The Social Cost of Near-Rational Investment

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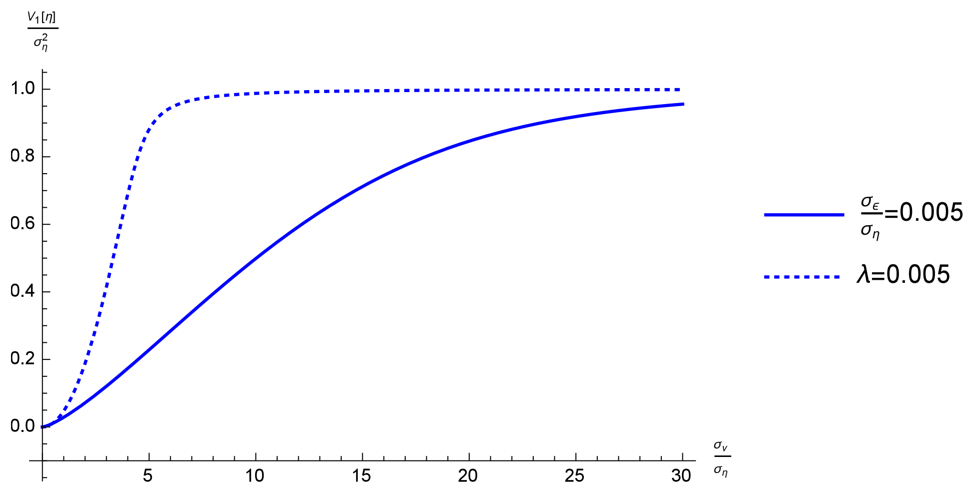
Online Appendix



Appendix Figure 1. : Ratio of the conditional variance of η to its unconditional variance plotted over the level of dispersion of private information. The graph compares the case of an endogenous σ_ϵ with fixed λ to case of a fixed σ_ϵ using $\kappa = 2$ and $\rho = 5$.

Appendix to Section I

1. *Derivation of* (14), (15), *and* (27)

Plugging (13) back into (10) and matching coefficients with (11) yields

$$\pi_0 = \frac{\alpha_0 + \rho V_1[\eta]\kappa(1-\alpha_2)}{(1-\alpha_2)(1+\rho V_1[\eta]\kappa)}, \tag{A1}$$

$$\pi_1 = \frac{\alpha_1}{(1-\alpha_2)(1+\rho V_1[\eta]\kappa)}, \tag{A2}$$

$$\gamma = \frac{1}{(1-\alpha_2)(1+\rho V_1[\eta]\kappa)}. \tag{A3}$$

Using (2) and (11), the vector (η, s_i, Q) has unconditional expectation $(\bar{\eta}, \bar{\eta}, \pi_0 + \pi_1\bar{\eta})$ and the following variance covariance matrix:

$$\Sigma = \begin{pmatrix} \sigma_\eta^2 & \sigma_\eta^2 & \pi_1\sigma_\eta^2 \\ \sigma_\eta^2 & \sigma_\eta^2 + \sigma_\nu^2 & \pi_1\sigma_\eta^2 \\ \pi_1\sigma_\eta^2 & \pi_1\sigma_\eta^2 & \pi_1^2\sigma_\eta^2 + \gamma^2\sigma_\epsilon^2 \end{pmatrix}.$$

Thus, by the property of the conditional variance of the multi-normal distribution,

$$\begin{aligned} V_1[\eta] &= \sigma_\eta^2 - \begin{pmatrix} \sigma_\eta^2 & \pi_1\sigma_\eta^2 \end{pmatrix} \begin{pmatrix} \sigma_\eta^2 + \sigma_\nu^2 & \pi_1\sigma_\eta^2 \\ \pi_1\sigma_\eta^2 & \pi_1^2\sigma_\eta^2 + \gamma^2\sigma_\epsilon^2 \end{pmatrix}^{-1} \begin{pmatrix} \sigma_\eta^2 \\ \pi_1\sigma_\eta^2 \end{pmatrix} \\ &= \frac{1}{\sigma_\eta^{-2} + \left(\pi_1^2\gamma^{-2}\sigma_\epsilon^{-2} + \sigma_\nu^{-2}\right)}. \end{aligned} \tag{A4}$$

Plugging (A2) and (A3) into this expression gives (14).
Similarly, by the properties of the multi-normal distribution,

$$E[\eta|s_i,Q]=\bar{\eta}+\begin{pmatrix}\sigma_\eta^2 & \pi_1\sigma_\eta^2\end{pmatrix}\begin{pmatrix}\sigma_\eta^2+\sigma_\nu^2 & \pi_1\sigma_\eta^2\\ \pi_1\sigma_\eta^2 & \pi_1^2\sigma_\eta^2+\gamma^2\sigma_\epsilon^2\end{pmatrix}^{-1}\begin{pmatrix}s_i-\bar{\eta}\\ Q-(\pi_0+\pi_1\bar{\eta})\end{pmatrix}.$$

Replacing Q by (10) and plugging in (13) and (A4) gives (15).
Matching the coefficients of (15) with (12)

$$\begin{pmatrix}\alpha_1\\ \alpha_2(1+\rho V_1[\eta]\kappa)\end{pmatrix}=\begin{pmatrix}\sigma_\eta^2 & \pi_1\sigma_\eta^2\end{pmatrix}\begin{pmatrix}\sigma_\eta^2+\sigma_\nu^2 & \pi_1\sigma_\eta^2\\ \pi_1\sigma_\eta^2 & \pi_1^2\sigma_\eta^2+\gamma^2\sigma_\epsilon^2\end{pmatrix}^{-1},$$

and solving for α_1, α_2 yields

$$\alpha_1=\frac{\gamma^2\sigma_\eta^2\sigma_\epsilon^2}{\gamma^2\sigma_\nu^2\sigma_\epsilon^2+\sigma_\eta^2\left(\pi_1^2\sigma_\nu^2+\gamma^2\sigma_\epsilon^2\right)}, \tag{A5}$$

$$\alpha_2=\frac{\pi_1\sigma_\eta^2\sigma_\nu^2}{\left(\gamma^2\sigma_\nu^2\sigma_\epsilon^2+\sigma_\eta^2\left(\pi_1^2\sigma_\nu^2+\gamma^2\sigma_\epsilon^2\right)\right)(1+\rho V_1[\eta]\kappa)}. \tag{A6}$$

Combining (A6) with (A2), (A3), and (14) yields (27).

2. *Proof of Lemma 1*

Use the law of total variance and (11) and (12) to get

$$\begin{aligned}\sigma_\eta^2 &= V_1[\eta]+V_0[E_{1i}[\eta]]\\ &= V_1[\eta]+V_0[\alpha_1\nu_i+(\alpha_1+\alpha_2\pi_1(1+\rho V_1[\eta]\kappa))\,\eta+\alpha_2\gamma\epsilon(1+\rho V_1[\eta]\kappa)]\\ &= V_1[\eta]+\alpha_1^2\sigma_\nu^2+(\alpha_1+\alpha_2\pi_1(1+\rho V_1[\eta]\kappa))^2\sigma_\eta^2+\alpha_2^2\gamma^2\sigma_\epsilon^2(1+\rho V_1[\eta]\kappa)^2. \qquad \text{(A7)}\end{aligned}$$

Now note from (17) and (18) that

$$\alpha_1^2\sigma_\nu^2+\alpha_2^2\gamma^2\sigma_\epsilon^2(1+\rho V_1[\eta]\kappa)^2=\frac{V_1[\eta]^2}{\sigma_\nu^2}+\frac{V_1[\eta]^4}{\sigma_\nu^4\sigma_\epsilon^2}=\frac{V_1[\eta]^2}{\sigma_\nu^2}+\frac{V_1[\eta]^2\alpha_1^2}{\sigma_\epsilon^2},$$

and from (14) that $\frac{\alpha_1^2}{\sigma_\epsilon^2}=\frac{1}{V_1[\eta]}-\left(\sigma_\eta^{-2}+\sigma_\nu^{-2}\right)$ such that

$$\alpha_1^2\sigma_\nu^2+\alpha_2^2\gamma^2\sigma_\epsilon^2(1+\rho V_1[\eta]\kappa)^2=V_1[\eta]-V_1[\eta]^2\sigma_\eta^{-2}.$$

In addition, using (A2) we can show that

$$\begin{aligned}(\alpha_1+\alpha_2\pi_1(1+\rho V_1[\eta]\kappa))^2 &= ((1-\alpha_2)\pi_1(1+\rho V_1[\eta]\kappa)+\alpha_2\pi_1(1+\rho V_1[\eta]\kappa))^2\\ &= \pi_1^2(1+\rho V_1[\eta]\kappa)^2.\end{aligned}$$

Substituting these two expressions back into (A7) yields

$$\sigma_\eta^2=2V_1[\eta]-V_1[\eta]^2\sigma_\eta^{-2}+\pi_1^2(1+\rho V_1[\eta]\kappa)^2\sigma_\eta^2.$$

Solving this expression for $V_1[\eta]$ gives

$$V_1[\eta] = \frac{\sigma_\eta^2(1-\pi_1)}{1+\kappa\rho\pi_1\sigma_\eta^2}. \tag{A8}$$

Now take the market-clearing condition (8), plug in (7) and (9) on the left-hand side and (1) on the right to get

$$\frac{\int_0^1 E_{1i}[\eta]di - Q + \epsilon}{\rho V_1[\eta]} = \kappa(Q-1).$$

Take the unconditional expectation on both sides:

$$E_0[\eta - Q] = \rho\kappa V_1[\eta](E_0[Q]-1).$$

Now note from (11) that $E_0[Q] = \pi_0 + \pi_1\bar{\eta}$ and therefore:

$$-\pi_0 + (1-\pi_1)\bar{\eta} = \rho\kappa V_1[\eta](E_0[Q]-1).$$

Solving for π_0 and plugging in (A8) yields

$$\pi_0 = \frac{(1-\pi_1)(\bar{\eta}+\kappa\rho\sigma_\eta^2)}{1+\kappa\rho\sigma_\eta^2}. \tag{A9}$$

Similarly, from (A2), (A3), and (17), it follows that

$$\gamma = \pi_1\frac{\sigma_\nu^2}{V_1[\eta]}. \tag{A10}$$

Again plugging in (A8) yields

$$\gamma = \frac{\pi_1\sigma_\nu^2\left(1+\kappa\rho\pi_1\sigma_\eta^2\right)}{\sigma_\eta^2(1-\pi_1)}. \tag{A11}$$

To solve for π_1, substitute (17), (18), and (A10) into (A2) to get

$$\pi_1 = \sigma_\nu^{-2}\left(V_1[\eta]^{-1} - \frac{V_1[\eta]^2}{\pi_1\sigma_\nu^4\sigma_\epsilon^2} + \rho\kappa\right)^{-1}. \tag{A12}$$

Combining this expression with (A8) and solving yields (20). Plugging (20) into (A9) and (A10) separately gives (19) and (21). And substituting α_1 using (17) in (14) yields (22).

3. *Details on Amplification from Cost Function*

Lemma 4 *There is a one-to-one mapping between the cost parameter λ and the size of the error σ_ϵ where the derivative $\frac{d\sigma_\epsilon}{d\lambda}$ is strictly positive. In the limit in which $\lambda \to 0$, all households behave fully rationally with $\sigma_\epsilon \to 0$.*

To show the magnitude of near-rational errors, take the first-order condition with respect to the optimal choice of μ_i

$$\frac{dE_0[w_{2i}]}{d\mu_i} - \frac{\rho}{2} V_1[w_{2i}] + \lambda |\bar{U}| = 0 \tag{A13}$$

where we plug in for wealth from equation (4), use $V_1[w_{2i}] = z_i^2 V_1[\eta]$, the optimal portfolio choice (9), and the definition of near-rational expectations (5). We arrive at

$$\frac{\mu_i \sigma_\epsilon^2}{\rho V_1[\eta]} + \frac{E_0[E_1[\eta]\epsilon]}{\rho V_1[\eta]} = \lambda |\bar{U}|. \tag{A14}$$

Now we plug in the equilibrium choice $\mu_i = 1$ and recognize that

$$E_0[E_1[\eta]\epsilon] = \frac{\alpha_2}{1-\alpha_2} \sigma_\epsilon^2 \tag{A15}$$

from (12) and (13). After substituting α_2 by plugging (17) into (18), we get that the size of the errors relates to the costs via

$$\sigma_\epsilon^2 = \frac{V_1[\eta](\lambda |\bar{U}| \sigma_\nu^2 - V_1[\eta])}{\sigma_\nu^2}. \tag{A16}$$

