Online Appendix: "Power Dynamics in Organizations," Jin Li, Niko Matouschek, and Michael Powell

This appendix is divided into two sections. Appendix A contains proofs for the results describing the optimal relational contract in the baseline model. Appendix B contains proofs for the model with public opportunities.

Appendix A: Optimal Relational Contract in the Baseline Model

LEMMA A1. *Without loss of generality, along the equilibrium path, $k_t = m_t$ for all t and $e_{it} = 1$ for $i = A, P$ for all t.*

Proof of Lemma A1. Take an equilibrium with $k_t \neq m_t$ for some t. Consider another strategy profile in which the agent's recommendation is k_t instead of m_t. This change does not affect any player's payoffs, so it does not affect any constraints. It follows that this new strategy profile is an equilibrium.

Consider any strategy profile in which for some t, $e_{At} = 1$ and $e_{Pt} = 0$, and consider an alternative strategy profile that coincides with the original strategy profile but for which $e_{At} = 0$. Under this strategy profile, the Principal's payoff is unaffected, the public outcome is unaffected, and the agent's payoff is strictly higher. This means that the original strategy profile cannot be an equilibrium. An identical argument shows that any equilibrium strategy profile cannot have $e_{Pt} = 1$ and $e_{At} = 0$ for any t. Therefore, $e_{At} = e_{Pt}$ for all t in any equilibrium.

Consider a strategy profile in which for some t, $e_{At} = e_{Pt} = 0$, and consider an alternative strategy profile that coincides with the original strategy profile but for which $k_t = D$ is chosen in that period. This change does not affect players' payoffs, and it does not affect any constraints, so it is also an equilibrium.■

We now use the techniques developed by Abreu, Pearce, and Stacchetti (1990) to characterize the PPE payoff set and, in particular, its frontier. For this purpose, we define the payoff frontier as

$$\pi(u) \equiv \sup \{\pi' : (u, \pi') \in \mathcal{E}\},$$

where $\mathcal{E}$ is the PPE payoff set.

We can now state our first lemma, which establishes several properties of the PPE payoff set.

LEMMA A2. *The PPE payoff set $\mathcal{E}$ has the following properties:* (*i.*) *it is compact;* (*ii.*) $\pi(u)$ *is concave;* (*iii.*) $\inf\{u : (u, \pi) \in \mathcal{E}\} = 0$ and $\sup\{u : (u, \pi) \in \mathcal{E}\} = B$.

Proof of Lemma A2: Part (i.): Note that there are finite number of actions the players can take, and standard arguments then imply that the PPE payoff set $\mathcal{E}$ is compact. Part (ii.): the concavity of π follows immediately from the availability of the public randomization device. Part (iii.): Notice that 0 is the agent's maxmin payoff. Moreover, $(0, 0)$ is an equilibrium payoff for the stage game, sustained by the strategy that players always choose the default project or, if they choose any other project, they both choose $e_i = 0$. It then follows that $\inf\{u : (u, \pi) \in \mathcal{E}\} = 0$. Also notice that B is the maximal feasible payoff for the agent. Moreover, (B, b) can be sustained as an equilibrium payoff in which the players choose entrenchment along the equilibrium path in every period. To see that this can be sustained as an equilibrium, notice that the agent does not have incentive to deviate since the equilibrium provides him with the highest feasible payoff. Any deviation by the principal would be an off-schedule deviation. The deviation can either be the choice of the default project, in which case the Principal would receive $0 < b$, or it can be the choice not to choose the agent's recommended project, in which case, it can be punished with both players choosing $e_i = 0$ in the implementation phase, in which case again, the Principal would receive $0 < b$.■

LEMMA A3. *For any payoff $(u, \pi(u))$ on the frontier, the equilibrium continuation payoffs remain on the frontier.*

Proof of Lemma A3 : To show that for each payoff $(u, \pi(u))$ on the frontier, the equilibrium continuation payoffs remain on the frontier, it suffices to show that this is true if $(u, \pi(u))$ is supported by a pure action. Suppose $(u, \pi(u))$ is supported by centralization. Let (u_C, π_C) be the associated continuation payoff. Suppose to the contrary of the claim that $\pi_C < \pi(u_C)$. Now consider an alternative strategy profile that also specifies centralization but in which the continuation payoff is given by $(u_C, \widehat{\pi}_C)$, where $\widehat{\pi}_C = \pi_C + \varepsilon$ and where $\varepsilon > 0$ is small enough such that $\pi_C + \varepsilon \leq \pi(u_C)$. It follows from the promise-keeping constraints $\mathrm{PK}_{\mathrm{C}}^{\mathrm{P}}$ and $\mathrm{PK}_{\mathrm{C}}^{\mathrm{A}}$ that under this alternative strategy profile the payoffs are given by $\widehat{u} = u$ and $\widehat{\pi}_C = \pi(u) + \delta\varepsilon > \pi(u)$. It can be checked that this alternative strategy profile satisfies all the constraints and therefore constitutes a PPE. Since $\widehat{\pi}_C > \pi(u)$, this contradicts the definition of $\pi(u)$, thus it must be that $\pi_C = \pi(u_C)$. The argument is identical when $(u, \pi(u))$ is supported by other actions. ■

LEMMA A4. If any payoff pair $(u, \pi(u))$ is supported by a pure action, it is supported by an action $j \in \{C, E_R, E_C, E_U\}$.

Proof of Lemma A4. It is without loss of generality to show that if a payoff pair (u, π) is supported by $m_t = A$ when P yields $(-\infty, \infty)$ and by $m_t = D$ when P yields (b, B), then either $\pi < \pi(u)$ or there is a payoff-equivalent equilibrium in which the players randomize between choosing C or E_U. To see this, suppose that (u, π) is supported in this way. Define $u_1 = (1-\delta)\,0 + \delta u_D$ and $u_2 = (1-\delta)\,B + \delta u_A$, where u_D is the continuation payoff associated with $k_t = D$ and u_A is the payoff associated with $k_t = A$. The agent's equilibrium utility is therefore

$$u = pu_1 + (1-p)\,u_2,$$

and the principal's is

$$\begin{aligned}
\pi &= p\left((1-\delta)\,0 + \delta\pi(u_D)\right) + (1-p)\left((1-\delta)\,b + \delta\pi(u_A)\right) \\
&\leq p\pi(u_1) + (1-p)\,\pi(u_2)\,.
\end{aligned}$$

Therefore, either $\pi < \pi(u)$ or (u, π) can be supported by randomization between $(u_1, \pi(u_1))$, where C is chosen in period t with probability 1, and $(u_2, \pi(u_2))$, where A is chosen in period t with probability 1.■

For any action in $j \in \{C, E_R, E_C, E_U\}$, define $u_j(u)$ as the agent's continuation payoff when the equilibrium gives the agent payoff u. Note that for $j \in \{C, E_U\}$, the agent's continuation payoff is deterministic and is given by the corresponding promise-keeping constraints. The next lemma describes the agent's continuation payoff under restricted empowerment and cooperative empowerment.

LEMMA A5. The following hold.

(*i.*) *If* $(u, \pi(u))$ *is supported by restricted empowerment, there exists a payoff-equivalent equilibrium in which the agent's continuation payoffs are* $\delta u_{E_R,h}(u) = \delta u_{E_R,\ell}(u) = (u - (1-\delta)pb) \equiv \delta u_{E_R}(u)$.

(*ii.*) *If* $(u, \pi(u))$ *is supported by cooperative empowerment, there exists a payoff-equivalent equilibrium in which the agent's continuation payoffs are:*

$$\begin{aligned} \delta u_{E_C,h}(u) &= u - (1-\delta)b; \\ \delta u_{E_C,\ell}(u) &= u - (1-\delta)B. \end{aligned}$$

Proof of Lemma A5: For part ($i.$), let $(u, \pi(u))$ be associated with the continuation payoffs $(u_{E_R,h}, \pi(u_{E_R,h}))$ and $(u_{E_R,\ell}, \pi(u_{E_R,\ell}))$. Suppose to the contrary that $u_{E_R,h} \neq u_{E_R,\ell}$. Consider an alternative strategy profile with continuation payoffs given by $(\hat{u}_{E_R,h}, \pi(\hat{u}_{E_R,h}))$ and $(\hat{u}_{E_R,\ell}, \pi(\hat{u}_{E_R,\ell}))$, where

$$\hat{u}_{E_R,h} = \hat{u}_{E_R,\ell} = pu_{E_R,h} + (1-p)u_{E_R,\ell}.$$

Under this new strategy profile, $\mathrm{PK}^{\mathrm{A}}_{\mathrm{E_R}}$ and $\mathrm{IC}_{\mathrm{E_R}}$ still hold. This new profile gives the principal a payoff of

$$\begin{aligned} \hat{\pi} &= p\left[(1-\delta)B + \delta\pi(\hat{u}_{E_R,h})\right] + (1-p)\delta\pi(\hat{u}_{E_R,\ell}) \\ &\geq p\left[(1-\delta)B + \delta\pi(u_{E_R,h})\right] + (1-p)\delta\pi(u_{E_R,\ell}), \end{aligned}$$

where the inequality holds because π is concave. By $\mathrm{PK}^{\mathrm{A}}_{\mathrm{E_R}}$, it then follows that $\delta u_{E_R,h} = \delta u_{E_R,\ell} = u - (1-\delta)pb$. We define this value to be δu_{E_R}.

