Serial Default and Debt Renegotiation*

[Job Market Paper]

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Abstract

Emerging countries that have defaulted on their debt repayment obligations in the past are more likely to default again in the future than are non-defaulters even with the same debt-to-GDP ratio. This paper explains this stylized fact within a dynamic stochastic general equilibrium framework by explicitly modeling renegotiations between a defaulting country and its creditors. The quantitative analysis of the model reveals that the equilibrium probability of default for a given debt-to-GDP level is weakly increasing with the number of past defaults, consistent with empirical observations. The equilibrium of the model also accords with an additional observed fact: a country for which default terms require less than a 100 percent recovery rate tends to pay a higher rate of return (relative to a risk-free rate) on subsequently issued debt than do defaulting countries that agree to a full recovery rate.

JEL Classification Codes: E43; F32; F34; G12

Key words: Serial default; Debt renegotiation; Past credit history; Recovery rates; Interest spreads

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1 Introduction

Emerging countries that have defaulted on their debt repayment obligations in the past are more likely to default again in the future than are non-defaulters with the same debt-to-GDP ratio. This paper explains this stylized fact within a dynamic stochastic general equilibrium framework that explicitly models renegotiations between a defaulting country and its creditors. Specifically, the model extends the existing literature by allowing the defaulter and creditors to bargain not just over recovery rates, but also over the rate of return offered on newly-issued debt. Quantitative analysis of the model reveals that the equilibrium probability of default for a given debt-to-GDP level is weakly increasing with the number of past defaults, consistent with empirical observations. The equilibrium of the model also accords with an additional observed trend: a country for which default terms require less than a 100 percent recovery rate tend to pay a higher rate of return (relative to a risk-free rate) on debt that is issued subsequently than do defaulting countries that agree to a full recovery rate. These findings are robust to extensions that allow the renegotiation outcome to be modeled more flexibly.

This paper deals with endogenous debt renegotiation after default in a standard dynamic model of defaultable debt. The renegotiation process involves Nash bargaining between the defaulting debtor and creditors over both the recovery rate and increases in rates of return on new debt. Evidence suggests that the spread between the rate of return on new debt and the risk-free rate increases after default more for defaulters that pay less than a full recovery rate than for defaulters that agree to repay all of the defaulted debt (i.e. a 100 percent recovery rate). Thus, it appears that, at least implicitly, a country that defaults negotiates with its creditors both over recovery rates and over future rates of return. This reflects a trade-off for defaulting country: the defaulted debt can be repaid in the present at a high short-run cost in return for only a small or even negligible deterioration in long-term credit condition; or the short-run benefit of repaying the debt only partially will be offset by having to pay lenders a higher rate of return on future issuances. The trade-off for creditors is symmetric: if they are not appeased by a full recovery of funds in the short term, they can attempt to recoup their losses by demanding higher rates of return for holding the country's bonds in the future.

The present paper seeks to incorporate these trade-offs facing the debtor and creditors during renegotiations following defaults. In the model, the endogenously-determined terms of renegotiations following default present the observed pattern, i.e. lower recovery rates are associated with larger increases in yield spreads. An emerging country that defaults once therefore pay a penalty either through a large recovery rate in the short term or through higher borrowing costs in the long term. If it chooses to repay less than full recovery rates, it will face high borrowing costs, which lead to increase the risks that the country will default again in the future. This mechanism drives the equilibrium serial default behavior in the model, and it is a plausible explanation of the pattern of repeat defaults observed in the data. Hence, the model is able to jointly explain both stylized facts of debt renegotiations and repeat defaults.

We embed the debt renegotiation in a dynamic sovereign debt model with endogenous defaults where an emerging country is subject to exogenous income shocks. This part of the model builds on recent quantitative analysis of sovereign debt such as Aguiar and Gopinath (2006), Arellano (2008) and Tomz and Wright (2007) which are based on classical setup of Eaton and Gersovitz (1981). At the renegotiation, creditors and defaulting country bargain over increases in rate of return on new debt together with recovery rates. Outcomes of the renegotiation represent trade-offs of both defaulting country and creditors, as indicated above. Total spread between the rate of return on new debt and the risk-free rate, incorporates not only the probability of future default but also impacts on increases in rate of return on new debt agreed at the past renegotiations.

Our paper is most closely related with Yue (2006), in which a dynamic model of defaultable debt is argumented with an endogenous treatment of debt renegotiation after default. Our model differs from her model in that we incorporate the effects of increases in rate of return on new debt. At the renegotiation, both parties bargain not only over recovery rates, but also over increases in rate of return on new debt. Therefore, its credit condition, i.e. borrowing cost of the country after re-entry to the market, depends on how much the country pays at the debt renegotiation. Increase in borrowing costs accompanied by repaying the debt only partially will lead to increase future default probability. In special case where the country always repays in full the level of defaulted debt, increases in rate of return on new debt will be close to zero. As impacts of additional default premia are totally negligible, results will be quite similar to ones in Yue (2006).

The rest of the paper is organized as follows. Section 2 reviews two strands of literature. Section 3 overviews stylized facts of debt negotiations and serial defaults. We provide our stochastic dynamic general equilibrium model in Section 4. We define recursive equilibrium of the model in Section 5. Quantitative analysis of the model is shown in Section 6. Model implications are indicated in

Section 7. A short conclusion summarizes the discussion. The computation algorithm is provided in Appendix A.

2 Literature Review

This paper is related to the literature of serial defaults. Reinhart, Rogoff and Savastano (2003) and Reinhart and Rogoff (2005) both advocate the role of past credit history in debt intolerance. On contrary, Eichengreen, Hausmann, and Panizza (2003) show that countries with "original sin", inability to issue bonds in their domestic currencies, must pay an additional risk premium when they borrow, increasing their solvency risks since the financial market knows this inability is a source of financial fragility. However, none provides economic models describing how weak credit history or "original sin" features are associated with serial defaults. With stochastic dynamic model, Kovrijnykh and Szentes (2007) explain the equilibrium default cycles, but they do not derive any relation between default occurrences and outcomes of negotiations. This paper improves these papers by explaining how results of current debt renegotiation, such as additional components in interest spreads, lead to higher probability of next default in future.

The other strand of literature models the outcome of sovereign defaults as a game between a sovereign debtor and its creditors[1]. Yue (2006) treats debt renegotiation process using a one-round Nash bargaining game. Moreover, both Benjamin and Wright (2008) and Bi (2008) presume a multi-round bargaining to analyze delay in renegotiation[2]. Benjamin and Wright (2008) assume that debtor and representative creditor randomly alternate in their ability to propose a bargaining outcome with changes in the probability of making future proposals serving to capture changes in bargaining power, while Bi (2008) supposes that lenders have an option to "pass" proposing to the debtor. Furthermore, Pitchford and Wright (2007) regard multi-creditor renegotiation process as a series of bilateral bargaining games to explain delays in renegotiation. Similarly, Kovrijnykh and Szentes (2007) also study multi-creditor renegotiation and makes the time of exclusion from the financial market endogenously and potentially long. Our paper differs from this literature in that we concentrate on the observed

[1] Our borrowing environment, besides the debt renegotiation, is a version of the Eaton and Gersovitz (1981) model of defaultable debt, which has been used recently by a number of authors including Arellano (2008), Aguiar and Gopinath (2006), and Tomz and Wright (2007).

[2] Trebesch (2008) investigates empirically determinants of delay in sovereign restructurings and suggests that political risk and government behavior might be more important reasons for restructuring delays than creditor behaviour.

pattern that lower recovery rates at the renegotiation are highly associated with larger increases in yield spreads[3].

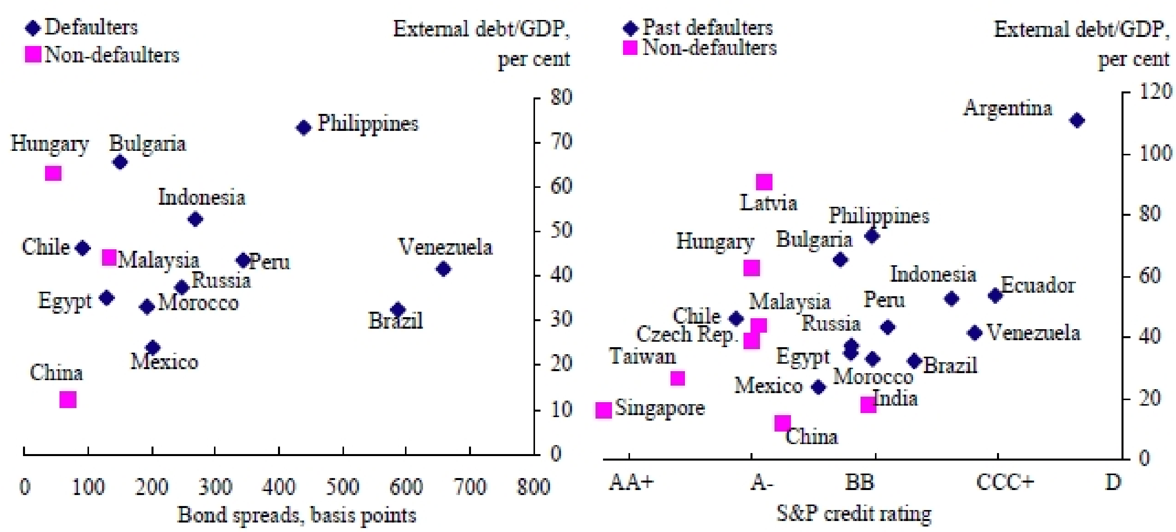
3 Stylized facts

Evidence of serial defaults reflects that past defaulters are more likely to default in the future than are non-defaulters given the debt-to-GDP ratio. Moreover, from recent debt renegotiation episodes, we observe that lower recovery rates at the renegotiation are highly associated with larger increases in yield spreads between the rate of return on new debt and the risk-free rate.

3.1 Evidence on serial defaults

In this subsection, we cover stylized facts of serial defaults, especially some features differing by countries' history of defaults.

Figure 1: External debt/GDP, bond spreads, and credit ratings, average 2003-2005



Source: De Paoli, Hoggarth and Saporta (2006)

Figure 1 reports external debt-to-GDP ratio, bond spreads and credit ratings. Bond spreads of past defaulters, except Chile and Egypt, are higher than those of non-defaulters given external debt-to-GDP ratio[4]. Past defaulters tend to suffer higher spreads on the newly issued bonds in the future

[3] We assume that debt renegotiation takes place only once for each default.

[4] One possible explanation for low spreads for Chile and Egypt with history of the default, is that neither of them has defaulted in the last two decades according to the survey of sovereign defaults in Celasun, Debrun and Ostry (2006).

after default, even if they have the same level of foreign debt relative to GDP as before. Similarly, past defaulters have lower credit ratings than non-defaulters, reflecting higher default probability.

Furthermore, Reinhart, Rogoff and Savastano (2003) show that countries with a weak credit history may become more vulnerable even at much lower levels of external debt, relative to countries with a sound credit history. Table 1 illustrates predicted Institutional Investor ratings and debt intolerance regions for Argentina and Malaysia[5].

[Insert Table 1 here]

It is apparent that precarious debt intolerance situation of Argentina is more severe than one of Malaysia[6]. Since Argentina is representative of many countries with a weak credit history and Malaysia is representative of countries with a sound credit history, this result reflects that the debt thresholds of countries with a weak credit history are lower than that of countries with a sound credit history. In other words, the default probability of countries with a weak credit history is higher than one of countries with a sound credit history, given the same level of debt-to-GNP.

3.2 Recent sovereign debt renegotiations

We start with an overview of recent debt renegotiation episodes. Table 2 summarizes 15 cases of expost-default and preemptive restructurings in the ten years from 1998 to 2007[7]. We present default year, defaulted debts, recovery rates, and increases in interest spreads for each episode. One feature which stands out is that recovery rates vary depending on the cases.

[Insert Table 2 here]

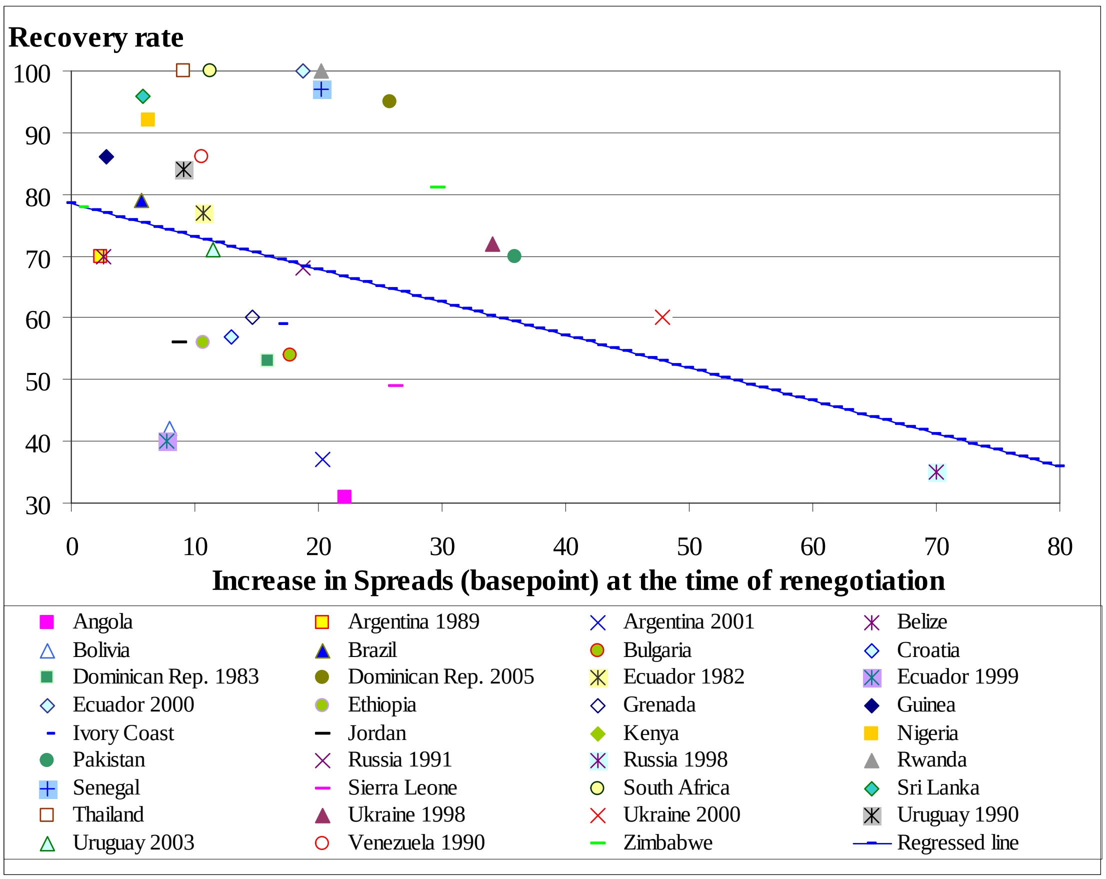
[5] In order to address this point, Reinhart, Rogoff and Savastano (2003) use the estimated coefficients from the regression which analyzes the role of history and "club" in Institutional Investor Ratings (IIR), together with actual values of external debt/GNP, to predict values of the IIR for varying ratios of external debt/GNP for two countries, Argentina and Malaysia, which were member of "club B" based on their classifications.

