Reputational Concern with Endogenous Information Acquisition

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Abstract

We develop a reputational cheap talk model to characterize the essential activities of an expert: information acquisition and information transmission. The decision maker, who has the authority to take actions, is in lack of relevant information and is uncertain about the expert's preference. The expert, who acquires and conveys information, may be biased that he is in favor of a particular action, or may be aligned that he cares about the decision maker's payoff and has reputational concern. Our main insight shows that an aligned expert's reputational concern may have a non-monotonic effect on his information acquisition incentive: he acquires better information if and only if his reputational concern is moderate. Another main insight describes that the possible existence of biased experts may actually increase the decision maker's payoff and social surplus, which differs from Stephen Morris (2001), Ely and Valimaki (2003) and Ely, Fudenberg and Levine (2008) substantially. Regarding delegation, unlike the result in Aghion and Tirole (1997), we show that delegation may reduce the aligned expert's information acquisition incentive. Finally, our analysis illustrates that the decision maker prefers communication to delegation whenever informative communication is feasible, which is opposite to Wouter Dessein (2002).

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1. Introduction

Information acquisition and information transmission are essential activities in various organizations, markets and societies. Uninformed principals, who have the authority to make decisions, have to rely on these activities by experts for the sake of decision optimalities. For instance, government officials often seek advice from social scientists when they are making policies, and for the advice to be valuable, it's necessary for the scientists to acquire the relevant information first. Similarly, investors may ask for suggestions from their financial consultants if they are uncertain about the value of the projects, and it's the consultants' discretion about how informative their reports should be. A pervasive feature lying behind these interactions is that advice and suggestions are often non-verifiable and explicit contracts contingent solely on them are infeasible. Instead, the experts may be self-incentivized by their reputations for being aligned, say, how they are perceived as caring about the decision makers' payoffs. This is quite common in practice, and helps to explain how these scientists, consultants, analysts and politicians are motivated and rewarded.

The literature on reputational cheap talk has characterized how reputational concerns may influence experts' incentives to convey information. More precisely, it's shown that if the decision maker believes there is positive probability that the expert is in favor of a particular action, or say he is biased, information transmission may be distorted even by an aligned expert who shares the preference about the optimal current action with the decision maker, if he has strong incentive to separate from the biased type. This is so called "political correctness" in Stehpen Morris (2001) and "bad reputation" in Ely and Valimaki (2003) and Ely, Fudenberg and Levine (2008). A common feature in these papers is, the expert's information is exogenously given, so his decision only involves whether to reveal the information truthfully or not. Apparently, to have a full understanding about how reputational concerns may affect experts' behaviors, it's necessary to have a model incorporating both information acquisition and information transmission. Is it possible for the aligned expert to signal his type solely by acquisition decision? Would expert's incentive to acquire better information enhance or mitigate his incentive on truthful information revelation? When should the decision maker delegate her decision rights to the expert? In the papers mentioned above, the existence of biased expert is detrimental to the decision maker's payoff, and so is to social welfare. Is it still true in our model, or we may have a more positive result?

We construct a reputational cheap talk model with information acquisition in this paper. An uninformed decision maker has access to a potentially informed expert, but she is uncertain about the expert's preference. The expert, may be aligned or biased, first decides whether costly but more accurate information should be acquired, after that he receives signals and sends messages to the decision maker. Based on her inferences about the state of the nature and the type of the expert, the decision maker takes an action that is payoff-relevant to both parties. We introduce reputational concern into the aligned expert's payoff to capture the idea that, experts may be motivated by how they are perceived by others, instead of some explicit compensation schemes. For simplicity, the biased expert is assumed to be myopic and only cares about the current action taken by the decision maker. But this could be easily modified without any significant change of the qualitative results, for instance, let the biased expert also has reputational concern.

Our first result shows that reputational concern may have a non-monotonic effect on the aligned expert's information acquisition incentive: he acquires better information if and only if his reputational concern is moderate. The intuition is, when reputational concern is relatively low, the information acquisition cost outweighs the acquisition benefit even though the aligned expert reveals his information truthfully, so his attempt to acquire better information is restricted; on the other hand, when reputational concern is relatively high, the aligned expert knows that, in order to separate from the biased type and capture the large reputational gain, he is ready to send the same message regardless of the information he has, so better information is worthless to him. Only in the moderate range of reputational concern, the gain from truthful and increased accuracy of message sending exceeds the information acquisition cost for this expert.

Another main insight we derive in this model is, the possible existence of biased experts may actually be beneficial to the decision maker, and to social welfare. This departs from the papers about "political correctness" and "bad reputation" substantially, see Stephen Morris (2001), Ely and Valimaki (2003) and Ely, Fudenberg and Levine (2008). Precisely, if the probability that an expert is biased is positive but lower than a threshold, and the aligned expert acquires better information in equilibrium, then the decision maker's payoff is improved by the potential presence of biased experts. The reason is, in case it's certain that the expert is aligned, the expert reveals his information truthfully, but he has no incentive to acquire better information since there is no additional reputational gain. With the possible existence of biased experts, the decision maker suffers an information loss from this type of expert, but now she may benefit from the aligned expert's information acquisition. If the probability to encounter a biased expert is sufficiently low, this benefit dominates the information loss, and the decision maker's payoff is augmented.

The observability of the information acquisition decision affects the biased expert's acquisition incentive sharply, but it has no effect on the aligned expert's incentive. We show in our setup such that, if the acquisition decision is observable, the biased expert has the same incentive as the aligned expert to acquire better information in equilibrium, but he never acquires if this decision is unobservable. Roughly, since the message sent by the biased expert is uninformative, the accuracy of his signal is irrelevant to the decision maker's optimal action, so the only reason for the biased expert to acquire better information is to avoid type-separation by the acquisition decision, but this can only happen when this decision is observable. This implies there is no interim belief updating about the expert's type based on the acquisition decision in equilibrium, so the aligned expert's incentive is unaffected and the decision maker's optimal actions are unchanged.

We also analyze what is the effect of delegation on the expert's decisions and when should the decision maker grant her decision rights to the expert. For instance, a regulator may delegate the rights of pricing to the regulated firm, and a financial consultant may have much discretion on investment decisions. Two scenarios are considered: unrestricted delegation and restricted delegation; for the latter, the decision maker can optimally design the delegation set. Under non-delegation, we have seen that the decision maker suffers information distortion that may be introduced by both types of experts; under delegation, since there is perfect type-separation based on the actions taken by the expert, the decision maker suffers action distortion introduced by the biased expert. This is the central trade off that the decision maker is concerned. Besides, there is another potential disadvantage under delegation: reputational concern brings no additional gain to the aligned expert, so his incentive to acquire better information may be weakened. Our main finding is, if there exists informative equilibrium in the non-delegation situation (whether with or without information acquisition), then non-delegation dominates delegation for the decision maker regardless whether the delegation is optimally restricted or not. Interestingly, this result is opposite to the finding in Wouter Dessein (2002), in which the decision maker prefers delegation to non-delegation whenever informative communication is feasible. Besides this finding, we also show that limiting the expert's decision rights may enhance his acquisition incentive, which is similar to the result in Szalay (2005).

The rest of the paper is organized as follows. Section 2 gives an overview of the related literature and section 3 describes the model formally. The main insights of this paper is derived in section 4 with observable acquisition decision and in section 5 with unobservable acquisition decision. Section 6 considers the delegation issues. Finally, we conclude this paper in section 7.

2. Literature

This paper belongs to the growing literature on cheap talk initiated by Crawford and Sobel (1982). Joel Sobel (1985) first derives a reputational cheap talk model, in which the aligned expert is non-strategic, but the biased expert has attempts to appear aligned. Benabou and Laroque (1992) modify Sobel's model with imperfect signals. Reputational concerns in these two papers restrict the biased expert's incentive to manipulate information, so they are "good". Most closely, we build on and borrow from Stephen Morris (2001). Morris (2001) endogenizes both types of experts' reputational concerns, and shows that if the biased expert is in favor of a particular message then the aligned expert may have incentive to avoid sending this message, in order to benefit from the reputation building, which he refers as "political correctness". Ely and Valimaki (2003) and Ely, Fudenberg and Levine (2008) show that reputational concern for a long-run player interacting with a sequence of short-run players could be unambiguously bad, leading to market shut down and loss of surplus, and this is so called "bad reputation". While it's assumed that the expert's information is exogenously given in these papers, we allow the expert to optimally decide whether costly but more accurate information should be acquired, and mainly focus on reputational concern's effect on expert's acquisition incentive.

Some recent papers introduce a third party into the reputational cheap talk setup. Wei Li (2010) considers a model in which there is an intermediary between the expert and the decision maker, and shows that the biased expert and the biased intermediary's reporting truthfulness are strategic complements. Durbin and Iyer (2009) and Li and Mylovanov (2008) develop similar models such that the expert may acquire information by himself or follow the recommendation from an interest group in exchange for an access fee, which endogenize the source of the expert's bias. Another strand of the literature on reputational cheap talk, not so closely to ours, focuses on the situation that the expert has incentive to build reputation about his ability, for instance, Scharfstein and Stein (1990), Brandenburger and Polak (1996), Gilat Levy (2004), Andrea Prat (2005), Ottaviani and Sorensen (2006a, b), Gentzkow and Shapiro (2006), Wei Li (2007), Giuseppe Moscarini (2007), etc.

Our paper also relates to the literature on information acquisition. Dezso Szalay (2005) studies a model in which the expert acquires costly information and then chooses an optimal action. He shows that it may be desirable for the principal to restrict the expert's discretion in order to improve the information acquisition incentive, even the expert is perfectly aligned. Hao Li (2001) derives a similar insight within a group decision framework such that optimally designed conservatism increases experts' incentives to collect evidence and improves the quality of the group decision. Dur and Swank (2005), Gerardi and Yariv (2008) and Che and Kartik (2009) show that it could be optimal for the decision maker to hire experts with different preferences and opinions, since these experts have stronger incentives to collect relevant information. None of these papers is concerned about the expert's reputation, so they are quite different from our model.

Finally, our paper refers to the issues about delegation. Aghion and Tirole (1997) note that delegation increases the agent's information acquisition incentive and generally the principal trades off between the loss of control and the gain of information. Interestingly, our result is opposite to theirs, say, delegation reduces the expert's acquisition incentive. Wouter Dessein (2002) finds that the decision maker prefers delegation whenever informative communication is feasible, but what we show in out setup is again the opposite: as long as there exists informative equilibrium in the non-delegation situation, non-delegation dominates delegation from the perspective of the decision maker. Alonso and Matouschek (2008) generalize the delegation literature and characterize the properties of optimal delegation. They show that for certain conditions, optimal delegation has the form that the delegation set is an interval, which holds in our model. For more papers regarding delegation, see Holmstrom (1977, 1984), Melumad and Shibano (1991), Tymofiy Mylovanov (2008), Krishna and Morgan (2008), Kovac and Mylovanov (2009), etc.