In a last step, we sign the derivative $\frac{d\sigma_\epsilon^2}{d\lambda}$. Therefore, we re-arrange the expression we just derived to

$$\rho \lambda |\bar{U}| = \frac{V_1[\eta]^2 + \sigma_\epsilon^2 \sigma_\nu^2}{V_1[\eta] \sigma_\nu^2} \tag{A17}$$

where we call $c = \lambda |\bar{U}|$. Taking the total derivative of (A16) with respect to c and recognizing that $V_1[\eta]$ is a function of σ_ϵ^2 delivers the following expression

$$\frac{d\sigma_\epsilon^2}{dc} = \frac{\rho V_1[\eta] \sigma_\nu^2}{2V_1[\eta] \frac{dV_1[\eta]}{d\sigma_\epsilon^2} + (1 - c\frac{dV_1[\eta]}{d\sigma_\epsilon^2})\sigma_\nu^2}. \tag{A18}$$

Using (A17) and the total derivative of (22) with respect to σ_ϵ^2 delivers the lengthy expression

$$\frac{d\sigma_\epsilon^2}{dc} = \frac{T(V_1[\eta])}{c\sigma_\nu^2 \left(\sigma_\eta^4 \sigma_\nu^2 V_1(\eta)^3 + \sigma_\eta^4 V_1(\eta)^4 + 2\sigma_\eta^2 \sigma_\nu^4 V_1(\eta)^2 \sigma_\epsilon^2 + \sigma_\eta^4 \sigma_\nu^4 V_1(\eta) \sigma_\epsilon^2 + 2\sigma_\eta^4 \sigma_\nu^2 V_1(\eta)^2 \sigma_\epsilon^2 + 2\sigma_\eta^2 \sigma_\nu^6 \sigma_\epsilon^4 + \sigma_\eta^4 \sigma_\nu^4 \sigma_\epsilon^4 + \sigma_\nu^8 \sigma_\epsilon^4\right)} \tag{A19}$$

where

$$\begin{aligned} T(V_1[\eta]) = \quad & \sigma_\eta^4 V_1(\eta)^6 + \sigma_\nu^8 V_1(\eta)^2 \sigma_\epsilon^4 + 4\sigma_\eta^2 \sigma_\nu^6 V_1(\eta)^2 \sigma_\epsilon^4 + 2\sigma_\eta^4 \sigma_\nu^6 V_1(\eta) \sigma_\epsilon^4 + 2\sigma_\eta^2 \sigma_\nu^4 V_1(\eta)^4 \sigma_\epsilon^2 + 2\sigma_\eta^4 \sigma_\nu^4 V_1(\eta)^3 \sigma_\epsilon^2 \\ & + 3\sigma_\eta^4 \sigma_\nu^4 V_1(\eta)^2 \sigma_\epsilon^4 + 3\sigma_\eta^4 \sigma_\nu^2 V_1(\eta)^4 \sigma_\epsilon^2 + 2\sigma_\eta^2 \sigma_\nu^8 \sigma_\epsilon^6 + \sigma_\eta^4 \sigma_\nu^6 \sigma_\epsilon^6 + \sigma_\nu^{10} \sigma_\epsilon^6 \end{aligned}$$

which is clearly positive.

4. *Deriving (23) and (24)*

Jointly solving (22) and (A16) for $V_1[\eta]$ and picking the only real solution for the conditional variance yields the two closed-form solutions

$$\sigma_\epsilon = \frac{1}{\sqrt{2}\sigma_\nu}\sqrt{\sqrt{\sigma_\eta^4\left(\lambda\rho\bar{U}+1\right)^2\left(2\lambda\rho\bar{U}\sigma_\eta^2\sigma_\nu^2\left(\lambda\rho\bar{U}-1\right)+\sigma_\eta^4\left(\lambda\rho\bar{U}+1\right)^2+\lambda^2\rho^2\bar{U}^2\sigma_\nu^4\right)}+\lambda\rho\bar{U}\sigma_\eta^2\sigma_\nu^2\left(1-\lambda\rho\bar{U}\right)+\sigma_\eta^4\left(-\left(\lambda\rho\bar{U}+1\right)^2\right)} \tag{A20}$$

and

$$V_1[\eta] = \frac{-\sqrt{\sigma_\eta^4\left(\lambda\rho\bar{U}+1\right)^2\left(2\lambda\rho\bar{U}\sigma_\eta^2\sigma_\nu^2\left(\lambda\rho\bar{U}-1\right)+\sigma_\eta^4\left(\lambda\rho\bar{U}+1\right)^2+\lambda^2\rho^2\bar{U}^2\sigma_\nu^4\right)}+\lambda\rho\bar{U}\sigma_\eta^2\sigma_\nu^2\left(\lambda\rho\bar{U}+1\right)+\sigma_\eta^4\left(\lambda\rho\bar{U}+1\right)^2}{2\sigma_\eta^2\left(\lambda\rho\bar{U}+1\right)}. \tag{A21}$$

The solution for the conditional variance can be re-written as

$$V_1[\eta] = -\frac{1}{2}\sigma_\eta^2\sqrt{\frac{\lambda\rho\bar{U}\left(2\sigma_\eta^2\sigma_\nu^2\left(\lambda\rho\bar{U}-1\right)+\lambda\rho\bar{U}\sigma_\eta^4+\lambda\rho\bar{U}\sigma_\nu^4+2\sigma_\eta^4\right)}{\sigma_\eta^4}+1}+\frac{1}{2}\lambda\rho\bar{U}\left(\sigma_\eta^2+\sigma_\nu^2\right)+\frac{\sigma_\eta^2}{2}. \tag{A22}$$

From this form, it can be directly seen that the limit of $\lambda \to 0$ results in a conditional variance of zero. Plugging this into (20) yields (24).

5. *Proof of Proposition 1*

Solve (A8) for π_1 and differentiate with respect to $V_1[\eta]$ to get

$$\frac{\partial \pi_1}{\partial V_1[\eta]} = -\frac{1+\kappa\rho\sigma_\eta^2}{\sigma_\eta^2(1+\kappa\rho V_1[\eta])^2} < 0. \tag{A23}$$

In addition, differentiate both sides of (22) with respect to σ_ϵ and rearrange to get

$$\frac{\partial V_1[\eta]}{\partial \sigma_\epsilon} = \frac{2V_1\left[\eta\right]^4}{2\sigma_\epsilon^2 V_1\left[\eta\right]^3+\sigma_\epsilon^3\sigma_\nu^4} > 0.$$

Then, using Lemma 1, the fact that $\frac{\partial \pi_1}{\partial \lambda} = \frac{\partial \pi_1}{\partial V_1[\eta]}\frac{\partial V_1[\eta]}{\partial \sigma_\epsilon}\frac{\partial \sigma_\epsilon}{\partial \lambda}$ yields (25), and applying (23) yields (26).

6. *Proof of Proposition 2*

For the first part of the proposition, differentiate both sides of (22) with respect to σ_ν and rearrange to get

$$\frac{\partial V_1[\eta]}{\partial \sigma_\nu} = \frac{2\sigma_\nu^2\sigma_\epsilon^2 V_1\left[\eta\right]^2+4V_1\left[\eta\right]^4}{\sigma_\nu^5\sigma_\epsilon^2+2\sigma_\nu V_1\left[\eta\right]^3} > 0. \tag{A24}$$

Combing this with (A23) proves the first equality and the inequality for strictly positive σ_ϵ. The proof of the case $\lambda \to 0$ follows directly from (23).
We start with the solution for the conditional variance in (A21) and re-write it in the form

$$V_1[\eta] = \frac{1}{2}\left(-\sqrt{2\lambda\rho\bar{U}\sigma_\eta^2\sigma_\nu^2\left(\lambda\rho\bar{U}-1\right)+\sigma_\eta^4\left(\lambda\rho\bar{U}+1\right)^2+\lambda^2\rho^2\bar{U}^2\sigma_\nu^4}+\sigma_\eta^2\left(\lambda\rho\bar{U}+1\right)+\lambda\rho\bar{U}\sigma_\nu^2\right). \tag{A25}$$

First, note that, when taking the limit, the term under the square root independent of σ_ν can be left out and further note that the following limit is true

$$\lim_{x\to\infty} a_0 + a_1 x^2 - x\sqrt{a_3 + a_1^2 x^2} = a_0 - \frac{a_3}{2a_1}. \tag{A26}$$

Apply this relationship to $\lim_{\sigma_\nu\to\infty} V_1[\eta]$ where we plug (A25) in for the conditional variance and we get $\lim_{\sigma_\nu\to\infty} V_1[\eta] = \sigma_\eta^2$ if $\lambda > 0$. The result from the proposition immediately follows. For the case where $\lambda = 0$, note that the conditional variance is zero independent of information dispersion.

7. *Proof of Proposition 3*

Differentiating (A16) with respect to σ_ν^2 and plugging in the derivative of the conditional variance which we obtain by totally differentiating (22), we get

$$\frac{d\sigma_\epsilon^2}{d\sigma_\nu^2} = \frac{\sigma_\eta^4 V_1(\eta)^7 + \sigma_\eta^4\sigma_\nu^6 V_1(\eta)^2\sigma_\epsilon^4 + \sigma_\nu^4 V_1(\eta)^3\sigma_\epsilon^4\left(\sigma_\eta^2+\sigma_\nu^2\right)^2 + 2\sigma_\eta^2\sigma_\nu^2 V_1(\eta)^5\sigma_\epsilon^2\left(\sigma_\eta^2+\sigma_\nu^2\right) + \sigma_\eta^4\sigma_\nu^8\sigma_\epsilon^6}{\sigma_\nu^4 V_1(\eta)\left(\sigma_\eta^4\sigma_\nu^2 V_1(\eta)^3 + \sigma_\eta^4 V_1(\eta)^4 + \sigma_\eta^4\sigma_\nu^4 V_1(\eta)\sigma_\epsilon^2 + 2\sigma_\eta^2\sigma_\nu^2 V_1(\eta)^2\sigma_\epsilon^2\left(\sigma_\eta^2+\sigma_\nu^2\right) + \sigma_\nu^4\sigma_\epsilon^4\left(\sigma_\eta^2+\sigma_\nu^2\right)^2\right)}. \tag{A27}$$

8. *Proof of Proposition 4*

Plug (11) into (1) to get

$$K = \kappa\left(\pi_0 + \pi_1\eta + \gamma\epsilon - 1\right). \tag{A28}$$

Taking time-zero expectations of (A28) and using (19) and (20) to substitute for π_0 yields

$$E_0\left[K\right] = \kappa\left(\bar\eta - 1\right)\left(\frac{1+\pi_1\kappa\rho\sigma_\eta^2}{1+\kappa\rho\sigma_\eta^2}\right). \tag{A29}$$

In addition, from (A28) and (28), we have

$$Cov_0\left(K,\eta\right) = \kappa\pi_1\sigma_\eta^2. \tag{A30}$$

It follows directly that $\frac{\partial E_0[K]}{\partial\pi_1} > 0$ and $\frac{\partial Cov_0[K,\eta]}{\partial\pi_1} > 0$. The remainder of the proof follows from Proposition 1.

9. *Proof of Lemma 2*

Combine (2), (5), (8), (7), (9), (12), and (17) to show that

$$z_i - K = \frac{\nu_i}{\rho\sigma_\nu^2}. \tag{A31}$$

From (1), equilibrium profits are

$$\Pi = \kappa\frac{(Q-1)^2}{2}. \tag{A32}$$

Taking (3), plugging in (4), and substituting Π using (A32) and (1) yields

$$U_i = z_i(\eta - Q) + \frac{K^2}{2\kappa} - \frac{\rho}{2}z_i^2 V_1[\eta].$$

Replacing $z_i = \frac{\nu_i}{\rho\sigma_\nu^2} + K$, applying the definition $w_a = K(\eta - Q) + \frac{K^2}{2\kappa}$, and taking time-zero expectations on both sides yields

$$E_0[U_i] = E_0\left[\frac{\nu_i}{\rho\sigma_\nu^2}(\eta - Q) + K(\eta - Q) + \frac{K^2}{2\kappa}\right] - \frac{\rho}{2}E_0\left[\left(\frac{\nu_i}{\rho\sigma_\nu^2} + K\right)^2\right]V_1[\eta].$$

The second equality in (29) follows from the fact that $E_0[\nu_i] = 0$ and ν_i is uncorrelated with η, K, and Q. As a result, the first term in the left square brackets drops out and $E_0\left[2\frac{\nu_i}{\rho\sigma_\nu}K\right] = 0$ in the right square brackets.

The first equality follows from noting that $E_0[U_i]$ does not depend on ν_i. It is thus independent of i, and we have that

$$SWF \equiv \int_0^1 E_0[U_i]\,di = E_0[U_i].$$

Finally, use (1) to substitute Q out of (4):

$$w_a = K(\eta - 1) - \frac{K^2}{2\kappa} = (K\eta - E_0[K]\bar{\eta}) + (E_0[K]\bar{\eta} - K) - \frac{K^2}{2\kappa}. \tag{A33}$$

Taking time-zero expectations on both sides yields (30).

