Pareto-Improving Firing Costs?[*]

Bilgehan Karabay[**]
John McLaren[***]
University of Virginia

July 2008

Abstract. We examine self-enforcing contracts between risk-averse workers and risk-neutral firms (the 'invisible handshake') in a labor market with search frictions. Employers promise as much wage smoothing as they can, consistent with incentive conditions that ensure they will not renege during low-profitability times. Equilibrium is inefficient if these incentive constraints bind, with risky wages for workers and a risk premium that employers must pay. Mandatory firing costs can help, by making it easier for employers to promise credibly not to cut wages in low-profitability periods. We show that firing costs are more likely to be Pareto-improving if they are not severance payments, or (for affluent economies) if the economy is open.

JEL codes: F10, F16, J63. Keywords: Implicit Contracts, Invisible Handshake, Firing Costs.

---

[*]The views expressed in this paper are those of the authors and are not necessarily reflective of views of the Central Bank of the Republic of Turkey. All errors are our own.

[**]The Central Bank of the Republic of Turkey, Research and Monetary Policy Department, İstiklal Cad. No. 10, Ulus, 06100 Ankara, Turkey. Email: bilgehan.karabay@tcmb.gov.tr.

[***]Department of Economics, University of Virginia, P.O. Box 400182, Charlottesville, VA 22904-4182. Email: jmclaren@virginia.edu.

Many countries impose costs on employers who wish to dismiss a worker. These can take several forms, including restrictions on how and when a worker can be fired, and severance costs that must be paid to the worker. These firing costs, also called employment protection, are imposed to benefit workers by providing enhanced job security, but they can also hurt workers by distorting employers' incentives in unintended ways.

A rich literature studies these side effects. Bentolila and Bertola (1990) look at hiring and firing behavior in a single firm facing a stochastic environment and find that for realistic parameters firing costs cause a small increase in employment, by depressing hiring but reducing firings rather more; but Hopenhayn and Rogerson (1993) embed these effects in an industry equilibrium with firm entry and exit and find that firing costs significantly decrease firm entry. As a result, the overall effect of firing costs on labor demand is negative – hurting workers as well as employers. Kambourov (2006) studies the effects of firing costs on the adjustment to trade liberalization, and finds that they slow intersectoral reallocation, significantly diminishing the gains from trade. Utar (2007) shows adverse effects of firing costs on the demand for labor and on productivity in a structural empirical model of manufacturing in Colombia. Heckman and Pagés (2003) quantify firing costs for a panel of many countries and find negative effects on employment and growth. Ljungqvist and Sargent (2008) show how firing costs can help explain the high European unemployment of recent decades (even while they can help explain *low* European unemployment in the past). Kugler and Saint-Paul (2004) show that firing costs can exacerbate adverse selection problems in labor markets, harming the unemployed. Overall, firing costs have been shown to have negative allocative effects, discouraging exit from or hiring into the affected sector, and distorting the allocation of workers across firms within the sector as well.

We offer a contrasting view: Despite this inventory of negative effects, this paper offers one possible *benefit* that firing costs may bring. In a world of incomplete contracts when incentive problems within the firm are solved by repeated interactions in long-term employment relationships, anything that penalizes severing the relationship can loosen incentive constraints by also, indirectly, penalizing any misbehavior that leads to separation. This can lead to efficiency improvements from firing costs that have no counterpart in a world of complete contracts.

Other authors have also examined the effects of firing costs in incomplete-contracting environments. Alvarez and Veracierto (2001) study a model in which labor contracts are assumed to be rigid, implying that employers have an excessive incentive to shed workers in a downturn. Pissarides (2001) shows how severance payments can provide workers with insurance against layoffs, which firms may not be able to provide on their own (since *ex-post* incentives to pay a worker who is being dismissed are weak). Saint-Paul (1995) and Fella (2000) show how a firm that pays efficiency wages has an incentive to try to convince workers that it will not lay off workers except in severe downturns, thus loosening the workers' no-shirking constraints and lowering the required efficiency wage. If the firm is unable to commit to a layoff policy, required severance payments can help convince workers that the layoff probability is indeed low, thus reducing the required efficiency wage and raising profits. Matouschek, Ramezzana, and Robert-Nicoud (2008) show that bargaining under private information within a firm following an adverse shock can result in excessively high separation, and mandated firing costs can result in less aggressive bargaining and a rate of separation closer to the optimum. In Blanchard and Tirole (2008), if severance payments are not enforceable in private contracts, government-provided unemployment insurance gives employers an incentive to layoff workers more often than is socially optimal, and this distortion can be corrected by imposing a tax on layoffs.

In all of these analyses, the absence of complete contracts creates inefficiencies that can be ameliorated by firing costs, and depending on parameter values, firing costs can be welfare-improving. However, using a model of optimal dynamic self-enforcing labor contracts similar to Thomas and Worrall (1988), we focus on an issue that these others have not focused on: *The nature of employment contracts, and the behavior of wages within an employment relationship*. In particular, we focus on a type of incomplete contract that has been shown to have wide importance in practice, often called the 'invisible handshake.'[1] This is an employment relationship in which the employer assures the worker that the wage paid will not decline in low-profitability states, in effect bundling the sale of insurance with its purchase of labor. Since the worker is risk averse, this allows the employer to attract the worker for a lower expected wage than would otherwise be required. However, for various reasons, such a contract may not be enforceable by third parties, requiring enforcement by reputation within the firm (or in other words, self-enforcing contracts). If discount factors are not high enough to sustain the first-best contract, in low-profitability states the firm's incentive-compatibility constraint will bind, and insurance will be incomplete. The wage will fluctuate somewhat, forcing the employer to pay a risk premium, reducing its profits.

The point is that the only punishment in such relationships if the employer reneges on its wage commitment is a broken reputation, ending the trust that is required for the relationship to be productive. The relationship being thus worthless following a reneging, both employer and

[1]See Beaudry and DiNardo (1991) and McDonald and Worswick (1999) for evidence on the importance of the invisible handshake in practice. Bertrand (2004) provides evidence on how the invisible handshake can be weakened by international competition or by bankruptcy risks.

employee must look for new partners. Since it is difficult for an outside authority to distinguish between a firing and any other kind of separation, firing costs will tend to penalize the employer for such a breakdown. But this can be helpful to the employer, because *a harsher punishment for reneging on its wage commitment makes it easier for the employer to make wage commitments credibly*. Thus, firing costs can allow the employer to promise less volatile wages, thus attracting workers with a lower expected wage, raising profits.

This incentive effect must be traded off against the direct cost that the firing cost imposes when, from time to time, separations are inevitable for exogenous reasons, but we show conditions under which the incentive effect dominates. When it does, *firing costs are Pareto improving*. They benefit employers by making lower expected wages possible, and this does not harm workers because it simply represents the elimination of a risk premium due to the reduction in wage risk. Paradoxically, we show that firing costs that impose a deadweight cost on employers without any severance payment to workers are more likely to be Pareto improving than mandated severance payments, partly because the latter worsen workers' incentives. In addition, we show that firing costs are more likely to be beneficial in an open economy than a closed one.

We must emphasize forcefully that we are not making a policy prescription, not arguing for the implementation of firing costs, or arguing that the allocative effects of firing costs studied in the rich literature on the subject are unimportant. We are merely adding a potentially important additional effect, the benefit from strengthening of the invisible handshake, which has so far escaped notice, but which ought to be a part of a complete cost-and-benefit analysis. This argument also suggests a potentially fruitful empirical agenda: The model suggests that certain kinds of firing costs are likely to strengthen implicit contracts, reducing the variance of wages within an employment relationship. This is testable (perhaps using state-level variation in policy as in Kugler and Saint-Paul (2004)), although such an exercise is far beyond the scope of this paper.

The next section lays out the invisible handshake model. The following section derives optimal contracts in equilibrium. Section 3 introduces firing costs and analyzes their effects. Section 4 examines the implications of these effects for globalization. The final section summarizes.

1. The Model.

In this section, we will describe the key features of the model. The set-up is a simplified version of the model in Karabay and McLaren (2008), and we will rely heavily on results developed there.[2] First, we will review the model without firing costs, and then we will introduce different types of firing cost and analyze their effect on welfare.

(i) Production.

There are two types of agent, 'workers,' of which there are a measure L, and 'employers,' of which there are a measure E. In order for production to occur, a worker must team up with an employer. We will call a given such partnership a 'firm.' Workers without an employer and employers without a worker are 'unemployed' and 'with vacancy' respectively. In each period, a

---

[2] That paper is a two-sector model of imperfect labor contracts with trade and offshoring, with no firing costs. Here, we simplify by eliminating one of the sectors (which in the notation of the other paper is equivalent to setting ω^y equal to zero). One technical difference that results from this is that we do not need any additional assumptions for the first-period wage to be independent of the state, unlike in the other model.

worker and employer must both put in one unit of non-contractible effort. Workers suffer a disutility from effort equal to $k > 0$, while employers suffer no such disutility.[3] Within a given employment relationship, denote the effort put in by agent i by $e^i \in \{0, 1\}$, where $i = W$ indicates the worker and $i = E$ denotes the employer. The output generated in that period is then equal to $R = x_\epsilon e^W e^E$, where ϵ is an idiosyncratic iid random variable that takes the value $\epsilon = G$ or B with respective probabilities π_ϵ, where $\pi_G + \pi_B = 1$ and $x_G > x_B > 0$. The variable ϵ indicates whether the current period is one with a good state or a bad state for the firm's profitability. The average revenue is denoted by $\bar{x} \equiv \pi_G x_G + \pi_B x_B$.

(ii) Search.

Workers seeking an employer and employers seeking a worker search until they have a match. Search follows a specification of a type used extensively by Pissarides (2000). If a measure n of workers and a measure m of employers search in a given period, then $\Phi(n, m)$ matches occur, where Φ is a concave function increasing in all arguments and homogeneous of degree 1, with $\Phi(n, m) \leq \min(n, m)$ and $\Phi_{nm} = \Phi_{mn} > 0 \; \forall \; n, m$. It is convenient to denote by Q^E the steady-state probability that a searching employer will find a worker in any given period, or in other words, $Q^E = \Phi(n, m)/m$, where n and m are set at their steady-state values. Similarly, denote by $Q^W = \Phi(n, m)/n$ the steady-state probability that searching worker will find a job in any given period. Search has no direct cost, but it does have an opportunity cost: If an agent is searching for a new partner, then she is unable to put in effort for production with her existing partner if she has

[3] This is a simplification only. Adding a disutility to the employer would add to notation without substantively changing the result.

one.

There is also a possibility in each period that a worker and employer who have been together in the past will be exogenously separated from each other. This probability is given by a constant $(1-\rho) \in (0, 1)$.

(iii) Preferences.

There is no storage, saving or borrowing, so an agent's income in a given period is equal to that agent's consumption in that period.

The workers are risk-averse, with increasing, differentiable and strictly concave utility function μ, while the employers are risk-neutral. There is a finite lower bound, $\mu(0)$, to workers' utility (or, equivalently, there is some exogenous source of consumption on which workers can rely even if they are unemployed). Workers maximize expected discounted lifetime utility, and employers maximize expected discounted lifetime profits. All agents discount the future at the constant rate $\beta \in (0, 1)$.

(iv) Sequence of events.

