Online Appendix to

"Fiscal Backing for Price Stability in a Monetary Union"

Bartosz Maćkowiak
European Central Bank and CEPR

Sebastian Schmidt
European Central Bank and CEPR

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A Intertemporal budget constraints (Section 2)

First, we derive the intertemporal budget constraint of each household i (equation (18) in the paper). To do that, we solve forward the flow budget constraint of the household, equation (3) in the paper. Household i satisfies first-order conditions stated in the paper as equations (13)-(14). Using these first-order conditions, we arrive at

$$\frac{\sum_{j=1}^{I}\left(1+\rho_j Q_{jt}\right)B_{ijt-1}^{H}+R_{t-1}H_{it-1}}{P_t}=\sum_{k=0}^{T}E_t\left[\Theta_{itk}\left(\tilde{P}_{it+k}C_{it+k}-\tilde{W}_{it+k}Y_{it+k}+\tilde{S}_{it+k}\right)\right]$$
$$+\quad E_t\left[\Theta_{itT}\left(\frac{\sum_{j=1}^{I}Q_{jt+T}B_{ijt+T}^{H}+H_{it+T}}{P_{t+T}}\right)\right]. \tag{A.1}$$

Here, as in the paper, $\tilde{P}_{it}\equiv P_{it}/P_t$, $\tilde{W}_{it}\equiv W_{it}/P_t$, and $\tilde{S}_{it}\equiv \tilde{W}_{it}S_{it}$. We take the limit of both sides of equation (A.1) as time goes to infinity. Each household satisfies the transversality condition stated in the paper as equation (16). The transversality condition says that the limit of the second term on the right-hand side of equation (A.1) equals zero. Hence, we obtain

$$\frac{\sum_{j=1}^{I}\left(1+\rho_j Q_{jt}\right)B_{ijt-1}^{H}+R_{t-1}H_{it-1}}{P_t}=\sum_{k=0}^{\infty}E_t\left[\Theta_{itk}\left(\tilde{P}_{it+k}C_{it+k}-\tilde{W}_{it+k}Y_{it+k}+\tilde{S}_{it+k}\right)\right] \tag{A.2}$$

which holds in equilibrium for each household i.

Second, we derive the intertemporal budget constraint of the public sector in the union (equation (19) in the paper). To do that, we solve forward the flow budget constraint of the public sector in the union, equation (8) in the paper. Using equations (13)-(14) in the paper, we arrive at

$$\frac{\sum_j (1+\rho_j Q_{jt})\left(\sum_i B^H_{ijt-1}\right) + R_{t-1}\sum_i H_{it-1}}{P_t}$$
$$= \sum_{k=0}^{T} E_t\left[\Theta_{htk}\left(\sum_i \tilde{S}_{it+k}\right)\right] + E_t\left[\Theta_{htT}\left(\frac{\sum_i\sum_j Q_{jt+T}B^H_{ijt+T} + \sum_i H_{it+T}}{P_{t+T}}\right)\right]. \qquad (A.3)$$

Equation (A.3) holds for *each* household $h = 1, \ldots, I$; in other words, Θ_{htk} for $k = 0, \ldots, T$ can be the stochastic discount factor of *any* household h. We now show that the limit as time goes to infinity of the second term on the right-hand side of equation (A.3) equals zero. By moving one of the summation symbols, we have

$$E_t\left[\Theta_{htT}\left(\frac{\sum_i\sum_j Q_{jt+T}B^H_{ijt+T} + \sum_i H_{it+T}}{P_{t+T}}\right)\right] = E_t\left[\sum\nolimits_i \Theta_{htT}\left(\frac{\sum_j Q_{jt+T}B^H_{ijt+T} + H_{it+T}}{P_{t+T}}\right)\right].$$

Let X_{t+k} denote the nominal value of a portfolio of bonds and reserves in any period $t+k$, $k \geq 0$, divided by the price level of the union in that period. No-arbitrage implies that $E_t\left(\Theta_{itk}X_{t+k}\right) = E_t\left(\Theta_{jtk}X_{t+k}\right)$ for any pair of households i and j. This no-arbitrage condition for period $k = T$ implies that

$$E_t\left[\sum\nolimits_i \Theta_{htT}\left(\frac{\sum_j Q_{jt+T}B^H_{ijt+T} + H_{it+T}}{P_{t+T}}\right)\right] = E_t\left[\sum\nolimits_i \Theta_{itT}\left(\frac{\sum_j Q_{jt+T}B^H_{ijt+T} + H_{it+T}}{P_{t+T}}\right)\right]. \qquad (A.4)$$

On the left-hand side, we have the stochastic discount factor of a single arbitrary household h, Θ_{htT}, appearing I times (one time for each $i = 1, \ldots, I$). On the right-hand side, for each $i = 1, \ldots, I$ we have the stochastic discount factor of household i, Θ_{itT}. Each household satisfies the transversality condition stated in the paper as equation (16). Summing this equation across the households yields equation (17) in the paper. Equation (17) in the paper says that the limit of the right-hand side of equation (A.4) equals zero. It follows that the limit of the second term on the right-hand side of equation (A.3) equals zero. Taking the limit of both sides of equation (A.3), we obtain

$$\frac{\sum_j (1+\rho_j Q_{jt})\left(\sum_i B^H_{ijt-1}\right) + R_{t-1}\sum_i H_{it-1}}{P_t} = \sum_{k=0}^{\infty} E_t\left[\Theta_{htk}\left(\sum_i \tilde{S}_{it+k}\right)\right]. \qquad (A.5)$$

Equation (A.5) holds in equilibrium for each household $h = 1, \ldots, I$; in other words, Θ_{htk} for $k \geq 0$ can be the stochastic discount factor of any household h.

Third, we solve forward the flow budget constraint of the public sector in any single country i, equation (7) in the paper. Using equations (13)-(14) in the paper, we arrive at

$$\frac{\left(1+\rho_i Q_{it}\right) B_{it-1}-\sum_j\left(1+\rho_j Q_{jt}\right) B_{ijt-1}^{CB}+R_{t-1}\left(H_{it-1}-\sum_j T_{ijt-1}\right)}{P_t}=\sum_{k=0}^{T} E_t\left[\Theta_{htk}\left(\tilde{S}_{it+k}\right)\right]$$
$$+\quad E_t\left[\Theta_{htT}\left(\frac{Q_{it+T} B_{it+T}-\sum_j Q_{jt+T} B_{ijt+T}^{CB}+H_{it+T}-\sum_j T_{ijt+T}}{P_{t+T}}\right)\right]. \tag{A.6}$$

Equation (A.6) holds in equilibrium for each household h. However, neither the transversality condition of an individual household (equation (16) in the paper) nor its sum across the households (equation (17) in the paper) determines what happens to the second term on the right-hand side of equation (A.6) as time goes to infinity. In the limit, this term may equal zero (in which case "the intertemporal budget constraint" of the public sector in country i holds) or not. Households' optimization by itself does not determine if the limit equals zero or not. Fiscal policy *may* imply that the limit equals zero (this is the case with the fiscal policy configuration in Section 4; this is not the case with the fiscal policy configuration in Section 3). In Section 3, after an asymmetric fiscal expansion in country 2 and after a monetary policy rate increase with the assumed asymmetric debt duration, the limit is negative for country 1 and positive for country 2 (in country 1, the real value of net public liabilities falls short of the present value of the national primary surplus; in country 2, the real value of net public liabilities exceeds the present value of the national primary surplus).

Fourth, we solve forward the flow balance-of-payments constraint of country i. The flow balance-of-payments constraint of country i follows from combining equations (3), (4), and (6) in the paper and reads:

$$\sum_j\left(1+\rho_j Q_{jt}\right)\left(B_{ijt-1}^H+B_{ijt-1}^{CB}\right)+R_{t-1}\sum_j T_{ijt-1}-\left(1+\rho_i Q_{it}\right) B_{it-1}$$
$$=P_{it} C_{it}-W_{it} Y_{it}+\sum_j Q_{jt}\left(B_{ijt}^H+B_{ijt}^{CB}\right)+\sum_j T_{ijt}-Q_{it} B_{it}.$$

Using equations (13)-(14) in the paper, we arrive at

$$\frac{\sum_j (1+\rho_j Q_{jt}) \left(B^H_{ijt-1} + B^{CB}_{ijt-1}\right) + R_{t-1}\sum_j T_{ijt-1} - (1+\rho_i Q_{it}) B_{it-1}}{P_t}$$
$$= \sum_{k=0}^{T} E_t \left[\Theta_{htk}\left(\tilde{P}_{it+k}C_{it+k} - \tilde{W}_{it+k}Y_{it+k}\right)\right]$$
$$+E_t\left[\Theta_{htT}\left(\frac{\sum_j Q_{jt+T}\left(B^H_{ijt+T} + B^{CB}_{ijt+T}\right) + \sum_j T_{ijt+T} - Q_{it}B_{it+T}}{P_{t+T}}\right)\right]. \qquad \text{(A.7)}$$

Equation (A.7) can also be obtained by combining equation (A.1) for an individual household and equation (A.6) for the public sector in the same country. It follows from the analysis of equations (A.1) and (A.6) that the limit as time goes to infinity of the second term on the right-hand side of equation (A.7) may equal zero or not. If "the intertemporal budget constraint" of the public sector in country i fails to hold, then "the intertemporal balance-of-payments constraint" of country i fails to hold (the second term on the right-hand side of equation (A.7) does not equal zero in the limit).

B Analytical results for Section 3

We prove Proposition 1 stated in Section 3 of the paper (existence and uniqueness of equilibrium). We also state and prove another proposition which characterizes the determinants of consumption and relative price changes (we refer to this proposition in Section 3).

B.1 Proof of Proposition 1

We first show that there exists a unique perfect-foresight equilibrium with time-invariant consumption and relative prices. Equation (20) in the paper with $\tilde{S}_{it} = \tilde{S}_i > 0$, $i = 1, 2$, $t \geq 0$, pins down P_0. Equation (23) with $R = \Pi/\beta$ pins down $P_t = \Pi^t P_0$, $t \geq 1$. The bond prices are time-invariant, $Q_i = 1/(R - \rho_i)$, $i = 1, 2$. It remains to be shown that there exists a unique vector of time-invariant consumption and relative prices, $\{C_1, C_2, \tilde{P}_1, \tilde{P}_2, \tilde{W}_1, \tilde{W}_2\}$, that solves the equations stated in the definition of the perfect-foresight equilibrium in Section 3.

The relevant equations are reproduced here, assuming $\theta = 1$:

$$\frac{\sum_j (1+\rho_j Q_j) B^H_{1j,-1} + R_{-1}H_{1,-1}}{P_0} = \frac{\tilde{P}_1 C_1 - \tilde{W}_1 Y_1 + \tilde{S}_1}{1-\beta} \qquad \text{(B.1)}$$

$$Y_1 = \gamma_{11}\left(\frac{\tilde{W}_1}{\tilde{P}_1}\right)^{-1} C_1 + \gamma_{21}\left(\frac{\tilde{W}_1}{\tilde{P}_2}\right)^{-1} C_2 \tag{B.2}$$

$$Y_2 = \gamma_{12}\left(\frac{\tilde{W}_2}{\tilde{P}_1}\right)^{-1} C_1 + \gamma_{22}\left(\frac{\tilde{W}_2}{\tilde{P}_2}\right)^{-1} C_2. \tag{B.3}$$

$$\tilde{P}_1 = \tilde{W}_1^{\gamma_{11}} \tilde{W}_2^{\gamma_{12}} \tag{B.4}$$

$$\tilde{P}_2 = \tilde{W}_1^{\gamma_{21}} \tilde{W}_2^{\gamma_{22}} \tag{B.5}$$

$$1 = \tilde{P}_1^{n_1} \tilde{P}_2^{n_2} \tag{B.6}$$

Note that equations (B.2)-(B.3) imply a union-wide resource constraint, $\tilde{W}_1 Y_1 + \tilde{W}_2 Y_2 = \tilde{P}_1 C_1 + \tilde{P}_2 C_2$. The union-wide resource constraint, equation (20) in the paper, and equation (B.1) together imply that the intertemporal budget constraint of household 2 holds.

We can reduce this system of equilibrium conditions to one equation and one unknown. Define

$$\Lambda_1 \equiv \frac{\sum_j \left(1+\rho_j Q_j\right) B^H_{1j,-1} + R_{-1} H_{1,-1}}{P_0}(1-\beta) - \tilde{S}_1$$

and write equation (B.1) as

$$\Lambda_1 - \tilde{P}_1 C_1 + \tilde{W}_1 Y_1 = 0. \tag{B.7}$$

Next, express $\tilde{P}_1 C_1$ as a function of $\tilde{W}_1$. Solve equation (B.3) for $\tilde{P}_2 C_2$ and substitute it into equation (B.2). Solving for $\tilde{P}_1 C_1$, we have

$$\tilde{P}_1 C_1 = \frac{\gamma_{22}}{\gamma_{11}\gamma_{22} - \gamma_{12}\gamma_{21}}\left(\tilde{W}_1 Y_1 - \frac{\gamma_{21}}{\gamma_{22}} \tilde{W}_2 Y_2\right) \tag{B.8}$$

where one can show that with home bias $\gamma_{22}/\left(\gamma_{11}\gamma_{22} - \gamma_{12}\gamma_{21}\right) > 1$. To substitute out $\tilde{W}_2$, combine equations (B.4)-(B.6) and solve for $\tilde{W}_2$:

$$\tilde{W}_2 = \tilde{W}_1^{-\frac{\gamma_{11} n_1 + \gamma_{21} n_2}{\gamma_{12} n_1 + \gamma_{22} n_2}} \tag{B.9}$$

Substituting equation (B.9) into equation (B.8), we have expressed $\tilde{P}_1 C_1$ as a function of $\tilde{W}_1$:

$$\tilde{P}_1 C_1 = \frac{\gamma_{22}}{\gamma_{11}\gamma_{22} - \gamma_{12}\gamma_{21}}\left(\tilde{W}_1 Y_1 - \frac{\gamma_{21}}{\gamma_{22}} \tilde{W}_1^{-\frac{\gamma_{11} n_1 + \gamma_{21} n_2}{\gamma_{12} n_1 + \gamma_{22} n_2}} Y_2\right) \tag{B.10}$$

Finally, substitute equation (B.10) into equation (B.7), and use $\gamma_{11} = 1 - n_2 \nu$, $\gamma_{12} = n_2 \nu$, $\gamma_{21} = n_1 \nu$, $\gamma_{22} = 1 - n_1 \nu$:

$$\Lambda_1 - \frac{\nu}{1-\nu} n_2 \tilde{W}_1 Y_1 + \frac{\nu}{1-\nu} n_1 \tilde{W}_1^{-\frac{n_1}{n_2}} Y_2 = 0. \tag{B.11}$$

This is an equation with one unknown, $\tilde{W}_1$.

