Online Appendix

for

Either or Both Competition: A "Two-sided" Theory of Advertising with Overlapping Viewerships

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This Online Appendix provides supplemental material for the paper "Either or Both Competition: A "Two-sided" Theory of Advertising with Overlapping Viewerships". Specifically, it presents an analysis of a two-stage game in which outlets simultaneously make offers and afterwards all agents simultaneously make their choices, and provides the conditions for outcome-equivalence to the four-stage game considered in the paper. It also demonstrates that the effects identified in the paper are also at work in a game with two incumbent outlets and one entrant. Finally, the Online Appendix presents an analysis of heterogeneous advertisers and relates it to the analysis of homogeneous advertisers presented in the paper.

I. Two-stage game

Consider the following two-stage game: In stage 1, outlets simultaneously offer menus of contracts to advertisers of the form $(t_i, m_i) \in \mathbb{R}^2_+$. After observing these contracts, viewers and advertisers simultaneously choose which outlet(s) to join and which contract(s) to accept, respectively.

In addition, consider the following assumptions:

$A1$ Outlets are symmetric.

$A2$ For any $\alpha \in [0,1]$, the following inequality holds

$$t_i^\star(1-\alpha) > \alpha\Big\{d_i(\alpha\tilde{n}_i)\phi_i^S(\tilde{n}_i) - d_i((1-\alpha)n_i^\star)\phi_i^S(n_i^\star)\Big\}, \tag{1}$$

where $d_i(\cdot) := D_i(\cdot) + D_{12}(\cdot)$, $\tilde{n}_i = \arg\max_{n_i} d_i(\alpha n_i)\phi_i^S(n_i)$, $n_i^\star$ is implicitly defined by equation (3) of the paper and $t_i^\star$ is given by $D_i(n_i^\star, n_j^\star)\phi_i^S(n_i^\star) +$

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$D_{12}(n_i^\star, n_j^\star)\left(\phi_{12}(n_i^\star, n_j^\star) - \phi_j^M(n_j^\star)\right)$.

We provide a discussion of these assumptions after the proof of the following proposition. There we explain that $A1$ can be weakened while $A2$ is a relatively natural assumption in our framework.

Proposition *Suppose that $A1$ and $A2$ hold. Then, there is an equilibrium in the two-stage game game with posted contracts, that is outcome-equivalent to the equilibrium of the game defined in Section 3 of the paper.*

Proof:

Suppose that in the two-stage game with posted contracts each outlet offers a contract with $n_i = n_i^\star$, where $n_i^\star$ is implicitly defined by equation (3) of the paper, and a transfer

$$t_i^\star = D_i(n_i^\star, n_j^\star)\phi_i^S(n_i^\star) + D_{12}(n_i^\star, n_j^\star)\left(\phi_{12}(n_i^\star, n_j^\star) - \phi_j^M(n_j^\star)\right).$$

By the same argument as we used for the original model, these contracts will be accepted by all advertisers. As this is anticipated by viewers, viewerships are $D_i(n_i^\star, n_j^\star)$ and $D_{12}(n_i^\star, n_j^\star)$. Since advertising levels are the same as in the equilibrium of the original model, viewerships are also the same. Therefore, this candidate equilibrium is outcome-equivalent to the equilibrium of the original model.

Let us now consider if there exists a profitable deviation from this candidate equilibrium. We first show that there can be no profitable deviation contract of outlet i that still induces full advertiser participation on outlet j but a smaller participation on outlet i. Let x_i denote the fraction of advertisers who accept the offer of outlet i.

Consider a candidate contract (n_i, t_i). Suppose that outlet i's equilibrium profit from this contract is $t_i x_i$. Now consider the following alternative contract: $(x_i n_i, x_i t_i)$. Note that total advertising on outlet i is still equal to $x_i n_i$. So outlet i is at least as attractive as with the candidate equilibrium contract. Note moreover that because ϕ_i^S and ϕ_{12} are strictly concave in n_i, the incremental value of accepting offer $(x_i n_i, x_i t_i)$ must exceed $x_i t_i$ for all levels of advertiser participation. So all advertisers would accept $(x_i n_i, x_i t_i)$ regardless. It follows that outlet i can marginally increase $x_i t_i$ while still getting full participation. Therefore, profits would strictly increase. It follows that no offer inducing a level of participation $x_i < 1$ can be part of a best reply.