The sequence of events within each period is as follows. (i) Any existing matched employer and worker learn whether or not they will be exogenously separated this period. (ii) The profitability state ϵ for each firm is realized. Within a given employment relationship, this is immediately common knowledge. The value of ϵ is not available to any agent outside of the firm, however. (iii) The wage, if any, is paid (a claim on the firm's output at the end of the period). (iv) The employer and worker simultaneously choose their effort levels e^i. At the same time, the search mechanism operates. Within a firm, if $e^i = 0$, then agent i can participate in search. (v) Each firm's revenue, R, is realized, and profits and consumption are realized.[4] (vi) For those agents who have found a new potential partner in this period's search, new partnerships with a new self-enforcing agreement are formed. This is achieved by a take-it-or-leave-it offer made by the employer to the worker.

We will focus on steady-state equilibria. In such an equilibrium, the expected lifetime discounted profit of an employer with vacancy is denoted V^{ES} and the expected lifetime discounted utility of an unemployed worker is denoted V^{WS}, where the S indicates the state of searching. Similarly, we can denote by V^{ER} and V^{WR} the lifetime payoffs to employers and workers respectively evaluated at the beginning of a cooperative relationship. Naturally, we must have $V^{WR} \geq V^{WS}$ in equilibrium, or no worker will accept a job. The values V^{ij} are endogenous, as they are affected by the endogenous probability of finding a match in any given period and by the endogenous value of entering a relationship once a match has been found. However, any employer will take them as given when designing the wage agreement. We can write:

[4]Strictly speaking, there is the possibility, off of the equilibrium path, that the firm's output will be zero because one or the other party has shirked, raising the question of how the wage claim issued in sub-period (iii) can be redeemed. This issue could be eliminated by assuming that, rather than zero output, the employer is able to produce some positive output, say, $x_{min} > 0$, even without a worker. The wages can be paid out of that output at the end of the period. The interpretation of x_G and x_B is, then, the *additional* output that is produced in cooperation with a non-shirking worker. This would require carrying this additional piece of notation throughout the analysis, but would not change any of our qualitative results.

$$V^{WS} = \mu(0) + Q^{W}\rho\beta\, V^{WR} + Q^{W}(1 - \rho)\beta\, V^{WS} + (1 - Q^{W})\beta\, V^{WS}, \text{ and}$$

$$V^{ES} = Q^{E}\rho\beta\, V^{ER} + Q^{E}(1 - \rho)\beta\, V^{ES} + (1 - Q^{E})\beta\, V^{ES}. \tag{1}$$

The unemployed worker's payoff from search is the zero wage plus the continuation values if the worker finds work and is not immediately separated, finds work and is immediately separated, or fails to find work. The payoff from search for an employer with vacancy is given by the continuation value if the employer finds a worker who is not immediately separated, finds a worker who is immediately separated, or fails to find a worker. If an employed worker or an employer with a worker chooses to search, that agent's payoff will be as given in (1), but with the first-period income changed.

Given those values, a self-enforcing agreement between a worker and an employer is simply a sub-game perfect equilibrium of the game that they play together. We assume that the employer has all of the bargaining power, so the agreement chosen is simply the one that gives the employer the highest expected discounted profit, subject to incentive constraints. Without loss of generality, we will assume that the 'grim punishment' is used, meaning here that if either agent defects from the agreement at any time, the relationship is severed and both agents must search for new partners. Thus, the payoff following a deviation would be V^{WS} for a worker and V^{ES} for an employer.

To sum up, risk-neutral employers search for risk-averse workers, and when they find each other, the employer offers the worker the profit-maximizing self-enforcing wage contract, which then remains in force until one party reneges or the two are exogenously separated. This pattern provides a steady flow of workers and employers into the search pool, where they receive endogenous payoffs V^{ES} and V^{WS}. These values are then parameters that constrain the optimal wage contract.

We now turn to the form of optimal contracts.

2. The form of optimal contracts.

In general, optimal incentive-constrained agreements in problems of this sort can be quite complex because the specified actions depend on the whole history of shocks and not only the current one. (See Thomas and Worrall (1988) and Kocherlakota (1996).) In analyzing the equilibrium, it is useful to note that in our model the employment contracts offered by employers always take one of two very simple forms. Derivation of this property is the purpose of this section.

The equilibrium can be characterized as the solution to a recursive optimization problem. Denote by Ω (W) the highest possible expected present discounted profit the employer can receive in a subgame-perfect equilibrium, conditional on the worker receiving an expected present discounted payoff of at least W. Arguments parallel to those in Thomas and Worrall (1988) can be used to show that Ω is defined on an interval [W_{min} , W_{max}] and is decreasing, strictly concave, and differentiable, where W_{min} and W_{max} are respectively the lowest and highest worker payoffs consistent with a subgame-perfect equilibrium of the game played by an employer-worker pair. This function must satisfy the following functional equation:

$$\Omega\left(W\right) = \max_{\{\omega_\varepsilon, \tilde{W}_\varepsilon\}} \sum_{\varepsilon = G, B} \pi_\varepsilon \left(x_\varepsilon - \omega_\varepsilon + \rho\beta\Omega\left(\tilde{W}_\varepsilon\right) + \left(1-\rho\right)\beta V^{ES}\right) \qquad (2)$$

subject to

$$x_{\epsilon} - \omega_{\epsilon} + \rho\beta\Omega(\tilde{W}_{\epsilon}) + (1-\rho)\beta V^{ES} \geq V^{ES}, \tag{3}$$

$$\mu(\omega_{\epsilon}) - k + \rho\beta\tilde{W}_{\epsilon} + (1-\rho)\beta V^{WS} \geq \mu(\omega_{\epsilon}) - \mu(0) + V^{WS}, \tag{4}$$

$$\sum_{\epsilon=G,B} \pi_{\epsilon}\left[\mu(\omega_{\epsilon}) - k + \rho\beta\tilde{W}_{\epsilon} + (1-\rho)\beta V^{WS}\right] \geq W, \tag{5}$$

$$W_{min} \leq \tilde{W}_{\epsilon} \leq W_{max}, \text{ and} \tag{6}$$

$$\omega_{\epsilon} \geq 0. \tag{7}$$

The right-hand side of (2) is the maximization problem solved by the employer. She must choose a current-period wage ω_{ϵ} for each state ϵ, and a continuation utility $\tilde{W}_{\epsilon}$ for the worker for subsequent periods following that state. Constraint (3) is the employer's incentive compatibility constraint: If this is not satisfied in state ϵ, then the employer will in that state prefer to renege on the promised wage, understanding that this will cause the worker to lose faith in the relationship and sending both parties into the search pool. Constraint (4) is the worker's incentive compatibility constraint. The left-hand side is the worker's payoff from putting in effort in the current period, collecting the wage, and continuing the relationship. The right-hand side is the payoff from shirking and searching, in which case the worker's payoff is the same as it would be if she were unemployed except that in the current period her income is ω_{ϵ} instead of zero. If this constraint is not satisfied, the worker will prefer to shirk by searching instead of working.[3] Constraint (5) is the target-utility

[3]Throughout, we will assume that it is optimal to induce the worker to exert effort in each state as long as the employment relationship continues. This is clearly the case in a substantial portion of the parameter space, and so we are implicitly restricting attention to that portion. For a

constraint. In the first period of an employment relationship, the employer must promise at least as much of a payoff to the worker as remaining in the search pool would provide. Thus, in that case, denoting the target utility at the beginning of the relationship by W_0, we have $W = W_0 = V^{WS}$ (and so $V^{ER} = \Omega\,(V^{WS})$). Thereafter, the employer will in general be bound by promises of payoffs she had made to the worker in the past. Finally, (6) and (7) are natural bounds on the choice variables.

Constraint (4) can be replaced by the more convenient form:

$$\tilde{W}_\epsilon \geq \tilde{W}^*, \text{ where } \tilde{W}^* \equiv [(1 - (1 - \rho)\beta\,)V^{WS} - \mu(0) + k]\,/\rho\beta. \tag{4$'$}$$

The value $\tilde{W}^*$ is the minimum future utility stream that must be promised to the worker in order to convince the worker to incur effort and forgo search. Given that $V^{WS} \leq V^{WR}$ in equilibrium, it is easy to see from (1) that $\tilde{W}^* > V^{WS}$.

Let the Kuhn-Tucker multiplier for (3) be denoted by ψ_ϵ, the multiplier for (4)′ by ν_ϵ, and the multiplier for (5) by λ. The first-order conditions with respect to ω_ϵ and $\tilde{W}_\epsilon$ respectively are:

$$-\pi_\varepsilon + \lambda\pi_\varepsilon\mu'(\omega_\varepsilon) - \psi_\varepsilon \leq 0 \tag{8}$$

$$\rho\beta\pi_\varepsilon\Omega'(\tilde{W}_\varepsilon) + \rho\beta\lambda\pi_\varepsilon + \rho\beta\psi_\varepsilon\Omega'(\tilde{W}_\varepsilon) + \nu_\varepsilon \leq 0 \tag{9}$$

(Condition (8) is an inequality to allow for the possibility that $\omega_\epsilon = 0$ at the optimum, and (9) is an

---

more detailed discussion, see Karabay and McLaren (2008).

inequality to allow for the possibility that $\tilde{w}_\epsilon = W_{min}$ at the optimum. It is easy to verify that $\tilde{w}_\epsilon = W_{max}$ is never an optimal choice, and so we will ignore that case.)

The following lemma is proven in the appendix:

Lemma 1. $W_{min} = V^{WS}$.

In other words, it is feasible for the employer to push the worker's payoff down to the opportunity payoff at the beginning of the employment relationship. Since it is in the interest of the employer to do so, Lemma 1 makes clear that employed workers receive the same payoff that they would receive when they are unemployed, or in other words, $V^{WR} = V^{WS}$. From (1), this immediately tells us:[4]

$$V^{WS} = \mu\,(0) / (1 - \beta), \tag{10}$$

and $\tilde{w}^*$ can be rewritten as:

[4]Of course, this implies that, in equilibrium, unemployed workers are indifferent between searching and not searching, so if a small search cost were imposed, there would be no search (this is a version of the Diamond search paradox). However, this feature would disappear if any avenue were opened up to allow workers to capture some portion of rents that employer-worker pair generate. For example, for simplicity, we have assumed that employers have all of the bargaining power, but this could be relaxed. In addition, we have assumed that *k* is common knowledge, but it would be reasonable to assume that different workers have different values of *k*, and while employers know the distribution of this parameter, they do not know any given worker's value of it. Either of these modifications would very substantially increase the complexity of the model, but would give employed workers some portion of the rents and thus avoid the Diamond paradox.

$$\tilde{W}^* \equiv \mu\,(0)\,/\,(1-\beta) + k\,/\rho\beta. \tag{11}$$

Further, since $\tilde{W}^* > V^{WS}$, Lemma 1 tells us that (6) is redundant, so it will be ignored henceforth. As a result, (9) will always hold with equality.

To sum up, in each period the employer maximizes (2), subject to (3), (4)′, (5), and (7). In the first period of the relationship, the worker's target utility $W = W_0$ is given by V^{WS}, but in the second period it is determined by the values of $\tilde{W}_\epsilon$ chosen in the first period and by the first-period state, and similarly in later periods it is determined by choices made at earlier dates.

We can now prove that the equilibrium always takes the same simple form: A one-period 'apprenticeship' in which zero wage is paid, followed by a time- and history-invariant but perhaps state-dependent wage. The key idea is that it is never optimal to promise more future utility than is required to satisfy the worker's incentive constraint (4)′, so after the first period of the relationship, the worker's target utility is always equal to $\tilde{W}^*$. This means that after the first period, the optimal wage settings by the firm are stationary. We can now establish a detailed proof through the following proposition.

