Supplemental appendix for 'Time to Say Goodbye: The Macroeconomic Implications of Termination Notice'

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Online Appendix E Recursive Stationary Equilibrium

Recursive stationary equilibrium in the model economy consists of household value functions $V_i^z(a)$ for each $i \in \Gamma$ and $z \in \{1,\ldots,\zeta\}$; household policy functions $c_i^z(a), x_i^z(a)$; stationary probability density functions $h_i^z(a)$; firm value functions $J_j(a)$ for each $j \in \{V,H,E,N\}$; policy functions for capital $k_E^\star, k_N^\star$; price of equity P; net return on assets r; wage rate w; aggregate vacancies v; aggregate effort X; labour market tightness θ; tax rate τ; and dividends d, which jointly satisfy the following.

1. Consumer optimization - Given the per effort unit job finding rate λ_f; net return r; wage rate w; benefits b; tax rate τ, the policy functions $c_i^z(a)$ and $x_i^z(a)$ solve the optimization problems given by (1), (2), (3), (4), and (5) with the value functions $V_i^z(a)$ and induce the transition matrix $\Lambda^z(a)$ which takes $x_i^z(a)$ as the effort level exerted by each household of labour market status i, discount rate type z, and asset level a.

2. Firm optimization - Given the rental rate $r_g = r + \delta$ and the bargained wage w, the firms optimally choose $k_E^\star, k_N^\star$ by solving the optimization problems in (8), (9). Given labour market tightness θ, and the implied population composition given by $h_i^z(a)$, the rental rate r_g, and the policy functions $x_i^z(a)$, the function J_V satisfies (12).

3. Free entry - The number of vacancies is consistent with free entry of firms such that $J_V = 0$ and Equation (14) holds.

4. Asset market clears in nominal terms and Equation (7) holds such that $A = K + P$, where A is the total steady-state desire of households to hold assets given the population composition and policy rules, K is the total

capital demanded by the firms and P is the price of the equity, existing in unit net supply, such that the no-arbitrage condition in (6) must hold.

5. Matching - The transition probabilities are consistent with the matching function.

6. Wage setting - The wage w is the median solution for the Nash bargaining problems between the worker and the firm as expressed in (16).

7. Government budget is balanced as in Equation (17).

8. Consistency - The distributions $h_i^z(a)$ are the stationary distributions implied by the transition matrix $\Lambda^z(a)$ and the policy functions $c_i^z(a)$ and $x_i^z(a)$.

Online Appendix F Model Solution in General Equilibrium

This appendix details the algorithm used to solve the model presented in Section 2. The algorithm owes much to the works of Krusell et al. (2010) and Achdou et al. (2021).

F.1 Solution Algorithm for the Steady State

The solution boils down to solving for the zero of a system of four equations in four unknowns, namely, $U(r, \theta, \tau, w) = 0$. The explicit system is given in stage 9 of the algorithm and follows the definition of recursive stationary equilibrium. As such, the solution algorithm proceeds as follows:

1. **Initialization** Provide a grid for assets, parameter values for the model, and initial guesses for r, θ, τ, and w.

2. **Compute benefits** Given the guess for the wage level and the calibrated replacement rate, determine b.

3. **Solve household block** Solve the household optimisation problem given the guesses and parameter values using the algorithm for solving the HJB equations and the Kolmogorov forward equations developed by

Achdou et al. (2021).[71] This will allow us to obtain the distributions $h_i^z(a)$, the policy functions $c_i^z(a)$ and $x_i^z(a)$, total assets held by households A the equilibrium masses m_i and the aggregate effort level X as well as X_N and X_U.

4. **Solve firm block** Use the first-order condition for capital and the relationship $r_g = r + \delta$ to solve for the capital choice of the firm and flow profit at each state by using $k_E^\star = \left(\frac{\alpha}{r_g}\right)^{\frac{1}{1-\alpha}}$ and $k_N^\star = \left(\frac{\alpha\epsilon^{1-\alpha}}{r_g}\right)^{\frac{1}{1-\alpha}}$. Given these values, the firm's value functions can be obtained from Equations (10), (11) and (13).

5. **Compute dividends** Compute the dividends using the flow profits, the vacancy stock $v = X\theta$, and Equation (15). Given the net return, compute the price of equities P.

6. **Compute capital demand** Combine the masses m_E, m_{N1} and m_{N2} from 3 with the capital solutions from 4 to obtain the aggregate capital demand by the firms K.

7. **Conduct wage bargaining** Use the procedure detailed in Appendix F.3 to compute a vector of Δws, which are the distances at each asset grid point of the guessed wage w from being the solution to the approximated Nash problem given in Appendix F.3.

8. **Find median worker** Use the resulting vector of Δw and the distributions $h_E^z(a)$ to find the median value of Δw or $MED(\Delta w)$.

9. **Market clearing** Compute $U(r, \theta, \tau, w)$ where U is given by the following system:

   - Asset market clearing condition:

$$U_1 = A - (K + P). \tag{F.1}$$

[71] The only meaningful adjustment I need to apply this algorithm is to use the first-order condition for the effort level at each iteration given the current guess for the value functions. This means that at each iteration, the guess for the policy functions for search effort $\mathrm{x}_{N1}^z(a)$,$\mathrm{x}_{U1}^z(a)$, and $\mathrm{x}_{U2}^z(a)$ solves $\Psi'\left(\mathrm{x}_{N1}^z(a)\right) = \lambda_f\left(\mathrm{V}_{N2}^z(a) - \mathrm{V}_{N1}^z(a)\right)$, $\Psi'\left(\mathrm{x}_{U1}^z(a)\right) = \lambda_f\left(\mathrm{V}_E^z(a) - \mathrm{V}_{U1}^z(a)\right)$, and $\Psi'\left(\mathrm{x}_{U2}^z(a)\right) = \lambda_f\left(\mathrm{V}_E^z(a) - \mathrm{V}_{U2}^z(a)\right)$ using the previous iteration's value functions. It is also useful to solve the consumption-saving problem first without the effort choice to provide a good initial guess for the value functions of the full model.

- Free entry:

$$U_2 = -\kappa + q\left[\mathrm{J}_E\frac{\mathrm{X}_U}{X} + \mathrm{J}_H\frac{\mathrm{X}_N}{X}\right]. \tag{F.2}$$

- Government deficit:

$$U_3 = \tau\left(w\left(m_E + m_{N1} + m_{N2}\right) + bm_{U1}\right) - bm_{U1}. \tag{F.3}$$

- Wage consistency:

$$U_4 = MED\left(\Delta w\right) \tag{F.4}$$

10. If the system U is sufficiently close to zero, stop. Else, update the initial guess accordingly, and repeat from 2 until convergence is achieved.

Solver A solver based on Newton-Raphson or Broyden's method can solve the model. In practice, a solver that combines both performs well and converges faster. The Jacobian matrix is computed using finite differences. It is helpful to relax the updated solution in the Newton direction, such that at the new guess, the value of r lies between zero and the maximum value of $\max_z\left\{\rho^z + \lambda_D\right\}$, and that the wage levels and labour market tightness are non-negative. I use backtracking to choose the largest relaxation parameter from a pre-specified set of values (all less than one), so the new guess is well within these bounds. If the bounds are already violated, which can occur, I use a pre-set relaxation parameter, which, in many cases, leads the algorithm to return to its normal bounds. If the solver is unsuccessful, a new guess is randomised, and the procedure begins anew.

Stopping criterion and normalizations A convergence criterion of $\max(|U|) < 10^{-4}$ yields fast results and performs well. All equations described in stage 9 of the algorithm are solved after normalisation to obtain a meaningful stopping criterion. The first two equations are solved in the form of relative errors, e.g., $1 - \frac{A}{K+P} = 0$ instead of (F.1). Thus, the error is interpreted as percentage deviations from equilibrium. The government budget constraint is set such that the deficit divided by output is close enough to zero. The equation concerning the median wage update is solved as $\frac{MED(\Delta w)}{w} = 0$.

Grid for assets The asset grid is a $n = 200$ grid points for assets for each labour market status and discount rate type. The grid is not uniform such that most grid points are concentrated near the borrowing constraint. The maximum value for assets is set at $a = 3,000$, corresponding to asset holdings equivalent to around ninety years of unconsumed wages. I set the asset vector $\bar{a}$ such that it has monotonically increasing increments as follows

$$\bar{a} = a_{\max}\frac{(0,1,\ldots,n-1)^4}{(n-1)^4}, \tag{F.5}$$

while having a grid point situated exactly on the borrowing constraint with a positive mass of households. This point is treated throughout as a Dirac mass.

F.2 Solution Algorithm with Transition Dynamics

The policy shock I assume that the reform from the baseline policy set T to a policy set T' occurs at time zero and is unanticipated ('MIT shock'). Unlike in an Aiyagari model, where asset holdings represent physical capital alone, in my model, asset holdings correspond to nominal asset positions that are the joint value of capital and equity. Recall that equity in the model is defined as claims on aggregate firm profits. Thus, given a policy reform that may affect future firm profitability and perfect foresight, the equity value will respond on impact and immediately affect the aggregate asset holdings in the economy. This immediate asset valuation effect changes aggregate asset positions by a factor of $\alpha_v = \frac{K_{t=0}+P_{t=0}(T')}{K_{t=0}+P_{t=0}(T)}$. With heterogeneous asset positions in the model, it is important to take a stand on who holds which asset, capital or equity, at the instant of the reform. Given the no-arbitrage condition, I assume that all households have a uniform portfolio composition, i.e., every unit of a held is equally affected by the asset valuation effect of the policy. To conclude, at the time of impact, the entire asset distribution will scale by a factor of α_v, which will be endogenously determined.[72]

[72]The existence of equity as an asset with a positive duration creates these capital gains or losses at the time of the reform. A similar issue arises in Kaplan et al. (2018), where the solution also involves changing the asset positions of agents upon impact; see the computational appendix for details of this. Unlike in Kaplan et al. (2018), in the present model, the reform might increase or decrease asset positions on impact; thus, I use the scaling factor α_v to handle this issue. To verify that the numerical error resulting from this change is small, I

The solution now requires solving for the zero of a system of four equations in four unknowns per period, plus one for α_v. Thus, one needs to specify a discretized time vector $\bar{t}$ with n_t periods and solve for $4 \times n_t + 1$ equations in $r(t), \theta(t), \tau(t), w(t)$ and α_v. The explicit system is given in stage 10 of the algorithm. The algorithm proceeds as follows.

1. **Initialization** Solve for the steady state under initial policy vector T and the steady state under the new policy vector T'. Provide a grid for assets, time, parameter values for the model, and initial guesses for the values of $r(t), \theta(t), \tau(t), w(t)$ and α_v.

2. **Compute benefits** Given the initial guess for the wage level and the calibrated replacement rate, determine $b(t)$.

3. **Adjust terminal condition** Given the guessed for valuation factor α_v, solve again for the steady-state value functions using the scaled grid to serve as a consistent terminal condition at the next step.

4. **Solve household block** Solve the household optimisation problem given the guesses and the calibrated parameters using the algorithm for solving the HJB equations and the Kolmogorov forward equations developed by Achdou et al. (2021), with the modification introduced in Appendix F.1. This time, the HJB equation is solved backwards in time using the steady state under the policies T' as the terminal condition for the value functions and yields the policy functions $c_i^z(a,t)$ and $x_i^z(a,t)$. Next, using the initial steady-state population composition and the newly obtained time-dependent policies, solve the Kolmogorov equation forward and use them to compute the distributions $h_i^z(a,t)$, the masses, aggregate asset holdings $A(t)$ and the aggregate effort level $X(t)$ as well as $X_N(t)$ and $X_U(t)$.

5. **Solve firm block** Use the first-order condition for capital and the relationship $r_g(t) = r(t) + \delta$ to solve for the capital choice of the firm and flow profit at each state by using $k_E^\star(t) = \left(\frac{\alpha}{r_g(t)}\right)^{\frac{1}{1-\alpha}}$ and $k_N^\star(t) =$

compare the value of steady-state welfare at the terminal condition using both the original and the scaled grid and find similar values. In all the optimal policy scenarios summarized in Table 5, the difference from scaling is at least an order of magnitude lower than the total welfare gain or loss.

