Selection, Growth and Learning

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Big Picture

- Firm behavior crucially depends on age (e.g. Evans '87)
  - Young firms grow faster, more likely to exit (even conditional on size)

- Large part of young firm growth due to demand (vs productivity)
  - Haltiwanger et al '09: evidence from homogenous good industries
  - EEKT, Albornoz et al '09: large growth for new exporters into individual destinations

What about idiosyncratic productivity shocks?

- Stochastic productivity essential modeling component
  - Arkolakis '08 extending Luttmer '07, Hopenhayn '92
    - Explain US cohort turnover & *growth*
    - Explains dependence of growth and turnover on firm *size*
- *Cannot* explain dependence of firm growth & turnover on *age*
  - Reason: one state Markov structure

This Paper: Quantitative Framework of Firm Demand-Learning

- Revisit findings of Evans '87 using Colombian plant data

- Develop a benchmark framework of firm learning and productivity
  - Learning generates age dependent turnover & growth (Jovanovic '82)
  - Approach related to Ruhl & Willis '09. Also to EEKKT '09
    - Simpler framework, going further in characterizing model implications

Agenda: is learning the missing link?

- Learning has a number of advantages vs e.g. financial constraints
  - Tractability
  - Results largely independent of productivity shock structure (Cooley & Quadrini '01)
  - Demand explanation: useful to model growth in individual markets

- Develop a benchmark framework of firm learning & productivity
  - SR: Estimate importance of firm learning vs productivity
  - MR: Perform counterfactual policy experiments
  - LR: Understand how learning affects trade

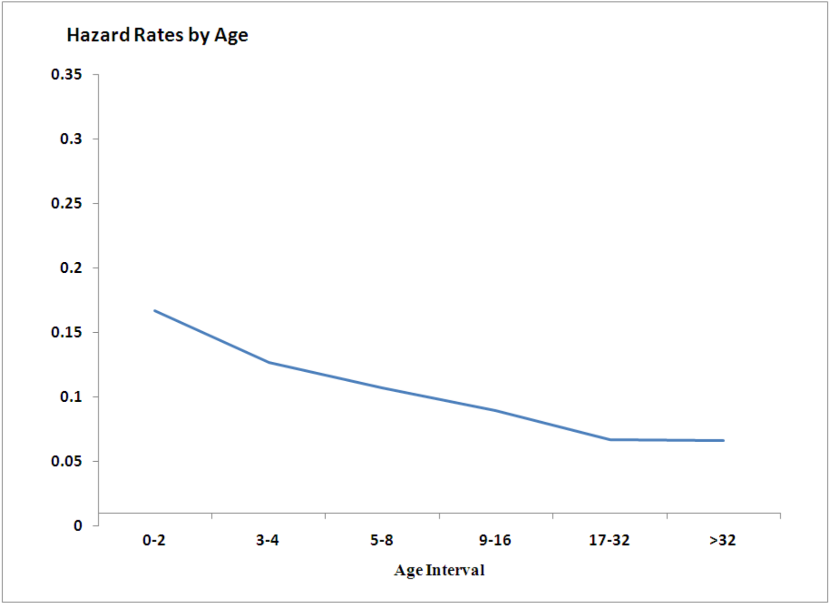
The data

Data

- Colombian data (DANE survey)
  - Dataset covers all plants with 10+ employees

- Look real production 83-91, treat each plant-year as an observation
  - Yearly turnover and growth

