Online Appendix - Not for Publication

Wealth Inequality, Aggregate Consumption, and Macroeconomic Trends under Incomplete Markets

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A Data appendix

A.1 Wealth, income, and consumption data in the PSID

This appendix covers the details of the variables constructed using the Panel Study of Income Dynamics (PSID) data, which are analyzed in Sections III and IV.C. Below, I also separately discuss the micro data used in Section IV.A.

(a) **Sample periods.** The PSID wealth data are available in the 1984, 1989, 1994, and 1999 waves and every two years after that.

(b) **PSID waves and calendar years.** I consider variables in the PSID 1984 wave to be observations from the calendar year of 1983. The same timing convention is used for the other waves.

(c) **Data cleaning.** I use the number of family members to compute the per capita net wealth and income variables. I dropped imputed data and households in which the head's age is less than 20 years or greater than 65 years.

(d) **Definitions of variables.**

- **Wealth.** Net wealth is defined as the sum of the values of a farm or business, checking or savings accounts, money market funds, certificates of deposit, government bonds or treasury bills, real estate, shares of stock in publicly held

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corporations, stock mutual funds or investment trusts, private annuities or IRAs, other assets, and the net value of any cars, trucks, motor homes, trailer or boat, and other assets less the sum of liabilities from a farm, a business, or real estate, credit card or store card debt, student loan debt, medical bills, legal bills, loans from relatives, and other debt.

- **Income.** Income includes taxable income, transfers, and social security income.
- **Wealth inclusive of income.** Wealth, inclusive of income, is the sum of net wealth and total income.
- **Consumption.** As in Attanasio and Pistaferri (2014), consumption includes expenditures on food, rent, home insurance, utilities, car insurance, car repairs, transportation, school, child care, health insurance, and out-of-pocket medical expenses. I use the data from the replication package for Attanasio and Pistaferri (2014) for years between the PSID 1999 and 2011 waves. For the data since the PSID 2013 wave, I construct the consumption variable using the same definition as Attanasio and Pistaferri (2014).

(e) **Income process data in Section IV.A.** I directly use the data and codes in the replication package for Blundell, Pistaferri and Preston (2008). The two differences from Blundell, Pistaferri and Preston (2008) are the definition of income and the usage of the SEO sample. First, I include financial income in addition to earnings and transfers, although Blundell, Pistaferri and Preston (2008) excluded financial income. Second, I try specifications with or without the SEO sample because the SEO sample features more observations at the lower part of the income distribution, where the nonlinearity of the conditional mean function is more significant (Figure 1).

A.2 Macroeconomic trends in the US economy

This appendix presents the details of the empirical time series data used in Section V.

(a) **Consumption-to-wealth ratios.** Following Laibson (1999), aggregate consumption includes personal consumption expenditure (U.S. Bureau of Economic Analysis, 2024*b*) and government consumption (U.S. Bureau of Economic Analysis, 2024*a*). Aggregate wealth, inclusive of income, is the sum of net national wealth (Board of Governors of the Federal Reser 2024*b*), K_t, and net national income (Board of Governors of the Federal Reserve System (US), 2024*a*), $Y_t - \delta K_t$. I obtain consumption data from the National Income and Product Accounts and wealth and national income series from Tables B.1 and S.1 in the Financial Accounts of the United States - Z.1.

(b) **Real interest rates.** I use the natural rate of interest estimated by Laubach and Williams (2003, 2016).

(c) **Capital-to-net-national-income ratios.** I use the time series computed by Piketty and Zucman (2014*a*,*b*) for the US economy. An updated series is retrieved from the World Inequality Database (WID) (2022).

B Computational algorithms

This section covers the computational details. In Section B.1, I explain how the stationary distributions of wealth, x, and the quintile shares are computed. Section B.2 illustrates how I computed the CDF of income, y, in Section IV.D. Finally, Section B.3 shows how I calculate the equilibrium values of other macroeconomic variables in Section V.

B.1 Cross-sectional distributions of $x_{i,t}$

This section illustrates how I computed the stationary cross-sectional distribution of cash on hand. I first discretize the support of the distribution and compute the probability mass function (pmf) on a grid. Then, I use a Pareto interpolation at the top to improve the accuracy of the computed quintile shares, especially the top 20% wealth shares.

I denote the logarithm of cash on hand, augmented with the borrowing limit, as a:

$$a_{i,t} = \log(x_{i,t} + \eta).$$

Note that a is well defined (Proposition 2). Additionally, $a_{i,t+1} = a_{i,t} + z_{i,t+1}$, conditional on survival (Proposition 3), and $a_{i,t+1} = \omega$, otherwise. Thus,

$$f_{a'}(q) = f_a(q) = p_d \delta_a(q) + (1 - p_d) f_{a+z'}(q) \quad \text{for all} \quad q \in \mathbb{R},$$

where $f_{a'}$, f_a, and $f_{a+z'}$ are the probability density functions (pdf) of the cross-sectional distributions of $a_{i,t+1}$, $a_{i,t}$, and $a_{i,t} + z_{i,t+1}$, respectively. δ_a is a Dirac delta function around $\log(\omega + \eta)$.

Discretization. Next, I set up a grid, $\{a_1, \ldots, a_N\}$, centered around $\log(\omega + \eta)$. I use an equispaced grid with $a_1 = \log(\omega + \eta) - 20$, $a_N = \log(\omega + \eta) + 20$, and $N = 3,001$. Let Δ_a be the pmf version of δ_a. Δ_a is an N-dimensional vector, where its $(N+1)/2$-th element is one, and all the other elements are zero. Similarly, I denote the pmf version of f_a as $f \in \mathbb{R}^N$.