$\left(\frac{\alpha\epsilon^{1-\alpha}}{r_g(t)}\right)^{\frac{1}{1-\alpha}}$. Given these values, the firm's value functions can be obtained from Equations (B.15), (B.16), and (B.17) using a finite difference approximation for the temporal derivative in the modified value functions, and the new steady state values of the firms' value functions as the terminal condition. Since the solution is obtained using a terminal condition, the time-dependent values should be solved backwards in time. An example of the exact iterative procedure is shown in Appendix F.3 when discussing the treatment of the wage derivatives out of steady state.

6. **Compute dividends** Compute the dividends using the flow profits, the vacancy stock $v(t) = X(t)\theta(t)$, and Equation (B.20). Given the net return, solve for the price of equities $P(t)$, again using the new steady state value of P as a terminal condition and iterating backwards in time.

7. **Compute aggregate capital demand** Combine the masses from 4 with the capital solutions from 5 to obtain the aggregate capital demand $K(t)$.

8. **Conduct wage bargaining** Use the procedure detailed in Appendix F.3 to compute a vector of $\Delta w(t)$s, which are the distances at each asset grid point of the guessed wage w from being the solution to the approximated Nash problem given in Appendix F.3.

9. **Find median worker** Use the resulting vector of $\Delta w(t)$ and the distributions $h_E^z(a,t)$ to find the median value of $\Delta w(t)$ or $MED(\Delta w(t))$.

10. **Market clearing** Compute $U(r(t), \theta(t), \tau(t), w(t))$ where U is a system of $4 \times n_t$ equations that is given by the following (subscripts denote equation numbers, e.g., U_{1-n_t} denotes equations one through n_t):

    - Asset market clearing condition:

$$U_{1-n_t} = A(t) - (K(t) + P(t)). \tag{F.6}$$

    - Distance from free entry:

$$U_{n_t+1-2n_t} = -\kappa + q(t)\left[\mathrm{J}_E(t)\frac{\mathrm{X}_U(t)}{X(t)} + \mathrm{J}_H(t)\frac{\mathrm{X}_N(t)}{X(t)}\right].$$

- Government deficit:

$$U_{2n_t+1-3n_t} = \tau(t)\,(w(t)\,(m_E(t) + m_{N1}(t) + m_{N2}(t)) + b(t)m_{U1}(t)) - b(t)m_{U1}(t). \tag{F.7}$$

- Wage consistency:

$$U_{3n_t+1-4n_t} = MED\,(\Delta w(t))\,. \tag{F.8}$$

- Valuation factor consistency:

$$U_{4n_t+1} = \alpha_v - \frac{K_{t=0} + P_{t=0}(T')}{K_{t=0} + P_{t=0}(T)} \tag{F.9}$$

11. If the system U is sufficiently close to zero, stop. Else, update the initial guess accordingly, and repeat from 1 until convergence is achieved.

Solver As in the previous case, a solver based on Newton-Raphson or Broyden's method can solve the model. Again, I use a solver that combines both methods. The Jacobian matrix is computed using finite differences. As it is for the steady state algorithm, it is helpful to relax the updated solution in the Newton direction using backtracking so that the new guess is still positive and economically possible. The new steady-state values perform well as guesses for the values of $r(t)$, $\theta(t)$, $\tau(t)$, and $w(t)$, and given that the immediate valuation effects are relatively small $\alpha_v = 1$ serves as a good initial guess for the valuation factor.

Stopping criterion I apply the same normalisations as in the solution algorithm for the steady-state values. A convergence criterion of $\max(|U|) < 10^{-4}$ performs well. However, this criterion requires solving for the initial steady state and the new steady state using a higher accuracy, for which I set the stopping criterion of the steady state solution to 10^{-5}.[73]

Grid for time Given the high computational cost of solving the transition dynamics and the accuracy needed for welfare maximisation, I use non-uniform

[73] In cases where solving to this accuracy proves difficult, I allow the solver to use a less restrictive tolerance of 0.5×10^{-4} for the terminal condition in step 3 of the algorithm.

grids for assets and time. The asset grid is the same one as in the steady-state solution algorithm. I use a non-equispaced grid for the time grid with more points at the beginning and a few at the end to save the computational cost. Recall that a time period in the model is set to one month. I compute the transition dynamics for two hundred years in the future, so $t_{\min} = 0$ and $t_{\max} = 12 \times 200$ are the time vector's minimum and maximum values. I use twenty time periods $n = 20$ and set the time vector $\bar{t}$ such that increments are monotonically increasing as follows

$$\bar{t} = t_{\max} \frac{(1, 2, \ldots, n_t)^p}{n_t^p}, \tag{F.10}$$

where p sets the grid curvature. Since transition dynamics involve iterating forward on the distribution, unsuitable grids might yield unstable results. I compute a total of eight optimal policy scenarios, each involving the computation of several thousand transition paths. For all scenarios that use termination notice and UIB, I set $p = 2$ (scenarios 1, 3, 5, and 7 in Table 5). The other four scenarios studied involve switching off termination notice and allowing all workers to be entitled to severance pay; this immediate change in policies and masses requires setting a more curved grid. Thus, I set $p = 5$ in the other four scenarios (scenarios 2, 4, 6, and 8 in Table 5).

F.3 Solving the Wage Bargaining Problem

The bargaining problems require maximising the Nash product, which is an objective function of the form

$$\left(\tilde{\mathrm{V}}_E^z(a, w) - \tilde{\mathrm{V}}_{N1}^z(a, w)\right)^{\beta} \left(\tilde{\mathrm{J}}_E(w) - \tilde{\mathrm{J}}_N(w)\right)^{1-\beta}, \tag{F.11}$$

at every value of a. For ease of notation, let $\Delta V^z(a, w) = \tilde{\mathrm{V}}_E^z(a, w) - \tilde{\mathrm{V}}_{N1}^z(a, w)$ and $\Delta J(w) = \tilde{\mathrm{J}}_E(w) - \tilde{\mathrm{J}}_N(w)$. Since the value functions are not solved as explicit functions of two state variables a, w, but for values of a and a given level of w, I use an approximation method to solve the problem. The Nash product for a given level of assets a can be approximated using the guessed wage level at each iteration of the solution algorithm as

$$\begin{aligned}&\left(\Delta V^z(a,w)\right)^{\beta}(\Delta J(w))^{1-\beta} \approx \\ &\left(\Delta V^z(w,a) + \frac{\partial \Delta V^z(a,w)}{\partial w}\Delta w\right)^{\beta}\left(\Delta J(w) + \frac{\partial \Delta J(w)}{\partial w}\Delta w\right)^{1-\beta}.\end{aligned} \tag{F.12}$$

I exploit this convenient approximation to analyse the approximated bargaining problem:

$$\max_{\Delta w} \quad \left(\Delta V^z(w,a) + \frac{\partial \Delta V^z(a,w)}{\partial w}\Delta w\right)^{\beta}\left(\Delta J(w) + \frac{\partial \Delta J(w)}{\partial w}\Delta w\right)^{1-\beta}. \tag{F.13}$$

Observe that the above problem has a straightforward analytical solution since it is an unconstrained problem in one variable Δw, which is given by:

$$\Delta w = -\frac{\beta \frac{\partial \Delta V^z(a,w)}{\partial w}\Delta J(w) + \left(1-\beta\right)\frac{\partial \Delta J(w)}{\partial w}\Delta V^z(w,a)}{\frac{\partial \Delta V^z(a,w)}{\partial w}\frac{\partial \Delta J(w)}{\partial w}}. \tag{F.14}$$

Note that bargaining takes place in partial equilibrium with labour market conditions, unemployment insurance benefits, tax rates, and prices all fixed. Thus, $\Delta V^z(a,w)$ is increasing in w and $\Delta J(w)$ is decreasing in w. Therefore, there will be a single solution to the problem for each level of a, i.e., a single value of w, which maximises the Nash product in Equation (F.11). Hence, the wage level consistent with Nash bargaining will be found when Δw will be close enough to zero.

All that remains is to compute $\frac{\partial(\Delta^z V(a,w))}{\partial w}$ and $\frac{\partial(\Delta J(w))}{\partial w}$ which means computing the derivatives of the value functions with respect to the wage as

$$\frac{\partial\left(\Delta V^z(a,w)\right)}{\partial w} = \frac{\partial \mathrm{V}^z_E(a)}{\partial w} - \frac{\partial \mathrm{V}^z_{N1}(a)}{\partial w}, \quad \frac{\partial\left(\Delta J(w)\right)}{\partial w} = \frac{\partial J_E}{\partial w} - \frac{\partial J_N}{\partial w}.$$

These can be computed by applying the envelope theorem to the value functions obtained at stages 3 and 4 of the solution algorithm. For the firm, this derivation is simple and can be done with pencil and paper from Equations

(10) and (11)

$$\frac{\partial J_N}{\partial w} = -\frac{1}{\phi + r + \lambda_D}, \tag{F.15}$$

$$\frac{\partial J_E}{\partial w} = \frac{-1 + \lambda_s \frac{\partial J_N}{\partial w}}{\lambda_s + r + \lambda_D}. \tag{F.16}$$

For the households, the derivation will be more complex. Start by applying the envelope theorem to the household's value functions. The derivatives are given by

$$\left(\rho^z + \lambda_D + \phi\right) \frac{\partial \mathrm{V}^z_{N2}(a)}{\partial w} = (1-\tau) \frac{\partial \mathrm{V}^z_{N2}(a)}{\partial a} + \mathrm{s}^z_{N2}(a) \frac{\partial^2 \mathrm{V}^z_{N2}(a)}{\partial w \partial a}, \tag{F.17}$$

$$\left(\rho^z + \lambda_D + \phi + \lambda_f \mathrm{x}^z_N(a)\right) \frac{\partial \mathrm{V}^z_{N1}(a)}{\partial w} = \tag{F.18}$$
$$(1-\tau) \frac{\partial \mathrm{V}^z_{N1}(a)}{\partial a} + \lambda_f \mathrm{x}^z_N(a) \frac{\partial \mathrm{V}^z_{N2}(a)}{\partial w} + \frac{\partial^2 \mathrm{V}^z_{N1}(a)}{\partial w \partial a} \mathrm{s}^z_{N1}(a),$$

$$\left(\rho^z + \lambda_D + \lambda_s\right) \frac{\partial \mathrm{V}^z_E(a)}{\partial w} = \tag{F.19}$$
$$(1-\tau) \frac{\partial \mathrm{V}^z_E(a)}{\partial a} + \lambda_s \frac{\partial \mathrm{V}^z_{N1}(a)}{\partial w} + \frac{\partial^2 \mathrm{V}^z_E(a)}{\partial w \partial a} \mathrm{s}^z_E(a),$$

where $\mathrm{s}^z_i = w(1-\tau) + ra - \mathrm{c}^z_i(a), \forall i \in \{E, N1, N2\}$ is introduced to streamline notation. The derivatives $\frac{\partial \mathrm{V}^z_i(a)}{\partial a}$ are computed from the first-order conditions of the household's problem $\frac{\partial \mathrm{V}^z_i(a)}{\partial a} = \frac{\partial u}{\partial c}$ using the policy functions from stage 3 of the solution algorithm. The cross-partial derivatives are slightly more complicated to compute. However, the discretisation method from Achdou et al. (2021) used in stage 3 of the solution algorithm provides a straightforward computation strategy. In discretising the household's HJB equation, I use an upwind finite difference approximation of the form $\frac{\partial \mathrm{V}^z_i(a)}{\partial a} \mathrm{s}^z_i(a) \approx {D_i}^z {V_i}^z$ where D^z_i is a square matrix with the same size as the asset grid which is the finite difference operator multiplied by the suitable approximation of $\mathrm{s}^z_i(a)$ given the guess for V^z_i, i.e., ${D_i}^z$ approximates the operator $\frac{\partial}{\partial a} \mathrm{s}^z_i(a)$. At the end of stage 3, I have the matrices D^z_i readily computed. Observe that

$\frac{\partial^2 \mathrm{V}_i^z(a)}{\partial w \partial a}\,\mathrm{s}_i^z(a)$ can be expressed as

$$\frac{\partial^2 \mathrm{V}_i^z(a)}{\partial w \partial a}\mathrm{s}_i^z(a) = \frac{\partial}{\partial a}\frac{\partial}{\partial w}\left[\mathrm{V}_i^z(a)\right]\mathrm{s}_i^z(a) \approx D_i{}^z \frac{\partial}{\partial w}\left[V_i{}^z\right] = D_i{}^z \frac{\partial V_i{}^z}{\partial w},$$

which means that each of the derivatives of the household's value functions with respect to the wage can be numerically solved by substituting in this approximation. To illustrate the computation, using $\frac{\partial \mathrm{V}_{N2}^z(a)}{\partial w}$, the approximation results in solving the system

$$\frac{\partial \mathrm{V}_{N2}^z(a)}{\partial w} = \left[\left(\rho^z + \lambda_D + \phi\right)I - D_{N2}{}^z\right]^{-1}(1-\tau)\frac{\partial \mathrm{V}_{N2}^z(a)}{\partial a},$$

where I is the identity matrix. This concludes all the requirements for computing (F.14).