Now suppose outlet i deviates from the candidate equilibrium in such a way that it induces a fraction α of the advertisers to single-home on its outlet while the remaining fraction $1-\alpha$ single-homes on outlet j. Using the definition $d_i(\cdot) := D_i(\cdot) + D_{12}(\cdot)$, the largest possible transfer that outlet i can ask is then bounded above by

$$t_i^d = d_i(\alpha\tilde{n}_i)\phi_i^S(\tilde{n}_i) - u_{shj},$$

where $\tilde{n}_i$ denotes the optimal deviation advertising level and u_{shj} denotes the payoff of an advertiser who chooses to reject the contract of outlet i and instead single-homes on outlet j. To determine u_{shj} we determine the advertiser's payoff when accepting only outlet j's contract, which is the outlet's equilibrium contract after outlet i has deviated to induce a fraction α of advertisers to single-home on outlet i. We obtain

$$u_{shj} = d_j((1-\alpha)n_j^\star, \alpha\tilde{n}_i)\phi_j^S(n_j^\star) - t_j^\star =$$

$$d_j((1-\alpha)n_j^\star, \alpha\tilde{n}_i)\phi_j^S(n_j^\star) - D_j(n_j^\star, n_i^\star)\phi_j^S(n_j^\star) - D_{12}(n_i^\star, n_j^\star)\left(\phi_{12}(n_i^\star, n_j^\star) - \phi_i^M(n_i^\star)\right).$$

Outlet i's profit is then $\alpha\tilde{t}_i$. Hence, deviating is not profitable if

$$\alpha\Big\{d_i(\alpha\tilde{n}_i)\phi_i^S(\tilde{n}_i) - d_j((1-\alpha)n_j^\star)\phi_j^S(n_j^\star) + D_j(n_j^\star, n_i^\star)\phi_j(n_j^\star) + D_{12}(n_i^\star, n_j^\star)\left(\phi_{12}(n_i^\star, n_j^\star) - \phi_i^M(n_i^\star)\right)\Big\}$$

$$< D_i(n_i^\star, n_j^\star)\phi_i^S(n_i^\star) + D_{12}(n_i^\star, n_j^\star)\left(\phi_{12}(n_i^\star, n_j^\star) - \phi_j^M(n_j^\star)\right).$$

Now suppose that the two outlets are symmetric. Then the above condition reduces to

$$\alpha\Big\{d_i(\alpha\tilde{n})\phi^S(\tilde{n}) - d_i((1-\alpha)n^\star)\phi(n^\star)\Big\} - (1-\alpha)\left(D_i(n^\star, n^\star)\phi(n^\star) + D_{12}(n^\star, n^\star)\left(\phi_{12}(n^\star, n^\star)\phi^M(n^\star)\right)\right) < 0,$$

where $n_i^\star = n_j^\star = n^\star$, $\tilde{n}_i = \tilde{n}^d$, $\phi_i^M(\cdot) = \phi_j^M(\cdot) = \phi^M(\cdot)$, and $\phi_i^S(\cdot) = \phi_j^S(\cdot) = \phi^S(\cdot)$. This can be rewritten as

$$t_i^\star(1-\alpha) > \alpha\Big\{d_i(\alpha\tilde{n}_i)\phi^S(\tilde{n}_i) - d_i((1-\alpha)n_i^\star)\phi^S(n_i^\star)\Big\}.$$

which is fulfilled by $A2$. As a consequence, a deviation is not profitable. ■

We now shortly explain why the assumptions $A1$ and $A2$ are not very restrictive in our framework. First, consider $A1$. Since the game is continuous, $A1$ can be relaxed to some extent without affecting the result, implying that the proposition still holds if outlets are not too asymmetric. Now consider $A2$. It is evident from (1), that the assumption is fulfilled for α low enough. In this case the right-hand side is close to 0, while the left-hand side is strictly positive. Now consider the opposite case, i.e., $\alpha \to 1$. In that case the left-hand side goes to zero, while the right-hand side goes to $d_i(\tilde{n}_i)\phi^S(\tilde{n}_i) - d_i(0)\phi^S(n_i^\star)$. Evidently, $d_i(0) > d_i(\tilde{n}_i)$. Hence, the right-hand side is negative if $\phi^S(\tilde{n}_i)$ is not much larger than $\phi^S(n_i^\star)$. In general, $n_i^\star$ can be larger or smaller than $\tilde{n}_i$, implying that the difference can be either positive or negative. However, even in case $\tilde{n}_i > n_i^\star$, if the slope of the advertising functions ϕ_i^S and ϕ_{12} is relatively small, the difference between $n_i^\star$ and $\tilde{n}_i$ will be small, implying that the right-hand side is negative. Finally, consider intermediate values of α. Again, if the difference between $n_i^\star$ and $\tilde{n}_i$ is relatively small, the term in the bracket on the right-hand side of (1) is close to zero. Since the left-hand side is strictly positive, $A2$ is then fulfilled as well.

