Hayek Vs Keynes: Dispersed Information and Market Prices in a Price-Setting Model*

Christian Hellwig[†] Venky Venkateswaran[‡]

September 2011

Abstract

We examine the role of dispersed knowledge about fundamentals in the presence of market-generated information. Our main theoretical result is a "Hayekian benchmark", defined by conditions under which dispersed information has no effect on outcomes. In a nominal price-setting context, these conditions are met when firms set prices every period after having seen contemporaneous market-generated information. When other frictions (nominal frictions and/or information lags) make the firm's decision problem dynamic, departures from this benchmark arise to the extent there are strategic interactions in firm's pricing decisions or differences in the persistence of various shocks. We examine the empirical significance of these results using a calibrated menu cost model. We document a novel interaction between nominal and informational frictions. Firms attribute aggregate nominal shocks to idiosyncratic factors, which are relatively less persistent and so, make smaller price adjustments. Quantitatively, however, this channel does not substantially increase monetary non-neutralities.

*We thank Alex Monge-Naranjo, Ariel Burstein, Pierre-Olivier Weill and participants at Annual Meeting of the Society for Economic Dynamics, 2011 and the European Economic Association 2011 for helpful discussions and comments. Hellwig gratefully acknowledges financial support from the European Research Council (starting grant agreement 263790).

[†]Toulouse School of Economics. Email: christian.hellwig@univ-tlse1.fr

[‡]Pennsylvania State University. Email: vuv10@psu.edu

1 Introduction

> *..in a system where knowledge of the relevant facts is dispersed, prices can act to coordinate.....The most significant fact about the system is the economy of knowledge with which it operates, how little the individual participants need to know in order to be able to take the right action.*
>
> Hayek (1945)

> *...Thus, certain classes of investment are governed by the average expectation of those who deal on the stock market, as revealed in the price, rather than by the genuine expectations of the professional entrepreneur.*
>
> Keynes (1936)

The two quotes above illustrate contrasting views on the functioning of markets in a world of uncertainty and limits to the perception of current and future economic conditions. Keynes' (1936) famous beauty contest analogy of investment decisions suggests that under such conditions markets cease to function well because the market participants' concerns with the views and decisions of others takes precedence over their views regarding fundamental economic conditions. The resulting herding behavior inefficiently amplifies fluctuations and induces a positive role for stabilization policies. The polar opposite view is expressed in the influential essay by Hayek (1945), which argues that markets are particularly effective at dealing with the limits to information and perception that are inherent in a market environment with a large number of participants. Hayek emphasizes the "parsimony of knowledge" with which the competitive price system guides individual participants to take decisions that are not only in their own best interest, but ultimately lead to a socially efficient allocation of resources, despite the lack of centrally organization and communication of the relevant information to market participants.

Our objective in this paper is to reconcile these contrasting and seemingly incompatible views, and to ask whether we can discriminate between them empirically. Specifically, we consider a class of dynamic, stochastic equilibrium economies of nominal price adjustment with monopolistically competitive firms, who hire labor to produce their output, and have limited information on the stochastic market-specific and aggregate conditions. The equilibrium interactions under limited perceptions give agents a beauty contest motive of guessing the behavior of others. To incorporate the Hayekian idea, we assume that firms update their beliefs based on the information they gather from their own market transactions.

We then show the existence of a "Hayekian benchmark", defined by conditions under which the market economy with limited information leads to equilibrium allocations that are identical to those that result if information about all shocks was perfect, thus validating the Hayekian argument for informational efficiency of markets. Departures from this benchmark in turn offer some validity to the Keynesian argument. As we shall see, both the sufficient conditions for the Hayekian benchmark, and the departures from this benchmark are directly interpretable in terms of the market information, and strategic interactions that were originally emphasized by Hayek and Keynes, respectively.

We first consider a case where imperfect information is the only potential source of departures from the flexible price benchmark. We show that in this case, where firms' pricing decisions are based on a static trade-off between marginal costs and revenues, the conditions for the Hayekian benchmark are particularly simple and powerful: whenever firms are able to respond to the information conveyed concurrently through their market transactions, they will be able to perfectly adjust prices in a fashion that replicates the full information outcome. Imperfect information is completely irrelevant for equilibrium allocations. This result is based on two simple insights. First, in any Bayesian game, the equilibrium outcome that obtains with perfect information remains an equilibrium outcome with imperfect information, whenever the information structure is sufficiently rich to allow all agents to infer their best responses to the actions of the other players - or in our case, whenever firms are able to perfectly figure out their optimal prices, even though they may still remain highly uncertain about aggregate conditions, or about the prices set by other firms. Second, the concurrent market information allows the firms to do just that, because the signals the firms obtain from their transactions in input and output markets offer them concise signals of their marginal costs and revenues, respectively. This result fully displays the logic of Hayek's argument.

Second, we consider the case where imperfect information interacts with other frictions in price adjustment, such as information lags, adjustment lags (Calvo pricing), or menu costs. We show that departures from the Hayekian benchmark only obtain to the extent that the additional frictions generate a motive for disentangling different types of idiosyncratic and aggregate shocks. We show that, for a number of adjustment frictions/lag specifications commonly used macroeconomics, these motives arise only to the extent that there are differences in time series properties (in particular, the persistence) of various shocks or there are significant strategic complementarities in pricing decision. The intuition is similar to the benchmark case - in the absence of strategic complementarities and differences in persistence, firms' current market signals are sufficient for the firms' best forecast of profit-maximizing prices in future periods. In such a scenario, the incompleteness of a firm's information set does not have any implications for its decision. This is the dynamic analogue of the Hayekian benchmark in the static case discussed above. Significant departures from this benchmark then require (i) the existence of a beauty contest motive (strategic interactions/complementarities) and differential degrees of persistence between different types of shocks. The Keynesian beauty contest argument thus retains its validity in a foreward-looking environment, in which firms try to forecast future actions by others, and need to disentangle different types of shocks to assess the persistence of their effects.

Finally, we examine the empirical significance of these results using a calibrated model. The model, a standard menu cost price-setting framework, is calibrated to match key facts both at the micro and macro level in order to incorporate reasonable estimates for the extent of strategic complementarity and stochastic properties of the shocks. Solving this model presents three major technical challenges. The first is the well-known 'curse of dimensionality' which arises in models where the cross-sectional distribution becomes a relevant state variable. Here, this problem is compounded by the so-called 'infinite regress' problem highlighted by Townsend (1983). When actions are strategically linked, firms' optimal decisions depend on the entire structure higher-order expectations (i.e. their beliefs about others' beliefs, their beliefs about others beliefs about their beliefs...). Thus, the entire structure of higher-order beliefs becomes an additional state variable. The second difficulty stems from the non-linearity in policy functions introduced by menu costs. This makes aggregation quite challenging - in particular, the direct aggregation property of linear models with Gaussian information structures, employed by almost the entire literature on heterogeneous information, is no longer available. Finally, the presence of a dynamic filtering problem with endogenous signals makes it difficult to directly apply Kalman filter techniques. In our numerical solution technique, we address these challenges by combining the approximation techniques in Krusell and Smith (1998) with standard filtering techniques. The key insight that enables this combination is that we can use the same low-order ARMA representation of aggregate dynamics to get around the Krusell-Smith curse of dimensionality, and the infinite regress issue in the filtering problem. This allows us to capture the complex multi-dimensional heterogeneity with only a small number of state variables and compute the non-linear value and policy functions directly using an iterative procedure.

Our main finding in the numerical analysis is that the departure from the benchmark takes the form of a novel interaction between nominal and informational frictions. Without menu costs, our model setup satisfies the conditions of the static Hayekian benchmark i.e. dispersed information has no effect on allocations. With menu costs, however, strategic complementarities and differences in persistence start to play a role. The 'market' signals observed by the firms are combinations of an aggregate nominal shocks and idiosyncratic demand/cost disturbances. In a calibrated model, evidence from the micro data points to the latter being an order of magnitude larger than aggregate shocks. As a result, the solution to the firms' inference problem leads them to attribute most of the changes in their signals, including those coming from aggregate shocks, to idiosyncratic factors. However, these idiosyncratic shocks, while positively autocorrelated, are relatively less persistent than innovations to aggregate money supply. Since, with positive menu costs, the firm expects to leave its price unchanged for a few periods, a less persistent shock leads to a smaller response. Therefore, an aggregate nominal shock generates a smaller price response under dispersed information compared to the full information case. Quantitatively, however, this channel is not sufficiently strong in our calibration to substantially increase monetary non-neutralities relative to a menu cost model with perfect information (or relative to the Hayekian benchmark without menu costs).[1]

[1] An alternative calibration strategy that uses firm dynamics rather than pricing facts to calibrate firm-specific shocks arrives at the same conclusion even more starkly: by matching aggregate consumption to a random walk, and firm-specific shocks to be consistent with Gibrat's Law, we find that both sources of shocks are very close to a random walk, which is very much in line with the Hayekian benchmark conditions.

