

Online Appendix for “The Political Economy of
Municipal Pension Funding”

Jeffrey Brinkman

Federal Reserve Bank of Philadelphia

Daniele Coen-Pirani

University of Pittsburgh

Holger Sieg

University of Pennsylvania and NBER

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A Proofs of Lemmas, Propositions, and Numerical Results

Note on the notation: equations with arabic numerals refer to the equations in the paper.

A.1 Inequality in Equation 33

In order to derive the inequality in equation (33), consider the price of land after a deviation, $\tilde{Q}(f, \tilde{f}'; f^*)$, given by equation (20). Replace the future price of land $Q(\tilde{f}'; f^*)$ into this equation:

$$\begin{aligned} \tilde{Q}(f, \tilde{f}'; f^*) &= (\psi + \beta)w - w^g - \frac{\tilde{f}'b^g}{R} - b^g(1 - f) + \\ &+ \frac{\kappa}{R} \left[(\psi + \beta)w - w^g - b^g - \frac{f^*b^g}{R} + b^g\tilde{f}' + \frac{\kappa}{R}Q(f^*; f^*) \right], \end{aligned} \quad (\text{A.1})$$

where $Q(f^*; f^*)$ is given by:

$$Q(f^*; f^*) = \frac{(\psi + \beta)w - w^g - b^g}{1 - \frac{\kappa}{R}} + b^gf^* \frac{1 - \frac{1}{R}}{1 - \frac{\kappa}{R}}.$$

Replace $Q(f^*; f^*)$ into (A.1) and collect terms to obtain:

$$\tilde{Q}(f, \tilde{f}'; f^*) = [(\psi + \beta)w - w^g - b^g] \frac{R}{R - \kappa} - \frac{b^g}{R}\tilde{f}'(1 - \kappa) + b^gf - b^gf^* \frac{\kappa}{R} \left(\frac{1 - \kappa}{R - \kappa} \right). \quad (\text{A.2})$$

Impose the condition that the downpayment constraint binds for all values of \tilde{f}' and f (equation 24), taking into account the equilibrium downpayment in equation (25):

$$\tilde{Q}(f, \tilde{f}'; f^*) > \frac{\beta R}{1 - \kappa}w. \quad (\text{A.3})$$

Notice that

$$\tilde{Q}(f, \tilde{f}'; f^*) \geq \tilde{Q}(f_{\min}, 1; f^*),$$

so it is necessary and sufficient for (A.3) to hold that:

$$\tilde{Q}(f_{\min}, 1; f^*) > \frac{\beta R}{1 - \kappa} w.$$

Replace $\tilde{Q}(f_{\min}, 1; f^*)$ from equation (A.2) in the inequality above and simplify to obtain equation (33). Q.E.D.

A.2 Parameters in the Example of Section II.C.

We have already set $R = 1.22$, $\kappa = 0.80$, and $b^g/R = \$24,000$. According to the U.S. Census, median household income in Chicago in 2010-2014 was $\$47,831$.¹ Nationally, mean income is about 1.3 times median income, so we assume that average household income in Chicago is $w = \$47,831 \times 1.3 = \$62,180$.² It follows that:

$$\frac{b^g}{w} = \frac{\$24,000}{\$62,180} \times R = 0.39 \times R.$$

To compute w^g/w , we make the conservative assumption that public sector employees are paid the same as private ones and that in Chicago they account for about 12 percent of aggregate non-farm employment.³ Hence we set $w^g/w = 0.12$. In the inequality (33) we also consider the case $f^* = 1$ which requires a higher cut-off value for ψ . We also notice that, according to the model (see equation (21)), a young household spends a fraction

$$\psi + \beta = \frac{l_t((1 + \tau_t)q_t - \kappa q_{t+1}/R)}{w}$$

of its income as a downpayment on housing. The average value of a house in Chicago is about $\$260,000$ (Davis and Palumbo, 2007) and property taxes are about 2 percent. If a

¹This figure is obtained from the U.S. Census' Quick Facts.

²The ratio 1.3 is from the Federal Reserve Bank of St Louis' FRED Blog, May 28, 2015.

³This figure is taken from the Bureau of Labor Statistics, Chicago Area Summary (2016).

household is able to borrow 80 percent of the value of the house, the downpayment is

$$\begin{aligned} \text{downpayment} &= \$260,000 (1.02) (0.2) \\ &= \$53,040. \end{aligned}$$

As a fraction of household income, the downpayment is:

$$\psi + \beta = \frac{\$53,040}{\$62,180} = 0.85.$$

Replacing these numbers in the inequality (33), we conclude that it is satisfied as long as the parameter ψ satisfies the condition $\psi > 0.764$.

A.3 Proof of Proposition 3

Assume that preferences take the form:

$$U(c_{yt}, l_{yt}, l_{ot+1}, c_{ot+1}) = (1 - \psi - \beta(1 + \theta)) \ln c_{yt} + \psi \ln l_{yt} + \beta (\ln c_{ot+1} + \theta \ln l_{ot+1}).$$

A.3.1 Frictionless Asset Market

In this case the budget constraint takes the form:

$$w = c_{yt} + \frac{c_{ot+1}}{R} + \left((1 + \tau_t) q_t - \frac{q_{t+1}}{R} - r_t \right) l_t + r_t l_{yt} + \frac{r_{t+1} l_{ot+1}}{R}.$$

The optimal choices of the agent are:

$$c_{yt} = (1 - \psi - \beta(1 + \theta)) w, \tag{A.4}$$

$$l_{yt} = \frac{\psi w}{r_t}, \tag{A.5}$$

$$c_{ot+1} = \beta R w, \tag{A.6}$$

$$l_{ot+1} = \frac{\beta R \theta w}{r_{t+1}}, \tag{A.7}$$

and the optimal choice of l_t must imply that in equilibrium:

$$(1 + \tau_t) q_t - \frac{q_{t+1}}{R} = r_t. \quad (\text{A.8})$$

Use equations (A.5) and (A.7) into the rental land market clearing condition $l_{yt} + l_{ot} = 1$ to obtain:

$$\frac{\psi w}{r_t} + \frac{\beta R \theta w}{r_t} = 1,$$

which pins down $r_t = r$ for all t . This implies that the indirect utility of a young agent is a constant. Replace r into (A.8) to get the equilibrium user cost of land. This is a version of equation (55) and the analysis that follows that equation in the main text applies. In particular, the price of land q_t is independent of a locality's pension funding policy.

