Industrial Dynamics, International Trade, and Economic Growth

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Abstract

This paper presents a dynamic general equilibrium model to illustrate how international trade and dynamic trade policies affect industrialization, industrial upgrading, and economic growth in a two-country world, where there is an infinite number of possible industries different in their capital intensities. Analytical solutions are obtained to fully characterize the endowment-driven industrialization and inverse-V-shaped life cycle of each underlying industry along the aggregate growth path. We show that industrial upgrading and aggregate growth can be facilitated or hampered by the investment-specific technology progress in the trade partner, depending on whether the *intertemporal elasticity of substitution* is larger than unity. This is because it determines whether the intertemporal terms-of-trade effect dominates the intertemporal market-size effect. We also analytically characterize the growth effect of any arbitrary dynamic trade policies. Accelerating trade liberalization is shown to have a non-monotonic impact on the speed of industrial upgrading and economic growth, again depending on the magnitude of the intertemporal elasticity of substitution.

Key Words: Industrial Dynamics, International Trade, Economic Growth, Structural Change, Trade Policies

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1 Introduction

How is economic growth affected by international trade and trade liberalization? How is structural change at the disaggregated industry level (industrial dynamics) affected by international trade and trade liberalization? In this paper we develop a dynamic general equilibrium model to address these two important questions in a unified and tractable framework.

The first question has been intensively studied but still far from being settled.[1] Empirically, while some researchers claim that their cross-country regression results support that international trade and trade liberalization help increase the income level and/or boost economic growth (see, for example, Sachs and Warner (1995), Edwards (1998), Frankel and Romer (1999), Wacziarg and Welch (2008)), others cast doubt on the legitimacy of such claims by contending that there are serious flaws in the methodologies, indexes, data sets, or interpretations of the regression results in those analyses. The most notable critiques are perhaps Rodriguez and Rodrik (2001).

The first goal of this paper, therefore, is to shed some new lights on this debate by showing that the impact of trade and trade policies on economic growth can be non-monotonic. In particular, in a free-trade dynamic world, both economic convergence and divergence are shown to be possible, depending on whether the trade partner has a faster investment-specific technological change (or ISTC thereafter) *a la* Greenwood, Hercowitz and Krusell (1997). Moreover, the output growth can be facilitated or hampered when the rate of ISTC increases in the foreign country, depending on whether the *intertemporal elasticity of substitution* is larger than one. This is because there are two competing effects when the rate of foreign ISTC increases. One is the intertemporal terms-of-trade effect, which tends to raise the saving rate and output growth rate because imports become increasingly cheaper over time. The second effect is the market-size expansion effect, which tends to increase the domestic consumption and lower the saving rate and output growth rate because the household income in the home country increases as the export market expands. These two effects exactly cancel out when the intertemporal elasticity of substitution is equal to one. When it is larger than unity, the intertemporal terms-of-trade effect is dominant and hence the home country's output growth is facilitated by the ISTC in the foreign country, *vice versa*. We also characterize the impact on growth of any arbitrary dynamic tariff adjustment. Speed of tariff adjustment matters. In particular, we show that a time-invariant tariff rate has no growth effect, but accelerating trade liberalization would first boost economic growth and then hurt economic growth when the intertemporal substitution elasticity is larger than one. In a more general model where tariff affects the expenditure share of imports, a unilateral tariff reduction may increase or decrease the growth rates of consumption and output, depending on (1) whether the intertemporal elasticity of

[1] Wonderful theoretical treatment and surveys include Grossman and Helpman (1991) and Ventura (2005). Edwards (1993) and Baldwin (2004) provide nice surveys on the empirical literature.

substitution is larger than unity; (2) whether the home country has a higher ISTC rate, and (3) whether the marginal change in the expenditure share on imports is sufficiently sensitive to a tariff reduction in the foreign country. To our knowledge, this is the first paper that highlights the importance of the intertemporal elasticity of substitution in determining the trade impact on growth in the literature.

The second, perhaps also more important, goal of this paper is to explore the impact of trade and dynamic trade policies on the structural change at a disaggregated industry level. Existing literature of structural change mainly focuses on the Kutnetz facts, which refer to the composition shift in the three aggregate sectors (namely, agriculture, industry, and service). For example, Mastuyama (2008) constructs a three-sector growth model with trade to show that, contrary to the predictions of a closed-economy model, now the productivity increase in the manufacturing sector of a country does not necessarily imply a decline of that sector. Yi and Zhang (2010) introduce the Eaton-Kortum trade with heterogeneous firms into the three-sector model, showing how international trade affects the structural change. Complementary to these studies, we investigate the industrial dynamics at the more disaggregated industry level and have both capital and labor as production factors. In our model, there are infinite industries with different capital intensities, capturing the fact that even the manufacturing sector alone covers a wide spectrum of sub-industries ranging from the labor-intensive apparels and textiles up to very capital-intensive aircraft and precision equipment. This setting allows us to study the Heckscher-Ohlin dynamic trade and industrial dynamics, while the aforementioned papers study the Ricardian trade with labor as the only input. Dornbusch, Fischer and Samuelson (1980) study the HO model with a continuum of industries. Bernard, Redding and Schott (2007) introduce two factors and two industries into the Melitz (2003) model with infinite heterogeneous firms. Burstein and Vogel (2011) also study the relative factor prices in a very general trade setting. However, all of these models are static.

Changes in the aggregate output and industrial development are shown to be positively synchronized. That is, the underlying industries upgrade faster as the aggregate output grows faster. Therefore, the aforementioned non-monotonicity results for the aggregate growth also apply for the speed of structural change at the disaggregated industry level.[2] In addition, we obtain closed-form solutions to characterize how international trade and trade policies affect the timing of industrialization and the whole inverse-V-shaped life cycle of each industry along the aggregate growth path: as capital accumulates endogenously and reaches certain threshold, a new industry appears, booms, reaches the peak, and eventually declines and is ultimately replaced by an even more capital-intensive new industry, *ad infinitum.* The model generates the inverse-V-shaped pattern of output and export for each

[2] McMillan and Rodrik (2011) show empirically that trade openness causes desirable structural transformation in some countries in the sense that labor moves into the sectors with higher productivities, but trade openness results in "undesirable" structural transformation in some other countries, where the relative high-TFP sector (industry) is destroyed by trade and labor moves to the sectors with low productivies and unemployment also rises.

industry, which is qualitatively consistent with the empirical pattern of industrial dynamics (for example, see Chenery *et al* (1986) and Schott (2003)).[3]

Our analysis highlights the role of endogenous capital accumulation in driving the life-cycle dynamics of all the alternating disaggregated industries along the aggregate growth path in an open environment. Our analysis is closely related to the dynamic Heckscher-Ohlin literature. Ventura (1997) constructs a two-sector growth model to show that trade causes factor price equalization and hence sustains a high return to capital in the developing countries, which helps those countries save more and thus converge to the rich countries. Recently, Bajona and Kehoe (2010) argue that factor price equalization may not hold in each period once some restrictive assumptions in Ventura (1997) are relaxed. In addition, they show that both convergence and divergence are possible, depending on the elasticity of substitution between the traded goods. Caliendo (2011) analytically characterizes the whole dynamics when the production technology is Cobb-Douglas in the Bajona-Kehoe two-sector world. He shows that the specialization pattern is not monotonic and countries are most likely to diverge. Different from the two-sector models in the literature, the infinite-industry setting in our model allows us to derive the endless industrial upgrading process and characterize the complete inverse-V-shaped life cycle of each industry along the balanced growth path. Again, we emphasize the role of intertemporal elasticity of substitution instead of the substitution elasticity between tradables highlighted in the existing literature.[4]

From the methodological perspective, it is technically challenging to fully characterize the whole dynamics even for a trade model with only two sectors (see, for example, Chen 1992, Caliendo, 2011, Nishimura and Shimomura, 2002, Boldrin and Deneckere, 1990). Now we have infinite industries in an infinite-horizon general equilibirum trade environment. The form of the aggregation production function itself may change endogenously as a consequence of the endogenous structural change in the underlying industries. Ultimately we must deal with a Hamiltonian system with endogenously switching state equations subject to trade interdependence. Despite all these complicating elements, fortunately, we still obtain a closed-form solution to fully characterize the whole dynamic system including the initial transitional process of industrialization and the inverse-V-shaped industrial dynamics of each individual industry along the aggregate growth path.

There is a huge literature addressing the roles of innovation and technology adoption (diffusion) in driving the industrial dynamics, product cycles, and economic growth. For example, Krugman (1979) formalizes Vernon's product-cycle ideas by constructing a horizontal innovation and imitation model to show that the South converges to the North if and only if the imitation speed exceeds the innovation speed. Grossman and Helpman (1989, 1991) and Eaton and Kortum (2001) present

[3] Ju, Lin and Wang (2010) document the data pattern of the inverse-V-shaped industrial dynamics with the US data of the manufacturing sector at six digit industry level covering 473 industries from 1958 to 2005. The cross-country evidence on the inverse-V-shaped industrial pattern based on the UNIDO data sets is provided in Haraguchi and Rezonja (2010).

[4] For dynamic Hechscher-Ohlin models in a small open economy, please refer to Findlay (1970), Mussa (1978), Atkeson and Kehoe (2000).

the multi-country growth and trade models with endogenous horizontal innovation and imitation. Flam and Helpman (1987) and Stokey (1991) are two static models that examine the vertical product differentiation and innovation in trade. Our model complements these studies by focusing on the mechanism of endowment-driven industrial dynamics and growth. As capital becomes more abundant and cheaper, industries tend to shift to those that use capital more intensively.[5] The consumption growth is shown to be always facilitated by its trade partner's capital accumulation due to the terms-of-trade effect, although the output growth and industrial upgrading *might* slow down. Ederington and McCalman (2009) study how international trade affects industrial evolution when firms make strategic dynamic decisions on technology choices as the production cost (again, labor is the only input) exogenously decreases over time. In our model, the production cost changes endogenously over time depending on the capital accumulation.

The paper is organized as follows. In Section 2, we set up a static general equilibrium model of two-country international trade. In Section 3 and Section 4, we develop an endogenous growth model with free trade. Section 5 examines the role of static and dynamic trade policies when the expenditure share on imports is exogenous and fixed. Section 6 examines the robustness of the main results in a more general setting. The last section concludes.

2 Static Trade Model

2.1 Environment

Consider a world with two countries indexed by $i = 1, 2$. There is a unit mass of identical households in each country. Each household in country i is endowed with L_i units of labor and E_i units of capital. The aggregate output of country i is produced with the following technology

$$X_i = \sum_{n=0}^{\infty} \lambda_n x_{i,n},$$

where $x_{i,n}$ denote the output of intermediate good n in country i and λ_n is the productivity coefficient for good n, where $n \geq 0$. Each intermediate good represents an industry, so there are infinite possible industries in each country.[6] We require $x_{i,n} \geq 0$ for any n.

[5] Acemoglu (2007) shows that technical change is biased toward using more abundant production factors.

[6] The assumption of perfect substitutability across different industries in the final output is adopted mainly for analytical simplicity, which is quite usual in the growth literature. For example, the agriculture Malthus production and the modern Solow production are two linearly additive components for the total output in Hansen and Prescott (2002). Also see Lucas (2009). It can be shown that the main qualitative features will remain valid when the substitution is imperfect, but closed-form characterization becomes infeasible. For more details, please see the closed-economy model in Ju, Lin and Wang (2010).

Consumers love the diversity of consumption goods and hence want to consume the aggregate goods produced by both the home country and the foreign country. For simplicity, we follow Acemoglu and Ventura (2002) by adopting the Armington assumption (Armington, 1966). That is, the final consumption good in country i is defined as

$$C_i = C_{i,1}^{\alpha} C_{i,2}^{\beta}, \tag{1}$$

where $C_{i,j}$ denotes country i's consumption of the aggregate consumption good produced by country j, where $i, j \in \{1, 2\}$. Assume $\alpha \geq 0$, $\beta \geq 0$, and $\alpha + \beta = 1$.[7] Intermediate goods are only useful in the domestic production so they are not traded. Capital and labor can move freely across different industries within a country but cannot move internationally. Since the law of one price holds under free trade, there will be no trade in the final good. We set the final good as the numerare.

The utility function of a representative household in country i is $CRRA$:

$$U_i = \frac{C_i^{1-\sigma} - 1}{1 - \sigma}, \text{ where } \sigma \in (0, \infty). \tag{2}$$

All the production technologies exhibit constant returns to scale. In particular, intermediate good 0 is produced with labor only. One unit of labor produces one unit of good 0. For any other intermediate good $n \geq 1$, the production function is Leontief: [8]

$$F_n(k, l) = \min\{\frac{k}{a_n}, l\}, \tag{3}$$

where a_n is the capital requirement to produce one unit of good n. Intermediate good 0 may be interpreted as a traditional "Malthusian" sector in the sense of Hansen and Prescott (2002) because the output grows only when the population grows. All the goods $n \geq 1$ as a whole may be interpreted as a modern "Solow" sector.

Without loss of generality, we assume a_n increases with n. Empirical evidence suggests that the productivity of the more capital-intensive intermediate inputs is generally higher (presumably as it embodies better technology), so we assume λ_n also increases in n. Therefore, a higher-indexed good has a higher productivity and is also more capital intensive. To obtain analytical solutions, we assume the following simplest parametric forms:

$$\lambda_n = \lambda^n, \ a_n = a^n, \tag{4}$$

$$\lambda > 1 \text{ and } a - 1 > \lambda. \tag{5}$$

[7] Later on, we will generalize our analysis by allowing α and β to be country-specific and endogenous to the trade policies.

[8] It drastically simplifies the dynamic strucutral analysis by giving us a lot of linearities. We can show that the main results remain valid with Cobb-Douglas production function, but the dynamic analysis will be much more complex. Houthakker (1956) shows that Leontief production functions with Pareto-distribution heterogenous paratermers can aggregate into Cobb-Douglas production functions. Lagos (2006) constructs another distribution that can aggregate heterogenous Leontief functions into CES production functions. These may be helpful in understanding how firm heterogeneities may affect our results, which we leave for future research.

With Leotief production functions and perfectly substitutable intermediate goods, the last inequality in (5) must be imposed to rule out the trivial case that only the good with the highest productivity is produced and to take care of good 0, which requires labor alone. But none of these parametric assumptions are crucial for the main qualitative results.