A important contribution of this paper is the development of a unified framework to test the validity - both theoretical and empirical - of the Hayekian and Keynesian views on the role of markets. This is particularly significant in the context of the existing literature on the subject. Both viewpoints have been extremely influential, but have been invoked or studied in environments that are not directly comparable. Hayek (1945) referred to a 'price system' without being explicit about the underlying market structure. His insight is implicitly invoked in models where the information structure is left unspecified, but has not been articulated in a formal model with a well-defined market structure. At the other extreme, the Keynesian beauty contest metaphor plays an important role in a large and growing work using models with heterogeneous information to study business cycles, asset pricing and financial crises[2]. However, the environments used by much of this literature cannot be directly applied to the questions studied here. A common approach to modeling information in this literature is an abstract one, where signals are modeled as noisy observations of the exogenous shocks themselves[3]. Therefore, by assumption, market prices and allocations do not have any part to play in the aggregation and transmission of information. As discussed earlier, our approach addresses this challenge by focusing on a class of models with non-trivial dispersed knowledge about fundamentals in combination with the crucial elements underlying both viewpoints - market-generated information and strategic interactions.

Our results also have implications for an important branch of the dispersed information literature - one that studies the welfare effects of additional information. In Morris and Shin (2002), Hellwig (2005) and Angeletos and Pavan (2007), additional information can reduce social welfare, due to misalignment of social and private incentives for coordination. Our analysis suggests that these insights may not be applicable to a market economy if the conditions for the Hayekian benchmark are satisfied.

[2]This is too large a body of work to cite every worthy paper. The papers most closely related to ours stem from the seminal work of Phelps (1970) and Lucas (1972) on the role of informational frictions in generating aggregate fluctuations. A few recent examples are Amador and Weill (2010), Angeletos and La'O (2010), Hellwig and Venkateswaran (2009), Lorenzoni (2009), Mackowiak and Wiederholt (2009, 2010) and Woodford (2003).

[3]Important exceptions are Amador and Weill (2010), Hellwig and Venkateswaran (2009) and Graham and Wright (2010). Mackowiak and Wiederholt (2009) also consider endogenous signals in an extension to their baseline model.

The price-setting environment used to illustrate the main ideas is very similar to the one used extensively in the New Keynesian literature to study the dynamics of price adjustment. Our work, particularly the results in section 3, complements this literature by analyzing the interaction of informational frictions with various assumptions about nominal rigidity, including both time-dependent (as in Taylor, 1980 or Calvo, 1983) and state-dependent models (as in Caplin and Leahy, 1991 and Golosov and Lucas, 2007). Gorodnichenko (2010) also analyzes a similar interaction in a model with both endogenous information choice and nominal frictions, but focuses on externalities affecting information acquisition/nominal adjustment. In his paper, the prospect of learning from market prices, albeit with a lag, reduces firms' incentives to acquire costly information. If the conditions of our Hayekian benchmark(s) are satisfied, this trade-off can be particularly extreme - markets provide firms with all the information they need and so additional information is worthless to them and will not be acquired at a positive cost in equilibrium[4].

Finally, for the numerical analysis in section 4, we draw on recent work documenting price adjustment at the micro level using large scale data sets of individual price quotes. The moments we target in our calibration - on the cross-sectional dispersion and time series properties of prices as well as the frequency and magnitude of price changes - are taken from the work of Bils and Klenow (2004), Nakamura and Steinsson (2008), Klenow and Krvystov (2008), Burstein and Hellwig (2007) and Midrigan(2011).

The rest of the paper proceeds as follows. Section 2 introduces the baseline model with static decisions and derives the benchmark Hayekian result. Section 3 extends the model - and the theoretical result - to a dynamic context. Section 4 presents the model used for the numerical analysis, along with the calibration strategy and the results. Finally, Section 5 presents a brief conclusion.

[4]Note the connection to Grossman and Stiglitz (1980). It is important to note that our arguments do not rely on market prices being fully revealing. In fact, even when the Hayekian benchmark obtains, firms' signals could be very poor indicators of the true nature of shocks hitting the economy. The key insight is that, despite this, they tend to provide an extremely accurate indication of optimal decisions.

2 A Static Price-Setting Model

We present our full model in two steps. This section focuses on the production side of the economy. It describes out the problem faced by a monopolistically competitive firm, setting nominal prices every period, subject to both aggregate and idiosyncratic shocks. It turns out that, in order to arrive at our main result on the implications for dispersed information for static decisions, we need to impose very little structure on the rest of the economy. In particular, no assumptions about household preferences, wages or the stochastic processes followed by the underlying shocks are necessary. The next section will present the rest of the model and extend the analysis to dynamic price-setting problems.

An economy has a single final good, which is produced using a continuum of intermediate goods.

$$Y_t = \left(\int B_{it}^{\frac{1}{\theta}} Y_{it}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}$$

where B_{it} is an idiosyncratic demand shock and the parameter $\theta > 1$ is the elasticity of substitution. Final goods production is undertaken by a competitive firm, leading to a demand function for intermediate good i of the form

$$Y_{it} = B_{it} Y_t \left(\frac{P_{it}}{P_t} \right)^{-\theta} \tag{1}$$

where P_t is an aggregate price index given by

$$P_t = \left(\int B_{it} P_{it}^{1-\theta} di \right)^{\frac{1}{1-\theta}}$$

Each intermediate good is produced by with labor of type i as the sole input, according to a decreasing- returns to scale production function:

$$Y_{it} = \frac{1}{\delta} N_{it}^{\frac{1}{\delta}}$$

Intermediate Producer's Problem: Each period, the intermediate goods producer i sets a nominal price P_{it} to maximize expected profits (weighted by the representative household's stochastic discount factor)

$$\max_{P_{it}} \quad \mathbb{E}_{it} \left[\lambda_t \left(P_{it} Y_{it} - W_{it} N_{it} \right) \right] \tag{2}$$

where λ_t is the stochastic discount factor and $\mathbb{E}_{it}$ is the expectation conditional of firm i's information set, i.e. $\mathbb{E}_{it} \equiv \mathbb{E}(\cdot|\mathcal{I}_{it}$. By setting a nominal price, the firm commits to supplying any quantity demanded by the final goods producer.

Household: The representative household maximizes

$$\mathbb{E} \quad \sum_{s=0}^{\infty} \quad \beta^s u(C_{t+s}, \{N_{it+s}\}, \mathcal{M}_{t+s}, P_{t+s})$$

subject to a standard budget constraint. The term $\mathcal{M}_{t+s}$ is a vector of aggregate shocks (to be specified later). Since our focus is the decision problem of firm under various informational assumptions, throughout the paper, we will maintain the assumption that representative household has access to a complete contingent claims market and operates under full information.

Equilibrium: An equilibrium consists of sequences of pricing strategies $\{P_{it}\}$ for intermediate goods producers, as functions of the information set $\mathcal{I}_{it}$, prices for final good P_t, prices of contingent claims, production choices by the final $\{Y_{it}, Y_t\}$ and consumption and labor supply choices of the household such that the pricing strategies solve 2, the production choices are consistent with maximization by the final goods producer and the household choices are optimal.

2.1 An Irrelevance Result

We are now ready to present our first theoretical result. We begin with some definitions of dispersed information and a natural notion of its relevance to allocations.

Definition 1 *1. Firms have access to contemporaneous information, if they make decisions in period t after observing information for period t.*

2. In an economy with dispersed information, firms only observe (histories of) signals generated by their market activities - in particular, their sales Y_{it} and wages W_{it}.

3. In an economy with full information, all firms observe (histories of) all shock processes and the prices set by other firms.

4. *Dispersed information is said to be relevant if prices and quantities are different in an economy with dispersed information compared to the one with full information.*

Note that, in general, knowledge about fundamentals (e.g. aggregate/idiosyncratic shocks or aggregates) will be very different in the two economies (except in the special case where the market signals allow the firm to infer the underlying shocks exactly). However, that feature by itself does not make informational frictions relevant in the sense of the above definition. The definition requires that dispersed information leads to equilibrium prices and quantities that are different from those under full information.

The following proposition presents the main result of this section - a Hayekian benchmark for static decisions.

