Online Appendix
Mind the gap! Stylized dynamic facts and structural models.

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March 17, 2021

Appendix B: The optimality conditions of the NK model with a reduced number of endogenous variables

1) Theory with $z_t = (o_t, \pi_t, n_t, r_t)$.

$$
\begin{aligned}
\chi_t &= E_t\chi_{t+1} - \frac{1}{1-h} E_t(a_{t+1} + o_{t+1} - o_t) + \frac{h}{1-h}(a_t + o_t - o_{t-1}) + r_t - E_t\pi_{t+1} \\
\pi_t &= E_t\pi_{t+1}\,\beta + k_p \left(\frac{h}{1-h}(a_t + o_t - o_{t-1}) + (1+\sigma_n)\, n_t\right) + k_p\,(\mu_t - \chi_t) \\
o_t &= \zeta_t + (1-\alpha)\, n_t \\
r_t &= \rho_r\, r_{t-1} + (1-\rho_r)\,(\phi_y\,(a_t + o_t - o_{t-1}) + \phi_p\pi_t\,) + \varepsilon_{mp_t}
\end{aligned}
$$

2) Theory with $z_t = (o_t, \pi_t, n_t)$.

$$
\begin{aligned}
(1+\rho_r)\chi_t - \rho_r E_t\chi_{t-1} &= \chi_{t+1} - \frac{1}{1-h} E_t(a_{t+1} + o_{t+1} - o_t) + (\frac{h+\rho_r}{1-h} + (1-\rho_r)\phi_y)\,(a_t + o_t - o_{t-1}) \\
&- (\frac{h\rho_r}{1-h})\,(a_{t-1} + o_{t-1} - o_{t-2}) + (\rho_r + (1-\rho_r)\phi_p)\,\pi_t + e_{mp_t} - E_t\pi_{t+1} \\
\pi_t &= E_t\pi_{t+1}\,\beta + k_p \left(\frac{h}{1-h}(a_t + o_t - o_{t-1}) + (1+\sigma_n)\, n_t\right) + k_p\,(\mu_t - \chi_t) \\
o_t &= \zeta_t + (1-\alpha)n_t
\end{aligned}
$$

3) Theory with $z_t = (r_t, \pi_t, n_t)$.

$$
\begin{aligned}
\chi_t &= \chi_{t+1} - \frac{1}{1-h}(a_{t+1} + \zeta_{t+1} - \zeta_t + (1-\alpha)\,(n_{t+1} - n_t)) \\
&+ \frac{h}{1-h}(a_t + \zeta_t - \zeta_{t-1} + (1-\alpha)\,(n_t - n_{t-1})) + r_t - \pi_{t+1} \\
\pi_t &= \pi_{t+1}\,\beta + k_p \left(\frac{h}{1-h}(a_t + \zeta_t - \zeta_{t-1} + (1-\alpha)\,(n_t - n_{t-1})) + (1+\sigma_n)\, n_t\right) + k_p\,(\mu_t - \chi_t) \\
r_t &= \rho_r\, r_{t-1} + (1-\rho_r)\,(\phi_y\,(a_t + \zeta_t - \zeta_{t-1} + (1-\alpha)\,(n_t - n_{t-1})) + \phi_p\,\pi_t) + \varepsilon_{mp_t}
\end{aligned}
$$

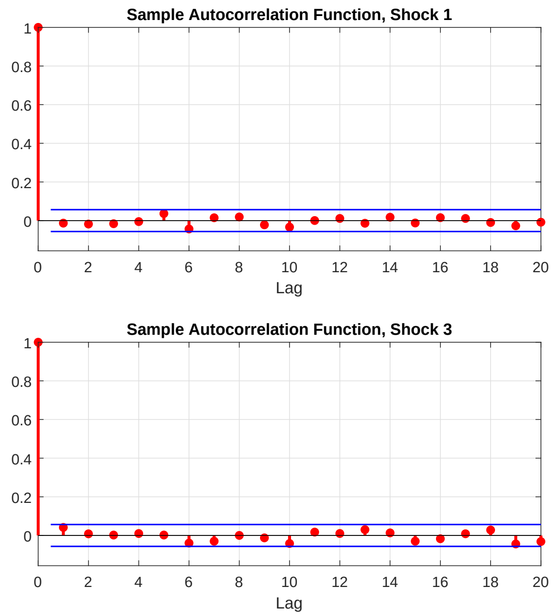
4) Theory with $z_t = (o_t, r_t)$.