2.2 Market Equilibrium

All the markets are perfectly competitive. Let P_i denote the price of aggregate good X_i for country $i = 1, 2$. Let $p_{i,n}$ denote the price of intermediate good n in country i. Let r_i denote the rental price of capital and w_i denote the wage rate in country i. A profit-maximizing firm in country i solves

$$\max_{x_{i,n} \geq 0} \left[P_i \sum_{n=0}^{\infty} \lambda^n x_{i,n} - \sum_{n=0}^{\infty} p_{i,n} x_{i,n} \right],$$

which implies

$$p_{i,n} = \lambda^n P_i = \begin{cases} w_i + a^n r_i, & \text{when} \quad n \geq 1 \\ w_i, & \text{when} \quad n = 0 \end{cases}. \tag{6}$$

The total income of a representative household in country i is $w_i L_i + r_i E_i$, which is also equal to the total value added $P_i X_i$. The household problem in country i is to maximize (2) subject to the following budget constraint

$$P_1 C_{i,1} + P_2 C_{i,2} = P_i X_i. \tag{7}$$

Goods markets clear internationally:

$$C_{1,1} + C_{2,1} = X_1;\ C_{1,2} + C_{2,2} = X_2.$$

In the equilibrium we have

$$C_1 = \alpha X_1^{\alpha} X_2^{\beta},\ C_2 = \beta X_1^{\alpha} X_2^{\beta}. \tag{8}$$

That is, the aggregate consumption of each country is a Cobb-Douglas function of the aggregate goods produced by the two countries. (8) also implies that the aggregate consumption ratio of the two countries is equal to the ratio of their expenditure shares on the domestic aggregate goods. We can show that in the equilibrium at most two intermediate goods will be produced in each country, and if two, they must be adjacent in capital intensities. More precisely, given the capital and labor endowment of the two countries $\{E_i, L_i\}_{i=1}^2$, there exists a unique competitive equilibrium, which is summarized in Table 1.

Table 1: Static Trade Equilibrium

$0 \le E_i < aL_i$	$a^n L_i \le E_i < a^{n+1} L_i$ for $n \ge 1$
$x_{i,0} = L_i - \frac{E_i}{a}$	$x_{i,n} = \frac{L_i a^{n+1} - E_i}{a^{n+1} - a^n}$
$x_{i,1} = \frac{E_i}{a}$	$x_{i,n+1} = \frac{E_i - a^n L_i}{a^{n+1} - a^n}$
$x_{i,,j} = 0$ for $\forall j \neq 0, 1$	$x_{i,j} = 0$ for $\forall j \neq n, n+1$
$\frac{r_i}{w_i} = \frac{\lambda - 1}{a}$	$\frac{r_i}{w_i} = \frac{\lambda - 1}{a^n (a - \lambda)}$
$X_i = L_i + (\lambda - 1) \frac{E_i}{a}$	$X_i = \frac{\lambda^{n+1} - \lambda^n}{a^{n+1} - a^n} E_i + \frac{\lambda^n (a - \lambda)}{a - 1} L_i$
$\Leftrightarrow E_{i,(0,1)} = \frac{a}{\lambda - 1} (X_i - L_i)$	$\Leftrightarrow E_{i,(n,n+1)} = \left[X_i - \frac{\lambda^n (a - \lambda)}{a - 1} L_i \right] \frac{a^{n+1} - a^n}{\lambda^{n+1} - \lambda^n}.$

Proposition 1 *In the static trade world, there exists a unique equilibrium, in which for any country $i \in \{1, 2\}$, the industrial and aggregate output are given by Table 1. Consumption C_i is given by (8). The equilibrium wage rate w_i, rental rate r_i, and prices for each intermediate good $p_{i,n}$ and final good P_i are given by (6) and the following:*

$$
\begin{aligned}
P_1 &= \alpha \left(\frac{X_2}{X_1} \right)^{\beta} \text{ and } P_2 = \beta \left(\frac{X_1}{X_2} \right)^{\alpha}, \\
w_i &= \begin{cases} P_i & \text{when} \quad 0 \le E_i < aL_i \\ \frac{\lambda^n (a - \lambda)}{a - 1} P_i & \text{when} \quad a^n L_i \le E_i < a^{n+1} L_i, \forall n \ge 1 \end{cases}, \\
r_i &= \begin{cases} \frac{\lambda - 1}{a} P_i & \text{when} \quad 0 \le E_i < aL_i \\ \frac{\lambda^{n+1} - \lambda^n}{a^{n+1} - a^n} P_i & \text{when} \quad a^n L_i \le E_i < a^{n+1} L_i, \forall n \ge 1 \end{cases}.
\end{aligned}
$$

Proof. For table 1, please refer to the proof of the closed-economy equilibrium in Proposition 1 in Ju, Lin and Wang (2010). The prices are derived from (6) together with the normalization assumption for the ultimate consumption good:

$$\left(\frac{P_1}{\alpha} \right)^{\alpha} \left(\frac{P_2}{\beta} \right)^{\beta} = 1$$

and the term of trade is

$$\frac{P_1}{P_2} = \frac{\alpha X_2}{\beta X_1}, \tag{9}$$

which is derived from the balanced trade condition. ∎

This proposition suggests that *generically* there exist only two industries in each country, and the capital intensities of these two industries are the closest to the capital-labor ratio of the economy. As the capital-labor ratio increases, the industries also become more and more capital intensive with the labor-intensive industries gradually replaced by the more capital-intensive ones. This can be illustrated more intuitively by Figure 1.

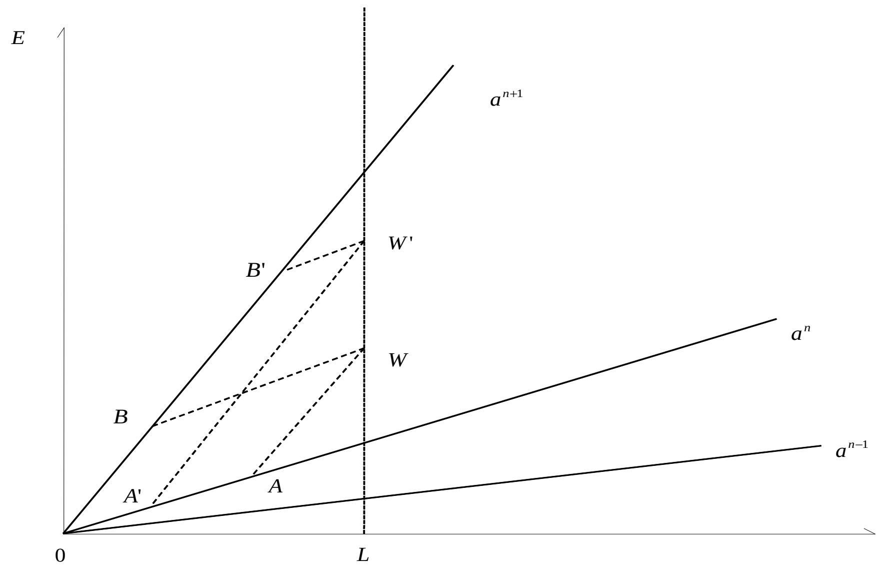

Figure 1. How Factor Endowment Determines the Optimal Industries in an Open Economy

The horizontal and vertical axes are labor and capital, respectively. Point O is the origin and Point $W = (L, E)$ denotes the endowment of the economy. When $a^n L < E < a^{n+1} L$, as shown in the current case, only goods n and $n+1$ are produced. The factor market clearing conditions determine the equilibrium allocation of labor and capital in industries n and $n+1$, which are represented by vector OA and vector OB, respectively, in the parallelogram $OAWB$. The equilibrium output c_n is the X-coordinate of point A and c_{n+1} is the X-coordinate of point B. If capital increases so the endowment point moves from W to W', the new equilibrium becomes parallelogram $OA'W'B'$ so that c_n decreases but c_{n+1} increases. When $E = a^n L$, only good n is produced. Similarly, if $E = a^{n+1} L$, only good $n+1$ is produced.

Table 1 states that the final output of each country is a linear function of its capital and labor endowments. Moreover, the form of the aggregate production function changes when the capital-labor ratio shifts across different diversification cones, reflecting the structural change in the underlying industries. The rental-wage ratio weakly decreases as the capital-labor ratio increases. The term of trade deteriorates when the capital endowment becomes larger holding labor endowment fixed. This property holds whenever the substitution elasticity between the two tradables in the Armington aggregate is finite. Notice, however, the production decision in each country is not affected by the way how the two country-specific final goods are aggregated in the Armington final good.

3 Dynamic Model with Free Trade

Now we develop a dynamic model to characterize the complete industrial dynamics. Without loss of generality, we focus on the problem in country 1. By the second welfare theorem, we can characterize the competitive equilibrium by resorting to the following social planner problem:

$$\max_{C_1(t)} \int_0^\infty \frac{C_1(t)^{1-\sigma}-1}{1-\sigma} e^{-\rho t} dt$$

subject to

$$\dot{K}_1 = \xi_1 K_1(t) - E(X_1(t)) \tag{10}$$

$$X_1(t) = \left[\frac{C_1(t)}{\alpha X_2^\beta(t)}\right]^{\frac{1}{\alpha}} \tag{11}$$

$$K_1(0) \text{ is given,}$$

where ρ is the time discount rate, and $K_1(t)$ is the *stock* of working capital at t, which cannot be traded or used for direct consumption. At each time, the capital inherited from the past can be transformed into new working capital using the standard learning-by-doing AK technology and ξ_1 is the technology parameter that measures the investment-specific technological change rate (Greenwood, Hercowitz, and Krusell, 1997).[9] All the new working capital can be used to either produce the consumption good or to save (invest). $E(X_1(t))$ is the total capital *flow* used to produce the aggregate good $X_1(t)$ and then fully depreciates. (11) comes from (8), which links the two countries together. All the consumption goods are non-storable. The total labor endowment L_1 is constant over time.[10] Following the pertinent literature, to ensure a positive consumption growth and to exclude the explosive solution, we assume

$$0 < \xi_i - \rho < \sigma \xi_i, \forall i = 1, 2. \tag{12}$$

Table 1 indicates that $E(X_i)$ is a strictly increasing, continuous, piecewise linear function of X_i. It is not differentiable at $X_i = \lambda^n L_i$, for any $n = 0, 1, ...$ Therefore, the above dynamic problem may involve changes in the functional forms of the state equation. That is, (10) can be rewritten as

$$\dot{K}_1 = \begin{cases} \xi_1 K_1, & \text{when} \quad X_1 < L_1 \\ \xi_1 K_1 - E_{1,(0,1)}(X_1), & \text{when} \quad L_1 \le X_1 < \lambda L_1 \\ \xi_1 K_1 - E_{1,(n,n+1)}(X_1), & \text{when} \quad \lambda^n L_1 \le X_1 < \lambda^{n+1} L_1, \text{ for } \forall n \ge 1 \end{cases},$$

where $E_{1,(n,n+1)}(X_1)$ is defined in Table 1 for any $n \ge 0$, denoting how much capital is used to produce X_1 when only industries n and $n+1$ coexist in country 1.

[9] A standard endogenous-growth interpretation for the AK model is that the productivity A is endogenouly determined by the amount of production as measured by the capital input, subject to decreasing return to scale. That is, $A(K) = \xi K^\alpha$. It captures the learning by doing. The production function for the final output is also subject to decreasing return to scale conditional on the productivity: $Y = A(K)K^{1-\alpha}$, thus the total output ultimately equals ξK , ensuring the sustainable growth.

[10] This setting is convenient to examine how exogenous changes in the "effetive labor" or population growth (for example, let $L(t) = L_0 e^{\gamma t}$ for some $\gamma > 0$) may affect the economic dynamics.

We can verify that, in this dynamic optimization problem, the objective function is strictly increasing, differentiable and strictly concave while the constraint set forms a continuous convex-valued correspondence, hence the equilibrium must exist and also be unique. The optimization problem for country 2 can be written symmetrically. For simplicity, international borrowing is prohibited so that trade is balanced at each time point, therefore the value of total import into country 1 equals to the value of total import into country 2:

$$\beta P_1(t) X_1(t) = \alpha P_2(t) X_2(t), \forall t. \tag{13}$$

3.1 Economic Growth

For any $i = 1, 2$, let $t_{i,0}$ denote the endogenous *final* time point when the aggregate output equals L_i in country i, which is also the starting time of industrialization because the output per capita will grow afterwards. Let $t_{i,n}$ denote the first time point when $X_i = \lambda^n L_i$ for any $n \geq 1$, that is the time when industry n reaches its peak. It turns out that aggregate consumption $C_1(t)$ is monotonically increasing over time in the equilibrium (to be verified soon), hence the problem can be rewritten as

$$\max_{C_1(t)} \int_0^{t_{1,0}} \frac{C_1(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt + \sum_{n=0}^{\infty} \int_{t_{1,n}}^{t_{1,n+1}} \frac{C_1(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt$$

subject to

$$\dot{K}_1 = \begin{cases} \xi_1 K_1 & \text{when} & 0 \leq t < t_{1,0} \\ \xi_1 K_1 - E_{1,(0,1)}(X_1), & \text{when} & t_{1,0} \leq t < t_{1,1} \\ \xi_1 K_1 - E_{1,(n,n+1)}(X_1), & \text{when} & t_{1,n} \leq t < t_{1,n+1}, \text{ for } n \geq 1 \end{cases},$$

$K_1(0)$ is given.

According to Table 1, when $t_{1,0} \leq t < t_{1,1}$, goods 0 and 1 are produced and the capital requirement function is given by $E_{1,(0,1)}(X_1) = \frac{a}{\lambda-1}(X_1 - L_1)$. When $t_{1,n} \leq t < t_{1,n+1}$ for any $n \geq 1$, goods n and $n+1$ are produced and $E_{1,(n,n+1)}(X_1) = \left[X_1 - \frac{\lambda^n(a-\lambda)}{a-1} L_1\right] \frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n}$. If $K_1(0)$ is sufficiently small (to be more precise below), then there exists a time period $[0, t_{1,0}]$ in which only good 0 is produced so that $E(t) = 0$ when $0 \leq t \leq t_{1,0}$. If $K_1(0)$ is sufficiently large, on the other hand, the economy may start with producing good $\widetilde{n}$ and $\widetilde{n}$ +1 for some $\widetilde{n} \geq 1$, then $t_{1,n}$ is not defined for any $n = 0, 1, .., \widetilde{n}$.

For the future reference, we introduce the following notations for the consumption growth rate and the output growth rate:

$$\theta_i(t) \equiv \frac{\dot{C}_i(t)}{C_i(t)}; \ h_i(t) \equiv \frac{\dot{X}_i(t)}{X_i(t)}, \text{ for } i = 1, 2.$$

Proposition 2 *In the dynamic free-trade equilibrium,*

$$\theta_1(t) = \theta_1(t) = \begin{cases} 0, & when \quad t < \min\{t_{1,0}, t_{2,0}\} \\ \frac{\xi_1 - \rho}{(\frac{\beta}{\alpha} + \sigma)}, & if \quad t_{1,0} \leq t < t_{2,0} \\ \frac{\xi_2 - \rho}{(\frac{\beta}{\alpha} + \sigma)}, & if \quad t_{2,0} \leq t < t_{1,0} \\ \frac{\alpha\xi_1 + \beta\xi_2 - \rho}{\sigma}, & when \quad t \geq \max\{t_{1,0}, t_{2,0}\} \end{cases} . \tag{14}$$

$$h_1(t) = \begin{cases} 0, & when \quad t < \min\{t_{1,0}, t_{2,0}\} \\ \frac{\xi_1 - \rho}{\beta + \alpha\sigma}, & if \quad t_{1,0} \leq t < t_{2,0} \\ 0, & if \quad t_{2,0} \leq t < t_{1,0} \\ \beta(\xi_1 - \xi_2) + \frac{\alpha\xi_1 + \beta\xi_2 - \rho}{\sigma}, & when \quad t \geq \max\{t_{1,0}, t_{2,0}\} \end{cases} , \tag{15}$$

$$h_2(t) = \begin{cases} 0, & when \quad t < \min\{t_{1,0}, t_{2,0}\} \\ 0, & if \quad t_{1,0} \leq t < t_{2,0} \\ \frac{\xi_2 - \rho}{\beta + \alpha\sigma}, & if \quad t_{2,0} \leq t < t_{1,0} \\ \alpha(\xi_2 - \xi_1) + \frac{\alpha\xi_1 + \beta\xi_2 - \rho}{\sigma}, & when \quad t \geq \max\{t_{1,0}, t_{2,0}\} \end{cases} , \tag{16}$$

where $t_{1,0}$ and $t_{2,0}$ are given by (19) in Lemma 2 below.