Define $Res(\tilde{W}_1)$ as the left-hand side of equation (B.11). For all $\tilde{W}_1 > 0$, Res is strictly decreasing in $\tilde{W}_1$:

$$\frac{\partial Res}{\partial \tilde{W}_1} = -\frac{\nu}{1-\nu} n_2 Y_1 - \frac{\nu}{1-\nu} \frac{n_1^2}{n_2} \tilde{W}_1^{-\frac{1}{n_2}} Y_2 < 0.$$

Hence, equation (B.11) has at most one root. Since

$$\lim_{\tilde{W}_1 \to 0} Res(\tilde{W}_1) = \infty$$

$$\lim_{\tilde{W}_1 \to \infty} Res(\tilde{W}_1) = -\infty$$

equation (B.11) has exactly one root: there is a unique $\tilde{W}_1$.

Given a unique $\tilde{W}_1$, equation (B.9) pins down $\tilde{W}_2$, $\tilde{P}_1$ and $\tilde{P}_2$ follow from equations (B.4)-(B.5), C_1 follows from equation (B.1), and C_2 follows from (B.2).

Thus, there exists a unique vector $\{C_1, C_2, \tilde{P}_1, \tilde{P}_2, \tilde{W}_1, \tilde{W}_2\}$, i.e., there exists a unique perfect-foresight equilibrium with time-invariant consumption and relative prices.

Next, we show that no other perfect-foresight equilibrium exists. With a time-invariant endowment and perfect foresight, it is optimal for each household i to keep consumption constant over time, $C_{it} = C_i$. Consider household i's saving choice between a one-period real bond, paying $1/\beta$ units of the endowment good, and reserves. No-arbitrage implies that

$$R_t = \frac{1}{\beta} \frac{W_{it+1}}{W_{it}}, \ i = 1, 2, \ t \geq 0. \tag{B.12}$$

With $R_t = \Pi/\beta$, equation (B.12) implies that $W_{it} = \Pi^t W_{i0}$, $t \geq 1$. Thus, in any equilibrium, W_{it} grows at rate Π and the relative price $\tilde{W}_{it}$ is time-invariant, $\tilde{W}_{it} = \tilde{W}_i$. By the same no-arbitrage argument, P_{it} also grows at rate Π and the relative price $\tilde{P}_{it}$ is time-invariant, $\tilde{P}_{it} = \tilde{P}_i$.

B.2 Changes in consumption and relative prices

When $\theta = 1$, we can characterize analytically how consumption and relative prices change in response to the type of fiscal shocks considered in Section 3 of the paper. We assume that $Y_i = n_i$, $i = 1, 2$, and that the initial share of assets held by household i conforms with the relative size of country i, in line with the baseline parameterization in the paper.

Lemma 1 *Baseline equilibrium: Suppose the common monetary authority sets a time-invariant interest rate on reserves, $R = \Pi/\beta$. Each fiscal authority i maintains a time-invariant primary surplus, $\tilde{S}_i = bY_i$, where $b \in (0,1)$. In the unique perfect-foresight equilibrium, $\tilde{W}_{it} = 1$, $\tilde{P}_{it} = 1$, and $C_{it} = n_i$, $t \geq 0$, $i = 1, 2$.*

Now suppose that in period 0 the primary surplus of at least one fiscal authority deviates from the baseline, $\tilde{S}_{i0} \neq \tilde{S}_i$, $i \in \{1, 2\}$.

Proposition 2 *Equilibrium with fiscal shocks:*

- *The percentage change in consumption of household i, C_{it}, in response to a pair of primary surplus shocks $\{\tilde{S}_{i0}, \tilde{S}_{j0}\}$ equals $(1-\beta)bn_j(2-\nu)\left(\frac{\tilde{S}_{j0}}{\tilde{S}_j} - \frac{\tilde{S}_{i0}}{\tilde{S}_i}\right)$, $t \geq 0$, $i \neq j$.*
- *The percentage change in the relative price index of country i, $\tilde{P}_{it}$, in response to a pair of primary surplus shocks $\{\tilde{S}_{i0}, \tilde{S}_{j0}\}$ equals $(1-\beta)bn_j\frac{(1-\nu)^2}{\nu}\left(\frac{\tilde{S}_{j0}}{\tilde{S}_j} - \frac{\tilde{S}_{i0}}{\tilde{S}_i}\right)$, $t \geq 0$, $i \neq j$.*
- *The percentage change in the relative price of endowment good i, $\tilde{W}_{it}$, in response to a pair of primary surplus shocks $\{\tilde{S}_{i0}, \tilde{S}_{j0}\}$ equals $(1-\beta)bn_j\frac{1-\nu}{\nu}\left(\frac{\tilde{S}_{j0}}{\tilde{S}_j} - \frac{\tilde{S}_{i0}}{\tilde{S}_i}\right)$, $t \geq 0$, $i \neq j$.*

A symmetric fiscal shock does not affect consumption or relative prices. An asymmetric shock affects consumption, and, when there is home bias, relative prices. An asymmetric fiscal expansion in country i ($\tilde{S}_{i0} < \tilde{S}_i$) raises consumption in country i. When there is home bias ($\nu < 1$), it also raises the relative price of good i and the relative price of the consumption basket in country i. The responses of consumption and the price index in country i increase with the relative size of country j (bigger n_j), home bias (smaller ν), and the size of the fiscal shock (bigger $\tilde{S}_{i0}/\tilde{S}_i$).

Proof: Let us slightly modify the definition of Λ_1 to allow for fiscal shocks in period 0

$$\Lambda_1 \equiv \frac{\sum_j (1+\rho_j Q_j)\, B^H_{1j,-1} + R_{-1}H_{1,-1}}{P_0}(1-\beta) - (1-\beta)\tilde{S}_{10} - \beta\tilde{S}_1.$$

Using equation (B.11), and dropping time subscripts for ease of exposition, the derivative of the implicit function $\tilde{W}_1(\Lambda_1)$ in the neighborhood of the baseline equilibrium is

$$\frac{d\tilde{W}_1}{d\Lambda_1} = \frac{1}{n_1}\frac{1-\nu}{\nu}. \tag{B.13}$$

It then follows from equations (B.4) and (B.9) that

$$\frac{d\tilde{P}_1}{d\Lambda_1} = \frac{1}{n_1}\frac{(1-\nu)^2}{\nu}. \tag{B.14}$$

Substituting $\tilde{P}_1 = \tilde{W}_1^{1-\nu}$ into equation (B.7), we have

$$\Lambda_1 - \left(\tilde{W}_1(\Lambda_1)\right)^{1-\nu} C_1 + \tilde{W}_1(\Lambda_1) n_1 = 0,$$

and, in the neighborhood of the baseline equilibrium

$$\frac{dC_1}{d\Lambda_1} = 2 - \nu. \tag{B.15}$$

Next, we use the intertemporal budget constraint of the public sector in the union, equation (20) in the paper, to substitute out P_0 in the definition of Λ_1, $\Lambda_1 =$

$$\frac{\sum_j (1+\rho_j Q_j)\, B^H_{1j,-1} + R_{-1} H_{1,-1}}{\sum_j (1+\rho_j Q_j)\left(\sum_i B^H_{ij,-1}\right) + R_{-1}\sum_i H_{i,-1}} \left((1-\beta)(\tilde{S}_{10}+\tilde{S}_{20}) + \beta(\tilde{S}_1+\tilde{S}_2)\right) - (1-\beta)\tilde{S}_{10} - \beta\tilde{S}_1$$

$$= n_1 n_2 b(1-\beta)\left[\frac{\tilde{S}_{20}}{\tilde{S}_2} - \frac{\tilde{S}_{10}}{\tilde{S}_1}\right],$$

where we made use of the assumption that the initial share of assets held by household i conforms with the relative size of country i, and that a primary surplus in the baseline is a constant fraction b of the country's endowment. Hence,

$$\frac{\partial\Lambda_1}{\partial(\tilde{S}_{10}/\tilde{S}_1)} = -n_1 n_2 b(1-\beta) \tag{B.16}$$

$$\frac{\partial\Lambda_1}{\partial(\tilde{S}_{20}/\tilde{S}_2)} = n_1 n_2 b(1-\beta). \tag{B.17}$$

Using (B.13) and (B.16)-(B.17), we have

$$\frac{d\tilde{W}_1}{d(\tilde{S}_{10}/\tilde{S}_1)}\frac{(\Delta\tilde{S}_{10})/\tilde{S}_1}{\tilde{W}_1} + \frac{\partial\tilde{W}_1}{\partial(\tilde{S}_{20}/\tilde{S}_2)}\frac{(\Delta\tilde{S}_{20})/\tilde{S}_2}{\tilde{W}_1}$$

$$= (1-\beta)bn_2\frac{(1-\nu)}{\nu}\left(\frac{\Delta\tilde{S}_{20}}{\tilde{S}_2} - \frac{\Delta\tilde{S}_{10}}{\tilde{S}_1}\right) = (1-\beta)bn_2\frac{(1-\nu)}{\nu}\left(\frac{\tilde{S}_{20}}{\tilde{S}_2} - \frac{\tilde{S}_{10}}{\tilde{S}_1}\right),$$

where $\Delta\tilde{S}_{i0} \equiv \tilde{S}_{i0} - \tilde{S}_i$, $i = 1, 2$. Using (B.14) and (B.16)-(B.17), we have

$$\frac{d\tilde{P}_1}{d(\tilde{S}_{10}/\tilde{S}_1)}\frac{(\Delta\tilde{S}_{10})/\tilde{S}_1}{\tilde{P}_1} + \frac{\partial\tilde{P}_1}{\partial(\tilde{S}_{20}/\tilde{S}_2)}\frac{(\Delta\tilde{S}_{20})/\tilde{S}_2}{\tilde{P}_1} = (1-\beta)bn_2\frac{(1-\nu)^2}{\nu}\left(\frac{\tilde{S}_{20}}{\tilde{S}_2} - \frac{\tilde{S}_{10}}{\tilde{S}_1}\right)$$

Using (B.15) and (B.16)-(B.17), we have

$$\frac{dC_1}{d(\tilde{S}_{10}/\tilde{S}_1)}\frac{(\Delta\tilde{S}_{10})/\tilde{S}_1}{C_1} + \frac{\partial C_1}{\partial(\tilde{S}_{20}/\tilde{S}_2)}\frac{(\Delta\tilde{S}_{20})/\tilde{S}_2}{C_1} = (1-\beta)bn_2(2-\nu)\left(\frac{\tilde{S}_{20}}{\tilde{S}_2} - \frac{\tilde{S}_{10}}{\tilde{S}_1}\right)$$

Using similar steps, we can derive the responses of $\tilde{W}_2$, $\tilde{P}_2$, and C_2.

C Solving the model (Sections 3-4)

We describe how we solve the equations stated in the definition of a perfect-foresight equilibrium in Section 3 in the paper. First, we consider the generic case where the trade elasticity θ may deviate from unity. We then describe a simpler solution procedure for the case of a unit trade elasticity.

Section 3. In the first step, we solve for P_0 from equation (20):

$$\frac{\sum_j (1+\rho_j Q_j)\left(\sum_i B_{ij,-1}\right) + R_{-1}\sum_i H_{i,-1}}{P_0} = \sum_i \tilde{S}_{i0} + \frac{\beta}{1-\beta}\sum_i \tilde{S}_i \tag{C.1}$$

where in the baseline equilibrium (in the absence of shocks) $\tilde{S}_{i0} = \tilde{S}_i$, $i = 1,2$. From the policy rule $R = \Pi/\beta$ we have $P_t = \Pi^t P_0$, $t \geq 1$. We also have $Q_i = 1/(R-\rho_i)$, $i = 1,2$. In the second step, we use the bisection method to find the vector $\{C_1, C_2, \tilde{P}_1, \tilde{P}_2, \tilde{W}_1, \tilde{W}_2\}$ that solves the remaining equilibrium conditions:

$$\frac{\sum_j (1+\rho_j Q_j) B_{1j,-1} + R_{-1}H_{1,-1}}{P_0} = \frac{\tilde{P}_1 C_1 - \tilde{W}_1 Y_1 + \beta\tilde{S}_1}{1-\beta} + \tilde{S}_{10} \tag{C.2}$$

$$\frac{\sum_j (1+\rho_j Q_j) B_{2j,-1} + R_{-1}H_{2,-1}}{P_0} = \frac{\tilde{P}_2 C_2 - \tilde{W}_2 Y_2 + \beta\tilde{S}_2}{1-\beta} + \tilde{S}_{20} \tag{C.3}$$

$$Y_1 = \gamma_{11}\left(\frac{\tilde{W}_1}{\tilde{P}_1}\right)^{-\theta} C_1 + \gamma_{21}\left(\frac{\tilde{W}_1}{\tilde{P}_2}\right)^{-\theta} C_2 \tag{C.4}$$

$$\tilde{P}_1 = \left[\gamma_{11}\left(\tilde{W}_1\right)^{1-\theta} + \gamma_{12}\left(\tilde{W}_2\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{C.5}$$

$$\tilde{P}_2 = \left[\gamma_{21}\left(\tilde{W}_1\right)^{1-\theta} + \gamma_{22}\left(\tilde{W}_2\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{C.6}$$

$$1 = \tilde{P}_1^{n_1}\tilde{P}_2^{n_2} \tag{C.7}$$

For any guess for $\tilde{P}_1$, equation (C.7) determines $\tilde{P}_2$. $\tilde{W}_1$ and $\tilde{W}_2$ follow from equations (C.5)-(C.6).[1] C_1 and C_2 follow from equations (C.2)-(C.3). Define

$$Res \equiv Y_1 - \left[\gamma_{11}\left(\frac{\tilde{W}_1}{\tilde{P}_1}\right)^{-\theta} C_1 + \gamma_{21}\left(\frac{\tilde{W}_1}{\tilde{P}_2}\right)^{-\theta} C_2\right].$$

Choose an interval for $\tilde{P}_1$. The bisection method calculates the value of Res for the two endpoints of the interval and for its midpoint. It then selects the subinterval for which the value of Res, when

[1] When $\theta = 1$, we replace equations (C.5) and (C.6) with equations (B.4) and (B.5).

evaluated at the endpoints of the subinterval, has opposite signs. The process is continued until the interval is sufficiently small.