3. The model.

We consider a game Γ, in which there are two players, an expert (E or he) and a decision maker (DM or she). The state of the world is binary, $\theta \in \{0,1\}$, and occurs with equal probabilities. Before he receives a signal $s \in \{0,1\}$, the expert has access to an information acquisition technology which can increase the accuracy p of the signal: with effort $e = 1$ and cost $c > 0$, the signal s will be equal to the state θ with probability $p = p_1$; with effort $e = 0$ and cost 0, the signal s will be equal to the state θ with probability $p = p_0$ such that $1/2 < p_0 < p_1$. After that, the expert receives a signal s and sends message $m \in \{0,1\}$ to the decision maker. Based on the expert's information acquisition and information transmission decisions, the decision maker takes an action $a \in [0,1]$ to maximize her payoff. For instance, this could be a decision about how much an investment should be implemented or what the optimal policy should be. Finally, the decision maker learns the state θ of the world and updates her belief about the the expert's type based on the information she has.

The decision maker's payoff depends on the true state of the world and her action. For simplicity, it's represented by the quadratic loss function $\Pi = -(a-\theta)^2$. Then it's easy to derive that the optimal action is equal to the probability she attaches to the state $\theta = 1$. There are two types of experts: aligned expert (A) with prior probability $\lambda \in (0,1)$ and biased expert (B) with prior probability $1-\lambda$. An aligned expert has the same preference about the current optimal action as the decision maker, and his payoff is given by $U_A = -(a-\theta)^2 + \mu\phi(\lambda) - ec$, in which subscript "$A$" represents "aligned", $\phi(\lambda)$ is the decision maker's posterior belief that the expert is aligned, and μ is the reputational concern weight attached to this belief by him. Besides this, depending on whether there is information acquisition to improve the signal accuracy p, say $e \in \{0,1\}$, an aligned expert pays cost ec. A biased expert prefers the action to be taken by the decision maker as large as possible, regardless of the true state, and his payoff is given by $U_B = \eta a - ec$, in which subscript "B" represents "biased", η is the weight attached to the action a, and ec is his information acquisition cost.

We look for Perfect Bayesian Equilibria (PBE) in this paper. For an expert of type $i \in \{A, B\}$, his strategy σ_i consists of five probabilities: $\sigma_i = \{\alpha_i, (x_i, w_i), (y_i, z_i)\}$. α_i is the probability that he acquires a better signal, x_i (w_i) is the truthful reporting probability that he sends message $m = 1$ ($m = 0$) when his signal is $s = 1$ ($s = 0$) conditional on the information acquisition decision $e = 0$; similarly, y_i (z_i) is the truthful reporting probability that he sends message $m = 1$ ($m = 0$) when his signal is $s = 1$ ($s = 0$) conditional on the information acquisition decision $e = 1$. We consider two different scenarios in this model: one is with observable information acquisition decision, and the other is with unobservable acquisition decision. If the acquisition decision is observable, the decision maker's strategy is to take an action $a_{em} \in [0, 1]$ based on her information about e and m. Also, she has interim belief updating given the expert's acquisition decision e: after observing $e = 1$, she attaches probability λ_1 for the expert to be aligned; correspondingly, after observing $e = 0$, her belief is adjusted to λ_0. If the acquisition decision is unobservable, the decision maker's action $a_m \in [0, 1]$ to be taken only depends on message m, besides, there is no interim belief updating about the expert's type. In a PBE, each player maximizes his/her expected payoff given the strategy of the other player, and the decision maker's posterior belief updating (also for interim belief λ_e if e is observable) follows Bayes's rule whenever possible.

Much of the analysis in the next sections is derived in the continuation games after the expert's information acquisition decision, for convenience, we define them here. Let Γ_0 be the continuation game such that the expert's acquisition decision is $e = 0$, similarly, Γ_1 is the continuation game after decision $e = 1$. We summarize the timing of this game in the following figure.

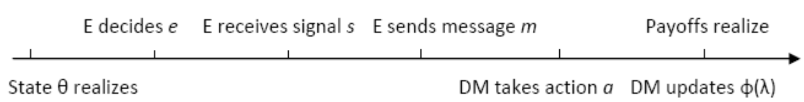

Figure 1. Timing of the game Γ

4. Equilibrium with observable information acquisition

In this section, we identify how reputational concern may affect expert's information acquisition and transmission decisions with the assumption that information acquisition is observable. This might be the situations such as, a financial consultant submits a report to his client with plentiful data and analysis, or a professor revises his student's paper with lots of detailed comments. For each case, the decision maker can easily infer the expert's effort decision. We delay our analysis with unobservable information acquisition to the next section.

Let $\sigma^* = \{\sigma_A^*, \sigma_B^*, a_{em}^*\}$ be an equilibrium strategy profile, in which $a_{em}^* = \Pr(\theta = 1|e, m)$ is the decision maker's optimal action given her information about the expert's decision e and message m. Also denote $\{\lambda_e^*, \phi^*(\lambda_e)\}$ be the decision maker's interim and posterior beliefs in equilibrium. Since our paper belongs to the literature on cheap talk, it's straightforward to see this game has a babbling equilibrium in which no information is revealed: the decision maker takes action $a^* = 1/2$ regardless of the messages she receives; there is no information acquisition, so $\alpha_A^* = \alpha_B^* = 0$, and both types of experts randomize 50-50 between sending message $m = 0$ and $m = 1$, whatever their acquisition decisions are and the signals they observe. Also, the interim and posterior beliefs keep unchanged both on and off equilibrium path, so $\lambda_0^* = \phi^*(\lambda_0^*) = \lambda_1^* = \phi^*(\lambda_1^*) = \lambda$. Given this strategy profile and belief updating system, no player has incentive to deviate. This babbling equilibrium guarantees the existence of equilibrium in this game.

Since we are mainly exploring the effects that aligned expert's reputational concern has on the incentives to acquire and convey information, informative equilibria have particular interest to us. The definition of informative equilibrium is given as follows.

Definition 1 *An equilibrium is an informative equilibrium (IE) if on the equilibrium path* $a_{em}^* \neq a_{em'}^*$ *for* $m \neq m'$.

This definition is consistent with the literature, see Stephen Morris (2001) and Wei Li (2010). Roughly speaking, for an equilibrium to be informative, different messages from the expert should induce different actions taken by the decision maker. Be specific to our model, it's necessary to restrict the definition of informativeness "on the equilibrium path". The reason is, without this restriction, it's possible to construct an equilibrium such that the decision maker's optimal actions are affected by the expert's messages only on an off-equilibrium path. But such an equilibrium is outcome equivalent to the babbling equilibrium.

Lemma 1 *For any informative equilibrium, if* Γ_0 *is the equilibrium continuation game, at most one of the following conditions holds for the aligned expert: (1)* $0 < x_A^* < 1$*, (2)* $0 < w_A^* < 1$*; only one of the following conditions holds for the biased expert: (1') $x_B^* = 0$ and $w_B^* = 1$, (2') $x_B^* = 1$ and $w_B^* = 0$. The symmetric argument holds if* Γ_1 *is the equilibrium continuation game.*

Proof. We prove this lemma for the continuation game Γ_0, the proof for Γ_1 is almost same, except the relevant notations should be changed.

For an equilibrium to be informative, it's necessary to have $a_{01}^* > a_{00}^*$ or $a_{01}^* < a_{00}^*$. To notice that, a_{em}^* is equal to the probability that the decision maker attaches to the state

$\theta = 1$ given her information. We consider $a_{01}^* > a_{00}^*$ first. Then, only (2') is true for the biased expert, since sending message $m = 1$ induces a higher action to be taken. For the aligned expert, given the signal $s = 1$, $a_{01}^* > a_{00}^*$ implies $E_\theta[-(a_{01}^* - \theta)^2] > E_\theta[-(a_{00}^* - \theta)^2]$. To have $0 < x_A^* < 1$, $E_\theta[\phi^*(\lambda_0|m = 1)] < E_\theta[\phi^*(\lambda_0|m = 0)]$ should be true. Now if the signal is $s = 0$, we have $E_\theta[-(a_{01}^* - \theta)^2] < E_\theta[-(a_{00}^* - \theta)^2]$. In order to have $0 < w_A^* < 1$, $E_\theta[\phi^*(\lambda_0|m = 1)] > E_\theta[\phi^*(\lambda_0|m = 0)]$ should be true. A contradiction.

Now consider the case with $a_{01}^* < a_{00}^*$. Since sending message $m = 0$ induces a higher action taken by the decision maker, the biased expert always sends this message regardless of his signal, so (1') is true. For the aligned expert, given signal $s = 1$, $a_{01}^* < a_{00}^*$ implies $E_\theta[-(a_{01}^* - \theta)^2] < E_\theta[-(a_{00}^* - \theta)^2]$ and to have $0 < x_A^* < 1$, $E_\theta[\phi^*(\lambda_0|m = 1)] > E_\theta[\phi^*(\lambda_0|m = 0)]$ should be true. On the other hand, given signal $s = 0$, $a_{01}^* < a_{00}^*$ implies $E_\theta[-(a_{01}^* - \theta)^2] > E_\theta[-(a_{00}^* - \theta)^2]$ and to have $0 < w_A^* < 1$, $E_\theta[\phi^*(\lambda_0|m = 1)] < E_\theta[\phi^*(\lambda_0|m = 0)]$ should be true. Again a contradiction. ■

The lemma shown above simplifies our analysis significantly. It says in any informative equilibrium, the biased expert always sends the same message $m = i$, while the aligned expert is truthful reporting on the signal $s = j$ such that $j \neq i$. Potentially, there are two classes of informative equilibria: one is with $a_{e1}^* > a_{e0}^*$, $x_B^* = 1 - w_B^* = 1$ (or $y_B^* = 1 - z_B^* = 1$) and $w_A^* = 1$ (or $z_A^* = 1$), the other is with $a_{e1}^* < a_{e0}^*$, $x_B^* = 1 - w_B^* = 0$ (or $y_B^* = 1 - z_B^* = 0$) and $x_A^* = 1$ (or $y_A^* = 1$). But for the second class, when receiving message $m = 0$, the decision maker has to correctly infer that the true state is more possible to be $\theta = 1$, so the meaning of the message is reversely understood. Since the game is symmetric, it is straightforward to show that for any informative equilibrium in the second class, there exists an equilibrium in the first class such that they are payoff equivalent. Intuitively, if there is a "reversely understood" equilibrium, then there is another "obversely understood" equilibrium. Thus, without loss of generality, we can simply focus on the first class of equilibria, in which, on the equilibrium path, the biased expert always sends message $m = 1$, and the aligned expert tells truth when his signal is $s = 0$.

