Trade in Intermediate Inputs and Business Cycle Comovement

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Abstract

The standard international real business cycle model struggles to replicate the strong empirical correlation between bilateral trade and output comovement. This paper explores whether trade in intermediate inputs resolves this puzzle. I integrate input trade into a many country, multi-sector model, and calibrate the model to match bilateral input-output linkages. I find that input trade fails to resolve the aggregate puzzle. The model fails in large part because it cannot match the observed correlation of services output across countries. In contrast, the model matches observed correlations of goods output well. Further, independent shocks across countries explain one-quarter of the trade-comovement relationship for gross output of goods. However, because independent shocks are transmitted through input linkages, they synchronize gross output, not value added. Using simulated data, I argue that caution is needed in interpreting trade-comovement regressions that include proxies for vertical linkages.

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1 Introduction

A large empirical literature suggests that international trade transmits shocks and synchronizes economic activity across borders. For example, bilateral trade is strongly (and robustly) correlated with bilateral GDP comovement.[1] Though standard international business cycle models predict a positive correlation between trade and comovement, they cannot replicate the quantitative magnitude of the empirical correlation. Kose and Yi (2006) have dubbed this the "trade comovement puzzle."[2]

In addressing this puzzle, recent empirical work has turned attention to the role of intermediate goods trade as a conduit for shocks. For example, Ng (2010) documents that proxies for bilateral production fragmentation predict bilateral GDP correlations, while Di Giovanni and Levchenko (2010) document that bilateral trade is more important in explaining output comovement for home and foreign sectors that use each other as intermediates. Further, Burstein, Kurz, and Tesar (2008) show that countries that intensively engage in intra-firm trade with United States multinational parents display higher manufacturing output correlations with the U.S.[3]

This focus on intermediate goods trade is potentially important, since trade in intermediate inputs accounts for as much as two thirds of international trade. This input trade links production processes across borders and opens relatively unexplored channels for shock transmission. To examine these channels, I develop a many country, multi-sector extension of the standard international real business cycles model that includes both trade in interme-

[1]See, for example, Frankel and Rose (1998), Imbs (2004), Baxter and Kouparitsas (2005), Kose and Yi (2005), Calderón, Chong, and Stein (2007), Inklaar, Jong-A-Pin, and Haan (2008), Di Giovanni and Levchenko (2010), and Ng (2010). The estimated partial correlation varies across studies, with differences in country samples and specifications, but trade is a significant predictor of comovement in all of them.

[2]This problem has been been long recognized, see Baxter (1995).

[3]In a related vein, Bergin, Feenstra, and Hanson (2009) find that Mexican export assembly (maquiladora) industries are twice as volatile as their US counterparts, suggesting possibly strong transmission of US shocks to Mexico via production sharing linkages.

diate and final goods. I then calibrate the model to data on bilateral final and intermediate goods trade flows for 22 countries and a composite rest-of-the-world region, and simulate model responses to sector-specific productivity shocks. Using the simulated data, I assess the ability of the model with intermediates to explain observed bilateral output correlations, with emphasis on evaluating the empirical importance of intermediate goods trade in generating output comovement.

In the model, intermediate goods trade transmits shocks across borders independent of, and in addition to, standard IRBC transmission mechanisms. In the cannonical IRBC model, productivity shocks are transmitted abroad via relative prices. Specifically, a positive shock in the home country raises home output and depreciates home's terms of trade, which induces increased labor supply and hence output abroad. Thus, the endogenous response of factor supply in response to relative price movements is essential to generating output comovement from idiosyncratic shocks.[4] In contrast, intermediate goods linkages generate comovement in gross output even if factor supply is exogenous.[5] With traded intermediates, productivity shocks are passed downstream through the production chain directly.[6] That is, a positive shock in the home country raises output in foreign countries that use home's goods as intermediates in production. Therefore, the production chain itself puts significant structure to how shocks are transmitted.

Both these mechanisms are operative in the general model. To quantify their importance, I calibrate the model to data on both final and intermediate goods trade. Specifically, I use

[4]Other recent attempts to solve the puzzle (not involving intermediate goods) have focused on lowering the short run elasticity of substitution between home and foreign goods, such as through introducing durable goods (Engel and Wang (forthcoming)) or search and matching frictions (Drozad and Nosal (2008)). Lowering the elasticity tends to amplify this channel.

[5]In the exogenous factor supply case, gross output comoves across countries, but value added does not. I discuss this distinction at length below.

[6]Productivity shocks travel unidirectionally downstream when intermediate goods are aggregated in a Cobb-Douglas fashion, the case considered in the benchmark model below. More generally, productivity shocks travel both downstream and upstream to supplies of intermediates.

data from national input-output tables combined with data on bilateral trade to construct a synthetic global input-output framework, as in Johnson and Noguera (2010).[7] One advantage of this approach is that the framework respects national accounts definitions of final and intermediate goods, and therefore is consistent with national accounts aggregates. Further, in distinguishing between final and intermediate goods, I improve on standard calibration procedures that ignore the "double counting" problem in trade statistics and implicitly assume that only final goods are traded.

Proceeding to the numerical analysis, input trade (in this model) does not solve the aggregate trade-comovement puzzle. Simulating the model with correlated productivity shocks across countries (estimated from data), bilateral GDP growth correlations in the model are positively correlated with GDP growth correlations in the data, yet predicted correlations account for a modest portion ($\approx 15\%$) of the variation in bilateral growth correlations observed in the data. Regressing bilateral GDP correlations on bilateral trade in the simulated data returns a coefficient equal 1/4-1/3 of that in the real data.[8] The aggregate model's poor performance is particularly disappointing because, following the literature, I allow shocks to be correlated across countries in the baseline simulation. Re-simulating the model with uncorrelated shocks across countries, trade-comovement coefficients approach zero, which indicates that the model's apparent positive trade-comovement relationship is nearly entirely driven by correlation in TFP shocks.

To better understand the origins of the puzzle, I turn to analyzing elements of the framework in greater detail. This yields two main results. The first is that the aggregate trade-

[7]The framework describes how individual sectors in each country source intermediate goods from both home and bilateral import sources, as well as how each country sources final goods.

[8]This is roughly in line with results in Kose and Yi (2006). The benchmark simulation assumes quasi-financial autarky, with constant aggregate trade balances. As documented in the literature (references below), restricting asset trade tends to raise comovement, so this represents an upper bound on the ability of the model to explain the data.

comovement puzzle is mostly due to services. The model generates comovement (with correlated shocks) for goods similar to that found in the data, however it completely misses on services. Regressing simulated output correlations on bilateral trade for goods producing sectors returns significant coefficients equal to 80% the size of similar coefficients in data for gross output (56% for real value added). In contrast, there no significant relationship between trade and comovement for services in the simulated data, at odds with the positive relationship in the data.[9]

Taking the results for goods one step further, the model generates appreciable comovement in gross output for many country pairs from uncorrelated productivity shocks across countries. For gross output, uncorrelated shocks account for nearly 30% of the positive trade-comovement coefficient in the simulated data, or almost one-quarter of the overall trade-comovement coefficient in data. There is an important caveat to these results, however.

Uncorrelated productivity shocks generate comovement for gross output, not value added. This is the second main result, and highlights the particular role of intermediate goods in the model. Specifically, gross output in the model is a composite of real value added and intermediate inputs. Therefore, gross output can be correlated across countries either because real value added is correlated, or because intermediate use is correlated. In the model, comovement following idiosyncratic shocks is primarily due to comovement in intermediate use. This is because intermediate trade is the primary conduit through which shocks travel in the model.

One advantage to simulating a many country model is that I generate an entire data set

[9]This aspect of the puzzle is hidden from view in previous work on the trade-comovement puzzle that uses one-sector models. In a one-sector model, the low correlation of productivity shocks in services across countries lowers aggregate TFP comovement, which lowers aggregate output comovement. Though I focus primarily on understanding results for goods trade in this paper, improving the performance of the model for services sectors remains a topic for future work.

similar to those used in empirical work. To exploit this, I use my simulated data to examine whether trade-comovement regressions that control for 'vertical linkages' or cross-border 'fragmentation' are capable of cleanly identifying the role of intermediates in generating comovement.[10] I argue that coefficients on proxies for production sharing in trade-comovement are difficult to interpret, as they appear to be correlated with omitted shocks driving output correlations.

In addition to the empirical work cited above, this paper is related to a number of recent attempts to incorporate production sharing into business cycle models. The closest antecedent to the model developed below is a two-country, two-sector IRBC model with intermediates by Ambler, Cardia, and Zimmerman (2002).[11] This paper is distinguished from Amber et al. in both scope and focus. Whereas Amber et al. focus on a stylized two country case, I calibrate and simulate a many country model to match data on bilateral production sharing relations. Further, I hone the empirical focus toward understanding the trade-comovement puzzle, in contrast to the focus on general business cycle properties of the model in Ambler et al. Lastly, my exposition and analysis of the basic mechanisms underlying international comovement differs substantially from to Ambler et al.[12]

This paper is also related in spirit to recent models by Burstein, Kurz, and Tesar (2008) and Arkolakis and Ramanarayan (2009). Burstein, Kurz, and Tesar (2008) specify a two sector IRBC model in which the production sharing sector has a lower elasticity of substitution between home and foreign goods than the non-production sharing sector, which

[10] In this draft, I focus on recent results by Di Giovanni and Levchenko (2010) regarding sector-level correlations and my own preferred specification that introduces a measure of "sourcing similarity" that emerges from the model. One could alternatively revisit results in Burstein et al. (2006) or Ng (2010).

[11] Both Ambler et al. and this paper are also related to Cole and Obstfeld (1991) who write down a two country model with intermediate linkages and full depreciation of capital in the spirit of Long and Plosser (1983). This seems to be an under-appreciated contribution of their paper.

[12] Ambler et al. devote attention to analyzing the role of investment frictions in their framework and explaining the differences between their empirical findings and those of Long and Plosser (1983) by appealing to different assumptions regarding capital depreciation.

effectively lowers the aggregate elasticity of substitution and raises comovement.[13] Arkolakis and Ramanarayan (2009) adopt a multi-stage production function in the spirit of Yi (2003), an approach that is significantly different and less tractable in a multi-country setting than the approach in this paper.

More broadly, the basic structure of the model in this paper has important characteristics in common with models of sectoral linkages within the domestic economy, such as those analyzed by Long and Plosser (1983), Horvath (1998, 2000), Dupor (1999), Shea (2002), Carvalho (2008), or Foerster, Sarte, and Watson (2008). These papers provide many insights into the role input-linkages play in translating idiosyncratic shocks into aggregate fluctuations that could be applied to understanding regional business cycles using the framework and data introduced below.

2 Mechanics of Output Comovement

I begin by articulating a stylized static model that isolates some key features of the full dynamic model. The general formulation of the static model combines international trade in both final and intermediate goods with endogenous factor supply. This framework nests two separate channels for transmitting shocks across borders and generating output comovement. To develop intuition, I compare two polar opposite cases of the framework that clearly separate the two channels.

In the first case, I assume that there is no trade in intermediate goods. This case corresponds to the static version of the standard multi-good international real business cycle model, in which comovement is driven by endogenous factor supply.[14] In the second case,

[13] In contrast to the model in this paper, the performance of the Burstein et al. model is identical regardless of whether they assume that goods cross borders only once or whether there is back-and-forth shipment of goods across the border associated with production sharing.