For part ($ii.$), let $(u, \pi(u))$ be associated with the continuation payoffs

$(u_{E_C,h}, \pi(u_{E_C,h}))$ and $(u_{E_C,\ell}, \pi(u_{E_C,\ell}))$. Suppose that for this PPE, $\mathrm{IC_{E_C}}$ is slack. That is, $(1-\delta)b + \delta u_{E_C,h} > (1-\delta)B + \delta u_{E_C,\ell}$. Now consider an alternative strategy profile with continuation payoffs given by $(\hat{u}_{E_C,h}, \pi(\hat{u}_{E_C,h}))$ and $(\hat{u}_{E_C,\ell}, \pi(\hat{u}_{E_C,\ell}))$, where $\hat{u}_{E_C,h} = u_{E_C,h} - (1-p)\varepsilon$ and $\hat{u}_{E_C,\ell} = u_{E_C,\ell} + p\varepsilon$ for $\varepsilon > 0$. It follows from the promise-keeping constraints $\mathrm{PK^P_{E_C}}$ and $\mathrm{PK^A_{E_C}}$ that, under this strategy profile, the payoffs are given by $\hat{u} = u$ and

$$\hat{\pi} = p\left[(1-\delta)B + \delta\pi(\hat{u}_{E_C,h})\right] + (1-p)\left[(1-\delta)b + \delta\pi(\hat{u}_{E_C,\ell})\right].$$

From the concavity of π it then follows that

$$\hat{\pi} \geq (1-\delta)b + \delta\left[(1-p)\pi(u_{E_C,\ell}) + p\pi(u_{E_C,h})\right] = \pi(u).$$

It can be checked that for sufficiently small ε this alternative strategy profile satisfies all the constraints and therefore constitutes a PPE. Since $\widehat{\pi} \geq \pi(u)$ this implies that for any PPE with payoffs $(\pi, u(\pi))$ for which IC is not binding there exists another PPE for which $\mathrm{IC_{E_C}}$ is binding and which gives the parties weakly larger payoffs. Notice that when $\mathrm{IC_{E_C}}$ is binding, we have $u_{E_C,h}(u) = (u - (1-\delta)b)/\delta$ and $u_{E_C,\ell}(u) = (u - (1-\delta)B)/\delta$. This proves part $(ii.)$. ■

Next, let $\pi_j(u)$ for $j \in \{C, E_R, E_C, E_U\}$ be the principal's highest equilibrium payoff given action j and agent's payoff u. We then have

$$\pi_C(u) = \delta\pi(u_C(u)),$$

$$\pi_{E_R}(u) = p\left[(1-\delta)B\right] + \delta\pi(u_{E_R}(u)),$$

$$\pi_{E_C}(u) = p\left[(1-\delta)B + \delta\pi(u_{E_C,h}(u))\right] + (1-p)\left[(1-\delta)b + \delta\pi(u_{E_C,\ell}(u))\right],$$

$$\pi_{E_U}(u) = (1-\delta)b + \delta\pi(u_{E_U}(u)).$$

LEMMA A6. *The PPE frontier $\pi(u)$ is the unique function that solves the* *following problem. For all* $u \in [0, B]$,

$$\pi(u) = \max_{\alpha_j \geq 0, u_j \in [0,B]} \sum_{j \in \{C, E_R, E_C, E_U\}} \alpha_j \pi_j(u_j)$$

such that

$$\sum_{j \in \{C, E_R, E_C, E_U\}} \alpha_j = 1$$

and

$$\sum_{j \in \{C, E_R, E_C, E_U\}} \alpha_j u_j = u.$$

Proof of Lemma A6: Since the frontier is Pareto efficient, by the APS bang-bang result, for any efficient payoff pair, only using the extreme points of the payoff set is sufficient. Replacing the sup with max is valid since the payoff set is compact. To establish the uniqueness, we just observe that the problem is now a maximization problem on a compact set, so that even if the maximizers are not unique, the maximum is. ■

LEMMA A7. *There exists a cutoff value* $\bar{u}_{E_C} < B$ *such that* $\pi(u)$ *is a straight line for* $u \geq \bar{u}_{E_C}$ *and* $\pi_{E_U}(u) = \pi(u)$ *if and only if* $u \in [(1-\delta)B + \delta\bar{u}_{E_C}, B]$.

Proof of Lemma A7: First, notice that $\pi_{E_U}(B) = \pi(B)$. Next, recall that $\pi_{E_U}(u) = (1-\delta)b + \delta\pi(u_{E_U}(u))$. Taking the right derivative, we have

$$\pi^+_{E_U}(u) = \delta\pi^+(u_{E_U}(u))\, u^+_{E_U}(u) = \pi^+(u_{E_U}(u)) \geq \pi^+(u),$$

where we used the fact that if $u < B$, $u_{E_U}(u) < u$, and therefore $\pi^+(u_{E_U}(u)) \geq \pi^+(u)$ by concavity of the frontier. Since $\pi^+_{E_U}(u) \geq \pi^+(u)$ for all $u < B$, there exists u^* such that $\pi_{E_U}(u) = \pi(u)$ if and only if $u \in [u^*, B]$.

Next, we show that $u^* < B$. That is, there exists some $u < B$ such that $\pi_{E_U}(u) = \pi(u)$. We prove this by contradiction. Suppose to the contrary that $\pi_{E_U}(u) < \pi(u)$ for all $u < B$. Choose a small enough $\varepsilon > 0$ such that $(B-\varepsilon, \pi(B-\varepsilon))$ cannot be supported by pure actions. Notice that such ε

exists, because by assumption $(B-\varepsilon, \pi(B-\varepsilon))$ is not supported by E_U, and if it were supported by any other pure action, the agent's continuation payoff must exceed B, leading to a contradiction. This implies that $(B-\varepsilon, \pi(B-\varepsilon))$ must be supported by randomization, and therefore the frontier is a straight line between $B-\varepsilon$ and B. Denote the slope of the payoff frontier between $(B-\varepsilon, \pi(B-\varepsilon))$ and (B, b) as s. It then follows that for all $u \in [B-\delta\varepsilon, B)$ (i.e. $u_{E_U}(u) \geq B-\varepsilon$), we have

$$\pi_{E_U}(u) = \pi(u) = b + s(u-B).$$

This contradicts the assumption that $\pi_{E_U}(u) < \pi(u)$ for all $u < B$.

The above shows that $\pi_{E_U}(u) = \pi(u)$ for $u \in [u^*, B]$, where $u^* < B$. It follows that for all $u \in (u^*, B]$, $\pi^-_{E_U}(u) = \pi^-(u_{E_U}(u)) = \pi^-(u)$. Since π is concave, this implies that the slope of π is constant for all $u \in (u^*, B)$. That is, $\pi(u)$ is a straight line on $[u^*, B]$. Define $(\bar{u}_{E_C}, \pi(\bar{u}_{E_C}))$ to be the left endpoint of the line segment.■

LEMMA A8. *$\pi(u)$ is a straight line for $u \in [0, pb]$ and $\pi(u) = Bu/b$.*

Proof of Lemma A8. Both $(0,0)$ and (pb, pB) are stage-game equilibrium payoffs. Moreover, recall that the agent will never choose $e=1$ for any project if the principal chooses $e=0$. This implies that all payoffs fall weakly below the line that includes $(0,0)$ and (pb, pB). As a result, the line segment connecting $(0,0)$ and (pb, pB) is on the frontier of the convex hull of the expected stage-game payoffs, which includes the PPE payoff set.■

LEMMA A9. $\pi_C(u) = \pi(u)$ if and only if $u \in [0, \delta pb]$.

Proof of Lemma A9. First, note that $\pi_C(0) = \pi(0)$. Clearly, for all $u \in [0, \delta pb]$, $\pi_C(u) = \pi(u)$, because we have established that $\pi(u)$ is a straight line between $(0,0)$ and (pb, pB). If there exists $u > \delta pb$, then $u_C(u) > pb$. Then $\pi^-_C(u) = \pi^-(u_C(u)) < B/b$ since $\pi(u) < Bu/b$ for u/pb. If so, then for $\varepsilon > 0$ small enough, $\pi_C(pb-\varepsilon) > \pi(pb-\varepsilon)$, which is a contradiction. We therefore have that $\pi_C(u) = \pi(u)$ only in $[0, \delta pb]$.■

LEMMA A10. *There exists a cutoff value $\underline{u}_{E_C}$ such that $\pi(u)$ is a straight line* *on* $\left[pb, \underline{u}_{E_C}\right]$ *and* $\pi_{E_R}(u) = \pi(u)$ for all $u \in \left[(1-\delta)pb, (1-\delta)pb + \delta\underline{u}_{E_C}\right]$.

Proof of Lemma A10. We first argue that if $\pi_{E_R}(u) = \pi(u)$ for any $u > pb$, then π is linear on $[pb, u]$. We define $\underline{u}_{E_C}$ as the right endpoint of this line segment and then show that for $u \in \left[pb, (1-\delta)pb + \delta\underline{u}_{E_C}\right]$, $\pi_{E_R}(u) = \pi(u)$ and for any $u' > (1-\delta)pb + \delta\underline{u}_{E_C}$, $\pi_{E_R}(u) < \pi(u)$. Finally, we show that $\pi_{E_R}(u) = \pi(u)$ for all $u \in [(1-\delta)pb, pb]$.

For the first step, suppose $\pi(u) = \pi_{E_R}(u)$ for some $u > pb$. Then $u_{E_R}(u) > u$ and

$$\pi^-_{E_R}(u) = \pi^-(u_{E_R}(u)) \leq \pi^-(u)$$

since π is concave. This implies that for all $u' \in [pb, u]$, $\pi_{E_R}(u') \geq \pi(u')$ and therefore $\pi_{E_R}(u') = \pi(u')$. Moreover, it must be the case that $\pi^-(u_{E_R}(u)) = \pi^-(u)$, so π must be linear on $[pb, u]$. Define the right endpoint of this line segment as $\underline{u}_{E_C}$. For any $u \in \left[pb, \underline{u}_{E_C}\right]$ such that $u_R(u) \leq \underline{u}_{E_C}$, since π is linear between $\left[pb, \underline{u}_{E_C}\right]$, we can write $\pi(u) = pB + s(u - pb)$ for some s. Moreover,

$$\begin{aligned}
\pi_{E_R}(u) &= (1-\delta)pB + \delta\pi(u_{E_R}(u)) = (1-\delta)pB + \delta(pB + s(u_{E_R}(u) - pb)) \\
&= pB + s(u - pb) = \pi(u).
\end{aligned}$$

Next, suppose that $u_{E_R}(u) > \underline{u}_{E_C}$ and $\pi_{E_R}(u) = \pi(u)$. Then, since $u_{E_R}(u) > \underline{u}_{E_C}$,

$$\pi^-_{E_R}(u) = \pi^-(u_{E_R}(u)) < \pi^-(u).$$

Now, consider $\hat{u} = u - \varepsilon$ for ε small. Then $\pi_{E_R}(\hat{u}) > \pi(\hat{u})$, so it must be the case that $\pi_{E_R}(u) < \pi(u)$ for all u such that $u_{E_R}(u) > \underline{u}_{E_C}$.

Finally, since $\pi(u) = Bu/b$ on $[0, pb]$ and $0 \leq u_{E_R}(u) \leq pb$ whenever $(1-\delta)pb \leq u \leq pb$,

$$\pi_{E_R}(u) = (1-\delta)pB + \delta\pi(u_{E_R}(u)) = Bu/b = \pi(u).$$

This establishes that $\pi_{E_R}(u) = \pi(u)$ on $[(1-\delta)pb, pb]$.■

LEMMA A11. For all $u \in \left[\underline{u}_{E_C}, \bar{u}_{E_C}\right]$, $\pi_{E_C}(u) = \pi(u)$.

