Online Appendix for Relational Knowledge Transfers

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Here we present supplementary material for Section 5 in the text.

1 Alternative timing in the discrete-time model

Here we show that, in the model considered in Section 5.1 in the text, Lemmas 1-2 and Proposition 1 remain valid.[1] Recall that period t output is now $f(X_t + x_t) = f(X_{t+1})$, while all other aspects of the model are unchanged. The novice's outside option remains equal to $\frac{1}{1-\delta} f(X_t)$.[2]

Other than the modifications listed below, the proofs of Lemmas 1-2 and Proposition 1 are identical to before. (In these proofs, y_t still denotes $f(X_t)$. Therefore, period-t output is now $y_{t+1} = f(X_t + x_t)$.)

Proof Lemma 1. In period k, $\Pi_k(\mathcal{C}')$ is now equal to $y'_{k+1} - w'_k = y_{\sup} - w'_k = y_{\sup} + \frac{1}{r} y_{\sup} - V_k(\mathcal{C}')$ and $\Pi_k(\mathcal{C})$ is now equal to $\sum_{\tau=k}^{\infty} \delta^{\tau-k} y_{\tau+1} - V_k(\mathcal{C})$. Therefore, as before, $\Pi_k(\mathcal{C}') > \Pi_k(\mathcal{C}) \geq 0$ and, for all $t < k$,

$$\Pi_t(\mathcal{C}') - \Pi_t(\mathcal{C}) = \delta^{k-t} \left[\Pi_k(\mathcal{C}') - \Pi_k(\mathcal{C})\right] > 0.$$

Proof Lemma 2. Step 1. For all t, $\Pi_t(\mathcal{C}')$ is now equal to $\sum_{\tau=t}^{\infty} \delta^{\tau-t} y'_{\tau+1} - V_t(\mathcal{C}')$ and therefore $\Pi_t(\mathcal{C}') \geq \frac{1}{1-\delta} y'_{t+1} - V_t(\mathcal{C}') \geq \frac{1}{1-\delta} y'_t - V_t(\mathcal{C}') = 0$ (as required in footnote ??). In addition, $\Pi_0(\mathcal{C}') - \Pi_0(\mathcal{C})$ is now equal to $\sum_{t=0}^{\infty} \delta^t \left[y'_{t+1} - y_{t+1}\right]$. Since $y'_t \geq y_t$ for all t, $\Pi_0(\mathcal{C}') - \Pi_0(\mathcal{C}) \geq 0$. Moreover, once $V_{t^*}(\mathcal{C}') > V_{t^*}(\mathcal{C})$, and therefore $y'_{t^*} > y_{t^*}$, we have $\Pi_0(\mathcal{C}') - \Pi_0(\mathcal{C}) \geq \delta^{t^*-1} [y'_{t^*} - y_{t^*}] > 0$.

Step 2. The expert's profits are now

$$\Pi_0(\mathcal{C}) = \sum_{t=0}^{s} \delta^t y_{t+1} - \delta^s w_s.$$

[1] The only difference is that, in Proposition 1, the additional knowledge that the novice learns in period t is now equal to his opportunity cost of working for the expert (i.e. $f(X_t)$), rather than the actual output he produces for the expert (i.e. $f(X_t + x_t)$). Therefore, equation (4) remains valid.

[2] The required genericity assumption is now $(1-\delta) n \neq \delta$ for all $n \in \mathbb{N}$.

Therefore, after substituting for y_{t+1} (where $y_{s+1} = f(1)$ and, for all $t < s$, y_{t+1} is obtained from the novice's binding incentive constraint), and rearranging terms, we obtain

$$\Pi_0(\mathcal{C}) = \delta^{s-1} w_s \left[s(1-\delta) - \delta \right] + \textit{ constant,}$$

where *constant* $= (s+1)\,\delta^s f(1)$. By assumption, $(1-\delta)\,n \neq \delta$ for all $n \in \mathbb{N}$, and therefore $[(1-\delta)\,s - \delta] \neq 0$. Since the expert is free to vary w_s in the range $[0, f(1)]$, the optimality of $\mathcal{C}$ requires that $w_s \in \{0, f(1)\}$. As a result, $\mathcal{C}$ belongs to $\mathcal{D}$, as desired.

Proof Proposition 1. The expert's profits are now $\Pi_0(\mathcal{C}) = \sum_{t=0}^{\infty} \delta^t [y_{t+1} - w_t] = \sum_{t=0}^{T-1} \delta^t y_{t+1}$, where $y_T = f(1)$.