Proposition 1 *Suppose firms set prices every period and have access to contemporaneous information. Then, dispersed information is not relevant.*

To explain the intuition behind this striking result, we proceed in 2 steps. First, we argue that the equilibrium in an economy with information can be sustained under an information set $\mathcal{I}_{it}$ if it allows every firm to infer its own full information best response. To see this, note that, under this information set, every firm will act as if it were perfectly informed. By definition of an equilibrium, actions in the full information equilibrium are mutual best responses. It then follows that the equilibrium under full information is also an equilibrium under $\mathcal{I}_{it}$. Second, we show that firms' information sets in the economy with dispersed information satisfies this property. The full information optimal price of firm i is characterized by the following first order condition:

$$\underbrace{\Phi_1 \, P_{it}^{-\theta} \; P_t^{\theta} B_{it} C_t}_{\text{Marginal Revenue}} = \underbrace{\Phi_2 \, P_{it}^{-\theta\delta-1} (P_t^{\theta} B_{it} C_t)^{\delta} \quad W_{it}}_{\text{Marginal Cost}} \tag{3}$$

Under dispersed information, the firm is assumed to have access to two contemporaneous signals - its own sales Y_{it} and wage rate W_{it}. From (1), it is easy to see that the former is informationally equivalent to $P_t^{\theta} B_{it} C_t$. Along with the directly observed wage signal, this gives the firm all the information it needs to accurately forecast its own marginal revenues/costs and therefore, infer its best response. By the earlier argument, the full information equilibrium is obtained.

The above result has a number of implications for models with heterogeneous information. First, note that, so long as the firms has access to contemporaneous market signals, any additional information about the aggregate economy, or even direct information about the shocks themselves, is irrelevant for the firm's decision. This holds irrespective of precision or the public versus private nature of that additional source of information. It then follows that the results in Morris and Shin (2002) or Angeletos and Pavan (2007) about the welfare implications of additional information do not apply in an environment with market-generated information. Second, Proposition 1 also implies that heterogeneity in beliefs about aggregate conditions by itself may not be indicative of the relevance of informational frictions in an economy. In the above environment, learning from market signals could induce a considerable amount of cross-sectional dispersion in firms' beliefs about the aggregate economy, but as we have shown, that would have little implication for outcomes.

Needless to say, this result is at odds with the findings of the rather large body of work on heterogeneous information models. The main reason for this difference is the information structure. A common approach to modeling information in this literature is an abstract one - signals are modeled as arbitrary combinations of fundamental shocks and observational noise. Therefore, the Hayekian role of markets is ruled out by assumption. Here, on the other hand, information arrives through endogenous objects - prices, quantities etc. The result in Proposition 1 states that, in the absence of any lags in the arrival of information or frictions in the adjustment process, markets play an extremely effective informational role. Exactly as Hayek conjectured, they give each agent precisely the information she needs to know her optimal decision.

As mentioned in the introduction, we are not the first to introduce market information in models of dispersed knowledge. Amador and Weill (2010), Hellwig and Venkateswaran (2009) and Graham and Wright (2010) all use signals based on market prices and/or allocations. However, all of them restrict the information sets of agents in subtle ways - by eliminating certain forms of market interaction or introducing lags. For example, in Amador and Weill (2010), agents make labor supply decisions in a non-market setting (i.e. without wages to guide them). In Hellwig and Venkateswaran (2009), decisions have to made *before* current signals are observed and in Graham and Wright (2010), the key decision is an investment choice - an intertemporal optimization problem to which Proposition 1 does not apply directly. In the following section, we will show that, under certain conditions, the conclusion in Proposition 1 extends to an environment with dynamic decisions, but the models in these two papers do not meet these conditions.

3 A Dynamic Model

The key insight from the previous section is that a combination of static decisions and contemporaneously observed market information ensures that the Hayekian viewpoint holds exactly i.e. the lack of common knowledge about the sources of uncertainty does not have any implications for prices or quantities. This suggests two natural modifications to induce departures from this benchmark - dynamic decision problems and/or lags in the arrival of information. Both these changes can interfere with the firm's ability to infer its full information best response from the information available to it. The objective of this section is identify the conditions under which this happens i.e. the irrelevance result in Proposition 1 does not extend to an environment with adjustment/information lags. For concreteness, we will focus on the following four types of frictions:

- **Case I:** Prices set every period, but information observed with an N-period lag.
- **Case II:** Prices set once every N periods, but information observed contemporaneously.
- **Case III:** Prices set as in Calvo, with information observed contemporaneously.
- **Case IV:** Prices set subject to fixed menu costs, but information observed contemporaneously.

The formal description of the firm's problem in each of the cases is described below.

Case I: Prices set every period, but information observed with a lag of N: Here, the firm's problem is the same as (2),

$$\max_{P_{it}} \quad \mathbb{E}_{it}\left[\lambda_t\left(P_{it}Y_{it} - W_{it}N_{it}\right)\right] \ .$$

The only change is in the information available to the firm. At the time of setting period t prices, the firm has access to information N-period old. The formal definition of the information set in the full information economy is

$$\mathcal{I}_{it}^{\text{Full}} = \{M_{t-N-s}, P_{t-N-s}, B_{it-N-s}, Z_{it-N-s}\}_{s=0}^{\infty}$$

In the economy with dispersed information, we have

$$\mathcal{I}_{it}^{\text{Disp}} = \{Y_{it-N-s}, W_{it-N-s}\}_{s=0}^{\infty} .$$

Case II: Prices set once every N periods, but information observed contemporaneously: In every reset period t, the firm solves

$$\max_{P_{it}} \quad \mathbb{E}_{it} \quad \sum_{s=0}^{N} \quad \beta^s \left[\lambda_{t+s} \left(P_{it} Y_{it+s} - W_{it+s} N_{it+s}\right)\right] \tag{4}$$

The information sets are contemporaneous, as in the static case, i.e.

$$\begin{aligned}\mathcal{I}_{it}^{\text{Full}} &= \{M_{t-s}, P_{t-s}, B_{it-s}, Z_{it-s}\}_{s=0}^{\infty} \\ \mathcal{I}_{it}^{\text{Disp}} &= \{Y_{it-s}, W_{it-s}\}_{s=0}^{\infty}\end{aligned}$$

Case III: Prices set as in Calvo (1983), but information observed contemporaneously: Every period, with probability ξ, the firm can change its price. This probability is independent over time and across firms. Thus, the probability that the firm's price remains unchanged for exactly T periods is given by $(1-\xi)^{T-1}\xi$. In every reset period, the firm solves

$$\max_{P_{it}} \quad \mathbb{E}_{it} \quad \sum_{T=1}^{\infty} \; (1-\xi)^{T-1}\xi \; \sum_{s=0}^{T-1} \quad \beta^s \quad \left[\lambda_{t+s} \left(P_{it} Y_{it+s} - W_{it+s} N_{it+s}\right)\right] \tag{5}$$

The information sets are contemporaneous, as in Case II above.

Case IV: Prices set subject to fixed menu costs, but information observed contemporaneously: Let $V(P_{it-1}, \mathcal{I}_{it})$ denote the value of firm which starts period t with a price P_{it-1} and an information set $\mathcal{I}_{it}$. The Bellman equation below characterizes $V(\cdot)$:

$$V(P_{it-1}, \mathcal{I}_{it}) = \max \quad \{ \ \mathbb{E}_{it}[\lambda_t \Pi(P_{it-1}, M_t, P_t, B_{it}, Z_{it}) + \beta V(P_{it-1}, \mathcal{I}_{it+1})],$$
$$\max_P \ \mathbb{E}_{it}[\lambda_t \Pi(P, M_t, P_t, B_{it}, Z_{it}) - \lambda_t \mathcal{C} + \beta V(P, \mathcal{I}_{it+1})]\} \qquad (6)$$

where $\mathcal{C}$ is the fixed cost of changing prices. The information sets are contemporaneous, as in Case II above.

3.1 Closing the Model

At this stage, we need to impose additional structure on the model. To see why this is necessary, note that in all the cases, firms need to forecast future revenues and costs using information in their current signals[5]. To describe their laws of motion, we need to specify both their relationship to the underlying shocks as well as the laws of motion for the underlying shocks themselves. Our strategy for the first of these issues is to adopt a flexible specification, which allows us to retain tractability and at the same time, capture key aspects of commonly used preference-technology assumptions in macroeconomic models. In particular, we assume that the preferences are such[6] that the following relationships hold:

$$P_t C_t^{\psi} = \Psi M_t \qquad (7)$$
$$\lambda_t = M_t^{-\zeta} \qquad (8)$$
$$W_{it} = \Upsilon \ M_t^{\eta} P_t^{1-\eta} \ Z_{it} \ N_{it}^{\kappa} \qquad (9)$$

where Z_{it} is an idiosyncratic cost shifter and M_t is the process for aggregate money supply.

Next, we describe the stochastic processes of the aggregate and idiosyncratic shocks. Recall that there are thus 3 sources of uncertainty faced by firm i - one aggregate (money supply) and two idiosyncratic (the cost shock Z_{it} and the demand shock B_{it} in (1)). We assume that all these processes follow AR(1) processes (in logs) with normally distributed innovations:

[5]Or equivalently, forecast current marginal revenues/costs with past signals.