Proof. Refer to the Appendix 1. ■

This proposition states that the aggregate consumption of the two countries will grow at the same rate, which generally depends on the technology parameters of both countries. Obviously, when $\alpha = 1$ ($\beta = 0$), country 1 becomes a closed economy, which is characterized in Ju, Lin and Wang (2010). Similar argument applies to country 2 when $\alpha = 0$ ($\beta = 1$). The result of equal consumption growth comes from the assumption that the two countries have the same Armington Cobb-Douglas production function in (1). If the two countries have different expenditure shares on the imports and exports (because of home bias, for example) in their total consumption budget, then the final consumption growth rates are generally different, which we will show later.[11] This proposition shows that the output growth rates are generally different for the two countries. Before $t_{i,0}$, country i is in the "Malthusian" regime in the sense that the total output must be equal to its labor (population) endowment and thus output per capita stays constant over time. Let i^* (or i^{**}) denote the index of the country which starts to produce good 1 earlier (or later), where $i^*, i^{**} \in \{1, 2\}$. The following two figures depict the time path of the output of the total consumption goods in country i^* and country i^{**}, respectively.

[11] Notice that the share of import or export in the total GDP is endogenous, not necessarily fixed, as GDP incorporates both consumption goods and capital goods.

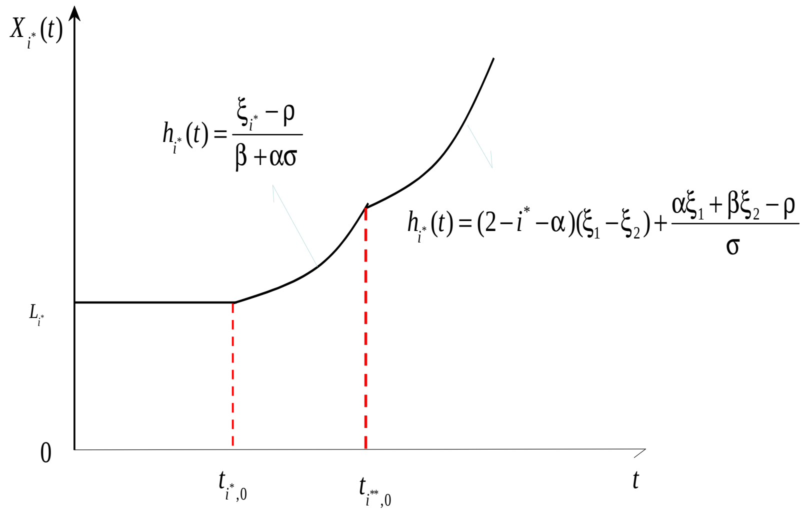

Figure 2. The time path of output X in country i^*, which starts to produce good 1 earlier than its trade partner.

Figure 2 shows that after $t_{i^*,0}$ country i^* enters the "Solow" regime with positive per capita output growth. More specifically, the output growth rate changes twice in the country which "industrializes" (that is, to start producing good 1) earlier than its trade partner. The first turning point is $t_{i^*,0}$, when industrialization takes place in the home country. The second time is $t_{i^{**},0}$, when industrialization occurs in the foreign country. Under assumption (12), the growth rate becomes strictly larger at $t_{i^*,0}$, but the growth rate at $t_{i^{**},0}$ may or may not change, depending on the parameters. For example, when $i^* = 1$, the output growth rate of country 1 strictly increases at $t_{2,0}$ if and only if

$$[\sigma - (\beta\sigma + \alpha)(\beta + \alpha\sigma)]\, \xi_1 > (\beta + \alpha\sigma)(\beta - \beta\sigma)\xi_2 + (\sigma - \beta - \alpha\sigma)\rho.$$

Thus the growth rate would not change if $\sigma = 1$, independent of ξ_1 and ξ_2. By contrast, for the country which industrializes later, the output growth rate changes only once, as is depicted in Figure 3. In addition, the growth rate can become negative after the change.

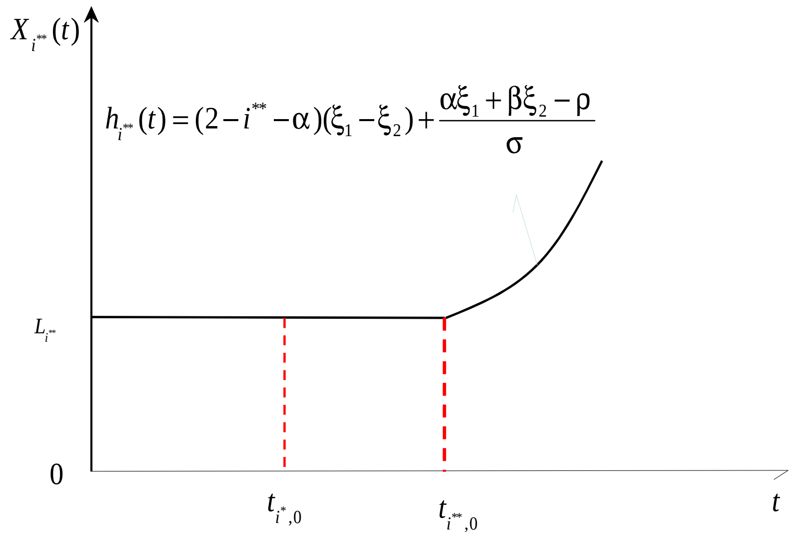

Figure 3. The time path of output X in country i^{**}, which starts to produce good 1 later than its trade partner.

(14) implies that, eventually (namely, when $t \geq \max\{t_{1,0}, t_{2,0}\}$), the aggregate

consumption growth is faster when the investment-specific technology parameter of the trade partner increases. As an immediate implication of this proposition, we have the following comparative statics results.

Corollary 1 *[1]* $\frac{\partial h_1}{\partial \xi_1} > 0, \frac{\partial h_2}{\partial \xi_2} > 0$ *for any* $\sigma > 0$*; [2]*$\frac{\partial h_1}{\partial \xi_2} \geq 0, \frac{\partial h_2}{\partial \xi_1} \geq 0$*, when* $\sigma \in (0, 1]$*, "=" only if* $\sigma = 1$*;[3]*$\frac{\partial h_1}{\partial \xi_2} < 0, \frac{\partial h_2}{\partial \xi_1} < 0$*, when* $\sigma > 1$.

Part [1] is intuitive. Parts [2] and [3] point to the importance of the intertemporal elasticity of substitution in determining how the output growth rate is affected by the trade partner's efficiency in the capital good production. The output growth is mainly determined by how fast capital accumulates via the endogenous saving decision. Suppose the production efficiency of capital good increases in country 2 (that is, ξ_2 becomes larger), it will generate two opposite effects. First, the dynamic terms-of-trade effect implies that households in country 1 should substitute today's consumption for tomorrow's consumption as imports become increasingly cheaper. This intertemporal substitution effect means that country 1 should save more capital today and hence will have a faster output growth. Second, the intertemporal market-size effect implies that country 1 should consume more because its export revenue grows faster as the market size of the trade partner increases faster. More consumption implies less saving, so the output growth is slowed down.

When the intertemporal elasticity of substitution is larger, the dynamic terms-of-trade effect becomes stronger because consumers are willing to save more today. In particular, when the intertemporal elasticity of substitution is unity ($\frac{1}{\sigma} = 1$), the dynamic terms-of-trade effect and the market-size effect exactly cancel out. In other words, the investment-specific technological progress of the trade partner will enhance domestic output growth ($\frac{\partial h_1}{\partial \xi_2} > 0$ and $\frac{\partial h_2}{\partial \xi_1} > 0$) if and only if the intertemporal elasticity of substitution is larger than unity ($\frac{1}{\sigma} > 1$). The following lemma summarizes the dynamics of the prices and terms of trade.

Lemma 1 *For any $t \geq 0$,*

$$\frac{\overset{\bullet}{P_1}(t)}{P_1(t)} = \beta\left[h_2(t) - h_1(t)\right]; \; \frac{\overset{\bullet}{P_2}(t)}{P_2(t)} = \alpha\left[h_1(t) - h_2(t)\right], \tag{17}$$

where $h_1(t)$ and $h_2(t)$ are given by Proposition 2.

Proof. Please refer to Appendix 2. ■

In particular, the lemma implies that we must have $\frac{\overset{\bullet}{P_1}(t)}{P_1(t)} = \beta(\xi_2 - \xi_1)$ and $\frac{\overset{\bullet}{P_2}(t)}{P_2(t)} = \alpha(\xi_1 - \xi_2)$ after both countries industrialize. That is, the terms of trade deteriorate when a country has a higher capital production efficiency. It is because a more efficient technology of capital good production leads to a faster industrial upgrading and hence a larger output, which worsens the terms of trade as the substitution elasticity between the domestic and foreign goods are finite. Theoretically speaking, output growth can be even negative when the trade partner has a higher productivity in the capital goods sector and the intertemporal elasticity of substitution is smaller than unity: According to (16), $h_2 < 0$ when $\xi_1 > \xi_2$ and $\frac{1}{\sigma}$ is sufficiently small. In that case, country 2 still enjoys a positive consumption growth despite the negative output growth, because the terms of trade become increasingly favorable for country 2. This "immiserizing growth" result is mainly due to the Armington assumption with finite substitution elasticity, a feature shared by Acemoglu and Ventura (2002), who also provide the empirical evidences for this immiserizing growth. However, we will focus on the industrial upgrading with $h_1(t)$ and $h_2(t)$ both strictly positive. Industrial degrading, however, can be analyzed in the same spirit. To satisfy the transversality condition, we further impose

$$a < \lambda^{\frac{\xi_i}{h_i}} \tag{18}$$

for both $i = 1$ and 2. Intuitively, if the capital intensity parameter a is sufficiently large such that (18) is violated, then capital accumulation is not sustainable to ensure a positive consumption and output growth. Please refer to Appendix 3 for further discussion on the necessity of (18).

3.2 Industrial Dynamics

Now we derive the industrial dynamics in the two countries for the entire period. Proposition 1 implies that at most two industries can coexist in any country at any time. This is useful in deriving the critical time points$\{t_{i,n}\}_{n=0}^{\infty}$ for $i = 1$ and 2.

Lemma 2 *Suppose at time* 0*, only good* 0 *is produced in country* $i \in \{1, 2\}$*, then*

$$t_{i,0} = \frac{\log \frac{\vartheta_{i,0}}{K_{i,0}}}{\xi_i}. \tag{19}$$

where $\vartheta_{i,0}$ *is given by (22) in Proposition 5 below. For any* $n \geq 1$*, we have*

$$t_{i,n} = \frac{\log \frac{\lambda^n L_i}{X_i(0)}}{h_i}, \tag{20}$$

where $X_i(0)$ *is unique and given by Proposition 6. If at time* 0 *both good* 0 *and good* 1 *are produced, then (20) holds for any* $n \geq 0$*. If, instead, at time* 0 *both good* $\widetilde{n}$ *and good* $\widetilde{n}$ $+1$ *are produced for some* $\widetilde{n} \geq 1$*, (20) still holds for any* $n \geq \widetilde{n} + 1$*.*

The proof is straightforward. Since $h_i > 0$, $t_{i,n}$ must be weakly increasing in n for both $i = 1$ and 2. Define $m_{i,n} \equiv t_{i,n+1} - t_{i,n}$, which measures how long goods n and $n+1$ coexist in country i, or in other words, the duration for the diversification cone containing goods n and $n+1$ in country i. Except for the "truncated" diversification cone at the initial period, we must have

$$m_{i,n} = m_i \equiv \frac{\log \lambda}{h_i}, \forall n \geq \widetilde{n} + 1, \tag{21}$$

where $\widetilde{n}$ denotes the index of the less capital intensive industry in the two coexisting industries at time 0 and h_i is given by (15) or (16). Thus industry $n+1$ first appears in country i at time $t_{i,n} = t_{i,\widetilde{n}+1} + (n - \widetilde{n} - 1)\, m_i$ for any $n \geq \widetilde{n} + 1$.

Proposition 3 *In the dynamic equilibrium with free trade, all the industries (except for the initial industries) in country* i *will exist for an equal period* $2m_i$*, and the industrial upgrading speed in country* i *(measured by* $\frac{1}{m_i}$*) increases with its output growth rate* h_i *but decreases with* λ*.*

This proposition states that the complete life cycle of each industry in the same country will be equally long (equal to $2m_i$). The length depends on the characteristics of both countries via trade. Since the industrial upgrading speed is proportional to the output growth rate, Corollary 1 immediately implies that the industrial upgrading of a country can be either facilitated or hampered by its trade partner's investment-specific technology progress, depending on whether the intertemporal elasticity of substitution is larger than one for the same intuition explained before. Moreover, the country with a larger investment-specific technology parameter (higher ξ) will have a faster industrial upgrading than its trade partner. When λ increases, the productivities of the neighboring industries both increase (recall assumptions (4) and (5)), which creates two opposite effects. The productivity increase in the higher-indexed industry induces a faster upgrading while the productivity increase in the lower-indexed industry induces a longer stay at the current industry.

The assumption $a-1>\lambda$ in (5) dictates that the second effect dominates, therefore a larger λ implies a lower speed of industrial upgrading.

The following proposition analytically characterizes the dynamics of each industry along the aggregate balanced growth path.

Proposition 4 *In the dynamic trade equilibrium , each country has an inverse-V-shaped industrial evolution path. More precisely, for any country $i=1$ or 2, suppose $K_i(0)$ is sufficiently small such that the economy starts by producing in industry 0 only, then we have*

$$x_{i,n}^*(t)=\begin{cases}\frac{L_ie^{h_i(t-t_{i,0})}}{\lambda^n-\lambda^{n-1}}-\frac{L_i}{\lambda-1} & \text{when} \quad t\in[t_{i,n-1},t_{i,n}]\\ -\frac{L_ie^{h_i(t-t_{i,0})}}{\lambda^{n+1}-\lambda^n}+\frac{\lambda L_i}{\lambda-1}, & \text{when} \quad t\in[t_{i,n},t_{i,n+1}]\\ 0, & \text{otherwise}\end{cases}, \text{ for all } n\geq 2$$

$$x_{i,1}^*(t)=\begin{cases}\frac{L_ie^{h_i(t-t_{i,0})}-L_i}{\lambda-1}, & \text{when} \quad t\in[t_{i,0},t_{i,1}]\\ -\frac{L_ie^{h_i(t-t_{i,0})}}{\lambda^2-\lambda}+\frac{\lambda L_i}{\lambda-1}, & \text{when} \quad t\in[t_{i,1},t_{i,2}]\\ 0, & \text{otherwise}\end{cases},$$

$$x_{i,0}^*(t)=\begin{cases}L_i-\frac{L_ie^{h_i(t-t_{i,0})}-L_i}{\lambda-1}, & \text{when} \quad t\in[t_{i,0},t_{i,1}]\\ L_i, & \text{when} \quad t\in[0,t_{i,0}]\end{cases},$$

where the critical time point $t_{i,n}$ is given by Lemma 2 for any $i=1,2$ and $n=0,1,2,...$.