$$\begin{aligned}
\chi_t &= (1+\beta)\chi_{t+1} - \beta\chi_{t+2} - \frac{1}{1-h}\left(a_{t+1} + o_{t+1} - o_t\right) + \frac{\beta}{1-h}\left(a_{t+2} + o_{t+2} - o_{t+1}\right) \\
&+ \left(\frac{h}{1-h}\right)\left(a_t + o_t - o_{t-1}\right) - \left(\frac{h\beta}{1-h}\right)\left(a_{t+1} + o_{t+1} - o_t\right) + r_t - \beta r_{t+1} \\
&- k_p\left(\frac{h}{1-h}\left(a_{t+1} + o_{t+1} - o_t\right) + (1+\sigma_n)\ \frac{1}{1-\alpha}(o_{t+1} - \zeta_{t+1})\right) - k_p\ (\mu_{t+1} - \chi_{t+1}) \\
r_t &= \beta r_{t+1} + \rho_r r_{t-1} - \beta\rho_r r_t\ + (1-\rho_r)\phi_y((a_t + o_t - o_{t-1}) - \beta(a_{t+1} + o_{t+1} - o_t)) \\
&+ (1-\rho_r)\phi_\pi\left(k_p\left(\frac{h}{1-h}\left(a_t + o_t - o_{t-1}\right) + (1+\sigma_n)\ \frac{1}{1-\alpha}(o_t - \zeta_t)\right) + k_p\ (\mu_t - \chi_t)\right) \\
&+ \epsilon_{mp_t} - \beta\epsilon_{mp_{t+1}}
\end{aligned}$$

5) Theory with $z_t = (g_t, n_t)$ (assuming $\beta^{-1} = (1-\rho_r)\phi_\pi\pi + \rho_r$).

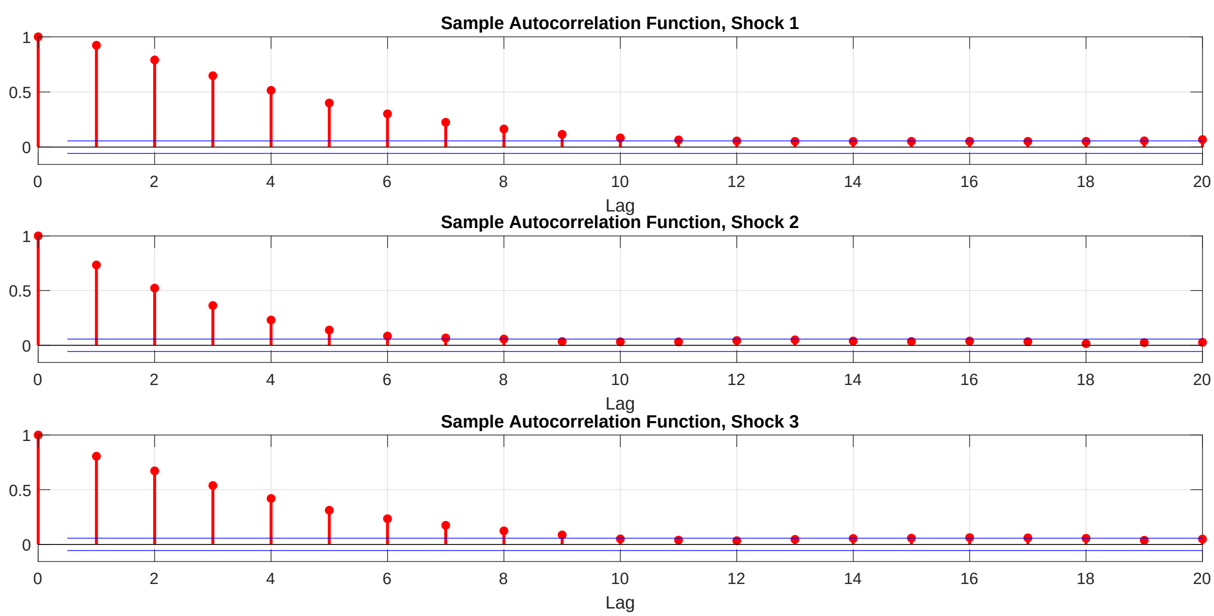
$$\begin{aligned}
(1+\rho_r)\chi_t &= \rho_r\chi_{t-1} + \chi_{t+1} + \frac{1}{1-h}g_{t+1} + \left(\frac{\rho_r + h}{1-h} + (1-\rho_r)\phi_y\right)g_t \\
&- \frac{h\rho_r}{1-h}g_{t-1} + \epsilon_{mp_t} + \kappa_p\left(\frac{h}{1-h}g_t + (1+\sigma_n)n_t\right) + \kappa_p(\mu_t - \chi_t) \\
g_t &= a_t + \zeta_t + (1-\alpha)n_t - \zeta_{t-1} - (1-\alpha)n_{t-1}
\end{aligned}$$