Modifications required outside of steady state The algorithm can be equally applied to the bargaining problem outside of the steady state, with only one modification. It is essential to include the temporal derivative in the households' and firms' value functions. Thus, when one applies the envelope theorem to obtain their derivatives, a new cross partial is introduced, which requires special attention.

As before, the firm's side is easier to handle. The firm with a worker under notice has the value function given by Equation B.16. Deriving it with respect to w results in

$$r(t)\frac{\partial J_N}{\partial w} = -1 - \left(\phi + \lambda_D\right)\frac{\partial J_N}{\partial w} + \frac{\partial}{\partial w}\frac{\partial J_N}{\partial t}. \tag{F.20}$$

This equation is much easier to handle after changing the order of differentiation to yield

$$r(t)\frac{\partial J_N}{\partial w} = -1 - \left(\phi + \lambda_D\right)\frac{\partial J_N}{\partial w} + \frac{\partial}{\partial t}\frac{\partial J_N}{\partial w}. \tag{F.21}$$

This equation can be discretised using a simple finite difference scheme of the form:

$$r(t-1)\frac{\partial J_N{}^{t-1}}{\partial w} = -1 - \left(\phi + \lambda_D\right)\frac{\partial J_N{}^{t-1}}{\partial w} + \frac{\frac{\partial J_N{}^{t}}{\partial w} - \frac{\partial J_N{}^{t-1}}{\partial w}}{\Delta t}. \tag{F.22}$$

The terminal condition for this equation is that at the last period, period s, the derivative is $\frac{\partial {J_N}^s}{\partial w} = \frac{\partial J_N}{\partial w} = \frac{-1}{\phi + r^s + \lambda_z}$ where r^s is the new steady-state net return. By using this terminal condition, the temporal derivatives can be computed recursively by a formula for the form

$$\left(r(t-1) + \phi + \lambda_D + \frac{1}{\Delta t}\right)\frac{\partial {J_N}^{t-1}}{\partial w} = -1 + \frac{1}{\Delta t}\frac{\partial {J_N}^{t}}{\partial w}. \tag{F.23}$$

Similarly, using the same discretisation and the analogous terminal condition, the formula for $\frac{\partial J_E}{\partial w}$ is given by

$$\left(r(t) + \lambda_s + \lambda_D + \frac{1}{\Delta t}\right)\frac{\partial {J_E}^{t-1}}{\partial w} = -1 + \lambda_s \frac{\partial {J_N}^{t-1}}{\partial w} + \frac{1}{\Delta t}\frac{\partial {J_E}^{t}}{\partial w}. \tag{F.24}$$

I apply the same idea to the household's value functions and will illustrate its use on the value function in state $N2$. The derivative with respect to the wage is given by

$$\left(\rho^z + \lambda_D + \phi\right)\frac{\partial \mathrm{V}^z_{N2}(a)}{\partial w} = \tag{F.25}$$

$$(1-\tau)\frac{\partial \mathrm{V}^z_{N2}(a)}{\partial a} + \mathrm{s}^z_{N2}(a)\,\frac{\partial^2 \mathrm{V}^z_{N2}(a)}{\partial w \partial a} + \frac{\partial^2 \mathrm{V}^z_{N2}(a)}{\partial w \partial t}. \tag{F.26}$$

Using the same discretisation notation as before with respect to the treatment of the expression $\mathrm{s}^z_{N2}(a)\,\frac{\partial^2 \mathrm{V}^z_{N2}(a)}{\partial w \partial a}$, and the same discretisation for the temporal derivative as in the firm's value function above, results in the following discretisation scheme for the wage derivative

$$\left[\left(\rho^z + \lambda_D + \phi + \frac{1}{\Delta t}\right)I - {D_{N2}}^{z,t-1}\right]\frac{\partial \mathrm{V}^{z,t-1}_{N2}(a)}{\partial w} = \tag{F.27}$$
$$(1-\tau)\frac{\partial \mathrm{V}^{z,t-1}_{N2}(a)}{\partial a} + \frac{1}{\Delta t}\left[\frac{\partial \mathrm{V}^{z,t}_{N2}(a)}{\partial w}\right],$$

whereas in the firm's case, the new steady state is used as a terminal condition for the value of $\frac{\partial \mathrm{V}^z_{N2}(a)}{\partial w}$.

Introducing severance pay In the model that features severance pay as introduced in Appendix C, severance pay involves modifying the derivative of J_N as follows

$$\frac{\partial J_N}{\partial w} = -\frac{1}{r + \lambda_D + \phi} - \frac{\phi \lambda_{SP}}{r + \lambda_D + \phi}. \tag{F.28}$$

For the household in states $N1$ and $N2$ the adjustment is as follows

$$\left(\rho^z+\lambda_D+\phi\right)\frac{\partial \mathrm{V}^z_{N2}(a)}{\partial w}= \tag{F.29}$$
$$(1-\tau)\frac{\partial \mathrm{V}^z_{N2}(a)}{\partial a}+\mathrm{s}^z_{N2}(a)\,\frac{\partial^2 \mathrm{V}^z_{N2}(a)}{\partial w\partial a}+\frac{\partial \mathrm{V}^z_{E}(a)}{\partial a}\phi\lambda_{SP}(1-\tau)$$

$$\left(\rho^z+\lambda_D+\phi+\lambda_f\mathrm{x}^z_{N1}(a)\right)\frac{\partial \mathrm{V}^z_{N1}(a)}{\partial w}=(1-\tau)\frac{\partial \mathrm{V}^z_{N1}(a)}{\partial a}+ \tag{F.30}$$
$$\mathrm{s}^z_{N1}(a)\,\frac{\partial^2 \mathrm{V}^z_{N1}(a)}{\partial w\partial a}+\lambda_f\mathrm{x}^z_{N1}(a)\,\frac{\partial \mathrm{V}^z_{N2}(a)}{\partial w}+\lambda_{SP}(1-\tau)\left[\phi\frac{\partial \mathrm{V}^z_{U1}(a)}{\partial a}\right]$$

Grids used for the wage solution In Krusell et al. (2010), a multi-grid structure is utilised to improve efficiency while solving for the wage function. The asset grid was finer than the grid used for wage bargaining (1,000 points vs 125 points on the same support), and cubic-spline interpolation was used to connect the two. This has a speed advantage over using the same grid for both needs and, in practice, can smooth out minor numerical errors that would occur in a very fine grid, thus resulting in a smooth wage function. In my case, however, the wage is a scalar, and the main source of inaccuracies lies in computing the median for a coarse distribution, which may result in small jumps in the solution that would hinder convergence. To mitigate this problem, I use a non-uniform asset grid of 200 points for the household and a finer grid on the same support with equidistant 10^4 points to update the wage. The distributions $h_i(a)$, the value functions, and their derivatives are interpolated using a cubic spline to the finer grid. This setup is practical since solving for the wage in the abovementioned method involves no optimisation, just operations on vectors that would yield Δw by Equation (F.14).

This approximation method described in this appendix can be used whenever the derivatives of the value functions can be characterised analytically. The method saves many computational resources because there is no need to use optimisation at each grid point. After the derivatives are computed, the entire bargaining procedure collapses into a few lines of code, requiring only vector operations.

Online Appendix G Calibrating the Model

G.1 Calibration Strategy and Targets

Calibration Strategy My calibration strategy uses six free parameters ψ, ψ_0, η, κ, λ_s and Δ_ρ to minimise the model's distance from three scalar moments and two distributions: the unemployment rate; vacancy rate; average duration elasticity to benefits; the unemployment duration distribution; and wealth distribution measured as shares of total wealth held by each decile. Since there is a difference between targeting a scalar and minimising distance from a distribution, I use a different distance metric for each in constructing the objective function. For scalar targets, I use squared relative errors as a distance measure. For targeted distributions, I use the Kolmogorov-Smirnov distance between the model-implied distribution and the empirically observed one to measure distance.

Scalar targets As discussed in the main text, I target an unemployment rate of 4.6%,[74] and a vacancy rate of 3.27%.[75] I also target the elasticity of unemployment duration with respect to benefits. I target a value of −0.5, an accepted value in the literature, taken from Chetty and Finkelstein (2013). To do so, I target the average elasticity $\frac{\partial x^z_{U1}}{\partial b}\frac{b}{x_{U1}}$ among those households eligible for UIB in the model to be −0.5. To compute this elasticity, I compute counterfactual policy functions x_{U1} resulting from increasing b by one per cent, holding all prices constant and weighting these changes by the population distribution in the baseline model. I cap the effort levels that households can exert such that no household may have an expected unemployment duration of less than one month at each instant when choosing effort, i.e., $\lambda_f x^z_i(a) \leq 1$ to avoid degeneracies in the distribution.

Matching labour-market dynamics Calibrating the model poses several challenges which merit a short discussion. First, while calibrating simple search and matching models, one may directly calibrate the job-finding and separa-

[74]The average unemployment rate for persons between ages 25 to 54 in Israel for 2012 - 2019.

[75]The average value from the Bank of Israel series taken at a monthly frequency for the years 2012 - 2019

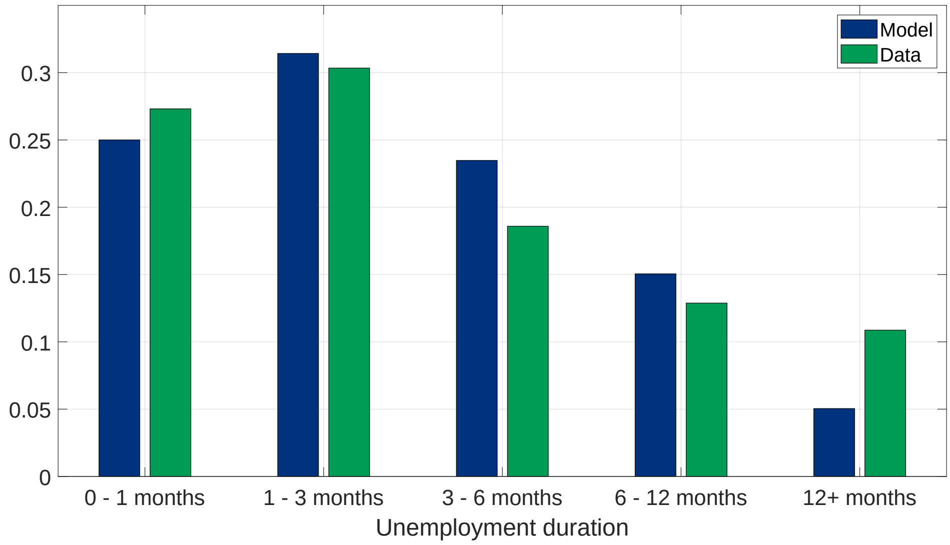
tion rates and obtain the unemployment rate as a result. However, my model features a non-degenerate, endogenous heterogeneity in the job-finding rates, making a direct calibration of the job-finding rate impossible. Second, the separation rate cannot be directly calibrated by setting the value of λ_s, as some shock realisations will result in job-to-job transitions and not in an unemployment spell. Thus, calibrating for job flows by directly setting hazard rates is infeasible in the current setup.

Instead, I fit the model's aggregate outcomes and job flows to the data as follows. I use the internally calibrated parameters to obtain the best fit to the unemployment duration distribution, thus capturing the overall severity of the risk of unemployment to a household's income and consumption. In so doing, I target the distribution of job flows without externally setting the hazard rates directly, as one would do in the simple case of a DMP model. Data on this distribution is available in the form of five bins, which consist of the proportion of unemployed persons unemployed for less than one month, between one and three months, between three to six months, between six to twelve months, and over twelve months.[76] Since this is not a linear hazard model, I simulate the steady-state unemployment duration distribution by iterating forward on the laws of motion obtained from the model solution on a uniform asset grid with 100 points. The policies, distributions and laws of motion are interpolated using a cubic spline. Given the instability of forward simulations, I use very small time steps of 0.01 months while simulating this distribution. Figure 11 shows the targeted and resulting distributions.