[6]See Hellwig (2005), Hellwig and Venkateswaran (2009) and Nakamura and Steinsson (2008) for micro-founded general equilibrium models where similar relationships are derived.

$$m_t = \rho_m \cdot m_{t-1} + u_t$$

$$b_{it} = \rho_b \cdot b_{it-1} + u_{it}$$

$$z_{it} = \rho_z \cdot z_{it-1} + v_{it}$$

where u_t, u_{it}, v_{it} are mean-zero, normally distributed random variables[7]. As is standard in the heterogeneous information literature, the structure of the economy is assumed to be common knowledge (which includes all the parameters and the variances of the shock processes.).

Before commencing our analysis of the specific frictions listed above, it is worth reiterating our notion of relevance of dispersed information. For each of the cases, we will compare the behavior of the dispersed information economy under the corresponding friction to an identical economy subject to the same friction but under full information (i.e. assuming the realizations of the underlying shocks and aggregate conditions are common knowledge). The following proposition presents two benchmark cases in which both economies show identical outcomes i.e. dispersed information turns out to be irrelevant.

Proposition 2 *Consider economies subject to the frictions listed in cases I through IV above. Suppose* $\theta = \frac{1}{\psi}$ *and* $\eta = 1$.

1. *Then, dispersed information is irrelevant if all shocks are permanent.*

2. *Suppose* $\zeta = 0$ *so* λ_t *is a constant. Then, dispersed information is irrelevant if all shocks are equally persistent.*

Recall that in the static case studied in the previous section, dispersed information turned out to be irrelevant because the firms' signals allowed it to perfectly infer its (marginal) revenues and costs. Two complications arise in extending that logic to an intertemporal decision-making environment. First, firms now have to forecast future revenues and costs

[7]The natural logs of capital-lettered variables, e.g. for any variable X, we write $x = \ln X$ are denoted by the corresponding small letters.

using current signals[8]. The second complication arises because profits are weighted by an aggregate stochastic discount factor. For the informational friction to be irrelevant, the incompleteness of firms' information sets in the dispersed information economy (relative to that of their counterparts in the full information economy) must not affect their ability to forecast future profits or the relative weight attached to them. The conditions in the above proposition achieve this by eliminating strategic considerations and imposing additional structure on the stochastic processes of the shocks. To see this more clearly, we use equations (7) and (9) to rewrite revenues and costs as follows:

$$\text{Total Revenue}_t = \Phi_1 \; P_{it}^{1-\theta} \; (P_t^{\theta} B_{it} C_t) = \Phi_1 \; P_{it}^{-\theta} \; (P_t^{\theta - \frac{1}{\psi}} B_{it} M_t) \tag{10}$$

$$\text{Total Cost}_t = \Phi_2 \; P_{it}^{-\theta\delta} (P_t^{\theta} B_{it} C_t)^{\delta} \;\; W_{it} = \Phi_2 \; P_{it}^{-\theta\delta} (P_t^{\theta - \frac{1}{\psi}} B_{it} M_t)^{\delta} \;\; (M_t^{\eta} P_t^{1-\eta} Z_{it}) \;\; N_{it}^{\kappa} \tag{11}$$

The two key unknown objects - $P_t^{\theta - \frac{1}{\psi}} B_{it} M_t$ and $M_t^{\eta} P_t^{1-\eta} Z_{it}$ - are thus combinations of the shocks (M_t, B_{it} and Z_{it}) and the aggregate price level, P_t. If $\theta = \frac{1}{\psi}$ and $\eta = 0$, then the aggregate price level no longer has any direct effect on the firm's profits or on the firm's signals. In other words, both expected future profits and the signals are now functions solely of the underlying shocks in case. In particular, revenues and costs are functions of $B_{it} M_t$ and $M_t Z_{it}$ respectively. Then, given the lognormality and AR(1) assumptions, it is then easy to see that, when shocks are equally persistent, the most recent realization of $B_{it} M_t$ is a sufficient statistic for characterizing the conditional distribution of $B_{it+s} M_{t+s}$. Therefore, the additional information available to firms in the full information economy (i.e. the realizations of the underlying shocks themselves) is irrelevant for the purposes of the price-setting decision.

We now turn to the issue of the discount factor. The second part of proposition 2 directly eliminates this source of strategic interaction. In the first part, future innovations to the discount factor are iid and so firms in both economies have identical expectations about the relative weight of current versus future profits. In combination with the equal persistence condition, this ensures that firms make identical decisions under both informational assumptions.

[8]Or equivalently, forecast current revenues/costs using past signals.

To summarize, we have established two important theoretical benchmarks in assessing the role of dispersed information in a market economy. First, in the absence of information and adjustment lags, markets play a very effective role in the transmission of information. Second, introducing lags or frictions induces departures from this benchmark only through strategic interactions in decisions or differences in the dynamic properties of underlying shock processes. An obvious next step is to the investigate the empirical significance of these results. The next section spells out our strategy for this exercise and presents some preliminary results.

4 Numerical Analysis

As mentioned earlier, the objective of this section is an quantitative evaluation of the role of informational frictions. Towards this end, we use a modified version of the general environment considered above. This version will incorporate all the essential features we need for our analysis - dispersed information arising from market signals, dynamic decisions and strategic considerations - in a flexible and numerically tractable framework. This will allow us to calibrate the model to key facts both at the micro and macro level and then, examine whether dispersed information leads to *quantitatively* different outcomes in a calibrated model.

4.1 A Simple Menu Cost Model

An economy populated by a continuum of firms, indexed by i. The 'target' nominal price[9] of firm i is given by:

$$p_{it}^{*} = \gamma \cdot b_{it} + (1-r)m_t + rp_t \tag{12}$$

where γ and r are parameters. The target is influenced by two exogenous processes - an idiosyncratic shock b_{it} and aggregate money supply, m_t. It is also affected by the overall price level, denoted $p_t \equiv \int p_{it}di$, which is the only source of strategic interaction in this version.

[9] A very similar expression can be derived from (3), the equation characterizing the static optimum, by imposing conditional lognormality and the equilibrium conditions (7)-(9).

The parameter r captures the strength of this strategic interaction. Both the exogenous shocks are normally distributed AR(1) processes, i.e.

$$m_t = \rho_m \cdot m_{t-1} + u_t$$
$$b_{it} = \rho_b \cdot b_{it-1} + u_{it}$$

The flow payoff from a arbitrary price p_{it} in period t is given by

$$\pi_{it} = -(p_{it} - p^*_{it})^2$$

which is discounted at a constant rate β. The firm is subject to a fixed menu cost, denoted $\mathcal{C}$, if it decides to change its price in any given period. The firm's value function satisfies this Bellman equation:

$$\begin{aligned} V(p_{it}, \mathcal{I}_{it}) = \max \quad \{ \; & \mathbb{E}_{it}[-(p_{it} - p^*_{it})^2 + \beta V(p_{it-1}, \mathcal{I}_{it+1})], \\ & \max_p \;\; \mathbb{E}_{it}[-(p - p^*_{it})^2 - \mathcal{C} + \beta V(p, \mathcal{I}_{it+1})]\} \end{aligned} \tag{13}$$

where $\mathcal{I}_{it}$ denotes the information set of the firm at the time of making the period t decision.

Finally, the rest of the economy is summarized by a simple aggregate quantity equation:

$$y_t = m_t - p_t \ ,$$

where y_t denotes fluctuations in real output.

Information: Under full information, the firm observes the entire history of shocks m_t and b_{it} as well as the average price level p_t. Under dispersed information, the firm is assumed to observe a noisy signal of its current target:

$$s_{it} = p^*_{it} + v_{it} \ ,$$

where v_{it} is an idiosyncratic noise term, $\sim \mathcal{N}(0, \sigma^2_v)$.

The key properties of the models considered in the previous sections can be shown in this simplified setup as well. For example, setting the menu cost, $\mathcal{C}$ and the noise in the signal

σ_v^2 both to 0 implies that the firm makes static decisions after observing a perfect signal of its optimal choice. This corresponds exactly to the static case studied in Section 2, where the firm's wage and sales signals allowed it to infer the static optimal price perfectly. In this case, the insight of Proposition 1 applies directly - dispersed information does not have any effect on actions or outcomes in this economy.

Eliminating complementarities and signal noise, i.e. $r = 0$ and $\sigma_v^2 = 0$, allows us to see the intuition behind Proposition 2 at work. Under these assumptions, profits only depend on the chosen price and a particular combination of the underlying shocks $(\gamma b_{it} + m_t)$. If, in addition, both shocks are assumed to be equally persistent, the most recent signal is a sufficient statistic for forecasting future profits, making dispersed information irrelevant.