Evidence from Colombian Data



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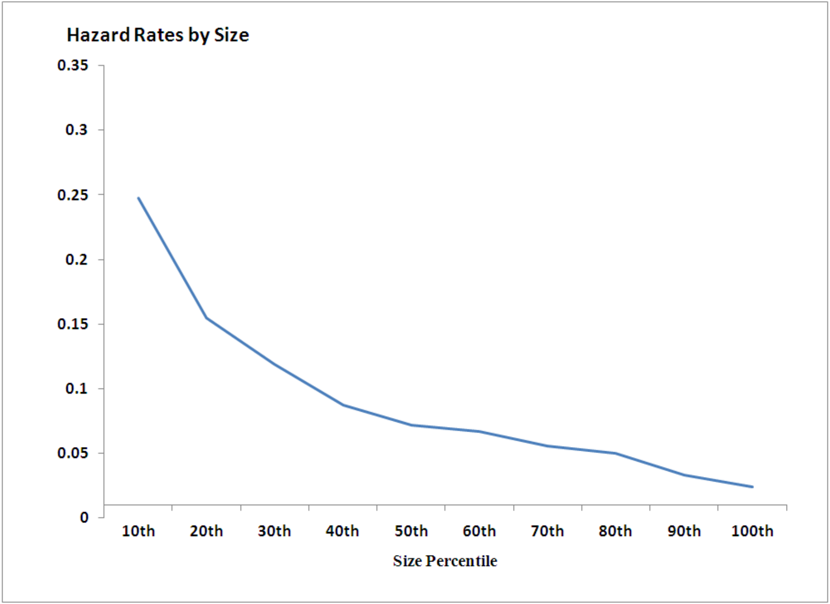

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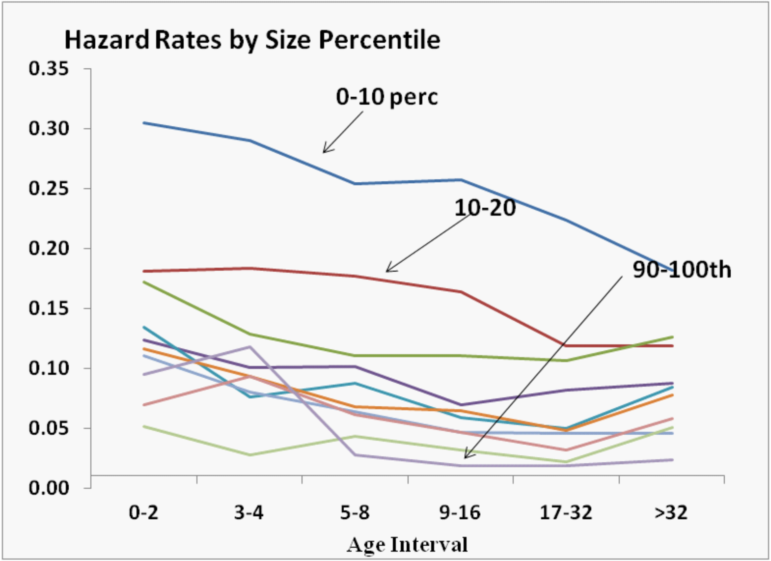

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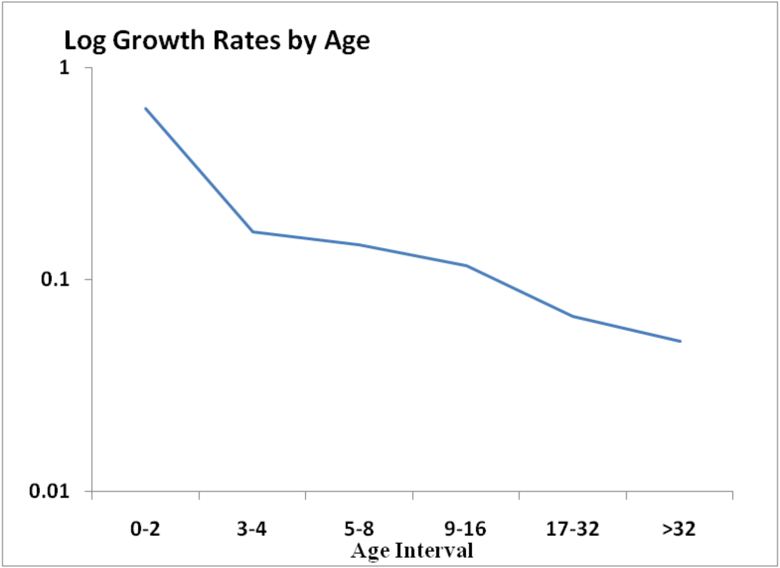