A.3.2 Binding Downpayment Constraint

In this case, the budget constraints are:

$$\begin{aligned} w &= c_{yt} + r_t l_{yt} + (d_t - r_t) l_t, \\ c_{ot+1} + r_{t+1} l_{ot+1} &= q_{t+1} l_t, \end{aligned}$$

where we have already incorporated the restriction that $\kappa = 0$. Replace consumption when young and old in the objective function:

$$\begin{aligned} U(c_{yt}, l_{yt}, l_{ot+1}, c_{ot+1}) &= (1 - \psi - \beta(1 + \theta)) \ln(w - r_t l_{yt} - (d_t - r_t) l_t) + \psi \ln l_{yt} \\ &\quad + \beta (\ln(q_{t+1} l_t - r_{t+1} l_{ot+1}) + \theta \ln l_{ot+1}). \end{aligned}$$

The first-order condition for land consumption when young is:

$$\begin{aligned} l_{yt} &: \frac{(1 - \psi - \beta - \beta\theta) r_t}{w - r_t l_{yt} - (d_t - r_t) l_t} = \frac{\psi}{l_{yt}} \Rightarrow \\ r_t l_{yt} &= \frac{\psi}{1 - \beta - \beta\theta} (w - (d_t - r_t) l_t). \end{aligned} \quad (\text{A.9})$$

Similarly, land consumption when old satisfies:

$$l_{ot+1} : r_{t+1} l_{ot+1} = \frac{\theta}{1 + \theta} q_{t+1} l_t. \quad (\text{A.10})$$

The demand for land as an asset is such that:

$$l_t : \frac{(1 - \psi - \beta - \beta\theta)(d_t - r_t)}{w - r_t l_{yt} - (d_t - r_t) l_t} = \frac{\beta q_{t+1}}{q_{t+1} l_t - r_{t+1} l_{ot+1}}.$$

Replacing the first-order conditions (A.9) and (A.10), and simplifying we obtain:

$$l_t = \frac{\beta(1 + \theta)w}{d_t - r_t}.$$

Replace into the budget constraint when young and old to obtain:

$$\begin{aligned} c_{yt} &= w - r_t l_{yt} - (d_t - r_t) l_t = w(1 - \psi - \beta(1 + \theta)), \\ c_{ot+1} &= \frac{1}{1 + \theta} q_{t+1} l_t. \end{aligned}$$

Equilibrium in the market for land ownership ($l_t = 1$) pins down $d_t - r_t$:

$$\frac{\beta(1 + \theta)w}{d_t - r_t} = 1 \rightarrow d_t - r_t = \beta(1 + \theta)w. \quad (\text{A.11})$$

Equilibrium in the market for land consumption ($l_{yt} + l_{ot} = 1$) together with the above expression implies:

$$\frac{\psi w}{r_t} + \frac{\theta}{1 + \theta} \frac{q_t}{r_t} = 1$$

so we can solve for r_t as function of q_t (equation (45)):

$$r_t = \psi w + \frac{\theta}{1 + \theta} q_t. \quad (\text{A.12})$$

Combine (A.11), (A.12), and the definition of d_t to derive the dynamic equation for land

prices:

$$(1 + \tau_t) q_t = \psi w + \frac{\theta}{1 + \theta} q_t + \beta (1 + \theta) w.$$

Replace in it the government's budget constraint (8):

$$q_t \left(1 - \frac{\theta}{1 + \theta} \right) = \psi w + \beta (1 + \theta) w - w^g - \frac{f_{t+1} b^g}{R} - b^g (1 - f_t).$$

Simplify to obtain equation (44) in the main text:

$$q_t = (1 + \theta) \left\{ (\psi + \beta (1 + \theta)) w - w^g - \frac{f_{t+1} b^g}{R} - b^g (1 - f_t) \right\}.$$

Using this equation, it is straightforward to show that a policy deviation that increases pension funding leads to a smaller land price today and a higher land price in the following period. Replacing the optimal choices into the utility function, we derive the utilities of old and young agents in period t (up to some irrelevant constants):

$$\begin{aligned} V_t^{\text{old}} &\approx (1 + \theta) \ln q_t - \theta \ln r_t, \\ V_t^{\text{young}} &\approx -\psi \ln r_t + \beta V_{t+1}^{\text{old}}. \end{aligned}$$

We now show that the utility of the old is strictly increasing in q_t . Use equation (A.12) to replace r_t into V_t^{old} :

$$\begin{aligned} V_t^{\text{old}} &= (1 + \theta) \ln q_t - \theta \ln r_t \\ &= (1 + \theta) \ln q_t - \theta \ln \left(w\psi + \frac{\theta}{1 + \theta} q_t \right). \end{aligned}$$

Now, take its derivative with respect to q_t :

$$\frac{\partial V_t^{\text{old}}}{\partial q_t} = \frac{1 + \theta}{q_t} - \frac{\theta}{w\psi + \frac{\theta}{1 + \theta} q_t} \frac{\theta}{1 + \theta}.$$

This is strictly positive if and only if the following condition holds:

$$\frac{(1 + \theta)}{q_t} > \frac{\theta}{w\psi + \frac{\theta}{1+\theta}q_t} \frac{\theta}{1 + \theta},$$

which can be re-written as:

$$(1 + \theta)^2 w\psi + (1 + \theta) \theta q_t > \theta^2 q_t.$$

The latter always holds. Using the same argument, it is straightforward to show that the young's utility is strictly increasing in the future price of land and strictly decreasing in its current price.

A.4 Proof of Proposition 4

Assume that there is no borrowing, so $\kappa = 0$.

