Retirement Savings and Portfolio Choices in Taxable and Tax-Deferred Accounts

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Abstract

This study attempts to explain the observed asset allocation and location decisions for households making taxable and tax-deferred investment. I incorporate employer matching policies and other pension account characteristics into a life-cycle model of optimal intertemporal consumption and portfolio choice, which includes a taxable saving account and a tax-deferred retirement saving account. The model is estimated using data from the Surveys of Consumer Finances (1992 to 2007), and the structural parameters are recovered by the Method of Simulated Moments. After modeling the features of U.S. pension system, the predicted policy rules are able to explain the observed portfolio patterns of American households, who are more likely to hold equities in the tax-deferred pension account. The estimates and results show that employer matching induces higher proportion of total wealth held in the pension account, so investors tend to boost equity investments in the tax-deferred account to enjoy the higher returns and reduce equity holdings in the taxable account for precautionary saving purpose. I find that an increase of 10 percent in employer match rate makes investors boost the average equity proportion in the pension account by 10 percent and reduce those holdings in the taxable account by 22 percent. In contrast, since the employer stock match exposes the households to a riskier situation in the pension account than the cash match, it causes households voluntarily to hold less equity in that account, resulting in an average decrease of 4 percent in the equity ownership and 3 percent in the conditional equity proportion. Moreover, the policy experiment reveals that a deletion of Social Security taxes and payments makes the pension account the only source of retirement income, so households tend to put a higher proportion of savings in the tax-deferred account, and they are likely to invest conservatively and hold about 25 percent more of pension wealth in relatively safe assets.

Keywords: Private pension, life cycle, portfolio choice, saving, method of simulated moments

JEL Classification: J32, G11, D14, D91, C61,C63

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1 Introduction

The growth of Defined Contribution (DC) Pensions (401k, 403b, and IRAs)[1] provides households new choices of which types of assets and how much of each asset to hold in conventional taxable accounts and tax-deferred pension accounts. This problem involves making both an optimal asset allocation decision (i.e. deciding how much of each asset to hold) and an optimal asset location decision (i.e. deciding which assets to hold in the taxable and tax-deferred accounts). Households have the opportunity to save and invest in the pension account on a pretax basis, but their behavior is also affected by the pension account characteristics, such as employer matching policies, liquidity constraints, contribution limit, and transaction costs. Investors not only would like to reduce the tax burden of owning financial assets, but also take advantages of pension account design.

This paper investigates households' optimal saving and asset allocation and location decisions in the presence of the employer matching policies and other aspects of pension account design. I estimate a dynamic stochastic model of life-cycle intertemporal consumption and portfolio choice, in which households save and invest in a taxable account together with a tax-deferred pension account. The pension account is characterized by the design of U.S. Defined Contribution plans. I focus on explaining observed household saving and investment decisions over the life time, and analyzing how the employer matching strategy and its interaction with other pension account features influence households' optimal behavior.

This study is motivated by two observations. First, although the asset allocation and location decision is crucial to the wealth accumulation and the welfare of households over the life time, only limited guidance is available. Tepper (1981) and Black (1980) suggest that people should hold all higher-taxed asset (bonds, in the case of the U.S.) in the tax-deferred account to shelter the tax burden of owning financial assets. Only in cases when desired holdings of higher-taxed assets exceed the capacity of tax-deferred accounts can some of them spill over into taxable accounts. These portfolio allocations have acquired the label of "tax-efficient", and have been shown optimal in a dynamic intertemporal choice model (Dammon et al., 2004). However, that advice is sharply at odds with observed portfolio choices of American households, who commonly keep both equities and bonds in each account type, and they often maintain higher equity positions inside their retirement accounts (Poterba and Samwick, 1997; Bergstresser and Poterba, 2004). For instance, as reported in the Survey of Consumer Finances (SCF) from 1992 to 2007, among households with a positive balance in both taxable and tax-deferred accounts,

[1]The recent expansion of defined contribution plans in the United States has included Individual Retirement Accounts (IRAs), which are available to all taxpayers with earned income, 401(k) plans, which are employer-provided defined contribution plans available at some firms, 403(b) plans, which are similar to 401(k) plans but are available to employees at nonprofit institutions, and a number of other smaller programs. At the end of the second quarter in 2008, assets in those retirement accounts totaled nearly 8.8 trillion dollars, with US $4.3 trillion in Individual Retirement Accounts (IRA) and US $4.5 trillion in 401(k)-type pension plans.

a significant percentage of households have all of their equities in the pension account (29.43%). The survey also shows that investors are more likely to hold equities in the tax-deferred account. There are about 77.09% of the households having equities in the pension account, while only 58.32% of them holding positive equities in the taxable account. The evidence indicates that tax treatment may not be the only explanation of household portfolio problems. Some theoretical studies (Shoven and Sialm, 2004; Huang, 2001; Amromin, 2003; Dammon et al., 2004) also suggest that other aspects of the household decision-making environment must be taken into account in order to explain the observed patterns of portfolio choices.[2]

Second, the employer matching policy has been shown empirically important in the problems of retirement saving and portfolio choices. In most types of defined contribution plans, employers usually make some contributions to match the employees' savings in their individual retirement accounts, which is considered to be one type of employees' benefits. And in some DC plans, mainly 401(k) type, the employer match is in the form of company stock instead of cash. Company stock is considered to be a more risky investment because it is positively correlated with labor income risk. Therefore, a worker with company stock investment in the pension accounts may face the risk of losing job and the loss of retirement wealth at the same time. Those employer matching strategies, especially in the form company stocks, tend to affect the returns and riskiness of the pension accounts, and therefore the saving and investment behavior of the households. Engelhardt (2004) suggests that employer matches tend to increase the retirement savings of the participants in the pension account, and Benartzi (2001), VanDerhei (2002), Liang and Weisbenner (2002), and Papke (2004) also empirically show that individuals with employer matching benefit are more likely to hold equities in the retirement account. However, those account features have not been fully analyzed in a structural model of households' savings and portfolio choices.

In this study, I incorporate the employer matching policies and other pension account characteristics into an intertemporal consumption and portfolio choice model over the life-cycle, where assets are accumulated in a taxable saving account and a tax-deferred pension account. The solution rules suggest that the employer matching benefit in the pension account causes households to increase the tax-deferred savings, resulting in a higher proportion of wealth held in the pension account. Therefore, in the presence of uninsurable labor income risks and imperfect liquidity of pension wealth, borrowing-constraint households would like to hold more safe assets in the taxable account for precautionary saving purpose (Amromin, 2003), and they tend to boost equity holdings in the tax-deferred account to maintain an op-

[2]Some recent theoretical studies produce a mix holding of equities and bonds in the tax-deferred account by considering tax-exempt investment opportunity, consumption and labor income shocks, and liquidity needs. For instance, Shoven and Sialm (2004) extends the asset set to tax-exempt municipal bonds; Huang (2001) analyzes the borrowing and short-selling constraints and liquidity needs; Amromin (2003) emphasizes the joint importance of uninsurable labor income risk and imperfect liquidity of pension account; Dammon et al. (2004) adds consumption shocks to their model.

timal portfolio mix and enjoy the higher returns of equity investment. This asset allocation and location strategy ends up with a lower equity investment in the taxable account, but a relative high equity holding in the tax-deferred pension account.

The model is estimated using the data from the Surveys of Consumer Finances (1992 to 2007), and the structural parameters are recovered by the Method of Simulated Moments, which has been used by Garkidis (1998), French (1998), Gourinchas and Parker (2002), and Cagetti (2003) to study consumption, labor supply and retirement behavior problems. Considering the size of this problem, parallel programming techniques as implemented in Message Passing Interface (MPI) are applied to make the problem computationally feasible (Swann, 2000, 2001). The estimated model is used to re-interpret the households saving and investment decision rules over the life time, and to analyze the impact of different pension account designs on households' behavior.

With the pension account features, the fitted model is able to match the observed portfolio age-patterns in both the taxable and tax-deferred saving accounts. I find that equity ownership in the taxable account is increasing with age, from around 25% at age 25 to about 60% right before retirement, while in the tax-deferred account, the equity ownership rate remains at a very high level through out the whole working life. The average conditional equity share among the investors is also slightly higher in the pension account than that in the taxable account. The estimated results show that there are a certain amount of households who only hold equities in the pension account, with all their taxable wealth invested in safe assets (bonds), which is also consistent with the fact observed by the data. This study suggests that the observed household portfolio dynamics in the taxable and tax-deferred accounts are optimal decisions with respect to pension account designs.

In addition to explaining the data, the estimates and results of this study yield the following findings. First, the estimated model suggests that the employer matching strategy makes households boost equity investments in the tax-deferred account and hold more safe assets in the taxable account. I find that the average conditional equity proportion in the pension account is increasing by 10% after a 0.1 increase in the estimated employer matching rate, while the equity proportion in the taxable account is decreasing by 22% on average. Moreover, households at different age stages respond differently to employer match changes. It is shown that young households (age $25-45$) are more sensitive to the change in employer matching policies, yielding an average elasticity of equity ownership with respect to the match rate at 2.6 in the pension account, but older households who are more concerned with retirement wealth are less sensitive to the policy changes, yielding a matching rate elasticity of equity ownership at 0.04 in the tax-deferred investment.

Second, the estimates show that the employer stock match exposes the households to a riskier situation in the pension account than the cash match, so it causes the households voluntarily to hold less equities in the pension account, with an average decrease of 4% in equity ownership and 3% in conditional equity proportions. However, when the risky asset return is positively correlated with labor income shock, the situation is quite different. The precautionary saving concern during the young ages increases the proportion of wealth held in the taxable account and further reduce the equity investment in that account, which in turn results in a even higher concentrated equity holding in the pension account. This result is consistent with the empirical findings in Benartzi (2001) and Liang and Weisbenner (2002), who show that employer company stock matches cause participants to hold more company stocks in the pension account.

Finally, this model provides a framework to analyze the consequences of transforming the Social Security system to a tax-deferred retirement saving account. I analyze two cases in this study. The first one is to remove the Social Security taxes during the working time and payments during the retirement. The results suggest that households do not change their behavior except during the periods right before retirement. Since the saving limit in the pension account remains the same, a higher proportion of savings is allocated to the taxable account. In the second case, besides removing the Social Security taxes and payments I increase the saving limit in the pension account accordingly. I find that households would like to increase their retirement savings to meet the new limit, which consequently ends up with little wealth in the taxable account. Additionally, as the larger size of pre-tax savings increases the relative importance of pension account to total wealth, households invest conservatively and hold about 25% more of pension wealth in the safe assets.

This paper is related to two main strands of literatures. First, my work builds on previous studies that analyze the dynamic decisions of taxable and tax-deferred investments. Campbell et al. (2001) studies the interaction between asset choices for retirement savings and taxable accounts. Dammon et al. (2004) and Shoven and Sialm (2004) show that the households have preference of holding taxable bonds (higher-taxed asset) in the tax-deferred pension account. Huang (2001) and Amromin (2003) emphasizes the importance of liquidity needs of the taxable wealth, which may generate a mix holding of equities and bonds in the tax-deferred account. My approach goes beyond those studies by characterizing the pension account designs in the analysis, and estimating a structural model of retirement savings and portfolio choices.

Second, my study extends the discussions of employer matching policies and other pension account designs. Choi et al. (2004) summarizes the pension account features that may influence individuals' or households' behavior. Engelhardt (2004), Benartzi (2001), VanDerhei (2002), Liang and Weisbenner (2002), and Papke (2004) empirically show the impact of employer cash matches and stock matches on the saving and investment decisions of households. This riskiness of company stock investments is also addressed by Meulbroek (2002), Ramaswamy (2003), Poterba (2003), Campbell and Viceira (1999), Davis and Willen (2000a,b), and Heaton and Lucas (2000). In this paper, I incorporate the employer matching strategy into a structural model of portfolio choices with taxable and tax-deferred investments, and investigate the different effects of cash match and stock match on households' optimal behavior.

The remainder of the paper is structured as follows. Section 2 lays out an empirically tractable dynamic model of optimal household savings and portfolio choices. Section 3 describes the data set and the construction of life-cycle profiles of asset allocation and location. The fourth section introduces the estimation method that is used to recover the structural parameters. In section 5, I present the estimated results, analyze the effects of pension account designs, and explore the consequences of the changes of some parameters. Finally, the conclusion and future extensions are discussed in section 6. Appendices contain more detailed descriptions of the theoretical model, the numerical optimization, and the econometric procedure.

2 Model and Numerical Solutions

In this section, I build an intertemporal life-cycle model of optimal consumption and portfolio choice, by incorporating a tax-deferred (pension) savings account together with a taxable savings account. Households receive stochastic income, and make decisions of how much to consume, how much to save in each account, and how much of each asset to hold in each account. They are confronted with specific characteristics of the retirement account, tax policies on different assets, riskiness in the financial and labor market, as well as uncertainties in life expectancy.

2.1 The Model

The model assumes that each household is one decision unit, for most consumption and investment decisions are made jointly by members within one family. The head of the household makes decisions annually starting age 25 and lives until age 85, during which he works the first 40 years. The maximum age $T = 85$ and the retirement age $K = 65$ are set exogenous and fixed. Households derive utility from the consumption of a single good C_t in every period. During the working life, $t = 25, \dots, K-1$, each household receives a stochastic annual income Y_t, [3] chooses how much to consume, and makes saving and investment decisions in both a pension account (such as 401(k), 403(b), and IRAs) and a conventional taxable account. Their decisions are conditional on the tax and pension account policies. I label the wealth in the pension account as W^P, and that in the taxable account as W^A. When the head of the household retires, he liquids the pension wealth, receives the expected value of retirement payments

[3] Although households do not make an endogenous labor-leisure choice in my model, I interpret the income as labor income, which is related to the saving decisions in the pension account.

from Social Security pension system,[4] and consumes those assets until the last period of life.

The features of the pension account make it a special investment vehicle for households, otherwise the investor would be indifferent between putting a dollar into retirement savings or into regular savings. In my model the pension account differs from the taxable account in the following ways. The first difference is tax benefit — the savings and investment gains in the pension account are tax-deferred. So the total wealth of a household during working life, W, is

$$W_t = W_t^A + (1-\tau^b)W_t^P, t = 25\dots K-1. \tag{1}$$

where τ^b is the ordinary income tax rate imposed when the household liquidize the pension wealth. The second difference is illiquid constraint — pre-retirement withdrawal from the pension account is subject to penalty. Without illiquid constraint, tax benefit yielding a higher effective rate of return on retirement savings would cause the household to hold all wealth in the retirement account. I set the penalty of pre-retirement withdrawal as pnl, a percentage of the amount withdrawn, and the age at which one can liquidize the pension funds without penalty is the retirement age K.[5] Thirdly, the contributions into the pension account during working life are subject to a upper limit. I define that the savings in the tax-deferred account, s_t^P, cannot exceed a certain percentage of current income, $q \cdot Y_t$, where q denotes the maximal proportion of income could be allocated to the pension account.[6] Fourthly, in most employer-sponsored DC plans, the employer usually match some or all of the employees' contributions. The model assumes the employer will provide some matchings whenever the employee saves a positive amount in the pension account. Those matchings are placed into the tax-deferred account of the employee, and can be invested in many ways. The ratio of the contribution from the employer to that from the employee is defined as matching rate, m.[7] Lastly, there are usually less transaction cost in the pension account than that in the taxable account. Investors have to pay certain transaction fees whenever they trade the assets in the taxable account, but they do not need to pay any fee for the transactions when first enrolled in the pension account, and only slight fees for the future transactions. So I insert a parameter, tr, in the

[4]Social Security is a public pension system in the United States, which collects taxes during the working life and pays out benefit during retirement. It is a *pay-as-you-go* system. In this study, I treat it as an additional source of retirement income.

[5]Currently, the age that one can withdraw retirement funds in regular IRA without penalty is $59\frac{1}{2}$. I set it at the retirement age for simplicity.