[14] See Backus, Kehoe, and Kydland (1994) or Baxter (1995).

I assume that there is no trade in final goods and that factor supply is exogenous. This case isolates the role of intermediate goods linkages in generating output comovement, and highlights an important distinction between comovement in gross output versus value added.

2.1 A Benchmark Model

Consider a static world economy with many countries ($i, j \in \{1, \ldots, N\}$). Country i produces a single tradable Armington differentiated good using labor L_i and composite intermediate good X_i, which is a CES aggregate of intermediate goods produced by different source countries. The aggregate production function is Cobb-Douglas in the domestic factor and the composite intermediate:

$$
\begin{aligned}
& Q_i = Z_i \left(X_i\right)^{\theta} L_i^{1-\theta} \\
& \text{with} \quad X_i = \left(\sum_j \omega_{ji}^{x} X_{ji}^{\rho}\right)^{1/\rho},
\end{aligned}
\tag{1}
$$

where X_i is a CES aggregate of intermediate inputs produced in j and shipped to i (with technology weights ω_{ji}^{x}), θ is the intermediate input share in production, and Z_i is exogenous productivity.

Each country is populated by a representative consumer. The consumer is endowed with labor that it supplies to firms and consumes final goods. The consumer has preferences:

$$
\begin{aligned}
& U_i(C_i, L_i) = \log(C_i) - \frac{\chi\epsilon}{1+\epsilon} L_i^{(1+\epsilon)/\epsilon} \\
& \text{with} \quad C_i = \left(\sum_j \omega_{ji}^{c} C_{ji}^{\gamma}\right)^{1/\gamma},
\end{aligned}
\tag{2}
$$

where C_i is a CES aggregate of final goods produced in j and shipped to i (with preference weights ω^c_{ji}), χ measures the disutility of working, and ϵ is the Frisch elasticity of labor supply.

For simplicity, I assume there exists a social planner. The planner maximizes a social welfare function that is the weighted sum of utility of consumers from each country: $\sum_i \mu_i U_i(C_i, L_i)$, where μ_i is the welfare weight assigned to the consumer in country i. The social planner is constrained by the following adding-up condition for output from each country: $Q_i = \sum_j C_{ij} + X_{ij}$. This states that output in each country equals the sum of shipments of final and intermediate goods from country i to all destinations j.

The social planners problem is then to choose $\{\{C_{ji}, X_{ji}\}_{\forall j}, L_i\}_{\forall i}$ to solve:

$$\begin{aligned} \max \quad & \sum_i \mu_i \left[\log(C_i) - \frac{\chi\epsilon}{1+\epsilon} L_i^{(1+\epsilon)/\epsilon} \right] \\ & \text{s.t.} \quad Q_i = Z_i \left(X_i\right)^\theta L_i^{1-\theta} \\ & \text{and} \quad Q_i = \sum_j C_{ij} + X_{ij}, \end{aligned} \tag{3}$$

where C_i and X_i are defined above.

2.2 Case One: No Intermediate Goods Trade

The standard international real business cycle (IRBC) model omits cross-border intermediate goods linkages. Trade in the standard IRBC model should be thought of as trade in quasi-final goods, wherein each good crosses an international border only once. Put differently, the standard model does not admit international multi-stage production processes in which imports are used to produce exports.

In the general framework above, the production function in (1) combined with the resource constraint represents a multi-stage production process with an effectively infinite number of production stages, where value is added at each stage in a decreasing geometric sequence. Because production requires both domestic and imported intermediates, this implies that gross trade will be a multiple over actual value exchanged between countries, as goods cross borders many times throughout the production process. This contrasts with the standard IRBC model in which the ratio of value added embodied in exports to gross exports is equal to one.

To mimic the IRBC framework, I assume here that there are no intermediate goods in the model, setting $\theta = 0$, which necessarily eliminates trade in intermediates.[15] In this event, the production function is linear in labor: $Q_i = Z_i L_i$. As such, if productivity innovations are independent across countries, output in country i is correlated with output in country j only if factor supplies L_i and L_j co-move.

To understand when these factor supplies co-move, we can turn to the first-order conditions for the social planners problem in this case. Using the first-order condition for labor, we can write factor supply in country i as:

$$L_i = \left(\frac{\lambda_i Z_i}{\chi \mu_i} \right)^{\epsilon}, \tag{4}$$

where λ_i is the shadow price of output in country i. Labor supply here is increasing in productivity and the shadow price of output in country i, as both raise the marginal revenue product of labor. Using the production function, then output can be written as:

$$Q_i = Z_i^{1+\epsilon} \lambda_i^{\epsilon} (\chi \mu_i)^{-\epsilon}. \tag{5}$$

[15] A natural alternative assumption would be that each country uses only its own good as an intermediate. This yields similar results to assuming that there are no intermediates in the model.

Given a productivity innovation in country i, the resulting change in output is given by:

$$\hat{Q}_i = (1 + \epsilon)\hat{Z}_i + \epsilon\hat{\lambda}_i. \tag{6}$$

Obviously, the shadow price of output λ_i itself depends on productivity, but this formulation is instructive because it highlights three channels for understanding the effect of productivity on output. First, a productivity shock directly raises output. Second, a productivity shock raises the amount of labor supplied to firms. Third, a productivity shock will tend to drive down the shadow price of output (λ_i), which will attenuate the amount by which labor supply (and hence output) rises.

In this formulation, a productivity shock spills across borders via relative (shadow) prices. As productivity rises in country i, the relative price of output in country i falls, equivalently the relative price of output in country j rises. As the relative price of output in country j rises, this induces the representative consumer in j to supply more labor, which raises country j's output. Thus, output in country i rises due to the direct effect of productivity on output and the indirect effect of productivity in raising labor supply, while output in country j rises because terms of trade movements raise the return to supplying labor.

In this version of model, endogenous factor supply is the basic mechanism that drives comovement, as in IRBC models more generally.[16] The strength with which productivity shocks spill across borders then depends on: (a) how responsive relative prices are to the underlying shocks; (b) the elasticity of factor supply. In the extreme, when labor supply is inelastic and productivity shocks are independent across countries, there is no output

[16]With capital, factor supply continues to play an important role. However, the "resource shifting effect" whereby agents reallocate capital to the country with the positive productivity shock and falls in other countries attenuates output comovement. Specifically, resource shifting induces a negative correlation in capital across countries which offsets the positive correlation in labor supply across countries that arises due to terms of trade effects. See Kose and Yi (2006) for additional analysis of these issues.

comovement across countries.

2.3 Case Two: No Final Goods Trade, Exogenous Factor Supply

Traded intermediate goods serve to synchronize output movements across countries, independent of the standard endogenous factor supply mechanism discussed above. To illustrate this point, I consider a second case of the general framework in which I shut down endogenous factor supply entirely and assume labor supply is exogenous, set to $\bar{L}_i$ in country i. Further, I assume there is no trade in final goods to focus attention on intermediate goods linkages. This can be thought of as a restriction that $\omega^c_{ji} = 0 \; \forall j \neq i$ and $\omega^c_{ii} = 1$. Then output from each country is allocated across uses to satisfy: $Q_i = C_{ii} + \sum_j X_{ij}$.

With this set-up, the social planner chooses $\{C_{ii}, \{X_{ji}\}_{\forall j}\}_{\forall i}$ to maximize $\sum_i \mu_i \log(C_i)$. The first order conditions are then:

$$\frac{\mu_i}{C_{ii}} = \lambda_i \tag{7}$$

$$\lambda_i \theta Q_i X_i^{-\rho} \omega^x_{ji} X_{ji}^{\rho-1} = \lambda_j. \tag{8}$$

These first order conditions along with the technology and resource constraints can be linearized around the equilibrium to analyze the effects of a productivity shock. It is convenient to stack the equilibrium conditions to generate the following system:

$$\hat{C} = -\hat{\lambda} \tag{9}$$

$$\hat{\mathbb{X}} = \frac{1}{1-\rho}\left[M_\lambda \hat{\lambda} + M_Q \hat{Q} + M_Q \hat{X}\right] \tag{10}$$

$$\hat{X} = W\hat{\mathbb{X}} \tag{11}$$

$$\hat{Q} = S_X \hat{\mathbb{X}} + S_C \hat{C} \tag{12}$$

$$\hat{Q} = \hat{Z} + \theta \hat{X}, \tag{13}$$

where $\hat{\mathbb{X}} = [\hat{X}_{11}, \hat{X}_{12}, \ldots, \hat{X}_{1N}, \hat{X}_{21}, \hat{X}_{22}, \ldots]'$ is an $(N^2 \times 1)$ vector and $\{\hat{C}, \hat{\lambda}, \hat{Q}, \hat{X}, \hat{Z}\}$ are $(N \times 1)$ vectors of the underlying variables for each country. The matrices are defined as follows:

$$M_\lambda \equiv \mathbf{1}_{N\times 1} \otimes I_{N\times N} - I_{N\times N} \otimes \mathbf{1}_{N\times 1},$$

$$M_Q \equiv \mathbf{1}_{N\times 1} \otimes I_{N\times N},$$

$$W \equiv [diag(w_1), diag(w_2), \ldots] \quad \text{with} \quad w_i = [w_{i1}, \cdots, w_{iN}] \quad \text{and} \quad w_{ij} \equiv \frac{\omega_{ij}^x X_{ij}^\rho}{X_j^\rho},$$

$$S_X \equiv \begin{pmatrix} s_1^x & \mathbf{0} & \cdots \\ \mathbf{0} & s_2^x & \cdots \\ \vdots & \cdots & \ddots \end{pmatrix} \quad \text{with} \quad s_i^x = [s_{i1}^x, \cdots, w_{iN}] \quad \text{and} \quad s_{ij}^x = \frac{X_{ij}}{Q_i},$$

$$S_C \equiv [diag(s^c)] \quad \text{with} \quad s_i^c = \frac{C_{ii}}{Q_i}.$$

Equations (9)-(13) can be solved to yield a reduced from expression that relates output in each country to productivity shocks in all other countries via intermediate goods linkages. Rather than analyze this general case, I turn to an analytically elegant special case.

