2\right)\lambda_t^{RC} - \frac{\phi}{\theta}\left(\Pi_t - (\Pi^*)^h\right)\Pi_t\lambda_t^{PC} = 0$$

together with the private sector behavioral constraints. $\lambda_t^{EE}, \lambda_t^{RC}, \lambda_t^{PC}, \lambda_t^{LB}$ are the Lagrange multipliers associated with the constraints.

H Extension: Analyses based on the fully non-linear model without fiscal stabilization policy

This section presents analyses based on the fully non-linear version of the baseline model without fiscal stabilization policy. The model is described in Section G. Here, we consider the case without fiscal stabilization policy, and assume $G_t = 0$ for all t.

H.1 Monetary policy frameworks

We consider the two monetary policy frameworks that are also analyzed in the main body of the paper, a non-zero inflation target and inflation conservatism. The central bank has the following objective

$$V_t^{CB} = (1-\alpha)\left[\frac{C_t^{1-\frac{1}{\sigma}}-1}{1-\frac{1}{\sigma}} - \chi_Y \frac{N_t^{1+\eta}}{1+\eta}\right] + \alpha\left[-\frac{(\Pi_t-\Pi^*)^2}{2}\right] + \beta\delta_t \mathrm{E}_t V_{t+1}^{CB}, \tag{H.1}$$

where $\alpha \in [0,1]$ and Π^* are policy parameters to be set by society when designing the central bank's objective function. When $\alpha = 0$, the central bank's objective function coincides with society's objective function (**??**).

When we analyze the effect of alternative degrees of inflation conservatism, we will set $\Pi^* = 1$ and vary α. When we analyze the effect of alternative values of the inflation target, we will set $\alpha = 1$ and vary Π^*.

The problem of the central bank under discretion at time t is to maximize V_t^{CB} subject to the private-sector equilibrium conditions—summarized in the previous section—taking as give the value and policy functions at time $t+1$.

Let λ_t^{EE}, λ_t^{PC}, λ_t^{RC}, and λ_t^{LB} be the Lagrange multipliers on the Euler equation, the Phillips Curve, the aggregate resource constraint, and the lower bound constraint, where $\lambda_t^{LB} > 0$ when the lower bound is binding, and $\lambda_t^{LB} = 0$ otherwise. The first order necessary conditions of the central bank's problem under discretion are given by

$$\begin{aligned}\frac{\partial L}{\partial C_t}: \quad & 0 = (1-\alpha)C_t^{-\frac{1}{\sigma}} - \frac{1}{\sigma}C_t^{-\frac{1}{\sigma}-1}R_t^{-1}\lambda_t^{EE} \\ & + (1-\frac{1}{\sigma})\phi(\Pi_t-(\Pi^*)^h)\Pi_t C_t^{-\frac{1}{\sigma}}\lambda_t^{PC} + \theta Y_t(-\frac{1}{\sigma})C_t^{-\frac{1}{\sigma}-1}\lambda_t^{PC} \\ & - \left(1+\frac{\phi}{2}(\Pi_t-(\Pi^*)^h)^2\right)\lambda_t^{RC}\end{aligned} \tag{H.2}$$

$$\begin{aligned}\frac{\partial L}{\partial Y_t}: \quad & 0 = -(1-\alpha)\chi_Y Y_t^{\eta} + \theta C_t^{-\frac{1}{\sigma}}\lambda_t^{PC} \\ & - (1+\eta)\theta\chi_Y Y_t^{\eta}\lambda_t^{PC} + \lambda_t^{RC}\end{aligned} \tag{H.3}$$

$$\begin{aligned}\frac{\partial L}{\partial \Pi_t}: \quad & 0 = -\alpha(\Pi_t-\Pi^*) + \phi\left(2\Pi_t-(\Pi^*)^h\right)C_t^{1-\frac{1}{\sigma}}\lambda_t^{PC} \\ & - \phi(\Pi_t-(\Pi^*)^h)C_t\lambda_t^{RC}\end{aligned} \tag{H.4}$$

$$\frac{\partial L}{\partial R_t}: \quad 0 = -C_t^{-\frac{1}{\sigma}}R_t^{-2}\lambda_t^{EE} + \lambda_t^{LB} \tag{H.5}$$

H.2 Sunspot shock and sunspot equilibrium

As in the semi-loglinear model, we assume that the sunspot shock follows a two-state Markov process, $\xi_t \in (\xi_L, \xi_H)$. We refer to state ξ_L and ξ_H as the low- and high-confidence state, respectively. In the model with a sunspot shock, we set $\delta_H = \delta_L = 1$, because there is no fundamental shock. Let p_H and p_L be the persistence of high and low confidence states. We use p_H and p_L to denote the persistence of high- and low-confidence states, respectively.

The sunspot equilibrium with occasionally liquidity traps is defined as a vector $\{C_H$, Y_H, Π_H, R_H, λ_H^{EE},λ_H^{PC}, λ_H^{RC}, λ_H^{LB}, V_H, V_H^{CB}, C_L, Y_L, Π_L, R_L, λ_L^{EE}, λ_L^{PC}, λ_L^{RC}, λ_L^{LB}, V_L, $V_L^{CB}\}$ satisfying the following system of non-linear equations and inequality constraints:

(For the high-confidence state)

$$C_H^{-\frac{1}{\sigma}} R_H^{-1} = \beta\delta_H \Big[p_H C_H^{-\frac{1}{\sigma}} \Pi_H^{-1} + (1-p_H) C_L^{-\frac{1}{\sigma}} \Pi_L^{-1}\Big], \tag{H.6}$$

$$\begin{aligned} &\phi\left(\Pi_H - (\Pi^*)^h\right)\Pi_H C_H^{1-\frac{1}{\sigma}} + \theta Y_H C_H^{-\frac{1}{\sigma}} - \theta\chi_Y Y_H^{1+\eta} \\ = \beta\delta_H \Big[&p_H \phi\left(\Pi_H - (\Pi^*)^h\right)\Pi_H C_H^{1-\frac{1}{\sigma}} + (1-p_H)\phi\left(\Pi_L - (\Pi^*)^h\right)\Pi_L C_L^{1-\frac{1}{\sigma}}\Big], \end{aligned} \tag{H.7}$$

$$Y_H = C_H + \frac{\phi}{2}\left[\Pi_H - (\Pi^*)^h\right]^2 C_H, \tag{H.8}$$

$$R_H > 1. \tag{H.9}$$

$$\begin{aligned} 0 = &(1-\alpha) C_H^{-\frac{1}{\sigma}} - \frac{1}{\sigma} C_H^{-\frac{1}{\sigma}-1} R_t^{-1} \lambda_H^{EE} \\ &+ (1-\frac{1}{\sigma})\phi(\Pi_H - (\Pi^*)^h)\Pi_H C_H^{-\frac{1}{\sigma}} \lambda_H^{PC} + \theta Y_H (-\frac{1}{\sigma}) C_H^{-\frac{1}{\sigma}-1} \lambda_H^{PC} \\ &- \left(1 + \frac{\phi}{2}(\Pi_H - (\Pi^*)^h)^2\right) \lambda_H^{RC} \end{aligned} \tag{H.10}$$

$$\begin{aligned} 0 = &-(1-\alpha)\chi_Y Y_H^\eta + \theta C_H^{-\frac{1}{\sigma}} \lambda_H^{PC} \\ &- (1+\eta)\theta\chi_Y Y_H^\eta \lambda_H^{PC} + \lambda_H^{RC} \end{aligned} \tag{H.11}$$

$$\begin{aligned} 0 = &-\alpha(\Pi_H - \Pi^*) + \phi\left(2\Pi_H - (\Pi^*)^h\right) C_H^{1-\frac{1}{\sigma}} \lambda_H^{PC} \\ &- \phi(\Pi_H - (\Pi^*)^h) C_H \lambda_H^{RC} \end{aligned} \tag{H.12}$$

$$0 = -C_H^{-\frac{1}{\sigma}} R_H^{-2} \lambda_H^{EE} + \lambda_H^{LB} \tag{H.13}$$

$$0 = \lambda_H^{LB} \tag{H.14}$$

(For the low-confidence state)

$$C_L^{-\frac{1}{\sigma}} R_L^{-1} = \beta\delta_L \Big[(1-p_L) C_H^{-\frac{1}{\sigma}} \Pi_H^{-1} + p_L C_L^{-\frac{1}{\sigma}} \Pi_L^{-1}\Big], \tag{H.15}$$

$$\begin{aligned}&\phi \left(\Pi_L - (\Pi^*)^h\right) \Pi_L C_L^{1-\frac{1}{\sigma}} + \theta Y_L C_L^{-\frac{1}{\sigma}} - \theta \chi_Y Y_L^{1+\eta} \\ = \beta\delta_L \Big[(1-p_L)\phi &\left(\Pi_H - (\Pi^*)^h\right) \Pi_H C_H^{1-\frac{1}{\sigma}} + p_L \phi \left(\Pi_L - (\Pi^*)^h\right) \Pi_L C_L^{1-\frac{1}{\sigma}}\Big],\end{aligned} \tag{H.16}$$