Similar to the backward induction approach, we derive the players' equilibrium behaviors in the continuation games first.

4.1. Equilibrium analysis in the continuation game Γ_0.

Suppose we are on the equilibrium path with $e = 0$. Given interim belief λ_0 and both types' strategies, the decision maker's optimal actions are:

$a^*_{01} = \Pr(\theta = 1|e = 0, m = 1) = \frac{(1-\lambda_0)+\lambda_0 p_0 x_A}{2(1-\lambda_0)+\lambda_0 x_A}$ and $a^*_{00} = \Pr(\theta = 1|e = 0, m = 0) = \frac{1-p_0 x_A}{2-x_A}$

Also, after observing the state θ, the posterior beliefs are:

$\phi^*(\lambda_0|m = 0, \theta = 0) = 1$ and $\phi^*(\lambda_0|m = 0, \theta = 1) = 1$

$\phi^*(\lambda_0|m = 1, \theta = 0) = \frac{(1-p_0)x_A\lambda_0}{(1-p_0)x_A\lambda_0+(1-\lambda_0)}$ and $\phi^*(\lambda_0|m = 1, \theta = 1) = \frac{p_0 x_A\lambda_0}{p_0 x_A\lambda_0+(1-\lambda_0)}$

To notice that, since the biased expert always sends message $m = 1$, sending message $m = 0$ perfectly signals the aligned expert's type, so the posterior beliefs in the first line are 1 regardless of the states; on the other hand, sending message $m = 1$ pools the aligned expert with the biased expert, so the beliefs in the second line are no larger than λ_0.

Some properties of these actions and beliefs are useful, so we summarize them in the next remark.

Remark 1 *For any* $p_0 \in (\frac{1}{2}, 1)$, $x_A \in [0, 1]$ *and* $\lambda_0 \in [0, 1]$, $\frac{1}{2} \leq a^*_{01} \leq p_0$, $\frac{\partial a^*_{01}}{\partial x_A} > 0$, $\frac{\partial a^*_{01}}{\partial p_0} > 0$ *and* $\frac{\partial a^*_{01}}{\partial \lambda_0} > 0$; $1 - p_0 \leq a^*_{00} \leq \frac{1}{2}$, $\frac{\partial a^*_{00}}{\partial x_A} < 0$, $\frac{\partial a^*_{00}}{\partial p_0} < 0$ *and* $\frac{\partial a^*_{00}}{\partial \lambda_0} = 0$.

It's easy to see that the continuation game Γ_0 is informative if and only if $x_A > 0$, say, at least the aligned expert tells truth with positive probability when his signal is $s = 1$. In that case, $a^*_{01} > 1/2 > a^*_{00}$. Now we are going to check when x_A is actually larger than 0.

Consider the case that $s = 1$. Let v_1 be the aligned expert's continuation payoff by sending $m = 1$ and v_0 be his continuation payoff by sending $m = 0$. Then,

$v_0 = -p_0(a^*_{00} - 1)^2 - (1 - p_0)(a^*_{00} - 0)^2 + \mu$

$v_1 = -p_0(a^*_{01} - 1)^2 - (1 - p_0)(a^*_{01} - 0)^2 + \mu\{p_0\phi^*(\lambda_0|m = 1, \theta = 1) + (1 - p_0)\phi^*(\lambda_0|m = 1, \theta = 0)\}$

And we have

$v_1 - v_0 = (a^*_{00} - a^*_{01})(a^*_{00} + a^*_{01} - 2p_0) + \mu\{p_0\phi^*(\lambda_0|m = 1, \theta = 1) + (1 - p_0)\phi^*(\lambda_0|m = 1, \theta = 0) - 1\}$

For notational simplicity, we define $\Omega_0 = (a^*_{00} - a^*_{01})(a^*_{00} + a^*_{01} - 2p_0)$ and $\Omega_0(1)$ is the value of Ω_0 when $x_A = 1$. Similarly, we define $\Delta_0 = p_0\phi^*(\lambda_0|m = 1, \theta = 1) + (1 - p_0)\phi^*(\lambda_0|m = 1, \theta = 0)$ and $\Delta_0(1)$. We also summarize some properties of Ω_0 and Δ_0 in the following remark.

Remark 2 *For any* $p_0 \in (\frac{1}{2}, 1)$, $x_A \in [0, 1]$ *and* $\lambda_0 \in [0, 1]$, $\Omega_0 > 0$ *if* $a^*_{00} \neq a^*_{01}$ *and* $\Omega_0 = 0$ *if* $a^*_{00} = a^*_{01}$; $\frac{\partial \Omega_0}{\partial x_A} > 0$, $\frac{\partial \Omega_0}{\partial p_0} > 0$, $\frac{\partial \Omega_0}{\partial \lambda_0} > 0$ *if* $a^*_{01} < p_0$ *and* $\frac{\partial \Omega_0}{\partial \lambda_0} = 0$ *if* $a^*_{01} = p_0$. $\Delta_0 > 0$, $\frac{\partial \Delta_0}{\partial x_A} > 0$, $\frac{\partial \Delta_0}{\partial p_0} > 0$, $\frac{\partial \Delta_0}{\partial \lambda_0} > 0$.

Let $\mu_0(\lambda_0) = \frac{\Omega_0(1)}{1-\Delta_0(1)} = [p_0^3+(1-p_0)^3-p_0(\frac{1-\lambda_0 p_0}{2-\lambda_0})^2-(1-p_0)(\frac{1-\lambda_0+\lambda_0 p_0}{2-\lambda_0})^2]/[1-\frac{p_0^2\lambda_0}{p_0\lambda_0+(1-\lambda_0)}-\frac{(1-p_0)^2\lambda_0}{(1-p_0)\lambda_0+(1-\lambda_0)}]$ if $\lambda_0 < 1$ and $\mu_0(\lambda_0) = +\infty$ if $\lambda_0 = 1$.

Now it's easy to see that, for any $\mu > \mu_0(\lambda_0)$, $v_1 - v_0 < 0$ for any $x_A \in [0,1]$, sending message $m = 1$ results the aligned expert with lower payoff than sending message $m = 0$, so $x_A^* = 0$. For $\mu \leq \mu_0(\lambda_0)$, apparently, $x_A^* = 1$ is a feasible solution. But to notice that, for any $\mu < \mu_0(\lambda_0)$, there also exists another x_A' such that $v_1 - v_0 = 0$ and $0 < x_A' < 1$, $\partial x_A'/\partial\mu > 0$ for this range of μ.

These results imply, if the continuation game Γ_0 is on the equilibrium path, for an equilibrium to be informative, the aligned expert's reputational concern should be not too large, say $\mu \leq \mu_0(\lambda_0)$. The intuition here is, if the aligned expert cares so much to be perceived as "aligned", he has strong incentive to send message $m = 0$ whatever his true signal is, since message $m = 0$ can separate him from the biased expert perfectly. Only when reputational concern is relatively unimportant compared with the current payoff, truthful revealing of information and inducing a correctly action is in the aligned expert's interest. For the reason of $\partial x_A'/\partial\mu > 0$, since with the increase of μ when $\mu < \mu_0(\lambda_0)$, the aligned expert's incentive to send message $m = 0$ increases, in order to be indifferent between these two messages, sending message $m = 1$ should result him with higher current payoff by improving the decision maker's optimal action, so x_A' should also be increased.

4.2. Equilibrium analysis in the continuation game Γ_1.

Now suppose we are on the equilibrium path with $e = 1$. Repeat the analysis shown above, we have the optimal actions and posterior belief updating as follows:

$a_{11}^* = \Pr(\theta = 1|e = 1, m = 1) = \frac{(1-\lambda_1)+\lambda_1 p_1 y_A}{2(1-\lambda_1)+\lambda_1 y_A}$ and $a_{10}^* = \Pr(\theta = 1|e = 1, m = 0) = \frac{1-p_1 y_A}{2-y_A}$

Also, after observing the state θ, the posterior beliefs are:

$\phi^*(\lambda_1|m = 0, \theta = 0) = 1$ and $\phi^*(\lambda_1|m = 0, \theta = 1) = 1$

$\phi^*(\lambda_1|m = 1, \theta = 0) = \frac{(1-p_1)y_A\lambda_1}{(1-p_1)y_A\lambda_1+(1-\lambda_1)}$ and $\phi^*(\lambda_1|m = 1, \theta = 1) = \frac{p_1 y_A\lambda_1}{p_1 y_A\lambda_1+(1-\lambda_1)}$

Similarly, consider the case that $s = 1$. Let u_1 be the aligned expert's continuation payoff by sending $m = 1$ and u_0 be his continuation payoff by sending $m = 0$. Then,

$u_0 = -p_1(a_{10}^* - 1)^2 - (1 - p_1)(a_{10}^* - 0)^2 + \mu$

$u_1 = -p_1(a_{11}^* - 1)^2 - (1 - p_1)(a_{11}^* - 0)^2 + \mu\{p_1\phi^*(\lambda_1|m = 1, \theta = 1) + (1 - p_1)\phi^*(\lambda_1|m = 1, \theta = 0)\}$

And we have

$u_1 - u_0 = (a_{10}^* - a_{11}^*)(a_{10}^* + a_{11}^* - 2p_1) + \mu\{p_1\phi^*(\lambda_1|m = 1, \theta = 1) + (1-p_1)\phi^*(\lambda_1|m = 1, \theta = 0) - 1\}$

Define Ω_1, $\Omega_1(1)$, Δ_1,$\Delta_1(1)$ similarly as in the case of continuation game Γ_0.

Let $\mu_1(\lambda_1) = \frac{\Omega_1(1)}{1-\Delta_1(1)} = [p_1^3 + (1-p_1)^3 - p_1(\frac{1-\lambda_1 p_1}{2-\lambda_1})^2 - (1-p_1)(\frac{1-\lambda_1+\lambda_1 p_1}{2-\lambda_1})^2]/[1 - \frac{p_1^2\lambda_1}{p_1\lambda_1+(1-\lambda_1)} - \frac{(1-p_1)^2\lambda_1}{(1-p_1)\lambda_1+(1-\lambda_1)}]$ if $\lambda_1 < 1$ and $\mu_1(\lambda_1) = +\infty$ if $\lambda_1 = 1$.