Proof of Lemma A11. By Lemmas A7, A9, and A10, for all $u \in \left[\underline{u}_{E_C}, \bar{u}_{E_C}\right]$, which is a subset of $\left[(1-\delta) pb + \delta \underline{u}_{E_C}, (1-\delta) B + \delta \bar{u}_{E_C}\right]$, if $(u, \pi(u))$ is supported by a pure action, then it must be supported by cooperative empowerment. Next, since $\underline{u}_{E_C}$ and $\bar{u}_{E_C}$ are extremal points, they must be supported by a pure action, and therefore $\pi_{E_C}(u) = \pi(u)$ for $u = \underline{u}_{E_C}$ and $u = \bar{u}_{E_C}$. Take any $u \in \left(\underline{u}_{E_C}, \bar{u}_{E_C}\right)$. If $(u, \pi(u))$ is supported by randomization, it is supported by randomization between two points $(u_1, \pi(u_1))$ and $(u_2, \pi(u_2))$, $u_1 < u_2$, which are each supported by pure actions. If either $u_1 < \underline{u}_{E_C}$ or $u_2 > \bar{u}_{E_C}$, we can replace the left (right) endpoint of this randomization with $\left(\underline{u}_{E_C}, \pi\left(\underline{u}_{E_C}\right)\right)$ $\left(\left(\bar{u}_{E_C}, \pi\left(\bar{u}_{E_C}\right)\right)\right)$, and this new randomization generates higher payoffs for the principal. Thus, if $(u, \pi(u))$ is supported by randomization, it is supported by randomization between two points that are each supported by cooperative empowerment.

Define the function $f(u) = \pi(u) - \pi_{E_C}(u)$ on $\left[\underline{u}_{E_C}, \bar{u}_{E_C}\right]$. $f(u)$ is continuous and therefore achieves a maximum on $\left[\underline{u}_{E_C}, \bar{u}_{E_C}\right]$. Suppose $f(u^*) = \max_{u \in \left[\underline{u}_{E_C}, \bar{u}_{E_C}\right]} f(u) > 0$. Then at $u = u^*$, $\pi(u^*) > \pi_{E_C}(u^*)$, and therefore $(u^*, \pi(u^*))$ is supported by randomization between two points $(u_1, \pi(u_1)), (u_2, \pi(u_2))$, each of which is supported by cooperative empowerment. But then $f(u^*) = \alpha f(u_1) + (1-\alpha) f(u_2) = 0$, which implies that $\pi(u^*) = \pi_{E_C}(u^*)$.■

LEMMA A12. $\bar{u}_{E_C} = (1-\delta) b + \delta B$.

Proof of Lemma A12. Suppose $u_{E_C,h}(\bar{u}_{E_C}) < B$. Then, since by Lemma A11, $\pi_{E_C}(\bar{u}_{E_C}) = \pi(\bar{u}_{E_C})$, we have that

$$\pi^{+}(\bar{u}_{E_C}) = \pi^{+}_{E_C}(\bar{u}_{E_C}) = (1-p)\,\pi^{+}(u_{E_C,\ell}(\bar{u}_{E_C})) + ps,$$

where s is the slope of the line segment between $(\bar{u}_{E_C}, \pi(\bar{u}_{E_C}))$ and (B, b). Since $u_{E_C,\ell}(\bar{u}_{E_C}) < \bar{u}_{E_C}$, $\pi^{+}(u_{E_C,\ell}(\bar{u}_{E_C})) > s$. Take $\hat{u} = \bar{u}_{E_C} + \varepsilon$ for $\varepsilon > 0$ small. Then $\pi_{E_C}(\hat{u}) > \pi(\hat{u})$, which is a contradiction. Finally, since $u_{E_C,h}(\bar{u}_{E_C}) = B$, we have that $\bar{u}_{E_C} = (1-\delta) b + \delta B$.■

LEMMA A13. $\underline{u}_{E_C} \in [(1-\delta) B, (1-\delta) B + \delta pb]$.

Proof of Lemma A13. Suppose that $u_{E_C,\ell}\left(\underline{u}_{E_C}\right) > pb$. Then, since by Lemma A9, $\pi_{E_C}\left(\underline{u}_{E_C}\right) = \pi\left(\underline{u}_{E_C}\right)$, we have that $\pi^{-}\left(\underline{u}_{E_C}\right) = \pi^{-}_{E_C}\left(\underline{u}_{E_C}\right) =$

$(1-p)\,s+p\pi^{-}\left(u_{E_C,h}\left(\underline{u}_{E_C}\right)\right)$, where s is the slope of the line segment between (pb,pB) and $\left(\underline{u}_{E_C},\pi\left(\underline{u}_{E_C}\right)\right)$. Since $u_{E_C,h}\left(\underline{u}_{E_C}\right)>\underline{u}_{E_C}$, $\pi^{-}\left(u_{E_C,h}\left(\underline{u}_{E_C}\right)\right)<s$. Take $\hat{u}=\underline{u}_{E_C}-\varepsilon$ for $\varepsilon>0$ small. Then $\pi_{E_C}(\hat{u})>\pi(\hat{u})$, which is a contradiction. Since $u_{E_C,\ell}\left(\underline{u}_{E_C}\right)\in[0,pb]$, we have that $\underline{u}_{E_C}\in[(1-\delta)\,B,(1-\delta)\,B+\delta pb]$.

LEMMA A14. $b\leq\underline{u}_{E_C}\leq\max\{b,(1-\delta)\,B+\delta pb\}$.

Proof of Lemma A14. First, suppose that $\underline{u}_{E_C}<b$. Then we have $u_{E_C,\ell}\left(\underline{u}_{E_C}\right)<u_{E_C,h}\left(\underline{u}_{E_C}\right)<\underline{u}_{E_C}$. Since by Lemma A11, $\pi_{E_C}\left(\underline{u}_{E_C}\right)=\pi\left(\underline{u}_{E_C}\right)$, we have that $\pi^{+}\left(\underline{u}_{E_C}\right)=\pi^{+}_{E_C}\left(\underline{u}_{E_C}\right)=(1-p)\,\pi^{+}\left(u_{E_C,l}\left(\underline{u}_{E_C}\right)\right)+p\pi^{+}\left(u_{E_C,h}\left(\underline{u}_{E_C}\right)\right)\geq s$, where s is the slope of the line segment between (pb,pB) and $\left(\underline{u}_{E_C},\pi\left(\underline{u}_{E_C}\right)\right)$. Take $\widetilde{u}=\underline{u}_{E_C}+\varepsilon$ for $\varepsilon>0$ small. It then follows that $\pi(\widetilde{u})\geq\pi_{E_C}(\widetilde{u})\geq\pi(u)+s\varepsilon$. This contradicts the definition of $\underline{u}_{E_C}$ as the right end point of the line segment that includes (pb,pB) and $\left(\underline{u}_{E_C},\pi\left(\underline{u}_{E_C}\right)\right)$. This proves that $\underline{u}_{E_C}\geq b$.

Next, suppose $(1-\delta)\,B+\delta pb>b$ and suppose that $u_{E_C,\ell}\left(\underline{u}_{E_C}\right)>pb$. Then, since by Lemma A11, $\pi_{E_C}\left(\underline{u}_{E_C}\right)=\pi\left(\underline{u}_{E_C}\right)$, we have that

$$\pi^{-}\left(\underline{u}_{E_C}\right)=\pi^{-}_{E_C}\left(\underline{u}_{E_C}\right)=(1-p)\,s+p\pi^{-}\left(u_{E_C,h}\left(\underline{u}_{E_C}\right)\right),$$

where recall again that s is the slope of the line segment between (pb,pB) and $\left(\underline{u}_{E_C},\pi\left(\underline{u}_{E_C}\right)\right)$. Since $u_{E_C,h}\left(\underline{u}_{E_C}\right)>\underline{u}_{E_C}$, which follows from the agent's promise-keeping constraint and that $(1-\delta)\,B+\delta pb>b$, we have $\pi^{-}\left(u_{E_C,h}\left(\underline{u}_{E_C}\right)\right)<s$. Take $\hat{u}=\underline{u}_{E_C}-\varepsilon$ for $\varepsilon>0$ small. Then $\pi_{E_C}(\hat{u})>\pi(\hat{u})$, which is a contradiction. This proves that if $(1-\delta)\,B+\delta pb>b$, we have $u_{E_C,\ell}\left(\underline{u}_{E_C}\right)\leq pb$, and therefore, we have that $\underline{u}_{E_C}\leq(1-\delta)\,B+\delta pb$.

Finally, suppose $(1-\delta)\,B+\delta pb\leq b$, and suppose that $\underline{u}_{E_C}>b$. Then we have $u_{E_C,\ell}\left(\underline{u}_{E_C}\right)\in\left(pb,\underline{u}_{E_C}\right)$ and $u_{E_C,h}\left(\underline{u}_{E_C}\right)>\underline{u}_{E_C}$. The same argument above implies that $\pi^{-}\left(\underline{u}_{E_C}\right)=\pi^{-}_{E_C}\left(\underline{u}_{E_C}\right)=(1-p)\,s+p\pi^{-}\left(u_{E_C,h}\left(\underline{u}_{E_C}\right)\right)$. Again take $\hat{u}=\underline{u}_{E_C}-\varepsilon$ for $\varepsilon>0$ small, we have $\pi_{E_C}(\hat{u})>\pi(\hat{u})$, which is a contradiction.■

PROPOSITION 1. *The optimal relational contract satisfies the following:*
*First period***:** *The agent's and the principal's payoffs are* $u^{*}\in\left[\underline{u}_{E_C},\bar{u}_{E_C}\right]$ *and* $\pi(u^{*})=\pi_{E_C}(u^{*})$. *The parties engage in cooperative empowerment. If*

the agent chooses the principal's preferred project, his continuation payoff increases, and it falls otherwise.