$$Y_L = C_L + \frac{\phi}{2}\left[\Pi_L - (\Pi^*)^h\right]^2 C_L, \tag{H.17}$$

$$R_L = 1. \tag{H.18}$$

$$\begin{aligned}0 = &(1-\alpha) C_L^{-\frac{1}{\sigma}} - \frac{1}{\sigma} C_L^{-\frac{1}{\sigma}-1} R_L^{-1} \lambda_L^{EE} \\ &+ (1-\frac{1}{\sigma})\phi(\Pi_L - 1)\Pi_L C_L^{-\frac{1}{\sigma}} \lambda_L^{PC} + \theta Y_L (-\frac{1}{\sigma}) C_L^{-\frac{1}{\sigma}-1} \lambda_L^{PC} \\ &- \left(1 + \frac{\phi}{2}(\Pi_L - (\Pi^*)^h)^2\right) \lambda_L^{RC}\end{aligned} \tag{H.19}$$

$$\begin{aligned}0 = &-(1-\alpha)\chi_Y Y_L^{\eta} + \theta C_L^{-\frac{1}{\sigma}} \lambda_L^{PC} \\ &- (1+\eta)\theta\chi_Y Y_L^{\eta} \lambda_L^{PC} + \lambda_L^{RC}\end{aligned} \tag{H.20}$$

$$\begin{aligned}0 = &-\alpha(\Pi_L - \Pi^*) + \phi\left(2\Pi_L - (\Pi^*)^h\right) C_L^{1-\frac{1}{\sigma}} \lambda_L^{PC} \\ &- \phi(\Pi_L - (\Pi^*)^h) C_L \lambda_L^{RC}\end{aligned} \tag{H.21}$$

$$0 = -C_L^{-\frac{1}{\sigma}} R_L^{-2} \lambda_L^{EE} + \lambda_L^{LB} \tag{H.22}$$

$$0 < \lambda_L^{LB} \tag{H.23}$$

$$V_H = u_H + \beta p_H V_H + \beta(1-p_H)V_L \tag{H.24}$$

$$V_L = u_L + \beta(1-p_L)V_H + \beta p_L V_L \tag{H.25}$$

$$V_H^{CB} = u_H^{CB} + \beta p_H V_H^{CB} + \beta(1-p_H)V_L^{CB} \tag{H.26}$$

$$V_L^{CB} = u_L^{CB} + \beta(1-p_L)V_H^{CB} + \beta p_L V_L^{CB}, \tag{H.27}$$

where u is the period utility flow of households and $u^C B$ is the period utility flow of the central bank.

Welfare is measured by the unconditional expectations of society's value function. In the model with a sunspot shock, welfare is given by

$$W = \frac{1}{1-\beta}\left[\frac{1-p_L}{1-p_L+1-p_H}u_H + \frac{1-p_H}{1-p_L+1-p_H}u_L\right] \tag{H.28}$$

H.3 Fundamental shock and fundamental equilibrium

As in the semi-loglinear model, we assume that the fundamental shock follows a two-state Markov process, $\delta_t \in (\delta_L, \delta_H)$ with $\delta_N = 1$ and $\delta_C > \frac{1}{\beta}$. We refer to state δ_L and δ_H as the low- and high-fundamental state, respectively. We will also refer to the low- and high-fundamental states as the crisis and normal states, respectively. Let p_C and p_N be the persistence of crisis and normal states.

The fundamental equilibrium with occasional liquidity traps is defined as a vector $\{C_N$, Y_N, Π_N, R_N, λ_N^{EE},λ_N^{PC}, λ_N^{RC}, λ_N^{LB}, V_N, V_N^{CB}, C_C, Y_C, Π_C, R_C, λ_C^{EE}, λ_C^{PC}, λ_C^{RC}, λ_C^{LB}, V_C, $V_C^{CB}\}$ satisfying the following system of non-linear equations and inequality constraints:

(For the normal (high-fundamental) state)

$$C_N^{-\frac{1}{\sigma}}R_N^{-1} = \beta\delta_N\Big[p_N C_N^{-\frac{1}{\sigma}}\Pi_N^{-1} + (1-p_N)C_C^{-\frac{1}{\sigma}}\Pi_C^{-1}\Big], \tag{H.29}$$

$$\begin{aligned}&\phi\left(\Pi_N - (\Pi^*)^h\right)\Pi_N C_N^{1-\frac{1}{\sigma}} + \theta Y_N C_N^{-\frac{1}{\sigma}} - \theta\chi_Y Y_N^{1+\eta}\\ =\ &\beta\delta_N\Big[p_N\phi\left(\Pi_N - (\Pi^*)^h\right)\Pi_N C_N^{1-\frac{1}{\sigma}} + (1-p_N)\phi\left(\Pi_C - (\Pi^*)^h\right)\Pi_C C_C^{1-\frac{1}{\sigma}}\Big],\end{aligned} \tag{H.30}$$

$$Y_N = C_N + \frac{\phi}{2}\left[\Pi_N - (\Pi^*)^h\right]^2 C_N, \tag{H.31}$$

$$R_N > 1. \tag{H.32}$$

$$
\begin{aligned}
0 &= (1-\alpha)C_N^{-\frac{1}{\sigma}} - \frac{1}{\sigma}C_N^{-\frac{1}{\sigma}-1}R_t^{-1}\lambda_N^{EE} \\
&+ (1-\frac{1}{\sigma})\phi(\Pi_N - (\Pi^*)^h)\Pi_N C_N^{-\frac{1}{\sigma}}\lambda_N^{PC} + \theta Y_N(-\frac{1}{\sigma})C_N^{-\frac{1}{\sigma}-1}\lambda_N^{PC} \\
&- \left(1 + \frac{\phi}{2}(\Pi_N - (\Pi^*)^h)^2\right)\lambda_N^{RC}
\end{aligned}
\tag{H.33}
$$

$$
\begin{aligned}
0 &= -(1-\alpha)\chi_Y Y_N^{\eta} + \theta C_N^{-\frac{1}{\sigma}}\lambda_N^{PC} \\
&- (1+\eta)\theta\chi_Y Y_N^{\eta}\lambda_N^{PC} + \lambda_N^{RC}
\end{aligned}
\tag{H.34}
$$

$$
\begin{aligned}
0 &= -\alpha(\Pi_N - \Pi^*) + \phi\left(2\Pi_N - (\Pi^*)^h\right)C_N^{1-\frac{1}{\sigma}}\lambda_N^{PC} \\
&- \phi(\Pi_N - 1)C_N\lambda_N^{RC}
\end{aligned}
\tag{H.35}
$$

$$
0 = -C_N^{-\frac{1}{\sigma}}R_N^{-2}\lambda_N^{EE} + \lambda_N^{LB} \tag{H.36}
$$

$$
0 = \lambda_N^{LB} \tag{H.37}
$$

(For the crisis (low-fundamental) state)

$$
C_C^{-\frac{1}{\sigma}}R_C^{-1} = \beta\delta_C\Big[(1-p_C)C_N^{-\frac{1}{\sigma}}\Pi_N^{-1} + p_C C_C^{-\frac{1}{\sigma}}\Pi_C^{-1}\Big], \tag{H.38}
$$

$$
\begin{aligned}
&\phi\left(\Pi_C - (\Pi^*)^h\right)\Pi_C C_C^{1-\frac{1}{\sigma}} + \theta Y_C C_C^{-\frac{1}{\sigma}} - \theta\chi_Y Y_C^{1+\eta} \\
&= \beta\delta_C\Big[(1-p_C)\phi\left(\Pi_N - (\Pi^*)^h\right)\Pi_N C_N^{1-\frac{1}{\sigma}} + p_C\phi\left(\Pi_C - (\Pi^*)^h\right)\Pi_C C_C^{1-\frac{1}{\sigma}}\Big],
\end{aligned}
\tag{H.39}
$$

$$
Y_C = C_C + \frac{\phi}{2}\left[\Pi_C - (\Pi^*)^h\right]^2 C_C, \tag{H.40}
$$

$$
R_C = 1. \tag{H.41}
$$

$$
\begin{aligned}
0 &= (1-\alpha)C_C^{-\frac{1}{\sigma}} - \frac{1}{\sigma}C_C^{-\frac{1}{\sigma}-1}R_C^{-1}\lambda_C^{EE} \\
&+ (1-\frac{1}{\sigma})\phi(\Pi_C - (\Pi^*)^h)\Pi_C C_C^{-\frac{1}{\sigma}}\lambda_C^{PC} + \theta Y_C(-\frac{1}{\sigma})C_C^{-\frac{1}{\sigma}-1}\lambda_C^{PC} \\
&- \left(1 + \frac{\phi}{2}(\Pi_C - (\Pi^*)^h)^2\right)\lambda_C^{RC}
\end{aligned}
\tag{H.42}
$$

$$0 = -(1-\alpha)\chi_Y Y_C^{\eta} + \theta C_C^{-\frac{1}{\sigma}} \lambda_C^{PC}$$
$$- (1+\eta)\theta\chi_Y Y_C^{\eta} \lambda_C^{PC} + \lambda_C^{RC} \tag{H.43}$$

$$0 = -\alpha(\Pi_C - \Pi^*) + \phi \left(2\Pi_C - (\Pi^*)^h\right) C_C^{1-\frac{1}{\sigma}} \lambda_C^{PC}$$
$$- \phi(\Pi_C - (\Pi^*)^h) C_C \lambda_C^{RC} \tag{H.44}$$

$$0 = -C_C^{-\frac{1}{\sigma}} R_C^{-2} \lambda_C^{EE} + \lambda_C^{LB} \tag{H.45}$$

$$0 < \lambda_C^{LB} \tag{H.46}$$

$$V_N = u_N + \beta p_N V_N + \beta(1-p_N)V_C \tag{H.47}$$
$$V_C = u_C + \beta(1-p_C)V_N + \beta p_C V_C \tag{H.48}$$
$$V_N^{CB} = u_N^{CB} + \beta p_N V_N^{CB} + \beta(1-p_N)V_C^{CB} \tag{H.49}$$
$$V_C^{CB} = u_C^{CB} + \beta(1-p_C)V_N^{CB} + \beta p_C V_C^{CB} \tag{H.50}$$

