Domestic Policies and Sovereign Default

Online Appendix

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This material is for a separate, on-line appendix and not intended to be printed with the paper.

A Theory

A.1 Derivations

In order to characterize the solution to the household's problem, let χ and ψ denote the Lagrange multipliers associated with constraints (7) and (2), respectively. The necessary first-order conditions with respect to (c^N, c^T, h, m') for an interior solution are

$$u_N - p^N(\chi + \psi) = 0, \tag{33}$$

$$u_T - e\chi = 0, \tag{34}$$

$$-v_\ell + \chi(1-\tau)w = 0, \tag{35}$$

$$\beta\mathbb{E}\left[V'_m | B, \mathscr{I}, s\right] - \chi(1+\mu) = 0, \tag{36}$$

where V_m denotes the partial derivative of V with respect to the individual state variable, m. The corresponding envelope condition implies that $V_m = \chi + \psi$. From (33) and (34) we can solve for the Lagrange multipliers,

$$\chi = \frac{u_T}{e}, \tag{37}$$

$$\psi = \frac{u_N}{p^N} - \frac{u_T}{e}. \tag{38}$$

Replacing these expressions in (35) and (36) yields (8). Using (6) to replace e in (38) and imposing $\psi \geq 0$ yields (12).

The government budget constraint

Take the government budget constraint (5), multiply both sides by $F_N c^N$, use the market clearing for labor, $h = F(y^N, y^T)$, and conditions (9), (10) and (11) to obtain

$$\tau F(y^N, y^T) + F_N(\mu c^N - g - \gamma) - (F_T/p^T)(p^T y^T - c^T) \geq 0.$$

Next, replace the tax rate, τ, using (13) and the money growth rate, μ, using (14) to obtain the government budget constraint in a competitive equilibrium,

$$[1 - (F_T/p^T)(v_\ell/u_T)]F(c^N + g, y^T) - F_N(c^N + g + \gamma) + \beta(F_T/p^T)\mathbb{E}\left[u'_N c^{N\prime} | B, \mathscr{I}, s\right]/u_T$$
$$- (F_T/p^T)(p^T y^T - c^T) \geq 0.$$

Since $F(y^N, y^T) = F_N y^N + F_T y^T = F_N(c^N + g) + F_T y^T$ we obtain

$$(F_T/p^T)\left\{c^T - (v_\ell/u_T)F(c^N + g, y^T) + \beta\mathbb{E}\left[u'_N c^{N\prime} | B, \mathscr{I}, s\right]/u_T\right\} - \gamma F_N \geq 0,$$

which after multiplying both sides by $u_T(p^T/F_T)$ implies (15).

A.2 Value Functions and Extreme Value Shocks

Given the policy functions, we can define the value functions $V^P(B,s)$, $V^D(s)$ as follows:

$$
\begin{aligned}
V^P(B,s) &= u(\mathscr{C}^N(B,s),\mathscr{C}^T(B,s)) + v(1-F(\mathscr{C}^N(B,s)+\mathscr{G}(B,s),\mathscr{Y}^T(B,s))) + \vartheta(\mathscr{G}(B,s)) \\
&\quad + \beta\mathbb{E}[\mathscr{V}(\mathscr{B}(B,s),s')|s], \\
V^D(s) &= u(\bar{\mathscr{C}}^N(s),\bar{\mathscr{C}}^T(s)) + v(1-F(\bar{\mathscr{C}}^N(s)+\bar{\mathscr{G}}(s),\bar{\mathscr{Y}}^T(s))) + \vartheta(\bar{\mathscr{G}}(s)) \\
&\quad + \beta\mathbb{E}[\delta\mathscr{V}(B^D,s') + (1-\delta)V^D(s')|s],
\end{aligned}
$$

for all (B,s).

As described in the main text, the utility shocks ε^P and ε^D affect the government's decision to repay or default. The assumptions on the distribution of these shocks imply that their difference has mean zero and is distributed logistic; i.e., $\varepsilon = \varepsilon^P - \varepsilon^D$ follows

$$
\mathscr{F}(\varepsilon) = \frac{\exp[\varepsilon/\kappa]}{1+\exp[\varepsilon/\kappa]},
$$

where $\kappa > 0$ is the scale parameter of the distribution, which will be useful to control the variance of the ε shocks.

Recall that the repayment probability $\mathscr{P}(B,s)$ can be expressed as

$$
\mathscr{P}(B,s) = \Pr(V^P(B,s) - V^D(s) \geq \varepsilon^D - \varepsilon^P).
$$

Following McFadden (1973), this expectation has the following closed-form expression:

$$
\mathscr{F}(V^P(B,s)-V^D(s)) = \frac{\exp[(V^P(B,s)-V^D(s))/\kappa]}{1+\exp[(V^P(B,s)-V^D(s))/\kappa]}
$$

and so,

$$
\mathscr{F}(V^P(B,s)-V^D(s)) = \frac{\exp[V^P(B,s)/\kappa]}{\exp[V^P(B,s)/\kappa]+\exp[V^D(s)/\kappa]}.
$$

Therefore,

$$
\mathscr{P}(B,s) = \frac{\exp[V^P(B,s)/\kappa]}{\exp[V^P(B,s)/\kappa]+\exp[V^D(s)/\kappa]}, \tag{39}
$$

which, in turn, implies

$$
\frac{\partial\mathscr{P}(B,s)}{\partial B} = \frac{\partial V^P(B,s)}{\partial B}\frac{\mathscr{P}(B,s)(1-\mathscr{P}(B,s))}{\kappa}. \tag{40}
$$

Next, we can derive a closed-form expression for the expectation of the value function with respect to the utility shocks:

$$
\mathscr{V}(B,s) = E_\varepsilon[\hat{\mathscr{V}}(B,s,\varepsilon^P,\varepsilon^D)] = \kappa\ln\left\{\exp[V^P(B,s)/\kappa]+\exp[V^D(s)/\kappa]\right\}. \tag{41}
$$

Using this expression, we can easily see that

$$\frac{\partial \mathscr{V}(B,s)}{\partial B} = \mathscr{P}(B,s)\frac{\partial V^P(B,s)}{\partial B}. \tag{42}$$

We use (39) and (41) in the formulation of the government's problem. We use (40) and (42) when characterizing the choice of debt.

A.3 Full characterization of the government's problem

We characterize the problem of the government allowing for transfers to be positive, i.e., $\gamma \geq 0$, and the non-negativity constraint (19) to be potentially bind. These more general assumptions do not alter the characterization of debt choice, (29). To simplify some of the notation below, let $\Gamma(c^N, c^T, y^T, g; s) \equiv u_T p^T(F_N/F_T)$, which is an expression that shows up in the government budget and non-negativity constraints. Note that $\Gamma_T = d\Gamma/dc^T = \Gamma \times (u_{TT}/u_T) < 0$, while the convexity of F implies that $\Gamma_N = \Gamma_g = \Gamma \times (F_{NN}/F_N - F_{NT}/F_T) > 0$ and $\Gamma_y = \Gamma \times (F_{NT}/F_N - F_{TT}/F_T) < 0$. Recall that $\Phi \equiv v_\ell - v_{\ell\ell}F(c^N + g, y^T) > 0$.