Next we derive the different channels of utility. The utility specification in (3) embeds a preference for early resolution of uncertainty. To see this, note that disutility from variance stems only from conditional variance and thus the timing of the arrival of information matters for welfare. More technically, we can write $E_0[U_i] = -\frac{1}{\rho}\mathbb{E}_0[\log(\mathbb{E}_1[exp(-\rho w)])]$. The concave transformation through the logarithm favors volatile expectations and thus gives rise to a preference for early resolution of uncertainty.

To show the economically relevant channels of information aggregation on welfare, we make two rearrangements to the specification of utility. First, we apply the law of total variance to the variance term $V_1[w_{2i}] = V_0[w_{2i}] - V_0[E_{1i}[w_{2i}]]$. Second, we recognize that

$$w_{2i} - w_{2a} = \frac{\nu_i}{\rho\sigma_\nu^2}(\eta - Q).$$

and, since ν_i is independent of all other shocks, the covariance $Cov_0[w_{2i} - w_{2a}, w_{2a}]$ is zero. As a result,

$$\begin{aligned} V_0[w_{2i}] &= V_0[w_{2a} + w_{2i} - w_{2a}] \\ &= V_0[w_{2a}] + V_0[w_{2i} - w_{2a}]. \end{aligned}$$

The different channels by which near-rationality influences social welfare can now better been seen when we rearrange the second term as follows

$$\text{SWF} = E_0[U_i] = \underbrace{E_0[w_{2a}]}_{\text{Level effect}} - \underbrace{\frac{1}{2}\rho V_0[w_{2a}]}_{\text{Variance effect}} - \underbrace{\frac{1}{2}\rho V_0[w_{2i} - w_{2a}]}_{\text{Dispersion effect}} + \underbrace{\frac{1}{2}\rho V_0[E_1[w_{2i}]]}_{\text{Early resolution of uncertainty}}$$

10. *Proof of Proposition 5*

It follows directly from the envelope theorem that

$$\lim_{\lambda \to 0} \left[-\frac{\partial E_0[U_i]}{\partial \mu_i} \right] = 0.$$

For the second and third equality, note that the social welfare function (29) depends on three terms: the level of expected wealth, the idiosyncratic component in the expected volatility of portfolio returns, and the aggregate component in the expected volatility of portfolio returns. We first solve each of the three components as a function of the parameters of the model and π_1. Equating (14) and (A8) gives

$$\gamma = \sqrt{\frac{(1-\pi_1)\,\pi_1^2 \sigma_\nu^2 \sigma_\eta 2}{\sigma_\epsilon^2 \left(\pi_1 \sigma_\nu^2 + \sigma_\eta^2 \left(\pi_1 \sigma_\nu^2 \kappa\rho + (\pi_1 - 1)\right)\right)}}. \tag{A34}$$

Squaring both sides of (A28) and taking expectations gives $E_0[K^2]$. Plugging $E_0[K^2]$, (A29), and (A30) into (30) and substituting in (A9) and (A34) yields

$$\begin{aligned} E_0\left[w_a\right] \quad = \quad & -\frac{1}{2}\kappa\{2\bar{\eta}\left(1 - \frac{(\pi_1 - 1)^{\,2}\left(\bar{\eta} + \kappa\rho\sigma_\eta^2\right)}{\kappa\rho\sigma_\eta^2 + 1}\right) + \frac{(\pi_1 - 1)^{\,2}\left(\bar{\eta} + \kappa\rho\sigma_\eta^2\right)^{\,2}}{\left(\kappa\rho\sigma_\eta^2 + 1\right)^{\,2}} \\ & + (\pi_1 - 2)\,\pi_1^2 \bar{\eta}^2 + \frac{(1-\pi_1)\,\pi_1^2 \sigma_\nu^2 \sigma_\eta^2}{\pi_1 \sigma_\nu^2 + \sigma_\eta^2 \left(\pi_1 \left(\sigma_\nu^2 \kappa\rho + 1\right) - 1\right)} + (\pi_1 - 2)\,\pi_1 \sigma_\eta^2 - 1\}. \end{aligned}$$

We can then show that

$$\lim_{\sigma_\epsilon \to 0} \left[\frac{\partial E_0\left[w_a\right]}{\partial \sigma_\epsilon}\right] = \lim_{\sigma_\epsilon \to 0} \left[\frac{\partial E_0\left[w_a\right]}{\partial \pi_1}\frac{\partial \pi_1}{\partial \sigma_\epsilon}\right] = \frac{\kappa\sigma_\eta^2}{2\left(1 + \kappa\rho\sigma_\eta^2\right)} \lim_{\sigma_\epsilon \to 0} \left[\frac{\partial \pi_1}{\partial \sigma_\epsilon}\right] < 0,$$

where the last equality uses (25). Using (25) and (A23) from Proposition 1,

$$\lim_{\sigma_\epsilon \to 0} \left[-\frac{1}{2\rho\sigma_\nu^2}\frac{\partial V_1\left[\eta\right]}{\partial \sigma_\epsilon}\right] = -\frac{1}{2\rho\sigma_\nu^2} \lim_{\sigma_\epsilon \to 0} \left(\frac{\partial \pi_1}{\partial V_1[\eta]}\right)^{-1} \lim_{\sigma_\epsilon \to 0} \left[\frac{\partial \pi_1}{\partial \sigma_\epsilon}\right] < 0.$$

Similarly, taking time-zero expectations of the third term and plugging in (A9), (A34), and (A8) yields

$$E_0\left[K^2\right] V_1[\eta] = (1-\pi_1)\,\kappa^2\sigma_\eta^2 \frac{\sigma_\eta^2 \left(\frac{\kappa\rho\left(\pi_1^2(\bar{\eta}-1)^2 - 1\right)}{\kappa\rho\sigma_\eta^2 + 1} + \frac{\kappa\rho\left(-(\pi_1 - 2)\pi_1(\bar{\eta}-1)^2 - 1\right)}{\left(\kappa\rho\sigma_\eta^2 + 1\right)^2} - \frac{(\pi_1 - 1)\pi_1^2\sigma_\nu^2}{\sigma_\eta^2\left(\pi_1\left(\sigma_\nu^2\kappa\rho + 1\right) - 1\right) + \pi_1\sigma_\nu^2} + \pi_1^2\right) + \frac{(\bar{\eta}-2)\bar{\eta}}{\left(\kappa\rho\sigma_\eta^2 + 1\right)^2} + 1}{\pi_1\kappa\rho\sigma_\eta^2 + 1}.$$

Again taking the derivative with respect to σ_ϵ, taking the limit as σ_ϵ goes to zero and using (25) yields

$$\lim_{\sigma_\epsilon \to 0} \left[-\frac{\rho}{2}\frac{\partial E_0\left[K^2\right] V_1[\eta]}{\partial \sigma_\epsilon}\right] = \frac{\kappa^2\rho\sigma_\eta^2\left((1-\bar{\eta})^2 + \sigma_\eta^2\right)}{2\left(1 + \kappa\rho\sigma_\eta^2\right)} \lim_{\sigma_\epsilon \to 0} \left[\frac{\partial \pi_1}{\partial \sigma_\epsilon}\right] < 0$$

and concludes the proof.

11. Alternative information environments

Dispersed Information with an Exogenous Public Signal

This section solves the model with public signals introduced in section I.D. We may guess that the solution for Q is some linear function of η, ϖ, and ϵ:

$$Q = \pi_0 + \pi_1 \eta + \pi_2 \varpi + \gamma \epsilon,$$

where the rational expectation of η given Q and the private and public signals is

$$E_{it}\left(\eta_{t+1}\right) = \alpha_0 + \alpha_1 s_i + \alpha_2 Q + \alpha_3 g.$$

A matching coefficients approach parallel to that in section I.A gives

$$\pi_1 = \frac{\alpha_1 + \alpha_3}{1-\alpha_2},\ \pi_2 = \frac{\alpha_3}{1-\alpha_2},\ \gamma = \frac{1}{1-\alpha_2}. \tag{A35}$$

The amplification of near-rational errors is thus influenced only in so far as the presence of public information may induce households to put less weight on the market price of capital when forming their expectations.

The vector (η, s_i, Q, g) has the following variance covariance matrix:

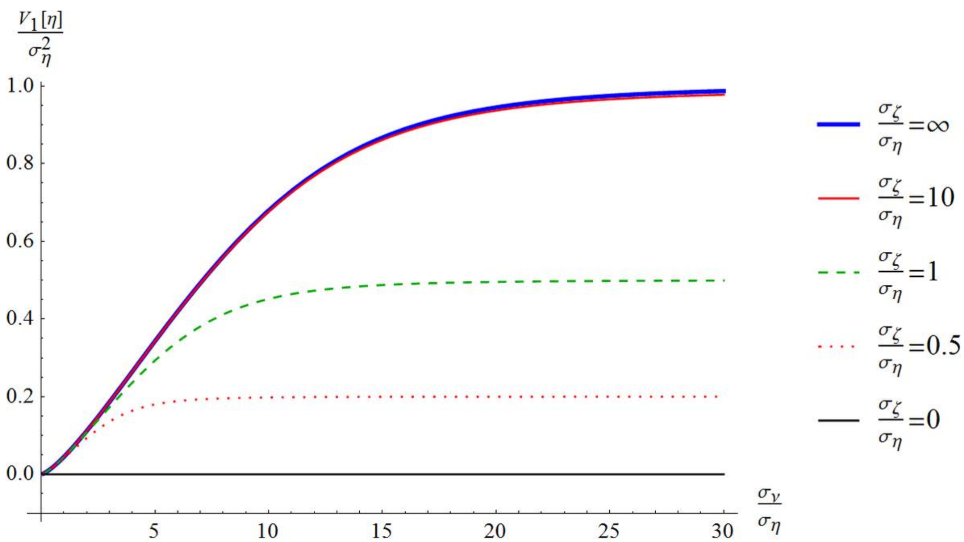
$$\begin{pmatrix} \sigma_\eta^2 & \sigma_\eta^2 & \pi_1\sigma_\eta^2 & \sigma_\eta^2 \\ \sigma_\eta^2 & \sigma_\eta^2+\sigma_\nu^2 & \pi_1\sigma_\eta^2 & \sigma_\eta^2 \\ \pi_1\sigma_\eta^2 & \pi_1\sigma_\eta^2 & \pi_2^2\sigma_\varpi^2+\pi_1^2\sigma_\eta^2+\gamma^2\sigma_\epsilon^2 & \pi_2\sigma_\varpi^2+\pi_1\sigma_\eta^2 \\ \sigma_\eta^2 & \sigma_\eta^2 & \pi_2\sigma_\varpi^2+\pi_1\sigma_\eta^2 & \sigma_\varpi^2+\sigma_\eta^2 \end{pmatrix}.$$

Solving the signal-extraction problem returns

$$\begin{aligned} \alpha_1 &= \frac{\gamma^2\sigma_\varpi^2\sigma_\eta^2\sigma_\epsilon^2}{\sigma_\varpi^2\left(\sigma_\eta^2(\gamma^2\sigma_\epsilon^2+(\pi_1-\pi_2)^2\sigma_\nu^2)+\gamma^2\sigma_\nu^2\sigma_\epsilon^2\right)+\gamma^2\sigma_\eta^2\sigma_\nu^2\sigma_\epsilon^2} \\ \alpha_2 &= \frac{(\pi_1-\pi_2)\sigma_\varpi^2\sigma_\eta^2\sigma_\nu^2}{\sigma_\varpi^2\left(\sigma_\eta^2(\gamma^2\sigma_\epsilon^2+(\pi_1-\pi_2)^2\sigma_\nu^2)+\gamma^2\sigma_\nu^2\sigma_\epsilon^2\right)+\gamma^2\sigma_\eta^2\sigma_\nu^2\sigma_\epsilon^2} \\ \alpha_3 &= \frac{\sigma_\eta^2\sigma_\nu^2\left(\gamma^2\sigma_\epsilon^2+\pi_2(\pi_2-\pi_1)\sigma_\varpi^2\right)}{\sigma_\varpi^2\left(\sigma_\eta^2(\gamma^2\sigma_\epsilon^2+(\pi_1-\pi_2)^2\sigma_\nu^2)+\gamma^2\sigma_\nu^2\sigma_\epsilon^2\right)+\gamma^2\sigma_\eta^2\sigma_\nu^2\sigma_\epsilon^2} \end{aligned}. \tag{A36}$$

Based on these results, Figure A1 plots the conditional variance of η for the rational and near-rational expectations equilibrium and for varying levels of precision of the public signal.

In the absence of near-rational behavior, the provision of public information makes no difference, because households are already fully informed from the outset. When households are near-rational, the presence of the public signal is relevant only insofar as a collapse of information aggregation affects only the subset of information that is dispersed across households and not the information that is publicly available. If the public information provided is relatively precise, $\frac{V_1[\eta]}{\sigma_\eta^2}$ now converges to values less than 1 as σ_ν goes to infinity.



Appendix Figure A1. : Ratio of the conditional variance of the productivity shock to its unconditional variance plotted over the level of dispersion of information, σ_ν/σ_η, and for varying precisions of the public signal. In each case, $\sigma_\epsilon/\sigma_\eta$ is set to 0.01.