Proof. Using Table 1 and the fact that $X_i(t)=L_i$ for any $t\leq t_{i,0}$ and $X_i(t)=L_ie^{h_i(t-t_{i,0})}$ for any $t\geq t_{i,0}$. ■

This proposition can be illustrated more intuitively by the following inverse-V-shaped life cycle of different industries in Figure 4.

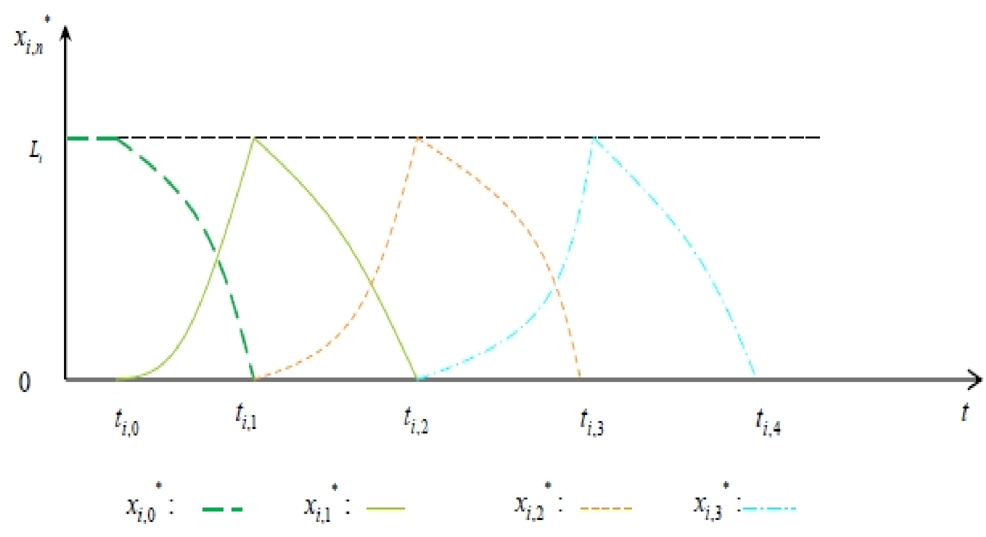

Figure 4. Industrial Dynamics in Country i with International Trade when $t_{i,0}>0$.

Before "industrialization", only good 0 is produced and the total output per capita is stagnant. The economy escapes this "Malthusian trap" and enters the "Solow regime" at time $t_{i,0}$, after which the per capita output growth rate h_i is strictly positive. Beneath this sustainable aggregate growth path, the underlying industries are shifting endogenously and their outputs follow an inverse-V-shaped pattern. The quantity of export follows the same pattern, as implied by (13). These dynamic patterns are consistent with the empirical facts documented in the literature (see Schott, 2003; Chenery et al, 1986; Haraguchi and Rezonja, 2010; Ju, Lin and Wang, 2010).

When $K_i(0)$ is such that both good 0 and good 1 are produced at time 0, the output of each industry is given by

$$x^*_{i,n}(t) = \begin{cases} \frac{X_i(0)e^{h_i t}}{\lambda^n - \lambda^{n-1}} - \frac{L_i}{\lambda - 1} & \text{when} \quad t \in [t_{i,n-1}, t_{i,n}] \\ -\frac{X_i(0)e^{h_i t}}{\lambda^{n+1} - \lambda^n} + \frac{\lambda L_i}{\lambda - 1}, & \text{when} \quad t \in [t_{i,n}, t_{i,n+1}] \\ 0, & \text{otherwise} \end{cases}, \text{ for all } n \geq 2$$

$$x^*_{i,1}(t) = \begin{cases} \frac{X_i(0)e^{h_i t} - L_i}{\lambda - 1}, & \text{when} \quad t \in [0, t_{i,1}] \\ -\frac{X_i(0)e^{h_i t}}{\lambda^2 - \lambda} + \frac{\lambda L_i}{\lambda - 1}, & \text{when} \quad t \in [t_{i,1}, t_{i,2}] \\ 0, & \text{otherwise} \end{cases},$$

$$x^*_{i,0}(t) = \begin{cases} L_i - \frac{X_i(0)e^{h_i t} - L_i}{\lambda - 1}, & \text{when} \quad t \in [0, t_{i,1}] \\ 0, & \text{otherwise} \end{cases},$$

which means that the diversification cone for good 0 and good 1 is "truncated", as shown in Figure 5.

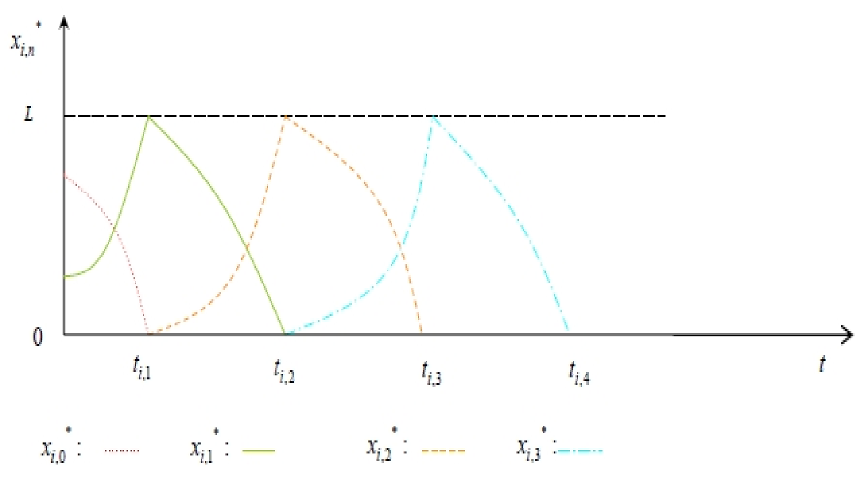

Figure 5. Industrial Dynamics in Country i with International Trade when $t_{i,0} = 0$ and $t_{i,1} > 0$.

Similarly, when $K_i(0)$ is such that both good $\widetilde{n}$ and good $\widetilde{n}+1$ are produced at time 0 for some $\widetilde{n} \geq 1$, then the industrial dynamics is given by

$$x_{i,n}^*(t) = \begin{cases} \frac{X_i(0)e^{h_i t}}{\lambda^n - \lambda^{n-1}} - \frac{L_i}{\lambda - 1} & \text{when} \quad t \in [t_{i,n-1}, t_{i,n}] \\ -\frac{X_i(0)e^{h_i t}}{\lambda^{n+1} - \lambda^n} + \frac{\lambda L_i}{\lambda - 1}, & \text{when} \quad t \in [t_{i,n}, t_{i,n+1}] \\ 0, & \text{otherwise} \end{cases}, \text{ for all } n \geq \widetilde{n} + 2$$

$$x_{i,\widetilde{n}+1}^*(t) = \begin{cases} \frac{X_i(0)e^{h_i t}}{\lambda^{\widetilde{n}+1} - \lambda^{\widetilde{n}}} - \frac{L_i}{\lambda - 1}, & \text{when} \quad t \in [0, t_{i,\widetilde{n}+1}] \\ -\frac{X_i(0)e^{h_i t}}{\lambda^{\widetilde{n}+2} - \lambda^{\widetilde{n}+1}} + \frac{\lambda L_i}{\lambda - 1}, & \text{when} \quad t \in [t_{i,\widetilde{n}+1}, t_{i,\widetilde{n}+2}] \\ 0, & \text{otherwise} \end{cases},$$

$$x_{i,\widetilde{n}}^*(t) = \begin{cases} -\frac{X_i(0)e^{h_i t}}{\lambda^{\widetilde{n}+1} - \lambda^{\widetilde{n}}} + \frac{\lambda L_i}{\lambda - 1}, & \text{when} \quad t \in [0, t_{i,\widetilde{n}+1}] \\ 0, & \text{otherwise} \end{cases},$$

$x_{i,n}^*(t) = 0$ for any $t \geq 0$ and any $n \leq \widetilde{n} - 1$.

It can be illustrated graphically as follows.

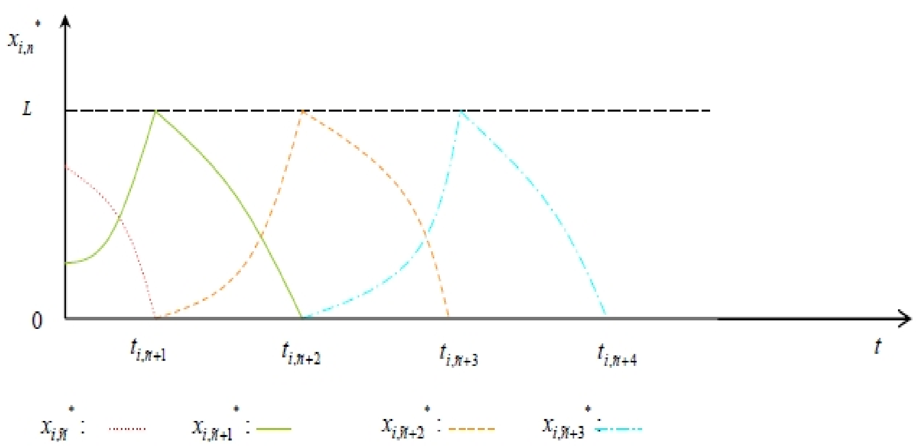

Figure 6. Industrial Dynamics in Country i with International Trade when $t_{i,\widetilde{n}} = 0$ and $t_{i,\widetilde{n}+1} > 0$ for some $\widetilde{n} \geq 1$.

The following proposition tells how the initial industries and $X_i(0)$ are determined for country $i \in \{1, 2\}$.

Proposition 5 *Given $K_i(0) = K_{i,0}$ for both $i = 1$ and 2, there exists a unique and increasing sequence of strictly positive numbers, $\vartheta_{i,0}, \vartheta_{i,1}, \cdots, \vartheta_{i,n}, \vartheta_{i,n+1}, \cdots$, such that if $0 < K_{i,0} \leq \vartheta_{i,0}$, country i will start by producing good 0 only; if $\vartheta_{i,n} < K_{i,0} \leq \vartheta_{i,n+1}$, the economy will start by producing goods n and $n+1$, for any $n \geq 0$. In* *addition,*

$$\vartheta_{i,0} \equiv \frac{ah_i\left(1-\lambda^{1-\frac{\xi_i}{h_i}}\right)(1-\lambda^{\frac{\xi_i}{h_i}})}{(\lambda-1)\,\lambda^{\frac{\xi_i}{h_i}}\xi_i\,(h_i-\xi_i)\left(1-a\lambda^{\frac{-\xi_i}{h_i}}\right)}L_i, \tag{22}$$

$$\vartheta_{i,n} \equiv \frac{a^n\left[\xi_i\left(a-\lambda^{\frac{\xi_i}{h_i}}\right)(\lambda-1)+h_i(a-\lambda)\left(1-\lambda^{\frac{\xi_i}{h_i}}\right)\right]}{(\lambda-1)\,\lambda^{\frac{\xi_i}{h_i}}\xi_i\,(h_i-\xi_i)\left(1-a\lambda^{\frac{-\xi_i}{h_i}}\right)}L_i, \text{ for any } n \geq 1.$$

Proof. Refer to Appendix 3. ■

Observe that the threshold values for capital are proportional to the domestic labor endowment. They also depend on the trade partner's technology parameter, but independent of the initial capital endowment. In particular, substituting (22) into (19), we obtain the explicit expression for the time of industrialization in country i:

$$t_{i,0} = \frac{1}{\xi_i}\left[\log\frac{ah_i\left(1-\lambda^{1-\frac{\xi_i}{h_i}}\right)(1-\lambda^{\frac{\xi_i}{h_i}})}{(\lambda-1)\,\lambda^{\frac{\xi_i}{h_i}}\xi_i\,(h_i-\xi_i)\left(1-a\lambda^{\frac{-\xi_i}{h_i}}\right)}-\log\frac{K_{i,0}}{L_i}\right], \tag{23}$$

which is affected by the trade partner's technology parameter via h_i given by (15) or (16). (23) reveals that industrialization occurs later if the initial capital-labor ratio is smaller ($\frac{\partial t_{i,0}}{\partial\left(\frac{K_{i,0}}{L_i}\right)} < 0$) or if capital requirement parameter a is larger ($\frac{\partial t_{i,0}}{\partial a} > 0$). It remains to characterize how $X_i(0)$ and the time path of capital stock $K_i(t)$ are determined for country $i \in \{1,2\}$. For the convenience of exposition, define

$$\widetilde{B}_i \equiv \frac{L_i}{\lambda-1}\left[\frac{\lambda^{\frac{\xi_i}{h_i}}-\lambda}{h_i-\xi_i}+\frac{(a-\lambda)\left(\lambda^{\frac{\xi_i}{h_i}}-1\right)}{\xi_i(a-1)}\right] < 0. \tag{24}$$

Define, for any $n \geq 1$,

$$\alpha_{i,n} = -\frac{a^n(a-\lambda)L_i}{\xi_i\,(\lambda-1)}, \tag{25}$$

$$\beta_{i,n} = -\left(\frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n}\right)\frac{X_i(0)}{(h_i-\xi_i)}, \tag{26}$$

$$\gamma_{i,n} = \left[\frac{\lambda^n L_i}{X_i(0)}\right]^{\frac{-\xi_i}{h_i}}\left\{\vartheta_{i,n}+\frac{\left(a^{n+1}-a^n\right)L_i}{\lambda-1}\left[\frac{1}{(h_i-\xi_i)}+\frac{(a-\lambda)}{\xi_i\,(a-1)}\right]\right\}. \tag{27}$$

Proposition 6 *For any country $i \in \{1,2\}$, given $K_i(0) = K_{i,0}$, $X_i(0)$ and $K_i(t)$*

for any $t \geq 0$ are uniquely determined as follows: [1]When $K_{i,0} \in (0, \vartheta_{i,0}]$,

$$X_i(0) = L_i,$$

and the capital accumulation function is

$$K_i(t) = \begin{cases} K_{i,0}e^{\xi_i t}, & \textit{for} \quad t \in [0, t_{i,0}] \\ \frac{-\frac{aL_i}{\lambda-1}}{h_i-\xi_i}e^{h_i t} + \frac{-aL_i}{\xi_i(\lambda-1)} + \left[\vartheta_{i,0} + \frac{\frac{aL_i}{\lambda-1}}{h_i-\xi_i} + \frac{aL_i}{\xi_i(\lambda-1)}\right]e^{\xi_i t} & \textit{for} \quad t \in [t_{i,0}, t_{i,1}] \\ \alpha_{i,n} + \beta_{i,n}e^{h_i t} + \gamma_{i,n}e^{\xi_i t}, & \textit{for} \quad \begin{array}{c} t \in [t_{i,n}, t_{i,n+1}], \\ \textit{any } n \geq 1 \end{array} \end{cases};$$