A.4.1 Notation

To save on notation, define:

$$\delta \equiv (\psi + \beta) \frac{w}{b^g} - \frac{w^g}{b^g} - 1,$$

and re-write equations (28) and (29) as:

$$Q(f; F) = \delta b^g - \frac{F(f) b^g}{R} + b^g f, \tag{A.13}$$

$$\tilde{Q}(f, \tilde{f}'; F) = \delta b^g - \frac{\tilde{f}' b^g}{R} + b^g f. \tag{A.14}$$

A.4.2 Equilibrium Given Policy Rule

Guess that the policy rule takes the affine form:

$$F(f) = \lambda + \phi f,$$

for some parameters (λ, ϕ) , which will have to be determined in equilibrium. Similarly, guess that the equilibrium land pricing function takes the form:

$$Q(f; F) = \pi b^g + \omega b^g f, \quad (\text{A.15})$$

for some constants (π, ω) to be determined. To solve for the latter, replace (A.15) into (A.13):

$$Q(f; F) = \delta b^g - \frac{(\lambda + \phi f) b^g}{R} + b^g f.$$

Collect the constant terms and those in f :

$$Q(f; F) = \left(\delta - \frac{\lambda}{R} \right) b^g + b^g \left(1 - \frac{\phi}{R} \right) f.$$

Impose consistency with the guess (A.15). First the slope:

$$\omega = 1 - \frac{\phi}{R}. \quad (\text{A.16})$$

Then, the intercept:

$$\pi = \delta - \frac{\lambda}{R}. \quad (\text{A.17})$$

A.4.3 Politico-Economic Equilibrium

Now solve for the equilibrium policy rule parameters (λ, ϕ) . The policymaker maximizes the weighted average of the old and young utilities:

$$\alpha \ln \tilde{Q}(f, \tilde{f}'; F) + (1 - \alpha) \beta \ln Q(\tilde{f}'; F), \quad (\text{A.18})$$

where:

$$\tilde{Q}(f, \tilde{f}'; F) = \delta b^g - \frac{\tilde{f}' b^g}{R} + b^g f, \quad (\text{A.19})$$

and

$$Q(\tilde{f}'; F) = \pi b^g + \omega b^g \tilde{f}'. \quad (\text{A.20})$$

The interior first-order condition is:

$$-\frac{\alpha}{\tilde{Q}(f, \tilde{f}'; F)} \frac{1}{R} + \frac{(1-\alpha)\beta\omega}{Q(\tilde{f}'; F)} = 0.$$

Simplify this equation to:

$$\alpha Q(\tilde{f}'; F) = (1-\alpha)\beta R\omega \tilde{Q}(f, \tilde{f}'; F).$$

Replace $Q(\tilde{f}'; F)$ and $\tilde{Q}(f, \tilde{f}'; F)$ from equations (A.20) and (A.19):

$$\alpha \left(\pi + \omega \tilde{f}' \right) = (1-\alpha)\beta R\omega \left(\delta + f - \frac{\tilde{f}'}{R} \right).$$

Solve for \tilde{f}' :

$$\alpha\pi + \alpha\omega\tilde{f}' = (1-\alpha)\beta R\omega\delta + (1-\alpha)\beta R\omega f - (1-\alpha)\beta R\omega\frac{\tilde{f}'}{R}$$

or

$$\tilde{f}'\omega(\alpha + (1-\alpha)\beta) = (1-\alpha)\beta R\omega\delta - \alpha\pi + (1-\alpha)\beta R\omega f$$

or

$$\tilde{f}' = \frac{(1-\alpha)\beta R\omega\delta - \alpha\pi}{\omega(\alpha + (1-\alpha)\beta)} + \frac{(1-\alpha)\beta R}{\alpha + (1-\alpha)\beta} f.$$

A.4.4 Impose Consistency

Now, impose consistency to obtain (λ, ϕ) :

$$\lambda = \frac{(1-\alpha)\beta R\omega\delta - \alpha\pi}{\omega(\alpha + (1-\alpha)\beta)}, \tag{A.21}$$

$$\phi = \frac{(1-\alpha)\beta R}{\alpha + (1-\alpha)\beta}. \tag{A.22}$$

Solve equation (A.21) after replacing (A.16) and (A.17):

$$\lambda = \frac{(1 - \alpha) \beta R \left(1 - \frac{\phi}{R}\right) \delta - \alpha \left(\delta - \frac{\lambda}{R}\right)}{\left(1 - \frac{\phi}{R}\right) (\alpha + (1 - \alpha) \beta)}.$$

Simplify:

$$\lambda \left(1 - \frac{\phi}{R}\right) (\alpha + (1 - \alpha) \beta) = (1 - \alpha) \beta R \left(1 - \frac{\phi}{R}\right) \delta - \alpha \delta + \frac{\alpha}{R} \lambda$$

or

$$\lambda \left[\left(1 - \frac{\phi}{R}\right) (\alpha + (1 - \alpha) \beta) - \frac{\alpha}{R} \right] = (1 - \alpha) \beta R \left(1 - \frac{\phi}{R}\right) \delta - \alpha \delta,$$

thus:

$$\lambda = \frac{((1 - \alpha) \beta (R - \phi) - \alpha) \delta}{\left(1 - \frac{\phi}{R}\right) (\alpha + (1 - \alpha) \beta) - \frac{\alpha}{R}}.$$

Replace ϕ :

$$\lambda = \frac{\left((1 - \alpha) \beta \left(R - \frac{(1 - \alpha) \beta R}{\alpha + (1 - \alpha) \beta}\right) - \alpha\right) \delta}{\left(1 - \frac{(1 - \alpha) \beta}{\alpha + (1 - \alpha) \beta}\right) (\alpha + (1 - \alpha) \beta) - \frac{\alpha}{R}}$$

and simplify again:

$$\lambda = \frac{\left(\frac{R\beta(1-\alpha)}{\alpha+(1-\alpha)\beta} - 1\right) \delta}{(1 - 1/R)}$$

or

$$\lambda = \frac{R\delta}{R - 1} \left(\frac{R\beta(1 - \alpha)}{\alpha + (1 - \alpha) \beta} - 1 \right).$$

More succinctly, using (A.22), it can be written as:

$$\lambda = \frac{R\delta}{R - 1} (\phi - 1).$$

A.4.5 Equilibrium Policy Rule

The equilibrium policy rule is therefore as follows:

$$f' = \frac{R\delta}{R-1} (\phi - 1) + \phi f,$$

where ϕ is defined in equation (A.22).