2.3.1 Cobb-Douglas Intermediate Goods Aggregator

To develop intuition regarding how comovement depends on the input sourcing structure, I assume now that the intermediate goods aggregator takes a Cobb Douglas form. The production function is then:

$$Q_i = Z_i X_i^{\theta} \bar{L}_i^{1-\theta}$$
$$\text{with} \quad X_i = \prod_j (X_{ji})^{\theta_{ji}/\theta}, \tag{14}$$

with $\sum_j \theta_{ji} = \theta$. With this assumption, equation (10) is replaced by:

$$\hat{\mathbb{X}} = M_\lambda \hat{\lambda} + M_Q \hat{Q}. \tag{15}$$

Using this alternative first order condition, one can show that the proportional change in output following productivity innovations:

$$\hat{Q} = \Theta' \hat{Q} + \hat{Z}. \tag{16}$$

The Θ matrix is a global input-output matrix that summarizes flows of intermediate goods across countries, with elements θ_{ij} equal to the share of expenditure on intermediates that j directly purchases from i as a fraction of the value of output in country j. Rearranging this equation, I write the change in log output as a reduced form function of productivity innovations:

$$\hat{Q} = [I - \Theta']^{-1} \hat{Z}. \tag{17}$$

The matrix $[I - \Theta']^{-1}$ provides a set of weights that indicate how production in country i responds to productivity shocks in country j. The weights can be interpreted as the total cost share of intermediates from j in production in country i, taking into account both direct and indirect purchases of inputs from j. These cost shares reflect global production sharing relationships. This is intuitive, since a positive productivity shock in country k benefits countries that use country k goods as inputs. This is true whether they use k goods directly or whether they rely on country k goods indirectly, in the sense that they source intermediates from some third country that itself relies heavily on inputs from country k. This has the implication that output will be correlated for country i and country j when they have similar overall sourcing patterns.[17]

2.3.2 Three Country Example

To make these ideas concrete, consider a simple three country version of the Cobb-Douglas version of the model above in which there are no domestic intermediates ($\theta_{ii} = 0$). Then the solution for the vector of output growth takes the form:

$$\hat{Q} = [I - \Theta']^{-1}\hat{Z} \quad \text{with} \quad \Theta = \begin{pmatrix} 0 & \theta_{12} & \theta_{13} \\ \theta_{21} & 0 & \theta_{23} \\ \theta_{31} & \theta_{32} & 0 \end{pmatrix}. \tag{18}$$

This solution can be rewritten as:

$$\hat{Q} = M \begin{pmatrix} 1 - \theta_{32}\theta_{23} & \theta_{21} + \theta_{23}\theta_{31} & \theta_{31} + \theta_{32}\theta_{21} \\ \theta_{12} + \theta_{13}\theta_{32} & 1 - \theta_{31}\theta_{13} & \theta_{32} + \theta_{31}\theta_{12} \\ \theta_{13} + \theta_{12}\theta_{23} & \theta_{23} + \theta_{21}\theta_{13} & 1 - \theta_{21}\theta_{12} \end{pmatrix} \hat{Z}, \tag{19}$$

where $M = \frac{1}{\det[I - \Theta']^{-1}}$. There are two points to note regarding this solution.

[17]There are two distinct elements to differences in sourcing patterns. First, the overall level of trade will differ across countries. Second, conditional on overall openness, bilateral trade patterns also differ.

First, for each country, the loadings on foreign country shocks are a function both of parameters associated with bilateral trade as well as trade with third countries. For example, the impact of a productivity innovation in country 2 on country 1's output is: $M(\theta_{21} + \theta_{23}\theta_{31})\hat{z}_2$. This effect is a function of both the intensity with which country 1 sources intermediates from country 2 (θ_{21}) and the compound term $\theta_{23}\theta_{32}$. This compound term picks up the indirect effect of country 2 productivity shocks operating via country 1's sourcing intermediates from country 3. Specifically, a shock in country 2 raises the supply of the country 2 intermediate good. This benefits country 1 directly because it uses this intermediate in production, but also benefits it indirectly because it uses intermediates from country 3 and country 3 intermediates are in turn produced using country 2 goods. Thus, the structure of the entire production chain matters, not just bilateral input sourcing patterns.[18]

Second, there is multiplier effect that controls the magnitude of effect of shocks on each country. To see this clearly, I re-write output growth for country 1 as:

$$\hat{Q}_1 = M_1 \left[\hat{Z}_1 + \left(\frac{\theta_{21} + \theta_{23}\theta_{31}}{1 - \theta_{32}\theta_{23}} \right) \hat{Z}_2 + \left(\frac{\theta_{31} + \theta_{32}\theta_{21}}{1 - \theta_{32}\theta_{23}} \right) \hat{Z}_3 \right], \tag{20}$$

where I define $M_1 = \frac{1-\theta_{32}\theta_{23}}{\det[I-\Theta']^{-1}}$ to be country 1's multiplier. M_1 summarizes how much country 1 output increases with shocks to its own productivity and is generally greater than one. Thus, the sensitivity of output to shocks for different countries can be decomposed into a country specific effect M_i and a vector of weights on different shocks that varies across countries.

[18] In a concrete example, the U.S. benefits from productivity increases in China not only because it imports from China, but also because the U.S. sources intermediates from Japan and Japan uses Chinese goods as inputs in production.

2.3.3 Gross Output versus Value Added

Thus far, I have implicitly focused the discussion of comovement via intermediate goods linkages on comovement in gross output. This is because there is an important distinction between gross output and value added in models with intermediate goods that does not arise in standard IRBC models without intermediates. To make this distinction explicit, I rewrite the production function in equation (1) as:

$$\begin{aligned} &Q_i = V_i^{1-\theta} X_i^{\theta} \\ &\text{with} \quad V_i \equiv Z_i^{\frac{1}{1-\theta}} L_i. \end{aligned} \tag{21}$$

The quantity V_i is real value added. Real value added in this framework is a sub-function of gross output, which itself a composite of productivity and factor inputs (labor). Gross output then is a composite, homogeneous of degree one, function of real value added and intermediate goods.[19] This set-up implies that real value added can be computed using the "double-deflation" method, the current best practice in sector-level national accounts. Under double deflation, nominal output and nominal input purchases for each sector are deflated via their own price indices. Real value added growth is then equal to: $\hat{V}_i = \frac{1}{(1-\theta)}\left(\hat{Q}_i - \theta \hat{X}_i\right)$, where $\hat{Q}_i$ and $\hat{X}_i$ are directly measured in the national accounts.[20]

One important implication of this distinction between real value added and gross output is that output comoves across countries for two reasons. First, real value added may comove across countries. Second, input use may comove across countries. In this section with exogenous factor supply, value added comoves across countries if and only if productivity

[19] To generalize the definition of real value added, consider a general production function (supressing country subscripts and time indexation): $Q = f(K, L, X)$. Then if the production function is weakly separable in capital and labor, it can be rewritten as: $Q = f(h(K, L), X)$. The sub-function $h(K, L)$ is then "real value added."

[20] Of course, input shares θ are also measured in national accounts.

shocks are correlated across countries. On the other hand, gross output can comove across countries even if productivity shocks are uncorrelated if input use is correlated. Intermediate goods linkages imply that input use will in fact be correlated, most intensely so for countries that either have strong bilateral production sharing linkages or are exposed to common shocks originating in an input supplier to both countries.

With endogenous factor supply, the logic is obviously more complicated, as one layers this mechanism on top of the standard IRBC transmission of shocks via relative prices and factor supply. However, distinguishing output and value added comovement in this special case yields important intuition regarding mechanics that I will exploit below.

3 Dynamic Many Country, Multi-Sector Sector Model

The full model extends the benchmark model in a number of directions. First, the full model includes both transmission channels discussed above: endogenous factor supply (capital and labor) and intermediate goods linkages. The model admits both trade in final and intermediate goods, as well as dynamic adjustment of capital. Second, the full model includes multiple sectors. Disaggregating the model is important because sectors differ substantially in both overall openness and integration into cross-border production chains. In specifying equilibrium in the full model, I need to take a stand on financial market structure. In what follows, I focus on the case of financial autarky (equivalently, balanced trade) on the grounds that financial autarky has been shown to generate terms of trade movements and cross-country correlations that align more closely with data.[21]

[21] For example, see Heathcote and Perri (2002) and Kose and Yi (2006). Financial autarky tends to deliver stronger comovement because it shuts down "resource-shifting" effects where in capital is reallocated toward countries with positive productivity shocks. It is straightforward to consider complete financial markets in future drafts.

3.1 Production

Consider a multi-period world economy with many countries ($i, j \in \{1, \ldots, N\}$). Country i produces a tradable differentiated good in sector s using capital $K_{it}(s)$, labor $L_{it}(s)$, and composite intermediate good $X_{it}(s)$, which is an aggregate of intermediate goods produced by different source countries. The aggregate production function is Cobb-Douglas in the domestic factor and the composite intermediate:

$$\begin{aligned} & Q_{it}(s) = Z_{it}(s) K_{it}(s)^{\alpha_i(s)} X_{it}(s)^{\theta_i(s)} L_{it}(s)^{1-\alpha_i(s)-\theta_i(s)} \\ & \text{with} \quad X_{it}(s) = X_i(\ldots, X_{jit}(s', s), \ldots; s) \end{aligned} \tag{22}$$

where $X_i(\cdot; s)$ is an aggregator of intermediate inputs for sector s in country i, $X_{jit}(s', s)$ is the quantity of intermediate goods from sector s' in country j used by sector s in country i, $\{\theta_i(s), \alpha_i(s)\}$ are the intermediate input and capital shares in production for sector s and country i, and $Z_{it}(s)$ is exogenous sector-specific productivity.

Output is produced under conditions of perfect competition. A representative firm in country i, sector s takes the prices for it's output and inputs as given, and the firm rents capital and hires labor to solve:

$$\begin{aligned} \max \quad & p_{it}(s) Q_{it}(s) - w_{it} L_{it}(s) - r_{it} K_{it}(s) - \sum_{j=1}^{N} \sum_{s'=1}^{S} p_{jt}(s') X_{jit}(s', s) \\ \text{s.t.} \quad & L_{it}(s), K_{it}(s), X_{jit}(s', s) > 0 \end{aligned} \tag{23}$$

where $p_{it}(s)$ denotes the price of output, w_{it} is the wage, r_{it} is the rental rate for capital, and the production function for $Q_{it}(s)$ is given above by (22).

Labor, capital, and intermediate goods choices for production in country i satisfy:

$$\alpha_i(s)p_{it}(s)Q_{it}(s) = r_{it}K_{it}(s) \tag{24}$$

$$\left(\frac{\theta_i(s)p_{it}(s)Q_{it}(s)}{X_{it}(s)}\right)\frac{\partial X_{it}(s)}{\partial X_{jit}(s',s)} = p_{jt}(s') \tag{25}$$

$$(1-\alpha_i(s)-\theta_i(s))p_{it}(s)Q_{it}(s) = w_{it}L_{it}(s). \tag{26}$$

Output is used as an intermediate good in production and to produce a composite final good for consumption and investment. Within each sector, perfectly competitive firms aggregate final goods from all sources to form a sector-level composite using production function: $F_{it}(s) = F_i(\ldots, F_{jit}(s), \ldots; s)$. These sector composites are then aggregated to form an aggregate final good via a Cobb-Douglas technology: $F_{it} = \prod_s F_{it}(s)^{\gamma_i(s)}$, where $\gamma_i(s)$ is the expenditure share on final goods of type s in country i. Note that I assume that there is no value added at this stage to be consistent with the accounting conventions in my input-output data which records the value of retail and distribution services as production of a separate services sector.

A representative final goods firms maximizes:

$$\max \quad p^f_{it}F_{it} - \sum_{j=1}^{N}\sum_{s=1}^{S} p_{jt}(s)F_{jit}(s), \tag{27}$$

where p^f_{it} is the price of the composite final good and F_{it} is defined above. Purchases of individual final goods F_{jit} for aggregation into the final good satisfy:

$$\left(\frac{\gamma_i(s)p^f_{it}F_{it}}{F_{it}(s)}\right)\frac{\partial F_{it}(s)}{\partial F_{jit}(s)} = p_{jt}(s). \tag{28}$$

Aggregate final goods are used for consumption and investment: $F_{it} = C_{it} + I_{it}$.[22] Gross output equals total purchases used as intermediates and to produce final composite goods: $Q_{it}(s) = \sum_{j=1}^{N} \sum_{s'=1}^{S} F_{ijt}(s) + X_{ijt}(s, s')$.