When $\theta = 1$, there is a simpler solution procedure. We use equation (20) to solve for P_0 as before. We then solve equation (B.11) for $\tilde{W}_1$ using either a guess-and-verify method or a root-finding algorithm, where we have to augment the definition of Λ_1 when there is a shock to the surplus of national fiscal authority 1 in period 0, $\Lambda_1 = \left(\sum_j (1+\rho_j Q_j) B_{1j,-1} + R_{-1} H_{1,-1}\right)(1-\beta)/P_0 - \beta\tilde{S}_1 - (1-\beta)\tilde{S}_{10}$. Given $\tilde{W}_1$, equation (B.9) pins down $\tilde{W}_2$, $\tilde{P}_1$ and $\tilde{P}_2$ follow from equations (B.4)-(B.5). C_1 follows from equation (B.1), where the right-hand side becomes $\left(\tilde{P}_1 C_1 - \tilde{W}_1 Y_1 + (1-\beta)\tilde{S}_{10} + \beta\tilde{S}_1\right)/(1-\beta)$ when there is a shock to the surplus of national fiscal authority 1 in period 0, and C_2 follows from (B.2).

Section 4. We use equation (20) in the paper to solve for P_0, as in Section 3; the right-hand side depends on the sum of the surpluses in each period, and the sum of the surpluses is exogenous here as in Section 3. In equation (21) the present value of an individual surplus enters, and an individual surplus now follows a feedback rule. We use the flow budget constraint of national fiscal authority i (equation (4)) and feedback rule (27) to compute the surplus of national fiscal authority i in each period $t = 0, 1, \ldots, T$, where T is a very large number. We calculate the period 0 present value and use the result as an input in equation (21). We use the flow budget constraint of the common fiscal authority (equation (28)) and feedback rule (29) to compute the common surplus in each period $t = 0, 1, \ldots, T$. We calculate the period 0 present value and use the result as an input in equation (21). We use the bisection method to solve the rest of the model.

D The cross-country wealth transfers (Section 3)

Net foreign assets. In Section 3, the equilibrium after an asymmetric fiscal expansion in country 2 and the equilibrium after a monetary policy rate increase with the assumed asymmetric debt duration involve the public sector in country 1 making a transfer to the public sector in country 2 (and, ultimately, household 1 making a transfer to household 2). This cross-country wealth transfer shows up as growing net foreign assets in country 1 and shrinking net foreign assets in country 2 (Figures 3 and 4 in the paper). When discounted with β, the net claims of country 1 on country 2 converge to a strictly positive number. To see this, consider the model as in Section 3 (in particular, the monetary union consists of two countries). Let NFA_{it} denote the net foreign assets of country i in period t, denominated in the common currency and divided by the price level of the union. Combining the flow budget constraints of household i and the public sector in country i, we obtain

$$\begin{aligned} NFA_{it} &\equiv \frac{Q_{jt}(B^H_{ijt}+B^{CB}_{ijt})+T_{ijt}-Q_{it}(B^H_{jit}+B^{CB}_{jit})}{P_t} \\ &= \tilde{W}_{it}Y_{it}-\tilde{P}_{it}C_{it}+\frac{(1+\rho_j Q_{jt})(B^H_{ijt-1}+B^{CB}_{ijt-1})+R_{t-1}T_{ijt-1}-(1+\rho_i Q_{it})(B^H_{jit-1}+B^{CB}_{jit-1})}{P_t}, \end{aligned}$$

where $j \neq i$. In the equilibria in Section 3, consumption, relative prices, the interest on reserves, and the bond prices are constant from period $t=0$ onward, $C_{it}=C_i$, $\tilde{W}_{it}=\tilde{W}_i$, $\tilde{P}_{it}=\tilde{P}_i$, $R_t=R=\Pi/\beta$, and $Q_{it}=Q_i=1/(R-\rho_i)$, $t\geq 0$, $i\in\{1,2\}$. Assuming that initial net foreign assets equal zero, we have $NFA_{i0}=\tilde{W}_iY_i-\tilde{P}_iC_i$ for $t=0$ and $NFA_{it}=\tilde{W}_iY_i-\tilde{P}_iC_i+(1/\beta)\,NFA_{it-1}$ for $t>0$. Let $\widehat{NFA}_{it}\equiv\beta^t NFA_{it}$ be the discounted value of the net foreign asset position in period $t\geq 0$ as of period $t=0$. Then

$$\widehat{NFA}_{it}=\sum_{k=0}^{t}\beta^k\left(\tilde{W}_iY_i-\tilde{P}_iC_i\right)$$

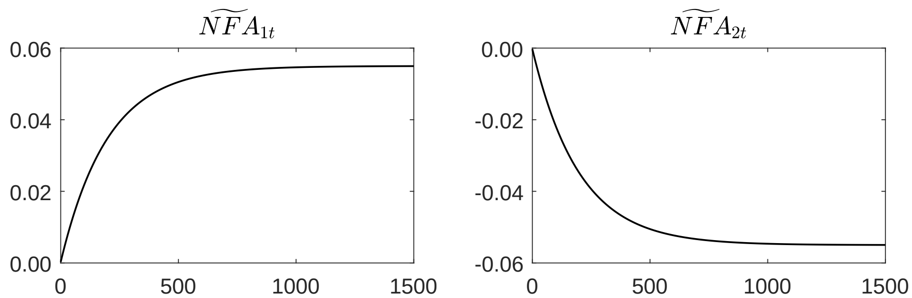
and

$$\lim_{t\to\infty}\widehat{NFA}_{it}=\frac{1}{1-\beta}\left(\tilde{W}_iY_i-\tilde{P}_iC_i\right).$$

Figure D.1 plots the impulse response of $\widehat{NFA}_{it}$ for country $i=1,2$ in the equilibrium with the asymmetric fiscal expansion in country 2 from Section 3. In this equilibrium, the impulse response converges to 0.055 for country 1 and -0.055 for country 2, as a fraction of GDP of the union (GDP of the union equals 1).

Gross foreign assets. How can the limit of the second term on the right-hand side of equation (A.6) not equal zero for both countries while, at the same time, the transversality condition holds for both households? Let us focus on the case when in equilibrium the public sector in country 1 is making a transfer to the public sector in country 2, as in Section 3. In this case, the limit of the second term on the right-hand side of equation (A.6) is negative for country 1 and positive for country 2. Consider the numerator in the second-term on the right-hand side of equation (A.6) for country 1 and for country 2, respectively (we specialize to the case of perfect foresight with two countries, we write T instead of $t+T$ in the subscripts, and we use the relation $B_i =$

Figure D.1: **Asymmetric fiscal expansion, discounted net foreign assets**



$\sum_j B^H_{ji} + \sum_j B^{CB}_{ji}$):

$$Q_{1T}\left(B^H_{11,T} + B^H_{21,T} + B^{CB}_{21,T}\right) - Q_{2T}B^{CB}_{12,T} + H_{1T} - T_{12,T}$$

$$Q_{2T}\left(B^H_{12,T} + B^H_{22,T} + B^{CB}_{12,T}\right) - Q_{1T}B^{CB}_{21,T} + H_{2T} + T_{12,T},$$

together with the numerator in the transversality condition for household 1 and for household 2, respectively (see equation (16) in the paper):

$$Q_{1T}B^H_{11,T} + Q_{2T}B^H_{12,T} + H_{1T}$$

$$Q_{1T}B^H_{21,T} + Q_{2T}B^H_{22,T} + H_{2T}.$$

By inspecting these equations, we can see that there are five feasible outcomes for gross foreign assets positions (in addition, a linear combination of the five outcomes is feasible). The five feasible outcomes are:

(1) $\beta^T T_{12,T}/P_T$ is strictly positive in the limit: NCB 1 is lending to NCB 2; in parallel fiscal authority 1 accepts lower remittances from NCB 1 or makes a transfer to NCB 1 (either way, a transfer occurs from the public sector in country 1 to the public sector in country 2). In the context of the euro area, this case corresponds to a net TARGET2 claim between the NCBs that is rolled over forever.

(2) $\beta^T Q_{2T}B^H_{12,T}/P_T$ is strictly positive in the limit: household 1 is lending to fiscal authority 2; in parallel, fiscal authority 1 is lending to household 1, i.e., the limit of $\beta^T Q_{1T}B^H_{11,T}/P_T$ equals minus one times the limit of $\beta^T Q_{2T}B^H_{12,T}/P_T$.

(3) $\beta^T Q_{1T}B^H_{21,T}/P_T$ is strictly negative in the limit: household 2 is borrowing from fiscal authority 1; in parallel, household 2 is lending to fiscal authority 2, i.e., the limit of $\beta^T Q_{2T}B^H_{22,T}/P_T$

equals minus one times the limit of $\beta^T Q_{1T} B^H_{21,T}/P_T$. Cases (2) and (3) involve a national fiscal authority maintaining a positive net claim on private agents, domestically or in another country in the union.

(4) $\beta^T Q_{2T} B^{CB}_{12,T}/P_T$ is strictly positive in the limit: NCB 1 is lending to fiscal authority 2; in parallel, fiscal authority 1 accepts lower remittances from NCB 1 or makes a transfer to NCB 1. This case involves an NCB maintaining a positive net claim on a national fiscal authority in another country in the union. Currently in the euro area an NCB does not hold marketable debt of a national fiscal authority in another country. However, the ECB does hold marketable debt of national fiscal authorities, with gains or losses shared symmetrically among the NCBs. If such debt is rolled over forever *asymmetrically* (e.g., in the two-country case, if the limit of $\beta^T Q_{iT} B^{CB}_{iT}/P_T$ for $i = 2$, where B^{CB}_i now denotes the debt of fiscal authority *i held by the ECB*, is strictly positive and greater than the same limit for $i = 1$), then the public sector in country 1 will be making a transfer to the public sector in country 2 via the Eurosystem.

(5) $\beta^T Q_{1T} B^{CB}_{21,T}/P_T$ is strictly negative in the limit: fiscal authority 1 is lending to NCB 2; in parallel, fiscal authority 2 receives higher remittances from NCB 2. This case involves a national fiscal authority maintaining a positive net claim on an NCB in another country.

Table D.1 summarizes the five feasible outcomes for the equilibrium with the asymmetric fiscal expansion in country 2 from Section 3.

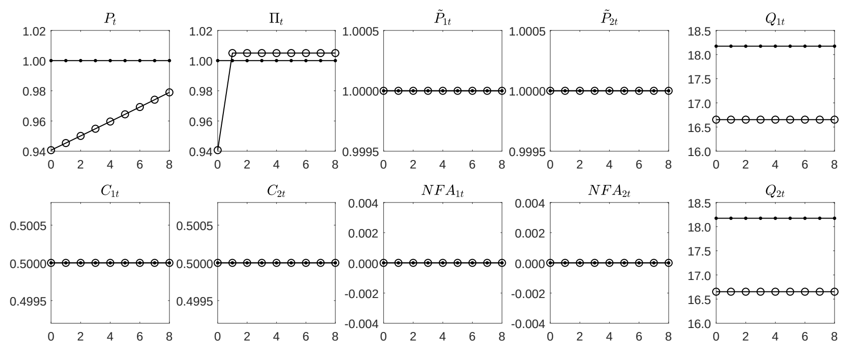
Table D.1: **Asymmetric fiscal expansion, feasible outcomes**

Case	Limit of						
	$\beta^T \frac{T_{12,T}}{P_T}$	$\beta^T \frac{Q_{2T} B^H_{12,T}}{P_T}$	$\beta^T \frac{Q_{1T} B^H_{11,T}}{P_T}$	$\beta^T \frac{Q_{1T} B^H_{21,T}}{P_T}$	$\beta^T \frac{Q_{2T} B^H_{22,T}}{P_T}$	$\beta^T \frac{Q_{2T} B^{CB}_{12,T}}{P_T}$	$\beta^T \frac{Q_{1T} B^{CB}_{21,T}}{P_T}$
(1)	0.055	0	0	0	0	0	0
(2)	0	0.055	-0.055	0	0	0	0
(3)	0	0	0	-0.055	0.055	0	0
(4)	0	0	0	0	0	0.055	0
(5)	0	0	0	0	0	0	-0.055

E Additional results for Sections 3 and 4

An increase in the monetary policy rate with symmetric debt duration (Section 3). In period 0, the central bank raises its inflation target from $\Pi = 1$ to $\Pi = 1.005$. To implement the new inflation objective, the central bank increases the interest rate on reserves from $R = 1/\beta$ to $R = 1.005/\beta$ permanently ($R_t = R$, $t \geq 0$). Figure E.1 shows the equilibrium for the benchmark passive-money active-fiscal policy mix from Section 3 in the case when public debt maturity is the *same* for both countries (lines with circles). From period 1, the union inflation rate equals the central bank's new, higher inflation target, $\Pi_t = 1.005$ for $t \geq 1$. In period 0, the union price level falls. There is no change in consumption or relative prices.

Figure E.1: **Monetary policy shock with symmetric debt duration**

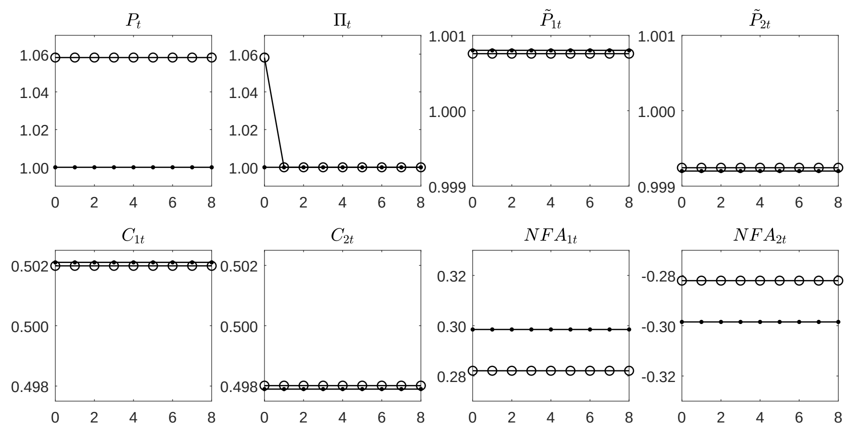


Lines with points: baseline (no shocks). Lines with circles: increase in monetary policy rate. Benchmark passive-money active-fiscal policy mix from Section 3.

Non-zero initial net foreign assets: symmetric fiscal expansion and asymmetric fiscal expansion (Section 3). We consider the symmetric fiscal expansion and the asymmetric fiscal expansion by country 2 from Section 3, except that we relax the assumption that initial net foreign assets equal zero. We suppose that in period -1 country 1 is a net creditor of country 2: relative to Section 3, we increase $B^H_{12,-1}$ and decrease $B^H_{22,-1}$ holding their sum constant; in particular, initial net foreign assets (liabilities) of country 1 (country 2) equal 15 percent of annual GDP. The other numerical assumptions are as in Section 3.

Figure E.2 reports the baseline equilibrium in the absence of shocks (lines with points). Compared with the case of zero initial net foreign assets (Figure 2 in the paper, lines with points), consumption of household 1 (the net creditor) is higher, consumption of household 2 (the net debtor) is lower, the price level in country 1 is higher, and the price level in country 2 is lower. The price level of the union is unchanged.