[2]when $K_{i,0} \in (\vartheta_{i,0}, \vartheta_{i,1}]$, $X_i(0)$ is uniquely determined by

$$\begin{aligned} &\left[K_{i,0} + \frac{aL_i}{\xi_i(\lambda-1)} + \frac{aX_i(0)}{(\lambda-1)(h_i-\xi_i)}\right]\left(\frac{\lambda L_i}{X_i(0)}\right)^{\frac{\xi_i}{h_i}} \\ = \quad &\frac{aL_i}{\xi_i(\lambda-1)} + \frac{a\lambda L_i}{(\lambda-1)(h_i-\xi_i)} - \frac{a(a-1)\widetilde{B}_i\lambda^{\frac{-\xi_i}{h_i}}}{1-a\lambda^{\frac{-\xi_i}{h_i}}}; \end{aligned}$$

and

$$K_i(t) = \begin{cases} \frac{-\frac{aX_i(0)}{\lambda-1}}{h_i-\xi_i}e^{h_i t} + \frac{-aL_i}{\xi_i(\lambda-1)} + \left[K_{i0} + \frac{\frac{aX_i(0)}{\lambda-1}}{h_i-\xi_i} + \frac{aL_i}{\xi_i(\lambda-1)}\right]e^{\xi_i t} & \textit{when} \quad t \in [0, t_{i,1}] \\ \alpha_{i,n} + \beta_{i,n}e^{h_1 t} + \gamma_{i,n}e^{\xi_i t} & \textit{when} \quad \begin{array}{c} t \in [t_{i,n}, t_{i,n+1}] \\ \textit{for any } n \geq 1 \end{array} \end{cases};$$

[3]when $K_{i,0} \in (\vartheta_{i,m}, \vartheta_{i,m+1}]$, for any $m \geq 1$, $X_i(0)$ is uniquely determined by

$$\begin{aligned} &\left[K_{i,0} + \frac{a^m(a-\lambda)L_i}{\xi_i(\lambda-1)} + \frac{a^{m+1}-a^m}{\lambda^{m+1}-\lambda^m}\frac{X_i(0)}{(h_i-\xi_i)}\right]\left[\frac{\lambda^{m+1}L_i}{X_i(0)}\right]^{\frac{\xi_i}{h_i}} \\ = \quad &\frac{a^m(a-\lambda)L_i}{\xi_i(\lambda-1)} + \left(\frac{a-1}{\lambda-1}\right)\frac{a^m\lambda L_i}{(h_i-\xi_i)} - \frac{a(a-1)\widetilde{B}_i\lambda^{\frac{-\xi_i}{h_i}}}{1-a\lambda^{\frac{-\xi_i}{h_i}}}, \end{aligned}$$

and

$$K_i(t) = \begin{cases} \alpha_{i,m} + \beta_{i,m}e^{h_i t} + \left[K_i(0) + \frac{a^m(a-\lambda)L_i}{\xi_i(\lambda-1)} + \frac{a^{m+1}-a^m}{\lambda^{m+1}-\lambda^m}\frac{X_i(0)}{(h_i-\xi_i)}\right]e^{\xi_i t}, & \textit{when} \quad t \in [0, t_{i,m+1}] \\ \alpha_{i,n} + \beta_{i,n}e^{h_i t} + \gamma_{i,n}e^{\xi_i t}, & \textit{when} \quad \begin{array}{c} t \in [t_{i,n}, t_{i,n+1}] \\ \textit{for any } n \geq m+1 \end{array} \end{cases},$$

where $\widetilde{B}_i$ is given by (24) and $\alpha_{i,n}, \beta_{i,n}$ and $\gamma_{i,n}$ are defined by (25)-(27), respectively. For all the above three different cases, $t_{i,n}$ is given by lemma 2 for any i and any n; and $\vartheta_{i,n}$ is given by the previous proposition for any i and any n.

Proof. See Appendix 3. ■

The functional forms of the capital accumulation are changing over time, reflecting the structural changes in the underlying industries in both the home country and the foreign country. Once $X_1^*(0)$ and $X_2^*(0)$ are determined, the initial aggregate consumptions are also determined by

$$C_1^*(0) = \alpha X_1^{*\alpha}(0) X_2^{*\beta}(0);\ C_2^*(0) = \beta X_1^{*\alpha}(0) X_2^{*\beta}(0).$$

Since $X_i(t) = L_i$ for any $t \leq t_{i,0}$ and $X_i(t) = L_i e^{h_i(t-t_{i,0})}$ for any $t \geq t_{i,0}$, we can also uniquely determine the consumption for any country at any time:

$$C_1^*(t) = \alpha X_1^{*\alpha}(t) X_2^{*\beta}(t);\ C_2^*(t) = \beta X_1^{*\alpha}(t) X_2^{*\beta}(t).$$

This completes all the characterization of the free trade dynamic economy. Observe that $\vartheta_{i,n} \equiv K_i(t_{i,n})$ for any i and any n as long as $t_{i,n} > 0$. So long as the initial capital endowment is not too small (such that only good 0 is produced initially), different initial capital levels only translate into different levels of the initial aggregate consumption and initial industrial structures, but they cannot affect the speed of consumption and output growth.

3.3 Summary

In this section, we obtain closed-form solutions to fully characterize the whole dynamic path of each industry as well as the aggregate economy for both countries in the general equilibrium world with free trade. We show that in both countries industrial development demonstrates an inverse-V-shaped life-cycle pattern: capital-intensive industries gradually replace the labor-intensive industries as the economy grows. The endogenous change in the underlying industrial structures translates into different functional forms of the aggregate production function and capital accumulation function.

Different from the closed economy studied in Ju, Lin and Wang (2010), now the speed of industrial upgrading and the growth rates of output and consumption in a country are all affected by its trade partner's initial endowment and technology parameters. The initial endowment has a level effect on industrial development and total output, but it has no speed effect after the industrialization. Pareto optimality is achieved because the first welfare theorem applies. However, nothing ensures output convergence between the two trading countries. In particular, we see that convergence occurs in the long run if and only if the less developed country has a faster investment-specific technological progress than its trade partner. Moreover, a higher speed of the investment-specific technological progress in a country will result in a faster industrial upgrading and a more rapid economic growth of its trade partner, if and only if the intertemporal elasticity of substitution is larger than one, because the dynamic terms-of-trade effect dominates the dynamic market-size effect in that case. In other words, free trade does not necessarily speed up the industrial upgrading in a country.

Naturally, one may ask what happens if there exist some trade barriers, which is addressed next.

4 Trade Policy and Industrial Upgrading

4.1 Static World with Protectionist Trade Policy

First consider trade policies in a static model. Suppose country 1 imposes tariff τ_2 on the import from country 2 and all the tariff revenue T_1 is given to the domestic households as a lump-sum transfer. Similarly, country 2 imposes tariff τ_1 on the import from country 1 and all the tariff revenue T_2 is also transferred to the domestic households in a lump-sum fashion. The equilibrium is characterized in the following lemma.

Lemma 3 *In the static trade equilibrium, the total consumptions are given by*

$$C_1(\tau_1, \tau_2) = C_{1,1}^{\alpha} C_{1,2}^{\beta} = \alpha \left[\frac{(1+\tau_2)}{(1+\alpha\tau_2)} \right]^{\alpha} \left[\frac{1}{(1+\beta\tau_1)} \right]^{\beta} X_1^{\alpha} X_2^{\beta}, \tag{28}$$

and

$$C_2(\tau_1, \tau_2) = C_{2,1}^{\alpha} C_{2,2}^{\beta} = \beta \left[\frac{1}{(1+\alpha\tau_2)} \right]^{\alpha} \left[\frac{(\tau_1+1)}{1+\beta\tau_1} \right]^{\beta} X_1^{\alpha} X_2^{\beta}, \tag{29}$$

while the equilibrium term of trade is given by

$$\frac{P_1}{P_2} = \frac{\alpha(1+\alpha\tau_2)X_2}{\beta(1+\beta\tau_1)X_1}. \tag{30}$$

where X_1 and X_2 are provided in Table 1.

Proof. See Appendix 4. ■

(28) and (29) indicate that the total consumption of a country increases with the tariff rate on the import but decreases with the foreign tariff imposed on its export. This is due to the endogenous terms of trade effect shown in equation (30): A fixed expenditure share on imports implies that the after-tariff price of import must increase relative to the export price when the tariff rate increases, as output X_1 and X_2 are fixed. Moreover, the consumption ratio of the two countries is given by

$$\frac{C_1}{C_2} = \frac{\alpha(1+\tau_2)^{\alpha}}{\beta\,(1+\tau_1)^{\beta}}, \tag{31}$$

which is independent of the total output. It also indicates that the protectionist trade policy favors domestic consumption in the world consumption distribution.

In the model, the supply side is immune from this particular type of international trade policies because the tariff is imposed on the aggregate good instead of some specific industries, therefore neither the marginal rate of change nor the equilibrium relative prices across different industries is altered by the industry-neutral trade policies within the same country. Profit-maximization of all the competitive firms plus the factor market clearing conditions would therefore lead to the same quantity of output as in a free-trade static economy. What is changed by this trade policy is the relative output prices across different countries but not the relative output prices across different industries within a country.

4.2 Dynamic Trade Policy

Consider the effect of an arbitrary dynamic trade policy. Imagine the gross tariff rates behave as follows:

$$\frac{\dot{\tau}_1(t)}{\tau_1(t)+1} = \phi_1(t), \frac{\dot{\tau}_2(t)}{\tau_2(t)+1} = \phi_2(t), \tag{32}$$

where $\phi_1(t)$ and $\phi_2(t)$ are the change rates of the gross tariff rates. Both of them are exogenous and arbitrary functions. The following lemma characterizes how an arbitrary dynamic trade policy may affect the consumption growth and output growth (hence the speed of industrial upgrading).

Lemma 4 *For any exogenous dynamic trade policies specified as (32), the consumption and output growth rates for the two countries are given by*

$$\theta_1(t) = \frac{\alpha\xi_1+\beta\xi_2-\rho}{\sigma} - \frac{\beta}{\sigma}\left\{\alpha\phi_2(t)\left[\frac{\alpha\tau_2(t)}{1+\alpha\tau_2(t)}-\sigma\right]+\beta\phi_1(t)\left[\sigma+\frac{\alpha\tau_1(t)}{1+\beta\tau_1(t)}\right]\right\}, \tag{33}$$

$$h_1(t) = \frac{\alpha\xi_1+\beta\xi_2-\rho}{\sigma}+\beta\left(\xi_1-\xi_2\right)+\beta\left\{\begin{array}{c}\phi_2(t)\alpha\left[\frac{\tau_2(t)\left[\frac{1}{\beta}-\left(\frac{\alpha}{\sigma}+\beta\right)\right]+\frac{1}{\beta}}{1+\alpha\tau_2(t)}-\left(\frac{\alpha}{\beta}+\sigma\right)\right] \\ -\phi_1(t)\left[\frac{\left[(1+\beta(\sigma-1))\frac{\beta}{\sigma}\tau_1(t)+1\right]}{1+\beta\tau_1(t)}-\left(1+\beta\left(\sigma-1\right)\right)\right]\end{array}\right\}, \tag{34}$$

$$\theta_2(t) = \frac{\alpha\xi_1+\beta\xi_2-\rho}{\sigma} - \frac{\alpha}{\sigma}\left[\alpha\phi_2(t)\left(\frac{\beta\tau_2(t)}{1+\alpha\tau_2(t)}+\sigma\right)+\beta\phi_1(t)\left(\frac{\beta\tau_1(t)}{1+\beta\tau_1(t)}-\sigma\right)\right], \tag{35}$$

$$h_2(t) = \frac{\alpha\xi_1+\beta\xi_2-\rho}{\sigma}+\alpha\left(\xi_2-\xi_1\right)+\alpha\left\{\begin{array}{c}\phi_1(t)\beta\left[\frac{\tau_1(t)\left[\frac{1}{\alpha}-\left(\frac{\beta}{\sigma}+\alpha\right)\right]+\frac{1}{\alpha}}{1+\beta\tau_1(t)}-\left(\frac{\beta}{\alpha}+\sigma\right)\right] \\ -\phi_2(t)\left[\frac{\left[(1+\alpha(\sigma-1))\frac{\alpha}{\sigma}\tau_2(t)+1\right]}{1+\alpha\tau_2(t)}-\left(1+\alpha\left(\sigma-1\right)\right)\right]\end{array}\right\}. \tag{36}$$

Proof. See Appendix 5. ■

From this lemma, we can see that the net growth impact of dynamic trade policies is summarized in the last term of each of the above four expressions. Thus we immediately obtain the following corollary.

Corollary 2 *A time-invariant tariff rate (i.e., when $\phi_1(t) = \phi_2(t) = 0$, $\forall t$) does not affect the long-run equilibrium growth rate and industrial upgrading speed. In addition, the consumption and output growth rates under a time-invariant tariff are exactly the same as in the free trade characterized by (14)-(16) in Proposition 1.*

The absence of growth effect results from the fact that time-invariant tariffs do not distort the production activities within each country. This is because tariffs can only distort the terms of trade, but not the marginal rate of transformation within each country. Consequently, tariffs result in deadweight loss only in terms of consumption and welfare, as indicated by (28) and (29), but not in production. When tariff rates are not constant over time, consumption and output growth rates

may change because of the intertemporal terms of trade effect and the market-size effect. For concreteness, consider the growth effect of gradual trade liberalization in country 1 (that is, $\phi_2(t) \leq 0$). Then the previous lemma yields the following result.

Proposition 7 *When $\sigma \in (0,1)$, the following is true: [1]* $\frac{\partial\theta_1(t)}{\partial|\phi_2(t)|}\begin{cases} >0, & when \quad \tau_2(t) > \tau_2^* \\ =0, & when \quad \tau_2(t) = \tau_2^* \\ <0, & when \quad \tau_2(t) < \tau_2^* \end{cases}$, $\frac{\partial h_1(t)}{\partial|\phi_2(t)|}\begin{cases} >0, & when \quad \tau_2(t) > \tau_2^{**} \\ =0, & when \quad \tau_2(t) = \tau_2^{**} \\ <0, & when \quad \tau_2(t) < \tau_2^{**} \end{cases}$ *, where $\tau_2^* \equiv \frac{\sigma}{\alpha(1-\sigma)}$ and $\tau_2^{**} \equiv \frac{\beta(1-\sigma)}{\frac{\alpha\beta}{\sigma}+\alpha\sigma-2\alpha\beta}$. [2] When $\sigma \in [1,\infty)$, the following is true: $\frac{\partial\theta_1(t)}{\partial|\phi_2(t)|} < 0$ whenever $\tau_2(t) \geq 0$, $\frac{\partial h_1(t)}{\partial|\phi_2(t)|} \geq 0$, "$=$ " holds only when $\sigma = 1$; [3] For any $\sigma \in (0,\infty)$, we have $\frac{\partial\theta_i(t)}{\partial|\phi_i(t)|} > 0$ and $\frac{\partial^2\theta_i(t)}{\partial|\phi_i(t)|\partial t} < 0$ for $i = 1,2$.* $\frac{\partial h_i(t)}{\partial|\phi_i(t)|}\begin{cases} >0, & when \quad \sigma \in (0,1) \\ =0, & when \quad \sigma = 1 \\ <0, & when \quad \sigma \in (1,\infty) \end{cases}$.