Since the problems in repayment and default are functionally identical with respect to (c^N, c^T, y^T, g), we focus on (PP)–(19). Let ξ, λ and ζ be the Lagrange multipliers associated with the constraints (17), (18) and (19), respectively. The necessary first-order conditions with respect to (c^N, c^T, y^T, g) are

$$u_N - v_\ell F_N - \lambda(F_N\Phi + \gamma\Gamma_N) + \zeta(u_{NN} - \Gamma_N) = 0, \tag{43}$$

$$u_T - \xi + \lambda(u_T + u_{TT}c^T - \gamma\Gamma_T) - \zeta\Gamma_T = 0, \tag{44}$$

$$-v_\ell F_T + \xi p^T - \lambda(F_T\Phi + \gamma\Gamma_y) - \zeta\Gamma_y = 0, \tag{45}$$

$$-v_\ell F_N + \vartheta_g - \lambda(F_N\Phi + \gamma\Gamma_g) - \zeta\Gamma_g = 0. \tag{46}$$

Since $\Gamma_N = \Gamma_g$, (43) and (46) imply

$$\zeta = -\frac{(u_N - \vartheta_g)}{u_{NN}} \tag{47}$$

Suppose $\zeta = 0$. Then, (47) implies $u_N = \vartheta_g$ and (43) and (45) solve for the remaining Lagrange multipliers

$$\begin{aligned} \lambda &= \frac{u_N - v_\ell F_N}{F_N\Phi + \gamma\Gamma_N}, \\ \xi &= \frac{u_N F_T \Phi + \gamma[v_\ell F_T\, \Gamma_N + (u_N - v_\ell F_N)\Gamma_y]}{p^T(F_N\Phi + \gamma\Gamma_N)}, \end{aligned}$$

Hence, (44) implies

$$F_T\Phi(u_N - \Gamma) - \gamma\left[(u_T p^T - v_\ell F_T)\Gamma_N - (u_N - v_\ell F_N)\Gamma_y\right] = p^T(u_N - v_\ell F_N)(u_T + u_{TT}c^T - \gamma\Gamma_T), \tag{48}$$

where we used the definition of Γ to simplify the expression.

When $\zeta > 0$, (47) implies $u_N > \vartheta_g$. Conditions (43) and (45) solve for the remaining Lagrange multipliers

$$\lambda = \frac{\vartheta_g - v_\ell F_N + (u_N - \vartheta_g)(\Gamma_N/u_{NN})}{F_N\Phi + \gamma\Gamma_N}$$

$$\xi = \frac{1}{p^T(F_N\Phi + \gamma\Gamma_N)}\left[\vartheta_g(F_T\Phi + \gamma\Gamma_y) - \gamma v_\ell(F_N\Gamma_y - F_T\Gamma_N) + \left(\frac{u_N - \vartheta_g}{u_{NN}}\right)(F_T\Gamma_N - F_N\Gamma_y)\Phi\right]$$

Hence, (44) implies

$$\begin{aligned} & u_T(F_N\Phi + \gamma\Gamma_N) + (\vartheta_g - v_\ell F_N)(u_T + u_{TT}c^T - \gamma\Gamma_T) \\ & + \left(\frac{u_N - \vartheta_g}{u_{NN}}\right)\left[\Gamma_N(u_T + u_{TT}c^T - \gamma\Gamma_T) - (F_T\Gamma_N - F_N\Gamma_y)(\Phi/p^T) + (F_N\Phi + \gamma\Gamma_N)\Gamma_T\right] \\ & = \frac{1}{p^T}\left[\vartheta_g(F_T\Phi + \gamma\Gamma_y) - \gamma v_\ell(F_N\Gamma_y - F_T\Gamma_N)\right] \end{aligned} \tag{49}$$

A.4 Proofs

Proof of Proposition 1. (i) Consider the case in which $\gamma = 0$. Note that (19) implies $u_N - \Gamma \geq 0$. Since $\Phi > 0$, the left-hand side of (27) is non-negative. Next, $\lambda > 0$ implies $u_N - v_\ell F_N > 0$. Hence, $u_N - \Gamma > 0$ if and only if $u_T + u_{TT}c^T > 0$, while $u_N - \Gamma = 0$ if and only if $u_T + u_{TT}c^T = 0$. If preferences are such that $u_T + u_{TT}c^T < 0$, then (27) cannot be satisfied—a contradiction. In this case, $\zeta > 0$ and therefore, (19) binds.

(ii) Consider the case in which $\gamma > 0$ suppose $\zeta = 0$. From (24) we can write $\xi = u_T + \lambda(u_T + u_{TT}c^T - \gamma\Gamma_T)$ and then rearrange (25) as follows

$$F_T(v_\ell + \lambda\Phi) - p^T u_T = p^T\lambda[u_T + u_{TT}c^T - \gamma(\Gamma_T + \Gamma_y)]. \tag{50}$$

From (23) $F_T(v_\ell + \lambda\Phi) = (u_N - \lambda\gamma\Gamma_N)(F_T/F_N)$. Thus, (50) implies

$$u_N(F_T/F_N) - p^T u_T = p^T\lambda[u_T + u_{TT}c^T + \gamma[\Gamma_N(F_T/F_N) - \Gamma_T - \Gamma_y]. \tag{51}$$

Recall that $\Gamma_N > 0$, $\Gamma_T < 0$ and $\Gamma_y < 0$. Hence, if $\frac{-u_{TT}c^T}{u_T} \leq 1$, then $u_T + u_{TT}c^T \geq 0$ and so, the right hand-side of (51) is strictly positive. Then, $u_N(F_T/F_N) - p^T u_T > 0$ and (19) is satisfied with strict inequality.

Now suppose (19) is satisfied with equality while $\zeta = 0$. Then (51) implies $u_T + u_{TT}c^T = -\gamma[\Gamma_N(F_T/F_N) - \Gamma_T - \Gamma_y] < 0$ and so, $\frac{-u_{TT}c^T}{u_T} > 1$. Therefore, it follows by continuity that there exists some $\hat{\sigma}^T > 1$ such that (19) is satisfied with equality for all $\frac{-u_{TT}c^T}{u_T} \leq \hat{\sigma}^T$. In this case, policy is away from the Friedman rule if $\frac{-u_{TT}c^T}{u_T} < \hat{\sigma}^T$. □

Proof of Proposition 2. The envelope condition of problem (PP) implies $\frac{\partial V^P(B,s)}{\partial B} = -\xi$. The derivative of (16) with respect to B' is:

$$\frac{\partial[Q(B',s)B']}{\partial B'} = \mathbb{E}\left\{\frac{\mathscr{P}(B',s')}{1+r}\left[1 + \frac{\partial V^P(B',s')}{\partial B'}\frac{(1-\mathscr{P}(B',s'))(B' - Q^D(s')B^D)}{\kappa}\right]\Bigg| s\right\}. \tag{52}$$

Using (40)–(42) and (52), we obtain the following expressions for the first and second terms of (28)

$$\begin{aligned}\frac{\partial \mathbb{E}[\mathscr{V}(B',s')|s]}{\partial B'} &= -\mathbb{E}[\mathscr{P}(B',s')\xi'|s] \\ \frac{\partial Q(B',s)B'}{\partial B'} &= \mathbb{E}\left\{\frac{\mathscr{P}(B',s')}{1+r}\left[1-\frac{(1-\mathscr{P}(B',s'))(B'-Q^D(s')B^D)\xi'}{\kappa}\right]\Big|s\right\}.\end{aligned}$$