Proposition 1. In the first period of an equilibrium employment relationship, the wage is set equal to zero in each state and the continuation payoff for the worker in each state is set equal to $\tilde{W}^*$. In the second period and all subsequent periods of the employment relationship, there is a pair of values ω^*_ϵ for $\epsilon = G, B$ such that regardless of history (provided neither partner has shirked), the wage is equal to ω^*_ϵ in state ϵ. In addition, the worker's continuation payoff is always equal to $\tilde{W}^*$. Further, after the first period there are three possible cases:

(i) The employer's incentive compatibility constraint (3) never binds, and $\omega_G^* = \omega_B^*$.

(ii) The employer's incentive compatibility constraint (3) binds in the bad states but not in the good states, and $\omega_G^* > \omega_B^*$.

(iii) The employer's incentive compatibility constraint (3) always binds, and $x_G - \omega_G^* = x_B - \omega_B^*$.

Proof. See Appendix.

To sum up, if the employer's incentive constraint does not bind, the worker goes through an 'apprenticeship period' at the beginning of the relationship, followed by a constant wage. This yields an equilibrium with *wage-smoothing*. On the other hand, if the employer's constraint ever binds, then it binds only (and always) in the bad state, resulting in a *fluctuating-wage equilibrium*.

It is useful to introduce the following lemma for the upcoming section.

Lemma 2. The value of the Kuhn-Tucker multiplier on the employer's bad-state incentive constraint, ψ_B, satisfies:

$$\psi_B = \pi_B \left(\frac{\mu'(\omega_B)}{\mu'(\omega_G)} - 1 \right).$$

Proof. See appendix.

In other words, ψ_B is a measure of how poor the wage insurance provided by the employer is. With perfect insurance, the wages in the two states would be equal, yielding a value of zero for ψ_B. With imperfect insurance, the good-state wage exceeds the bad-state wage, yielding $\psi_B > 0$.

The determination of the equilibrium wages can be summarized in Figure 1. The horizontal axis shows the good-state wage, ω_G, and the vertical axis shows the bad-state wage, ω_B. Given that the employer has all of the bargaining power, workers are indifferent between accepting a job and continuing to search, implying that $V^{WS} = V^{WR} = \mu(0) / (1 - \beta)$. Using $\tilde{w}^*$ as the target utility for workers in (5), and using equations (10) and (11) to simplify then implies that (5) becomes:

$$E_\epsilon \mu(\omega_\epsilon) \geq \mu(0) + k/\rho\beta.$$

This is represented as the downward-sloping curve *WW* in the figure; any wage combination must lie on or above this curve to satisfy the worker's incentive constraint. At the same time, using (1), the employer's bad-state incentive constraint (3) can be written as:

$$\omega_B \leq [-\rho\beta\, \pi_G \omega_G + x_B + \rho\beta\,(1 - Q^E)\pi_G (x_G - x_B)] / [1 - \rho\beta\,(\pi_G - Q^E)].$$

(A detailed derivation can be found in Karabay and McLaren (2008).) This is the downward-sloping line *EE* in the figure. Notice that it is shifted down by an increase in Q^E. Any wage combination must lie on or below this line in order to satisfy the employer's incentive constraint. The employer wishes to get as close as possible to the 45°-line, in order to minimize the expected wage, resulting in wages ω_B^* and ω_G^* as shown. If the equilibrium has volatile wages, as in the figure, anything that shifts the *EE* line upward, such as a reduction in Q^E, moves wages closer to the 45°-line, lowering the variance of wages and relaxing the employer's bad-state incentive constraint, thus reducing ψ_B. Further, a sufficient reduction in Q^E will bring *EE* above the intersection of *WW* with the 45°-line, making wage-smoothing the equilibrium.

Clearly, the value of Q^E is critical in determining the nature of the equilibrium. In Karabay and McLaren (2008), it is shown that Q^E is a decreasing function of the ratio E/L of employers to workers (this follows from writing the law of motion for searching workers and employers, and solving for the steady state). It follows that a rise in E/L, by lowering the steady-state value of Q^E, shifts the *EE* curve up. There is therefore a critical value, $(E/L)^{crit}$, of this ratio such that if $E/L > (E/L)^{crit}$, wage smoothing occurs and if $E/L < (E/L)^{crit}$, wages fluctuate. The reason is that the more abundant workers are, the easier it is for a shirking employer to find a new worker (the higher is Q^E), and so the weaker is the employer's punishment for shirking, and the harder it is to make a credible promise of wage smoothing. Further, if $E/L < (E/L)^{crit}$, a reduction in E/L will raise the good-state wage and lower the bad-state wage, increasing the variance of wages and the expected wage in the process. As a result, it is clear that ψ_B is a decreasing function of E/L, taking a value of zero if and only if $E/L > (E/L)^{crit}$. We will use this observation several times in analyzing firing costs.

A final observation about equilibrium without firing costs is that, since we have assumed that the employers have all of the bargaining power, workers are pushed to their reservation utility, $V^{WS} = \mu(0) / (1 - \beta)$. Introducing firing costs cannot possibly push them below this level of utility, so if it turns out that firing costs raise the welfare of employers, they are also Pareto improving.[5] This will

---

[5]Strictly speaking, we are using the term 'Pareto improvement' loosely, since we are comparing steady states.

simplify the welfare analysis in what follows.

3. Firing costs.

In this section, we will consider the effect of firing costs on the equilibrium. Governments impose many different forms of firing cost, such as administrative procedures for termination, advance notification, indemnities for dismissal, seniority pay and the legal costs of a trial if workers contest dismissals; see Heckman and Pagés (2003) for an extensive catalogue for Latin America. We will focus on two stylized types, in order to make our main points as simply as possible: Firing costs (such as administrative procedures and legal costs) that must be born by the employer but do not result in a payment to the worker, and therefore constitute a deadweight loss *per se*; and firing costs (such as indemnities and seniority pay) that do result in a payment to the worker, and thus constitute a mere transfer *per se*. We call the first type a nuisance cost and the second a severance payment. Ironically, we will find that the nuisance cost is more likely to be socially beneficial than the severance payment. We will formally define each in turn and analyze it in detail, but before we do so, we will consider as a thought experiment a hypothetical tax on shirking by employers, which is not implementable but is easy to understand, and helps to understand the mechanics of the firing costs studied subsequently. We call this thought experiment 'divine intervention.'

3.1. Divine Intervention.

This case is not realistic, but is a useful thought experiment for comparison with the two subsequent (and more realistic) cases to see how changes in incentive constraints change welfare. In this case, an employer pays a firing cost, denoted $c > 0$, if it shirks and thus terminates the relationship with the worker. (We assume that this cost is paid one period after the shirking.) In order to analyze this case, in the recursive optimization problem, equation (3) has to be replaced by the following:

$$x_{\epsilon} - \omega_{\epsilon} + \rho\beta\,\Omega\,(\tilde{W}_{\epsilon}) + (1-\rho)\beta\,V^{ES} \geq V^{ES} - \beta\,c\,. \qquad (3)(DI)$$

(The notation (*DI*) indicates 'divine intervention.') The only change is the last term on the right hand side, the penalty for shirking. All other constraints, and the objective function, are as in the original problem. Given this change, we have the following proposition on the effect of firing costs on employer's welfare.

Proposition 2. Under the fluctuating-wage equilibrium (in other words, if $E/L < (E/L)^{crit}$), firing costs modeled as divine intervention always improve the employer's welfare, whereas under the wage-smoothing equilibrium (in other words, if $E/L > (E/L)^{crit}$), they have no effect on the employer's welfare.

Proof. See appendix.

When there is a fluctuating-wage equilibrium, increasing the firing costs relaxes the employer's bad-state incentive constraint and thus helps the employer get closer to the wage-smoothing equilibrium where the employer has a higher payoff. In other words, they shift the *EE* curve up, moving the good-state and bad-state wages closer together, and therefore lowering the expected wage, raising expected profits. On the other hand, under the wage-smoothing equilibrium, the incentive constraint is not binding anyway, so firing costs do not improve the employer's welfare.

Note that, in this divine intervention case, the firing costs are never paid in equilibrium, because there is never any shirking in equilibrium. As a result, in the fluctuating-wage case, the higher c is, the higher the *EE* line is shifted up, and the higher is employer welfare, until at last the *EE* line crosses the *WW* curve above the 45°-line and a wage-smoothing equilibrium has been obtained. Note as well that there is no effect of c on workers' utility because the surplus is still all absorbed by employers.

3.2. Nuisance firing costs.

Of course, devices such as divine intervention are not possible in the real world, not least because third parties cannot identify which party has shirked, if any, when a worker and employer part company. However, we will now show that a simple but more realistic form of firing cost can have much the same effect. We introduce *nuisance firing costs*, in which an employer must pay a cost, denoted by $c > 0$, whenever its relationship with a worker terminates. This can be interpreted as paperwork and administrative procedures the firm must go through (or even a fine) whenever a worker leaves the firm.[6]

---

[6]This kind of firing cost is referred by a variety of terms by different authors, such as 'firing taxes' (as in Alvarez and Veracierto (2001)), 'dismissal costs' (Fella (2000)) or 'purely administrative costs' (Pissarides, 2001). An important difference, however, is that we assume that the enforcement agency cannot distinguish between quits and firings, so that the employer must pay the firing cost no matter why the worker leaves the firm – and yet, as we will see, the firing cost can

For simplicity, we assume that regulators cannot distinguish between the various reasons a worker may leave the firm,[7] so the cost must be paid whenever a worker leaves for any reason. If regulators *were* able to distinguish different reasons for worker separations (as in the divine intervention case), that would only strengthen the case for firing costs.

Compared to the original recursive optimization problem, we need to change the equations (2) and (3) as follows:

$$\Omega(W) = \max_{\{\omega_\epsilon, \tilde{W}_\epsilon\}} \sum_{\epsilon = G, B} \pi_\epsilon \left(x_\epsilon - \omega_\epsilon + \rho\beta\Omega(\tilde{W}_\epsilon) + (1-\rho)\beta\left(V^{ES} - \beta c\right)\right), \text{ and} \quad (2)(NC)$$

$$x_\epsilon - \omega_\epsilon + \rho\beta\,\Omega(\tilde{W}_\epsilon) + (1-\rho)\beta\,(V^{ES} - \beta\, c) \geq V^{ES} - \beta\, c\,. \quad (3)(NC)$$

(The notation (*NC*) denotes 'nuisance costs.') Comparing these with the case of divine intervention, we see that (3)(*NC*) has the same last term as (3)(*DI*), indicating the favorable effect of the firing cost on the employer's incentives. However, note two disadvantages compared with divine intervention. First, the employer's objective function (2)(*NC*) contains a term for the cost incurred from exogenous separations. Second, that same cost occurs in (3)(*NC*) (as the last term on the left-hand side), attenuating the beneficial effect of the firing cost on the employer's incentives. The net effect on

---

still be Pareto-improving.

[7]'Firing' in this model takes two interpretations. If the employer shirks, she has in effect chosen to end the relationship; this is, in effect, firing without cause. If the worker shirks, then when the employer discovers this fact at the end of the period, the employer will terminate the relationship. This is, in effect, firing with cause. We assume that regulators cannot distinguish either between these two, or between firing and exogenous separations.

welfare can be summarized as follows.

Proposition 3. There is a critical value of ψ_B, denoted $\psi_B^{NC} \equiv \rho / (1-\rho)$, such that the welfare of employers is improved by a small firing cost iff $\psi_B > \psi_B^{NC}$. Equivalently, there is a critical value of E/L, say $(E/L)^{NC} < (E/L)^{crit}$, such that employer welfare is improved by a small firing cost iff $E/L < (E/L)^{NC}$. Precisely, the derivatives of $V^{ER} = \Omega\,(V^{WS})$ and $\Omega\,(\tilde{w}^*)$ (respectively, an employer's welfare in the first period of an employment relationship and in subsequent periods) with respect to c are strictly positive at $c = 0$ iff $E/L < (E/L)^{NC}$.