Figure E.2: **Symmetric fiscal expansion, non-zero initial net foreign assets**



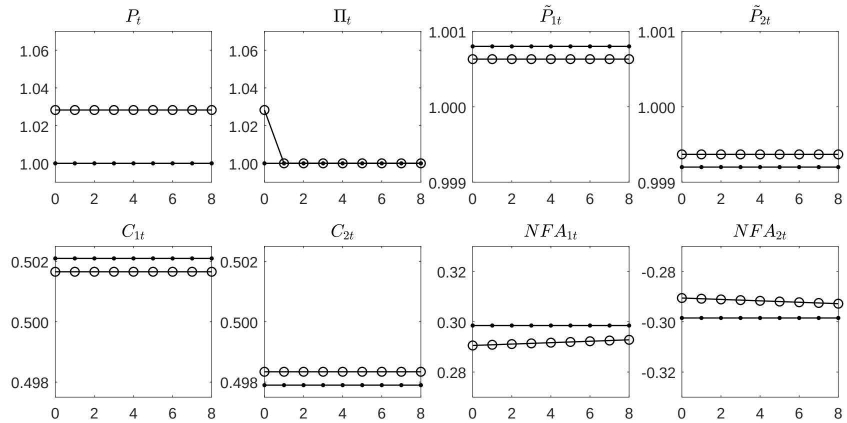
Lines with points: baseline (no shocks). Lines with circles: symmetric fiscal expansion.

Figure E.2 also shows the equilibrium given the same *symmetric* fiscal expansion as in Section 3 (lines with circles). Compared with the case of zero initial net foreign assets (Figure 2 in the paper, lines with circles), there is now a wealth transfer from household 1 (the net creditor) to household 2 (the net debtor). Consumption in country 1 falls and consumption in country 2 rises, permanently. The relative price levels change accordingly. There is a one-time change in the net foreign assets in period 0 in favor of the net debtor country. This is the standard revaluation effect in a model with nominal debt and imperfect risk sharing. Notice that "the intertemporal budget constraint" of the public sector, and therefore also "the intertemporal balance-of-payments constraint," hold in each country in this equilibrium.

Figure E.3 reports the equilibrium given the same *asymmetric* fiscal expansion by country 2 as in Section 3 (lines with circles). Compared with the case of zero initial net foreign assets (Figure 3 in the paper, lines with circles), country 2 gains for two reasons: the standard revaluation effect due to the initial non-zero net foreign asset position, and the wealth transfer due to the benchmark passive-money active-fiscal policy mix. In Figure E.3, the net foreign assets of country 2 rise on impact (the first effect) and then fall persistently (the second effect). In Figure 3 in the paper only the second effect is present because of the zero initial net foreign asset position.

National debt and the central bank's balance sheet (Section 3). The model determines

Figure E.3: **Asymmetric fiscal expansion, non-zero initial net foreign assets**



Lines with points: baseline (no shocks). Lines with circles: fiscal expansion in country 2.

the equilibrium portfolio $\sum_j Q_j B_{ijt}^H + H_{it}$ for each household i in every period $t \geq 0$. The model does not determine more details of the households' portfolios (bonds of the different national fiscal authorities are perfect substitutes; reserves and bonds are perfect substitutes). We can compute a more detailed path of bond and reserve holdings in equilibrium if we make some assumptions about the specifics of the central bank's balance sheet policy.

Let us give an example. Suppose that the common monetary authority chooses bond holdings B_{it}^{CB} for each i and $t \geq 0$ and instructs the NCBs to implement this path of bond holdings; consider the policy rule "$B_{it}^{CB} = \max\left(\delta B_{it}, 0\right)$, where $\delta \in (0, 1)$ is a parameter, and NCB i holds only debt of fiscal authority i ($B_{ijt}^{CB} = 0$ for $i \neq j$ in every period)." We use the max operator because we have not restricted B_{it} to be non-negative, while reserves must be non-negative. We make δ, the fraction of government bonds held by the central bank, constant over time but one could make it time-varying. The assumption "NCB i holds only debt of fiscal authority i" is consistent with the government bond purchase programs of the Eurosystem.

Next, suppose that the remittance from NCB i to fiscal authority i is governed by the following rule: let $\tilde{Z}_{it} \equiv \tilde{W}_{it} Z_{it}$ denote the remittance expressed in units of GDP of the union; from any period in which there is a shock, a time-invariant remittance $\tilde{Z}_i$ is paid such that the present value of the remittance equals that period's after-the-shock value of the NCB's assets minus liabilities,

$$\tilde{Z}_i = (1 - \beta) \frac{\left(1 + \rho_i Q_{ik}\right) B_{ii,k-1}^{CB} - R_{k-1} \left(H_{i,k-1} - T_{ij,k-1}\right)}{P_k} \tag{E.1}$$

where k denotes a period in which there is a shock (in most of the paper, $k = 0$; in the extension with default risk, shocks occur in two periods, $k = 0$ and $k = T$). This remittance rule has the property that all profits (or losses, $\tilde{Z}_i$ can be strictly negative) of the national central bank are absorbed by the national treasury. In the extension with currency, the central bank earns seigniorage revenue from currency issuance – we then add the seigniorage revenue on the right-hand side of equation (E.1).

Lastly, as an initial condition we can set $\sum_j (1 + \rho_j Q_j) B_{ij,-1}^{CB} = R_{-1} H_{i,-1}$ and $T_{ij,-1} = 0$ for each i (i.e., the initial capital of NCB i equals zero).

Having made such assumptions, we can calculate the path of debt of each fiscal authority i, B_{it}, using the solution for the variables stated in the definition of a perfect-foresight equilibrium (consumption levels and prices) and the flow budget constraint of fiscal authority i, equation (4) in the paper. We can also calculate the path of $\sum_i H_{it}$ from the flow budget constraint of the common monetary authority, equation (5) in the paper. This computation yields a path of bond and reserve holdings that is consistent with the equilibrium we solved for before.[2]

We can also solve the model assuming that the central bank holds private debt. In the setup of Section 3, suppose the central bank holds private debt and does not hold government bonds ($B_{ijt}^{CB} = 0$ for each i, j, and t). Specifically, NCB i makes one-period loans to household i at the interest rate R. Let D_i denote debt of household i held by the central bank. The equilibrium conditions are the same as before, except that in each equation where initial *net* claims of household i on the public sector appear one must subtract $RD_{i,-1}$. In particular, one must subtract $R \sum_i D_{i,-1}$ from the numerator on the left-hand side of equation (20) in the paper. If we adjust the initial conditions appropriately (household i owns the government bonds previously held by the central bank, but household i is also indebted to the central bank so that the household's net assets are

[2]We need to make another assumption to compute the path of reserves for each household i (each NCB i), H_{it}, separately (as opposed to the sum $\sum_i H_{it}$). For example, if we suppose that $T_{ijt} = 0$ for each i, j and $t \geq 0$ (no net claims between the NCBs), we can calculate H_{it} for each i and $t \geq 0$ from the flow budget constraint of NCB i, equation (6) in the paper, and we can compute $\sum_j B_{ijt}^H$ for each i and $t \geq 0$ from the flow budget constraint of household i, equation (3) in the paper. Even then the model determines the sum $\sum_j B_{ijt}^H$ but not $B_{i1t}^H, \ldots, B_{iIt}^H$ individually. To pin down all asset holdings, in future work one could assume that reserves and government bonds provide liquidity services in different amounts (and bonds of fiscal authority i provide a different convenience yield to household i than to household j).

unchanged), the equilibrium paths of the price level of the union, the relative price levels, and consumption in each country are *the same* as in Section 3. It is sometimes claimed that active fiscal policy requires the central bank to "monetize" government bonds in equilibrium. This claim is incorrect. The central bank need not hold any government bonds in equilibrium.

Asymmetric fiscal expansion, additional experiments (Section 4). We consider the same asymmetric fiscal expansion by country 2 as in Section 4 of the paper, except that we change parameter values. First, suppose that the fiscal authority in country 2 responds more aggressively to the national debt-to-GDP ratio than the fiscal authority in country 1. Replace equation (27) in the paper with

$$\tilde{S}_{it} = \phi_{it} + \phi_{Bi} \frac{Q_{it-1} B_{it-1}}{P_{t-1}} \tag{E.2}$$

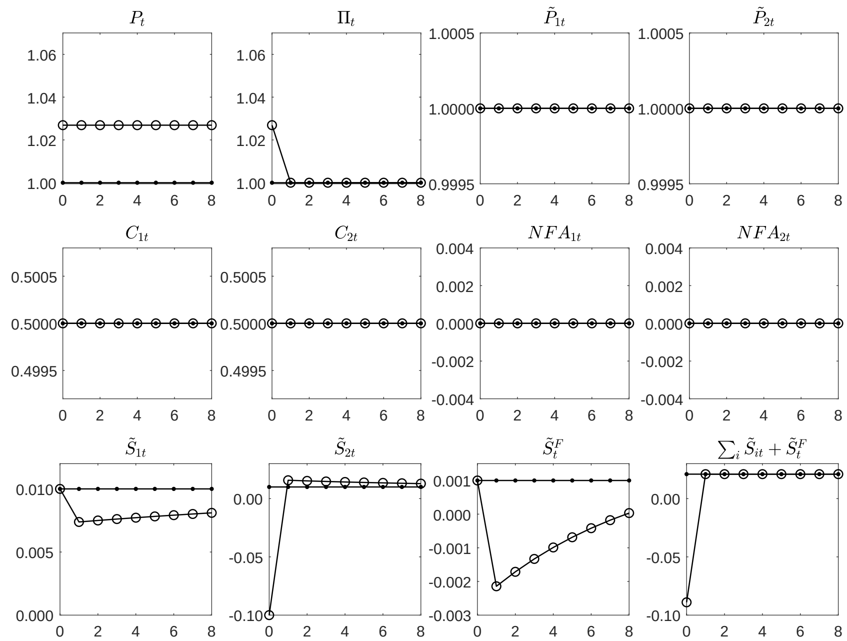
where $0 < \beta^{-1} - \phi_{Bi} < 1$ for each i, and replace equation (29) in the paper with

$$\tilde{S}_t^F = \phi_t^F - \frac{\sum_i \phi_{Bi} Q_{it-1} B_{it-1}}{P_{t-1}}. \tag{E.3}$$

Set $\phi_{B1} = 0.05$, $\phi_{B2} = 0.1$ (compare with $\phi_{B1} = \phi_{B2} = 0.05$ in Section 4). Figure E.4 (lines with circles) shows the equilibrium after the same asymmetric fiscal expansion by country 2 as in Section 4. The responses of the union price level, consumption, and relative prices are the same as in Section 4 (compare with Figure 5 in the paper). The changes in the surplus of country 2 from period 1 double compared with Section 4 (a maximum of 1.58 percent of national GDP, compared with the steady-state surplus of 1 percent and with a maximum of 1.29 percent in Section 4). To maintain a constant sum of the surpluses, the changes in the common surplus from period 1 increase (a maximum deficit of 0.21 percent of union GDP, compared with the steady-state surplus of 0.1 percent and with a minimum surplus of 0.074 percent in Section 4).

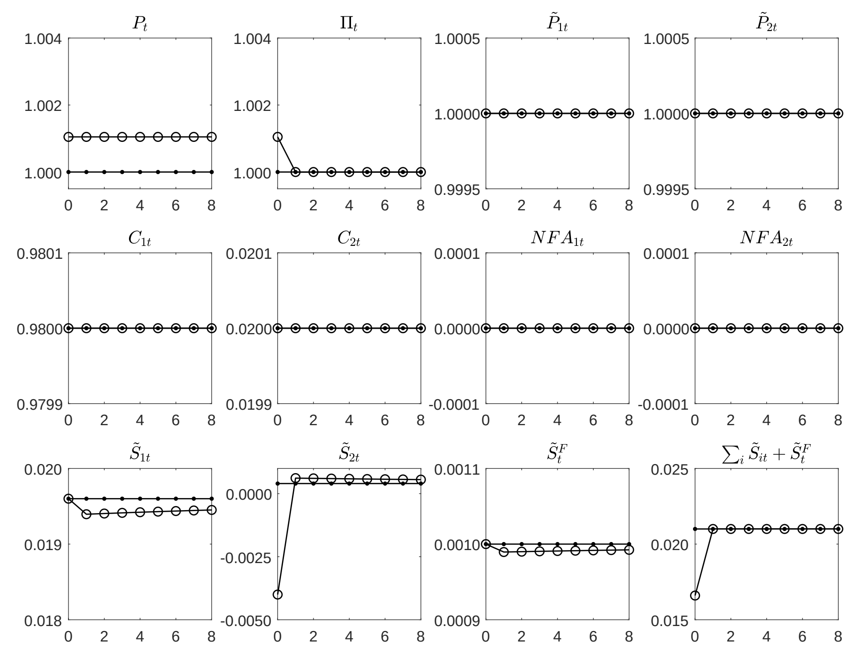
Second, suppose that country 2 is small. Specifically, consider the small-country parameterization from Table 2 in the paper ($n_1 = 0.98$, $n_2 = 0.02$, $\nu = 0.4$, $\theta = 0.5$, the fifth row in Table 2). Set $\phi_{B1} = \phi_{B2} = 0.05$. Figure E.5 (lines with circles) shows the equilibrium after the same asymmetric fiscal expansion by country 2 as in Section 4. The union inflation rate in period 0 is essentially the same as in the fifth row of Table 2, but there are no changes in consumption or relative prices. Furthermore, the changes in the surplus of country 1 and in the common surplus from period 1 are very small because of the small size of the expanding country.

Figure E.4: **Asymmetric fiscal expansion with Eurobonds**



Lines with points: baseline (no shocks). Lines with circles: fiscal expansion in country 2.

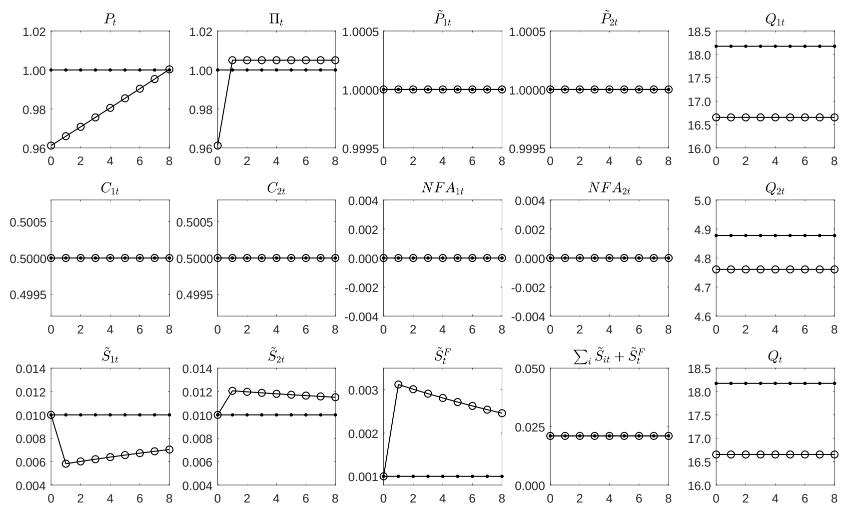
Figure E.5: **Asymmetric fiscal expansion with Eurobonds**



Lines with points: baseline (no shocks). Lines with circles: fiscal expansion in country 2.