Welfare is measured by the unconditional expectations of society's welfare function. In the model with a fundamental shock, welfare is given by

$$W = \frac{1}{1-\beta}\left[\frac{1-p_C}{1-p_C+1-p_N} u_N + \frac{1-p_N}{1-p_C+1-p_N} u_C\right] \tag{H.51}$$

H.4 Parameter values and model solution

The calibration of the parameters that are unrelated to the shocks is the same as in the main body of the paper (see Table 1), except that we set the discount factor β equal to 0.99375 (rather than 0.9975), which facilitates the solution of the model with the fundamental shock. As in Section 6 of the paper, we calibrate the price-adjustment cost parameter ϕ such that the slope of the Phillips curve, when log-linearized around the intended steady state, is identical to the one in the baseline model setup, and we set $\chi_Y = 1$. When assessing the effect of alternative inflation targets on allocations and welfare, we set the indexation parameter h equal to 0.5. For the sunspot shock, we assume $p_H = 0.995$ and $p_L = 0.99$. For the fundamental shock, we assume $p_N = 0.99$, $p_C = 0.75$, and we have $\delta_N = 1$, $\delta_C = 1.025$. All the takeaways from the non-linear analysis are robust to alternative parameter values. We solve the system of non-linear equations using Matlab's fsolve function.

H.5 Results: Inflation target

Figure H.1 shows how allocations in high- and low-confidence states vary with the inflation target in the sunspot equilibrium of the non-linear model with a sunspot shock. Consistent with the analysis based on the baseline semi-loglinear model, in the sunspot equilibrium, a higher inflation target increases inflation and consumption in the high-confidence state and decreases inflation and consumption in the low-confidence state. Higher (more positive) inflation in the high-confidence state and lower (more negative) inflation in the low-confidence state both contribute to an increase in the amount of resources used for price adjustment. Higher consumption above the efficient level—and inefficiently high labor supply associated with it—in the high-confidence state and lower consumption below the efficient level—and inefficiently low labor supply associated with it—in the low-confidence state are both associated with welfare reductions.

These considerations contribute to making the optimal inflation target negative, as shown in Figure H.2. This welfare result is consistent with the result in the semi-loglinear model that the optimal inflation target can be negative in the model with a sunspot shock, as discussed in the main text.

Figure H.3 shows how allocations in high- and low-fundamental states vary with the inflation target in the non-linear model with a fundamental shock. Also consistent with the analysis based on the semi-loglinear model, in the model with a fundamental shock, a higher inflation target increases inflation in the high-fundamental state and increases inflation (mitigate deflation) in the low-fundamental state. Because a higher inflation target in the high-fundamental state is associated with higher price adjustment costs, society faces a trade-off when choosing the value of the inflation target.

All in all, the optimal inflation target is positive, as shown in Figure H.4. The optimality of a positive inflation target in the non-linear model is consistent with the result in the semi-logilinear model described in the main text.

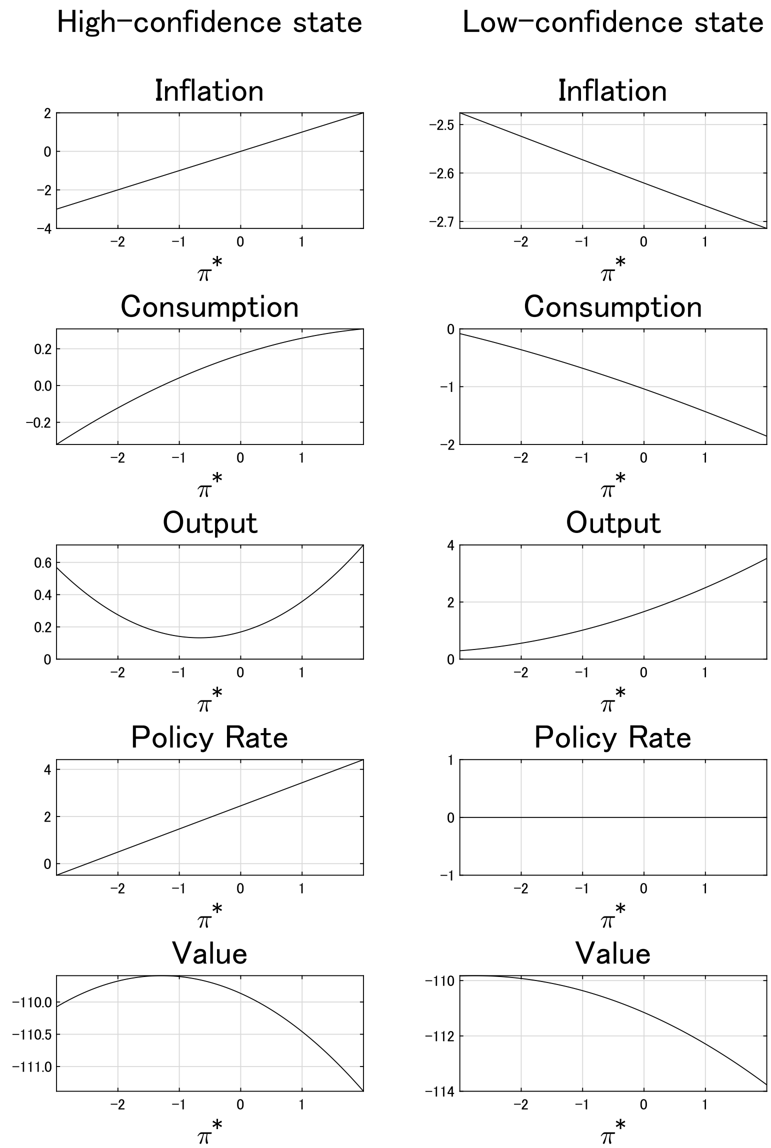
H.6 Results: Inflation conservatism

Figure H.5 shows how allocations in high- and low-confidence states vary with the degree of inflation conservatism in the sunspot equilibrium of the non-linear model with a sunspot shock. Consistent with the analysis based on the semi-loglinear model, in the sunspot equilibrium, a higher weight on the inflation volatility term increases inflation in the high-confidence state and lowers inflation in the low-confidence state. As a result, the welfare implication of putting more weight on the inflation volatility term is ambiguous.

Under our parameterization, the optimal weight on the inflation volatility falls short of one, as shown in Figure H.6. This welfare result is consistent with the result in the semi-loglinear model that it is not optimal to put a full weight on the inflation volatility term, as shown in the main text.

Figure H.7 shows how allocations in high- and low-fundamental states vary with the degree of inflation conservatism in the non-linear model with a fundamental shock. Also consistent with the

Figure H.1: Allocations in the non-linear model with a sunspot shock:
The effects of non-zero inflation target

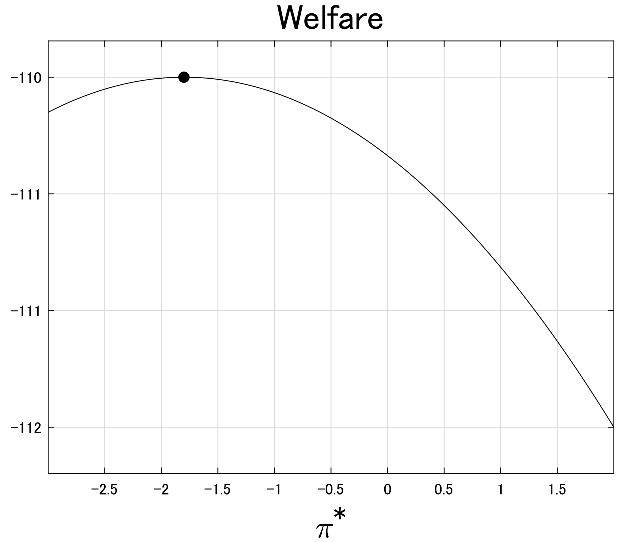


Note: The horizon axis shows the (net) inflation target (annualized percent). Inflation and the policy rate (both in net terms) are expressed as annualized percent. Consumption and output are expressed as percentage deviation from the efficient steady state.

analysis based on the semi-loglinear model, in the model with a fundamental shock, a higher weight on the inflation volatility term increases inflation in both high- and low-fundamental states.