Proof. See appendix.

This can be understood as follows. There are two competing effects. First, increasing firing costs relaxes the employer's constraint (as seen in (3)(*NC*)). This in turn decreases the incentive to shirk, increasing the amount of insurance to which the employer can credibly commit, and thus reducing the expected wage paid. This raises the employer's profits just as the shirking penalty in the divine intervention case did, and for the same reason. However, at the same time, when exogenous separation occurs, the employer pays the firing cost even though it is blameless (as seen in (2)(*NC*) and (3)(*NC*)). As a result, firing costs are Pareto improving only when insurance is sufficiently bad (in other words, when ψ_B is large enough).

In summary, because the nuisance cost must be paid whenever a worker leaves for any reason, Proposition 3 shows that this form of firing cost is less likely to be beneficial to the employers than divine intervention was. Further, it is clear for the same reason that the optimal level of c from the point of view of the employers is finite. (And, of course, in the case of a wage-smoothing equilibrium, since ψ_B is zero, the optimal value of c is zero.) Since the cost is paid in equilibrium with a frequency that is governed by the exogenous ρ, if c is increased without bound then eventually it will be impossible for an employer to make a profit.

Note that once again, workers' utility is unaffected by the firing costs.

3.3. Firing costs modeled as severance payment.

Now, suppose that the firing costs that the employer incurs whenever its relation with a worker terminates are paid to the worker. We will call this type of firing cost a *severance payment*. In the employer's recursive optimization problem, we keep equations (2)(*NC*) and (3)(*NC*) from the nuisance-cost case and change constraints (4) and (5) as follows:

$$\mu(\omega_\epsilon) - k + \rho\beta\,\tilde{W}_\epsilon + (1-\rho)\beta\,[V^{WS} + \beta\,(\mu(c)) - \mu(0))] \geq V^{WS} + \mu(\omega_\epsilon) - \mu(0) + \beta\,(\mu(c)) - \mu(0)), \quad (4)(SP)$$

$$\sum_{s=G,B} \pi_s \left[\mu(\omega_s) - k + \rho\beta\tilde{W}_s + (1-\rho)\,\beta\left[V^{WS} + \beta(\mu(c) - \mu(0))\right]\right] \geq W. \qquad (5)(SP)$$

(Here, the notation *SP* denotes 'severance payment.') The left-hand side of (4)(*SP*) shows the payoff to a worker from putting in effort. This includes the possibility of exogenous separation from the job, which would put the worker back into the search pool, giving a payoff equal to the payoff of any other searching worker except that in the first period of search her income would be c instead of 0. The right-hand side of (4)(*SP*) shows the payoff to a worker from shirking, which is the same as in (4) except that the worker's income in the first period after being fired is c instead of 0. This is important, in that it is a reward for shirking that is not present in other forms of firing cost, and of course it makes it harder for the employer to satisfy the worker's incentive constraint. We can call this the *worker's incentive effect* of severance payments. Constraint (4)(*SP*) can be replaced by the following more convenient form, which makes this effect more stark:

$$\tilde{w}_{\epsilon} \geq \tilde{w}^{*}, \text{ where } \tilde{w}^{*} \equiv [(1-(1-\rho)\beta)[V^{WS}+\beta(\mu(c)-\mu(0))]-\mu(0)+k]/\rho\beta . \qquad (4)'(SP)$$

The other changed equation, (5)(*SP*), is the same as the original target-utility constraint (5) except that (as above) the first-period income for a worker who has been exogenously separated is equal to c instead of 0. Furthermore, for the same reason the equation for V^{WR} is given by:

$$V^{WR} = \mu(0) - k + \rho\beta\,\tilde{w}^{*} + (1-\rho)\beta\,[V^{WS}+\beta(\mu(c)-\mu(0))] . \qquad (12)$$

If we plug $\tilde{w}^{*}$ from (4)′(*SP*) into (12), we get:

$$V^{WR} = V^{WS} + \beta(\mu(c)-\mu(0)) . \qquad (13)$$

Notice the difference that the presence of firing costs generates. Before, without firing costs, we have $V^{WR} = V^{WS}$, whereas with firing costs we have the extra term $\beta(\mu(c)-\mu(0))$ which represents the additional gain obtained by currently being employed rather than unemployed. This shows that a newly-employed worker has a strictly higher payoff than an unemployed worker, unlike in the previous cases, despite the fact that we have assigned all of the bargaining power to the employer. This is important, because it means that severance costs impart *rents* to employed workers, raising the share of the surplus captured by the worker. This *rent effect* is a second reason severance payments are costly to employers in a way in which the other forms of firing cost were not.

Both the incentive effect and the rent effect make it much less likely that employers will benefit from severance costs than from firing costs of the nuisance-cost variety. This is established in the following proposition:

Proposition 4. There is a critical value of ψ_B, denoted $\psi_B^{SP} > \psi_B^{NC}$, such that the welfare of employers is improved by a small firing cost of the severance-cost type iff $\psi_B > \psi_B^{SP}$. Precisely, the derivatives of $V^{ER} = \Omega\,(V^{WS})$ and $\Omega\,(\tilde{w}^*)$ (respectively, an employer's welfare in the first period of an employment relationship and in subsequent periods) with respect to c are strictly positive at $c = 0$ iff $\psi_B > \psi_B^{SP}$.

Proof. See appendix.

It is not mathematically impossible for severance costs to be Pareto improving, because the beneficial effect on employer's incentive is still there, but it is far less likely (meaning that $\psi_B^{SP} > \psi_B^{NC}$) because of the rent effect and the worker's incentive effect.

On the other hand, these two additional costs of severance payments can be neutralized with a scheme that taxes some portion of the severance payments and returns them to employers. Suppose that a fraction θ of severance payment received by each worker after exiting an employment relationship is taxed away and placed in a pool to be distributed evenly to employers. For each employer, the payment received out of the pool is lump-sum income. Clearly, in the limit as θ

approaches 1, the employer's objective function and all constraints become identical to what they were in the nuisance-cost case, except for an additional term of lump-sum income accruing to employers in each period. Clearly, then, if small firing costs of the nuisance-cost variety are Pareto -improving, a small firing cost of the severance-cost type combined with a sufficiently aggressive tax-and-transfer scheme will also be Pareto-improving. Retracing the steps of the proof of Proposition 3, and using *SPT* to denote 'severance payments with transfers,' we can establish:

Proposition 5. There is a critical value of ψ_B, denoted $\psi_B^{SPT} < \psi_B^{NC}$, such that the welfare of employers is improved by a small firing cost of the severance cost type, coupled with sufficiently aggressive transfers, iff $\psi_B > \psi_B^{SPT}$. Precisely, if $\psi_B > \psi_B^{SPT}$, then there is a value of θ, $\bar{\theta}$, such that if $\theta > \bar{\theta}$ the derivatives of $V^{ER} = \Omega\,(V^{WS})$ and $\Omega\,(\tilde{w}^*)$ (respectively, an employer's welfare in the first period of an employment relationship and in subsequent periods) with respect to c are strictly positive at $c = 0$.

Thus, we see that ironically, a nuisance-cost form of firing costs, in which the cost is *per se* deadweight loss, is more likely to be Pareto-improving than a severance payment, which is *per se* just a transfer. The reason is that the severance payments worsen workers' incentives by subsidizing shirking, and transfer rents to them as well, raising in effect their bargaining power within the firm. However, both of these effects can be neutralized with a simple tax-and-transfer scheme, with the result that severance payments are *more* likely to be Pareto-improving than the nuisance costs.

4. Globalization and Firing Costs.

A simple extension of the model can allow us to say something about the relationship between firing costs and international openness. Here we will focus on the case of nuisance-cost type firing costs, which is the simplest to analyze. Suppose that there are two countries, the US and India, which are identical except for the number of employers and workers. Denote respectively the number of employers and workers in the US by E and L, and the numbers in India by E^* and L^*. Suppose that the ratio of employers to workers is higher in the US than in India: $E/L > E^*/L^*$.

We can analyze the equilibrium in the case of a closed economy and compared it with the case of an integrated world economy. Since this is a one-good economy, there is no reason for goods trade, but labor market integration is possible. This could be interpreted as either immigration or offshoring, meaning hiring a worker from another country. Under full integration of the two labor markets, the world equilibrium would be the same as a closed-economy equilibrium with an employer-worker ratio of $(E + E^*) / (L + L^*)$. For the US, that is, of course, a drop in the ratio, and for India it is a rise in the ratio.

Proposition 3 shows that a small nuisance-type firing cost is Pareto-improving provided that $\psi_B > \psi_B^{NC}$, or equivalently, $E/L < (E/L)^{NC}$. From this, it is immediate that if a small nuisance cost is Pareto-improving in the US as a closed economy, then it will be so in the integrated economy as well. Further, if $(E/L)^{NC}$ is between and $(E + E^*) / (L + L^*)$ and E/L, then firing costs are not Pareto improving for the US economy before integration but they are so after integration. This can be summarized as follows:

Proposition 6. For the labor-scarce economy, firing costs of the nuisance type are more likely to be Pareto-improving after integration of the world economy than before. The opposite is true in the labor-abundant economy.

The reason is that these firing costs are most likely to be beneficial to employers the greater is the trust problem faced by employers (hence the higher is the multiplier, ψ_B, on the employer's incentive constraint). This trust problem is greater, the easier it is for an employer to find a new worker after reneging on a commitment to an existing employee and severing the agreement. The more plentiful workers are, the easier it is to find a new one, and so the worse the problem of trust becomes. Labor market integration with a labor-rich economy makes workers, in effect, more plentiful, thus providing the result.

This result may be surprising to readers familiar with the literature on firing costs. Kambourov (2006), for example, studies firing costs in a model of trade liberalization and finds that the gains from trade liberalization are reduced by the presence of firing costs because firing costs slow the reallocation of labor that is necessary to realize the gains from liberalization. Therefore, the efficiency argument against firing costs is stronger in an open economy than in a closed one. Utar (2007) develops a similar line of reasoning.

Here, the efficiency argument *in favor of* firing costs is *stronger* in an open economy, so the relationship between the two spheres of policy is very different indeed than it is in the previous papers. The point is not that either approach is wrong, but rather that the allocative effects of firing costs and their effects on the nature of contracting are very different, and a full analysis of firing costs in practice ought to allow for both.

5. Conclusion.

We have shown that in a world of 'invisible handshakes,' where firms offer wage insurance as part of an employment relationship but are constrained by lack of trust, firing costs imposed by government can be helpful, and indeed Pareto improving. The reason is that an employer's credibility is better, the worse is the punishment it faces from reneging on its wage promises, and if the punishment is to lose the worker and also face a firing cost, that is more severe than losing the worker alone. Thus, firing costs improve the employers' ability to make credible wage promises, allowing it to promise less of a wage cut in low-profitability times, and thus to get away with paying workers a smaller risk premium. We show that this phenomenon of Pareto-improving firing costs is, ironically, less likely with severance payments than with firing costs that are *per se* deadweight loss, unless the severance payment policy is combined with an appropriate tax-and transfer scheme. Finally, for an affluent economy, firing costs are more likely to be Pareto-improving with more international integration than for a closed economy.