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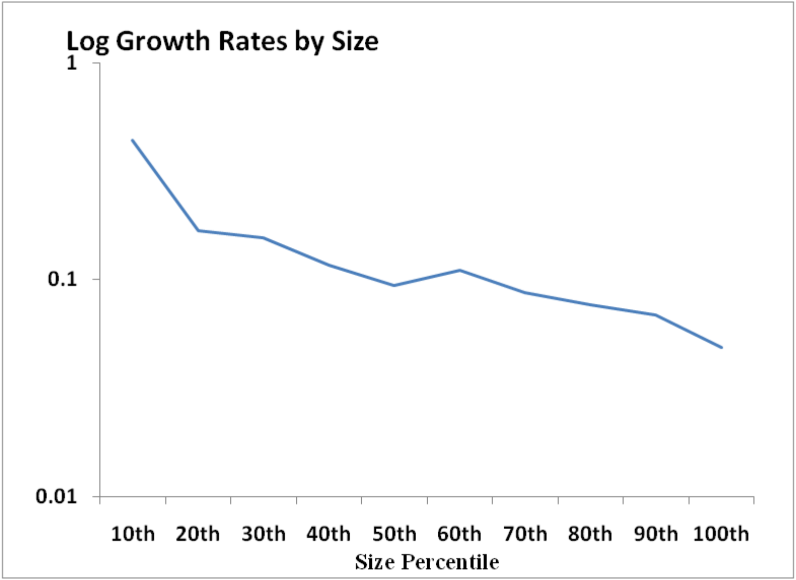

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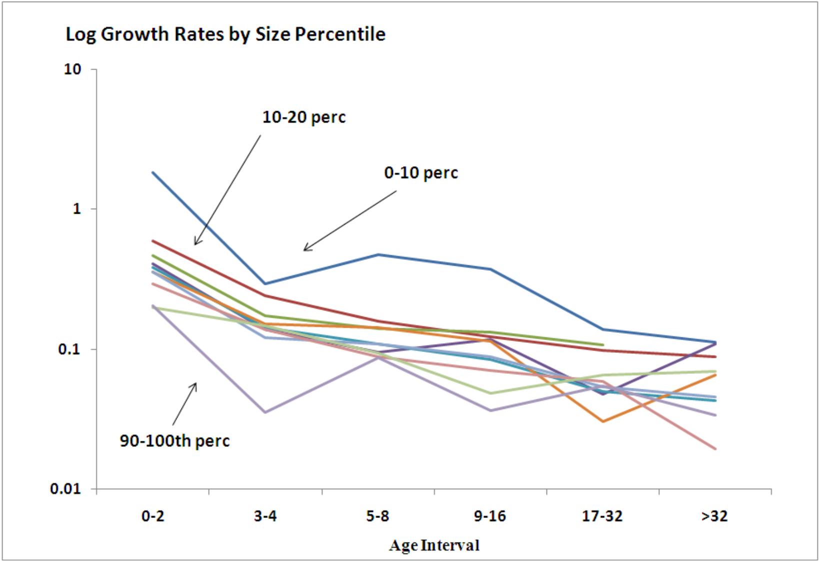


The model

Consumer Preferences

- Unit mass of consumers with preferences over a composite good, C_t:

$$E_t \left(\sum_{t=0}^{+\infty} \beta C_t^{\frac{\gamma-1}{\gamma}} dt \right)^{\frac{\gamma}{\gamma-1}}$$

where

$$(C_t)^{\rho} = \int_{\omega \in \Omega} \left[e^{a_t(\omega)} \right]^{1-\rho} q_t(\omega)^{\rho} \, d\omega$$

- $e^{a_t(\omega)}$: good ω idiosyncratic demand component
- $q_t(\omega)$: quantity consumed from good ω

Consumer Demand

- Modeling of representative consumer is parsimonious

- Implies demand for good ω

$$q_t(\omega) = e^{a_t(\omega)} \frac{p_t(\omega)^{-\sigma}}{P_t^{1-\sigma}}$$

  where w_t is worker wage , P_t is the CES price index, $\sigma = \frac{1}{1-\rho} > 1$ is the elasticity of substitution.