To discretize $f_{a+z'}$, note that for any $j = 1, \ldots, N$,

$$f_{a+z'}(a_j) = \int f_a(a) f_z(a_j - a) da \approx \sum_{i=1}^{N} f_i f_z(a_j - a_i),$$

where f_z is the pdf of z, f_i is the i-th element of f, and $\int f_z(a_j - a) da = 1$. Thus, I construct a matrix, $\mathcal{M}$, representing $f_z(a_j - a_i)$ in the following manner. First, $\mathcal{M}$ is an N by N matrix. Second, its j-th row is given by

$$(f_z(a_j - a_1), \ldots, f_z(a_j - a_N))/\sum_{i=1}^{N} f_z(a_j - a_i)$$

for all $j = 1, \ldots, N$. Note that each row of $\mathcal{M}$ adds up to one, reflecting the fact that $\int f_z(a_j - a)da = 1$ for all j. Given $\mathcal{M}$, $(f_{a+z'}(a_1), \ldots, f_{a+z'}(a_N))'$ is approximated by $\mathcal{M}f$. Finally, the stationary distribution, f, can be obtained as follows:

$$\begin{aligned} & f = p_d \Delta_a + (1 - p_d)\mathcal{M}f \\ \Rightarrow \quad & f = p_d(I_N - (1 - p_d)\mathcal{M})^{-1}\Delta_a, \end{aligned}$$

where I_N is an N-dimensional identity matrix.

Pareto interpolation for computing quintile shares. It is known that an upper tail of $x + \eta$ is approximated by a Pareto distribution, given the random growth dynamics and death shocks in the model. Beare and Toda (2022) showed that the Pareto α coefficient satisfies the following equation:

$$(1 - p_d)\psi(\alpha) = 1,$$

where ψ is the moment generating function (mgf) of z. When $z \sim \mathcal{N}(\mu_z, \sigma_z^2)$, $\psi(\alpha) = \exp(\mu_z \alpha + 0.5\sigma_z^2 \alpha^2)$. Then,

$$\alpha = \frac{-\mu_z + \sqrt{\mu_z^2 - 2\sigma_z^2 \log(1 - p_d)}}{\sigma_z^2}.$$

I approximate f_a using the pmf f below the 95th percentile of x. Then, above the 95th percentile, I assume that $x + \eta$ has a Pareto distribution with this α coefficient. For example, the aggregate cash on hand above the 95th percentile of x, denoted by $\underline{x}$, equals

$$0.05 \times \left[\frac{\alpha}{\alpha - 1}(\underline{x} + \eta) - \eta\right].$$

B.2 Cross-sectional distributions of $y_{i,t}$

Figure 4 in Section IV.D shows the logarithm of the tail function (i.e., 1 - cumulative distribution function (cdf)) against the logarithm of income. Here, I explain how the cdf of income is computed using the pmf of wealth, f, above.

To simplify the notation, I drop the individual subscript i. Also, time $t+1$ variables are distinguished from time t variables using prime marks. Equation (3), the definition of income ($y' = r(x-c) + m'$), and Propositions 2 and 3 imply that

$$\begin{aligned} y' = y_{i,t+1} &= x_{i,t+1} - (x_{i,t} - c_{i,t}) \\ &= x' - (x - c) \\ &= \exp(z')(x+\eta) - \eta - x + \zeta(x+\eta)^{\xi} \\ &= \exp(z')\exp(a) - \exp(a) + \zeta\exp(\xi a). \end{aligned}$$

Note that

$$\begin{aligned} F_y(\bar{y}) \equiv Pr(y' \leq \bar{y}) &= \int Pr(y' \leq \bar{y}|a) f_a(a)da \\ &= \int Pr\left(\exp(z') \leq \frac{\bar{y} + \exp(a) - \zeta\exp(\xi a)}{\exp(a)}|a\right) f_a(a)da. \end{aligned}$$

Because z' is independent of a,

$$\begin{aligned} & Pr\left(\exp(z') \leq \frac{\bar{y} + \exp(a) - \zeta\exp(\xi a)}{\exp(a)}|a\right) \\ &= \begin{cases} F_z\left(\log(\bar{y} + \exp(a) - \zeta\exp(\xi a)) - a\right), & \text{if} \quad \bar{y} + \exp(a) - \zeta\exp(\xi a) > 0 \\ 0, & \text{otherwise} \end{cases}, \end{aligned}$$

where F_z is the cdf of z.

For Figure 4, I set up a grid, $\{y_1, \ldots, y_M\}$, with $\log y_1 = -3$, $\log y_M = 3$, and equispaced

$\log y_i$s in between. Then, for $i = 1, \dots, M$,

$$
\begin{aligned}
F_y(y_i) &\approx \sum_{j=1}^{N} Pr\left(\exp(z') \leq \frac{y_i + \exp(a_j) - \zeta \exp(\xi a_j)}{\exp(a_j)} | a_j\right) f_j \\
&= \sum_{j=1}^{N} F_z\left(\log(y_i + \exp(a_j) - \zeta \exp(\xi a_j)) - a_j\right) \mathbb{I}(y_i + \exp(a_j) - \zeta \exp(\xi a_j) > 0) f_j,
\end{aligned}
$$

where $\mathbb{I}(\cdot)$ is an indicator function.

B.3 Aggregate variables

This section illustrates how the values of macroeconomic variables in a stationary equilibrium are computed in Section V.