Welfare increases as the weight on the inflation volatility term increases, and the optimal design is to focus on the inflation volatility only, as shown in Figure H.8. The optimality of a strict inflation-conservative central bank in the non-linear model is consistent with the result in the semi-loglinear model.

Figure H.2: Welfare in the non-linear model with a sunspot shock:
The effects of non-zero inflation target



I Extension: Analyses based on the fully non-linear model with fiscal stabilization policy

This section presents analyses based on the fully non-linear version of the baseline model with fiscal stabilization policy. The model is described in Section G.

I.1 Policy framework

The monetary-fiscal policymaker has the following objective

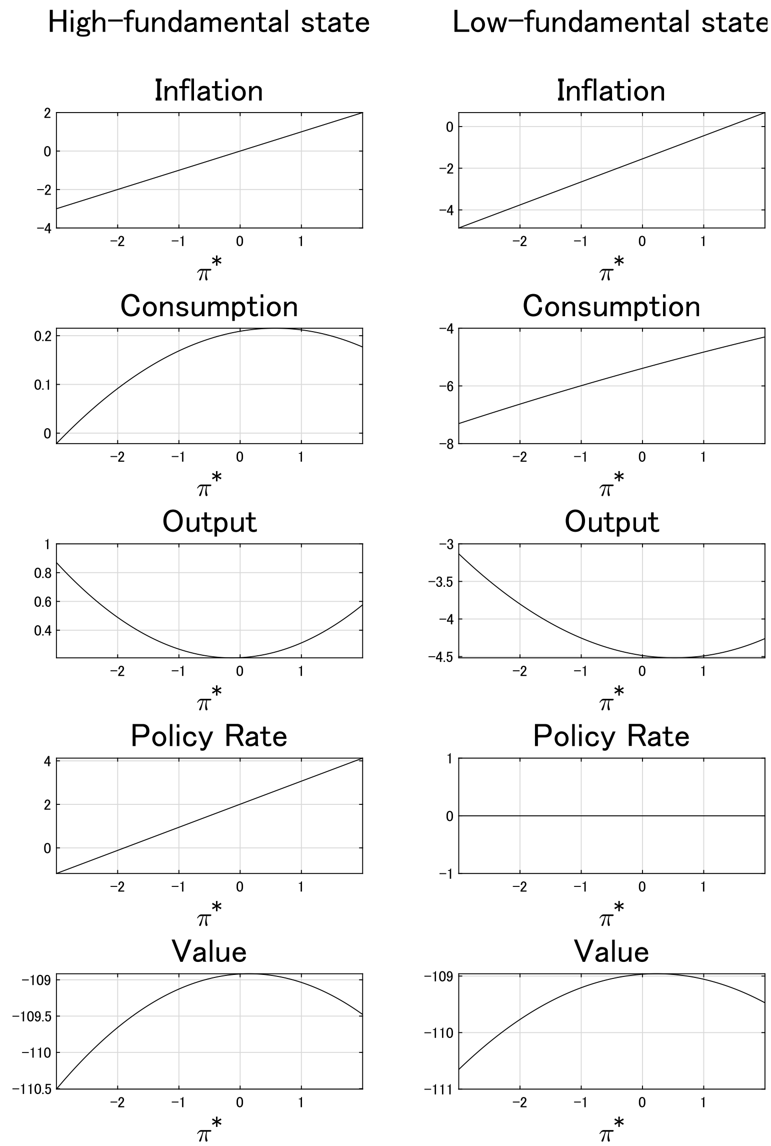
$$V_t^{MF} = (1-\alpha)\left[\frac{C_t^{1-\frac{1}{\sigma}}-1}{1-\frac{1}{\sigma}} - \chi_Y \frac{N_t^{1+\eta}}{1+\eta} + \chi_G \frac{G_t^{1-\frac{1}{\nu}}-1}{1-\frac{1}{\nu}}\right] + \alpha\left[-\frac{(G_t-G^*)^2}{2}\right] + \beta\delta_t \mathrm{E}_t V_{t+1}^{MF} \quad \text{(I.1)}$$

where G^* is the efficient level of government spending that would prevail at the efficient steady state, and $\alpha \in [0,1]$ is the parameter governning the degree of fiscal activism. If $\alpha = 0$, the policymaker has the same objective function as society. A higher α means that the policymaker cares more about stabilizing government spending around its efficient steady state. That is, a higher α means that the policymaker is less fiscally active. If $\alpha = 1$, the policymaker only cares about the stabilization of the government spending.[55]

The problem of the policymaker under discretion at time t is to maximize V_t^{MF} subject to the private-sector equilibrium conditions—summarized in Section G—taking as given the value and policy functions at time $t+1$.

[55]Note that, because $\alpha \in [0,1]$, the most fiscally active case we consider in this formualtion is when $\alpha = 0$, that is, when the policymaker's objective function is the same as that of society.

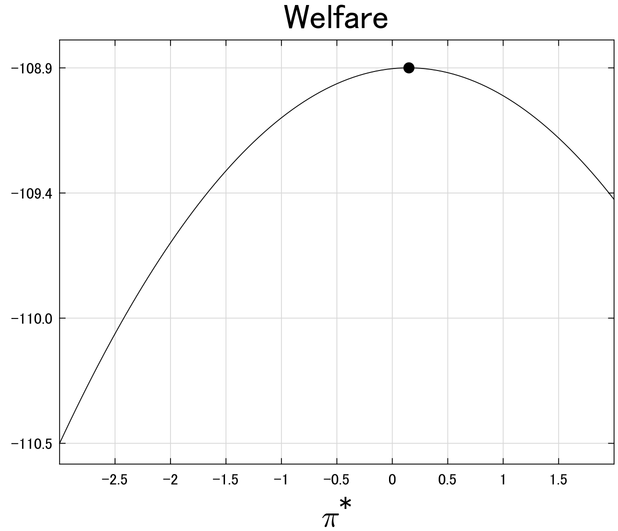
Figure H.3: Allocations in the non-linear model with a fundametal shock:
The effects of non-zero inflation target



Note: The horizon axis shows the inflation target (annualized percent). Inflation and the policy rate are expressed as annualized percent. Consumption and output are expressed as percentage deviation from the efficient steady state.

Let λ_t^{EE}, λ_t^{PC}, λ_t^{RC}, and λ_t^{LB} be the Lagrange multipliers on the Euler equation, the Phillips Curve, the aggregate resource constraint, and the lower bound constraint. The first order necessary

Figure H.4: Welfare in the non-linear model with a fundametal shock:
The effects of non-zero inflation target



conditions of the monetary-fiscal policymaker's problem under discretion are given by

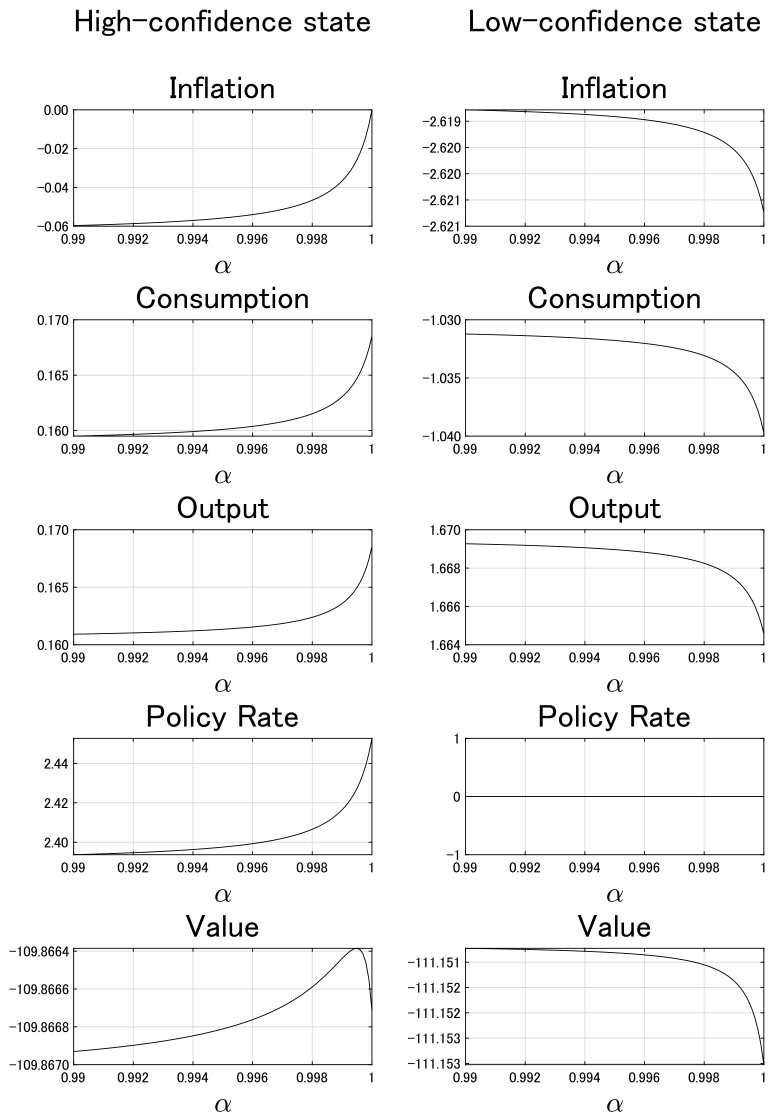
$$\begin{aligned}\frac{\partial L}{\partial C_t}: \quad 0 = &(1-\alpha)C_t^{-\frac{1}{\sigma}} - \frac{1}{\sigma}C_t^{-\frac{1}{\sigma}-1}R_t^{-1}\lambda_t^{EE} \\ &- \frac{1}{\sigma}\phi(\Pi_t - 1)\Pi_t G_t C_t^{-\frac{1}{\sigma}-1}\lambda_t^{PC} \\ &+ (1-\frac{1}{\sigma})\phi(\Pi_t - 1)\Pi_t C_t^{-\frac{1}{\sigma}}\lambda_t^{PC} + \theta Y_t(-\frac{1}{\sigma})C_t^{-\frac{1}{\sigma}-1}\lambda_t^{PC} \\ &- \left(1 + \frac{\phi}{2}(\Pi_t - 1)^2\right)\lambda_t^{RC}\end{aligned} \tag{I.2}$$