Appendix C: Additional graphs for the NK model



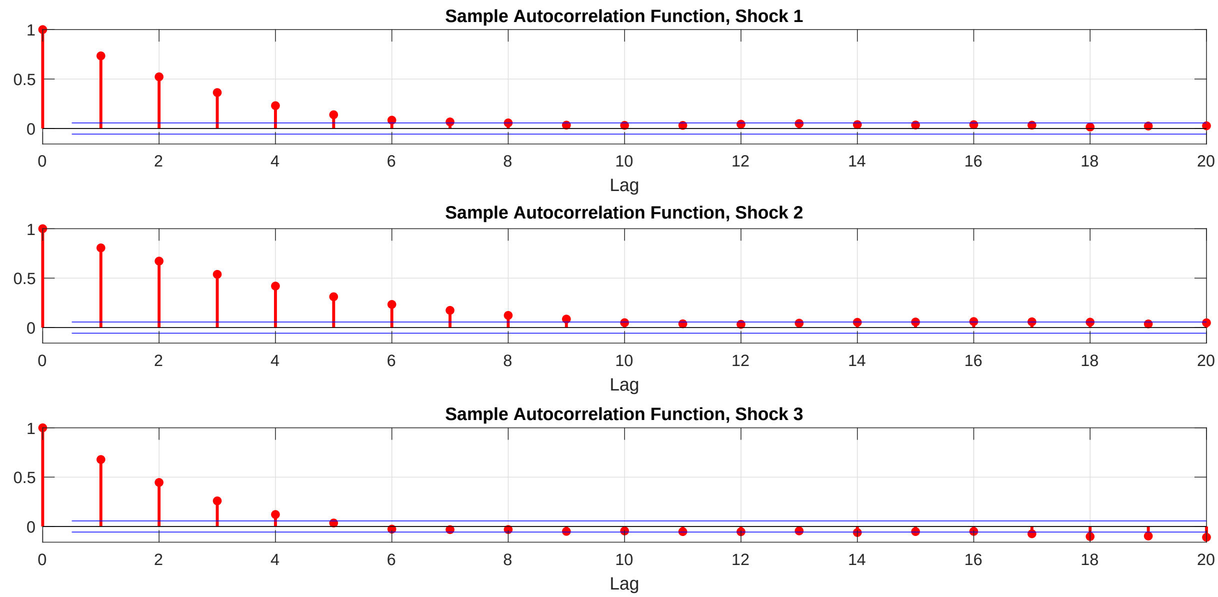
Note: Parallel lines describe 95 % asymptotic tunnel for the hypothesis of zero autocorrelations.

Figure C.1: Autocorrelation function, innovations in (o_t, π_t, n_t, r_t) system.



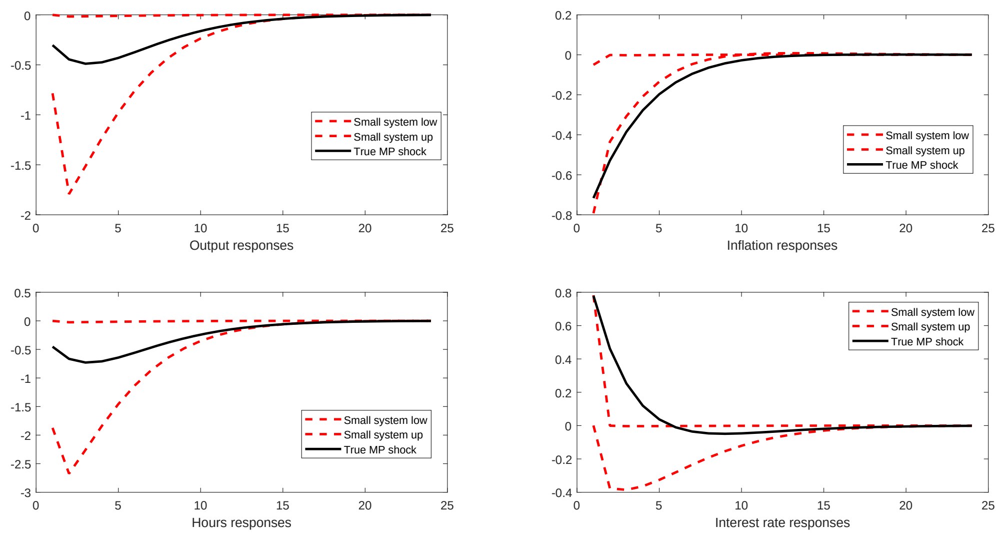
Note: Parallel lines delimit the 95 % asymptotic tunnel for the hypothesis of zero autocorrelations.