Matching the wealth distribution As discussed in the main text, I use data on the shares of aggregate wealth held by deciles to discipline the model wealth distribution. This data is available from the Credit Suisse 'global wealth report' databook of 2019 (Credit Suisse Research Institute, 2019). The wealth shares and their model counterparts are reported in Figure 12.

[76]This data is publicly available at https://stats.oecd.org/ under 'unemployment by duration'.

Figure 11: Model Fit - Unemployment Durations Distribution



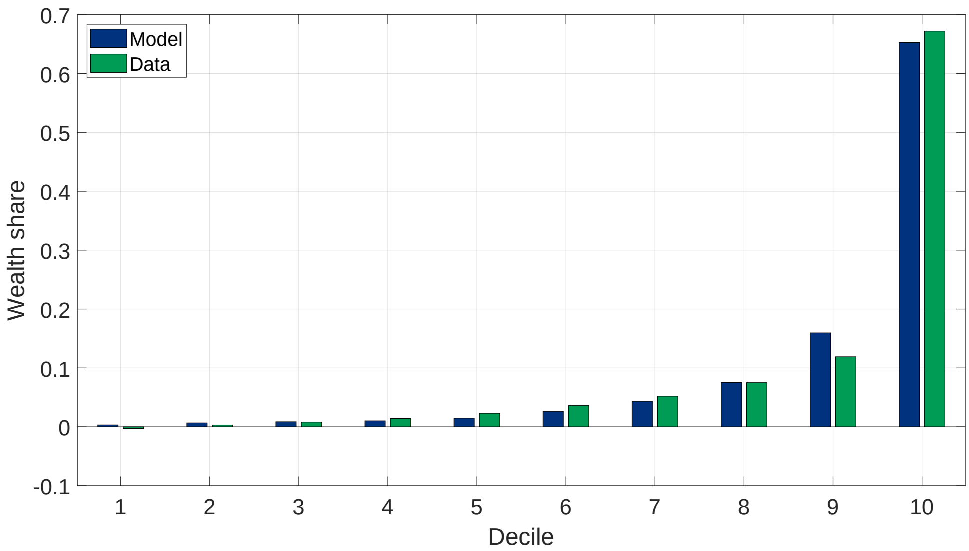
Note: The green bars correspond to the distribution of unemployment durations for all persons aged 25 to 54. I report averages for each bin for the years 2012 - 2019. The model counterpart of this distribution implied by the parametrisation in Table 2 is presented in blue.

G.2 Numerical Procedure

Objective Function I minimise the model's distance from three scalar targets and two distributions. I measure the distance from the scalar targets using squared relative errors; thus, for a scalar target G_i and a parameter vector $\vartheta \in \mathbb{R}^6$, the distance metric is given by $\hat{S}^2(\vartheta) = \left(\frac{G_i^{\text{model}}(\vartheta)}{G_i^{\text{target}}} - 1\right)^2$. For the distributions, I measure distance using the Kolmogorov–Smirnov distance S^{KS}, which is a distance metric between the two discretised cumulative distributions. Let F_k^{target} be the targeted cumulative distribution at bin k in the data, and $F_k^{\text{model}}(\vartheta)$ be its model counterpart. The Kolmogorov–Smirnov distance for parameter vector ϑ is given by $S_i^{KS}(\vartheta) = \max_k \left| F_k^{\text{target}} - F_k^{\text{model}}(\vartheta) \right|$.

Formally, for a parameter vector ϑ, the total distance from the five targets, the unemployment rate, vacancy rate, duration elasticity, unemployment du-

Figure 12: Model Fit - Wealth Shares



Note: The green bars correspond to the wealth shares at each decile in Israel from the Credit Suisse 'global wealth report' databook of 2019 (Credit Suisse Research Institute, 2019). The model counterpart implied by the parametrisation in Table 2 is presented in blue.

ration distribution, and wealth shares is given by

$$SSE(\vartheta) = \sum_{i=1}^{3} \hat{S}_i^2(\vartheta) + \sum_{i=1}^{2} S_i^{KS^2}(\vartheta), \tag{G.1}$$

where the squared Kolmogorov–Smirnov distance is used to make the distances commensurable. To illustrate, a ten per cent deviation from the unemployment rate, which will increase the SSE by 0.01, will be weighted equivalent to a Kolmogorov–Smirnov distance of 0.1 in the unemployment duration distribution. Alternatively, a maximum deviation between bins of the cumulative distributions of 0.1 contributes to the SSE just as much as a relative distance from the targeted unemployment rate of 0.1 contributes to it.

Optimisation Routine I employ the cross-entropy method (CEM) as developed in de Boer et al. (2005). Specifically, I use the Beta as my class of parametric distributions as is done in Mannor et al. (2003). I choose Beta distributions since a bounded support is helpful in this type of exercise. It prevents the algorithm from choosing extreme parameter values that yield no solutions and, thus, only result in costly evaluations that deliver no information. The algorithm proceeds as follows:

1. **Initialize the algorithm** Choose a number of evaluations N_{eval}, a smoothing parameter r_s, a size for the elite sample N_{elite}, tolerances ϵ_T, and ϵ_{sd}, prior distributions, and bounds for each parameter. Let the vector of lower bounds be $\underline{B} \in \mathbb{R}^6$ and the vector of upper bounds be $\overline{B} \in \mathbb{R}^6$. Set the iteration counter $x = 1$.

2. **Draw a sample** Draw N_{eval} independent random draws from the prior for each parameter to form a sample of N_{eval} parametrisations. Each one of the six model parameters is drawn from the uniform interval using its corresponding prior distribution and then rescaled into its corresponding bounds, i.e., the interval $\left[\underline{B}_k, \overline{B}_k\right]$ where $k \in \{1, 2, \ldots 6\}$.

3. **Evaluate** Let $\vartheta^j \in \mathbb{R}^6$ denote the j-th parametrization out of N_{eval} in the current iteration. For each j, evaluate $SSE(\vartheta^j)$. If the evaluation fails, use $SSE(\vartheta^j) = 9999999$.

4. **Find elite sample** Find the best N_{elite} parametrisation, those for whom $SSE(\vartheta^j)$ is the smallest, and use them as the elite sample. Also, find the best parametrisation, $\vartheta^\star_x$, minimising the SSE among those sampled at the current iteration x.

5. **Compute stopping criteria** Within the elite sample, for each one of the six model parameters $\vartheta_k \in \mathbb{R}$, compute its mean $\overline{\vartheta_k} = \frac{\sum_{i=1}^{N_{\text{elite}}} \vartheta_{k,i}}{N_{\text{elite}}}$. Proceed by computing the standard deviation of the mean-divided parameter $st.dev(\frac{\vartheta_k}{\overline{\vartheta_k}})$ for each parameter.

6. **Test for convergence** If $\max st.dev(\frac{\vartheta_k}{\overline{\vartheta_k}}) < \epsilon_{sd}$, $x > 1$, and the marginal improvement of the best iteration of the current iteration relative to the best of the previous iteration is smaller then ϵ_T or $\left|SSE(\vartheta^\star_{x-1}) - SSE(\vartheta^\star_x)\right| \leq \epsilon_T$ stop the loop and choose the best draw $\vartheta^\star_x$ as a solution.

7. **Update distributions** If convergence was not reached in the last step, for each parameter, use the elite sample to compute the method of moment estimates of a_{elite} and b_{elite}, where these are the parameters of

a new Beta distribution $Beta(a_{\text{elite}}, b_{\text{elite}})$. This distribution is the one that is most likely to generate the values in the elite sample.

8. **Repeat** Set for each of the six parameters a new distribution such that $Beta^k_{x+1}(a_x(1-r_s)+r_s a_{\text{elite}}, b_x(1-r_s)+r_s b_{\text{elite}})$, for each $k \in \{1,2,\dots 6\}$ update the iteration counter such that $x = x+1$, and repeat from 2.

Specifics of the procedure and resulting calibrations I implement the above algorithm using the uniform distribution or $Beta(1,1)$ as a prior for each of the six parameters. Each CEM iteration samples $N_{\text{eval}} = 5,000$ potential calibrated versions of the models, of which $N_{\text{elite}} = 40$ are chosen as the elite sample. The smoothing parameter is set to $r_s = 0.7$ and the tolerances are $\epsilon_T = 0.1$ and $.\epsilon_{sd} = 0.01$. The parameter values resulting from this exercise for the baseline calibration are reported in Table 2. The details of the model fit for the baseline calibration are presented in Table 1 of the main text and in Figures 11 and 12 of this appendix. This paper uses four different calibrated versions of the model. Table 6 reports all calibrations along with their fit and the bounds used to conduct the optimisation exercise.

Table 6: Summary of Calibrations Used in the Paper and Their Fit

	Target	(1)	(2)	(3)	(4)
Externally calibrated parameters					
ν - CRRA parameter		1	1	2	2
ϵ		0.0000	0.6500	0.0000	0.6500
$\bar{\rho}$		0.0036	0.0036	0.0036	0.0036
λ_D		0.0021	0.0021	0.0021	0.0021
β		0.5000	0.5000	0.5000	0.5000
α		0.3300	0.3300	0.3300	0.3300
δ		0.0067	0.0067	0.0067	0.0067
λ_{U1}		0.2500	0.2500	0.2500	0.2500
ϕ		1.0000	1.0000	1.0000	1.0000
R		0.6000	0.6000	0.6000	0.6000
Internally calibrated parameters					
Δ_ρ		0.00087	0.000878	0.001527	0.001541
ψ_0		9.6754	11.7563	1.7875	0.9674
ψ		0.2096	0.2003	0.3274	0.3341
λ_s		0.0147	0.0142	0.0181	0.0184
κ		12.4896	13.7684	7.8116	8.8446
η		0.7508	0.6342	0.6022	0.9057
Model fit					
	Target				
Unemp. rate	0.0460	0.0465	0.0465	0.0460	0.0460
Vacancy rate	0.0327	0.0329	0.0328	0.0327	0.0327
Average duration elasticity	-0.5000	-0.5004	-0.5025	-0.4991	-0.4992
S^{KS} unemp. duration	0.0000	0.0583	0.0452	0.0120	0.0129
S^{KS} wealth shares	0.0000	0.0204	0.0208	0.0198	0.0199
SSE		0.0040	0.0026	0.0005	0.0006
Bounds for internally calibrated parameters					
Δ_ρ		$[0, 0.00324]$			
ψ_0		$[0.5, 20]$	$[0.5, 20]$	$[0.01, 5]$	$[0.01, 5]$
ψ		$[0.01, 0.5]$			
λ_s		$[0.0137, 0.0416]$			
η		$[0.1, 2]$			
κ		$[5, 30]$	$[5, 30]$	$[2, 20]$	$[2, 20]$

Note: This table reports all the calibrations used in the main text and the appendices, their fit with respect to each calibration target, the value of the objective function, and the bounds used to obtain them.

Most of the bounds used are derived from trial and error, and the solution is situated well within them. The exceptions are the bounds for Δ_ρ and λ_s. The limits on Δ_ρ come from its definition as a positive increment, which sets the lower limit at zero, and from being related in size to $\bar{\rho}$. Thus, I set the upper limit of Δ_ρ to $0.9\bar{\rho}$. λ_s, unlike the other parameters, can be partially observed in reality. λ_s is the hazard of an idiosyncratic shock hitting the employer-employee pair and causing termination notice to be delivered. Thus, the value of $\frac{1}{\lambda_s}$ is the expected duration of a match net of the termination notice. This duration is bounded above by the expected duration of an employment spell, which gives a lower-bound value for λ_s. Using a GMM estimation, detailed below, of the Israeli unemployment duration that is based on a two-state model (employment and unemployment) for the 25-54 age cohort, I determine that for the relevant years, the average separation hazard into unemployment for an employed person is 0.0137. Therefore, I use 0.0137 as the lower bound value of λ_s. This lower bound figure means that a shock hits on average every 73 months. The upper bound is set to an expected duration of 24 months. The resulting value of λ_s for the baseline calibration corresponds to shocks arriving, on average, once in 68.03 months.

