A Pyrrhic Victory? –
Bank Bailouts and Sovereign Credit Risk[1]

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Abstract

We develop a model in which financial sector bailout and sovereign credit risk are intimately linked. The bailout ameliorates the under-investment problem of the financial sector. However, as the bailout is ultimately funded through taxation of the future profits of the non-financial sectors, it weakens their incentives to invest. This can adversely affect the sovereign's own credit risk which severely limits the size of the efficient bailout. In the short-run, the bailout is funded through issuance of government bonds, which erodes the value of existing bonds held by the financial sector and further reduces the size of the efficient bailout. The model provides testable implications concerning the relation between the credit risk of the sovereign and its financial sector. We provide supporting empirical evidence using data from the credit default swaps market and bank stress tests conducted during the financial and sovereign crises of 2007-10.

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1 Introduction

On September 30, 2008 the government of Ireland announced that it had guaranteed all deposits of the six of its biggest banks. The immediate reaction that grabbed newspaper headlines the next day was whether such a policy of a full savings guarantee was anti-competitive in the Euro area. However, there was something deeper manifesting itself in the credit default swap (CDS) markets for purchasing protection against the sovereign credit risk of Ireland and that of its banks. Figure 1 shows that while the cost of purchasing such protection on Irish banks – their CDS fee – fell overnight from around 400 basis points to 150 basis points, the CDS fee for the Government of Ireland's credit risk rose sharply. Over the next month, this rate more than quadrupled to over 100 basis points and within six months reached 400 basis points, the starting level of its financial firms' CDS. While there was a general deterioration of global economic health over this period, the event-study response in Figure 1 suggests that the risk of the financial sector had been substantially transferred to the government balance sheet, a cost that Irish taxpayers must eventually bear.

Viewed as of the Fall of 2010, this cost has risen to dizzying heights prompting economists to wonder if the precise manner in which bank bailouts were awarded have rendered the financial sector rescue exorbitantly expensive. Just one of the Irish banks, Anglo Irish, has cost the government up to Euro 25 billion (USD 32 billion), amounting to 11.26% of Ireland's Gross Domestic Product (GDP). Ireland's finance minister Brian Lenihan justified the propping up of the bank "to ensure that the resolution of debts does not damage Ireland's international credit-worthiness and end up costing us even more than we must now pay." However, rating agencies and credit markets revised Ireland's ability to pay future debts significantly downward. The original bailout cost estimate of Euro 90 billion was re-estimated to be 50% higher and the Irish 10-year bond spread over German bund widened significantly as well, ultimately leading to a bailout of Irish government by the stronger Eurozone countries such as Germany.[1]

This episode is not isolated to Ireland though it is perhaps the most striking case. In fact, a number of Western economies that bailed out their banking sectors in the Fall of 2008 have experienced, in varying magnitudes, similar risk transfer between their financial sector and government balance-sheets. Our paper develops a theoretical model and provides

[1] See "Ireland's banking mess: Money pit – Austerity is not enough to avoid scrutiny by the markets", the Economist, Aug 19th 2010; "S&P downgrades Ireland" by Colin Barr, CNNMoney.com, Aug 24th 2010; and, "Ireland stung by S&P downgrade", Reuters, Aug 25th, 2010.

empirical evidence that help understand this interesting phenomenon. Our results call into serious question the assumption, implicit in much of the banking literature, that government resources are vastly deep and that the main problem posed by bailouts is primarily that of moral hazard – that is, the distortion of future financial sector incentives. While the moral hazard cost is certainly pertinent, our conclusion is that bailout costs are not just in the future, but are tangible right around the timing of bailouts and priced into the sovereign's credit risk and cost of borrowing. Aggressive bailout packages that stabilize financial sectors in the short run but ignore the ultimate taxpayer cost might end up being a Pyrrhic victory.

Our theoretical model consists of two sectors of the economy – "financial" and "corporate" (more broadly, also the household and other non-financial parts of the economy), and a government. The two sectors contribute jointly to produce aggregate output: the corporate sector makes productive investments and the financial sector invests in intermediation "effort" (e.g., information gathering and capital allocation) that enhance the return on corporate investments. Both sectors, however, face a potential under-investment problem. The financial sector is leveraged (in a crisis, it may in fact be insolvent) and under-invests in its contributions due to the well-known debt overhang problem (Myers, 1977). The corporate sector is un-levered for simplicity. However, if the government undertakes a "bailout" of the financial sector, in other words, makes a transfer from the rest of the economy that results in a net reduction of the financial sector debt, then the transfer must be funded in the future (at least in part) through taxation of corporate profits. Such taxation, assumed to be proportional to corporate sector output, induces the corporate sector to under-invest.

A government that is fully aligned with maximizing the economy's current and future output determines the optimal size of the bailout. In particular, it determines the size of a transfer funded through future proportional corporate tax rates that balances the trade off of reducing the financial sector's under-investment problem against aggravating under-investment by the corporate sector. We show that tax proceeds that can be used to fund the bailout, in general, have a *Laffer curve* property, so that the optimal bailout size and tax rate are interior. The optimal tax rate that the government is willing to undertake for the bailout is greater when the financial sector's debt overhang is higher and its relative contribution (or size) in output of the economy is larger.

In practice, governments fund bailouts in the short run by borrowing or issuing bonds. Since new issuance must be repaid in future by taxation, our qualitative insights carry through. There are, however, two interesting results that emerge. One, the greater is the legacy debt of the government, the lower is its ability to undertake a bailout. This is because the Laffer curve of tax proceeds leaves lesser room for the government to increase tax rates for repaying its bailout-related debt. Second, the announcement of the bailout lowers the price of government debt due to the anticipated dilution from newly issued debt. Now, if the financial sector of the economy has assets in place that are in the form of government bonds, the bailout is in fact associated with some "collateral damage" for the financial sector itself. The possibility of such a two-way feedback in which bailouts cause government bonds to lose value further limits the size of the optimal bailout.

All of the above results hold in a model even with no uncertainty about future output of the economy. We extend the model to allow for uncertainty in the output of the corporate sector, keeping financial sector's outcomes as deterministic. This introduces a possibility of default on government debt. We assume that there are some deadweight costs of such default, for example, due to international sanctions or from being unable to borrow in debt markets for some time. Then, the greater the uncertainty of economic output, the greater is the quantity of bonds the government needs to issue in order to undertake a given bailout transfer and greater is the rise in its risk of default. As before, greater legacy debt of the government also limits optimal bailout size and raises sovereign credit risk more upon the bailout announcement.

Interestingly, due to the deadweight costs of default, there may be a precautionary component to government taxation. And, finally, given the collateral damage channel, an increase in uncertainty about sovereign's economic output not only lowers its own debt values but also increases the financial sector's risk of default (as some of its assets in place, e.g., government bond holdings, fall in value). The latter induces a post-bailout co-movement between financial sector's credit risk and that of the sovereign even though the immediate effect of the bailout is to lower financial sector's credit risk and raise that of the sovereign.

Our empirical work analyzes financial sector bailouts in Western economies during the financial crisis of 2007-10 and corroborates the theory. In our non-parametric analysis, we examine sovereign and banks CDS in the period from 2007 to 2010 and find three distinct periods. The first period covers the start of the financial crisis in January 2007 until the bankruptcy of Lehman Brothers. Across all Western economies, we see a large rise in bank CDS but sovereign CDS remain small. This evidence is consistent with a significant increase in the default risk of the financial sector with little effect on sovereigns in the pre-bailout period.

The second period covers the banks bailouts starting with the announcement of a bailout in Ireland in late September 2008 and ending with a bailout in Sweden in late October 2008.

During this one-month period, we find a significant decline in bank CDS across all countries and a corresponding increase in sovereign CDS. This evidence suggests that bank bailouts transferred the default risk from the financial sector to the sovereign. The third period covers the period after the bank bailouts until early 2010. We find that both sovereign and bank CDS are increasing during this period and the increase is larger for countries with significant public debt. This evidence suggests that banks and sovereigns share the default risk after the announcement of banks bailouts and that the risk is increasing in the relative size of countries' public debt.

In our parametric analysis, we examine the impact of sovereign credit risk on bank credit risk. We use two separate data sets for our analysis. First, we collect bank-level data on holdings of different sovereign government bonds released as part of the bank stress tests conducted for Europeans banks in 2010. We find that on average a 10% decrease in the value of a bank's sovereign holdings is associated with a 1.4% increase in the bank's credit risk measured using CDS prices. This result is robust to controlling for market-wide fluctuations and excluding home-country sovereign bonds. This evidence suggests that changes in the value of sovereign holdings have a statistically and economically important effect on bank credit risk.

Second, we collect credit ratings data as an independent measure of a bank's expected profitability. Controlling for credit ratings, we find that a 10 basis point increase in sovereign CDS is associated with a 5.4 basis point increase in bank CDS in 2010. The association is stronger for banks with lower credit ratings. Hence, banks of similar quality have higher credit risk if they are located in a country with higher sovereign credit risk. This evidence suggests that sovereign credit risk directly affects bank credit risk.

To summarize, we consider the appearance of meaningful sovereign credit risk as an important cost of bank bailout. This cost is a reflection of the future taxation (or inflation) risk imposed on corporate and household sectors of the economy. Such an *ex-post* cost of bailouts has received little theoretical attention and has also not been analyzed much empirically (except for some recent papers we cite in Section 2). Taking cognizance of this ultimate cost of bailouts has important consequences for the future resolution of financial crises, the design of fiscal policy, and the nexus between the two. Finally, Reinhart and Rogoff (2009a, b) and Reinhart and Reinhart (2010) document, for instance, that economic activity remains in deep slump "after the fall" (that is, after a financial crisis), and private debt shrinks significantly while sovereign debt rises, especially beyond a threshold of 90% debt to GDP ratio of the sovereign. These effects are potentially all consistent with our model of how financial sector bailouts affect sovereign credit risk and economic growth.

The remainder of the paper is organized as follows. Section 2 discusses the related literature. Section 3 presents our theoretical analysis, starting with a benchmark model without uncertainty about solvency of the economy and next analyzing the model with uncertainty. Section 5 provides evidence supporting the model's implications using sovereign and financial firm credit default swaps data around the financial crisis of 2007–09. Section 6 concludes. All proofs not in the main text of the paper are contained in the Appendix.

2 Related literature

Our paper is related to three different strands of literature: (i) the theoretical literature on bank bailouts; (ii) the literature on costs of sovereign default; and, (iii) the recent empirical literature on effects of bank bailouts on sovereigns.

The theoretical literature on bank bailouts has mainly focused on *how* to structure bank bailouts efficiently. While the question of *how* necessarily involves an optimization with some frictions, the usual friction assumed is the inability to resolve failed bank's distress entirely due to agency problems. This could be due to under-investment problem as in our setup (e.g., Philippon and Schnabl, 2009), adverse selection (e.g., Gorton and Huang, 2004), risk-shifting or asset substitution (e.g., Acharya, Shin and Yorulmazer, 2008, Diamond and Rajan, 2009), or tradeoff between illiquidity and insolvency problems (e.g., Diamond and Rajan, 2005). Some other papers (Philippon and Schnabl, 2010, Bhattacharya and Nyborg, 2010, among others) focus on specific claims through which bank bailouts can be structured to limit these frictions.

A large body of existing literature in banking considers that bank bailouts are inherently a problem of time consistency and induce moral hazard at individual-bank level (Mailath and Mester, 1994) and at collective level through herding (Penati and Protopapadakis, 1988, Acharya and Yorulmazer, 2007). Aghion, Bolton and Fries (1999) consider the cost that bank debt restructuring can in some cases delay the recognition of loan losses. Brown and Dinc (2009) show empirically that the governments are more likely to rescue a failing bank when the banking system, as a whole, is weak.

A small part of this literature, however, does consider *ex-post* costs of bailouts. Notably, Diamond and Rajan (2005, 2006) study how bank bailouts can take away a part of the aggregate pool of liquidity from safe banks and endanger them too. Acharya and Yorulmazer (2007, 2008) model, in a reduced-form manner, a cost of bank bailouts to the government or regulatory budget that is increasing in the quantity of bailout funds. They provide taxation-related fiscal costs as a possible motivation. Panageas (2010a,b) considers the optimal taxation to fund bailouts in a continuous-time dynamic setting, also highlighting when banks might be too big to save.

In the theoretical literature on sovereign defaults, Bulow and Rogoff (1989a, 1989b) initiated a body of work that focused on ex-post costs to sovereigns of defaulting on external debt, e.g., due to reputational hit in future borrowing, imposition of international trade sanctions and conditionality in support from multi-national agencies. Broner and Ventura (2005), Broner, Martin and Ventura (2007), Acharya and Rajan (2010) and Gennaioli, Martin and Rossi (2010), among others, consider a collateral damage to the financial institutions and markets when a sovereign defaults. They employ this as a possible commitment device that gives the sovereign "willingness to pay" its creditors. Our model considers both of these effects, an ex-post deadweight cost of sovereign default in external markets as well as an internal cost to the financial sector through bank holdings of government bonds.

Some recent empirical work focuses on the distortionary design of bank bailout packages. Acharya and Sundaram (2009) document how the loan guarantee program of the Federal Deposit Insurance Corporation in the Fall of 2008 was charged in a manner that favored weaker banks at the expense of safer ones, producing a downward revision in CDS spreads of the former. Veronesi and Zingales (2009) conduct an event study and specifically investigate the U.S. government intervention in October 2008 through TARP and calculate the benefits to banks and costs to taxpayers. They find that the government intervention increased the value of banks by over $100 billion, primarily of bank creditors, but also estimate a tax payer cost between $25 to $47 billion. Panetta et al. (2009) and King (2009) assess the Euro zone bailouts and reach the conclusion that while bank equity was wiped out in most cases, bank creditors were backstopped reflecting a waiting game on part of bank regulators and governments.

Finally, our empirical work relating financial sector and sovereign credit risk during the ongoing crisis shares some similarity to the very recent papers on this theme. Sgherri and Zoli (2009) and Attinasi, Checherita and Nickel (2009) focus on the effect of bank bailout announcements on sovereign credit risk measured using CDS spreads. Some of their evidence mirrors our descriptive evidence. Dieckmann and Plank (2009) analyze sovereign CDS of developed economies around the crisis and document a significant rise in co-movement following the collapse of Lehman Brothers. Demirguc-Kunt and Huizinga (2010) do an international study of equity prices and CDS spreads around bank bailouts and show that some large banks may be too big to save rather than too big to fail. Our analysis corroborates and complements some of this work. In particular, our empirical investigation of banking sector holdings of government debt and how this introduces a linkage between bank CDS and sovereign CDS is novel.

3 Model

We first sketch in words the setup and timing of the model and then present it formally. The productive economy consists of two parts, a financial sector and a non-financial sector. In addition, there is a government and a representative consumer. All agents are risk-neutral. The government has two policy instruments available to it: a proportional tax rate and a wealth transfer (injection) into the financial sector. The transfer is accomplished by issuing government bonds and giving them to the financial sector. The objective of the government is to maximize the welfare of the consumer, who consumes the output of the economy net of investment costs.

The financial sector has both liabilities and assets on its books. If, at maturity, the liabilities are greater than the combined value of its assets and the profits it generates, the financial sector is liquidated. In that case, the financial sector receives no payoff and its efforts go unrewarded. The financial sector generates operating revenues by providing financial services to the non-financial sector in a competitive market. Since it is liquidated if its assets' payoffs plus profits are not great enough, the financial sector suffers from debt-overhang. In other words, when the liabilities of the financial sector are large, the efforts of the financial sector are likely to go unrewarded, since under liquidation, the operating revenues generated by those efforts will be captured by its creditors.

The non-financial sector produces output by combining financial services and its own capital. It must decide how much financial services to buy, given the cost of financial services. It must also decide how much capital investment to make, taking into account the proportional tax rate levied on the investment's future investment payoffs by the government.

The government wants to maximize the total output of the economy, and hence the welfare of the consumer, by reducing the debt-overhang problem of the financial sector. This will spur greater effort by the financial sector and increase total output. To do this, it can increase the assets of the financial sector by making a transfer of government bonds to the balance sheet of the financial sector. The government funds these treasuries with a proportional tax on the future payoffs of the investment of the non-financial sector. This new debt issuance adds to the existing stock of debt that the government has accumulated from past activities. Hence, the government is faced with two decisions. First, it must choose the optimal tax rate to funds its new and existing treasuries. Second, it must choose how many new treasuries to issue to make the transfer to the non-financial sector.

Finally, the representative consumer chooses his portfolio, which consists of his holdings of government bonds, and he consumes the output generated by the non-financial sector.

3.1 Setup

There are three time periods in the model: $t = 0$, 1, and 2.

At $t = 0$, the operator of the financial sector faces the following problem, which involves the choice of the amount of financial services to supply at $t = 0$ in order to maximize the expected value of its net payoff at $t = 1$:

$$\max_{s_0^s} E_0 \left[\left(w_s s_0^s - L_1 + \tilde{A}_1 + A_G + T_0 \right) \times 1_{\left\{-L_1+\tilde{A}_1+A_G+T_0>0\right\}} \right] - c(s_0^s) \tag{1}$$

where s_0^s is the amount of financial services supplied by the financial sector at $t = 0$. The financial sector earns revenues at the rate of w_s per unit of output, with w_s being determined in equilibrium. To produce s_0 units, the operator of the financial sector needs to expend $c(s_0)$ units of effort. We assume that $c'(s_0) > 0$ and $c''(s_0) > 0$. L_1 denotes the liabilities of the financial sector, which are due (mature) at $t = 1$. We distinguish between two types of assets held by the financial sector, denoted $\tilde{A}_1$ and A_G. A_G is the value of the financial sector's holdings of a fraction k_A of outstanding government bonds, while $\tilde{A}_1$ represents the payoff of the other assets held by the financial sector.[2] We model the payoff $\tilde{A}_1$ as a continuously valued random variable that is realized at $t = 1$ and takes values in $[0, \infty)$. The payoff and value of government bonds is discussed below. Finally, T_0 represents the value of the time 0 transfer made by the government to the financial sector.