Then we have results: for any $\mu > \mu_1(\lambda_1)$, $u_1 - u_0 < 0$ for any $y_A \in [0,1]$, sending message $m = 1$ results the aligned expert with lower payoff than sending message $m = 0$, so $y_A^* = 0$. For $\mu \leq \mu_1(\lambda_1)$, $y_A^* = 1$ is a feasible solution. Also, for any $\mu < \mu_1(\lambda_1)$, there exists y_A' such that $u_1 - u_0 = 0$ and $0 < y_A' < 1$, $\partial\, y_A'/\partial\mu > 0$ for this range of μ.

The analysis shown so far describes the expert's equilibrium behavior in the continuation game of information transmission, and explores the necessary condition for there to exist informative equilibrium. We summarize these results in the following proposition.

Proposition 1 *For there to be informative equilibrium, it's necessary that the aligned expert's reputational concern μ is relatively low: if Γ_0 is the equilibrium continuation game, it's necessary that $\mu \leq \mu_0(\lambda_0^*)$, and if Γ_1 is the equilibrium continuation game, it's necessary that $\mu \leq \mu_1(\lambda_1^*)$.*

Proof. As the analysis shown above, and change λ_e with λ_e^* in $\mu_e(\lambda_e^*)$, since they are equilibrium interim beliefs. ■

Roughly speaking, this is so called "political correctness" in Morris (2001) and "bad reputation" in Ely and Valimaki (2003) and Ely, Fudenberg and Levine (2008). Since the biased expert is in favor of a particular direction, say, he always sends message $m = 1$ or always says an engine should be replaced, in order to separate from this type, the aligned expert with reputational concern endogenously generates an incentive to bias toward the opposite direction, this induces further distortion in the information transmission process and may cause the market to be shut down, as in Ely and Valimaki (2003). Apparently, given the settings as in these models and in our analysis thus far, the existence of the biased type is detrimental to the payoffs of the aligned expert and the decision maker. But then a question arise: is it possible that the existence of the biased type may actually be beneficial to the decision maker? We show in the following that, when there is an information acquisition stage before the transmission stage, the answer could be yes.

4.3. Equilibrium analysis about the information acquisition decision.

Now we are going to explore when the expert has incentive to acquire better information, say when $e = 1$ is an equilibrium decision. Define $E_\theta U_B^{NI}$ as the biased expert's expected payoff without information acquisition, and $E_\theta U_B^I$ is his expected payoff with information acquisition. Since he always sends message $m = 1$ in the continuation game, we have:

$E_\theta U_B^{NI} = \eta a_{01}^* = \eta \frac{(1-\lambda_0)+\lambda_0 p_0 x_A^*}{2(1-\lambda_0)+\lambda_0 x_A^*}$ and $E_\theta U_B^I = \eta a_{11}^* - c = \eta \frac{(1-\lambda_1)+\lambda_1 p_1 y_A^*}{2(1-\lambda_1)+\lambda_1 y_A^*} - c$

To make things interesting, we introduce the next assumption.

Assumption 1: $c < \eta \frac{\lambda(p_1 - p_0)}{2-\lambda}$.

It's easy for this assumption to satisfy if η is large enough, say, the biased expert cares much about the current action to be taken by the decision maker. The exact meaning of this assumption is, if the interim beliefs are unchanged, so $\lambda_0 = \lambda_1 = \lambda$, and the aligned expert always tells truth in the continuation game, so $x_A^* = y_A^* = 1$, then $E_\theta U_B^I > E_\theta U_B^{NI}$ and the biased expert has incentive to acquire better information with cost c.

Define $E_\theta U_A^{NI}$ and $E_\theta U_A^I$ similarly for the aligned expert. Also let $a_{em}^*(1)$ be the value of a_{em}^* when $x_A^* = y_A^* = 1$ Then,

$$E_\theta U_A^{NI} = \left\{ \begin{array}{cc} -\frac{1}{4} + \mu & \text{if } x_A^* = 0 \\ \frac{1}{2}\{-p_0(a_{00}^* - 0)^2 - (1-p_0)(a_{00}^* - 1)^2 + \mu\} & \\ +\frac{1}{2}\{-p_0(a_{01}^* - 1)^2 - (1-p_0)(a_{01}^* - 0)^2 & \text{if } x_A^* > 0 \\ +\mu[p_0\phi^*(\lambda_0 | m = 1, \theta = 1) + (1-p_0)\phi^*(\lambda_0 | m = 1, \theta = 0)]\} & \end{array} \right\}$$

Since if $0 < x_A^* < 1$, the aligned expert should be indifferent between sending message $m = 0$ and $m = 1$, so without loss of generality, we can represent his expected payoff with the payoff by sending $m = 0$, that's why we have the simple form of $-\frac{1}{4} + \mu$. For the case with $x_A^* = 1$, with probability 1/2, the aligned expert receives a signal $s = 0$, by sending $m = 0$ and inducing $a_{00}^*(1)$, his expected payoff is the term in the first $\{\cdot\}$; with probability 1/2, his signal is $s = 1$, and by sending $m = 1$, his expected payoff is the term in the second $\{\cdot\}$.

The aligned expert's expected payoff with information acquisition is:

$$E_\theta U_A^I = \left\{ \begin{array}{cc} -\frac{1}{4} + \mu - c & \text{if } y_A^* = 0 \\ \frac{1}{2}\{-p_1(a_{10}^* - 0)^2 - (1-p_1)(a_{10}^* - 1)^2 + \mu\} & \\ +\frac{1}{2}\{-p_1(a_{11}^* - 1)^2 - (1-p_1)(a_{11}^* - 0)^2 & \text{if } y_A^* > 0 \\ +\mu[p_1\phi^*(\lambda_1 | m = 1, \theta = 1) + (1-p_1)\phi^*(\lambda_1 | m = 1, \theta = 0)]\} - c & \end{array} \right\}$$

The meaning of these terms are similar as in the case without information acquisition.

Condition 3 *We restrict our attention on the set of (potential) Informative Equilibrium such that, if* Γ_0 *is on the equilibrium path, then* $x_A^* = 1$*; if* Γ_1 *is on the equilibrium path, then* $y_A^* = 1$*.*

Reasons:

Lemma 2 *In any informative equilibrium with assumption 1, the biased expert has weakly stronger incentive to acquire better information than the aligned expert, more precisely, (1)* $\alpha_B^* = \alpha_A^* = 0$ *if* $\alpha_A^* = 0$*; (2)* $\alpha_B^* = \alpha_A^* = 1$ *if* $\alpha_A^* = 1$*; (3)* $\alpha_B^* > \alpha_A^*$ *if* $\alpha_A^* \in (0,1)$*.*

Proof. For (1), suppose not. Then we have $\alpha_A^* = 0$ and $\alpha_B^* > 0$. Since $\lambda_1^* = 0 < \lambda < \lambda_0^*$ in this case, $a_{11}^* = \frac{1}{2}$ so $E_\theta U_B^I = \frac{\eta}{2} - c$. But since $a_{01}^* \geq \frac{1}{2}$ always holds, $E_\theta U_B^{NI} \geq \frac{\eta}{2} > E_\theta U_B^I$. So $\alpha_B' = 0$ is a profitable deviation. A contradiction.

For (3), suppose not. Then we have $\alpha_B^* \leq \alpha_A^* < 1$. By lemma 2, it's necessary that $y_A^* = 1$. So $a_{11}^* = \frac{1-\lambda_1^*+\lambda_1 p_1}{2-\lambda_1^*}$ and $E_\theta U_B^I = \eta\frac{1-\lambda_1^*+\lambda_1^* p_1}{2-\lambda_1^*} - c$. We also have $a_{01}^* = \frac{(1-\lambda_0^*)+\lambda_0^* p_0 x_A^*}{2(1-\lambda_0^*)+\lambda_0^* x_A^*}$ and $E_\theta U_B^{NI} = \eta\frac{(1-\lambda_0^*)+\lambda_0^* p_0 x_A^*}{2(1-\lambda_0^*)+\lambda_0^* x_A^*}$. Since $\alpha_A^* \geq \alpha_B^*$ implies $\lambda_1^* \geq \lambda \geq \lambda_0^*$, by remark 1 and assumption 1, we have $\eta\frac{1-\lambda_1^*+\lambda_1^* p_1}{2-\lambda_1^*} - c > \eta\frac{1-\lambda+\lambda p_1}{2-\lambda} - c > \eta\frac{1-\lambda+\lambda p_0}{2-\lambda} \geq \eta\frac{(1-\lambda_0^*)+\lambda_0^* p_0 x_A^*}{2(1-\lambda_0^*)+\lambda_0^* x_A^*}$, so by deviating to $\alpha_B' = 1$, the biased expert has higher expected payoff. A contradiction.

For (2), suppose not. Then we have $\alpha_B^* < \alpha_A^* = 1$. Almost repeat the proof in the above paragraph, it's straightforward to show $\alpha_B^* < \alpha_A^* = 1$ can not be part of equilibrium strategy profile. ■

It's reasonable to expect that the biased expert has incentive to mimic the aligned expert's strategy in order to avoid type-separation in the information acquisition stage, our result in this lemma is even stronger, for instance, the biased expert could be "over incentivized".

Definition 2 *An informative equilibrium (IE) is a most-informative equilibrium (MIE), if for the equilibrium strategy profile* $\sigma^* = \{\sigma_A^*, \sigma_B^*, (a_{e0}^*, a_{e1}^*)\}$*, there exists no equilibrium strategy profile* $\sigma' = \{\sigma'_A, \sigma'_B, (a'_{e0}, a'_{e1})\}$ *such that* $\sigma'_A \geq \sigma_A^*$*,* $\sigma'_B \geq \sigma_B^*$*,* $a'_{e1} \geq a_{e1}^*$*,* $a'_{e0} \leq a_{e0}^*$ *and the inequality is strict for at least one of them.*

This definition simply says for an informative equilibrium with strategy profile σ^*, if it's impossible to improve upon the information quality either by information acquisition or by information transmission on the equilibrium path, then this equilibrium has reached it's most informativeness. Since we are focusing on the possible IE set in which the biased expert always sends message $m = 1$, by lemma 3 and the decision maker's optimal action rule, what is relevant in this definition is the aligned expert's equilibrium strategy σ_A^*.