Figure C.2: Autocorrelation function, innovations in (o_t, π_t, n_t) system.



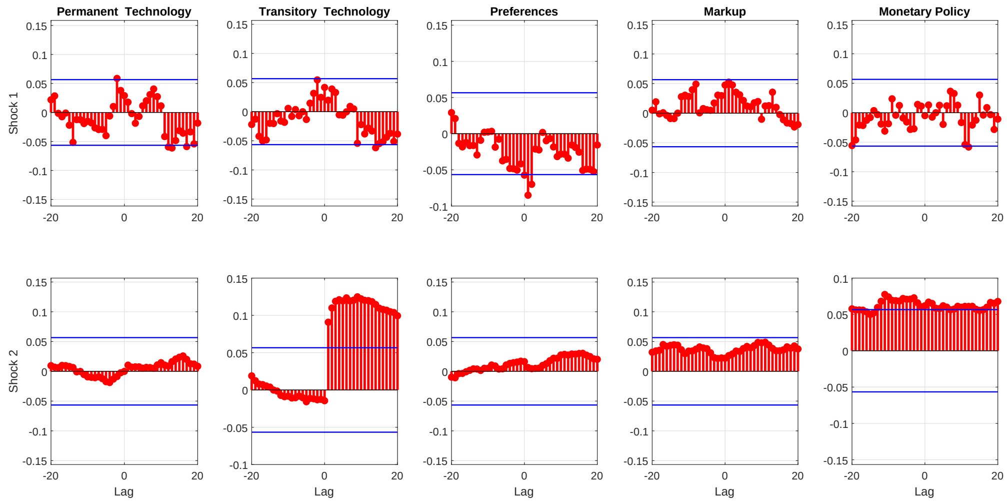
Note: Parallel lines delimit the 95 % asymptotic tunnel for the hypothesis of zero autocorrelations.

Figure C.3: Autocorrelation function, innovations in (π_t, n_t, r_t) system.



Note: The dashed regions report the profile of the identified set. The solid line reports the responses in the DGP.

Figure C.4: Responses to monetary policy shocks, (y_t, π_t, n_t, r_t) system.



Note: Parallel lines describe the 95 % asymptotic tunnel for the hypothesis of zero cross correlations.

Figure C5: Cross correlation function, innovations in (g_t, n_t) system and structural shocks.

Appendix D: The (linearized) equations of the extended Iacoviello model

$$rr_t = r_t - pi_{t+1}$$

$$\begin{aligned}
y_t &= c_y c_t + (1 - c_y - cii_y - i_y) ci_t + cii_y cii_t + i_y i_t \\
ci_t &= ci_{t+1} - rr_t \\
i_t - k_{t-1} &= \gamma(i_{t+1} - k_t) + \frac{(1-\gamma(1-\delta)}{\psi}(y_{t+1} - x_{t+1} - k_t) + (\frac{1}{\psi}(c_t - c_{t+1})
\end{aligned}$$