An increase in the monetary policy rate with asymmetric debt duration (Section 4). Figure E.6 (lines with circles) shows the equilibrium, described in Section 4, after an increase in the monetary policy rate with $\rho_1 = 0.95$, $\rho_2 = 0.8$ (longer public debt in country 1 than in country 2), and $\rho = 0.95$ (unchanged duration of Eurobonds). The union price level and the national bond prices follow identical paths as in Section 3 (Figure 4 in the paper), but there are no changes in consumption or relative prices. In period 0, the real value of public liabilities falls in country 1 and rises in country 2, due to the different duration of bonds, as in Section 3. Following the feedback rules assumed in Section 4, the surplus in country 1 decreases from period 1, whereas the surplus in country 2 and the common surplus increase (the latter reaches a maximum of 0.31 percent of union GDP, compared with the steady-state of 0.1 percent).

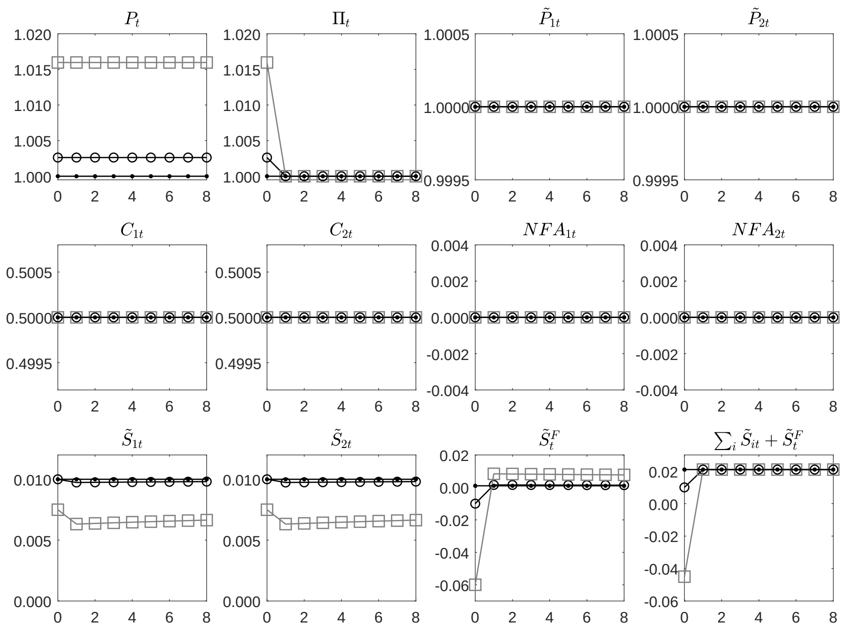
Figure E.6: **Monetary policy shock with asymmetric debt duration and Eurobonds**



Lines with points: baseline (no shocks). Lines with circles: increase in monetary policy rate. Passive-money active-fiscal policy mix with Eurobonds from Section 4.

An expansion by the common fiscal authority (Section 4). In the policy configuration of Section 4, consider a one-time unanticipated shock to ϕ_0^F such that $\tilde{S}_0^F = -0.01$ (the ratio of the period 0 common deficit to the steady-state common surplus is $-\tilde{S}_0^F/\tilde{S}^F = 10$). This is an expansion by the common authority on its own. Figure E.7 shows the equilibrium (lines with circles). The period 0 union inflation rate depends on the change in the sum of the surpluses of all fiscal authorities, the common authority *and* the national authorities. See equation (30)

Figure E.7: **Expansion by common fiscal authority**



Lines with points: baseline (no shocks). Lines with circles: expansion by the common fiscal authority. Lines with squares: same expansion with a higher steady-state primary surplus.

in the paper. Here, $P_0^{-1} = \beta + (1-\beta)\,0.01/0.021 = 1.0026^{-1}$ because $\tilde{S}_0^F + \sum_i \tilde{S}_i = 0.01$ and $\tilde{S}^F + \sum_i \tilde{S}_i = 0.021$. The inflation rate (0.26 percent) is *much smaller* than after the symmetric fiscal expansion in Section 4 (where it equals 5.82 percent). The reason is that the steady-state common surplus is small, 5 percent of the sum of the national surpluses.

Additional Eurobonds could be issued and common taxes raised in the steady state. In the real world, however, an increase in steady-state taxation would be distortionary (and we would be contemplating a sizable increase in steady-state taxation). Consider a different route, without a change in steady-state taxation. Recall that in the baseline parameterization in the paper the national debt-to-GDP ratio equals 100 percent in each country, and suppose that the central bank initially holds 25 percent of national public debt. Now imagine that additional Eurobonds, worth 25 percent of annual GDP of the union, are issued and swapped with the central bank against its holdings of national debt; furthermore, coincident with the bond swap, the common fiscal authority acquires the right to tax directly in order to back the newly issued Eurobonds while the national fiscal authorities stop making payments on their bonds held by the common fiscal authority. Thus, the common surplus increases to $\tilde{S}^F = 0.001 + 0.005 = 0.006$, where 0.005 is 25 percent of the sum of the national surpluses before the swap, and the national surpluses decrease by 25 percent. The equilibrium in the absence of shocks is unchanged, except that Eurobonds equal 30 percent instead of 5 percent and national debt equals 75 percent instead of 100 percent, as a ratio of annual GDP.[3] Suppose that $\tilde{S}_0^F = -0.06$ (again, the ratio of the period 0 common deficit to the steady-state common surplus is $-\tilde{S}_0^F/\tilde{S}^F = 10$). Figure E.7 shows the equilibrium (lines with squares). The inflation rate increases, $P_0^{-1} = \beta - (1-\beta)\, 0.045/0.021 \simeq 1.016^{-1}$ (1.6 percent vs. 0.26 percent before). The reason is that the sum of the period 0 surpluses of all fiscal authorities is much smaller, $\tilde{S}_0^F + \sum_i \tilde{S}_i = -0.045$, for the same ratio of the period 0 common deficit to the steady-state common surplus.

F Other ways to implement active fiscal policy (Section 4)

In this appendix, we consider additional ways to implement active fiscal policy in a monetary union. We continue to assume that the central bank pegs the interest rate on reserves, $R = \Pi/\beta$.

F.1 The Sims rule

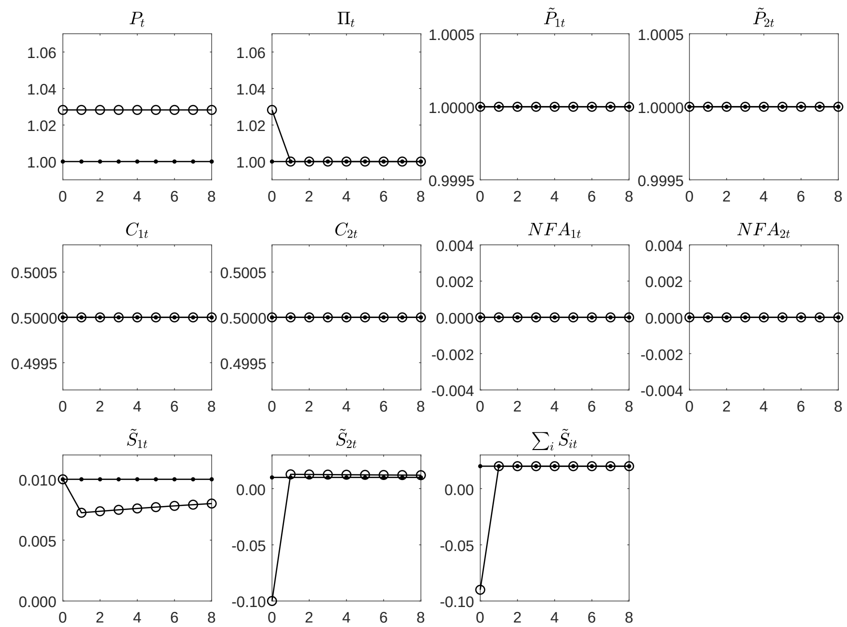
We return to the setup without a common fiscal authority as in Section 3. We assume that each national fiscal authority follows a feedback rule similar to Sims (1997), Section VI. Specifically, fiscal authority $i = 1, 2$ sets its primary surplus according to

$$\tilde{S}_{it} = \phi_{it} + \phi_B \left(\frac{Q_i B_{it-1}}{P_{t-1}} - \eta_i \frac{\sum_j Q_j B_{jt-1}}{P_{t-1}} \right) \tag{F.1}$$

where $\eta_i > 0$, $\sum_i \eta_i = 1$, and $0 < \beta^{-1} - \phi_B < 1$. The intercept in equation (F.1) is time-invariant, $\phi_{it} = \phi_i > 0$, except that in period 0 the intercept may be subject to a one-time unanticipated shock. The Sims rule requires each fiscal authority to respond, not to the real value of its own debt, but to its deviation from an intended share in the sum for the union. Equation (F.1) implies that $\sum_i \tilde{S}_{it} = \sum_i \phi_{it}$, which makes fiscal policy active at the level of the union. We set $\eta_i = \left(Q_i B_{i,-1} / \sum_j Q_j B_{j,-1} \right)$, $\phi_i = 0.02 n_i$, $i = 1, 2$, and $\phi_B = 0.05$. The equilibrium in the absence of shocks is identical to the baseline equilibrium in Section 3.

[3] We decrease the value of ϕ_i (equation (27) in the paper) by 25 percent for each i.

Figure F.1: **Asymmetric fiscal expansion with the Sims rule**



Lines with points: baseline (no shocks). Lines with circles: fiscal expansion in country 2.

Suppose that $\phi_{i0} = -0.2n_i$, $i = 1, 2$. This is the same symmetric fiscal expansion as in Section 3. We find that the equilibrium is identical to Section 3 (Figure 2 in the paper, lines with circles).[4]

Suppose that $\phi_{20} = -0.2n_2$. This is the same asymmetric fiscal expansion as in Section 3 (Figure 3 in the paper). Figure F.1 shows the equilibrium (lines with circles) which is similar to Section 4. The surplus in country 2 rises for some time, starting in period 1, to pay for a part of the period 0 expansion. In parallel, the surplus in country 1 falls, as country 1 responds to the expansion in country 2 by expanding itself. Consequently, in Figure F.1 the union price level follows the same path as in Figure 3, but no cross-country wealth transfer occurs. (If fiscal authority 1 tightens in response to the expansion in the other country, $\phi_{10} > \phi_1$, then the period 0 inflation rate falls relative to Figure F.1 and there is still no cross-country wealth transfer.)

The Sims rule is attractive. Fiscal policy in the union as a whole is active, implying price level determinacy, and asymmetric deficits and surpluses do not cause cross-country wealth transfers. However, feedback rule (F.1) is quite different from standard passive fiscal policy. Feedback rule

[4] To solve the model with the Sims rule, we proceed as in Section 4 (see Appendix C) except that to compute the surplus of each fiscal authority i we use feedback rule (F.1).

(F.1) requires a country to vary its primary surplus in response to a surplus or deficit in another country *even if* the real value of the country's own debt is unchanged. Furthermore, in the euro area with twenty countries, *all twenty* national fiscal authorities would need to respond appropriately to one another's budget balance.

F.2 One active fiscal agent

We consider the case when a single fiscal authority maintains an exogenous primary surplus and the other fiscal authorities are passive.

There are two subcases to consider. In the first subcase, the active fiscal agent is the common fiscal authority. We are in the setup of Section 4 except that the common authority maintains an exogenous surplus, $\tilde{S}_t^F = \tilde{S}^F > 0$. We set $\tilde{S}^F = 0.001$ (the stock of Eurobonds equals 5 percent of annual GDP of the union). The equilibrium in the absence of shocks is identical to the baseline equilibrium in Section 4. In the second subcase, the active fiscal agent is a national fiscal authority, for example fiscal authority 2 (for simplicity, there is no common fiscal authority). Fiscal authority 1 follows the passive feedback rule given by equation (27) in the paper. We adopt a parameterization such that the equilibrium in the absence of shocks is the same as the baseline equilibrium in Section 3.[5]

Consider an expansion by the active fiscal authority. In the first subcase, the common surplus falls in period 0, $\tilde{S}_0^F = -0.01$ (the ratio of the period 0 deficit to the steady-state surplus is $-\tilde{S}_0^F/\tilde{S}^F = 10$). In the second subcase, the surplus in country 2 falls in period 0, $\tilde{S}_{20} = -0.2n_2$ (again, the ratio of the period 0 deficit to the steady-state surplus is $-\tilde{S}_{20}/\tilde{S}_2 = 0.2n_2/0.02n_2 = 10$). In *either* case, the period 0 union inflation rate is the same as after the *symmetric* fiscal expansion in Section 3 (Figure 2 in the paper, lines with circles), $P_0^{-1} = \beta - (1-\beta)\,\tilde{S}_0^F/\tilde{S}^F = \beta - (1-\beta)\,10 \simeq 1.06^{-1}$. The inflation rate depends only on the *ratio* of the period 0 deficit to the steady-state surplus of the active fiscal authority. The inflation rate is independent of the steady-state size of the common fiscal authority (the first subcase) or of the size of country 2 (the second subcase). For any other P_0, households' holdings of Eurobonds rise (or fall) in real terms, violating the

[5] To solve the model with a single active fiscal agent, we guess P_0. As in Section 4 (see Appendix C), we compute the path of the surplus of any fiscal authority that follows a feedback rule. We calculate the period 0 present value of the sum of the surpluses, and verify the guess for P_0 using equation (20) in the paper. We solve the rest of the model as before.

transversality condition (equation (16) in the paper), regardless of the initial size of such holdings in relation to initial households' wealth (in the first subcase; in the second subcase, the same statement applies to households' holdings of country 2 bonds).

The fiscal theory focuses on the wealth effect of fiscal policy (a change in the present value of the budget surplus makes households richer or poorer at a given price level). In this model, outside of this Appendix F.2, the wealth effect always depends on households' wealth *as a whole*. In Appendix F.2, by contrast, households react to a component of their wealth that can be arbitrarily small. Households recognize that even an arbitrarily small component of their wealth will *eventually* become very large if they fail to adjust spending – and they adjust spending without delay which causes inflation in the union also without delay.