Dispersed Information with Aggregate Noise in Private Signal

This subsection solves the model with aggregate noise in the private signal introduced in section I.D. We may guess that

$$Q = \pi_0 + \pi_1 (\eta + \zeta) + \gamma\epsilon,$$

where both the expectation (12) and the coefficients π_0, π_1, and γ are the ones given in the main text. However, the variance-covariance matrix of the vector (η, s_i, Q) changes to

$$\begin{pmatrix} \sigma_\eta^2 & \sigma_\eta^2 & \pi_1\sigma_\eta^2 \\ \sigma_\eta^2 & \sigma_\zeta^2+\sigma_\eta^2+\sigma_\nu^2 & \pi_1\left(\sigma_\zeta^2+\sigma_\eta^2\right) \\ \pi_1\sigma_\eta^2 & \pi_1\left(\sigma_\zeta^2+\sigma_\eta^2\right) & \pi_1^2\left(\sigma_\zeta^2+\sigma_\eta^2\right)+\gamma^2\sigma_\epsilon^2 \end{pmatrix}.$$

Applying the projection theorem yields

$$\begin{aligned} \alpha_1 &= \frac{\gamma^2\sigma_\eta^2\sigma_\epsilon^2}{\sigma_\zeta^2\left(\gamma^2\sigma_\epsilon^2+\pi_1^2\sigma_\nu^2\right)+\sigma_\eta^2\left(\gamma^2\sigma_\epsilon^2+\pi_1^2\sigma_\nu^2\right)+\gamma^2\sigma_\nu^2\sigma_\epsilon^2} \\ \alpha_2 &= \frac{\pi_1\sigma_\eta^2\sigma_\nu^2}{\sigma_\zeta^2\left(\gamma^2\sigma_\epsilon^2+\pi_1^2\sigma_\nu^2\right)+\sigma_\eta^2\left(\gamma^2\sigma_\epsilon^2+\pi_1^2\sigma_\nu^2\right)+\gamma^2\sigma_\nu^2\sigma_\epsilon^2} \end{aligned} \tag{A37}$$

and

$$V_1[\eta] = \frac{\sigma_\eta^2\left(\sigma_\zeta^2\left(\gamma^2\sigma_\epsilon^2+\pi_1^2\sigma_\nu^2\right)+\gamma^2\sigma_\nu^2\sigma_\epsilon^2\right)}{\sigma_\zeta^2\left(\gamma^2\sigma_\epsilon^2+\pi_1^2\sigma_\nu^2\right)+\sigma_\eta^2\left(\gamma^2\sigma_\epsilon^2+\pi_1^2\sigma_\nu^2\right)+\gamma^2\sigma_\nu^2\sigma_\epsilon^2}.$$

The key insight is that aggregate noise does not get amplified. Figure 3 illustrates this result. The thick blue line plots the now familiar effect of a small common error in household expectations with $\frac{\sigma_\epsilon}{\sigma_\eta} = 0.01$. The red horizontal line plots the effect of an identical amount of small common noise in the private signal (i.e. $\frac{\sigma_\zeta}{\sigma_\eta} = 0.01$). The red line has an intercept of 0.01^2 and is perfectly horizontal. The common noise in the private signal is not amplified, and does the fact that an individual household observes a signal with common noise does not have an external effect on the market's capacity to aggregate information. The effect of common noise in the private signal is thus invariant to how dispersed information is in the economy.

The broken lines in Figure 3 show the same comparative static, but in the presence of large common noise in the private signal ($\frac{\sigma_\zeta}{\sigma_\eta} = 1$). Both lines retain their shape but now have a higher intercept, reflecting the fact that less information is now available to aggregate, even if the stock price is fully revealing. However, for the remaining dispersed information, the information externality of near-rational behavior operates in the same way as in the model in section I. The externality is thus relevant whenever financial markets play an important role in aggregating dispersed information, regardless of the exact information structure.

Formal proof of Proposition 6

Since the expressions are shorter for the case of aggregate noise in the private signal, we start with the proof for this case first. Combining (A2), (A3), and (A37) yields

$$\pi_1 = \sigma_\eta^2 \left(\sigma_\zeta^2 + \sigma_\eta^2\right)^{-1} - 2^{1/3} 3^{-1/3} \sigma_\nu^2 \left(\sigma_\zeta^2 + \sigma_\eta^2 + \sigma_\nu^2\right) \sigma_\epsilon^2 \Phi_\zeta^{-1} + \frac{2^{-1/3} 3^{-2/3} \Phi_\zeta}{\left(\sigma_\zeta^2 + \sigma_\eta^2\right)^3}, \tag{A38}$$

where

$$\Phi_\zeta = \left(-9\sigma_\eta^2 \left(\sigma_\zeta^2 + \sigma_\eta^2\right)^5 \sigma_\nu^4 \sigma_\epsilon^2 + \sqrt{3}\sqrt{\left(\sigma_\zeta^2 + \sigma_\eta^2\right)^9 \sigma_\nu^6 \sigma_\epsilon^4 \left(27\sigma_\eta^4 \left(\sigma_\zeta^2 + \sigma_\eta^2\right) \sigma_\nu^2 + 4\left(\sigma_\zeta^2 + \sigma_\eta^2 + \sigma_\nu^2\right)^3 \sigma_\epsilon^2\right)}\right)^{1/3}.$$

Rewriting this expression in order form of σ_ϵ yields

$$\pi_1 = O(1) - 2^{1/3} 3^{-1/3} O(1) O(\sigma_\epsilon^2) \Phi_\zeta^{-1} + 2^{-1/3} 3^{-2/3} O(1) \Phi_\zeta$$

and

$$\Phi_\zeta = \left(-9O(\sigma_\epsilon^2) + \sqrt{3\left(O(\sigma_\epsilon^2) + 4O(\sigma_\epsilon^6)\right)}\right)^{\frac{1}{3}} = O(\sigma_\epsilon),$$

where we denote $y = O(x)$ if $\frac{y}{x} = \text{const}$ as $\sigma_\epsilon \to 0$. Taking the derivative with respect to σ_ϵ yields

$$\frac{\partial \pi_1}{\partial \sigma_\epsilon} = -2^{4/3} 3^{-1/3} O(\sigma_\epsilon) \Phi_\zeta^{-1} + 2^{1/3} 3^{-1/3} O(\sigma_\epsilon^2) \Phi_\zeta^{-2} \frac{\partial \Phi_\zeta}{\partial \sigma_\epsilon} + 2^{-1/3} 3^{-2/3} O(1) \frac{\partial \Phi_\zeta}{\partial \sigma_\epsilon}$$

and

$$\frac{\partial \Phi_\zeta}{\partial \sigma_\epsilon} = \frac{1}{3} \Phi_\zeta^{-2} \left(-18O(\sigma_\epsilon) + \frac{1}{2}\left(O(\sigma_\epsilon^2) + 4O(\sigma_\epsilon^6)\right)^{-\frac{1}{2}} \left(2O(\sigma_\epsilon) + 24O(\sigma_\epsilon^5)\right)\right).$$

Cancelling coefficients and taking the limit on both sides yields the proof of the first statement:

$$\begin{aligned}
\lim_{\sigma_\epsilon \to 0} \frac{\partial \pi_1}{\partial \sigma_\epsilon} &= -\lim_{\sigma_\epsilon \to 0} O(\sigma_\epsilon)\Phi_\zeta^{-1} + \lim_{\sigma_\epsilon \to 0} O(\sigma_\epsilon^2)\Phi_\zeta^{-2}\frac{\partial \Phi_\zeta}{\partial \sigma_\epsilon} + \lim_{\sigma_\epsilon \to 0} \frac{\partial \Phi_\zeta}{\partial \sigma_\epsilon} \\
&= -\lim_{\sigma_\epsilon \to 0} O(\sigma_\epsilon)O(\sigma_\epsilon^{-1}) + \lim_{\sigma_\epsilon \to 0} \left(O(\sigma_\epsilon^2)O(\sigma_\epsilon^{-4}) + 1\right) O(\sigma_\epsilon^{-2}) \left(-O(\sigma_\epsilon) + O(\sigma_\epsilon^2)\right) \\
&= -\infty.
\end{aligned}$$

The result now follows from the chain rule since $\partial\sigma_\epsilon/\partial\lambda > 0$.

Similarly, rewriting (A38) in order form of σ_ζ yields

$$\pi_1 = O(1)O(\sigma_\zeta^{-2}) - 2^{1/3}3^{-1/3}O(1)O(\sigma_\zeta^2)\Phi_\zeta^{-1} + 2^{-1/3}3^{-2/3}O(\sigma_\zeta^{-6})\Phi_\zeta$$

and

$$\Phi_\zeta = \left(-9O(1)O(\sigma_\zeta^{10}) + \sqrt{3\left(27O(1)O(\sigma_\zeta^{20}) + 4O(\sigma_\zeta^{24})\right)}\right)^{\frac{1}{3}} = O(\sigma_\zeta^4).$$

Taking the derivative with respect to σ_ζ yields

$$\begin{aligned}
\frac{\partial \pi_1}{\partial \sigma_\zeta} &= -O(1)O(\sigma_\zeta^{-3}) - 2^{4/3}3^{-1/3}O(1)O(\sigma_\zeta)\Phi_\zeta^{-1} + 2^{1/3}3^{-1/3}O(1)O(\sigma_\zeta^2)\Phi_\zeta^{-2}\frac{\partial \Phi_\zeta}{\partial \sigma_\zeta} \\
&\quad -2^{2/3}3^{1/3}O(\sigma_\zeta^{-7})\Phi_\zeta + 2^{-1/3}3^{-2/3}O(\sigma_\zeta^{-6})\frac{\partial \Phi_\zeta}{\partial \sigma_\zeta}
\end{aligned}$$

and

$$\frac{\partial \Phi_\zeta}{\partial \sigma_\zeta} = \frac{1}{3}\Phi_\zeta^{-2}\left(-90\ O(1)O(\sigma_\zeta^9) + \frac{\sqrt{3}}{2}\left(27\ O(1)O(\sigma_\zeta^{20}) + 4\ O(\sigma_\zeta^{24})\right)^{-1/2}\left(540\ O(1)O(\sigma_\zeta^{19}) + 96O(\sigma_\zeta^{23})\right)\right).$$

The proof of the second statement follows from applying L'Hopital's rule to this expression. Because the analytical expressions become rather cumbersome, we refer the reader to the Mathematica file provided on the authors' websites for the remainder of the proof of the second statement.

For the case of the public signal, we start by combining (A36) and (A35) to get

$$\begin{aligned}
\pi_1 =& 27\sigma_\zeta^6\sigma_\eta^6\sigma_\nu^2\Phi_\varpi^3 + 2\sigma_\epsilon^2\left(\sigma_\zeta^2\Phi_\varpi\left(\sigma_\eta^2+\sigma_\nu^2\right) + \sigma_\eta^2\sigma_\nu^2\Phi_\varpi\right)^3 + 2\sigma_\epsilon^4\left(\sigma_\zeta^2\left(\sigma_\eta^2+\sigma_\nu^2\right)+\sigma_\eta^2\sigma_\nu^2\right)^6 \\
&\times \frac{\sqrt[3]{2}\sqrt{3}\sigma_\zeta\sigma_\eta^2\sigma_\nu^4\sigma_\epsilon^{4/3}\left(2\sqrt[3]{2}\sqrt{3}\Phi_\varpi^2 + 2\left(\sqrt{3}+3i\right)\sigma_\epsilon^{4/3}\left(\sigma_\zeta^2\left(\sigma_\eta^2+\sigma_\nu^2\right)+\sigma_\eta^2\sigma_\nu^2\right)^2\right)}{\Phi_\varpi^7\left(27\sigma_\zeta^6\sigma_\eta^6\sigma_\nu^2 + \Phi_\varpi^3 + 2\sigma_\epsilon^2\left(\sigma_\zeta^2\left(\sigma_\eta^2+\sigma_\nu^2\right)+\sigma_\eta^2\sigma_\nu^2\right)^3\right)}
\end{aligned} \tag{A39}$$

where

$$\Phi_\varpi = \sqrt[3]{3\sigma_\zeta^3\sigma_\eta^3\sigma_\nu\left(\sqrt{81\sigma_\zeta^6\sigma_\eta^6\sigma_\nu^2 + 12\sigma_\epsilon^2\left(\sigma_\nu^2\left(\sigma_\zeta^2+\sigma_\eta^2\right)+\sigma_\zeta^2\sigma_\eta^2\right)^3} - 9\sigma_\zeta^3\sigma_\eta^3\sigma_\nu\right) - 2\sigma_\epsilon^2\left(\sigma_\zeta^2\left(\sigma_\eta^2+\sigma_\nu^2\right)+\sigma_\eta^2\sigma_\nu^2\right)^3}.$$

The remaining steps mirror those of the proof for the case with aggregate noise in the private signal. In either case, the expressions are long and we refer the reader to the Mathematica file provided on the authors' websites.