[6]Most of all employer-sponsored plans impose a upper limit of retirement savings, for instance, 20 percent of pretax earnings. IRAs also have limit on the contributions, but whether those savings are tax deductible depends on many things, such as AGI (Adjusted Gross Income), whether having a employer-sponsored retirement plan, and how much have been saved in those pension accounts. Based on different situations, the contributions in IRAs could range from totally tax deductible to none tax deductible. Therefore, in this study I set the limit to 20 percent of pretax earnings, which is almost the same as the reality.

[7]The matching policies of the pension plans vary a lot from one employer to another. Some of them make matches whenever the employees have some contribution by themselves, the case in my model, while some of them make contributions unconditional on the employee's behavior. The matching rate also varies a lot across employers, from less than 50 percent to over 200 percent. For the purpose of numerical analysis, I choose to build the model under the former condition only, and assume all employers make matches at one level.

taxable account to capture the transaction costs difference between those two types of accounts.

In addition, the model assumes investors can only trade two types of assets in the financial market, one of which is risk free bond with a pretax return, r^b, [8] and the other is risky stock with a stochastic pretax return, r. The riskless bonds are taxed at the rate, τ^b, which is also the ordinary income tax rate. The total return on equities is comprised of dividends and capital gains. Dividends (and interest payments), d, are realized automatically and are taxed at τ^d, while realization of capital gains, g, is normally distributed, $g \rightarrow N(\mu_r, \sigma_r^2)$, and depends on timing choices of the investor. When realized, capital gains are taxed at τ^s.[9] Therefore, the after-tax return on equity is given by, $r^\tau = (1-\tau^d)d + (1-\tau^s)g$.

If the tax rates on different assets were the same, then investors would have the same portfolios in both tax-deferred pension account and regular taxable account. But bond is a higher-taxed security in the United States, so I set $\tau^s < \tau^b$, and $\tau^b = \tau^d$, as the case in reality. The conventional account taxes all earnings as soon as they are realized, and the pension account defers taxation on returns that accumulate on pre-tax contributions. Consequently, one-period return of \$1 portfolio in taxable account with α_t in equities is,

$$\begin{aligned} R_t &= \alpha_t(1+r^\tau) + (1-\alpha_t)(1+r^b(1-\tau^b)) \\ &= (1+r^b(1-\tau^b)) + \alpha_t(r^\tau - r^b(1-\tau^b)), \end{aligned} \tag{2}$$

and the return of a \$1 in pension account with voluntary α_t^P in stocks is given by,

$$\begin{aligned} R_t^P &= \alpha_t^P(1+r) + (1-\alpha_t^P)(1+r^b) \\ &= (1+r^b) + \alpha_t^P(r-r^b). \end{aligned} \tag{3}$$

If the employer matching rate is positive and in form of cash, then the return in the pension account can be rewritten as,

$$R_t^P(m) = \left(1 + \frac{m \cdot s_t^P}{W_t^P + s_t^P}\right) \cdot R_t^P, \tag{4}$$

where $m \cdot s_t^P/(W_t^P + s_t^P)$ is the fraction of employer matchings out of total pension wealth. When $m = 0$, the pre-tax return in the pension account is the same as before, $R_t^P(m) = R_t^P$, but when $m > 0$, the return of the pension wealth in Equation 3 is augmented by factor $(1 + \frac{m \cdot s_t^P}{W_t^P + s_t^P})$. If the employer matches stocks

[8] Another kind of bond is tax-exempt municipal bond, which returns less than risk-free bond. Most empirical data sets do not distinguish the difference between municipal bond and risk-free bond, so I categorize these two as the same in this model.

[9] In the U.S., a distinction is made between short-term capital gains (on equities held less than a year), which are taxed at τ^d, and long-term gains taxed at τ^s. The model assumes that all capital gains are long-term.

rather than cash, the return in the pension account can be written as,

$$R_t^P(m) = R_t^P + \frac{m \cdot s_t^P}{W_t^P + s_t^P}(1+r).$$

In the rest part of this paper, unless otherwise noted, I assume the employer matches are taking the form of cash.

From period K to T, the household retires, and chooses consumption based on initial retirement wealth W_K. Let $\psi(W_K)$ be the expected present value of the utility for the second part of life, as viewed at period K. Then the objective function of each household can be expressed as,

$$max_{(C_t, s_t^P, \alpha_t^P, \alpha_t)_{t=0}^{K-1}} E\left(\sum_{t=25}^{K-1} \beta^t [F(t)U(C_t) + (F(t-1) - F(t))B(W_t)] + \beta^K F(K)\psi(W_K)\right), \tag{5}$$

where $U(\cdot)$ denotes the investor's utility function, β is the subjective discount factor of the utility, $B(\cdot)$ is the bequest function, and $F(t)$ is the probability of living through period t. I assume that the household's preference can be expressed as the Constant Relative Risk Aversion (CRRA) form,

$$u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma},$$

where γ is the consumer's relative risk aversion coefficient. In addition, the bequest will be expressed as a function of total wealth only, and has the same coefficient as the utility function,

$$B(W_t) = bq \cdot \frac{W_t^{1-\gamma}}{1-\gamma}.$$

The total wealth at each date is defined as the sum of the taxable wealth and after-tax value of the retirement account balance (see Equation (1)).[10] At the time of death, the head of household liquidate total wealth and distribute it as a bequest to his beneficiary. For simplicity, the degree of altruism is dictated by the parameter, bq. When $bq = 0$, bequests are accidental, generated by the fact that the life span is uncertain. A higher value of bq indicates a stronger bequest motive for the head of the household. In addition, the value of $F(t)$ is given by

$$F(t) = exp\left(-\sum_{j=0}^{t} \vartheta_j\right),$$

where $\vartheta_j > 0$ is the single-period hazard rate for period j with $\vartheta_{T+1} = \infty$. In practice, the household's

[10] According to the *reset (or step-up) provision* of the current U.S. tax code, the tax bases of all inherited assets to be costlessly reset to current market prices at the time of the investor's death. This means the assets in the taxable account can be inherited without incurring a capital gains tax. Separating the capital gains of the equities from their dividends turn out to be a hard task in this model, so I will assume that all the gains from equities have to be taxed as a bequest.

annual mortality rates are calibrated to match those for the U.S. population.

The components of wealth evolve as following during working life ($t = 25, \dots K-1$),

$$W_t = W_t^A + (1-\tau^b)W_t^P, \tag{6}$$

$$W_{t+1}^A = (W_t^A + Y_t(1-\tau) - C_t - s_t^P(1-\tau-h\times I(s_t^P<0,t<K)) \cdot (1-tr\times I(\alpha_t>0)) \cdot R_t, \tag{7}$$

$$W_{t+1}^P = (W_t^P + s_t^P) \cdot R_t^P(m), \tag{8}$$

$$s_t^P \leq q \cdot Y_t, \tag{9}$$

$$C_t \geq 0, W_t^P \geq 0, W_t^A \geq 0, \tag{10}$$

$$0 \leq \alpha_t^P, \alpha_t \leq 1. \tag{11}$$

Equation (8) describes the wealth accumulation in the retirement account on a pre-tax basis, while (7) displays the taxable account, and total wealth is shown in (6). In equation (7), $Y_t(1-\tau)$ is the after tax income which are used for consumption, and s_t^P is the savings in the pension account. In the multiplier of s_t^P , $(1-\tau)$ indicates the contribution in retirement account is pre-tax, and $pnl \times I(s_t^P < 0, t \leq K)$ is the penalty imposed on households when they withdraw the tax-deferred wealth before the retirement age ($t = K$), where pnl is the parameter measuring penalty size and $I(s_t^P < 0, t < K)$ is the indicator. As Social Security public pension is available to everyone, the agents have to pay Social Security tax on the earning except for the tax-favored savings, and they will receive benefit payments during retirement. Therefore, I define the tax rate τ as the summation of ordinary income tax τ^b and Social Security tax (FICA) τ^{ss}. In the taxable account, when household trade stocks $\alpha_t > 0$, they need to pay a transaction cost tr, which is a positive proportion of the taxable wealth.[11] In addition, the contribution to the retirement account s_t^P cannot exceed a certain proportion, q, of the current stochastic income Y_t (equation (9)). R_t and $R_t^P(m)$ are the stochastic gross return rates earned on assets in the taxable and tax-deferred account respectively, and m in $R_t^P(m)$ is the parameter that measures the size of employer matching rate in the pension account. Furthermore, no borrowing and short selling are allowed in this model (see equation (10) and (11)).

Throughout the analysis I assume that the ratio of household labor income to their contemporaneous total wealth (taxable plus tax-deferred wealth) prior to retirement can be expressed as a function of age t.

[11] There should be many ways in modeling transaction cost in the portfolio choice problems (see Constantinides (1976) and Liu and Loewenstein (2002) for details). One example would be incurring transaction cost whenever the actual shares held in this period is different from those in the last period, but this method requires the shares of stock in the previous period as a state variable. This change will not only make my model a lot more complicated, but also the normalization procedure in the next section impossible. Another example is to model the transaction cost as a fixed value rather than a proportion of wealth. However, this method, on one hand, is still inaccurate if not combined with the actual trading shares, and on the other hand, is not possible for normalization, which will make this model huge in the computation. Therefore, I decide to treat the transaction cost as a proportion of the taxable wealth in this setting.

This assumption is needed in the numerical analysis to keep the problem homogeneous in wealth and to limit the number of state variables. Define $y_t = Y_t/W_t$, then the stochastic labor income can be expressed as,

$$y_t = e^{f(t)} \cdot u_t,$$

where $f(t)$ is a function derived from age t, taking the form

$$f(t) = l_0 + l_1 \cdot t + l_2 \cdot t^2. \tag{12}$$

And u_t is an idiosyncratic transitory shock, taking value 0 with probability $0 \leq pu_t < 1$, otherwise independently and identically log-normally distributed as $N(0, \sigma_u^2)$. The parameter pu_t presents the age-dependent unemployment rate, and defined as

$$pu_t = \frac{e^{(\lambda_0+\lambda_1 t)}}{1+e^{(\lambda_0+\lambda_1 t)}}. \tag{13}$$

Going from the model to the data, I need to make some assumptions about the post-retirement periods. First, different from working life, people after retiring will no longer make contributions to the pension account, and all the special features of pension account that affect household's behavior vanish after retirement. The asset "location and allocation" problem will look different from the working life because of the new uncertainty they face, such as medical expenses and the timing of death. Although these sources of uncertainty are also present to some extent in the last working years, labor income and asset return uncertainties are the dominant source of uncertainty when young. Since I know too little about the form that uncertainty takes after retirement to use this methodology and draw inferences from post-retirement behavior, I assume that households liquid the tax-deferred wealth at the time of retiring ($t = K$), and receive a fixed rate of return on wealth in the rest part of life.[12] Second, as the case in the United States, besides of tax-deferred saving account, most working families will receive Social Security payments after retirement, which is based on the taxes collected during the working time.[13] The amount of payment is calculated from the earning history of each individual and the years of service in the Social Security system. I assume that household will receive an expected endowment of all possible future payments, H_K, at the beginning of retirement, which is proportional to the total wealth right before retirement, $H_K = h \cdot W_{K-1}$.[14] This parameter can also capture the payments from employer-sponsored

[12] An alternative way to liquid pension account would be withdrawing tax-deferred wealth as an annuity, or a fixed proportion, such as the reciprocal of life expectancy in Dammon et al. (2001, 2004). However, as long as the rate of return of the taxable account is the same as that of the tax-deferred account, all the withdrawing methods will generate the same answer. Since I assume no asset allocation choice during retirement periods for simplicity, there would be no difference in using any way.

[13] The current tax rate of Social Security is 12.4 percent of pre-tax earnings up to \$106,800 in 2009, among which half is charged to employees and the other half to employers.

[14] An alternative way to model Social Security payments is to treat it as an additional saving account with a fixed contribution

Defined Benefit plans, and some other smaller pension funds. Because of the presence of Social Security system, it would be optimal for some low-income households to hold no wealth in the retirement saving account. Therefore, this part of the model is crucial to explain the empirical fact that a certain proportion of the population do not have any positive savings in the tax-deferred account.

Then, we can write the Bellman equation for the above maximization problem during working life ($t = 25, \ldots, K-1$) as follows:

$$V_t(W_t^A, W_t^P, Y_t) = max_{(C_t, s_t^P, \alpha_t, \alpha_t^P)} \left(e^{-\vartheta_t} U(C_t) + (1 - e^{-\vartheta_t}) B(W_t) + e^{-\vartheta_t} \beta E_t [V_{t+1}(W_{t+1}^A, W_{t+1}^P, Y_{t+1})] \right),$$

subject to equations (6)–(11). When $t = K-1$, the last period of working, the value function of the next period is defined as,

$$V_K(W_K^A, W_K^P, Y_K) \equiv V_K(W_K, 0) = \psi(W_K),$$

where

$$W_K = W_K^A + (1 - \tau^b) W_K^P + H_K.$$

and $\psi(W_K)$ is the expected present value of the utility for the second part of life as defined above. A complete description of the retirement value function is provided in the Appendix $A.1$.

The setup of this model combined with the particular choice of retirement value function makes the problem homogeneous of degree $(1-\gamma)$ in the total wealth (i.e., the value of the taxable account plus the after-tax value of the retirement account). I simplify the optimization problem by normalizing by the household total wealth W_t. Let $p_t = W_t^P(1-\tau)/W_t$ be the fraction of the investor's total wealth that is held in the retirement account, $s_t = s_t^P/W_t$ be the ratio of contribution in pension account to the total wealth, $c_t = C_t/W_t$ be consumption-wealth ratio, and let $y_t = Y_t/W_t$ be the income-wealth ratio as previously defined. The detailed steps of normalization are explained in Appendix $A.1$ and $A.2$.

After normalizing by total wealth, the household's intertemporal consumption and portfolio problem involves the following control variables during working time: the consumption-wealth ratio, c_t, the savings-wealth ratio in the pension account, s_t, the fraction of taxable wealth allocated to equity, α_t, and the fraction of tax-deferred pension wealth allocated to equity, α_t^P. The relevant state variables for the normalized problem are the fraction of the household's incoming total wealth that is held in the retirement account p_t, income-wealth ratio y_t and age t. After retiring, the consumption-wealth ratio c_t is the only control variable, and age is the state variable. Then the above problem can be solved numerically from the last period of life. Since the size and complexity of this problem implie heavy computational

rate and an implicit rate of return, which can be liquidized at retirement. However, modeling Social Security as a different saving account requires an additional continuous state variable, which is computationally intractable considering the current size of this problem. Therefore I assume a fixed proportion of labor income is set aside by a public sector, and a payment is given when the agent retires.

demands for the estimation process, I will follow Swann (2000, 2001) in solving this type of problem using parallel programming methods as implemented in the Message Passing Interface (MPI). The detailed solution method is explained in the Appendix $A.3$.

2.2 Numerical Solutions of Optimal Choices

After solving this model numerically using the baseline parameters,[15] I have discovered the following properties of optimal household saving and investment decisions that stem from different wealth and income levels:[16]

- The saving in the tax-deferred pension account (s_t) is increasing with normalized income level (y_t). Because of the existence of labor income shock and penalty for early withdrawal, the retirement savings in the pension account can be as low as zero when normalized income is small, and as the pretax saving cannot exceed a limit ($q \cdot y_t$), the saving constraint in the pension account is always binding when normalized income is sufficient high. In addition, when normalized income is low (especially unemployed) and pension wealth accounts for a substantial proportion of total wealth, it is optimal for households to withdrawal a positive amount from retirement account to fund current consumption.

- In the case of a relative low pension-wealth ratio (p_t), households have sufficient wealth in the taxable account for regular consumption needs. Since bonds have a relative high tax burden, it is optimal to hold taxable bonds in the tax-deferred account, and equities in the taxable account. As the normalized income increases, more wealth are held in the taxable account, and the optimal equity proportion in the taxable account increases. Because of the prohibition on borrowing, the proportion of equity in the taxable account is bounded above by 100%, so equities will spill over to tax-deferred account when income is sufficiently high. This portfolio mix is tax-efficient.