- Each firm is a monopolist of one good. Takes demand as given

Information Frictions

- The demand realization for the good of a firm ω is given by:

$$a_t(\omega) = \theta(\omega) + \varepsilon_t(\omega) \quad , \ \varepsilon_t(\omega) \sim N(0, \sigma^2) \ \text{i.i.d}$$

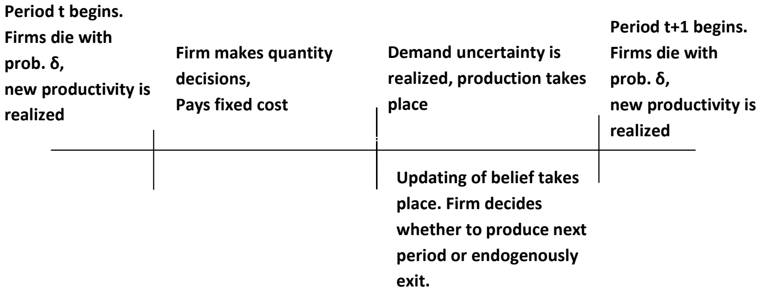
- Permanent demand realization $\theta(\omega)$ unobserved by the firm
  - Drawn from normal with known mean & variance.
  - Firm observes $a_t(\omega)$, updates beliefs for $\theta(\omega)$ in Bayesian fashion.

Firm Production and Equilibrium Conditions

- Firms use a CRS production function, productivity z
- We assume free entry condition to close the model.
  - Firms enter with a productivity drawn from $g_e(z)$
- Labor market clears

Timing of Firm Actions

- Timing



- Firm updates beliefs (learns) even if there is very little production
  - Firm optimization wrt to quantities is in fact static
  - But beliefs do affect quantity and entry-exit decisions

Firm Optimization

- Firm chooses quantity, q_t to maximize expected profits:

$$\pi_t\left(z, \overline{a_t}, n\right) =$$
$$\max_{q_t} \int \left[p_t q_t - q_t \frac{w_t}{z}\right] g_a\left(da_t | \overline{a_n}, n\right) - w_t f$$

subject to:

$$q_t = e^{a_t} \frac{p_t^{-\sigma}}{P_t^{1-\sigma}}$$

where $g_a\left(\cdot | \overline{a_n}, n\right)$ is the pdf of the firm beliefs at t regarding the realization a_t, conditional on having n signals with mean $\overline{a_n}$.

Characterization of learning

- $(\overline{a_n}, n)$ is a sufficient statistic for firm beliefs at t regarding a_t.

- Define firm expected demand,
  $b_t = E_t\left[e^{a_t}\right] = \int \left(e^{a_t}\right)^{\frac{1}{\sigma}} g_a\left(da_t | \overline{a_n}, n\right)$
  - Turns out that also (b_t, n) is a sufficient statistic for firm learning.
  - Firm state is (z, b_t, n).

Characterization of a Stationary Equilibrium

- Optimal choice of quantity for a firm(z, b)

$$q_t\left(z,b\right)=\frac{\left(\frac{\sigma}{\sigma-1}\frac{w}{z}\right)^{-\sigma}}{\left(P^{\sigma-1}Lw\right)^{-1}}\left(b\right)^{\sigma}$$

- Market clearing price:

$$p\left(z,b\right)=\frac{\sigma}{\sigma-1}\frac{w}{z}\frac{\left(e^{a}\right)^{\frac{1}{\sigma}}}{b}$$

Firm Growth

- **Proposition**: The growth rate of the sales is higher for Young firms ($n < +\infty$) versus Old firms ($n \to \infty$) (assuming there is no exit).