The financial sector operator maximizes the expected payoff at $t = 1$ net of the effort cost required to produce the financial services it sells at $t = 0$, as indicated in (1) by the time-0

[2]While we refer to government claims principally as government bonds, a broader interpretation can include claims on quasi-governmental agencies (e.g., Fannie Mae, Freddie Mac) and perhaps also the value of explicit and implicit government guarantees or support.

expectation. Note that the financial sector operator only receives a positive payoff at $t = 1$ if $-L_1 + \tilde{A}_1 + A_G + T_0 > 0$. In other words, there is a payoff only if the financial sector is solvent at time 1. In case of insolvency, debtholders receive ownership of all financial sector assets and wage revenue.[3]

The non-financial sector comes into $t = 0$ with an existing capital stock K_0. Its objective is to maximize the sum of the expected value of its net payoffs at times 1 and 2:

$$\max_{s_0^d,\, K_1} E_0 \left[f(K_0, s_0^d) - w_s s_0^d + (1-\theta_0)\tilde{V}(K_1) - (K_1 - K_0) \right] \tag{2}$$

The function f is the production function of the non-financial sector, which takes as inputs the financial services it demands (buys), s_0^d, and its capital stock K_0. Using these inputs, it produces consumption goods as output at $t = 1$. The output from this function is deterministic. Moreover, we assume that f is increasing in both arguments and concave. At $t = 1$, the non-financial sector is faced with a decision of how much capital K_1 to invest in a project $\tilde{V}$, whose payoff is realized at $t = 2$. This project represents the future or continuation value of the non-financial sector and is in general subject to uncertainty. The expectation at $t = 1$ of this payoff is $V(K_1) = E_1[\tilde{V}(K_1)]$ and, as indicated, is a function of the investment K_1. Moreover, we assume that $V'(K_1) > 0$ and $V''(K_1) < 0$, so that the expected payoff is increasing but concave in investment. A proportion θ_0 of the payoff of the continuation project is taxed by the government to pay its debt, both new and outstanding. The tax rate θ_0 is set by the government at $t = 0$ (though the tax is taken at $t = 2$ upon realization of the project's payoff). We assume that the government credibly commits to this tax rate. Finally, the incremental cost of investing K_1 in the continuation project is $K_1 - K_0$.

As mentioned above, the government issues bonds to make the transfer to the financial sector. These bonds are paid out from the taxes levied on the non-financial sector at a tax-rate of θ_0. Let N_D denote the number of bonds that the government has issued in the past – its outstanding stock of debt. For simplicity, we let bonds have a face value of one, so the face value of outstanding debt equals the number of bonds, N_D. The government issues N_T new bonds to accomplish the transfer to the financial sector. Hence, at $t = 2$ the government receives realized taxes equal to $\theta_0 \tilde{V}(K_1)$ and then uses them to pay bondholders $N_T + N_D$.

[3]Note that we could include the wage revenues in the solvency indicator function, which would provide an additional channel for wages to feedback into the probability of solvency. Although such a channel would reinforce the mechanism at work in the model, we choose to abstract from this to avoid the additional complexity.

We assume that if there are still tax revenues left over (a surplus), the government spends them on programs for the representative consumer, or equivalently, just rebates them to the consumer. On the other hand, if tax revenues fall short of $N_T + N_D$, then the government defaults on its debt. In that case, it pays only a fraction $\tilde{m} < 1$ of the debt's face value and gives back any remaining tax funds to the consumer.[4] We assume that the government credibly commits to a payout policy (a policy for $\tilde{m}$) and that this policy is known. We further assume that default incurs a deadweight loss. In case of default, the sovereign incurs a fixed deadweight loss of D. Hence, default is costly and there is an incentive to avoid it.[5] Finally, P_0 denotes the price of government bonds, which is determined in equilibrium. Hence, we have that the value of financial sector holdings of government bonds is $A_G = k_A P_0 N_D$.

The government's objective is to maximize the expected utility of the representative consumer, who consumes the combined output of the financial and non-financial sector. Hence, the government faces the following problem:

$$\max_{\theta_0, N_T} E_0 \left[f(K_0, s_0) + \tilde{V}(K_1) - c(s_0) - (K_1 - K_0) - 1_{def} D + \tilde{A}_1 \right] \tag{3}$$

where s_0 is the equilibrium provision of financial services. This maximization is subject to the budget constraint: $T_0 = P_0 N_T$ and subject to the choices made by the financial and non-financial sectors. Note that 1_{def} is an indicator function that equals 1 if the government defaults (if $\theta_0 \tilde{V}(K_1) < N_T + N_D$) and 0 otherwise.

Finally, the representative consumer solves a simple consumption and portfolio choice problem by allocating his wealth W between consumption, government bond holdings, and equity in the financial and non-financial sectors. For our purposes this provides the pricing condition for government bonds, which is simtle since the representative consumer is assumed to be risk-neutral. Let $P(i)$ and $\tilde{P}(i)$ denote the price and payoff of asset i, respectively. At $t = 0$, the consumer chooses optimal portfolio allocations, $\{n_i\}$, that solve the following problem:

$$\max_{n_i} E_0 \left[\Sigma_i n_i \tilde{P}(i) + (W - \Sigma_i n_i P(i)) \right] \tag{4}$$

[4] If upon default the government pays out all of the tax revenue raised to bond holders then $\tilde{m} = \theta_0 \tilde{V}(K_1)/(N_T + N_D)$.

[5] Although D here is obviously reduced-form, one can think of the deadweight cost in terms of loss of government reputation internationally, loss of domestic government credibility, degradation of the legal system and so forth. If a country's reputation is already weak, it will have less to lose from default.

Then the consumer's first order condition and market-clearing give the standard result that for each asset the equilibrium price equals the expected value of the payoff, $P(i) = E_0[\tilde{P}(i)]$.

4 Equilibrium Outcomes

We begin by examining the maximization problem of the financial sector. Let $p(\tilde{A})$ denote the probability density of $\tilde{A}$. Furthermore, let $\underline{A}_1$ be the minimum realization of $\tilde{A}_1$ for which the financial sector does not default: $\underline{A}_1 = L_1 - A_G - T_0$. The first order condition of the financial sector can now be written as:

$$\frac{\partial}{\partial s_0^s}\left(\int_{\underline{A}_1}^{\infty}\left(w_s s_0^s - L_1 + \tilde{A}_1 + A_G + T_0\right) p(\tilde{A}_1) d\tilde{A}_1\right) - c'(s_0^s) = 0 \ .$$

This evaluates to

$$w_s \int_{\underline{A}_1}^{\infty} p(\tilde{A}_1) d\tilde{A} - c'(s_0^s) = 0 \ . \tag{5}$$

We denote the quantity $\int_{\underline{A}_1}^{\infty} p(\tilde{A}_1) d\tilde{A}$, which is the probability that the financial sector is solvent at $t = 1$, by p_{solv}. We assume that at the optimal $\hat{s}_0^s$ the first-order condition is satisfied.

The second-order condition of the financial sector's problem is:

$$- c''(s_0^s) < 0 \ . \tag{6}$$

The parametric choice we will use below for $c(s_0)$ is $c(s_0) = \beta \frac{1}{m} s_0^m$ where $m > 1$.[6]

Consider now the problem of the non-financial sector at $t = 0$. Its demand for financial services, $\hat{s}_0^d$, is determined by its first-order condition:

$$\frac{\partial f(K_0, s_0^d)}{\partial s_0^d} = w_s \ . \tag{7}$$

Since f is concave in its arguments, the second order condition is satisfied:

$$\frac{\partial^2 f(K_0, s_0^d)}{\partial^2 s_0^d} < 0 \ . \tag{8}$$

[6]For this choice, it is the case that $\hat{s}_0^s \in (0, \frac{w_s}{s_0}^{1/(m-1)}]$. To see this, note that the derivative of the objective function of the financial sector is negative on (w_s, ∞), so the maximum must lie on $[0, w_s]$. Moreover, if there is any probability of solvency, the derivative of the objective at $s_0 = 0$ is positive, so the optimal s_0 lies in the interior, $(0, w_s]$.

Henceforth, we will parametrize f as Cobb-Douglas with the factor share of financial services given by ϑ: $f(K_0, s_0) = \alpha K_0^{1-\vartheta} s_0^{\vartheta}$.

In equilibrium the demand and supply of financial services are the same: $\hat{s}_0^d = \hat{s}_0^s$. From here on, we drop the superscripts on s_0 and denote the equilibrium quantity of financial services by s_0.

4.1 Transfer Reduces Underprovision of Financial Services

Taken together, the first-order conditions of the financial sector (5) and non-financial sector (7) show how debt-overhang impacts the provision of financial services by the financial sector. The marginal benefit of an extra unit of financial services to the economy is given by w_s, while the marginal cost, $c'(s_0)$, is less than w_s if there is a positive probability of insolvency. This implies that the equilibrium allocation is sub-optimal. The reason is that the possibility of liquidation $p_{solv} < 1$ drives a wedge between the marginal benefit that increased provision of financial services provides to economy and the marginal benefit it provides to the financial sector. The result is that as long as $p_{solv} < 1$, there is an under-provision of financial services relative to the first-best case ($p_{solv} = 1$). By making the transfer T_0, the government increases p_{solv} and reduces this debt-overhang problem.

Lemma 1. *An increase in the transfer T_0 increases the probability that the financial sector is solvent at $t = 1$, leading to an increase in the provision of financial services s_0 in equilibrium (equivalently, a decrease in the under-provision of financial services).*

Proof: See Appendix A.1.

4.2 Tax Revenues: A Laffer Curve

Next, to understand the government's problem, we first look at how expected tax revenue responds to the tax rate, θ_0. Let the expected tax revenue, $\theta_0 V(K_1))$, be denoted by $\mathcal{T}$. Raising taxes has two effects. On the one hand, an increase in the tax rate θ_0 captures a larger proportion of the future value of the non-financial sector, thereby raising tax revenues. On the other hand, this reduces the incentive of the non-financial sector to invest in its future, thereby leading to reduced investment, K_1. At the extreme, when $\theta_0 = 1$, the tax distortion eliminates the incentive for investment and tax revenues are reduced to zero. Hence, tax revenues are non-monotonic in the tax rate and revenues are maximized by a tax rate strictly less than 1.

Formally, the impact on tax revenue of an increase in the tax rate is given by:

$$\frac{\partial \mathcal{T}}{\partial \theta_0} = V(K_1) + \theta_0 V'(K_1)\frac{dK_1}{d\theta_0}$$

Note that at $\theta_0 = 0$, an increase in the tax rate increases the tax revenue at a rate equal to $V(K_1)$, the future value of the non-financial sector. It can be shown that since the production function $V(K_1)$ is concave, as taxes are increased, the incentive to invest is decreased by the tax rate, which reduces the marginal revenue of a tax increase. This is given by the second term on the right-hand side of the expression. To see this, consider the first-order condition for investment of the non-financial sector at $t = 1$:

$$(1 - \theta_0)V'(K_1) - 1 = 0 \tag{9}$$

Since $V''(K_1) < 0$, the second-order condition holds. Taking the derivative with respect to θ_0, using the Implicit Function theorem, and solving for $\partial K_1/\partial \theta_0$ gives:

$$\frac{dK_1}{d\theta_0} = \frac{V'(K_1)}{(1 - \theta_0)V''(K_1)} < 0$$

which shows that as the tax rate is increased, the non-financial sector reduces investment. In fact, since we know that at $\theta_0 = 1$ the tax revenue is zero, it must be the case that as the tax rate is increased, the marginal tax revenue decreases until it eventually becomes negative.

To summarize, tax revenues satisfy the Laffer curve property as a function of the tax rate:

Lemma 2. *The tax revenues, $\theta_0 V(K_1)$, increase in the tax rate, θ_0, as it increases from zero (no taxes), and then eventually decline.*

Henceforth, we parameterize V with the functional form $V(K_1) = K_1^\gamma$, $0 < \gamma < 1$.[7] As Appendix A.3 shows, (9) then implies that $\mathcal{T} = \theta_{t+1}\gamma^{\frac{\gamma}{1-\gamma}}(1 - \theta_{t+1})^{\frac{\gamma}{1-\gamma}}$. It can then be shown that:

Lemma 3. *$\mathcal{T}$ is maximized at $\theta_0^{max} = 1 - \gamma$. Furthermore $\mathcal{T}$ is increasing ($d\mathcal{T}/d\theta_0 > 0$) and concave ($d^2\mathcal{T}/d\theta_0^2 < 0$) on $[0, \theta_0^{max})$, and decreasing ($d\mathcal{T}/d\theta_0 < 0$) on $(\theta_0^{max}, 1)$.*

[7]This functional form is a natural choice for an increasing and concave function of K_1. Appendix A.2 provides a more structural motivation for this choice based on the calculation of a continuation value under our choice of production function. This calculation suggests that the continuation value implied by a multiperiod model should take a similar functional form.

4.3 Optimal Transfer Under Certainty and No Default

Next, we analyze the government's decision starting first with a simplified version of the general setup. We make two simplifying assumptions: (1) we set to zero the variance of the realized future value of the non-financial sector, so that $\tilde{V}(K_1) = V(K_1)$, (2) we force the government to remain solvent. In subsequent sections, we remove these assumptions. Since the government must remain solvent, it can only issue a number of bonds N_T that it can pay off in full, given its tax revenue. By assumption (1), the tax revenue is known exactly (it is equal to $\mathcal{T}$), and hence by assumption (2), $N_T + N_D = \mathcal{T}$. Moreover, since every bond has a sure payoff of 1, we know that the bond price is $P_0 = 1$.

Under the two simplifying assumptions, we have that the transfer is $T_0 = \theta_0 V(K_1) - N_D$ and there is no probability of default, $E[1_{def}] = 0$. Hence, the only choice variable for the government in this case is the tax rate. Since there is no change in the non-financial sector's investment opportunities between $t = 0$ and $t = 1$, the government's information regarding expected tax revenue is the same at $t = 0$ as at $t = 1$, and we can consider the problem directly at $t = 0$. Appendix A.4 shows that the first-order condition for the government can be written as:

$$\left[\frac{\partial f(K_0, s_0)}{\partial s_0} - c'(s_0)\right] \frac{ds_0}{dT_0} + \left[V'(K_1) - 1\right] \frac{dK_1}{d\mathcal{T}} = 0 \tag{10}$$

which expresses the first-order condition in terms of the choice of transfer size and expected tax revenue, rather than in terms of the tax rate. As we explain below, this condition is intuitive since it equates the marginal benefit and marginal cost of increasing tax revenue.

4.3.1 Gain From Increased Provision of Financial Services

The first term on the left side of (10) is the marginal gain to the economy of increasing expected tax revenue. This gain is due to an increase in the provision of financial services that occurs as the debt-overhang of the financial sector is mitigated by an increase in the transfer. Let $\mathcal{G}$ be the gain from the transfer relative to the no-transfer economy. Then $\mathcal{G} = (f(K_0, s_0) - c(s_0)) - (f(K_0, s(T_0 = 0)) - c(s(T_0 = 0))$, where $s(T_0 = 0)$ is the equilibrium level of financial services provided in the economy if there is no transfer. The marginal gain due to an increase in $\mathcal{T}$ is then given by $d\mathcal{G}/d\mathcal{T} = \left[\frac{\partial f(K_0,s_0)}{\partial s_0} - c'(s_0)\right] \frac{ds_0}{dT_0}$, which is the first left-hand side term.

From (5) and (7), we have that $\left[\frac{\partial f(K_0,s_0)}{\partial s_0} - c'(s_0)\right] = \frac{\partial f(K_0,s_0)}{\partial s_0}(1 - p_{solv})$, which is greater than zero as long as $p_{solv} < 1$. This term represents the wedge created by debt-overhang between the private and social benefit of increasing the provision of financial services. The result of this distortion in incentives is the under-supply of financial services. Since, as shown above, $ds_0/dT_0 > 0$, this under-supply is alleviated by the transfer.Thus, the marginal gain from increasing tax revenue (and hence the transfer) will be large when this term is large. This will be the case when p_{solv} is low – in other words, when the financial sector is at high risk of insolvency and debt-overhang is significant. The graph of $d\mathcal{G}/d\mathcal{T}$ is given by the solid curve in the top panel of Figure 2. As the graph indicates, and as the proof to Proposition 1 shows, $\mathcal{G}$ is concave in $\mathcal{T}$ since the marginal gain from increasing tax revenues (to increase the transfer) is decreasing.

4.3.2 Under-Investment Loss Due to Taxes

The second term on the left side of (10) is the marginal underinvestment loss to the economy due to a marginal increase in expected tax revenue. This loss is a direct result of the distortion induced by taxes on the non-financial sector's incentive to invest. To see this, let $\mathcal{L}$ be the loss due to underinvestment. Then $\mathcal{L} = (V(K_1) - K_1) - (V(K_1^*) - K_1^*)$, where K_1^* is the first-best (distortion-free) level of investment. The marginal loss due to an increase in $\mathcal{T}$ is then given by: $d\mathcal{L}/d\mathcal{T} = [V'(K_1) - 1]\,\frac{dK_1}{d\mathcal{T}}$, as in (10).

Equation (9) shows that $[V'(K_1) - 1]$ equals $\theta_0 V'(K_1)$, so the marginal underinvestment loss is greater than zero as long as the tax rate is non-zero. Since $dK_1/d\mathcal{T} < 0$, then $d\mathcal{L}/d\mathcal{T} < 0$. Furthermore, as shown in Appendix A.5, this marginal loss due to underinvestment worsens as $\mathcal{T}$ is increased, i.e., $d^2\mathcal{L}/d\mathcal{T}^2 < 0$.[8] A graph of $-d\mathcal{L}/d\mathcal{T}$ is given by the upward-sloping dashed curve in the top panel of Figure 2. The curve shows that $-\mathcal{L}$ is convex as raising additional tax revenues incurs an increasingly large marginal underinvestment loss.

4.3.3 The Optimal Tax Rate and Level of Debt under Certainty and No Default

The following proposition, which describes the solution to the government's problem under assumptions 1 (certainty) and 2 (no default), is proven in Appendix A.6.

Proposition 1. *Let $m \geq 2\vartheta$. Then there is a unique optimal tax rate, $\hat{\theta}_0$, which is strictly less than θ_0^{max}. Let $\hat{\mathcal{T}}$ represent the associated tax revenues. Then newly issued sovereign*

[8] The concavity of $\mathcal{L}$ in $\mathcal{T}$ is one reason to analyze the first-order condition in terms of $\mathcal{T}$ rather than θ_0. The first order condition in terms of the tax rate contains the term $dK_1/d\theta_0$, which is *convex*, thereby making it difficult to characterize the sign of the second derivative with respect to the tax rate. This difficulty disappears when we look at the first-order condition in terms of $\mathcal{T}$.

debt has face value $N_T = \hat{\mathcal{T}} - N_D$ and a price of $P_0 = 1$. Moreover, the following relationships hold:

1. *The optimal tax rate and revenue are **increasing** in L_1, the financial sector liabilities (the severity of debt-overhang), and in N_D, the outstanding government debt.*

2. *The face value of newly issued sovereign debt (the transfer) is **increasing** in the financial sector liabilities L_1, but **decreasing** in the amount of existing government debt N_D. Moreover, the* gross *transfer, $T_0 + k_A N_D$. is also **decreasing** in N_D.*

3. *If also $m \leq 2$, then the optimal tax rate, revenue, and newly issued sovereign debt, are **increasing** in the factor share of the financial sector.*

The optimal tax rate is less than θ_0^{max} due to the Laffer-curve property of tax revenues, whereby the marginal underinvestment loss induced by raising revenue becomes infinite as the tax rate rises to θ_0^{max}. In addition, if there is any debt-overhang (i.e., $p_{solv} < 1$), then the optimal tax rate will be strictly greater than zero, since at a zero tax rate there is a marginal benefit to having a transfer but no marginal cost.