Subsequent periods: The agent's and the principal's expected payoffs are given by $u \in \{0\} \cup \{pb\} \cup [\underline{u}_{E_C}, \bar{u}_{E_C}] \cup \{B\}$ *and* $\pi(u)$*. Their actions and continuation payoffs depend on what region* u *is in:*

(i.) If $u = 0$*, the parties choose centralization. The agent's continuation payoff is given by* $u_C(0) = 0$*.*

(ii.) If $u = pb$*, the parties choose restricted empowerment. The agent's continuation payoff is given by* $u_{E_R}(pb) = pb$*.*

(iii.) If $u \in [\underline{u}_{E_C}, \bar{u}_{E_C}]$*, the parties choose cooperative empowerment. If the agent chooses the principal's preferred project, his continuation payoff is given by* $u_{E_C,h}(u) > u$*. If, instead, he chooses his own preferred project, his continuation payoff is given by* $u_{E_C,\ell}(u) < u$*.*

(iv.) If $u = B$*, the parties engage in unrestricted empowerment. The agent's continuation payoff is given by* $u_{E_U}(B) = B$*.*

Proof of Proposition 1. The preceding lemmas characterize the payoff frontier, the associated actions, and their continuation payoffs. It remains only to show that in the first period, parties engage in cooperative empowerment. Given our assumption that $b > pB$, it suffices to show that there exists an equilibrium payoff, sustained by cooperative empowerment, that gives the principal a payoff that exceeds b. In particular, consider $\pi(\bar{u}_{E_C})$, where recall that $\bar{u}_{E_C} = (1-\delta)b + \delta B$. Notice that $u_{E_C,\ell}$ is decreasing in δ. It suffices to show that if $\pi(\bar{u}_{E_C}) \geq b$ when $u_{E_C,\ell}(\bar{u}_{E_C}) < pb$ for any $\hat{\delta}$, then $\pi(\bar{u}_{E_C}) \geq b$ for all $\delta \geq \hat{\delta}$. By Lemma A11,

$$\pi(\bar{u}_{E_C}) = p[(1-\delta)B + \delta b] + (1-p)\left[(1-\delta)b + \delta\frac{B}{b}\left(B - \frac{1-\delta}{\delta}(B-b)\right)\right]$$

It follows that

$$\frac{\pi(\bar{u}_{E_C}) - b}{b} = \frac{B-b}{b}\left[p(1-\delta) + \delta(1-p) + (2\delta - 1)\frac{B}{b}(1-p)\right].$$

Notice that this expression is always positive if $2\delta \geq 1$. When $2\delta < 1$, Assumption (iii.) ensures that it is positive. Also, the statement of Proposition 1 is true if we replace Assumption (ii.) with the assumption that $B/b > 1/p$ and Assumption (iii.) with

$$\left(\frac{B}{b}-1\right)\left[p\left(1-\delta\right)+\delta\left(1-p\right)+\left(2\delta-1\right)\frac{B}{b}\left(1-p\right)-1\right]+\frac{B}{b}\left(1-p\right)>0,$$

since this value is positive if and only if $\pi\left(\bar{u}_{E_C}\right) > pB > b$, which again implies that play begins with cooperative empowerment.■

PROPOSITION 2. *In the optimal relational contract, the principal chooses cooperative empowerment for the first τ periods, where τ is random and finite with probability one. For $t > \tau$, the relationship results in unrestricted empowerment, restricted empowerment, or centralization forever. Both unrestricted empowerment and restricted empowerment are chosen with positive probability on the equilibrium path. Specifically, if $B/b < \left(1-\delta p\right)/\left(1-\delta\right)$, only restricted empowerment and unrestricted empowerment are chosen, and if $\bar{u}_{E_C} < \left(1-\delta\right)B+\delta pb$, restricted empowerment, unrestricted empowerment, and centralization are chosen with positive probability.*

Proof of Proposition 2. Let $u^* = \text{argmax}_{u\in[0,B]}\,\pi\left(u\right)$ denote the agent's equilibrium utility when the principal's equilibrium utility is maximized. By Proposition 1, the relationship begins with cooperative empowerment, and therefore $u^* \geq \underline{u}_{E_C} \geq b$.

First, we will show that relationship settles in unrestricted empowerment with positive probability. To see this, first notice that $u^* > b$. Suppose to the contrary that $u^* = b$. Denote by s the slope of the payoff frontier between (pb, pB) and $(b, \pi\left(b\right))$. Then

$$\pi^+\left(b\right) = \pi^+_{E_C}\left(b\right) = \left(1-p\right)\pi^+\left(u_{E_C,\ell}\left(b\right)\right) + p\pi^+\left(b\right).$$

As a result, $\pi^+\left(b\right) = \pi^+\left(u_{E_C,\ell}\left(b\right)\right) \geq s > 0$, which contradicts the assumption that π is maximized at b. Next, given that $u^* > b$, we have that $u_h\left(u\right) - u > \frac{1-\delta}{\delta}\left(u^* - b\right)$ for all $u \in \left[u^*, \bar{u}_{E_C}\right]$. Then there exists an $N > 0$ such that if the principal's preferred project is available in the first N periods, the agent's continuation payoff has to exceed $\bar{u}_{E_C}$ with probability, and therefore, with positive probability, the relationship settles in unrestricted empowerment.

Next, we provide conditions for which centralization is never chosen on the equilibrium path. Suppose $B/b < [1-\delta p] / [1-\delta]$. Then, $u_{E_C,\ell}(b) > pb$, which means that for all $u \geq \underline{u}_{E_C}$, $u_{E_C,\ell}(b) > pb$. It follows that if u is ever below $\underline{u}_{E_C}$, it will be above pb, and therefore centralization is reached with probability zero.

We now provide conditions for which centralization is chosen with positive probability on the equilibrium path. If $\bar{u}_{E_C} < (1-\delta) B + \delta pb$, then $u_{E_C,\ell}(\bar{u}_{E_C}) < pb$, which implies that wherever cooperative empowerment is used, the agent's continuation payoff falls below pb with positive probability, and therefore centralization is reached with positive probability.

Finally, by standard arguments, the agent's continuation payoff converges with probability one.■

Appendix B: Optimal Relational Contract with Public Opportunities

Just as in the main section, we solve the game recursively by characterizing the PPE payoff sets. Define $\mathcal{E}_{pre}$ as the PPE payoff set of the pre-opportunity phase and $\mathcal{E}_{post}$ as the PPE payoff set of the post-opportunity phase. Let $\pi_i(u)$, $i \in \{pre, post\}$, be the associated payoff frontier. As in the baseline model, we can simplify our analysis by noting the following.

LEMMA B0. *Without loss of generality, along the equilibrium path,* $k_t = m_t$ *for all* t *and* $e_{it} = 1$ *for* $i = A, P$ *for all* t.

LEMMA B1. *For* $i \in \{pre, post\}$, *the PPE payoff set* $\mathcal{E}_i$ *has the following properties: (i.) it is compact; (ii.)* $\pi_i(u)$ *is concave; (iii.)* $\inf\{u : (u, \pi) \in \mathcal{E}\} = 0$ and $\sup\{u : (u, \pi) \in \mathcal{E}\} = B$.

The proofs for these results are essentially the same as in the baseline model, and they are omitted here. Next, we list the actions that are used to sustain the equilibrium payoff set and the associated constraints.

Constraints in the Post-Opportunity Phase

We first list the set of constraints for supporting the PPE payoff set $\mathcal{E}_{post}$. Consider a PPE payoff pair $(u, \pi) \in \mathcal{E}_{post}$. As in the baseline model, we can restrict attention to the following arrangements: centralization, restricted empowerment, cooperative empowerment, unrestricted empowerment, opportunity, and strategic opportunity. As we will show, the optimal relational contract can be sustained without making use of any other arrangement. The first four arrangements are the same as in the baseline model.

Centralization Under centralization, the agent recommends the default project, and the principal chooses the default project. A payoff pair (u, π) can be supported by centralization if the following constraints are satisfied.

(i.) Feasibility: For the continuation payoffs to be feasible, they also need to be PPE payoffs. The continuation payoffs $u_{post,C}$ and $\pi_{post,C}$ that the parties realize under centralization therefore have to satisfy the self-enforcement constraint

$$(u_{post,C}, \pi_{post,C}) \in \mathcal{E}_{post}. \tag{SE$_{\mathrm{post,C}}$}$$

(ii.) No Deviation: As in the baseline model, the principal and the agent never want to deviate off schedule, and there are no feasible on-schedule deviations. In contrast to off-schedule deviations, on-schedule deviations are privately observed. Since the principal does not have any private information, and the agent does not get to choose a project, there are no on-schedule deviations under centralization.

(iii.) Promise Keeping: Finally, the consistency of the PPE payoff decomposition requires that the parties' payoffs are equal to the weighted sum of current and future payoffs. The promise-keeping constraints

$$\pi = \delta \pi_{post,C} \tag{PK$^{\mathrm{P}}_{\mathrm{post,C}}$}$$

and

$$u = \delta u_{post,C} \tag{PK$^{\mathrm{A}}_{\mathrm{post,C}}$}$$

ensure that this is the case.

Unrestricted Empowerment Under unrestricted empowerment, the agent always recommends his own preferred project, and the principal rubberstamps this recommendation. A payoff pair (u, π) can be supported by unrestricted empowerment if the following constraints are satisfied.

(i.) Feasibility: We denote by $(u_{post,E_U}, \pi_{post,E_U})$ the continuation payoffs under unrestricted empowerment. The self-enforcement constraint is then given by

$$(u_{post,E_U}, \pi_{post,E_U}) \in \mathcal{E}_{post}. \qquad (\mathrm{SE}_{\mathrm{post,E_U}})$$

(ii.) No Deviation: As in the baseline model, the principal and the agent never want to deviate off schedule, and there are no feasible on-schedule deviations.

(iii.) Promise Keeping: The promise-keeping constraints are now given by

$$\pi = (1 - \delta)\, b + \delta \pi_{post,E_U} \qquad (\mathrm{PK}^{\mathrm{P}}_{\mathrm{post,E_U}})$$

for the principal and

$$u = (1 - \delta)\, B + \delta u_{post,E_U} \qquad (\mathrm{PK}^{\mathrm{A}}_{\mathrm{post,E_U}})$$

for the agent.

Cooperative Empowerment Under cooperative empowerment, the agent recommends the principal's preferred project when it is available and his own preferred project otherwise, and the principal rubberstamps the agent's recommendation. A payoff pair (u, π) can be supported by cooperative empowerment if the following constraints are satisfied.

(i.) Feasibility: For the continuation payoffs to be feasible, they also need to be PPE payoffs. Let $(u_{post,E_C,\ell}, \pi_{post,E_C,\ell})$ denote the parties' continuation payoffs if the agent chooses his own preferred project, and let $(u_{post,E_C,h}, \pi_{post,E_C,h})$ denote their payoffs if he chooses the principal's preferred project. The self-enforcement constraint is then given by

$$\left(u_{post,E_C,\ell},\pi_{post,E_C,\ell}\right),\left(u_{post,E_C,h},\pi_{post,E_C,h}\right)\in\mathcal{E}_{post}. \tag{$\mathrm{SE_{post,E_C}}$}$$

(ii.) No Deviation: The principal and the agent never want to deviate off schedule, and the principal has no on-schedule deviations. The agent, however, can deviate on schedule by recommending his preferred project when the principal's preferred project is available. The incentive constraint

$$(1-\delta)\,b+\delta u_{post,E_C,h}\geq(1-\delta)\,B+\delta u_{post,E_C,\ell} \tag{$\mathrm{IC_{post,E_C}}$}$$

ensures that he does not want to do so.