This particular result may not be robust to removing the assumption of perfect forward-looking behavior. As an example, Appendix F.3 considers a simple backward-looking model, inspired by Sims (2016), in which a different result arises.[6] We analyze the same expansion by the common fiscal authority as in Appendix F.2. Recall that in the forward-looking model studied so far, the inflation rate depends only on the *ratio* of the period 0 deficit to the steady-state surplus of the active fiscal authority; the union inflation rate is independent of the steady-state size of the active fiscal authority (its steady-state surplus). We find that this result no longer holds in the backward-looking model. In the backward-looking model, the period 0 union inflation rate is increasing in the steady-state surplus of the common authority. In other words, the short-run inflationary impact of the common authority is greater if Eurobonds are a larger component of households' wealth to begin with, which stands in contrast to – and seems more realistic than – the forward-looking model. The details are directly below, in Appendix F.3.

We conclude that in this policy configuration the common authority may need to be sizable in steady state in order to have a meaningful effect, in contrast to Section 4 in the paper.[7]

[6]Like Sims (2016), we assume backward-looking consumption behavior. Sims assumes a Phillips curve and works in a single-country model, whereas we assume flexible prices and work in a monetary-union model.

[7]Jarociński and Maćkowiak (2018) and Bianchi, Melosi, and Rogantini Picco (2023) work with monetary-union models in which the common fiscal authority is the active fiscal agent. The model of Bianchi, Melosi, and Rogantini Picco (2023) is much richer (it is a TANK model with government consumption and distorting taxes), and therefore the wealth effect is only one of multiple affects of fiscal policy.

F.3 One active fiscal agent in a backward-looking model

We study a model of a monetary union that differs from the model in the paper in that households are backward-looking. In the spirit of Sims (2016), suppose that household i's desired consumption in period t is a function of the real value of its assets and its after-tax endowment income:

$$\log(C_{it}/Y_i) = \gamma_Y \log\left(\frac{Y_{it} - T_{it}}{Y_i - T_i}\right) + \gamma_A \log\left(\frac{A_{it}/P_t}{a_i}\right), \tag{F.2}$$

where A_{it} are the nominal assets held by household i at the end of period t, and T_{it} are taxes paid by household i in period t. Variables without a time subscript denote steady state values, and $\gamma_Y, \gamma_A, a_i > 0$ are parameters.

Apart from this assumption, the model follows the model in the paper. We focus on the case where the monetary union consists of two countries, $i = 1, 2$, and endowments are constant over time. For ease of exposition, we assume that the endowment goods are perfect substitutes (hence, the national price indices coincide with the price level for the union).

The public sector consists of the common central bank, the two national fiscal authorities, and a common fiscal authority, as in Section 4 of the paper. Household i pays taxes to national fiscal authority i and to the common authority, $T_{it} = S_{it} + S^F_{it}$. We assume that the fiscal authorities issue one-period nominal bonds, which pay the constant gross interest rate R, and reserves are in zero supply. Thus, the assets of household i consist of government bonds and Eurobonds. The flow budget constraints of households and public sector authorities, equations (3)-(6) and (28) in the paper, are adjusted accordingly.

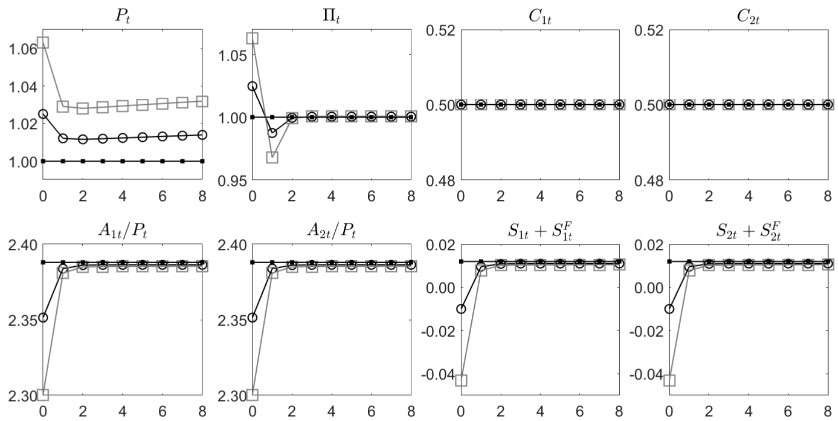
We assume that the common fiscal authority maintains an exogenous primary surplus, $S^F_t = S^F$, where $S^F_{it} = n_i S^F_t$, whereas the national fiscal authorities follow passive fiscal feedback rules

$$S_{it} = \phi_{0i} + \phi_B \frac{B_{it-1}}{P_{t-1}}, \tag{F.3}$$

$0 < \beta^{-1} - \phi_B < 1$, $\phi_{0i} = (1/\beta - 1 - \phi_B)B_{i,-1}$, $i = 1, 2$.

Given endowments, and initial conditions for government bonds and Eurobonds, we can solve for consumption C_{it} and the price level P_t, $i = 1, 2$, $t \geq 0$, using households' consumption function (F.2), the fiscal rules determining the surpluses, the households' flow budget constraints, the fiscal authorities' flow budget constraints, and the goods market clearing condition $Y_1 + Y_2 = C_{1t} + C_{2t}$.

Figure F.2: **Expansion by active fiscal authority in the backward-looking model**



Lines with points: baseline (no shocks). Lines with circles: asymmetric fiscal expansion by common authority. Lines with squares: asymmetric fiscal expansion by common authority, larger common authority.

To the extent possible, we follow the parameterization from the paper. In particular, $\beta = 0.995$, $\Pi = 1$, $S_i = 0.02n_i$, $Y_i = n_i$, $n_i = 0.5$, and $\phi_B = 0.05$. The initial ratio of national government debt to annual GDP equals 100%, and the initial ratio of Eurobonds to annual GDP of the union equals 20% (implying $S^F = 0.004$). Let $A_{i,-1} = B_{i,-1} + n_i F_{-1}$, $i = 1, 2$. Household i holds the government bonds issued by national fiscal authority i and a fraction of the Eurobonds that is proportional to the size of country i. We set $\gamma_Y = 0.7$, $\gamma_A = 2$ (as in Sims, 2016), and $a_i = A_{i,-1}$.

Figure F.2 plots the equilibrium paths of key variables. In the absence of shocks (lines with points), the price level and consumption are constant over time, and consumption of household 1 equals consumption of household 2.

Suppose that the common fiscal authority lowers its surplus in period 0, $S_0^F/S^F = -10$. In the resulting equilibrium (lines with circles), the price level jumps in period 0, $\Pi_0 = 1.025$, while consumption remains constant. Both households seek to consume more in response to the tax cut. In order to realign aggregate demand with the constant supply of the endowment good, the price level has to increase sufficiently so as to lower the real value of households' end-of-period assets. In period 1, the surplus of the common authority increases back to the steady state, but the national fiscal authorities lower their surpluses in response to the decline in the real value of national government debt. The price level falls but remains above the initial level. In subsequent periods, the price level increases gradually. In the long run, it converges to a constant (not shown), where $\lim_{T\to\infty} P_T > P_0$.

Suppose the initial ratio of Eurobonds to annual GDP of the union rises from 20% to 50%.[8] Figure F.2 plots the equilibrium when the common authority reduces its period 0 surplus (lines with squares). The size of the tax cut is the same proportion of the common authority's steady-state surplus ($S_0^F/S^F = -10$). With a larger steady-state surplus, the initial jump in the price level is larger, $\Pi_0 = 1.063$ (compare with $\Pi_0 = 1.025$). Households' after-tax income increases by more, and therefore a bigger jump in the price level is necessary for aggregate demand to match aggregate supply. As in the previous case, in the long run the price level converges to a constant where $\lim_{T\to\infty} P_T > P_0$. The ratio between the price level in period 0 and the long-run price level is bigger than in the previous case, i.e., with a larger common authority more of the adjustment in the price level takes place in the short run.

G Extensions (Section 5)

This appendix presents four extensions of the model.

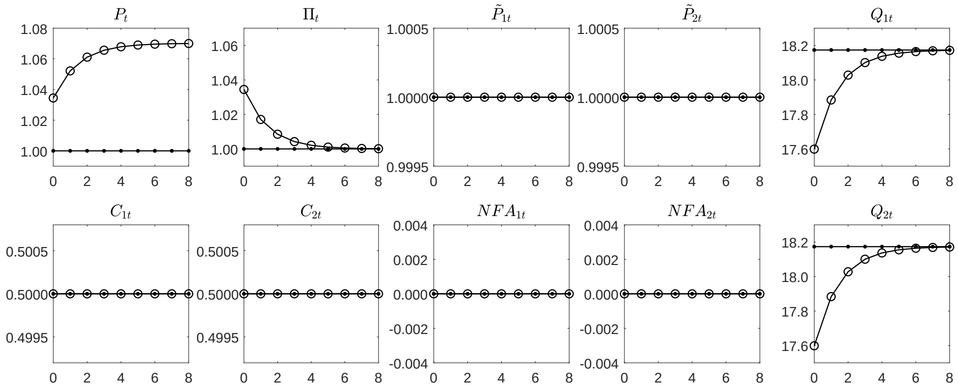
G.1 An interest-rate feedback rule

Suppose that instead of pegging the nominal interest rate, the common monetary authority follows a feedback rule for interest-rate setting, $R_t = \max\left\{\left(\Pi^{1-\alpha}/\beta\right)\Pi_t^{\alpha}, 1\right\}$, where $\alpha \leq 1$ so that monetary policy remains passive. The interest-rate feedback rule includes a lower bound, which is formally justified in the version of the model with currency that does not pay interest (see Appendix G.3). Fiscal policy is as in Section 3 of the paper.

Figure G.1 shows the effects of the same symmetric fiscal expansion as in Section 3 (Figure 2 in the paper) with identical numerical assumptions and $\alpha = 0.5$. There is a persistent increase in inflation and the bond prices decrease. Since the adjustment involves both a period 0 price level jump and a fall in the bond prices, the initial price level jump is smaller than in Section 3. Furthermore, the initial price level jump is smaller if debt maturity is longer (this can be either because the central bank holds a smaller fraction of government bonds, or because the parameter ρ_i

[8] We reduce the initial amount of national government debt symmetrically so that the total amount of households' initial assets is the same as in the previous case with the smaller common authority.

Figure G.1: **Symmetric fiscal expansion with an interest rate feedback rule**



Lines with points: baseline (no shocks). Lines with circles: symmetric fiscal expansion.

is larger). Another interesting feature of this equilibrium is that the central bank makes a capital loss in period 0. The central bank has long-term assets and short-term liabilities, and, since the shock causes persistent inflation, the real value of the assets falls more than the real value of the liabilities.

To solve this version of the model, we begin by guessing the period 0 bond prices Q_{i0}, $i = 1, 2$. We compute P_0 from equation (20) in the paper as usual. We use the interest-rate feedback rule, $R_t = \max\left\{\left(\Pi^{1-\alpha}/\beta\right)\Pi_t^{\alpha}, 1\right\}$, and the equilibrium condition $P_{t+1} = \beta R_t P_t$ to compute the price level of the union in each period $t = 1, 2, \ldots, T$, where T is a very large number. Using the terminal condition that by period T the bond prices have returned to their pre-shock levels, we employ the equilibrium condition $Q_{it} = \beta(1 + \rho_i Q_{it+1})P_t/P_{t+1}$ to solve backwards for the path of the bond prices Q_{it}, $i = 1, 2$, $t = T - 1, \ldots, 1, 0$, and we verify the guess for the period 0 bond prices. We use the bisection method to solve the rest of the model.

G.2 Default risk

We study the effects of partial default by a national fiscal authority. To make the analysis more interesting, we abandon perfect foresight and suppose that agents attach a strictly positive probability to default in the future, and then in one state of the world default occurs in equilibrium while in another state of the world default is avoided. The central bank pegs the interest rate on reserves.

To introduce default into the model, think of an agent who enters period $t \geq 0$ holding one unit of a bond of national fiscal authority j. If default occurs in period t, the bond "shrinks" to Δ_{jt} where $\Delta_{jt} \in (0,1)$ ($\Delta_{jt} = 1$ if there is no default). The bond price Q_{jt} now reflects default risk: the term $1 + \rho_j Q_{jt+1}$ inside the expectation operator on the right-hand side of optimality condition (14) in the paper gets multiplied by Δ_{jt+1}. Each term in the sum $\sum_j (1 + \rho_j Q_{jt})$ in the numerator on the left-hand side of equation (19) in the paper gets multiplied by Δ_{jt}.

The two national fiscal authorities run a constant-surplus policy as in Section 3 (for simplicity, there is no common fiscal authority). Let us modify the asymmetric fiscal expansion from Section 3 as follows. In period 0, a news shock arrives and agents learn that in period $T \geq 1$ the surplus in country 2 will fall, $\tilde{S}_{2T} = -0.5n_2$ (this is a deficit more than twice the size of the deficit in Section 3). Let $k \in \{D, N\}$ denote the state in period T, where D is the state in which fiscal authority 2 defaults and N is the state in which it does not default. The probability of state D is $d \in (0,1)$. In period T all uncertainty is resolved.

Equation (19) in period T in state k can then be written

$$\frac{(1+\rho_1 Q_1)\sum_i B^H_{i1,T-1} + (1+\rho_2 Q_2)\,\Delta_k \sum_i B^H_{i2,T-1} + R\sum_i H_{iT-1}}{P_{T|k}} = \sum_{t=T}^{\infty} \beta^{t-T} \sum_i \tilde{S}_{it} \tag{G.1}$$

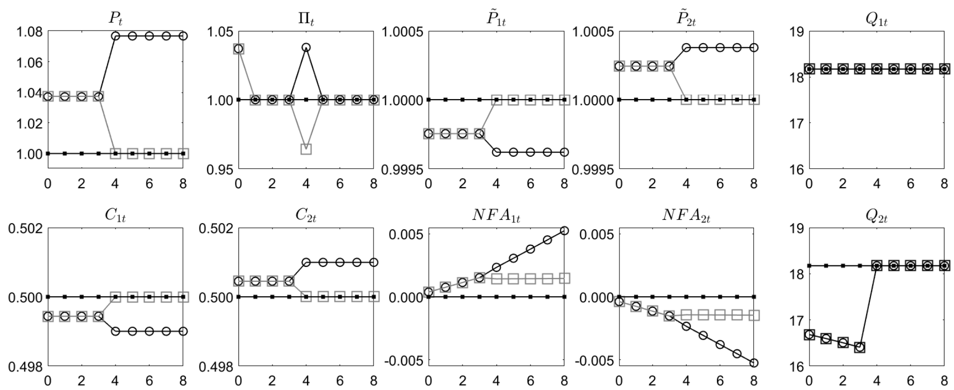
where $P_{T|k}$ is the price level of the union in period T in state k, $\Delta_k \equiv \Delta_{jt|k}$ for $j = 2$, $t = T$, and state k (hence, $\Delta_k = \Delta_D \in (0,1)$ in state D and $\Delta_k = \Delta_N = 1$ in state N), and Q_i is the price of bond i in periods $t \geq T$ for $i = 1, 2$ (Q_i does not depend on k because the interest rate on reserves is constant and the probability of default at $t \geq T+1$ is zero). We set $T = 4$ and $d = 0.5$. We must also specify a value of Δ_D. The arrival of the news about the deficit in country 2 sets off a process of deliberation and bargaining, which we do not model, that takes time and has an uncertain outcome. In state N the deficit is "accepted" by policy makers in the union, including the central bank. In state D the deficit is "rejected," in which case fiscal authority 2 defaults with a haircut implying that the price level of the union in period 4 equals the baseline in the absence of any disturbances. Given the parameterization in Table 1 in the paper, this yields $\Delta_D = 0.812$. One could make another assumption. One could assume that in the default state the policy makers "allow" some inflation relative to the baseline (they "accept" a fraction of the country 2 deficit).