The Appendix shows that the economic gain $\mathcal{G}(\mathcal{T})$ and loss $\mathcal{L}(\mathcal{T})$ from the transfer are both concave in $\mathcal{T}$. Therefore, the optimal government action is to increase the transfer via an increase in tax revenue and outstanding debt, until the marginal gain from the transfer no longer exceeds the associated marginal loss due to underinvestment. This point is given in the top panel of Figure 2 by the intersection point of the two curves, the x-coordinate of which represents $\hat{\mathcal{T}}$. The bottom panel of Figure 2 graphs the value of the government's objective function, whose slope is given by (10). As the graph illustrates, the objective is concave in $\mathcal{T}$ and the unique optimum occurs at $\hat{\mathcal{T}}$, corresponding to the intersection point in the top panel.

For any level of transfer, the marginal gain available is greater the more severe is the debt-overhang, since a lower probability of solvency increases the distortion in the provision of financial services. This represents an upward shift in the marginal gain curve. Therefore, as (1) and (2) of Proposition 1 state, an increase in L_1 (more severe debt-overhang) leads to a higher tax rate, more tax revenue, and greater issuance of new sovereign debt to fund a larger transfer.

If the level of pre-existing government debt (N_D) is increased, there is again an upward shift in the marginal gain curve, as shown by the dash-dot curve in the top panel of Figure 2.

The reason now is that for any level of tax revenue, the effective transfer (T_0) is smaller, and therefore the probability of solvency is lower. As is clear from the new intersection point in the top panel, and as (1) of Proposition 1 states, this pushes the government to increase the optimal tax rate, tax revenue, and overall amount of sovereign debt. The comparative static is displayed in the top panel of Figure 3, which shows how tax revenue (and hence total sovereign debt) is increasing in pre-existing debt, holding the other exogenous parameters constant.

However, as (2) of Proposition 1 shows, the rate of increase in total sovereign debt is less than the increase in N_D. Hence, under the no-default and certainty assumptions, an increase in pre-existing government debt corresponds to a decrease in newly issued sovereign debt and a smaller transfer T_0. This relationship is displayed in the bottom panel of Figure 3. The reason for this decrease is that the underinvestment cost of raising additional tax revenues is increasing. Later, we show that both the possibility of default and uncertainty alter this result.

Finally, Proposition 1 shows that, ceteris paribus, a larger factor share of the financial sector in aggregate production implies that the government will issue a greater amount of new debt and a larger transfer. As the factor share is the fraction of aggregate output captured by the financial sector, this result says that the greater is the importance of the financial sector for aggregate production, the larger is the optimal transfer. Intuitively, if the financial sector's output is a more important input into production, then there will be a greater marginal gain from an increase in the provision of financial services due to the transfer. The factor share is given by ϑ. Note, however, that the comparative static is not simply to vary ϑ, but to hold total output constant while doing so. Equivalently, we may think about comparing the ratio to total output of our variable of interest while varying the factor share.

4.4 Default Under Certainty

Now we allow the government to deviate from the no-default choice of setting $N_T = \mathcal{T} - N_D$. Recall that the transfer T_0 equals $P_0 N_T$, where $P_0 = \max(1, \mathcal{T}/(N_T + N_D))$ is the price of the government bond. If there is some probability of financial sector insolvency, and hence a positive marginal gain from the transfer, then for any given level of tax revenues $\mathcal{T}$, the government will choose to issue face value of debt N_T of *at least* $\mathcal{T} - N_D$. It will not want to issue less than the amount $\mathcal{T} - N_D$ (holding $\mathcal{T}$ constant), because that will decrease the transfer. On the other hand, increasing N_T above this threshold has both an associated cost and benefit. The benefit is that this can increase the transfer to the financial sector. The cost is that when $N_T > \mathcal{T} - N_D$, the government will not be able to fully cover its obligations. In that case, $P_0 < 1$ and the government will default, triggering the default dead-weight loss of D.

Hence, the government's decision on how many new bonds to issue, N_T, splits the parameter space into two regions, separated by a default boundary:

1. $N_T = \mathcal{T} - N_D$ and $1_{def} = 0$ (No Default)

2. $N_T > \mathcal{T} - N_D$ and $1_{def} = 1$ (Default)

Let W_1 denote the maximum value of the government's objective function conditional on being in Region 1 (no-default), W_2 denote the maximum value conditional on being in Region 2 (default), and $W = \max(W_1, W_2)$ be the unconditional maximum (given by (3)). The government's optimal policy conditional on being in Region 1 is characterized in Proposition 1. The following lemma uses this result to characterize the optimal choice when default is allowed.

Lemma 4. *In the default region, it is optimal to set $N_T \to \infty$ (and hence $P_0 \to 0$). This implies that:*

- $W_2 = W_1\big|_{N_D=0} - D$*: the maximum value of the government's objective function in Region 2 is given by subtracting the dead-weight cost of D from the maximum value of the government's objective attainable in Region 1 when pre-existing debt is set to zero.*

- *Default is optimal if* $W_1\big|_{N_D=0} - D > W_1$*. Relative to the no-default optimum, under default the tax rate is lower,* $\hat{\theta}_0^{def} < \hat{\theta}_0^{no\_def}$*, while, assuming* $k_A N_D < \hat{\mathcal{T}}^{def}$*, the gross transfer is bigger, e.g.,* $\hat{T_0}^{def} > \hat{T_0}^{no\_def} + k_A N_D$*, and there is a greater equilibrium provision of financial services,* $\hat{s_0}^{def} > \hat{s_0}^{no\_def}$*.*

Appendix A.7 contains the formal proof. The reasoning is straightforward. First, the benefit of increasing N_T beyond the default boundary is that this 'dilutes' the pre-existing debt and therefore allows the sovereign to capture a larger share of tax revenue for the transfer. The cost of doing this is the dead-weight loss of D. If $N_D = 0$, then there is no benefit to crossing the default boundary and incurring this default cost, so default is never optimal. On the other hand, if $N_D > 0$, and the choice to default is made, then it is optimal for the government to issue an infinite amount of new debt in order to fully dilute existing debt (P_0 becomes 0) and hence capture all tax revenues towards the transfer. The resulting situation is the same as if pre-existing debt N_D had been set to zero, hence the first bullet point in the lemma. Therefore, to determine whether defaulting is optimal, the government evaluates the comparison in the second bullet point.

4.4.1 Default Boundary

As Lemma 4 indicates, the tradeoff involved in default is the deadweight cost D, versus the larger transfer and reduced taxes made possible by diluting pre-existing debt. The net benefit of this tradeoff can be written as follows:

$$\int_{\hat{T_0}^{no\_def}}^{\hat{T_0}^{def}-kN_D} \frac{d\,\mathcal{G}}{dT_0} dT_0 + \int_{\hat{\mathcal{T}}^{no\_def}}^{\hat{\mathcal{T}}^{def}} \frac{d\mathcal{L}}{d\mathcal{T}} d\mathcal{T} - D \tag{11}$$

where the first integral is the gain due to increasing the (gross) transfer, while the second integral is the reduction in underinvestment loss due to reducing tax revenue. Note that $d\mathcal{G}/dT_0$ here is evaluated at the no-default values. If (11) is positive, it is optimal for the sovereign to choose default, while if it is negative then no-default is optimal. Figure 4 displays the resulting default boundary in $L_1 \times N_D$ space along with the No-Default and Default regions. The following proposition characterizes how a number of factors move the 'location' of the sovereign relative to the default-boundary, or shift the default-boundary itself, by changing the net benefit of defaulting.

Proposition 2. *The benefit to defaulting* (11) *is:*

1. ***increasing*** *in the financial sector liabilities L_1 (severity of debt-overhang), the amount of existing government debt N_D, and in the factor share of the financial sector*

2. ***decreasing*** *in the dead-weight default cost D, and in the fraction of existing government debt held by the financial sector k_A*

Furthermore, the benefit to defaulting (11) *is convex in N_D.*

Appendix A.8 provides the proof, which is established using (11). Consider a worsening of the financial sector's health, leading to a decreased provision of financial services. This increases the marginal gain from further government transfer, as the under-provision of financial services is exacerbated. This then increases the gain to the sovereign from defaulting, since it is then able to achieve a greater optimal transfer. This is represented by a move towards the right in Figure 4, decreasing the distance to the default-boundary. A similar kind of result holds if the factor share of financial services is increased, since the marginal gain of further transfer is higher at every level of transfer. An increase in existing debt implies a bigger spread between the optimal transfer and optimal tax revenue with and without default. Both the extra transfer and decreased underinvestment represent benefits to defaulting. This is represented by a move upwards in Figure 4, again decreasing the distance to the default-boundary. Moreover, since the marginal loss from funding extra debt is increasing, this benefit is convex in N_D. It is clear that a increase in the deadweight loss raises the threshold for default. If the sovereign has a lot to lose from defaulting (think a sovereign with strong domestic credibility or international reputation) then the net benefit to default will be relatively lower. Finally, an increase increase in the fraction of existing sovereign debt held by the financial sector also raises the threshold for default since the act of defaulting, which is aimed at freeing up resources towards the transfer, causes collateral damage to the financial sector balance sheet. From the vantage point of Figure 4, both an increase in D and k cause an outwards shift in the default boundary.

4.4.2 Two-way Feedback

Propositions 1 and 2 show that there is a two-way feedback between the solvency situation of the financial sector and the sovereign. Although the introduction of uncertainty will allow a more refined analysis of this feedback, the channels involved are apparent even in the current context. First, by Proposition 1, a severe deterioration in the financial sector's probability of solvency (e.g., an increase in L_1) leads to a large expansion in debt (N_T) by the sovereign, as it acts to mitigate the under-provision of financial services. As the marginal cost of raising the tax revenue ($d\mathcal{L}/d\mathcal{T}$) to fund this debt expansion is increasing, the sovereign is pushed closer to the decision to default (Proposition 2), as well as is its maximum debt capacity (lemma 3). Hence, a financial sector crisis pushes the sovereign towards distress and potentially a 'crisis'.

Going in the other direction, by Proposition 1, a distressed sovereign, e.g., one with high existing debt (N_D), will, all else equal, have a financial sector with a worse solvency situation. This is because it is very costly for such a sovereign to fund increased debt to make the transfer to the financial sector. Hence, a more distressed sovereign will tend to correspond to a more distressed financial sector (lower post-transfer p_{solv}). Increasing debt without funding (e.g., default) is an avenue for a distressed sovereign to free debt capacity for additional transfer. However, large holdings of sovereign debt (k) by the financial sector means that taking this avenue simultaneously causes collateral damage to the balance sheet of the financial sector, limiting the benefit from this option (Proposition 2). In this case, a distressed sovereign is further incapacitated in its ability to strengthen the solvency of its financial sector.

4.5 Uncertainty, Default, and Pricing

We now consider the case where the variance of $\tilde{V}(K_1)$ is nonzero. This means there is uncertainty about the size of future output (i.e., growth) and hence realized tax revenues, which depend on how well the investment of the non-financial sector performs in the future. An implication of this is that the probability of default is no longer binary, but depends in a continuous fashion on both the stock of outstanding government debt and the tax rate. Rather than making a binary default vs. no-default decision, the government must now optimally choose a probability of default. The cost and benefit of this choice are analogous to those governing the binary default vs. no-default choice under certainty: the cost of a greater expected deadweight loss of default versus the benefit of being able to increase the transfer to the financial sector. The degree to which issuing greater amounts of debt and therefore accepting a higher probability of sovereign default is optimal, depends on factors similar to those that were important under certainty. However, now the choice of default probability is no longer binary, so the mapping of economic factors to default probability, as well as the price/credit spread on the sovereign's bonds, is continuous. This continuous mapping demonstrates how the sovereign may be willing to partially sacrifice its own creditworthiness as a means of achieving a larger transfer to the banking sector–a policy which implies a positive correlation between financial sector credit spreads and sovereign credit spreads.

As in (3), θ_0 and N_T, are the variables the government directly controls in maximizing its objective. While these are perhaps the immediate variables that come to mind in formulating the problem, it will actually be more enlightening to map these one-to-one into two other control variables. The first is $\mathcal{T}$, the expected tax revenue, which equals $\theta_0 V(K_1)$. The second is given by:

$$H = \frac{N_T + N_D}{\mathcal{T}}$$

In words, H is the ratio of outstanding debt to expected tax revenue. This measures the ability of the sovereign to cover its outstanding debt obligations at face value. Since $V(K_1)$

is the expected future output from investment, H can also be viewed as $1/\theta_0$ times the ratio of outstanding debt to expected future economic output, i.e., a forward-looking debt/gdp ratio. Since the mapping from θ_0 to $\mathcal{T}$ is invertible on $[0, \theta_0^{max}]$ (as before, we can limit our concern to this region), and since, given $\mathcal{T}$, the mapping from H to N_T is invertible, these alternative control variables map uniquely to the original ones. We can therefore focus on the choice of $\mathcal{T}$ and H instead without any loss.

Consider the expressions for the price of a government bond, P_0, and the probability of

$$P_0 = E_0 \left[\min \left(1, \frac{\theta_0 \tilde{V}(K_1)}{N_T + N_D} \right) \right]$$
$$p_{def} = \text{prob} \left(\theta_0 \tilde{V}(K_1) < N_T + N_D \right)$$

We can write $\tilde{V}(K_1) = V(K_1)\tilde{R}_V$, where $\tilde{R}_V \geq 0$ represents the shock to $\tilde{V}(K_1)$. We assume that the distribution of $\tilde{R}_V$ is independent of the control variables K_1, θ_0, and N_T. By construction we also have $E[\tilde{R}_V] = 1$. Using the definition of H, we can now rewrite the expressions for P_0 and p_{def} as follows:

$$P_0 = E_0 \left[\min \left(1, \frac{1}{H} \tilde{R}_V \right) \right] \tag{12}$$
$$p_{def} = \text{prob} \left(\tilde{R}_V < H \right) \tag{13}$$

Since $N_T = (\mathcal{T} - N_D/H)H$, we can also write an expression for $T_0 = N_T P_0$ in terms of $\mathcal{T}$ and H:

$$T_0 = (\mathcal{T} - \frac{N_D}{H}) E_0 \left[\min \left(H, \tilde{R}_V \right) \right] \tag{14}$$

Thus, we have expressed these three quantities in terms of only the new control variables and the exogenous quantities. Moreover, note that the variable H is sufficient for determining P_0 and p_{def}. That is, these quantities do not change with $\mathcal{T}$ when H is held constant.

Using this revised formulation, we derive analytical properties in the Appendix for optimal tax revenue and the optimal probability of government default under uncertainty. The following is a key insight:

Proposition 3. *Let $\Phi_0 = (L_1, N_D, D, \vartheta, k, \ldots)$ denote the vector of parameters and $X(\Phi_0) = (\hat{\mathcal{T}}(\Phi_0), \hat{H}(\Phi_0))$ be the corresponding optimal government choices. Assume that the first-order condition for the government's problem* (3) *holds at $X(\Phi_0)$ for Φ_0 in a region of the* *parameter space Φ, i.e, the solution is interior on Φ. Then $\hat{\mathcal{T}}$ and $\hat{H}$ are **increasing** in L_1 (the severity of debt-overhang) and $\hat{H}$ is **increasing** in N_D (outstanding government debt), **increasing** in ϑ (the factor share of the financial sector) and **decreasing** in D (the deadweight cost of default).*

Appendix A.12 provides the proof of this result. The result is established using the first-order conditions for $\mathcal{T}$ and H, which are derived in Appendix A.9 and Appendix A.10. The two margins the government has for increasing the transfer are to increase H and increase $\mathcal{T}$. As Appendix A.9 shows, the first-order condition for $\mathcal{T}$ involves the same trade-offs as under certainty (with adjustments to account for the level of H). Varying the level of H involves a new trade-off. Increasing H while holding $\mathcal{T}$ constant means increasing the value of outstanding debt without increasing the resources to which it is a claim. By diluting the claim of existing bondholders, this increases the size of the transfer. At the same time, this action has a cost as it increases the probability that realized future tax revenues fall below the face value of outstanding debt and hence trigger sovereign default.

The top panel of Figure 5 illustrates the marginal gain (solid line) and loss (dashed line) incurred by increasing H for a fixed level of $\mathcal{T}$. The dash-dot line represents the marginal gain curve at a higher level of L_1 than for the solid line. The figure is generated from a parametrization of the model where $\tilde{R}_V$ is assumed to have a uniform distribution and illustrates a number of important points. First, as shown in Appendix A.10, the marginal gain curve is downwards sloping, i.e. $\mathcal{G}$ is concave in H. An important factor causing this is that as H increases, the marginal effectiveness of further dilution in achieving a larger transfer is decreased.

The marginal cost of an increase in H is the rise in expected dead-weight default cost. Due to the uniform distribution assumption for $\tilde{R}_V$, this cost is a flat line up to the upper end of the support of the distribution and then falls to zero beyond that point. This point for H represents sure default ($pdef = 1$) since default occurs if $\tilde{R}_V < H$. For a given level of (expected) tax revenue, $\mathcal{T}$, there are two potential candidates for $\hat{H}$. The first is at the intersection of the gain and loss curves. The second, letting $H \to \infty$, represents total default and full dilution of existing bondholders. The bottom panel of Figure 5 plots the corresponding value of the government's objective as a function of H. The plot shows that for this configuration a relatively small value of H achieves the optimum, which occurs at the intersection of the gain and loss curves in the top panel. Notice that beyond the upper end of the support of $\tilde{R}_V$, the objective function rises in H. This is just an implication of the fact that once debt issuance is so large that the probability of default is 1, it is optimal for the government to fully dilute existing bondholders to achieve the largest possible transfer. This is analogous to full-default in the certainty case. While this is not optimal in the Figure, the optimal level of H *does* still deteriorate somewhat the creditworthiness of the sovereign as it corresponds to a non-zero probability of default. This important possibility did not exist in the case of certainty.