- When the fraction of wealth in the pension account is high, there is relative low level of wealth held in the taxable account. Under the consideration of labor income uncertainties and illiquid constraint of pension wealth, households tend to have precautionary savings and hold safe assets (bonds) in the taxable account. Since households try to maintain an optimal overall portfolio mix, their equity holdings in the pension account increase correspondingly, thus resulting in a tax-inefficient portfolio. This phenomenon is more apparent when the normalized income is low, and diminishes when households have a sufficiently high income level.

[15] Please refer to Table 1 and 2 for a detailed description of benchmark parameters.

[16] The policy rules are derived by assuming that there is no employer matching benefit ($m = 0$) and no transaction costs difference between taxable and tax-deferred account ($tr = 0$).

- From the life-cycle prospective, young households, who expect to have a longer working life, are more likely to have precautionary savings and hold less equities in the taxable account. In the dynamic solutions, a higher level of taxable wealth is needed for young households to hold equities in the taxable account than that for old households.[17]

Based on those features of the policy rules, the model will be able to generate wealth accumulation and portfolio choices over the life time. However, the realist household behavior also depends on the pension account characteristics, such as the employer matching rate and the transaction costs on equity trading. In the next several paragraphs, I will discuss the role of those pension account designs in shaping households' saving and investment decisions.

One of the critical policies that affect household decisions is the employer matching strategy. As an important element in the employee's pension package, the employer matching rate are empirically tested to have substantial impacts on household saving and portfolio choices (see discussions in Engelhardt (2004), Benartzi (2001), VanDerhei (2002), Liang and Weisbenner (2002), and Papke (2004)), but its effect is not clear in a dynamic choice model. I am therefore interested in assessing the effect of changes in matching rate (m) on the retirement saving and asset allocation and location decisions.

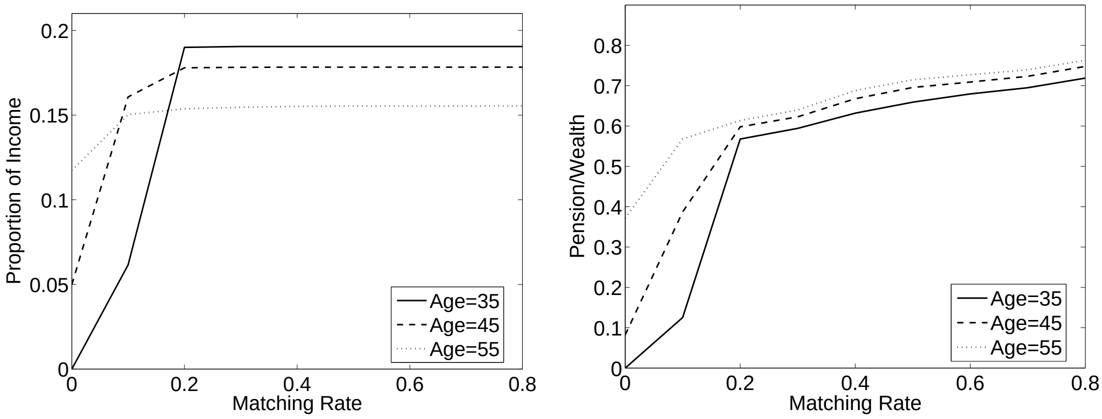
To investigate the impact of parameters on the pattern of saving and portfolio choices, I generate a number of simulated policy rules over the life-cycle for a given set of parameters and record the average level of pretax savings in the pension account and the distribution of equity investment in both taxable and tax-deferred accounts, and then I report the behavior changes at three age periods, $t = 35$, 45, and 55.[18] Figure 1 illustrates the effect of changes in employer matching rate on retirement saving decisions, with panel (a) displaying the saving rate in the pension account and panel (b) the fraction of total wealth held in the pension account. As shown in Figure 1(a), an increase in the matching rate makes household increase the proportion of income allocated in the pension account. The effect on saving rate is larger when m increasing from zero to 0.2, and since the savings are bounded by upper limit, the effect becomes quite small after 0.2. Figure 1(b) shows the effect of changes in employer matching rate m on pension-wealth ratio. The fraction of wealth held in the pension account increasing in m. The increased proportion of wealth in the pension account includes two components, one of which is the increase of households' own tax-deferred savings (Figure 1(a)), and the other is the increased contribution from the employers. In addition, the marginal effect (slope) of matching rate on tax-deferred savings and pension-wealth ratio is larger for young households than that for old ones, which may reflect the fact that the purpose of young households' pretax saving is to enjoy the pension-favored benefit, so they are

[17] For instance, households at age 35 will hold positive equities in the taxable account only when the fraction of wealth in the pension account is lower than 0.5, while households at age 50 will hold equities in the taxable account when the pension-wealth ratio is not higher than 0.9.

[18] All parameters, except for m, are set as those in Table 1 and 2.

more likely to be affected by policy changes.

Figure 1: The Role of Employer Matches in Retirement Savings

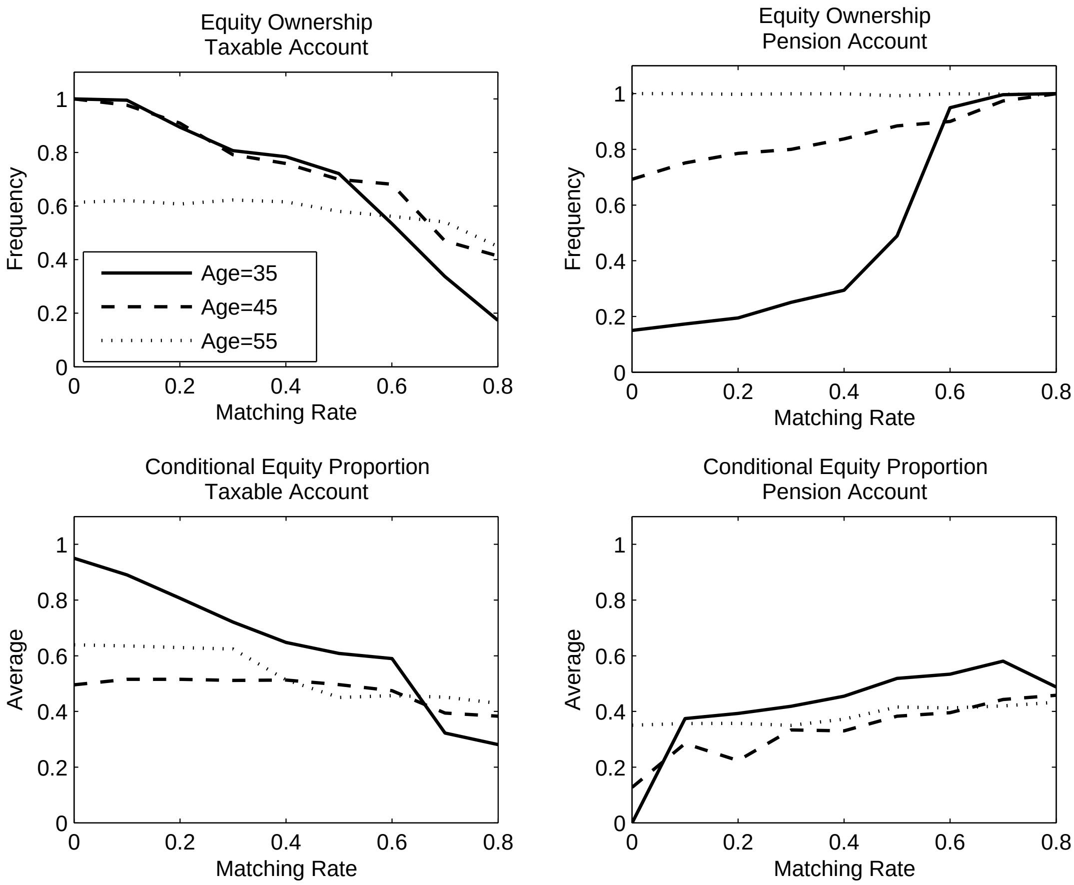


(a) Retirement Saving Rate (b) Pension Wealth Ratio

Besides saving behavior, the employer matching policies also have significant impact on asset allocation and location decisions. Figure 2 illustrates the effects of changes in employer matching rate on equity holdings in both taxable and tax-deferred account for different age groups. When there is no employer matches ($m = 0$), the figure shows that households tend to hold most equities in the taxable account for that equities are much less valuable when held in the pension account. However, when employer matches are positive, the optimal equity holding in the taxable account decreases with matching rate and the equity investment in the pension account increases correspondingly. This change can be explained as a consequence of the change in saving behavior. As the employer matches increases, a higher proportion of income are allocated to pension account, resulting in a higher fraction of wealth in the pension account. Therefore two implication can be derived: (1) A lower level of taxable wealth, interacted with labor income shocks and illiquid pension wealth, makes households reduce equity investment for precautionary saving motives; (2) To enjoy the higher returns from equity investment and to maintain an optimal portfolio mix in overall asset holdings, households tend to hold more equities in the pension account. Similarly as saving behavior, the effect of employer matching rate on asset allocation and location decisions is stronger for young households.

In addition to employer matches, the other pension account design that has impact on portfolio choices is transaction costs. The transaction cost parameter (tr) which is imposed on the taxable but not the tax-deferred investment captures the transaction cost difference in those two types account. A higher value of tr indicates a larger difference in the transaction fees of taxable and tax-deferred equity investment. Since the taxable account has a higher cost in the equity trading than the tax-deferred account, an increase in tr will decrease the equity holdings in the taxable account and increase those investment in the pension account accordingly. This effect is displayed in Figure 3. Although the transaction costs parameter has a similar role in the portfolio decisions with employer matching policies, they are different

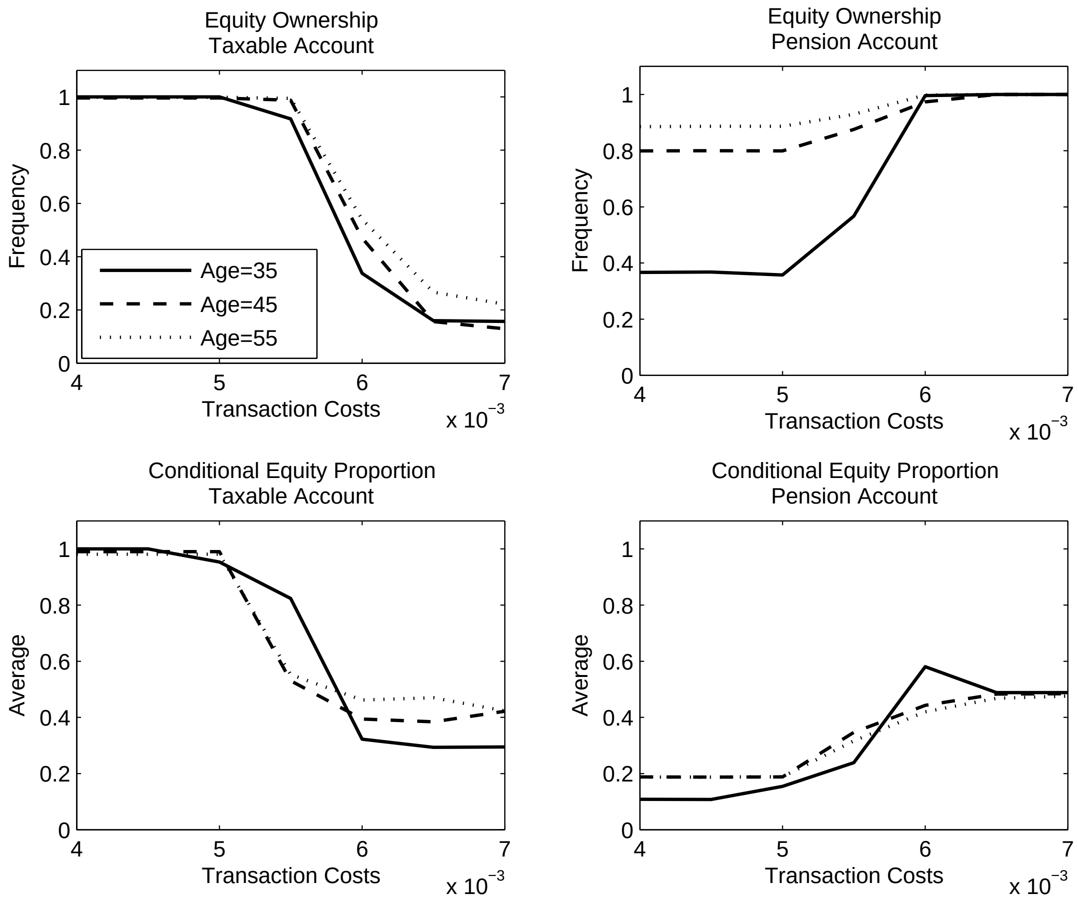
Figure 2: The Role of Employer Matches in Portfolio Choices



in two important ways. First, employer matching rate significantly influence the household retirement saving decisions, while transaction cost does not. Second, the marginal effect of transaction costs on portfolio choices is similar across different ages, but the effect of employer matches are more prominent for young households (compare taxable investment in Figure 2 and 3). Therefore, the transaction fees in the taxable account is crucial to explain the relative lower equity investment in the taxable account of the older households .

The numerical solutions illustrate a preference of holding equity in the tax-deferred pension account and taxable bonds in the taxable account under the condition of employer matching policies and other pension account characteristics. I also have discussed how the overall asset allocation depends upon the fraction of total wealth held in the pension account, income-wealth ratio and ages. In the results section, I will investigate the estimated time-series profiles of the households' optimal saving and portfolio allocation decisions using simulation analysis.

Figure 3: The Role of Transaction Costs in Portfolio Choices



3 Data

The data used in this study come from the latest Surveys of Consumer Finances (SCF), conducted in 1992, 1995, 1998, 2001, 2004 and 2007. The SCFs are sponsored by the Federal Reserve Board and have been conducted by survey research professional at the University of Michigan and the National Opinion Research Center triennially since 1983. Because of limitations in the pension investment questionnaire, I choose the waves starting in 1992. There are 3906, 4299, 4305, 4442, 4519, and 4418 households, respectively, in the surveys studied here. Since this study focuses on the working life of households, I drop all households with male heads younger than 25 and older than 70. The total number of households in the analysis becomes 21379.

The Survey data in the SCFs are the most complete data source on households' balance sheets in the United States. The surveys ask a wide array of questions on every aspect of the household financial situations - amount and type of liquid and illiquid assets, asset location and allocation, pension plan characteristics, sources of earnings, demographic information, and so on.[19] Of particular value for

[19]The SCFs attempt to uncover precise composition of household financial portfolios. Unfortunately, in the surveys conducted in 1992, 1995, 1998, and 2001, information on allocations to narrowly defined asset classes exists only for funds kept in taxable investment accounts. By contrast, the composition of holdings in tax-deferred retirement accounts, both individual

studies of household portfolio composition is the fact that the SCF oversamples wealthy households, which tend to have richer portfolio structures. Each survey makes available a set of sampling factors that allow one to re-weight the sample to produce population statistics. Unless otherwise noted, all descriptive statistics utilize population weights. In addition, I put all data into real 1992 dollars using Consumer Price Index (CPI) employed by the SCFs on all the surveys. [20]

I construct measures of wealth and portfolios that match the concepts in the theoretical model. I define investable household wealth as total quasi-liquid financial assets that can be explicitly allocated between investments with equity- or bond-like properties. The taxable account includes nearly all financial instruments, such as directly held bonds, stocks, mutual funds, and saving accounts. It specifically excludes checking accounts on the grounds that they are used primarily for transaction purposes, as well as housing, proprietary business wealth, and human capital wealth. The tax-deferred retirement wealth consists of employer-sponsored defined contribution plans (i.e., 401*k*, 403*b*) and individual retirement accounts (such as IRA and Keoghs), but it omits imputed values of future guaranteed pension income (Social Security, and defined benefit plans). In order to obtain a high quality sample that has the required information, I further drop 1156 households that report no financial wealth, since the solution of my model requires a normalization over total wealth. Thus among all the observations in sample (20223 households), there are 13243 respondents (65.5 percent of the whole sample) having positive taxable wealth and positive pension wealth,[21] and this is the sample of those who can make asset allocation and location decisions in taxable and tax-deferred accounts.