The last term in (28) requires additional work. Given that $\mathscr{P}(B',s')$ is the probability of transitioning from $\mathscr{I}=P$ to $\mathscr{I}'=P$ for all (B',s'), we can write

$$\mathbb{E}\left[u_N'c^{N\prime}|P,s\right] = \mathbb{E}\left[\mathscr{P}(B',s')u_N'\mathscr{C}^{N\prime} + (1-\mathscr{P}(B',s'))\bar{u}_N'\bar{\mathscr{C}}^{N\prime}\big|s\right],$$

where $u_N'\mathscr{C}^{N\prime}$ corresponds to the repayment state tomorrow, $\mathscr{I}'=P$, and $\bar{u}_N'\bar{\mathscr{C}}^{N\prime}$ corresponds to the default state tomorrow, $\mathscr{I}'=D$. Note that the expectation on the right-hand side (only conditional on s) is taken with respect to s'. We can take the derivative of the expression above with respect to B' to obtain

$$\frac{\partial \mathbb{E}\left[u_N'c^{N\prime}|P,s\right]}{\partial B'} = \mathbb{E}\left[\mathscr{P}(B',s')(u_N'+u_{NN}'\mathscr{C}^{N\prime})\mathscr{C}_B^{N\prime} + (u_N'\mathscr{C}^{N\prime}-\bar{u}_N'\bar{\mathscr{C}}^{N\prime})\mathscr{P}_B'\big|s\right],$$

where $\mathscr{C}_B^{N\prime}$ and $\mathscr{P}_B'$ denote the derivatives of $\mathscr{C}^N(B',s')$ and $\mathscr{P}(B',s')$ with respect to B'. Recall that, when in default, allocations are not a function of B, i.e., $\bar{C}^N(s)$ and so $\bar{C}_B^{N\prime}=0$. From (40), we have an analytical expression for $\mathscr{P}_B'$ and from the envelope condition, $\frac{\partial V^P(B,s)}{\partial B}=-\xi$. Thus, we obtain

$$\frac{\partial \mathbb{E}\left[u_N'c^{N\prime}|P,s\right]}{\partial B'} = \mathbb{E}\left\{\mathscr{P}(B',s')\left[(u_N'+u_{NN}'\mathscr{C}^{N\prime})\mathscr{C}_B^{N\prime} - \frac{(u_N'\mathscr{C}^{N\prime}-\bar{u}_N'\bar{\mathscr{C}}^{N\prime})(1-\mathscr{P}(B',s'))\xi'}{\kappa}\right]\Big|s\right\}.$$

We now have all the elements to write the equation characterizing debt choice. □

Proof of Proposition 3. Consider the EG real allocation $(\hat{B}',\hat{c}^N,\hat{c}^T,\hat{y}^T,\hat{g})$ that solves the problem (PPEP) where lump-sum, unconstrained taxes $\mathscr{T}$ make the government budget constraint becomes (31). In order to prove this result, we first solve the problem (PPEP) and then we construct the monetary policy and taxes $(\hat{\mu},\hat{\tau})$ as well as the prices $(\hat{p}^N,\hat{e},\hat{w})$ that support this allocation as an equilibrium in our setting.

The necessary first-order conditions characterizing the EG real allocation are

$$\hat{u}_N = \hat{v}_\ell\hat{F}_N, \tag{53}$$

$$\hat{v}_\ell\hat{F}_T = p^T\hat{u}_T, \tag{54}$$

$$\hat{\vartheta}_g = \hat{v}_\ell\hat{F}_N, \tag{55}$$

which imply $\frac{\hat{u}_N\hat{F}_T}{p^T}=\hat{u}_T\hat{F}_N$; i.e., the non-negative constraint (19), which we ignore to derive the

EG real allocation, is satisfied with equality. The balance of payment implies

$$p^T \hat{y}^T - \hat{c}^T + \hat{Q}(B', s)\hat{B}' - \hat{B} \quad = \quad 0. \tag{56}$$

We now construct the policies and prices that support the EG real allocation, we have that the price of non-tradable goods and wages are determined by

$$\hat{p}^N = \frac{1}{\hat{c}^N},$$
$$\hat{w} = \frac{\hat{p}^N}{\hat{F}_N},$$

while the exchange rate is determined by

$$\hat{e} = \frac{\hat{p}^N}{p^T}\frac{\hat{F}_T}{\hat{F}_N},$$

The monetary policy has to be tailored so that

$$\hat{\mu} = \frac{\beta \mathbb{E}\left[\hat{u}'_N \hat{c}^{N\prime} | B, \mathscr{I}, s\right]}{\hat{u}_T \hat{c}^N p^T (\hat{F}_N / \hat{F}_T)} - 1,$$

as it has to decentralize money holdings such that $m' = m = 1$. Since $\frac{\hat{u}_N \hat{F}_T}{p^T} = \hat{u}_T \hat{F}_N$, we obtain

$$\hat{\mu} = \frac{\beta \mathbb{E}\left[\hat{u}'_N \hat{c}^{N\prime} | B, \mathscr{I}, s\right]}{\hat{u}_N \hat{c}^N} - 1. \tag{57}$$

On the other hand, taxes are given by

$$\hat{\tau} = 1 - \frac{\hat{v}_\ell}{\hat{u}_T}\frac{\hat{F}_T}{p^T} = 0.$$

Finally, lump-sum transfers are designed to make the budget constraint of the government (31) hold so that

$$\hat{\mathscr{T}} = \hat{p}^N(\hat{g} + \gamma) - \hat{\mu} + \hat{e}(p^T \hat{y}^T - \hat{c}^T).$$

□

Proof of Proposition 4. As shown in Proposition 3, the real EG allocation implies zero labor taxes when monetary policy is given by (32). Thus, combining the balance of payments with the government budget constraint when lump-sum taxes are not available implies

$$\hat{e}[\hat{Q}(\hat{B}', s)\hat{B}' - B] = \hat{p}^N(\hat{g} + \gamma) - \hat{\mu}, \tag{58}$$

i.e., a non-linear first-order difference equation in domestic debt, B.

First, consider a steady state in an environment with no aggregate shocks s. Since $\hat{g} + \gamma \geq 0$ and $\hat{\mu} \leq 0$ (see (57) above), imply $\hat{e} > 0$ and $\hat{Q}(\hat{B}', s) < 1$, it follows that any steady state would require $B < 0$; i.e., the government must accumulate a sufficiently large amount of assets to finance its expenditures. This asset position would never be reached, as (58) implies that the amount of B is strictly increasing and positive when the initial stock of debt is positive.