The dash-dot curves in the two panels correspond to an increase in L_1 (the severity of debt-overhang) relative to the solid lines. This worsening of the financial sector solvency increases the marginal gain of an increase in H, pushing up the marginal gain curve in the top panel. By lowering overall welfare, it pushes down the curve in the bottom panel. As is apparent in the top panel, the optimal response of the sovereign is to increase $\hat{H}$–by issuing more debt–in order to increase the transfer. Of course this comes at a cost to the creditworthiness of the sovereign since the probability of its default rises. As can be seen from the bottom panel, total default is still suboptimal. However, it is apparent that total default could become optimal with some further increase in L_1.

4.5.1 Comparative Statics Under Uncertainty

For the comparative static in Figure 5, $\mathcal{T}$ is held constant. However, in general, the optimal choice can involve adjustment of both $\hat{H}$ and $\hat{\mathcal{T}}$. Indeed, Proposition 3 shows that an increase in L_1 implies an increase in both $\hat{\mathcal{T}}$ and $\hat{H}$. The reason is that the government raises the transfer by optimally adjusting the two margins at its disposal. When the marginal benefit of additional transfer is large and the marginal loss of taxation becomes high enough, the sovereign will begin to 'sacrifice' its own creditworthiness to generate additional transfer. From this point on it will begin to push up both margins in tandem, up to the point where total default becomes optimal. This is illustrated in Figure 6, which separately plots comparative statics of the equilibrium quantities as L_1 is varied. The discontinuity in the graphs, which is indicated with a dotted line, represents the point at which total default becomes optimal.

As Figure 6 shows, $\mathcal{T}$ increases in L_1 up to the point where the sovereign chooses total default. Looking at the corresponding plot for H, we see that for low levels of L_1, H held constant. This corresponds to the lower end of the support of $\tilde{R}_V$ so that the probability of sovereign default remains at 0. However, for sufficiently high L_1 (implying a severe financial crisis), the government increases H, sacrificing its creditworthiness in order to achieve a larger transfer. Indeed the subplot for the transfer shows that T_0 increases faster in L_1 once H begins to increase. The damage to the sovereign's credit is apparent in the plot for P_0 and p_{def}. Once the sovereign begins to raise H, the probability of default rises with L_1, while the price P_0 of the bonds falls. This illustrates clearly the feedback of the financial sector crises onto that of the sovereign debt–that is, a crisis in the financial sector (as captured by, for example, a rise in financial sector CDS) leads to a rise in the risk of sovereign debt.

As noted previously, when the financial sector situation is severe enough, the optimal response can become a total default. The outcome of a total default is illustrated in the plots at the point of the dotted line. As in the certainty case, total default fully dilutes existing bondholders, freeing extra capacity for the sovereign to achieve the transfer. As indicated in the plots, this leads to a jump down in $\mathcal{T}$ and a jump up in T_0, while p_{def} becomes equal to 1 and P_0 drops to 0.

Finally, Figure 7 shows the comparative static for existing government debt, N_D. As above, it is apparent that for low levels of debt, the sovereign keeps H constant at a low level implying a 0 probability of default and a price of 1 for its bonds. It funds this debt and the transfer through increases in tax revenues. As under certainty, the transfer is decreasing in N_D over this range. However, at sufficiently high levels of N_D, it is very costly to continue increasing tax revenue and the sovereign begins to increase H to fund the transfer. Consequently, the price of its bonds begin to fall and the probability of default rises. Interestingly, in this range the combination of increased H and $\mathcal{T}$ imply that the transfer is *increasing* in N_D. A reason for this is that when N_D high, dilution of existing bondholders is a more effective mechanism for increasing the transfer. As above, for high enough N_D, total default becomes optimal.

4.5.2 Two-way Feedback

We can now revisit in a more refined way the two-way feedback between the solvency of the financial sector and the sovereign. As illustrated by Proposition 3 and the discussion of Figure 6, a severe financial sector crises leads to an increase in H and deterioration of the sovereign's creditworthiness. The increase in H depends continuously on the severity of the financial sector crises up to a point where total default is optimal. As shown by 7, for a given situation in the financial sector, the creditworthiness of the sovereign is decreasing in the level of its pre-existing debt.

In the other direction, a sovereign with high pre-existing debt (N_D) will, for a given level of L_1, correspond to a more distressed financial sector (lower p_{solv}). Under uncertainty there is an additional important channel at work. If the bank has holdings of existing sovereign debt (k), then a deterioration in the sovereign credit has a direct adverse effect on the solvency of the financial sector. *Moreover*, funding the transfer by increasing H causing collateral damage to the existing sovereign debt held by the financial sector as P_0 decreases. The dilution channel therefore becomes decreasingly effective and larger debt issuance is needed to achieve a given transfer. Hence, a higher level of k implies a weaker (post-transfer) financial sector. In all cases, the post-transfer financial sector may be very sensitive to low realizations of $\tilde{R}_V$ since with H set high, the price of the sovereign's bonds are now sensitive to such shocks.

5 Empirical evidence on two-way feedback

In this section, we provide empirical evidence that corroborates our model. The focus is on the two-way feedback between sovereign and bank credit risk during the financial crisis of 2007-2010.

5.1 Data and Summary Statistics

Our empirical analysis uses data on bank and sovereign credit risk during the financial crisis. We construct two data sets for our analysis. The first data set uses information from the the European bank stress conducted in the first half of 2010. We collect the data from websites of national bank regulators in Europe. The data consists of bank characteristics and holdings of European sovereign bonds. A total of 91 banks participated in the bank stress tests. These banks represent about 70 percent of bank assets in Europe. For all banks, we search for CDS prices in the database Datastream. Using bank names, we match 51 banks to CDS prices. Unmatched banks are mostly smaller banks located in Spain and Eastern Europe that do not have publicly quoted CDS prices. For each bank, we match sovereign holdings to sovereign CDS and compute exposure to sovereign risk.

Panel A of Table 1 presents summary statistics for all banks that participated in the European bank stress tests. As of March 2010, the average bank had risk-weighted assets of 126 billion euros and a Tier 1 capital ratio of 10.2%. The average holdings of gross and net European sovereign bonds are 20.6 billion euros and 19.7 billion euros, respectively. Hence, the average bank holds about one sixth of risk-weighted assets in sovereign bonds. Banks have a strong home bias in their sovereign holdings: about 69.4% of bonds are issued by the country in which a bank is headquartered. These summary statistics are supportive of the model's assumption that banks are exposed to home-country sovereign risk through their holdings of government bonds.

The second data set uses information on bank credit ratings. We use Bankscope to identify the 200 largest banks by assets in OECD countries as of the end of fiscal year 2006. We then search for CDS prices in the database Datastream. We find CDS prices for 99 banks and match CDS prices to bank characteristics from Bankscope. Next, we search for investment grade credit ratings using S&P Ratings Express. We find credit ratings for 86 banks and match these data to CDS prices and bank characteristics. Finally, we match these data to sovereign CDS of bank headquarters and OCED Economic data on public debt.

Panel A of Table 2 presents summary statistics for all banks with CDS prices and investment grade credit ratings. As of January 2008, the average bank had assets of \$591 billion and equity of \$27 billion. About 39.8% of banks have a credit rating of 'AA', 55.4% have a credit rating of 'A', and the remainder have a credit rating of 'BBB'. The average bank CDS is 63 basis points and the average sovereign CDS is 11 basis points. These summary statistics suggests that banks were considered a low credit risk as of January 2008.

5.2 Impact of bank bailouts on sovereign credit risk

We start with a non-parametric analysis of the impact of bank bailouts on sovereign credit risk. We divide the financial crisis in three periods. We choose the periods based on the announcement dates of bank bailouts. The first bank bailout announcement in Western Europe was on September 30, 2008 in Ireland. We therefore define the first period as the period from March 2007, prior to the start of the financial crisis, until September 29, 2008. We note that this period includes the bankruptcy of Lehman Brothers in mid September 2008 and the period immediately afterwards. Hence, this period captures the immediate effect of Lehman's bankruptcy on other banks prior to the Ireland announcement. We choose this period to capture the increase in bank credit risk before the bank bailouts.

We compute the change in sovereign CDS and bank CDS during this period for all countries in our data set. We compute the change in bank CDS as the unweighted average of all the banks with CDS prices. We omit countries for which either sovereign CDS or banks CDS are not available.

Figure 8 presents the results. For each country, the first column depicts the change in sovereign CDS and the second column depicts the change in bank CDS. As shown in the figure, there is a large increase in banks CDS prior to the bank bailouts. For example, the average bank CDS in Ireland increases by 300 basis points over this period. However, there is almost no change in sovereign CDS. Overall, the figure suggests that the Lehman bankruptcy (and prior events) negatively affected the credit risk of the financial sector but had little effect on sovereign credit risk.

We note that some investors may have expected bank bailouts even before the first official announcement on September 30, 2008. An expectation of bank bailouts would reduce the observed change in bank CDS by shifting credit risk from the financial sector to the sovereign. Hence, to the extent that investors held such expectations prior to September 30, 2008, they can explain the small rise in sovereign CDS before Lehman's bankruptcy but the impact seems quantitatively small.

Next, we examine the change in bank CDS and sovereign CDS immediately after the bank bailouts. Almost every Western European country announced a bank support program in October 2008. Most bank support programs consisted of asset purchase programs, debt guarantees, and equity injections or some combination therefore. Several countries made more than one announcement during this period. As noted above, the first country to make a formal announcement was Ireland on September 30, 2010. Many other countries soon followed Ireland's example, partly to offset flows from their own financial sectors to newly secured financial sectors. As a result, the bank bailout announcements were not truly independent since sovereigns partly reacted to other sovereigns' announcements. We therefore define the second period as the period in which the bank bailout announcements occured. We choose a four-week period because almost all Western European countries made an announcement of bank bailouts by the end of this period.

Figure 9 plots the average change in bank CDS and sovereign CDS during the period of bank bailout announcements. As shown in the figure, bank CDS significantly decreased over this one-month period. For example, the average bank CDS in Ireland decreased by about 200 basis points. Similarly, most other countries had a significant decrease in bank CDS, especially the ones that had a large increase in the previous period. However, at the same time, there is a significant increase in sovereign CDS. For example, the sovereign CDS of Ireland increased by about 50 basis points. Most other countries exhibit a similar pattern with decreasing bank CDS and increasing sovereign CDS.

This evidence is consistent with our model. In particular, the evidence suggests that the bank bailout announcements shifted credit risk from the financial sector to the sovereign.

Hence, this analysis provides empirical support for our model's trade-off between sovereign credit risk and bank credit risk.

We then examine the change in bank CDS and the change in sovereign CDS after the announcement of bank bailouts. We define the third period as the period from the end of the bank bailouts until the end of March 2010. We choose March 2010 because this is the date for which the European bank stress data results were released. Our results are robust to using other cut-off dates in 2010.

Figure 10 plots the results. As shown in the figure, both sovereign CDS and bank CDS increased across all countries. More generally, we see that bank CDS and sovereign CDS move together after the bank bailouts. This result suggests that bank and sovereign CDS are affected by common factors such as common credit risk. However, we acknowledge that the bank bailouts are not the only explanation for co-movement. Other factors, such as changes in economic conditions, can also affect both sovereign and bank CDS directly.

To analyze the importance of the sovereign's financial position, we examine the correlation between sovereign CDS and public debt during the financial crisis. We measure public debt using gross liabilities as a percentage of total GDP as provided by the OECD database.

Figure 11 shows the correlation of sovereign CDS and public debt for each quarter from January 2007 to July 2010. There is no discernible relationship between the two variables prior to 2010. Starting in 2010, we find a positive and statically significant relationship between the two variables. For example, countries with high public debt, such as Greece, have significantly higher sovereign CDS than countries with little public debt, such as Norway. This evidence suggests that sovereign credit risk emerged as an economically significant factor in 2010.

5.3 Impact of sovereign credit risk on bank credit risk

Our model suggests than an increase in sovereign credit risk raises bank credit risk. We examine this hypothesis in two ways. First, we examine whether sovereign credit risk affects bank credit risk controlling for a bank's expected profitability. Second, we analyze whether changes in the value of sovereign bond holdings affect bank credit risk.

5.3.1 Credit ratings data set

Our first approach uses the credit ratings data set. We interpret credit ratings as a comprehensive measure of a bank's expected likelihood to repay its liabilities. We assign each bank the sovereign CDS of the country where the bank is headquartered. For each quarter from 2007 to 2010, we double-sort our data set by credit rating and sovereign credit risk. Specifically, we first group banks in five country quintiles using their respective sovereign CDS. We then compute average banks CDS by credit ratings and by country quintile.

Figure 12 presents the results. The top row shows the results for the four quarters in 2007. We find that bank CDS were almost equally low for all credit ratings and country quintiles. In 2007, there is no discernible relationship across credit ratings and country quintiles. The second row shows the four quarters in 2008. We find that bank CDS started to increase in early 2008 and the increase was larger for lower credit ratings. The bottom two rows show the results for the four quarters in 2009 and the first three quarters in 2010. We find two main results. First, average bank CDS increases with the bank's credit rating. Second, and more importantly, for a given credit rating, the average bank CDS increases with sovereign CDS. Put differently, banks of similar quality have higher credit risk if they are located in a country with higher sovereign risk.

To illustrate these results, consider the example of the Spanish bank Santander and the German bank WestLB. On June 1, 2010, Santander had a long-term bond rating of 'AA' and a CDS price of 207 basis points. Spain had a CDS of 247 basis points (highest country quintile). On the same day, West LB had a long-term rating of "BBB+" and a CDS price of 158 basis points. Germany had a CDS of 43 basis points (second lowest country quintile). Hence, even though credit ratings suggest that the expected profitability of Santander is higher than the expected profitability of WestLB, the credit risk of Santander is higher than the credit risk of WestLB. More generally, as shown in the bottom row of Figure 12, banks with credit ratings of 'AA' and 'A' in the highest country quintile (e.g., Spain in June 2010) have on average higher CDS prices than banks with credit ratings of "BBB" in the lower three country quintiles. These results suggest that an increase in sovereign CDS increases bank credit risk even after controlling for bank profitability.

We confirm this result using standard regression analysis. Our discussion above suggests that we can restrict our analysis to the period starting in 2008 because there is little variation in bank CDS prior to 2008. For each quarter, we use ordinary least squares to estimate:

$$B_{it} = \alpha_t + \sum \beta_j R_{ijt} + \varepsilon_{it}$$

where B_{it} is the CDS of bank i at time t, R_{ijt} are indicator variables equal to one if bank i is rated j at time t and zero otherwise, and α_t are day fixed effects. The omitted credit rating category is 'AA'. The coefficients β_j capture the average difference in CDS prices between a bank with credit rating j and a bank with credit rating 'AA'. We cluster standard errors at the bank-level.

Table 3 presents the results. Columns (1) to (3) show coefficients for the first three quarters of 2008. In each quarter, banks with lower credit ratings have higher CDS prices but the difference is not statistically significant. As shown in columns (4) to (11), the coefficients become statistically significant after the fourth quarter of 2008 and remain statistically significant thereafter. CDS prices of banks with a credit rating of 'A' or 'BBB' are, on average, 50 to 150 basis points higher than CDS prices of banks with a credit rating of 'AA'.

Next, we examine the impact of sovereign CDS on bank CDS. For each quarter, we use ordinary least squares to estimate:

$$B_{it} = \alpha_t + \sum \gamma_j R_{ijt} * P_{ct} + \sum \beta_j R_{jt} + \delta P_{ct} + \varepsilon_{it}$$

where P_{ct} is the sovereign CDS of the country c at time t where bank i is headquartered and the other variables remain unchanged. The coefficients γ_j capture the impact of sovereign CDS on bank CDS interacted with an indicator variable for rating j. The omitted credit rating category is 'AA'. We cluster standard errors at the bank-level.

Table 4 presents the results. As shown in columns (1) to (8), there is no statistically significant relationship between bank CDS and sovereign CDS in the years 2008 and 2009. As shown in columns (9) to (11), there is a positive and statistically significant relationship in all three quarters in 2010. For example, in the third quarter of 2010, a 10 basis point increase in sovereign risk is associated with a 5.4 basis point increase in bank credit risk. Moreover, the association is stronger for lower credit ratings although this result is not always statistically significant. These results suggests that, starting in 2010, sovereign credit risk had an economically and statistically significant association with bank credit risk.

One possible concern with a causal interpretation of these results is that credit ratings may not accurately control for bank profitability. In particular, if rating agencies adjust credit ratings with a significant delay, then our results may suffer from omitted variable bias because worsening local economic conditions may affect both bank and sovereign credit risk directly. We think this is unlikely given the magnitude of our effects. Moreover, the timing of our effects coincide with the emergence of sovereign credit risk in 2010 as documented in the previous section. However, we acknowledge the possibility of an omitted variable bias and therefore also consider a second approach for identifying the impact of sovereign risk on bank credit risk.

5.3.2 European bank stress data set

In this section, we estimate the impact of bank credit risk on sovereign risk using data from the European bank stress test conducted in 2010. We first test whether bank CDS prices co-move with sovereign CDS prices of the country where the bank is headquartered.

Specifically, denote ΔB_{it} as the log change in bank CDS of bank i headquartered in country j from day $t-1$ to day t and denote ΔC_{it} as the log change in the country CDS of country j from day $t-1$ to t. We use can use ordinary least square to estimate:

$$\Delta B_{it} = \alpha + \beta \Delta C_{jt} + \varepsilon_{it} \tag{15}$$

The coefficient β captures the association between sovereign CDS and bank CDS prices.

However, there are two issues with a causal interpretation of this regression. First, estimation of equation (15) may suffer from omitted variable bias. For example, a rise in the unemployment rate in country j can increase both the CDS of banks headquartered in country j and the CDS of country j. Hence, changes in economic variables can generate a positive coefficient β even in the absence of a direct effect of sovereign CDS on bank CDS. Second, estimation of equation (15) may suffer from reserve causality. For example, if banks gain concessions from the government for additional support, a decline in bank CDS may cause an increase in country CDS (similar to our interpretation of Figure 9). Hence, changes in the likelihood of government support can generate a negative coefficient β even in the absence of a direct effect of sovereign CDS on bank CDS.

We address this empirical challenge by using data on sovereign bond holdings of European banks. As part of the European bank stress test, European bank regulators released detailed data on sovereign holdings for 91 large European banks as of March 31, 2010.[9] Each bank reported its gross and net holdings of sovereign bonds of each European country.