(iii.) Promise Keeping: The promise-keeping constraints are now given by

$$\pi=p\left[(1-\delta)\,B+\delta\pi_{post,E_C,h}\right]+(1-p)\left[(1-\delta)\,b+\delta\pi_{post,E_C,\ell}\right] \tag{$\mathrm{PK^P_{post,E_C}}$}$$

and

$$u=p\left[(1-\delta)\,b+\delta u_{post,E_C,h}\right]+(1-p)\left[(1-\delta)\,B+\delta u_{post,E_C,\ell}\right]. \tag{$\mathrm{PK^A_{post,E_C}}$}$$

Restricted Empowerment Under restricted empowerment, the agent recommends the principal's preferred project when it is available and the default project otherwise, and the principal always rubberstamps the agent's recommendation. A payoff pair (u,π) can be supported by restricted empowerment if the following constraints are satisfied.

(i.) Feasibility: For the continuation payoffs to be feasible, they also need to be PPE payoffs. Let $(u_{post,E_R,\ell},\pi_{post,E_R,\ell})$ denote the parties' continuation payoffs if the agent recommends the default project, and let $(u_{post,E_R,h},\pi_{post,E_R,h})$ denote their payoffs if he recommends the principal's preferred project. The self-enforcement constraint is then given by

$$\left(u_{post,E_R,\ell},\pi_{post,E_R,\ell}\right),\left(u_{post,E_R,h},\pi_{post,E_R,h}\right)\in\mathcal{E}_{post}. \tag{$\mathrm{SE_{post,E_R}}$}$$

(ii.) No Deviation: The principal never wants to deviate off schedule. The agent can deviate off schedule by recommending his own project. If he does so, he receives $(1-\delta)\,B$ this period followed by 0. To prevent the agent from deviating off schedule, we need that

$$u \geq (1-\delta)\,B. \tag{$\mathrm{IC}^{\mathrm{Off}}_{\mathrm{post,E_R}}$}$$

The agent can also deviate on schedule by recommending the default project when the principal's preferred project is available. The incentive constraint

$$(1-\delta)\,b + \delta u_{post,E_R,h} \geq \delta u_{post,E_R,\ell} \tag{$\mathrm{IC}^{\mathrm{On}}_{\mathrm{post,E_R}}$}$$

ensures that he does not want to do so.

(iii.) Promise Keeping: The promise-keeping constraints are now given by

$$\pi = p\left[(1-\delta)\,B + \delta\pi_{post,E_R,h}\right] + (1-p)\,\delta\pi_{post,E_R,\ell} \tag{$\mathrm{PK}^{\mathrm{P}}_{\mathrm{post,E_R}}$}$$

and

$$u = p\left[(1-\delta)\,b + \delta u_{post,E_R,h}\right] + (1-p)\,\delta u_{post,E_R,\ell}. \tag{$\mathrm{PK}^{\mathrm{A}}_{\mathrm{post,E_R}}$}$$

Definite Adoption Under definite adoption (A_D), the agent recommends the new project, and the principal chooses the new project. Note that a payoff pair (u,π) can be supported by definite adoption if the following constraints are satisfied.

(i.) Feasibility: We denote continuation payoffs under opportunity by $(u_{post,A_D},\pi_{post,A_D})$. The self-enforcement constraint is then given by

$$(u_{post,A_D},\pi_{post,A_D}) \in \mathcal{E}_{post}. \tag{$\mathrm{SE}_{\mathrm{post,A_D}}$}$$

(ii.) No Deviation: As in the case of centralization, the principal and the agent never wants to deviate off schedule, and there are no feasible on-schedule deviations.

(iii.) Promise Keeping: The promise-keeping constraints are now given by

$$\pi = (1-\delta)\,\Pi_N + \delta\pi_{post,A_D} \tag{$\mathrm{PK}^{\mathrm{P}}_{\mathrm{post,A_D}}$}$$

for the principal and

$$u = (1 - \delta) U_N + \delta u_{post,A_D} \tag{$\mathrm{PK}^{\mathrm{A}}_{\mathrm{post,A_D}}$}$$

for the agent.

Probabilistic Adoption Under probabilistic adoption (A_P), the agent recommends the principal's preferred project when it is available and the new project otherwise, and the principal rubberstamps the agent's recommendation. A payoff pair (u, π) can be supported by probabilistic adoption if the following constraints are satisfied.

(i.) Feasibility: For the continuation payoffs to be feasible, they also need to be PPE payoffs. Let $(u_{post,A_P,\ell}, \pi_{post,A_P,\ell})$ denote the parties' continuation payoffs if the agent recommends the new project, and let $(u_{post,A_P,h}, \pi_{post,A_P,h})$ denote their payoffs if he recommends the principal's preferred project. The self-enforcement constraint is then given by

$$(u_{post,A_P,\ell}, \pi_{post,A_P,\ell}), (u_{post,A_P,h}, \pi_{post,A_P,h}) \in \mathcal{E}_{post}. \tag{$\mathrm{SE}_{\mathrm{post,A_P}}$}$$

(ii.) No Deviation: The principal never wants to deviate off schedule. The agent can deviate off schedule by recommending his own project. If he does so, he receives $(1 - \delta) B$ this period followed by 0. To prevent the agent from deviating off-schedule, we need that

$$u \geq (1 - \delta) B. \tag{$\mathrm{IC}^{\mathrm{Off}}_{\mathrm{post,A_P}}$}$$

The agent can also deviate on schedule by recommending the new project when the principal's preferred project is available. The incentive constraint

$$(1 - \delta) b + \delta u_{post,A_P,h} \geq (1 - \delta) U_N + \delta u_{post,A_P,\ell} \tag{$\mathrm{IC}^{\mathrm{On}}_{\mathrm{post,A_P}}$}$$

ensures that he does not want to do so.

(iii.) Promise Keeping: The promise-keeping constraints are now given by

$$\pi = p\left[(1-\delta)\,B + \delta\pi_{post,A_P,h}\right] + (1-p)\left[(1-\delta)\,\Pi_N + \delta\pi_{post,A_P,\ell}\right] \quad (\mathrm{PK}^{\mathrm{P}}_{\mathrm{post,A_P}})$$

and

$$u = p\left[(1-\delta)\,b + \delta u_{post,A_P,h}\right] + (1-p)\left[(1-\delta)\,U_N + \delta u_{post,A_P,\ell}\right]. \quad (\mathrm{PK}^{\mathrm{A}}_{\mathrm{post,A_P}})$$

Randomization Finally, a payoff pair (u, π) can be supported by randomization. In this case, there exist at most three distinct PPE payoffs $(u_i, \pi_i) \in \mathcal{E}_{post}$, $i = 1, 2, 3$ such that

$$(u, \pi) = \alpha_1\,(u_1, \pi_1) + \alpha_2\,(u_2, \pi_2) + \alpha_3\,(u_3, \pi_3)$$

for some $\alpha_1, \alpha_2, \alpha_3 \geq 0$ and $\alpha_1 + \alpha_2 + \alpha_3 = 1$.

Constraints in the Pre-Opportunity Phase

We now list the set of constraints for supporting the PPE payoff set $\mathcal{E}_{pre}$. Consider a PPE payoff pair $(u, \pi) \in \mathcal{E}_{pre}$. Again as in the baseline model, we can restrict our attention to the following arrangements: centralization, restricted empowerment, cooperative empowerment, and unrestricted empowerment. In contrast to the baseline model, we now need to specify the continuation payoffs both when the opportunity has arrived and when it has not.

Centralization Under centralization, the agent recommends the default project, and the principal chooses the default project. A payoff pair (u, π) can be supported by centralization if the following constraints are satisfied.

(i.) Feasibility: For the continuation payoffs to be feasible, they also need to be PPE payoffs. Let $(u_{pre,C}, \pi_{pre,C})$ be the associated continuation payoffs if the opportunity does not arrive next period and $(u_{trans,C}, \pi_{trans,C})$ be the associated continuation payoffs when the game transitions to the post-opportunity phase for the first time next period. The continuation payoffs therefore have to satisfy the self-enforcement constraint

$$(u_{pre,C}, \pi_{pre,C}) \in \mathcal{E}_{pre} \text{ and } (u_{trans,C}, \pi_{trans,C}) \in \mathcal{E}_{post} \tag{SE$_{\text{pre,C}}$}$$

(ii.) No Deviation: As in the baseline model, the principal and the agent never want to deviate off schedule, and there are no feasible on-schedule deviations. Since the principal does not have any private information, and the agent does not get to choose a project, there are no on-schedule deviations under centralization.

(iii.) Promise Keeping: Finally, the consistency of the PPE payoff decomposition requires that the parties' payoffs are equal to the weighted sum of current and future payoffs. The promise-keeping constraints

$$\pi = \delta\left[(1-q)\,\pi_{pre,C} + q\pi_{trans,C}\right]; \tag{PK$^{\text{P}}_{\text{pre,C}}$}$$

$$u = \delta\left[(1-q)\,u_{pre,C} + qu_{trans,C}\right]. \tag{PK$^{\text{A}}_{\text{pre,C}}$}$$

Unrestricted Empowerment Under unrestricted empowerment, the agent always recommends his own preferred project, and the principal rubberstamps this recommendation. A payoff pair (u, π) can be supported by unrestricted empowerment if the following constraints are satisfied.

(i.) Feasibility: For the continuation payoffs to be feasible, they also need to be PPE payoffs. Let $(u_{pre,E_U}, \pi_{pre,E_U})$ be the associated continuation payoffs if the opportunity does not arrive next period and $(u_{trans,E_U}, \pi_{trans,E_U})$ be the associated continuation payoffs when the new opportunity arrives. The continuation payoffs therefore have to satisfy the self-enforcement constraint

$$(u_{pre,E_U}, \pi_{pre,E_U}) \in \mathcal{E}_{pre} \text{ and } (u_{trans,E_U}, \pi_{trans,E_U}) \in \mathcal{E}_{post} \tag{SE$_{\text{pre,E}_\text{U}}$}$$

(ii.) No Deviation: As in the case of centralization, the principal and the agent never want to deviate off schedule, and there are no feasible on-schedule deviations.

(iii.) Promise Keeping: The promise-keeping constraints are now given by

$$\pi = (1-\delta)\, b + \delta \left[(1-q)\, \pi_{pre,E_U} + q \pi_{trans,E_U}\right] \tag{$\mathrm{PK}^{\mathrm{P}}_{\mathrm{pre,E_U}}$}$$

for the principal and

$$u = (1-\delta)\, B + \delta \left[(1-q)\, u_{pre,E_U} + q u_{trans,E_U}\right] \tag{$\mathrm{PK}^{\mathrm{A}}_{\mathrm{pre,E_U}}$}$$

for the agent.