$$\begin{aligned}\frac{\partial L}{\partial Y_t}: \quad 0 = &-(1-\alpha)\chi_Y Y_t^{\eta} + \theta C_t^{-\frac{1}{\sigma}}\lambda_t^{PC} \\ &- (1+\eta)\theta\chi_Y Y_t^{\eta}\lambda_t^{PC} + \lambda_t^{RC}\end{aligned} \tag{I.3}$$

$$\begin{aligned}\frac{\partial L}{\partial \Pi_t}: \quad 0 = &\phi\left(2\Pi_t - 1\right)C_t^{1-\frac{1}{\sigma}}\lambda_t^{PC} \\ &+ \phi\left(2\Pi_t - 1\right)G_t C_t^{-\frac{1}{\sigma}}\lambda_t^{PC} \\ &- \phi(\Pi_t - 1)(C_t + G_t)\lambda_t^{RC}\end{aligned} \tag{I.4}$$

$$\begin{aligned}\frac{\partial L}{\partial G_t}: \quad 0 = &(1-\alpha)\chi_G G_t^{-\frac{1}{\nu}} - \alpha(G_t - G^*) + \phi\left(\Pi_t - 1\right)\Pi_t C_t^{-\frac{1}{\sigma}}\lambda_t^{PC} \\ &- \left(1 + \frac{\phi}{2}(\Pi_t - 1)^2\right)\lambda_t^{RC}\end{aligned} \tag{I.5}$$

Figure H.5: Allocations in the non-linear model with a sunspot shock:
The effects of inflation conservatism



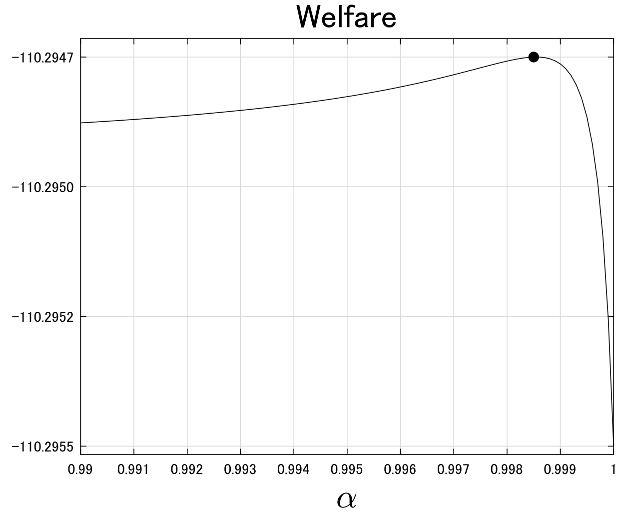
Note: The horizon axis shows the value of α. Inflation and the policy rate are expressed as annualized percent. Consumption and output are expressed as percentage deviation from the efficient steady state.

$$\frac{\partial L}{\partial R_t}: \quad 0 = -C_t^{-\frac{1}{\sigma}} R_t^{-2} \lambda_t^{EE} + \lambda_t^{LB} \tag{I.6}$$

If the lower bound constraint is binding,

$$0 = \lambda_L^{LB} \tag{I.7}$$

Figure H.6: Welfare in the non-linear model with a sunspot shock:
The effects of inflation conservatism



Otherwise,

$$0 \leq \lambda_L^{LB} \tag{I.8}$$

I.2 Sunspot shock and sunspot equilibrium

The process governing the sunspot shock is the same as in the model without fiscal stabilization policy.

The sunspot equilibrium with occasional liquidity traps is defined as a vector $\{C_H,\ Y_H,\ G_H,\ \Pi_H,\ R_H,\ \lambda_H^{EE}, \lambda_H^{PC},\ \lambda_H^{RC},\ \lambda_H^{LB},\ V_H,\ V_H^{CB},\ C_L,\ Y_L,\ G_L,\ \Pi_L,\ R_L,\ \lambda_L^{EE},\ \lambda_L^{PC},\ \lambda_L^{RC},\ \lambda_L^{LB},\ V_L,\ V_L^{CB}\}$ satisfying the following system of non-linear equations and inequality constraints:

(For the high-confidence state)

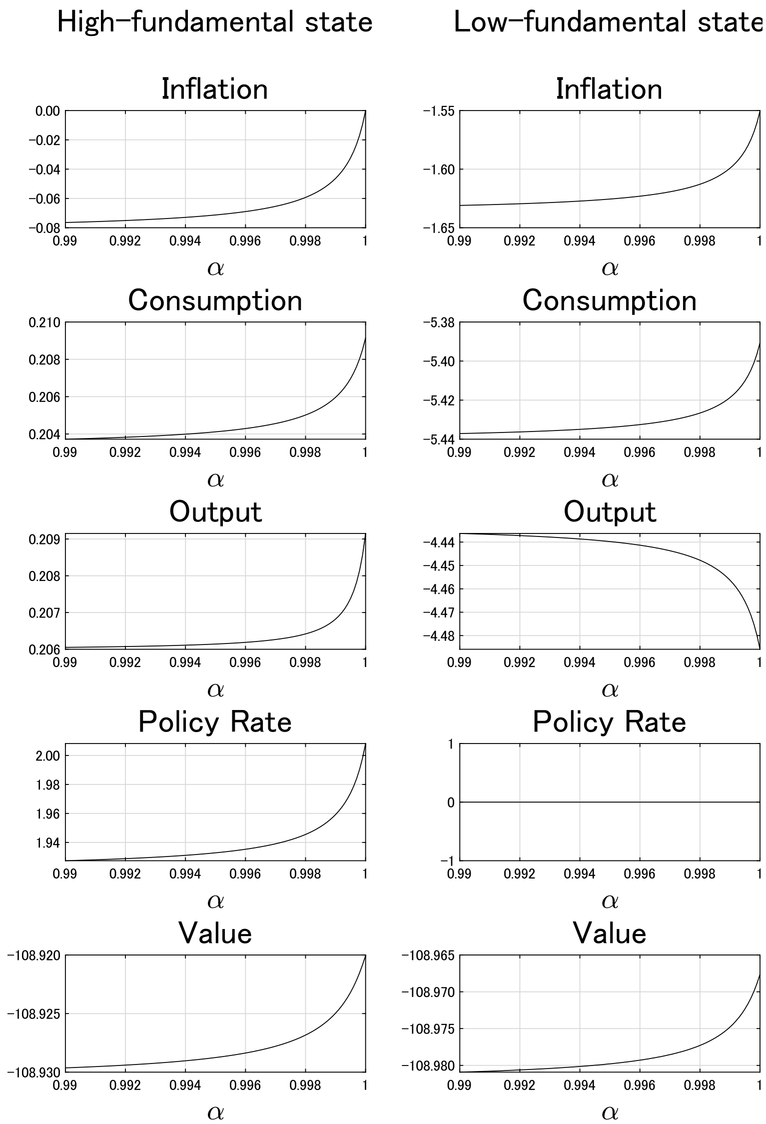
$$C_H^{-\frac{1}{\sigma}} R_H^{-1} = \beta\Big[p_H C_H^{-\frac{1}{\sigma}} \Pi_H^{-1} + (1-p_H) C_L^{-\frac{1}{\sigma}} \Pi_L^{-1}\Big], \tag{I.9}$$

$$\begin{aligned} &\phi\left(\Pi_H - 1\right)\Pi_H (C_H + G_H) C_H^{-\frac{1}{\sigma}} + \theta Y_H C_H^{-\frac{1}{\sigma}} - \theta \chi_Y Y_H^{1+\eta} \\ = \beta\Big[&p_H \phi\left(\Pi_H - 1\right)\Pi_H (C_H + G_H) C_H^{-\frac{1}{\sigma}} + (1-p_H)\phi\left(\Pi_L - 1\right)\Pi_L (C_L + G_L) C_L^{-\frac{1}{\sigma}}\Big], \end{aligned} \tag{I.10}$$