We use these data to construct a bank-specific exposure variable. Let S_{ij} be the sovereign holdings of country j of bank i. We construct the bank exposure E_{it} as follows:

$$E_{it} = \sum_{i \neq j} S_{ij} * C_{jt}.$$

We note that the exposure variable does not include sovereign holdings from the country where the bank is headquartered. As a result, we can avoid the empirical challenges described above. First, changes in local economic conditions are not captured in E_{it}. Second, changes

[9] A few banks reported holdings data as of May 31, 2010.

in the likelihood of domestic government support do not affect the exposure variable. We can use ordinary least square to estimate:

$$\Delta B_{it} = \delta + \gamma \Delta E_{it} + \eta_{it}. \tag{16}$$

One possible concern with equation (16) is that European-wide changes in economic conditions may affect both changes in bank CDS and changes in the exposure variable. We can address this concern by including time fixed effects. Hence, we can modify the regression equation and estimate

$$\Delta B_{it} = \delta_t + \gamma \Delta E_{it} + \eta_{it}. \tag{17}$$

In equation (17) we identify the coefficient γ from the bank-specific deviation of the change in the exposure variable *after* controlling for market-wide flucations. Hence, we control for any common shock that affects both the exposure variable and bank CDS.

We estimate these regression using the period one month before and one month after the reporting date of sovereign bond holdings. Our regressions therefore implicitly assume that the holdings are known to the marginal investor in the CDS market. This assumption is reasonable because many banks disclose detailed sovereign holdings to investors. We compute log changes to minimize the effect of outliers. We note that 51 banks (out of 91 banks) have publicly quoted CDS prices. We cluster standard errors at the bank-level to allow for correlation of error terms within banks over time.

We report our results in Table 5. Column (1) shows a positive and statistically significant coefficient (elasticity) of 0.325. This result suggests that a one-standard deviation increase in the change in the exposure variable leads to an increase of about half of a one-standard deviation in the change of the bank CDS. Column (2) controls for bank fixed effects and the coefficient remains unchanged. This result suggests that there are no bank-specific trends that may confound the analysis.

Column (3) controls for week fixed effects. The coefficient of interest decreases from 0.325 to 0.261. This result suggests that common shocks affect both the change in bank CDS and the change in the exposure variable. Column (4) controls for day fixed effects. This coefficient is identified only off the cross-sectional variation in sovereign holdings. The coefficient decreases to 0.141 but remains statistical significant at the 1%-level. This result suggests that sovereign risk has an economically important effect on bank risk.

To further check for robustness, Column (5) controls for bank fixed effects. The controls have no effect on the coefficient of interest. Column (6) estimates the same regressions as in Column (5) but excludes the holdings of German bonds from the construction of the exposure variable. The concern is that Germany may provide bailouts to other countries, or banks in other countries, which could generate reverse causality. Again, we find no effect on the coefficient of interest.

Overall, our results suggest an important empirical relationship between sovereign and bank credit risk. Bank bailouts generate sovereign credit risk and, as a result, changes in sovereign credit risk affect bank credit risk. This evidence is consistent with the two-way feedback mechanism described in our model.

6 Conclusion

This paper examines the link between bank bailouts and sovereign credit risk. We develop a model in which the government faces an important trade-off: bank bailouts ameliorate the under-investment problem in the financial sector but reduce investment incentives in the non-financial sector due to costly future taxation. In the short-run, bailouts are funded through the issuance of government bonds, which dilutes the value of existing government bonds and creates a two-way feedback mechanism because financial firm hold government bonds for liquidity purposes. These results hold even in a model without uncertainty about future economic output. If we introduce uncertainty, we find that bank bailouts generate a co-movement between financial sector and sovereign credit risk. We provide supporting evidence for our model using data from the financial crisis of 2007-10. In particular, we document that developed country governments transferred credit risk from the financial sector to taxpayers during the height of the crisis in October 2008. Using credit ratings data and data on sovereign bond holdings from the European bank stress test in May 2010, we find that sovereign credit risk in turn affected bank credit risk.

Our paper highlights that bank bailout policy is intimately tied to fiscal policy of governments, which in turn has direct influence on economy-wide incentives to invest and generate economic growth. While the financial sector is generally believed to have a salubrious effect on economic growth, ensuring its survival through bailout of its creditors can produce countervailing effect due to bailout costs being borne by rest of the economy. This integrated view of financial sector and fiscal policies is worthy of further research, and can potentially embed monetary policy too. Expansionary monetary policy represents a transfer from savers to borrowers. While the borrowers may help propel one leg of the economy, the losses to savers can reduce their demand and hold back the other leg. Much like bailouts, we conjecture that overly accommodating monetary policy in wake of financial sector stress also runs the risk of ending up being a Pyrrhic victory.

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Figure 1: Sovereign CDS and bank CDS of Ireland

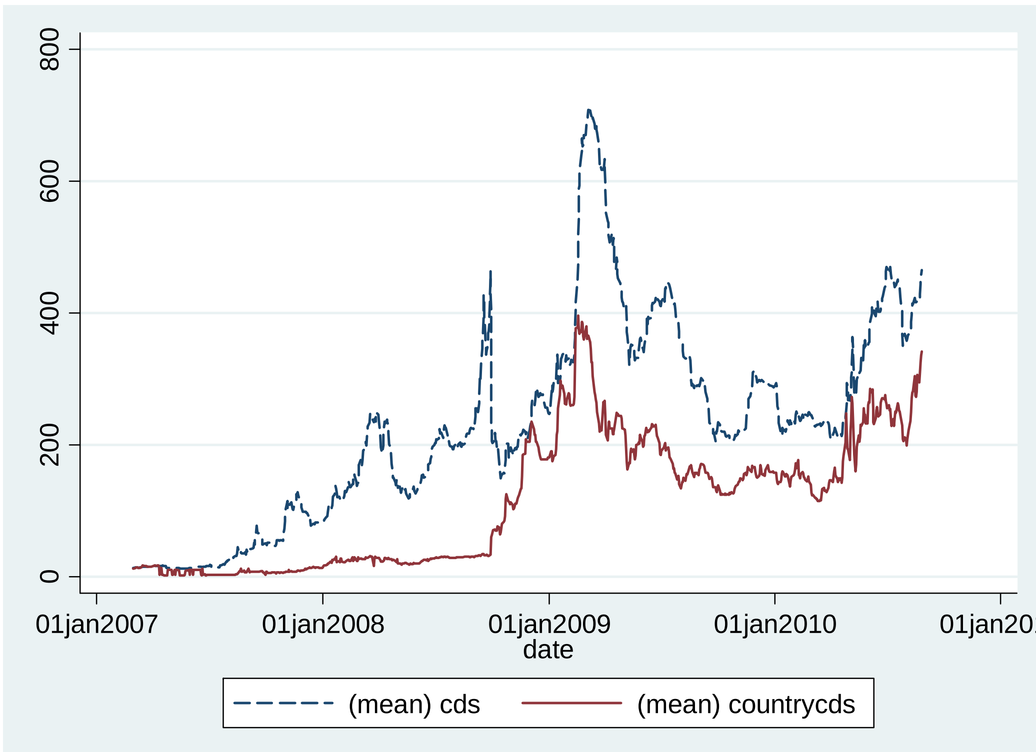
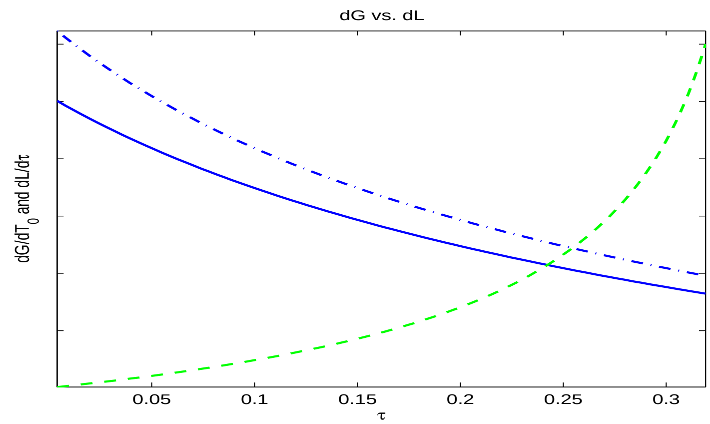
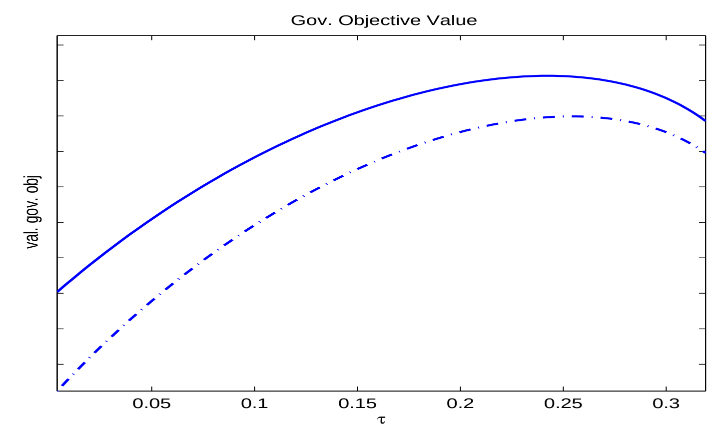


Figure 1 plots the sovereign CDS and bank CDS for Ireland in the period from 3/1/2007 to 8/31/2010. The bank CDS is computed as the unweighted average of bank CDS for banks headquartered in Ireland (Allied Irish Bank, Anglo Irish Bank, Bank of Ireland, and Irish Life and Permanent). The data are from Datastream.

Figure 2: Marginal Gain and Loss of Raising $\mathcal{T}$ (Certainty Case)



Value of Government's Objective Function



The top panel of Figure 2 plots the marginal gain ($d\mathcal{G}/d\mathcal{T}$) of raising tax revenues (solid line and dash-dot line) and the marginal loss ($d\mathcal{L}/d\mathcal{T}$, dashed line) as functions of $\mathcal{T}$ for the certainty model of Section 4.3. The dash-dot line corresponds to a higher level of existing government debt, N_D, than the solid line. The bottom panel of the Figure shows the resulting value of the government's objective function (equation (3)), with the the solid and dash-dot line corresponding to their counterparts in the top panel. The plots correspond to a parameterization of the model where $\tilde{A}_1 \sim U[0,1]$, $L_1 = 0.5$, $\alpha = 1$, $\vartheta = 0.3$, $\gamma = 0.2$, $\beta = 0.5$, $m = 1.3$, $D = 0.02$, $k = 0$, and $N_D = 0.25$ (solid line).

Figure 3: Existing Debt vs. Tax Revenue and Transfer (Certainty Case)

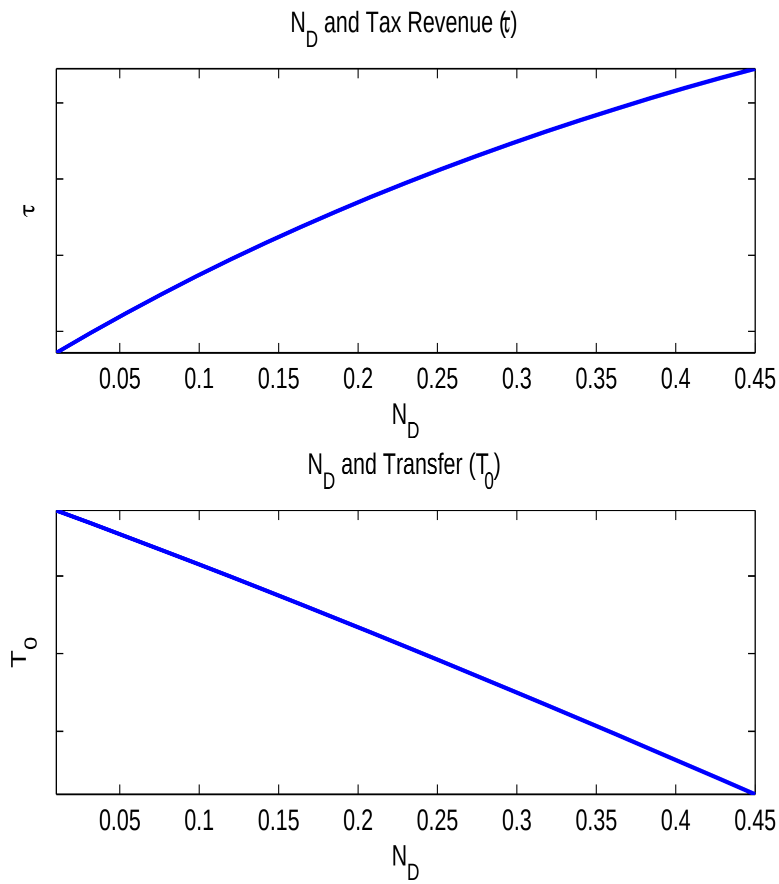


Figure 3 plots the optimal level of tax revenues $\hat{\mathcal{T}}$ (top panel) and corresponding transfer $\hat{T}_0$ (bottom panel) as a function of pre-existing government debt, N_D. The plots correspond to the certainty model parameterized as in Figure 2.

Figure 4: The Default Boundary (Certainty Case)

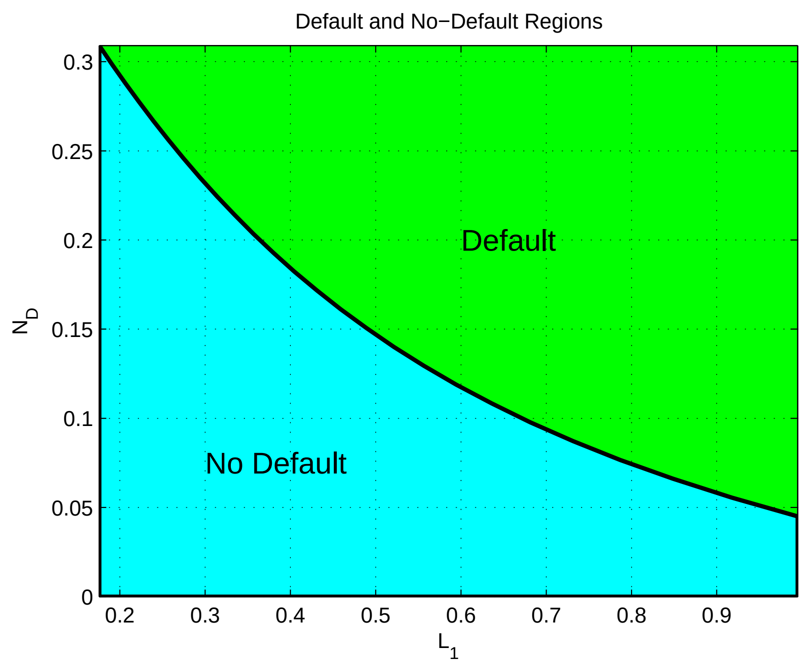
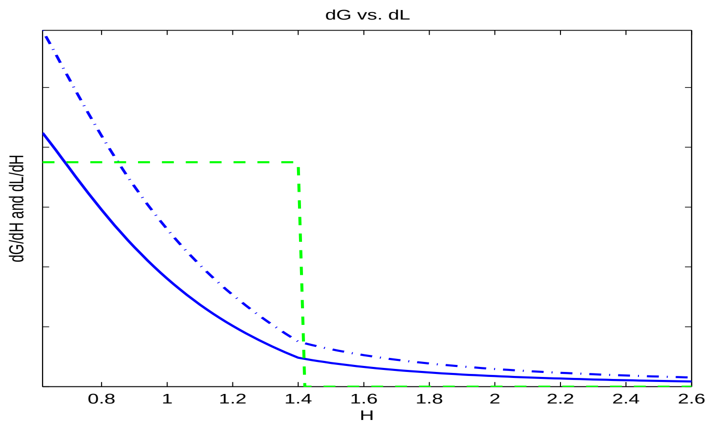
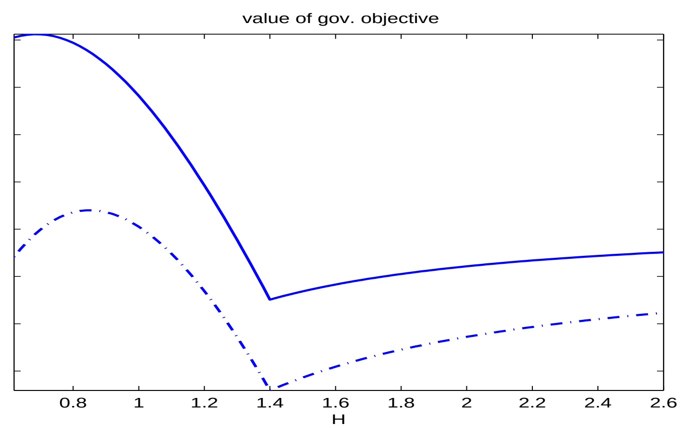


Figure 4 shows the Default and No-Default Regions in the space of $L_1 \times N_D$ (financial sector leverage/debt overhang $\times$ pre-existing sovereign debt) for the certainty model parameterized as in Figure 2. The black curve separating these two regions gives the Default Boundary.

Figure 5: Marginal Gain and Loss of Increasing H



Value of Government's Objective Function



The top panel of Figure 5 plots the marginal gain of increasing H while holding constant $\mathcal{T}$, $d\mathcal{G}/dH$ (solid line and dash-dot line) and the resulting marginal increase in expected dead-weight default cost $D\frac{dpdef}{dH}$ (dashed line). Uncertainty over growth/tax revenues, $\tilde{R}_V$, is assumed to have a uniform distribution. The dash-dot line corresponds to a higher level of L_1 than for the solid line. The bottom panel of the Figure shows the resulting value of the government's objective function, with the the solid and dash-dot line corresponding to their counterparts in the top panel. The plots correspond to a parameterization of the model where $\tilde{R}_V \sim U[0.6, 1.4]$, $\tilde{A}_1 \sim U[0, 1]$, $L_1 = 0.5$ (solid line), $\alpha = 1$, $\vartheta = 0.3$, $\gamma = 0.2$, $\beta = 0.5$, $m = 1.3$, $D = 0.06$, $k = 0$, and $N_D = 0.25$.