Cooperative Empowerment Under cooperative empowerment, the agent recommends the principal's preferred project when it is available and his own preferred project otherwise, and the principal rubberstamps the agent's recommendation. A payoff pair (u, π) can be supported by cooperative empowerment if the following constraints are satisfied.

(i.) Feasibility: For the continuation payoffs to be feasible, they also need to be PPE payoffs. If the new opportunity does not arrive next period, let $(u_{pre,E_C,\ell}, \pi_{pre,E_C,\ell})$ denote the parties' continuation payoffs if the agent chooses his own preferred project, and $(u_{pre,E_C,h}, \pi_{pre,E_C,h})$ denote their payoffs if he chooses the principal's preferred project. Define $(u_{trans,E_C,\ell}, \pi_{trans,E_C,\ell})$ and $(u_{trans,E_C,h}, \pi_{trans,E_C,h})$ accordingly. The self-enforcement constraint is then given by

$$\begin{aligned} (u_{pre,E_C,\ell}, \pi_{pre,E_C,\ell}) &\in \mathcal{E}_{pre}, (u_{pre,E_C,h}, \pi_{pre,E_C,h}) \in \mathcal{E}_{pre}; \\ (u_{trans,E_C,\ell}, \pi_{trans,E_C,\ell}) &\in \mathcal{E}_{Post}, (u_{trans,E_C,h}, \pi_{trans,E_C,h}) \in \mathcal{E}_{Post.} \end{aligned} \tag{$\mathrm{SE}_{\mathrm{pre,E_C}}$}$$

(ii.) No Deviation: The principal and the agent never want to deviate off schedule, and the principal has no on-schedule deviations. The agent, however, can deviate on schedule by recommending his preferred project when the principal's preferred project is available. The incentive constraint

$$\begin{aligned} & (1-\delta)\, b + \delta \left((1-q)\, u_{pre,E_C,h} + q u_{trans,E_C,h}\right) \\ \geq\ & (1-\delta)\, B + \delta \left((1-q)\, u_{pre,E_C,\ell} + q u_{trans,E_C,\ell}\right) \end{aligned} \tag{$\mathrm{IC}_{\mathrm{pre,E_C}}$}$$

ensures that he does not want to do so.

(iii.) Promise Keeping: The promise-keeping constraints are now given by

$$\begin{aligned} \pi &= p\left[(1-\delta) B+\delta\left((1-q) \pi_{pre,E_C,h}+q \pi_{trans,E_C,h}\right)\right] \qquad (\mathrm{PK}^{\mathrm{P}}_{\mathrm{pre,E_C}}) \\ &+(1-p)\left[(1-\delta) b+\delta\left((1-q) \pi_{pre,E_C,\ell}+q \pi_{trans,E_C,\ell}\right)\right], \end{aligned}$$

and

$$\begin{aligned} u &= p\left[(1-\delta) b+\delta\left((1-q) u_{pre,E_C,h}+q u_{trans,E_C,h}\right)\right] \qquad (\mathrm{PK}^{\mathrm{A}}_{\mathrm{pre,E_C}}) \\ &+(1-p)\left[(1-\delta) B+\delta\left((1-q) u_{pre,E_C,\ell}+q u_{trans,E_C,\ell}\right)\right]. \end{aligned}$$

Restricted Empowerment Under Restricted Empowerment, the agent recommends the principal's preferred project when it is available and the default project otherwise, and the principal always rubberstamps the agent's recommendation. A payoff pair (u,π) can be supported by restricted empowerment if the following constraints are satisfied.

(i.) Feasibility: For the continuation payoffs to be feasible, they also need to be PPE payoffs. If the new opportunity does not arrive next period, let $(u_{pre,E_R,\ell},\pi_{pre,E_R,\ell})$ denote the parties' continuation payoffs if the agent chooses his own preferred project, and $(u_{pre,E_R,h},\pi_{pre,E_R,h})$ denote their payoffs if he chooses the principal's preferred project. Define $(u_{trans,E_R,\ell},\pi_{trans,E_R,\ell})$ and $(u_{trans,E_R,h},\pi_{trans,E_R,h})$ accordingly. The self-enforcement constraint is then given by

$$\begin{aligned} (u_{pre,E_R,\ell},\pi_{pre,E_R,\ell}) &\in \mathcal{E}_{pre}, (u_{pre,E_R,h},\pi_{pre,E_R,h}) \in \mathcal{E}_{pre}; \qquad (\mathrm{SE}_{\mathrm{pre,E_R}}) \\ (u_{trans,E_R,\ell},\pi_{trans,E_R,\ell}) &\in \mathcal{E}_{Post}, (u_{trans,E_R,h},\pi_{trans,E_R,h}) \in \mathcal{E}_{Post.} \end{aligned}$$

(ii.) No Deviation: The principal and the agent never want to deviate off schedule, and the principal has no on-schedule deviations. The agent, however, can deviate on schedule by recommending the default project when the principal's preferred project is available. The incentive constraint

$$(1-\delta)\,b+\delta\left((1-q)\,u_{pre,E_R,h}+qu_{trans,E_R,h}\right)\geq\delta\left((1-q)\,u_{pre,E_R,\ell}+qu_{trans,E_R,\ell}\right) \tag{$\mathrm{IC}_{\mathrm{pre,E_R}}$}$$

ensures that he does not want to do so.

(iii.) Promise Keeping: The promise-keeping constraints are now given by

$$\begin{aligned}\pi &= p\left[(1-\delta)\,B+\delta\left((1-q)\,\pi_{pre,E_R,h}+q\pi_{trans,E_R,h}\right)\right] \qquad (\mathrm{PK}^{\mathrm{P}}_{\mathrm{pre,E_R}})\\ &\quad+(1-p)\left[(1-\delta)\,b+\delta\left((1-q)\,\pi_{pre,E_R,\ell}+q\pi_{trans,E_R,\ell}\right)\right],\end{aligned}$$

and

$$\begin{aligned}u &= p\left[(1-\delta)\,b+\delta\left((1-q)\,u_{pre,E_R,h}+qu_{trans,E_R,h}\right)\right] \qquad (\mathrm{PK}^{\mathrm{A}}_{\mathrm{pre,E_R}})\\ &\quad+(1-p)\left[(1-\delta)\,B+\delta\left((1-q)\,u_{pre,E_R,\ell}+qu_{trans,E_R,\ell}\right)\right].\end{aligned}$$

Randomization Finally, a payoff pair (u,π) can be supported by randomization. In this case, there exist at most three distinct PPE payoffs $(u_i,\pi_i)\in\mathcal{E}_{Pre}$, $i=1,2,3$ such that

$$(u,\pi)=\alpha_1\,(u_1,\pi_1)+\alpha_2\,(u_2,\pi_2)+\alpha_3\,(u_3,\pi_3)$$

for some $\alpha_1,\alpha_2,\alpha_3\geq 0$ and $\alpha_1+\alpha_2+\alpha_3=1$.

Properties of π_{post}

To focus our analysis but to allow for sufficient generality, we make the following assumptions.

ASSUMPTION B1. $pb<U_N\leq b$.

ASSUMPTION B2. $(1-\delta)\,B\leq pb+(1-p)\,U_N$.

ASSUMPTION B3. $B<\Pi_N\leq\min\left\{B-U_N,\left(p^{-1}+1-p\right)B\right\}$.

We will refer to the set (U_N,Π_N) that satisfy assumptions B1, B2, and B3 as N. Lemma B2 shows that $\pi_{post}\,(u)$ shares similar features as the PPE payoff frontier in the main section.

LEMMA B2. *For any payoff* $(u, \pi_{Post}(u))$ *on the frontier, the equilibrium continuation payoffs remain on the frontier. For all* $(U_N, \Pi_N) \in N$, *the following hold.*

(i.) If $(u, \pi_{post}(u))$ *is supported with centralization, the agent's continuation payoff is given by*

$$\delta u_{post,C}(u) = u.$$

(ii.) If $(u, \pi_{post}(u))$ *is supported with unrestricted empowerment, the agent's continuation payoff is given by*

$$\delta u_{post,E_U}(u) = u - (1-\delta) B.$$

(iii.) If $(u, \pi_{post}(u))$ *is supported with cooperative empowerment, the agent's continuation payoff can be chosen to be*

$$\begin{aligned} \delta u_{post,E_C,h}(u) &= u - (1-\delta) b; \\ \delta u_{post,E_C,\ell}(u) &= u - (1-\delta) B. \end{aligned}$$

(iv.) If $(u, \pi_{post}(u))$ *is supported with restricted empowerment, the agent's continuation payoff is given by*

$$\delta u_{post,E_R,h}(u) = \delta u_{post,E_R,\ell}(u) = u - (1-\delta) pb.$$

(v.) If $(u, \pi_{post}(u))$ *is supported with definite adoption, the agent's continuation payoff is given by*

$$\delta u_{post,A_D}(u) = u - (1-\delta) U_N.$$

(vi.) If $(u, \pi_{post}(u))$ *is supported with probabilistic adoption, the agent's continuation payoff is given by*

$$\delta u_{post,A_P,h}(u) = \delta u_{post,A_P,\ell}(u) = u - (1-\delta)(pb + (1-p) U_N).$$

Proof of Lemma B2: Parts (i.)–(iv.) are proven in the same way as in the proof of the baseline model. Part (v.) follows directly from the agent's promise-keeping condition ($\mathrm{PK}^{\mathrm{A}}_{\mathrm{post,A_D}}$). Part (vi.) follows from the agent's promise-keeping condition ($\mathrm{PK}^{\mathrm{A}}_{\mathrm{post,A_P}}$) and the condition that $b \geq u_N$, which ensures that the agent's on-schedule IC constraint is satisfied when $\delta u_{post,A_P,h}(u) = \delta u_{post,A_P,\ell}(u)$.■

Just as in the main section, let $\pi_{post,j}(u)$ for $j \in \{C, E_R, E_C, E_U, A_D, A_P\}$ be the highest equilibrium payoff for the principal when the agent's payoff is u and action j is chosen. Lemma B2 implies that

$$
\begin{aligned}
\pi_{post,C}(u) &= \delta\pi_{post}(u_{post,C}(u)); \\
\pi_{post,E_R}(u) &= (1-\delta)pB + \delta\pi_{post}(u_{post,E_R}(u)); \\
\pi_{post,E_C}(u) &= p[(1-\delta)B + \delta\pi_{post}(u_{post,E_C,h}(u))] \\
&\quad + (1-p)[(1-\delta)b + \delta\pi_{post}(u_{post,E_C,\ell}(u))]; \\
\pi_{post,E_U}(u) &= (1-\delta)b + \delta\pi_{post}(u_{post,E_U}(u)); \\
\pi_{post,A_D}(u) &= (1-\delta)\Pi_N + \delta\pi_{post}(u_{post,A_D}(u)); \\
\pi_{post,A_P}(u) &= (1-\delta)(pB + (1-p)\Pi_N) + \delta\pi_{post}(u_{post,A_P}(u)).
\end{aligned}
$$

The characterization of π_{post} is similar to the analysis in the baseline model. It is worth noting that if $(U_N, \Pi_N) \in N$, restrictive empowerment is no longer used to support any payoff pair $(u, \pi_{post}(u))$.