II. Entry in case of two incumbent outlets

Consider the case of two incumbents and entry of a third outlet. After entry, the profit of outlet i is

$$\Pi_i(n_1,n_2,n_3) = D_i(n_1,n_2,n_3)\phi_i^S(n_i) + D_{ij}(n_1,n_2,n_3)\left(\phi_{ij}(n_i,n_j) - \phi_j^M(n_j)\right)$$

$$+D_{ik}(n_1,n_2,n_3)\left(\phi_{ik}(n_i,n_k) - \phi_k^M(n_k)\right) + D_{123}(n_1,n_2,n_3)\left(\phi_{ijk}(n_i,n_j,n_k) - \phi_{jk}(n_j,n_k)\right)$$

As in the case of entry of a second outlet, we can rewrite this profit function as the profit without entry plus a negative correction term. This leads to (dropping arguments)

$$\Pi = (D_i + D_{ik})\phi_i + (D_{ij} + D_{ijk})(\phi_{ij} - \phi_j^M)$$
$$-D_{ik}(\phi_i^S + \phi_k^M - \phi_{ik}) - D_{ijk}\left(\phi_{ij} - \phi_j^M - (\phi_{ijk} - \phi_{jk})\right).$$

The first two terms are the profit in duopoly. Note that without entry D_{ik} did not exist since there was no outlet k and so outlet i could get ϕ_i for these viewers due to the fact that they were single-homing on outlet i. Similarly, D_{ijk} did not exist and these viewers were multi-homing in outlets i and j. The last two terms are the negative correction terms.

Taking the derivative with respect to n_i yields

$$\frac{\partial \Pi}{\partial n_i} = \frac{\partial \Pi^d}{\partial n_i} + D_{ik}(\phi_i^S + \phi_k^M - \phi_{ik})\left[E_{D_{ik}} - E_{\phi_i^S + \phi_k^M - \phi_{ik}}\right]$$

$$+D_{ijk}\left(\phi_{ij} - \phi_j^M - (\phi_{ijk} - \phi_{jk})\right)\left[E_{D_{ijk}} - E_{\phi_{ij} - \phi_j^M - (\phi_{ijk} - \phi_{jk})}\right] = 0,$$

where $\partial \Pi^d / \partial n_i$ is the derivative with respect to n_i of an outlet's profit in case of duopolistic competition. So we obtain that for $E_{D_{ik}} > E_{\phi_i^S + \phi_k^M - \phi_{ik}}$ and $E_{D_{ijk}} > E_{\phi_{ij} - \phi_j^M - (\phi_{ijk} - \phi_{jk})}$, the business-sharing effect dominates the duplication effect. The formula now consists of two additional terms since entry of a third outlet leads to changes in two viewer groups, namely, the exclusive ones and the overlapping ones before entry. Each term is multiplied by the absolute profits of the respective viewer group. In a similar vain, the analysis can be extended to any number of incumbent outlets.

III. Heterogeneous Advertisers

We discuss how the trade-off characterized in Proposition 1 extends to advertisers with heterogeneous product values, as in Anderson and Coate (2005). As we will show, the key insights obtained in the analysis with homogeneous advertisers carries through to heterogeneous advertisers. In particular, outlet competition is also characterized by the tension between the duplication and business-sharing effect. This holds although the analysis is more involved compared to homogeneous advertisers, as we need to characterize an entire contract schedule (i.e., the optimal screening contracts) offered by outlets, instead of only a single transfer-quantity pair.[1]

Consider the following extension of our baseline model. The value of informing a viewer, ω, is distributed according to a smooth c.d.f. F with support $[\underline{\omega}, \overline{\omega}]$, $0 < \underline{\omega} \leq \overline{\omega}$, that satisfies the monotone hazard rate property. The value ω is private information to each advertiser. The timing of the game is the same as before. In the first stage, each outlet i announces its total advertising level n_i. Afterwards, consumers decide which outlet to join. Given these decisions, each outlet offers a menu of contracts consisting of a transfer schedule $t_i := [0, \overline{m}] \rightarrow \mathbb{R}$ defined over a compact set of advertising levels. $t_i(m)$ is the transfer an advertiser has to pay to get an advertising intensity m from outlet i. In the final stage, as before, advertisers decide which outlet to join. In what follows, we define $n = (n_1, n_2)$.