Lemma 3 *For any informative equilibrium with strategy profile σ^* such that $0 < \alpha_A^* < 1$, there exists a most-informative equilibrium with strategy profile σ' such that $\alpha_A' = 1$. Equilibrium strategy σ' payoff improves upon σ^* for each player.*

Proof. First we construct a new strategy profile σ' such that $\alpha_A' = \alpha_B' = 1$, $x_B' = y_B' = 1$, $w_B' = z_B' = 0$, $w_A' = y_A' = z_A' = 1$, $x_A' = 1$ if $\mu \leq \mu_0(\lambda)$ and $x_A' = 0$ if $\mu > \mu_0(\lambda)$. a_{em}' follows the decision maker's optimal action rule given σ_A' and $\sigma_{B.}'$ Finally, the belief updating system $\{\lambda_e', \phi'(\lambda_e)\}$ follows Bayes's rule whenever possible.

Given an informative equilibrium with $0 < \alpha_A^* < 1$, by lemma 2 and lemma 3, we have $y_A^* = 1$ and $\lambda_1^* < \lambda < \lambda_0^*$. As $y_A^* = 1$ implies $\mu \leq \mu_1(\lambda_1^*)$ by proposition 1 and $\lambda_1^* < \lambda$ implies $\mu_1(\lambda_1^*) < \mu_1(\lambda)$, we have $\mu < \mu_1(\lambda)$, so $y_A' = 1$ is equilibrium strategy in the new continuation game Γ_1.

On the other hand, $0 < \alpha_A^* < 1$ implies $E_\theta U_A^I(\sigma^*) = E_\theta U_A^{NI}(\sigma^*)$, and given $y_A^* = y_A' = 1$, $E_\theta U_A^I$ increases when λ_1^* increases to λ and $E_\theta U_A^{NI}$ weakly decreases when λ_0^* decreases to λ, so $E_\theta U_A^I(\sigma') > E_\theta U_A^I(\sigma^*) = E_\theta U_A^{NI}(\sigma^*) > E_\theta U_A^{NI}(\sigma')$, $\alpha_A' = 1$ is actually part of the aligned expert's equilibrium strategy for the new profile.

For the biased expert, $\alpha_B^* > \alpha_A^* > 0$ implies $E_\theta U_B^I(\sigma^*) \geq E_\theta U_B^{NI}(\sigma^*)$, similarly, as $E_\theta U_B^I$ increases when λ_1^* increases to λ and $E_\theta U_B^{NI}$ weakly decreases when λ_0^* decreases to λ, we have $E_\theta U_B^I(\sigma') > E_\theta U_B^I(\sigma^*) = E_\theta U_B^{NI}(\sigma^*) > E_\theta U_B^{NI}(\sigma')$, so $\alpha_B' = 1$ is part of the biased expert's equilibrium strategy for the new profile.

Now it is easy to check that the new strategy profile and belief updating system consist an informative equilibrium. Since this equilibrium has maximized the information acquisition and truthful information transmission, it is a most-informative equilibrium by checking the definition.

Finally, in the proof above we have shown $E_\theta U_A^I(\sigma') > E_\theta U_A^I(\sigma^*)$ and $E_\theta U_B^I(\sigma') > E_\theta U_B^I(\sigma^*)$, so the new equilibrium payoffs improve upon the old one's for both types of experts. Also as a_{11}' is larger than both a_{11}^* and a_{01}^*, and a_{10}' is smaller than both a_{00}^* and a_{10}^*

the decision maker's actions become more accurate with respecting to the true state, and it's easy to show $E_\theta\Pi(\sigma') > E_\theta\Pi(\sigma^*)$. So the new equilibrium strategy profile σ' payoff improves upon σ^*. ■

Lemma 4 *For any most-informative equilibrium, the equilibrium outcome is pareto efficient.*

Proof. By the results shown before, if this game has most-informative equilibria, then the equilibrium outcome is unique: either has the form $\alpha_B^* = \alpha_A^* = 0$ and in the equilibrium continuation game Γ_0, the aligned expert truthfully reveals his information, but the biased always sends $m = 1$; or has the form $\alpha_B^* = \alpha_A^* = 1$ and in the equilibrium continuation game Γ_1, the aligned expert reveals information truthfully and the biased expert sends $m = 1$. By this uniqueness, it's straightforward to conclude that the equilibrium outcome is pareto efficient. ■

These lemmas show for our seeking for potential informative equilibria, it may be more interesting to focus on the equilibria with most informativeness, since these equilibria maximize information acquisition and transmission, and have higher payoff efficiency. There is another advantage with this lemma: since all the potential most-informative equilibria either have $\alpha_B^* = \alpha_A^* = 0$ or $\alpha_B^* = \alpha_A^* = 1$, there is no type separation by the information acquisition decision e, so the thresholds in the continuation games Γ_0 and Γ_1 has the property such that $\mu_0(\lambda_0^*) = \mu_0(\lambda) < \mu_1(\lambda) = \mu_1(\lambda_1^*)$.

Now we are going to see when there exists most-informative equilibrium with information acquisition. Assumption 2 is introduced here. To save notations, we define some terms first. Let $W = (p_1 - p_0)(p_1 + p_0 - 1)\frac{2+\lambda}{2-\lambda}$, $W_1 = E_\theta\phi^*(\lambda|e = 1, m = 1) = \frac{p_1^2\lambda}{p_1\lambda+1-\lambda} + \frac{(1-p_1)^2\lambda}{(1-p_1)\lambda+1-\lambda}$, and $W_0 = E_\theta\phi^*(\lambda|e = 0, m = 1) = \frac{p_0^2\lambda}{p_0\lambda+1-\lambda} + \frac{(1-p_0)^2\lambda}{(1-p_0)\lambda+1-\lambda}$. Apparently, $W > 0$, $W_1 > W_0 > 0$ by remark 2.

Assumption 2: $W < c < W + \frac{\mu_0(\lambda)}{2}[W_1 - W_0]$.

We describe our first main result here.

Proposition 2 *With assumptions* 1 *and* 2*, there exists most-informative equilibrium with information acquisition if and only if* $\mu \in [\mu_0^*(\lambda), \mu_1^*(\lambda)]$ *such that* $\mu_0^*(\lambda) \in (0, \mu_0(\lambda))$ *and* $\mu_1^*(\lambda) \in (\mu_0(\lambda), \mu_1(\lambda))$.

Proof. First, consider the situation such that $\mu = 0$, then most informativeness implies $x_i^* = y_i^* = 1$ for $i \in \{A, B\}$, since no one has incentive to distort the information when $s = 1$.

Then, $E_\theta U_A^I - E_\theta U_A^{NI} < 0$ by assumption $W < c$. The aligned expert has no incentive to acquire better information, so $\alpha_A^* = 0$. By lemma 3, $\alpha_B^* = 0$.

Second, consider the situation such that $\mu = \mu_0(\lambda)$. Again, most informativeness implies $x_i^* = y_i^* = 1$ for $i \in \{A, B\}$, since μ satisfies $\mu \leq \mu_0(\lambda)$ and $\mu < \mu_1(\lambda)$. Here, $E_\theta U_A^I - E_\theta U_A^{NI} > 0$ by assumption $c < W + \frac{\mu_0(\lambda)}{2}[W_1 - W_0]$. This implies the aligned expert acquires better information in this case, so $\alpha_A^* = 1$ and $\alpha_B^* = 1$ by lemma 3.

Third, consider the situation such that $\mu = \mu_1(\lambda)$. Here, most informativeness implies $y_A^* = 1$ and $x_A^* = 0$, since $\mu_0(\lambda) < \mu \leq \mu_1(\lambda)$. Since the threshold $\mu_1(\lambda)$ is solved by the aligned expert's indifference between send $m = 1$ with $y_A = 1$ and $y_A = 0$, we have $E_\theta U_A^I = -\frac{1}{4} + \mu - c < -\frac{1}{4} + \mu = E_\theta U_A^{NI}$. Again, the aligned expert has no incentive to acquire better information, and $\alpha_B^* = \alpha_A^* = 0$.

Now as $E_\theta U_A^I - E_\theta U_A^{NI}$ is continuous with μ, and is increasing with μ for $\mu \leq \mu_0(\lambda)$ and is decreasing with μ for $\mu \geq \mu_0(\lambda)$ there exists $\mu_0^*(\lambda) \in (0, \mu_0(\lambda))$ and $\mu_1^*(\lambda) \in (\mu_0(\lambda), \mu_1(\lambda))$ such that $E_\theta U_A^I - E_\theta U_A^{NI} = 0$ if $\mu = \mu_0^*(\lambda)$ or $\mu = \mu_1^*(\lambda)$. This shows if and only if $\mu \in [\mu_0^*(\lambda), \mu_1^*(\lambda)]$, we have most-informative equilibrium with information acquisition, say $\alpha_A^* = \alpha_B^* = 1$. This ends the proof. ■

With proper parameter ranges, there is a non-monotonic relationship between the aligned expert's reputational concern and his incentive to acquire better information. When his reputational concern is relatively low, the information acquisition cost outweighs the benefit from improved information accuracy and optimal actions to be taken, thus his incentive is limited. On the other hand, when his reputational concern is relatively high, he is ready to send the same message $m = 0$ regardless of his information, so more accurate information is worthless to him. Only if his concern is moderate, truthful revealing the better information can cover the information cost.

Fix the relevant parameters, figure 1 shown below provides a full description about the aligned expert's information acquisition and transmission decisions without the restriction imposed by assumption 2. If the cost is so high such that $c > W + \frac{\mu_0(\lambda)}{2}[W_1 - W_0]$, then none type of the experts has incentive to acquire better information, although whether the equilibrium is informative or not depends on the aligned expert's reputational concern. These are represents by regions R_1 and R_2 respectively. If the cost is relatively low such that $c < W$, then the aligned expert acquires better information and truthfully reveals it if and only if his reputational concern is not too high, and this is partially described by region R_3. For our interest, we restrict our attention on the moderate cost range, say $W < c < W + \frac{\mu_0(\lambda)}{2}[W_1 - W_0]$.