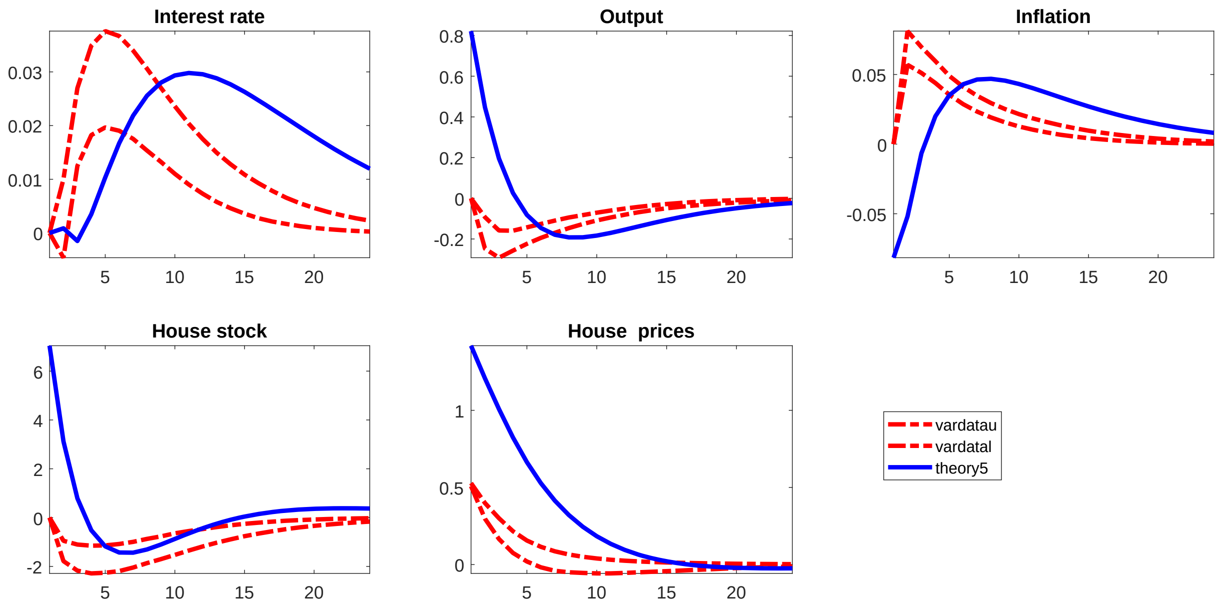
$$\begin{aligned}
q_t &= \gamma_E q_{t+1} + (1-\gamma_E)(y_{t+1} - x_{t+1} - h_t) - m\beta\ rr_t - i_{1,t} - (1-m\beta)(c_{t+1} - c_t) \\
&- \phi_E(h_t - h_{t-1} - \gamma(h_{t+1} - h_t)) \\
q_t &= \gamma_H q_{t+1} + (1-\gamma_H)(j_t - hii_t) - mii\beta\ rr + (1 - mii\beta)(cii_t - \omega cii_{t+1}) \\
&- \phi_H(hii_t - hii_{t-1} - \beta_{ii}(hii_{t+1} - hii_t) \\
q_t &= \beta q_{t+1} + (1-\beta) j_t + \iota h_t + \iota_{ii} hii_t + ci_t - betaci_{t+1} + \frac{phi_H}{hi}(h(h_t - h_{t-1}) \\
&+ hii(hii_t - hii_{t-1}) - \beta h(h_{t+1} - h_t) - \beta hii(hii_{t+1} - hii_t))
\end{aligned}$$

$$\begin{aligned}
b_t &= q_{t+1} + h_t - rr_t + i_{1,t} \\
bii_t &= q_{t+1} + hii_t - rr_t
\end{aligned}$$

$$\begin{aligned}
y_t &= \frac{\eta}{\eta - (1-\nu-\mu)}(a_t + \nu h_{t-1} + \mu k_{t-1}) - \frac{1-\nu-\mu}{\eta-(1-\nu-\mu)}(x_t + \alpha ci_t + (1-\alpha) cii_t) \\
\pi_t &= \beta\pi_{t+1} - \kappa x_t + u_t
\end{aligned}$$

$$\begin{aligned}
k_t &= \delta i_t + (1-\delta) k_{t-1} \\
b_y b_t &= c_y c_t + qh_y(h_t - h_{t-1}) + i_y i_t + \frac{b_y}{\beta}(r_{t-1} + b_{t-1} - \pi_t) - (1 - si - sii)(y_t - x_t) \\
bii_y bii_t &= cii_y cii_t + qhii_y(hii_t - hii_{t-1}) + \frac{bii_y}{\beta}(bii_{t-1} + r_{t-1} - \pi_t) - sii(y_t - x_t)
\end{aligned}$$

$$\begin{aligned}
r_t &= (1-\rho_R)(1+\rho_\pi)\pi_{t-1} + \rho_y(1-\rho_R) y_{t-1} + \rho_R r_{t-1} + e_R \\
j_t &= \rho_j j_{t-1} + e_t^j \\
u_t &= \rho_u u_{t-1} + e_t^u \\
a_t &= \rho_a a_{t-1} + e_t^a \\
i_{1,t} &= \rho_1 i_{1,t-1} + e_t^{i1} \\
tc_t &= c_y c_t + (1 - c_y - cii_y - i_y) ci_t + cii_y cii_t \\
th_t &= h_t + hii_t
\end{aligned}$$



Note: The solid blue line represents responses to preference shocks in a theory with 5 disturbances. The red dashed lines represent the 68% highest posterior responses to an identified house price shocks in a VAR with five variables and data simulated from the model with 5 disturbances.