We should emphasize that this is not a clarion call for firing costs, as even in our model they are Pareto-improving in some circumstances but not in others, and further, our model has been constructed in such a way as to exclude the well-known allocative effects of firing costs, which could lead to a net negative effect. However, these implicit-contracting effects ought to be part of a full cost-benefit analysis, and this model further shows the way to a possibly useful empirical agenda. Our model predicts that some kinds of firing cost should reduce the variance of wages a given worker will receive within a relationship with a given employer, and this can be tested. It would be of interest to know whether or not firing costs at least satisfy this necessary condition for Pareto improvement, and this would shed light not only on firing costs but on the workings of the invisible handshake in practice.

Appendix.

In order to clarify the exposition, from now on, we leave the notation of the multipliers for the constraints in the first period intact (i.e. ψ_ϵ , ν_ϵ , λ), whereas we change the notation of the multipliers for the constraints in subsequent periods by adding a hat symbol (i.e. $\psi_\epsilon^{\wedge}$, $\nu_\epsilon^{\wedge}$, $\lambda^{\wedge}$).

Proof of Lemma 1.

First, observe that since a worker will never accept employment with payoff below V^{WS}, we must have $W_{min} \geq V^{WS}$. We will show that $W_{min} = V^{WS}$ by showing through contradiction that it is not possible to have $W_{min} > V^{WS}$. First, however, it will be useful to demonstrate that $W_{min} \leq \tilde{w}^*$, which will allow us to ignore the constraint $W_{min} \leq \tilde{w}_\epsilon$ and treat (9) as an equality.

Suppose, then, that $W_{min} > \tilde{w}^*$. In this case, the worker's incentive constraint (4)′ can never bind, and so $\nu_\epsilon = 0$ for $\epsilon = B, G$. Consider the first-period decision. Then if the lower-bound constraint in (6) does not bind for state ϵ, then the first order condition (9) holds with equality, so from (9), we have $-\lambda = \Omega'(W_0) \leq \Omega'(\tilde{w}_\epsilon) < 0$, so $W_0 \geq \tilde{w}_\epsilon > W_{min}$. But this would be a suboptimal choice by the employer, as the employer could choose a first-period wage and future worker payoffs to give the worker a current payoff W_0 equal to W_{min}; this would realize a higher profit, and would also satisfy the first-period target utility constraint (5) since $W_{min} \geq V^{WS}$. Therefore, we conclude that the lower-bound constraint in (6) must bind in the first-period decision for both states, and so $\tilde{W}_\epsilon = W_{min}$ for $\epsilon = B, G$. Now, suppose that the target utility constraint (5) binds in the first period, recalling that the first-period target utility level W is equal to V^{WS}. This immediately yields a contradiction, as it implies that a worker payoff of V^{WS} can be realized in equilibrium, which contradicts the maintained assumption that $W_{min} > \tilde{W}^*$ (since $\tilde{W}^* > V^{WS}$). Therefore, the target utility constraint does not bind in the first period, and so $\lambda = 0$. But then the first-order condition (8) for the wage cannot be satisfied for any positive value of the wage, implying a wage of zero in the first period in both states. This implies a first-period payoff for the worker equal to:

$$\mu(0) - k + \rho\beta\, W_{min} + (1 - \rho)\beta\, V^{WS} \leq \mu(0) - k + \beta\, W_{min},$$

since $W_{min} \geq V^{WS}$. This is strictly less than W_{min}, since W_{min} cannot be less than $\mu(0)/(1 - \beta)$ (which is the utility from permanent zero consumption; in no case could the worker receive lower utility than that).

But this is a contradiction, since by definition it is not possible to give a worker a payoff less than W_{min}. Therefore, $W_{min} \leq \tilde{W}^*$. As a result, the constraint $W_{min} \leq \tilde{W}_\epsilon$ is redundant, and can be removed without changing the solution. Consequently, we can treat (9) as an equality.

Now, suppose that $W_{min} > V^{WS}$. Then if the target utility constraint (5) binds in the first period, recalling that the first-period target utility level W is equal to V^{WS}, then we have a contradiction as before, so suppose that the target utility constraint does not bind in the first period. Then $\lambda = 0$, so the first-order condition (8) for the wage cannot be satisfied for any positive value of the wage, implying a wage of zero in the first period in both states. Note that with $\lambda = 0$, the first-order condition (9) cannot be satisfied with equality unless $\nu_\epsilon > 0$, so that the workers' incentive constraint binds, and so $\tilde{w}_\epsilon = \tilde{w}^*$. (We already know that the lower-bound constraint in (6) is redundant, because we have shown above that $W_{min} \leq \tilde{w}^*$.) This implies a first-period payoff for the worker equal to:

$$\begin{aligned} &\mu(0) - k + \rho\beta\, \tilde{w}^* + (1-\rho)\beta\, V^{WS} \\ &= V^{WS} \\ &< W_{min}, \end{aligned}$$

since we assume that $W_{min} > V^{WS}$. Again, this is a contradiction, since it is not possible to give a worker a payoff less than W_{min}. We conclude that $W_{min} \leq V^{WS} < \tilde{w}^*$. Since we already know that $W_{min} \geq V^{WS}$, we conclude that $W_{min} = V^{WS}$. **Q.E.D.**

Proof of Proposition 1.

Consider the first period problem. First, assume that $\lambda = 0$. Then, the first order condition (8) for the wage cannot be satisfied for any positive value of the wage, implying a wage of zero in the first period in both states. We know from Lemma 1 that the first order condition (9) holds with equality. This in turn requires that $\nu_\epsilon > 0$, meaning $\tilde{w}_\epsilon = \tilde{w}^*$. As a result, $\omega_\epsilon = 0$ and $\tilde{w}_\epsilon = \tilde{w}^*$ in the first period. Alternatively, assume that $\lambda > 0$. If $\nu_\epsilon > 0$, then $\tilde{w}_\epsilon = \tilde{w}^*$ and from the target utility constraint, we must have $\omega_\epsilon = 0$ and we are done. On the other hand, if $\nu_\epsilon = 0$, then (9) becomes:

$$(\pi_\varepsilon + \psi_\varepsilon)\Omega'\left(\tilde{W}_\varepsilon\right) = -\pi_\varepsilon \lambda, \text{ or}$$

$$\Omega'\left(\tilde{W}_\varepsilon\right) = \left(\frac{\pi_\varepsilon}{\pi_\varepsilon + \psi_\varepsilon}\right)\Omega'\left(W_0\right),$$

since by the envelope theorem $\Omega'(W_0) = -\lambda$. The concavity of Ω implies that $\tilde{W}_\epsilon \leq W_0 = V^{WS}$. But since $V^{WS} < \tilde{W}^*$, this implies that the worker's incentive compatibility constraint in (4)′ will be violated, a contradiction. Therefore, we conclude that $\omega_\epsilon = 0$ and $\tilde{W}_\epsilon = \tilde{W}^*$ in the first period.

Consider now the second-period problem. We know that the target continuation payoff for the worker is $\tilde{W}^*$. We claim that the choice of next-period continuation payoff $\tilde{W}_\epsilon$ will be equal to $\tilde{W}^*$ for $\epsilon = G, B$. If $\hat{\nu}_\epsilon > 0$, then complementary slackness implies that $\tilde{W}_\epsilon = \tilde{W}^*$. Therefore, suppose that $\hat{\nu}_\epsilon = 0$. This implies that (9) becomes:

$$\Omega'\left(\tilde{W}_\varepsilon\right) = \left(-\lambda^{\wedge}\right)\left(\frac{\pi_\varepsilon}{\pi_\varepsilon + \hat{\psi}_\varepsilon}\right).$$

Since, by the envelope theorem, $-\lambda^{\wedge} = \Omega'(W)$, and as we recall for the second-period problem the worker's target utility $W = \tilde{W}^*$, this becomes:

$$\Omega'\left(\tilde{W}_\varepsilon\right) = \Omega'\left(\tilde{W}^*\right)\left(\frac{\pi_\varepsilon}{\pi_\varepsilon + \hat{\psi}_\varepsilon}\right). \tag{14}$$

If $\psi^{\wedge}_{\epsilon} = 0$, this implies through the strict concavity of Ω that $\tilde{w}_{\epsilon} = \tilde{w}^*$, and we are done. On the other hand, if $\psi^{\wedge}_{\epsilon} > 0$, (14) then implies that $0 > \Omega'(\tilde{w}_{\epsilon}) > \Omega'(\tilde{w}^*)$, implying that $\tilde{w}_{\epsilon} < \tilde{w}^*$. However, this violates (4)$'$. Therefore, all possibilities either imply that $\tilde{w}_{\epsilon} = \tilde{w}^*$ or lead to a contradiction, and the claim is proven.

Since $\tilde{w}_{\epsilon} = \tilde{w}^*$, the optimization problem in the third period of the relationship is identical to that of the second period. By induction, the target utility for the worker in every period after the first, regardless of history, is equal to $\tilde{w}^*$, and so the wage chosen for each state in every period after the first, regardless of history, is the same.

Now, to establish the three possible outcomes, we consider each possible case in turn. Consider the optimization problem (2) at any date after the first period of relationship. First, suppose that the employer's constraint does not bind in either state. In this case, $\psi^{\wedge}_{\epsilon} = 0$ for $\epsilon = G, B$. Condition (8) now becomes:

$$-\pi_{\varepsilon} + \lambda^{\wedge}\pi_{\varepsilon}\mu'(\omega_{\varepsilon}) \leq 0. \tag{15}$$

If this holds with strict inequality for some ϵ, then $\omega_{\epsilon} = 0$. This clearly cannot be true for both values of ϵ, because that would imply a permanent zero wage, and it would not be possible to satisfy (5). (To see this, formally, substitute $W = \tilde{w}^*$, the expression for V^{WS}, and $\omega_G = \omega_B = 0$ into (5), and note that the constraint is violated.) Therefore, for at most one state, say ϵ', can the inequality in (15) be strict. Denote by ϵ'' the state with equality in (15). Then $\mu'(0) < 1/\lambda^{\wedge} = \mu'(\omega_{\epsilon''})$. However, given that $\omega_{\epsilon''}$ is non-negative and μ is strictly concave, this is impossible. We conclude that (15) must hold with equality in both states, and therefore $\omega_G = \omega_B$.

Next, suppose that we have $\hat{\psi}_G > 0$ and $\hat{\psi}_B = 0$, so that the employer's constraint binds only in the good state. We will show that this leads to a contradiction. Recall from the previous proposition that $\tilde{W}_\epsilon = \tilde{W}^*$ for both states, and note that, by assumption, (3) is satisfied with equality for $\epsilon = G$. Since $x_B < x_G$, we now see that (3) must be violated for $\epsilon = B$ if $\omega_G \leq \omega_B$. Therefore, $\omega_G > \omega_B \geq 0$. This implies that (8) holds with equality in the good state. Applying (8), then, we have:

$$\mu'(\omega_G) = \frac{1}{\hat{\lambda}}\left(1 + \frac{\hat{\psi}_G}{\pi_G}\right) > \frac{1}{\hat{\lambda}} \geq \mu'(\omega_B),$$

which contradicts the requirement that $\omega_G > \omega_B$. This shows that it is not possible for the employer's constraint to bind in the good state.

Now suppose that we have $\hat{\psi}_G = 0$ and $\hat{\psi}_B > 0$, so that the employer's constraint binds only in the bad state. We now wish to prove that in this case $\omega_G > \omega_B$. Suppose to the contrary that $\omega_G \leq \omega_B$. This implies that $\omega_B > 0$ (since, as shown earlier, after the first period it is not possible to have zero wage in both states), so that (8) holds with equality in the bad state. Then, from (8):

$$\mu'(\omega_B) = \frac{1}{\hat{\lambda}}\left(1 + \frac{\hat{\psi}_B}{\pi_B}\right) > \frac{1}{\hat{\lambda}} \geq \mu'(\omega_G),$$

which implies that $\omega_G > \omega_B$. Therefore, we have a contradiction, and we conclude that $\omega_G > \omega_B$.