Figure 6: Comparative Statics for L_1

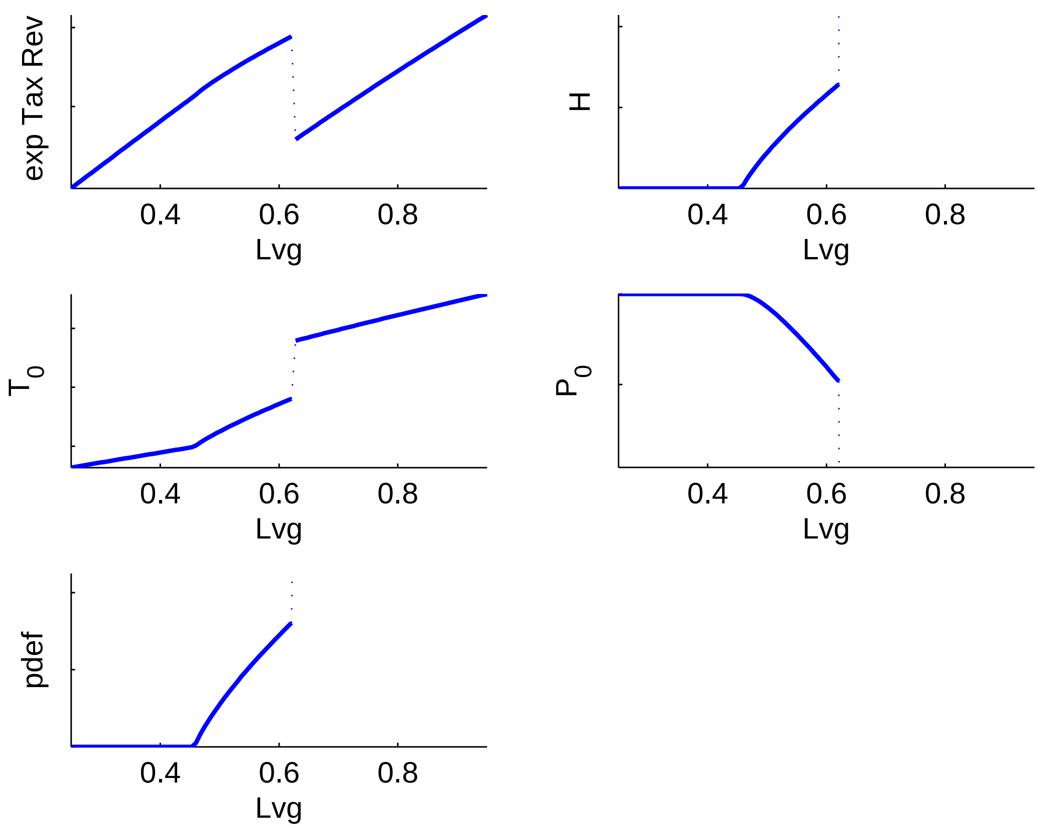


Figure 6 plots the equilibrium values of $\mathcal{T}$ (expected tax revenue), H, T_0 (the transfer), p_{def} (probability of sovereign default), and P_0 (price of sovereign bond) as L_1 is varied. The dotted line in the plots represents the point at which the government chooses a total default ($H \to \infty$), which results in a discontinuity in the plot. The parameters of the model correspond to those in Figure 5.

Figure 7: Comparative Statics for N_D

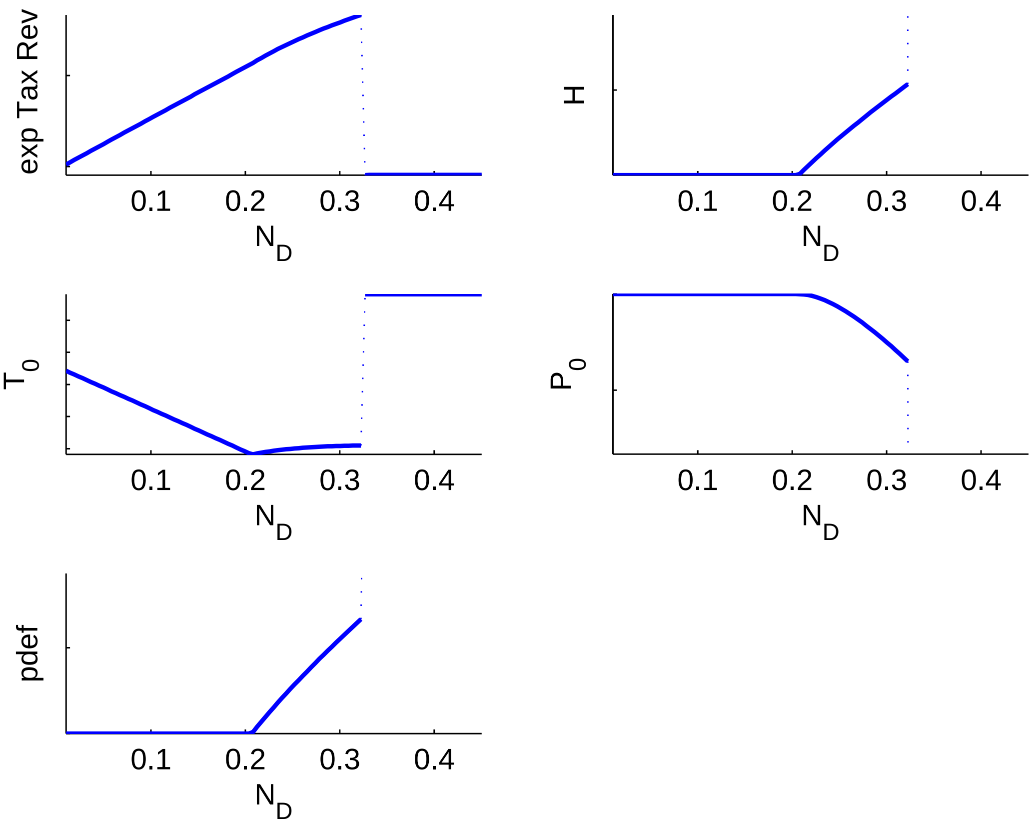


Figure 7 plots the equilibrium values of $\mathcal{T}$ (expected tax revenue), H, T_0 (the transfer), p_{def} (probability of sovereign default), and P_0 (price of sovereign bond) as N_D is varied. The dotted line in the plots represents the point at which the government chooses a total default ($H \to \infty$), which results in a discontinuity in the plot. The parameters of the model correspond to those in Figure 5.

Figure 8: Change in Sovereign and Bank CDS before Bank Bailouts

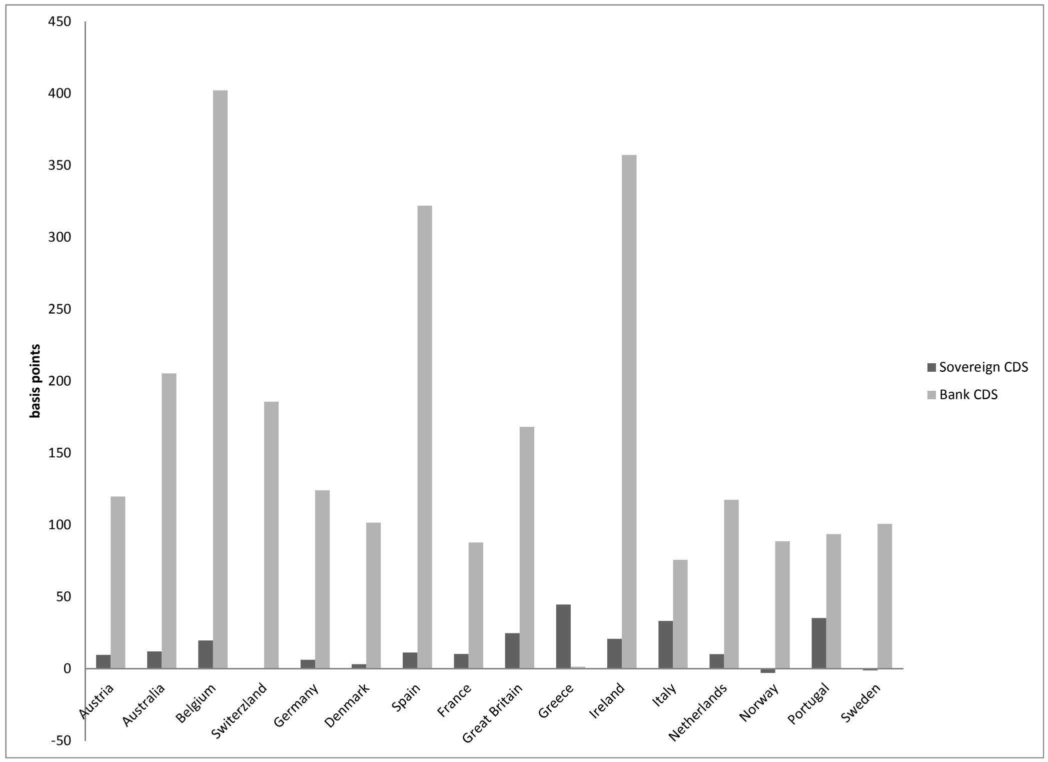


Figure 8 plots the sovereign CDS and bank CDS for Ireland in the period from 3/1/2007 to 8/31/2010. The bank CDS is computed as the unweighted average of bank CDS for banks headquartered in Ireland (Allied Irish Bank, Anglo Irish Bank, Bank of Ireland, and Irish Life and Permanent). The data are from Datastream.

Figure 9: Change in Sovereign and Bank CDS during the Period of Bank Bailouts

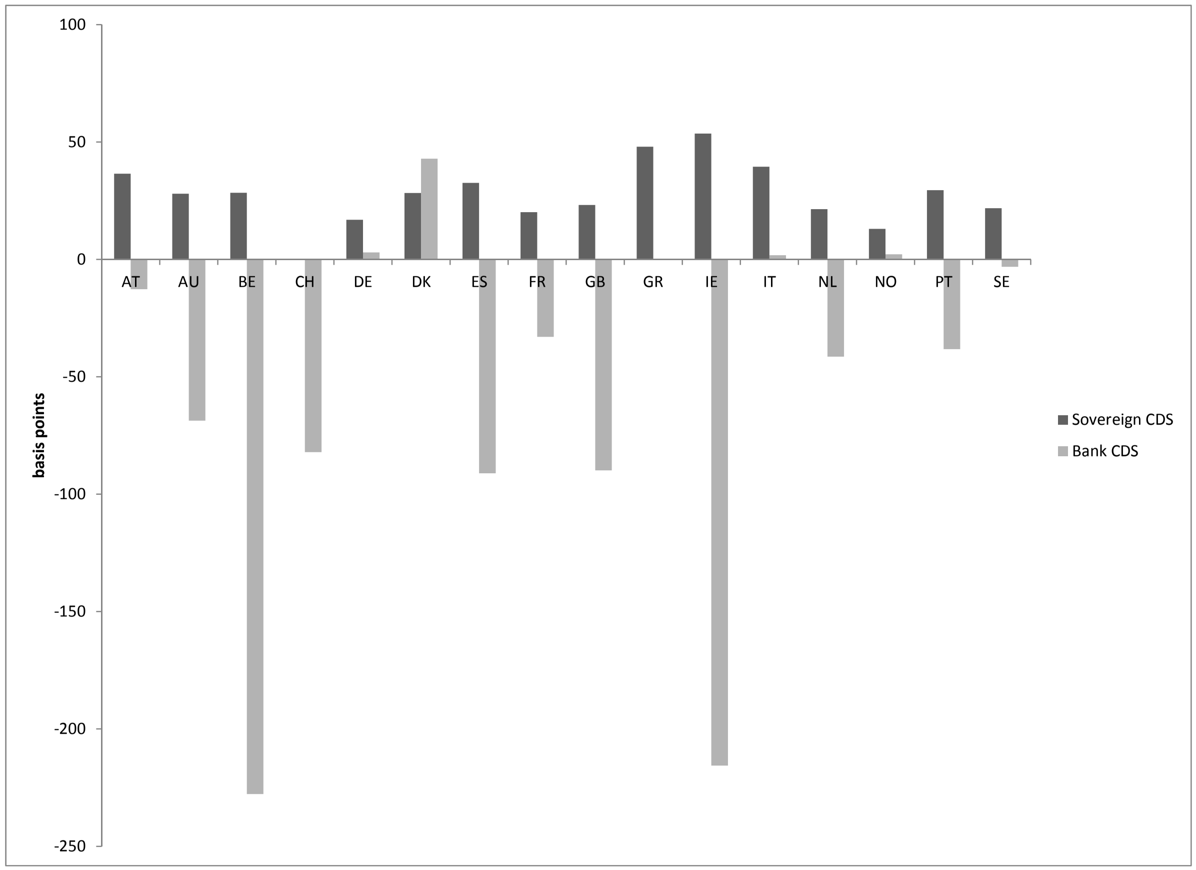


Figure 9 plots the change in average bank CDS and sovereign CDS for Western European countries in the period from 9/26/2008 to 10/21/2008. The bank CDS is computed as the unweighted average of bank CDS for banks headquartered in that country. The data are from Datastream (no data available for Switzerland Country CDS and Greek banks CDS during this period).

Figure 10: Change in Sovereign and Bank CDS after Bank Bailouts

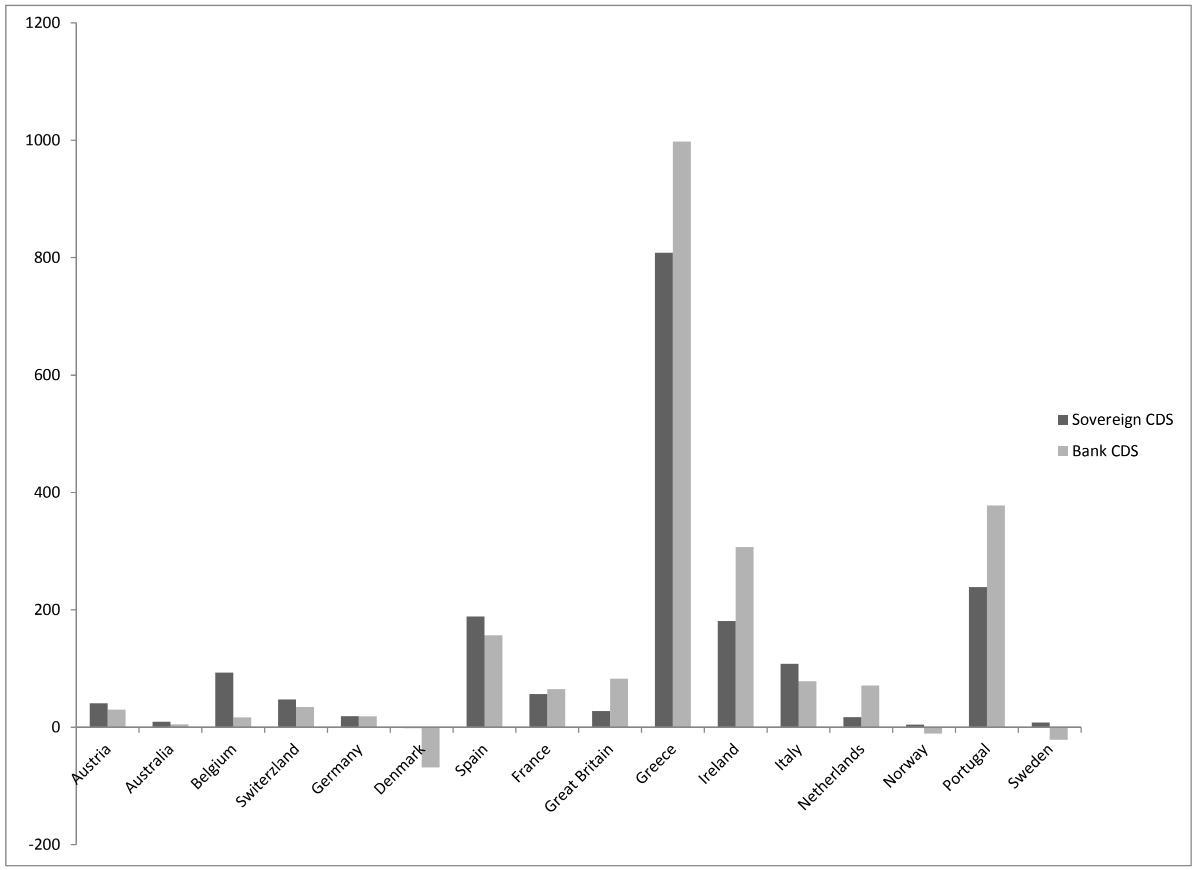


Figure 10 plots the change in average bank CDS and sovereign CDS for Western European countries in the period from 10/22/2008 to 6/30/2010. The bank CDS is computed as the unweighted average of bank CDS for banks headquartered in that country. The data are from Datastream.

Figure 11: Correlation between Sovereign CDS and Public Debt from 2007 to 2010

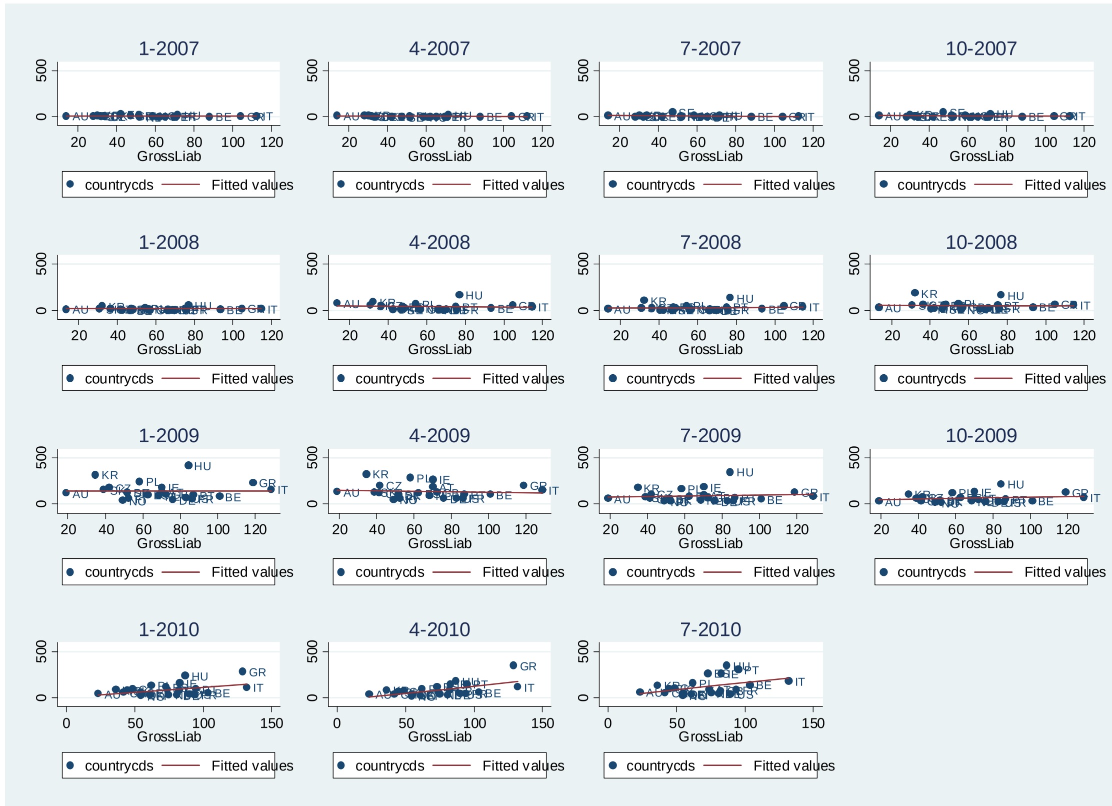


Figure 11 shows the correlation between sovereign CDS and public liabilities as a percentage of GDP at the beginning of each quarter from 2007 to 2010. We exclude outliers with CDS higher than 500 basis points. The data are from Datastream and the OECD Economic database.