A.5 Proof of Lemma 1

Apply the implicit function theorem to equation (53) to get:

$$g'(q_{t+1}) = (1 - \kappa) \frac{v'(q_{t+1}(1 - \kappa)) + v''(q_{t+1}(1 - \kappa)) q_{t+1}(1 - \kappa)}{u_1(w - d_t, 1) - d_t u_{11}(w - d_t, 1) + u_{21}(w - d_t, 1)}. \quad (\text{A.23})$$

Notice that:

$$g'(q_{t+1}) < (1 - \kappa) \frac{v'(q_{t+1}(1 - \kappa))}{u_1(w - d_t, 1) - d_t u_{11}(w - d_t, 1) + u_{21}(w - d_t, 1)} < (1 - \kappa) \frac{v'(q_{t+1}(1 - \kappa))}{u_1(w - d_t, 1)},$$

where the first inequality follows from the facts that $v''(q_{t+1}(1 - \kappa)) < 0$ and the denominator of equation (A.23) is positive, while the second inequality follows from the fact that $-u_{11}(w - d_t, 1) \geq 0$ and $u_{21}(w - d_t, 1) \geq 0$. Q.E.D.

A.6 Proof of Proposition 5

Take the derivative of the indirect utility function in equation (56) with respect to q_{t+1} :

$$\frac{\partial U(w - g(q_{t+1}), 1, (1 - \kappa) q_{t+1})}{\partial q_{t+1}} = -u_1(w - g(q_{t+1}), 1) g'(q_{t+1}) + (1 - \kappa) v'((1 - \kappa) q_{t+1}).$$

Notice that this is strictly positive due to Lemma 1. Q.E.D.

A.7 Proof of Proposition 7

It follows directly from the analysis in the text. Q.E.D.

B Additional Results for the Case of a General Utility Function

B.1 Irrelevance of Pension Funding Policy Under Perfect Capital Markets

Formally, the basic statement is:

Proposition 1 *Without a downpayment constraint (or when the latter never binds), both the price of land and the indirect utility offered by a municipality are independent of the one-period deviation \tilde{f}' from f^* . As a result, both young and old agents are indifferent about alternative pension funding policies.*

Consider an arbitrary utility function $U(c_{yt}, l_t, c_{ot+1})$ with the standard properties. An agent's lifetime budget constraint is

$$c_{yt} + l_t (q_t (1 + \tau_t) - q_{t+1}/R) + c_{ot+1}/R = w.$$

Optimal choices, including land $L(u_t)$, depend on the user cost of land:

$$u_t \equiv (q_t (1 + \tau_t) - q_{t+1}/R).$$

Land market equilibrium requires that $L(u_t) = 1$, pinning down $u_t = u^*$ uniquely because $L(u_t)$ is strictly decreasing in u_t . It follows that an agent's lifetime utility, which depends only on u_t , is also a constant independent of pension funding. To verify that the current

price of land is also independent of a locality's pension funding policy, write the user cost in recursive form and solve for the current land price:

$$\tilde{Q}(f, \tilde{f}'; f^*) = u^* - w^g - \tilde{f}'b^g/R - b^g(1 - f) + Q(\tilde{f}'; f^*)/R. \quad (\text{B.1})$$

It is straightforward to verify that the price of land tomorrow is such that:

$$Q(\tilde{f}'; f^*) = u^* - w^g - f^*b^g/R - b^g(1 - \tilde{f}') + Q(f^*; f^*)/R$$

Replace $Q(\tilde{f}'; f^*)$ into equation (B.1) and simplify we obtain:

$$\tilde{Q}(f, \tilde{f}'; f^*) = u^* - w^g - b^g(1 - f) + (u^* - w^g - b^g - f^*b^g/R + Q(f^*; f^*)) / R.$$

This is independent of the policy deviation \tilde{f}' . Hence, the utility of the old generation is independent of the locality's pension funding policy. Q.E.D.

B.2 General Utility Function: Properties of the Demand for Land and the Indirect Utility Function

After replacing the budget constraints (2)-(3) into the utility function, the optimization problem of an agent is:

$$\max_{l_t, b_{t+1}} U \left(w - (1 + \tau_t) q_t l_t - \frac{b_{t+1}}{R}, l_t, q_{t+1} l_t + b_{t+1} \right), \quad (\text{B.2})$$

subject to the downpayment constraint (4). If the downpayment constraint binds ($b_{t+1} = -\kappa q_{t+1} l_t$), the optimization problem takes the form in equation (51). Formally, the land demand function $L(d_t, q_{t+1})$ and the associated indirect utility function $V(d_t, q_{t+1})$ have the properties summarized in the following proposition.

Proposition 2 (Properties of the demand for land and of the indirect utility function)

(a) *There exists a unique land demand function $L(d_t, q_{t+1})$ that solves problem (51).*

(b) *If the Inada conditions $u_1(c_y, l) \rightarrow +\infty$ as $c_y \rightarrow 0$ and $u_2(c_y, l) \rightarrow +\infty$ as $l \rightarrow 0$ hold, then the land demand function $L(d_t, q_{t+1})$ satisfies the first-order condition for l :*

$$-d_t u_1(w - d_t l, l) + u_2(w - d_t l, l) + v'(q_{t+1}(1 - \kappa)l) q_{t+1}(1 - \kappa) = 0. \quad (\text{B.3})$$

(c) *Under the assumptions in part (b), the downpayment constraint binds if and only if the Euler equation for consumption holds as an inequality:*

$$u_1(w - d_t L(d_t, q_{t+1}), L(d_t, q_{t+1})) > v'(q_{t+1}(1 - \kappa)L(d_t, q_{t+1})) R. \quad (\text{B.4})$$

(d) *Under the assumptions in part (b), the land demand function $L(d_t, q_{t+1})$ is strictly decreasing in d_t . The effect of q_{t+1} on the demand for land is ambiguous.⁴*

(e) *Under the assumptions in part (b), the indirect utility function $V(d_t, q_{t+1})$, defined in (52), is strictly decreasing in d_t and strictly increasing in q_{t+1} .*

(a) The function $U(w - d_t l, l, q_{t+1}(1 - \kappa)l)$ is continuous in l on the interval $[0, d_t/w]$. Therefore it achieves a maximum in this interval.