12. Comparison with Noise-Trader Model

Consider two modifications to the model in section I: First, households have rational expectations:

$$\mu_i = 0 \ \forall i.$$

Second, in addition to the unit interval of rational households, the economy is inhabited by a unit interval of noise traders $j \in [0,1]$ inhabit the economy. Noise traders are identical to rational households in that they have the same preferences (3), budget constraint (4), and information set (they receive the signal (2) and observe the equilibrium stock price Q). However, when making their portfolio decisions, noise traders do not maximize their utility but exogenously and inelastically demand

$$z_j = \mu_j \vartheta, \tag{A40}$$

where $\vartheta \sim N\left(0, \sigma_\vartheta^2\right)$. This behavior makes the supply of stocks stochastic from the perspective of rational households.

Because $\kappa = 0$ implies $K = 0$, market clearing requires that the sum of rational households' and noise traders' stock demands equals zero:

$$\int_0^1 z_i di + \int_0^1 \mu_j \vartheta \ dj = 0, \tag{A41}$$

where $\mu_j = 1 \ \forall j$.

Proposition 7 *Shocks to noise-trader demand lower the utility of noise traders but raise the welfare of rational households. Noise traders' demand shocks thus represent a positive externality on rational households:*

$$\frac{\partial SWF}{\partial \sigma_\vartheta} > 0 \ \forall \sigma_\vartheta > 0 \ \textit{and} \ \frac{\partial E_0[U_j]}{\partial \mu_j} < 0 \ \forall \mu_j > 0.$$

PROOF:

See Appendix A.12.

The intuition behind this result is a redistribution of wealth between the two types of agents in the model. Although rational households incur some losses due to the increased variability of their portfolios, the market compensates them for the higher risk they take in the form of a higher risk premium. Their welfare increases because they can "lean against" noise traders' demand and thus earn higher expected returns on their investments.[21] Noise-trader demand shocks thus represent a positive rather than negative externality on the welfare of rational households.

In addition, the size of this externality shrinks to 0 in the limit in which noise-trader demand shocks become small.

Proposition 8 *As the standard deviation of noise-trader demand approaches 0, its marginal*

[21] With endogenous capital accumulation ($\kappa > 0$), there also exist parameter combinations for which the deadweight loss from distortions in the capital stock outweighs the redistribution of wealth from noise traders to rational households such that the marginal effect on rational households' utility becomes negative.

effect on the elasticity of the stock price with respect to productivity goes to 0

$$\lim_{\sigma_\vartheta \to 0} \frac{\partial \pi_1}{\partial \sigma_\vartheta} = 0.$$

PROOF:
See Appendix A.12.

To see the intuition for this result, replace K with ϑ in (I.D). Noise-trader demand shocks are multiplied with $\frac{\rho V_1[\eta]}{1-\alpha_2}$. For small σ_ϑ, both the numerator and the denominator go to 0, such that the fraction as a whole remains a finite number. (In Appendix A.12, we show that the multiplier on noise traders' demand shocks is always strictly smaller than $\rho\sigma_\nu^2$.) Small common shocks to noise traders' demand thus have no first-order effect on the equilibrium informativeness of stock prices. As a result, they affect neither noise traders' own utility nor the welfare of rational households. We show in the appendix that

$$\lim_{\sigma_\vartheta \to 0} \left[\frac{\partial SWF}{\partial \sigma_\vartheta} \right] = \lim_{\sigma_\vartheta \to 0} \left[\frac{\partial E_0[U_j]}{\partial \mu_j} \right] = 0.$$

Small shocks to noise traders' demand thus do not give rise to the type of externality we derive in section I. In addition, allowing for large shocks to noise-trader demand actually gives rise to a positive rather than a negative externality.

Proof of Proposition 7

Because households are now fully rational, their demand schedule is

$$z_i = \frac{E_{1i}[\eta] - Q}{\rho V_1[\eta]}. \tag{A42}$$

Taking time-zero expectations of (3), plugging in (4) and (A42), and simplifying by law of iterated expectations yields

$$\begin{aligned} E_0\left[U_i\right] &= E_0\left[\frac{E_{1i}[\eta - Q](\eta - Q)}{\rho V_1[\eta]}\right] - \frac{\rho}{2} E_0\left[\frac{\left(E_{1i}[\eta - Q]\right)^2}{\rho^2 V_1[\eta]}\right] = \frac{1}{2} E_0\left[\frac{\left(E_{1i}[\eta - Q]\right)^2}{\rho V_1[\eta]}\right] \\ &= \frac{1}{2\rho V_1[\eta]}\left(V_0\left[E_{1i}[\eta - Q]\right] + \left(E_0\left[\eta - Q\right]\right)^2\right), \end{aligned}$$

where we have used that $\Pi = 0$ when $\kappa = 0$. Using the law of total variance, we can then replace $V_0\left[E_{1i}[\eta - Q]\right] = V_0\left[\eta - Q\right] - V_1\left[\eta\right]$ and simplify to get

$$E_0\left[U_i\right] = \frac{\left(E_0[\eta - Q]\right)^2 + V_0[\eta - Q]}{2\rho V_1[\eta]} - \frac{1}{2\rho} = SWF,$$

where the second equality uses the fact that $E_0\left[U_i\right]$ is no longer a function of i and thus $SWF = \int E_0\left[U_i\right] di = E_0\left[U_i\right]$.
Plugging in (11) and the expressions from (A44) yields

$$SWF = \frac{1}{2}\sigma_\nu^2 \sigma_\vartheta^2 \rho - \frac{1}{2} \frac{\sigma_\nu^6 \sigma_\vartheta^4 \rho^3}{\sigma_\nu^4 \sigma_\vartheta^2 \rho^2 + \sigma_\eta^2\left(\sigma_\nu^2 \sigma_\vartheta^2 \rho^2 + 1\right)}.$$

It follows immediately that

$$\frac{\partial SWF}{\partial \sigma_\vartheta} = \frac{\sigma_\nu^8 \sigma_\vartheta^5 \rho^6 \sigma_\eta^2 + \sigma_\vartheta \sigma_\eta^4 \left(\sigma_\nu^3 \sigma_\vartheta^2 \rho^3 + \sigma_\nu \rho\right)^2}{\rho \left(\sigma_\eta^2 \left(\sigma_\nu^2 \sigma_\vartheta^2 \rho^2 + 1\right) + \sigma_\nu^4 \sigma_\vartheta^2 \rho^2\right)^2} > 0.$$

To calculate expected utility of noise traders, again take time-zero expectations of (3), plug in (4) and (A40), and simplify to get

$$\begin{aligned} E_0[U_j] &= E_0\left[\mu_j \vartheta(\eta - Q)\right] - \frac{\rho}{2} E_0\left[\mu_j^2 \vartheta^2\right] V_1[\eta] \\ &= -\mu_j \gamma \sigma_\vartheta^2 - \frac{\rho}{2} \mu_j^2 \sigma_\vartheta^2 V_1[\eta]. \end{aligned}$$

Taking the derivative with respect to μ_j yields

$$\frac{\partial E_0[U_j]}{\partial \mu_j} = -\gamma \sigma_\vartheta^2 - \rho \mu_j \sigma_\vartheta^2 V_1[\eta] < 0. \tag{A43}$$

Proof of Proposition 8

Substituting $E_{1i}[\eta]$ in (A42) with $E_{1i}[\eta] = \alpha_0 + \alpha_1 s_i + \alpha_2 Q$ and (2), plugging the resulting expression into (A41), and simplifying yields

$$\alpha_0 + \alpha_1 \left(\eta + \int_0^1 \nu_i di\right) + (\alpha_2 - 1) Q = \rho V_1[\eta] \vartheta.$$

Solving this expression for Q and matching coefficients with (11) yields

$$\pi_0 = \frac{\alpha_0}{1 - \alpha_2}, \ \pi_1 = \frac{\alpha_1}{1 - \alpha_2}, \ \gamma = \frac{\rho V_1[\eta]}{1 - \alpha_2}.$$

Note that the expressions π_0 and π_1 are identical to (A1) and (A2). Similarly, repeating the steps in section I.A, we find that the expressions for (16), 17, and (18) are identical to those in the near-rational model. However, the expression for γ is now multiplied with $\rho V_1[\eta]$ relative to its counterpart in (A3). Solving the system yields

$$\pi_0 = \frac{\sigma_\eta^{-2} \bar{\eta}}{\sigma_\eta^{-2} + \sigma_\nu^{-2} + \rho^{-2} \sigma_\vartheta^{-2} \sigma_\nu^{-4}}, \ \pi_1 = \frac{\sigma_\nu^{-2} + \rho^{-2} \sigma_\vartheta^{-2} \sigma_\nu^{-4}}{\sigma_\eta^{-2} + \sigma_\nu^{-2} + \rho^{-2} \sigma_\vartheta^{-2} \sigma_\nu^{-4}}, \ \gamma = \rho \sigma_\nu^2 \pi_1. \tag{A44}$$

Taking the derivative of π_1 with respect to σ_ϑ in (A44) and simplifying yields

$$\frac{\partial \pi_1}{\partial \sigma_\vartheta} = -\frac{2 \sigma_\nu^4 \sigma_\vartheta \rho^2 \sigma_\eta^2}{\left(\sigma_\eta^2 \left(\sigma_\nu^2 \sigma_\vartheta^2 \rho^2 + 1\right) + \sigma_\nu^4 \sigma_\vartheta^2 \rho^2\right)^2}.$$

As σ_ϑ approaches 0 the denominator approaches σ_η^4 while the numerator approaches 0.

13. Errors about Higher Moments

Rather than making near-rational errors about the conditional mean of η, we may consider a model identical to the one in section I, but in which households make a small common error about the second conditional moment rather than about the first conditional moment. We could then rewrite the market clearing condition as

$$\frac{\alpha_0 + \alpha_1 \int s_i di + \alpha_2 Q - Q}{\rho V_1[\eta] + \epsilon_V} = K.$$

Solving for Q yields

$$\frac{\alpha_0 - K\rho V_1[\eta]}{1-\alpha_2} + \frac{\alpha_1}{1-\alpha_2}\eta - \frac{K}{1-\alpha_2}\epsilon_V = Q.$$

In a model with an exogenous and strictly positive supply of capital, near-rational errors about the first and second conditional moments are thus isomorphic. However, with an endogenous capital stock, errors about the second conditional moment break the Gaussian structure of the model and are more complicated to analyze.

14. Benefits of Observing Mistakes

A guiding principle in our analysis of a near-rational household's incentive to become fully rational in section I was that households have the same information set, regardless of whether they behave fully rationally or near-rationally. In particular, a rational household can condition its decisions on s_i and Q, but does not know the small correlated error it would have made, had it been near-rational.

We can relax this assumption by considering the willingness to pay of a rational household at $t = 0$ for observing $\epsilon + \hat{\epsilon}_i$ at $t = 1$. A rational household can benefit from observing this error by extracting the information it conveys about η (and equivalently about the common component in the error, ϵ). Using (11), we can define

$$\text{(A45)} \qquad \hat{s}_i \equiv \frac{Q - \gamma\left(\epsilon + \hat{\epsilon}_i\right) - \pi_0}{\pi_1} = \eta - \frac{\gamma\hat{\epsilon}_i}{\pi_1},$$

where $\hat{s}_i$ is the un-biased signal about η conveyed by $\epsilon + \hat{\epsilon}_i$.

Proposition 9 *As the standard deviation of the near-rational error goes to 0, a rational household's willingness to pay to observe the near-rational error it would have made had it been near-rational goes to*

$$\text{(A46)} \qquad \lim_{\sigma_\epsilon \to 0} \left[E_0\left[U_i|_{\mu_i=0,\hat{s}_i}\right] - E_0\left[U_i|_{\mu_i=0}\right]\right] = \frac{1}{2\hat{\mu}^2}.$$

PROOF:

See Appendix A.14.

The potential gain of observing this additional signal thus goes to one half of the ratio of common variance to idiosyncratic variance in the error in household expectations. Since none of the results in section I place restrictions on $\hat{\mu}$, the potential incentive to observe $\epsilon + \hat{\epsilon}_i$ is thus small for a large range of plausible parameters.