4.2 Solution Algorithm

Even with a simplified version, solving the model numerically presents several challenges. The first of these is the well-known curse of dimensionality. Since each firm's payoffs are linked to other firms actions, the cross-sectional distribution of prices becomes a relevant state variable for the firm. This problem is compounded by the dispersed nature of information. This implies that, in order to forecast other firms actions, firms need to form forecasts about their forecasts (and their forecasts about the others forecasts and so on). In a one-shot game, all these higher-order beliefs are functions of a single random variable. This often allows the use of a simple method of undetermined coefficients to solve the problem. With more periods, higher-order beliefs depend in a complicated way on the history of signals. As a result, the set of relevant state variables can become quite large as the number of periods increases. If past realizations are never revealed, strategic linkages lead to the well-known "infinite regress" problem (Townsend, 1983). The evolution of the economy depends on the realizations of an infinite history of signals, making the problem generally intractable.

The most common approach for dealing with the first problem is to use the method proposed in Krusell and Smith (1998), which involves approximating the entire distribution using a small number of moments[10]. The heterogeneous information literature has dealt with this problem either by restricting attention to special cases where the relevant history can

[10] See Nakamura and Steinsson(2008) for an application of this approach to a menu cost setting.

be summarized in a finite dimensional state variable (e.g. Woodford, 2003), by assuming that shock become commonly known after a finite number of periods (e.g. as in Hellwig and Venkateswaran (2009)), by truncating the dependence of equilibrium actions on higher order beliefs (e.g. Graham and Wright, 2010 or Nimark, 2008) or by modeling the history dependence using finite-order ARMA processes (e.g. Sargent, 1991 or Mackowiak and Wiederholt, 2010).

Our solution strategy combines the approximation technique in Krusell and Smith (1998) with the recursive formulation techniques in Sargent (1991) and others. We start by conjecturing that firms fit low-order ARMA processes to capture the effect of the aggregate price level on their signals. Given this conjecture, the information extraction problem of a firm can be cast in recursive form using a Kalman filter and only a small number of state variables. The value and policy functions are then directly computed using standard iterative procedures. The policy functions are used to simulate data and verify the initial conjecture about the aggregate price level.

Formally, define

$$X_{it} \equiv (b_{it} \qquad m_t \qquad p_t - m_t)' \ .$$

The state vector for the firm is then

$$(\mathbb{E}_{it} X_{it} \ , p_{it-1})'$$

The algorithm has the following 4 main steps:

- Conjecture a law of motion for $p_t - m_t$
- Derive law of motion for $\mathbb{E}_{it} X_{it}$ using a Kalman filter
- Use value function iteration to solve firm's problem
- Simulate data and verify the conjectured law of motion

Appendix B contains more details for each of these steps.

4.3 Calibration

A period in the model is set equal to a week. The parameters governing the stochastic process for m_t are based on the values in Golosov and Lucas (2007). In particular, we set $\rho_m = 0.995$

and $\sigma_m^2 = (0.0018)^2$. The parameter γ, the coefficient of the idiosyncratic component in the target price is normalized to 1. Rather than pick a single value for r, the degree of strategic complementarity, we will present results for various values of r, recalibrating the remaining parameters to target the same set of moments. For now, we set σ_v^2, the noise in the signal to 0. This will allow us to connect our numerical analysis directly with the benchmark results[11]

This leaves 3 parameters to be picked - the persistence and variance of the idiosyncratic shock, i.e. (ρ_b, σ_b^2) and the menu cost, $\mathcal{C}$. We choose parameter combinations to target the following four moments in both the full information and dispersed information economies:

- Monthly frequency of price changes: 20-25%
- Average absolute price change: 10-14 %
- Monthly autocorrelation of prices: 0.68
- Standard deviation of prices: 6-10 %

The ranges for these targets are drawn from the recent literature documenting the properties of prices in the US, using various micro-level data sets. The frequency and size of price changes are consistent with the estimates of Bils and Klenow (2004), Nakamura and Steinsson (2008), Klenow and Krvystov (2008) and Burstein and Hellwig (2007). The autocorrelation target is in line with the monthly serial correlation estimate reported by Midrigan(2011). Our target for price dispersion is derived from the statistics reported by Burstein and Hellwig (2007) for the Dominick's scanner price data.[12]

The results of the calibration procedure are presented in Table 1. Two features of the calibration will play an important in our results. The large size of price changes relative to aggregate nominal disturbances points to idiosyncratic shocks, that are an order of magnitude

[11] If, on the other hand, firms in the dispersed information economy were assumed to observe only a noisy signal of their target, then even with static decisions and contemporaneous signals, informational frictions will affect allocations. We will return to this idea later in the paper.

[12] Burstein and Hellwig find price dispersion measures of roughly 10%. We did not find direct measures of relative price dispersion in papers using other data sources, but this level seems consistent with the widely reported numbers on the magnitude of price changes (Klenow and Kryvtsov2008, Bils and Klenow 2004, Nakamura and Steinsson 2008).

	Description	Value
	Parameters	
	Time period	1 week
β	Discount factor	0.996
γ	Coefficient of idio. shock	1
$\mathcal{C}$	Menu cost	0.0285
ρ_b	Persistence of b_{it}	0.875
σ_b	Std devn of u_{it}	0.035
ρ_m	Persistence of m_t	0.998
σ_b	Std devn of u_t	0.0018
	Moments	
	Monthly frequency of price changes	23 %
	Average absolute price change	10 %
	Monthly autocorrelation of prices	0.68
	Standard deviation of prices	6 %

Table 1: Calibration Summary

larger than innovations to aggregate nominal demand. These shocks are less persistent than aggregate shocks - an implication of the relatively modest autocorrelation in prices[13].

4.4 Results

We use impulse response functions of real output (y_t) to innovations in aggregate money supply to highlight the differences between the full information and dispersed information cases. To generate these functions, we averaged the impulse response functions across 1000 runs. In each run[14], we simulated an economy with 10000 firms for 1200 periods, with the realization of the aggregate shock for the 1000^{th} period fixed at 0.0072. For each case, we

[13]Our estimates for ρ_b is slightly higher than, but in the same ballpark as, the baseline calibration of the persistence of idiosyncratic productivity shocks in Golosov and Lucas (2007). Their procedure does not target the autocorrelation or the unconditional dispersion of prices. To the extent that differences in persistence between aggregate and idiosyncratic shocks are an important source of departures from the Hayekian benchmark, our baseline calibration makes it harder for informational frictions to have any effects.

[14]We varied these parameters to verify that our results were not particularly sensitive to changes in the simulation methodology.

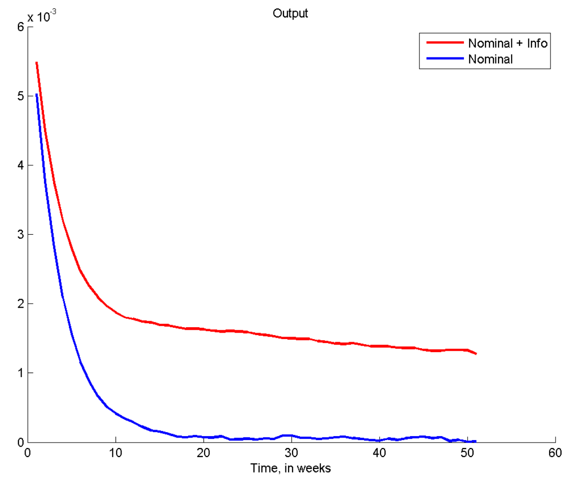

Figure 1: Response of Real Output, $r = 0$

show the response of output for both the full and dispersed information economies, calibrated according to the strategy discussed above.

Figure 1 shows the results for the case without complementarities, i.e. $r = 0$. The line in blue shows the full information case. As the graph reveals, the real effects in this case are quite short-lived. Within 10 weeks, prices have reflected more than 90% of the innovation to money supply, leading to very modest effects on real output. This is consistent with the findings in Golosov and Lucas (2007) and other papers, which show that a calibrated menu cost model does not generate persistent real effects from nominal shocks. The intuition for this result is the well-known selection effect - aggregate shock affect the distribution of firms who choose to change their price. When a positive aggregate shock hits the economy, firms whose current prices are below their target are more likely to change prices. Therefore, their adjustments - which are large and positive - account for a big chunk of overall price adjustments in the economy, increasing the aggregate price level. In a calibrated model, this effect is quite strong, leading to transitory effects on real variables.