(b) The Inada conditions on the derivatives at $l = d_t/w$ and $l = 0$ rule out corner solutions in which the optimal demand for land is either d/w or zero. Therefore, the solution must be interior and satisfy the first-order condition (B.3):

$$-d_t u_1(w - d_t l_t, l_t) + u_2(w - d_t l_t, l_t) + v'(q_{t+1}l(1 - \kappa)) q_{t+1}(1 - \kappa) = 0. \quad (\text{B.5})$$

This equation admits a solution because of the Assumptions in part (b) of Proposition 2. Notice that the objective function $U(w - d_t l, l, q_{t+1}(1 - \kappa)l)$ is strictly concave in l_t because

⁴Specifically, it is strictly decreasing in q_{t+1} if and only if the absolute value of the elasticity of $v'(c_o)$ with respect to c_o is strictly larger than one.

its second derivative with respect to l_t is negative:

$$\Delta \equiv d_t^2 u_{11}(w - d_t l_t, l_t) - 2d_t u_{12}(w - d_t l_t, l_t) + u_{22}(w - d_t l_t, l_t) + v''(q_{t+1} l (1 - \kappa)) [q_{t+1} (1 - \kappa)]^2 < 0. \quad (\text{B.6})$$

This is true because, by Assumption 1:

$$\begin{aligned} u_{11}(w - d_t l_t, l_t) &\leq 0, \\ u_{22}(w - d_t l_t, l_t) &\leq 0, \\ u_{12}(w - d_t l_t, l_t) &\geq 0, \\ v''(q_{t+1} l (1 - \kappa)) &\leq 0, \end{aligned}$$

and at least one of the own-second derivatives is strictly negative. Thus, the solution to the first-order condition is unique.

(c) Consider the agent's optimization problem (B.2), without imposing that the downpayment constraint binds. Let μ denote the Lagrange multiplier associated with the constraint $b_{t+1} + \kappa q_{t+1} l_t \geq 0$. The interior first-order conditions of the problem are:

$$l_t : -(1 + \tau_t) q_t U_{1t} + U_{2t} + U_{3t} q_{t+1} + \mu \kappa q_{t+1} = 0, \quad (\text{B.7})$$

$$b_{t+1} : -U_{1t}/R + U_{3t} + \mu = 0. \quad (\text{B.8})$$

The downpayment constraint binds if and only if $\mu > 0$. Thus, according to equation (B.8), the Euler equation holds as an inequality (equation (B.4)) if and only if the downpayment constraint binds.

(d) The properties of the demand function can be proved using the implicit function theorem as applied to equation (B.5). First:

$$\frac{\partial L(d_t, q_{t+1})}{\partial d} = \frac{u_1(w - d_t l_t, l_t) - d_t l_t u_{11}(w - d_t l_t, l_t) + u_{12}(w - d_t l_t, l_t) l_t}{\Delta} < 0$$

because the numerator of this expression is positive and $\Delta < 0$. The other derivative is:

$$\frac{dL(d_t, q_{t+1})}{\partial q} = \frac{v'(q_{t+1}l(1-\kappa)) + v''(q_{t+1}l(1-\kappa))q_{t+1}l(1-\kappa)}{-\Delta} (1-\kappa)$$

which can be expressed as:

$$\frac{\partial L(d_t, q_{t+1})}{\partial q} = \frac{v'(q_{t+1}l(1-\kappa)) [1 - \varepsilon(q_{t+1}l(1-\kappa))]}{-\Delta} (1-\kappa), \quad (\text{B.9})$$

where

$$\varepsilon(q_{t+1}l(1-\kappa)) \equiv -\frac{v''(q_{t+1}l(1-\kappa))q_{t+1}l(1-\kappa)}{v'(q_{t+1}l(1-\kappa))} > 0$$

is the (positive) elasticity of $v'(c_{ot+1})$ with respect to c_{ot+1} . It follows that the derivative (B.9) is negative if and only if $\varepsilon(q_{t+1}l(1-\kappa)) > 1$. In terms of κ , we can write:

$$\frac{\partial L(d_t, q_{t+1})}{\partial \kappa} = \frac{\partial L(d_t, q_{t+1})}{\partial d_t} \frac{\partial d_t}{\partial \kappa} + \frac{v'(q_{t+1}l(1-\kappa)) + v''(q_{t+1}l(1-\kappa))q_{t+1}l(1-\kappa)}{\Delta} q_{t+1}.$$

Replacing $\partial L(d_t, q_{t+1})/\partial d_t$ from above and noticing that $\partial d_t/\partial \kappa = -q_{t+1}/R$, leads to

$$\begin{aligned} & \frac{\partial L(d_t, q_{t+1})}{\partial \kappa} \\ = & \frac{q_{t+1}}{R(-\Delta)} \left\{ \begin{array}{l} u_1(w - d_t l_t, l_t) - d_t l_t u_{11}(w - d_t l_t, l_t) + u_{12}(w - d_t l_t, l_t) l_t - Rv'(q_{t+1}l(1-\kappa)) \\ -Rv''(q_{t+1}l(1-\kappa))q_{t+1}l(1-\kappa) \end{array} \right\}, \end{aligned}$$

where $(-\Delta) > 0$. The term in brackets is also positive because the agent is constrained and so $u_1(w - d_t l_t, l_t) > Rv'(q_{t+1}l(1-\kappa))$.

(e) By the envelope theorem:

$$\begin{aligned} \frac{\partial V(d_t, q_{t+1})}{\partial d_t} &= -u_1(w - d_t L(d_t, q_{t+1}), L(d_t, q_{t+1})) L(d_t, q_{t+1}) < 0, \\ \frac{\partial V(d_t, q_{t+1})}{\partial q_{t+1}} &= v'(q_{t+1}L(d_t, q_{t+1})(1-\kappa)) L(d_t, q_{t+1})(1-\kappa) > 0. \end{aligned}$$

Q.E.D.

B.3 Binding Downpayment Constraint

The analysis of Section III proceeds under the assumption that the downpayment constraint in equation (5) is always binding both in equilibrium and following a deviation. As we had mentioned at the beginning of the section, outside of special cases, such as the logarithmic example of Section II, there are no simple conditions on the model's parameters that guarantee that this is indeed the case. It is, however, feasible to provide *sufficient* conditions that can be verified given specific utility functions and parameter values, such that the downpayment constraint is guaranteed to always be binding. These are contained in the following proposition.