[6] Argentina only remains in the relatively safe "region **1**" as long as its external debt is below 15 percent of GNP, whereas Malaysia stays in "region **1**" up to a debt-to-GNP ratio of 30 percent, and it is still in the relatively safe "region **2**" with a debt of 35 percent of GNP.

[7] We exclude the cases of swap agreement and delay in payment such as Venezuela in 1995, 1998 and 2005, Peru in 2000 and Paraguay in 2003.

Furthermore, Figure 2 displays recovery rates and increase in spreads for 35 sovereign debt renegotiation episodes for 1986-2007[8][9]. We focus only on expost-default and preemptive renegotiation episodes in the sample periods, and we exclude examples of delays in payment such as Paraguay in 2003, and Venezuela in 1995, 1998, 2005, and swap agreement for Peru in 2000. We define "Increase in spreads" as the difference in spreads between the time of renegotiation and one year before the renegotiation.

Figure 2: Recovery rate and increase in spreads for recent debt renegotiations



[8] For 6 cases such as Argentina 2001, Ecuador 1999, Pakistan, Russia 1998, Ukraine 1998, Uruguay 2003, we use recovery rates in Sturzeneger and Zettlemeyer (2007). Recovery rates for Grenada, Dominican Rep.2005 and Belize are from Bedford, Penalver and Salomon (2005). The remaining cases are based on Benjamin and Wright (2008).

[9] Strurzenegger and Zettelmeyer (2005) define recovery rates as the market value of the new instruments, plus any cash payment received, relative to the net present value (NPV) of the remaining contractual payments on the old instruments (inclusive of any principal or interest arrears). They attempt to compare the value of the new instruments to the value of the old debt in a situation in which the sovereign would not have defaulted. Contrary to that, Bedford, Penalver and Salomon (2005) and Benjamin and Wright (2008) define recovery rates as the market value of the new debt and cash received to the sum of *outstanding face value* of the old debt and past due interest. The justification for using the face value - apart from the fact that it makes recovery rates much easier to compute, since it is based only on the total volume of outstanding debt, not the payments terms of the old bonds - is that in a default situation, payments due under the old bonds are usually accelerated, so that the contractual right of the creditor shifts from being entitled to a future payment stream to the right to immediate payment of the principal.

Source: Bedford, Penalver and Salomon (2005), Benjamin and Wright (2008), Datastream, and Sturzenegger and Zettelmeyer (2006 and 2007)

The fitted line is obtained by regressing recovery rates on increases in spreads as indicated in Table 3. This negative relationship is robust even controlling for the detrended GDP. These results reflect that lower recovery rates at the renegotiation are associated with larger increases in yield spreads between the rate of return on new debt and the risk-free rate. This presents a trade-off for defaulting countries; if the countries pay higher fractions of debt at the renegotiations, long-term borrowing costs will be smaller. At the same time, we can interpret it as a trade-off of creditors. If the creditors recover only a small fraction of defaulted debt, they can recoup their losses by demanding higher rates of return for the newly issued bonds.

Table 3: Regression results

Variable	OLS	OLS
Increase in spreads	-0.52*	-0.49*
	(0.27)	(0.28)
Detrended GDP^{*1}	-	1.40
	-	(2.52)
Sample	35	34

Note: Standard errors are in parentheses. OLS=ordinary least squares. * significantly different from 0 at the 10 percent level. ** significantly different from 0 at 5 percent level. *** significantly different from 0 at 1 percent level. *1: We use an HP filter with a smoothing parameter of 1600 quarterly frequencies to obtain the detrended GDP.

4 Model environment

The basic structure of the model follows previous work that extends the model of sovereign default by Eaton and Gersovitz (1981) and applies its quantitative analysis. Among these studies, the closest reference to our paper is Yue (2006). The distinctive feature in our model with respect to her model is that we introduce effects of increases in rate of return on new debt after the re-entry to the market. Since both recovery rates and increases in rate of return on new debt are determined endogenously, how much the country pays at the renegotiation will affect its credit condition in the future, i.e. borrowing costs of the country after re-entry to the market, which will have impacts on default probability[10].

4.1 General points

The model analyzes sovereign default and negotiation in a stochastic dynamic equilibrium model. We consider a risk-averse country that can't affect world risk-free interest rate. The country's preference is given by following utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

where $0 < \beta < 1$ is a discount factor, c_t denotes consumption in period t, and $u(.)$ is its one-period utility function, which is continuous, strictly increasing and strictly concave and satisfies the Inada conditions. A discount factor reflects both pure time preference and probability that the current sovereignty will survive into next period.

In each period, the country starts with its credit history h_t, which satisfies $h_t \in H$ where $H = [0, 1, 2, ..., h_{\max}]$. The credit history expresses number of debt renegotiations the country has experienced in the past. The reason why we assume multi-state credit history rather than two-state credit history as in Yue (2006) is to analyze how the outcomes of past debt renegotiations associated with defaults affect the probability of next default. Moreover, we assume that the credit history reverts with exogenous probability χ conditional on that the country chooses to pay the spread returns after defaults[11].

The country receives an exogenous income shock y_t. Income shock (y_t) is stochastic, drawn from a compact set $Y = [y_{\min},\ y_{\max}] \subset \mathbb{R}_+$. $\mu(y_{t+1}|y_t)$ is the probability distribution of a shock y_{t+1} conditional on previous realization y_t.

There is an infinite number of investors who are risk-neutral and behave competitively in the international capital market. They have perfect information on the country's assets, credit history,

[10] On the other hand, Yue (2006) has not taken into account impacts of increases in rate of return on new debt. In her model, both parties negotiate over only recovery rate after default. The reason why effects of increases in rate of return on new debt are missing in her model is that the country's credit condition will always return to the same level irrelevant to recovery rate which is determined at the renegotiation.

[11] Following the consumer defaults as in Chatterjee et al (2007), we assume that the record of the recent default remains on the country's credit history for only a finite number of years.

income shocks and additional components in spreads agreed to at previous debt renegotiation. We also assume that they can borrow or lend as much as needed at a constant risk-free interest rate (r) in the market. Since they are symmetric and similarly ranked, we can interpret them as "a representative investor" lending money to the country. The country borrows the money from the same representative investor even after it defaults[12]. Thus, "a representative investor" has a bargaining power at the renegotiation in order to impose higher spreads on future bonds, though its bargaining power is low compared to that of country. Moreover, we assume that all the investors behave in the same manner: they all lend the money to the country every time the country issues bonds, and there is no sub-group of investors who behave differently from the majority of investors such as they still lend money to the country even if the country defaults and refuse to negotiate with the majority of investors.

The international capital market is incomplete. The country and foreign investors can borrow and lend only via one-period zero-coupon bonds where b_{t+1} denotes amount of bonds to be repaid next period. When the country purchases bonds, $b_{t+1} > 0$, and when it issues new bonds, $b_{t+1} < 0$. The set of amount of bonds is $B = [b_{\min},\ b_{\max}] \subset \mathbb{R}$ where $b_{\min} \leq 0 \leq b_{\max}$. The upper bound is the highest level of assets that the country can accumulate and the lower bound is the highest level of debts that the country can hold. We assume $q(b_{t+1}, h_t, y_t)$ is the price of a bond with asset position (b_{t+1}), credit history (h_t), and income level (y_t). The bond price will be determined in equilibrium.

We assume that foreign investors always commit to repay their debts. However, the country is free to decide whether to repay its debt or to default. If the country chooses to repay its debt, it will preserve access to the international capital market next period.

If the country chooses not to pay its debt, it is subject to both exclusion from the international capital markets and direct output cost[13][14]. When a default occurs, the country and foreign investors negotiate reduction of unpaid debt via Nash bargaining. At the renegotiation, both recovery rates and additional components in spreads on the newly issued bonds are agreed to by both parties. The

[12] It implies that the country borrows the same share of money from each individual investor whenever it issues new bonds after the re-entry to the capital market.

[13] There are several estimates for output loss at the time of default. Sturzeneger (2002) esimates output loss as around 2% of GDP. On the other hand, De Paoli, Hoggarth, and Saporta (2006) suggest that the output loss in the wake of sovereign default apprears to be very large - around 7% a year on the median measure - as well as long lasting.

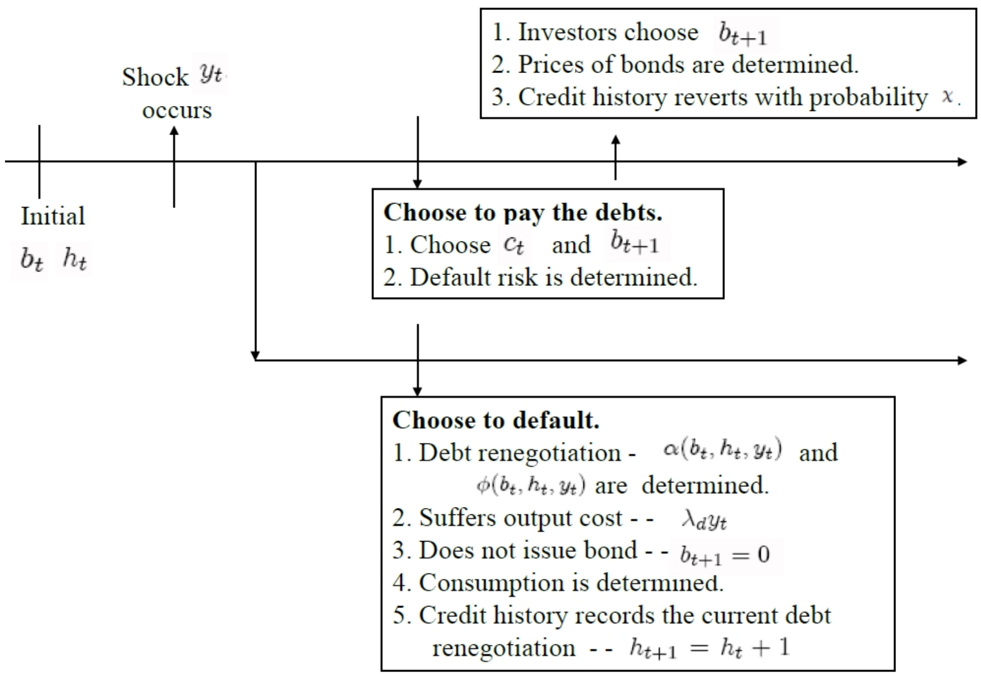
[14] Mendoza and Yue (2008) explain that output cost associated sovereign default is efficiency loss of production through two channels: inefficient production using domestic inputs which are imperfect substitutable with imported inputs, and labor reallocation away from final good production.

country regains access to the market in the next period, but the country's credit history records the current debt renegotiation. In order to avoid permanent exclusion from the international capital market, the country has an incentive to negotiate over haircut rates (recovery rates) and additional default premia. From foreign investors' point of view, Foreign investors want to maximize the payment from recovered debt and spread returns on newly issued bonds after default, so they are also willing to negotiate over reduction of unpaid debt.

4.2 Timing of the model

Timing of decisions within each period is summarized in Figure 3.

Figure 3: Timing of the model



The country starts the current period with initial asset position (b_t) and credit history (h_t). After observing the current income shock (y_t), the country chooses either to pay the debt or to default. If the country decides to pay the debt, given the bond price schedule, the country chooses next period assets (b_{t+1}) and consumption (c_t). Then the default probability and price of bond are determined by the market equilibrium. Given the price of bonds, foreign investors choose b_{t+1} consistent with belief of default probability. Its credit history will be upgraded with exogenous probability χ.

If the country chooses to default, the country and foreign investors negotiate a debt reduction. Both recovery rates $\alpha(b_t, h_t, y_t)$, and additional components in spreads $\phi(b_t, h_t, y_t)$ are agreed to by both sides. After negotiation, the country pays the recovered debt $\alpha(b_t, h_t, y_t)b_t$ and suffers direct output cost due to default, $\lambda_d y_t$. The country can not raise funds in the international capital market this period ($b_{t+1} = 0$), but will regain access to the market next period. The consumption level is $c_t = (1 - \lambda_d)\, y_t + \alpha(b_t, h_t, y_t)b_t$. The country's credit history records the current debt renegotiation $h_{t+1} = h_t + 1$.

5 Recursive Equilibrium

In this section, we define stationary recursive equilibrium of the model.

5.1 Sovereign country's problem

The country's problem is to maximize its expected lifetime utility. The country makes its default decision and determines its assets for next period (b_{t+1}), given its current asset position (b_t), credit history (h_t), and income shock (y_t). Let $V(b_t, h_t, y_t)$ be one value function of the country that starts the current period with initial asset (b_t), credit history (h_t), and income (y_t).