Finally, suppose that the employer's constraint binds in both states. Given that $\tilde{W}_\epsilon = \tilde{W}^*$ in both states, equality in both states for (3) requires that short-term profits $x_\epsilon - \omega_\epsilon^*$ are equal in the two states.

We have thus eliminated all possibilities aside from those listed in the statement of the proposition. **Q.E.D.**

Proof of Lemma 2.

In order to prove Lemma 2, we will first state and prove the following lemma.

Lemma A1. The multiplier on the target utility constraint, $\hat{\lambda}$, satisfies $\hat{\lambda} = 1/\mu'(\omega_G)$.

Proof. We know from Proposition 1 that after the first period, it is not possible to have zero wage in both states and $\omega_B \leq \omega_G$. As a result, $\omega_G > 0$ and equation (8) holds with equality in the good state. In addition, again from Proposition 1, we know that $\hat{\psi}_G = 0$ (except a single case where employer's incentive compatibility constraint (3) always binds. We can ignore this point by continuity). Then in the good state, (8) becomes:

$$-\pi_G + \hat{\lambda}\pi_G \mu'(\omega_G) = 0 \text{ or } \hat{\lambda} = 1/\mu'(\omega_G). \qquad \textbf{Q.E.D.}$$

We are now ready to prove Lemma 2. The first order condition of the optimization problem with respect to ω_ϵ holds with equality for an interior solution (exceptions are ruled out by continuity). Then, in the bad state, (8) is given by:

$$-\pi_B + \hat{\lambda}\pi_B \mu'(\omega_B) - \hat{\psi}_B = 0. \tag{16}$$

Then, using Lemma A1, the result follows immediately. **Q.E.D.**

The proof of the following lemma will be useful in order to determine the region in which the employer benefits from the presence of the firing costs.

Lemma A2. In any equilibrium, $\psi_{\hat{B}} \leq (1-\rho\beta\,(1-Q^E))/\rho\beta$. Equality holds only on a set of measure zero of the parameter space.

Proof. Recalling Figure 1, the equations for the *EE* line and the *WW* curve are respectively:[8]

$$\omega_B = [-\rho\beta\,\pi_G\,\omega_G + x_B + \rho\beta\,(1 - Q^E)\pi_G\,(x_G - x_B)] \,/\, [1-\rho\beta\,(\pi_G - Q^E)], \text{ and}$$

$$\mathrm{E}_\epsilon\,\mu(\omega^*_\epsilon) = \mu(0) + k/\rho\beta.$$

Consider changes to the equilibrium as we vary Q^E, holding other parameters constant. An increase in Q^E shifts the *EE* line down. If Q^E is low enough that the *WW* curve intersects the 45°-line below the *EE* line, the equilibrium features wage smoothing, and as a result (from Lemma 2) $\psi_{\hat{B}} = 0$. On the other hand, if Q^E is large enough that the *WW* curve lies everywhere above the *EE* line there is no equilibrium with positive output. In between those extremes, the equilibrium has volatile wages at the intersection of *EE* and *WW* closest to the 45°-line, and any increase in Q^E moves the

[8]The introduction of firing costs will shift *EE* and *WW* without changing their slopes.

intersection down and to the right along the *WW* curve. From Lemma 2, in this range an increase in Q^E will therefore raise $\psi_B^{\hat{}}$. As a result, the highest possible value of $\psi_B^{\hat{}}$ will result from the highest value of Q^E that is consistent with an equilibrium with positive output, or the value for which *EE* is tangent to *WW*. Setting the slopes of *EE* and *WW* equal yields:

$$-\rho\beta\,\pi_G/(1-\rho\beta\,(\pi_G - Q^E)) = -\pi_G\,\mu'(\omega_G)/\pi_B\,\mu'(\omega_B). \tag{17}$$

From Lemma A1, we know that $\lambda^{\hat{}} = 1/\mu'(\omega_G)$. The lower bound then becomes:

$$-\pi_B + \lambda^{\hat{}}\,\pi_B\,\mu'(\omega_B) \leq (1-\rho\beta\,(1-Q^E))/\rho\beta\,.$$

Then using (16), we have:

$$\psi_B^{\hat{}} \leq (1-\rho\beta\,(1-Q^E))/\rho\beta\,.$$

Clearly, this holds with equality only where *WW* and *EE* are tangent. This occurs at a single value of Q^E, conditional on all other parameter values, and so equality occurs only on a zero-measure portion of the parameter space. **Q.E.D.**

Proof of Proposition 2.

We now need to condition the value function on *c*, and so we write it as $\Omega\,(W, c)$. In this section, we first show that if firing costs improve the welfare of employers in the first period, then they also improve employers' welfare in subsequent periods.[9] Then, we prove that firing costs indeed improve the welfare of employers in the first period.

First, we write the expression for V^{ER} (recalling that $V^{ER} = \Omega\,(V^{WS}, c)$ and $\omega_{\epsilon} = 0$ in the first period):[10]

$$V^{ER} = \bar{x} + \rho\beta\,\Omega\,(\tilde{w}^*, c) + (1-\rho)\beta\, V^{ES}, \tag{18}$$

where $\bar{x} \equiv \pi_G\, x_G + \pi_B\, x_B$. In the above equation, we use the fact that $\tilde{w}_{\epsilon} = \tilde{w}^*$. If we take the total derivative of both sides in (18) with respect to firing costs, we have:

$$d\,V^{ER} / d\,c = \rho\beta\,(d\Omega\,(\tilde{w}^*, c) / d\,c) + (1-\rho)\beta\,(d\,V^{ES} / d\,c). \tag{19}$$

In the divine intervention case, V^{ES} is still given by (1). Hence, using (1), one can obtain $d\,V^{ES} / d\,c$ as:

$$d\,V^{ES} / d\,c = [Q^E\rho\beta \,/\, (1-\beta\,(1-Q^E\rho))](d\,V^{ER} / d\,c). \tag{20}$$

If we plug (20) into (19) and rearrange, we get:

$$d\,V^{ER} / d\,c = \{\rho\beta\,(1-\beta\,(1-Q^E\rho)) \,/\, [(1-\beta\,)(1+\rho\beta\,Q^E\,) + \rho^{\,2}\beta^{\,2}\,Q^E]\}(d\Omega\,(\tilde{w}^*, c) / d\,c).$$

---

[9]Recall that in this model equilibrium contracts specify the same behavior in all periods after the first.

[10]In the first period of the employment relationship, $W = W_0 = V^{WS}$.

As a result, whenever $d\,V^{ER}/d\,c$ (hence $d\Omega\,(V^{WS},c)/d\,c$) is positive, $d\Omega\,(\tilde{w}^*,c)/d\,c$ is positive as well. In other words, whenever the firing costs benefit the employer in the first period, they also benefit the employer in subsequent periods. This is because the benefit of lower expected wages occurs in the second and subsequent periods, not the first period.

Our next task to show is under what circumstances firing costs benefit employers in the first period. If we take the total derivative of (2), (3)(*DI*), (4)′, (5) and (7) with respect to *c* in the first period, we have:[11,12]

$$d\Omega\,(V^{WS},c)/d\,c = (\pi_G+\pi_B)[\rho\beta\,(d\Omega\,(\tilde{w}^*,c)/d\,c) + (1-\rho)\beta\,(d\,V^{ES}/d\,c)]\,, \quad (21)$$

where $\pi_G+\pi_B=1$ and $d\,V^{ES}/d\,c$ is given by (20). In order to evaluate the above derivative we need to determine $d\Omega\,(\tilde{w}^*,c)/d\,c$, which is the effect of the firing costs on the employer's welfare in the second period. To do so, we take the total derivative of (2), (3)(*DI*), (4)′, (5) and (7) with respect to *c* in the second period:

$$\begin{aligned} d\Omega\,(\tilde{w}^*,c)/d\,c = {} & [\rho\beta\,(d\Omega\,(\tilde{w}^*,c)/d\,c) + (1-\rho)\beta\,(d\,V^{ES}/d\,c)] + \\ & (\psi_{\hat{G}}+\psi_{\hat{B}})[\rho\beta\,(d\Omega\,(\tilde{w}^*,c)/d\,c) - (1-(1-\rho)\beta)(d\,V^{ES}/d\,c) + \beta\,]\,. \quad (22) \end{aligned}$$

---

[11]We use the fact that in the first period, employer's IC constraint does not bind in either state.

[12]In taking the total derivative, we use the envelope theorem, so the derivative with respect to ω_ϵ and $\tilde{w}_\epsilon$ are 0.

Then, using the fact that the employer's good state IC constraint does not bind in any period, i.e., $\hat{\psi}_G = 0$ and rearranging, (22) becomes:

$$d\Omega\,(\tilde{W}^*, c) / d\,c = \{[(1-\rho)\beta - (1- (1-\rho)\,\beta\,)\hat{\psi}_B] /(1-\rho\beta\,(1+\hat{\psi}_B))\}(d\,V^{ES} / d\,c) + \beta\,\hat{\psi}_B/(1-\rho\beta\,(1+\hat{\psi}_B)). \tag{23}$$

If we plug (23) into (21), we obtain:

$$d\Omega\,(V^{WS}, c) / d\,c = \{\beta\,[1-\rho\,(1+\hat{\psi}_B)] /(1-\rho\beta\,(1+\hat{\psi}_B))\}(d\,V^{ES} / d\,c) + \rho\beta^2\,\hat{\psi}_B / (1-\rho\beta\,(1+\hat{\psi}_B)). \tag{24}$$

Finally, using (20) and the fact that $\Omega\,(V^{WS}, c) = V^{ER}$ (hence, $d\Omega\,(V^{WS}, c)/ d\,c = d\,V^{ER} / d\,c$), equation (24) becomes:

$$d\Omega\,(V^{WS}, c) / d\,c = [\rho\beta^2\,(1-\beta\,(1-Q^E\rho) / (1-\beta\,)(1-\rho\beta\,(1+\hat{\psi}_B - Q^E))]\hat{\psi}_B.$$

From the above expression, we can easily see that $d\Omega\,(V^{WS}, c)/ d\,c > 0$ as long as:

$$0 < \hat{\psi}_B < (1-\rho\beta\,(1-Q^E))\,/\rho\beta\,.$$

Notice that in a fluctuating wage equilibrium, employer's bad-state IC constraint binds, so necessarily $0 < \hat{\psi}_B$. In addition, Lemma A2 shows us that in order to have an equilibrium,

$\psi_{\hat{B}} < (1-\rho\beta\ (1-Q^{E}))\ /\rho\beta$. (We are disregarding the set of measure zero where in place of the inequality we would have an equality.) Thus, we conclude that in any period for the divine intervention case, under the fluctuating wage equilibrium, increasing the firing costs benefits the employer, whereas under the wage-smoothing equilibrium (where $\psi_{\hat{B}} = 0$), firing costs have no effect on employer's welfare in any period. **Q.E.D.**

Proof of Proposition 3.

First, as in the proof of Proposition 2, we show that if firing costs improve the welfare of employers in the first period, then they also improve the welfare of employers in subsequent periods. Again, this is because the benefit of lower expected wages occurs in the second and subsequent periods, not the first period. Then, we show the conditions under which firing costs improve the welfare of employers in the first period.