G.3 GMM Estimation Using Israeli Labour Market Data

Source data description To provide a lower bound for λ_s, I utilise data on labour force size and unemployment by duration available for the years 1995-2019 for all persons aged 25 to 54.[77] The choice of ages is done to be consistent with the rest of the calibration in Section 3, which also leads me to focus solely on the years 2012 - 2019. The data consists of the total number of persons in the labour force and the number of persons at each unemployment duration bin for each year. Bins are available for duration groups with unemployment durations of less than one month, between one to three months, more than three and less than six months, more than six months and less than a year, over than a year of unemployment, and persons for whom duration data is unavailable.

[77] Data was retrieved from https://stats.oecd.org/

Data transformation I first assume that duration data is missing at random and distribute the number of persons for whom duration is missing proportionally into the other five bins. Following this, each bin is divided by the total size of the labour force such that summing all the bins yields the unemployment rate for this year, and the population size is normalised to unity within each year.

Structural assumptions I assume the standard two-states representation of employment E and unemployment U that features the following law of motion:

$$\frac{\mathrm{d}U}{\mathrm{d}t} = s(1-U) - fU, \tag{G.2}$$

where $E = 1 - U$ and s and f denote the separation rate and the job-finding rate correspondingly, which are the objects of interest for this estimation. The system has a unique steady-state with $U^\star = \frac{s}{s+f}$. At this steady state, the flow from employment to unemployment and from employment to unemployment is fixed at $z = \frac{fs}{s+f}$.

The law of motion above means job-finding occurs at a constant hazard of f. The survival function in unemployment is $S(t) = e^{-ft}$. Thus, the total number of persons unemployed with duration τ is $zS(\tau)$.

The normalised number of persons in each bin is given by:

$$u_i = \frac{fs}{f+s}\int_a^b e^{-ft}dt, \tag{G.3}$$

where the i-th bin is the one which includes durations of anywhere from a to b months.

Moment Conditions and Estimation For each unemployment duration bin, I compute its average size for the sample duration $\bar{u_{a-b}}$. The estimation is carried out by solving

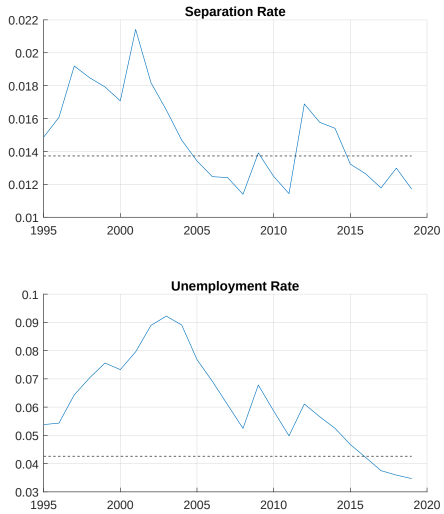
$$\min_{s,f} \sum_{i=1}^{4}\left(1 - \frac{\overline{u_i}}{\hat{u}_i\left(s,f\right)}\right)^2 \tag{G.4}$$

where $\hat{u}_i\left(s,f\right)$ is the value computed using the Equation (G.3) for a given pair s, f. I use the identity as a weighting matrix as I will not conduct inferences on these estimates.

The procedure and especially the moment conditions described here owe much to the insights in the work of Hobijn and Sahin (2009). Modifications arise from differences in identifying assumptions and data availability. Namely, Hobijn and Sahin (2009) have data on employment and unemployment by duration, which allows for two separate estimations, one for each hazard in an independent fashion, using a Gompertz hazard model. As such, their model includes an additional scale parameter in the survival function that, due to the limited data availability, my set-up would not be able to identify. As in Hobijn and Sahin (2009), I omit the bin, which includes only persons unemployed for over a year.

Results The estimates which minimise the moment conditions are monthly hazards of $f = 0.3083$ and $s = 0.0137$. To illustrate the fit of these numbers to the long-term behaviour of the Israeli labour market, see the figure at the end of this appendix. The upper panels of the figure present a replication of the above estimation but for each year separately to give a range of values for s and f. The lower panel plots each year's implied steady-state unemployment rate against the implied steady-state obtained from estimating the flow rates from 2012 to 2019. The obtained value of s is used to discipline the lower bound of λ_s.

Figure 13: Estimated Israeli Labour Market Flow Hazards



Note: The upper two panels plot the results from estimating s and f annually using the above-described procedure, with the long-term estimates in the dashed lines. The lower panel plots the actual unemployment rate with the unemployment rate implied by the long-term estimation results of s and f in the dashed line.

Online Appendix H Optimal Policy Optimization

All the optimization exercises in Section 6 are done using grid search. This method is suitable and feasible as the social planner wishes to set only three policy parameters in each scenario presented in Table 3. When severance pay is considered instead of notice, termination notice is set at the moment of reform to 0.1 weeks or $\phi = 43$. Since all exercises here consider policy combinations of three parameters, I will denote a potential policy vector as $T \in \mathbb{R}^3_+$, and a particular element thereof as $T_i, i \in \{1,2,3\}$. The procedure is as follows

1. **Initialization** - Set the iteration counter $j = 1$. For each policy parameter, define upper and lower bounds $\left[L_i^j, U_i^j\right]$ such that $D^j = \left[L_1^j, U_1^j\right] \times \left[L_2^j, U_2^j\right] \times \left[L_3^j, U_3^j\right]$ is the domain in iteration j where $\times$ denotes the Cartesian product. Define also tolerances for each range as Δ_i. The bounds for ϕ and λ_{U1} are expressed in terms of duration, and not hazard, so the variable chosen is $\frac{1}{\phi}$ and $\frac{1}{\lambda_{U1}}$. For the replacement rate, the

natural bounds are $R \in [0,1]$. Bounds for notice duration, UIB duration, and severance pay eligibility are chosen by trial and error to ensure the solution is internal.

2. **Discretizing the domain** - For each policy parameter, create a grid consisting of 10 equispaced increments for each policy parameter between its lower and upper bound such that the first point is the lower bound and the last is the upper bound, let this grid for policy parameter i be denoted by $G_i^j = \left\{g_1^j, g_2^j, \ldots, g_{10}^j\right\}$. This results in 10^3 potential policy combinations in a grid $G^j = G_1^j \times G_2^j \times G_3^j$.

3. **Evaluation** - For each one of the 10^3 policy vectors $T \in G^j$ considered, solve the model's transition path as a result of the policy reform using the algorithm presented in Appendix F.2 and compute the welfare $\Omega(T)$ at time of impact. If an evaluation fails, use the value of $\Omega(T) = -\infty$ for welfare on this point.

4. **Find the maximum** - Find the best combination of policy parameters $T_j^\star$ such that $T_j^\star = \arg\max_{T \in G^j} \Omega(T)$ and find the indices of corresponding to it for each policy variable. Let p_i denote the index on the grid $p \in \{1, 2, \ldots, 10\}$ of the optimal value of policy parameter i, corresponding to the i^{th} element of $T_j^\star$.

5. **Refine the domain**

   (a) **Compute new domain** - Set the new bounds $\left[L_i^{j+1}, U_i^{j+1}\right]$ for the $j+1$ iteration as follows $L_i^{j+1} = g_{p_i-1}^j$ and $U_i^{j+1} = g_{p_i+1}^j$. These new bounds bracket $T_j^\star$ between its closest grid points. If $p_i = 1$, then $L_i^{j+1} = L_i^j$ and if $p_i = 10$ then $U_i^{j+1} = U_i^j$. The new candidate domain is $D^c = \left[L_1^{j+1}, U_1^{j+1}\right] \times \left[L_2^{j+1}, U_2^{j+1}\right] \times \left[L_3^{j+1}, U_3^{j+1}\right]$.

   (b) **Check for failed evaluations** - Ensure that the objective function was successfully evaluated in the current iteration j for every T that lies on the edge of the new domain. I.e., verify that for all policy vector $T \in D^c$ such that its i^{th} element is equal in either the lower or upper bounds, the value of $\Omega(T)$ was successfully evaluated in step 3. If so, set $D^{j+1} = D^c$. Otherwise, expand the grid by one increment of the grid G^j along the i^{th} dimension and update

Table 7: Bounds for Policy Variables in Optimal Policy Exercises

	ν - CRRA	Scenario	Termination notice duration (weeks)	Severance pay generosity	UIB eligibility duration (weeks)	Replacemen rate
(1)	1	TN & UIB, $\epsilon = 0$	$[0.043, 30]$	none	$[0.043, 260]$	$[0, 1]$
(2)	1	SP & UIB, $\epsilon = 0$	none	$[0.043, 12]$	$[0.043, 260]$	$[0, 1]$
(3)	1	TN & UIB, $\epsilon = 0.65$	$[0.043, 80]$	none	$[0.043, 260]$	$[0, 1]$
(4)	1	SP & UIB, $\epsilon = 0.65$	none	$[0.043, 12]$	$[0.043, 260]$	$[0, 1]$
(5)	2	TN & UIB, $\epsilon = 0$	$[0.043, 40]$	none	$[0.043, 260]$	$[0, 1]$
(6)	2	SP & UIB, $\epsilon = 0$	none	$[0.043, 12]$	$[0.043, 260]$	$[0, 1]$
(7)	2	TN & UIB, $\epsilon = 0.65$	$[0.043, 104]$	none	$[0.043, 500]$	$[0, 1]$
(8)	2	SP & UIB, $\epsilon = 0.65$	none	$[0.043, 12]$	$[0.043, 260]$	$[0, 1]$

D^c accordingly. Repeat this step until there are no failed runs on the edges.

6. **Check domain size for convergence** If the new bounds bracket a region such that $U_i^{j+1} - L_i^{j+1} < \Delta_i$ for every i stop and use $T_j^\star$ as the solution. Otherwise, repeat from step 2 until this condition is satisfied.

The objective function in all cases appears to be well-behaved and single-peaked in the chosen domain, and the maximum in each exercise is an internal one for all policy parameters. The reason to check for failed evaluations is a contingency to prevent the program from ruling out a part of the domain concerning which information is unavailable. Thus, the contingency in 5 (b) prevents the newly chosen domain from including a blind spot. These failures are uncommon in practice. Regardless, it is still necessary to verify that the end result is indeed internal to the first specified domain D^1; otherwise, one needs to restart with new bounds. For each of the eight optimal policy exercises conducted in the paper, four in Section 6, and an additional four in Appendix D, the bounds are reported in Table 7. Since setting the durations to exactly zero implies using an infinite hazard rate, which is impractical, I use a lower bound of 0.01 months duration or 0.043 weeks.

Online Appendix I Stylized Model of Termination Notice

In what follows, I present a stylized model illustrating the effects of termination notice on job creation and wage bargaining in a DMP style environment. This model is intentionally simplified compared to the full scale model presented in the main text. It does not feature a consumption-savings decision nor an intensive margin search effort choice. The model can serve as an intuitive basis for understanding the effects of termination notice on vacancy posting and wage bargaining in the model of Section 2. Additionally, I will use the model to demonstrate some of the differences between severance pay and termination notice mandates.

I.1 A Stylized Model of Termination Notice

I begin the analysis by considering the textbook search and matching model from Pissarides (2000) and extend it to allow for mandated termination notice as follows. When an idiosyncratic productivity shock hits an employer-employee match, they do not separate immediately but enter into a period of termination notice. The shock causes the match to separate by reducing the production value of the job to a fraction ϵ of its original value p. This production decline is assumed severe enough to merit termination of the employment relationship.[78] The worker can use the notice period to search for a new job, which increases their value from the employment contract.

Formally, let the population have a unit measure composed of four types of households, the employed E, the unemployed U, those employed with termination notice and are searching $N1$, and those who had found a job $N2$, the mass of each type i is denoted by m_i.[79]

Matching The matching function $\mu(m_U + m_{N1}, v)$ is monotonic and increasing in both arguments and is homogeneous of degree one. Rather than having the unemployed and job vacancies v as inputs, the unemployed are now replaced by the total searching population $m_{N1} + m_U$. As such, labour-market

[78] This is equivalent to assuming exogenous separation or endogenous separation with two levels of match quality such that one of them is strictly below the reservation level.