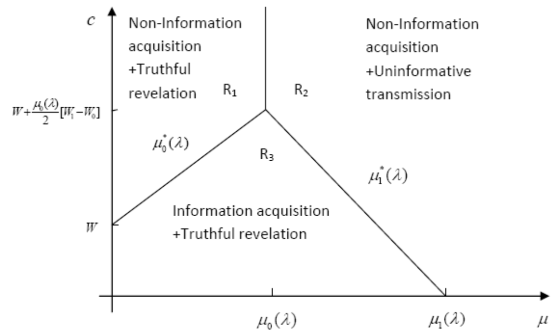

Figure 1. Information acquisition and transmission decisions

4.4. Welfare analysis.

When the existence of the biased type may turn out to be beneficial to the decision maker? Proposition 1 shows there are two kinds of information distortion in the transmission stage given the presence of the aligned type: one is exorgenously induced by the biased type, and the other is endogenously induced by the aligned type if his reputational concern is high. These information distortion could be seen as the cost side. Proposition 2 shows there may be better information acquired in the acquisition stage if the aligned expert's reputational concern is moderate. This information acquisition could be seen as the benefit side. Apparently, the answer would depend on the trade-off between the cost side and the benefit side.

Let $\lambda^* = [1 + 4(p_0^2 - p_0)]/[1 + 2(p_0^2 - p_0 + p_1^2 - p_1)]$, we have $0 < \lambda^* < 1$.

Proposition 3 *With assumptions* 1 *and* 2*, compared with the non-existence of the biased expert, the decision maker's (possible) most-informative equilibrium payoff is larger with the existence of the biased type if and only if* $\lambda \in (\lambda^*, 1)$ *and* $\mu \in [\mu_0^*(\lambda), \mu_1^*(\lambda)]$.

Proof. Without the existence of the biased type, say $\lambda = 1$, the decision maker's most-informative equilibrium payoff is $E_\theta \Pi = p_0^2 - p_0$, which involves the aligned expert's perfect information revelation but no information acquisition.

Given any $\lambda \in (0, 1)$, when $\mu \notin [\mu_0^*(\lambda), \mu_1^*(\lambda)]$, by proposition 2, there is no most-informative equilibrium with information acquisition. So $E_\theta \Pi = -\frac{1-\lambda}{2(2-\lambda)} + \frac{\lambda}{2-\lambda}(p_0^2 - p_0)$ if $\mu < \mu_0^*(\lambda)$ and $E_\theta \Pi = -\frac{1}{4}$if $\mu > \mu_0^*(\lambda)$ for any possible most-informative equilibrium. It's straightforward to check that none of these values is larger than $p_0^2 - p_0$ for $\lambda \in (0,1)$ and $p_0 \in (1/2,1)$. So $\mu \in [\mu_0^*(\lambda),\mu_1^*(\lambda)]$ is necessary.

Now consider $\lambda \in (0,1)$ with $\mu \in [\mu_0^*(\lambda),\mu_1^*(\lambda)]$. By definition 2 and proposition 2, any most-informative equilibrium involves information acquisition. Then $E_\theta \Pi = -\frac{1-\lambda}{2(2-\lambda)} + \frac{\lambda}{2-\lambda}(p_1^2 - p_1)$ and increases with λ. Since λ^* solves $-\frac{1-\lambda}{2(2-\lambda)} + \frac{\lambda}{2-\lambda}(p_1^2 - p_1) = p_0^2 - p_0$, it's easy to check that if and only if $\lambda \in (\lambda^*, 1)$ with $\mu \in [\mu_0^*(\lambda),\mu_1^*(\lambda)]$, the decision maker's payoff is larger with the existence of the biased type. ■

When the expert's preference is certain to be aligned, he has no attempt to distort the information he has, since there is no additional reputational gain. But because of the information cost, his incentive to acquire better information is restricted. Although the existence of the biased type introduces potential information distortion, with moderate reputational concern, the aligned expert's information acquisition incentive now is generated, complemented by truthful revelation. If the initial probability to be an aligned expert is higher enough, the gain from the aligned expert's information acquisition outweighs the biased expert's information distortion, results the decision maker with higher expected payoff.

The argument can be modified to analyze the effect on social welfare, if a social welfare function is properly adopted. For instance, suppose weights β_{DM} β_A β_B are attached to the decision maker, to the aligned expert and to the biased expert accordingly. Then, when λ is sufficiently high, the social welfare may be improved upon with the existence of biased type of experts.

5. Equilibrium with unobservable information acquisition.

Although in many situations it's possible for the decision maker to observe or infer the expert's information acquisition decision, this may be quite difficult or impossible in many other situations. This section derives the possible effects that reputational concern has on the expert's decisions with the assumption that information acquisition is unobservable.

The type $i \in \{A, B\}$ expert's strategy $\sigma_i = \{\alpha_i, (x_i, w_i), (y_i, z_i)\}$ is unchanged; but the decision maker's action to take now only depends on the message m she receives, represented by a_m. Also, there is no interim belief updating. Since the definitions of PBE, IE, MIE are almost the same as in the last section, we omit these definitions here. For our interest, we still focus on the possible existence of most-informative equilibrium.

Since the biased expert prefers the action to be taken by the decision maker as larger as possible, if a most-informative equilibrium exists with $a_m^* > a_{m'}^*$ for $m \neq m'$, then on the equilibrium path he always sends message m regardless of his information. With slightly modification, lemma 1 holds for the situation here. Moreover, if the expert's messages in the equilibrium are "reversely understood", say $a_0^* > a_1^*$, then there exists a payoff equivalent equilibrium such that the messages are "obversely understood", say $a_0' < a_1'$. This follows the same argument as in the last section. Without loss of generality, we identify the equilibria in which the biased expert always sends message $m = 1$ and the aligned expert tells truth when his signal is $s = 0$.

Given the strategies with α_A, α_B, x_A and y_A, we have the decision maker's optimal actions as:

$a_0^* = \Pr(\theta = 1|m = 0) = \frac{\alpha_A(1-p_1 y_A)+(1-\alpha_A)(1-p_0 x_A)}{\alpha_A(2-y_A)+(1-\alpha_A)(2-x_A)}$

$a_1^* = \Pr(\theta = 1|m = 1) = \frac{1-\lambda+\lambda\{\alpha_A p_1 y_A+(1-\alpha_A)p_0 x_A\}}{2-2\lambda+\lambda\{\alpha_A y_A+(1-\alpha_A)x_A\}}$

And the decision maker's posterior beliefs are:

$\phi^*(\lambda|m = 0, \theta = 0) = \phi^*(\lambda|m = 0, \theta = 1) = 1$

$\phi^*(\lambda|m = 1, \theta = 0) = \frac{\lambda\{\alpha_A(1-p_1)y_A+(1-\alpha_A)(1-p_0)x_A\}}{\lambda\{\alpha_A(1-p_1)y_A+(1-\alpha_A)(1-p_0)x_A\}+1-\lambda}$

$\phi^*(\lambda|m = 1, \theta = 1) = \frac{\lambda\{\alpha_A p_1 y_A+(1-\alpha_A)p_0 x_A\}}{\lambda\{\alpha_A p_1 y_A+(1-\alpha_A)p_0 x_A\}+1-\lambda}$

Similarly we have $1 - p_1 \leq a_0^* \leq 1/2 \leq a_1^* \leq p_1$ and $\phi^*(\lambda|m = 1, \theta = 0) \leq \phi^*(\lambda|m = 1, \theta = 1) \leq \lambda$. Because of the biased expert's push of his agenda, sending message $m = 1$ pools the aligned expert with the biased expert and results him with a reputational loss. On the other hand, sending message $m = 0$ perfectly separates the experts' types and brings the aligned expert with a reputational gain.

How the expert's incentive to acquire better information and to convey information would be changed? We derive a useful lemma here.

Lemma 5 *For any informative equilibrium, the biased expert has no incentive to acquire better information, so $\alpha_B^* = 0$.*

Proof. Suppose not, then there exists an informative equilibrium such that $\alpha_B^* > 0$. Fix the aligned expert's equilibrium strategy σ_A^* and the decision maker's optimal action rule a_m^*, without information acquisition, the biased expert's expected payoff is $E_\theta U_B^{NI} = \eta a_1^* = \eta\frac{1-\lambda+\lambda\{\alpha_A^* p_1 y_A^*+(1-\alpha_A^*)p_0 x_A^*\}}{2-2\lambda+\lambda\{\alpha_A^* y_A^*+(1-\alpha_A^*)x_A^*\}}$, with information acquisition, his expected payoff is $E_\theta U_B^I = \eta a_1^* - c = \eta\frac{1-\lambda+\lambda\{\alpha_A^* p_1 y_A^*+(1-\alpha_A^*)p_0 x_A^*\}}{2-2\lambda+\lambda\{\alpha_A^* y_A^*+(1-\alpha_A^*)x_A^*\}} - c$. Apparently, $E_\theta U_B^{NI} > E_\theta U_B^I$. So $\alpha_B' = 0$ is a profitable deviation. ■

The intuition here is, regardless of the biased expert's information acquisition decision, his message is uninformative, so it will be filtered out by the decision maker. This could be seen from the decision maker's optimal action a_m^*, such that the accuracy p of the signal only matters with the combination of the aligned expert's strategy elements α_A, x_A and y_A. From the perspective of the biased expert, the only role of his acquisition decision is to avoid any type-separation, and this results his behaviors quite differently given the observability of the acquisition decision: lemma 3 shows that he has the same incentive to acquire better information as the aligned expert in any possible most-informative equilibrium if the acquisition decision is observable, but here, he never acquires since it's optimal for him to save the effort cost c.

We have derived the biased expert's strategy in any possible most-informative equilibrium, say he involves no information acquisition and always sends message $m = 1$. How would the aligned expert's strategy be changed if acquisition decision is unobservable? Surprisingly, the following proposition shows observability of information acquisition has no effect on the aligned expert's equilibrium strategy.

Proposition 4 *With assumptions* 1 *and* 2*, observability of information acquisition has no effect on the aligned expert's equilibrium strategy, so there exists most-informative equilibrium with information acquisition if and only if* $\mu \in [\mu_0^*(\lambda),\mu_1^*(\lambda)]$.