Proposition 3 (Sufficient conditions for binding downpayment constraint) *Consider a politico-economic equilibrium characterized by the constant funding rule $f^* = F(f)$. Then, sufficient conditions for the downpayment constraint to be always binding, both in equilibrium and following a deviation from it, are:*

(a) *If the function $g(q_{t+1})$ is weakly increasing in q_{t+1} :*

$$u_1(w - g(Q(f_{\min}; f^*)), 1) > Rv'(Q(f_{\min}; f^*)(1 - \kappa)).$$

(b) *If the function $g(q_{t+1})$ is weakly decreasing in q_{t+1} :*

$$u_1(w - g(Q(1; f^*)), 1) > Rv'(Q(f_{\min}; f^*)(1 - \kappa)).$$

As proved in Proposition 2, part (c), the downpayment constraint is binding when

$$u_1(w - d_t, 1) > v'(q_{t+1}(1 - \kappa))R,$$

where we are considering the local economy in a situation in which the land market is in equilibrium ($l_t = 1$). Replace d_t from equation (54) and re-write this equation using the

recursive notation:

$$u_1 \left(w - g \left(Q \left(\tilde{f}'; f^* \right) \right), 1 \right) > Rv' \left(Q \left(\tilde{f}'; f^* \right) (1 - \kappa) \right).$$

For the downpayment constraint to always be binding we need the inequality above to hold for all $\tilde{f}' \in [f_{\min}, 1]$, which includes the politico-economic equilibrium case $\tilde{f}' = f^*$. Notice that, since $Q \left(\tilde{f}'; f^* \right)$ is strictly increasing in \tilde{f}' and $v'' \leq 0$, we can write:

$$v' \left(Q \left(f_{\min}; f^* \right) (1 - \kappa) \right) \geq v' \left(Q \left(\tilde{f}'; f^* \right) (1 - \kappa) \right).$$

(a) Since $u_{11} \leq 0$, if the function $g(\cdot)$ is increasing in its argument, we know that:

$$u_1 \left(w - g \left(Q \left(\tilde{f}'; F \right) \right), 1 \right) \geq u_1 \left(w - g \left(Q \left(f_{\min}; f^* \right) \right), 1 \right)$$

for all $\tilde{f}' \in [f_{\min}, 1]$. Therefore, if the function $g(\cdot)$ is increasing, we can write:

$$\begin{aligned} u_1 \left(w - g \left(Q \left(\tilde{f}'; F \right) \right), 1 \right) &\geq u_1 \left(w - g \left(Q \left(f_{\min}; f^* \right) \right), 1 \right) > Rv' \left(Q \left(f_{\min}; f^* \right) (1 - \kappa) \right) \\ &\geq Rv' \left(Q \left(\tilde{f}'; f^* \right) (1 - \kappa) \right). \end{aligned}$$

Thus, as long as the middle inequality holds, the agent is constrained.

(b) Conversely, if the function $g(\cdot)$ is decreasing in its argument, we know that:

$$u_1 \left(w - g \left(Q \left(\tilde{f}'; F \right) \right), 1 \right) \geq u_1 \left(w - g \left(Q \left(1; f^* \right) \right), 1 \right)$$

and the same argument applies with:

$$\begin{aligned} u_1 \left(w - g \left(Q \left(\tilde{f}'; F \right) \right), 1 \right) &\geq u_1 \left(w - g \left(Q \left(1; f^* \right) \right), 1 \right) > Rv' \left(Q \left(f_{\min}; f^* \right) (1 - \kappa) \right) \\ &\geq Rv' \left(Q \left(\tilde{f}'; f^* \right) (1 - \kappa) \right). \end{aligned}$$

Q.E.D.

For example, if the utility function takes the quasi-linear form:

$$U = c_{yt} + \phi(l_t) + \beta c_{ot+1}, \tag{B.10}$$

with $\phi'(l) > 0$ and $\phi''(l) < 0$, the marginal utility of consumption when young is a constant equal to 1, while the marginal utility of consumption when old is simply β . Thus, the downpayment constraint is always binding if $\beta R < 1$.⁵

C Additional Extensions and Results

In this appendix we consider two extensions of the model. In the first one we consider alternative forms for the downpayment constraint. In the second we consider a version of the model with geographic mobility.

C.1 Alternative Forms of the Downpayment Constraint

In this section we consider and analyze two alternative formalizations of the downpayment constraint considered in the main text. The first one makes the constraint a function of the current price of land, instead of its future price. In the second one, agents face a higher interest rate when borrowing than when lending. In the former case, we are able to solve for equilibrium under a binding constraint only if utility is logarithmic. In the latter case we are able to solve for equilibrium for a general utility function. In this case, differently from the benchmark model, young agents' utility is always independent of the locality's funding policy. Old agents always prefer to maximize current land prices by setting pension funding at its lower possible level.

⁵For this quasi-linear case the function $g(q_{t+1}) = \phi'(1) + \beta(1 - \kappa)q_{t+1}$. Notice that, in this example, the condition $\beta R < 1$ is also necessary for the downpayment constraint to bind.

C.1.1 Downpayment Constraint with Current Land Price (Log Utility Case)

The downpayment constraint is now:

$$b_{t+1} \geq -\kappa q_t l_t.$$

For the case of log utility, the analysis in the main text goes through after redefining

$$d_t = (1 + \tau_t - \kappa/R) q_t. \quad (\text{C.1})$$

Land market clearing ($l_t = 1$) then pins down $d_t = d^*$, which can be used to solve for q_t :

$$q_t = \frac{d^*}{1 - \kappa/R} - \frac{\tau_t q_t}{1 - \kappa/R}.$$

Replacing the government's budget constraint (8) for $\tau_t q_t$ we obtain:

$$q_t = \frac{d^*}{1 - \kappa/R} - \frac{w^g}{1 - \kappa/R} - \frac{1}{1 - \kappa/R} \frac{f_{t+1} b^g}{R} - \frac{1}{1 - \kappa/R} b^g (1 - f_t).$$

The analysis of the politico-economic equilibrium is then straightforward and confirms our results. Specifically, the current price of land decreases in pension funding (f_{t+1}), while the future price increases.

The lifetime utility of a young agent can be written as:

$$V_t^{\text{young}} = (1 - \psi - \beta) \ln(1 - \psi - \beta) w + \beta \ln(q_{t+1} - \kappa q_t).$$

It follows that young agents prefer the maximum funding policy in order to maximize q_{t+1} and minimize q_t .