In addition, the definition of ”bonds” and ”stocks” is clearly important too. Typically, ”bonds” have been interpreted to be corporate, municipal, and government bonds traded on financial markets.[22] Since I focus on the asset location and allocation problem, I augment this set of assets with money market and saving accounts which face the same tax treatment as conventional bonds, as described in Amromin (2005). Consequently, the share of ”equities” held in taxable accounts is defined as the sum of directly held stocks and stock mutual funds divided by total investable taxable wealth, while the share of ”equities” in tax-deferred account would be the sum of stocks and stock mutual funds divided by total retirement wealth. Furthermore, the labor income is defined as the sum of all pre-tax wages earned within a family. If the member of a household is self-employed, the labor income is the earnings from

(like IRA and Keoghs) and employer-sponsored (i.e. 401k, 403b) has to be inferred from categorical responses. I applied a mapping similar as Amromin (2005) to construct the retirement portfolios in those years.

[20]The detailed explanations of the inflators can be found in SCFs bulletin (e.g. *http://www.federalreserve.gov/pubs/oss/oss2/2007/scf2007home.html#bullart*). Note that since some of the variables in the theoretical analysis are normalized values by total wealth, the inflation-adjusted step may not be needed.

[21]Among the sample in analysis, 6870 households report having no pension wealth. This is the sample of those who do not have tax-deferred savings, or have liquid the tax-deferred balance. There is another 110 households only have positive wealth in the pension account, which could be the case that those households consume up the taxable wealth in a certain period due to short of income or savings. I consider both of the cases could be explained by the theoretical model, so I leave those respondents in the analysis.

[22]I exclude the tax-exempt municipal bonds for that they have different tax treatment with taxable bonds.

business. Since the theoretical model treat each year as a time period, all the empirical variables are evaluated on a annual basis.

In the estimation, I use the average of life-cycle portfolio choices in both taxable accounts and tax-deferred accounts as the sample moments. Generally speaking, those average age-profiles of portfolio choices can be constructed by averaging the data across households at each age. However, it is not possible to ignore cohort and time effects, because asset investments have changed with generations and time periods. In order to reconstruct the household portfolio data uncontaminated by cohort and time effects, I apply a method similar to that described in Gourinchas and Parker (2002). In particular, four profiles are estimated using household level data from age 25 to 64: the equity ownership in the taxable account; the equity share in the taxable account, conditional on flows to equity being positive; the equity ownership in the tax-deferred account; [23] and the conditional equity share in the tax-deferred account. For each of these profiles, an equation is regressed over a list of age, time, and cohort indicators. As discussed in (Ameriks and Zeldes, 2004), it is not possible to separately identify the linear component of the age, time, and cohort effects. Due to the fact that individuals who witnessed poor stock market returns early in their life would choose to be less exposed to stocks later in the life, I choose the identifying restriction such that cohort effects are the history of financial returns experienced during the lifetime and captured by the average real stock return experienced by the head of the household from age 15 to age 25 (age $15-25$ return). [24] Then the life-cycle portfolio choices are constructed by removing the time and cohort effects. In practice, the ownership equations (based on a dichotomous choice variable) are estimated using a probit procedure and the portfolio share equations (based on a continuous variable) using simple OLS.[25] Other age-dependent profiles, such as pension-wealth proportion and income-wealth ratio, are also constructed by the same method.

The reconstructed life-cycle (working time) asset location and allocation profiles are displayed in Figure 4. The evidence is consistent with that observed in other studies (Poterba and Samwick, 1997; Bergstresser and Poterba, 2004; Amromin, 2005) — households commonly keep both equities and bonds in tax-deferred and taxable accounts, and they often maintain a higher equity position inside the retirement accounts. The top two graphs show the proportion of the population with positive equity holdings in each type of account. It can be clearly observed that the equity ownership in the taxable account is in-

[23]The equity ownership in the tax-deferred account is evaluated based on the condition that the tax-deferred wealth is positive.

[24]This formulation assumes that cohort effects are not important for the evidence in portfolio choices. The effects of age and time may be sensitive to this assumption.

[25]Specifically, each dependent portfolio decision is regressed on a complete set of age dummies, time dummies (less year 1992), cohort indicators (age $15-25$ returns), and a retirement dummy that is equal to 1 if the respondent is retired. With these estimates, a predicted value is obtained by setting the time and retirement dummies to zero, and the cohort indicator to a average value (10 percent, estimated from the historical data). The retirement variable is removed because this study focuses on the working population that have labor income and make saving decisions. Thus the constructed portfolio data represent the behavior of the observed households facing the financial market like that in 1992, and not retired.

creasing with age, while that in the retirement account remains at a higher level throughout the life time. The lower two graphs illustrate the average equity share conditional on ownership in both accounts. The average proportion in the taxable account is slightly increasing with age, while that in the retirement account is slightly decreasing with age. This is the version of sample moments used in the structural estimation discussed in the next section.

4 Estimation

I recover those structural parameters of the theoretical model using the Method of Simulated Moments (MSM). I assume that households start off the retirement saving with an initial pension-wealth ratio, and for each set of parameter I compute the policy rules that solve the dynamic problem and use these rules to generate simulated portfolio choices over the life-cycle. Then at each iteration of the parameters I construct a measure of distance between the observed and simulated moments, namely the age-dependent saving and investment behavior in the taxable account and tax-deferred account. The parameter estimates of the theoretical model are those that minimize this distance.

On thing that needs to be noted before the discussion of estimation process is that some of the parameters are pre-determined in the model. This is due to the observable features of those parameters and the fact that they may not be identified separately from one or another. Table 1 provides a summary of those determined parameters. First, I estimate the riskless return on taxable bonds from the average real return on Moody's AAA Corporate Bonds, and the risky stock returns from the S&P500 index. Over the period January 1990 to December 2007, which is the duration of the data used in the empirical analysis, the pre-tax interest rate on the riskless taxable bond is $r^b = 7\%$ per year; the dividend yield on the stock index is $d = 2\%$ per year; and the annual capital gains return on the stock index is nominally distributed with a mean of $\mu_r = 9.81\%$ and standard deviation of $\sigma_r = 16.9\%$. Because the pre-tax expected return on the stock index is given by $\bar{r^s} = (1+\mu_r)(1+d) - 1$, the annual pre-tax equity risk premium is 5%. Because I use the most recent financial information, this equity risk premium is relatively low compared to the historical average risk premium of about 8%, but it is consistent with most of recent studies (Dammon et al., 2004; Shoven, 1999). In addition, I assume that the tax rate on dividends and interest is, $\tau^d = 36\%$, the tax rate on realized capital gains is $\tau^s = 20\%$, and Social Security tax is $\tau^{ss} = 6\%$. In reality, the tax rate on income and capital gains is progress, but I will simplify it to a flat rate in the analysis, and this rate is about the actual tax paid by middle-class population. I further set the cap of retirement savings at the rate $q = 20\%$, and the penalty on pre-retirement withdrawal as $pnl = 10\%$. Both of these two values addict to the true values in the real life.

The rest of the parameters will be recovered from estimation. I assume all households start off at age $t = 25$, with a pension-wealth ratio p_{25} drawn from a lognormal distribution, $ln(p_{25}) \sim N(\mu_p, \sigma_p^2)$. The

Table 1: Pre-determined Parameter Values

Parameters	Notation	Base-line Value
Asset Returns		
Riskless pre-tax return	r^b	7.0%
Dividend yield on equity	d	2.0%
Expected capital gain return on equity	μ_r	9.81%
Standard deviation of capital gain return	σ_r	16.9%
Tax Rates		
Dividend (Bonds) tax rate	τ^d	36%
Capital gain tax rate	τ^s	20%
Social Security tax rate	τ^{ss}	6%
Retirement savings		
Cap of retirement saving rate	q	20%
Penalty on pre-retirement withdrawal	pnl	10%

parameters to estimate are then the followings: Retirement Saving Parameters $(m, tr, h, \mu_p, \sigma_p^2)$, Preference Parameters (β, γ, bq),and Labor Market Parameters $(l_0, l_1, l_2, \sigma_u, \lambda_0, \lambda_1)$. The parameters of labor income process, such as the marginal effect of time (l_0, l_1, l_2) and labor income shocks $(\sigma_u, \lambda_0, \lambda_1)$, can be identified from the aggregate income profile.[26] Given the wage distribution, asset stochastic returns and life time uncertainty, the parameters of a a standard intertemporal consumption and portfolio choice model can be identified by the asset levels and portfolio choices over time, including discount factor β, risk aversion rate γ and bequest factor bq (see Gourinchas and Parker (2002)) and Cagetti (2003)). In addition, the data of separate wealth accumulation paths in the taxable and tax-deferred accounts can identify structural parameters of retirement saving propensities, such as initial wealth distribution (μ_p, σ_p^2), and the fraction of Social Security payment to total wealth level h, which is a crucial income to households who do not have significant assets in the pension account. Furthermore, the pension account characteristics, such as employer matching rate m and the level of transaction cost in equity trading tr, could be identified from their unique effects on the observed saving and the asset location and allocation decisions in each year during working time. Specifically, although both employer matching rate and lower transaction cost in the tax-deferred pension account tend to increase equity holdings in that account, their effects differ in two important ways: (1) while transaction cost does not show clear effect on the saving behavior, the employer matching policies additionally influence pre-tax saving decisions, which in turn affect the share of total wealth held in the pension account; (2) when households are getting older, the role of employer matching strategies in the retirement saving and investment decisions becomes smaller, but the impact of transaction costs remains the same as that for young households.[27] Therefore, the parameters m and tr can be identified by the dynamic data of saving and portfolio choices.

[26]Ideally, one can identify the income transition parameters with a complete record of individual earnings over time. Although I do not have a longitudinal data, I can still estimate the wage distribution at each age by the cross-section observations.

[27]See detailed discussion about Figure 2 and 3 in section 2 .

Given the value of those parameters, one can simulate the model for a large number of agents and compute the distribution of choice variables for each agent at age t. One way to estimate the parameters is therefore to choose the ones that generate a simulated distribution that matches some aspects of the empirical distribution. In particular, the moments used in this estimation are the cell-by-cell expectations for the following distributions over 40 years: the equity ownership probability and the conditional equity share in the taxable account, the equity ownership probability and the conditional equity share in the tax-deferred account, and the fraction of wealth held in the pension account and income-wealth ratio distribution. [28]

Thus, there are $J = 240$ (6×40 years) moments and $N = 14$ parameters in the estimation. Let the parameter set be denoted by θ, and the observation of each moment j for household i by ζ_i^j, $j = 1, \ldots, J$. I seek to estimate the model from the following moment conditions:

$$E[\zeta_i^j - \hat{\zeta}^j(\theta_0)] = 0,$$

where θ_0 is the true parameter vector, and $\hat{\zeta}^j$ is the predicted counterpart.

Therefore the estimation procedure is then a method of simulated moments (MSM) (see details in Pakes and Pollard (1989) and Duffie and Singleton (1993)) that minimizes over θ:

$$S(\theta) = g(\theta)' W g(\theta), \tag{14}$$

where $g(\theta) = (g_1, \ldots, g_J)'$, with the jth element as,

$$g_j(\theta) = \frac{1}{I_j} \sum_{i=1}^{I_j} \zeta_{i,j} - \hat{\zeta}_j(\theta), \tag{15}$$

where I_j is the number of observations for the jth moment. W in equation (14) is a positive definite weighting matrix, which can be chose so that the MSM estimator $\tilde{\theta} = argmin[S(\theta)]$ is both consistent and asymptotically normally distributed (see Gourinchas and Parker (2002) and Cagetti (2003)).

In practice, I simulate life-cycle portfolio profiles by generating a sequence of 5000 income processes and risky asset returns over 40 years, and computing in each year the associated asset allocations in each type of account. Once the optimum is found, the asymptotic standard errors are evaluated numerically using the gradient of the moment vector. Considering the size of this problem, parallel programming techniques as implemented in Message Passing Interface (MPI) are applied to make the problem computationally feasible. The extensive explanation of the MSM estimator, the derivation of

[28] Because of the lack of the information in the surveys, consumption-wealth ratio and saving-wealth ratio cannot be used as moments in the estimation. The state variables moments, pension-wealth ratio and income-wealth ratio, are taken in the estimation to capture the saving behavior.

Table 2: Structural Estimation Results

Parameters	Notation	Coef.	(Std.Err.)
Retirement Savings			
Employer matching rate	m	0.6768	(0.0510)
Transaction cost (percentage of taxable wealth)	tr	0.0063	(0.0004)
Social Security payments (percentage of W_K)	h	3.7114	(0.2615)
Initial pension-wealth ratio, Mean	μ_p	−0.6412	(0.0477)
Initial pension-wealth ratio, S.D.	σ_p	0.5490	(0.0449)
Utility and Bequest Functions			
Discount factor	β	0.9838	(0.0739)
Relative risk aversion	γ	3.9607	(0.2811)
Bequest parameter	bq	0.0888	(0.0067)
Labor income equation			
Constant	l_0	6.9976	(0.5595)
Coefficient of age	l_1	−0.1328	(0.0085)
Coefficient of age^2 ×100	l_2	0.0512	(0.0038)
Base unemployment rate	λ_0	−3.0505	(0.1925)
Unemployment rate growth×10	λ_1	−0.0080	(0.0005)
S.D. of income shock	σ_u	0.4552	(0.0306)
Criterion Function[1]	χ^2	48.1174	

[1] Note: The last row reports a test of the overidentifying restrictions distributed as Chi-squared with 226 degrees of freedom. The critical value at 5% is 262.07.

the standard error, and the procedure of parallel programming are provided in the Appendix A.4.

5 Results

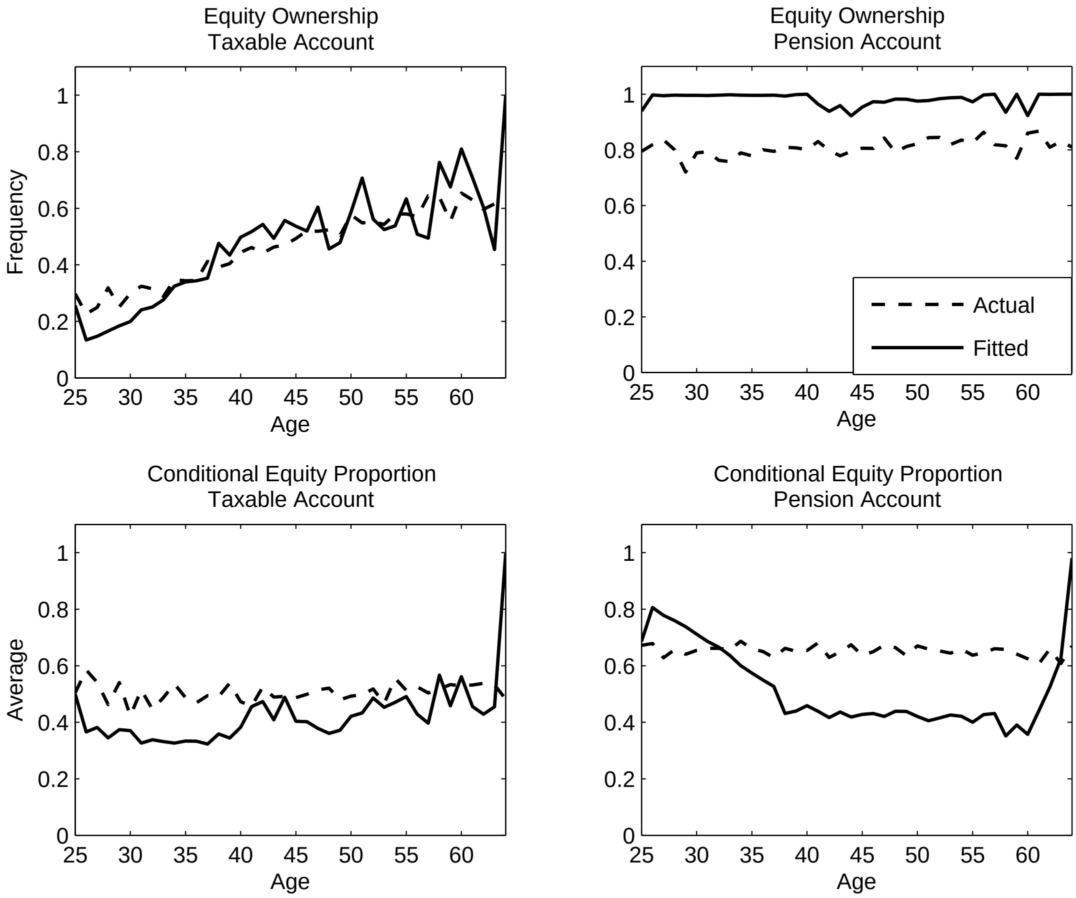
In this section, I first estimate the model for the average household and discuss the implications of the fitted structural model for retirement saving behavior and portfolio choices. And then, I turn to the results of the characteristics in the tax-deferred pension account, and evaluate the impact they have on the households' behavior. Lastly, I analyze the consequences of household behavior that are caused by removing the Social Security tax and payments.