3.2 Consumption and Labor Supply

Each country is populated by a representative consumer. The consumer is endowed with labor (with time endowment normalized to one) that it supplies to firms and consumes final goods. The representative consumer also owns the capital stock in her country and makes investment decisions. The capital stock evolves according to: $K_{it+1} = I_{it} + (1 - \delta)K_{it}$, where $K_{it} = \sum_{s=1}^{S} K_{it}(s)$. Under financial autarky (balanced trade), expenditure on final goods must equal income in each period for the consumer: $p_{it}^{f} F_{it} = w_{it}L_{it} + r_{it}K_{it}$, where $L_{it} = \sum_{s=1}^{S} L_{it}(s)$.

The consumer chooses $\{C_{it}, L_{it}, K_{it+1}\}$ to solve:

$$\begin{aligned} \max \quad & E_0 \sum_{t=0}^{\infty} \beta^t U_i(C_{it}, L_{it}) \\ & \text{s.t.} \quad p_{it}^{f}(C_{it} + I_{it}) = w_{it}L_{it} + r_{it}K_{it} \\ & \text{and} \quad K_{it+1} = I_{it} + (1-\delta)K_{it}. \end{aligned} \tag{29}$$

The Euler equation and first-order condition for labor supply are then:

$$\frac{\partial U_i(C_{it}, L_{it})}{\partial C_{it}} = \beta E_t \left[\frac{\partial U_i(C_{it}, L_{it})}{\partial C_{it+1}} \left(\frac{r_{it+1}}{p_{it+1}^{f}} + (1-\delta) \right) \right] \tag{30}$$

$$\frac{\partial U_i(C_{it}, L_{it})}{\partial L_{it}} = \frac{\partial U_i(C_{it}, 1 - L_{it})}{\partial C_{it}} \frac{w_{it}}{p_{it}^{f}}. \tag{31}$$

[22] Note that this assumption implies that the aggregator is the same for consumption goods and investment goods. This assumption could be relaxed.

3.3 Equilibrium

Given a stochastic process for productivity, an equilibrium in the model is a collection of quantities $\{C_{it}, F_{it}\}$ for each country, $\{Q_{it}(s), K_{it}(s), L_{it}(s), \{F_{jit}(s)\}_j, \{X_{jit}(s', s)\}_{j,s'}\}_{i,s}$ for each country-sector, and prices $\{r_{it}, w_{it}, p^f_{it}, \{p_{it}(s)\}_s\}_i$. These must satisfy the producers' first order conditions (24)-(26) and (28) and the consumer's Euler equation (30) and first-order condition for labor supply (31). They must also satisfy market clearing conditions $Q_{it} = \sum_j (F_{ijt} + X_{ijt})$ and $F_{it} = C_{it} + K_{it+1} - (1-\delta)K_{it}$, the budget constraint $p^f_{it} F_{it} = w_{it} L_{it} + r_{it} K_{it}$, and the production function (22). The equilibrium conditions are collected explicitly in the appendix [to be completed].

3.4 Calibration

3.4.1 Functional Forms

To calibrate the model, I need to specify functional forms for preferences, the final goods aggregator, and the intermediate goods aggregator. I assume that preferences are given by: $U_i(C_{it}, L_{it}) = \log(C_{it}) - \frac{\chi\epsilon}{1+\epsilon} L_{it}^{(1+\epsilon)/\epsilon}$. Further, I assume that the final goods are produced via a CES production function: $F_{it}(s) = \left(\sum_j \omega^f_{ji}(s) F_{jit}(s)^\rho\right)^{1/\rho}$, where $\{\omega^f_{ji}(s)\}$ and ρ are parameters to be calibrated.

In the benchmark calibration, I assume that the intermediate goods aggregator is Cobb-Douglas: $X_{it}(s) = \prod_j \prod_{s'} (X_{jit}(s', s))^{\theta_{ji}(s',s)/\theta_i(s)}$, where $\{\theta_{ji}(s', s)\}$ are parameters be calibrated. If the elasticity of substitution between final goods is greater than one, this Cobb-Douglas assumption implies that the elasticity of substitution within intermediates is lower than that between final goods. This is consistent with existing work such as Burstein, Kurz, and Tesar (2008) or Jones (2010), among others, who argue that the scope for substitution across intermediate goods is lower than for final goods.

With these assumptions, I need values for the following parameters: $\{\beta, \epsilon\}$ for preferences and $\{\alpha_i(s), \theta_i(s), \{\theta_{ji}(s', s)\}, \rho, \{\omega_{ji}^f(s)\}, \delta\}$ for the technology.[23]

3.4.2 Technology and Preferences

I set $\rho = .33$, $\delta = .1$, $\beta = .96$, and $\epsilon = 1$ based on standard values in the literature. I use data from input-output tables to calibrate the remaining technology and preference parameters.

To calibrate $\{\alpha_i(s), \theta_i(s), \{\theta_{ji}(s', s)\}, \rho, \{\omega_{ji}^f(s)\}$, my primary data source is the GTAP 7.1 Data Base assembled by the Global Trade Analysis Project at Purdue University.[24] The data set includes internally consistent bilateral trade statistics combined with domestic and import input-output tables for 94 countries plus 19 composite regions covering 57 sectors in 2004. I retain country level data for 22 countries, covering roughly 80% of world GDP, and aggregate the remaining countries to form a composite "rest-of-the world" region. The choice of countries is determined primarily by the availability of time series data on gross production and productivity data (see below).

A key part of the calibration is accurate data on bilateral intermediate and final goods flows. I construct these data by combining input-output tables with data on bilateral trade, as in Johnson and Noguera (2010).[25] From the GTAP database, I extract disaggregate input use tables for domestic input purchases and imported inputs. I then use bilateral trade data to split imported input use across bilateral partners, assuming that input purchases from each source are proportional to bilateral trade shares within a given sector. I split final goods

[23]Note, some parameters are not needed to simulate the model. For example, χ governs the level of labor supplied in the steady state, but model dynamics are independent of this value due to the constant elasticity of labor supply.

[24]This data is compiled based on three main sources: (1) World Bank and IMF macroeconomic and Balance of Payments statistics; (2) United Nations Commodity Trade Statistics (Comtrade) Database; and (3) input-output tables from national statistical sources. To reconcile data from these different sources, GTAP researchers adjust the input-output tables to be consistent with international data sources.

[25]Similar approaches have been used by Daudin, Rifflart, and Schweisguth (2009), Koopman, Powers, Wang, and Wei (2010), and Trefler and Zhu (forthcoming).

imports across source countries using trade shares in a similar way. This yields bilateral final and intermediate goods shipments for 57 sectors. I then aggregate data on sectoral production, trade, final and intermediate shipments across sectors to form two composite sectors, defined as "goods" (including agriculture, natural resources, and manufacturing) and "services."

These data allow me to fit the model to replicate data for output, value added, and domestic/foreign shipments exactly. I calculate the intermediate goods share of output in each country and sector $\theta_i(s)$. The median intermediate share for goods producing sectors is 0.65 for my country sample, while the corresponding share for services is 0.46. Then, I calculate the capital share in gross output as $\alpha_i(s) = (1/3) * (1 - \theta_i(s))$, equal to an assumed capital share in value added (1/3) times the value added to output ratio $(1 - \alpha_i(s))$.

The bilateral intermediate and final goods shipments serve as data targets for $\{\theta_{ji}(s', s)\}$ and $\{\omega^f_{ji}(s)\}$. According to the producer's first order condition (25), $\theta_{ji}(s', s)$ is the ratio of expenditure on inputs from country j to gross output:

$$\theta_{ji}(s', s) = \frac{p_j(s') X_{ji}(s', s)}{p_i(s) Q_i(s)}. \tag{32}$$

To calibrate $\{\omega^f_{ji}(s)\}$, note that the final goods producer's first order condition (28) can be rewritten in share form as:

$$\frac{p_i(s) F_{ij}(s)}{p^f_j F_j} = \gamma_j(s) \omega^f_{ij}(s) \left(\frac{p_i(s)}{p^f_j(s)} \right)^{-\rho/(1-\rho)}, \tag{33}$$

where $\frac{p_i(s) F_{ij}(s)}{p^f_j F_j}$ is the share of final goods of sector s sourced from country i in total final goods expenditure in j. The share of final expenditure on goods of sector s – $\gamma_j(s)$ – can be computed directly in the data. Then, choosing quantity units so that the price of gross output and the final goods are equal to one in the steady state, $\{\omega_{ij}^f\}$ can computed by combining these expenditure shares.

In the data, trade is unbalanced. Therefore, in calibrating the model, I allow steady state trade to be unbalanced as well to recover 'true' preference and technology parameters. I then solve for dynamics in the model by linearizing around this unbalanced steady state, assuming that trade imbalances are constant.[26] The linearized equilibrium conditions are included in the appendix [to be completed].

3.4.3 Productivity

To estimate stochastic processes for productivity, I use sectoral productivity data from the Groningen Growth and Development Centre's EU KLEMS and 10-Sector databases. Though ideally one would like estimate the productivity process using data on TFP, data constraints prevent this for many countries over long periods of time. Therefore, as is standard, I estimate the productivity process using data on labor productivity.[27] Availability of labor productivity data in the Groningen data limits the number of countries included in the simulation to 22 countries, covering approximately 80% of world GDP over the period 1970-2007. I take sectoral labor productivity growth for 19 OECD countries from the EU KLEMS data, where labor productivity growth is computed as the difference between real value

[26] An alternative approach would be to calibrate the model to the unbalanced steady state, then solve for and linearize around the corresponding balanced trade equilibrium. In practice, the differences in behavior of the model linearized around balanced steady state versus imbalanced steady states are small.

[27] The main data constraint is that estimates of sector level capital stocks and/or labor quality are difficult to obtain. Though motivated by data constraints, using labor productivity in place of TFP implicitly assumes that capital and/or labor quality dynamics do not drive variation in labor productivity at business cycle frequencies. This assumption is common in the aggregate IRBC literature: see Backus, Kehoe, and Kydland (1992), Heathcote and Perri (2002), or Kose and Yi (2006) for example. Examining countries in the Groningen data for which both TFP and labor productivity growth rates are available for specific periods, the year-on-year growth rates of TFP and labor productivity are roughly proportional, which suggests this assumption is innocuous.

added growth and growth in hours worked for each sector.[28] I turn to the 10-Sector data to compute productivity growth rates for three large emerging markets – Brazil, India, and Mexico.[29] Productivity in this data is measured as the difference between real value added growth less growth in the number of workers employed.

For each country and sector, I estimate univariate, trend stationary productivity process. Suppressing constants and time trends, the estimating equation is:

$$\log LP_{it}^{VA}(s) = \lambda_i(s) \log LP_{it-1}^{VA}(s) + \epsilon_{it}(s), \tag{34}$$

where $LP_{it}^{VA}(s)$ is the level labor productivity (measured using value added) and $\lambda_i(s)$ is the persistence parameter. In a modest departure from the existing literature, I restrict cross-country spillovers to be equal to zero and further assume that there are no spillovers across sectors withing a country.[30] The correlation of productivity shocks $\epsilon_{it}(s)$ is unrestricted. To compute this correlation, I estimate equation 34 for each country and sector separately, recover regression residuals $\hat{\epsilon}_{it}(s)$, and then construct the covariance matrix of the shocks as: $\Sigma \equiv \frac{1}{T}\sum_t \hat{\epsilon}_t \hat{\epsilon}_t'$.[31]

To simulate the model, I need to convert the covariance matrix Σ, constructed using

[28] Countries include Australia, Austria, Belgium, Canada, Denmark, Spain, Finland, France, Germany, Greece, Ireland, Italy, Japan, Korea, Netherlands, Portugal, Sweden, United Kingdom, and the United States. I omit most Central and Eastern European countries in the data with short time series starting in the mid-1990s.