Let us start with the monopoly case. With an abuse of notation we still use $\omega u(m_i, n_i)$ to denote the surplus of advertiser type ω from advertising intensity m_i. The overall utility of an advertiser depends on the transfer schedule in addition to the surplus. If $m_i(\omega)$ denotes the optimal intensity chosen by type ω, then outlets i's problem in case of monopoly is

$$\Pi = \max_{t_i(\cdot)} \int_{\underline{\omega}}^{\overline{\omega}} t_i(m_i(\omega)) dF(\omega). \tag{2}$$

By choosing the optimal menu of contracts, the monopolist determines which advertiser types to exclude, that is, $m_i(\omega) = 0$ for these types, and which advertiser types will buy a positive intensity. We denote the marginal advertiser by ω_0^m. Problem (2) can be expressed as a standard screening problem:

$$\Pi = \max_{\omega_0^m, m_i(\omega)} \int_{\omega_0^m}^{\overline{\omega}} t_i(m_i(\omega)) dF(\omega)$$

$$\begin{aligned} \text{subject to} \quad & m_i(\omega) = \arg\max\nolimits_{m_i} v_i^m(m_i, \omega, n_i) - t_i(m_i), \\ & v_i^m(m_i(\omega), \omega, n_i) - t_i(m_i(\omega)) \geq 0 \ \text{ for all } \omega \geq \omega_0^m, \\ & \textstyle\int_{\omega_0^m}^{\overline{\omega}} m_i(\omega) dF(\omega) \leq n_i, \end{aligned}$$

where $v_i^m(m_i, \omega, n_i) := \omega d_i(n_i) \phi_i^S(m_i)$ denotes the net value of advertising intensity m_i to type ω in the monopoly case. The first constraint is the incentive-compatibility constraint and the second one the participation constraint. The third one is the capacity constraint specifying that the aggregate advertising level cannot exceed the one specified by the outlet in the first stage. Provided that the function $v_i^m(m_i, \omega, n_i)$ satisfies the standard regularity conditions in the screening

[1]Our results also hold when outlets can perfectly discriminate between advertisers. In that case, the results for each type are the same as the ones in case of homogeneous advertisers.

literature, we can apply the canonical screening methodology Our assumptions on the viewer demand $d_i(n_i)$ and on the advertising technology $\phi_i^S(m_i)$ ensure that v_i^m is continuous and increasing in ω. It also has strictly increasing differences in (m, ω).

Evidently, the capacity constraint will be binding at the optimal solution since it can never be optimal for the monopolist to announce a strictly larger advertising level than the one it uses. Applying the above-mentioned methodology, we can transform the maximization problem to get

$$\Pi = \max_{\omega_0^m, m_i(\omega)} \int_{\omega_0^m}^{\overline{\omega}} \left(\omega - \frac{1-F(\omega)}{f(\omega)}\right) d_i(n_i)\phi_i^S(m_i(\omega))dF(\omega)$$

subject to $n_i = \int_{\omega_0^m}^{\overline{\omega}} m_i(\omega)dF(\omega)$.

We show at the end of this section that the optimal advertising level n_i can be characterized by the following equation:

$$\int_{\omega_0^m}^{\overline{\omega}} \left(\omega - \frac{1-F(\omega)}{f(\omega)}\right)\left(\tilde{d}_i \frac{\partial \phi_i^S}{\partial m_i} + \frac{\partial d_i}{\partial n_i}\phi_i^S\right) dF(\omega) = 0, \tag{3}$$

with $\tilde{d}_i := (1 - F(\omega_0^m))d_i$. We can compare this characterization with the one for homogeneous advertisers given by equation (4) of the paper. Due to the information rent that is required for incentive compatibility, the outlet can no longer extract the full rent from advertisers but only a fraction of it. This is expressed by the first bracket in the integral. Inspecting the second bracket, the expression is analogous to the one with homogeneous advertisers. Note that in the latter case $m_i = n_i$ implies that the derivative was taken with respect to n_i in both terms. The above expression instead accounts for the fact that the optimal allocation $m_i(\omega)$ is heterogeneous across types. A second difference comes from the first term in the second bracket where we have $\tilde{d}_i$ instead of d_i. When changing m_i, only those advertisers who participate are affected. This is only a mass of $1 - F(\omega_0^m)$. By contrast, with homogeneous advertisers all of them are active in equilibrium.