5.1 Estimation Results

The results of estimating the structural model are shown in Table 2, and the associated household portfolio choices are displayed in Figure 4. The parameter estimates comprise of those that characterize retirement savings, utility and bequest functions, and labor income process.

The estimated parameters of retirement savings are shown in the top panel of Table 2. The pension account characteristics are the most important factors in this study, which are considered to have significant impact on households' saving decisions and portfolio choices. Those features of retirement saving accounts are captured by two parameters, employer matching rate (m) and transaction cost (tr). The employer matching rate is estimated at 0.6768, which can be interpreted as one dollar retirement saving by

Figure 4: Portfolio Choices over the Life-Cycle



the employee is associated with 67.68 cents contribution from the employer. It is worth noting that the estimated employer matching rate is within the reasonable range. According to the Survey of Consumer Finances, the self-reported employer matching rate[29] by the households has a mean of 0.534, with a standard deviation of 2.484. In addition, Engelhardt (2004) also show similar evidence from the Health and Retirement Study (HRS). By using the Summary of Plan Description in HRS, he shows that about 43% of the 209 employer-sponsored defined contribution plans have a matching rate at 0.5, which cover 38% of all the individuals in analysis, and additional 35% of those plans provide a matching rate ranging from 0.51 to 1.00, which cover 42% of all the respondents. The estimate of the employer matching rate in this study shows the average level that the households may face when they make saving and portfolio decisions.

Besides employer matching rate, the transaction cost is also an important factor in household portfolio decisions. The estimated transaction cost parameter is about 0.0063, which shows the size of fees that incurred whenever the equity holdings in the taxable account is positive. Since this factor is only imposed on the taxable account but not on the pension account, it captures the transaction costs differ-

[29] Since employers differ a lot in the matching policies, I simply define the employer matching rate as the ratio of employer's contribution to that of the employee's.

ence between those two types of accounts. Therefore, accurately speaking, the estimate indicates that households pay 0.63% of their taxable wealth more as transaction fees when they make equity trading in the conventional taxable accounts than that they pay in the tax-deferred accounts. In particular, for instance, if an agent trades 100 dollars stocks in the pension account without paying any fees, he has to pay 63 cents as transaction fees when he make the trading in the taxable account. Based on the reality, although different transaction companies charge different fees, this estimate is in the reasonable range of actual data.[30]

In addition to the tax-deferred retirement saving opportunity, the households can also receive retirement benefits from Social Security, the public pension system in the United States. The estimate implies a ratio of Social Security payments to total wealth before retirement as 3.7114, which suggests that Social Security is still the main retirement income source for most households.[31] Moreover, the procedure also give an estimated distribution of the initial pension-wealth ratio, which is log-normally distributed as $N(\mu_p = -0.6412, \sigma_p^2 = 0.5490^2)$. This distribution implies that the mean of the ratio of pension wealth to total wealth, $exp[\mu_p + \sigma_p^2/2]$, is about 0.612, a little higher than the mean level reported by the Survey of Consumer Finances, which provides a average pension-wealth ratio of 0.433 for households with positive balance in the pension account.

The second panel of Table 2 shows the estimate of parameters in the utility and bequest functions. The discount factor (β) is estimated at around 0.98, the risk aversion rate at a level just below 4.0, and the bequest parameter less than 0.1. The results are consistent with the findings in Cagetti (2003). The higher discount factor in this study (compared with Gourinchas and Parker (2002)) indicates a relative higher degree of patience for the households, who are willing to save more and earlier for retirement purpose. This saving tendency will be shown in the following discussions. The estimate of risk aversion rate may connect with the uncertainties in labor income and financial assets, which is in the reasonable range of most studies of portfolio choices (for instance, Campbell and Viceira (1999, 2006)).

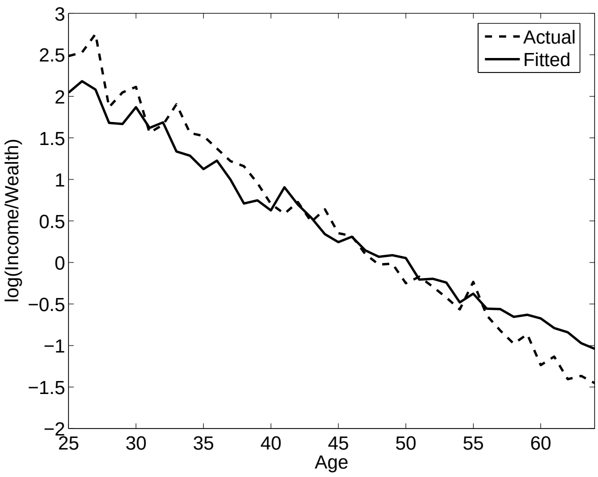
The estimates of the labor income process are displayed in the bottom panel of Table 2. The coefficients are estimated from Equation 12 and 13 in the Section 2.1. The unemployment rate at age 25, $pu_{25} = \frac{e^{(\lambda_0+\lambda_1 25)}}{1+e^{(\lambda_0+\lambda_1 25)}}$, is about 4.43%, and slightly decreases across the life time ($\lambda_1 < 0$), until reaching $p_{64} = 4.30\%$ at the last period of working. While the estimated unemployment rate is at a reasonable

[30]For example, Fidelity charge a \$8 − \$19.95 flat commission for each trade in regular taxable account. E∗TRADE also has a fee ranging from \$6.99 to \$12.99 per trade.

[31]Two experiments are done to test the accuracy of this estimate. First, consider a household with annual pre-tax income 50,000 save 1,000 every year for 40 years, and the wealth accumulate with a constant interest rate 0.0344 (historical average of the stock market). Based on the current Social Security policy, they can get monthly payment as 1,363 after retirement. By roughly calculation, if the household receive payment for 20 years, then the ratio of Social Security payments to total wealth is 3.922. However, this calculation is sensitive to many factors, such as years of retirement, saving rate, interest rate, and so forth. Second, assume that the Social Security taxes are put aside in a separate saving account and accumulated with a risk fee return of 0.5, and then based on the settings of this model, the ratio of Social Security payments to total wealth is 3.65. This high risk free return may reflect the fact of inflation in the real economy.

level, the estimated labor income shock, σ_u, is relative high at 0.455. One possible interpretation for a higher σ_u is that some factors with important impact on labor income may not be captured by Equation 12, such as schooling and experiences of the workers.[32] Figure 5 shows how the predicted income process matches the data. The actual life cycle income-wealth ratio is obtained from SCF by log-averaging across households. As explained in Section 3, the life-cycle profile are corrected by eliminating cohort and time effects. It is worth noting that the estimated income-wealth profile fit the data well. The income-wealth profile implies a life-cycle trend of wealth accumulation. At the beginning of working life, most workers start with a lower wage, and the households do not have sufficient savings out of consumption, so the annual income is usually greater than accumulated wealth, thus the ratio of income to wealth is greater than one. This ratio can be as large as 15 during the twenties, but will dramatically decrease with age, and reach two or three around age 35. As the working experience accumulates, the earnings will increase, and so will the balance in the savings account. Therefore, the income-wealth ratio decreases with age gradually after age 35, and the income becomes lower than wealth after age 45. After mid of fifties, the earnings may start to decline, and the income-wealth ratio continue decreasing to around 0.2.

Figure 5: Log Income-Wealth Ratio over the Life-Cycle



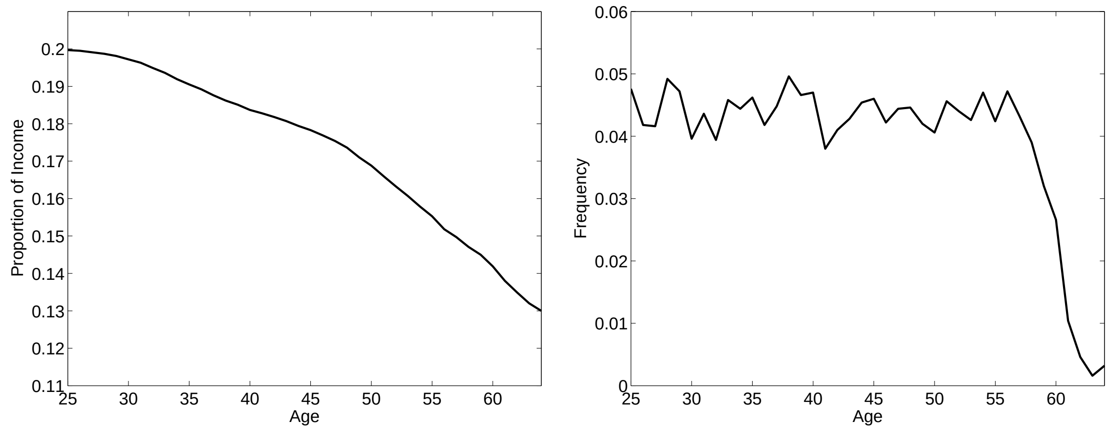
With the estimates in hand, one can address how well the structural model fits the life-cycle portfolio choices. Figure 4 plots the simulated and actual asset allocation and location decisions over the life time, among which the two figures on the left display the equity ownership probability and average conditional equity shares in the conventional taxable account, and the two figures on the right show the equity holding distribution in the tax-deferred pension account. The model with pension account characteristics does break the "tax-efficient" portfolio rules, and produce life-cycle paths that mimic

[32]A complete formulation of labor income process may require the inclusion of time, schooling and experience (such as Belzil and Hansen (2002)). However, adding more variables into the labor income equation may request more state variables in the model, which in turn causes huge computational burden. Therefore, I choose the simplified version of labor income equation.

the observed household investment behavior. With the fitted model, the probability that households participate in the stock market in the taxable account is increasing in age, from around 25% at age 25 to about 60% before retirement, and the conditional shares of equity in that account have a mean around 50% over the life time, while in the tax-deferred account, the equity ownership rate remains at a very high level through out the whole working life, and conditional mean of equity shares is slightly decreasing in age from 70% to 40%. The estimated results suggest that there are a large amount of households only hold equities in the pension account, with all their taxable wealth invested in safe assets, and, during a certain period of life (from age 25 to 40), the equity shares in the pension account are higher than that in the taxable account. Therefore, the model successfully explain the observed evidence of asset allocation and location problem: households tend to hold stocks in both taxable and tax-deferred account, and they usually maintain a higher equity investment level in the pension account.

While the predicted distribution of asset allocation replicate the actual data closely in the taxable account, however, the model overpredicts the equity ownership in the tax-deferred pension account, and a relative lower equity share level during the old ages. These differences may reflect a weakness of the data, rather than the model. In some waves of SCF used in this study, the reported asset allocation in the pension account are in the type of categories. The questions in the survey have three choices: no stock at all, half in stocks and all in stocks. This design may have such consequence that part of the observations who have little equity investment report no stock at all. This tendency produce a lower observed equity participation rate, but a relative high equity conditional share. Apart from these features, however, the tight structure imposed by the model produces good predictions in terms of portfolio dynamics.

Figure 6: Retirement Savings over the Life-Cycle



(a) Retirement Saving Rate (b) Early Withdrawal Probability

I now turn to the question of how household save for retirement over the life cycle. Figure 6 describes the household saving behavior in the tax-deferred pension account. The Figure 6(a) on the left shows the contribution rate (retirement savings as a percentage of income) into the tax-deferred account conditional on that the retirement saving is positive, and the Figure 6(b) plots the probability of pre-retirement withdrawal from the pension account, which is subject to the pre-withdrawal penalty 10%. From Figure 6(a), we can see that the average contribution rate into the tax-deferred account is slightly decreasing in age, but remains at a relative high level in most life time. This trend suggests that most households start saving for retirement at very young age, and they put a lot of savings in the retirement account. The high saving rate in the pension account reflects two things: (1) the tax benefits and other pension account advantages (such as employer matches) and (2) the relative high average return earned on the assets (more equity investment) held in the retirement account. Moreover, because of the labor income shocks, households on average never reach the saving limit (20%) in the tax-deferred account, and they also pre-withdraw some funds occasionally for regular consumptions (see Figure 6(b)). The positive pre-retirement withdrawal rate may be a result of the high level of retirement savings. As the household approaches retirement, the pre-retirement withdrawal rate dramatically decreases to almost zero, reflecting the high propensity of retirement savings during these ages.

Figure 7: Retirement Wealth over the Life-Cycle

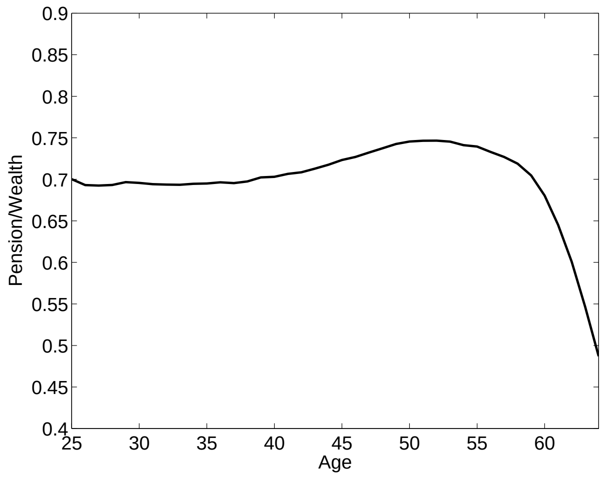


Figure 7 displays the predicted pension-wealth ratio over the life cycle. The average fraction of total wealth held in the pension account remains at a high level during young ages, and reaches its maximum of 75% at age 50, well before the head of the household reaches retirement age. The decline in the fraction of total wealth held in the retirement account in the years prior to retirement may result from the lower fraction of total savings allocated to the retirement account at these ages. It has to be noted that almost all households in the simulation have positive retirement wealth over the life time, but there are about 30% of the households in the data do not have any savings in the pension account. This difference between simulated and observed profiles might be explained by the assumption in the theoretical model: all households face the same type of pension account characteristics. The estimated employer matching rate and transaction costs benefit in the tax-deferred pension account make it a very popular choice for most households, thus making the retirement saving rate and accumulated retirement wealth at a relative high level. Holding other constant, if the employer does not provide any matching benefit, the savings in the retirement account will decrease to zero during the young ages and increase gradually with age. Moreover, there are about 90% of the households having no pension wealth between age 25 to 35, and 30% of them no pension wealth between 35 to 45. How will the households react when the pension account policies change is explained in the next several sections.[33]

5.2 The Role of Employer Matching Policy over the Life-Cycle

In the Defined Contribution pension plans, employer matching policy is one of the most important designs. Almost in all employer sponsored DC pensions (such as 401(k) and 403(b)), employers match some or all of employees' contributions. This phenomenon raises the question of what is the role that employer matching strategy plays in the household behavior. This section explains the impact of employer matching policies on household retirement saving decisions and portfolio choices.