Since we have abandoned perfect foresight, we need to make some additional assumptions to solve the model. The stochastic discount factor of household i in equilibrium no longer depends only on β and on time (footnote 17 in the paper). Therefore, we assume a particular utility function, $U(C_{it}) = \ln(C_{it})$, $i = 1, 2$. Furthermore, the bond price of country 2 is now affected by default risk, which implies that initial gross foreign asset positions matter for the equilibrium outcome. For simplicity, we suppose that initial gross foreign asset positions equal zero. In addition, since there is now a shock in period T in addition to a shock in period 0, in principle gross foreign asset positions in period T also matter. We make an assumption about how these evolve between period 0 and period $T-1$, for a given path of the current account determined by the model. Specifically, we assume that if the current account is unbalanced between period 0 and period $T-1$, the imbalance is financed by debt of fiscal authority 2 held by household 1, B^H_{12t}.

The lines with circles in Figure G.2 show the path of the economy if state N occurs; the lines with squares show the path of the economy if state D occurs. Since the realization of the state is determined in period $T = 4$, the two paths diverge in period 4 (as usual, the lines with points show the baseline equilibrium in the absence of shocks). To gain intuition, note that state N is qualitatively the same as the asymmetric fiscal expansion from Section 3. Accordingly, in period $T = 4$ in state N the price level of the union rises, consumption of household 1 and the relative price level in country 1 fall, and consumption of household 2 and the relative price level in country 2 increase; furthermore, the net claims of country 1 on country 2 grow at rate β^{-1}. In period 0, agents understand that all this will happen with probability $1 - d$. As a consequence, *already in period zero*, the price level of the union rises, consumption of household 1 and the relative price level in country 1 fall to some extent, and consumption of household 2 and the relative price level in country 2 increase to some extent; furthermore, the net claims of country 1 on country 2 begin to grow.

The model now features a government bond spread: the bond price Q_{2t} declines relative to the bond price Q_{1t} at $t = 0$ because of the default risk. Default, when it occurs, triggers *deflation* in the union. Moreover, in the default state there is a wealth transfer from country 1 to country 2 for the usual reason: household 1 holds some bonds of fiscal authority 2 (a current account imbalance in favor of country 1 between period 0 and period $T-1$ has been financed by household 1 lending to fiscal authority 2); the default makes household 1, who is a creditor, poorer than in the baseline. In this example, however, household 1 is richer in state D than in state N: household 1 holds only a small fraction of bonds of fiscal authority 2, and consequently the household's capital loss in

Figure G.2: **Asymmetric fiscal expansion with default risk**



Lines with points: baseline (no shocks). Lines with circles: expansion in country 2 with no default. Lines with squares: same expansion with default.

state D turns out to be smaller than its loss in state N. Since the cross-country wealth transfer is smaller in state D than in state N, state D undoes, to some extent, the relative price changes and consumption changes that took place in period 0, whereas state N exacerbates them. The general lesson here is that, while we are used to thinking of sovereign default as triggering a cross-country wealth transfer, replacing default with inflation as the adjustment mechanism in a monetary union can *also* lead to a cross-country wealth transfer, possibly a *greater* one. It is also interesting that NCB 2, which holds bonds of fiscal authority 2, makes a capital loss in period 0 due to the decline in the bond price; and in period T makes another capital loss from default in state D or a capital gain in state N when the bond price rises.

Let us describe the details of the solution. Recall that: (i) in period $T \geq 1$ with probability d the state is "default" (D), and with probability $1-d$ the state is "no default" (N); (ii) $U(C_{it}) = \ln(C_{it})$, $i = 1, 2$; and (iii) if there is a current account imbalance between period 0 and period $T-1$, it is financed by debt of fiscal authority 2 held by household 1, B^H_{12t} ($B^H_{21t} = 0$, $B^{CB}_{ijt} = 0$, and $T_{ijt} = 0$ for $t = 0, \ldots, T-1$ and $i \neq j$).

We begin by guessing the asset holdings in period $T-1$ and (if $T > 1$) in period $T-2$. We solve the model iteratively, starting at T and going backwards to 0. Finally, we verify the guess.

Period $\mathbf{t = T}$. This is the period in which all uncertainty is resolved: either there is default or not. Given B^H_{ijT-1} and H_{iT-1}, $i, j = 1, 2$, we use equation (G.1) to solve for Δ_D (assuming $P_{T|D} = 1$) and for $P_{T|N}$ (assuming $\Delta_N = 1$). We then use the bisection method to find the vector

$\{C_{1T|k}, C_{2T|k}, \tilde{P}_{1T|k}, \tilde{P}_{2T|k}, \tilde{W}_{1T|k}, \tilde{W}_{2T|k}\}$ that solves the system of equilibrium conditions

$$\frac{(1+\rho_1 Q_1)\, B^H_{i1T-1} + (1+\rho_2 Q_2)\, \Delta_k B^H_{i2T-1} + RH_{iT-1}}{P_{T|k}}$$

$$= \sum_{m=T}^{\infty} \beta^{m-T} \left(\tilde{P}_{im|k} C_{im|k} - \tilde{W}_{im|k} Y_i + \tilde{S}_{im}\right),\ i = 1,2$$

$$Y_j = \sum_i \gamma_{ij} \left(\frac{\tilde{W}_{jT|k}}{\tilde{P}_{iT|k}}\right)^{-\theta} C_{iT|k},\ j = 1,2$$

$$\tilde{P}_{iT|k} = \left(\gamma_{i1} \tilde{W}^{1-\theta}_{1T|k} + \gamma_{i2} \tilde{W}^{1-\theta}_{2T|k}\right)^{\frac{1}{1-\theta}},\ i = 1,2$$

$$1 = \tilde{P}^{n_1}_{1T|k} \tilde{P}^{n_2}_{2T|k},$$

for $k = D, N$, where Q_i is the price of bond i at $t \geq T$. The right-hand side of the first equation can be simplified since consumption and relative prices in each country are constant from T onward.

Period $\mathbf{t = T-1}$. This is the period before the one in which all uncertainty is resolved. Given B^H_{ijT-2} and H_{iT-2}, $i, j = 1,2$, we use the MATLAB routine *fsolve* to find the vector $\{P_{T-1}, C_{1T-1}, C_{2T-1}, \tilde{P}_{1T-1}, \tilde{P}_{2T-1}, \tilde{W}_{1T-1}, \tilde{W}_{2T-1}, Q_{1T-1}, Q_{2T-1}\}$ that solves the system of equilibrium conditions

$$\frac{\sum_j (1+\rho_j Q_{jT-1}) B^H_{ijT-2} + RH_{iT-2}}{P_{T-1}} = \tilde{P}_{iT-1} C_{iT-1} - \tilde{W}_{iT-1} Y_{iT-1} + \tilde{S}_i$$

$$+ d\beta \frac{\tilde{P}_{iT-1} C_{iT-1}}{\tilde{P}_{iT|D} C_{iT|D}} \left(\tilde{P}_{iT|D} C_{iT|D} - \tilde{W}_{iT|D} Y_i + \tilde{S}_{iT}\right)$$

$$+ (1-d)\beta \frac{\tilde{P}_{iT-1} C_{iT-1}}{\tilde{P}_{iT|N} C_{iT|N}} \left(\tilde{P}_{iT|N} C_{iT|N} - \tilde{W}_{iT|N} Y_i + \tilde{S}_{iT}\right)$$

$$+ \frac{\beta}{1-\beta} \left(d\beta \frac{\tilde{P}_{iT-1} C_{iT-1}}{\tilde{P}_{iT|D} C_{iT|D}} \left(\tilde{P}_{iT|D} C_{iT|D} - \tilde{W}_{iT|D} Y_i + \tilde{S}_i\right) \right.$$

$$\left. + (1-d)\beta \frac{\tilde{P}_{iT-1} C_{iT-1}}{\tilde{P}_{iT|N} C_{iT|N}} \left(\tilde{P}_{iT|N} C_{iT|N} - \tilde{W}_{iT|N} Y_i + \tilde{S}_i\right) \right),\ i = 1,2$$

$$Y_j = \gamma_{1j} \left(\frac{\tilde{W}_{jT-1}}{\tilde{P}_{1T-1}}\right)^{-\theta} C_{1T-1} + \gamma_{2j} \left(\frac{\tilde{W}_{jT-1}}{\tilde{P}_{2T-1}}\right)^{-\theta} C_{2T-1},\ j = 1,2$$

$$\frac{1}{P_{T-1}} = d\beta \frac{\tilde{P}_{jT-1} C_{jT-1}}{\tilde{P}_{jT|D} C_{jT|D}} \frac{R}{P_{T|D}} + (1-d)\beta \frac{\tilde{P}_{jT-1} C_{jT-1}}{\tilde{P}_{jT|N} C_{jT|N}} \frac{R}{P_{T|N}},\ j = 1,2$$

$$\frac{Q_{1T-1}}{P_{T-1}} = d\beta \frac{\tilde{P}_{jT-1}C_{jT-1}}{\tilde{P}_{jT|D}C_{jT|D}} \frac{1+\rho_1 Q_1}{P_{T|D}} + (1-d)\beta \frac{\tilde{P}_{jT-1}C_{jT-1}}{\tilde{P}_{jT|N}C_{jT|N}} \frac{1+\rho_1 Q_1}{P_{T|N}},\ j=1,2$$

$$\frac{Q_{2T-1}}{P_{T-1}} = d\beta \frac{\tilde{P}_{jT-1}C_{jT-1}}{\tilde{P}_{jT|D}C_{jT|D}} \frac{(1+\rho_2 Q_2)\Delta_D}{P_{T|D}} + (1-d)\beta \frac{\tilde{P}_{jT-1}C_{jT-1}}{\tilde{P}_{jT|N}C_{jT|N}} \frac{(1+\rho_2 Q_2)\Delta_N}{P_{T|N}},\ j=1,2$$

$$\tilde{P}_{jT-1} = \left(\gamma_{j1}\tilde{W}_{1T-1}^{1-\theta} + \gamma_{j2}\tilde{W}_{2T-1}^{1-\theta}\right)^{\frac{1}{1-\theta}},\ j=1,2$$

$$1 = \tilde{P}_{1T-1}^{n_1}\tilde{P}_{2T-1}^{n_2}.$$

Period $\mathbf{t < T-1}$ (relevant if $\mathbf{T > 1}$). Consider period $t = T-2$. We want to find a vector $\{P_{T-2}, C_{1T-2}, C_{2T-2}, \tilde{P}_{1T-2}, \tilde{P}_{2T-2}, \tilde{W}_{1T-2}, \tilde{W}_{2T-2}, Q_{1T-2}, Q_{2T-2}\}$ that solves the system of equilibrium conditions

$$\tilde{P}_{jT-2} = \left(\gamma_{j1}\tilde{W}_{1T-2}^{1-\theta} + \gamma_{j2}\tilde{W}_{2T-2}^{1-\theta}\right)^{\frac{1}{1-\theta}},\ j=1,2 \tag{G.2}$$

$$1 = \tilde{P}_{1T-2}^{n_1}\tilde{P}_{2T-2}^{n_2} \tag{G.3}$$

$$Y_j = \gamma_{1j}\left(\frac{\tilde{W}_{jT-2}}{\tilde{P}_{1T-2}}\right)^{-\theta} C_{1T-2} + \gamma_{2j}\left(\frac{\tilde{W}_{jT-2}}{\tilde{P}_{2T-2}}\right)^{-\theta} C_{2T-2},\ j=1,2 \tag{G.4}$$

$$\frac{Q_{1T-2}}{P_{T-2}} = \beta \frac{\tilde{P}_{jT-2}C_{jT-2}}{\tilde{P}_{jT-1}C_{jT-1}} \frac{1+\rho_1 Q_{1T-1}}{P_{T-1}},\ j=1,2 \tag{G.5}$$

$$\frac{Q_{2T-2}}{P_{T-2}} = \beta \frac{\tilde{P}_{jT-2}C_{jT-2}}{\tilde{P}_{jT-1}C_{jT-1}} \frac{1+\rho_2 Q_{2T-1}}{P_{T-1}},\ j=1,2 \tag{G.6}$$

$$\frac{1}{P_{T-2}} = \beta \frac{\tilde{P}_{jT-2}C_{jT-2}}{\tilde{P}_{jT-1}C_{jT-1}} \frac{R}{P_{T-1}},\ j=1,2 \tag{G.7}$$

$$1 = \frac{\tilde{P}_{jT-2}C_{jT-2}}{\tilde{P}_{jT-1}C_{jT-1}},\ j=1,2 \tag{G.8}$$

The last equation is the stochastic discount factor (normalized by β): since there is no uncertainty in period $T-2$ about period $T-1$, we have $\Theta_{jT-2,1} = \beta$ for $j = 1,2$. Since $R = \Pi/\beta$, it follows from equations (G.7) and (G.8) that $P_{T-2} = P_{T-1}\Pi^{-1}$. From equations (G.5), (G.6) and (G.8), it then follows that $Q_{iT-2} = \beta\Pi^{-1}(1 + \rho_i Q_{iT-1})$, for $i = 1,2$. Finally, it is straightforward to verify that the solution for consumption and relative prices in period $T-1$ also solves equations (G.2)-(G.4) and (G.8). That is, $C_{jT-2} = C_{jT-1}$, $\tilde{P}_{jT-2} = \tilde{P}_{jT-1}$ and $\tilde{W}_{jT-2} = \tilde{W}_{jT-1}$ for $j = 1,2$. We use the same procedure in any period $t < T-1$.