Figure D1: Responses to a Cholesky identified q_t innovations in a 5 variable VAR and theoretical preferences disturbances.

Appendix E: The equations of extended Basu and Bundik model

$$
\begin{aligned}
y_t + fixedcost &= productionconstant(z_t n_t)^{1-\alpha}(u_t k_{t-1})^{\alpha} \\
c_t + leverageratio(k_t/rr_t) &= w_t n_t + de_t + leverageratio * k_{t-1} \\
w_t &= ((1-\eta)/\eta)c_t/(1-n_t) \\
vf_t &= [utilityconstant * a_t(c_t^{\eta}(1-n_t)^{1-\eta})^{(1-\sigma)/\theta_{vf}} \\
&+ \beta * expvf1sigma_t^{1/\theta_{vf}}]^{\theta_{vf}/(1-\sigma)} \\
expvf1sigma_t &= vf_{t+1}^{1-\sigma} \\
w_t n_t &= (1-\alpha)(y_t + fixedcost)/\mu_t \\
rrk_t * u_t * k_{t-1} &= \alpha(y_t + fixedcost)/\mu_t \\
q_t * deltauprime_t * u_t * k_{t-1} &= \alpha(y_t + fixedcost)/\mu_t \\
k_t &= [(1-deltau_t) - \phi_k/2(inv_t/k_{t-1} - \delta_0)^2]k_{t-1} + inv_t \\
deltau_t &= \delta_0 + \delta_1(u_t - 1) + \delta_2/2(u_t-1)^2 \\
deltauprime_t &= \delta_1 + \delta_2(u_t - 1) \\
sdf_t &= \beta(a_t/a_{t-1})((c_t^{\eta}(1-n_t)^{1-\eta})/(c_{t-1}^{\eta}(1-n_{t-1})^{1-\eta}))^{\frac{1-\sigma}{\theta_{vf}}} \\
&* (c_{t-1}/c_t)(vf_t^{1-\sigma}/expvf1sigma_{t-1})^{1-1/\theta_{vf}} \\
1 &= rr_t * sdf_{t+1} \\
pie_{t+1} &= r_t * sdf_{t+1} \\
1 &= sdf_{t+1}(u_{t+1} * rrk_{t+1} + q_{t+1}[(1-deltau_{t+1}) - \phi_k/2(inv_{t+1}/k_t - \delta_0)^2 \\
&+ \phi_k(inv_{t+1}/k_t - \delta_0)(inv_{t+1}/k_t)])/q_t \\
1 &= sdf_{t+1}(de_{t+1} + pe_{t+1})/pe_t \\
\log r_t &= (1-\rho_r)(\log(rss) + \rho_{pie}\log(pie_t/piess) + \rho_y \log(y_t/y_{t-1})) + \rho_r \log(r_{t-1}) + e_t \\
de_t &= y_t - w_t n_t - inv_t - \phi_p/2(pie_t/piess - 1)^2 y_t \\
&- leverageratio * (k_{t-1} - k_t/rr_t) \\
q_t^{-1} &= 1 - \phi_k(inv_t/k_{t-1} - \delta_0) \\
\phi_p(pie_t/piess - 1)(pie_t/piess) &= (1-\theta_\mu) + \theta_\mu/\mu_t + sdf_{t+1}\phi_p(pie_{t+1}/piess - 1)(y_{t+1}/y_t)(pie_{t+1}/piess) \\
profit_t &= (\mu_t - 1)y_t - fixedcost \\
expre_t &= (de_{t+1} + pe_{t+1})/pe_t \\
expre2_t &= (de_{t+1} + pe_{t+1})^2/pe_t^2 \\
varexpre_t &= expre2_t - expre_t^2 \\
a_t &= (1-\rho_a)ass + \rho_a a_{t-1} + vola_{t-1}ea_t \\
vola_t &= (1-\rho_{vola})volass + \rho_{vola}vola_{t-1} + volvola * evola_t \\
z_t &= (1-\rho_z)zss + \rho_z z_{t-1} + volz * ez_t \\
e_t &= (1-\rho_e)ess + \rho_b e_{t-1} + voless * ee_t
\end{aligned}
$$