Part [1] of the proposition states that, when the intertemporal elasticity of substitution is larger than unity ($\sigma \in (0,1)$), the consumption growth rate in country 1, $\theta_1(t)$, first increases with the speed of trade liberalization in country 1 ($\frac{\partial\theta_1(t)}{\partial|\phi_2(t)|} > 0$) until τ_2 reaches the value $\tau_2^* \equiv \frac{\sigma}{\alpha(1-\sigma)}$, after which the consumption growth rate strictly decreases with the speed of trade liberalization (when $\tau_2 < \tau_2^*$). In other words, the consumption growth rate will be increased by accelerating the trade liberalization if and only if the tariff rate is sufficiently high. In particular, when τ_2^*, the consumption growth rate is the same as in the long-run free trade equilibrium. Similarly, the output growth rate in country 1, $h_1(t)$, also first increases when trade liberalization accelerates, but then declines with the speed of trade liberalization once the tariff rate is below τ_2^{**}. Observe that $\tau_2^{**} < \tau_2^*$ when $\sigma \in (0,1)$, implying that the consumption growth starts to decline earlier than the output growth if trade liberalization accelerates.

The intuition for this non-monotonic impact is the following. As the tariff rate increasingly declines over time, imports for country 1 becomes increasingly cheaper, therefore the intertemporal substitution effect causes consumers to substitute today's consumption of imports for tomorrow through saving, which in turn increases the consumption growth rate. On the other hand, the real income becomes increasingly larger as the import price becomes increasing cheaper, and this positive income effect tends to increase the consumption and decreases the saving, which in turn tends to lower the consumption growth rate. When the tariff rate is sufficiently high, the substitution effect dominates the income effect, therefore accelerating trade liberalization increases the growth rate of consumption. However, the substitution effect becomes increasingly weaker as the tariff level goes down. Eventually, when the tariff rate is sufficiently small, the income effect dominates the substitution effect, so the growth rate of consumption starts to decline. Symmetrically, if trade liberalization decelerates in country 1 (that is, $|\phi_2(t)|$ decreases over time), then the consumption growth rate will first decline and then increase. A similar pattern also applies for the output growth for similar intuitions.

To understand the impact on the output growth, first note that there are two competing effects on the domestic output when the tariff rate on imports decreases. One is the substitution effect which tends to decrease the domestic demand for domestic output. The second effect is the positive income effect due to the rise of real income as a result of tariff reduction. The income effect tends to increase the demand for domestic output. The net impact on the domestic output is positive because the substitution elasticity between imports and outputs is unity (Cobb-Douglas function). The competitive market force dictates that output of domestic goods will increase, only partly offsetting the decline of the relative price of imports because of the balanced trade constraint (30). Consequently, an acceleration in the trade liberalization leads to an increase in the output growth rate ($\frac{\partial h_1(t)}{\partial|\phi_2(t)|} > 0$). As only a fraction of total output is used for domestic consumption, therefore the impact on consumption growth is smaller than that on the output growth. This is why consumption growth rate declines earlier than the output growth ($\tau_2^{**} < \tau_2^*$).

By contrast, Part [2] of the proposition states that the non-monotonicity result disappears when $\sigma \in (1, \infty)$: Accelerating trade liberalization will strictly decrease the domestic consumption growth rate but strictly increase the output growth rate. The impact on consumption growth is negative because the intertemporal elasticity is now smaller than one, so the intertemporal substitution effect is always dominated by the intertemporal income effect, therefore the growth rate of consumption monotonically decreases as the trade liberalization accelerates. The positive impact on output growth is mainly due to the increase in the external demand from the trade partner (country 2), because the trade partner can sell more to country 1 after the trade liberalization and hence its income grows faster.

The previous argument is consistent with Part [3] of the proposition, which states that, for any $\sigma \in (0, \infty)$, the consumption growth rate always increases ($\frac{\partial \theta_i(t)}{\partial|\phi_i(t)|} > 0$) when the trade partner unilaterally accelerates trade liberalization , but the marginal change in the consumption growth rate declines over time ($\frac{\partial^2 \theta_i(t)}{\partial|\phi_i(t)|\partial t} < 0$) due to the tariff reduction. However, accelerating trade liberalization by the trade partner may increase or decrease the output growth rate, depending on whether the intertemporal elasticity of substitution is larger or smaller than one.

The above proposition reveals that the speed of trade liberalization matters for our understanding of the growth effects of trade liberalization. Furthermore, the intertemporal elasticity of substitution also matters! The same policy change may affect the output or consumption growth in the opposite directions, depending on whether the intertemporal elasticity of substitution is larger or smaller than unity.

5 Further Discussion

Our previous analysis assumes that the two countries have the same expenditure shares on the two country-specific goods. What happens when the expenditure shares are country-specific? Suppose

$$C_i = C_{i,1}^{\alpha_i} C_{i,2}^{\beta_i}, \ \alpha_i \geq 0, \beta_i \geq 0, \alpha_i + \beta_i = 1 \text{ for } i \in \{1,2\}. \tag{37}$$

Also assume $\alpha_1 \geq \alpha_2$ to capture the home bias effect. In the long-run dynamic free-trade equilibrium, we have

$$h_1(t) = \frac{(1-\sigma)\left[\rho\beta_2 - \beta_2\xi_1 + \beta_1(\xi_2 - \rho)\right] + \xi_1 - \rho}{\sigma\left[\alpha_2 + \beta_1(1-\sigma) + (1-\alpha_2)\sigma\right]}; \tag{38}$$

$$h_2(t) = \frac{\rho(\alpha_1 - \alpha_2)(1-\sigma) - \rho + (1-\sigma)(\alpha_2\xi_1 - \alpha_1\xi_2) + \xi_2}{\sigma\left[\alpha_2 + \beta_1 + (\alpha_1 - \alpha_2)\sigma\right]}; \tag{39}$$

$$\theta_1(t) = \frac{\left[\alpha_1 - (1-\sigma)(\alpha_1 - \alpha_2)\right]\xi_1 + \beta_1\xi_2 - \rho\left[1 - (1-\sigma)(\alpha_1 - \alpha_2)\right]}{\sigma\left[\alpha_2 + \beta_1 + (\alpha_1 - \alpha_2)\sigma\right]}; \tag{40}$$

$$\theta_2(t) = \frac{\alpha_2\xi_1 + \left[\beta_2 - (1-\sigma)(\alpha_1 - \alpha_2)\right]\xi_2 - \rho\left[1 - (1-\sigma)(\alpha_1 - \alpha_2)\right]}{\sigma\left[\alpha_2 + \beta_1 + (\alpha_1 - \alpha_2)\sigma\right]}. \tag{41}$$

When $\alpha_1 = \alpha_2$ (therefore $\beta_1 = \beta_2$), the above equations degenerate to (14)-(16). It can be verified that, again, consumption growth rates are strictly increasing in both the domestic and foreign capital goods productivities. In addition, the output growth increases with the trade partner's capital goods productivity if and only if the intertemporal elasticity is larger than one. So the key results derived before are still valid in this more general setting.

Until now we implicitly assume that the expenditure shares are fixed and unaffected by any trade policies. However, this may not be realistic. Suppose the Armington trade assumption in (1) is changed to the general CES function, raising tariff typically leads to a decrease in the expenditure share on imports. This is, for example, supported by Eaton and Kortum (2001), where the expenditure shares are endogenously affected by trade barriers. We adopt a reduced-form approach here by simply assuming $\beta_1'(\tau_2) < 0$, that is, an increase in the tariff on imports from country 2 leads to a decrease in country 1's expenditure share on total imports. Also assume $\alpha_2'(\beta_1) > 0$ to capture the usual general equilibrium effect that, when a country imports more (hence consumes a larger fraction of the income on imports), its trade partner will import more as well because a larger fraction of its own output is now exported. We can easily obtain the following

$$
\begin{aligned}
\frac{\partial \theta_1(t)}{\partial \beta_1} &\propto (\xi_2 - \xi_1)\left[\alpha_2 + (1-\alpha_2)\,\sigma - (1-\sigma)\,\beta_1 \alpha_2'(\beta_1)\right], \\
\frac{\partial^2 \theta_1(t)}{\partial \beta_1^2} &\propto (\xi_2 - \xi_1)(\sigma - 1)\left[\alpha_2'(\beta_1) + \beta_1 \alpha_2''(\beta_1)\right], \\
\frac{\partial h_1(t)}{\partial \beta_1} &\propto (1-\sigma)(\xi_2 - \xi_1)\left[\alpha_2 + (1-\alpha_2)\,\sigma - (1-\sigma)\,\beta_1 \alpha_2'(\beta_1)\right], \\
\frac{\partial^2 h_1(t)}{\partial \beta_1^2} &\propto -(1-\sigma)^2(\xi_2 - \xi_1)\left[\alpha_2'(\beta_1) + \beta_1 \alpha_2''(\beta_1)\right].
\end{aligned}
$$

First of all, this result immediately means that, when the two countries have the same technical change rate ($\xi_1 = \xi_2$) or when $\alpha_2 + (1-\alpha_2)\,\sigma - (1-\sigma)\,\beta_1 \alpha_2'(\beta_1) = 0$, neither the consumption growth rate nor the output growth rate is affected by the change in the expenditure share on imports.

Second, when $\xi_1 \neq \xi_2$ and $\alpha_2 + (1-\alpha_2)\,\sigma - (1-\sigma)\,\beta_1 \alpha_2'(\beta_1) \neq 0$, the consumption growth rate and the output growth rate will change with the import share in the same direction when $\sigma \in (0,1)$, but in the opposite direction when $\sigma \in (1,\infty)$. When $\sigma = 1$, the output growth rate does not depend on the import share, but the consumption growth rate may either increase with the import share or decrease with it, depending on whether the foreign technology parameter is larger than the domestic one or not.

More specifically, suppose $\xi_1 > \xi_2$ and $\sigma \in (1,\infty)$. The consumption growth rate strictly decreases with the import share ($\frac{\partial \theta_1(t)}{\partial \beta_1} < 0$) while the output growth strictly increases with it ($\frac{\partial h_1(t)}{\partial \beta_1} > 0$). Together with $\beta_1'(\tau_2) < 0,$ this result states that when the intertemporal elasticity of substitution is smaller than one, a tariff reduction will lead to an increase in the output growth but a decrease in the consumption growth for the country that has a larger capital goods production efficiency than its trade partner. The opposite is true for its trade partner. To understand the intuition, observe that there are several competing effects working in the opposite directions following a tariff reduction. First, since a larger fraction of consumption expenditure will be on foreign imports, the saving decision of country 1 will respond more to the intertemporal change in the imports. Since $\xi_2 < \xi_1$, imports become increasingly more expensive relative to its own output, which has two effects. One is the intertemporal substitution effect which tends to substitute future consumption for today, hence lower the saving rate and output growth rate. The second effect is the negative income effect. The output revenue decreases and therefore consumers tend to lower the consumption and save more, which tends to increase the output growth. But the first effect always dominates the second effect, so the net effect is to lower the output growth. The third effect comes from the export market expansion for the output in country 1 as captured by $\alpha_2'(\beta_1) > 0$. It tends to increase the domestic income level. However, since $\xi_2 < \xi_1$, the market in country 2 grows more slowly, the export revenues also increase more slowly, which tends to raise the current saving and lower consumption, so the output growth rate increases. The intertemporal substitution effect is dominated when the intertemporal elasticity of substitution is less than 1, so the output growth rate ultimately increases with the tariff reduction. Since the imports occupy a larger share of total consumption and imports grow relatively slowly as $\xi_2 < \xi_1$, the consumption growth rate is decreasing. All these results will be reversed when $\xi_1 < \xi_2$.

By contrast, now suppose $\sigma \in (0,1)$ while we continue to assume that $\xi_1 > \xi_2$. In this case, both the consumption growth rate and the output growth rate increase with the import share when $\beta_1\alpha_2'(\beta_1) > \alpha_2(\beta_1) + \frac{\sigma}{1-\sigma}$, but the opposite is true when $\beta_1\alpha_2'(\beta_1) < \alpha_2(\beta_1) + \frac{\sigma}{1-\sigma}$. In other words, both the consumption growth rate and the output growth reach a local maximum when

$$\beta_1\alpha_2'(\beta_1) = \alpha_2(\beta_1) + \frac{\sigma}{1-\sigma}. \tag{42}$$

In addition, suppose $\phi(\beta_1) > 1, \forall\beta_1 \in (0,1)$, where $\phi(\beta_1) \equiv -\frac{\beta_1\alpha_2''(\beta_1)}{\alpha_2'(\beta_1)}$. Economically, $\phi(\beta_1)$ is the elasticity of marginal change in country 2's expenditure share on imports relative to the change in country 1's expenditure share on imports. Then both the consumption growth rate and the output growth rate are strictly concave functions of the import share β_1. Furthermore, suppose $\alpha_2(\beta_1)$ satisfies the following Inada-like condition:

$$\lim_{\beta_1\to 0}\beta_1\alpha_2'(\beta_1) > \alpha_2(\beta_1) + \frac{\sigma}{1-\sigma} > \alpha_2'(1), \tag{43}$$

then both $h_1(t)$ and $\theta_1(t)$ reach the unique global maximum when $\beta_1 = \beta_1^*$, where β_1^* is the unique solution to (42) and $\frac{\partial\beta_1^*}{\partial\sigma} < 0$. Suppose β_1 can reach any value in the interval $(0,1)$ by choosing some finite (possibly negative) τ_2 as $\beta_1'(\tau_2) < 0$, the result means that there exists a finite non-zero tariff (subsidy) rate at which both the consumption and output growth rates are the largest. Furthermore, the larger the intertemporal elasticity of substitution, the smaller the growth-maximizing tariff rate.

6 Conclusion

This paper develops a highly tractable dynamic two-country growth model with trade to study how international trade affects the dynamics of underlying industries and the aggregate economic growth. We obtain closed-form solutions to characterize the endowment-driven inverse-V-shaped life cycle dynamics of each underlying industries, which are different in their capital intensities, along the sustained growth path of the aggregate economy. We find that the intertemporal elasticity of substitution is a crucial parameter, which determines how industrialization, industrial upgrading, and aggregate growth are affected by the trade partner's technological progress and the trade policies. This is mainly because it determines whether the intertemporal terms-of-trade effect dominates the dynamic market-size (income) effect as these two effects have opposite impact on the endogenous saving decision and industrial upgrading. These two competing effects exactly cancel out when the intertemporal elasticity of substitution equals one. We also find that the magnitude of the intertemporal elasticity of substitution also affects the growth impact of trade liberalization. In particular, when the intertemporal elasticity of substitution is larger than one, accelerating trade liberalization may first increase the rates of consumption growth, industrial upgrading, and economic growth when the tariff rate is sufficiently large, but the effect is exactly the opposite when the tariff rate is sufficiently low. Moreover, the growth impact is not monotonic. When the important shares are functions of the tariff rate, we show that a country may achieve the fastest industrial upgrading and output growth by choosing an optimal finite and positive tariff rate, which, again, depends on the intertemporal elasticity of substitution.