[79] Unlike in the main text, this model will not feature a UIB eligibility margin.

tightness is now defined as $\theta = \frac{v}{m_{N1}+m_U}$, and the filling rate and the job finding rate are given by $q = \frac{\mu(m_U+m_{N1},v)}{v}, \lambda_f = \frac{\mu(m_U+m_{N1},v)}{m_U+m_{N1}}$. I assume that matches require positive masses of vacancies and job seekers, i.e., $\mu(m_U+m_{N1},0) = \mu(0,v) = 0$.

Households Households in the economy are risk neutral and maximize lifetime utility which is discounted at rate ρ. The unemployed gain flow value b and search for work, which is a costless activity in this stylized model. Its value function V_U is thus given by

$$\rho V_U = b + \lambda_f (V_E - V_U). \tag{I.1}$$

The employed person receives a wage w, faces a termination risk with arrival rate λ_s, and has the value function V_E given by

$$\rho V_E = w + \lambda_s (V_{N1} - V_E). \tag{I.2}$$

While on termination notice, the worker is entitled by legislation to receive her previous wage and can search for a new job. The expected termination notice duration is $\frac{1}{\phi}$ and the value function of a person under notice V_{N1} is thus given by

$$\rho V_{N1} = w + \phi(V_U - V_{N1}) + \lambda_f (V_{N2} - V_{N1}). \tag{I.3}$$

As in the main text, if the worker finds a job during the notice period which occurs with hazard λ_f, the new match is 'on hold', and the worker has to wait for the end of the notice period to switch employers. The value from being under notice with a new job lined-up is

$$\rho V_{N2} = w + \phi(V_E - V_{N2}). \tag{I.4}$$

This set-up, and especially Equations (I.3) and (I.4), assumes that the worker cannot force a direct transition to a new job. See discussion of this assumption in Section 2.1.

The firms The firms can post a vacant job which is matched with a job seeker at a rate q. A vacancy has a flow cost of κ and, once filled, will generate a value of J_E. If the job seeker is unemployed, the firm and the worker commence production immediately. However, if matched with a worker under termination notice, the firm has a job 'on hold'. The value of a job vacancy is given by

$$\rho J_V = -\kappa + q\frac{m_U}{m_U + m_{N1}}(J_E - J_V) + q\frac{m_{N1}}{m_U + m_{N1}}(J_H - J_V). \tag{I.5}$$

The value of a job 'on hold' comes only from its potential to become a producing job with hazard ϕ and is given by

$$\rho J_H = \phi(J_E - J_H). \tag{I.6}$$

I assume free entry so that at every point in time, $J_V = 0$, which results in the free entry condition

$$J_E = \frac{\kappa}{q\left[\frac{m_U}{m_U+m_{N1}} + \frac{m_{N1}}{m_U+m_{N1}}\frac{\phi}{r+\phi}\right]} = \frac{\kappa}{q l_c}. \tag{I.7}$$

The difference between the model laid out here, and the textbook model in terms of job creation is the labour-composition term l_c, as it depends on the population masses. Substituting in the values of the steady-state masses yields that in steady state $l_c = \frac{\phi}{(\rho+\phi)}\frac{(\rho+\phi+\lambda_f)}{(\phi+\lambda_f)}$.[80] This ratio is bounded between zero as $\phi \to 0$ and unity for $\phi \to \infty$. However, ρ is usually calibrated to around 4% annually, which is equivalent to a Poisson hazard that arrives on average once every twenty-five years. Since we usually consider the notice period to be several months long and job finding rates corresponding to unemployment durations of several months to a year the only it is thus reasonable to consider that l_c will be very close to unity since ϕ and λ_f will be orders of magnitude larger then ρ.

Once in active production, the filled job produces p and has to pay a wage rate of w. The value of a filled job is

$$\rho J_E = p - w + \lambda_s(J_N - J_E), \tag{I.8}$$

where J_N is the value of the job during termination notice. After the impact of the shock λ_s, the job produces a fraction $\epsilon \in [0,1)$ of its previous production

[80] For explicit derivation of this expression, see Appendix I.5.

value. The value of the job under notice is thus

$$\rho J_N = \epsilon p - w + \phi(J_V - J_N). \tag{I.9}$$

Wage bargaining Since employment protection provisions are in place, the surplus that governs hiring and the renegotiation of wages differ. The outsider's wage is solved from the standard problem, which is

$$w^0 = \arg\max (V_E - V_U)^{\beta}(J_E - J_V)^{1-\beta}. \tag{I.10}$$

As is standard for cases with employment protection policies, an insider-outsider dynamic of the labour markets emerges. One surplus level would govern job creation, and yet another would govern future wage renegotiation.[81] The insider's problem is given by

$$w = \arg\max (\mathrm{V}_E - \mathrm{V}_{N1})^{\beta}(\mathrm{J}_E - \mathrm{J}_N)^{1-\beta}. \tag{I.11}$$

The most important feature of this problem is that the value of each party's outside option is a function of the solution to the problem itself. The solution to the problem 16 presented in the main text, is simply the median solution to problems that are identical to I.11 but for workers with varying degrees of patience and asset holdings. Unlike in the main text, I keep the insider outsider structure here to make this model comparable with the textbook analysis in Pissarides (2000), and to maintain tractability.

Model solution The above bargaining problems, together with the free entry condition in Equation (I.7), allow me to characterize the solution to this system by using two equations.[82] First, the wage solution is given by

$$w = \underbrace{\left[\beta\left[p + \frac{\theta\kappa}{l_c}\right] + (1-\beta)b\right]}_{\text{Standard DMP wage}} + \underbrace{\rho\beta\frac{p(1-\epsilon)}{\phi}}_{\text{Threat}} + \underbrace{\frac{\rho}{\rho+\phi+\lambda_f}\beta\frac{\theta\kappa}{l_c}}_{\text{Search on notice}}. \tag{I.12}$$

[81]The model is set in continuous time so the wage w^0 would not be paid to any worker as renegotiation is immediate. However, since this instantaneous first wage reflects the sharing rule for the surplus that governs job creation, it will have a bearing on the solution.

[82]For a step by step derivation see Appendix I.5.

This wage solution is the classical DMP wage with the addition of the threat of reduced production during the notice period which is give by $\rho\frac{p(1-\epsilon)}{\phi}$ and the added value of search during the notice period that is given by $\frac{\rho}{\rho+\phi+\theta\,\mathrm{q}(\theta)}\frac{\theta pc}{l}$. Second, the job-creation condition in the model is

$$\left[p\underbrace{\left(1+\frac{\lambda_s}{\rho+\phi}\epsilon\right)}_{\text{extra production value}}-w\underbrace{\left(1+\frac{\lambda_s}{\rho+\phi}\right)}_{\text{longer wage contract}}\right]\frac{ql_c}{\rho+\lambda_s}=\kappa. \tag{I.13}$$

This equation is derived from the definitions of J_E, J_N and the free entry condition in Equation (I.7). It deviates from the textbook model by allowing for the different horizons of production and wages along the lifetime of a job. These two equations along with the steady-state value of l, determine the equilibrium pair of (θ, w) in this model.

I.2 Discussion of the Model and its Policy Implications

In this section, I elaborate on the model's properties, discuss its assumptions and explore its implications for the use of termination notice.

Comparison to the literature The present model nests the textbook search and matching model as a special case. If there were no termination notice, we would have that $\phi \to \infty$. From the steady-state value of l_c, we can see that in this special case, $l_c \to 1$ and the wage solution and job-creation condition collapse into the standard textbook equations.

My extension builds on earlier models that introduce termination notice into the search and matching literature, all of which try to understand aggregate employment fluctuations. Garibaldi (1998) was the first to introduce termination notice into a search and matching model. His modelling approach was adopted and extended by Bentolila et al. (2012) to consider the effects of the 2008 financial crisis on France and Spain. These two works abstract from the feedback that termination notice introduces into the outside option in the wage bargaining problem (I.11).[83] The model of Ben Zeev and Ifergane

[83] Garibaldi (1998) abstracts from this feedback directly by assuming that the firm can extract the full rent from the worker, and the wage is equal to the outside option. Bentolila et al.

(2022) accounts for this feedback but, like the previous works, assumes no search takes place during termination notice.

Production during termination notice I earlier defined that $\epsilon \in [0,1)$. I.e., I assume the terminated workers neither cause damage in the place of employment nor do they suddenly become more productive after termination notice was given.

In fact, the upper limit on ϵ is even more restrictive. For the firing decision to be internally consistent, the firm should still be willing to let the worker go, given the lower production value. Recall that a job on termination notice yields a profit of $\epsilon p - w$ to the firm every period. Thus, the highest production value for which termination is internally consistent would be $\bar{\epsilon} = \frac{w}{p}$. Under perfect competition this value would be $\bar{\epsilon} = 1$ but in the presence of search frictions $\bar{\epsilon} < 1$.[84]

I.3 Termination Notice and Job Creation

Proposition 3. *Increasing the duration of termination notice (lowering ϕ) will lower labour market tightness if $\epsilon < \bar{\epsilon}$.*

The intuition behind this conclusion is as follows (for a formal proof, see Appendix I.6). Increasing the duration of termination notice improves the worker's bargaining position and shifts the wage curve to the left and upwards in the (θ, w) plain, which raises the wage for every value of θ. It would also add to the value of a job $\epsilon p - w$ for every added instant of termination notice which is strictly negative since $\epsilon < \bar{\epsilon}$. Therefore, termination notice reduces the incentive to create new jobs and shifts the job creation curve inwards, which lowers labour-market tightness. A corollary to this result is that the

(2012) calibrate their model such that the average wage in the economy is the prevailing one during the notice period, and assume that the firm knows this wage and takes it as a known cost.

[84]This argument implicitly assumes that the wage cannot be adjusted during the notice period itself. Allowing the wage to adjust would require additional assumptions regarding the potential duration of the match, its future production value, and the regulatory requirements imposed upon it. Such analysis would not contribute much to what follows. Note that the critical value that would result if the firm were to renegotiate wages after the shock hits would likely be smaller since the wage reduction would make realisations worse off than $\bar{\epsilon}$ acceptable to the firm.

impact on the wage is ambiguous.[85]

Comparison with UIB UIB would be mapped to the model as the value of the outside option b. Observe that changing the level of UIB would influence only the wage equation and not the job creation curve. Thus increasing the generosity of UIB would lower job creation and increase wages in the model economy, which is different from what increasing termination notice duration would do. From a simpler, spot-labour-market perspective, increasing UIB generosity will reduce labour supply, while a longer termination notice period will reduce labour demand.

I.4 Relationship to Severance Pay

To consider the relationship between termination notice and severance pay, I derive the job creation equation and the wage curve of the stylised model with severance pay and compare them to those of the model with termination notice instead. I briefly lay down the model equations, which will be very close to those in Appendix I.1. Most notations are identical to those in I.1 and will not be re-stated. The model is simpler than the one in Appendix I.1 as it has only two states, namely employment and unemployment, without the interim state of a termination notice and the associated job creation delay. As such, the modified model will still have a unit measure of households with the following value functions:

$$\rho V_U = b + \lambda_f (V_E - V_U), \tag{I.14}$$

$$\rho \mathrm{V}_E = w + \lambda_s \big(\mathrm{V}_U + S_p - \mathrm{V}_E\big), \tag{I.15}$$

where S_p denotes the severance pay received at termination, which is a single transfer from employer to employee. Note that now the termination rate λ_s is identical to the separation rate.

[85] Contrast to the result in Figure 5 of Pissarides (2001), where the job creation curve shifts in the same manner but the wage curve shifts in the opposite direction.

Accordingly, the firm's value functions are given by

$$\rho J_V = -\kappa + qJ_E, \tag{I.16}$$

$$\rho J_E = p - w + \lambda_s\big(J_V - J_E - S_p\big). \tag{I.17}$$

Observe that in this case, the free entry condition results in the standard expression $J_E = \frac{\kappa}{q,}$.

As in Appendix I.1, there is an insider outsider dynamic for the workforce, with the outsider facing the standard problem of

$$w^0 = \text{arg max} \quad (V_E - V_U)^{\beta}(J_E - J_V)^{1-\beta}, \tag{I.18}$$

which is identical to the one in I.1. The insider's bargaining problem is given by:

$$w = \text{arg max} \quad \big(\mathrm{V}_E - \big(V_U + S_p\big)\big)^{\beta}\big(\mathrm{J}_E - \big(J_V - S_p\big)\big)^{1-\beta}, \tag{I.19}$$

where S_p will be indexed to the wage, I assume that $S_p = \lambda_{SP} w$ where λ_{SP} denotes the number of wages the worker is entitled to receive from her employer when they separate.