The red line, the dispersed information case, shows a very different profile. The selection effect is still active - the aggregate shock enters the firms signals and thus, affects the distribution of firms changing prices. However, the overall price adjustment is more muted than in the full information case, particularly a few quarters out. To see why this occurs,

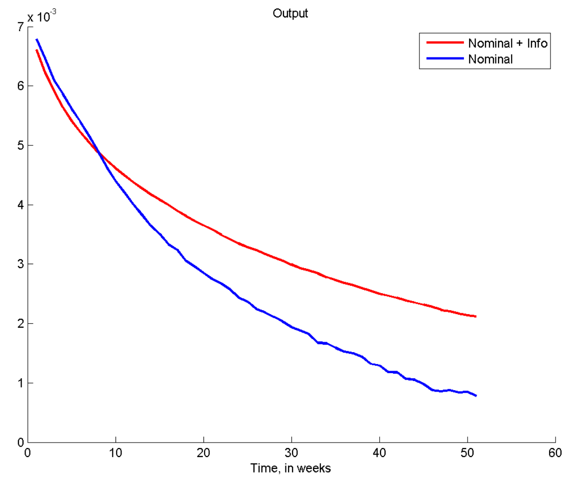

Figure 2: Response of Real Output, $r = 0.7$

recall that in our calibration idiosyncratic factors were an order of magnitude larger than aggregate shocks. As a result, the firms filtering problem causes them to attribute aggregate shocks to idiosyncratic factors, at least in the short run. Since idiosyncratic shocks are more transitory than aggregate disturbances, they call for a smaller price response. This dampens the response of prices, leading to more persistent real effects. Ultimately, firms learn the true nature of the shock, but, as the graph shows, the delay can be quite significant.

Next, we examine the role of complementarities. Figure 2 repeats the analysis for the case with $r = 0.7$. This is towards the higher end of estimates in the sticky price literature[15]. The overall message is the same as that of Figure 1 - dispersed information leads to more persistent real effects from nominal shocks. However, quantitatively, there are some differences from the profile in Figure 1. First, even in the full information case, real output shows more persistent effects from the aggregate shock. This is because the strong pricing complementarity reduces the incentives of firms to respond immediately to an aggregate shock. This channel mutes the selection effect and makes the response of aggregate prices more sluggish. Second, the difference between the full and dispersed information cases is less pronounced than in Figure 1.

Finally, Figure 3 highlights the importance of differences in persistence by plotting im-

[15]See discussion in Burstein and Hellwig (2007).

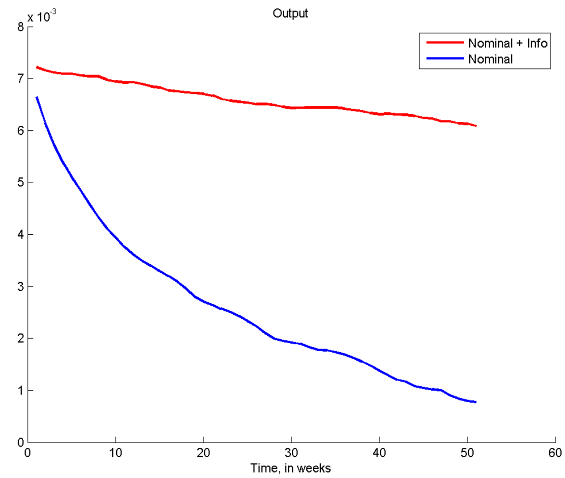

Figure 3: Response of Real Output, $\rho = 0$

pulse response functions for the case with completely transitory idiosyncratic shocks, i.e. $\rho_b = 0$, with all the other parameters fixed at the same values as in Figure 1. As we would expect, under this parameter combination, the model cannot match the micro facts targeted under the baseline calibration. Since price changes much less frequent and smaller under this parameterization, the selection effect is much weaker - aggregate shocks still change the composition of the price changers, but the strength of this channel is much less than that in the baseline calibration. As a result, nominal shocks now have much more persistent effects on real output. But, the more striking difference with Figure 1 is that the gap between the full and dispersed information economies is much bigger now. This is intuitive - firms attribute aggregate shocks to completely transitory idiosyncratic factors and so adjust their prices even less. This in turn leads to a slower aggregate price response and more persistent effects on real output.

5 Conclusion

TO BE ADDED.

References

Amador, Manuel, and Pierre-Olivier Weill. 2010. "Learning from Prices: Public Communication and Welfare." *Journal of Political Economy.* forthcoming.

Angeletos, George-Marios, and Jennifer La'O. 2010. "Noisy Business Cycles." In *NBER Macroeconomics Annual 2009, Volume 24*, NBER Chapters, 319–378. National Bureau of Economic Research, Inc.

Angeletos, George-Marios, and Alessandro Pavan. 2007. "Efficient Use of Information and Social Value of Information." *Econometrica* 75 (4): 1103–1142.

Bils, Mark, and Peter J. Klenow. 2004. "Some Evidence on the Importance of Sticky Prices." *Journal of Political Economy* 112 (5): 947–985.

Burstein, Ariel T., and Christian Hellwig. 2007. "Prices and Market Shares in a Menu Cost Model." NBER Working Paper No. 13455.

Calvo, Guillermo A. 1983. "Staggered prices in a utility-maximizing framework." *Journal of Monetary Economics* 12 (3): 383 – 398.

Caplin, Andrew, and John Leahy. 1991. "State-Dependent Pricing and the Dynamics of Money and Output." *The Quarterly Journal of Economics* 106 (3): pp. 683–708.

Golosov, Mikhail, and Robert E. Lucas Jr. 2007. "Menu Costs and Phillips Curves." *Journal of Political Economy* 115 (2): 171–199.

Gorodnichenko, Yuriy. 2010. "Endogenous Information, Menu Costs and Inflation Persistence." NBER Working Paper No. 14184.

Graham, Liam, and Stephen Wright. 2010. "Information, heterogeneity and market incompleteness." *Journal of Monetary Economics* 57 (2): 164 – 174.

Grossman, Sanford J., and Joseph E. Stiglitz. 1980. "On the Impossibility of Informationally Efficient Markets." *The American Economic Review* 70 (3): pp. 393–408.

Hayek, F. A. 1945. "The Use of Knowledge in Society." *The American Economic Review* 35 (4): pp. 519–530.

Hellwig, Christian. 2005. "Heterogeneous Information and the Welfare Effects of Public Information Disclosures." Mimeo, UCLA.

Hellwig, Christian, and Venky Venkateswaran. 2009. "Setting the right prices for the wrong reasons." *Journal of Monetary Economics* 56 (Supplement 1): S57 – S77.

Keynes, J. M. 1936. *The General Theory of Employment, Interest and Money.* London: McMillan.

Klenow, Peter J., and Oleksiy Kryvstov. 2008. "State-Dependent or Time-Dependent Pricing: Does It Matter for Recent U.S. Inflation?" *Quarterly Journal of Economics* 123 (3): 863–904.

Krusell, Per, and Jr. Smith, Anthony A. 1998. "Income and Wealth Heterogeneity in the Macroeconomy." *Journal of Political Economy* 106 (5): pp. 867–896.

Lorenzoni, Guido. December 2009. "A Theory of Demand Shocks." *The American Economic Review* 99:2050–2084(35).

Lucas, Robert Jr. 1972. "Expectations and the neutrality of money." *Journal of Economic Theory* 4 (2): 103–124 (April).

Mackowiak, Bartosz Adam, and Mirko Wiederholt. 2009. "Optimal Sticky Prices Under Rational Inattention." *American Economic Review* 99 (2): 769–803.

———. 2010. "Business Cycle Dynamics Under Rational Inattention." Mimeo, European Central Bank/Nothwestern University.

Midrigan, Virgiliu. 2011. "Menu Costs, Multiproduct Firms, and Aggregate Fluctuations." *Econometrica* 79 (4): 1139–1180.

Morris, Stephen, and Hyun Song Shin. 2002. "Social Value of Public Information." *The American Economic Review* 92:1521–1534(14).

Nakamura, Emi, and Jon Steinsson. 2008. "Five Facts About Prices: A Reevaluation of Menu Cost Models." *Quarterly Journal of Economics* 123 (4): 1415–1464.

Nimark, Kristoffer. 2008. "Dynamic pricing and imperfect common knowledge." *Journal of Monetary Economics* 55 (2): 365 – 382.

Phelps, Edmund S. 1970. "Introduction: The New Microeconomics in Employment and Inflation Theory." In *Microeconomic Foundations of Employment and Inflation Theory*, edited by Edmund et al. Phelps. New York: Norton.

Sargent, Thomas J. 1991. "Equilibrium with signal extraction from endogenous variables." *Journal of Economic Dynamics and Control* 15 (2): 245 – 273.

Taylor, John B. 1980. "Aggregate Dynamics and Staggered Contracts." *Journal of Political Economy* 88 (1): pp. 1–23.

Townsend, Robert M. 1983. "Forecasting the Forecasts of Others." *Journal of Political Economy* 91 (4): 546–588.

Woodford, Michael D. 2003. "Imperfect Common Knowledge and the Effects of Monetary Policy." In *Knowledge, Information and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*, edited by Phillipe Aghion et al.