Aggregate wealth and the calibration of μ_z. A newly born agent is endowed with ω. Conditional on survival, their cash on hand, augmented with the borrowing limit (η), grows according to z shocks. Therefore, by taking the cross-sectional average of $x_{i,t}$ for agents with different ages and aggregating them, I obtain the following equation:

$$
X + \eta = p_d(\omega + \eta) + (1 - p_d)p_d\psi(1)(\omega + \eta) + (1 - p_d)^2 p_d[\psi(1)]^2(\omega + \eta) + \dots,
$$

where $\psi(\cdot)$ is the mgf of z, and p_d is the population share of newly born agents, whose average wealth, augmented with the borrowing limit, is given by $\omega + \eta$. Similarly, $(1 - p_d)p_d$ is the population share of 1-period-old agents, and their average wealth, augmented with the borrowing limit, equals $\psi(1)(\omega + \eta)$. With the normalization that $X = 1$, I obtain the following moment condition:

$$
\psi(1) = \frac{1 - \frac{p_d(\omega+\eta)}{1+\eta}}{1 - p_d}. \tag{OA.1}
$$

Note that this condition determines μ_z, given σ_z, when $z \sim \mathcal{N}(\mu_z, \sigma_z^2)$:

$$
\mu_z = \log\left(\frac{1 - \frac{p_d(\omega+\eta)}{1+\eta}}{1 - p_d}\right) - \frac{1}{2}\sigma_z^2.
$$

Government's budget constraint and the calibration of G. After the death and birth of the affected agents, the aggregate wealth, X, is one by normalization. Then, over the next period, individual wealth, $x+\eta$, randomly grows with the average gross growth rate of $\psi(1)$. Thus, before death shocks occur, the aggregate wealth equals $\psi(1)(X+\eta)-\eta$, implying that the accidental bequests confiscated by the government equals $p_d[\psi(1)(X+\eta)-\eta]$. Then, the government's budget constraint (12) yields the following equation:

$$G + p_d\omega = p_d[\psi(1)(X+\eta)-\eta].$$

This equation for G and Equation (OA.1) pin down the value of the government consumption:

$$\begin{aligned} G &= p_d[\psi(1)(1+\eta)-\eta] - p_d\omega \\ &= \frac{p_d}{1-p_d}\left[1+\eta-p_d(\omega+\eta)-(1-p_d)(\eta+\omega)\right] \\ &= \frac{p_d}{1-p_d}(1-\omega). \end{aligned} \tag{OA.2}$$

Aggregate consumption. A newly born agent's consumption is $\zeta(\omega+\eta)^{\xi}$ given the consumption function (7). Similar to the wealth case, the random growth results for consumption in Proposition 3 implies that the average consumption of 1-period-old households equals $\psi(\xi)\zeta(\omega+\eta)^{\xi}$. By aggregating across different age groups, I obtain the following result:

$$C = p_d\zeta(\omega+\eta)^{\xi} + (1-p_d)p_d\psi(\xi)\zeta(\omega+\eta)^{\xi} + (1-p_d)^2 p_d[\psi(\xi)]^2\zeta(\omega+\eta)^{\xi} + \dots.$$

Thus, the aggregate consumption is given by:

$$C = \frac{p_d\zeta(\omega+\eta)^{\xi}}{1-(1-p_d)\psi(\xi)}. \tag{OA.3}$$

Other macroeconomic variables. Because $x_{i,t} = c_{i,t} + k_{i,t+1}$ for all i, in a stationary equilibrium where $K_{t+1} = K_t = K$,

$$K = X - C = 1 - \frac{p_d\zeta(\omega+\eta)^{\xi}}{1-(1-p_d)\psi(\xi)}. \tag{OA.4}$$

Next, the aggregate resource constraint (13) determines Y:

$$\begin{aligned} Y &= C + \delta K + G \\ &= C + \delta(1-C) + G \\ &= \delta + (1-\delta)\frac{p_d \zeta(\omega+\eta)^{\xi}}{1-(1-p_d)\psi(\xi)} + \frac{p_d}{1-p_d}(1-\omega). \end{aligned} \tag{OA.5}$$

Then, the aggregate labor in efficiently unit, L, follows from the production function (9):

$$L = \left(\frac{Y}{K^{1-\alpha}}\right)^{1/\alpha}, \tag{OA.6}$$

where K and Y are given by (OA.4) and (OA.5).

The wage rate per efficiency unit, W, is computed using the labor demand equation (11):

$$W = \alpha \frac{Y}{L}, \tag{OA.7}$$

where Y and L are given by (OA.5) and (OA.6).

Finally, I calculate the (gross) real interest rate in the following manner.

$$\begin{aligned} R = 1 + r &= 1 + (1-\alpha)\frac{Y}{K} - \delta \\ &= 1 + (1-\alpha)\frac{C + \delta(1-C) + G}{1-C} - \delta \\ &= \frac{1-\alpha}{1-C}(C+G) + \delta(1-\alpha) + 1 - \delta \\ &= \frac{1-\alpha}{1-C}(1+G) - (1-\alpha) + \delta(1-\alpha) + 1 - \delta \\ &= \frac{1-\alpha}{1 - \frac{p_d\zeta(\omega+\eta)^{\xi}}{1-(1-p_d)\psi(\xi)}} \frac{1-p_d\omega}{1-p_d} + \alpha(1-\delta). \end{aligned} \tag{OA.8}$$

I used Equation (10) in the first line. The second line follows from Equations (OA.4) and (OA.5). The third and fourth lines are based on algebra. The last line relies on Equations (OA.2) and (OA.3).

Variables in Table 5. There are three variables in Table 5. First, consumption-to-wealth ratios, $\frac{C+G}{X}$, are given by $C + G$ because of the normalization that $X = 1$. Then, C and G follow from Equations (OA.2) and (OA.3). Second, real interest rates, r, equal $R - 1$, where

R can be calculated using Equation (OA.8). Finally, I compute capital-to-(net-national-)income ratios, $\frac{K}{Y-\delta K}$, by noting that $\frac{K}{Y-\delta K} = \frac{1-C}{C+G}$ and using C and G in Equations (OA.2) and (OA.3).