LEMMA B3. *For each $(U_N, \Pi_N) \in N$, there exist two cutoffs $\underline{u}_{post,E_C}$ and $\bar{u}_{post,E_C}$ such that the PPE payoff frontier $\pi_{post}(u)$ is divided into at most five regions:*

(i.) For $u \in (0, U_N)$, $\pi_{post}(u)$ is supported by randomization between centralization and definite adoption. $\pi_{post}(0) = 0$ and $\pi_{post}(U_N) = \Pi_N$.

(ii.) For $u \in (U_N, pb + (1-p)U_N]$, $\pi_{post}(u)$ is supported by randomization between definite adoption and probabilistic adoption. $\pi_{post}(pb + (1-p)U_N) = pB + (1-p)\Pi_N$.

(iii.) For $u \in \left[pb + (1-p)\,U_N, \underline{u}_{post,E_C}\right]$*,* $\pi_{post}(u)$ *is supported by randomization between probabilistic adoption and cooperative empowerment.*

(iv.) For $u \in \left[\underline{u}_{post,E_C}, \bar{u}_{post,E_C}\right]$*,* $\pi_{post}(u)$ *is supported by cooperative empowerment.*

(v.) For $u \in [\bar{u}_{post,E_C}, B]$*,* $\pi_{post}(u)$ *is supported by randomization between cooperative empowerment and unrestricted empowerment.*

In addition, $\bar{u}_{post,E_C} = (1-\delta)\,b + \delta B$ and

$$b \le \underline{u}_{post,E_C} \le \max\{b, (1-\delta)\,B + \delta\,(pb + (1-p)\,U_N)\}\,.$$

The payoff frontier π_{post} *is maximized at* U_N*.*

Proof of Lemma B3: To see part (i.), note that $(0,0)$ and (U_N, Π_N) are stage-game equilibrium payoffs. Recall that the agent will never choose $e = 1$ for any project if the principal chooses $e = 0$. This implies that all equilibrium payoffs lie weakly below the line segment hat connects $(0,0)$ and (U_N, Π_N). As a result, the line segment connecting $(0,0)$ and (U_N, Π_N) is on the frontier of the convex hull of the expected stage-game payoffs, which includes the PPE payoff set. For part (ii.), notice that $(pb + (1-p)\,U_N, pB + (1-p)\,\Pi_N)$ is a stage-game equilibrium expected payoff given that $(1-\delta)\,B \ge pb + (1-p)\,U_N$. Notice that $(pb + (1-p)\,U_N, pB + (1-p)\,\Pi_N)$ is on the line segment between (U_N, Π_N) and (b, B). This line segment is on the frontier of the convex hull of the expected stage-game payoffs, which includes the PPE payoff set. For the remaining part of the lemma, notice that for the proof of parts (iii.)–(v.), the value of $\bar{u}_{post,E_C}$ and the bounds on $\underline{u}_{post,E_C}$ follow from the same analysis as in the baseline model. Finally, since (U_N, Π_N) is an equilibrium payoff, and Π_N is the highest stage-game payoff for the principal, it is immediate that π_{post} is maximized at U_N.■

Properties of π_{pre}

Now we characterize the payoff frontier of the pre-opportunity game. Unlike the analysis of the baseline model or of the post-opportunity game, there are no explicit expressions for the agent's continuation payoffs. Instead, they are pinned down by the following two conditions. First, their expected value is determined by the promise-keeping condition (with the same expressions as those in the baseline model). Second, we have $\pi'_{pre}\left(u_{pre,j}\left(u\right)\right)=\pi'_{post}\left(u_{trans,j}\left(u\right)\right)$ for $j=\left\{C,E_R,E_U,\left(E_C,h\right),\left(E_C,\ell\right)\right\}$ when the payoff frontiers are differentiable. The next lemma provides the details.

LEMMA B4. *For any payoff $(u,\pi_{pre}(u))$ on the frontier, the equilibrium continuation payoffs remain on the frontier. In addition, the following holds.*

(i.) If $(u,\pi_{pre}(u))$ is supported by centralization, the agent's continuation payoff satisfies

$$\delta q u_{trans,C}\left(u\right)+\delta\left(1-q\right)u_{pre,C}\left(u\right)=u.$$

In addition,

$$\pi^+_{pre}(u_{pre,C}\left(u\right))\leq\pi^-_{post}(u_{trans,C}\left(u\right));\ \pi^+_{post}(u_{trans,C}\left(u\right))\leq\pi^-_{pre}(u_{pre,C}\left(u\right)).$$

(ii.) If $\left(u,\pi_{pre}\left(u\right)\right)$ is supported by restricted empowerment, the agent's continuation payoff satisfies $u_{pre,E_R,\ell}\left(u\right)=u_{pre,E_R,h}\left(u\right)\equiv u_{pre,E_R}\left(u\right)$, $u_{trans,E_R,\ell}\left(u\right)=u_{trans,E_R,h}\left(u\right)\equiv u_{transt,E_R}\left(u\right)$

$$\delta\left[q u_{trans,E_R}\left(u\right)+\left(1-q\right)u_{pre,E_R}\left(u\right)\right]=u-\left(1-\delta\right)pb.$$

In addition,

$$\pi^+_{pre}(u_{pre,E_R}\left(u\right))\leq\pi^-_{post}(u_{trans,E_R}\left(u\right));\ \pi^+_{post}(u_{trans,E_R}\left(u\right))\leq\pi^-_{pre}(u_{pre,E_R}\left(u\right)).$$

(iii.) If $(u,\pi_{pre}(u))$ is supported by cooperative empowerment, the agent's continuation payoff can be chosen to satisfy

$$\begin{aligned}\delta q u_{trans,E_C,l}\left(u\right)+\delta\left(1-q\right)u_{pre,E_C,l}\left(u\right)&=u-\left(1-\delta\right)B;\\ \delta q u_{trans,E_C,h}\left(u\right)+\delta\left(1-q\right)u_{pre,E_C,h}\left(u\right)&=u-\left(1-\delta\right)b.\end{aligned}$$

In addition, for $j\in\{h,\ell\}$,

$$\pi^+_{pre}(u_{pre,E_C,j}\left(u\right))\leq\pi^-_{post}(u_{trans,E_C,j}\left(u\right));\quad\pi^+_{post}(u_{trans,E_C,j}\left(u\right))\leq\pi^-_{pre}(u_{pre,E_C,j}\left(u\right)).$$

(iv.) If $(u, \pi_{pre}(u))$ *is supported by unrestricted empowerment, the agent's continuation payoff is given by*

$$\delta q u_{trans,E_U}(u) + \delta(1-q)\, u_{trans,E_U}(u) = u - (1-\delta) B.$$

In addition,

$$\pi^{+}_{pre}(u_{pre,E_U}(u)) \leq \pi^{-}_{post}(u_{trans,E_U}(u));\ \pi^{+}_{post}(u_{trans,E_U}(u)) \leq \pi^{-}_{pre}(u_{pre,E_U}(u)).$$

Proof of Lemma B4: This is proven in the same way as that in the baseline model. The additional inequality constraints arise, because at the optimum, for a given expected continuation payoff for the agent, it has to be optimal for the principal not to increase or decrease the agent's state-contingent continuation payoff.■

Now we can prove proposition 3.

PROPOSITION 3. *For each* $(U_N, \Pi_N) \in N$,
(i.) There exists $\bar{\Pi}(U_N)$ *and* $\overline{q}(U_N, \Pi_N)$ *such that for all* $\Pi_N \leq \bar{\Pi}(U_N)$ *and* $q \leq \overline{q}(U_N, \Pi_N)$, *there exists a public history* h^T *such that* $\Pr\left(u_T = U_N | h^T\right) < 1$, *where* T *is the first period in the post-opportunity phase.*
(ii.) There exists a $\hat{\delta}$ *and* $\hat{q}(U_N, \Pi_N)$ *such that for all* $\delta \leq \hat{\delta}$ *and* $q \leq \hat{q}(U_N, \Pi_N)$, *there exists a public history* h^T *such that* $\Pr\left(u_t = U_N | h^T\right) = 0$ *for all* $t \geq T$.

Proof of Proposition 3: Denote $\pi^q_{pre}(u)$ to be the payoff frontier in the pre-opportunity game with parameter q, and notice that $\pi^0_{pre}(u) = \pi(u)$, which is the frontier of the baseline model. By Berge's maximum theorem, $\lim_{q \to 0} \pi^q_{pre}(u) = \pi(u)$ for each u. Define

$$\bar{u}^q_{pre,E_C} = \max\left\{u : \pi^q_{pre,}(u) = \pi^q_{pre,E_C}(u)\right\}.$$

Then, $\lim_{q\to 0} \bar{u}^q_{pre,E_C} = \bar{u}^0_{pre,E_C} = \bar{u}_{E_C}$ and $\pi^q_{pre}(u)$ is sustained by randomization on the interval $\left(\bar{u}^q_{pre,E_C}, \tilde{u}^q_{pre,E_C}\right)$ for some $\tilde{u}^q_{pre,E_C} > \bar{u}^q_{pre,E_c}$. Denote s^q to be the slope of π^q_{pre} on this interval, and denote by s^0 the slope of π on this interval. It follows that $\lim_{q\to 0} s^q = s^0$.