Consider now the general case in a stochastic environment. Observe that since $\hat{p}^N(\hat{g}+\gamma) - \hat{\mu} \geq 0$, then $B > 0$ implies that $B' > 0$ as

$$\frac{\hat{B}'}{(1+r)} \geq \hat{Q}(\hat{B}',s)\hat{B}' = B + \frac{\hat{p}^N}{\hat{e}}(\hat{g}+\gamma) - \frac{\hat{\mu}}{\hat{e}}, \tag{59}$$

and so $\hat{B}' \geq (1+r)B + (1+r)\left(\frac{\hat{p}^N}{\hat{e}}(\hat{g}+\gamma) - \frac{\hat{\mu}}{\hat{e}}\right)$. Therefore, as $r > 0$, the sequence of debt for this allocation is strictly increasing and unbounded as long as $B_0 > 0$. We argue that this cannot be an equilibrium path. To see this, define $\bar{y}^T$ as the unique solution to $F(0,\bar{y}^T) = 1$, i.e., the highest level of the tradable good that can be produced as $h = 1$ and $\hat{y}^N = \hat{c}^N + \hat{g} = 0$.

Let $\bar{p^T} = \max p^T$ and conjecture that $Q(B',s) = 0$ for all $B' \geq \frac{(1+r)}{r}\bar{p^T}\bar{y}^T$ and all s. From the balance of payments, non-default tradable consumption can be written as

$$\hat{c}^T = \hat{Q}(\hat{B}',s)\hat{B}' + p^T\hat{y}^T - B \leq \max_{B'}\{\hat{Q}(\hat{B}',s)\hat{B}'\} + \bar{p}^T\bar{y}^T - B \leq \frac{1}{1+r}\frac{(1+r)}{r}\bar{p^T}\bar{y}^T + \bar{p}^T\bar{y}^T - B. \tag{60}$$

Therefore, if $B = \frac{(1+r)}{r}\bar{y}^T$, then (60) implies that non-default tradable consumption cannot be positive and leads to a contradiction, as the EG allocation is interior and consequently the outcome must be default; i.e., $Q\left(\frac{(1+r)}{r}\bar{y}^T, s\right) = 0$ for all s. Therefore, since Q is decreasing, $Q(B',s) = 0$ for all $B' \geq \frac{(1+r)}{r}\bar{y}^T$ and all s and validates the conjecture.

To conclude the proof, observe that as the sequence of debt would be strictly increasing and unbounded, it would be larger than $\frac{(1+r)}{r}\bar{y}^T$ in finite time and thus contradicts (59) since $Q(B',s) = 0$ for all $B' \geq \frac{(1+r)}{r}\bar{y}^T$ and all s . □

A.5 General cash-in-advance constraint

In this section we consider a version of our model with a more general cash-in-advance constraint. Following the rationale provided by Lucas (1980) one could think of an agent in our economy as being split in three: a shopper for non-tradable goods, an importer and a worker. Shopper and worker set off at the same time, which means that the shopper cannot use money acquired in the period to make purchases, i.e., needs to use money acquired in previous periods as rationalized by constraint (2). In our benchmark model, the importer is assumed to trade on credit, so money is not necessary to purchase imported goods. Alternatively, if the importer sets out after the worker comes back or if imports are purchased on credit but required to be settled in domestic money by the end of the period, then a cash-in-advance constraint on the purchase of imported goods would not bind, as some money would still need to be carried over to purchase non-tradable goods at the beginning of the following period. The budget constraint would thus be sufficient to guarantee that the household has enough money to purchase and settle imports. Hence, for a cash-in-advance constraint to also be relevant for imported goods, we must assume that the importer sets out at the same time as the shopper and is required to settle purchases in domestic currency immediately and not on credit.[34]

In the analysis that follows, we assume that proportions θ^N and θ^T of expenditures on non-tradable and imported goods, respectively, must be supported by previously accumulated money balances. These proportions could be lower, equal or greater than one. One important caveat is that θ^N and θ^T must be different since, otherwise, the problem is not well defined. For the symmetric case when $\theta^N = \theta^T$, we study the limiting case as $\theta^T \to \theta^N = \theta$. We find that this case implies that monetary policy follows the Friedman rule, which would be inconsistent with the data on monetary policy and inflation in emerging markets.

Consider then a general version of the cash-in-advance constraint:

$$\theta^N p^N c^N + \theta^T e c^T \leq m. \tag{61}$$

where $\theta^N \geq 0$, $\theta^T \geq 0$ and $\theta^N \neq \theta^T$.

The solution to the representative household's problem must necessarily satisfy

$$\frac{(1-\tau)w}{\theta^N - \theta^T}\left(\frac{u_T\theta^N}{e} - \frac{u_N\theta^T}{p^N}\right) = v_\ell \tag{62}$$

$$(1+\mu)\left(\frac{u_T\theta^N}{e} - \frac{u_N\theta^T}{p^N}\right) = \beta\mathbb{E}\left[\frac{u'_N(1-\theta^T)}{p^{N\prime}} - \frac{u'_T(1-\theta^N)}{e'}\Big|B,\mathscr{I},s\right] \tag{63}$$

plus constraints (61) and the budget constraint (7).

In equilibrium, we obtain:

$$p^N = \frac{F_N}{\theta^N c^N F_N + \theta^T c^T F_T/p^T}. \tag{64}$$

$$w = \frac{1}{\theta^N c^N F_N + \theta^T c^T F_T/p^T}. \tag{65}$$

$$e = \frac{F_T/p^T}{\theta^N c^N F_N + \theta^T c^T F_T/p^T}. \tag{66}$$

Finally, the Lagrange multiplier associated with the cash-in-advance constraint must be non-negative, which implies the following equilibrium condition:

$$\left(\frac{1}{\theta^N - \theta^T}\right)\left(\frac{u_N}{F_N} - \frac{u_T}{F_T/p^T}\right) \geq 0. \tag{67}$$

Note that this condition would not be well-defined if $\theta^T = \theta^N = \theta$.

[34]Note that the argument still applies if imports are purchased with foreign currency but currency exchange markets are subject to similar timing and settlement assumptions we described.

The expressions for the tax rate and the money growth rate are, respectively,

$$\tau = 1 - \frac{(\theta^N - \theta^T)v_\ell}{\theta^N p^T(u_T/F_T) - \theta^T(u_N/F_N)}, \tag{68}$$

$$\mu = \frac{\beta \mathbb{E}\left\{\left[\theta^N c^{N\prime} F_N' + \theta^T c^{T\prime}(F_T'/p^{T\prime})\right]\left[(1-\theta^T)(u_N'/F_N') - (1-\theta^N)p^{T\prime}(u_T'/F_T')\right] \big| B, \mathscr{I}, s\right\}}{\left[\theta^N c^N F_N + \theta^T c^T (F_T/p^T)\right]\left[\theta^N p^T(u_T/F_T) - \theta^T(u_N/F_N)\right]} - 1. \tag{69}$$

Finally, following the steps described in Section A.1 for the benchmark case, we obtain the government budget constraint in a monetary equilibrium:

$$\begin{aligned}&\left[\theta^N p^T(u_T/F_T) - \theta^T(u_N/F_N)\right]\left[(1-\theta^N)F_N c^N + (1-\theta^T)(F_T/p^T)c^T - F_N\gamma\right] - (\theta^N - \theta^T)v_\ell F(y^N, y^T) \\ &+ \beta \mathbb{E}\left\{\left[\theta^N c^{N\prime} F_N' + \theta^T c^{T\prime}(F_T'/p^{T\prime})\right]\left[(1-\theta^T)(u_N'/F_N') - (1-\theta^N)p^{T\prime}(u_T'/F_T')\right] \big| B, \mathscr{I}, s\right\} \geq 0. \end{aligned} \tag{70}$$

To summarize, policy rates τ and μ are given by (68) and (69), respectively. Implementability is restricted by the non-negativity constraint, (67), the government budget constraint (70) and the balance of payment (4).