C.1.2 Higher Borrowing Rate (General Utility Case)

Suppose that preferences take the form:

$$U(c_{yt}, l_t, c_{ot+1}) = u(c_{yt}, l_t) + v(c_{ot+1}).$$

Agents face an interest rate R/κ when borrowing and R when lending (with $\kappa < 1$). Consider, for the sake of the argument, that a young agent would like to borrow, in which case the budget constraint is:

$$w = c_{yt} + d_t l_t + \frac{c_{ot+1}}{\kappa^{-1}R},$$

where the d_t is defined as in the main text:

$$d_t = (1 + \tau_t) q_t - \frac{q_{t+1}}{\kappa^{-1}R}.$$

Replace the budget constraints into the utility function:

$$U(c_{yt}, l_t, c_{ot+1}) = u\left(w - d_t l_t - \frac{c_{ot+1}}{\kappa^{-1}R}, l_t\right) + v(c_{ot+1}).$$

The first-order condition for land and consumption when old are:

$$\begin{aligned} l_t &: -d_t u_1\left(w - d_t l_t - \frac{c_{ot+1}}{\kappa^{-1}R}, l_t\right) + u_2\left(w - d_t l_t - \frac{c_{ot+1}}{\kappa^{-1}R}, l_t\right) + v'(c_{ot+1}) = 0, \\ c_{ot+1} &: -u_1\left(w - d_t l_t - \frac{c_{ot+1}}{\kappa^{-1}R}, l_t\right) \frac{1}{\kappa^{-1}R} + v'(c_{ot+1}) = 0. \end{aligned}$$

Imposing land market clearing $l_t = 1$, we obtain:

$$\begin{aligned} l_t &: -d_t u_1\left(w - d_t - \frac{c_{ot+1}}{\kappa^{-1}R}, 1\right) + u_2\left(w - d_t - \frac{c_{ot+1}}{\kappa^{-1}R}, 1\right) + v'(c_{ot+1}) = 0, \\ c_{ot+1} &: -u_1\left(w - d_t - \frac{c_{ot+1}}{\kappa^{-1}R}, 1\right) \frac{1}{\kappa^{-1}R} + v'(c_{ot+1}) = 0. \end{aligned}$$

One can solve for c_{ot+1} from the second equation as a function of d_t :

$$c_{ot+1} = C(d_t)$$

and replace the resulting function in the first equation:

$$-d_t u_1 \left(w - d_t - \frac{C(d_t)}{\kappa^{-1}R}, 1 \right) + u_2 \left(w - d_t - \frac{C(d_t)}{\kappa^{-1}R}, 1 \right) + v'(C(d_t)) = 0.$$

This equation pins down the equilibrium downpayment:

$$d_t = d^*.$$

Thus, in equilibrium:

$$(1 + \tau_t) q_t - \frac{q_{t+1}}{\kappa^{-1}R} = d^*,$$

and the analysis follows in the same way as in the main text. Specifically, the current land price increases as pension funding declines and an old policymaker would want to set pension funding to the minimum allowed level. Differently from the main text, however, the lifetime utility of a young agent is:

$$V^{\text{young}}(d^*) = u \left(w - d^* - \frac{C(d^*)}{\kappa^{-1}R}, 1 \right) + v(C(d^*)),$$

so it is not affected by the locality's funding policy.

C.2 Geographic Mobility

Intuitively, geographic mobility should act as a force that dampens the effect of reducing pension funding on young agents' utility and, consequently, on the price of land. This intuition is correct within the context of our model. In particular, with perfect mobility, lower pension funding leads to a smaller increase in land prices in a locality than with a geographically fixed population. In addition, young agents are insulated from the effect of

reduced pension funding by a locality on utility. However, young agents are *not* insulated if *all* localities follow the same policy, as they do in the symmetric general equilibrium of the model. In such case, a policy intervention by a higher authority that dictates minimum funding levels produces the same effect as in Section 4.2.2, i.e. it increases the welfare of the young at the expense of the utility of the old generation.

We consider the case in which young agents' labor mobility is perfect, in the sense that a locality would not be able to attract *any* young agents if it offered less than some lifetime utility V^* . We start by considering one locality in isolation taking V^* as given, but later endogenize V^* by imposing an economy-wide market clearing condition for the young population. The timing of events is as follows. A location funding policy is chosen first, then young agents choose where to reside, and how much land and consumption to demand. Finally, the market for land clears. Thus, the policymaker fully anticipates the effect of her choices on the measure of young agents that choose to reside in the locality. Given that young agents are fully mobile and always attain lifetime utility V^* , they are indifferent about the pension funding policy of any locality they are considering as potential place of residence. Thus, we only focus on the case in which the policymaker is an old agent.

Relative to the case of exogenous population, with endogenous young population the land market equilibrium condition becomes:

$$n_t L(d_t, q_{t+1}) = 1, \tag{C.2}$$

where n_t denotes the endogenous measure of young agents who are attracted to the location. In equilibrium, the young have to be indifferent between living in the locality or living elsewhere and obtaining lifetime utility V^* :

$$V(d_t, q_{t+1}) = V^*, \tag{C.3}$$

where the indirect utility function is defined in equation (52). Since the indirect utility function is strictly decreasing in d_t (Proposition 2, part (e)), equation (C.3) can be written

as:

$$d_t = h(q_{t+1}). \quad (\text{C.4})$$

The function $h(\cdot)$ plays the same role as the function $g(\cdot)$ introduced in Section 4 for the case of fixed population. Apply the implicit function theorem to equation (C.3) to compute the derivative of $h(\cdot)$:

$$h'(q_{t+1}) = (1 - \kappa) \frac{v'((1 - \kappa)q_{t+1}L(d_t, q_{t+1}))}{u_1(w - d_tL(d_t, q_{t+1}), L(d_t, q_{t+1}))}. \quad (\text{C.5})$$