Labor market clearing condition. To clear the labor market, the aggregate labor in efficiency unit, L, should equal the aggregated idiosyncratic labor productivity, $\int \varepsilon_{i,t} di$. Note that $m' = W\varepsilon'$. Thus,

$$\begin{aligned} WL &= W \int \varepsilon' \text{ di} = \int m' \text{ di} \\ &= \int R\zeta(\tilde{x}+\eta)^{\xi} + [\exp(z') - R](\tilde{x}+\eta) + r\eta \text{ di} \\ &= C + (\psi(1) - R)(X+\eta) + r\eta \\ &= -R(X-C) + \psi(1)(X+\eta) - \eta. \end{aligned}$$

I used the fact that $\tilde{x} = x$ in the third line. Because $K = X - C$, $Y = WL + (r+\delta)K$, $Y = C + \delta K + G$, and $X = C + K$,

$$\begin{aligned} \psi(1)(X+\eta) &= WL + (r+\delta)K + (1-\delta)K + \eta \\ &= Y - \delta K + K + \eta \\ &= C + G + K + \eta \\ &= X + \eta + G. \end{aligned}$$

Then, because of Equation (OA.1),

$$\begin{aligned} G = (\psi(1) - 1)(1+\eta) &= \frac{1+\eta - p_d(\omega+\eta)}{1-p_d} - (1+\eta) \\ &= \frac{1+\eta - p_d(\omega+\eta) - (1-p_d)(1+\eta)}{1-p_d} \\ &= \frac{p_d}{1-p_d}(1-\omega), \end{aligned}$$

which is identical to Equation (OA.2). Thus, the labor market clearing condition does not restrict the equilibrium quantities in addition to the other conditions above in this section.

Euler equation and the calibration of β. Given the random growth result of consumption in Proposition 3, the Euler equation simplifies to Assumption 1(EE2):

$$1 = \beta R \psi(-\xi\gamma).$$

Using Equation (OA.8) for R, I calibrate β to satisfy the Euler equation:

$$\begin{aligned}\beta &= R^{-1}[\psi(-\xi\gamma)]^{-1} \\ &= \left[\frac{1-\alpha}{1-\frac{p_d\zeta(\omega+\eta)^{\xi}}{1-(1-p_d)\psi(\xi)}}\frac{1-p_d\omega}{1-p_d}+\alpha(1-\delta)\right]^{-1}[\psi(-\xi\gamma)]^{-1}. \end{aligned} \tag{OA.9}$$

C Supplementary results

This section presents supplementary tables and figures to the results in the main text. Section C.1 conducts similar analyses to Section IV using different income concepts (m instead of y) and different distributional assumptions for z (symmetric or asymmetric Laplace distributions instead of Gaussian distributions). Section C.2 revisits the aggregate implications in Section V when a symmetric or an asymmetric Laplace distribution is assumed for z.

C.1 Different income processes

Definitions of Laplace distributions. Consider an asymmetric Laplace random variable $z \sim \mathcal{AL}(\theta_z, \sigma_z^2, \kappa_z)$. The pdf of z is given by:

$$f_z(z) = \frac{\sqrt{2}}{\sigma_z} \frac{1}{\kappa_z + 1/\kappa_z} \begin{cases} \exp\left(-\frac{\sqrt{2}}{\sigma_z \kappa_z}|z - \theta_z|\right), & z \geq \theta_z \\ \exp\left(-\frac{\sqrt{2}\kappa_z}{\sigma_z}|z - \theta_z|\right), & z < \theta_z \end{cases}.$$

Note that I use $1/\kappa_z$ in the place of κ_z in Kotz, Kozubowski and Podgórski (2001) to make an increase in κ_z represent an increase in the skewness of a distribution. Its mgf $\psi(s)$ equals

$$\psi(s) = \frac{e^{\theta_z s}}{1 - \frac{1}{2}\sigma_z^2 s^2 - \mu_z s},$$

where $\mu_z = \frac{\sigma_z}{\sqrt{2}}(\kappa_z - 1/\kappa_z)$ and $-\frac{\sqrt{2}}{\sigma_z}\kappa_z < s < \frac{\sqrt{2}}{\sigma_z \kappa_z}$.

When $\kappa_z = 1$, $\mathcal{AL}(\theta_z, \sigma_z^2, 1)$ is a symmetric Laplace distribution with mean θ_z and variance σ_z^2.

Calibration of the distributions of z. I similarly calibrate the distributional parameters by matching quintile shares of wealth in the data and the model. When an asymmetric Laplace distribution is assumed, I adjust σ_z and κ_z to match the data, whereas θ_z follows from the normalization that $X = 1$. Similarly, when a symmetric Laplace distribution is used, I adjust σ_z to match the data with θ_z being determined by the same normalization that $X = 1$.