Given with the bond market price $q(b_{t+1}, h_t, y_t)$, debt recovery rates $\alpha(b_t, h_t, y_t)$, and additional components in spreads $\phi(b_t, h_t, y_t)$, the country solves its optimization problem. We assume both the debt recovery rates and additional components in interest spreads determined at current debt negotiation depend on these state variables.

For simplicity, we consider the problem with $h_t = 0$, indicating that the country has never defaulted in the past. Later, we consider the problem with general cases $h_t \geq 1$.

For $b_t \geq 0$ ($h_t = 0$), the country has savings. The country receives payments from foreign investors and determines its next period asset position b_{t+1} and its consumption c_t to maximize utility, given the price of bond $q(b_{t+1}, 0, y_t)$. Thus the value function is

$$V(b_t, 0, y_t) = \max_{c_t, b_{t+1}} u(c_t) + \beta \int_Y V(b_{t+1}, 0, y_{t+1}) d\mu(y_{t+1}, y_t) \tag{1}$$

$$s.t. \quad c_t + q(b_{t+1}, 0, y_t)b_{t+1} = y_t + b_t$$

For $b_t < 0$ ($h_t = 0$), the country has the debt. If the country decides to pay its debt, it chooses its next-period asset position b_{t+1} and consumption c_t. On the other hand, if the country chooses to default, it become financial autarky for this period and its credit history deteriorates to $h_{t+1} = 1$ next period. Due to agreement in debt renegotiation, the country must pay $-\alpha(b_t, 0, y_t)b_t$ in current period, and it regains access to the international capital market next period with history $h_{t+1} = 1$. With credit history $h_{t+1} = 1$, when the country issues new bonds, it must pay interests on newly issued bonds equal to the sum of the risk-free rate (r) and the component agreed at the last renegotiation ($\phi(b_{t+1}, 0, y_{t+1})$). Thus, the price of bonds after default $q(b_{t+2}, 1, y_{t+1})$ incorporates $\phi(b_{t+1}, 0, y_{t+1})$.

Given the option to default, $V(b_t, 0, y_t)$ satisfies

$$V(b_t, 0, y_t) = \max\left[V^R(b_t, 0, y_t), V^D(b_t, 0, y_t; \alpha(b_t, 0, y_t), \phi(b_t, 0, y_t))\right] \tag{2}$$

where $V^R(b_t, 0, y_t)$ is the value associated with paying debt:

$$V^R(b_t, 0, y_t) = \max_{c_t, b_{t+1}} u(c_t) + \beta \int_Y V(b_{t+1}, 0, y_{t+1}) d\mu(y_{t+1}, y_t) \tag{3}$$

$$s.t. \quad c_t + q(b_{t+1}, 0, y_t)b_{t+1} = y_t + b_t$$

and $V^D(b_t, 0, y_t; \alpha(b_t, 0, y_t), \phi(b_t, 0, y_t))$ is the value associated with default given with debt recovery schedule $\alpha(b_t, 0, y_t)$, and additional component on spreads $\phi(b_t, 0, y_t)$ which will be determined at negotiation after current default.

$$V^D(b_t, 0, y_t; \alpha(b_t, 0, y_t), \phi(b_t, 0, y_t)) = u\left((1 - \lambda_d)y_t + \alpha(b_t, 0, y_t)b_t\right) + \beta \int_Y V(0, 1, y_{t+1}) d\mu(y_{t+1}, y_t) \tag{4}$$

where $V(0, 1, y_{t+1})$ is value function next period with credit history $h_{t+1} = 1$ defined below in general cases with $h_t \geq 1$ and $-\alpha(b_t, 0, y_t)b_t$ is the amount of defaulted debt which the country repays at the debt negotiation and $\lambda_d y_t$ denotes output costs which the country suffers due to defaults.

Next we consider the problem with $h_t \geq 1$ expressing that the country has experienced default at least once in the past.

For $b_t \geq 0$ ($h_t \geq 1$), the country has savings. The country receives payments from foreign investors and determines its next period asset position (b_t) and its consumption (c_t) to maximize utility. Thus the value function is

$$V(b_t, h_t, y_t) = \max_{c_t, b_{t+1}} u(c_t) + \beta \int\limits_Y V(b_{t+1}, h_t, y_{t+1}) d\mu(y_{t+1}, y_t) \tag{5}$$

$$s.t. \quad c_t + q(b_{t+1}, h_t, y_t) b_{t+1} = y_t + b_t$$

Note that credit history remains unchanged in next period $h_{t+1} = h_t$.

For $b_t < 0$ ($h_t \geq 1$), the country has the debt. The country can borrow money from the foreign investors, but the country needs to pay not only the risk-free interest rate (r), but also the sum of additional components in spreads $\sum_{i=0}^{h_t-1} \phi(b_t, i, y_t)$ which were agreed to by both the country and foreign investors at the time of past debt renegotiations. Thus, the price of bonds $q(b_{t+1}, h_t, y_t)$ is different from the one with history $h_t = 0$, defined as $q(b_{t+1}, 0, y_t)$, as it incorporates the effects of sum of additional default premia associated with deteriorated credit history. As in the case of history $h_t = 0$, the country chooses either to pay the debt or to default. The values are as before:

$$V(b_t, h_t, y_t) = \max \left[V^R(b_t, h_t, y_t), V^D \left(b_t, h_t, y_t; \alpha(b_t, h_t, y_t), \phi(b_t, h_t, y_t) \right) \right] \tag{6}$$

where $V^R(b_t, h_t, y_t)$ is the value associated with paying debt with history $h_t \geq 1$,

$$V^R(b_t, h_t, y_t) = \max_{c_t, b_{t+1}} u(c_t) + \beta \begin{bmatrix} (1-\chi) \int\limits_Y V(b_{t+1}, h_t, y_{t+1}) d\mu(y_{t+1}, y_t) \\ +\chi \int\limits_Y V(b_{t+1}, h_t - 1, y_{t+1}) d\mu(y_{t+1}, y_t) \end{bmatrix} \tag{7}$$

$$s.t. \quad c_t + q(b_{t+1}, h_t, y_t) b_{t+1} = y_t + b_t$$

Note that with exogenous probability χ, the country's credit history next period will revert due to limited memory of the investors as $h_{t+1} = h_t - 1$. Otherwise, it remains constant as $h_{t+1} = h_t$.

$V^D(b_t, h_t, y_t; \alpha(b_t, h_t, y_t), \phi(b_t, h_t, y_t))$ is the value associated with default given with debt recovery schedule $\alpha(b_t, h_t, y_t)$, and additional components in spreads agreed after current default $\phi(b_t, h_t, y_t)$

which are defined below:

$$V^D\left(b_t, h_t, y_t; \alpha(b_t, h_t, y_t), \phi(b_t, h_t, y_t)\right) = u\left((1-\lambda_d)y_t + \alpha(b_t, h_t, y_t)b_t\right) + \beta \int_Y V\left(0, h_t+1, y_{t+1}\right) d\mu(y_{t+1}, y_t) \tag{8}$$

where $V(0, h_t + 1, y_{t+1})$ is the value function next period with credit history $h_{t+1} = h_t + 1$ and $-\alpha(b_t, h_t, y_t)b_t$ is amount of defaulted debt which the country recovers after negotiation.

Every time (at period t) the country defaults, its credit history records the current debt renegotiation $h_{t+1} = h_t + 1$. Thus, the credit condition, i.e. borrowing costs of the country after re-entry to the market depends on how much the country pays at the renegotiation. When the country issues new bonds after it defaults, it must pay returns based on the risk-free rate and the sum of additional components in spreads, which are determined at the previous debt renegotiations.

The country's default policy can be characterized by default sets $D(b_t, h_t) \subset Y$, defined as the set of income shock y's for which default is optimal given the debt position b_t, and credit history h_t.

$$D(b_t, h_t) = \left\{ y_t \in Y : V^R(b_t, h_t, y_t) < V^D\left(b_t, h_t, y_t; \alpha(b_t, h_t, y_t), \phi(b_t, h_t, y_t)\right)\right\} \tag{9}$$

Furthermore, we define an indicator of non-defaulting given initial asset position ($b_t < 0$), credit history (h_t), and income level (y_t) as follows;

$$I(b_t, h_t, y_t) = \begin{Bmatrix} 1 & if & y_t \notin D(b_t, h_t) \\ 0 & if & y_t \in D(b_t, h_t) \end{Bmatrix}$$

Finally, based on the policy function of asset position derived above ($b_{t+1}(b_t, h_t, y_t)$) and non-defaulting indicator $I(b_t, h_t, y_t)$, we define discounted value of expected amounts of debts which will be paid to investors next period as:

$$P(b_t, h_t, y_t) = \frac{1}{1+r} \int_Y I\left(b_{t+1}(b_t, h_t, y_t), h_t, y_{t+1}\right) b_{t+1}(b_t, h_t, y_t) d\mu(y_{t+1}, y_t) \tag{10}$$

Note that we use the discount factor for foreign investors ($\frac{1}{1+r}$), not the discount factor for the country (β).

5.2 Debt renegotiation problem

The debt renegotiation takes a form of generalized Nash bargaining game. Not only the recovery rate, but also additional components in spreads are agreed to by both parties. This is because foreign investors will obtain interest returns every time the country issues new bonds after current default as long as the country does not default again. From the country's perspective, it has to pay interests on bonds every time it issues new bonds after renegotiation, unless it chooses to remain in the financial autarky permanently.

After debt renegotiation, the country pays a fraction $\alpha(b_t, h_t, y_t)$ of defaulted debt. The value of the country after the renegotiation is defined above;

$$V^D\left(b_t, h_t, y_t; \alpha(b_t, h_t, y_t), \phi(b_t, h_t, y_t)\right) = u\left((1-\lambda_d)y_t + \alpha(b_t, h_t, y_t)b_t\right) + \beta \int\limits_Y V\left(0, h_t+1, y_{t+1}\right) d\mu(y_{t+1}, y_t)$$

Needless to say, this value takes into account the impact of both debt reduction to $-\alpha(b_t, h_t, y_t)b_t$, and additional component in spread $\phi(b_t, h_t, y_t)$ which will be agreed at current debt negotiation.

Foreign investors obtain the present value of the reduced debt $-\alpha(b_t, h_t, y_t)b_t$ and interests on newly issued bonds after debt negotiation. The present value of expected payment of bonds which investors receive in the future after the country's re-entry to the market, can be defined in the following recursive form:

$$R(b_t, h_t, y_t) = P(b_t, h_t, y_t) + \frac{1}{1+r} \int\limits_Y R(b_{t+1}, h_t, y_{t+1}) d\mu(y_{t+1}, y_t) \tag{11}$$

$$s.t. \quad b_{t+1} = b^*_{t+1}(b_t, h_t, y_t),$$

where $P(b_t, h_t, y_t)$ is the discounted value of expected amount of bonds which are returned in next period defined in equation (10) and $b^*_{t+1}(b_t, h_t, y_t)$ is policy function of the country if it chooses not to default ($h_{t+i} = h_t$).

We assume that debt negotiation takes place only once for each default event. The threat point of the bargaining game is that the country stays in permanent autarky and the foreign investors get nothing. Moreover, we assume that impose direct sanctions $\lambda_s y_t$ on the country, which is in addition to the defaulting country's direct output cost $\lambda_d y_t$ if the country chooses not to negotiate. The expected value of autarky for the country, $V^{AUT}(y_t)$ is given by following expression;

$$V^{AUT}(y_t) = u((1-\lambda_s-\lambda_d)y_t) + \beta \int_Y V^{AUT}(y_{t+1}) d\mu(y_{t+1,}y_t) \tag{12}$$

We consider one-round bargaining since one-round bargaining keeps the model tractable as there is no need to consider multiple rounds of bargaining or the debt arrears based on different reduction schedules[15].

For any debt recovery rate a_t and additional component in interest spreads sp_t, we denote the country's surplus in Nash bargaining by $\Delta^B(a_t, sp_t; b_t, h_t, y_t)$, which is the difference between the value of accepting a proposal of debt recovery rate a_t and additional components in interest spreads sp_t, and the value of rejecting it, given the country's debt level (b_t), credit history (h_t), and income level (y_t).

$$\Delta^B(a_t, sp_t; b_t, h_t, y_t) = V^D(b_t, h_t, y_t; \alpha(b_t, h_t, y_t), \phi(b_t, h_t, y_t)) - V^{AUT}(y_t) \tag{13}$$

The surplus to the country comes from two sources. First, the country will be able to issue bonds again from the following period, though its credit history deteriorates. Also, the direct cost to output is smaller under renegotiations because no sanctions are imposed.

On the other hand, the surplus to investors is the present value of the sum of recovered debt and interest returns on newly issued bonds after renegotiation:

$$\Delta^L(a_t, sp_t; b_t, h_t, y_t) = -a_t b_t - sp_t \left[\sum_{i=h_{t+1}}^{h_{\max}} R(b_t, i, y_t) \right] \tag{14}$$

where interest returns are evaluated with expected payment incorporating the future default choices of the country as in equation (11).

We assume that the country has a bargaining power θ and foreign investors have a bargaining

[15] Bi (2008) and Benjamin and Wright (2008) analyze multi-round bargaining to consider delay in renegotiation. Based on the assumption that the lenders have an option to "pass" proposing to the debtor, Bi (2008) argues that both parties can be better off by waiting and dividing a larger "cake" as it takes time for the economy to recover. On contrary, Benjamin and Wright (2008) assume that the debtor and representative creditor randomly alternate in their ability to propose a bargaining outcome with a changes in the probability of making future proposals serving to capture changes in bargaining power. They find that both parties find it optimal to postpone renegotiation until future default risk is low since the debtor's ability to share the future surplus created by by a debt renegotiation is limited by future default risk.

power $(1-\theta)$. A bargaining power parameter θ summarizes the institutional arrangement of debt negotiation. To ensure that the bargaining problem is well defined, we define the bargaining power set $\Theta \subset [0,1]$ such that for $\theta \in \Theta$ the negotiation surplus has a unique optimum for any asset position $(b_t < 0)$, its history (h_t), income level (y_t).