Proof. For the aligned expert, given the biased expert's strategy and the decision maker's optimal action rule, without information acquisition, his payoff is:

$$E_\theta U_A^{NI} = \left\{ \begin{array}{cc} -\frac{1}{4} + \mu & \text{if } x_A^* < 1 \\ \frac{1}{2}\{-p_0(a_0^*(1) - 0)^2 - (1 - p_0)(a_0^*(1) - 1)^2 + \mu\} & \\ +\frac{1}{2}\{-p_0(a_1^*(1) - 1)^2 - (1 - p_0)(a_1^*(1) - 0)^2 & \text{if } x_A^* = 1 \\ +\mu[p_0\phi^*(\lambda|m = 1, \theta = 1) + (1 - p_0)\phi^*(\lambda|m = 1, \theta = 0)]\} & \end{array} \right\}$$

With information acquisition, his payoff is:

$$E_\theta U_A^{I} = \left\{ \begin{array}{cc} -\frac{1}{4} + \mu - c & \text{if } y_A^* < 1 \\ \frac{1}{2}\{-p_1(a_0^*(1) - 0)^2 - (1 - p_1)(a_0^*(1) - 1)^2 + \mu\} & \\ +\frac{1}{2}\{-p_1(a_1^*(1) - 1)^2 - (1 - p_1)(a_1^*(1) - 0)^2 & \text{if } y_A^* = 1 \\ +\mu[p_1\phi^*(\lambda|m = 1, \theta = 1) + (1 - p_1)\phi^*(\lambda|m = 1, \theta = 0)]\} - c & \end{array} \right\}$$

There formulas are almost the same as in the last section when information acquisition is observable, except the actions are only based on message m and there is no interim belief updating. By comparing the payoffs $E_\theta U_A^{NI}$ and $E_\theta U_A^{I}$ with various ranges of reputational concern μ, as the proof in proposition 2, it's straightforward see there exists most-informative equilibrium with information acquisition if and only if $\mu \in [\mu_0^*(\lambda),\mu_1^*(\lambda)]$ such that $\mu_0^*(\lambda) \in (0, \mu_0(\lambda))$ and $\mu_1^*(\lambda) \in (\mu_0(\lambda), \mu_1(\lambda))$. ■

The effect of the aligned expert's acquisition decision on the decision maker's optimal actions could be decomposed as two parts: direct effect such that the accuracy p of information enters the optimal action rule, and indirect effect such that interim belief may be changed based on the acquisition decision and this belief also enters the optimal action rule. But by the results we already have, in equilibrium this interim belief is unchanged whether acquisition decision is observable or not, so only the direct effect plays a role. Then the aligned expert's incentives to acquire better information and truthfully transmit it are the same in both situations, and they are determined solely by the importance of his reputational concern μ. This is the intuition lies behind proposition 4.

With the characterization of the players' equilibrium strategies, it's easy to see whether their payoffs are affected or not. We summarize the results in the following corollary.

Corollary 1 *The aligned expert and the decision maker's payoffs are unchanged whether the acquisition decision is observable or not; when the acquisition decision is unobservable, the biased expert's payoff and social welfare weakly increase because of the possible saving of acquisition cost.*

To some extent, the analysis in this section could be seen as the robustness examination of the last section, and it's reasonable to expect that if the decision maker observes the expert's acquisition decision with some positive probability, the aligned expert's acquisition incentive would be unaffected. Regarding to the acquisition cost, since the biased expert's acquisition decision has no essential influence on the decision maker's optimal actions, it's better to save this cost from the perspective of social welfare. Alternatively, if the decision maker has to share the acquisition cost, it becomes the dominant strategy for the decision maker to restrict her observability on the acquisition decision, or say, she would better keep "arm's length relationship" from the expert and commit that she would not monitor his acquisition activities.

6. Delegation.

Instead of eliciting relevant information from the experts and take actions by themselves, in many organizations the decision makers may delegate their decision rights to the experts. For example, government officials make regular decisions on behalf of the public, and company managers have significant discretion that is granted by shareholders. Moreover, in order to further limit the agency costs, in many situations it's possible for the decision makers to optimally restrict the experts' decision sets, as frequently seen in reality such that regulated firms are permitted to set prices only below some price caps. What are the pros and cons of delegating decision rights to the expert in our setup with reputational concern and information acquisition? How should the decision maker optimally design the delegation set? Our analysis in this section captures these issues.

Two scenarios are considered: unrestricted delegation such that the delegation set is $S = [0, 1]$, so the expert can take any action in the original action space; restricted delegation such that the delegation set S^* is optimally designed, so the expert's discretion is limited. Being consistent with the analysis shown in the sections before, we also look for Perfect Bayesian Equilibrium here. For the expert of type $i \in \{A, B\}$, his strategy $\sigma_i = \{\alpha_i, \{a_{ies}\}_{e,s\in\{0,1\}}\}$ consists of two parts: information acquisition decision α_i, and action a_{ies} to take given his decision e and signal s. For the decision maker, since we suppose the delegation structure is predetermined, she takes no action during this game, so we only have to derive her posterior belief updating $\phi(\lambda)$ based on her observed information. The definition of equilibrium is standard and consists of equilibrium strategy profile σ^* and belief updating $\phi^*(\lambda)$. At first sight the aligned expert should have stronger incentive to acquire better information under delegation since now he has full charge of how to use his information most efficiently. As our analysis unfolds, it becomes clear that the actual situation is opposite.

Scenario 1: unrestricted delegation.

This scenario serves as the benchmark for our analysis with delegation. For the sake of a full description, we relax assumption 2 and consider all the positive values of acquisition cost c in these subsections. The following lemma identifies the expert's information acquisition decision and optimal actions to take.

Lemma 6 *In any equilibrium, the biased expert has no incentive to acquire better information, and he takes action* 1 *regardless of his signal; for* $c \leq (p_1 - p_0)(p_1 + p_0 - 1)$*, the aligned expert acquires better information and he takes action* p_1 $(1-p_1)$ *if his signal is* $s = 1$ $(s = 0)$*, for* $c > (p_1 - p_0)(p_1 + p_0 - 1)$*, the aligned expert does not acquire better information and he takes action* p_0 $(1 - p_0)$ *if his signal is* $s = 1$ $(s = 0)$*.*

Proof. Given the biased expert's payoff function $U_B = \eta a - ec$, strategy $\alpha_B = 0$ and $a_{B0s} = 1$

is his strictly dominant strategy for any $s \in \{0, 1\}$, so this is his equilibrium strategy. Now for the aligned expert, given signal $s = 1$ ($s = 0$), his optimal current action to take is p_1 ($1 - p_1$) if his acquire better information, and is p_0 ($1 - p_0$) if he does not. Since all these actions differ from the biased expert's action 1, there is full type-separation solely based on these actions and reputational concern is irrelevant to the information acquisition decision. For $c \leq (p_1 - p_0)(p_1 + p_0 - 1)$, $E_\theta U_A^{NI} \leq E_\theta U_A^I$; for $c > (p_1 - p_0)(p_1 + p_0 - 1)$, $E_\theta U_A^{NI} > E_\theta U_A^I$. This proves the argument in the lemma. ■

Since the biased expert is myopic and has no concern about the state of the world, it's intuitive to expect that he takes the largest action in the delegation set, and this action is 1 under this unrestricted scenario. But action 1 never is the aligned expert's optimal current action, this implies there is full type-separation based on the actions taken by the expert. So reputational concern has no effect on the aligned expert's information acquisition decision, what is relevant to his acquisition decision is the trade off between the gain from the more accurate actions and the acquisition cost.

Scenario 2: restricted delegation.

Now consider the scenario that the decision maker can optimally design the delegation set. As the biased expert still has attempt to take the largest permissible action, the decision maker has to balance two opposite effects induced by her restricted delegation: the gain from limiting the biased expert's discretion and the loss from limiting the aligned expert's discretion. Define this optimal set as S^*. The following lemma describes the structure of S^* and the expert's optimal decisions.

Lemma 7 *If $c \leq (p_1 - p_0)(p_1 + p_0 - 1)\frac{2+\lambda}{2-\lambda}$, then $S^* = [0, \frac{1-\lambda+\lambda p_1}{2-\lambda}]$ and in equilibrium $\alpha_B^* = 0$ and $a_{B0s}^* = \frac{1-\lambda+\lambda p_1}{2-\lambda}$ for $s \in \{0, 1\}$, $\alpha_A^* = 1$ and $a_{A10}^* = 1 - p_1$, $a_{A11}^* = \frac{1-\lambda+\lambda p_1}{2-\lambda}$. If $c > (p_1 - p_0)(p_1 + p_0 - 1)\frac{2+\lambda}{2-\lambda}$, then $S^* = [0, \frac{1-\lambda+\lambda p_0}{2-\lambda}]$ and in equilibrium $\alpha_A^* = \alpha_B^* = 0$, $a_{B0s}^* = \frac{1-\lambda+\lambda p_1}{2-\lambda}$ for $s \in \{0, 1\}$, and $a_{A00}^* = 1 - p_0$, $a_{A01}^* = \frac{1-\lambda+\lambda p_0}{2-\lambda}$.*

Proof. Given the delegation set S^*,let a^* defines the largest element in this set. Again it's a strictly dominant strategy for the biased expert to choose $\alpha_B^* = 0$ and $a_{B0s}^* = a^*$. For the aligned expert, given his acquisition decision e and signal s, if his optimal action a in S^* differs from a^*, then a is chosen and there is full type-separation; if $a = a^*$, then he can choose $a - \epsilon$ such that ϵ is arbitrarily small (suppose $a - \epsilon$ is in this set, this will be proved in the next paragraph), then in equilibrium the limit a^* is chosen and again there is full type-separation. So reputational concern has no effect on the aligned expert's information acquisition decision.

Since the biased expert always takes the largest element a^* in S^*, it's weakly dominant for the decision maker to have any $a \in [0, a^*)$ in the set S^*, so without loss of generality, S^* takes the form $S^* = [0, a^*]$. The decision maker's reduced problem is to determine what is the optimal a^*. Apparently, a^* is the solution to the following problem:

$$\max_a \lambda\{\tfrac{1}{2}(p^2 - p) + \tfrac{1}{2}[-p(a-1)^2 - (1-p)a^2]\} + (1-\lambda)\{-\tfrac{1}{2}(a-1)^2 - \tfrac{1}{2}a^2\}$$

First order condition shows that $a^* = (1 - \lambda + \lambda p)/(2 - \lambda)$. So, if the decision maker can expect that the aligned expert acquires better information, then a^* should be $(1 - \lambda + \lambda p_1)/(2 - \lambda)$, otherwise it is $(1 - \lambda + \lambda p_0)/(2 - \lambda)$.