Table OA.1 shows the results. Clearly, the fit of the normal, asymmetric Laplace, and symmetric Laplace models for the data are similar. Note that regardless of the properties of the distributional family of z, the random growth results for a and i.i.d. death shocks

Table OA.1: Quintile shares of wealth in the data and the model

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Quintile Shares of Wealth, Inclusive of Income (x, %)							
	1st	2nd	3rd	4th	5th	θ_z	σ_z	κ_z
PSID 1984 wave:								
Data	1	5	9	17	67			
Model $\mathcal{N}(\theta_z, \sigma_z^2)$	3	6	9	15	67	0.010	0.090	
Model $\mathcal{AL}(\theta_z, \sigma_z^2, \kappa_z)$	3	6	9	15	67	0.011	0.090	0.997
Model $\mathcal{L}(\theta_z, \sigma_z^2)$	3	6	9	15	67	0.010	0.090	1
PSID 2001 wave:								
Data	1	4	7	15	73			
Model $\mathcal{N}(\theta_z, \sigma_z^2)$	1	4	9	14	74	0.006	0.125	
Model $\mathcal{AL}(\theta_z, \sigma_z^2, \kappa_z)$	1	5	8	14	73	0.040	0.122	0.824
Model $\mathcal{L}(\theta_z, \sigma_z^2)$	1	3	9	14	73	0.006	0.125	1
PSID 2015 wave:								
Data	0	2	5	12	80			
Model $\mathcal{N}(\theta_z, \sigma_z^2)$	-1	3	6	12	80	0.002	0.159	
Model $\mathcal{AL}(\theta_z, \sigma_z^2, \kappa_z)$	-1	3	6	12	80	0.003	0.159	0.992
Model $\mathcal{L}(\theta_z, \sigma_z^2)$	-1	3	6	12	80	0.001	0.159	1

Notes: Columns (1)-(5) in this table show quintile shares of wealth, inclusive of income, in the selected waves of the PSID. The model parameters are calibrated by matching the quintile shares of wealth in the data and a stationary equilibrium of the model for each year. Despite the parsimonious structures, all three models generate a reasonable fit for the data.

lead to double Pareto tails in the cross-sectional distribution of $x + \eta$ (see Kotz, Kozubowski and Podgórski, 2001; Toda, 2014, Theorem 15). This result helps the model to replicate the concentration of wealth at the top.

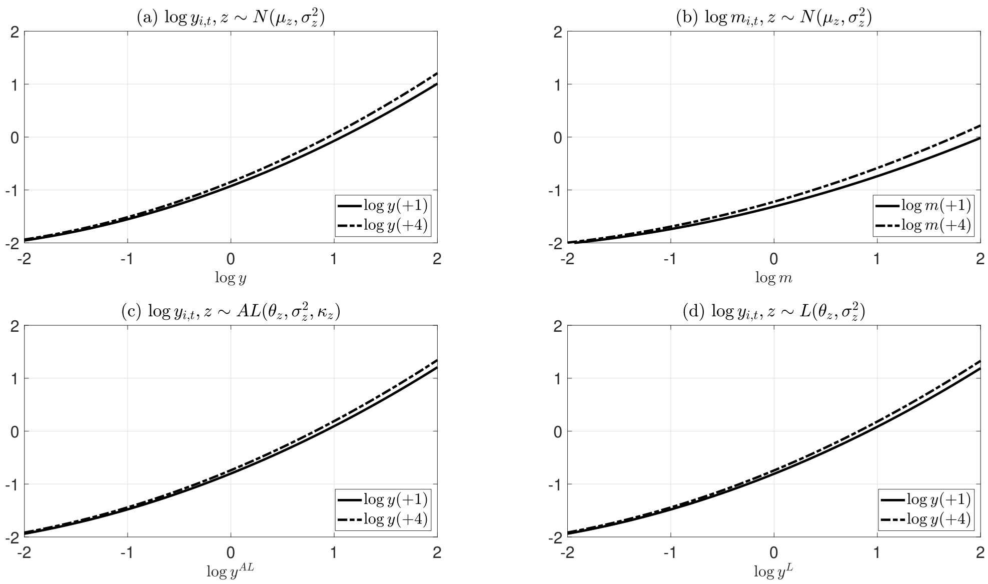

Figure OA.1: Conditional mean function of $\log y_{i,t+1}$ and $\log y_{i,t+4}$ on $\log y_{i,t}$

Notes: This figure shows the conditional mean function of $\log y_{i,t+4}$ and $\log y_{i,t+1}$ on $\log y_{i,t}$, computed by regressing the future income on $\log y_{i,t}$, $(\log y_{i,t})^2$, and the intercept. Panel (a) is based on the benchmark model with a Gaussian shock $\{z\}$. Panel (b) uses the same model but a different definition of income, $m_{i,t}$, excluding the risk-free return from $y_{i,t}$. Panels (c) and (d) assume asymmetric and symmetric Laplace income shocks, respectively. Parameters in Table OA.1 are used for computation. The results are robust; the conditional mean function is nonlinear, and the individual income process is estimated to be persistent in all cases.

Conditional mean function of $\log y_{i,t+1}$ and $\log y_{i,t+4}$ on $\log y_{i,t}$. Similar to Figure 1, Figure OA.1 shows the conditional mean function of $\log y_{i,t+4}$ and $\log y_{i,t+1}$ on $\log y_{i,t}$, computed by regressing the future income on $\log y_{i,t}$, $(\log y_{i,t})^2$, and the intercept. Panel (a) is based on the benchmark model with a Gaussian shock $\{z\}$. Panel (b) uses the same model but a different definition of income, $m_{i,t}$, excluding the risk-free return from $y_{i,t}$. Panels (c) and (d) assume asymmetric and symmetric Laplace income shocks, respectively. Parameters in Table OA.1 are used for computation. The results are robust; the conditional mean function is nonlinear, and the individual income process is estimated to be persistent in all cases.

