ONLINE APPENDIX FOR CORRELATION MISPERCEPTION IN CHOICE

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Appendix A. Additional results

Proposition A.1. *The preference $\succsim$ has basic correlation representation if and only if it has a PCR.*

Proof. It is easy to see that if $\succsim$ has a basic representation, it has a PCR with $\mathcal{U} = \{\{a\} : a \in \mathcal{A}\}$. Suppose $\succsim$ has a PCR $(\mathcal{U}, \pi, u)$. For every $a \in \mathcal{A}$, choose $C_a \in \mathcal{U}$ with $a \in C_a$. Pick any $B = \{a_1, ..., a_n\} \subset \mathcal{A}$. Define

$$\pi_B(\{\vec{\tau} \in \Omega^B : \tau_i \in E_i \ \forall i\}) = \pi(\{\vec{\omega} \in \Omega^{\mathcal{U}} : \omega^{C_{a_i}} \in E_i \forall \ i\})$$

where $E_i \in \sigma(a_i)$ for $i = 1, ..., n$. This π_B is clearly a measure defined on the π-system that generates $\otimes_{i=1}^n \sigma(a_i)$ and so can be uniquely extended to it. Moreover, the collection $\{\pi_B\}$ is Kolmogorov consistent and so by Kolmogorov's extension theorem, we can define π_0 on Σ_A to agree with every π_B. Thus $\succsim$ has a basic correlation representation with probability π_0 and utility u. $\square$

For a PCR $(\mathcal{U}, \pi, u)$ and finite $B \subseteq \mathcal{U}$, let π_B denote the marginal distribution over the copies of Ω assigned to understanding classes in B. Note that the utility of any profile consisting of n actions is determined by some π_B with $\#B \leq n$.

Theorem A.1. *If $\succsim$ has a rich PCR $(\mathcal{U}, \pi, u)$ and u is a polynomial of degree N, then it also has a PCR $(\mathcal{U}, \mu, u)$ if and only if $\mu_B = \pi_B$ for any $B \subseteq \mathcal{U}$ with $\#B \leq N$.*

Recall that $S_N(x_1, x_2, ..., x_N) = \sum_{Q \subseteq \{1,...,N\}} (-1)^{[N-\#Q]} u(\sum_{i \in Q} x_i)$. From our observation in the proof of Theorem 2, if u is continuous, then $S_N(x_1, x_2, ..., x_N) = 0$ for all $x_1, ..., x_N$ if and only if u is a polynomial of degree $N - 1$. From primitives, $S_N(x_1, ..., x_N) = 0$ for all $x_1, ..., x_n$ if and only if $p_N^E \sim p_N^O$ where

$$p_N^O = \Big(2^{-(N-1)}, \sum_{x \in Q} x\Big)_{\#Q \ odd} \text{ and } p_N^E = \Big(2^{-(N-1)}, \sum_{x \in Q} x\Big)_{\#Q \ even}$$

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and Q ranges over all subsets (including $\emptyset$) of $\{x_1, ..., x_N\}$. When $x_i > 0$ for each i, a result in Eeckhoudt et al. (2009) implies p_N^O N-order stochastically dominates p_N^E. Therefore, the result follows from the below Proposition.

Proposition A.2. *If the preference $\succsim$ has a rich PCR $(\mathcal{U}, \pi, u)$, and*

$$N^* = \inf\{N : S_N(\vec{x}) = 0 \text{ for all } \vec{x}\},$$

then the PCR $(\mathcal{U}, \mu, u)$ also represents $\succsim$ if and only if $\mu_B(E) = \pi_B(E)$ for every $B \subseteq \mathcal{U}$ with $\#B < N^*$.

Proof. Sufficiency follows from exactly the same arguments used in Thoerem 2. To see necessity, suppose that $S_N(\vec{x}) = 0$ for all $\vec{x}$ and that π agrees with μ on any rectangle for B when $\#B < N - 1$. Consider any profile $\langle a_i \rangle_{i=1}^m$, and assume WLOG that each a_i belongs to a distinct understanding class C_i; we show that

$$V_\pi(\langle a_i \rangle_{i=1}^m) = V_\mu(\langle a_i \rangle_{i=1}^m).$$

This is trivially true if $m < N$. The claim is proved if we show that, when $m \geq N$, we can replace each $V_\pi(\langle a_i \rangle_{i=1}^m)$ and $V_\mu(\langle a_i \rangle_{i=1}^m)$ with the (possibly negatively) weighted sum of the utilities of "sub-profiles" of $\langle a_i \rangle_{i=1}^m$ with at most $N - 1$ elements. Rearranging the equation $S_N(x_1, ..., x_N) = 0$,

$$u(\sum_{i=1}^N x_i) = - \sum_{Q \subseteq \{1,...,N\}, \#Q < N} (-1)^{[N - \#Q]} u(\sum_{i \in Q} x_i). \tag{A.1}$$

for any $x_1, ..., x_N$. Now,

$$V_\pi(\langle a_i \rangle_{i=1}^m) = \int u\left(\sum_{i=1}^m a_i(\omega^{C_i})\right) d\pi,$$

so by (A.1) where $x_i = a_i(\omega^{C_i})$, $i = 1, ..., N - 1$, and $x_N = \sum_{i=N}^m a_i(\omega^{C_i})$, each term

$$u\left(\sum_{i=1}^m a_i(\omega^{C_i})\right) = u\left(\sum_{i=1}^{N-1} a_i(\omega^{C_i}) + [\sum_{i=N}^m a_{C_i}(\omega^{C_i})]\right)$$

can be written as the sum of utilities where each argument contains the sum of at most $m - 1$ terms. We can repeat this procedure until the arguments of each $u(\cdot)$ contain the sum of at most $N - 1$ terms. Naturally, the exact same procedure can be applied to V_μ. This establishes the result. □

References

Eeckhoudt, Louis, Harris Schlesinger, and Ilia Tsetlin (2009), "Apportioning of risks via stochastic dominance." *Journal of Economic Theory*, 144, 994–1003.