Given the country's asset level $(b_t < 0)$, its credit history (h_t), and income level (y_t) together with sum of additional components in spreads which are agreed at past debt renegotiations $\sum_{i=0}^{h_{t-1}} \phi(b_t, i, y_t)$ recovery rates $\alpha(b_t, h_t, y_t)$ and additional components in spreads $\phi(b_t, h_t, y_t)$ solve the following bargaining problem:

$$\begin{Bmatrix} \alpha(b_t, h_t, y_t) \\ \phi(b_t, h_t, y_t) \end{Bmatrix} = \arg\max_{a_t, sp_t} \left[\left(\Delta^B \left(a_t, sp_t; b_t, h_t, y_t\right)\right)^{\theta} \left(\Delta^L \left(a_t, sp_t; b_t, h_t, y_t\right)\right)^{1-\theta} \right] \tag{15}$$

$$s.t. \quad \Delta^B(a_t, sp_t; b_t, h_t, y_t) \geq 0$$

$$s.t. \quad \Delta^L(a_t, sp_t; b_t, h_t, y_t) \geq 0$$

Since the set of both debt recovery schedule and additional components in spreads that maximize total negotiation surplus conditional on the country's asset level, credit history, and income level, negotiation outcome provides better insurance to the country in the case of default.

5.3 Foreign investors' problem

For the cases with $h_t \geq 1$, our derived bond price incorporates the effects of additional components in spreads agreed at previous debt renegotiations, which are the new elements in our model. First we consider foreign investors' problem given the country's credit history $h_t = 0$.

With the country's credit history $h_t = 0$, taking the bond price function as given, foreign investors choose the amount of asset (b_{t+1}) that maximizes their expected profit $\pi(b_{t+1}, 0, y_t)$, given by

$$\pi(b_{t+1}, 0, y_t) = \begin{Bmatrix} q(b_{t+1}, 0, y_t) b_{t+1} - \frac{1}{1+r} b_{t+1} & if \quad b_{t+1} \geq 0 \\ \frac{[1 - p(b_{t+1}, 0, y_t) + p(b_{t+1}, 0, y_t)\gamma(b_{t+1}, 0, y_t)]}{1+r}(-b_{t+1}) - q(b_{t+1}, 0, y_t)(-b_{t+1}) & otherwise \end{Bmatrix} \tag{16}$$

where $p(b_{t+1}, 0, y_t)$ and $\gamma(b_{t+1}, 0, y_t)$ are the expected default probability and expected recovery rates respectively for country with asset position $(b_{t+1} < 0)$, credit history $(h_t = 0)$, income level (y_t), and r is risk-free rate.

Since we assume that the market for new sovereign bonds is completely competitive, foreign investors' expected profit is zero in equilibrium. Using the zero expected profit condition, we get

$$q(b_{t+1},0,y_t) = \left\{ \begin{array}{ll} \frac{1}{1+r} & if \quad b_{t+1} \geq 0 \\ \frac{[1-p(b_{t+1},0,y_t)+p(b_{t+1},0,y_t)\gamma(b_{t+1},0,y_t)]}{1+r} & otherwise \end{array} \right\} \tag{17}$$

When the country buys bonds from foreign investors $b_{t+1} \geq 0$, the sovereign bond price is equal to the price of risk-free bond, $\frac{1}{1+r}$. When the country issues bonds to foreign investors $b_{t+1} < 0$, there is default risk, and the bond is priced to compensate foreign investors for this. Since $0 \leq p(b_{t+1},0,y_t) \leq 1$ and $0 \leq \gamma(b_{t+1},0,y_t) \leq 1$, the bond price $q(b_{t+1},0,y_t)$ lies in $\left[0, \frac{1}{1+r}\right]$.

Next, we consider foreign investors' problem for general cases with the country's history $h_t \geq 1$. Note that the borrowing costs of the country is denoted by $1+r+\sum_{i=0}^{h_t-1}\phi(b_t,i,y_t)$ which include the sum of additional components in spreads agreed at the previous debt renegotiations. Given the borrowing costs, together with the bond price $q(b_{t+1},h_t,y_t)$, foreign investors maximize their expected profit $\pi(b_{t+1},h_t,y_t)$, given by

$$\pi(b_{t+1},h_t,y_t) = \left\{ \begin{array}{ll} q(b_{t+1},h_t,y_t)b_{t+1} - \frac{1}{1+r}b_{t+1} & if \quad b_{t+1} \geq 0 \\ \frac{[1-p(b_{t+1},h_t,y_t)+p(b_{t+1},h_t,y_t)\gamma(b_{t+1},h_t,y_t)]}{1+r+\sum_{i=0}^{h_t-1}\phi(b_t,i,y_t)}(-b_{t+1}) - q(b_{t+1},h_t,y_t)(-b_{t+1}) & otherwise \end{array} \right\} \tag{18}$$

where $p(b_{t+1},h_t,y_t)$ and $\gamma(b_{t+1},h_t,y_t)$ are as above. Using the zero profit condition, we obtain

$$q(b_{t+1},h_t,y_t) = \left\{ \begin{array}{ll} \frac{1}{1+r} & if \quad b_{t+1} \geq 0 \\ \frac{[1-p(b_{t+1},h_t,y_t)+p(b_{t+1},h_t,y_t)\gamma(b_{t+1},h_t,y_t)]}{1+r+\sum_{i=0}^{h_t-1}\phi(b_t,i,y_t)} & otherwise \end{array} \right\} \tag{19}$$

When the country issues bonds to foreign investors, the bond price $q(b_{t+1},h_t,y_t)$ lies in $\left[0, \frac{1}{1+r+\sum_{i=0}^{h_t-1}\phi(b_t,i,y_t)}\right]$ since $0 \leq p(b_{t+1},h_t,y_t) \leq 1$ and $0 \leq \gamma(b_{t+1},h_t,y_t) \leq 1$. Thus, the bond price incorporates the sum of the additional default premia $\sum_{i=0}^{h_t-1}\phi(b_t,i,y_t)$ due to the past debt renegotiations; the price of bonds decreases as additional components in spreads increases.

Moreover, for any credit history (h_t), interest rate on sovereign bonds is defined as follows; $r^S(b_{t+1},h_t,y_t) = \frac{1}{q(b_{t+1},h_t,y_t)} - 1$. It is bounded below by the risk-free rate (r). We define the country's total spreads which is a difference between country's interest rate and the risk-free rate,

$$s(b_{t+1}, h_t, y_t) = \frac{1}{q(b_{t+1}, h_t, y_t)} - 1 - r \tag{20}$$

5.4 Recursive equilibrium

We define a stationary recursive equilibrium of the model.

Definition 1 *:A* ***recursive equilibrium*** *is a set of functions for, (A) the country's value function $V^*(b_t, h_t, y_t)$ (together with $V^{*R}(b_t, h_t, y_t)$ and $V^{*D}(b_t, h_t, y_t)$), asset position $b^*_{t+1}(b_t, h_t, y_t)$, consumption $c^*_t(b_t, h_t, y_t)$, default set $D^*(b_t, h_t)$, discounted expected payment $P^*(b_t, h_t, y_t)$, (B) recovery rate $\alpha^*(b_t, h_t, y_t)$, additional components in interest spreads $\phi^*(b_t, h_t, y_t)$, and (C) bond price function $q^*(b_{t+1}, h_t, y_t)$, and total spread $s^*(b_{t+1}, h_t, y_t)$ such that*

[1]. *Given the bond price function $q^*(b_{t+1}, h_t, y_t)$, recovery rate $\alpha^*(b_t, h_t, y_t)$ and additional component in interest spreads $\phi^*(b_t, h_t, y_t)$, the country's value function $V^*(b_t, h_t, y_t)$ (together with $V^{*R}(b_t, h_t, y_t)$ and $V^{*D}(b_t, h_t, y_t)$), asset position $b^*_{t+1}(b_t, h_t, y_t)$, consumption $c^*_t(b_t, h_t, y_t)$, default set $D^*(b_t, h_t)$ satisfy the country's optimization problem (1)-(10).*

[2]. *Given the bond price function $q^*(b_{t+1}, h_t, y_t)$, the country's value function $V^*(b_t, h_t, y_t)$ (together with $V^{*R}(b_t, h_t, y_t)$ and $V^{*D}(b_t, h_t, y_t)$), discounted expected payment $P^*(b_t, h_t, y_t)$, the recovery rate $\alpha^*(b_t, h_t, y_t)$ and additional components in interest spreads $\phi^*(b_t, h_t, y_t)$ solve debt renegotiation problem (15).*

[3]. *Given recovery rate $\alpha^*(b_t, h_t, y_t)$ and additional spread on bonds $\phi^*(b_t, h_t, y_t)$, the bond price function $q^*(b_{t+1}, h_t, y_t)$, total spread $s^*(b_{t+1}, h_t, y_t)$ and satisfy optimal conditions of foreign investors' problem (17), (19) and (20).*

In equilibrium, default probability $p^*(b_{t+1}, h_t, y_t)$ is defined by using the country's default decision:

$$p^*(b_{t+1}, h_t, y_t) = \int_{D^*(b_{t+1}, h_t)} d\mu(y_{t+1}, y_t) \tag{21}$$

The expected recovery rate $\gamma^*(b_{t+1}, h_t, y_t)$ in equilibrium is given by

$$\gamma^*(b_{t+1}, h_t, y_t) = \frac{\int_{D^*(b_{t+1},h_t)} \alpha^*(b_{t+1}, h_t, y_{t+1}) d\mu(y_{t+1}, y_t)}{\int_{D^*(b_{t+1},h_t)} d\mu(y_{t+1}, y_t)}$$

$$= \frac{\int_{D^*(b_{t+1},h_t)} \alpha^*(b_{t+1}, h_t, y_{t+1}) d\mu(y_{t+1}, y_t)}{p^*(b_{t+1}, h_t, y_t)} \tag{22}$$

The numerator is expected proportion of the debt which the country will repays at renegotiation, and the denominator is default probability.

6 Quantitative Analysis

This section provides quantitative analysis of the model. We set parameters and functional forms of the model and discuss equilibrium properties of the model. Simulation results based on equilibrium distribution of the model are presented in Section 6.3. Finally, we summarize main implications of quantitative analysis.

6.1 Parameters and functional forms

We use most of parameters and functional forms specified in Yue (2006). There are three new elements in our model: (1) the maximum level of additional component in spreads, (2) the maximum level of credit history and (3) probability of upgrading in credit history. The rationale of the upper limits of both additional component in spreads and credit history is to satisfy the stationarity of the model; if we do not set the upper limits, the country will face high borrowing costs and repeat defaults in short periods leading to higher spreads, and investors will not be able to receive spread payments. Reflecting the fact that the record of defaults remains on the country's credit history for only a finite number of years rather than infinite periods, we assume the probability of upgrading in credit history.

We define each period as a quarter. The following constant relative risk-aversion (CRRA) utility function is used in numerical simulations:

$$u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} \tag{23}$$

where σ expresses degree of risk aversion. We set σ equal to 2, which is a common value used in real business cycle studies. Following Arellano (2008), the risk-free rate is equal to 1.7%. The baseline output loss parameter λ_d is set to 2% based on Sturzenegger's (2002) estimate.

We follow the same stochastic process for output used in Yue (2006). She models the output growth rate as AR(1) process to capture the stochastic trend in GDP of Argentina as;

$$\log(y_t) = (1-\rho_g)\log(1+\mu_g) + \rho_g \log(y_{t-1}) + \epsilon_t^g \tag{24}$$

where growth rate is $g_t = \frac{y_t}{y_{t-1}}$, growth shock is $\epsilon_t^g \sim^{i.i.d.} N(0, \sigma_g^2)$, and $\log(1+\mu_g)$ is expected log gross growth rate of the country's endowment. We set $\mu_g = 0.0042$, $\sigma_g = 0.0253$, and $\rho_g = 0.41$, and approximate this stochastic process as a discrete Markov chain of 21 equally spaced grids by using the quadrature method in Tauchen (1986).

Since a realization of the growth shock permanently affects endowment and the model economy is nonstationary, we detrend the model by dividing by the lagged endowment level y_{t-1}. The detrended counterpart of the any variable x_t is thus $\hat{x}_t = \frac{x_t}{y_{t-1}}$. The equilibrium value function, bond price function, recovery rate and interest spreads are evaluated based on the detrended variables.

Concerning time discount factor β, the baseline country's bargaining power θ, and direct sanction parameter λ_s, we again follow Yue (2006) by setting $\beta = 0.74$, $\theta = 0.83$, and $\lambda_s = 0.012$. It uses these parameter values to obtain its average default frequency 2.78% annually or 0.69% quarterly[16]. For interest spreads, we set the maximum level of additional components in interest spreads ($\phi_{\max}$) as 0.01 in order to make our results consistent with the evidence in Figure 2 that increase in spreads is less than 0.01 (100 basis points). Lastly, taking into account 3 defaults of Argentina in the period from 1901-2002 indicated in Reinhart, Rogoff, and Savastano (2003), we specify the maximum level of credit history ($h_{\max}$) as 3. The probability of upgrading χ, which governs the average length of time that a recent default remains on the country's credit history is set to 0.025, reflecting the punishment by investors on recent defaults last for 10 years. Table 4 summarizes the model parameters.

Our computation algorithm is shown in Appendix A.

[16] Later in Appendix E, a case which bargaining power increases with credit history (h_t) is analyzed.