Next, I regress $\log y_{i,t+1}$ (panel A) and $\log y_{i,t+4}$ (panel B) on $\log y_{i,t}$, $(\log y_{i,t})^2$, and the intercept. Table OA.2 shows the regression coefficients, which confirm the graphical illustration in Figure OA.1. Columns (1)-(3) are identical to the same columns in Table 2. Columns (5)-(6) are based on asymmetric and symmetric Laplace shocks, corresponding to panels (c)-(d) in Figure OA.1, respectively. The results are similar to the benchmark

Table OA.2: Conditional mean functions of $\log y_{i,t+1}$ and $\log y_{i,t+4}$ on $\log y_{i,t}$

	(1) Data w/ SEO	(2) Data w/o SEO	(3) $y_{i,t}$	(4) $m_{i,t}$	(5) $y_{i,t}^{AL}$	(6) $y_{i,t}^{L}$
Panel A. Conditional mean of $\log y_{i,t+1}$ *on* $\log y_{i,t}$						
$\log y_{i,t}$	0.79	0.80	0.74	0.50	0.79	0.78
(s.e.)	(0.02)	(0.02)				
$(\log y_{i,t})^2$	0.10	0.08	0.11	0.07	0.11	0.11
(s.e.)	(0.02)	(0.05)				
Obs.	1,779	1,154				
Panel B. Conditional mean of $\log y_{i,t+4}$ *on* $\log y_{i,t}$						
$\log y_{i,t}$	0.75	0.76	0.79	0.55	0.82	0.81
(s.e.)	(0.03)	(0.04)				
$(\log y_{i,t})^2$	0.09	0.14	0.12	0.08	0.11	0.11
(s.e.)	(0.04)	(0.07)				
Obs.	1,535	1,015				

Notes: I regress $\log y_{i,t+1}$ (panel A) and $\log y_{i,t+4}$ (panel B) on $\log y_{i,t}$, $(\log y_{i,t})^2$, and the intercept. This table shows the regression coefficients, which confirm the graphical illustration in Figure OA.1. Columns (1)-(3) are identical to the same columns in Table 2. Columns (5)-(6) are based on asymmetric and symmetric Laplace shocks, corresponding to panels (c)-(d) in Figure OA.1, respectively. The results are similar to the benchmark Gaussian shock case (column (3)). Finally, income without the risk-free return, m, is less persistent than the other cases, although the results are qualitatively similar (column (4)).

Gaussian shock case (column (3)). Finally, income without the risk-free return, m, is less persistent than the other cases, although the results are qualitatively similar (column (4)).

Table OA.3: Serial correlations of income growth

	(1) $\log y_{i,t+1} - \log y_{i,t}$	(2) $\log y_{i,t+2} - \log y_{i,t+1}$	(3) $\log y_{i,t+3} - \log y_{i,t+2}$	(4) $\log y_{i,t+3} - \log y_{i,t}$	(5) $\log y_{i,t+1} - \log y_{i,t}$	(6) $\log y_{i,t+2} - \log y_{i,t}$	(7) $\log y_{i,t+3} - \log y_{i,t}$	(8) $\log y_{i,t+3} - \log y_{i,t}$
	Panel A. $y_{i,t}$				*Panel B.* $m_{i,t}$			
$\Delta \log y_{i,t}$	-0.49	0.00	0.00	-0.49	-0.49	0.00	0.00	-0.49
$(\Delta \log y_{i,t})^2$	0.11	0.00	0.00	0.11	0.11	0.00	0.00	0.11
	Panel C. $y_{i,t}^{AL}$				*Panel D.* $y_{i,t}^{L}$			
$\Delta \log y_{i,t}$	-0.49	0.00	0.00	-0.49	-0.49	0.00	0.00	-0.49
$(\Delta \log y_{i,t})^2$	0.11	0.00	0.00	0.11	0.11	0.00	0.00	0.11

Notes: This table shows the characteristics of income growth dynamics in the benchmark Gaussian-shock model for y and m (panels A-B) and the asymmetric and symmetric Laplace-shock cases (panels C-D). In all cases, I regress leads of short and long differences of $\log y_{i,t}$ (or $\log m_{i,t}$ for panel B) on $\Delta \log y_{i,t}$, $(\Delta \log y_{i,t})^2$, and the intercept. Across all models, the regression coefficients are almost identical. Thus, all models generate similar serial correlations of income growth.

Serial correlations of income growth. Table OA.3 shows the characteristics of income growth dynamics in the benchmark Gaussian-shock model for y and m (panels A-B) and the asymmetric and symmetric Laplace-shock cases (panels C-D). In all cases, I regress leads of short and long differences of $\log y_{i,t}$ (or $\log m_{i,t}$ for panel B) on $\Delta \log y_{i,t}$, $(\Delta \log y_{i,t})^2$, and the intercept. Across all models, the regression coefficients are almost identical. Thus, all models generate similar serial correlations of income growth.

Table OA.4: Moments of the residual components

	(1) Data w/ SEO	(2) Data w/o SEO	(3) $y_{i,t}$	(4) $m_{i,t}$	(5) $y_{i,t}^{AL}$	(6) $y_{i,t}^{L}$
St. dev.	0.28	0.26	0.58	0.60	0.55	0.55
Skewness	-0.44	-0.27	-1.40	-1.64	-1.73	-1.67
Kurtosis	6.22	7.01	12.94	13.16	16.59	15.90
Obs.	1,779	1,154				

Notes: I compute the moments of the residual components in the regression of $\log y_{i,t+1}$ on the intercept, $\log y_{i,t}$, and $(\log y_{i,t})^2$. This table shows the residuals' standard deviations, skewness coefficients, and kurtosis coefficients. Empirically, this residual component is left-skewed and features fatter tails than normal distributions. Income with or without risk-free return (columns (3)-(4)) in this paper replicates this pattern at least qualitatively, although the income shocks, $z_{i,t}$, are assumed to be normally distributed. When z is an asymmetric Laplace or a symmetric Laplace random variable (columns (5)-(6)), the qualitative pattern survives.