The expression for V^{ER} in the nuisance cost case is given by (recalling that $V^{ER} = \Omega\ (V^{WS}, c)$ and $\omega_{\epsilon} = 0$ in the first period):

$$V^{ER} = \bar{x} + \rho\beta\ \Omega\ (\tilde{w}^{*}, c) + (1-\rho)\beta\ (V^{ES} - \beta\ c), \tag{25}$$

where $\bar{x} \equiv \pi_G\ x_G + \pi_B\ x_B$. In the above equation, we use the fact that $\tilde{w}_{\epsilon} = \tilde{w}^{*}$. Taking the total derivative of both sides in (25) with respect to c, we have:

$$d\ V^{ER} / d\ c = \rho\beta\ (d\Omega\ (\tilde{w}^{*}, c) / d\ c) + (1-\rho)\beta\ (d\ V^{ES} / d\ c - \beta\ ). \tag{26}$$

In the nuisance cost case, the V^{ES} expression changes as:

$$V^{ES} = Q^E\rho\beta\, V^{ER} + Q^E(1-\rho)\beta\,(V^{ES} - \beta\, c) + (1-Q^E)\beta\, V^{ES}. \tag{27}$$

Then, using (27), we can obtain $d\,V^{ES} / d\,c$ as:

$$d\,V^{ES} / d\,c = [Q^E\rho\beta / (1-\beta\,(1-Q^E\rho))](d\,V^{ER} / d\,c) - [Q^E(1-\rho)\beta^2 / (1-\beta\,(1-Q^E\rho))]. \tag{28}$$

If we plug (28) into (26) and rearrange, we have:

$$d\,V^{ER} / d\,c = \{\rho\beta\,(1-\beta\,(1-Q^E\rho)) / [(1-\beta\,)(1+\rho\beta\,Q^E) + \rho^2\beta^2 Q^E]\}(d\Omega\,(\tilde{w}^*, c) / d\,c) -$$
$$(1-\rho)\beta^2(1-\beta\,(1-Q^E)) / [(1-\beta\,)(1+\rho\beta\,Q^E) + \rho^2\beta^2 Q^E]. \tag{29}$$

As a result, whenever $d\,V^{ER} / d\,c$ is positive, we necessarily have $d\Omega\,(\tilde{w}^*, c) / d\,c > 0$ (notice that the second term on the right hand side of (29) is negative).

Now, we are ready to analyze under what circumstances firing costs benefit employers in the first period. If we take the total derivative of (2)(*NC*), (3)(*NC*), (4)′, (5) and (7) with respect to c in the first period, we have:

$$d\Omega\,(V^{WS}, c) / d\,c = (\pi_G + \pi_B)[\rho\beta\,(d\Omega\,(\tilde{w}^*, c) / d\,c\,) + (1-\rho)\beta\,(d\,V^{ES} / d\,c - \beta\,)]. \tag{30}$$

where $\pi_G + \pi_B = 1$ and $d\,V^{ES} / d\,c$ is given by (28). In order to evaluate the above derivative we need to determine $d\Omega\,(\tilde{w}^*, c) / d\,c$, which is the effect of the firing costs on the employer's welfare in the second period. To do so, we take the total derivative of (2)(*NC*), (3)(*NC*), (4)′, (5) and (7) with respect to c in the second period:

$$d\Omega\,(\tilde{w}^*, c) / d\,c = [\rho\beta\,(d\Omega\,(\tilde{w}_\epsilon, c) / d\,c) + (1-\rho)\beta\,(d\,V^{ES} / d\,c - \beta\,)] + (\psi_{\hat{G}} + \psi_{\hat{B}})[\rho\beta\,(d\Omega\,(\tilde{w}_\epsilon, c) / d\,c) - (1-\,(1-\rho)\,\beta\,)(d\,V^{ES} / d\,c - \beta\,)]. \tag{31}$$

Then, using the fact that the employer's good state IC constraint does not bind in any period, i.e., $\psi_{\hat{G}} = 0$ and rearranging, (31) becomes:

$$d\Omega\,(\tilde{w}^*, c) / d\,c = \{[(1-\rho)\beta - (1-\,(1-\rho)\beta\,)\psi_{\hat{B}}] / (1-\rho\beta\,(1+\psi_{\hat{B}}))\}(d\,V^{ES} / d\,c - \beta\,). \tag{32}$$

If we plug (32) into (30), we obtain:

$$d\Omega\,(V^{WS}, c) / d\,c = \{\beta\,[1-\rho\,(1+\psi_{\hat{B}})] / (1-\rho\beta\,(1+\psi_{\hat{B}}))\}(d\,V^{ES} / d\,c - \beta\,). \tag{33}$$

Finally, using (28) and the fact that $\Omega\,(V^{WS}, c) = V^{ER}$ (hence, $d\Omega\,(V^{WS}, c) / d\,c = d\,V^{ER} / d\,c$), equation (33) becomes:

$$d\Omega\,(V^{WS}, c) / d\,c = -[\beta^2\,(1-\rho\,(1+\psi_{\hat{B}}))(1-\beta\,(1-Q^E\,)) / (1-\beta\,)(1-\rho\beta\,(1+\psi_{\hat{B}} - Q^E\,))].$$

From the above expression, we can easily see that $d\Omega\,(V^{WS}, c) / d\,c > 0$ as long as:

$$(1-\rho)/\rho < \psi_B^{\hat{}} < (1-\rho\beta(1-Q^E))/\rho\beta .$$

Lemma A2 shows us that in order to have an equilibrium, $\psi_B^{\hat{}} < (1-\rho\beta(1-Q^E))/\rho\beta$ (disregarding the set of measure zero where the constraint holds with equality). Therefore, there is a threshold value of $\psi_B^{\hat{}}$, denoted $\psi_B^{NC} \equiv \rho/(1-\rho)$, above which a small firing cost improves employer's welfare. In other words, there is a critical value of E/L, $(E/L)^{NC} < (E/L)^{crit}$, such that employer welfare is improved by a small firing cost iff $E/L < (E/L)^{NC}$. **Q.E.D.**

Proof of Proposition 4.

First, as in the proofs of Propositions 2 and 3, we show that if firing costs improve the welfare of employers in the first period of an employment relationship, then they also improve the welfare of employers who are in later stages of an employment relationship, because the benefit of lower expected wages occurs in the second and subsequent periods, not the first period. Then, we show the conditions under which firing costs improve the welfare of employers in the first period.

The expressions for V^{ER}, V^{ES} and hence dV^{ER}/dc in the severance payment case are the same as the ones in the nuisance cost case and given by (25), (27) and (29), respectively. As a result, by the same reasoning as in the nuisance cost case, we conclude that whenever dV^{ER}/dc is positive, we necessarily have $d\Omega(\tilde{w}^*, c)/dc > 0$. Thus, if employers in the first period of a relationship benefit, then employers in later periods do as well.

Now, we are ready to analyze under what circumstances firing costs benefit employers in the first period. If we take the total derivative of (2)(*NC*), (3)(*NC*), (4)′(*SP*), (5)(*SP*) and (7) with respect to c in the first period, we have:

$$d\Omega\,(V^{WS}, c)/\,d\,c = (\pi_G + \pi_B)[\rho\beta\,(d\Omega\,(\tilde{w}^*, c)\,/\,d\,c\,) + (1-\rho)\beta\,(d\,V^{ES}\,/\,d\,c - \beta)] -$$

$$[(1-\,(1-\rho)\beta\,)\,/\rho\beta\,](\nu_G + \nu_B)(d\,V^{WS}\,/\,d\,c + \beta\,\mu'(c)), \qquad (34)$$

where $\pi_G + \pi_B = 1$, and $d\,V^{ES}\,/\,d\,c$ is given by (28). In the above equation, we use the fact that in the first period, with the presence of firing costs, the target utility constraint does not bind, so $\lambda = 0$.[13] It is possible to simplify (34) further. We know that equation (9) holds with equality. In the first period, since $\lambda = 0$ and the employer's IC constraint does not bind in any state (i.e., $\psi_\epsilon = 0$), equation (9) becomes:

$$\nu_\epsilon = \rho\beta\,\pi_\epsilon\,\lambda^{\wedge}, \qquad (35)$$

where $\lambda^{\wedge} = -\Omega\,'(\tilde{w}^*)$ (from the envelope theorem). Using (35), (34) becomes:

$$d\Omega\,(V^{WS}, c)\,/\,d\,c = \rho\beta\,(d\Omega\,(\tilde{w}^*, c)\,/\,d\,c\,) + (1-\rho)\beta\,(d\,V^{ES}\,/\,d\,c - \beta) -$$

$$(1-\,(1-\rho)\beta\,)\,\lambda^{\wedge}\,(d\,V^{WS}\,/\,d\,c + \beta\,\mu'\,(c)). \qquad (36)$$

In order to evaluate the above equation, we still need to determine $d\,V^{WS}\,/\,d\,c$ and $d\Omega\,(\tilde{w}^*, c)\,/\,d\,c$. Notice that the V^{WS} expression in (1) changes as follows:

---

[13]It is easy to see this if we substitute the expression for $\tilde{w}^*$ (given by (11)) together with $\omega_\epsilon = 0$ into the target utility constraint ((5)(*SP*)) and recalling that in the first period $W = W_0 = V^{WS}$.

$$V^{WS} = \mu(0) + Q^{W}\rho\beta\, V^{WR} + Q^{W}(1-\rho)\beta\,[V^{WS} + \beta\,(\mu(c) - \mu(0))] + (1 - Q^{W})\beta\, V^{WS}. \tag{37}$$

If we substitute (13) into (37):

$$V^{WS} = [\mu(0) + Q^{W}\beta^{2}\,(\mu(c) - \mu(0))] \,/\, (1-\beta\,). \tag{38}$$

Then, we can easily find $d\,V^{WS} / d\,c$ by using (38):

$$d\,V^{WS} / d\,c = (Q^{W}\beta^{2} / (1-\beta\,))\mu'(c). \tag{39}$$

To determine $d\Omega\,(\tilde{w}^{*}, c) / d\,c$, we take the total derivative of (2)(*NC*), (3)(*NC*), (4)′(*SP*), (5)(*SP*) and (7) with respect to c in the second period:

$$\begin{aligned} d\Omega\,(\tilde{w}^{*}, c) / d\,c = {} & \rho\beta\,(d\Omega\,(\tilde{w}^{*}, c) / d\,c\,) + (1-\rho)\beta\,(d\,V^{ES} / d\,c - \beta\,) + \\ & (\psi_{\hat{G}} + \psi_{\hat{B}})[\rho\beta\,(d\Omega\,(\tilde{w}^{*}, c) / d\,c\,) - (1-(1-\rho)\beta\,)(d\,V^{ES} / d\,c - \beta\,)] - \\ & (\nu_{\hat{G}} + \nu_{\hat{B}})[(1-(1-\rho)\beta\,)/\rho\beta\,](d\,V^{WS} / d\,c + \beta\,\mu'(c)) + \lambda^{\wedge}(1-\rho)\beta\,(d\,V^{WS} / d\,c + \beta\,\mu'(c)) - \\ & \lambda^{\wedge}[(1-(1-\rho)\beta\,)/\rho\beta\,](d\,V^{WS} / d\,c + \beta\,\mu'(c)). \end{aligned} \tag{40}$$

Since from the envelope theorem, $\lambda^{\wedge} = -\Omega'(\tilde{w}^{*})$, equation (9) after the first period becomes:

$$\nu_{\hat{\epsilon}} = \rho\beta\,\psi_{\hat{\epsilon}}\,\lambda^{\wedge}. \tag{41}$$