2 Continuous-time model

Here we show that, in the continuous-time model considered in Section 5.2 in the text, the equivalent of Proposition 1 holds. When time is continuous (starting at $t = 1$), the expert's problem is

$$\max_{\mathcal{C}=(y_t,w_t)_{t=1}^{\infty}} \quad \Pi_1(\mathcal{C}) = \int_1^{\infty} \delta^{t-1} [y_t - w_t]\, dt$$

$$s.t.$$

$$\Pi_t(\mathcal{C}) \geq 0 \text{ for all } t, \qquad (IC_E)$$

$$V_t(\mathcal{C}) \geq \frac{1}{r_0} y_t \text{ for all } t, \qquad (IC_N)$$

$$\int_1^t (1+r)^{t-\tau} w_\tau d\tau \geq 0 \text{ for all } t, \qquad (L)$$

$$y_t \in [0, f(1)] \text{ and nondecreasing.}$$

Where $\delta = \frac{1}{1+r} = e^{-r_0}$, (IC_E) and (IC_N) are the expert's and novice's incentive constraints, (L) is the novice's liquidity constraint, $\Pi_t(\mathcal{C}) = \int_t^{\infty} \delta^{\tau-t} [y_\tau - w_\tau]\, d\tau$, and $V_t(\mathcal{C})$ $= \int_t^{\infty} \delta^{\tau-t} w_\tau d\tau$.

Proposition 4 *In the continuous-time model, every profit-maximizing contract has the following properties (which are the continuous-time equivalent of the properties stated in Proposition 1):*

1. *All knowledge is transferred in finite time.*

2. *During training, the novice earns zero wages and produces output* $y_t = \delta^{T-t} f(1)$, *where* T *is the date of graduation.*[3]

Proof. Let $\mathcal{C} = (y_t, w_t)_{t=1}^{\infty}$ be an arbitrary contract (satisfying all constraints) and let $\mathcal{C}' = (y'_t, w'_t)_{t=1}^{\infty}$ be the (unique) contract such that:

(a) The novice's payoff is equal under $\mathcal{C}$ and $\mathcal{C}'$, namely,

$$V_1(\mathcal{C}) = \int_1^{\infty} \delta^{t-1} w_t dt = \int_1^{\infty} \delta^{t-1} w'_t dt = V_1(\mathcal{C}').$$

(b) Wages are $w'_t = 0$ for all $t < S$ and $w'_t = f(1)$ for all $t \geq S$. As a result, S satisfies $\int_S^{\infty} \delta^{t-1} f(1)\, dt = V_1(\mathcal{C})$.

(c) Constraints (IC_N) hold with equality, namely, $V_t(\mathcal{C}') = \frac{1}{r_0} y'_t$.

Since $V_t(\mathcal{C}') = \frac{1}{r_0} f(1)$ for all $t \geq S$, and $V_t(\mathcal{C}') = \delta^{S-t} \frac{1}{r_0} f(1)$ for all $t < S$, we obtain:

$$y'_t = f(1) \text{ for all } t \geq S \text{ and } y'_t = \delta^{S-t} f(1) \text{ for all } t < S.$$

Therefore, S is the novice's graduation.

Notice that $\mathcal{C}'$ satisfies all constraints.[4] In addition, $\mathcal{C}'$ satisfies all properties in the Proposition (with graduation date S) and has the property that

$$V_t(\mathcal{C}) \leq V_t(\mathcal{C}') \text{ for all } t. \tag{S1}$$

For $t < S$, (S1) follows from the fact that $\int_1^t \delta^{\tau-1} w'_\tau d\tau = 0$ and $\int_1^t \delta^{\tau-1} w_\tau d\tau \geq 0$.[5] And, for $t \geq S$, (S1) follows from the fact that, owing to (IC_E), $V_t(\mathcal{C}) \leq \frac{1}{r_0} f(1) = V_t(\mathcal{C}')$.

Notice also that properties (c) and (S1) together imply that $y_t \leq y'_t$ for all t. Finally, since $\mathcal{C}$ and $\mathcal{C}'$ deliver the same payoff for the novice, we have

$$\Pi_1(\mathcal{C}') - \Pi_1(\mathcal{C}) = \int_1^{\infty} \delta^{t-1} [y'_t - y_t]\, dt \geq 0. \tag{S2}$$

[3] Formally, that the novice earns zero wages during training means that cumulative wages $\int_1^t \delta^{\tau-1} w_\tau d\tau$ are zero for all $t < T$. Since time is continuous, it is possible, though economically immaterial, that $w_t \neq 0$ during zero-measure moments of time.

[4] (IC_E) holds because y'_t is nondecreasing and, therefore, $\Pi_t(\mathcal{C}') = \int_t^{\infty} \delta^{\tau-t} y'_\tau d\tau - V_t(\mathcal{C}') \geq \frac{1}{r_0} y'_t - V_t(\mathcal{C}') = 0$. (IC_N) and (L) hold by construction.

[5] Indeed, $\delta^{t-1} V_t(\mathcal{C}') = V_1(\mathcal{C}') \geq V_1(\mathcal{C}) - \int_1^t \delta^{\tau-1} w_\tau d\tau = \delta^{t-1} V_t(\mathcal{C})$.