The residual components: skewness, kurtosis, and the conditional heteroskedasticity. I compute the moments of the residual components in the regression of $\log y_{i,t+1}$ on the intercept, $\log y_{i,t}$, and $(\log y_{i,t})^2$. Table OA.4 shows the residuals' standard deviations, skewness coefficients, and kurtosis coefficients. Empirically, this residual component is left-skewed and features fatter tails than normal distributions. Income with or without risk-free return (columns (3)-(4)) in this paper replicates this pattern at least qualitatively, although the income shocks, $z_{i,t}$, are assumed to be normally distributed. When z is an asymmetric Laplace or a symmetric Laplace random variable (columns (5)-(6)), the qualitative pattern survives.

Figure OA.2 illustrates the conditional heteroskedasticity of the residuals on the level of income. I estimate the conditional mean function of $\log y_{i,t+1}$ for $y_{i,t}$ being included in each quintile and then compute the standard deviation of the residual components for each income quintile. Across all models and the definition of income, the shape and the level of the graphs are similar, exhibiting a U-shaped graph against the level of income.

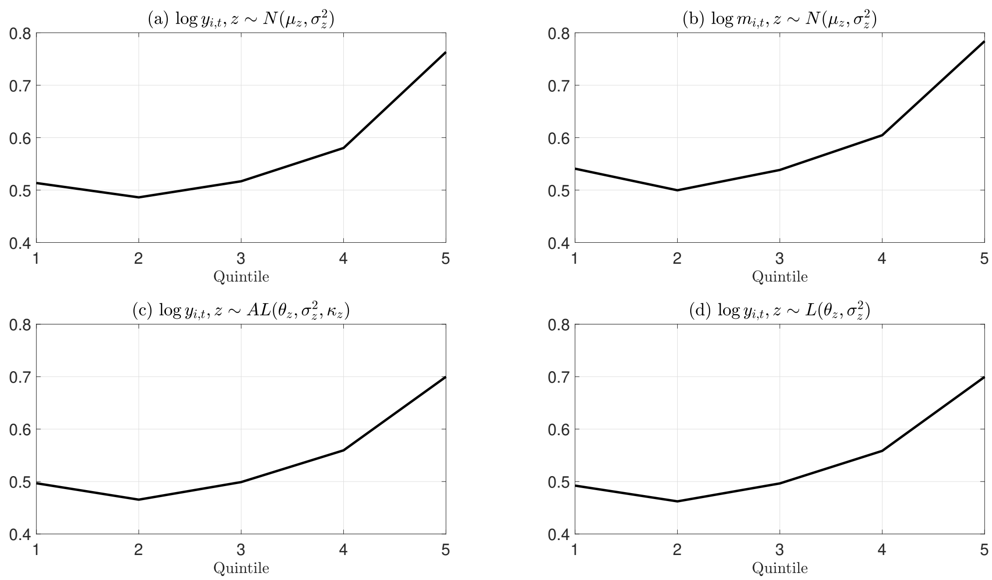

Figure OA.2: Conditional heteroskedasticity on $\log y_{i,t}$

Notes: This figure illustrates the conditional heteroskedasticity of the residuals on the level of income. I estimate the conditional mean function of $\log y_{i,t+1}$ for $y_{i,t}$ being included in each quintile and then compute the standard deviation of the residual components for each income quintile. Across all models and the definition of income, the shape and the level of the graphs are similar, exhibiting a U-shaped graph against the level of income.

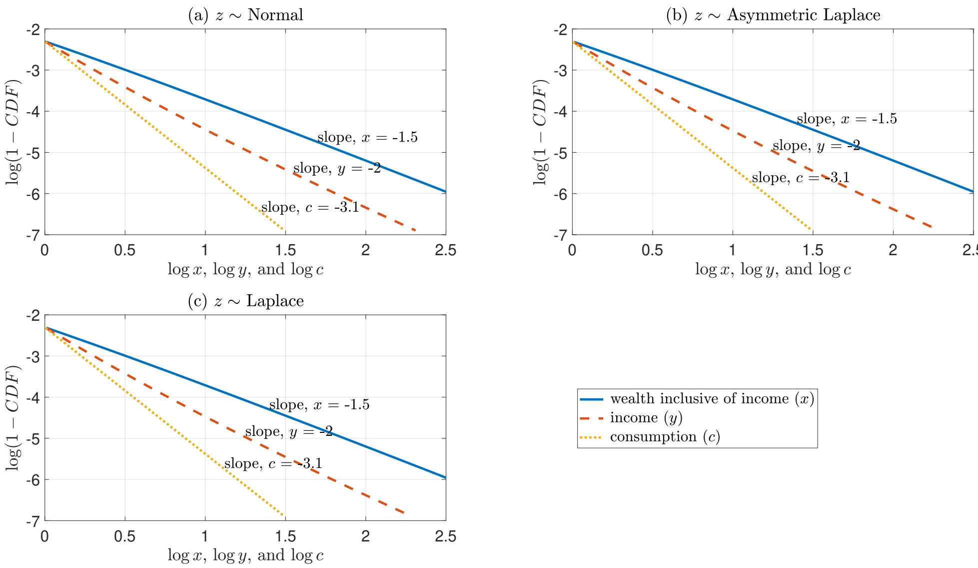

Figure OA.3: Pareto right tails of wealth, income, and consumption in different models

Notes: This figure shows the logarithm of the tail function against the logarithm of wealth x, income y, and consumption c greater than the 90th percentiles and less than the 99.9th percentiles when the model is calibrated to the US economy in 1983. Panel (a) is identical to Figure 4 in the main text. Panels (b) and (c) depict the corresponding graphs for the cases where income shocks have an asymmetric Laplace or a symmetric Laplace distribution, respectively. In all panels, all three lines are almost linear, implying that the right tails of wealth, income, and consumption distributions in the model are well approximated by Pareto tails. Furthermore, the degrees of tail thickness, represented by the estimated Pareto α coefficient shown in each panel, are almost the same across different models with different distributional assumptions for z.