Solving the model The outsider's problem has the following first-order condition:

$$\beta(\mathrm{J}_E) - \big(1-\beta\big)(\mathrm{V}_E - V_U) = 0, \tag{I.20}$$

which together with the definition of V_U and the free entry condition yield:

$$\rho V_U = b + \frac{\beta}{1-\beta}\kappa\theta. \tag{I.21}$$

The insider's problem has the following first-order condition

$$\beta\big(\mathrm{J}_E + S_p\big) - \big(1-\beta\big)\big(\mathrm{V}_E - V_U - S_p\big) = 0. \tag{I.22}$$

Multiplying by ρ and substituting in the definitions of V_E and J_E result in

$$\beta\big[p - w\big] + \beta\rho S_p - \big(1-\beta\big)\big[w - \rho V_U - \rho S_p\big] = 0, \tag{I.23}$$

which combined with Equation I.21, yields

$$w = \beta p + (1-\beta) b + \beta \kappa \theta + \rho S_p. \tag{I.24}$$

Substituting in $S_p = \lambda_{SP} w$ will allow us to obtain the wage equation

$$w = \frac{1}{1-\rho\lambda_{SP}} \left[\beta p + (1-\beta) b + \beta\kappa\theta\right], \tag{I.25}$$

which is the analogue of Equation I.12. Substituting the free entry condition $J_E = \frac{\kappa}{q}$ into the definition of J_E, combined with $S_p = w\lambda_{SP}$ yields the job creation condition

$$\frac{\kappa}{q} = \frac{p - (1+\lambda_s\lambda_{SP})w}{\rho + \lambda_s}. \tag{I.26}$$

Comparison of Severance Pay and Termination Notice in the Stylised Models Let us examine the two wage equations

$$w = \beta\left[p + \frac{\theta\kappa}{l_c}\right] + (1-\beta) b + \rho\beta p \frac{(1-\epsilon)}{\phi} + \frac{\rho}{\rho+\phi+\lambda_f}\beta\frac{\theta\kappa}{l_c}, \tag{w - TN}$$

$$w = \frac{1}{1-\rho\lambda_{SP}}\left[\beta p + (1-\beta) b + \beta\kappa\theta\right], \tag{w - SP}$$

where TN denotes termination notice and SP denotes severance pay. It is instructive to recall that in the textbook model without any policies, the wage curve would be $w = \beta p + \beta\kappa\theta + (1-\beta)b$. Thus, the wage in the case of severance pay is a mark-up over the wage in the textbook model. Although termination notice and severance pay encapsulate a similar mandated monetary transfer between firm and worker, there are three differences between these policies. First, termination notice introduces an interim period, during which the worker produced ϵp; thus, output in the economy is different even if total employment and unemployment are the same for every case in which $\epsilon > 0$. Second, production in this interim period serves as a threat in the bargaining problem, which is added slightly differently to the wage. Last, the interim period provides the worker time to search for a job, which generates value for the worker at the cost of forgoing the value of the outside option b, and thus affects the match surplus. Therefore, the surplus will not be the same under both policies even if θ is the same and there is no production value. The reason behind this result is that severance pay in the stylised model has no effect on the unemployment duration of the worker other than through its effect on θ.

Let us proceed now by analysing job creation under the two policies. In particular, let us examine the job creation conditions

$$\frac{\kappa}{q} = \frac{l_c}{\rho + \lambda_s}\left[p\left(1 + \frac{\lambda_s}{\rho + \phi}\epsilon\right) - w\left(1 + \frac{\lambda_s}{\rho + \phi}\right)\right], \tag{JC - TN}$$

$$\frac{\kappa}{q} = \frac{1}{\rho + \lambda_s}\left[p - (1 + \lambda_s \lambda_{SP}) w\right]. \tag{JC - SP}$$

As explained earlier while discussing the wage equations, the production value under notice ϵ will change the income flow from a new job which will factor into the job creation condition, which is essentially an asset price equation. As this paper assumes that $\epsilon = 0$, and this is a rather convenient case, let us begin the comparison using this case. Next, the job creation delay, l_c will make the comparison challenging as there is no delay inherent in job creation under severance pay. Let us assume for a moment that the delay is of a negligible magnitude, resulting in the convenient case of $l_c \to 1$. From there, one may construct, for every value of ϕ, a corresponding value of λ_{SP} such that the two job creation conditions are equivalent $\lambda^*_{SP}(\phi) = \frac{1}{\rho+\phi}$. If so, can we obtain that a under the same equilibrium value of θ and the suitable policies ϕ and $\lambda^*_{SP}(\phi)$, termination notice and severance pay would yield the same wage? Define the difference in wages under the two policies in this special case as

$$\Delta = w_{SP}\left(\lambda^*_{\text{SP}}(\phi)\right) - w_{TN}(\epsilon = 0, l = 1), \tag{I.27}$$

which, after some tedious algebra yields

$$\Delta = \frac{\rho}{\phi}\left[b(1 - \beta)\right] + \kappa\theta\beta\rho\left[\frac{1}{\phi} - \frac{1}{\rho + \phi + \lambda_f}\right] > 0. \tag{I.28}$$

Therefore, holding θ constant, the wage under severance pay would be higher in the simple case than under termination notice. The cause of this is the added value the worker receives from the match while searching during the notice period. However, since this expression is proportional to ρ, it will be quantitatively a negligible difference. However, this illustrates that there are slight differences that even the stylised model will not be able to entirely ignore when comparing these tools.

Moreover, stepping outside the bounds of the model and discussing the policies as they are applied in reality, the comparison above clarifies that the two policies have a different impact on aggregate production and unemployment. These differences occur even though the two policy tools are pretty similar in terms of the monetary transfer they entail. This result is not simply a modelling artefact but rather the implications of these policies in practice. Thus, despite the apparent similarities between these policies, it is essential to specify the exact policy instrument used to analyse it correctly.

To further drive this point home, note that without policies, the match surplus will be given by:

$$S = J_E + V_E - V_U. \tag{I.29}$$

Introducing severance pay into the economy will not affect the match surplus as

$$S_{SP} = J_E - \left(J_V - S_p\right) + V_E - \left(V_U + S_p\right) = J_E + V_E - V_U = S. \tag{I.30}$$

However, introducing termination notice into the economy will affect the match surplus

$$S_{TN} = J_E + V_E - V_{N1} - J_N \neq S. \tag{I.31}$$

To verify the last statement, one can obtain the following to derive the wage Equation (see Equation I.45)

$$\left(\rho + \phi\right)(V_{N1} + J_N) = \epsilon p + \phi V_U + \frac{\lambda_f \phi}{\rho + \phi + \lambda_f}(V_E - V_U). \tag{I.32}$$

Combining the fact that $\rho V_U - b = \lambda_f (V_E - V_U)$ with the above we have that

$$\left(\rho + \phi\right)(V_{N1} + J_N) = \epsilon p - b\frac{\phi}{\rho + \phi + \lambda_f} + \phi\left[\frac{\rho}{\rho + \phi + \lambda_f} + 1\right] V_U. \tag{I.33}$$

Finally, subtracting $\left(\rho + \phi\right) V_U$ from both sides and substituting in Equation I.47 as $\rho V_U = b + \lambda_f \frac{\beta}{1-\beta}\frac{\kappa}{q l_c}$ will yield

$$\begin{aligned} &\left(\rho + \phi\right)(\mathrm{V}_{N1} + J_N - V_U) = \\ &\epsilon p - b - \left[\frac{\rho + \lambda_f}{\rho + \phi + \lambda_f}\right]\left[\frac{\beta}{1-\beta}\frac{\kappa\theta}{l_c}\right]. \end{aligned} \tag{I.34}$$

The sign of the expression on the left-hand side of the equation will determine under which policy the surplus is larger. However, without knowing the parameter values, the sign of this expression is unknown. The reason behind this ambiguity is that during every period of termination notice, the pair forgoes b utility units and gains the values of production ϵp and of search during notice. The trade-off between the two will depend on the exact values of the parameters. As a side note, observe that in a model with lay-off taxes, which will not be fully derived here, the surplus S_{LT} will be larger with

$$S_{LT} = J_E + F + V_E - V_U \geq S, \tag{I.35}$$

where F is a firing tax.[86]

I.5 Model Solution and Additional Derivations

This appendix presents the explicit derivation of the equilibrium masses in the model, and the steady-state value of l, as well as Equations (I.12), and (I.13).

Population composition The following laws of motion govern the transitions in the model (dots denote temporal derivatives)

$$\begin{bmatrix} \dot{m}_U \\ \dot{m}_E \\ \dot{m}_{N1} \\ \dot{m}_{N2} \end{bmatrix} = \begin{bmatrix} -\lambda_f & 0 & \phi & 0 \\ \lambda_f & -\lambda_s & 0 & \phi \\ 0 & \lambda_s & -\phi-\lambda_f & 0 \\ 0 & 0 & \lambda_f & -\phi \end{bmatrix} \begin{bmatrix} m_U \\ m_E \\ m_{N1} \\ m_{N2} \end{bmatrix}. \tag{I.36}$$

Using the laws of motion in Equation (I.36) and the fact that the masses

[86] For a more comprehensive treatment of firing taxes in these frameworks see chapter 9 of Pissarides (2000).

sum up to unity, the steady-state masses can be derived as

$$m_U = \frac{\phi^2\lambda_s}{{\lambda_f}^2\lambda_s + \lambda_f\phi\lambda_s + \lambda_f\phi\big(\phi+\lambda_f\big) + \phi^2\lambda_s}, \tag{I.37}$$

$$m_{N1} = \frac{\lambda_f\phi\lambda_s}{{\lambda_f}^2\lambda_s + \lambda_f\phi\lambda_s + \lambda_f\phi\big(\phi+\lambda_f\big) + \phi^2\lambda_s}, \tag{I.38}$$

$$m_E = \frac{\lambda_f\phi\big(\phi+\lambda_f\big)}{{\lambda_f}^2\lambda_s + \lambda_f\phi\lambda_s + \lambda_f\phi\big(\phi+\lambda_f\big) + \phi^2\lambda_s}, \tag{I.39}$$

$$m_{N2} = \frac{{\lambda_f}^2\lambda_s}{{\lambda_f}^2\lambda_s + \lambda_f\phi\lambda_s + \lambda_f\phi\big(\phi+\lambda_f\big) + \phi^2\lambda_s}. \tag{I.40}$$

Combining these to the value of l_c as defined by Equation (I.7) yields:

$$l_c = \frac{m_U}{m_U + m_{N1}} + \frac{m_{N1}}{m_U + m_{N1}}\frac{\phi}{\rho+\phi} = \frac{\phi\big(\rho+\phi+\lambda_f\big)}{\big(\rho+\phi\big)\big(\phi+\lambda_f\big)}. \tag{I.41}$$

The wage solution To solve for the wage, one needs to start from the first order condition for the bargaining problem (I.11), which is:

$$\beta(J_E - J_N) = \big(1-\beta\big)(V_E - V_{N1}). \tag{I.42}$$

It is convenient to examine the problem in terms of the surplus level $S = V_E - V_{N1} + J_E - J_N$ associated with it, which after multiplying by ρ and substituting in the definitions for V_E, V_{N1}, J_E and J_N and results in

$$\big(\rho+\lambda_s\big)S = p - \rho \mathrm{V}_{N1} - \rho J_N. \tag{I.43}$$

The sum $J_N + V_N$ cab be expressed as

$$\rho(V_{N1} + J_N) = \phi(V_U - V_{N1}) + \lambda_f(V_{N2} - V_{N1}) + \epsilon p + \phi(J_V - J_N). \tag{I.44}$$

Subtracting V_{N1} from V_{N2} or Equation (I.3) from Equation (I.4) yields the following relationship

$$\big(\rho+\phi+\lambda_f\big)(V_{N2} - V_{N1}) = \phi(V_E - V_U),$$

which substituted into Equation (I.44) yields

$$\left(\rho+\phi\right)(V_{N1}+J_N)=\epsilon p+\phi V_U+\frac{\lambda_f\phi}{\rho+\phi+\lambda_f}(V_E-V_U). \tag{I.45}$$