We now assume that the contract $\mathcal{C}$ we began with is not only feasible, but also profit-maximizing (and therefore $y_{\sup} = f(1)$, where $y_{\sup} = \lim_{t\to\infty} y_t$).[6] We proceed in three steps. In each, we show that if $\mathcal{C}$ fails to satisfy a desired property, it delivers strictly lower profits than $\mathcal{C}'$ – a contradiction.

Step 1. Contract $\mathcal{C}$ prescribes $y_T = f(1)$ for some finite T.

Suppose not – namely, $y_t < f(1)$ for all t. Since $y'_t = f(1)$ for all $t \geq S$, we have $y_t < y'_t$ for all $t \geq S$. Consequently, it follows from (S2) that $\Pi_1(\mathcal{C}') - \Pi_1(\mathcal{C}) \geq \int_S^\infty \delta^{t-1}[y'_t - y_t]\,dt > 0$, a contradiction to the optimality of $\mathcal{C}$. *QED*

Step 2. For all $t < T$, contract $\mathcal{C}$ prescribes $\int_1^t \delta^{\tau-1} w_\tau d\tau = 0$.

Suppose not – namely, $\int_1^s \delta^{\tau-1} w_\tau d\tau > 0$ for some $s < T$. For expositional ease, we begin by assuming that, in addition, $w_t \geq 0$ for all $t < T$ (namely, all money transfers flow from expert to novice). As a result, we must have $\int_1^T \delta^{\tau-1} w_\tau d\tau > 0$.

Since $\int_1^S \delta^{\tau-1} w'_\tau d\tau = 0$, it follows from the construction of $\mathcal{C}'$ that $S < T$ (i.e. the novice graduates earlier in $\mathcal{C}'$ than in $\mathcal{C}$). Therefore, for all t in the interval $[S, T)$ we have $y'_t > y_t$. It follows from (S2) that $\Pi_1(\mathcal{C}') - \Pi_1(\mathcal{C}) \geq \int_S^T \delta^{t-1}[y'_t - y_t]\,dt > 0$, a contradiction to the optimality of $\mathcal{C}$.

We now allow for $w_t < 0$ (this hypothetical case includes instances in which the novice initially borrows money from the expert via positive wages and then pays this loan back via negative wages). Since $\int_1^s \delta^{\tau-1} w_\tau d\tau > 0$ for some $s < T$ (and $\int_1^t \delta^{\tau-1} w_\tau d\tau$ is continuous in t) we must have $\int_1^t \delta^{\tau-1} w_\tau d\tau > 0$ for all t in an interval (r, s), with $r < s$.

If, in addition, $\int_1^T \delta^{\tau-1} w_\tau d\tau > 0$, then $S < T$ and the argument above, for the case in which $w_t \geq 0$, continues to hold. If instead $\int_1^T \delta^{\tau-1} w_\tau d\tau = 0$, then $S = T$. Therefore, for all t in (r, s),

$$\delta^{t-1} V_t(\mathcal{C}') = V_1(\mathcal{C}') > V_1(\mathcal{C}) - \int_1^t \delta^{\tau-1} w_\tau d\tau = \delta^{t-1} V_t(\mathcal{C}),$$

and so, in this interval, $V_t(\mathcal{C}') > V_t(\mathcal{C})$ and $y'_t > y_t$. It follows from (S2) that $\Pi_1(\mathcal{C}') - \Pi_1(\mathcal{C}) \geq \int_r^s \delta^{t-1}[y'_t - y_t]\,dt > 0$, a contradiction. *QED*

Step 3. For all $t < T$, contract $\mathcal{C}$ prescribes $y_t = \delta^{T-t} f(1)$.

[6] If instead $y_{\sup} < f(1)$, the expert's profits can be raised, while satisfying all constraints, by scaling up all wages and output levels by $\frac{f(1)}{y_{\sup}} > 1$.

From steps 1 and 2, $\mathcal{C}$ and $\mathcal{C}'$ have the same graduation dates $(S = T)$ and continuation values for the novice. Indeed, for all $t < T$, we have $V_t(\mathcal{C}) = V_t(\mathcal{C}') = \delta^{T-t}\frac{1}{r_0}f(1)$, and so $y_t' = \delta^{T-t}f(1)$.

Now suppose toward a contradiction that $y_s < \delta^{T-s}f(1)$ for some $s < T$. Since y_t is nondecreasing, we must have $y_t < \delta^{T-t}f(1)$ for all t in an interval (r, s), with $r < s$. It follows from (S2) that $\Pi_1(\mathcal{C}') - \Pi_1(\mathcal{C}) \geq \int_r^s \delta^{t-1}[y_t' - y_t]\,dt > 0$, a contradiction. *QED*

The Proposition follows from combining Steps 1-3. ■