Now it's straightforward to show that if $c \leq (p_1 - p_0)(p_1 + p_0 - 1)\frac{2+\lambda}{2-\lambda}$, we have $E_\theta U_A^{NI} \leq E_\theta U_A^I$ for the aligned expert given $S^* = [0, \frac{1-\lambda+\lambda p_1}{2-\lambda}]$, this justifies $a^* = (1-\lambda+\lambda p_1)/(2-\lambda)$ is actually optimal, and $\alpha_A^* = 1$, $a_{A10}^* = 1 - p_1$, $a_{A11}^* = \frac{1-\lambda+\lambda p_1}{2-\lambda}$. If $c > (p_1 - p_0)(p_1 + p_0 - 1)\frac{2+\lambda}{2-\lambda}$, we have $E_\theta U_A^{NI} > E_\theta U_A^I$ for the aligned expert given $S^* = [0, \frac{1-\lambda+\lambda p_1}{2-\lambda}]$, this justifies $a^* = (1 - \lambda + \lambda p_0)/(2 - \lambda)$ is actually optimal, and $\alpha_A^* = 0$, $a_{A00}^* = 1 - p_0$, $a_{A01}^* = \frac{1-\lambda+\lambda p_0}{2-\lambda}$. This finishes the proof. ■

Several remarks can be drawn from these two lemmas. We show that the optimal delegation set is an interval[1], this is consistent with the theoretical analysis such as Holmstrom (1977, 1984) and Alonso and Matouschek (2008), and matches the widespread use of price cap regulation in practice. Besides, our result is similar to Szalay (2005), say, it's desirable for the decision maker to restrict the expert's decision rights in order to enhance his information acquisition incentive. This could be seen from the different thresholds of acquisition cost in the lemmas. Under unrestricted delegation, the aligned expert acquires better information if and only if $c \leq (p_1 - p_0)(p_1 + p_0 - 1)$, but the condition is $c \leq (p_1 - p_0)(p_1 + p_0 - 1)\frac{2+\lambda}{2-\lambda}$ under restricted delegation. Apparently there is positive measure of acquisition cost such that the aligned expert's acquisition incentive exists only because his delegation is restricted.

While Aghion and Tirole (1997) show that delegation ultimately increases the expert or agent's information acquisition incentive, the relationship in our setup is more ambiguous. Roughly, the reason is, under delegation the aligned expert's reputational concern has no influence on his acquisition decision, but under non-delegation his reputational concern may

[1] Although there are only two elements matter in this optimal delegation set, we argued in the proof that it's a weakly dominant strategy for the decision maker to include all the elements between 0 and the cap. Alternatively, if the expert's signal space is generalized to have more elements or to be continuous, the optimal delegation set would actually be an interval.

facilitate or may destroy his acquisition incentive. So the net effect depends. Especially, if cost c is in the range such that assumption 2 holds and $\mu \in [\mu_0^*(\lambda),\mu_1^*(\lambda)]$, then the aligned expert acquires better information under non-delegation, but he does not under delegation, whether his delegation set is restricted or not. This is because in the former situation the gain from reputation building further helps him to cover part of the acquisition cost, which relaxes the constraint on his acquisition decision.

Finally, we identify the effects of delegation on the decision maker's payoff. Compared with unrestricted delegation, it's not surprising to see that restricted delegation increases the decision maker's payoff. Essentially, there are two main advantages that restricted delegation possesses: the first one is that the biased expert's action distortion is limited; the second one is that the aligned expert has stronger motivation to acquire better information and uses it efficiently. For our purpose, we have particular interest in comparing the payoffs with and without delegation. The following proposition derives this.

Proposition 5 *If most-informative equilibrium exists under non-delegation (whether with or without information acquisition in equilibrium), the decision maker prefers non-delegation to delegation (regardless whether the delegation is restricted or not) for any $\lambda \in (0,1)$;*

If most-informative equilibrium does not exist under non-delegation, the decision maker prefers restricted delegation to non-delegation for any $\lambda \in (0,1)$, and prefers unrestricted delegation to non-delegation if and only if λ is high enough.

Proof. Under non-delegation, let $E_\theta\Pi^I$, $E_\theta\Pi^{NI}$ and $E_\theta\Pi^N$ be the decision maker's payoffs in the MIE with information acquisition, in the MIE without information acquisition, and in the non-informative equilibrium respectively. Under delegation, let $E_\theta\Pi^{UD-NI}$ and $E_\theta\Pi^{UD-I}$ be the decision maker's payoffs under unrestricted delegation without and with information acquisition in equilibrium, and $E_\theta\Pi^{RD-NI}$ and $E_\theta\Pi^{RD-I}$ be her payoffs under restricted delegation without and with information acquisition in equilibrium. Then, we have $E_\theta\Pi^I = -\frac{1-\lambda}{2(2-\lambda)} + \frac{\lambda}{2-\lambda}(p_1^2 - p_1)$, $E_\theta\Pi^{NI} = -\frac{1-\lambda}{2(2-\lambda)} + \frac{\lambda}{2-\lambda}(p_0^2 - p_0)$ and $E_\theta\Pi^N = -\frac{1}{4}$, $E_\theta\Pi^{UD-NI} = \lambda(p_0^2 - p_0) - \frac{1-\lambda}{2}$, $E_\theta\Pi^{UD-I} = \lambda(p_1^2 - p_1) - \frac{1-\lambda}{2}$, $E_\theta\Pi^{RD-NI} = -\frac{1-\lambda}{2(2-\lambda)} + \frac{\lambda}{2-\lambda}(p_0^2 - p_0)$ and $E_\theta\Pi^{RD-I} = -\frac{1-\lambda}{2(2-\lambda)} + \frac{\lambda}{2-\lambda}(p_1^2 - p_1)$. Direct compare shows $E_\theta\Pi^{RD-NI} > E_\theta\Pi^{UD-NI}$ and $E_\theta\Pi^{RD-I} > E_\theta\Pi^{UD-I}$ for any $\lambda \in (0,1)$, say, restricted delegation dominates unrestricted delegation.

Suppose there exists MIE under non-delegation. If $c \leq (p_1 - p_0)(p_1 + p_0 - 1)\frac{2+\lambda}{2-\lambda}$, then there is information acquisition both under non-delegation and under restricted delegation, so it's relevant to compare $E_\theta \Pi^I$ and $E_\theta \Pi^{RD-I}$. $E_\theta \Pi^I \geq E_\theta \Pi^{RD-I}$ implies the decision maker weakly prefers non-delegation. If $c > (p_1 - p_0)(p_1 + p_0 - 1)\frac{2+\lambda}{2-\lambda}$, then there is no information acquisition under restricted delegation, but may or may not have information acquisition under non-delegation, since $E_\theta \Pi^I > E_\theta \Pi^{NI} \geq E_\theta \Pi^{RD-NI}$, again the decision maker prefers non-delegation. This finished the proof for the first part in the proposition.

Now suppose there exists no MIE under non-delegation. Since $E_\theta \Pi^{RD-I} > E_\theta \Pi^{RD-NI} > E_\theta \Pi^N$, the decision maker prefers restricted delegation to non-delegation. Also, $E_\theta \Pi^{UD-NI} \geq E_\theta \Pi^N$ if and only if $\lambda \in [1/2 + 4(p_0^2 - p_0), 1)$ and $E_\theta \Pi^{UD-I} \geq E_\theta \Pi^N$ if and only if $\lambda \in [1/2 + 4(p_1^2 - p_1), 1)$, so the decision maker prefers unrestricted delegation to non-delegation if and only if λ is high enough. ■

The decision maker trades off two dimensions of gains and losses when she decides whether delegation should be employed or not. First, the effect of delegation on the expert's information acquisition incentive is indeterminate, so it may enhance but also may destroy this incentive; second, delegation can induce both types of expert to reveal their information truthfully, but it may suffer from the biased expert's distorted action. Intuitively, proposition 5 tells us that it's better for the decision maker to keep her authority whenever at least truthful revelation is guaranteed. On the other hand, if the expert's reputational concern is so high that informative communication becomes infeasible, it's optimal for the decision maker to decentralize her decision rights and to incentivize the expert based on his actions. But this simply implies that the expert's "real authority" should be weakly increasing with his reputational concern, which may shred some light on the optimal delegation in many real situations. Interestingly, the result derived in our setup is opposite to the finding in Wouter Dessein (2002), in which the author shows that the decision maker prefers delegation to communication whenever informative communication is feasible.

7. Conclusion.

A common property of many organizations is that experts are incentivized by their reputational concerns when contingent payment schemes are infeasible. In this paper we characterized how these concerns may affect their information acquisition incentives, which is still silent in the literature on reputational cheap talk. Our main finding shows that for proper effort cost, the aligned expert acquires better information if and only if his reputational concern is moderate. While in some papers the possible existence of biased expert was shown to be detrimental to the decision maker's payoff and to the social surplus, the result in our paper is more positive. The reason is, the decision maker now can benefit from the aligned expert's endogenous information acquisition when he is seeking to separate from the biased type. This enables us to contribute some new viewpoints to the understanding on these issues. As a robustness check, our result shows that the observability of information acquisition decision has no essential influence on the aligned expert's acquisition and transmission decisions, and the decision maker's payoff is unaffected.

Since with the increase of his reputational concern, the expert's attempt to distort information is stronger for the sake of reputation building, it becomes better for the decision maker to grant her decision rights to the expert. More precisely, our insight shows that the decision maker prefers communication to delegation whenever informative communication is feasible, which corresponds to the situation that the expert's reputational concern is not too high. Interestingly, we find that with more real authority or decision powers the aligned expert may turn out to limit his action on information acquisition, which is also novel to the results in existing papers.

One potential extension of this paper is to generalize the effort and accuracy levels. For instance, consider an effort cost function $c(p)$ such that $c(1/2) = 0$, $c(1) = +\infty$, $c' > 0$ and $c'' > 0$. We expect the qualitative results in our paper will be unchanged with this cost function, in particular, when the aligned expert's reputational concern increases, his optimal acquisition effort first increases strictly and smoothly, then decreases strictly and smoothly. Alternatively, we can enrich the signal space with more elements or let it be a continuum, this will help our justification on the optimal interval delegation.

Another extension is to allow the biased expert also have reputational concern μ_B. If μ_B is relatively unimportant compared with the current payoff weight η, it's natural to see that our findings still hold with this modification, since he is mainly interested in pushing his current agenda, instead of building reputation. On the other hand, if μ_B is quite large, things may be different. Consider the extreme case that $\eta = 0$ and $\mu_B > c$, then there is no type-separation in equilibrium, both types of expert's reveal information truthfully but the aligned expert's acquisition decision only depends on the trade off between current gain and acquisition cost. Generally, with the increase of the biased expert's reputational concern, it becomes harder for the aligned expert to build reputation, so his incentive to acquire better information is weakened. But his information distortion would also be reduced, and the net effect is much ambiguous. This implies that the decision maker's payoff is not necessary to be improved when the biased type of expert becomes more self-disciplined.

A dynamic setup that endogenize the expert's reputational concern may also be of interest, for instance, let the expert be a long-run player who interacts with a sequence of short-run decision makers. Also, the model can be adapted to consider the situation that the decision maker's uncertainty about the expert's preference is replaced with an uncertainty about the expert's ability. We leave this topics for future study.

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