[29] The 10-Sector database includes other smaller emerging markets that could be added to the analysis in future drafts.

[30] I restrict cross-country spillovers as a matter of necessity. With N countries and 2 sectors, there are too many unrestricted spillover parameters to estimate given the relatively short length of the time series available. I have experimented with estimation of cross-sector spillovers within countries. Point estimates for cross-sector spillovers are generally unstable across countries and imprecisely estimated (often indistinguishable from zero).

[31] For three of the forty-four country-sector pairs, the estimated persistence parameters exceed one. Examination of the data indicates that this is due to breaks in the trend for these country-sector time series. For these countries, I estimate productivity processes assuming that each experiences only aggregate productivity shocks (i.e., productivity growth in goods and services is equal to aggregate productivity growth). These three countries are Italy, India, and Mexico.

residuals from estimation of the process for productivity measured using real value added, into an equivalent covariance matrix for shocks to productivity measured on a gross output basis. The adjustment multiplies each residual by the ratio of value added to output: $\hat{\hat{\epsilon}}_{it}(s) \equiv (1-\theta_i(s))\hat{\epsilon}_{it}(s)$.

To understand this adjustment, recall the discussion in Section 2.3.3 about distinguishing gross output from real value added. At the sector level, gross output is a composite of real value added and intermediate inputs:

$$\begin{aligned} & Q_{it}(s) = V_{it}(s)^{1-\theta_i(s)} X_{it}(s)^{\theta_i(s)} \\ & \text{with} \quad V_{it}(s) \equiv Z_{it}^{\frac{1}{1-\theta_i(s)}} K_{it}(s)^{\frac{\alpha_i(s)}{1-\theta_i(s)}} L_{it}(s)^{\frac{1-\alpha_i(s)-\theta_i(s)}{1-\theta_i(s)}} \end{aligned} \tag{35}$$

Then, TFP measured using gross output is $\widehat{TFP}_{it}^{Q}(s) = \hat{Z}_{it}(s)$, while TFP measured using real value added is $\widehat{TFP}_{it}^{V}(s) = \frac{1}{1-\theta_i(s)}\hat{Z}_{it}(s)$. The two TFP measures are related by $\widehat{TFP}_{it}^{Q}(s) = (1-\theta_i(s))\widehat{TFP}_{it}^{V}(s)$, so shocks to productivity measured using value added will be larger than the corresponding shocks measured using gross output. This explains the need to adjust Σ and means that the correct covariance matrix for simulation is: $\widetilde{\Sigma} = \frac{1}{T}\sum_t \hat{\hat{\epsilon}}_t \hat{\hat{\epsilon}}'_t$. The persistence parameter $\lambda_i(s)$ obtained in estimation of 34 can be directly used in simulations, as it does not depend on which definition of productivity is used in the estimation.

In the simulations below, I will use this covariance matrix in two ways. One set of simulations will allow shocks to be correlated across countries, with correlations determined by the estimated covariance matrix. This is the standard approach in the literature. The shortcoming of this approach is that comovement in this set of simulations is driven both by transmission of shocks across countries via trade linkages and the direct correlation of the underlying shocks themselves.

To more cleanly identify the trade transmission mechanism, I will also simulate the model under the (counterfactual) assumption that shocks are uncorrelated across countries. To parameterize this counterfactual scenario, I zero out the "off-diagonal" elements of the covariance matrix.[32] Specifically, I impose $\text{cov}(Z_{it}(s), Z_{jt}(s')) = 0$ for all $i \neq j$. This allows shocks to be correlated across sectors within countries, but uncorrelated for any cross-country sector pairs. While this eliminates cross-country correlations in shocks, it should be noted that $\text{cov}(Z_{it}(s), Z_{it}(s'))$ is an upper bound to the size of the truly independent productivity shocks.[33] This implies that simulated shocks using this method will be somewhat too large relative to the truly idiosyncratic shocks that countries face. Thus, one should interpret simulation results using these idiosyncratic shocks as an upper bound on the ability of the model to generate comovement from true (correctly measured) idiosyncratic country shocks.

One last detail regarding the simulation is that I include a composite rest-of-the-world region in the simulations, but do not have directly measured productivity data for this composite region. Therefore, I assume that the rest-of-the-world experiences independent productivity shocks. I parameterize the persistence, variance, and cross-sector correlations of the shocks to this region based on median values in the data.

4 Results

To evaluate the model, I examine whether the model can match data on bilateral correlations and replicate the empirical trade-comovement relationship. I begin by presenting aggregate results for the multi-sector model, which may be compared to previous research focused on one-sector models. I then turn to disaggregated sector-level data to unpack these aggregate results, with emphasis on contrasting the performance of the model for goods versus ser-

[32] This approach is adapted from Horvath (1998).

[33] For example, suppose that there are global shocks and i.i.d. country shocks. Then $\text{cov}(Z_{it}(s), Z_{it}(s'))$ is equal to the sum of the variance of the global shock plus the variance of the idiosyncratic country shock, and hence an upper bound on the variance of the idiosyncratic shock.

vices sectors and for gross output versus value added. Finally, I use the simulated data to explore whether augmented trade-comovement regressions with vertical linkages are capable of identifying the causal influence of input linkages on comovement.

4.1 Aggregate Results

Figure 1 presents bilateral correlations of real value added for each country pair in the model and data.[34] Bilateral correlations are computed as the correlation of year on year growth rates. Correlations in the model are computed as averages over 500 replications of 35 years each, roughly the same period over which correlations are computed in the data, using the estimated covariance of shocks.

As is evident from the figure, model-based correlations are positively related to data-based correlations, though the fit is far from perfect. The regression line of best fit is $\rho_{ij}(\text{data}) = .26 + .46\rho_{ij}(\text{model})$ with standard error on the slope of .07 and $R^2 = .14$. Note that the model generally under predicts the average correlation in the data, which is quite reasonable given that there are other shocks outside the model (e.g., demand shocks) driving correlations in the data that may be on average positively correlated across countries. One possible candidate for these omitted shocks would be monetary shocks. Indeed, examining the model's fit for EU-pairs versus non-EU pairs in Figure 2, the model does a better job explaining variation in bilateral correlations for non-EU pairs than among EU-pairs.[35] While the model under predicts the average correlation by more and generates a shallower slope for EU-pairs than non-EU pairs, the slope is significantly positive within both sub-groups.

[34] I focus on real value added here because I do not have time series data on gross output for all countries. In the model and data, gross output is very highly correlated with real value added at the aggregate level.

[35] A similar result obtains if I look explicitly at Eurozone pairs versus non-Eurozone pairs. Since several non-Eurozone EU countries (e.g., Denmark) peg to the Euro, the EU versus non-EU comparison may be more appropriate. While the model does not fit EU-pairs in the aggregate, I show below that it does fit EU-pairs well for the goods sector. This is indirect evidence that demand shocks could be an important driver of services correlations observed in the data that cannot be explained by the model.

To evaluate the trade-comovement puzzle directly in the model, I regress bilateral correlations in the model and data on bilateral trade intensity. Bilateral trade intensity is measured as: $\log\left(\frac{EX_{ij}+EX_{ji}}{GDP_i+GDP_j}\right)$, computed for the benchmark 2004 year in my data.[36] Table 1 records the regression results for real value added in Panel A and gross output in Panel B.[37] The first column of each panel is based on the data, the second column is based on the model with correlated shocks, the third column is based on the model with uncorrelated shocks. Looking at the first column, comovement is clearly positively correlated with log bilateral trade in the data. In the model with correlated shocks, the correlation is significantly weaker, with the regression coefficients roughly 1/4 to 1/3 of the magnitudes in column one. Even this, however, perhaps overstates the role of trade per se in generating comovement in the model. Simulating the model with uncorrelated shocks, the regression coefficients decline substantially, and become insignificant for real value added. Thus, nearly all of the positive aggregate correlation in the model with correlated shocks appears to be due to the correlation of shocks themselves.

4.2 Disaggregate Results

The results above indicate that the aggregate trade comovement puzzle is alive and well, despite the introduction of intermediate goods into the model. To better understand the origins of the puzzle and mechanics of the model, I turn to disaggregating the results.

Figure 3 plots bilateral sector-level correlations in the data and model with correlated shocks for goods and services production separately. The upper panel contains the data

[36]Because trade shares are stable over time, results are not sensitive as to whether one computes bilateral trade intensity using trade data single year or averages bilateral trade over time prior to computing the metric. The basic results also hold if the level, rather than log, of bilateral trade intensity is used.

[37]Gross output correlations are computed using the Groningen EU KLEMS database, which implies that I cannot calculate correlations for pairs involving Brazil, India, and Mexico. Because I do have real value added data for these countries, I do include them in computing real value added correlations. This explains the differences in the number of observations across columns for gross output.

for each country's goods sector paired with a bilateral foreign goods sector, and the lower panel contains the same for services. The results are striking: the model with correlated shocks does a good job predicting gross output correlations for goods, but fails miserably for services. The correlation of model and data-based correlations is .47 for goods, and only .08 for services. Examining Table 2, this basic dichotomy – the model fits relatively well for goods and poorly for services – is borne out no matter whether one looks at gross output or real value added, or whether one simulates the model with correlated or uncorrelated shocks across countries.[38] Further, the model even does significantly better for cross-sector pairing (e.g., mixed services-goods pairs) than for services-services pairs.[39]

Not surprisingly, the good model fit for goods and poor fit for services manifests itself in trade-comovement regressions as well. In these sector level regressions, log bilateral trade intensity between sector s in country i and sector s' in country j is defined as: $\log(\frac{EX_{ij}(s)+EX_{ji}(s')}{GDP_i+GDP_j})$. In Table 3, the coefficient on log bilateral trade intensity in the model with correlated shocks is roughly 80% the size of the corresponding coefficient in the data for gross output among goods sector pairs. For services trade, the coefficient in data is nearly as large as that for goods (and highly significant), but is small and insignificantly different than zero for services. Even for cross sector pairs, the model-based coefficient is roughly 42%, significantly larger than the aggregate coefficients.

These results refine the trade-comovement puzzle. For the gross output of goods, there is no puzzle in the model with correlated shocks – the model generates correlations in line with data. For the gross output of services, the puzzle is severe – the model generates near zero correlation between bilateral trade and comovement, while there is a strong positive correlation in the data. Given the large size of the services sector in most OECD countries,

[38]The model actually generates negative correlations for services out of uncorrelated shocks.
[39]This is likely due to the fact that imported goods are used as an input in the production of services.

combining these results means that the *aggregate* trade-comovement puzzle for gross output is due in large part to the inability of the model to explain services sector correlations.[40]

There are two important caveats to this diagnosis of the trade-comovement puzzle. First, just because the model with correlated shocks replicates bilateral correlations for gross output does not mean that trade itself is a strong propagator of shocks. Trade may simply be a proxy for the correlation of shocks, in that countries that trade intensively may have shocks that are more correlated with one another. Second, the attentive reader will note that I have thus far focused primarily on the fit of the model for gross output correlations. With intermediates, real value added and gross output can behave differently in the model. In fact, quick inspection of Tables 1 or 3 suggest that the model seems to do less well for value added. I deal with these two issues in turn, focusing on explaining results for goods output in greater detail. Moreover, to hone in on the propagation mechanism for idiosyncratic shocks, I focus on results from simulating the model with uncorrelated shocks.