Appendix A Proofs of Propositions

A.1 Proposition 1:

The firm's optimality condition is given by

$$\underbrace{\Phi_1\ P_{it}^{-\theta}\ \mathbb{E}_{it}[\lambda_t P_t^{\theta} B_{it} C_t]}_{\text{Exp. Marginal Revenue}} = \underbrace{\Phi_2\ P_{it}^{-\theta\delta-1}\mathbb{E}_{it}[\lambda_t (P_t^{\theta} B_{it} C_t)^{\delta}\ \ W_{it}]}_{\text{Exp. Marginal Cost}} \tag{14}$$

Under dispersed information, the firm observes sales and its wage bill, which are informationally equivalent to observing $P_t^{\theta} B_{it} C_t$ and W_{it}. Then,

$$\mathbb{E}_{it}[\lambda_t P_t^{\theta} B_{it} C_t] = P_t^{\theta} B_{it} C_t \mathbb{E}_{it}[\lambda_t]$$
$$\mathbb{E}_{it}[\lambda_t (P_t^{\theta} B_{it} C_t)^{\delta}\ \ W_{it}] = (P_t^{\theta} B_{it} C_t)^{\delta}\ \ W_{it}\mathbb{E}_{it}[\lambda_t]$$

Substituting in the optimality condition yields the same expression as that of the firm in a full information economy. Since the two economies are identical in all other aspects, it follows that the full information equilibrium is also one under dispersed information.

A.2 Proposition 2:

If $\theta = \frac{1}{\psi}$ and $\eta = 1$, then total revenue and cost in period $t+s$ become

$$\text{Total Revenue}_t = \Phi_1\ P_{it+s}^{-\theta}\ B_{it+s} M_{t+s} \tag{15}$$

$$\text{Total Cost}_t = \Phi_2\ P_{it+s}^{-\theta\delta}(B_{it+s} M_{t+s})^{\delta}\ \ M_{t+s} Z_{it+s}\ \ N_{it+s}^{\kappa} \tag{16}$$

In other words, profits, marginal revenues and costs do not depend on P_t. We will exploit this property in our discussion of each of the cases below.

Case I: The first order condition of the firm is

$$\underbrace{\Phi_1 \; P_{it}^{-\theta} \; \mathbb{E}_{it-N}[\lambda_t B_{it} M_t]}_{\text{Exp. Marginal Revenue}} = \underbrace{\Phi_2 \; P_{it}^{-\theta\delta-1} \mathbb{E}_{it-N}[\lambda_t (B_{it} M_t)^{\delta} \;\; M_t Z_{it}]}_{\text{Exp. Marginal Cost}} \tag{17}$$

Substituting for λ_t, the optimal price given by

$$\begin{aligned}
P_{it}^{1-\theta+\theta\delta} &= \text{Constant} \; \cdot \; \frac{\mathbb{E}_{it-N}[(B_{it} M_t)^{\delta} \;\; M_t^{1-\zeta} Z_{it}]}{\mathbb{E}_{it-N}[B_{it} M_t^{1-\zeta}]} \\
&= \text{Constant} \; \cdot \; \frac{\mathbb{E}_{it-N} e^{\delta b_{it} + \delta m_t + m_t(1-\zeta) + z_{it}}}{\mathbb{E}_{it-N} e^{b_{it} + m_t(1-\zeta)}} \\
&= \text{Constant} \; \cdot \; \frac{\mathbb{E}_{it-N} e^{\delta(b_{it}+m_t) - \zeta m_t + (m_t + z_{it})}}{\mathbb{E}_{it-N} e^{(b_{it}+m_t) - \zeta m_t}} \\
&= \text{Constant} \; \cdot \; \frac{\mathbb{E}_{it-N} e^{\delta(b_{it}+m_t) - \zeta m_t + (m_t + z_{it})}}{\mathbb{E}_{it-N} e^{(b_{it}+m_t) - \zeta m_t}}
\end{aligned}$$

If shocks are equally persistent, i.e. $\rho_b = \rho_m = \rho$, this becomes

$$P_{it}^{1-\theta+\theta\delta} = \text{Constant} \; \cdot \; \frac{\mathbb{E}_{it-N} e^{\delta\rho^N(b_{it-N}+m_{t-N}) - \zeta\rho^N m_{t-N} + \rho^N(m_{t-N}+z_{it-N})} e^{\tilde{V}_{it}}}{\mathbb{E}_{it-N} e^{\rho^N(b_{it-N}+m_{t-N}) - \zeta\rho^N m_{t-N}} e^{\tilde{U}_{it}}}$$

where $\tilde{U}_{it}$ and $\tilde{V}_{it}$ are functions of $\{u_{t-s}, u_{it-s}, v_{it-s}\}_{s=0}^{N-1}$ and ρ. Under dispersed information, the signals in period $t-N$ allow the firm to perfectly infer the combinations $(b_{it-N} + m_{t-N})$ and $(z_{it-N} + m_{t-N})$. Obviously, these combinations are known to the firm in period $t-N$ under full information. Then, under both informational assumptions, the above expression can be written as

$$\begin{aligned}
P_{it}^{1-\theta+\theta\delta} &= \text{Constant} \; \cdot \; \frac{e^{\delta\rho^N(b_{it-N}+m_{t-N}) + \rho^N(m_{t-N}+z_{it-N})} \;\; \mathbb{E}_{it-N} e^{-\zeta\rho^N m_{t-N}} e^{\tilde{V}_{it}}}{e^{\rho^N(b_{it-N}+m_{t-N})} \;\; \mathbb{E}_{it-N} e^{-\zeta\rho^N m_{t-N}} e^{\tilde{U}_{it}}} \\
&= \text{Constant} \; \cdot \; \frac{e^{\delta\rho^N(b_{it-N}+m_{t-N}) + \rho^N(m_{t-N}+z_{it-N})} \mathbb{E}_{it-N} e^{-\zeta\rho^N m_{t-N}} \mathbb{E}_{it-N} e^{\tilde{V}_{it}}}{e^{\rho^N(b_{it-N}+m_{t-N})} \;\; \mathbb{E}_{it-N} e^{-\zeta\rho^N m_{t-N}} \;\; \mathbb{E}_{it-N} e^{\tilde{U}_{it}}} \\
&= \text{Constant} \; \cdot \; \frac{e^{\delta\rho^N(b_{it-N}+m_{t-N}) + \rho^N(m_{t-N}+z_{it-N})} \;\; \mathbb{E}_{it-N} e^{\tilde{V}_{it}}}{e^{\rho^N(b_{it-N}+m_{t-N})} \;\; \mathbb{E}_{it-N} e^{\tilde{U}_{it}}}
\end{aligned}$$

Under both full and dispersed information, the expression on the right hand side is identical (because the $\mathbb{E}_{it}$ terms in the numerator and denominator are constants). Therefore, it follows that outcomes will be identical in both economies, i.e. dispersed information is irrelevant[16].

Case II: When a single price is to be set for N periods at a time, the optimal price is characterized by

$$P_{it}^{1-\theta+\theta\delta} = \text{Constant} \cdot \frac{\sum_{s=0}^{N} \mathbb{E}_{it}\beta^s \ \left[(B_{it+s}M_{t+s})^{\delta} \ \ M_{t+s}^{1-\zeta} Z_{it+s}\right]}{\sum_{s=0}^{N} \beta^s \ \ \mathbb{E}_{it}[B_{it+s}M_{t+s}^{1-\zeta}]}$$

When shocks are equally persistent, we can rewrite the denominator as

$$\begin{aligned}\sum_{s=0}^{N} \ \beta^s \ \mathbb{E}_{it}[B_{it+s}M_{t+s}^{1-\zeta}] &= \sum_{s=0}^{N} \beta^s \ \ \mathbb{E}_{it} \ \ e^{\rho^{s-1}(b_{it}+m_t)} \ \ e^{-\rho^{s-1}\zeta m_t} \ \ e^{\hat{U}_{it+s}} \\ &= \sum_{s=0}^{N} \ \ \beta^s \ \ e^{\rho^{s-1}(b_{it}+m_t)} \ \ \mathbb{E}_{it} e^{-\rho^{s-1}\zeta m_t} \ \ \mathbb{E}_{it} e^{\hat{U}_{it+s}}\end{aligned}$$

where the second equality uses the fact that $b_{it} + m_t$ is in the firm's information set under both assumptions. In fact, it is easy to see that the informational friction only affects the second term, $\mathbb{E}_{it} e^{-\rho^{s-1}\zeta m_t}$. An identical term shows up in each term of the numerator. When $\rho = 1$, i.e. all shocks are permanent, or $\zeta = 0$, i.e. the discount factor is a constant, the term becomes independent of s and can be factored from both the numerator and denominator. In these two cases, which correspond to the two statements in Proposition 2, dispersed information has no effect on the firm's optimal pricing decision and therefore on allocations in this economy.