Therefore, with heterogeneous advertisers the equation characterizing n_i trades off the cost and benefits of increasing n_i over the whole mass of participating advertisers, implying that the average costs and benefits are important. However, the basic trade-off for homogeneous advertisers and heterogeneous advertisers is the same. In particular, the first term in the second bracket represents the average marginal profit from increased reach on infra-marginal consumers, whereas the second term represents the average loss from marginal consumers who switch off.

Let us now turn to the optimal advertising levels in duopoly. The goal is to characterize the best-reply tariff $t_i(m_i)$ given outlet j's choice $t_j(m_j)$. As in the monopoly case, it is possible to rewrite this problem as a standard screening problem. To this end, denote by $\omega u(m_1, m_2, n)$ the surplus of type ω from advertising intensities (m_1, m_2). If $m_i(\omega)$ denotes the optimal quantity chosen by type ω, then outlets i's optimization problem is

$$\Pi = \max_{\omega_0^i, m_i(\omega)} \int_{\omega_0^i}^{\overline{\omega}} t_i(m_i(\omega)) dF(\omega) \tag{4}$$

$$\begin{aligned} \text{subject to} \quad & m_i(\omega) = \arg\max\nolimits_{m_i} v_i^d(m_i, \omega, n) - t_i(m_i), \\ & v_i^d(m_i(\omega), \omega, n) - t_i(m_i(\omega)) \geq 0 \quad \text{for all } \omega \geq \omega_0^i, \\ & \textstyle\int_{\omega_0^i}^{\overline{\omega}} m_i(\omega) dF(\omega) \leq n_i, \end{aligned}$$

where $v_i^d(m_i, \omega, n) := \max_y \omega u(m_i, y, n) - t_j(y) - \max_{y'}(\omega u(0, y', n) - t_j(y'))$, with $u(m_i, y, n) := D_i(n_1, n_2)\phi_i^S(m_i) + D_j(n_1, n_2)\phi_j^S(y) + D_{12}(n_1, n_2)\phi_{12}(m_i, y)$.

Note that the sole difference with respect to the monopoly case is that each advertiser's outside option accounts for the possibility of accepting the rival's offer. Hence, $v_i^d(m_i, \omega, n)$ is larger than $v_i^m(m_i, \omega, n_i)$. Again, our assumptions about the viewer demands $D_i(n_1, n_2)$ and $D_{12}(n_1, n_2)$ and about the advertising technology $\phi_i^S(m_i)$ and $\phi_{12}(m_1, m_2)$ ensure that v_i^d is continuous and increasing in ω. It also has strict increasing differences in (m, ω).

In the derivation at the end of this section, we show by following the methodology of Martimort and Stole (2009) that it is possible to characterize the best-reply allocation as the solution to

(5)

$$\int_{\omega_0^i}^{\overline{\omega}} \left(\omega - \frac{1-F(\omega)}{f(\omega)}\right) \left(\tilde{d}_i \frac{\partial \phi_i^S}{\partial m_i} + \frac{\partial d_i}{\partial n_i}\phi_i^S + \tilde{D}_{12}\frac{\partial(\phi_{12} - \phi_i^S - \phi_j^M)}{\partial m_i} + \frac{\partial D_{12}}{\partial n_i}(\phi_{12} - \phi_i^S - \phi_j^M)\right) dF(\omega) + \kappa = 0,$$

with $\tilde{d}_i := (1 - F(\omega_o^i))d_i$, $\tilde{D}_{12} := (1 - F(\omega_o^i))D_{12}$, and κ defined in the derivation at the end of the section. Ignoring κ for the moment, it is evident that this optimal duopoly solution (5) is the analog of condition (5) of the paper accounting for the business sharing and duplication effect with heterogeneous advertisers.

Let us finally turn to κ. When changing the advertising intensity of type ω, outlet i has to take into account that such a different intensity also affects the advertisers' demand from the rival outlet, m_j, given the posted schedule $t_j(\cdot)$. Intuitively, the higher the number of advertising messages on outlet i, the lower the utility from one additional ad on outlet j. This channel brings in new competitive forces that are absent with homogeneous advertisers. These forces are specific to the contracting environment considered and *in addition* to the ones discussed so far. To stress this, we note that if the rival outlet were to offer a single quantity-transfer pair (or, in other words, were to implement an incentive compatible allocation flat across all active types) then $\kappa = 0$.