For gross output of goods, propagation of independent shocks explains at most one-third of the observed comovement in the data. Figure 4 plots actual gross output correlations for goods against those predicted by the model with uncorrelated shocks. There is a clear positive relationship, particularly among EU country pairs. The U.S.-Canada outlier is particularly instructive. The predicted correlation is roughly .23, while the actual correlation in the data is near .75, roughly a ratio of three to one. More generally, this magnitude is consistent with the overall spread in the data. Focusing on EU-pairs, predicted correlations vary in the range $(0, .15)$ while actual correlations lie in the range $(.25, .75)$, so the ratio of the ranges is roughly $.5/.15$ or three to one.[41]

[40]Further, the model's inability to generate correlated services output across countries also means that it struggles to explain aggregate correlations, as in Figure 1.

[41]In this comparison, I relate changes in comovement across pairs to changes in predicted model correlations for EU pairs. This obviously ignores the fact that the model grossly underestimates the median correlation. The median ratio of the model correlation with uncorrelated shocks to the actual correlation is $\approx 10\%$ for

These relationships are borne out in looking at the trade-comovement regressions for goods trade in Panel A of Table 3, where the coefficient generated by the model with uncorrelated shocks is just under 30% the size of the coefficient in the model with correlated shocks. Thus, while say 2/3 of the goods trade-comovement relationship for gross output is due to correlated shocks in the model, nearly 1/3 is explained entirely by the propagation of uncorrelated shocks across countries.

While the model generates comovement in gross output from idiosyncratic shocks, it generates much weaker comovement in real value added. To see this, I plot the correlation of gross output against the correlation for real value added for goods sector-pairs in Figure 5. The top two panels are the relationship between these alternative correlations in the data and in the model with correlated shocks, while the third panel is a depiction of this relationship in the model with uncorrelated shocks. Whereas correlations for value added and gross output track each other closely in the upper two panels, there are large differences between the two in the third panel. Further, the variance of correlations of real value added is much lower than the variance of correlations in gross output. These discrepancies in the model with uncorrelated shocks are in and of themselves interesting because they shed light on the role of intermediate goods in the model.

Recall from the discussion in previous sections that gross output is a composite of real value added and intermediate inputs, as in Equation (35). The correlation of gross output can them be decomposed into a weighted sum of the correlation of real value added across countries, the correlation of input use across countries, and the cross-correlation of real value added and input use:

$$\rho_{ij}(Q) = w_{ij}^{vv}\rho_{ij}(V) + w_{ij}^{xx}\rho_{ij}(X) + w_{ij}^{vx}\rho_{ij}(V,X) + w_{ij}^{xv}\rho_{ij}(V,X), \tag{36}$$

EU pairs.

where $w_{ij}^{vv}, w_{ij}^{xx}, w_{ij}^{vx}, w_{ij}^{xv}$ are the appropriate weighting terms for each correlation, themselves functions of the Cobb-Douglas share parameters and standard deviations of gross output, real value added, and input use. To provide a visual sense of how these correlations aggregate, I plot the correlations $\rho_{ij}(V)$ and $\rho_{ij}(X)$ for select country pairs in Figure 6. As is evident, the correlation in input use across countries dwarfs the correlation in real value added. Further, the correlation of output lies somewhere in between, near the simple average of these two correlations.[42] Thus, the correlation of gross output is high because intermediate use is highly correlated, not because value added is highly correlated.

The fact that intermediate use is highly correlated is direct evidence that productivity shocks are being forcefully transmitted through cross-border production chains in the model. Because the share of intermediates in gross output for goods is roughly 2/3, this translates into significant output comovement. On the other hand, value added comovement is not high, and this drags down overall comovement. Recall that one reason value added comoves in the model is that factor supply responds to relative prices. The low comovement of real value added indicates this channel is relatively weak in the model. To raise comovement in value added, one would need to strengthen this channel. In particular, the model would need to be adapted to translate the relatively strong comovement in intermediate use into stronger comovement in value added. I return to this point in the discussion below.

4.3 Vertical Linkages in Trade-Comovement Regressions

Recently, several papers have included proxies for bilateral vertical linkages in trade-comovement regressions. Di Giovanni and Levchenko (2010) and Ng (2010) both find evidence that vertical linkages at least partly explain why bilateral trade is correlated with comovement. In

[42] In practice, the weights on each term are approximately equal (roughly 1/4) and the typical cross-correlation ($\rho_{ij}(V,X)$ or $\rho_{ij}(V,X)$) is relatively close to $\rho_{ij}(Q)$, lying between the extremes of $\rho_{ij}(V)$ and $\rho_{ij}(X)$. Hence, the simple average of $\rho_{ij}(V)$ and $\rho_{ij}(X)$ approximates $\rho_{ij}(Q)$ quite well.

previous sections, we have seen that coefficients in trade-comovement regressions can be hard to interpret because they are likely to be correlated with unobserved shocks. This continues to be true in trade-comovement regressions with intermediates. It is an open question, then, whether trade-comovement regressions with vertical linkages can be interpreted as evidence of a causal relationship between vertical linkages and output comovement?

Because Di Giovanni and Levchenko (2010) examine sectoral data, it is straightforward to map their empirical exercise to my framework and therefore I focus on their work. Di Giovanni and Levchenko attack the identification problem by estimating trade-comovement regressions at the sector level, pooling across sectors, and adding fixed effects to absorb particular unobservable shocks.[43] Specifically, they construct a metric of bilateral vertical linkages at the sector level to capture the intensity with which exports from sector s in country i are used as intermediates by sector s' in country j (and vice versa). This takes the form: $\left[\mathrm{IO}(s,s') \times \mathrm{Exports}_{ij}(s) + \mathrm{IO}(s',s) \times \mathrm{Exports}_{ji}(s')\right]$, where $\mathrm{IO}(s,s')$ is a measure of input-output linkages between sectors s and s' taken from a single country's input-output table and $\mathrm{Exports}_{ij}(s) = \log\left(\frac{EX_{ij}(s)}{GDP_i + GDP_j}\right)$ is the log of exports from i to j in sector s normalized by the sum of value added in the source and destination countries.[44]

Then, Di Giovanni and Levchenko estimate the following regression:

$$\begin{aligned}\rho_{ij}(s,s') = &\alpha + \beta \mathrm{Trade}_{ij}(s,s') \\ &+ \gamma\left[\mathrm{IO}(s,s') \times \mathrm{Exports}_{ij}(s) + \mathrm{IO}(s',s) \times \mathrm{Exports}_{ji}(s')\right] \\ &+ \mathrm{FE} + \epsilon_{ij}(s,s'),\end{aligned} \tag{37}$$

where $\mathrm{Trade}_{ij}(s,s') \equiv \log\left(\frac{EX_{ij}(s) + EX_{ji}(s')}{GDP_i + GDP_j}\right)$ and FE denotes fixed effects that vary by spec-

[43] Alternative ways to deal with this problem used in the literature include adding additional controls to proxy for possible omitted variables or adopting instrumental variables strategies.

[44] I use the direct input-requirements $\mathrm{IO}(s,s')$ the U.S. to proxy for cross-sector input links. Di Giovanni and Levchenko also use input links for a single country.

ification. One specification includes sector-pair fixed effects, while a second specification includes sector-pair fixed effects and country-pair fixed effects. These fixed effects are introduced to address concerns about omitted common shocks. The sector pair effects control for worldwide sector-specific shocks (possibly correlated across sectors) that hit all countries simultaneously. The country pair fixed effects control for aggregate shocks that may be correlated across countries, but hit all sectors symmetrically within each country.[45]

I report the results of running these regressions in my data in Table 4. Focusing on results for gross output, the regression results in the actual data are generally consistent with those reported in Di-Giovanni Levchenko. Both bilateral trade and vertical linkages (Trade $\times$ IO) are positively correlated with bilateral sector-level comovement.[46] Examining results in the model with correlated shocks, vertical linkages remain significant and the coefficient magnitudes are the same or larger than those found in the data. Turning to the model with uncorrelated shocks, however, the magnitude of the coefficient on vertical linkages drops significantly, explaining only perhaps $17 - 33\%$ of the magnitude of the coefficients in the data.[47] Recall that the fixed effects are intended to control for correlated shocks driving correlations in the data. If these fixed effects adequately control for these shocks, one should expect that regression results in the model with uncorrelated shocks to be similar to those in the data (alternatively, the model with correlated shocks). Given that they are not, this suggests that vertical linkages proxies in the data may themselves be picking up shocks that vary by country-pair and sector-pair that the fixed effects cannot absorb.

[45] Ng (2010) embeds a vertical linkages metric into an aggregate trade-comovement regression, and therefore cannot use pair fixed effects to absorb common shocks. Instead, he includes other possible determinants of correlations (e.g., financial openness, output composition, etc.) directly as control variables in the regression.

[46] Though trade intensity is not significant when country fixed are included, vertical linkages are significant with both sets of fixed effects. One point to note is that my country sample is much smaller than Di Giovanni and Levchenko (2010), so lower significance levels may be expected.

[47] Note that looking at real value added, the model with correlated shocks continues to generate coefficients on vertical linkages similar to those in the data, though smaller in magnitude. However, the sign on vertical linkages actually flips sign in regressions in the simulated data with uncorrelated shocks.

In evaluating these results, one might be concerned that this coefficient in stability across the model and data might be related to the fact that the vertical linkages metrics used are not theoretically motivated. Therefore, let me make the same basic point using a convenient proxy for the role of vertical linkages in generating output comovement suggested by the model.

If we define $\tilde{\Theta} = [I - \Theta']^{-1}$, then equation (17) implies that the covariance of output growth in countries i and j is:

$$\mathrm{cov}(\hat{Q}_i, \hat{Q}_j) = \tilde{\Theta}(i,:)\, \Sigma_Z\, \tilde{\Theta}(j,:)', \tag{38}$$

where Σ_Z is the covariance matrix for productivity innovations $\hat{Z}$, and $\tilde{\Theta}(k,:)$ is the k^{th} row of $\tilde{\Theta}$. When productivity innovations are independent across countries with variance σ_k^2 for country k, then:

$$\mathrm{cov}(\hat{Q}_i, \hat{Q}_j) = \sum_k \tilde{\Theta}(i,k)\sigma_k^2 \tilde{\Theta}(j,k). \tag{39}$$

This covariance is increasing in the similarity between the weighting vectors for country i and country j. And these vectors of shock loadings are themselves functions of the input-output structure. Countries with more similar input sourcing and production sharing patterns will tend to have more similar exposure to shocks.

While the Cobb-Douglas, exogenous factor supply version of the model suggests that $[I - \Theta']^{-1}$ provides the correct sourcing weights, the general model generates more complicated weighting formulas, where input sourcing patterns are one key component among a variety of forces. Therefore, rather than relying too heavily on the sourcing weights from the Cobb-Douglas case, I focus on a simple and general metric for input sourcing similarity, based loosely on work by Conley and Dupor (2003).[48] I characterize each destination country

[48] Conley and Dupor (2003) model correlation across sectors within the domestic economy as a function

by an 2N-dimensional vector of cost shares on inputs from each source country. For each country pair, I compute a sourcing similarity index: $SS_{ij} = 1 - \|\Theta(:,i) - \Theta(:,j)\|$, where $\|\cdot\|$ is a Euclidean distance operator and high values of SS_{ij} indicate similar input sourcing patterns.[49] The basic prediction is that output comovement should be positively correlated with input sourcing similarity SS_{ij}.