Table 4: Model parameters

Parameter	Value	Sources
Risk aversion	$\sigma = 2$	RBC Literature
Risk-free interest rate	$r = 0.017$	Arellano (2008)
Baseline output loss in default	$\lambda_d = 0.02$	Sturzeneger (2002)
Average endowment growth	$\mu_g = 0.0042$	Yue (2006)
Standard deviation of endowment growth shock	$\sigma_g = 0.0253$	Yue (2006)
Endowment growth AR(1) coefficient	$\rho_g = 0.41$	Yue (2006)
Discount factor	$\beta = 0.74$	Yue (2006)
Baseline bargaining power	$\theta = 0.83$	Yue (2006)
Direct sanction	$\lambda_s = 0.012$	Yue (2006)
Maximum level of additional component in spreads	$\phi_{\max} = 0.01$	Computed
Maximum level of credit history	$h_{\max} = 3$	Computed
Probability of upgrading in credit history	$\chi = 0.025$	Chatterjee et al (2007)

6.2 Numerical results on equilibrium properties

In this subsection, we cover the equilibrium properties of the model. Figure 4 shows the relationship between increase in interest spreads and recovery rates. As in Section 3, we define increase in spreads as the difference between defaulting and non-defaulting. We calculate the spreads after default based on both expected recovery rates for next default and agreed additional components in spreads, and spreads with non-defaulting are measured with expected recovery rates for the current default. It is clear that there is a negative relationship between recovery rates and increase in interest spreads in the lowest, mean and highest mean income states. If the increase in spreads is high, recovery rate is low and vice versa. One interpretation is that if the country pays a higher fraction of its debt at the renegotiations, long-term borrowing costs will be smaller. The slope of the contract curve in the lowest income state is steeper than ones in both the mean or the highest income states.

Figure 4: Relationship between increase in interest spreads and recovery rates ($h_t = 0$)

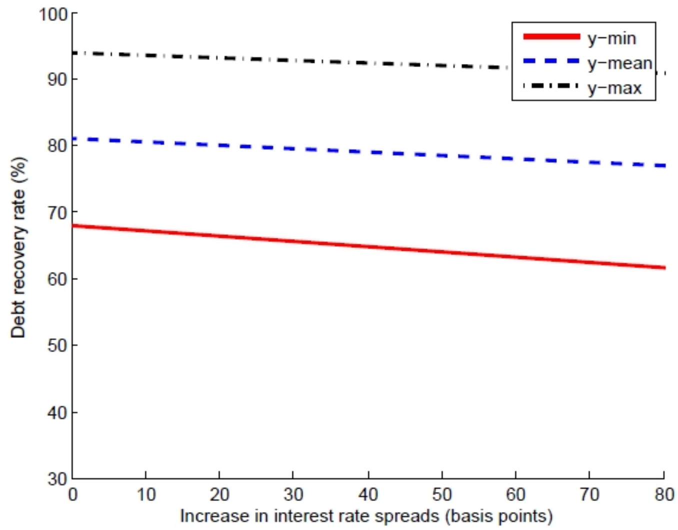


Figure 5: Default probability under baseline case

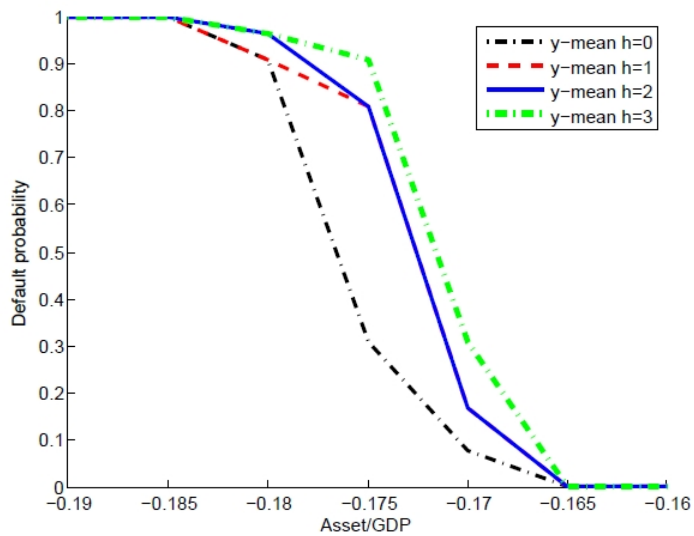
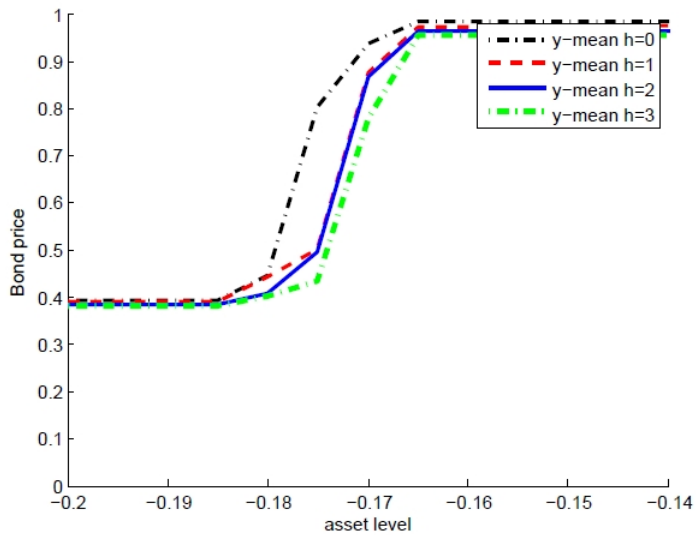


Figure 5 illustrates the baseline default probability at the mean output. It is apparent that the default probability is weakly increasing with the credit history. At the higher level of credit history, additional increase in spreads on the newly issued bonds, determined at the previous debt renegotiation, leads to higher cost for the country to borrow from investors compared with credit history $h_t = 0$.

Figure 6 presents that bond price is also weakly decreasing with respect to credit history. What play behind are the agreed additional components in spreads agreed at the debt renegotiations: as explained clearly in Section 6.4, the additional components in spreads decrease both directly and indirectly through default probability the bond price. Several figures under different levels of new debts and income are provided in Appendix C.

Figure 6: Bond price schedule under baseline case ($b_{t+1} = -0.16$)



6.3 Simulation results

We conduct 1000 rounds of simulations, with 2000 periods per round, and then extract the last 100 periods to analyze features in the stationary distribution. Our results are presented in Table 5.

Table 5: Model statistics for Argentina (annual-base)

	Data			Simulation	results
Target statistics	Argentina*1	Brady Bonds	EMBI Ave.*2	Model	Yue (2006)
Default probability	2.76%	n.a	n.a.	2.20%	1.98%
Average bond spreads	4.08%	5.78%	3.31%	4.00%	1.49%
Bond spreads std*3	1.68%	3.13%	0.78%	7.00%	3.27%

Source: Datastream and Yue (2006)

*1: In order to stress the comparison between results of the model and Yue (2006), we use the same annual data. *2: EMBI composite provided by Datasream, is computed with 20 EMBI bonds for the period from 1991Q1 to 2009Q1, including Africa, Asia, Argentina, Brazil, Bulgaria, Colombia, Ecuador, Egypt, Europe, Indonesia, Latin, Mexico, Morocco, Nigeria, Panama, Peru, the Philippines, Russia, South Africa, Turkey, and Ukraine. *3: The difference of standard deviation of bond spreads from one reported in Yue (2006) is due to a difference in simulation procedures.

The model simulation improves the accuracy of average bond spreads and default probability, compared with Yue (2006). Average bond spreads matches the data better, reflecting a large difference from the statistics in the comparison model. Moreover, the default probability gets closer to the data. Additional increase in spreads associated with past defaults drives these results as explained in details in next subsection.

On the other hand, the standard deviation of bond spreads increases from the one in the comparison model which leads a larger deviation from the data. The higher standard deviation is associated with additional increases in spreads at the past debt renegotiations. The standard deviation of Argentina sovereign bond is lower than that of the Brady bonds, but higher than that of the EMBI average.

6.4 Impacts of additional components in spreads

In this subsection, we explain how the additional components in spreads agreed at past debt renegotiations lead to the increase in current interest spreads, which distinguishes this paper with the previous work.

Based on equation (19) and (20), we can rewrite interest spreads for credit history $h_t \geq 1$ as follows.

$$s(b_{t+1}, h_t, y_t) = \left\{ \begin{array}{cc} 0 & if \quad b_{t+1} \geq 0 \\ \frac{1+r+\sum_{i=0}^{h_t-1} \phi(b_t, i, y_t)}{[1-p(b_{t+1},h_t,y_t)+p(b_{t+1},h_t,y_t)\gamma(b_{t+1},h_t,y_t)]} - (1+r) & otherwise \end{array} \right\} \tag{20a}$$

Given risk-free rate (r), total spreads can be decomposed into three factors:

(A) spread components based on impact of "pure" default probability,

(B) spread components based on direct impact of additional components in spreads, and

(C) spread components based on indirect impact of additional components in spreads (through increase in default probability).

The first factor, which is simply calculated based on "pure" probability of future defaults, is totally irrelevant to credit history. It is the measure of interest spreads used in Yue (2006). The second factor is how much the term $\sum_{i=0}^{h_t-1} \phi(b_t, i, y_t)$, increases total spreads given default probability used in (A). The last factor is how much increase in default probability increases total interest rate spreads. The sum of (B) and (C) can be regarded as spread components associated with the past default history.

Figure 7: The total spreads and spreads based on "pure" default probability

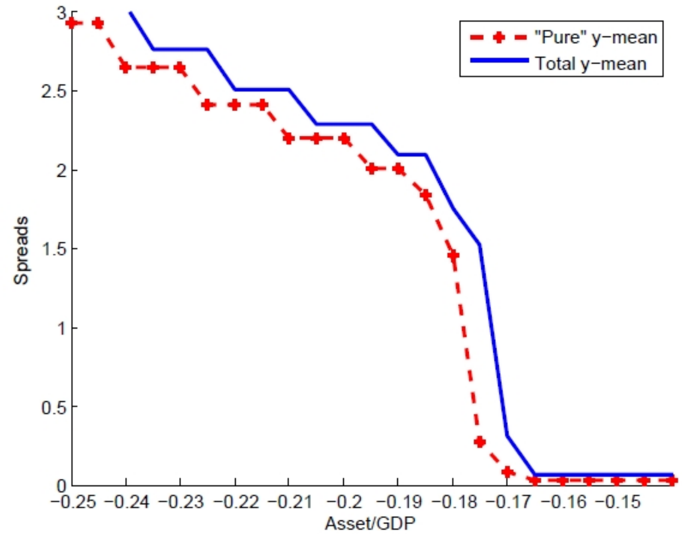


Figure 7 displays both the total spreads and spread components measured with "pure" default probability at credit history $h_t = 3$ under baseline case. The spread components measured with "pure" default probability is equal to (A). The total spreads is defined by equation (20a). The difference between these two corresponds to the sum of (B) and (C), which can be regarded as the impact associated with the past default history. It is clear that there is a difference between these two that increases as the asset-to-GDP ratio is below the threshold value -0.17 in the mean income state.

6.5 A brief summary of quantitative analysis

Our major findings are as follows. First of all, by incorporating additional components in interest spreads, the model accommodates an observed pattern of lower recovery rates associated with larger increases in yield spreads. Second, we show that default probability is weakly increasing with credit history, given the same debt-to-GDP ratio. As the additional default premia makes it more costly for the country to borrow from investors in the later periods, the country ends up with higher default probability compared with non-default history. Third, our simulation improves the predictions of the average bond spreads and the default probability significantly, compared with previous predictions in Yue (2006). Finally, interest spreads in our model can be decomposed into two parts; spread based on "pure" default probability, and spread associated with impacts of additional components in spreads due to past defaults. The latter component enlarges if the debt-to-GDP ratio is above the threshold value for each state.

7 Model implications

In this section, we explore the determinants of the slope of the contract curve, default probability and total spreads. Moreover, we consider possible implications derived from changes in length of punishments and upper limit of credit history. Finally, we describe welfare analysis after the debt renegotiation.

7.1 Determinants of the slope of the contract curve

First, we focus on factors which affect the value of the slope of the contract curve. Table 6 shows the values of the slope of the contract curve under different values for the discount factor, output cost, bargaining power, maximum level of additional components in spreads, and risk-free rate. The impacts of a change in one parameter, leaving all other parameters fixed are indicated respectively.

Table 6: Values of the slope of the contract curve under different parameter values

Data	-0.52				
Discount factor	Slope	Output cost	Slope	Bargaining power	Slope
$\beta = 0.80$	-0.06	$\lambda_d = 0.028$	-0.13	$\theta = 0.74$	-0.078
$\boldsymbol{\beta} = \mathbf{0.74}$	**-0.08**	$\boldsymbol{\lambda}_d = \mathbf{0.02}$	**-0.08**	$\theta = 0.80$	-0.065
$\beta = 0.63$	-0.09	$\lambda_d = 0.005$	-0.04	$\boldsymbol{\theta} = \mathbf{0.83}$	**-0.08**
Maximum level of additional components in spreads	Slope	Risk-free interest rate	Slope	$\theta = 0.86$	-0.04
				$\theta = 0.92$	-0.07
$\phi_{\max} = 0.03$	-0.02	$r = 0.005$	-0.05		
$\boldsymbol{\phi}_{\max} = \mathbf{0.01}$	**-0.08**	$\mathbf{r = 0.017}$	**-0.08**		
$\phi_{\max} = 0.008$	-0.14	$r = 0.03$	-0.05		

Note: all the values are the ones at the lowest income state.

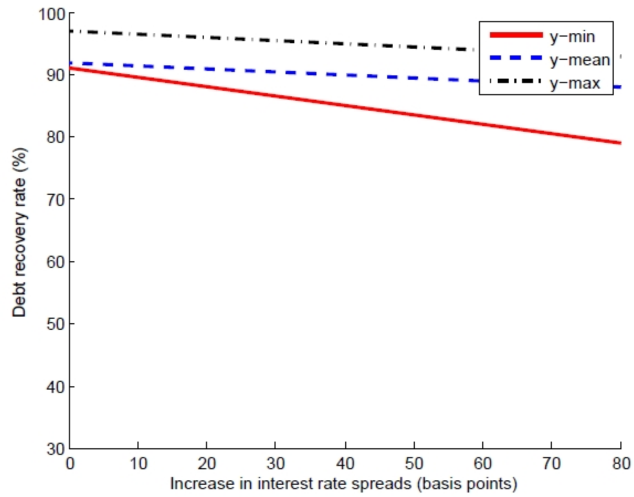
First, the slope gets steeper as the discount factor decreases. From the country's perspective, the cost of paying to one additional unit of defaulted debt at the renegotiation relative to the cost of facing one additional unit of increase in spreads, gets smaller as the discount factor decreases. On the other hand, when the maximum level of additional components in spreads is reduced to 80 basis points ($\phi_{\max} = 0.008$), the absolute value of the slope increases. Since the increase in spreads is limited to a lower level due to the lower maximum level of additional components in spreads, paying one additional unit of defaulted debt at the renegotiation is less costly relative to paying one additional unit of spread increases in the future period.