Proof of proposition 9

Lemma 5 *A rational household would pay*

$$\text{(A47)} \qquad E_0\left[U_i|_{\mu_i=0,\hat{s}_i}\right] - E_0\left[U_i|_{\mu_i=0}\right] = \frac{\pi_1^2\left(\left(\left(\pi_1 - 1\right)\bar{\eta} + \pi_0\right)^2 + \gamma^2\sigma_\epsilon^2 + \left(\pi_1 - 2\right)\pi_1\sigma_\eta^2 + \sigma_\eta^2\right)}{2\gamma^2\sigma_\epsilon^2\hat{\mu}^2}$$

to observe the near-rational error it would have made, had it been near-rational. PROOF:

First, a household using additional signal $\hat{s}_i$ has a conditional variance of

$$V[\eta|s_i, Q, \hat{s}_i] \equiv \hat{V}_1[\eta] = \left(\sigma_\eta^{-2} + \sigma_\nu^{-2} + \pi_1^2\gamma^{-2}\sigma_\epsilon^{-2}\left(1 + \hat{\mu}^{-2}\right)\right)^{-1} \tag{A48}$$

and holds the posterior expectation

$$E[\eta|s_i, Q, \hat{s}_i] \equiv \hat{E}_{i1}[\eta] = \frac{\sigma_\eta^{-2}\bar{\eta} + \sigma_\nu^{-2}s_i + \pi_1^2\sigma_\epsilon^{-2}\gamma^{-2}(\eta + \frac{\gamma}{\pi_1}\epsilon) + \pi_1^2\sigma_\epsilon^{-2}\gamma^{-2}\hat{\mu}^{-2}\hat{s}_i}{\hat{V}_1[\eta]^{-1}}. \tag{A49}$$

Second, plugging (4) *into* (3)*, taking time-zero expectations, and rearranging yields*

$$E_0\left[U_i|_{\mu_i=0,\hat{s}_i}\right] = E_0\left[z_i(\eta - Q) + \Pi\right] - \frac{\rho}{2}E_0\left[z_i^2\right]\hat{V}_1[\eta],$$

where $z_i = \frac{\hat{E}_{1i}[\eta]-Q}{\rho\hat{V}_1[\eta]}$ from (9)*. It follows that a rational household's willingness to pay to observe $\hat{s}_i$ is*

$$\begin{aligned} E_0\left[U_i|_{\mu_i=0,\hat{s}_i}\right] - E_0\left[U_i|_{\mu_i=0}\right] \quad = \quad & E_0\left[\frac{\hat{E}_{1i}[\eta] - Q}{\rho\hat{V}_1[\eta]}(\eta - Q) + \kappa\frac{(Q-1)^2}{2}\right] - \frac{\rho}{2}E_0\left[\left(\frac{\hat{E}_{1i}[\eta] - Q}{\rho\hat{V}_1[\eta]}\right)^2\right]\hat{V}_1[\eta] \\ & - \left(E_0\left[\frac{E_{1i}[\eta] - Q}{\rho V_1[\eta]}(\eta - Q) + \kappa\frac{(Q-1)^2}{2}\right] - \frac{\rho}{2}E_0\left[\left(\frac{E_{1i}[\eta] - Q}{\rho V_1[\eta]}\right)^2\right]V_1[\eta]\right). \end{aligned} \tag{A50}$$

Plugging in (2), (11), (14), (15), (A45), (A48), *and* (A49) *and applying the expectations operator yields the expression in the proof. Note that this calculation is somewhat involved.*

Using this lemma, we now proof the Proposition. From (A8), we have

$$1 - \pi_1 = \frac{V_1[\eta]\left(\kappa\rho\sigma_\eta^2 + 1\right)}{\sigma_\eta^2\left(\kappa\rho V_1[\eta] + 1\right)}. \tag{A51}$$

Solving (22) for $V_1[\eta]$ yields three roots, one of which is real and in the interval $[0, \sigma_\eta^2]$:

$$V_1[\eta] = \frac{\sqrt[3]{2}\left(9\sigma_\eta^6\sigma_\nu^4\sigma_\epsilon^2 + \sqrt{3}\sqrt{\sigma_\eta^6\sigma_\nu^6\sigma_\epsilon^4\left(27\sigma_\eta^6\sigma_\nu^2 + 4\sigma_\epsilon^2\left(\sigma_\eta^2 + \sigma_\nu^2\right)^3\right)}\right)^{2/3} - 2\sqrt[3]{3}\sigma_\eta^2\sigma_\nu^2\sigma_\epsilon^2\left(\sigma_\eta^2 + \sigma_\nu^2\right)}{6^{2/3}\sigma_\eta^2\sqrt[3]{9\sigma_\eta^6\sigma_\nu^4\sigma_\epsilon^2 + \sqrt{3}\sqrt{\sigma_\eta^6\sigma_\nu^6\sigma_\epsilon^4\left(27\sigma_\eta^6\sigma_\nu^2 + 4\sigma_\epsilon^2\left(\sigma_\eta^2 + \sigma_\nu^2\right)^3\right)}}}. \tag{A52}$$

From (A52), we have

$$V_1[\eta] = \frac{O(\sigma_\epsilon^2)}{O(\sigma_\epsilon)} - O(\sigma_\epsilon) = O(\sigma_\epsilon). \tag{A53}$$

Combining (A51) and (A53) yields $1 - \pi_1 = O(\sigma_\epsilon)$. Thus, using (A9) and (A34), we have

$$\begin{aligned} \pi_0 \quad &= \quad O(1 - \pi_1) \quad = \quad O(\sigma_\epsilon), \\ \gamma \quad &= \quad O(\sqrt{\frac{\pi_1(1 - \pi_1)}{\sigma_\epsilon^2}}) = O(\sqrt{\frac{\pi_1\sigma_\epsilon}{\sigma_\epsilon^2}}) \quad = \quad O(\pi_1^{\frac{1}{2}}\sigma_\epsilon^{-\frac{1}{2}}). \end{aligned}$$

With these two facts, taking the limit of (A47) of Lemma 5 as $\sigma_\epsilon \to 0$ yields

$$\begin{aligned}\lim_{\sigma_\epsilon\to 0}\left[E_0\left[U_i|_{\mu_i=0,\hat{s}_i}\right]-E_0\left[U_i|_{\mu_i=0}\right]\right] &= \lim_{\sigma_\epsilon\to 0}\frac{\pi_1^2\left(O\left(\sigma_\epsilon^2\right)+\left(\pi_1-2\right)\pi_1\sigma_\eta^2+\sigma_\eta^2\right)}{2\ O\left(\frac{\pi_1}{\sigma_\epsilon}\right)\sigma_\epsilon^2\hat{\mu}^2}+\lim_{\sigma_\epsilon\to 0}\pi_1^2\frac{1}{2\hat{\mu}^2}\\ &= \lim_{\sigma_\epsilon\to 0}\pi_1^2\frac{O\left(\sigma_\epsilon^2\right)}{2\ O(\pi_1\sigma_\epsilon)\hat{\mu}^2}+\lim_{\sigma_\epsilon\to 0}\pi_1^2\frac{\left(\pi_1-2\right)\pi_1\sigma_\eta^2+\sigma_\eta^2}{2\ O(\pi_1\sigma_\epsilon)\hat{\mu}^2}+\lim_{\sigma_\epsilon\to 0}\pi_1^2\frac{1}{2\hat{\mu}^2}\end{aligned}$$

Then using (24) and simply plugging in $\pi_1 = 1$ gives (A46).

Appendix to Section II

1. Equation of Motion for Capital

Plugging (48) into (42) and integrating over individuals on both sides with market-clearing conditions (44), (45), and (46) gives

$$Q_tK_{t+1} = Q_{t-1}R_tK_t - C_t + w_tN_t. \tag{B1}$$

Plugging in (B13), (43), and (47) yields

$$K_{t+1} = (1-\delta_k)K_t + I_t - G_tK_t. \tag{B2}$$

2. Deriving the Equilibrium Conditions

Definition 1 *Given a time path of shocks* $\left\{\eta_t^j, \epsilon_t^j, \varpi_t^j\{\nu_{it}^j, \hat{\epsilon}_{it}^j : i \in [0,1]\} : j = L, S\right\}_{t=0}^{\infty}$*, an equilibrium in this economy is a time path of quantities* $\{\{C_{it}, b_{it}, n_{it}, k_{it} : i \in [0,1]\}, C_t, N_t, K_t, Y_t, I_t, G_t, R_t, a_t, \omega_t\}_{t=0}^{\infty}$*, signals* $\{s_{it}^j, g_t^j : i \in [0,1]\}_{t=0}^{\infty}$*, and prices* $\{Q_t, r_t, d_t, w_t\}_{t=0}^{\infty}$ *with the following properties:*

1) $\{\{C_{it}\}, \{b_{it}\}, \{n_{it}\}, \{k_{it}\}\}_{t=0}^{\infty}$ *maximize households' lifetime utility* (39) *given the vector of prices, and the random sequences* $\left\{\epsilon_t^j, \varpi_t^j, \{\nu^j{}_{it}, \hat{\epsilon}_{it}^j\}\right\}_{t=0}^{\infty}$*;*

2) *The demand for capital and labor services solves the representative firm's maximization problem* (37) *given the vector of prices;*

3) $\{I_t\}_{t=0}^{\infty}$ *is the investment goods sector's optimal policy, maximizing* (38) *given the vector of prices;*

4) $\{w_t\}_{t=0}^{\infty}$ *clears the labor market,* $\{Q_t\}_{t=0}^{\infty}$ *clears the stock market,* $\{r_t\}_{t=0}^{\infty}$ *clears the bond market, and* $\{d_t\}_{t=0}^{\infty}$ *clears the market for capital services;*

5) $\{Y_t\}_{t=0}^{\infty}$ *is determined by the production function* (33)*, and* $\{K_t\}_{t=0}^{\infty}$*,* $\{G_t\}_{t=0}^{\infty}$*,* $\{a_t\}_{t=0}^{\infty}$*,* $\{R_t\}_{t=0}^{\infty}$*, and* $\{\omega_t\}_{t=0}^{\infty}$ *evolve according to (B2),* (36)*,* (34)*,* (43)*, and* (35)*, respectively;*

6) $\{C_t, N_t\}_{t=0}^{\infty}$ *are given by the identities*

$$X_t = \int_0^1 X_{it}di\ ,\ X = C, N. \tag{B3}$$

After taking the ratio of the first-order conditions with respect to labor and consumption, we get the marginal rate of substitution between labor and consumption:

$$\frac{1-o}{o}\frac{(1-n_{it})^{-1}}{C_{it}^{-1}} = w_t. \tag{B4}$$

The optimal choice of stock holdings is determined by the familiar asset-pricing equation,

$$\mathcal{E}_{it}[M_{it+1}R_{t+1}] = 1, \tag{B5}$$

where the stochastic discount factor $M_{i,t+1}$ is given by

$$M_{it+1} = \delta\left(\frac{C_{it+1}}{C_{it}}\right)^{-1}\left(\frac{\tilde{C}_{it+1}}{\tilde{C}_{it}}\right)^{1-\frac{1}{\psi}}\left(\frac{U_{it+1}}{\mathcal{E}_{it}\left[U_{it+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma}, \tag{B6}$$

and returns R_{t+1} are defined in (43).

Similarly, by combining the first-order and envelope conditions for bonds, the optimal choice of bonds holdings is determined by

$$\mathcal{E}_{it}[M_{it+1}](1+r_t) - \frac{\pi'(b_{it})}{o(1-\delta)(1-\frac{1}{\psi})\tilde{C}_{it}^{1-\frac{1}{\psi}}C_{it}^{-1}} = 1. \tag{B7}$$

Given these conditions of optimality, capital and labor markets clear when conditions (44) and (46) hold, and the optimal consumption follows from the household's budget constraint (42).