$$Y_H = C_H + G_H + \frac{\phi}{2}\left[\Pi_H - 1\right]^2 (C_H + G_H), \tag{I.11}$$

$$R_H > 1. \tag{I.12}$$

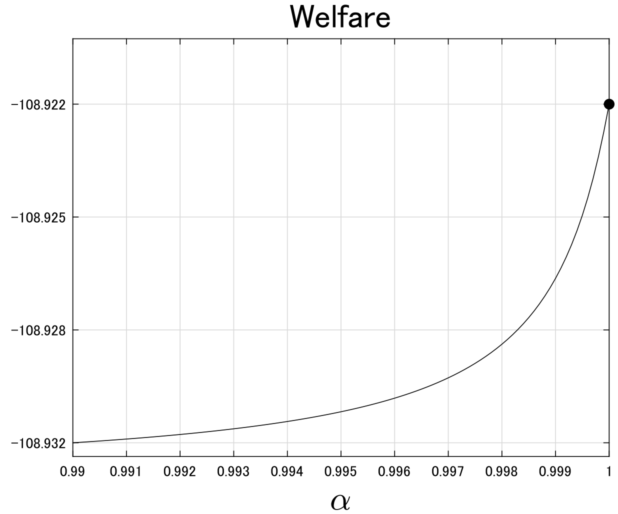
Figure H.7: Allocations in the non-linear model with a fundamental shock:
The effects of inflation conservatism



Note: The horizon axis shows the value of α. Inflation and the policy rate are expressed as annualized percent. Consumption and output are expressed as percentage deviation from the efficient steady state.

$$
\begin{aligned}
0 =\ & (1-\alpha)C_H^{-\frac{1}{\sigma}} - \frac{1}{\sigma}C_H^{-\frac{1}{\sigma}-1}R_H^{-1}\lambda_H^{EE} \\
& - \frac{1}{\sigma}\phi(\Pi_H - 1)\Pi_H G_H C_H^{-\frac{1}{\sigma}-1}\lambda_H^{PC} \\
& + (1-\frac{1}{\sigma})\phi(\Pi_H - 1)\Pi_H C_H^{-\frac{1}{\sigma}}\lambda_H^{PC} + \theta Y_H(-\frac{1}{\sigma})C_H^{-\frac{1}{\sigma}-1}\lambda_H^{PC} \\
& - \left(1 + \frac{\phi}{2}(\Pi_H - 1)^2\right)\lambda_H^{RC}
\end{aligned} \tag{I.13}
$$

Figure H.8: Welfare in the non-linear model with a fundamental shock:
The effects of inflation conservatism



$$
\begin{aligned}
0 = &-(1-\alpha)\chi_Y Y_H^{\eta} + \theta C_H^{-\frac{1}{\sigma}} \lambda_H^{PC} \\
&- (1+\eta)\theta\chi_Y Y_H^{\eta}\lambda_H^{PC} + \lambda_H^{RC}
\end{aligned}
\tag{I.14}
$$

$$
\begin{aligned}
0 = &\ \phi\left(2\Pi_H - 1\right) C_H^{1-\frac{1}{\sigma}} \lambda_H^{PC} \\
&+ \phi\left(2\Pi_H - 1\right) G_H C_H^{-\frac{1}{\sigma}} \lambda_H^{PC} \\
&- \phi(\Pi_H - 1)(C_H + G_H)\lambda_H^{RC}
\end{aligned}
\tag{I.15}
$$

$$
\begin{aligned}
0 = &\ (1-\alpha)\chi_G G_H^{-\frac{1}{\nu}} - \alpha(G_H - G^*) + \phi\left(\Pi_H - 1\right)\Pi_H C_H^{-\frac{1}{\sigma}} \lambda_H^{PC} \\
&- \left(1 + \frac{\phi}{2}(\Pi_H - 1)^2\right)\lambda_H^{RC}
\end{aligned}
\tag{I.16}
$$

$$
0 = -C_H^{-\frac{1}{\sigma}} R_H^{-2} \lambda_H^{EE} + \lambda_H^{LB} \tag{I.17}
$$

$$
0 = \lambda_H^{LB} \tag{I.18}
$$

(For the low-confidence state)

$$
C_L^{-\frac{1}{\sigma}} R_L^{-1} = \beta\Big[(1-p_L) C_H^{-\frac{1}{\sigma}} \Pi_H^{-1} + p_L C_L^{-\frac{1}{\sigma}} \Pi_L^{-1}\Big], \tag{I.19}
$$

$$\begin{aligned}
&\phi\left(\Pi_L-1\right)\Pi_L(C_L+G_L)C_L^{-\frac{1}{\sigma}}+\theta Y_L C_L^{-\frac{1}{\sigma}}-\theta\chi_Y Y_L^{1+\eta} \\
=\ &\beta\Big[(1-p_L)\phi\left(\Pi_H-1\right)\Pi_H(C_H+G_H)C_H^{-\frac{1}{\sigma}}+p_L\phi\left(\Pi_L-1\right)\Pi_L(C_L+G_L)C_L^{-\frac{1}{\sigma}}\Big],
\end{aligned} \tag{I.20}$$

$$Y_L=C_L+G_L+\frac{\phi}{2}\left[\Pi_L-1\right]^2(C_L+G_L), \tag{I.21}$$

$$R_L=1. \tag{I.22}$$

$$\begin{aligned}
0=\ &(1-\alpha)C_L^{-\frac{1}{\sigma}}-\frac{1}{\sigma}C_L^{-\frac{1}{\sigma}-1}R_L^{-1}\lambda_L^{EE} \\
&-\frac{1}{\sigma}\phi(\Pi_L-1)\Pi_L G_L C_L^{-\frac{1}{\sigma}-1}\lambda_L^{PC} \\
&+(1-\frac{1}{\sigma})\phi(\Pi_L-1)\Pi_L C_L^{-\frac{1}{\sigma}}\lambda_L^{PC}+\theta Y_L(-\frac{1}{\sigma})C_L^{-\frac{1}{\sigma}-1}\lambda_L^{PC} \\
&-\left(1+\frac{\phi}{2}(\Pi_L-1)^2\right)\lambda_L^{RC}
\end{aligned} \tag{I.23}$$

$$\begin{aligned}
0=\ &-(1-\alpha)\chi_Y Y_L^{\eta}+\theta C_L^{-\frac{1}{\sigma}}\lambda_L^{PC} \\
&-(1+\eta)\theta\chi_Y Y_L^{\eta}\lambda_L^{PC}+\lambda_L^{RC}
\end{aligned} \tag{I.24}$$

$$\begin{aligned}
0=\ &\phi\left(2\Pi_L-1\right)C_L^{1-\frac{1}{\sigma}}\lambda_L^{PC} \\
&+\phi\left(2\Pi_L-1\right)G_L C_L^{-\frac{1}{\sigma}}\lambda_L^{PC} \\
&-\phi(\Pi_L-1)(C_L+G_L)\lambda_L^{RC}
\end{aligned} \tag{I.25}$$

$$\begin{aligned}
0=\ &(1-\alpha)\chi_G G_L^{-\frac{1}{\nu}}-\alpha(G_L-G^*)+\phi\left(\Pi_L-1\right)\Pi_L C_L^{-\frac{1}{\sigma}}\lambda_L^{PC} \\
&-\left(1+\frac{\phi}{2}(\Pi_L-1)^2\right)\lambda_L^{RC}
\end{aligned} \tag{I.26}$$

$$0=-C_L^{-\frac{1}{\sigma}}R_L^{-2}\lambda_L^{EE}+\lambda_L^{LB} \tag{I.27}$$

$$0<\lambda_L^{LB} \tag{I.28}$$

Once allocations are computed, one can solve for V_H, V_L, V_H^{CB}, V_L^{CB} using

$$V_H = u_H + \beta p_H V_H + \beta(1 - p_H)V_L \tag{I.29}$$
$$V_L = u_L + \beta(1 - p_L)V_H + \beta p_L V_L \tag{I.30}$$
$$V_H^{CB} = u_H^{CB} + \beta p_H V_H^{CB} + \beta(1 - p_H)V_L^{CB} \tag{I.31}$$
$$V_L^{CB} = u_L^{CB} + \beta(1 - p_L)V_H^{CB} + \beta p_L V_L^{CB} \tag{I.32}$$

Welfare is measured by the unconditional expectations of society's value function. In the model with a sunspot shock, welfare is given by

$$W = \frac{1}{1-\beta}\left[\frac{1-p_L}{1-p_L+1-p_H}u_H + \frac{1-p_H}{1-p_L+1-p_H}u_L\right] \tag{I.33}$$

I.3 Fundamental shock and fundamental equilibrium

The process governing the fundamental shock is the same as in the model without fiscal stabilization policy.