Table 5 presents estimates from trade-comovement regressions controlling for sourcing similarity in the data and model for goods production.[50] In the data and model with correlated shocks, both trade and sourcing similarity are positively correlated with bilateral comovement, with roughly similar magnitudes in both cases. In contrast, the correlation of sourcing similarity and bilateral trade with comovement is much weaker in the model with uncorrelated shocks. While still significant, the partial correlation is only 11% the size of the correlation in the data. As in the previous regressions, a natural interpretation of this is that sourcing similarity is correlated with output comovement in the data primarily because it is correlated with omitted shocks.

5 Complementarity and Comovement

A recent strain of thought holds that disruptions in input-sourcing produce large output losses because inputs are complements in production.[51] There are several different formulations of this basic idea. First, inputs may complements to each other. In this instance,

of the similarity between sectors in terms of their input use, measured using input cost shares.

[49]In practice, this sourcing similarity metric is highly correlated with an alternative metric that takes the Euclidean distance between rows of the $[I - \Theta']^{-1}$ matrix.

[50]In the data-based regression, I exclude bilateral pairs involving Italy. Including the Italian data weakens the correlations in the data, as Italy in the data appears too have too high correlations in output with other countries given it's level of input sourcing similarlity. This is mostly due to Italy's agriculture and natural resources sector. If one looks at manufacturing correlations and sourcing similarity only (rather than all goods) Italy does not appear unusual.

[51]For example, see Burstein, Kurz, and Tesar (2008), Jones (2010), or the discussion in Di Giovanni and Levchenko (2010).

complementaries among inputs could be symmetric, or complementaries could vary among subsets of inputs (e.g., home and foreign inputs could be complements, while foreign inputs are substitutable among themselves). Second, inputs may be complementary to other factors of production. Put differently, inputs may be complementary to value added.

While the predominant view seems to be that the first type of complementarity is the most important, the results above seem to suggest that the second could play a large role. Specifically, the model generates substantial comovement in intermediate goods use, but it fails to translate this into comovement in value added. One way to induce additional comovement in value added is to assume that intermediates are complements to value added. This then motivates a quantitative exploration of the role of this form of complementarity in amplifying comovement in the model.

[**To Be Completed.**]

6 Conclusion

This paper uses a multi-sector, many country extension of the IRBC model with trade in both final and intermediate goods to dissect the trade-comovement puzzle. Using the model, I attempt to refine our understanding of the trade comovement puzzle along several dimensions.

First, input trade does not resolve the aggregate trade-comovement puzzle in a straightforward manner. That said, input trade does appear to play a role in explaining the relatively good fit of the model for the gross output of goods. Surprisingly, however, transmission of shocks through intermediate input channels does not appear to generate strong comovement in value added. This points the way toward adaptations of the standard framework, specifically introducing greater complementarity between real value added and input use, that may amplify the strength of this transmission channel.

Second, and more generally, the aggregate trade comovement puzzle is linked to the model's inability to generate a positive correlation between bilateral trade and comovement in services output across countries. This suggests that closing the gap between theory and data will require expanding the set of shocks considered beyond productivity to include shocks that synchronize services more forcefully. This is perhaps not as easy as it sounds, given that demand shocks tend to be concentrated on demand for goods, specifically durable goods.

Third, trade-comovement regressions are difficult to interpret because it is not generally possible to control unobservable common shocks. This is true for plain-vanilla specifications, as well as augmented specifications that include proxies for vertical linkages and/or employ sector-level data with fixed effects. Model simulations with uncorrelated shocks suggest that the "causal" role of bilateral trade and/or vertical linkages is much smaller than suggested by raw regression estimates.

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Table 1: Aggregate Trade-Comovement Regressions: Data and Model

Panel A: Real Value Added

	Data	Model (corr. shocks)	Model (uncorr. shocks)
Log Bilateral Trade	0.104***	0.025*	0.001
	(0.014)	(0.013)	(0.000)
N	231	231	231
R-sq	0.18	0.02	0.01

Panel B: Gross Output

	Data	Model (corr. shocks)	Model (uncorr. shocks)
Log Bilateral Trade	0.093***	0.035***	0.008***
	(0.012)	(0.013)	(0.001)
N	171	231	231
R-sq	0.25	0.03	0.36

Robust standard errors in parentheses. Significance levels: * $p < .1$, ** $p < .05$, *** $p < .01$. Constants included in all regressions.

Table 2: Correlation of Model Predicted vs. Actual Correlations, by Sector Pair

Panel A: Gross Output – Model with Correlated Shocks

Sector Pair	Correlation	N
Goods	0.47	171
Services	0.08	171
Cross	0.21	342

Panel B: Gross Output – Model with Uncorrelated Shocks

Sector Pair	Correlation	N
Goods	0.52	171
Services	-0.22	171
Cross	0.30	342

Panel C: Real Value Added – Model with Correlated Shocks

Sector Pair	Correlation	N
Goods	0.51	231
Services	0.25	231
Cross	0.42	462

Panel D: Real Value Added – Model with Uncorrelated Shocks

Goods	0.25	231
Services	-0.11	231
Cross	0.14	462

Table 3: Trade-Comovement Regressions, by Sector Pair

Panel A: Gross Output

	Data			Model (correlated shocks)			Model (uncorrelated shocks)		
	Goods	Services	Cross	Goods	Services	Cross	Goods	Services	Cross
Log Bilateral Trade	0.090***	0.071***	0.071***	0.072***	0.009	0.030***	0.020***	-0.002***	0.010***
	(0.013)	(0.013)	(0.009)	(0.012)	(0.012)	(0.008)	(0.001)	(0.001)	(0.001)
N	171	171	342	231	231	462	231	231	462
R-sq	0.23	0.15	0.16	0.13	0.00	0.03	0.56	0.03	0.33

Panel B: Real Value Added

	Data			Model (correlated shocks)			Model (uncorrelated shocks)		
	Goods	Services	Cross	Goods	Services	Cross	Goods	Services	Cross
Log Bilateral Trade	0.098***	0.054***	0.055***	0.055***	0.006	0.019**	0.003***	-0.003***	0.005***
	(0.012)	(0.013)	(0.009)	(0.012)	(0.012)	(0.008)	(0.001)	(0.001)	(0.000)
N	231	231	462	231	231	462	231	231	462
R-sq	0.22	0.07	0.08	0.09	0.00	0.01	0.10	0.09	0.17

Robust standard errors in parentheses. Significance levels: * $p < .1$, ** $p < .05$, *** $p < .01$. Constants included in all regressions.

Table 4: Disaggregate Trade-Comovement Regressions with "Vertical Linkages"

Panel A: Gross Output

	Sector-Pair Fixed Effects			Sector-Pair & Country-Pair Fixed Effects		
	Data	Model (corr. shocks)	Model (uncorr. shocks)	Data	Model (corr. shocks)	Model (uncorr. shocks)
Log Bilateral Trade	0.046***	0.007	0.005**	0.025	0.004	0.007***
	(0.013)	(0.013)	(0.002)	(0.017)	(0.015)	(0.002)
Trade x IO	0.059**	0.056**	0.010***	0.036**	0.061***	0.012***
	(0.024)	(0.024)	(0.004)	(0.018)	(0.018)	(0.003)
N	684	924	924	684	924	924
R-sq	0.23	0.08	0.44	0.68	0.65	0.64

Panel B: Real Value Added

	Sector-Pair Fixed Effects			Sector-Pair & Country-Pair Fixed Effects		
	Data	Model (corr. shocks)	Model (uncorr. shocks)	Data	Model (corr. shocks)	Model (uncorr. shocks)
Log Bilateral Trade	0.021	0.001	0.005***	-0.031**	0.003	0.008***
	(0.014)	(0.013)	(0.001)	(0.013)	(0.018)	(0.002)
Trade x IO	0.088***	0.048**	-0.006***	0.075***	0.054***	-0.005***
	(0.024)	(0.024)	(0.002)	(0.016)	(0.021)	(0.002)
N	924	924	924	924	924	924
R-sq	0.13	0.05	0.15	0.70	0.47	0.36

Robust standard errors in parentheses. Significance levels: * $p < .1$, ** $p < .05$, *** $p < .01$. Constants included in all regressions.

Table 5: Sourcing Similarity Regressions for Goods: Data and Model

	Data (1)	Model (correlated shocks) (2)	Model (uncorrelated shocks) (3)
Sourcing Similarity	0.523*	0.545**	0.061***
	(0.312)	(0.236)	(0.022)
Log Bilateral Trade	0.075***	0.062***	0.019***
	(0.015)	(0.013)	(0.002)
N	153	231	231
R-sq	0.23	0.15	0.57

Robust standard errors in parentheses. Significance levels: * $p < .1$, ** $p < .05$, *** $p < .01$. Constants included in all regressions.