Detailed Derivation

Agents maximize utility (39) subject to budget constraint (42). State variables in individual optimization are the holdings of capital and bonds, namely, $U_{it} = U_{it}(k_{it}, b_{it-1})$. We denote the derivatives of the value function with respect to k_{it} and b_{it-1} by U_{ikt} and U_{ibt} respectively. Thus the first-order conditions and envelope conditions are as follows:
First-order condition with respect to consumption:

$$(1-\delta)\tilde{C}_{it}^{-\frac{1}{\psi}}\tilde{C}_{it}oC_{it}^{-1} = \delta\mathcal{E}_{it}\left[U_{it+1}^{1-\gamma}\right]^{\frac{\gamma-\frac{1}{\psi}}{1-\gamma}}\mathcal{E}_{it}[U_{it+1}^{-\gamma}U_{ikt+1}\frac{1}{Q_t}]. \tag{B8}$$

First-order condition with respect to bonds:

$$\delta\mathcal{E}_{it}[U_{it+1}^{1-\gamma}]^{\frac{\gamma-\frac{1}{\psi}}{1-\gamma}}\mathcal{E}_{it}\left[U_{it+1}^{-\gamma}\left(U_{ikt+1}\frac{1}{Q_t} - U_{ibt+1}\right)\right] + (1-\frac{1}{\psi})^{-1}\pi'(b_{it}) = 0. \tag{B9}$$

First-order condition with respect to labor:

$$(1-\delta)\tilde{C}_{it}^{-\frac{1}{\psi}}\tilde{C}_{it}(1-o)(1-n_{it})^{-1} = \delta\mathcal{E}_{it}\left[U_{it+1}^{1-\gamma}\right]^{\frac{\gamma-\frac{1}{\psi}}{1-\gamma}}\mathcal{E}_{it}[U_{it+1}^{-\gamma}U_{ikt+1}\frac{w_t}{Q_t}]. \tag{B10}$$

Envelope condition for capital:

$$U_{ikt} = U_{it}^{\frac{1}{\psi}} \delta \mathcal{E}_{it} \left[U_{it+1}^{1-\gamma} \right]^{\frac{\gamma - \frac{1}{\psi}}{1-\gamma}} \mathcal{E}_{it}[U_{it+1}^{-\gamma} U_{ikt+1} \frac{Q_{t-1}}{Q_t} R_t]. \tag{B11}$$

Envelope condition for bonds:

$$U_{ibt} = U_{it}^{\frac{1}{\psi}} \delta \mathcal{E}_{it} \left[U_{it+1}^{1-\gamma} \right]^{\frac{\gamma - \frac{1}{\psi}}{1-\gamma}} \mathcal{E}_{it}[U_{it+1}^{-\gamma} U_{ikt+1} \frac{1}{Q_t} (1 + r_{t-1})]. \tag{B12}$$

Taking the ratio of first-order conditions with respect to labor (B10) and consumption (B8) gives (B4), where w_t is given by

$$w_t = (1 - \alpha) \frac{Y_t}{N_t}. \tag{B13}$$

The first-order condition with respect to capital pins down the rental rate as

$$d_t = \alpha \frac{Y_t}{K_t}. \tag{B14}$$

Plugging the first-order condition with respect to consumption (B8) into the right-hand side of the envelope condition for capital (B11) gives

$$U_{ikt} = U_{it}^{\frac{1}{\psi}} (1 - \delta) \tilde{C}_{it}^{1 - \frac{1}{\psi}} o C_{it}^{-1} Q_{t-1} R_t. \tag{B15}$$

Iterating (B15) to $t+1$, plugging $\frac{U_{ikt+1}}{Q_t}$ into the first-order condition with respect to consumption (B8), and rearranging yields

$$\tilde{C}_{it}^{-\frac{1}{\psi}} \tilde{C}_{it} o C_{it}^{-1} = \delta \mathcal{E}_{it} \left[U_{it+1}^{1-\gamma} \right]^{\frac{\gamma - \frac{1}{\psi}}{1-\gamma}} \mathcal{E}_{it}[U_{it+1}^{-\gamma} U_{it+1}^{\frac{1}{\psi}} \tilde{C}_{it+1}^{1 - \frac{1}{\psi}} o C_{it+1}^{-1} R_{t+1}]. \tag{B16}$$

Using (B6) in (B16) yields (B5).

Analogously, for bond holdings, combining first-order conditions with respect to bonds (B9) and consumption (B8) gives

$$(1 - \delta) \tilde{C}_{it}^{-\frac{1}{\psi}} \tilde{C}_{it} o C_{it}^{-1} = \delta \mathcal{E}_{it} [U_{it+1}^{1-\gamma}]^{\frac{\gamma - \frac{1}{\psi}}{1-\gamma}} \mathcal{E}_{it} \left[U_{it+1}^{-\gamma} U_{ibt+1} \right] - \frac{\pi'(b_{it})}{1 - \frac{1}{\psi}}. \tag{B17}$$

Combining the first-order condition with respect to consumption (B8) and the envelope condition for bond holdings (B12) gives

$$U_{ibt} = U_{it}^{\frac{1}{\psi}} (1 - \delta) \tilde{C}_{it}^{1 - \frac{1}{\psi}} o C_{it}^{-1} (1 + r_{t-1}). \tag{B18}$$

Substituting (B18) into (B17) for U_{ibt+1} simplifies to (B7).

3. Proof of Lemma 3

We proceed in three steps that demonstrate the consistency of the two statements in Lemma 3. To economize on notation, we show the equations in this section only for learning about one type of shock. The analysis readily extends to learning about short-run and long-run risk as carried out in our estimation.

First, individual state variables are functions of the set of commonly known state variables S_t as they would be in a representative agent economy. Furthermore, households form beliefs about next period's innovation to productivity using their private signal and the market price of capital. Any individual choice by households x_i (where x can be consumption c, labor n, or capital holdings k') is thus a function of the state space $x_i(S_{it})$, where $S_{it} = \{S_t, \hat{q}_t, \mathcal{E}_{it}[\eta_{t+1}]\}$. Plugging this structure into our equilibrium condition results in a form

$$g_l(S_{it}) = \mathcal{E}_{it}\left[g_r(S_{it}, S_{it+1})\right]. \tag{B19}$$

Note here that S_{it} contains all possible state variables in period t, and hence aggregate variables can be determined by a subset of this state vector as well.

Now we show that given the structure on the right-hand side of the equation, the left-hand side is a function of the state space S_{it}. We replace the function inside the expectation on the right-hand side by its Taylor series:

$$\begin{aligned} &g_r\left[S_{it}, K_{t+1}, \omega_t, \eta_{t+1}, \varphi_{t+1}, \hat{q}_{t+1}, \mathcal{E}_{it+1}\right] \\ &\qquad = \sum_{\boldsymbol{\iota}} \frac{c_{\boldsymbol{\iota}}(S_{it})}{\boldsymbol{\iota}!}(K_{t+1} - K_0)^{\iota_1}\omega_t^{\iota_2}\eta_{t+1}^{\iota_3}\varphi_{t+1}^{\iota_4}\hat{q}_{t+1}^{\iota_5}\mathcal{E}_{it+1}^{\iota_6}, \end{aligned} \tag{B20}$$

where K_0 is the level of capital at the deterministic steady state, $\mathcal{E}_{it} = \mathcal{E}_{it}[\eta_{t+1}]$, $c_{\mathbf{j}}(S_{it})$ denotes the (state-t dependent) coefficients of the Taylor series, and $\boldsymbol{\iota} = (\iota_1, \iota_2, \iota_3, \iota_4, \iota_5, \iota_6)$ a multi-index for the expansion.

Now we take near-rational expectations conditional on s_{it} and $\hat{q}_t$. As Lemma 6 shows, the conditional expectation is a sufficient statistic for the entire posterior distribution due to normality and a constant conditional variance. The terms depending on K_{t+1} and ω_t are known at time t and can thus be taken outside the expectations operator. Moreover, we get a series of terms depending on the conditional expectation of φ_{t+1}. Because φ_{t+1} is unpredictable for an investor at time t and all shocks are uncorrelated with each other, the first-order term is 0, and all the higher-order terms depending on $\mathcal{E}_{it}[\varphi_{t+1}]$ are just moments of the unconditional distributions of φ. The same is true for the terms depending on $\hat{q}_{t+1}$, and $\mathcal{E}_{it+1}$. The only terms remaining inside the expectations operator are then those depending on η_{t+1}. We can thus write

$$\begin{aligned} \mathcal{E}_{it}\left[g_r[S_{it}, S_{it+1}]\right] &= \sum_{\iota=0}^{\infty} \frac{\hat{c}_\iota(S_{it}, K_{t+1}, \rho\omega_{t-1} + \eta_t)}{\iota!}\mathcal{E}_{it}[\eta_{t+1}] \\ &= g_l(K_t, \omega_{t-1}, \eta_t, \varphi_t, \hat{q}_t, \mathcal{E}_{it}), \end{aligned} \tag{B21}$$

where the coefficients $\hat{c}_\iota(S_{it}, K_{t+1}, \omega_t)$ collect all the terms depending on the K_{t+1}, ω_t, and higher moments of the shocks η_{t+1} and $\mathcal{E}_{it+1}$. The third line follows from the second since all expectations of higher-order monomials of η_{t+1} are known. This step again follows from the conditional normality with constant variance and known (deterministic) higher moments. Hence we only need to keep track of the expectation of the innovation to productivity but its higher conditional moments are constant.

Finally, in deriving the set of individual state variables, we notice that contingent-claims trading eliminates any meaningful distribution of capital across time, and thus show the consistency of the individual state space.

Second, we show that aggregate quantities depend on known state variables as well as the average expectation of next period's innovation to productivity $\hat{q}$. Therefore, consider an aggregate variable of the form

$$\bar{X}(\bar{S}) = \int x_i(S_i)di, \tag{B22}$$

where $\bar{X}$ can represent labor (as in (46)), consumption (B3), or capital (44). Again, we plug in the Taylor series representation for individual state variables:

$$\int x_i(S_i)di = \int \sum_{\boldsymbol{\iota}} \frac{c_{\boldsymbol{\iota}}}{\boldsymbol{\iota}!}(K_t - K_0)^{\iota 1}\omega_{t-1}^{\iota 2}\eta_t^{\iota 3}\varphi_t^{\iota 4}\hat{q}_t^{\iota 5}\mathcal{E}_{it}^{\iota 6}di. \tag{B23}$$

Only the last term differs across households, and thus all other variables can be taken outside the integral. Integrating over individual expectations can be rewritten as

$$\int \mathcal{E}_{it}^{\iota}di = \int (\mathcal{E}_{it} - \hat{q}_t + \hat{q}_t)^{\iota}di = \sum_{k=0}^{\iota}\begin{pmatrix} \iota \\ k \end{pmatrix}\int(\mathcal{E}_{it} - \hat{q}_t)^k di \hat{q}_t^{\iota - k}. \tag{B24}$$

Again, all moments of $\mathcal{E}_{it} - \hat{q}_t$, which only depends on ν_{it}, are known and thus the integral only depends on $\hat{q}$. Therefore, equation (B22) holds.

Using these insights, we solve the model using standard perturbation techniques. Perturbation methods approximate equilibrium policy functions by their Taylor series around the deterministic steady state. To arrive at the coefficients of the Taylor series, we bring all equilibrium conditions into the appropriate form shown in equation (B19). Successively differentiating the equation, evaluating at the steady state, and solving the resulting system of equations for the coefficients in the Taylor series delivers the approximate solutions for the equilibrium policy functions and prices.

4. Details on Signal Extraction

Lemma 6 *Given Lemma 3 and Condition 1, households' equilibrium expectations of η_{t+1}^j for $j = S, L$ are independent of the aggregate dynamics of the model. Due to the normality of conditioning variables s_{it}^j and $\hat{q}_t^j$ for $j = S, L$ respectively, the resulting conditional distributions are Gaussian and identical to the linear Gaussian setup in section I.*

Proof Given Lemma 3 and Condition 1, households infer $\hat{q}_t^S$ and $\hat{q}_t^L$ from asset prices and macroeconomic quantities. It follows immediately that

$$E_{it}\left[\eta_{t+1}^j\right] = E\left[\eta_{t+1}^j | s_{it}^S, s_{it}^L, S_t\right] = E\left[\eta_{t+1}^j | s_{it}^j, \hat{q}_t^j\right] \quad \text{for } j \in \{S, L\}, \tag{B25}$$

where $\hat{q}_t^j$ is defined by (49).

We can thus guess that the rational expectation of η_{t+1}^j is the linear function

$$E_{it}[\eta_{t+1}^j] = \alpha_0^j + \alpha_1^j s_{it}^j + \alpha_2^j \hat{q}_t^j, \tag{B26}$$

where α_0^j, α_1^j, and α_2^j are the optimal weights on the prior, the private signal, and the average expectation, respectively. Substituting in (49), taking the integral across individuals, and solving for $\int E_{it}\left[\eta_{t+1}^j\right] di$ gives

$$\int E_{it}[\eta_{t+1}^j]di = \frac{\alpha_0^j}{1-\alpha_2^j} + \frac{\alpha_1^j}{1-\alpha_2^j}\eta_{t+1}^j + \frac{\alpha_2^j}{1-\alpha_2^j}\epsilon_t^j. \tag{B27}$$

Adding ϵ_t^j on both sides of the equation, substituting (49) and simplifying yields

$$\frac{1-\alpha_2^j}{\alpha_1^j}\hat{q}_t^j - \frac{\alpha_0^j}{\alpha_1^j} = \eta_{t+1}^j + \frac{1}{\alpha_1^j}\epsilon_t^j. \tag{B28}$$

Thus with the normality of the fundamental shock ϵ_t^j and the demand statistics $\hat{q}_t^j$, the forms for expectations and conditional variances following from Bayes' rule are identical to the linear setup.

Appendix to Section III

1. *Moment Generation and Standard Errors*

For the macroeconomic and financial moments listed in Table 3 we use annual data from 1929 to 2008. For the first five moments , concerned with the dynamics of expectations, we use quarterly data from 1969 to 2008.

In Table 3, $E[.]$, $\sigma(.)$, and $cor(.,.)$ denote time-series means, standard deviations, and correlations, respectively. d stands for the first difference in the time series (e.g., $\sigma(dy)$ stands for the standard deviation of output growth). $ACF[.]$ refers to the first-order autocorrelation. $E_{\mathrm{i}}[.]$ denotes the one-period-ahead forecast from forecaster i, $\bar{E}[.]$ denotes the cross-sectional average of $E_{\mathrm{i}}[.]$, and $\sigma_{xs}(.)$ denotes the time-series average of the cross-sectional standard deviation of one-period-ahead forecasts.