Case III: With Calvo pricing, the optimality condition is similar to that of Case II, with one difference. Instead of a deterministic number of periods, we now have a random number of periods for which the price will last. The probability that a price will remain unchanged for exactly T periods is given by $\xi(1-\xi)^{T-1}$, where the ξ is the (exogenous) probability of resetting prices in any given period. Then, both the numerator and the denominator are

[16]Note that, for this case, we have shown a stronger result than the statement of the proposition.

weighted sums, with the weights determined by this probability. It is easy to see that the logic of the proof for Case II goes through exactly for this case as well.

$$P_{it}^{1-\theta+\theta\delta} = \text{Constant} \ \cdot \ \frac{\sum_{T=1}^{\infty} \ (1-\xi)^{T-1}\xi \ \ \left(\sum_{s=0}^{T-1} \ \beta^s \ \mathbb{E}_{it}[(B_{it+s}M_{t+s})^{\delta} \ \ M_{t+s}^{1-\zeta} Z_{it+s}]\right)}{\sum_{T=1}^{\infty} \ (1-\xi)^{T-1}\xi \ \ \left(\sum_{s=0}^{T-1} \beta^s \ \mathbb{E}_{it}[B_{it+s}M_{t+s}^{1-\zeta}]\right)}$$

Case IV: Let us first consider the case where $\zeta = 0$ (i.e. the stochastic discount factor is a constant) and all shocks are equally persistent. Let V^* be the solution to the Bellman equation (6) under dispersed information i.e. under $\mathcal{I}_{it}^H \equiv \{B_{it-s}M_{t-s} \ , \ Z_{it-s}M_{t-s}\}_{s=0}^{\infty}$. We will show that V^* also solves the functional equation (6) under full information, i.e. under $\mathcal{I}_{it}^F \equiv \{B_{it-s}, M_{t-s}, Z_{it-s}\}_{s=0}^{\infty}$.

We begin with a guess that continuation values are the same under both informational assumptions:

$$\mathbb{E}[\ V(P, \mathcal{I}_{it+1}) \ |\mathcal{I}_{it}^F] = \mathbb{E}[\ V^*(P, \mathcal{I}_{it+1}) \ |\mathcal{I}_{it}^H] \tag{18}$$

where the set $\mathcal{I}_{it}^H$ contains only $\{B_{it-s}M_{t-s} \ , \ Z_{it-s}M_{t-s}\}_{s=0}^{\infty}$ corresponding to the full history in $\mathcal{I}_{it}^F$. Now, it is straightforward to show that $\mathcal{I}_{it}^H$ contains the sufficient statistics for forecasting current profits, i.e.

$$\mathbb{E}[\ \Pi(P, \cdot) \mid \mathcal{I}_{it}^F] = \mathbb{E}[\ \Pi(P, \cdot) \mid \mathcal{I}_{it}^H]$$

This is true because both revenues and costs are functions of particular combinations of the current realizations of the shocks and $\mathcal{I}_{it}^H$ contains exactly those combinations. Therefore, given the guess (18) about continuation values, the value of holding prices unchanged is the same under full and dispersed information. The same holds for the value of changing prices. Therefore, since both parts inside the max operator on the right hand side of (6) are the same, it follows that the maximized value is the same too. In other words, given the guess about continuation values, the value under full information is the same as under dispersed information, i.e. equal to $V^*(P_{it}, \mathcal{I}_{it}^H)$. Now, if we show that the $t-1$ expectation of this expression is the same under the two informational assumptions, we have verified the guess and thus, found a fixed point for the full information problem as well. To do this, we note that $V^*(P_{it}, \mathcal{I}_{it}^H)$ is a non-linear function of the price and the two sufficient statistics $B_{it}M_t$ and $Z_{it}M_t$. When all shocks are equally persistent, their corresponding $t-1$ realizations, $B_{it-1}M_{t-1}$ and $Z_{it-1}M_{t-1}$, are sufficient for characterizing the one-period ahead conditional distribution. It then follows that, when all shocks are equally persistent, the conditional expectation of $V^*(P_{it}, \mathcal{I}_{it}^H)$ in period $t-1$ under dispersed information must coincide with that under full information. Thus, we have shown that V^* also solves the functional equation (6) under full information. In other words, values, policies and therefore, allocations are identical under both informational assumptions.

Next, we turn to the case where $\zeta \neq 0$ i.e. λ_t is a random variable but all shocks are permanent ($\rho = 1$). We note that, under full information, the firm's problem has an alternative representation in the form of the following Bellman equation:

$$\begin{aligned}\tilde{V}(P_{it-1}, \mathcal{I}_{it}^F) = \max \quad \{ \; & [\Pi(P_{it-1}, M_t, B_{it}, Z_{it}) + \beta \mathbb{E}_{it} \frac{\lambda_{t+1}}{\lambda_t} \tilde{V}(P_{it-1}, \mathcal{I}_{it+1}^F)], \\ & \max_P \; [\Pi(P, M_t, B_{it}, Z_{it}) - \mathcal{C} + \beta \mathbb{E}_{it} \frac{\lambda_{t+1}}{\lambda_t} \tilde{V}(P, \mathcal{I}_{it+1}^F)]\} \end{aligned} \tag{19}$$

In particular, the policy function induced by the above formulation is identical to the one that emerges from solving (6) under full information. Now, using a very similar argument to the one laid out above, we can show that, if all shocks are permanent, then the function $\tilde{V}$ also solves the Bellman equation above under the corresponding $\mathcal{I}_{it}^H$ i.e.

$$\tilde{V}(P_{it-1}, \mathcal{I}_{it}^F) = \tilde{V}(P_{it-1}, \mathcal{I}_{it}^H)$$

This equality holds because when all shocks are permanent, $\frac{\lambda_{t+1}}{\lambda_t}$ is an iid log-normal random variable. Therefore, the joint distribution of all relevant $t+1$ variables is still summarized by the sufficient statistics in $\mathcal{I}_{it}^H$. If, on the other hand, λ_t was not a random walk, then the current realization of M_t would provide additional information about the growth rate of the discount factor and affect optimal decisions.

Given the equivalence of the value functions, the policy functions (under both informational assumptions) are functions only of elements in $\mathcal{I}_{it}^H$ and therefore, remain unchanged if both sides of (19) are multiplied by λ_t and expectations conditional on $\mathcal{I}_{it}^H$ are taken.

Rearranging, we see that this rescaling yields the original value function V in (6). Thus, we have shown that the policy functions under both informational assumptions are identical, making dispersed information irrelevant for prices and quantities.

Appendix B Solution Algorithm

Step 1: Conjecture about Aggregates: We begin with a conjecture that the aggregate price level follows

$$p_t = m_{t-1} + \rho_p(p_{t-1} - m_{t-1}) + (1 - \sigma_p)u_t \ ,$$

where the coefficients ρ_p and σ_p are to be determined. Rearranging,

$$p_t - m_t = \rho_p(p_{t-1} - m_{t-1}) - \sigma_p u_t \ . \tag{20}$$

Given this conjecture, the vector

$$X_{it} \equiv (b_{it} \quad m_t \quad p_t - m_t)'$$

has a law of motion of the form

$$X_{it} = F \cdot X_{it-1} + G \cdot (u_t \quad u_{it})'$$

Step 2: The Kalman Filter: The evolution of beliefs is given by

$$\mathbb{E}_{it}X_{it} = F\mathbb{E}_{it-1}X_{it-1} + K_t \cdot (s_{it} - H'F\mathbb{E}_{it-1}X_{it-1}) \ ,$$

where K_t is the Kalman gain matrix and $H' = [\gamma \quad 1 \quad r]$. We focus on the time-invariant case where $K_t = K$ for all t. Standard results from filtering theory can be used to characterize K. Then, using the properties of the filter and the laws of motion above, we can conditional distribution of one-step ahead beliefs, i.e.

$$\mathbb{E}_{it}X_{it} \ \sim \ \mathcal{N}\left(F\mathbb{E}_{it-1}X_{it-1}, \ \tilde{Q}\right)$$

Step 3: Value Function Iteration: We can rewrite the Bellman equation (13) as follows

$$V(p_{it-1}, \mathbb{E}_{it}X_{it}) = \max \quad \{ \ \mathbb{E}_{it}[-(p_{it} - p^*_{it})^2 + \beta V(p_{it}, \mathbb{E}_{it+1}X_{it+1})], \\ \max_p \ \ \mathbb{E}_{it}[-(p - p^*_{it})^2 - \mathcal{C} + \beta V(p, \mathbb{E}_{it+1}X_{it+1})]\} \tag{21}$$

Using a discrete grid for each of the states, this problem can be solved using standard iterative procedures.

Step 4: Simulation and Verification: Data are then simulated for 10000 firms for 1200 periods using the policy functions derived above. A regression of the form (20) is used to estimate the coefficients $\hat{\rho}_p$ and $\hat{\sigma}_p$. If they match the corresponding ones in the original conjecture, we are done ! If not, the conjecture is updated and the process repeated until convergence is obtained. The simulated data are then used to estimate impulse responses and other moments of interest.