Follow the same steps as in Section 4, to obtain the derivative of land prices with respect to pension funding \tilde{f}' for the case of endogenous young population:⁶

$$\frac{\partial \tilde{Q}(f, \tilde{f}', f^*)}{\partial \tilde{f}'} = \underbrace{-\frac{b^g}{R}}_{\text{current taxes}} + \underbrace{\frac{\kappa b^g}{R}}_{\text{borrowing}} + \underbrace{h'(Q(\tilde{f}', F))}_{\text{effect of resale value of land}} b^g, \quad (\text{C.6})$$

with $h'(\cdot)$ replacing $g'(\cdot)$. Since the derivative in equation (C.5) is strictly larger than $g'(q_{t+1})$ (Lemma 1), reducing pension funding leads to a smaller increase in current land prices when young agents are mobile than when they are not so. The intuition is that, following a reduction in the future price of land brought about by a decline in \tilde{f}' , the location becomes less attractive to prospective young residents. The loss of young population contributes to reduce current land prices. This additional mechanism reduces, but does not fully offset, the extent of the increase in current land prices following a reduction in pension funding. In order to compute the overall effect on land prices, replace equation (C.5) into (C.6), and rearrange:

$$\frac{\partial \tilde{Q}(f, \tilde{f}', f^*)}{\partial \tilde{f}'} = -\frac{b^g(1 - \kappa)}{R} (1 - R \times MRS_t^c) < 0, \quad (\text{C.7})$$

where the negative sign is due to the fact that $R \times MRS_t^c < 1$, since young agents are constrained. The only difference with respect to the case of a fixed population is that, with a mobile population, the derivative on the left-hand side of equation (C.7) is equal to - rather

⁶See the analog equation (4.13) for the case of fixed population.

than strictly less than - the term on the right-hand side.

Thus, as in the case of fixed population, the current land price declines in response to an increase in pension funding \tilde{f}' . The optimal policy deviation for an old policymaker is therefore to set $\tilde{f}' = f_{\min}$. This is the same result obtained when population is fixed (Proposition 7).

In general equilibrium, the measure of young agents born in each period needs to settle somewhere in the economy. Therefore, if all locations are homogeneous, each of them absorbs a measure one of young agents:

$$n_t = 1.$$

This condition pins down the equilibrium utility level V^* . That is, geographic mobility does not insulate young agents from the effects of reduced pension funding if *all localities* pursue the same funding policy. Specifically, since in general equilibrium $n_t = 1$, land market clearing requires that:

$$L(d_t, q_{t+1}) = 1.$$

Thus, the analysis of Section 4 applies and the lifetime utility achieved by a young agent must equal to

$$V^* = \bar{V}(g(q_{t+1}), q_{t+1}),$$

with $\bar{V}(g(q_{t+1}), q_{t+1})$ defined in equation (4.6). That is, the lifetime utility achieved by a young agent in this economy is increasing in the future price of land q_{t+1} (Proposition 5). In turn, the future price of land Q^* , defined in equation (4.10), is increasing in f^* . Recall that when the policymaker is old, $f^* = f_{\min}$. It follows that the equilibrium utility level V^* achieved by a young agent is lowest when old agents are in charge of setting pension funding policy. This is the same result as in the benchmark model with fixed population.

D Empirical Results: Robustness Analysis

In Table D.1 we report regression results corresponding to using as independent variable of interest the homeownership rate of households under 55 in 1990 instead of 2012.

Table D.1: Robustness: Lagged Measures of Age and Ownership

	UAAL/population		UAAL/income	
% owners under 55 (1990)	-79.0** (13.1)	-38.0** (12.7)	-0.37** (0.06)	-0.23** (0.06)
controls ^a	NO	YES	NO	YES
R^2	0.18	0.31	0.21	0.34
	UAAL/revenues		UAAL/house values	
% owners under 55 (1990)	-2.30** (0.66)	-1.66** (0.78)	-0.25** (0.04)	-0.22** (0.05)
controls ^a	NO	YES	NO	YES
R^2	0.06	0.16	0.20	0.36
Number of cities	160	160	160	160

Robust standard errors in parenthesis. ** p-value<0.05

^aControls include population density, liabilities per capita, the ratio of median income and median house values, log city population, percentage population change between 2000 and 2012, and regional dummies.

In Table D.2 we report regression results corresponding to alternative definitions of the explanatory variable of interest. In particular, we vary the age cutoff between young and old, and we also consider the role of home ownership. Each estimate represents a different regression with UAAL/population as the dependent variable but with different explanatory variables. All controls are included in each regression. The first column shows the sensitivity

of the baseline results to age cutoffs. The second column performs the same exercise, but considers the age distribution for all households instead of only homeowners. The third column reports the results of regressing the UAAL/population measure on the percent of households that are renters.

Table D.2: Robustness: Effect of age cutoffs and ownership on baseline estimates.

	Home owners	All households	Renters
% Under 35	-78.03* (45.85)	-14.42 (18.35)	-6.72 (18.85)
% Under 45	-58.81** (23.23)	-26.40 (20.52)	-3.08 (18.22)
% Under 55	-39.63** (14.00)	-44.10* (25.55)	2.35 (16.46)
% Under 65	-30.32** (11.93)	-52.64 (33.90)	10.47 (13.80)

This table shows the robustness of baseline parameter estimates to age cutoffs and home ownership. Each point estimate represents a separate regression using UAAL/population as the dependent variable and different dummy variables as proxies for the young population. Each regression includes controls include population density, liabilities per capita, the ratio of median income and median house values, log city population, percentage population change between 2000 and 2012, and regional dummies. Robust standard errors in parenthesis. ** p-value < 0.05 , * p-value<0.1

In Table D.3 we report regression results corresponding to dropping from the sample the 10 cities with the lowest homeownership rates. These are: Boston, Hartford, Jersey City, Miami, New Haven, New York City, Newark, Providence, Los Angeles, and San Francisco.

Table D.3: Robustness: Remove Low-Ownership Cities

	UAAL/population		UAAL/income	
% owners under 55	-51.2** (15.1)	-32.4** (13.2)	-0.26** (0.06)	-0.22** (0.06)
controls ^a	NO	YES	NO	YES
R^2	0.06	0.12	0.09	0.18
	UAAL/revenues		UAAL/house values	
% owners under 55	-0.78 (0.62)	-0.31 (0.71)	-0.18** (0.04)	-0.21** (0.05)
controls ^a	NO	YES	NO	YES
R^2	0.00	0.08	0.09	0.29
Number of cities	158	155	158	155

The ten cities with the lowest overall ownership rates were removed. Robust standard errors in parenthesis.

** p-value<0.05

^aControls include population density, liabilities per capita, the ratio of median income and median house values, log city population, percentage population change between 2000 and 2012, and regional dummies.