There are several interesting directions for future research. The most natural direction is to relax the Armington assumption by allowing each country to have access to the production technologies of any goods. Correspondingly, different industries should be imperfectly substitutable, which would allow us to explore the consequences of industry-specific trade policies more fruitfully. Dornbusch, Fischer and Samuelson (1980) is a desirable starting point, but tractability can be easily hurt not only because of the newly added nonlinearity, but also due to the curse of dimensionality as no industry dies out from the world in this new setting, which is different from the current model where the dimensionality problem is tremendously simplified due to the exit of industries. Numerical methods seem indispensable. To quantitatively match the data of industrial dynamics, we presumably need to introduce industry-specific productivity changes as well. Another interesting direction is to introduce the productivity heterogeneity of different firms into each industry by following Bernard, Redding and Schott (2007), which may shed new light on the firm dynamics together with the industrial dynamics.

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Appendix

Appendix 1. Proof of Proposition 2. To solve the above dynamic problem, following Kamien and Schwartz (1991), we set the *discounted-value* Hamiltonian in the interval of $t_{1,n} \le t \le t_{1,n+1}$ for any $n \ge 1$, and use subscripts "$n, n+1$" to denote all variables in this interval:

$$\begin{aligned} H_{n,n+1} &= \frac{C_1(t)^{1-\sigma}-1}{1-\sigma}e^{-\rho t} + \eta_{n,n+1}\left[\xi_1 K_1(t) - \left[\left[\frac{C_1(t)}{\alpha X_2^\beta(t)}\right]^{\frac{1}{\alpha}} - \frac{\lambda^n(a-\lambda)}{a-1}L_1\right]\frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n}\right] \\ &\quad + \zeta_{n,n+1}^{n+1}\left(\lambda^{n+1}L_1 - \left[\frac{C_1(t)}{\alpha X_2^\beta(t)}\right]^{\frac{1}{\alpha}}\right) + \zeta_{n,n+1}^{n}\left(\left[\frac{C_1(t)}{\alpha X_2^\beta(t)}\right]^{\frac{1}{\alpha}} - \lambda^n L_1\right) \end{aligned} \tag{44}$$

where $\eta_{n,n+1}$ is the co-state variable, $\zeta_{n,n+1}^{n+1}$ and $\zeta_{n,n+1}^{n}$ are the Lagrangian multipliers for the two constraints $\lambda^{n+1}L_1 - C_1(t) \ge 0$ and $C_1(t) - \lambda^n L_1 \ge 0$, respectively. The first order and K-T conditions are

$$\frac{\partial H_{n,n+1}}{\partial C_1} = C_1(t)^{-\sigma}e^{-\rho t} - \left(\eta_{n,n+1}\frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n} + \zeta_{n,n+1}^{n+1} - \zeta_{n,n+1}^{n}\right)\frac{1}{\alpha^2 X_2^\beta(t)}\left[\frac{C_1(t)}{\alpha X_2^\beta(t)}\right]^{\frac{1}{\alpha}-1} = 0 \tag{45}$$

$$\zeta_{n,n+1}^{n+1}\left(\lambda^{n+1}L_1 - \left[\frac{C_1(t)}{\alpha X_2^\beta(t)}\right]^{\frac{1}{\alpha}}\right) = 0;\ \zeta_{n,n+1}^{n+1} \ge 0, \lambda^{n+1}L_1 - \left[\frac{C_1(t)}{\alpha X_2^\beta(t)}\right]^{\frac{1}{\alpha}} \ge 0$$

$$\zeta_{n,n+1}^{n}\left(\left[\frac{C_1(t)}{\alpha X_2^\beta(t)}\right]^{\frac{1}{\alpha}} - \lambda^n L_1\right) = 0,\ \ \zeta_{n,n+1}^{n} \ge 0,\ \left[\frac{C_1(t)}{\alpha X_2^\beta(t)}\right]^{\frac{1}{\alpha}} - \lambda^n L_1 \ge 0.$$

We also have

$$\eta_{n,n+1}'(t) = -\frac{\partial H_{n,n+1}}{\partial K_1} = -\eta_{n,n+1}\xi_1. \tag{46}$$

In particular, when $\left[\frac{C_1(t)}{\alpha X_2^\beta(t)}\right]^{\frac{1}{\alpha}} \in (\lambda^n L_1, \lambda^{n+1}L_1)$, $\zeta_{n,n+1}^{n+1} = \zeta_{n,n+1}^{n} = 0$, and equation (45) becomes

$$C_1(t)^{-\sigma}e^{-\rho t} = \eta_{n,n+1}\frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n}\frac{1}{\alpha^2 X_2^\beta(t)}\left[\frac{C_1(t)}{\alpha X_2^\beta(t)}\right]^{\frac{1}{\alpha}-1}. \tag{47}$$

The left hand side is the marginal utility gain by increasing one unit of aggregate consumption, while the right hand side is the marginal utility loss due to the decrease in capital because of that additional unit of consumption, which by chain's rule can be decomposed into three multiplicative terms: the marginal utility of capital $\eta_{n,n+1}$, the marginal capital requirement for each additional unit of aggregate consumption $\frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n}$ and the terms of trade $\frac{1}{\alpha^2 X_2^\beta(t)}\left[\frac{C_1(t)}{\alpha X_2^\beta(t)}\right]^{\frac{1}{\alpha}-1}$. Taking log of both sides of equation (47) and differentiating with respect to t, we have:

$$(\frac{1}{\alpha}+\sigma-1)\frac{\dot{C_1}(t)}{C_1(t)}=\xi_1-\rho+\frac{\beta}{\alpha}\frac{\dot{X_2}(t)}{X_2(t)}, \tag{48}$$

for $t_{1,n} \le t \le t_{1,n+1}$ for any $n \ge 0$.

Symmetrically, we also have

$$(\frac{1}{\beta}+\sigma-1)\frac{\dot{C_2}(t)}{C_2(t)}=\xi_2-\rho+\frac{\alpha}{\beta}\frac{\dot{X_1}(t)}{X_1(t)}. \tag{49}$$

Recall we have (8) hold for any time, which implies

$$\frac{\dot{C_1}(t)}{C_1(t)}=\frac{\dot{C_2}(t)}{C_2(t)}=\alpha\frac{\dot{X_1}(t)}{X_1(t)}+\beta\frac{\dot{X_2}(t)}{X_2(t)}. \tag{50}$$

Therefore we obtain (14). For the completeness of the proof, observe that the strictly concave utility function implies that the optimal consumption flow $C_1(t)$ must be continuous and sufficiently smooth (with no kinks) throughout the time, hence from (50) we obtain:

$$C_1(t)=C_1(t_{1,0})e^{\frac{\alpha\xi_1+\beta\xi_2-\rho}{\sigma}(t-t_{1,0})} \text{ for any } t \ge t_{1,0}. \tag{51}$$

Following Kamien and Schwartz (1991), we have two additional necessary conditions at $t=t_{1,n+1}$:

$$\begin{aligned} H_{n,n+1}(t_{1,n+1}) &= H_{n+1,n+2}(t_{1,n+1}) && (52)\\ \eta_{n,n+1}(t_{1,n+1}) &= \eta_{n+1,n+2}(t_{1,n+1}) && (53) \end{aligned}$$

Substituting equations (52) and (53) into (44), we can verify that $K_1^-(t_{1,n+1}) = K_1^+(t_{1,n+1})$. In other words, $K_1(t)$ is indeed continuous.

When $t \le t_{1,0}$,

$$H_0=\frac{C_1(t)^{1-\sigma}-1}{1-\sigma}e^{-\rho t}+\eta_0\xi_1K_1(t)+\zeta_0(L_1-\left[\frac{C_1(t)}{\alpha X_2^{\beta}(t)}\right]^{\frac{1}{\alpha}}) \tag{54}$$

FOCs and K-T conditions:

$$\begin{aligned} C_1(t)^{-\sigma}e^{-\rho t} &= \frac{1}{\alpha}\zeta_0\left[\frac{C_1(t)}{\alpha X_2^{\beta}(t)}\right]^{\frac{1}{\alpha}-1}\frac{1}{\alpha X_2^{\beta}(t)}\\ \eta_0'(t) &= -\frac{\partial H_{n,n+1}}{\partial K_1}=-\eta_0\xi_1.\\ \zeta_0 &\ge 0, L_1-\left[\frac{C_1(t)}{\alpha X_2^{\beta}(t)}\right]^{\frac{1}{\alpha}}\ge 0 \text{ and } \zeta_0(L_1-\left[\frac{C_1(t)}{\alpha X_2^{\beta}(t)}\right]^{\frac{1}{\alpha}})=0. \end{aligned}$$

therefore, we have $\frac{\dot{X_1}(t)}{X_1(t)} = 0$, $\frac{\dot{C_1}(t)}{C_1(t)} = \beta\frac{\dot{X_2}(t)}{X_2(t)}$. And $\frac{\dot{\zeta_0}(t)}{\zeta_0(t)} = \rho - \beta(1-\sigma)\frac{\dot{X_2}(t)}{X_2(t)}$.

When $t \in (t_{1,0}, t_{1,1})$,

$$
\begin{aligned}
H_{0,1} \quad = \quad & \frac{C_1(t)^{1-\sigma}-1}{1-\sigma}e^{-\rho t} + \eta_{0,1}\left[\xi_1 K_1(t) - \frac{a}{\lambda - 1}\left[\left[\frac{C_1(t)}{\alpha X_2^\beta(t)}\right]^{\frac{1}{\alpha}} - L_1\right]\right] \\
& + \zeta_{0,1}^1(\lambda L_1 - \left[\frac{C_1(t)}{\alpha X_2^\beta(t)}\right]^{\frac{1}{\alpha}}) + \zeta_{0,1}^0(\left[\frac{C_1(t)}{\alpha X_2^\beta(t)}\right]^{\frac{1}{\alpha}} - L_1) \qquad (55)
\end{aligned}
$$

Optimality conditions state that

$$
C_1(t)^{-\sigma}e^{-\rho t} - \left(\eta_{0,1}\frac{a}{\lambda-1} + \zeta_{0,1}^1 - \zeta_{0,1}^0\right)\frac{1}{\alpha^2 X_2^\beta(t)}\left[\frac{C_1(t)}{\alpha X_2^\beta(t)}\right]^{\frac{1}{\alpha}-1} = 0 \qquad (56)
$$

$$
\zeta_{0,1}^1(\lambda L_1 - \left[\frac{C_1(t)}{\alpha X_2^\beta(t)}\right]^{\frac{1}{\alpha}}) = 0;\ \zeta_{0,1}^1 \geq 0, \lambda L_1 - \left[\frac{C_1(t)}{\alpha X_2^\beta(t)}\right]^{\frac{1}{\alpha}} \geq 0
$$

$$
\zeta_{0,1}^0(\left[\frac{C_1(t)}{\alpha X_2^\beta(t)}\right]^{\frac{1}{\alpha}} - L_1) = 0,\ \ \zeta_{0,1}^0 \geq 0,\ \left[\frac{C_1(t)}{\alpha X_2^\beta(t)}\right]^{\frac{1}{\alpha}} - L_1 \geq 0.
$$

We also have

$$
\eta_{0,1}'(t) = -\frac{\partial H_{0,1}}{\partial K_1} = -\eta_{0,1}\xi_1. \qquad (57)
$$

thus we have

$$
C_1(t)^{-\sigma}e^{-\rho t} = \eta_{0,1}\frac{1}{\lambda-1}\frac{1}{\alpha X_2^\beta(t)}\left[\frac{C_1(t)}{\alpha X_2^\beta(t)}\right]^{\frac{\beta}{\alpha}}
$$

implying

$$
(\frac{1}{\alpha} + \sigma - 1)\frac{\dot{C_1}(t)}{C_1(t)} = \xi_1 - \rho + \frac{\beta}{\alpha}\frac{\dot{X_2}(t)}{X_2(t)}.
$$

at the same time

$$
X_1(t) = \left[\frac{C_1(t)}{\alpha X_2^\beta(t)}\right]^{\frac{1}{\alpha}}
$$

If $t < t_{2,0}$ holds, $\frac{\dot{X_2}(t)}{X_2(t)} = 0$ and $\frac{\dot{C_2}(t)}{C_2(t)} = \alpha\frac{\dot{X_1}(t)}{X_1(t)}$. If $t > t_{2,0}$ holds, $\frac{\dot{X_2}(t)}{X_2(t)} = 0$ and $\frac{\dot{C_2}(t)}{C_2(t)} = \alpha\frac{\dot{X_1}(t)}{X_1(t)}$

Consequently, when $t < \min\{t_{1,0}, t_{2,0}\}$, we must have $X_1(t) = L_1; X_2(t) = L_2$;

$C_1(t) = \alpha L_1^\alpha L_2^\beta$; $C_2(t) = \beta L_1^\alpha L_2^\beta$. In other words,

$$\frac{\dot{C_1}(t)}{C_1(t)} = \frac{\dot{C_2}(t)}{C_2(t)} = \frac{\dot{X_1}(t)}{X_1(t)} = \frac{\dot{X_2}(t)}{X_2(t)} = 0.$$

Suppose, $t_{1,0} \neq t_{2,0}$, then when $t \in [t_{1,0}, t_{2,0}]$, then

$$(\frac{1}{\alpha} + \sigma - 1)\frac{\dot{C_1}(t)}{C_1(t)} = \xi_1 - \rho + \frac{\beta}{\alpha}\frac{\dot{X_2}(t)}{X_2(t)}.$$

at the same time, i

$$\frac{\dot{X_2}(t)}{X_2(t)} = 0, \frac{\dot{C_2}(t)}{C_2(t)} = \alpha\frac{\dot{X_1}(t)}{X_1(t)}, \frac{\dot{C_2}(t)}{C_2(t)} = \frac{\dot{C_1}(t)}{C_1(t)}$$

thus

$$\begin{aligned} \frac{\dot{C_1}(t)}{C_1(t)} &= \frac{\dot{C_2}(t)}{C_2(t)} = \frac{\xi_1 - \rho}{(\frac{\beta}{\alpha} + \sigma)} \\ \frac{\dot{X_1}(t)}{X_1(t)} &= \frac{\xi_1 - \rho}{(\beta + \alpha\sigma)}; \frac{\dot{X_2}(t)}{X_2(t)} = 0. \end{aligned}$$

Symmetrically, when $t \in [t_{2,0}, t_{1,0}]$, then

$$\begin{aligned} \frac{\dot{C_1}(t)}{C_1(t)} &= \frac{\dot{C_2}(t)}{C_2(t)} = \frac{\xi_2 - \rho}{(\frac{1}{\alpha} + \sigma - 1)} \\ \frac{\dot{X_2}(t)}{X_2(t)} &= \frac{\xi_2 - \rho}{\alpha(\frac{1}{\alpha} + \sigma - 1)}; \frac{\dot{X_1}(t)}{X_1(t)} = 0. \end{aligned}$$

Q.E.D.