Table 3: The Estimated Elasticity with respect to Matching Rate

	Taxable Account		Tax-Deferred Account		Retirement Savings
Age	Equity Ownership	Equity Share	Equity Ownership	Equity Share	Pension Wealth
25-29	-31.1535	-9.8060	3.8401	5.4377	0.2949
30-34	-18.7860	-12.0746	4.0875	5.1420	0.3633
35-39	-8.4524	-9.3201	2.7063	2.9442	0.3327
40-44	-2.9888	-3.0939	1.2829	1.2545	0.2863
45-49	-0.9422	-1.0152	0.3230	0.3964	0.2438
50-54	-0.3127	0.1588	0.0410	0.2354	0.2354
55-59	-0.8528	-0.0457	-0.0322	0.1972	0.2311
60-64	0.8109	0.3559	0.0051	-0.5683	0.2369
Total	-7.8347	-4.3551	1.5317	1.8799	0.2784

Given the estimates in hand, Table 3 shows the elasticities of portfolios and savings with respect to the change in employer matching rate. The numbers in the table are five-year average from age 25 to 64, with the total average on the bottom. The columns display the matching rate elasticity of equity investment in the taxable account and tax-deferred pension account, as well as the elasticity of retirement saving decisions over the life time. Consistent with the findings in Engelhardt (2004), the proportion of wealth in the pension account respond positively to the changes in matching rate,. Although inelastic, the life-cycle trend of the elasticity in pension wealth ratio suggests that younger households are more responsive to changes in employer matching policy. The portfolio choices are quite sensitive with respect to the changes in matching rate. During the ages 25 to 50, equity holdings

[33] A more complete analysis may involve different type of retirement plans, and households make decisions based on different account characteristics. Due to the consideration of the computational burden, I choose to set up the model with the simplified version.

in both types of saving accounts respond elastically to matching policy changes, with positive response in the tax-deferred account and negative in the taxable account. This finding is a optimal response to changes in the saving behavior. As a higher fraction of wealth held in the pension account, households tend to boost equity holdings in the retirement saving account for the higher returns and reduce those investments in the taxable account for precautionary saving purpose. Moreover, the life-cycle trend of the elasticity indicates that the impact of employer matching policy on portfolio choices is more apparent during the young ages. Households in ages $25-29$ have a matching rate elasticity of equity ownership in the taxable account at -31.15 and in the tax-deferred account at 3.84, but this elasticity is only about 0.81 in the taxable account during ages $60-64$ and 0.0051 in the pension account. One possible explanation of this phenomenon is that the purpose of young households saving in pension account is to enjoy higher asset returns from tax benefit and pension favored features, so their portfolios are more likely to get affected by policy changes, while older households who are more concerned with the accumulation of retirement wealth are less likely to get influenced.

To make the point clearer, Figure 8, 9 and 10 provide a graphical illustration of the impacts of employer matching strategies on household behavior. The four graphs in Figure 8 show the changes in life-cycle portfolio choices. There are two implications that can be derived. Firstly, the figure displays the change in household behavior when the matching rate decreases by 10% from 0.6768 to 0.5768. Consistent with the results in Table 3, the lower the matching rate implies a lower proportion of total wealth held in the pension account, thus it decreases both the equity ownership and conditional equity holdings in that account, with a larger effect on young households (e.g. at 30s, about 60% decrease in equity ownership and 50% decrease in equity shares) and smaller effect on older ones (e.g., at 50s, the equity ownership decreases about 5% and equity shares about 3%). In order to balance the riskiness in investment, households thus hold more equities in the taxable account. From age 25 to 45, the average change is about 52% increase in equity ownership and 45% increase in average equity shares, while the change during ages above 45 is relative small, around 2% increase in equity ownership and 0.5% increase in equity shares. The results further suggest that a change in the employer matching rate will significantly influence the young households' asset allocation and location decisions. Secondly, the figure also shows the change of household saving and investment behavior after a decrease of matching rate from 0.5768 to 0.4768. Although this 10% decrease still has the same effects on portfolio choices as the previous change, the magnitude of its influence is apparently smaller, which tells us that the impacts of employer matching strategies are not uniform across different levels. Any policy change or regulation should consider both the size of change and marginal effects.[34]

[34] Since different level of employer matching rates display different effects on household behavior, I have done an additional analysis on the household responses to an employer matching rate change from zero to 0.1. The results indicate that household retirement saving behavior (pension saving rate and accumulated pension wealth) reacts to this change significantly, especially during the young ages, but the investment behavior does not display significant difference. This finding suggest different

Figure 8: The Role of Employer Matching Policy in Portfolio Choices

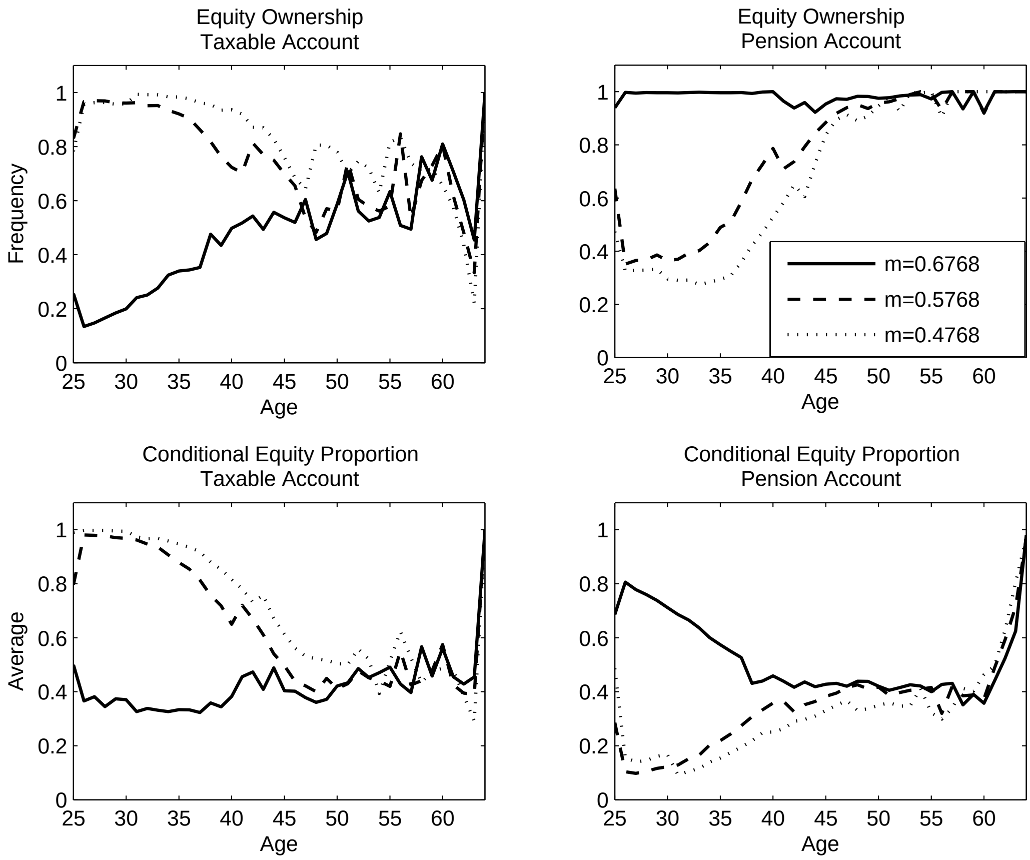


Figure 9: The Role of Employer Matching Policy in Overall Equity Holdings

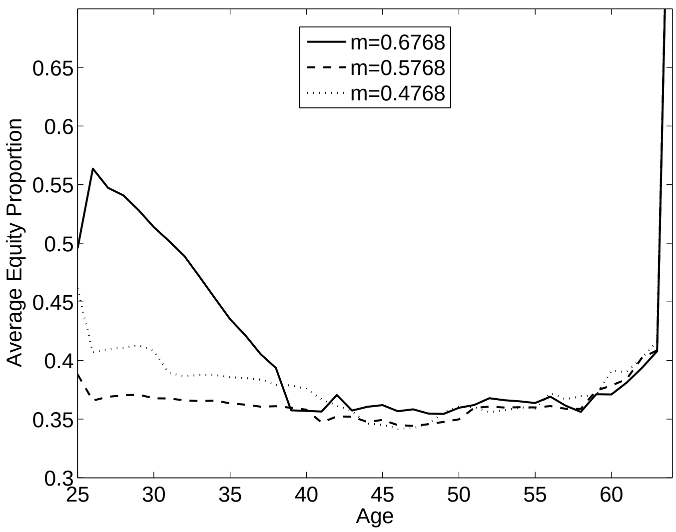


Figure 10: The Role of Employer Matching Policy in Retirement Savings

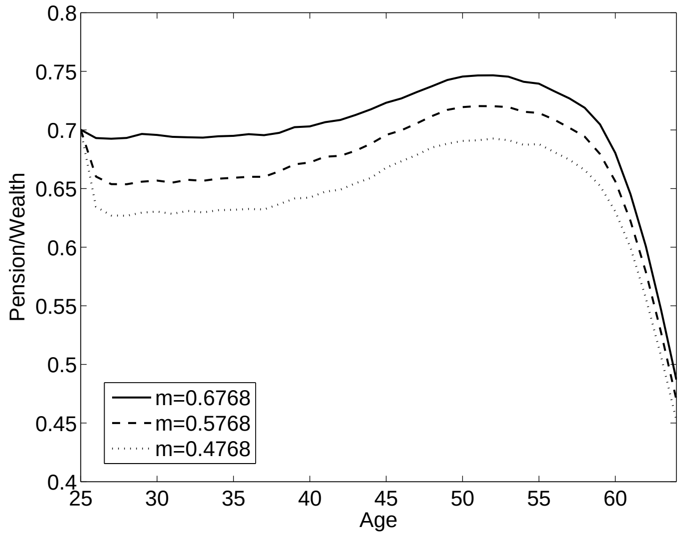


Figure 9 displays the change in overall equity holdings after the decrease in employer matching rate. Given that almost all households participate in the equity market, the conditional overall fitted equity proportion decreases in age. When employer matching rate decreases, the income the households can get implicitly decreases too, thus their equity investment becomes conservative, especially for the young households. The average equity proportion decreases about 10% during ages 25 to 35, as a response to a matching rate decrease from 0.6768 to 0.5768. The result suggests that a proper increase in the employer matching rate will encourage young households to hold more equities in the financial market. Figure 10 is about the change in pension-wealth ratio after a decrease in employer matching rate. The decrease in pension-wealth ratio comes from the deduction in the employer contribution and the decrease in the households' own savings. The relative larger decrease during the younger ages reflects a larger change in the retirement saving behavior during those ages.

5.3 The Role of Employer Stock Matches

In addition to the form of cash, some employer matches also take the form of company stock (mainly in 401(k) types). Meulbroek (2002), Ramaswamy (2003) and Poterba (2003) suggest that holding company stock is risky because it raises the volatility of the retirement wealth. Some other researchers (such as Campbell and Viceira (1999), Davis and Willen (2000a,b), and Heaton and Lucas (2000)) also point out that company stock is considered to be a more risky investment because it is likely positively correlated with labor income risk. Therefore, a worker with company stock investment in his pension accounts may face the risk of losing his job and his retirement wealth at the same time. Then the question would be, if putting inside a life-cycle model, how will company stock matches change households'

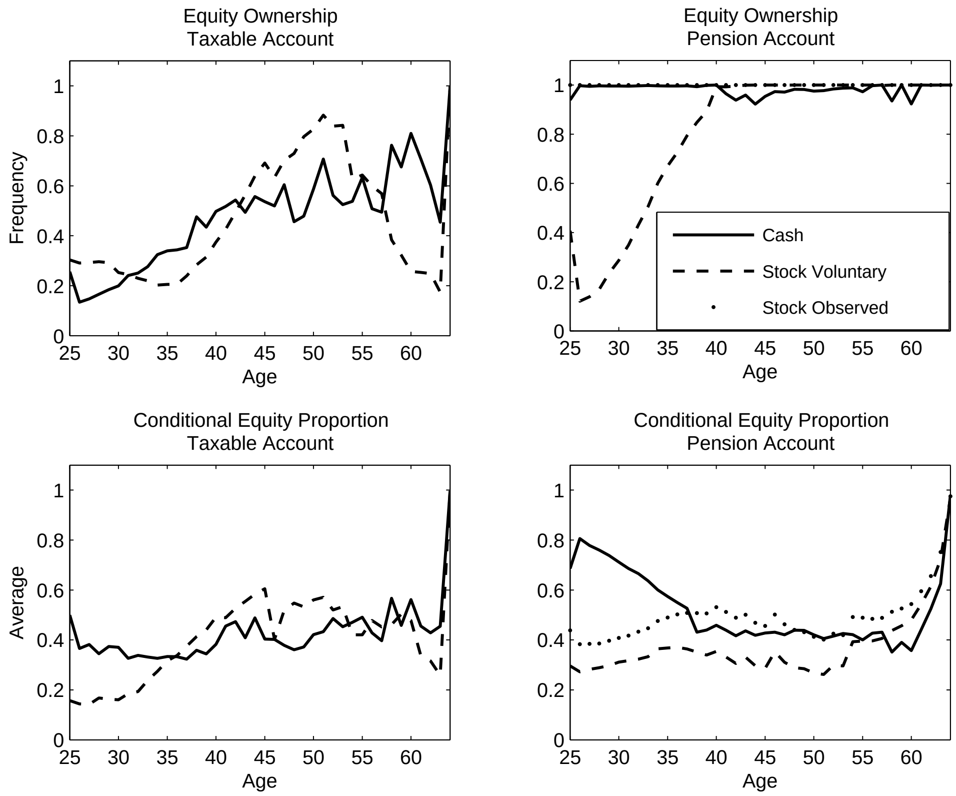
employer matching policies have different roles in explaining the household behavior.

investment decisions. In the following paragraphs, I study the consequences two scenarios: (1) changing the employer cash match to stocks, remaining at the estimated rate 0.6768; (2) under the condition of stock matches, adding the correlation between the equity return risk and the labor income shock into the model, and fixing the correlation at $corr(r,u) = 0.1$.

In the first scenario, after changing the employer cash matches to stock, the retirement saving rate and pension-wealth ratio do not deviate from the original predicted level, which suggests that the household actively change the portfolio combination in order to keep the optimal value of retirement wealth. Figure 11 compare the portfolio choices under the condition of the employer cash match with the stock match. There are two lines in the graphs of pension account representing the result of employer stock matches, one showing the household voluntary portfolio choice (the dash line), and the other observed equity distribution (the dot line), which is the summation of household voluntary equity investment and mandatory employer stock match. Because of the riskiness of stocks, employer stock matches tend to reduce the voluntary equity investment of the employees in the retirement account, resulting in decreases in both the equity ownership (about 4% on average) and conditional equity shares (about 3% on average). The effect is more significant for young households, who are more sensitive to pension account policies (a decrease of equity ownership up to 20% at age 20s). But the observed equity ownership in the pension account is very high, almost 100% for all ages. The portfolio choices in the taxable account under employer stock match does not differ much from those under employer cash match, with a slightly higher equity ownership and a slightly higher average equity proportion.