Fore example, $\sigma_{xs}(E_i[dy])$ is the time-series average of the cross-sectional standard deviation in forecasted GDP growth one period ahead. Because forecasts in the data are for the current quarter rather than the current month we divide these series by factor three for consistency. This scaling is not an issue for the remaining variables as they are all calculated as ratios or correlations.

Standard errors of the moments and moment ratios are calculated by block-bootstrapping the truncated dataset from 1969 to 2008 times across years (following defaults of Stata's "bootstrap" command). In robustness checks we have also experimented with GMM standard errors and obtained similar results.

2. Welfare Calculations

Lemma 7 *The share increase in lifetime consumption that makes a household indifferent with respect to the implementation of a given policy experiment at time 0 can be written as*

$$\lambda = \frac{\log\left(\hat{U}_0\right) - \log\left(\bar{U}_0\right)}{o},$$

where $\hat{U}_0 = E_0\left[U\left(\left\{\hat{C}_{it}, \hat{n}_{it}\right\}_{t=1}^{\infty}\right)\right]$, $\bar{U}_0 = E_0\left[U\left(\left\{\bar{C}_{it}, \bar{n}_{it}\right\}_{t=1}^{\infty}\right)\right]$, *and the sequences* $\left\{\hat{C}, \hat{n}\right\}$ *refer to the household's sequences of consumption and labor if the policy is implemented, and* $\left\{\bar{C}, \bar{n}\right\}$ *are the corresponding sequences if the policy is not implemented. PROOF:*

First note that the utility function (39) *is homogeneous of degree o in consumption:*

$$U\left(\left\{e^{\lambda} C_{it}, n_{it}\right\}_{t=1}^{\infty}\right) = e^{o\lambda} U\left(\left\{C_{it}, n_{it}\right\}_{t=1}^{\infty}\right).$$

Using this property, it follows that the share increase in consumption, λ, that compensates the household for not adopting the policy can be written as

$$\hat{U}_0 = e^{o\lambda} \bar{U}_0.$$

The lemma follows from solving this equation for λ.

3. Data Sources

Consumption (C_t). Per-capita consumption data are from the National Income and Product Accounts (NIPA) annual data reported by the Bureau of Economic Analysis (BEA). The data are constructed as the sum of consumption expenditures on nondurable goods and services (Table 1.1.5, Lines 5 and 6) deflated by corresponding price deflators (Table 1.1.9, Lines 5 and 6).

Physical Investment (I_t). Per-capita physical investment data are also from the NIPA tables. We measure physical investment by fixed investment (Table 1.1.5, Line 8) minus information-processing equipment (Table 5.5.5, Line 3) deflated by its price deflator (Table 1.1.9, Line 8). Information-processing equipment is interpreted as investment in intangible capital and is therefore subtracted from fixed investment.

Output (Y_t). It is the sum of total consumption and investment, that is, $C_t + I_t$. We exclude government expenditure and net export because they are not explicitly modeled in our economy.

Labor (N_t). It is measured as the total number of full-time and part-time employees as reported in the NIPA Table 6.4. Data are annual.

Stock market return (R_t) and **Risk-free rate**. (r_t) The stock market returns are from the Fama-French dataset available online on K. French′s webpage at
http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/ftp/F-F_Research_Data_Factors.zip.
The nominal risk-free rate is measured by the annual three-month T-bill return. The real stock market returns and risk-free rate are computed by subtracting realized inflation (annual CPI through FRED) from the nominal risk-free rate.

Tobin′s Q (Q_t). Data on Tobin′s Q are from the Flow of Funds (FoF) and are obtained directly from the St. Louis Fed by dividing the variable MVEONWMVBSNNCB (Line 35 of Table B.102 in the FoF report) by TNWMVBSNNCB (Line 32 of table B.102 in the FoF report).

Forecast Data GDP and consumption forecast data for the period 1969-2010 are downloaded from the Survey of Professional Forecasters provided by the Philadelphia Federal Reserve at https://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/historical-data/individual-forecasts We construct our forecasted GDP and consumption growth rates using the forecast for the current quarter, i.e. the quarter when the survey is conducted. The survey's questionnaires are usually sent out at the end of the first month of each quarter (after NIPA advance report), and the associated response deadlines are the second to third week of the middle month of each quarter. As a result the forecasters are essentially providing a 6-week ahead forecast. For more detail please see the documentation listed on the above URL. The forecast file contains data from many forecasters that appear to forecast only on an occasional basis and in particular these forecasts often appear highly volatile. To screen out these, potentially less serious, forecasters we consider only data from forecasts that are in the sample for at least 80 consecutive quarters and fulfill the basic requirement of a rational forecaster that over a long horizon the variance of the forecast be strictly smaller than the variance of the forecasted variable. These criteria leave us with a total of 38 time series of forecasts.

4. Estimating $\bar{\lambda}$ using tests of the Permanent Income Hypothesis

Fuchs-Schuendeln and Hassan (forthcoming) argue that although many studies reject the Permanent Income Hypothesis (PIH), it appears that households behave more rationally when the stakes are high. Specifically, when the welfare loss (as measured by equivalent variation) is economically large, studies tend to find support for the PIH. Fuchs-Schuendeln and Hassan (forthcoming) calculate this equivalent variation by comparing two households. The first rationally smooths a pre-announced income change (such as a bonus paid in December) over the course of the entire year. The second has the same baseline consumption, but consumes the extra income in the same period it is received. The equivalent variation is defined as the additional consumption amount that would have to be given to the second household to make it as well off as the first, expressed as a fraction of baseline consumption. In this appendix we replicate Fuchs-Schuendeln and Hassan's calculations using the same Epstein and Zin (1989) utility function used in the main text (39) and an intertemporal elasticity of substitution of $\psi = 2$ as in Table 1.

The first household's utility is given by:

$$U^{rational} = \left((1-\delta) \sum_{t=0}^{11} \delta^t (y + \frac{x}{12})^{1-\frac{1}{\psi}} + \delta^{12} \left(U^{rational} \right)^{1-\frac{1}{\psi}} \right)^{\frac{1}{1-\frac{1}{\psi}}}$$

Where y is the baseline consumption level and x is the extra amount of consumption received in a natural experiment. The second household has the same baseline consumption, but consumes the extra income in the same period it is received (December). Thus, its utility is given by:

$$U^{hand-to-mouth} = \left((1-\delta) \left(\sum_{t=0}^{10} \delta^t (y+z)^{1-\frac{1}{\psi}} + \delta^{11} (y+x+z)^{1-\frac{1}{\psi}} \right) + \delta^{12} \left(U^{hand-to-mouth} \right)^{1-\frac{1}{\psi}} \right)^{\frac{1}{1-\frac{1}{\psi}}}$$

where z is the additional amount of consumption we would have to give to the "hand-to-mouth" consumer such that $U^{rational} = U^{hand-to-mouth}$ and the equivalent variation as a percentage of permanent consumption is $z/y \times 100$.

In some studies, the increases in income are assumed to be permanent. In these cases we assume the change occurs in the middle of the year and the additional income x accrues in the last six months. In some other cases the additional income is paid over two or three months. In each case we assume that these payments are made at the end of the year.

The results of these calculations are given in Table C1. For each of 17 published studies it gives the size of the change in income (x), the baseline income (y), as well as the horizon over which the additional income is paid. The last two columns show whether the study rejects the PIH as well as the equivalent variation as a percentage of y.

Below we list for each study how the values for x and y are calculated (see Fuchs-Schuendeln and Hassan (forthcoming) for additional details).

Appendix Table C1—: Studies of the Permanent Income Hypothesis (PIH) Sorted by Equivalent Variation (EV) as a Percentage of Permanent Consumption

Paper	x	y	Paid over	Reject PIH	EV (%)
Parker (1999) (social security tax)	34.06	1449	6 months	yes	0.00
Shea (1995)	83.88	2330	6 months	yes	0.00
Souleles (2002)	234	$\frac{3587}{2}$	6 months	yes	0.00
Aaronson, Agarwal and French (2012)	474	2154	6 months	yes	0.01
Agarwal and Qian (2014)	511	6644	1 month	no	0.01
Johnson, Parker and Souleles (2006)	480	$\frac{47021}{12}$	1 month	yes	0.03
Agarwal, Liu and Souleles (2007)	300	1635	1 month	yes	0.06
Broda and Parker (2014)	898	$\frac{537000}{133}$	1 month	yes	0.09
Stephens (2008)	2436	3325	6 months	yes	0.09
Scholnick (2013)	4508.76	5379.58	6 months	yes	0.11
Parker et al. (2013)	970.8	$\frac{10601}{3}$	1 month	yes	0.13
Coulibaly and Li (2006)	1662	1785	6 months	no	0.14
Parker (1999) (social security cap)	990	1449	3 months	yes	0.22
Souleles (1999)	874	$\frac{3587}{2}$	1 month	yes	0.37
Browning and Collado (2001)	$\frac{817232}{7}$	222674	2 months	no	0.76
Souleles (2000)	−1960	777.79	6 months	no	1.40
Hsieh (2003)	2048	1786	1 month	no	1.63

- Parker (1999) (social security tax):
  Assume there is a permanent change in the social security tax rate in the middle of the year.
  x = 34.06 (Table 2, this is the average individual tax rates times the pre-tax monthly income of 2241 times six to calculate the value for half of a year)
  y=1449 (Table 2, average monthly expenditures of a household)

- Shea (1995):
  x = 83.88 (Table 2, expected wage growth due to education times annual income divided

by two to give increase to income in the middle of the year)
y = 2330 (Table 2, average annual household income deflated to 1982 US-dollars)

- Souleles (2002):
  x = 234 (Average change of quarterly withholding using the WHOLDP measure times two)
  y = 3587/2 (Table 1, real gross households earnings in 1983 dollars)
- Aaronson, Agarwal and French (2012):
  x = 474 (The permanent wage change increases earning by 237 dollar per quarter and we assume the consumer receives the wage increase in the middle of the year)
  y = 6462/3 (Table 2, average quarterly spending in 2006 dollars)
- Agarwal and Qian (2014):
  x = 511 (Table 1, Panel A, average monthly benefit of treatment group in experiment)
  y = 6644 (Table 1, Panel A, average monthly income of treatment group in 2016 dollars)
- Johnson and Parker (2006):
  x = 480 (Table 1, tax rebate for consumers with a positive tax rate)
  y = 47021/12 (Table 1, annual income divided by twelve)
- Agarwal, Liu and Souleles (2007):
  x = 300 (page 1, average monthly income for singles)
  y = 327 * 5 (The average consumer in this study uses 327 dollars of credit per month, but the authors cite Chimerine 1997 to indicate that credit is about 20 percent of spending)
- Broda and Parker (2014):
  x = 898 (Table 2, average tax rebate given the rebate is greater than zero)
  y = 179*30/7*100/19 (Table 2, average weekly spending multiplied by 30/7 to compute monthly spending. Y is scaled by 100/19 to adjust for the fact that data from the Nielson Consumer Panel does not capture all consumption goods)
- Stephens (2008):
  x = 2436 (Table 1, value of six months of vehicle loan payments)
  y = 3325 (Table 1, average annual after tax income per consumer paying off vehicle loan)
- Scholnick (2013):
  x = 4508.76 (Table 1, value of the average final mortgage payment times six)
  y = 5379.58 (Online Appendix, average income of treatment group families)
- Parker, Souleles, Johnson and McClelland (2013):
  x = 970.8 (Table 6, average tax rebate given the rebate is greater than zero)
  y = 10601/3 (Table 6, average quarterly consumption divided by three)
- Coulibaly and Li (2006):
  x = 1662 (Table 1, average payment multuplied by six to compute half a year of payments)
  y = 1785 (Table 1, average consumption from sample)
- Parker 1999 (Social Security Cap):
  Assume the household reaches the social security cap in the last three months of the year and does not pay any social security tax.
  x = 990 (The temporary increase in income for the last three months of the year)
  y = 1449 (Table 2, average monthly expenditures of a household)

- Souleles (1999):
  x = 874 (Table 1, Mean real refund for households in CEX data in 1982-1984 dollars)
  y = 3587/2 (Real gross annual earnings divided by twelve to compute monthly value from Souleles (2002))

- Browning and Collado (2001):
  x = 408616*4/14 (Table A2, a bonus of 1/14 of annual earnings is paid twice a year, so there are two months where 1/7 of annual income is received)
  y = 668022/2 (Table A2, total quarterly expenditures divided by 3)

- Souleles (2000):
  x = -1960 (Household expeditures for college when it is positive)
  y = 777.79 (There is an absence of expenditure and income data in the paper, so we calculate the equivalent variation using average quarterly spending from Johnson, Parker, and Souleles (2006).)

- Hsieh (2003):
  x = 2048 (Table 1, Alaska bonus in 1982-1984 dollars)
  y = ((713 + 1107) + (643 + 1109)/2) (Table 1, average monthly spending over two periods of time provided by author)

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