Figure 1: Aggregate GDP: Data vs. Model with Correlated TFP Shocks

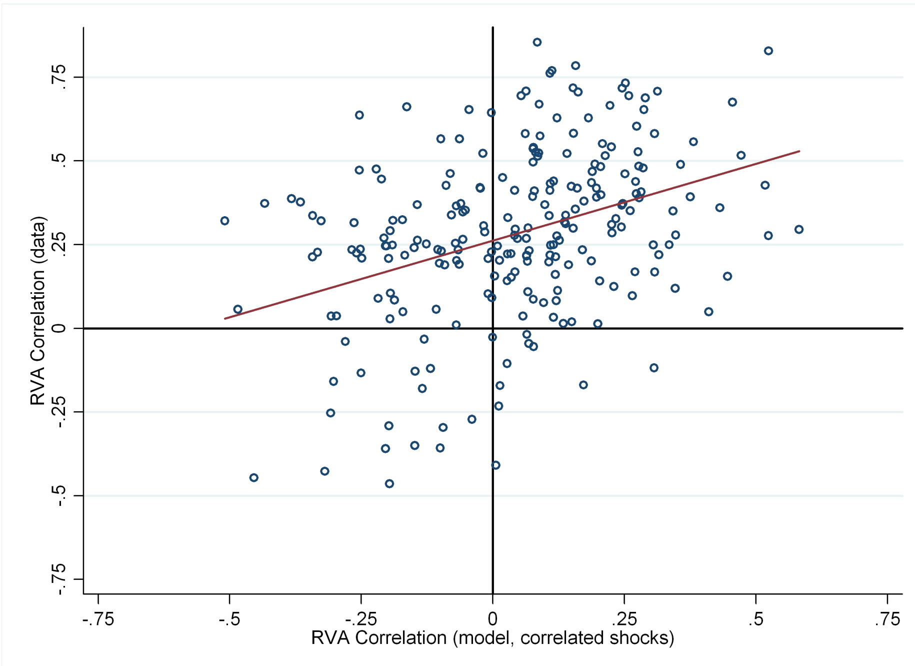


Figure 2: Aggregate GDP: Data vs. Model with Correlated TFP Shocks

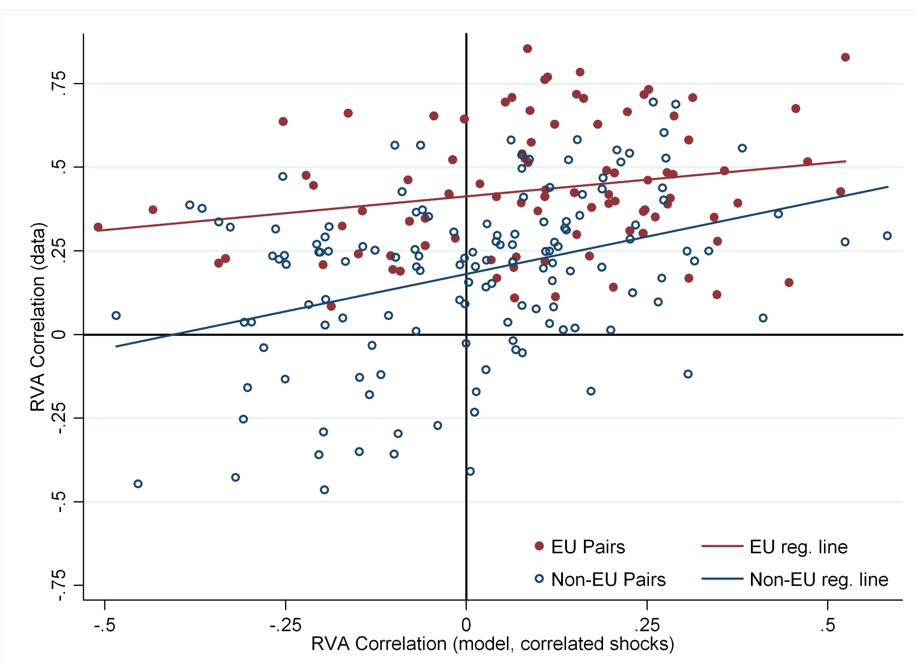
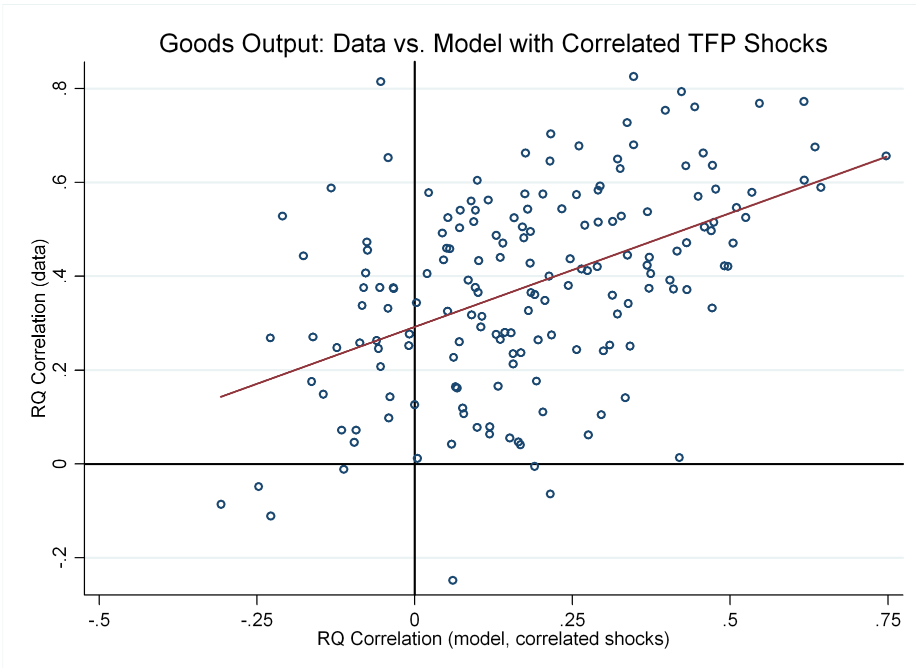


Figure 3: Correlations in Data vs. Correlations in Model with Correlated Shocks



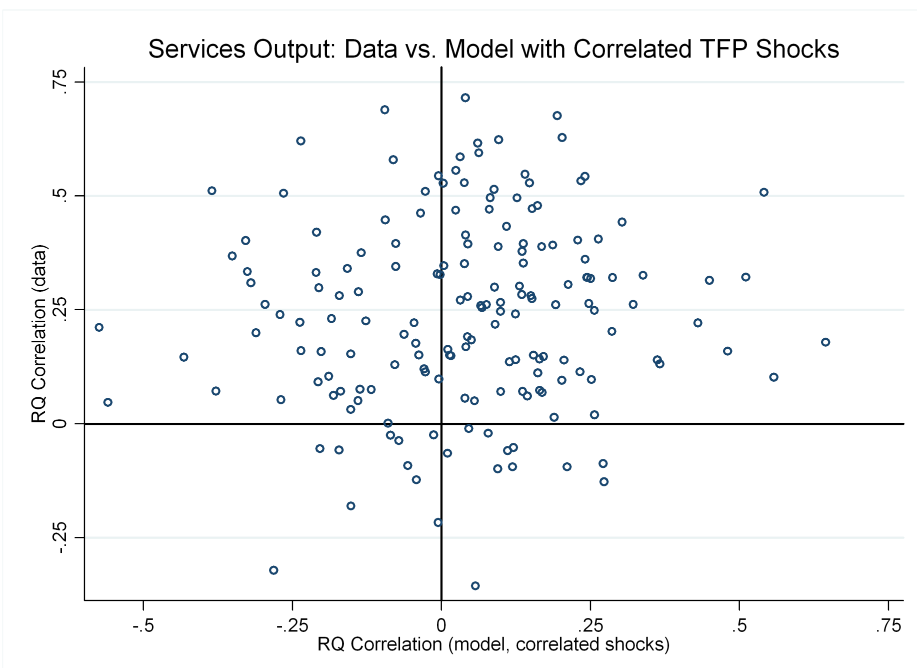


Figure 4: Correlations in Data vs. Correlations in Model with Uncorrelated Shocks

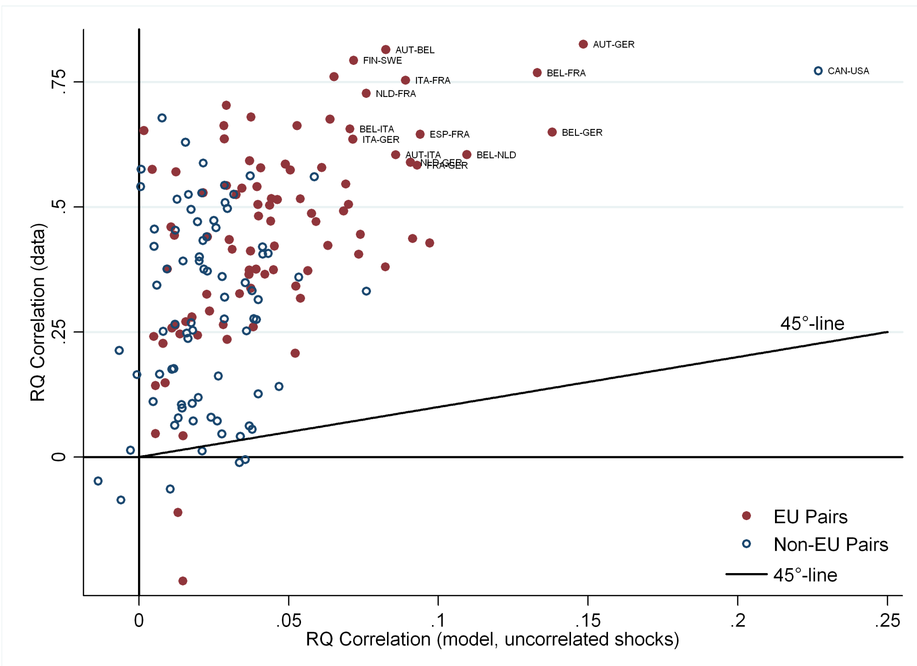


Figure 5: Correlations in Gross Output vs. Correlations in Value Added: Model and Data

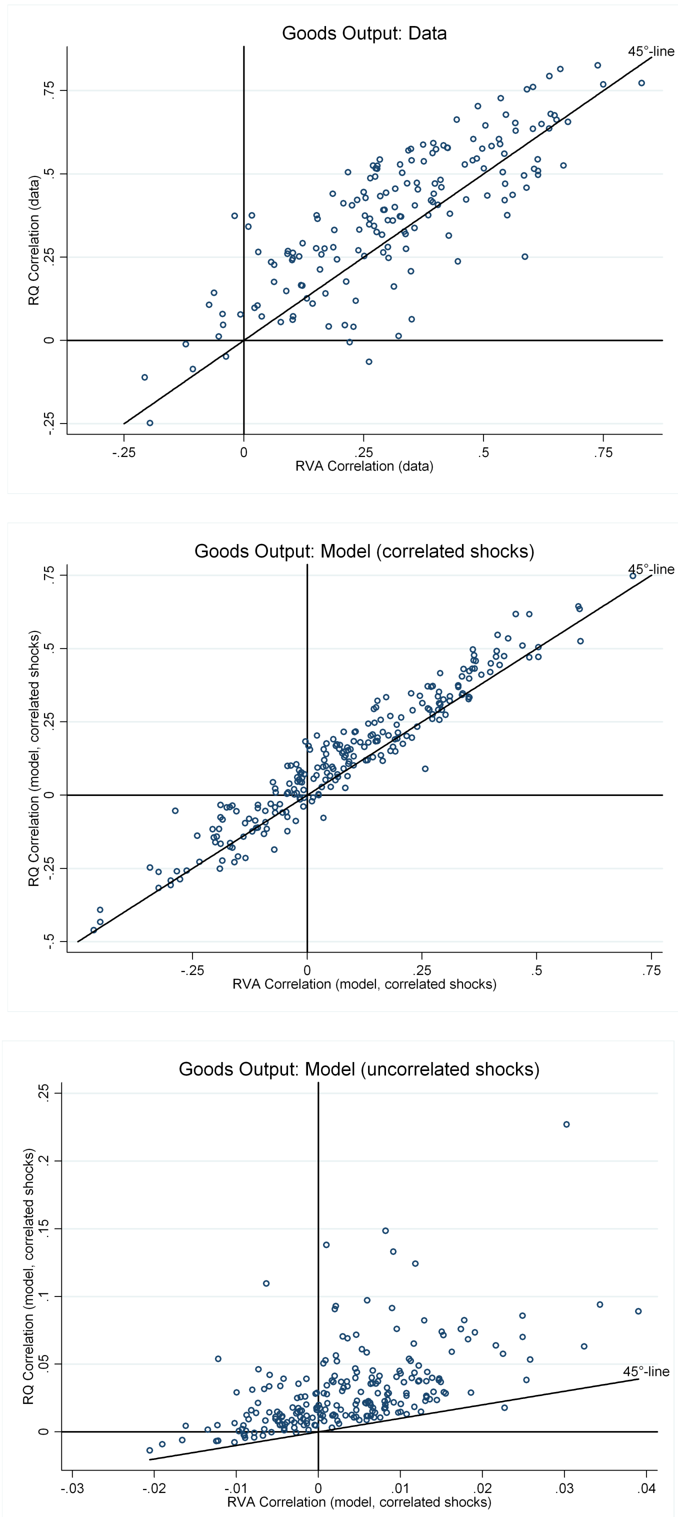


Figure 6: Correlations of Goods Output and Components in Model with Uncorrelated Shocks