Figure 12: : Quarterly Bank CDS by Credit Rating and Country CDS from 2007 to 2010

Figure 12 plots average banks CDS by credit rating and CDS country quintile for each quarter from 2007 to 2010. The country CDS quintile (1 to 5) are constructed separately for each quarter. The data are from Datastream and S&P.

Appendix

A Derivations

A.1 Proof of Lemma 1

Use (7) to substitute for w_s in the financial sector's first-order condition and then take the derivative with respect to the transfer T_0:.

$$\frac{d^2 f(K_0, s_0)}{ds_0^2}\frac{ds_0}{dT_0}p_{solv} + w_s\frac{dp_{solv}}{dT_0} - c''(s_0)\frac{ds_0}{dT_0} = 0$$
$$\frac{ds_0}{dT_0} = -w_s\frac{dp_{solv}}{dT_0}/\left(\frac{d^2 f(K_0, s_0)}{ds_0^2}p_{solv} - c''(s_0)\right) \tag{A.1.1}$$

Since $dp_{solv}/dT_0 = p(\underline{A}_1)$, this term is positive so long as $\underline{A}_1$ is in the support of $\tilde{A}_1$ and the transfer increases the probability of solvency by decreasing the solvency threshold $\underline{A}_1$. Hence the numerator of the right hand side in the second line is negative. That the denominator is also negative follows from the concavity of f and the convexity of c. This establishes that the right side is positive and hence $ds_0/dT_0 > 0$.

A.2 A Candidate for $V(K)$ based on $f(K, s)$

Consider the frictionless counterpart to our setting, with $p_{solv} = 1$. In a dynamic setting, the expression for V would reflect the value of future production of the non-financial sector as a function of its future capital, K. For simplicity, consider one extra period of output. The case of more than one future period should be similar as it is the sum of multiple one-period output. The output of the additional period is given by $\max_s f(K, s)$. It is natural then to let

$$V(K) = \max_s f(K, s) - w_s s$$

with w_s determined by the financial sector's first-order condition. With $f(K, s) = \alpha K^{1-\vartheta}s^{\vartheta}$, this implies that

$$V(K) = (1 - \vartheta)\alpha K^{1-\vartheta}s^{*\vartheta}$$

where s^* is the optimal choice of s.

Let $c(s) = \frac{1}{m}s^m$ for $m \geq 2$. Then the first-order condition of the financial sector implies that $w_s = s^{m-1}$ and the first-order condition of the non-financial sector implies that:

$$\vartheta\alpha K^{1-\vartheta}s^{\vartheta-1} = w_s = s^{m-1}$$

Solving for s^*, substituting into the expression above for $V(K)$, and simplifying gives:

$$s^* = (\vartheta\alpha)^{\frac{1}{m-\vartheta}} K^{\frac{1-\vartheta}{m-\vartheta}}$$

$$V(K) = (1-\vartheta)\alpha^{\frac{m}{m-\vartheta}} K^{\gamma} \quad \text{where} \quad \gamma = \frac{(1-\vartheta)}{1-\frac{\vartheta}{m}}$$

Hence, $V(K)$ has the power form that is used in the paper. Moreover,for $m \geq 2$ (which is assumed), $\gamma < 1$.

A.3 Properties of Expected Tax Revenue: $\mathcal{T}$

For the assumed parametric forms, we obtained the following results:

$$\mathcal{T} = \theta_0 \gamma^{\frac{\gamma}{1-\gamma}} (1-\theta_0)^{\frac{\gamma}{1-\gamma}}$$

$$\frac{d\mathcal{T}}{d\theta_0} = \gamma^{\frac{\gamma}{1-\gamma}} (1-\theta_0)^{\frac{\gamma}{1-\gamma}} - \theta_0 \frac{\gamma}{1-\gamma} \gamma^{\frac{\gamma}{1-\gamma}} (1-\theta_0)^{\frac{\gamma}{1-\gamma}-1} = \frac{\mathcal{T}}{\theta}\left(1 - \frac{\gamma}{1-\gamma}\frac{\theta_0}{1-\theta_0}\right)$$

$$\frac{d^2\mathcal{T}}{d\theta_0^2} = -2\frac{\gamma}{1-\gamma}\gamma^{\frac{\gamma}{1-\gamma}}(1-\theta_0)^{\frac{\gamma}{1-\gamma}-1} + \frac{\theta_0}{1-\theta_0}\left(\frac{\gamma}{1-\gamma} - 1\right)\frac{\gamma}{1-\gamma}\gamma^{\frac{\gamma}{1-\gamma}}(1-\theta_0)^{\frac{\gamma}{1-\gamma}-1}$$

The second line shows that $d\mathcal{T}/d\theta_0 > 0$ on $[0, \theta_0^{max})$ and $d\mathcal{T}/d\theta_0 < 0$ on $(\theta_0^{max}, 1)$ where θ_0^{max} solves: $\frac{\gamma}{1-\gamma}\frac{\theta_0^{max}}{1-\theta_0^{max}} = 1$. It is zero at θ^{max} and at 1 (where $\mathcal{T} = 0$).

The third line implies that $d^2\mathcal{T}/d\theta_0^2 < 0$ on $[0, \theta_0^{max}]$ so $\mathcal{T}$ is *increasing* but *concave* on this region. To see this, note that the third line can be rewritten as:

$$\frac{d^2\mathcal{T}}{d\theta_0^2} = \left(-2 + \frac{\gamma}{1-\gamma}\frac{\theta_0}{1-\theta_0} - \frac{\theta_0}{1-\theta_0}\right)\frac{\gamma}{1-\gamma}\gamma^{\frac{\gamma}{1-\gamma}}(1-\theta_0)^{\frac{\gamma}{1-\gamma}-1}$$

We know that $-1 + \frac{\gamma}{1-\gamma}\frac{\theta_0}{1-\theta_0} < 0$ on $[0, \theta_0^{max}]$ and so, on this region, the leading term in parenthesis is negative. Since the remaining terms are positive, $d^2\mathcal{T}/d\theta_0^2 < 0$ in this region.

A.4 The Government's First-Order Condition

From (3) we obtain the following first order condition of the government for the tax rate, θ_0:

$$\left[\frac{\partial f(K_0, s_0)}{\partial s_0} - c'(s_0)\right]\frac{ds_0}{dT_0}\frac{dT_0}{d\mathcal{T}}\frac{d\mathcal{T}}{d\theta_0} + [V'(K_1) - 1]\frac{dK_1}{d\theta_0} = 0 \qquad \text{(A.4.1)}$$

Note that the derivatives of s_0 and $\mathcal{T}$ here are total derivatives, since the government's choices are subject to the equilibrium choices of the financial and non-financial sectors.

As shown above, $d\mathcal{T}/d\theta_0$ is positive and decreasing (towards zero), but remains positive, on $[0, \theta_0^{max}]$. Therefore, the mapping from tax level (θ_0) to the marginal rate of transformation of taxes into tax revenue ($d\mathcal{T}/d\theta_0$), is invertible on this region. A high tax rate corresponds to a low marginal rate of transformation of taxes into tax revenue and vice versa. Note that the optimal tax rate must be in the region $[0, \theta_0^{max}]$, since any further increase in θ_0 beyond θ_0^{max} reduces tax revenue and investment. Hence, we can limit the consideration of the optimal tax rate to this region. Since $d\mathcal{T}/d\theta_0$ is positive and the mapping from θ_0 to $\mathcal{T}$ is invertible in this region, we can instead consider the government's first order condition with respect to $\mathcal{T}$, which turns out to be more intuitive for analyzing the government's problem. Dividing (A.4.1) through by $d\mathcal{T}/d\theta_0$, and rewriting $(dK_1/d\theta_0)/(d\mathcal{T}/d\theta_0) = dK_1/d\mathcal{T}$ we obtain this alternative first-order condition:

$$\left[\frac{\partial f(K_0, s_0)}{\partial s_0} - c'(s_0)\right]\frac{ds_0}{dT_0} + [V'(K_1) - 1]\frac{dK_1}{d\mathcal{T}} = 0 \tag{A.4.2}$$

where the term $dT_0/d\mathcal{T}$, which equals 1 under a no-default government policy, is omitted from the expression.

A.5 Under-Investment Loss Due to Taxes

We want to obtain an expression for the second term in (10), the transfer version of the government's first-order condition:

$$\frac{[V'(K_1) - 1]\frac{dK_1}{d\theta_0}}{\frac{d\mathcal{T}}{d\theta_0}}$$

The first-order condition for investment of the non-financial sector, (9), and the parametric form for V imply that:

$$\begin{aligned} V'(K_1) - 1 &= \theta_0 V'(K_1) \\ &= \theta_0 \gamma K^{\gamma - 1} \end{aligned}$$

Substituting in the parametric form also gives:

$$\frac{dK_1}{d\theta_0} = \frac{1}{1 - \theta_0}\frac{1}{\gamma - 1}K_1$$

Moreover, from (9) we can solve for the equilibrium K_1 as a function of θ_0:

$$K_1 = \gamma^{\frac{1}{1-\gamma}}(1 - \theta_0)^{\frac{1}{1-\gamma}}$$

We can obtain the numerator to our fraction of interest by multiplying the expressions for the two terms together:

$$\begin{aligned} [V'(K_1) - 1]\frac{dK_1}{d\theta_0} &= \frac{\theta_0 \gamma}{(1 - \theta_0)(\gamma - 1)}K^{\gamma} \\ &= \frac{\theta_0}{1 - \theta_0}\frac{\gamma}{\gamma - 1}\gamma^{\frac{\gamma}{1-\gamma}}(1 - \theta_0)^{\frac{\gamma}{1-\gamma}} \\ &= \frac{\mathcal{T}}{\theta_0}\frac{\theta_0}{1 - \theta_0}\frac{\gamma}{\gamma - 1} \end{aligned}$$

where the second line follows by substituting in the expression for K_0 and the third line follows by substituting in the expression for $\mathcal{T}$. Appendix A.3 derives $d\mathcal{T}/d\theta_0$. Dividing the expression for the numerator by the expression for $d\mathcal{T}/d\theta_0$ shows that the marginal loss per transfer is given by:

$$\frac{d\mathcal{L}}{d\mathcal{T}} = \frac{[V'(K_1)-1]\frac{dK_1}{d\theta_0}}{\frac{d\mathcal{T}}{d\theta_0}} = \frac{-\frac{\theta_0}{1-\theta_0}\frac{\gamma}{1-\gamma}}{1-\frac{\theta_0}{1-\theta_0}\frac{\gamma}{1-\gamma}}$$

From this it is clear that $d\mathcal{L}/d\mathcal{T} \to -\infty$ as $\theta_0 \to \theta^{max}$ (since at θ^{max} the denominator is 0). Additionally, we have:

$$\frac{d^2\mathcal{L}}{d\mathcal{T}^2} = \frac{d^2\mathcal{L}}{d\theta_0 d\mathcal{T}}\frac{d\theta_0}{d\mathcal{T}} < 0 \qquad \text{for} \quad \theta_0 \in [0, \theta^{max}) \quad .$$

Hence, the marginal loss to the economy is increasing in magnitude (getting worse) as the tax rate increases up to θ^{max} and expected tax revenue rises to $\mathcal{T}^{max}$. In other words, marginal tax revenues becomes increasingly expensive to raise as the marginal loss to the economy from underinvestment rises in the tax rate/level of tax revenues.

A.6 Proof of Proposition 1

We show sufficient conditions which, under certainty and no-default, make the government's problem (3) concave, so that optimum is given by the unique solution to (10). For $f(K,s)$ and $c(s)$ we use the functional forms given in the main text.

Substituting (7) into (5) and solving, we obtain the equilibrium level of s_0 (note that we refer to the *equilibrium* level of s_0 also as s_0, an abuse of notation intended to reduce clutter):

$$s_0 = \left(\frac{\vartheta\alpha}{\beta}\right)^{\frac{1}{m-\vartheta}} K_0^{\frac{1-\vartheta}{m-\vartheta}} p_{solv}^{\frac{1}{m-\vartheta}}$$

Now substitute this into the expression for $d\mathcal{G}/d\mathcal{T}$ to get:

$$\frac{d\mathcal{G}}{d\mathcal{T}} = \frac{\partial f(K_0,s_0)}{\partial s}(1-p_{solv})\frac{ds_0}{dT_0} = \frac{1}{m-\vartheta}\left(\vartheta\alpha K_0^{1-\vartheta}\right)^{\frac{m}{m-\vartheta}} \beta^{\frac{-\vartheta}{m-\vartheta}} p_{solv}^{\frac{\vartheta}{m-\vartheta}-1}(1-p_{solv})\frac{dp_{solv}}{dT_0}$$

Taking derivative again with respect to $\mathcal{T}$ shows that:

$$\frac{d^2\mathcal{G}}{d\mathcal{T}^2} \propto \left(\frac{\vartheta}{m-\vartheta}-1\right) p_{solv}^{\frac{\vartheta}{m-\vartheta}-2}(1-p_{solv})\frac{dp_{solv}}{dT_0} - p_{solv}^{\frac{\vartheta}{m-\vartheta}-1}\left(\frac{dp_{solv}}{dT_0}\right)^2 + p_{solv}^{\frac{\vartheta}{m-\vartheta}-1}(1-p_{solv})\frac{d^2p_{solv}}{dT_0^2}$$

where $dT_0/d\mathcal{T} = 1$ is omitted. Since the second term in the above expression is always negative, a sufficient condition to ensure that $d^2\mathcal{G}/d\mathcal{T}^2 < 0$ is to ensure that the first and third terms in the above expression are non-positive. The condition: $m - 2\vartheta \geq 0$ ensures that the first term is non-positive. The third term is negative if the slope of the probability density of $\tilde{A}_1$ at $\underline{A}_1$ is non-positive. Letting $\tilde{A}_1$ take a uniform distribution sets this term to zero.[10]

Since we have shown that both $\mathcal{G}$ and $\mathcal{L}$ are concave in $\mathcal{T}$, the government's problem is concave in $\mathcal{T}$. Furthermore, the optimum tax revenue, $\hat{\mathcal{T}}$, must correspond to a tax rate $\hat{\theta} < \theta^{max}$, because the first-order condition is *negative* at θ^{max}. To see that this is the case, note that $d\mathcal{L}/d\mathcal{T} \to \infty$ as $\theta \to \theta^{max}$ while $d\mathcal{G}/d\mathcal{T}$ is finite for $p_{solv} > 0$.

A.6.1 Impact of L_1 and N_D on $\mathcal{T}$

Let $x = L_1$ or N_D. Rewriting (10) using the gain and loss notation as $d\mathcal{G}/d\mathcal{T} + d\mathcal{L}/d\mathcal{T} = 0$ and then taking the derivative with respect to x gives:

$$\frac{d^2\mathcal{G}}{dx d\mathcal{T}} + \frac{d^2\mathcal{L}}{dx d\mathcal{T}} = 0 \tag{A.6.1}$$

Using the Implicit Function Theorem, the two terms on the right side evaluate to the following:

$$\begin{aligned}
\frac{d^2\mathcal{G}}{dx d\mathcal{T}} &= \frac{d}{dp_{solv}}\left(\frac{d\mathcal{G}}{d\mathcal{T}}\right)\left\{\frac{\partial p_{solv}}{\partial T_0}\left(\frac{\partial T_0}{\partial \mathcal{T}}\frac{d\mathcal{T}}{dx} + \frac{\partial T_0}{\partial x}\right) + \frac{\partial p_{solv}}{\partial x}\right\} \\
\frac{d^2\mathcal{L}}{dx d\mathcal{T}} &= \frac{d^2\mathcal{L}}{d\mathcal{T}^2}\frac{d\mathcal{T}}{dx}
\end{aligned}$$

Substituting into (A.6.1) and combining the terms multiplying $d\mathcal{T}/dx$ yields:

$$\frac{d\mathcal{T}}{dx}\left[\frac{d}{dp_{solv}}\left(\frac{d\mathcal{G}}{d\mathcal{T}}\right)\frac{\partial p_{solv}}{\partial T_0}\frac{\partial T_0}{\partial \mathcal{T}} + \frac{d^2\mathcal{L}}{d\mathcal{T}^2}\right] = -\frac{d}{dp_{solv}}\left(\frac{d\mathcal{G}}{d\mathcal{T}}\right)\left\{\frac{\partial p_{solv}}{\partial T_0}\frac{\partial T_0}{\partial x} + \frac{\partial p_{solv}}{\partial x}\right\} \tag{A.6.2}$$

Note for the left-hand side term in parenthesis:

$$\frac{d}{dp_{solv}}\left(\frac{d\mathcal{G}}{d\mathcal{T}}\right)\frac{\partial p_{solv}}{\partial T_0}\frac{\partial T_0}{\partial \mathcal{T}} + \frac{d^2\mathcal{L}}{d\mathcal{T}^2} = \frac{d^2\mathcal{G}}{d\mathcal{T}^2} + \frac{d^2\mathcal{L}}{d\mathcal{T}^2} < 0$$

For $x = N_D$:

$$\frac{\partial p_{solv}}{\partial T_0}\frac{\partial T_0}{\partial x} + \frac{\partial p_{solv}}{\partial x} = \frac{\partial p_{solv}}{\partial T_0}(k_A - 1) < 0$$

since $\partial T_0/\partial N_D = -1$ and $\partial p_{solv}/\partial N_D = (\partial p_{solv}/\partial T_0)k_A$.

[10] Using an exponential distribution would also be sufficient. For the log-normal distribution, this term will be negative for a range of values below a cutoff.

For $x = L_1$:

$$\frac{\partial p_{solv}}{\partial T_0}\frac{\partial T_0}{\partial x} = 0 \quad \text{and} \quad \frac{\partial p_{solv}}{\partial x} < 0$$

so for either value of x, the term in braces on the right side is negative. Finally, the intermediate steps in the proof of the concavity of G in $\mathcal{T}$ show that

$$\frac{d}{dp_{solv}}\left(\frac{d\mathcal{G}}{d\mathcal{T}}\right) < 0$$

Combining these results shows that $d\mathcal{T}/dx > 0$ for $x = L_1$ or N_D.