Next, an increase in output cost leads to an increase in the absolute value of the slope. As the cost of default is larger for the country, relative cost of paying one additional unit of defaulted debt at the renegotiation instead of facing one additional unit of increase in spreads decreases taking into account the cost of next default.

Note that the value of the slope does not change monotonically as the risk-free interest rate increases. The risk-free interest rate affects the value of the slope through two channels: for investors, an increase in risk-free interest rate makes receiving one additional unit of defaulted debt at the renegotiation more beneficial than receiving one unit of spreads returns in the future periods since investors are less patient. On the other hand, the total size of increase in spreads gets larger associated with an increase in risk-free interest rate. Given the constant change in recovery rate, it makes the slope of contract curve more flatter, indicating that from the country's perspective paying one additional unit of defaulted debt at the renegotiation is less costly than paying one additional unit of spread returns in the future periods. When the risk-free rate is low ($r = 0.005$), the first effect dominates. On the contrary, the second effect is larger than the first one when the risk-interest rate is around the baseline value.

Lastly, bargaining power has an ambitious impact on the slope of the contract curve. Rather than the slope, the intercept (recovery rates at 0 basis point increase in spreads) will be influenced by change in bargaining power. Figure 8 shows how the slope of the contract curve can change.

Figure 8: Improved slope of the contract curve with $\phi_{\max} = 0.008$, $\lambda_d = 0.027$ and $\beta = 0.72$ (slope: -0.163*)



* reflect the values at the lowest income state.

7.2 Determinants of default probability and spreads

Impacts of the change in parameter values on average spreads, standard deviations of spreads, and default probability under different parameter values are reported in Table 7. First, an increase in maximum level of additional components in spreads ($\phi_{\max} = 0.03$) increases both the average and standard deviation of spreads. At the renegotiation, both the country and investors choose higher level of additional components in spreads so the total spreads will be higher on average.

Next, a higher output cost ($\lambda_d = 0.027$) leads to higher average spreads and zero default probability. Since the default is too costly, the country is willing to avoid defaults. Higher average spreads is a little counter-intuitive, but the mechanism is the following; as the country will not choose to default, it will accumulate the debt. Thus in average, higher debt-to-GDP ratio relative to one with baseline cases, leads to higher spreads.

A higher risk-free interest rate ($r = 0.03$) is associated with the higher average and standard deviation of spreads. As long as the default probability is not zero, higher risk-free interest rate leads to an increase in spreads. This comes from equation (20a): non-zero default probability implies that denominator of the first component is smaller than 1 and a one unit increase in the risk-free interest rate increases the first component more than one unit, while only increasing the second component by one unit.

Table 7: Sensitivity analysis for default probability and spreads

	Average spreads	Std. spreads	Default probability
Data	4.08%	1.68%	2.76%
$\beta = 0.80$	4.00%	7.00%	2.20%
$\boldsymbol{\beta} = \mathbf{0.74}$	**4.00%**	**7.00%**	**2.20%**
$\beta = 0.63$	4.00%	7.00%	2.20%
$\phi_{\max} = 0.03$	5.13%	9.53%	2.20%
$\boldsymbol{\phi}_{\max} = \mathbf{0.01}$	**4.00%**	**7.00%**	**2.20%**
$\phi_{\max} = 0.005$	3.72%	6.46%	2.20%
$\lambda_d = 0.027$	14.91%	0.00%	0.00%
$\boldsymbol{\lambda}_d = \mathbf{0.02}$	**4.00%**	**7.00%**	**2.20%**
$\lambda_d = 0.005$	4.00%	7.00%	2.20%
$r = 0.03$	8.44%	14.64%	2.20%
$\mathbf{r = 0.017}$	**4.00%**	**7.00%**	**2.20%**
$r = 0.005$	1.44%	2.70%	2.20%
$\theta = 0.92$	4.00%	7.00%	2.20%
$\theta = 0.86$	4.00%	7.00%	2.20%
$\boldsymbol{\theta} = \mathbf{0.83}$	**4.00%**	**7.00%**	**2.20%**
$\theta = 0.80$	4.00%	7.00%	2.20%
$\theta = 0.74$	14.91%	0.00%	0.00%

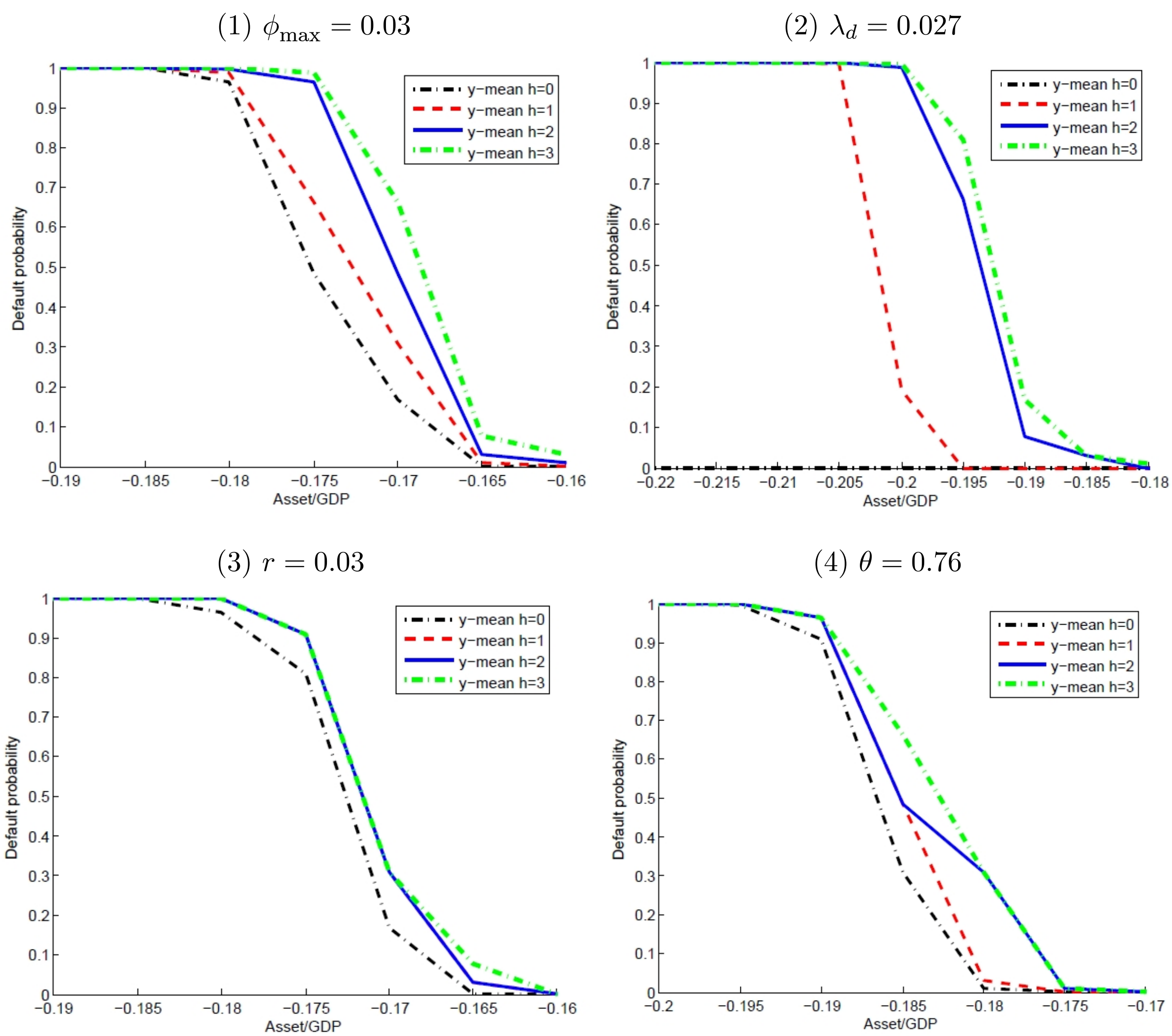
With lower bargaining power ($\theta = 0.74$), the default probability is zero and the average spread is high. Lower bargaining power at the renegotiation leads to a higher recovery rate, indicating higher default costs for the country. Thus, the country is less willing to default. If the country wants to avoid default, average spread will be higher as in the case of higher output cost mentioned above. Lastly, the discount factor (β) affects neither the average spread, standard deviation of spreads, nor the default probability.

Figure 9 displays the default probability of the country with respect to asset-to-GDP ratio under different parameter values. The left top panel shows that if the maximum level of the additional components in spreads is high ($\phi_{\max} = 0.03$), the default probability is higher than in the baseline case of Figure 5. As the country suffers a larger deterioration in long-run credit associated with past defaults due to the higher maximum level of additional components in spreads, it is more willing to default rather than paying higher spread returns.

Figure 9: Default probability under different parameter values

(1) $\phi_{\max} = 0.03$ (2) $\lambda_d = 0.027$

(3) $r = 0.03$ (4) $\theta = 0.76$



The right top panel displays that under higher output cost ($\lambda_d = 0.027$), default probability is lower than one in the baseline case, given the debt-GDP ratio. Since the country will face higher output loss associated with defaults, the country is less willing to choose to default.

The left bottom panel shows that an increase in risk-free interest rate ($r = 0.03$) leads to a higher default probability than one in the baseline case. As long as the default probability is non-zero, a higher risk-free interest rate leads to higher spreads as explained above. Due to higher spreads, it is more expensive for the country to pay back its debts, so it will be willing to default.

Finally, the right bottom panel indicates that under lower bargaining power ($\theta = 0.76$), it is more costly to default for the country since it needs to pay higher recovery rates at the renegotiation due to the lower bargaining power.

In all cases, default probability is weakly increasing respect to credit history, similar to Figure 5. Thus, the observation that the default probability is weakly increasing with respect to the country's credit history is robust under the above sensitivity analysis.

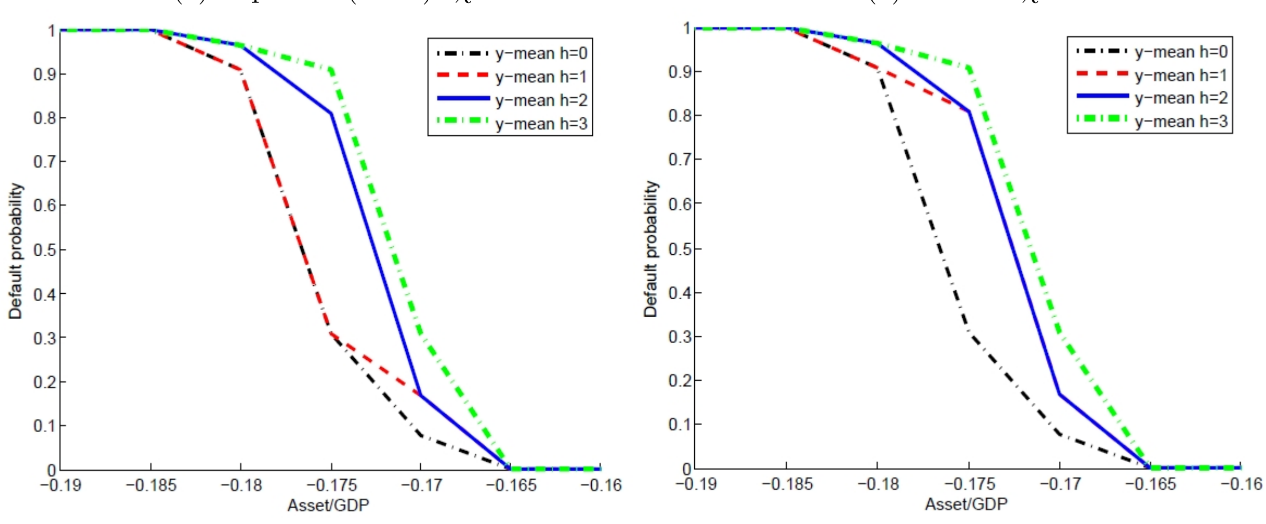
Moreover, one extension such as increasing bargaining power is discussed in Appendix E.

7.3 Limits of investors' memory and credit history

The discussion in this subsection centers on implications related to the length of investors' memory and upper limit of credit history. As apparent in Figure 10, the default probability does not differ from the baseline case if the credit history will never revert corresponding to the case $\chi = 0$. On the other hand, in the case that investors have the limited memory which lasts only 1 quarter and the credit history will be surely upgraded in the next period, the default probability with credit history $h_t = 1$ is lower than one in the baseline case. Moreover, the default probability with $h_t = 0, 1, 3$ is the same as the one in the baseline case. Still, the default probability is weakly increasing with respect to the credit history under both cases. Thus, the length of investors' memory does not matter much for the default probability as long as the country will be obliged to pay spread returns for at least one period.

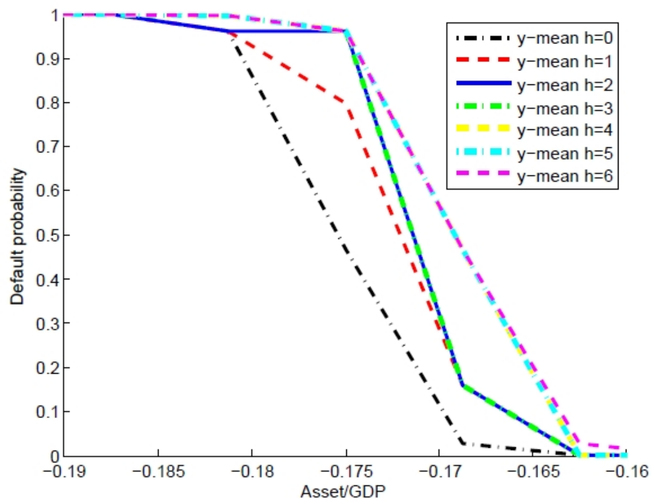
Figure 10: Default probability under short and infinite periods of investors' memory

(1) 1 quarter (short): $\chi = 1$ (2) Infinite: $\chi = 0$



Next, we increase the upper limit of credit history ($h_{\max}$). Figure 11 displays that as the credit history increases, the default probability weakly increases, given the debt-to GDP ratio as in the baseline case.