Then, using (41) and the fact that the employer's IC constraint does not bind in any period, i.e., $\hat{\psi}_G = 0$, (after rearranging) we have:

$$d\Omega(\tilde{W}^*, c) / dc = \{[(1-\rho)\beta - (1-(1-\rho)\beta)\hat{\psi}_B] / (1-\rho\beta(1+\hat{\psi}_B))\}(dV^{ES}/dc - \beta) -$$
$$\{[(1-(1-\rho)\beta)(1+\hat{\psi}_B) + (1-\beta)/\rho\beta] / (1-\rho\beta(1+\hat{\psi}_B))\}\hat{\lambda}(dV^{WS}/dc + \beta\mu'(c)). \quad (42)$$

If we plug (42) into (36), we obtain:

$$d\Omega(V^{WS}, c) / dc = [\beta(1-\rho(1+\hat{\psi}_B)) / (1-\rho\beta(1+\hat{\psi}_B))](dV^{ES}/dc - \beta) -$$
$$[(2(1-\beta) + \rho\beta) / (1-\rho\beta(1+\hat{\psi}_B))]\hat{\lambda}(dV^{WS}/dc + \beta\mu'(c)). \quad (43)$$

Finally, using (28), (39) and the fact that $\Omega(V^{WS}, c) = V^{ER}$ (hence, $d\Omega(V^{WS}, c)/dc = dV^{ER}/dc$), equation (43) becomes:

$$d\Omega(V^{WS}, c) / dc = -\beta^2(1-\rho(1+\hat{\psi}_B))(1-\beta(1-Q^E)) / (1-\beta)(1-\rho\beta(1+\hat{\psi}_B - Q^E)) -$$
$$[\beta(2(1-\beta) + \rho\beta)(1-\beta(1-Q^W))(1-\beta(1-\rho Q^E)) / (1-\beta)^2(1-\rho\beta(1+\hat{\psi}_B - Q^E))]\hat{\lambda}\mu'(c). \quad (44)$$

From the above expression, we can easily see that $d\Omega(V^{WS}, c)/dc > 0$ as long as:

$$(1-\rho)/\rho + [(2(1-\beta) + \rho\beta)(1-\beta(1-Q^W))(1-\beta(1-\rho Q^E)) / \rho\beta(1-\beta)(1-\beta(1-Q^E))](\mu'(c)/\mu'(\omega_G))$$
$$< \hat{\psi}_B <$$
$$(1-\rho\beta(1-Q^E))/\rho\beta.$$

Notice that for the lower bound, we use the result from Lemma A1 that $\lambda^{\wedge} = 1/\mu'(\omega_G)$. Lemma A2 shows us that in order to have an equilibrium with positive output, $\psi_B^{\wedge} < (1-\rho\beta\,(1-Q^E))\,/\rho\beta$ (disregarding the set of measure zero where in place of the inequality we have an equality). Therefore, there is a threshold value of $\psi_B^{\wedge}$, denoted $\psi_B^{SP} > \psi_B^{NC} \equiv \rho\,/\,(1-\rho)$, given by the lower bound, above which a small firing cost improves the employer's welfare. **Q.E.D.**

References.

Alvarez, Fernando and Marcelo Veracierto (2001). "Severance Payments in an Economy with Frictions." *Journal of Monetary Economics*, 47:3 (June), pp.477–498.

Amiti, Mary and Shang-Jin Wei (2005). "Fear of Service Outsourcing: Is it justified?" *Economic Policy*, 20:42 (April), pp. 308-347.

________ (2006). "Service Offshoring and Productivity: Evidence from the United States." NBER Working Paper No. 11926 (January).

Antràs, Pol, Luis Garicano and Esteban Rossi-Hansberg (2006). "Offshoring in a Knowledge Economy." *Quarterly Journal of Economics*, 121:1 (February), pp. 31-77.

Beaudry, Paul and John DiNardo (1991). "The Effects of Implicit Contracts on the Behavior of Wages over the Business Cycle." *Journal of Political Economy*, 99:4 (August), pp. 665-688.

Bentolila, Samuel, and Giuseppe Bertola (1990). "Firing Costs and Labour Demand: How Bad is Eurosclerosis?" *The Review of Economic Studies*, 57:3 (July), pp. 381-402.

Bertrand, Marianne (2004). "From the Invisible Handshake to the Invisible Hand? How Import Competition Changes the Employment Relationship." *Journal of Labor Economics*, 22:4 (October), pp. 723-765.

Blanchard, Olivier J. and Jean Tirole (2008). "The Joint Design of Unemployment Insurance and

Employment Protection: A First Pass." *Journal of the European Economic Association*, 6:1 (March), pp. 45–77.

Feenstra, Robert C. and Gordon H. Hanson (1996). "Foreign Investment, Outsourcing and Relative Wages." In R.C. Feenstra, G.M. Grossman and D.A. Irwin, eds., *The Political Economy of Trade Policy: Papers in Honor of Jagdish Bhagwati*, Cambridge, MA: MIT Press, pp. 89-127.

Fella, Giulio (2000). "Efficiency Wage and Efficient Redundancy Pay." *European Economic Review*, 44:8 (August), pp.1473-1490.

Gosselin, Peter G. (2004). "If America Is Richer, Why Are Its Families So Much Less Secure?" *Los Angeles Times*, October 24.

Gottschalk, Peter, Robert Moffitt, Lawrence F. Katz, and William T. Dickens (1994). "The Growth of Earnings Instability in the U.S. Labor Market." *Brookings Papers on Economic Activity* 1994:2, pp. 217-272.

Greenberg Quinlan Rosner Research (2004). "National Public Radio: Frequency Questionnaire, March 12-14."

Grossman, Gene M. and Elhanan Helpman (2005). "Outsourcing in a Global Economy." *Review of Economic Studies*, 72:1 (January), pp. 135-159.

Grossman, Gene M. and Esteban Rossi-Hansberg (2006). "Trading Tasks: A Simple Theory of Offshoring." NBER Working Paper No. 12721 (December).

Hacker, Jacob S. (2004). "False Positive: The So-Called Good Economy." *The New Republic*, August 16 and 23.

Heckman, James and Carmen Pagés (2003). "Law and Employment: Lessons from Latin American and the Caribbean." NBER Working Paper No. 10129 (December).

Holstein, William J. (2005). "Job Insecurity, From the Chief Down." *New York Times*, March 27.

Hopenhayn, Hugo and Richard Rogerson (1993). "Job Turnover and Policy Evaluation: A General Equilibrium Analysis." *The Journal of Political Economy*, 101:5 (October), pp. 915-938.

Karabay, Bilgehan, John McLaren (2008). "Trade, Offshoring, and the Invisible Handshake." Mimeo: University of Virginia (May).

Kambourov, Gueorgui (2006). "Labor Market Regulations and the Sectoral Reallocation of Workers: The Case of Trade Reforms." University of Toronto Working Paper (June).

Kocherlakota, Narayana R. (1996). "Implications of Efficient Risk Sharing without Commitment."

The Review of Economic Studies, 63:4 (October), pp. 595-609.

Kugler, Adriana D. and Gilles Saint-Paul (2004). "How Do Firing Costs Affect Worker Flows in a World with Adverse Selection?" *Journal of Labor Economics*, 22:3 (July), pp. 553-584.

Lazear, Edward P. (1979). "Why Is There Mandatory Retirement?" *Journal of Political Economy*, 87:6 (December), pp. 1261-1284.

Levine, Robert A. (2006). "Globalization's grave challenges for the West: An economic time bomb." *The International Herald Tribune*, May 10, p. 6.

Ljungqvist, Lars and Thomas J. Sargent (2008). "Two Questions about European Unemployment." *Econometrica*, 76:1 (January), pp. 1-29.

MacLeod, W. Bentley and James M. Malcomson (1989). "Implicit Contracts, Incentive Compatibility, and Involuntary Unemployment." *Econometrica*, 57:2 (March), pp. 447-480.

Malcomson, James M. (1999). "Individual Employment Contracts." Chapter 35 of O. Ashenfelter and D. Card, eds., *The Handbook of Labor Economics*, Vol. 3, Amsterdam: Elsevier, pp. 2291-2372.

Mann, Catherine L. (2003). "Globalization of IT Services and White Collar Jobs: The Next Wave of Productivity Growth." Institute for International Economics: International Economics Policy Brief PB03-11 (December).

Mataloni, Raymond J. (2004). "A Note on Patterns of Production and Employment by U.S. Multinational Companies." *Survey of Current Business*, 84:3 (March), pp. 52-56.

Matouschek, Niko, Paolo Ramezzana, and Frédéric Robert-Nicoud (2008). "Labor market reforms, job instability, and the flexibility of the employment relationship." Forthcoming, *European Economic Review*.

McDonald, J. T. and C. Worswick (1999). "Wages, Implicit Contracts and the Business Cycle: Evidence from Canadian Micro Data." *Journal of Political Economy*, 107:4 (August), pp. 884-892.

McLaren, John and Andrew Newman (2004). "Globalization and Insecurity." Mimeo: University of Virginia.

Meyerson, Harold (2006). "A call for a kinder, gentler capitalism." *Pittsburgh Tribune Review*, January 15.

Pissarides, Christopher A. (2001). "Employment protection." *Labour Economics*, 8:2 (May), pp. 131–159.

_____________ (2000). *Equilibrium Unemployment Theory.* (2nd edition). Cambridge, MA: MIT Press.

Ramey, Garey and Joel Watson (2001). "Bilateral Trade and Opportunism in a Matching Market." *Contributions to Theoretical Economics*, 1:1.

Rodrik, Dani (1997). *Has Globalization Gone Too Far?* Washington, DC.: Institute for International Economics.

Saint-Paul, Gilles (1995). "Efficiency Wage, Commitment and Hysteresis." *Annales D'Économie et De Statistique*, 37/38, pp.39-53.

Scheve, Kenneth F. and Matt Slaughter (2004). "Economic Insecurity and the Globalization of Production." *American Journal of Political Science*, 48:4 (October), pp. 662-674.

Sennett, Richard (2006). *The Culture of the New Capitalism.* New Haven: Yale University Press.

Shleifer, Andrei and Lawrence H. Summers (1988). "Breach of Trust in Hostile Takeovers." In Alan J. Auerbach, ed., *Corporate Takeovers: Causes and Consequences*, Chicago: The University of Chicago Press, pp. 33-56.

Sijabat, Ridwan Max (2006). "Outsourcing Still 'Slavery' to Unions." *Jakarta Post*, May 11, p. 9.

Spencer, Barbara J. (2005). "Presidential Address: International Outsourcing and Incomplete Contracts." *Canadian Journal of Economics*, 38:4 (November), pp. 1107-1135.

Thomas, Jonathan and Tim Worrall (1988). "Self-enforcing Wage Contracts." *Review of Economic Studies*, 55:4 (October), pp. 541-554.

Traca, Daniel (2005). "Globalization, Wage Volatility and the Welfare of Workers." *Review of International Economics*, 13:2 (May), pp. 237-249.

Uchitelle, Louis (2005). "Were the Good Old Days That Good?" *The New York Times*, July 3, Section 3, p. 1.

_____________ (2006). *The Disposable American: Layoffs and Their Consequences.* New York: Alfred A. Knopf.

Utar, Hale (2007). "Import Competition and Employment Dynamics." University of Colorado at Boulder Working Paper (October).

Valletta, Robert G. (1999). "Declining Job Security." *Journal of Labor Economics*, 17:4, pt. 2 (October), pp. S170-S197.

Zeile, William J. (2003). "U.S. Affiliates of Foreign Companies." *Survey of Current Business*, 83:8 (August), pp. 38-56, Bureau of Economic Analysis.