We now consider the limiting case, $\theta^T \to \theta^N = \theta$. The inequality condition (67) implies that

$$\lim_{\theta^T \to \theta^N} \left(\frac{u_N}{F_N} - \frac{u_T}{F_T/p^T}\right) = 0. \tag{71}$$

If this were not the case, the limiting Lagrange multiplier on the cash-in-advance constraint would not a finite number. We can now derive expressions for taxes and money growth. Starting with (68), we rewrite it as

$$\tau = 1 - \frac{(\frac{\theta^N}{\theta^T} - 1)v_\ell}{\frac{\theta^N}{\theta^T} p^T(u_T/F_T) - (u_N/F_N)}. \tag{72}$$

When $\frac{\theta^N}{\theta^T} = 1$, the equation above is not well-defined as (71) leads to $0/0$ when $\frac{\theta^N}{\theta^T} = 1$. To analyze the case with $\frac{\theta^N}{\theta^T} = 1$, we apply the L'Hôpital's rule, which implies that

$$\tau = 1 - \frac{v_\ell F_T}{p^T u_T} = 1 - \frac{v_\ell F_N}{u_N} \tag{73}$$

as $\theta^T \to \theta^N = \theta$. Note that this expression is the same as the one in the benchmark case, equation (13). Also, given (71), the tax rate may also be expressed as $\tau = 1 - \frac{v_\ell F_N}{u_N}$.

As with the tax rate, we can rewrite the expression for the money growth rate using (69) as follows,

$$\mu = \frac{\beta \mathbb{E}\left\{\left[\frac{\theta^N}{\theta^T} c^{N\prime} F_N' + c^{T\prime}(F_T'/p^{T\prime})\right]\left[(\frac{1}{\theta^T} - 1)(u_N'/F_N') - (\frac{1}{\theta^T} - \frac{\theta^N}{\theta^T})p^{T\prime}(u_T'/F_T')\right] \big| B, \mathscr{I}, s\right\}}{\left[\frac{\theta^N}{\theta^T} c^N F_N + c^T(F_T/p^T)\right]\left[\frac{\theta^N}{\theta^T} p^T(u_T/F_T) - (u_N/F_N)\right]} - 1.$$

When $\theta^T = \theta^N = \theta$, it follows that

$$\mu = \frac{\beta \mathbb{E}\left\{\left[c^{N\prime}F_N' + c^{T\prime}(F_T'/p^{T\prime})\right](\frac{1}{\theta}-1)\left[(u_N'/F_N') - p^{T\prime}(u_T'/F_T')\right]\big|B,\mathscr{I},s\right\}}{\left[c^N F_N + c^T(F_T/p^T)\right]\left[p^T(u_T/F_T) - (u_N/F_N)\right]} - 1.$$

which by (71) would not be well-defined. Hence, apply the L'Hôpital's rule for $\theta^T \to \theta^N = \theta$ and obtain

$$\mu = \frac{\beta \mathbb{E}\left[c^{N\prime}F_N' p^{T\prime}(u_T'/F_T')\big|B,\mathscr{I},s\right]}{c^N F_N p^T(u_T/F_T)} - 1$$

which, given (71), simplifies to

$$\mu = \frac{\beta \mathbb{E}\left[c^{N\prime}u_N'\big|B,\mathscr{I},s\right]}{c^N u_N} - 1.$$

As mentioned above, we find that monetary policy implements the Friedman rule when $\theta^N = \theta^T$.

As a final note, we made the derivations above using $\theta^T \to \theta^N = \theta$. The same results apply when instead using $\theta^N \to \theta^T = \theta$.

B Quantitative Results

B.1 Definition of macroeconomic aggregates

- Nominal GDP (in pesos, normalized by the money stock),

$$Y_t = e_t p_t^T y_t^T + p_t^N y_t^N.$$

- GDP in foreign currency (USD),

$$Y_t^{USD} = p_t^T y_t^T + \frac{1}{e_t} p_t^N y_t^N.$$

- The GDP deflator (in pesos, normalized by the money stock)

$$P_t^y = \left(\frac{e p^T y^T}{Y}\right) e_t p_t^T + \left(\frac{p^N y^N}{Y}\right) p_t^N$$

- Real GDP,

$$Y_t^R = \frac{Y_t}{P_t^y}.$$

- Consumption expenditures (in pesos, normalized by the money stock),

$$C_t = e_t c_t^T + p_t^N c_t^N.$$

- Consumption price index (in pesos, normalized by the money stock),

$$P_t^c = \left(\frac{e c^T}{C}\right) e_t + \left(\frac{p^N c^N}{C}\right) p_t^N.$$

- Inflation, measured as the change in the consumption price index,

$$\pi_t = \frac{P_t^C}{P_{t-1}^C}(1 + \mu_{t-1}) - 1.$$

- Currency depreciation

$$\Delta_t = \frac{e_t}{e_{t-1}}(1 + \mu_{t-1}) - 1$$

Note that inflation and currency depreciation are corrected by the money growth rate, since prices are normalized by the money stock.

B.2 Identification

To provide a heuristic proof of identification, we compute how each parameter would change if we change one target at a time by 10 percent. The results, presented in Table 8, justify the link between parameters and targets mentioned in the calibration section. The first column shows how each parameter changes when we target a default rate 10 percent larger, i.e., 1.1 percent instead of 1 percent. Note that the more significant change is for κ. By increasing κ 9.70 percent

Table 8: Percent change in each parameter when a target is increased by 10 percent (p^T shock)

	Target increased by 10 percent								
	Default	Debt	Haircut	G	Hours	Exports	Inflation	Transfers	Real GDP
κ	**9.70**	9.03	6.09	-11.61	54.59	6.22	7.09	-22.02	4.88
B^D	0.00	**10.00**	-4.39	0.00	0.00	-4.13	0.00	0.00	10.00
ω_1	3.35	9.34	**5.84**	-4.83	9.88	4.72	2.61	-9.39	10.00
α^G	0.02	0.24	0.01	**11.79**	-4.31	-7.97	-0.28	3.30	4.88
α^H	0.01	0.07	0.00	-1.81	**-27.33**	-6.86	0.33	-1.52	4.88
α^N	0.02	0.23	0.01	0.59	-4.31	**-9.38**	-0.28	3.30	0.00
β	-0.01	-0.09	0.00	-1.60	4.50	1.31	**0.66**	-3.20	0.00
γ	0.00	0.00	0.00	0.00	0.00	-0.15	0.00	**10.00**	10.00
A	0.00	0.00	0.00	0.00	-9.09	-1.03	0.00	0.00	**10.00**

Note: Each number represents the percentage change in the parameter when the target is increased by 10 percent.

and adjusting all the parameters (except ω_1) very slightly, the model can replicate all the targets perfectly. Thus, we selected κ as the critical parameter to obtain the default rate.