We verify the guess about asset holdings in the following way. Given the vector $\{P_0$, C_{10}, C_{20}, $\tilde{P}_{10}$, $\tilde{P}_{20}$, $\tilde{W}_{10}$, $\tilde{W}_{20}$, Q_{10}, $Q_{20}\}$, we can determine period 0 asset holdings. We use the balance of payments identity of country 1 to compute B^H_{120}. We use the flow budget constraint of fiscal authority i together with the remittance rule, equation (E.1), to determine B_{i0}, $i = 1, 2$. The balance sheet policy of the central bank, introduced in Section 3, $B^{CB}_{ii0} = \max(\delta B_{i0}, 0)$, $\delta \in (0, 1)$, pins down B^{CB}_{ii0} and B^H_{ii0}, $i = 1, 2$. The flow budget constraint of NCB i pins down H_{i0}, $i = 1, 2$. We use the same procedure to calculate asset holdings in any period $t = 0, \ldots, T-1$. We can then verify the guess about asset holdings, or update the guess if necessary.

G.3 Currency

We study a version of the model in which the monetary base consists of reserves and currency that provides liquidity services and does not pay interest.

Let $M_{it} > 0$ denote currency held by household i in period t. In the flow budget constraint of household i (equation (3) in the paper) we add the term M_{it-1} on the left-hand side and the term M_{it} on the right-hand side. We make the same changes in the flow budget constraint of NCB i (equation (6)). To motivate why households hold currency that does not pay interest, we proceed similarly to Del Negro and Sims (2015) and suppose that in country i consuming one unit requires spending $1 + \psi(V_{it})$ units of income, where $V_{it} \equiv P_{it}C_{it}/M_{it}$ is velocity and $\psi(\cdot)$ is a function that captures transactions costs. Thus, in equation (3) the term $P_{it}C_{it}$ gets multiplied by $1 + \psi(V_{it})$. Similarly, the resource constraint for each good i (equation (9)) now reads $Y_{it} = \sum_j C_{jit}[1 + \psi(V_{it})]$. We would like the $\psi(\cdot)$ function to have the following properties, which are empirically realistic: (i) at the lower bound $R_t = 1$, the demand for currency is finite, (ii) the demand for currency falls as the nominal interest rate rises, and (iii) in the limit as velocity goes to infinity (the demand for currency goes to zero), transactions costs are finite. A function that satisfies these properties is $\psi_0 V_{it}/(1 + V_{it}) + \psi_1/V_{it}$, where $\psi_0 > \psi_1 > 0$ are parameters.

Let us summarize how the optimality conditions stated in Section 2 change. Equation (12) now reads $1/C_{it} = \lambda_{it}[1 + \psi(V_{it}) + \psi'(V_{it})V_{it}]$, where we have assumed that $U(C_{it}) = \ln(C_{it})$. From the first-order condition with respect to M_{it} and equation (13) we can derive a new equilibrium condition, the liquidity preference relation $(R_t - 1)/R_t = \psi'(V_{it})V^2_{it}$ for each i. The transversality condition of household i now reads

$$\lim_{T\to\infty} E_t\left[\Theta_{itT}\left(\frac{\sum_j Q_{jT}B^H_{ijT} + H_{iT} + M_{iT}}{P_T}\right)\right] = 0,$$

instead of equation (16). When we solve forward the budget constraint of the public sector in the union, we obtain a version of equation (19) with $R_{t-1} \sum_i M_{it-1}$ added in the numerator on the left-hand side and the present value of seigniorage revenues added on the right-hand side. The seigniorage revenue term in period t is $\sum_i (R_{t-1} - 1) M_{it-1}/P_t$.

We focus on the perfect-foresight equilibrium with a time-invariant interest rate on reserves, $R_t = R$, and a time-invariant surplus in each country, $\tilde{S}_{it} = \tilde{S}_i > 0$. We now interpret a surplus as inclusive of seigniorage revenue. The definition of a perfect-foresight equilibrium is analogous to Section 3, with the equations $1/C_{it} = \lambda_{it} [1 + \psi (V_{it}) + \psi' (V_{it}) V_{it}]$ and $(R-1)/R = \psi' (V_{it}) V_{it}^2$, which hold for each i, added. With $I = 2$ we must solve for the path of four additional variables, λ_{1t}, λ_{2t}, V_{1t}, and V_{2t}, and we need to specify initial conditions $M_{1,-1}, M_{2,-1}$. If we adjust the other initial conditions appropriately (e.g, household i's initial holdings of currency rise from zero to a strictly positive number while the household's initial holdings of reserves fall, in such a way that the household's net assets are unchanged), the equilibrium paths of the price level of the union, the relative price levels, and consumption inclusive of the transactions costs in each country, $C_{it} [1 + \psi (V_{it})]$, are *the same* as in the model without currency. With seigniorage revenues ($R = \Pi/\beta > 1$), the lump-sum taxes are lower in equilibrium than in the model without currency.

Equilibrium velocity V_{it} for each i follows period by period from $(R-1)/R = \psi' (V_{it}) V_{it}^2$, where we assume that the central bank meets the currency demand in each country. This is consistent with the balance sheet policy from Appendix E, $B_{iit}^{CB} = \max (\delta B_{it}, 0)$, $\delta \in (0,1)$, so long as currency is not too large as a fraction of the monetary base and B_{it} is not too small. One can then solve for the path of P_t, $\tilde{P}_{it}$, the marginal utility of wealth λ_{it}, and C_{it} for each i in every period, following the same steps as in Section 3. Finally, one can use the equation $(R-1)/R = \psi' (V_{it}) V_{it}^2$ again to solve for M_{it}/P_{it} or M_{it}/P_t period by period.

G.4 Non-traded goods

We study a version of the model with a single traded good and I non-traded goods. The source of country heterogeneity, instead of home bias in consumption preferences, is a technological constraint that prevents some goods from being traded internationally.

The changes in the model relative to Section 2 are as follows. In every period, household i in country i receives an endowment of the traded good, $Y_{Tit} > 0$, and an endowment of non-traded good i, $Y_{Nit} > 0$. The household derives utility from consumption of the traded good and non-traded good i according to

$$C_{it} = \left[\gamma^{\frac{1}{\theta}} C_{Tit}^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} C_{Nit}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

where C_{Tit} (C_{Nit}) is consumption of the traded good (non-traded good i) by household i in period t, $\theta > 0$, and $\gamma \in (0,1)$. The consumption-based price index for country i is

$$P_{it} = \left[\gamma P_{Tt}^{1-\theta} + (1-\gamma) P_{Nit}^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

where P_{Tt} (P_{Nit}) is the price of the traded good (non-traded good i) in period t. Real GDP of the union is defined according to $Y_t = \left(\sum_i P_{it} Y_{it}\right) / P_t$, where $Y_{it} = \left(P_{Tt} Y_{Tit} + P_{Nit} Y_{Nit}\right) / P_{it}$ for each i and P_t is defined as before (equation (10) in the paper). The flow budget constraints are the same as in Section 2, except that the terms $W_{it}Y_{it}$, $W_{it}S_{it}$, and $W_{it}Z_{it}$ get replaced by $P_{it}Y_{it}$, $P_{it}S_{it}$, and $P_{it}Z_{it}$, respectively. The resource constraint (9) gets replaced by the resource constraint for the traded good $\sum_i Y_{Tit} = \sum_i C_{Tit}$; in addition, there is a resource constraint for each non-traded good i, $Y_{Nit} = C_{Nit}$. The optimality conditions are unchanged, except that instead of equation (15) we have for each household i

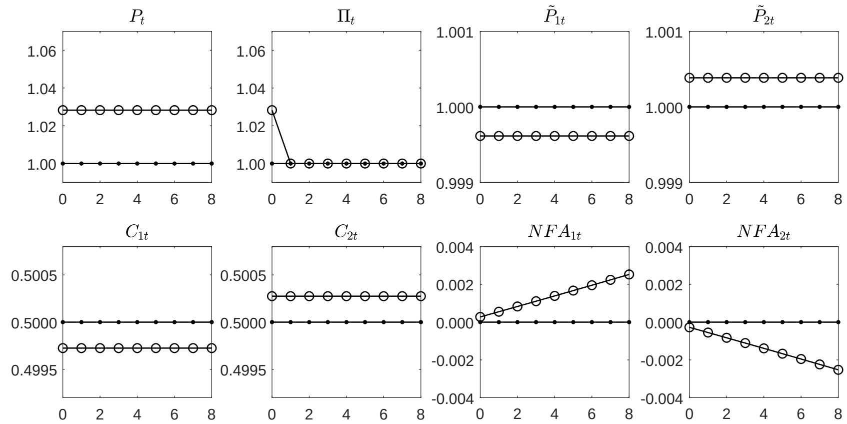
$$C_{Tit} = \gamma \left(\frac{P_{Tt}}{P_{it}} \right)^{-\theta} C_{it}$$

and

$$C_{Nit} = (1-\gamma) \left(\frac{P_{Nit}}{P_{it}} \right)^{-\theta} C_{it}.$$

Let $\tilde{S}_{it} \equiv \tilde{P}_{it} S_{it}$ denote the period t primary surplus of fiscal authority i expressed in units of GDP of the union. As in Section 3, we assume perfect foresight, a time-invariant interest rate on reserves, $R_t = R$, and a time-invariant primary surplus in each country, $\tilde{S}_{it} = \tilde{S}_i > 0$. When we solve this version of the model numerically, we obtain very similar results to Section 3. As an example, let us reconsider the fiscal expansion by country 2, $\tilde{S}_{20} = -0.2 n_2$. We assume $\gamma = 0.3$ and constant endowments $Y_{Ti} = \gamma n_i$ and $Y_{Ni} = (1-\gamma) n_i$. The rest of the parameterization is as in Table 1 in the paper. Figure G.3 shows the equilibrium, which can be compared with Figure 3 in the paper. The figures are almost identical.

Figure G.3: **Asymmetric fiscal expansion with non-traded goods**



Lines with points: baseline (no shocks). Lines with circles: fiscal expansion in country 2.

H Equilibria with passive fiscal policy (Section 6)

We study price level determinacy under passive fiscal policy in the version of the model with currency. We first assume active monetary policy and then passive monetary policy. We solve for perfect-foresight equilibria taking as given initial, period -1 asset holdings.

Each fiscal authority i sets its surplus according to feedback rule (27) in the paper, except that in this appendix we think of the period t surplus of fiscal authority i as inclusive of the period t remittance from NCB i, $\tilde{Z}_{it}$, where $\tilde{Z}_{it} = \tilde{W}_{it} Z_{it}$. Formally, $\tilde{S}_{it}$ equals the right-hand side of equation (27) in the paper plus $\tilde{Z}_{it}$. Let $K_{it} \equiv \sum_j Q_{jt} B_{ijt}^{CB} - H_{it} - M_{it} + \sum_j T_{ijt}$ denote the period t capital of NCB i, and let $k_{it} \equiv K_{it}/P_t$ denote its real value. We assume a remittance rule such that, irrespective of the path of the price level of the union, the limit as time goes to infinity of $\beta^t k_{it}$ equals 0. An example is the remittance rule stated in equation (E.1) augmented by adding on the right-hand side $1-\beta$ times the present value of NCB i's seigniorage revenues, where the period t seigniorage revenue of NCB i equals $(R_{t-1}-1) M_{it-1}/P_t$.

Let $b_{it} \equiv Q_{it} B_{it}/P_t$. We solve backward the flow budget constraint of fiscal authority i, equation (4), and conclude that in any perfect-foresight equilibrium, irrespective of the path of the price level of the union, b_{it} converges to $-\phi_i/\left(1-\left(\beta^{-1}-\phi_B\right)\right) > 0$ in the limit as time goes to infinity. Thus, the limit as time goes to infinity of $\beta^t b_{it}$ equals 0. Next, note that in a perfect-foresight equilibrium with currency, equation (17) in the paper changes to

$$\lim_{T\to\infty} \beta^T \frac{\sum_i \left(\sum_j Q_{jT} B^H_{ijT} + H_{iT} + M_{iT}\right)}{P_T} = 0.$$

Given the fiscal policy and the remittance rule assumed here, this equation holds irrespective of the path of the price level of the union. To arrive at this conclusion, one needs to take the limit as time goes to infinity of $\beta^t \sum_i (b_{it} - k_{it})$ and recall that $B_{it} = \sum_j \left(B^H_{jit} + B^{CB}_{jit}\right)$ and $\sum_i \sum_j T_{ijt} = 0$, $t \geq 0$.

Suppose that monetary policy is active. The central bank sets $R_t = \max\left\{\left(\Pi^{1-\alpha}/\beta\right)\Pi_t^\alpha, 1\right\}$, $\alpha > 1$. Then the initial price level P_0 is indeterminate and, in addition, the path of the inflation rate Π_t, $t \geq 0$, is indeterminate. It follows that the price level in each country in the union is also indeterminate. There exists a perfect-foresight equilibrium in which $\Pi_t = \Pi$, $t \geq 0$ (P_0 is indeterminate). There also exist perfect-foresight equilibria in which $\Pi_0 > \Pi$, $\Pi_t > \Pi_{t-1}$ for each $t \geq 1$, and Π_t goes to infinity in the limit as time goes to infinity. To construct such an equilibrium, pick any $\Pi_0 > \Pi$, compute $R_0 = \left(\Pi^{1-\alpha}/\beta\right)\Pi_0^\alpha$ and $\Pi_1 = \beta R_0$ (equation (23) in the paper), verify that $\Pi_1 > \Pi_0$, and iterate. There also exist perfect-foresight equilibria in which $\Pi_0 < \Pi$, $\Pi_0 \geq \beta$, $\Pi_t \leq \Pi_{t-1}$ for each $t \geq 1$, and Π_t converges to β in finite time. To construct such an equilibrium, pick any $\Pi_0 < \Pi$ with $\Pi_0 > \beta$, compute $R_0 = \left(\Pi^{1-\alpha}/\beta\right)\Pi_0^\alpha$ and $\Pi_1 = \beta R_0$, verify that $\Pi_1 < \Pi_0$, and iterate until $\Pi_t = \beta$ becomes a steady state.

Consider an equilibrium in which Π_t goes to infinity asymptotically. The demand for currency falls as the nominal interest rate rises along the equilibrium path. At some point, when the nominal interest rate is finite, the demand for currency goes to zero. With the transaction cost function from Appendix G, however, transaction costs remain finite and they equal $\psi_0 > 0$. There is no violation of transversality or other equilibrium conditions, though consumption is at a minimum for this model. In the steady state with $\Pi_t = \beta$, by contrast, the demand for currency is at a maximum and consumption is at a maximum.

Suppose that monetary policy is passive, $R_t = R = \Pi/\beta$. Then P_0 and Π_0 are indeterminate, and, from equation (23) in the paper, $\Pi_t = \Pi$ for each $t \geq 1$.

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