In the second scenario, households face a employer stock match, and the equity risky return is positively correlated with the labor income shock, namely a higher equity return more likely accompanied by a higher level of income. Figure 12 illustrates the differences in household behavior between two cases, one of which is that the shocks in equity returns and labor income are not correlated, and the other is that the shocks are correlated at a rate of 0.1. The positive correlation of risks exhibits different effects on household behavior for different life stages. During the younger ages (from age 25 to 45), the correlated risks indicate a higher risk in the equity investment. From the consideration of precautionary savings, people tend to save less for retirement purpose, remain a larger proportion of total wealth in the taxable account, and invest less equities in the taxable account. But the households would voluntarily hold a larger share of equities in the pension account to enjoy the higher returns. Therefore, during this period of time, an employer company stock match which is positively correlated with labor income may result in even higher concentrated equity investment in the pension account (nearly 100% equity holdings). This finding is consistent with the empirical results discussed in Benartzi (2001) and Liang and Weisbenner (2002), who show that employer company stock matches cause participants to hold more company stocks in the pension account. During the older ages (from age 45 to retirement), how-

Figure 11: Portfolio Choices over the Life-Cycle: The Role of Employer Stock Matching



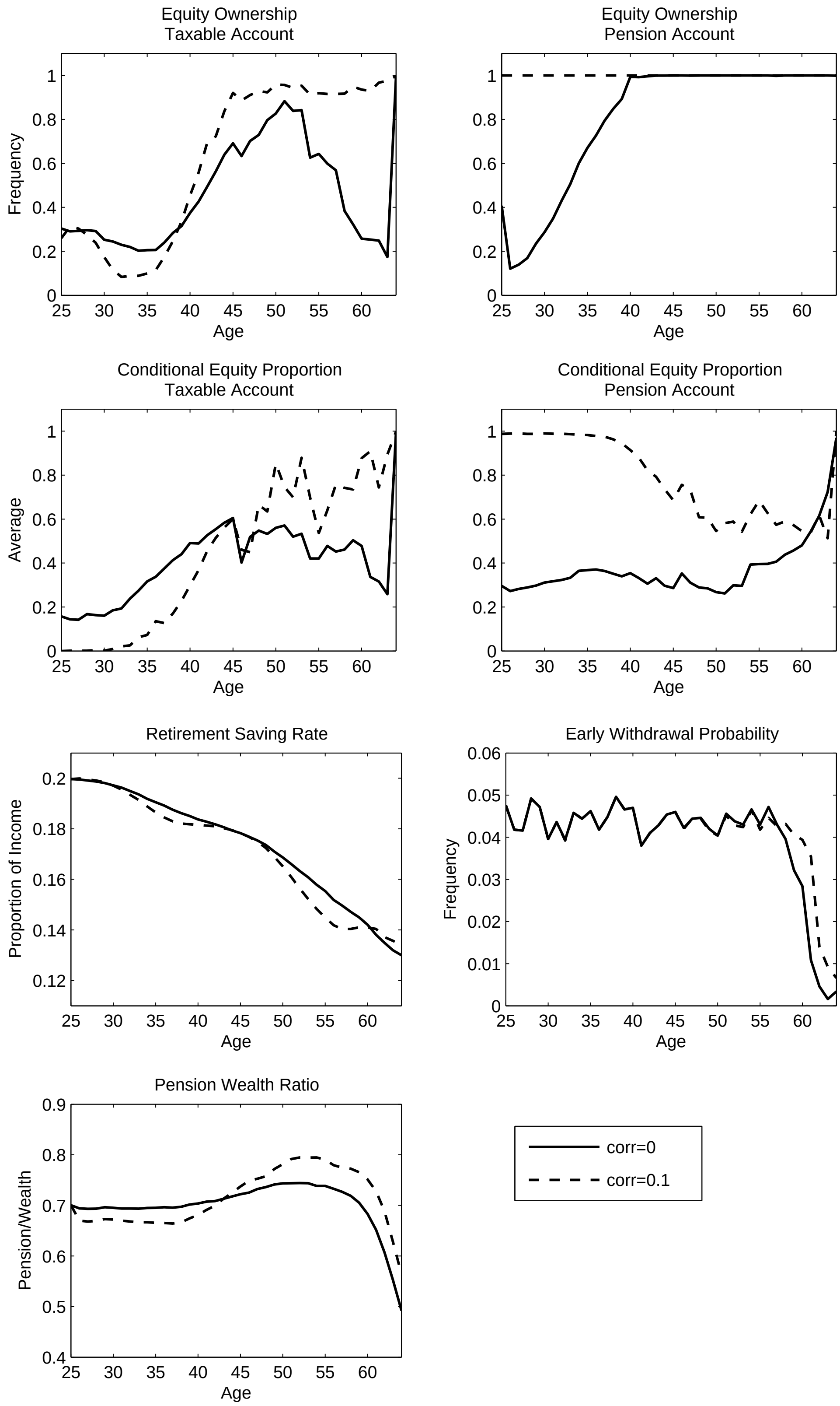
ever, in response to the retirement saving consideration, the investment behavior in the pension account becomes conservative, together with a higher equity holdings in the taxable account. Because of the bigger risk from the positive correlation, such investment strategy ends up with a even higher fraction of pension-wealth ratio during the old ages, and a higher rate of pre-retirement withdrawal from retirement wealth.

5.4 The Consequences of Change in Social Security

Social Security is an important income source for most retirees. However, it is forecast to face financial difficulties in the coming decades. Some researchers and investment advisors suggest privatizing some part of the current Social Security system, but few studies have quantitatively shown the possible effects of such change on equity investment and retirement wealth. In this section, I study a policy change by transforming the Social Security taxes and retirement payments into a tax-deferred retirement saving account during working life, and analyze the consequences of this change.

Before talking about the results, I have to point out the changes caused by removing Social Security taxes and payments. First, this modification makes the tax-deferred pension account the only income source during retirement. Households may also get resource from their regular savings in the taxable account, but they do not have additional income from any Defined Benefit type pensions. Second,

Figure 12: Portfolio Choices an Retirement Savings over the Life-Cycle: Correlation in uncertainties



the popularity of the retirement saving account is reduced due to the deduction in the tax rate (Social Security tax). Since the savings in the pension account is on a pretax basis, the higher the tax rate, the more attractive the retirement savings would be.

Figure 13 shows the consequences after removing Social Security payments, $h = 0$, and reducing total tax rate, τ, by 6%, which is around the current taxes collected for Social Security purpose. It is suggested by the figure that the effect of this change kick into household behavior only right before retirement, and both portfolio choices and retirement saving decisions remain unchanged before age 55. There are two implications that can be derived from the household behavior between age 55 to 64. First, as the retirement account becomes the only income source for retirement, households may value it more than before, so they hold more equities in that account for higher returns. The equity ownership in the pension account increases by 1% during age 55 to 64, and equity proportion in that account increases by 13% on average. In addition, in order to keep sufficient balance in the pension account, people reduce the early withdrawal rate by 0.6%, and they invest more in safer assets in the taxable account to fund consumption needs (an average decrease of 20% in both equity ownership and conditional proportion). Second, since the tax rate on earnings is reduced by 6%, and the retirement saving limit remains at the same level, more savings are held in the taxable account, and consequently the fraction of total wealth held in the pension account becomes lower.

Some people may argue that if there is no Social Security benefit, then households may want to save more for the retirement purpose. Therefore, I study an additional case by increasing the saving limit in the pension account to 26% of the earnings, which is 6 percent more than before, and the results are shown in Figure 14.[35] The households reaction to this change is quite large. They actively save more in the pension account to meat the new limit, which leave less savings in the taxable account, and thus result in a higher pension-wealth ratio through out the life time. In regard with portfolio choices, except for the ages right before retirement, most households maintain a "tax-efficient" asset allocation and location strategy, in which they hold high-taxed asset (i.e. bonds in the U.S. case) in the tax-deferred account, but low-taxed asset (equity) in the taxable account. Because of the relative low capacity of taxable account in this situation, the equity holdings spill over to the tax-deferred account. This phenomenon might reflect the fact that when the wealth in the pension account is relative high, the effect of tax benefit is larger than the role of employer matching policy and transaction costs difference, so household would like to hold high-taxed asset in the tax-deferred account. There are about 25% more of pension wealth are held in bonds. An alternative explanation of the lower equity investment in the pension account might be that since the pension account becomes the only retirement income, households tend to invest conservatively

[35]One thing need to note about the case of changing saving limit. In this analysis, I implicitly assume that the employer matches every dollar the employee saves in the pension account, up to the saving limit. This is a reasonable assumption given that every dollar tax contribution into the Social Security system by the employee is accompanied by the tax contribution from the employer too. The Social Security tax is collected by the method of 50/50 between employees and employers.

Figure 13: Portfolio Choices and Retirement Savings over the Life-Cycle: Change in Social Security

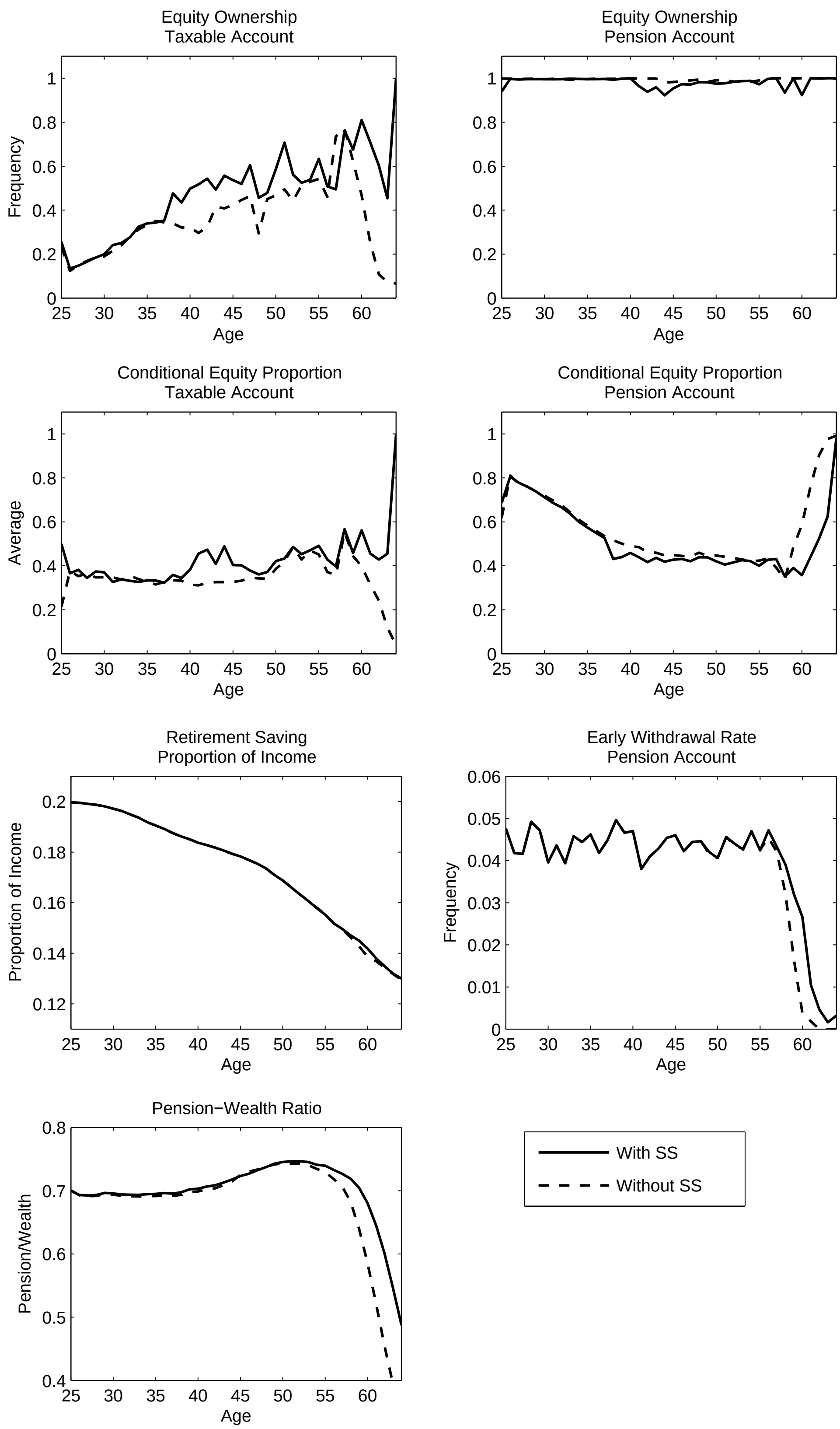


Figure 14: Portfolio Choices and Retirement Savings over the Life-Cycle: Changes in Saving Limit

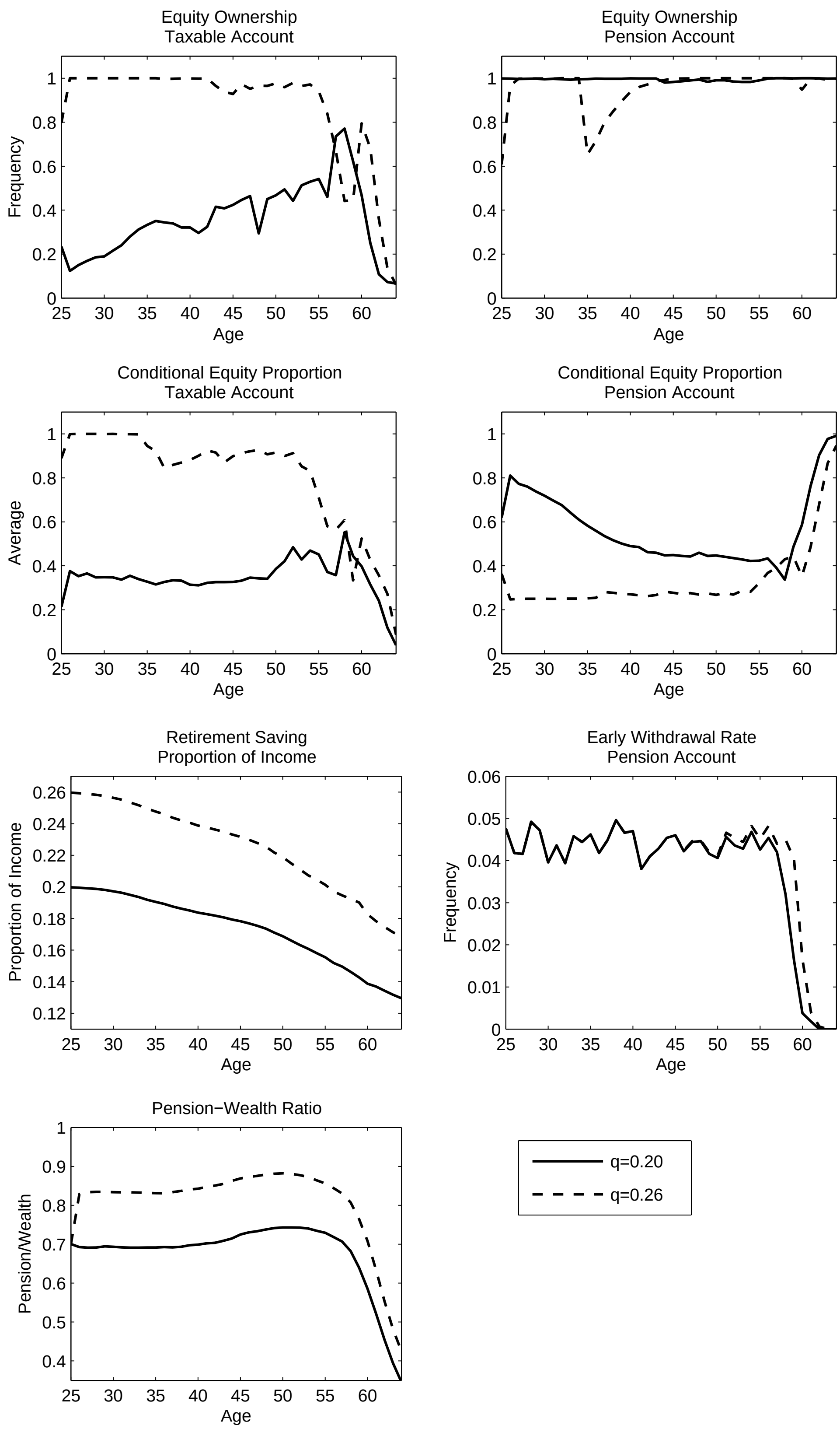
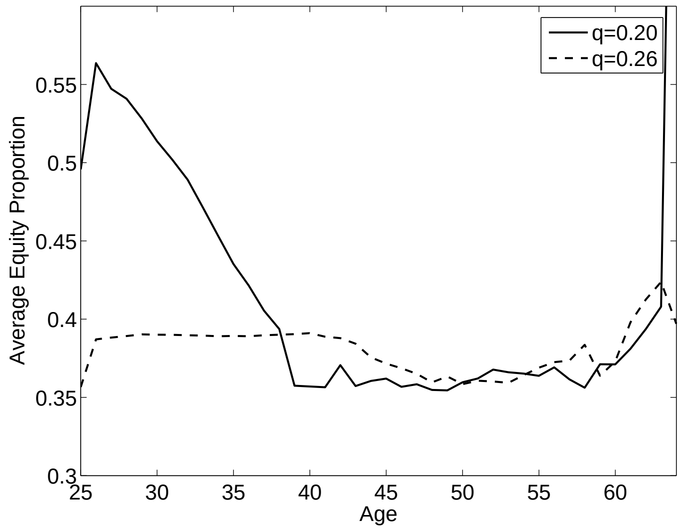


Figure 15: Overall Equity Proportion: Changes in Saving limit



and hold more safe assets. However, during the periods approaching retirement, the behavior of the households is similar to the case with lower saving limit. Considering the overall equity proportion, this change does not induce more shares of total wealth held in equities (see Figure 15). Since more wealth are held in the pension account, but less proportion of equities are invested in that account, the overall equity proportion decreases about 10% during the ages 25 to 35.