In the second column of Table 8, we present the percent change in each parameter that would allow the model to replicate a debt to GDP ratio 10 percent larger. In addition to the change in κ, which we already show is key to replicating the default rate, the most substantial change is in B^D followed by ω_1. Clearly, these parameters are important to determine debt because they determine the benefits and costs of default. We pick B^D for debt because its adjustment is larger and highlights ω_1 for matching haircuts because it is the larger adjustment to match the haircut among the remaining parameters.

Continuing with the same logic, we connect each parameter in Table 8 with a moment.

B.3 Calibration

When we calibrated the models with aggregate shocks we only targeted debt over GDP, the default probability, and the debt haircut when defaulting. As shown in Table 9, other moments are all very close to the data, except for inflation that it is equal to 4.425% for the model with TFP shocks, while it is 3.835% in the data.

B.4 Computational procedure

The equilibrium is solved globally, using the equations derived above. The algorithm uses 21 equally spaced gridpoints for debt, between 0 and 1.5, and 21 gridpoints for p^T or TFP estimated with the Tauchen method with a bandwidth of 2 (i.e., a multiple 2 of the unconditional standard deviation). To compute expectations we interpolate policy functions with a modified Akima piecewise cubic Hermite interpolation in a dense grid of 20001 equally spaced points in debt and 501 equally spaced points in p^T or TFP for which we estimate its corresponding

Table 9: Averages of simulated data in economies with aggregate shocks

	Data	Shocks to p^T		Shocks to TFP	
		Benchmark	Low default	Benchmark	Low default
Real GDP	1.000	0.996	0.998	0.992	0.995
Inflation, %	3.835	4.180	3.950	4.425	4.185
Transfers / GDP	0.117	0.118	0.117	0.118	0.118
Exports / GDP	0.209	0.208	0.207	0.209	0.209
Employment / Population	0.581	0.580	0.581	0.580	0.580
Gov Consumption / GDP	0.134	0.134	0.134	0.135	0.135
Debt / GDP	0.187	0.173	0.173	0.169	0.173
Default probability	0.020	0.022	0.008	0.021	0.008
Haircut, Share of Debt	0.305	0.257	0.259	0.231	0.259

transition matrix with the Tauchen method.[35] We experimented with different grid sizes, different interpolations schemes (e.g., linear), and different ways of computing the expectations (e.g., computing its corresponding integral). The final choice of grid points and methods is the most efficient allocation of computing time. For example, either computing the integral, or increasing the size of the grids, deliver the same solution but require more computing time.

B.5 The choice of ρ, σ, θ_N, and θ_T

This section discusses the choice of $\sigma^N = \sigma^T = 0.5$, $\rho = 1.5$, $\theta_N = 1$, and $\theta_T = 0$ by comparing the results for alternative parameters. In particular, we consider three alternatives: (i) $\sigma^N = \sigma^T = 1.5$; (ii) $\rho = 2$; and (iii) $\theta_N = 0.95$ and $\theta_T = 0.40$. Recall that we set $\sigma^N = \sigma^T = 0.5$ because $\sigma^T < 1$ is sufficient for the non-negativity constraint in the government's problem to be satisfied with strict inequality (it is also necessary when transfers γ are zero). The value of ρ determines the elasticity of substitution between y^N and y^T in the cost function and is set to 1.5. A number larger than 1 ensures that the production possibilities frontier is concave. The cash-in-advance constraint in our benchmark model assumes money is only used to finance the purchases of non-tradable goods. In Section A.5 we presented a generalized version of this constraint. Using the notation from the general version, our benchmark case sets $\theta_N = 1$ and $\theta_T = 0$.

We re-calibrate the model with p^T shocks to ensure that each alternative calibration fits the targeted moments well. Next, we evaluate the non-targeted statistics in Tables 4, 6 and 7. We present all the moments in Table 10, where we added at the bottom the average for all moments of the absolute percentage distance between data and model. This last statistic is revealing of how better our preferred calibration fits these moments. This measure of the fit of non-targeted moments in twice as large for the economy with $\rho = 2$ and a bit more in the economy with

[35]The interpolated value at a query point is based on a piecewise function of polynomials with degree at most three evaluated using the values of neighboring grid points in each respective dimension. The Akima formula is modified to avoid overshoots.

$\sigma = 1.5$. It is only slightly higher for the economy with $\theta_N = 0.95$ an $\theta_T = 0.40$.

We find that the economy with $\sigma = 1.5$ does the worse job a fitting data moments because it generates a positive correlation of trade balance/Y with output, a negative correlation of real expenditure with output, no correlation of real consumption and real output, and almost acyclical inflation. In the case of the economy with $\rho = 2$, the poorest fit is due to the fact that it generates a positive correlation of exports/Y with output and pro-cyclical tax rates.

Table 10: Business Cycles and Policy Statistics for Alternative Parameters (p^T model)

	Data	Benchmark	$\rho = 2.0$	$\sigma = 1.5$	$\theta_N = 0.95$ $\theta_T = 0.40$
Std. Dev. (y)	0.038	0.016	0.006	0.008	0.016
Std. Dev. (trade balance/Y)	0.035	0.017	0.008	0.007	0.018
Std. Dev. (c) / Std. Dev. (y)	1.130	2.502	3.947	1.776	2.321
Std. Dev. (spreads)	3.923	3.167	2.189	1.428	4.356
Std. Dev. (exports/Y)	0.052	0.021	0.014	0.013	0.023
Std. Dev. (depreciaton)	0.196	0.103	0.086	0.104	0.113
Correlation(trade balance/Y, y)	-0.358	-0.180	-0.288	0.289	0.064
Correlation(c,y)	0.844	0.589	0.728	-0.080	0.366
Correlation(spreads,y)	-0.358	-0.075	-0.129	0.239	0.033
Correlation(exports/Y,y)	-0.189	-0.143	0.107	0.317	-0.055
Correlation(depreciation,y)	-0.256	-0.230	-0.322	0.289	-0.005
Correlation(depreciation,spreads)	0.430	0.196	0.260	0.267	0.168
Corr. (inflation tax, y)	-0.214	-0.533	-0.674	0.113	-0.255
Corr. (real expenditure, y)	0.260	0.470	0.714	-0.287	0.285
Corr. (personal income tax, y)	-0.153	-0.123	0.263	0.381	-0.241
Average of absolute percentage distance to data	-	0.570	1.043	1.393	0.614

Note: Data is the average of the numbers Argentina, Brazil, Chile, Colombia, Mexico, Peru and Uruguay. The variable y is the cycle of GDP. The inflation tax is defined as in Vegh and Vuletin (2015): inflation/ (1 + inflation).

For the economy with $\theta_N = 0.95$ an $\theta_T = 0.40$, we find that the volatilities of the trade balance-to-GDP ratio, interest rate spreads and exports-to-GDP ratio are similar to the benchmark calibration. However, the alternative economy has counterfactual cyclicality of the trade balance, spreads and exports (i.e., the correlations of these variables with GDP have the opposite sign in the data).