A.6.2 Impact of N_D on T_0

To show how T_0 changes with N_D, begin by using the result above for $\mathcal{T}$. In particular, letting $x = N_D$ in (A.6.2) and simplifying the right-side expression using $\frac{\partial p_{solv}}{\partial T_0}\frac{\partial T_0}{\partial x} + \frac{\partial p_{solv}}{\partial x} = \frac{\partial p_{solv}}{\partial T_0}(k_A - 1)$ and $d^2\mathcal{G}/(dT_0 d\mathcal{T}) = d^2\mathcal{G}/d\mathcal{T}^2$ gives:

$$\frac{d\mathcal{T}}{dN_D}\left[\frac{d^2\mathcal{G}}{d\mathcal{T}^2} + \frac{d^2\mathcal{L}}{d\mathcal{T}^2}\right] = (1 - k_A)\frac{d^2\mathcal{G}}{d\mathcal{T}^2}$$

$$\frac{d\mathcal{T}}{dN_D} = \frac{(1 - k_A)\frac{d^2G}{d\mathcal{T}^2}}{\frac{d^2\mathcal{G}}{d\mathcal{T}^2} + \frac{d^2\mathcal{L}}{d\mathcal{T}^2}} \quad \Rightarrow \quad 0 < \frac{d\mathcal{T}}{dN_D} < 1 - k_A$$

Since $T_0 = \mathcal{T} - N_D$,

$$\frac{dT_0}{dN_D} = \frac{d\mathcal{T}}{dN_D} - 1 \quad \Rightarrow \quad -1 < \frac{dT_0}{dN_d} < -k_A$$

Moreover, this shows that $T_0 + k_A N_D$, the *gross* transfer to the financial sector, is *decreasing* in N_D.

A.6.3 Impact of Factor Share on $\mathcal{T}$

Next we examine the effect of the factor share of financial services on $\mathcal{T}$, while holding constant total output. To that end, we consider the impact of a change in ϑ while simultaneously adjusting α (the level of productivity) to keep output constant. Let $D(\cdot)$ be the following differential with respect to $d\vartheta$ and $d\alpha$

$$Dg = \frac{dg}{d\vartheta}d\vartheta + \frac{dg}{d\alpha}d\alpha$$

where the derivatives are taken holding $\mathcal{T}$ constant but include the change caused by $ds_0/d\vartheta$ and $ds_0/d\alpha$. Now let $d\alpha$ be set to keep total output constant, e.g., $Df = 0$, where f is equilibrium output. This implies $d\alpha = -(df/d\vartheta)/(df/d\alpha)d\vartheta$, which gives:

$$\frac{Dg}{d\vartheta} = \frac{dg}{d\vartheta} - \frac{dg}{d\alpha}\left(\frac{df/d\vartheta}{df/d\alpha}\right)$$

Hence, to find the impact of ϑ on $\mathcal{T}$ while holding output constant, we analyze $D\mathcal{T}/d\vartheta$. Applying this differentiation operator to the first-order condition for $\mathcal{T}$ and collecting terms gives:

$$\left(\frac{d^2\mathcal{G}}{d\mathcal{T}^2}+\frac{d^2\mathcal{L}}{d\mathcal{T}^2}\right)\frac{D\mathcal{T}}{d\vartheta}+\frac{D}{d\vartheta}\frac{dG}{d\mathcal{T}}+\frac{D}{d\vartheta}\frac{d\mathcal{L}}{d\mathcal{T}}=0 \tag{A.6.3}$$

Note that the application of the D operator is linear as it is simply a sum of two derivatives. Furthermore,

$$\frac{D}{d\vartheta}\left(\frac{d\mathcal{G}}{d\mathcal{T}}\right)>0$$
$$\frac{D}{d\vartheta}\left(\frac{d\mathcal{L}}{d\mathcal{T}}\right)=0$$

The first line is proved below, while the second line follows directly since $d\mathcal{L}/d\mathcal{T}$ is not a function of ϑ or α. Using the second-order condition, it follows that $D\mathcal{T}/d\vartheta>0$.

To prove the first line from above, note that the sign of this term in question is equal to the sign of $D(\partial f/\partial s_0\times ds_0/dT_0)/d\vartheta$. This follows from the expression for $d\mathcal{G}/d\mathcal{T}$ and that p_{solv} does not depend on ϑ or α. Substituting (7) into (5) and using the functional form of $c(s_0)$ shows that

$$\operatorname{sgn}\left(\frac{D}{d\vartheta}\frac{\partial f}{\partial s_0}\right)=\operatorname{sgn}\left((m-1)s_0^{m-2}\frac{Ds_0}{d\vartheta}\right)$$

Since $m>1$, this last term equals $\operatorname{sgn}(Ds_0/d\vartheta)$. To find $\operatorname{sgn}(Ds_0/d\vartheta)$, substitute (7) into (5), multiply both sides of the resulting expression by s_0, and substitute in the functional forms of f and $c(s_0)$ to obtain:

$$\vartheta f(K_0,s_0)p_{solv}=\beta s_0^m \quad .$$

Applying the D operator to both sides of this expression gives:

$$\frac{D(\vartheta f(K_0,s_0)p_{solv})}{d\vartheta}=f(K_0,s_0)p_{solv}$$
$$\frac{D(\beta s_0^m)}{d\vartheta}=ms_0^{m-1}\frac{Ds_0}{d\vartheta}$$

Since the right-hand side of the first line is positive, so must be the right-hand side of the second line, showing that $Ds_0/d\vartheta>0$ and hence, $\operatorname{sgn}\left(D(\partial f/\partial s_0)/d\vartheta\right)>0$.

It remains to find $\operatorname{sgn}\left(D(ds_0/dT_0)/d\vartheta\right)$, which can be found using (A.1.1). Using similar steps to those immediately above, it can be shown that if $m\leq 2$ then $\operatorname{sgn}\left(D(d^2f/s_0^2)/d\vartheta\right)\geq 0$. Moreover, direct differentiation and $Ds_0/d\vartheta>0$ show that if $m\leq 2$ then $\operatorname{sgn}(c''(s_0))<0$. It is then straightforward to show that $\operatorname{sgn}\left(D(ds_0/dT_0)/d\vartheta\right)>0$.

A.7 Proof of Lemma 4

The derivative of the government's objective with respect to N_T is given by:

$$\frac{d\mathcal{G}}{dT_0}\frac{dT_0}{dN_T}$$

When $N_T + N_D \geq \mathcal{T}$ (Region 2), then $T_0 = N_T P_0 = \frac{N_T}{N_T+N_D}\mathcal{T}$ and

$$\frac{dT_0}{dN_T} = P_0 + N_T\frac{dP_0}{dN_T} = P_0\left(\frac{N_D}{N_T + N_D}\right) \quad .$$

Therefore $dT_0/dN_T > 0$ if $N_D > 0$. Moreover, this implies that $N_T \to \infty$ is optimal in the default region. Alternatively, if $N_D = 0$, then increasing N_T into the default region provides no benefit but does incur the loss of D.

When $N_T \to \infty$, then $T_0 = \mathcal{T}$, as pre-existing bondholders are completely diluted. Note that $T_0 = \mathcal{T}$ is the same situation as if N_D were set to 0. Conditional on this, the government's problem reduces to the same problem it faces in Region 1. Therefore, to determine if default is optimal, the government needs to compare this optimum-cum-default-loss, $W_1\big|N_D = 0 - D$, with the maximum from region 1, W_1. Since the optimum within the default region can be found by setting $N_D = 0$, Appendix A.6.2 shows that the transfer will be bigger conditional on default. By Appendix A.1 this implies the equilibrium provision of financial services is greater.

A.8 Proof of Proposition 2

To prove point (1), take the derivative of (11) with respect to L_1 and simplify the resulting expression to obtain:

$$\int_{\hat{T_0}^{no\_def}}^{\hat{T_0}^{def}-k_A N_D} \frac{d}{dL_1}\left(\frac{d\mathcal{G}}{dT_0}\right) > 0$$

The intermediate steps in Appendix A.4 show that the derivative in the integrand is positive. As shown in Appendix A.6.2, the *gross* transfer is decreasing in N_D, so $T_0^{def} > k_A N_D + T_0^{no\_def}$ and hence the integral is positive.

To prove the statement for N_D, take the derivative of (11) with respect to N_D. Simplifying the derivative at the upper integration boundary gives $-k_A d\mathcal{G}/dT_0\big|_{\hat{T_0}^{def}-k_A N_D}$ while from the lower boundary we get we get $d\mathcal{G}/dT_0\big|_{\hat{T_0}^{no\_def}}$. The remaining part of the derivative is:

$$\begin{aligned}\int_{\hat{T_0}^{no\_def}}^{\hat{T_0}^{def}-k_A N_D} \frac{d}{dN_D}\left(\frac{d\mathcal{G}}{dT_0}\right) &= k_A \int_{\hat{T_0}^{no\_def}}^{\hat{T_0}^{def}-k_A N_D} \frac{d}{dT_0}\left(\frac{d\mathcal{G}}{dT_0}\right) \\ &= k_A\left(\frac{d\mathcal{G}}{dT_0}\bigg|_{\hat{T_0}^{def}-K_A N_D} - \frac{d\mathcal{G}}{dT_0}\bigg|_{\hat{T_0}^{no\_def}}\right)\end{aligned}$$

Combining the three parts of the derivatives gives: $(1-k_A)d\mathcal{G}/dT_0\big|_{\hat{T_0}^{no\_def}} > 0$. To show that the benefit of defaulting is convex in N_D, take a second derivative to obtain: $(1-k_A)d^2\mathcal{G}/dT_0^2\big|_{\hat{T_0}^{no\_def}} dT_0^{no\_def}/dN_D > 0$.

To prove the statement for factor share, apply the operator $D/d\vartheta$ (defined in Appendix A.6.3) to (11) and again simplify to get:

$$\int_{\hat{T_0}^{no\_def}}^{\hat{T_0}^{def}-k_AN_D} \frac{D}{d\vartheta}\left(\frac{d\mathcal{G}}{dT_0}\right) > 0$$

The integrand is positive as shown in Appendix A.6.3, so again the integral is positive.

Finally, taking the derivative with respect to k, we obtain $-(d\mathcal{G}/dT_0)N_D < 0$ at the upper integration boundary and 0 at the lower boundary. In the interior we obtain

$$\int_{\hat{T_0}^{no\_def}}^{\hat{T_0}^{def}-k_AN_D} \frac{d}{dk_A}\left(\frac{d\mathcal{G}}{dT_0}\right) = N_D \int_{\hat{T_0}^{no\_def}}^{\hat{T_0}^{def}-k_AN_D} \frac{d}{dT_0}\left(\frac{d\mathcal{G}}{dT_0}\right) < 0$$

so the derivative is negative.

A.9 Optimal Tax Revenue Under Uncertainty

The first order condition for the government's choice of $\mathcal{T}$ is given by:

$$\frac{d\mathcal{G}}{dT_0}\frac{dT_0}{d\mathcal{T}} + \frac{d\mathcal{L}}{d\mathcal{T}} = 0$$

Whereas under certainty $dT_0/d\mathcal{T}$=1, this is no longer the case. Taking the derivative of T_0 with respect to $\mathcal{T}$ in (14) (while holding H constant) and then using (12) to substitute into the resulting expression gives $dT_0/d\mathcal{T} = P_0H$. Therefore, the first-order condition for $\mathcal{T}$ is:

$$\frac{d\mathcal{G}}{\partial T_0}HP_0 + \frac{d\mathcal{L}}{d\mathcal{T}} = 0 \tag{A.9.1}$$

with T_0 given in (14). The loss due to underinvestment, $\mathcal{L}$, is the same as under certainty. Recall that it is concave, with the magnitude of the marginal loss, $d\mathcal{L}/d\mathcal{T}$, increasing in $\mathcal{T}$. Similarly, $d\mathcal{G}/dT_0$, the gain to the economy from the increased provision of financial services, remains the same with uncertainty and is decreasing in T_0. However, the rate at which T_0 increases in $\mathcal{T}$ is now HP_0 rather than 1. Note that this rate is a constant in $\mathcal{T}$, as P_0 is only a function of H, and is less than 1.[11] Finally, the second-order condition for $\mathcal{T}$ holds

$$\frac{d^2\mathcal{G}}{\partial T_0^2}(HP_0)^2 + \frac{d^2\mathcal{L}}{d\mathcal{T}^2} < 0$$

as $\mathcal{G}$ and $\mathcal{L}$ are concave and HP_0 is a function only of H.

[11] To see this, note that $HP_0 = E_0\left[\min\left(H, \tilde{R}_V\right)\right] < E_0[\tilde{R}_V] = 1$.

A.10 Optimal Probability of Default Under Uncertainty

Changing H affects two components of the government's objective. As can be seen from (14), increasing H changes T_0. Unlike the case with $\mathcal{T}$, however, increasing H does not have any effect on investment. Instead, the cost associated with increasing H is that it increases the probability of default, and so also the expected deadweight cost. The first-order condition for H shows this tradeoff:

$$\frac{d\mathcal{G}}{dT_0}\frac{dT_0}{dH} - D\frac{dp_{def}}{dH} = 0$$

From (13), it is clear that $dp_{def}/dH > 0$ and we can think of choosing H exactly as choosing the probability of default. The effect on $T_0 = P_0 N_T$ is less immediately clear, since increasing H increases N_T, but decreases P_0. However, (14) shows that $dT_0/dH > 0$. To see this we break up T_0 into two terms based on (14) and consider their derivatives:

$$d\,(\mathcal{T} - \frac{N_D}{H})/dH = \frac{N_D}{H^2} > 0 \tag{A.10.1}$$

$$d\,E_0\left[\min\left(H, \tilde{R}_V\right)\right]/dH = (1 - p_{def}) > 0 \tag{A.10.2}$$

Demonstrating the equivalence in the second line is straightforward, as shown in Appendix A.11. We refer to (A.10.1) as increasing the *dilution* of existing bondholders' claim, since the increase in H reduces the share of tax revenues that goes to the holders of the existing debt, N_D. We refer to (A.10.2) as reducing either the *default buffer* or *precautionary taxation*, since by increasing H, it increases the probability that $\tilde{R}_V < H$, in which case the government defaults. Hence, (A.10.1) and (A.10.2) show that increasing H (while holding $\mathcal{T}$ constant) increases T_0. It immediately follows that $d\mathcal{G}/dH > 0$ and there is a benefit to increasing H. Substituting in for dT_0/dH, the first-order condition becomes:

$$\frac{d\mathcal{G}}{dT_0}\left(\frac{N_D}{H^2}E_0\left[\min\left(H, \tilde{R}_V\right)\right] + (\mathcal{T} - \frac{N_D}{H})(1 - p_{def})\right) - D\frac{dp_{def}}{dH} = 0$$

Appendix A.11 also shows that as H increases, raising it further becomes decreasingly effective at increasing T_0:

$$\frac{d^2T_0}{dH^2} = \frac{-2N_D}{H^3}\int_0^H x p_{\tilde{R}_V}(x)dx - (\mathcal{T} - \frac{N_D}{H})p_{\tilde{R}_V}(H) < 0$$

where $p_{\tilde{R}_V}(x)$ denotes the probability density of $\tilde{R}_V$ evaluated at x. In other words, T_0 is concave in H. Together with the concavity of $\mathcal{G}$ in T_0, this implies that $\mathcal{G}$ is concave in H, e.g., $d^2\mathcal{G}/dH^2$.[12] The implication is that while increasing H provides a benefit to

[12] Note that in the first-order conditions, we have assumed that the government takes into account the (negative) impact of higher H on prices. Thus, we have *NOT* treated the government here as a price-taker. If we instead treat the government as a price-taker, the resulting conditions are simpler: $dT_0/dH = P_0\mathcal{T}$ (as dP_0/dH is omitted due to the price-taking assumption) and the first-order condition is: $d\mathcal{G}/dT_0(P_0\mathcal{T}) - D d\,p_{def}/dH = 0$. In this case, concavity of $\mathcal{G}$ in H still holds because $\mathcal{G}$ is concave in T_0.

the government by increasing the transfer through dilution and reduction of precautionary taxation, the marginal benefit is decreasing. Meanwhile, the government bears a cost for increasing H; the resulting increased likelihood of default increases the expected deadweight cost of default.

We assume that at the optimal choice of H, $d^2 p_{def}/d^2 H \geq 0$.

A.11 Uncertainty Calculations

To derive $d\,E_0\left[\min\left(H, \tilde{R}_V\right)\right]/dH$, rewrite the expectation as:

$$E_0\left[\min\left(H, \tilde{R}_V\right)\right] = \int_0^H x\, p_{\tilde{R}_V}(x)dx + H\int_H^\infty p_{\tilde{R}_V}(x)dx$$

Now taking the derivative with respect to H, one obtains:

$$\begin{aligned} d\,E_0\left[\min\left(H, \tilde{R}_V\right)\right]/dH =& H p_{\tilde{R}_V}(H) - H p_{\tilde{R}_V}(H) + \int_H^\infty p_{\tilde{R}_V}(x)dx \\ =& \int_H^\infty p_{\tilde{R}_V}(x)dx \\ =& (1 - p_{def}) \end{aligned}$$

The first line is just the derivative, while the last line follows by definition of p_{def}.

Using this result we have that:

$$\frac{dT_0}{dH} = \frac{N_D}{H^2}E_0\left[\min\left(H, \tilde{R}_V\right)\right] + \left(\mathcal{T} - \frac{N_D}{H}\right)(1 - p_{def})$$

Substituting in the expression above for $E_0\left[\min\left(H, \tilde{R}_V\right)\right]$, taking the derivative with respect to T_0, and simplifying gives:

$$\begin{aligned} \frac{d^2T_0}{dH^2} =& \frac{-2N_D}{H^3}\left[\int_0^H x\, p_{\tilde{R}_V}(x)dx + H\int_H^\infty p_{\tilde{R}_V}(x)dx\right] + \frac{N_D}{H^2}(1 - p_{def}) \\ &+ \frac{N_D}{H^2}(1 - p_{def}) - \left(\mathcal{T} - \frac{N_D}{H}\right)p_{\tilde{R}_V}(H) \\ =& \frac{-2N_D}{H^3}\left[\int_0^H x\, p_{\tilde{R}_V}(x)dx\right] - \left(\mathcal{T} - \frac{N_D}{H}\right)p_{\tilde{R}_V}(H) \end{aligned}$$

Since $(\mathcal{T} - N_D/H) = N_T/H > 0$, it is clear that $d^2T_0/dH^2 < 0$.

A.12 Proof of Proposition 3

TBC