- Intuition of the result: Jensen's inequality
  - Young firms: Chance to be superstar, production expected to increase
  - Old firms: no uncertainty of true $\theta(\omega)$, production roughly constant
  - Result does not depend on normality of $\theta(\omega)$

Firm Growth

- **Proposition**: The growth rate of the sales is higher for Young firms ($n < +\infty$) versus Old firms ($n \to \infty$) (assuming there is no exit).

- Furthemore, proposition is true for any prior distribution of $\theta\left(\omega\right)$

Firm Entry-Exit

- Each period the firm can either stay in the market or exit.
  - Its value function is given by:

$$V(z, b, n) = \pi(z, b) + \beta(1-\delta)\int \max\left[V(z, b', n), 0\right] g_b(db'|b, n)$$

where g_b distr. of next period b.

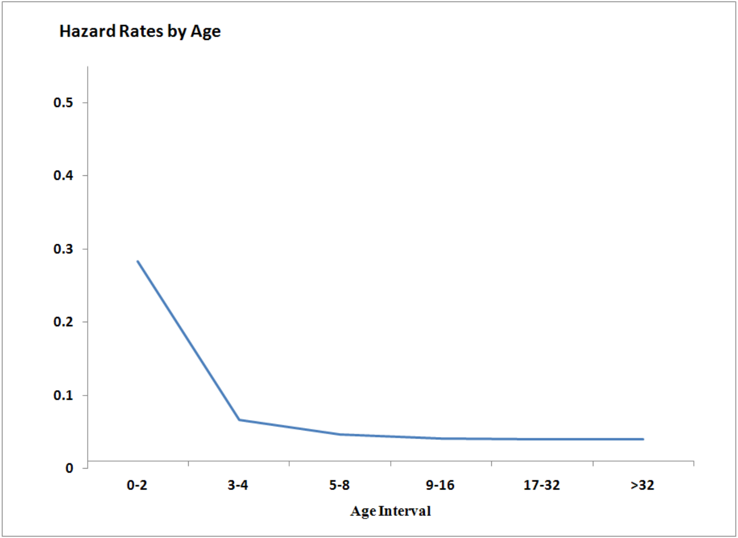
- **Proposition:**
  - Value function is unique.
  - Value function is increasing in z and b.
    - Thus, given n, z, $\exists\ b^*(z, n)$ s.th. $\forall b \geq b^*(z, n)$ firms operate

Numerical Simulations

- A stationary equilibrium exists
  - Belief process is positive recurrent

- Some quantitative preliminary results with homogeneous z
  - Model can deliver both age and size dependent growth
    - Consumer Paremeters: $\sigma = 6$, $\beta = 0.99$
    - demand shock true mean: $\sigma_\theta = 1$. Noise st.dev: $\sigma_\varepsilon = 0.5$
    - Exogenous death: $\delta = .03$

Model Simulation



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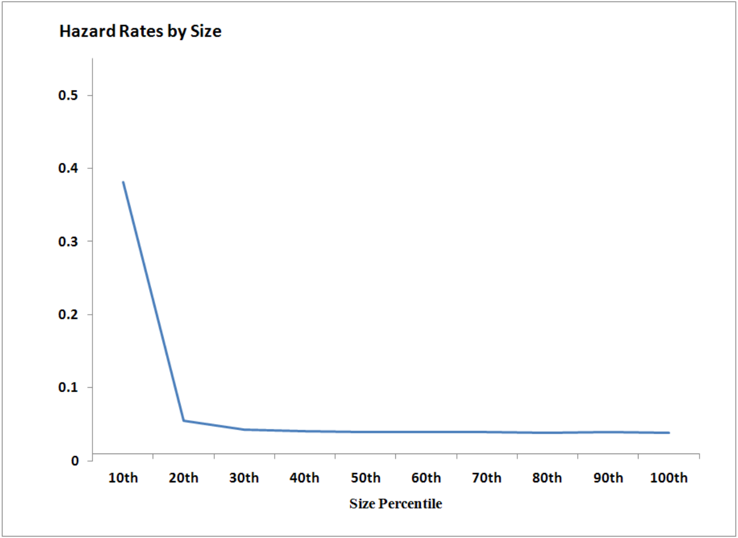