Figure 11: Default probability under higher maximum level of credit history ($h_{\max} = 6$)



Our quantitative result which the default probability is weakly increasing with respect to its credit history is robust to extensions related with limited periods of investors' memory and upper limit of credit history. Moreover, Table 8 shows that neither the probability of upgrading in credit history nor increase in limits of the credit history changes the average spreads, standard deviation of spreads and default probability.

Table 8: Sensitivity analysis for default probability and spreads

	Average spreads	Std. spreads	Default probability
Data	4.08%	1.68%	2.76%
$\chi = 0$	4.01%	7.01%	2.20%
$\boldsymbol{\chi = 0.025}$	**4.00%**	**7.00%**	**2.20%**
$\chi = 1$	4.01%	7.01%	2.20%
$\mathbf{h_{\max} = 3}$	**4.00%**	**7.00%**	**2.20%**
$h_{\max} = 4$	4.02%	7.01%	2.20%
$h_{\max} = 6$	4.02%	7.01%	2.20%

7.4 Welfare analysis

Lastly, we analyze the welfare of the country after the debt renegotiation. As mentioned above, the contract curve agreed to at the renegotiation presents a trade-off of the country; if the recovery rate is high implying that the country needs to pay back a higher amount in the current period, then it can reduce the increase in interest spreads, meaning lower deterioration in long-run credit. We evaluate the welfare of the country at the renegotiation along the contract curve quantitatively.

Figure 12: Contract curve and welfare of the country (in the highest income state) at the renegotiation

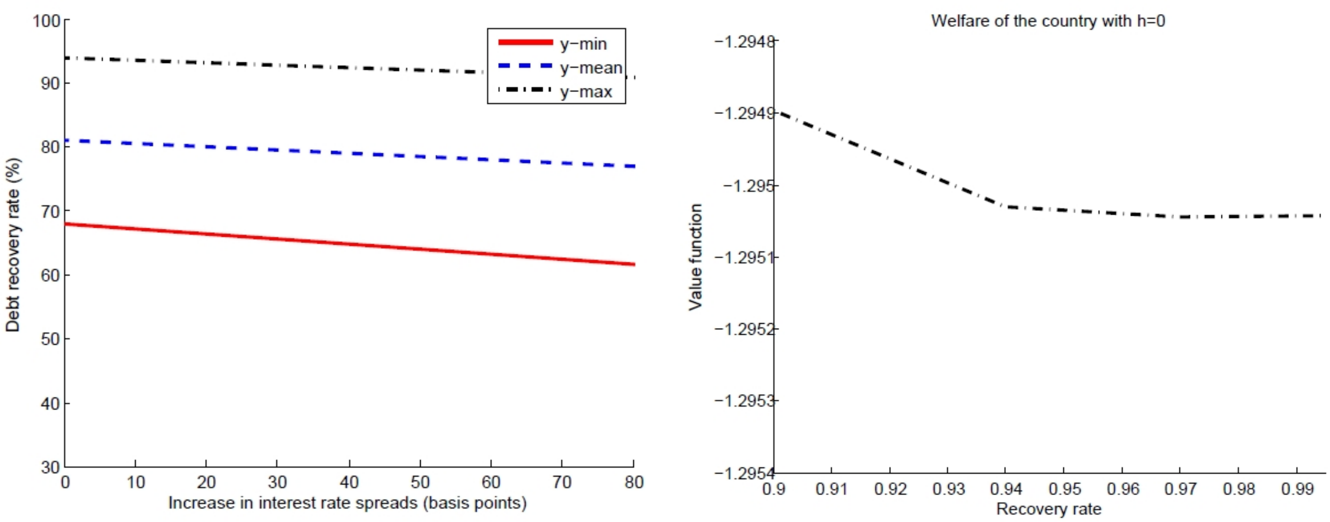


Figure 12 shows that welfare is decreasing with respect to the recovery rates determined at renegotiation. Equivalently, the welfare of the country with an increase in spread equal to 0.05 is higher than one with an increase in spread equal to 0.00. As shown in Figure A1, recovery rate is decreasing with respect to the debt-to-GDP ratio which is the driving force for this mechanism. In the highest income state, the recovery rate is close to 90% at the debt-to-GDP ratio -0.1095. On contrary, the recovery rate is near 100%, if the debt-to-GDP ratio is -0.10. Despite having suffered an increase in spreads in later periods, payments in the current period with a recovery rate of 90% (at the debt-to-GDP ratio -0.1095), are less than those with recovery rate 100% (at the debt-to-GDP ratio -0.10). This leads to a higher welfare for the country with a recovery rate of 90% than with recovery rate close to 100%.

8 Conclusion

Emerging countries that have defaulted on their debt repayment obligations in the past are more likely to default again in the future than are non-defaulters with the same debt-to-GDP ratio. This paper explains this stylized fact within a dynamic stochastic general equilibrium framework that explicitly models debt renegotiations between a defaulting country and its creditors. Specifically, the model extends the existing literature by allowing defaulters and creditors to bargain not just over recovery rates, but also over the rate of return offered on newly-issued debt. Quantitative analysis of the model reveals that the equilibrium probability of default for a given debt-to-GDP level is weakly increasing with the number of past defaults, consistent with empirical observations. The equilibrium of the model also corresponds with an additional observed trend: countries for which default terms require less than a 100 percent recovery rate tend to pay a higher rate of return (relative to the risk-free rate) on debt that is issued subsequently than do defaulting countries that agree to a full recovery rate. These findings are robust to extensions that allow for the negotiated outcome to be modeled more flexibly.

So far, we have considered the debt renegotiation under symmetric information across the country and investors. It might be possible that some of the information concerning the country's profile remains unrevealed to investors at the time of renegotiation, such as the country's government type as in Hachondo et al (2008), income process or actual level of output costs. A comparison of renegotiations between under perfect information and imperfect information could be a potential research topic in the future.

References

[1] Aguiar, M., and G. Gopinath, 2006, "Defaultable Debt, Interest Rates and the Current Account," *Journal of International Economics*, Vol.69(1), pp.64-83.

[2] Andrizky, J. R., 2005, "Default and Recovery Rates of Sovereign Bonds: A Case Study of the Argentine Crisis," *Journal of Fixed Income*, Vol.7, pp.97-107.

[3] Arellano, C., 2008, "Default Risk and Income Fluctuations in Emerging Economies," *American Economic Review*, Vol.98(3), pp.690-712.

[4] Bedford, P., Penalver, A., and C. Salmon, 2005, "Resolving the Sovereign Debt Crises: the Market-base Approach and the Role of the IMF," *Financial Stability Review*, Bank of England.

[5] Benjamin, D., and M. Wright, 2008, "Recovery Before Redemption? A Theory of Delays in Sovereign Debt Renegotiations," manuscript, University of Southhampton.

[6] Bi, R., 2008, ""Beneficial" Delays in Debt Restructuring Negotiations," IMF working paper No.WP/08/38, International Monetary Fund.

[7] Bulow, J., and K. Rogoff, 1989a, "LDC Debts: Is to Forgive or to Forget?" *American Economics Review*, Vol.79, pp.43-50.

[8] Bulow, J., and K. Rogoff, 1989b, "A Constant Recontracting Model of Sovereign Debt," *Journal of Political Economy*, Vol.97, pp.155-178.

[9] Chatterjee, S., Corbae, D., Nakajima, M, and J.V. Rios-Rull, 2007, "A Quantitative Theory of Unsecured Consumer Credit with Risk of Default," *Econometrica*, Vol.75(6), pp.1525-1589.

[10] De Paoli, B., Hoggarth, G., and V. Saporta, 2006, "Cost of Sovereign Defaults," *Financial Stability Paper* No.1, Bank of England.

[11] Eaton, J. and M. Gersovitz, 1981, "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *Review of Economic Studies*, Vol.48, pp.289-309.

[12] Eichengreen, B., Hausmann, R., and U. Panizza, 2003, "The Pain of Original Sin," in B. Eichengreen and R. Hausmann (eds.), *Debt Denomination and Financial Instability in Emerging Market Economies*, Chicago, University of Chicago Press.

[13] Finger H., and M. Mecagni, 2007, "Sovereign Debt Restructuring and Debt Sustainability - An Analysis of Recent Cross-Country Experience," IMF Occasional Paper No.255.

[14] Kovrinykh, N., and B. Szentes, 2007, "Equilibrium Default Cycles," *Journal of Political Economy*, Vol. 90(3), pp.621-639.

[15] Hachondo, J. C., Martinez, L., and H. Sapriza, 2008, "Heterogeneous Borrowers in Quantitative Models of Sovereign Default," forthcoming, *International Economic Review*.

[16] Mendoza. E., and V. Yue 2008, "A Solution to the Default Risk-Business Cycle Disconnect," NBER Working Paper No.13861.

[17] Moody's, 2007, "Sovereign Default and Recovery Rates, 1983-2006," *Moody's Special Comments*.

[18] Pitchford, R., and M. Wright, 2007, "Restructuring the Sovereign Debt Restructuring Mechanism," Research Department Staff Report, Federal Bank of Minneapolis.

[19] Reinhart, C., and K. Rogoff, 2004, "Serial Default and the "Paradox" of Rich-to-poor Capital Flows„" *American Economic Review*, Vol.94(2), pp.53-58.

[20] Reinhart, C., Rogoff, K., and M. Savastano, 2003, "Debt Intolerance," *Brookings Papers on Economic Activity*, Vol.1, Spring 2003, pp.1-74.

[21] Strurzeneger, F., 2002, "Default Episodes in the 90s; Fact Book Preliminary Lessons," manuscript, Universidad Torcuato Di Tella.

[22] Strurzeneger, F., and J. Zettelmeyer, 2006, *Debt defaults and lessons from a decade of crises*, MIT Press.

[23] Strurzeneger, F., and J. Zettelmeyer, 2007, "Haircuts: Estimating Investor Losses in Sovereign Debt Restructuring, 1998-2005," *Journal of International Money and Finance*, Vol.27, pp.780-805.

[24] Tauchen, G., 1986, "Finite State Markov-chain Approximations to Univariate and Vector Autoregressions," *Economic Letters*, Vol.20, pp.177-181.

[25] Tomz, M., and M. Wright, 2007, "Do Countries Default in "Bad" Times?" *Journal of the European Economic Association*, Vol.5(2-3), pp.352-360.

[26] Trebesch, C., 2008, "Delays in Sovereign Default Restructurings. Should we really blame the creditors?," manuscript, Free University Berlin.

[27] Yue, V., 2006, "Sovereign Default and Debt Renegotiation," manuscript, New York University.

A Computation Algorithm

The procedure to compute the equilibrium distribution of the model is the following.

(1) First, we set discrete grids on the space of credit history as $H = [0, 1, 2, 3]$ corresponding to $h_{\max} = 3$.

(2) Second, we set finite grids on the space of endowment and asset holdings as $B = [-0.3, ..., 0]$. The limits of the asset space are set to ensure that the limits do not bind in equilibrium. The limits of endowment space are big enough to include large deviations from the average value of shocks. We approximate the stochastic income process given by equation (24) using a discrete Markov chain of 21 equally spaced grids. Moreover, we calculate the transition matrix based on the probability distribution $\mu(y_{t+1}|y_t)$.

(3) Third, we set finite grids on the space of recovery rate and additional components in spreads. Limits of both recovery rates and additional components in spreads are set to ensure that they do not bind in equilibrium.

(4) Fourth, we set the initial values for equilibrium bond price, recovery rate, and interest rate spreads. We use the risk-free bond price ($q_1 = q^f = (1+r)^{-1}$) for the baseline value of equilibrium bond price. We use $\alpha_0 = 0.5$, and $\phi_0 = 0.01$ for the baseline recovery rate and interest rate spreads.

(5) Fifth, given the baseline equilibrium bond price ($q_0 = q^f$), recovery rate ($\alpha_0 = 0.5$), and spreads ($\phi_0 = 0.01$), we solve for the country's optimization problem for each credit history ($h_t = 0, 1, 2, ...$). This procedure finds the value function as well as the default decisions. We first guess the value function (V^0, $V^{D,0}, V^{R,0}$) and iterate it using the Bellman equation to find the fixed value (V^*, $V^{D*,}, V^{R*}$), given the baseline bond price, recovery rate, and spreads. By iterating the Bellman function, we also derive the optimal asset policy function for every value (a', a'^D, a'^R). For each credit history, we also obtain choices of default, which requires comparison of the values of defaulting and non-defaulting. By comparing the these two values, we calculate the corresponding default set.

(6) Sixth, using the default set in step (5), and the zero profit condition for foreign investors, we compute the new price of discounted bond (q_1). Then we iterate step (5) to have fixed value of equilibrium bond price.

(7) Seventh, given the value functions (V^*, $V^{D*,}, V^{R*}$), value of autarky (V^A), the payment of bonds (R^*) derived from the iterations above and the price of discounted bond (q^*), we solve the

bargaining problem and compute the new debt recovery schedule and interest rate spreads for every (b, h, y). Then, we iterate step (5), (6) to have the fixed optimal debt recovery rate (α^*), and the optimal interest rate (ϕ^*)

B Tables in Section 3

Table 1: Predicted Institutional Investor Ratings and Debt Intolerance Regions for Argentina and Malaysia

	Argentina		Malaysia	
External debt/GNP	Predicted *IIR*	Region	Predicted *IIR*	Region
0	51.4	**1**	61.1	**1**
5	49.3	**1**	59.0	**1**
10	47.3	**1**	57.0	**1**
15	45.2	**3**	54.9	**1**
20	43.2	**3**	52.9	**1**
25	41.1	**3**	50.8	**1**
30	39.1	**3**	48.8	**1**
35	37.0	**3**	46.7	**2**
40	34.9	**4**	44.7	**4**
45	32.9	**4**	42.6	**4**

Source: Reinhart, Rogoff, and Savastano (2003)

Note: 1. The Institutional Investor Ratings (IIR) are compiled twice a year, are based on information provided by economists and sovereign risk analysts at leading global banks and securities firms. The ratings grade each country on a scale from 0 to 100, with a ratings of 100 given to those countries perceived as having the lowest chance of defaulting on their government debt obligations.