B.6 The role of fiscal and monetary policy distortions

Consider a version of our model in which we remove the monetary policy distortion. Specifically, we modify the problem of the government by imposing that the non-negativity constraint (12) be satisfied with equality. This assumption eliminates the monetary policy distortion as it forces the government to always implement a slack cash-in-advance constraint (i.e., households are satiated with money balances). Analytically, this situation corresponds to the case $\zeta > 0$, derived in Appendix A.3.

We computed the deterministic steady state of the economy without monetary policy distortions and compared it to our benchmark case. The results displayed in Table 11 show the effect of monetary policy distortions for some key macroeconomic variables. Notably, inflation would be 17 percentage points lower if we remove monetary distortions. Consequently, the tax rate would be significantly higher (12.6 percentage points) in the economy without monetary distortions. Other variables also change as a consequence of the change in the mix of monetary and fiscal financing of government expenditures. The clearest example is that real GDP would be more than 3 percent lower in the economy without monetary distortions. This is a consequence of relying too heavily on taxation to finance government expenditures, i.e., of implementing a suboptimal policy mix.

Table 11: Steady-state statistics

	Benchmark	No monetary distortions	Lump-sum taxes
Real GDP	1.000	0.966	1.121
Inflation, %	3.835	-13.751	-13.755
Revenue / GDP	0.232	0.358	0.336
Exports / GDP	0.209	0.168	0.169
Gov. Consumption / GDP	0.134	0.136	0.130
Debt / GDP	0.187	0.176	0.165
Haircut, share of debt	0.305	0.304	0.358
Default probability, %	0.700	0.688	1.029

The last column in Table 11 presents steady-state moments for the economy with lump-sum taxes. As we show in Section 4.4.2, this economy has no fiscal or monetary policy distortions. Inflation and tax revenue are similar to those in the economy with no monetary policy distortions. The absence of any policy distortions implies the economy is significantly larger: real GDP is about 12% higher than in the benchmark economy. Interestingly, the economy with lump-sum taxes displays the lowest debt-over-GDP ratio and the highest debt haircut and default probability.

B.7 Seigniorage in emerging countries

The most consistent reference for seigniorage in emerging markets is the IMF. Following Aisen and Veiga (2008), seigniorage is measured as the change in reserve money (line 14a IFS) over nominal GDP (line 99a IFS). The first column in the following table provides the values for the countries for which we calibrated the model, taken directly from Aisen and Veiga (2008). On average, the measure they consider for seigniorage is 4.1 percent for the countries in our sample. There are important differences across countries, ranging from 1.9% in Colombia to over 6.9% in Chile.

The second column in Table 12 presents our replication and extension of the measure pro-

Table 12: Seigniorage in Emerging Markets

	Aisen and Veiga (2008) (original)	Aisen and Veiga (2008) (updated)	Kehoe et al. (2020) (original)
Argentina	0.060	0.045	0.022
Brazil	0.036	0.033	0.019
Chile	0.069	0.029	0.021
Colombia	0.019	0.016	0.010
Mexico	0.022	0.018	0.017
Peru	0.034	0.032	0.021
Uruguay	0.049	0.029	0.026
Average	0.041	0.029	0.019

vided by Aisen and Veiga (2008). We extended the sample period from their last observation, 1999, to the more recent observation for each country in the IFS data set. The average measure of seigniorage is now significantly lower, 2.9 percent, indicating that the use of seigniorage has declined over time for these countries. This is to be expected as these countries continue to develop and the quality/independence of their central banks improves. The last measure that we consider is from Kehoe, Nicolini, and Sargent (2020), whose average for the countries we consider is a bit lower than our updated number for Aisen and Veiga (2008).

C Data

C.1 Data Sources

This section lists the sources for all the variables used in the main body of the paper.

Variables in Table 2 and 3:

- "Inflation" is Inflation, consumer prices (annual %) from the World Bank. Indicator Code FP.CPI.TOTL.ZG.
- "Transfers/GDP" constructed as the product of two series from the World Bank. Subsidies and other transfers (% of expense) with indicator code GC.XPN.TRFT.ZS and Expense (% of GDP) with indicator code GC.XPN.TOTL.GD.ZS.
- "Exports/GDP" is Exports of goods and services (% of GDP) from the World Bank. Indicator code NE.EXP.GNFS.ZS.
- "Employment/Population" is Employment to population ratio, 15+, total (%) (modeled ILO estimate). Indicator code SL.EMP.TOTL.SP.ZS.
- "Gov. Consumption/GDP" is General government final consumption expenditure (% of GDP) from the World Bank. Indicator code NE.CON.GOVT.ZS.
- "Debt/GDP" is Public External Debt (%GDP) computed using the ratio of the following two variables from the World Bank. External debt stocks, public and publicly guaranteed (PPG) (DOD, current US$) with indicator code DT.DOD.DPPG.CD and GDP (current US$) with indicator code NY.GDP.MKTP.CD.
- "Haircut, Share of Debt" is the median "SZ haircut, HSZ" in Table 1 of Dvorkin et al. (2021).
- "Default rate" is obtained from Tomz and Wright (2013). They construct a database of 176 sovereign entities spanning 1820 to 2012. The frequency of default is sensitive to the sample being analyzed. They mention that their findings are "similar to the 2% default probability that is a target for many calibrated versions of the standard model," which is the number we use as well. The unconditional probability of a country with positive debt (a borrower) defaulting on debts owed to commercial creditors is 1.7% per year. Nevertheless, this probability is higher in developing countries. Note also in Figure 2 of Tomz and Wright (2013) that in a typical year, there are no defaults or there is one country in default. We considered this fact when calibrating a significantly lower default rate in the model with only ε shocks.

The sources for variables used in Table 4 and 7 are:

- "Real GDP growth" is GDP per capita (constant LCU) from the World Bank. Indicator Code NY.GDP.PCAP.KN.
- "Trade balance" is Trade balance (% GDP) computed using two variables from the World Bank. Trade (% of GDP) with indicator code NE.TRD.GNFS.ZS and the variable Exports of goods and services (% of GDP) mentioned above.
- "Spreads" is the J.P. Morgan Emerging Markets Bond Spread (EMBI+) obtained from the World Bank. Indicator Id: EMBIG.

The additional sources for Table 7 is Vegh and Vuletin (2015). For taxes, we use the file they made available, "data_AEJEP.dta", and using the variable "individual_tr," we follow the same detrending procedure to make the correlation more comparable. For this variable, Vegh and Vuletin (2015) present in Figure 11 the correlation in growth rates; i.e., the correlation between the change in the personal income tax rate and GDP growth. If we follow that procedure, our results confirm the similarity of the model and the data—we obtain -0.2319 in the model and -0.1009 in the data.

C.2 Estimation of a stochastic process for terms of trade and productivity

We use data on terms of trade from ECLAC - CEPALSTAT, Economic Indicators and statistics, External sector. The index is called "terms of trade and purchasing power of exports". We also use the time series of commodity prices used by Drechsel and Tenreyro (2017). Before estimating the autoregressive process, we take logs of the series and subtract the mean. Table 13 presents the results. The time period is 1980 to 2019. The coefficients ρ_p and σ_p are both similar for all the seven countries, so we use the average in our benchmark calibration. It is reassuring that the estimation results for the commodity price index presented at the bottom of Table 13 are also quite similar to the average. We did not de-trend the series before estimating the stochastic process so as to include long-duration cycles in the terms of trade (often referred to as "super-cycles") in our quantitative exercises and keep the model and the data more comparable. We have also estimated these stochastic processes after de-trending the time series for terms of trade. The main difference is that the resulting value of ρ_p is smaller, which implies that shocks are less persistent.