Derivation of (3) **and** (5)

We first determine the solution to the more complicated duopoly problem. (Solving the monopoly problem proceeds along very similar lines and we will describe it very briefly towards the end.) The problem of a duopolist i is to maximize its profits $\int_{\underline{\omega}}^{\overline{\omega}} t_i(m_i(\omega))dF(\omega)$ with respect to the transfer schedule, given its rival's choice $t_j(m_j)$. From the main text, this problem can be rewritten as in (4). Denote by $m_j^\star(m,\omega)$ the advertising intensity that type ω optimally buys from outlet j when buying intensity m from outlet i. Then, the net contracting surplus for type ω is

$$\begin{aligned} v_i^d(m,\omega,n) &= \max_y \left[\omega u(m,y,n) - t_j(y)\right] - (\max_{y'} \left[\omega u(0,y',n) - t_j(y')\right]) \\ &= \omega u(m, m_j^\star(m,\omega),n) - t_j(m_j^\star(m,\omega)) - \omega u(0,m_j^\star(0,\omega),n) + t_j(m_j^\star(0,\omega)) \end{aligned}$$

Incentive compatibility requires $m_i(\omega) = \arg\max_m v_i^d(m,\omega,n) - t_i(m)$, which implies

$$v_i^d(m_i(\omega),\omega,n) - t_i(m_i(\omega)) = \max_{y,y',m} \left\{\omega u(m,y,n) - t_j(y) - (\omega u(0,y',n) - t_j(y')) - t_i(m)\right\}$$

By the envelope theorem the derivative of the above with respect to ω is

$$u(m, m_j^\star(n_i(\omega),\omega),n) - u(0, m_j^\star(0,\omega),n)$$

Since this pins down the growth rate of the advertiser's payoff, we find that $\max_{\omega_0^i, m_i(\cdot)} \int_{\omega_0^i}^{\overline{\omega}} t_i(m_i(\omega))$ subject to the first two constraints of (2) equals

$$\begin{aligned} &\max_{\omega_0^i, m_i(\cdot)} \int_{\omega_0}^{\overline{\omega}} \Big\{\omega u(m_i(\omega), m_j^\star(m_i(\omega),\omega),n) - \omega u(0,m_j^\star(0,\omega),n) - t_j(m_j^\star(m_i(\omega),\omega)) + t_j(m_j^\star(0,\omega)) \\ &\qquad - \int_{\omega_0^i}^{\omega} \left[\omega u(m, m_j^\star(m_i(z),z),n) - \omega u(0,m_j^\star(0,z),n)\right] dz\Big\} dF(\omega) \\ &= \max_{\omega_0^i, m_i(\cdot)} \int_{\omega_0^i}^{\overline{\omega}} \Big\{ v_i^d(m_i,\omega,n) - \underbrace{\int_{\omega_0^i}^{\omega} \left[\omega u(m, m_j^\star(m_i(z),z),n) - \omega u(0,m_j^\star(0,z),n)\right] dz}_{\text{information rent}} \Big\} dF(\omega), \end{aligned}$$

Integrating the double integral by parts gives

$$\begin{aligned} &\max_{m_i(\cdot), \omega_0^i} \int_{\omega_0^i}^{\overline{\omega}} \omega u(m_i(\omega), m_j^\star(m_i(\omega),\omega),n) - \omega u(0,m_j^\star(0,\omega),n) - t_j(m_j^\star(m_i(\omega),\omega)) + t_j(m_j^\star(0,\omega)) + \\ &\qquad - \frac{1-F(\omega)}{f(\omega)} (u(m, m_j^\star(m_i(\omega),\omega),n) - u(0,m_j^\star(0,\omega),n))\ dF(\omega) \end{aligned}$$

The duopolist's best-reply allocation of advertising intensities $m_i^d(\omega)$ then solves

$$\max_{m_i(\cdot),\omega_0^i} \int_{\omega_0^i}^{\overline{\omega}} \left(\omega - \frac{1-F(\omega)}{f(\omega)}\right)\left(u(m_i(\omega), m_j^\star(m_i(\omega),\omega), n) - u(0, m_j^\star(0,\omega), n)\right)$$
$$- \left(t_j(m_j^\star(m_i(\omega),\omega)) - t_j(m_j^\star(0,\omega))\right) dF(\omega),$$
$$\text{subject to} \int_{\omega_0^i}^{\overline{\omega}} m_i(\omega')dF(\omega') \leq n_i.$$