2. For countries in club B (24.2$<$ IIR $<$67.7), the four regions (from least to most vulnerable) defined are : Least debt intolerant, Type **1** Region (45.9$\leq$IIR$\leq$67.7 and debt/GNP$<$35), quasi debt intolerant, Type **2** Region (45.9$\leq$IIR$\leq$67.7 and debt/GNP$>$35), quasi debt intolerant, Type **3** Region (25.2$\leq$IIR$\leq$45.9 and debt/GNP$<$35) and; most debt intolerant Type **4** Region (25.2$\leq$IIR$\leq$45.9 and debt/GNP$>$35).

Table 2: Stylized facts about sovereign debt renegotiations in 1998-2007*[1]

Country	Year*[2] of default	Defaulted debt ($ billions)	Defaulted debt*[2] (of GDP)	Recovery*[3] rates(%)	Increases*[7] in spreads
Expost-default					
Russia	1998	72.709	26.8%	35% *[4]	69.97
Ecuador	1999	6.604	39.6%	40% *[4]	7.73
Ecuador	2000	0.346	2.5%	100%	18.72
Ivory Coast	2000	15.6	148.3%	59%	16.84 *[8]
Argentina	2001	82.268	30.6%	37% *[4]	20.3
Grenada	2004	0.297	68.0%	60% *[5]	14.69
Moldova	2004	0.145	9.8%	42% *[6]	N.A.
Preemptive					
Pakistan	1998	1.627	2.7%	70% *[4]	35.87 *[8]
Ukraine	1998	1.271	3.9%	72% *[4]	34.05 *[8]
Ukraine	2000	1.064	3.4%	60%	47.85
Moldova	2002	0.04	2.4%	94% *[6]	N.A.
Dominica	2003	N.A.	N.A.	71%	N.A.
Uruguay	2003	5.744	51.3%	71% *[4]	11.54
Dominican Republic	2005	1.622	5.6%	95% *[5]	25.78
Belize	2006	0.242	19.9%	76% *[5]	2.59 *[8]

Source: Bedford, Penalver and Salomon (2005), Benjamin and Wright (2008), Datastream, Finger and Mecagni (2007) Moody's (2007) and Sturzeneger and Zettelmeyer (2007).

Note: *[1]We list only export-default and preemptive renegotiation episodes in 1998-2007. We exclude the cases of swap agreement or delay in payment such as Venezuela in 1995, 1998 and 2005, Peru in 2000 and Paraguay in 2003. *[2]Data (year of default and defaulted debt) is from Moody's (2007). The debt is total amount of sovereign bonds which the government defaulted on and does not include the private debt. *[3]Data for recovery rate is from Benjamin and Wright (2008). *[4]Recovery rates for Russia, Ecuador, Argentina, Pakistan, Ukraine, and Uruguay are from Strurzeneger and Zettelmeyer (2007). *[5]Recovery rates for Grenada, Dominican Rep. and Belize are from Bedford, Penalver and Salomon (2005). *[6]Recovery rate for Moldova 2002, 2004 is from Finger and Mecagni (2007). *[7]Data (spreads) is from J.P. Morgan's Emerging

Market Bond Index (EMBI) on Datastream and we define "increases in spreads" as a difference in spreads between at the time of renegotiations and one with one year before the renegotiations. *[8]Spread data for Pakistan and Ukraine is measured at 6/2002 and at 9/2001 respectively. Spread data for Ivory Coast and Belize is one of African composite sovereign bonds.

C Figures of bond prices

Bond prices are weakly decreasing respect to its new level of debts as shown in Figure A1 and also weakly increasing respect to its income as shown in Figure A2.

Figure A1: Bond price schedule with different levels of new debts

(a) $b_{t+1} = -0.22$ (b) $b_{t+1} = -0.12$

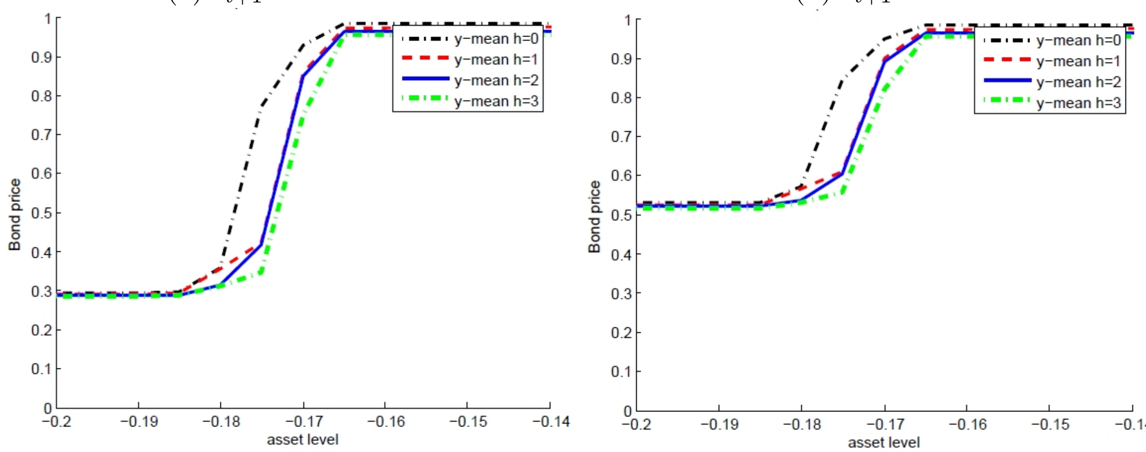
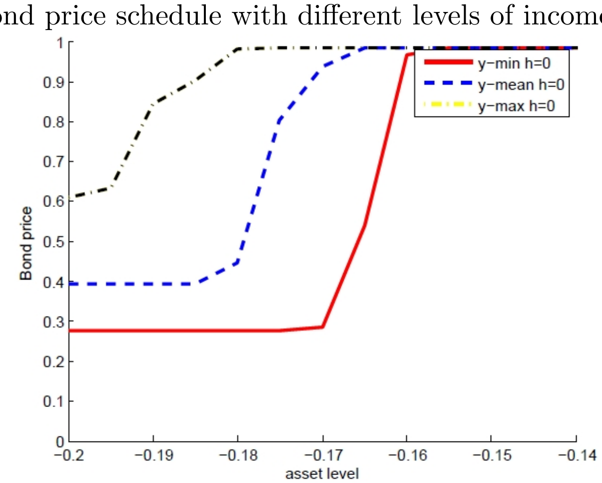


Figure A2: Bond price schedule with different levels of income ($b_{t+1} = -0.16$)



D Numerical results on equilibrium properties

Figure A3: Recovery rates and total spreads with credit history $h_t = 0$ and $h_t = 1$

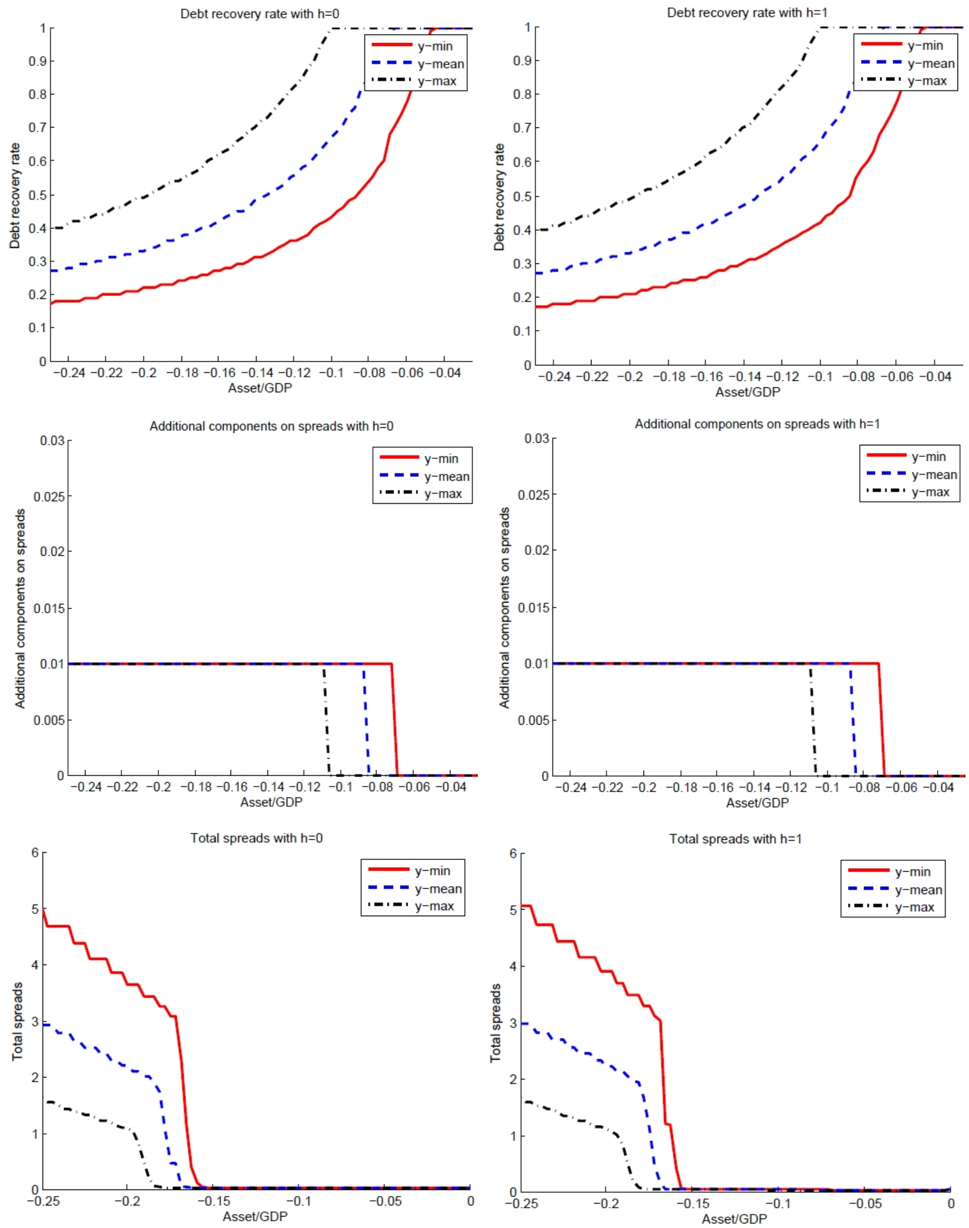


Figure A3 displays the recovery rates and total interest spreads after renegotiations conditional on defaults. For recovery rates, the general features are similar to previous papers. The recovery rates for credit history $h_t = 0$ and $h_t = 1$ are almost the same because that the bargaining power at renegotiation is constant for both credit history $h_t = 0$ and $h_t = 1$.

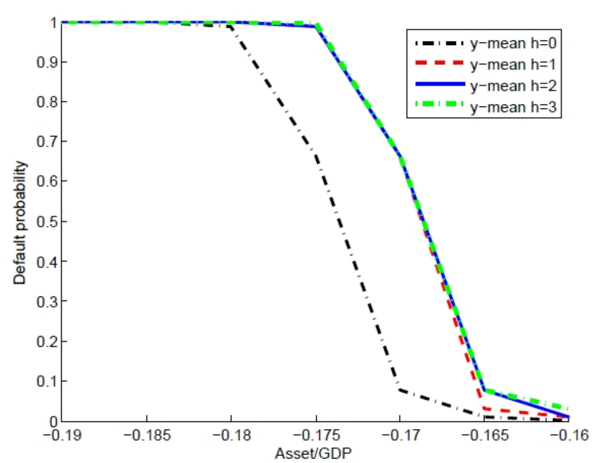
On the other hand, both additional components in spreads and total spreads are increasing functions of debts. Beyond the debt reduction threshold (-0.1 in the highest and -0.05 in the lowest income states), the total spread is zero. This implies that since the country recovers full amount of the defaulted debt, there is no need to pay additional spread returns for newly issued bonds leading to zero total interest rate spreads. Total interest rate spreads for credit history $h_t = 1$ are higher than for credit history $h_t = 0$. This is due to additional components in spreads agreed at negotiations which lead to higher probability of default at $h_t = 1$ than at $h_t = 0$.

E Default probability under different assumptions

In this subsection, we cover one extension case such that the bargaining power of the country increases as the credit history deteriorates. We assume the following set of the bargaining power, such as $\theta(h_t = 0) = 0.83$, and $\theta(h_t = i) = 0.85$ for $i = 1, 2$. Note the bargaining power increases as credit history deteriorates from $h_t = 0$ to $h_t = 1$[17].

Note that equilibrium default probability is increasing with credit history as shown in Figure A4. In the case of increasing bargaining power implying that the country has less a reputation to lose for consecutive defaults, lower recovery rates agreed at negotiation due to increase in bargaining power, leads to higher probability of default next time since it is less costly to defaults next time.

Figure A4. Default probability under increasing bargaining power



[17] Reinhart and Rogoff (2004) mention that serial defaulters such as countries which have defaulted several times in row, have less of a reputation to lose for the following defaults, while they lose large degree of a reputation at the first default. Sturzenegger and Zettelmeyer (2006) stress that the bargaining power of debtor may be larger when the economy is still in turmoil. These imply that at renegotiation of later defaults, the country will have higher degree of bargaining power since they have less of a reputation to lose, compared with the bargaining power at negotiation of the first default.