Now, we turn our attention to the outsider's problem (I.10), which has the first order condition

$$\beta(J_E-J_V)-\left(1-\beta\right)(V_E-V_U)=0. \tag{I.46}$$

It is again convenient to define the surplus level S^0 which is given by $J_E-J_V+V_E-V_U$, and using the free entry condition from Equation (I.7), the definition of V_U and the fact that $V_E-V_U=\frac{\beta}{1-\beta}J_E$ we obtain

$$\rho V_U=b+\lambda_f\beta S^0=b+\lambda_f\frac{\beta}{1-\beta}\frac{\kappa}{ql_c}. \tag{I.47}$$

This expression can be substituted into Equation (I.45), which along with the free entry condition in Equation (I.7), and the insight that $V_E-V_U=\frac{\beta}{1-\beta}J_E$ yields after some tedious algebra

$$\left(\rho+\phi\right)(V_{N1}+J_N)=\epsilon p+\frac{\phi}{\rho}b+\phi\lambda_f\left[\frac{1}{\rho+\phi+\lambda_f}+\frac{1}{\rho}\right]\frac{\beta}{1-\beta}\frac{\kappa}{ql_c}. \tag{I.48}$$

Using this expression in Equation (I.43), we can express the surplus as

$$\left(\rho+\lambda_s\right)S=p-\frac{\rho}{\rho+\phi}\left[\epsilon p+\frac{\phi}{\rho}b+\phi\lambda_f\left[\frac{1}{\rho+\phi+\lambda_f}+\frac{1}{\rho}\right]\frac{\beta}{1-\beta}\frac{\kappa}{ql_c}\right]. \tag{I.49}$$

The surplus can also be described as follows:

$$\left(\rho+\lambda_s\right)(J_E-J_N)=\left(\rho+\lambda_s\right)\left(1-\beta\right)S, \tag{I.50}$$

which combined with the fact that $\left(\rho+\lambda_s\right)J_E=p-w+\lambda_sJ_N$, and that $\left(\rho+\phi\right)J_N=\epsilon p-w$ allows us to write the surplus as

$$\left(\rho+\lambda_s\right)S=\frac{1}{1-\beta}\left[p-w-\rho\frac{\epsilon p-w}{\rho+\phi}\right]. \tag{I.51}$$

Equating the two expressions of the surplus as given by Equation (I.49) and

Equation (I.51) and using $\frac{\lambda_f}{q} = \theta$ allows us to finally obtain the wage solution given in Equation (I.12)

$$w = \underbrace{\left[\beta\left[p + \frac{\theta\kappa}{l_c}\right] + (1-\beta)b\right]}_{\text{Standard DMP wage}} + \underbrace{\rho\beta\frac{p(1-\epsilon)}{\phi}}_{\text{Threat}} + \underbrace{\frac{\rho}{\rho+\phi+\lambda_f}\beta\frac{\theta\kappa}{l_c}}_{\text{Search on notice}}. \tag{I.52}$$

The job-creation equation Combining the free entry relationship $\frac{\kappa}{ql_c} = J_E$, with $(\rho+\lambda_s)J_E = p - w + \lambda_s(J_N)$ and with $J_N = \frac{\epsilon p - w}{\rho+\phi}$, after some algebraic manipulation yields the job creation condition from Equation (I.13).

$$\left[p \underbrace{\left(1 + \frac{\lambda_s}{\rho+\phi}\epsilon\right)}_{\text{extra production value}} - w \underbrace{\left(1 + \frac{\lambda_s}{\rho+\phi}\right)}_{\text{longer wage contract}}\right]\frac{ql_c}{\rho+\lambda_s} = \kappa. \tag{I.53}$$

I.6 Proof of Proposition 3

Proof. Recall that the steady-state equilibrium of the system is given by the following three equations:

$$w = \beta\left[p + \frac{\theta\kappa}{l_c}\right] + (1-\beta)b + \rho\beta p\frac{(1-\epsilon)}{\phi} + \frac{\rho}{\rho+\phi+\lambda_f}\beta\frac{\theta\kappa}{l_c}. \tag{I.54}$$

$$w = \left(p - \kappa\frac{\rho+\lambda_s}{ql_c}\right)\frac{\rho+\phi}{\rho+\phi+\lambda_s} + \epsilon p\frac{\lambda_s}{\rho+\phi+\lambda_s}, \tag{I.55}$$

$$l_c = \frac{\phi(\rho+\phi+\lambda_f)}{(\rho+\phi)(\phi+\lambda_f)}. \tag{I.56}$$

where the job-creation curve is reordered such as to make the graphical explanation easier. Graphically it is constructive to examine the system, as is done in the standard search and matching representation, in the θ, w plane while treating l_c as a function of θ. As such, I first illustrate the behaviour of this function. One can show that l_c is a monotonically decreasing function of θ.[87] The function l_c is also bounded by $l_c(\theta = 0) = 1$ and by $\lim_{\theta\to\infty} l_c = \frac{\phi}{\phi+\rho}$. This behaviour of l_c, taken together with the observation that $\frac{\mathrm{d}q}{\mathrm{d}\theta} < 0$, means that

[87] To verify this statement, observe that $\frac{\partial l_c}{\partial\theta} = \frac{\phi}{(\rho+\phi)}\frac{-\rho}{(\phi+\lambda_f)^2}\frac{\mathrm{d}\lambda_f}{\mathrm{d}\theta} < 0$. This statement holds since $\lambda_f = \frac{\mu(m_U+m_{N1},v)}{m_U+m_{N1}} = \mu(1,\theta)$ and as μ is an increasing function of both arguments we obtain that $\frac{\mathrm{d}\lambda_f}{\mathrm{d}\theta} > 0$.

as in the standard search and matching model, the job creation curve slopes downwards on the θ, w plane and that the wage curve slopes upwards along the same plane, yielding a unique equilibrium. Finally, observe that holding the value of θ constant, the steady-state value of l_c will increase as ϕ increases as $\frac{\partial l_c}{\partial \phi} = \frac{(\rho+2\phi+\lambda_f)\rho\lambda_f}{(\phi+\lambda_f)^2(\rho+\phi)^2} > 0$. With these insights in mind, we can proceed to examine the θ, w plane.

The wage curve For a given value of θ, a decrease in ϕ, which also means a decrease in l_c, will unambiguously raise the wage. Thus, the wage curve will shift to the left. Intuitively speaking, increasing the duration of the notice period, holding labour market conditions constant, will strengthen the worker's bargaining position and weaken that of the employer, thus, resulting in a wage increase. The converse also holds, i.e., increasing ϕ will shift the curve to the right.

The job creation curve Two conflicting forces affect the job-creation curve. Since examining the job creation curve as w as a function of θ is more convenient, consider the reordered expression

$$w = \left(p - \kappa \frac{\rho+\lambda_s}{q l_c}\right) \frac{\rho+\phi}{\rho+\phi+\lambda_s} + \epsilon p \frac{\lambda_s}{\rho+\phi+\lambda_s}. \tag{I.57}$$

First, observe the special case of $\epsilon = 0$. If ϵ, the job creation condition is given by

$$w(\epsilon=0,\theta) = \underbrace{\frac{\rho+\phi}{\rho+\phi+\lambda_s}}_{\text{Duration wedge}} \cdot \underbrace{\left(p - \kappa \frac{\rho+\lambda_s}{q l_c}\right)}_{\text{Job-creation curve DMP}}. \tag{I.58}$$

This case can be simply interpreted because both expressions on the right-hand side of the equation are positive and increasing in ϕ for every equilibrium in which there is job creation. To see this, first, recall that ρ, ϕ, and λ_s are all positive, so the duration wedge is also positive. Second, dividing the second expression by $\rho + \lambda_s$ yields $\frac{p}{\rho+\lambda_s} - \frac{\kappa}{q l_c}$, which is the total discounted production value of a job minus the flow cost of job-creation pc divided by the job-filling rate. Recall from free entry that $J_E = \frac{\kappa}{q l_c}$. Thus, if we have that $\frac{p}{\rho+\lambda_s} - J_E < 0$, it means that there is no incentive to create jobs in this economy, for any positive wage rate, as only a negative wage will justify the firm's job creation cost. To summarize, for $\epsilon = 0$, decreasing ϕ, or increasing the duration of termination notice, shifts the job-creation curve to the left and lowers θ for every wage rate.

If ϵ were strictly positive, we could restate the job creation curve as:

$$w(\theta) = \frac{p\epsilon\lambda_s}{\rho + \phi + \lambda_s} + w(\epsilon = 0, \theta). \tag{I.59}$$

As explained above, lowering ϕ shifts $w(\epsilon = 0, \theta)$ to the left, but, this time it also creates a conflicting force that rises $\frac{p\epsilon\lambda_s}{\rho+\phi+\lambda_s}$ and shifts the job creation curve to the right. Which of these forces will prevail is a quantitative question. But, given that we know what would happen if ϵ were zero, and given that $\frac{p\epsilon\lambda_s}{\rho+\phi+\lambda_s}$ is monotonically increasing in ϵ we can conclude that there exists a level of ϵ such that above it, the job creation curve would shift to the right as a result of increasing the duration of termination notice.

Additionally, we have defined the internally consistent level of the production value during notice $\bar{\epsilon}$ as $\frac{w}{p}$. Since this is an upper limit on the value of production during termination notice in the model, it is also a useful reference case to examine. Substituting in the value of $\bar{\epsilon}$ into Equation (I.57) yields

$$w(\bar{\epsilon}) = \frac{p\frac{w(\bar{\epsilon})}{p}\lambda_s}{r + \phi + \lambda_s} + w(\epsilon = 0), \tag{I.60}$$

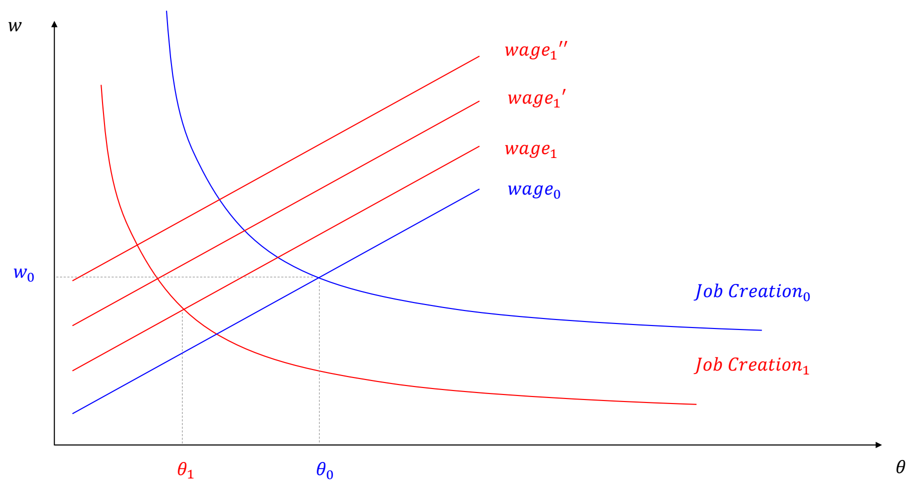
Or using Equation (I.58)

$$w(\bar{\epsilon}) = p - \kappa\frac{\rho + \lambda_s}{ql_c}. \tag{I.61}$$

This expression is increasing in ϕ or decreasing in the termination notice duration. As such, even at $\bar{\epsilon}$ the influence of $\frac{p\epsilon\lambda_s}{\rho+\phi+\lambda_s}$ is not sufficiently strong to push the job creation to the right in response to a decrease in ϕ. Therefore, for every value of $\epsilon \in [0, \bar{\epsilon}]$, the job creation curve shifts to the left in response to increasing the duration of termination notice.

To conclude, in response to an increased duration of termination notice, both the job creation condition and the wage equation shift to the left, leading to a decrease in labour-market tightness. The converse also holds, i.e., increasing ϕ will increase labour market tightness for sufficiently low values of ϵ. For a graphical representation of this proof, see Figure 14. Finally, note that as a corollary to this result, the effect of increased termination notice duration on wages is ambiguous. □

Figure 14: Varying the Duration of Termination Notice in the Stylized Model



Note: This figure presents the impact of increasing termination notice in the (θ, w) plain, where the intersection of the wage curve and the job creation curve determines the equilibrium pair. Increasing the duration of termination notice shifts the job creation curve towards the origin, while the wage curve shifts upwards and to the left.