From now on we will denote the integrand function by $\Lambda^d(m_i(\omega),\omega,n)$. Recall that solving a canonical screening problem usually involves maximizing the integral over all served types, where the integrand is the utility of type ω minus his information rent, expressed as a function of the allocation. The utility here is the incremental value $u(m_i(\omega), m_j^\star(m_i(\omega),\omega), n) - u(0, m_j^\star(0,\omega), n)$, minus the difference in transfers.

The maximization problem in the first stage with respect to n_i can be written as

$$\max_{n_i} \quad \left(\max_{m_i(\cdot),\omega_0} \quad \int_{\omega_0^i}^{\overline{\omega}} \Lambda^d(m_i(\omega),\omega,n)dF(\omega) \quad \text{s.t.} \quad n_i = \int_{\omega_0^i}^{\overline{\omega}} m_i(\omega)dF(\omega)\right). \tag{6}$$

Let us first determine $u(m_i(\omega), m_j^\star(m_i(\omega),\omega), n) - u(0, m_j^\star(0,\omega), n)$. Abbreviating $m_j^\star(m_i(\omega),\omega)$ by $m_j^\star$ and $m_j^\star(0,\omega)$ by $(m_j')^\star$ we can write

$$u(m_i(\omega), m_j^\star, n) - u(0, (m_j')^\star), n)$$

$$\begin{aligned}
&= D_i(n_1,n_2)\phi_i^S(m_i(\omega)) + D_j(n_1,n_2)\phi_j^S(m_j^\star) + D_{12}(n_1,n_2)\phi_{12}(m_i(\omega), m_j^\star) \\
&\quad - D_j(n_1,n_2)\phi_j^S((m_j')^\star) - D_{12}(n_1,n_2)\phi_j^S((m_j')^\star) \\
&= d_i(n_i)\phi_i^S(m_i(\omega)) + D_{12}(n_1,n_2)\left(\phi_{12}(m_i(\omega), m_j^\star) - \phi_i^S(m_i(\omega)) - \phi_j^M((m_j')^\star)\right) \\
&\quad + D_j(n_1,n_2)\left(\phi_j^S(m_j^\star) - \phi_j^M((m_j')^\star)\right),
\end{aligned}$$

where $\phi_{12}(m_i(\omega), m_j^\star) = \phi_i^M(m_i(\omega)) + \phi_j^M(m_j^\star) - \phi_i^M(m_i(\omega))\phi_j^M((m_j')^\star)$.

Adapting results from Martimort and Stole (2009), we know that at the optimal solution $m_i(\omega) = 0$ for all $\omega < \omega_0$ and that $m_i(\omega) = \arg\max_m \Lambda^d(m_i(\omega),\omega,n)$. By our assumptions about the demand and advertising function, the optimal solution involves a schedule $m_i(\omega)$ that is non-decreasing.

From (6), we can write the maximization problem with respect to the optimal allocation of advertising intensities, given n_i, as

$$\max_{m_i(\cdot),\lambda} \quad \int_{\omega_0^i}^{\overline{\omega}} \Lambda^d(m_i(\omega),\omega,n)dF(\omega) + \lambda\left(n_i - \int_{\omega_0^i}^{\overline{\omega}} m_i(\omega)dF(\omega)\right).$$

Pointwise maximization with respect to $m_i(\cdot)$ yields

$$\left(\omega - \frac{1-F(\omega)}{f(\omega)}\right)\left[d_i(n_i)\frac{\partial \phi_i^S}{\partial m_i} + D_{12}(n_1,n_2)\left(\frac{\partial\left(\phi_{12}((m_i,m_j^\star)) - \phi_i^S(m_i)\right)}{\partial m_i}\right)\right.$$

$$\left. + \left[D_j(n_1,n_2) - D_{12}(n_1,n_2)\right]\frac{\partial \phi_j^S}{\partial m_j^\star}\frac{\partial m_j^\star}{\partial m_i}\right] - \frac{\partial t_j}{\partial m_j^\star}\frac{\partial m_j^\star}{\partial m_i} = \lambda. \tag{7}$$