For productivity we use the Penn World Table version 10.0, variable *rtfpna* (TFP at constant national prices). Before estimating the autoregressive process, we take logs of the series and subtract the linear trend. Table 13 presents the results. The time period is the same as for terms of trade, 1980 to 2019. The coefficients ρ_{tfp} and σ_{tfp} are both similar for all the seven countries, so we use the average in our benchmark calibration.

Table 13: Estimation of process for shocks: p^T and tfp

		p^T		TFP	
	Number of years	ρ_p	σ_p	ρ_{tfp}	σ_{tfp}
Argentina	40	0.9303	0.0608	0.7759	0.0433
		(0.0568)	(0.0064)	(0.1072)	(0.0062)
Brazil	40	0.8746	0.0657	0.8158	0.0270
		(0.0760)	(0.0072)	(0.0575)	(0.0027)
Colombia	40	0.9187	0.0847	0.8796	0.0139
		(0.0585)	(0.0095)	(0.0905)	(0.0015)
Mexico	40	0.8216	0.0702	0.8765	0.0251
		(0.1339)	(0.0036)	(0.0556)	(0.0023)
Chile	40	0.9139	0.1021	0.9066	0.0289
		(0.0871)	(0.0106)	(0.0906)	(0.0024)
Peru	40	0.9329	0.0733	0.9171	0.0444
		(0.0729)	(0.0063)	(0.0533)	(0.0040)
Uruguay	40	0.7706	0.0717	0.8712	0.0369
		(0.0926)	(0.0072)	(0.0432)	(0.0031)
Average		0.8804	0.0755	0.8632	0.0314
Commodity price index	36	0.8757	0.0910		
		(0.0965)	(0.0134)		

Note: Standard errors in parenthesis.

C.3 Event Study Selection

We use quarterly data on EMBI+ Sovereign Spread data from Bloomberg to select the episodes of debt crisis. The series are in basis points and cover 13 emerging market countries and extend from 1997Q1 to 2021Q4, depending on the country. The countries are Argentina, Brazil, Colombia, Croatia, Hungary, Indonesia, Mexico, Peru, Philippines, Russia, South Africa, Turkey, and Ukraine.

Following Calvo et al. (2006), episodes are initially flagged if the EMBI+ spread is larger than the sample mean plus 2 standard deviations. The sample mean and standard deviation are calculated without spread observations above the 95th percentile to avoid increasing these values with extreme observations. Episodes are dropped if the EMBI spread never reaches 500 basis points or if the peak is small relative to previous country events (e.g. Ukraine in 2020). Additionally, episodes are dropped if the spread is within 100 basis points in the prior year without continual increases (up-down scenario). This is done to limit volatile events. For example, assume 2007Q3 is flagged as an event start for a country with a spread of 750 basis points. If the spread in 2007Q1 is 700 basis points and then drops to 600 basis points in 2007Q2 and then increases to 750 basis points in 2007Q3 then we remove the flag. Given the flag is time t=0, this procedure is applied to t=-4, t=-3, and t=-2, i.e., the quarters in the year prior to the event flag, except the one directly before. We do not include the quarter directly before the flag in question (t=-1) since it cannot be determined if the spread declined between the two values. To these episodes we added others for which four conditions are satisfied: the spreads increase to over 500 basis points, there are no “up-down” patterns, the peak is not small relative to previous country events, and there is a known event that occured (the 2007-09 GFC shock). This adds events in 2008 for Peru, Phillipines, and Russia. Once we have an episode, quarters leading to that episode are flagged as part of the event if spreads are significantly increasing at that point. This is measured by if the change in spread from previous quarter is greater than 90 basis points or the year-over-year growth rate is greater than 75 basis points. This is done to better capture the beginnings of some crises. If there are only 1 to 3 unflagged quarters between flagged quarters for a country, we count those two flags as the same event. After we have the episodes selected, the event start (t=0) is marked as the first quarter flagged. For example, if an episode has flags from 2005Q1 to 2007Q2 then t=0 would be 2005Q1.

The inflation series are end-of-quarter consumer price index values retreived from either IMF International Financial Statistics (IFS) or the country’s statistical agency via Haver Analytics, depending on availability. The indices are used to calculate year-over-year percent inflation. Croatia, Mexico, and Ukraine inflation data are from IFS. Argentina inflation data are from Cavallo and Bertolotto (2016). All others are from the Emerge database in Haver Analytics. The measure of inflation is truncated at +/- 50 percentage points to reduce the weight of extreme events.

For the table, we presented the difference in inflation (and depreciation) between one period before the shock (t=-1) and the period of the shock (t=0). Since the model is yearly and the data is quarterly, we take end-of-period inflation values over 4 quarters in the data. We take the 4 quarters preceding the event to be period -1, so the end-of period value is inflation in the quarter prior to the event start. Likewise, we take the 4 quarters including and directly after the event as period 0, so the end-of-period inflation is the value in the third quarter after the event. The same process is implemented to obtain end-of-period depreciation.

For the model, we need to adjust the criteria that the spread must be at least 500 basis points because the period is year. We take the average spread in selected events in the data from the quarter of the event (t=0) to the third quarter after the event (t=3). The minimum of these averages is 379 basis points, which we use for the model’s lower threshold for event selection.

C.4 Local projections

We consider four alternative left-hand-side variables: inflation, currency depreciation, EMBI spreads, and GDP growth (i.e., $ln(GDP_t) - ln(GDP_{t-1})$). We refer to these variables as y_t^i, where

i refers to the country and t to the year. The right-hand-side variable of interest is the log(terms of trade), or productivity, and we refer to this variable as lp_t^i.

The difference of a variable δ periods ahead with the same variable one period ago is $\Delta y_{t+\delta,t-1}^i = y_{t+\delta}^i - y_{t-1}^i$. The panel regression we run to obtain the response to terms of trade shocks is

$$\Delta y_{t+\delta,t-1}^i = \alpha^\delta + \beta^\delta \Delta lp_{t,t-1}^i + \text{controls.}$$

We run this regression 32 times: for each of the four alternative left-hand-side variables, for each of the two alternative right-hand-side variables, and for $\delta = \{0,1,2,3\}$. The controls consist of two lags of $\Delta y_{t+\delta,t-1}^i$, two lags of $\Delta lp_{t,t-1}^i$, and country fixed effects.

In Figure 5 and 6, we plot the coefficients β^δ multiplied by -10 to represent a 10 percent decline in the terms of trade or productivity. The standard errors showed by the shaded area in the figure are robust standard errors.

The time period for the regressions in 1980 to 2019 or the latest available observation. The most important exception is the regression for the EMBI spread, which starts in 1997 due to data availability of this variable.

We also conduct this comparison using contemporaneous regressions between these four variables and the terms of trade. The estimated semi-elasticities, similar to those in Drechsel and Tenreyro (2017), are quite similar in the model and the data and resemble the effect at time zero in the analysis presented here.