Pareto right tails of wealth, income, and consumption. Figure OA.3 shows the logarithm of the tail function against the logarithm of wealth (x), income (y), and consumption (c) greater than the 90th percentiles and less than the 99.9th percentiles when the model is calibrated to the US economy in 1983. Panel (a) is identical to Figure 4 in the main text. Panels (b) and (c) depict the corresponding graphs for the cases where income shocks have an asymmetric Laplace or a symmetric Laplace distribution, respectively. In all panels, all three lines are almost linear, implying that the right tails of wealth, income, and consumption distributions in the model are well approximated by Pareto tails. Furthermore, the degrees of tail thickness, represented by the estimated Pareto α coefficient shown in each panel, are almost the same across different models with different distributional assumptions for z.

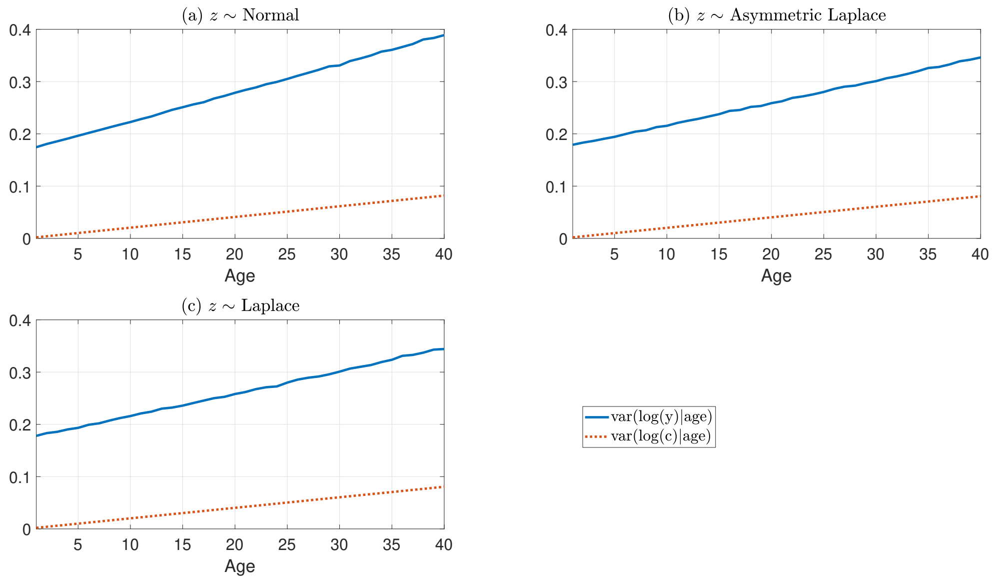

Figure OA.4: Within-cohort variance of log income and log consumption

Notes: This figure OA.4 shows the within-cohort variance of log income and consumption against age. Panel (a) is the same as Figure 5 in the main text. Panels (b) and (c) depict the corresponding graphs for the cases where income shocks have an asymmetric Laplace or a symmetric Laplace distribution, respectively. The results are almost identical across all models based on different distributional assumptions for income shocks.

Within-cohort income and consumption inequality. Finally, Figure OA.4 shows the within-cohort variance of log income and consumption against age. Panel (a) is the same as Figure 5 in the main text. Panels (b) and (c) depict the corresponding graphs for the cases where income shocks have an asymmetric Laplace or a symmetric Laplace distribution, respectively. The results are almost identical across all models based on different distributional assumptions for income shocks.

C.2 Aggregate implications when z has a Laplace distribution

In this section, I redo the analysis in Section V with $z \sim \mathcal{AL}(\theta_z, \sigma_z^2, \kappa_z)$ and $z \sim \mathcal{L}(\theta_z, \sigma_z^2)$. As in Section V, I compare the US economy in 1983 with that in 2014 based on the models using the calibrated parameters in Table OA.1.

Table OA.5 shows consumption-to-wealth ratios, $(C+G)/X$, real interest rates, r, and capital-to-net-national-income ratios, $K/(Y-\delta K)$, as in Table 5. Columns (1)-(4) are the same as columns (1)-(4) in Table 5 and are computed for the data and the benchmark model with Gaussian shocks. The remaining columns illustrate the results when z has asymmetric

Table OA.5: Macroeconomic trends in the data and different models

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Data		Model $\mathcal{N}$		Model $\mathcal{AL}$		Model $\mathcal{L}$	
	1983	2014	1983	2014	1983	2014	1983	2014
$\frac{C+G}{X}$ (%)	19.5	17.1	19.5	17.7	19.5	17.7	19.5	17.7
r (%)	3.30	0.21	3.48	2.44	3.48	2.43	3.48	2.42
$\frac{K}{Y-\delta K}$ (%)	390	449	422	474	422	475	422	475

Notes: This table shows consumption-to-wealth ratios, $(C+G)/X$, real interest rates, r, and capital-to-net-national-income ratios, $K/(Y-\delta K)$, as in Table 5. Columns (1)-(4) are the same as columns (1)-(4) in Table 5 and are computed for the data and the benchmark model with Gaussian shocks. The remaining columns illustrate the results when z has asymmetric and symmetric Laplace distributions. The aggregate implications of the model are robust to the distributional assumption for z. Indeed, the results in columns (4), (6), and (8) are almost identical.

and symmetric Laplace distributions. The aggregate implications of the model are robust to the distributional assumption for z. Indeed, the results in columns (4), (6), and (8) are almost identical.

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