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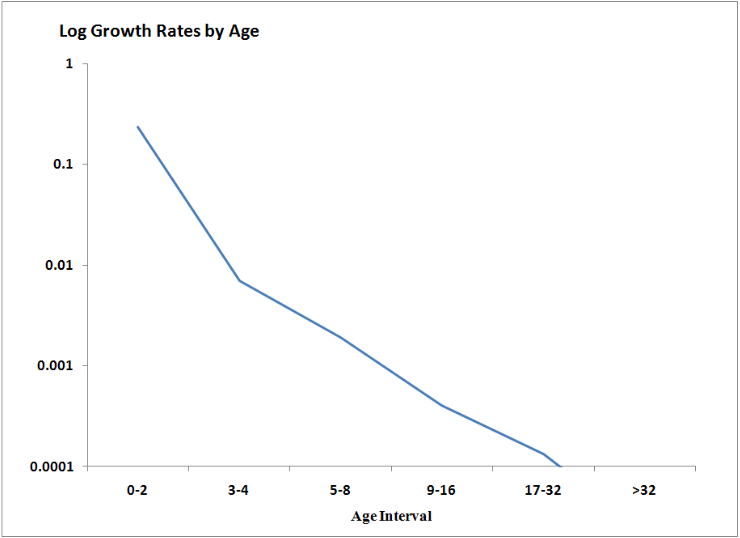

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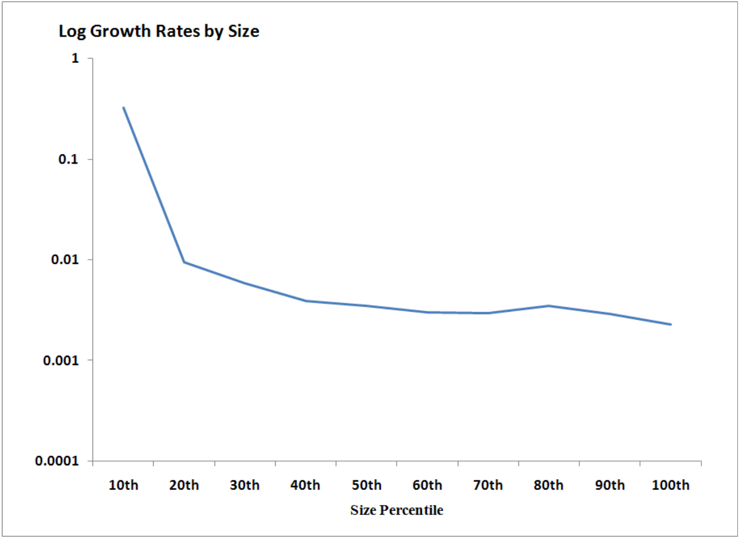

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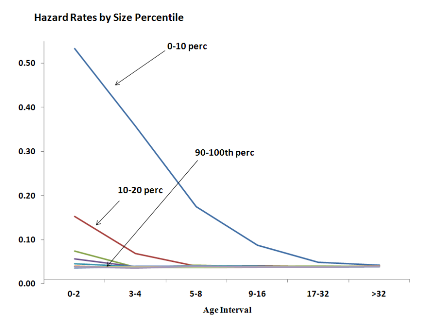

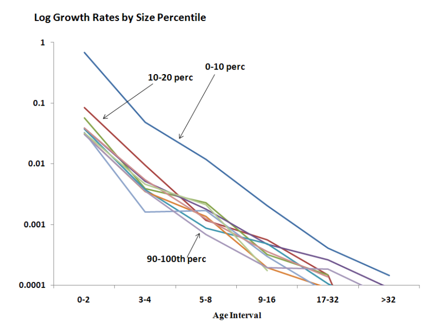

Summary

- Model of learning and productivity heterogeneity
  - Tractable framework, easy to extend to productivity dynamics

- Tractable framework.
  - Continuous time version would allow more tractability
  - Some positive preliminary results.

- Working on finding better data and on estimation
  - Trade extension (similar to Ruhl & Willis '09)