The fundamental equilibrium with occasional liquidity traps is defined as a vector $\{C_N,\ Y_N,\ G_N,\ \Pi_N,\ R_N,\ \lambda_N^{EE},\lambda_N^{PC},\ \lambda_N^{RC},\ \lambda_N^{LB},\ V_N,\ V_N^{CB},\ C_C,\ Y_C,\ G_C,\ \Pi_C,\ R_C,\ \lambda_C^{EE},\ \lambda_C^{PC},\ \lambda_C^{RC},\ \lambda_C^{LB},\ V_C,\ V_C^{CB}\}$ satisfying the following system of non-linear equations and inequality constraints:

(For the normal (high-fundamental) state)

$$C_N^{-\frac{1}{\sigma}} R_N^{-1} = \beta\delta_N\Big[p_N C_N^{-\frac{1}{\sigma}}\Pi_N^{-1} + (1-p_N)C_C^{-\frac{1}{\sigma}}\Pi_C^{-1}\Big], \tag{I.34}$$

$$\begin{aligned}&\phi\left(\Pi_N-1\right)\Pi_N(C_N+G_N)C_N^{-\frac{1}{\sigma}} + \theta Y_N C_N^{-\frac{1}{\sigma}} - \theta\chi_Y Y_N^{1+\eta}\\ = \beta\delta_N\Big[&p_N\phi\left(\Pi_N-1\right)\Pi_N(C_N+G_N)C_N^{-\frac{1}{\sigma}} + (1-p_N)\phi\left(\Pi_C-1\right)\Pi_C(C_C+G_C)C_C^{-\frac{1}{\sigma}}\Big],\end{aligned} \tag{I.35}$$

$$Y_N = C_N + G_N + \frac{\phi}{2}\left[\Pi_N - 1\right]^2(C_N+G_N), \tag{I.36}$$

$$R_N > 1. \tag{I.37}$$

$$\begin{aligned}0 = {}&(1-\alpha)C_N^{-\frac{1}{\sigma}} - \frac{1}{\sigma}C_N^{-\frac{1}{\sigma}-1}R_N^{-1}\lambda_N^{EE}\\ &- \frac{1}{\sigma}\phi(\Pi_N-1)\Pi_N G_N C_N^{-\frac{1}{\sigma}-1}\lambda_N^{PC}\\ &+ (1-\frac{1}{\sigma})\phi(\Pi_N-1)\Pi_N C_N^{-\frac{1}{\sigma}}\lambda_N^{PC} + \theta Y_N(-\frac{1}{\sigma})C_N^{-\frac{1}{\sigma}-1}\lambda_N^{PC}\\ &- \left(1+\frac{\phi}{2}(\Pi_N-1)^2\right)\lambda_N^{RC}\end{aligned} \tag{I.38}$$

$$
\begin{aligned}
0 = &-(1-\alpha)\chi_Y Y_N^{\eta} + \theta C_N^{-\frac{1}{\sigma}} \lambda_N^{PC} \\
&-(1+\eta)\theta\chi_Y Y_N^{\eta}\lambda_N^{PC} + \lambda_N^{RC}
\end{aligned} \tag{I.39}
$$

$$
\begin{aligned}
0 = &\,\phi\left(2\Pi_N - 1\right) C_N^{1-\frac{1}{\sigma}} \lambda_N^{PC} \\
&+ \phi\left(2\Pi_N - 1\right) G_N C_N^{-\frac{1}{\sigma}} \lambda_N^{PC} \\
&- \phi(\Pi_N - 1)(C_N + G_N)\lambda_N^{RC}
\end{aligned} \tag{I.40}
$$

$$
\begin{aligned}
0 = &\,(1-\alpha)\chi_G G_N^{-\frac{1}{\nu}} - \alpha(G_N - G^*) + \phi\left(\Pi_N - 1\right)\Pi_N C_N^{-\frac{1}{\sigma}} \lambda_N^{PC} \\
&- \left(1 + \frac{\phi}{2}(\Pi_N - 1)^2\right)\lambda_N^{RC}
\end{aligned} \tag{I.41}
$$

$$
0 = -C_N^{-\frac{1}{\sigma}} R_N^{-2} \lambda_N^{EE} + \lambda_N^{LB} \tag{I.42}
$$

$$
0 = \lambda_N^{LB} \tag{I.43}
$$

(For the crisis (low-fundamental) state)

$$
C_C^{-\frac{1}{\sigma}} R_C^{-1} = \beta\delta_C\Big[(1-p_C)C_N^{-\frac{1}{\sigma}}\Pi_N^{-1} + p_C C_C^{-\frac{1}{\sigma}}\Pi_C^{-1}\Big], \tag{I.44}
$$

$$
\begin{aligned}
&\phi\left(\Pi_C - 1\right)\Pi_C(C_C + G_C)C_C^{-\frac{1}{\sigma}} + \theta Y_C C_C^{-\frac{1}{\sigma}} - \theta\chi_Y Y_C^{1+\eta} \\
&= \beta\delta_C\Big[(1-p_C)\phi\left(\Pi_N - 1\right)\Pi_N(C_N + G_N)C_N^{-\frac{1}{\sigma}} + p_C\phi\left(\Pi_C - 1\right)\Pi_C(C_C + G_C)C_C^{-\frac{1}{\sigma}}\Big],
\end{aligned} \tag{I.45}
$$

$$
Y_C = C_C + G_C + \frac{\phi}{2}\left[\Pi_C - 1\right]^2 (C_C + G_C), \tag{I.46}
$$

$$
R_C = 1. \tag{I.47}
$$

$$
\begin{aligned}
0 = &\,(1-\alpha)C_C^{-\frac{1}{\sigma}} - \frac{1}{\sigma}C_C^{-\frac{1}{\sigma}-1} R_C^{-1}\lambda_C^{EE} \\
&- \frac{1}{\sigma}\phi(\Pi_C - 1)\Pi_C G_C C_C^{-\frac{1}{\sigma}-1}\lambda_C^{PC} \\
&+ (1 - \frac{1}{\sigma})\phi(\Pi_C - 1)\Pi_C C_C^{-\frac{1}{\sigma}}\lambda_C^{PC} + \theta Y_C(-\frac{1}{\sigma})C_C^{-\frac{1}{\sigma}-1}\lambda_C^{PC} \\
&- \left(1 + \frac{\phi}{2}(\Pi_C - 1)^2\right)\lambda_C^{RC}
\end{aligned} \tag{I.48}
$$

$$0 = -(1-\alpha)\chi_Y Y_C^{\eta} + \theta C_C^{-\frac{1}{\sigma}} \lambda_C^{PC}$$
$$- (1+\eta)\theta \chi_Y Y_C^{\eta} \lambda_C^{PC} + \lambda_C^{RC} \tag{I.49}$$

$$0 = \phi \left(2\Pi_C - 1\right) C_C^{1-\frac{1}{\sigma}} \lambda_C^{PC}$$
$$+ \phi \left(2\Pi_C - 1\right) G_C C_C^{-\frac{1}{\sigma}} \lambda_C^{PC}$$
$$- \phi(\Pi_C - 1)(C_C + G_C)\lambda_C^{RC} \tag{I.50}$$

$$0 = (1-\alpha)\chi_G G_C^{-\frac{1}{\nu}} - \alpha(G_C - G^*) + \phi \left(\Pi_C - 1\right) \Pi_C C_C^{-\frac{1}{\sigma}} \lambda_C^{PC}$$
$$- \left(1 + \frac{\phi}{2}(\Pi_C - 1)^2\right) \lambda_C^{RC} \tag{I.51}$$

$$0 = -C_C^{-\frac{1}{\sigma}} R_C^{-2} \lambda_C^{EE} + \lambda_C^{LB} \tag{I.52}$$

$$0 < \lambda_C^{LB} \tag{I.53}$$

Once allocations are computed, one can solve for V_N, V_C, V_N^{CB}, V_C^{CB} using,

$$V_N = u_N + \beta p_N V_N + \beta(1-p_N)V_C \tag{I.54}$$
$$V_C = u_C + \beta(1-p_C)V_N + \beta p_C V_C \tag{I.55}$$
$$V_N^{CB} = u_N^{CB} + \beta p_N V_N^{CB} + \beta(1-p_N)V_C^{CB} \tag{I.56}$$
$$V_C^{CB} = u_C^{CB} + \beta(1-p_C)V_N^{CB} + \beta p_C V_C^{CB} \tag{I.57}$$

Welfare is measured by the unconditional expectations of society's value function. In the model with a fundamental shock, welfare is given by

$$W = \frac{1}{1-\beta}\left[\frac{1-p_C}{1-p_C+1-p_N}u_N + \frac{1-p_N}{1-p_C+1-p_N}u_C\right] \tag{I.58}$$

I.4 Parameter values and model solution

The calibration of the parameters that are unrelated to the shocks is the same as in the main body of the paper (see Table 1), except that we set the discount factor β equal to 0.99375 (rather than 0.9975), which facilitates the solution of the model with the fundamental shock. As in Section 6 of the paper, we calibrate the price-adjustment cost parameter ϕ such that the slope of the Phillips curve, when log-linearized around the intended steady state, is identical to the one in the baseline model setup, and we set χ_Y and χ_G such that total output equals one in the intended steady state, and the steady-state ration of government spending to output equals 0.2. When assessing the effect of alternative inflation targets on allocations and welfare, we set the indexation parameter h equal to 0.5. For the sunspot shock, we assume $p_H = 0.995$ and $p_L = 0.99$. For the fundamental shock, we assume $p_N = 0.99$, $p_C = 0.75$, and we have $\delta_N = 1$, $\delta_C = 1.025$. All the takeaways from the non-linear analysis are robust to alternative parameter values. We solve the system of non-linear equation using Matlab's `fsolve` function.