To prove part (i.), it suffices to show that we cannot simultaneously have both $u^q_{trans,E_C,h}\left(\bar{u}^q_{pre,E_C}\right) = U_N$ and $u^q_{trans,E_C,\ell}\left(\bar{u}^q_{pre,E_C}\right) = U_N$. In order to get a contradiction, suppose to the contrary that $\left(\bar{u}^q_{pre,E_C}, \pi^q_{pre}\left(\bar{u}^q_{pre,E_C}\right)\right)$ is supported by cooperative empowerment and the pair of continuation payoffs $\left(u^q_{trans,E_C,h}\left(\bar{u}^q_{pre,E_C}\right), u^q_{trans,E_C,\ell}\left(\bar{u}^q_{pre,E_C}\right)\right)$ and $\left(u^q_{pre,E_C,h}\left(\bar{u}^q_{pre,E_C}\right), u^q_{pre,E_C,\ell}\left(\bar{u}^q_{pre,E_C}\right)\right)$, where

$$u^q_{trans,E_C,h}\left(\bar{u}^q_{pre,E_C}\right) = u^q_{trans,E_C,\ell}\left(\bar{u}^q_{pre,E_C}\right) = U_N.$$

Consider an alternative strategy profile that delivers equilibrium payoffs $(\hat{u}, \hat{\pi})$ on the frontier, and this point is sustained by cooperative empowerment with continuation payoffs given by $u_{trans,E_C,h} = u_{trans,E_C,\ell} = U_N + \varepsilon$ for some $\varepsilon > 0$ small and $u_{pre,E_C,h} = u^q_{pre,E_C,h}\left(\bar{u}^q_{pre,E_C}\right)$ and $u_{pre,E_C,\ell} = u^q_{pre,E_C,\ell}\left(\bar{u}^q_{pre,E_C}\right)$. The promise-keeping condition implies that

$$\hat{u} = \bar{u}^q_{pre,E_C} + \delta\varepsilon q$$

and, if we denote by r the slope between $(pb + (1-p)\,U_N, pB + (1-p)\,\Pi_N)$ and (U_N, Π_N),

$$\hat{\pi} = \pi^q_{pre}\left(\bar{u}^q_{pre,E_C}\right) + \delta q r \varepsilon.$$

Now, for any $U_N \le b$, there exists $\bar{\Pi}\,(U_N)$ such that for all $B < \Pi_N \le \bar{\Pi}\,(u_N)$, the slope $r > s^0/2$. Further, there exists $\bar{q}\,(U_N)$ such that for any $q \le \bar{q}\,(U_N)$, $s^q \in (3s^0/4, s^0)$. It then follows that

$$\hat{\pi} > \pi^q_{pre}\left(\bar{u}^q_{pre,E_C}\right) + \frac{1}{2}\delta q s^0 \varepsilon$$

and

$$\begin{aligned}
\pi^q_{pre}\left(\bar{u}^q_{pre,E_C} + \delta q \varepsilon\right) &= \pi^q_{pre}\left(\bar{u}^q_{pre,E_C}\right) + \delta q s^q \varepsilon \\
&\le \pi^q_{pre}\left(\bar{u}^q_{pre,E_C}\right) + \frac{3}{4}\delta q s^0 \varepsilon \\
&< \pi^q_{pre}\left(\bar{u}^q_{pre,E_C}\right) + \frac{1}{2}\delta q s^0 \varepsilon < \hat{\pi},
\end{aligned}$$

which implies that $(\hat{u}, \hat{\pi})$ lies above the point $\left(\hat{u}, \pi^q_{pre}(\hat{u})\right)$ because $s^0 < 0$, which is a contradiction.

To prove part (ii.), suppose that $\delta < \hat{\delta} = \frac{B-b}{2B-(1+p)b}$, so that $u_{E_C,\ell}(\bar{u}_{E_C}) < pb$. It suffices to show that for q sufficiently small, $u_{pre,E_C,h}\left(\bar{u}^q_{pre,E_C}\right) = B$. In order to get a contradiction, suppose that $u_{pre,E_C,h}\left(\bar{u}^q_{pre,E_C}\right) < B$ for all q. Define s^q as above. We know that $\lim_{q\to 0} s^q = s^0$. As above, suppose to the contrary that $\left(\bar{u}^q_{pre,E_C}, \pi^q_{pre}\left(\bar{u}^q_{pre,E_C}\right)\right)$ is supported by cooperative empowerment and continuation payoffs $\left(u^q_{trans,E_C,h}\left(\bar{u}^q_{pre,E_C}\right), u^q_{trans,E_C,\ell}\left(\bar{u}^q_{pre,E_C}\right)\right)$ and $\left(u^q_{pre,E_C,h}\left(\bar{u}^q_{pre,E_C}\right), u^q_{pre,E_C,\ell}\left(\bar{u}^q_{pre,E_C}\right)\right)$, where $u^q_{pre,E_C,h}\left(\bar{u}^q_{pre,E_C}\right) < B$. Consider an alternative strategy profile that delivers equilibrium payoffs $(\hat{u}, \hat{\pi})$ on the frontier, and this point is sustained by cooperative empowerment with continuation payoffs given by $\hat{u}_{trans,E_C,h}\left(\bar{u}^q_{pre,E_C}\right) = u_{trans,E_C,h}\left(\bar{u}^q_{pre,E_C}\right)$ and $\hat{u}_{trans,E_C,\ell}\left(\bar{u}^q_{pre,E_C}\right) = u_{trans,E_C,\ell}\left(\bar{u}^q_{pre,E_C}\right)$, for the transitional continuation payoffs, and $\hat{u}_{pre,E_C,h}\left(\bar{u}^q_{pre,E_C}\right) = u_{pre,E_C,h}\left(\bar{u}^q_{pre,E_C}\right) + \varepsilon$ and $\hat{u}_{pre,E_C,\ell}\left(\bar{u}^q_{pre,E_C}\right) = u_{pre,E_C,\ell}\left(\bar{u}^q_{pre,E_C}\right) + \varepsilon$ for the continuation payoffs that remain on the pre-opportunity frontier. This new strategy profile provides the agent with a payoff of

$$\hat{u} = \bar{u}^q_{pre,E_C} + \delta(1-q)\varepsilon$$

and the principal with a payoff of

$$\begin{aligned}\hat{\pi} \;=\; & \pi^q_{pre}\left(\bar{u}^q_{pre,E_C}\right) + \delta\left[p(1-q)\left(\pi^q_{pre}\left(\hat{u}_{pre,E_C,h}\right) - \pi^q_{pre}\left(u_{pre,E_C,h}\right)\right)\right] \\ & +\delta(1-p)(1-q)\left(\pi^q_{pre}\left(\hat{u}_{pre,E_C,\ell}\right) - \pi^q_{pre}\left(u_{pre,E_C,\ell}\right)\right).\end{aligned}$$

Moreover, this change preserves the agent's incentive constraint, and it is an equilibrium payoff.

Next, notice that

$$\pi^q_{pre}(\hat{u}) = \pi^q_{pre}(u) + s^q\delta(1-q)\varepsilon$$

and

$$\hat{\pi} \geq \pi^q_{pre}(u) + \delta(1-q)\varepsilon\left[p\pi^{q+}_{pre}\left(u_{pre,E_C,h}\left(\bar{u}^q_{pre,E_C}\right)\right) + (1-p)\pi^{q+}_{pre}\left(u_{pre,E_C,\ell}\left(\bar{u}^q_{pre,E_C}\right)\right)\right].$$

Therefore, we obtain a contradiction if

$$p\pi_{pre}^{q+}\left(u_{pre,E_C,h}\left(\bar{u}_{pre,E_C}^{q}\right)\right)+\left(1-p\right)\pi_{pre}^{q+}\left(u_{pre,E_C,\ell}\left(\bar{u}_{pre,E_C}^{q}\right)\right)>s^{q}.$$

Next, notice that there exists $\bar{q}\left(U_N,\Pi_N\right)$ such that if $q<\bar{q}\left(U_N,\Pi_N\right)$ and $\delta<\bar{\delta}\left(U_N,\Pi_N\right)$, then $\pi_{pre}^{q+}\left(u_{pre,E_C,\ell}\left(\bar{u}_{pre,E_C}^{q}\right)\right)=B/b$ and this inequality is satisfied if

$$p\pi_{pre}^{q+}\left(u_{pre,E_C,h}\left(\bar{u}_{pre,E_c}^{q}\right)\right)+\left(1-p\right)\frac{B}{b}>s^{0}.$$

The left-hand side of this inequality is weakly bigger than

$$p\pi_{pre}^{q-}\left(B\right)+\left(1-p\right)\frac{B}{b}\geq p\pi_{post}^{-}\left(B\right)+\left(1-p\right)\frac{B}{b},$$

so it suffices to show that $p\pi_{post}^{-}\left(B\right)+\left(1-p\right)\left(B/b\right)>s^{0}$.

By construction,

$$s^{0}=\frac{b-\pi_{pre}^{0}\left(\bar{u}_{E_C}\right)}{B-\bar{u}_{E_C}}\text{ and }\pi_{post}^{-}\left(B\right)=\frac{b-\pi_{post}\left(\bar{u}_{E_C}\right)}{B-\bar{u}_{E_C}}.$$

Since $\bar{u}_{E_C}=\left(1-\delta\right)b+\delta B$, we have that $B-\bar{u}_{E_C}=\left(1-\delta\right)\left(B-b\right)$. Further,

$$\pi_{post}\left(\bar{u}_{E_C}\right)-\pi_{pre}^{0}\left(\bar{u}_{E_C}\right)=\left(1-p\right)\delta\left[\pi_{post}\left(u_{post,E_C,\ell}\left(\bar{u}_{E_C}\right)\right)-\pi_{pre}^{0}\left(u_{E_C,\ell}\left(\bar{u}_{E_C}\right)\right)\right],$$

so the inequality becomes

$$\left(1-\delta\right)\left(B-b\right)\frac{B}{b}+\pi_{pre}^{0}\left(\bar{u}_{E_C}\right)-b>p\delta\left[\pi_{post}\left(u_{post,E_C,\ell}\left(\bar{u}_{E_C}\right)\right)-\pi_{pre}^{0}\left(u_{E_C,\ell}\left(\bar{u}_{E_C}\right)\right)\right].$$

By the proof of Proposition 1, $\pi_{pre}^{0}\left(\bar{u}_{E_C}\right)>b$. Finally, we can note that since $U_N>pb$ and $\Pi_N<\left(\frac{1}{p}+p-1\right)B$,

$$\pi_{post}\left(u_{post,E_C,\ell}\left(\bar{u}_{E_C}\right)\right)-\pi_{pre}^{0}\left(u_{E_C,\ell}\left(\bar{u}_{E_C}\right)\right)\leq\pi_{post}\left(pb\right)-pB<\left(\frac{1}{p}+p-1\right)B-pB;$$

further, because the incentive constraint holds with equality, and $u_{E_C,h}\left(\bar{u}_{E_C}\right)=$

B and $u_{E_C,\ell}(\bar{u}_{E_C}) \leq pb$, we have that

$$\frac{\delta}{1-\delta} \leq \frac{B-b}{B-pb}.$$

Combining these inequalities gives us the desired inequality.■