6 Conclusion

This study contributes to the analysis and understanding of household saving decisions and portfolio choices in the taxable and tax-deferred pension accounts, as well as the effects of the tax-deferred pension account designs. I develop a new approach by incorporating the pension account characteristics into a dynamic asset allocation and location problem. I model households' behavior conditional on employer matching strategies and other features in the tax-deferred account, and estimate structural parameters and household portfolio choices using the Method of Simulated Moments. The model fits the data well and provides a framework for explaining the role of employer matching policies in household optimal saving and portfolio decisions. This study can further entail the question of what would be the consequences of transforming the current Social Security system to an individual retirement account.

The results of my paper indicate that the higher equity concentration in the pension account is an optimal response to the pension account designs. In particular, the employer matches in the pension account cause a higher proportion of savings allocated into the pension account. Therefore, the relative less assets in the taxable account make the households invest more safe assets in that account for the precautionary purpose, and they are more likely to hold equities in the pension account to maintain an optimal portfolio mix. This asset allocation strategy ends up with a lower equity holding in the taxable account, but a relative high equity investment in the tax-deferred account. From the perspective of life cycle, younger households who save in the retirement account for pension-favored benefits are more likely to be affected by the change in matching rates, while older households who have higher propensity for retirement savings are less likely to be influenced. My results also show that the employer stock match makes the investment in the pension accounts riskier than a cash match, so the households with a stock match from the employer tend to voluntarily reduce their own equity holdings in the tax-deferred account. With the estimated model, I further find that households actively respond to the transformation of Social Security taxes and payments to a tax-deferred retirement saving account. Since the pension account becomes the only income source for retirement, households tend to save more for the retirement purpose and hold more safe assets (bonds) in that account.

There are two interesting avenues for future research. First, the method in this article could be extended to study richer household intertemporal choices with taxable and tax-deferred accounts, such as labor supply and retirement investment decisions, and the model could also be used to study the behavior of people with different occupation and education levels. With proper modification of the existing model, I think the effect of tax volatility and social insurance program can also be further analyzed. Second, employer matching policy is one complicated part of pension account designs. It comes with different formats, for instance, first dollar match, unconditional matches, contribution-based match, and matching caps. I expect that, with a complete data set about pension plan descriptions, this model could be extended to analyze the different marginal effects of detailed employer matching policies.

Appendix

A.1 Household Problem During Retirement

This part will describe the maximization problem during retirement. Each household retires in period $K = 65$, and live until the last period of life $T = 85$. I assume that the household liquidates the pension account at the beginning of retirement, and also receives an endowment H_K, which is the expected value of Social Security payment in the rest years of life. Then the total wealth of each household at period $t = K$ is

$$W_K = W_K^A + (1-\tau)W_K^P + H_K.$$

From period $t = K$ to $t = T$, Households make consumption decisions, and the wealth will accumulate at a fixed rate of return $\bar{r}$. In practise, I define $\bar{r} = \frac{1}{2}(1-\tau_b)r^b + \frac{1}{2}r^\tau$, which the after-tax return of a portfolio with half invested in stocks and half in bonds. The household's problem after retirement can be written as,

$$max_{C_{t=K}^T} E\left(\sum_{t=K}^{T} \beta^t [F(t)U(C_t) + (F(t-1) - F(t))B(W_t)]\right), \tag{A-1}$$

s.t.

$$W_{t+1} = (W_t - C_t)\cdot\bar{r}, \tag{A-2}$$

$$C_t \geq 0, W_t \geq 0. \tag{A-3}$$

Defining the value function for the household problem at time t as V_t, the Bellman equation for this maximization problem can be written as,

$$V_t(W_t) = max_{C_t}\left[e^{-\vartheta_t}U(C_t) + (1-e^{-\vartheta_t})B(W_t) + e^{-\vartheta_t}\beta E_t[V_{t+1}(W_{t+1})]\right] \tag{A-4}$$

subjective to equation (A-2) and (A-3).

The setup of the household problem during retirement makes the model homogeneous of degree $(1-\gamma)$ in the total wealth. After normalizing the problem by total wealth, the problem becomes,

$$v_t = max_{c_t}\left[e^{-\vartheta_t}U(c_t) + (1-e^{-\vartheta_t})\frac{ba}{1-\gamma} + e^{-\vartheta_t}\beta E_t[v_{t+1}\delta_{t+1}^{1-\gamma}]\right] \tag{A-5}$$

s.t.

$$\delta_{t+1} = (1-c_t)\cdot\bar{r}, \tag{A-6}$$

where $v_t = V_t(W_t)/W_t^{1-\gamma}$ is the normalized value function, $c_t = C_t/W_t$ is the consumption-wealth ratio, and $\delta_{t+1} = W_{t+1}/W_t$ one plus the growth rate in wealth from period t to period $t+1$.

The above problem has one state variable t, and one control variable c_t,consumption-wealth ratio, and can be solved using backward recursion. At the terminal date T, the household value function takes the know value,

$$v_T = \frac{bq}{1-\gamma}$$

The value function at date T is then used to solve for the optimal decision rules at date $T-1$. The procedure is repeated recursively for each time period until the solution for date $t = K$. The value function at the beginning of retirement, v_K, will be used to find solutions for the problem during working time.

A.2 Normalization of the Household Problem

I simplify the optimization problem by normalizing by the household total wealth W_t. Let $p_t = W_t^P(1-\tau)/W_t$ be the fraction of the investor's total wealth that is held in the retirement account, $s_t = s_t^P/W_t$ be the ratio of contribution to pension account to the total wealth, $c_t = C_t/W_t$ be consumption-wealth ratio, and let $y_t = Y_t/W_t$ be the income-wealth ratio as previously defined. Then equation (1) can be written as following dynamic equation:

$$W_{t+1} = [(1-p_t-c_t+y_t\cdot(1-\tau)-s_t(1-\tau-h\times I)\cdot(1-tr\times I(\alpha_t>0))R_t + (p_t+(1-\tau)s_t)R_t^P(m)]W_t \tag{A-7}$$

The linearity of the dynamic wealth equation and the assumption of CRRA preferences ensure that the model has the property that the consumption, saving and portfolio decision rules, $c_t, s_t, \alpha_t, \alpha_t^P$, are independent of total wealth, W_t. Furthermore, with the above normalization, the relevant state variables for the investor's problem become (p_t, y_t). Defining $v(p_t,y_t) = V(W_t^A, W_t^P, Y_t)/[W_t^{1-\gamma}]$ to be the normalized value function and $\delta_{t+1} = W_{t+1}/W_t$ to be one plus the growth rate in wealth from period t to period $t+1$, the household's problem in working life can restated as follows:

$$\begin{aligned} v(p_t,y_t) = {} & max_{(c_t,s_t,\alpha_t,\alpha_t^P)}\left(e^{-\vartheta_t}U(c_t) + (1-e^{-\vartheta_t})\frac{bq}{1-\gamma} + e^{-\vartheta_t}\beta E[v(p_{t+1},y_{t+1})\delta_{t+1}^{1-\gamma}]\right) \\ & t = 1,\ldots,K-1, \end{aligned} \tag{A-8}$$

s.t.

$$\begin{aligned} \delta_{t+1} = {} & (1-p_t-c_t+y_t\cdot(1-\tau)-s_t(1-\tau-h\times I))\cdot(1-tr\times I(\alpha_t>0))R_t \\ & + (p_t+(1-\tau)s_t)R_t^P(m) \end{aligned}$$

$$p_{t+1} = \frac{(p_t+(1-\tau)s_t)R_t^P(m)}{\delta_{t+1}}$$

$$y_{t+1} = e^{f(t+1)}\cdot u_{t+1} = e^{l_0+l_1(t+1)+l_2(t+1)^2}\cdot u_{t+1}$$

$$s_t \le q\cdot y_t$$

$$c_t \ge 0, p_t \ge 0, 0 \le \alpha_t^P, \alpha_t \le 1.$$

The value function of the first retirement period can be defined as,

$$v(p_K,y_K) \equiv v_k = V(W_K)/[W_K^{1-\gamma}]$$

and

$$\begin{aligned} \delta_K = {} & (1-p_{K-1}-c_{K-1}-(1-\tau)\cdot y_{K-1}-s_{K-1}(1-\tau-pnl\times I))\cdot(1-tr\times I(\alpha_{K-1}>0))R_{K-1} \\ & + (p_{K-1}+(1-\tau)s_{K-1})R_{K-1}^P(m) + h \end{aligned}$$

where h is the proportion of Social Security payment over total wealth defined in Section 2.

A.3 Numerical Algorithm

The problem during retirement can be derived analytically, while that during working years need to be solved numerically, so the rest of this section is a discussion of solutions during working life. Standing at each working date, one can rewrite the Bellman equation (A-8) as,

$$\begin{aligned} v(p_t, y_t) =& max_{(c_t, s_t, \alpha_t, \alpha_t^P)}\{e^{-\vartheta_t}U(c_t) + (1-e^{-\vartheta_t})\frac{bq}{1-\gamma} \\ &+ e^{-\vartheta_t}\beta(1-pu_t)\int\int v(p_{t+1}, y_{t+1})\delta_{t+1}^{1-\gamma}dF(r)dF(u) \\ &+ e^{-\vartheta_t}\beta pu_t \int v(p_{t+1}, 0)\delta_{t+1}^{1-\gamma}dF(r)\}. \end{aligned} \tag{A-9}$$

The above equation indicates that each household face two types of uncertainties, one of which is the risky asset, and the other is labor income shock. A two-dimensional Gauss-Hermite quadrature is performed to evaluate the expectation in (A-9).

To solve for the optimal saving and investment decisions, I discretize the endogenous state variables, (p_t, y_t), into a grid of (20×21) over the following ranges: pension-wealth ratio $p_t \in [0,1]$, and income-wealth ratio $y_t \in [0,10]$. In order to capture the curvature of the policy rules at low values of income-wealth ratio, the grid will be finer for $y_t \in [0,2]$. At each point in the state space, in order to find the optimal choices , I need to further discretize the choice space, $(s_t, \alpha_t, \alpha_t^P)$. The saving-wealth ratio, s_t is discretized into 20 grids over $[-1,2]$, where the upper bound is 20 percent of the maximum labor income (saving limit in the pension account), and the lower bound is the maximum pension-wealth ratio. The agent will withdraw some money out of the tax-deferred pension account when $s_t < 0$, have no activity when $s_t = 0$, and make some positive contributions otherwise. Both α_t and α_t^P are discretized into 5 grids over $[0,1]$, indicating the share of equities held in taxable account and tax-deferred account respectively. For each value in the state space, $(p_t, y_t)^j$, and each value in the consistent choice space, $(s_t, \alpha_t, \alpha_t^P)^{ij}$, I find consumption-wealth ratio, c_t^{ij}, that satisfies equation (A-9). Then the values in the choice space are compared to yield the optimal combination at the state point, $(c_t, s_t, \alpha_t, \alpha_t^P)^j = max_i[(c_t, s_t, \alpha_t, \alpha_t^P)^{ij}|(p_t, y_t)^j]$. A two-dimensional linear interpolation is used to calculate the value function for points in the state space that lie between the grid points, and extrapolation is applied when state values move out of the grid. Since extrapolation is much less precise than interpolation, I adopted a simple checking approach to assure that the state values in the simulations remains within the grid with probability 0.95.

In choosing the size and coarseness of the grid, I face the usual trade-off between precision and computing time. Adding points on the grid gives a finer approximation of the policy rules, but also increases the need to calculate equation (A-9). The current size of this problem takes more than half an hour on a 3.4GHz Intel Pentium IV Xeon CPUs.[36] After paralleling the program onto 80 processors using Message Passing Interface (MPI), the solution can be obtained in about 30 seconds. As the number of processors increases, the marginal time that can be saved may be compensated by the time spent in the message passing (see Swann (2000) for details). The number of processors in choice is around the point that marginal time gains equal to marginal time losses.

A.4 The Simulated Method of Moments

Suppose there are J moments in the estimation. Since each element of those moments is the expectation of one distribution, I define ζ^j is the variable for each distribution, $j = 1, \ldots, J$, and I_j the corresponding number of observations of that variable. The simulated expectation for variable j and parameter set θ is $\bar{\zeta}^j(\theta)$. Let $g(\theta) = [g_1(\theta), \ldots, g_J(\theta)]'$ be a vector in which $g_j(\theta) = \frac{1}{I_j}\sum_{i=1}^{I_j}\zeta_i^j - \bar{\zeta}^j(\theta)$, that is, the distance between the actual and predicted moments. The method of simulated moments minimizes the weighted average distance between the actual and predicted distributions $g(\theta)'Wg(\theta)$, in which W is a weighting matrix.

In the first stage, the identity weighting matrix is used to minimize,

$$g(\theta)'Ig(\theta),$$

with respect to θ. Using the computed $\hat{\zeta}$, an estimate variance-covariance matrix $\hat{\Omega}$ can be constructed,

$$\hat{\Omega} = E[(\zeta_i - \bar{\zeta}(\hat{\theta}))(\zeta_i - \bar{\zeta}(\hat{\theta}))']$$

with $\hat{\Omega}_j = \frac{1}{I_j}\sum_{i=1}^{I_j}(\zeta_i^j - \hat{\zeta}^j(\hat{\theta}))^2$ on the diagonal, and zeros off-diagonal.

$W = \hat{\Omega}^{-1}$ is the optimal weighting matrix, so in the second stage

$$S(\theta) = g(\theta)'\hat{\Omega}^{-1}g(\theta),$$

[36] The program is finished at Seawulf Cluster. Seawulf Cluster is a computational resource at Stony Brook University, equipped with 3.4GHz Intel Pentium IV Xeon CPUs, 470 processors, and a couple of MPI systems. (http://www.stonybrook.edu/seawulfcluster/index.shtml)

is minimized. The distribution of the resulting estimate $\tilde{\theta}$ is

$$\sqrt{I}(\tilde{\theta}-\theta_0)\underset{\longrightarrow}{d}N(0,Q),$$

where I is the number of observations, and letting ς be the ratio of the number of observations to the number of simulated points,

$$Q=(1+\varsigma)(G_\theta'\hat{\Omega}^{-1}G_\theta)^{-1}$$

where $G_\theta = E[\partial g(\tilde{\theta})/\partial\theta']$.

To test the overidentifying restrictions, one can use

$$\chi^2_{J-14} = Ig(\tilde{\theta})'\hat{\Omega}^{-1}g(\tilde{\theta}),$$

which is distributed asymptotically as Chi-squared with $J-14$ degrees of freedom.

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