I.5 Results: Fiscal activism

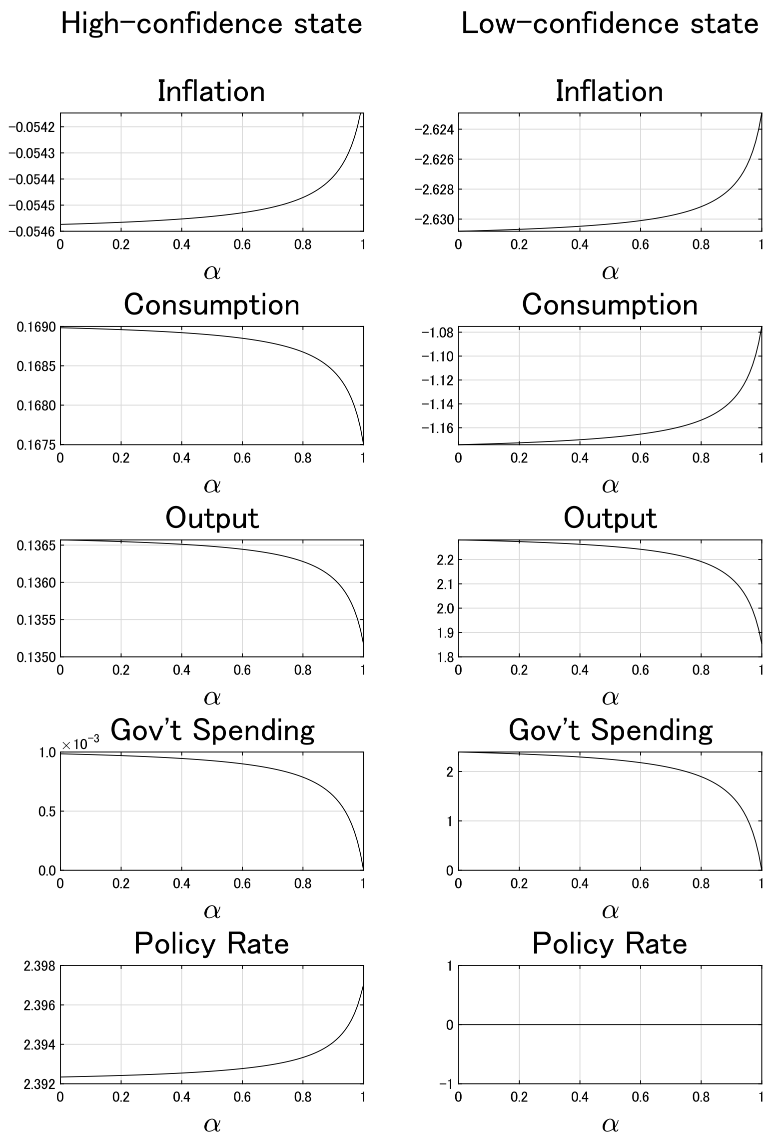
Figure I.1 shows how allocations in high- and low-confidence states vary with the relative weight on the government spending stabilization term α in the sunspot equilibrium of the non-linear model with a sunspot shock. Consistent with the analysis based on the baseline semi-loglinear model, in the sunspot equilibrium, a higher weight on the government spending stabilization term (less fiscal activism) results in a less aggressive use of government spending as a stabilization tool at the lower bound, higher inflation (less deflation) in the low-confidence state and higher inflation (less deflation) in the high-confidence state.

Thus, conditional on the existence of the sunspot equilibrium, it is optimal to focus on minimizing the volatility in government spending, as shown in Figure I.2. This result is consistent with the result in the semi-loglinear model shown in the main text.

Figure I.3 shows how allocations in high- and low-fundamental states vary with α in the non-linear model with a fundamental shock. Also consistent with the analysis based on the semi-loglinear model, in the model with a fundamental shock, a higher weight on the government spending stabilization term lowers government spending and inflation in the low-fundamental state, and increases the deflationary bias in the high-fundamental state.

Welfare declines as the weight on the government spending stabilization increases, and the optimal weight is zero, as shown in Figure I.4.

Figure I.1: Allocations in the non-linear model with a sunspot shock: The effects of fiscal activism



Note: The horizon axis shows the value of α. Inflation and the policy rate are expressed as annualized percent. Consumption, output, and government spending are expressed as percentage deviation from the efficient steady state.

Figure I.2: Welfare in the non-linear model with a sunspot shock:
The effects of fiscal activism

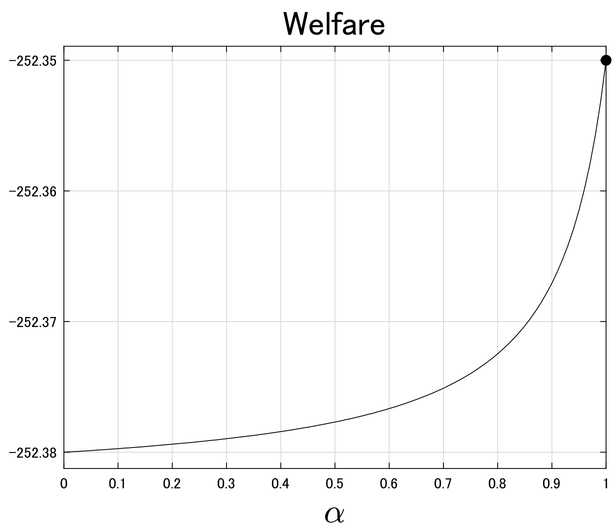
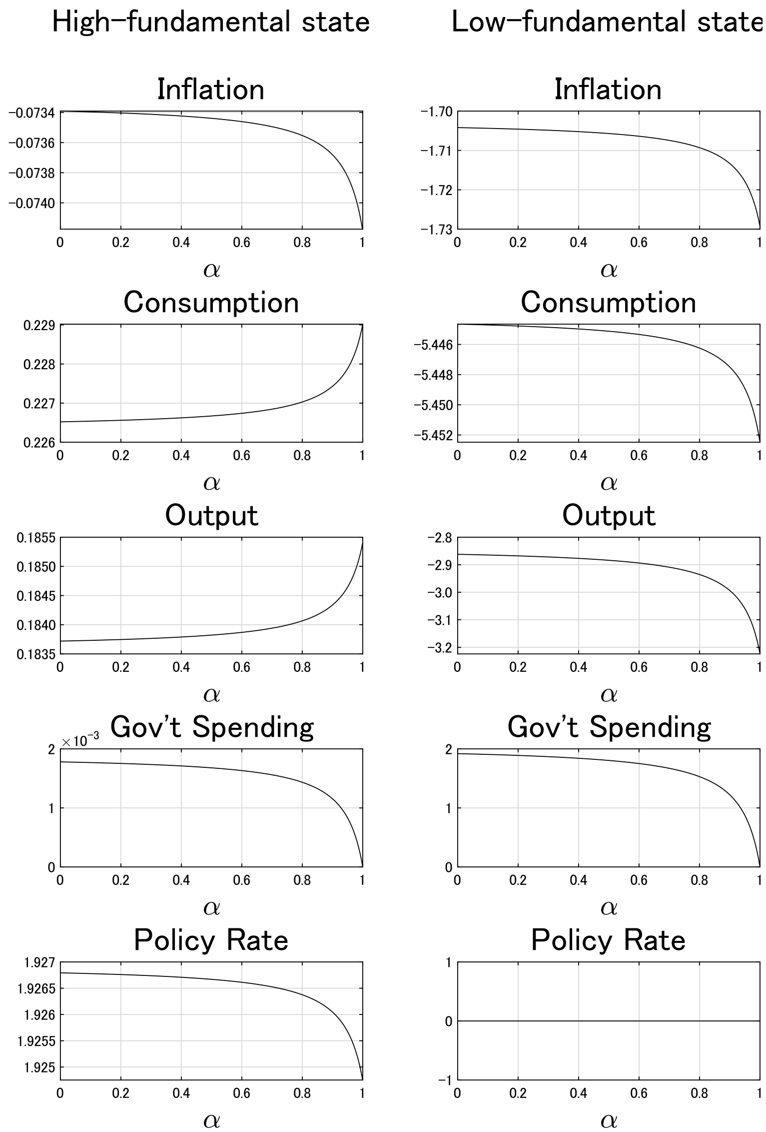


Figure I.3: Allocations in the non-linear model with a fundamental shock: The effects of fiscal activism



Note: The horizon axis shows the value of α. Inflation and the policy rate are expressed as annualized percent. Consumption, output, and government spending are expressed as percentage deviation from the efficient steady state.

Figure I.4: Welfare in the non-linear model with a fundamental shock:
The effects of fiscal activism