Denoting the left-hand side of (7) by ψ, and integrating both sides from ω_0^i to $\overline{\omega}$, we obtain

$$\frac{\int_{\omega_0^i}^{\overline{\omega}} \psi dF(\omega)}{1-F(\omega_0^i)} = \lambda.$$

The maximization problem of the first stage with respect to n_i is

$$\max_{m_i(\cdot),\lambda} \quad \int_{\omega_0^i}^{\overline{\omega}} \Lambda_i^d(\omega, m_i(\omega)^\star, n_i)dF(\omega) + \lambda\left(n_i - \int_{\omega_0^i}^{\overline{\omega}} m_i(\omega)^\star dF(\omega)\right).$$

Differentiating with respect to n_i and using the Envelope Theorem yields

$$\int_{\omega_0^i}^{\overline{\omega}}\left(\omega - \frac{1-F(\omega)}{f(\omega)}\right)\left[\frac{\partial d_i}{\partial n_i}\phi_i^S + \frac{\partial D_{12}}{\partial n_i}\left(\phi_{12}((m_i,m_j^\star)) - \phi_i^S(m_i) - \phi_j^M((m_j')^\star)\right)\right.$$

$$\left. + D_{12}\left[\frac{\partial \phi_{12}}{\partial m_j^\star}\frac{\partial m_j^\star}{\partial n_i} - \frac{\partial \phi_j^M}{\partial (m_j')^\star}\frac{\partial (m_j')}{\partial n_i}\right]\right] + D_j\left[\frac{\partial \phi_j^S}{\partial m_j^\star}\frac{\partial m_j^\star}{\partial n_i} - \frac{\partial \phi_j^M}{\partial (m_j')^\star}\frac{\partial (m_j')}{\partial n_i}\right]\Bigg] dF(\omega) \tag{8}$$

$$-\frac{\partial t_j}{\partial m_j^\star}\frac{\partial m_j^\star}{\partial n_i} + \frac{\partial t_j}{\partial (m_j')^\star}\frac{\partial (m_j')^\star}{\partial n_i} = -\lambda.$$

Combining (7) and (8) to get rid of λ yields expression (5) of the main text, where κ is defined as

$$\kappa \equiv \int_{\omega_0}^{\overline{\omega}}\left(\omega - \frac{1-F(\omega)}{f(\omega)}\right)\left\{\frac{1}{1-F(\omega)}(D_j - D_{12})\frac{\partial \phi_j^S}{\partial m_j^\star}\frac{\partial m_j^\star}{\partial m_i} + D_{12}\left[\frac{\partial \phi_{12}}{\partial m_j^\star}\frac{\partial m_j^\star}{\partial n_i} - \frac{\partial \phi_j^M}{\partial (m_j')^\star}\frac{\partial (m_j')^\star}{\partial n_i}\right]\right.$$

$$\left. + D_j\left[\frac{\partial \phi_j^S}{\partial m_j^\star}\frac{\partial m_j^\star}{\partial n_i} - \frac{\partial \phi_j^S}{\partial (m_j')^\star}\frac{\partial (m_j')^\star}{\partial n_i}\right] + \frac{\partial D_j}{\partial n_i}\left(\phi_j^S(m_j^\star) - \phi_j^S((m_j')^\star)\right) - \frac{\partial t_j}{\partial m_j^\star}\frac{\partial m_j^\star}{\partial n_i}\right\} dF(\omega)$$

$$-\frac{\partial t_j}{\partial m_j^\star}\frac{\partial m_j^\star}{\partial n_i}+\frac{\partial t_j}{\partial (m_j')^\star}\frac{\partial (m_j')^\star}{\partial n_i}.$$

It is evident that if outlet j offers a single transfer-intensity pair, then $m_j^\star$ equals $(m_j')^\star$ and both are invariant to changes in $m_i(\cdot)$ and n_i. This implies that $\kappa = 0$.

Proceeding in the same way for the monopoly outlet, we obtain that its profit function is given by

$$\max_{n_i} \left(\max_{m_i(\cdot),\omega_0^m} \int_{\omega_0^m}^{\overline{\omega}} \left(\omega - \frac{1-F(\omega)}{f(\omega)}\right) d_i(n_i)\phi_i^S(m_i(\omega))dF(\omega) \quad \text{s.t.} \quad n_i = \int_{\omega_0^m}^{\overline{\omega}} m_i(\omega)dF(\omega)\right).$$

The solution is then characterized by (3).

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