Appendix 2: Proof. of Lemma 3:First notice (13) implies

$$\frac{\dot{P_1}(t)}{P_1(t)} - \frac{\dot{P_2}(t)}{P_2(t)} = \frac{\dot{X_2}(t)}{X_2(t)} - \frac{\dot{X_1}(t)}{X_1(t)} = \xi_2 - \xi_1. \tag{58}$$

In addition, recall the price for the final good is normalized to unity at any time point, that is,

$$\left(\frac{P_1(t)}{\alpha}\right)^\alpha \left(\frac{P_2(t)}{\beta}\right)^\beta = 1,$$

which implies

$$\alpha\frac{\dot{P_1}(t)}{P_1(t)} + \beta\frac{\dot{P_2}(t)}{P_2(t)} = 0. \tag{59}$$

(58) and (59) jointly yield (17). **Q.E.D.**

Appendix 3

In this Appendix 3, we solve for the initial value of total consumption $X_i(0)$ when $\vartheta_{i,0} < K_i(0) \leq \vartheta_{i,1}$, and also show how to derive the threshold values for $\vartheta_{i,n}, \forall n = 0, 1, 2, ...$ Here we demonstrate how to characterize $X_1(0)$ and $\{\vartheta_{1,n}\}_{n=0}^{\infty}$. The values for country 2 can be derived similarly.

The transversality condition is derived from

$$\lim_{t\to\infty} H(t) = 0,$$

so

$$\lim_{t\to\infty}\left[\frac{C_1(t)^{1-\sigma}-1}{1-\sigma}e^{-\rho t}+\eta_{n(t),n(t)+1}\left[\xi_1 K_1(t) - E_{1,(n(t),n(t)+1)}(X_1(t))\right]\right] = 0.$$

Note that

$$\begin{aligned}
&\lim_{t\to\infty}\left[\frac{C_1(t)^{1-\sigma}-1}{1-\sigma}e^{-\rho t}+\eta_{n(t),n(t)+1}\left[\xi_1 K_1(t) - E_{1,(n(t),n(t)+1)}(X_1(t))\right]\right]\\
&=\lim_{t\to\infty}\left[\frac{C_1(0)^{1-\sigma}e^{\frac{(1-\sigma)(\alpha\xi_1+\beta\xi_2)-\rho}{\sigma}t}}{1-\sigma}+\eta_{n(t),n(t)+1}\left[\xi_1 K_1(t) - E_{1,(n(t),n(t)+1)}(X_1(t))\right]\right]\\
&=\lim_{t\to\infty}\eta_{n(t),n(t)+1}[\xi_1 K_1(t) - E_{1,(n(t),n(t)+1)}(X_1(t))]\\
&=\lim_{t\to\infty}\left\{\eta_{(0)}e^{-\xi_1 t}\left[\xi_1 K_1(t) - \left[X_1(0)e^{h_1 t} - \frac{\lambda^{n(t)}(a-\lambda)}{a-1}L_1\right]\frac{a^{n(t)+1}-a^{n(t)}}{\lambda^{n(t)+1}-\lambda^{n(t)}}\right]\right\}\\
&=\lim_{t\to\infty}\left\{\eta_{(0)}\left[\xi_1 K_1(t)e^{-\xi_1 t} - \left[-\frac{e^{-\xi_1 t}\lambda^{n(t)}(a-\lambda)}{a-1}L_1\right]\frac{a^{n(t)+1}-a^{n(t)}}{\lambda-1}\right]\right\}\\
&=\lim_{t\to\infty}K_1(t)e^{-\xi_1 t},
\end{aligned}$$

where the second equality is due to (12), the fourth equality comes from $\xi_1 > h_1$. Thus we must have $\lim_{t\to\infty} K_1(t)e^{-\xi_1 t} = 0$.

Now let's find the necessary and sufficient condition such that country 1 starts with industries 0 and 1. When $t \in [0, t_{1,1}]$,

$$E_1(t) = \frac{a}{\lambda-1}(X_1(t) - L_1) = \frac{a}{\lambda-1}(X_1(0)e^{h_1 t} - L_1),$$

Correspondingly,

$$\dot{K_1} = \xi_1 K_1(t) - E_1(C_1(t)) = \xi_1 K_1(t) - \frac{a}{\lambda-1}(X_1(0)e^{h_1 t} - L_1)$$

Solving this first-order differential equation with the condition $K_1(0) = K_{1,0}$, we obtain

$$K_1(t) = \frac{-\frac{aX_1(0)}{\lambda-1}}{h_1-\xi_1}e^{h_1 t} + \frac{-aL_1}{\xi_1(\lambda-1)} + \left[K_{1,0} + \frac{\frac{aX_1(0)}{\lambda-1}}{h_1-\xi_1} + \frac{aL_1}{\xi_1(\lambda-1)}\right]e^{\xi_1 t}, \tag{60}$$

which implies

$$K_1(t_{1,1}) = \frac{-\frac{a\lambda L_1}{\lambda-1}}{h_1-\xi_1} + \frac{-aL_1}{\xi_1(\lambda-1)} + \left[K_{1,0} + \frac{\frac{aX_1(0)}{\lambda-1}}{h_1-\xi_1} + \frac{aL_1}{\xi_1(\lambda-1)}\right]\left(\frac{\lambda L_1}{X_1(0)}\right)^{\frac{\xi_1}{h_1}}. \tag{61}$$

When $t \in [t_{1,n}, t_{1,n+1}]$ for $\forall n \geq 1$, we have

$$K_1(t) = -\frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n}\left[\frac{X_1(0)e^{h_1 t}}{h_1-\xi_1} + \frac{\lambda^n(a-\lambda)L_1}{\xi_1(a-1)}\right] + \theta_{n.n+1}e^{\xi_1 t}. \tag{62}$$

which, together with $X_1(0)e^{h_1 t_{1,n}} = X_1(t_{1,n}) = \lambda^n L_1$, determines

$$\theta_{n.n+1} = \left[\frac{\lambda^n L_1}{X_1(0)}\right]^{\frac{-\xi_1}{h_1}}\left\{K_1(t_{1,n}) + \frac{a^{n+1}-a^n}{\lambda-1}L_1\left[\frac{1}{h_1-\xi_1} + \frac{(a-\lambda)}{\xi_1(a-1)}\right]\right\} \tag{63}$$

Substituting $t = t_{1,n+1} = \frac{\log\frac{\lambda^{n+1}L_1}{X_1(0)}}{h_1}$ and (63) into (62), we obtain

$$K_1(t_{1,n+1}) = \lambda^{\frac{\xi_1}{h_1}}K_1(t_{1,n}) + \frac{a^{n+1}-a^n}{\lambda-1}L_1\left[\frac{\lambda^{\frac{\xi_1}{h_1}}-\lambda}{h_1-\xi_1} + \frac{(a-\lambda)(\lambda^{\frac{\xi_1}{h_1}}-1)}{\xi_1(a-1)}\right],$$

which can be used recursively to obtain

$$K_1(t_{1,n}) = \lambda^{\frac{(n-1)\xi_1}{h_1}}K_1(t_{1,1}) + (a-1)B\lambda^{\frac{(n-2)\xi_1}{h_1}}\frac{a\left[1-\left(a\lambda^{\frac{-\xi_1}{h_1}}\right)^{n-1}\right]}{1-a\lambda^{\frac{-\xi_1}{h_1}}}, \text{ for any } n \geq 2 \tag{64}$$

where parameter B is defined as

$$B \equiv \frac{L_1}{\lambda-1}\left[\frac{\lambda^{\frac{\xi_1}{h_1}}-\lambda}{h_1-\xi_1} + \frac{(a-\lambda)\left(\lambda^{\frac{\xi_1}{h_1}}-1\right)}{\xi_1(a-1)}\right].$$

(18) implies $B < 0$. Substituting (62), (63) and (64) into the transversality condition $\lim_{t\to\infty} K_1(t)e^{-\xi_1 t} = 0$ and by revoking (18), we obtain

$$\lambda^{\frac{-\xi_1}{h_1}}K_1(t_{1,1}) + (a-1)B\lambda^{-2\frac{\xi_1}{h_1}}\frac{a}{1-a\lambda^{\frac{-\xi_1}{h_1}}} = 0,$$

so

$$K_1(t_{1,1}) = -\frac{(a-1)B\lambda^{\frac{-\xi_1}{h_1}}a}{1-a\lambda^{\frac{-\xi_1}{h_1}}} > 0. \tag{65}$$

It can be verified that, without condition (18), the transversality condition cannot hold unless both B and $K_1(t_{1,1})$ are equal to zero, which is economically unreasonable because $K_1(t_{1,1}) > 0$ must hold due to the resource contraint as no international borrowing or lending is allowed.

According to (61), we have

$$\begin{aligned} &\left[K_{1,0} + \frac{aX_1(0)}{(\lambda-1)(h_1-\xi_1)} + \frac{aL_1}{\xi_1(\lambda-1)}\right]\left(\frac{\lambda L_1}{X_1(0)}\right)^{\frac{\xi_1}{h_1}} \\ = \quad & \frac{a\lambda L_1}{(\lambda-1)(h_1-\xi_1)} + \frac{aL_1}{\xi_1(\lambda-1)} - \frac{(a-1)B\lambda^{\frac{-\xi_1}{h_1}}a}{1-a\lambda^{\frac{-\xi_1}{h_1}}}. \end{aligned} \tag{66}$$

We can verify that the right hand side is strictly positive and that the left hand side is a strictly decreasing function of $X_1(0)$, therefore we can uniquely pin down the optimal $X_1^*(0)$. (66) immediately implies $\frac{\partial X_1^*(0)}{\partial K_{1,0}} > 0$ and $\frac{\partial X_1^*(0)}{\partial L_1} > 0$.

Note that (65) implies that $K(t_{1,1})$ does not depend on $K_1(0)$, therefore (64) tells that $K_1(t_{1,n})$ for all $n \geq 1$ are independent from $K_1(0)$. Since we assume good 0 and good 1 are produced at time 0, we need to ensure $L_1 < X_1^*(0) \leq \lambda L_1$.

To ensure $X_1^*(0) \leq \lambda L_1$, from (66) , it requires

$$K_{1,0} \leq \vartheta_{1,1} \equiv K_1(t_{1,1}) = -\frac{a\lambda^{\frac{-\xi_1}{h_1}}}{1-a\lambda^{\frac{-\xi_1}{h_1}}}\frac{L_1}{\lambda-1}\left[\frac{\xi_1\left(a-\lambda^{\frac{\xi_1}{h_1}}\right)(1-\lambda)+h_1(a-\lambda)\left(\lambda^{\frac{\xi_1}{h_1}}-1\right)}{(h_1-\xi_1)\,\xi_1}\right],$$

which is strictly positive due to (18). We also need to ensure $X_1^*(0) > L_1$, which, by revoking (66), requires

$$K_{1,0} > \vartheta_{1,0} \equiv \frac{a}{\left(1-a\lambda^{\frac{-\xi_1}{h_1}}\right)(\lambda-1)}\frac{h_1L_1}{(h_1-\xi_1)\,\xi_1}\left[\frac{\left(1-\lambda^{1-\frac{\xi_1}{h_1}}\right)(1-\lambda^{\frac{\xi_1}{h_1}})}{\lambda^{\frac{\xi_1}{h_1}}}\right] > 0. \tag{67}$$

Since $K_1(t_{1,1})$ is known (given by (65)), $K_1(t_{1,n})$ can be uniquely determined by (64) for any $n \geq 2$. Consequently, for any $t \geq 0$, $K(t)$ can be explicitly computed from (60) or (62) and (63), where $t_{i,n}$ is determined by (20) in Lemma 2 for any $n \geq 0$ because $X_1^*(0)$ is uniquely determined by (66).

Thus, using (62) and (60), we obtain, more generally, for any $i = 1, 2$,

$$K_i(t) = \begin{cases} \frac{-\frac{aX_i(0)}{\lambda-1}}{h_i-\xi_i} e^{h_i i t} + \frac{-aL_i}{\xi_i(\lambda-1)} + \left[K_{i0} + \frac{\frac{aX_i(0)}{\lambda-1}}{h_i-\xi_i} + \frac{aL_i}{\xi_i(\lambda-1)}\right] e^{\xi_i t} & \text{when} \quad t \in [0, t_{i,1}] \\ \alpha_{i,n} + \beta_{i,n} e^{h_1 t} + \gamma_{i,n} e^{\xi_i t} & \text{when} \quad t \in [t_{i,n}, t_{i,n+1}], \text{ for any } n \geq 1 \end{cases} \tag{68}$$

where $t_{i,n}$ is given by (20) and for any $n \geq 1$,

$$\begin{aligned} \alpha_{i,n} &= -\frac{a^n (a-\lambda) L_i}{\xi_i (\lambda - 1)}, \\ \beta_{i,n} &= -\left(\frac{a^{n+1} - a^n}{\lambda^{n+1} - \lambda^n}\right) \frac{X_i(0)}{(h_i - \xi_i)}, \\ \gamma_{i,n} &= \left[\frac{\lambda^n L_i}{X_i(0)}\right]^{\frac{-\xi_i}{h_i}} \left\{\vartheta_{i,n} + \frac{\left(a^{n+1} - a^n\right) L_i}{\lambda - 1} \left[\frac{1}{(h_i - \xi_i)} + \frac{(a-\lambda)}{\xi_i (a-1)}\right]\right\}. \end{aligned}$$

Note that

$$\vartheta_{i,1} \equiv K_i(t_{i,1}) = \frac{-\frac{a\lambda L}{\lambda-1}}{h_i - \xi_i} + \frac{-aL_i}{\xi_i (\lambda - 1)} + \left[K_{i0} + \frac{\frac{aX_i(0)}{\lambda-1}}{h_i - \xi_i} + \frac{aL_i}{\xi_i (\lambda - 1)}\right] \left(\frac{\lambda L_i}{X_i(0)}\right)^{\frac{\xi_i}{h_i}},$$

and $\{\vartheta_{i,n}\}_{n=2}^{\infty}$ are all constants, and $\vartheta_{i,n} \equiv K_i(t_{i,n})$ can be sequentially computed by applying (68) recursively with $K_i(t_{i,n-1})$ known. The initial output $X_1(0)$ is uniquely determined by (66) obtained from the transversality condition, $X_2(0)$ can be obtained using the same method..

Next, let us characterize what happens when $K_{1,0} \in (0, \vartheta_{1,0}]$, in which case country 1 must start by producing good 0 only.

$$\max_{C_1(t)} \int_0^{t_{1,0}} \frac{C_1(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt + \sum_{n=0}^{\infty} \int_{t_{1,n}}^{t_{1,n+1}} \frac{C_1(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt$$

subject to

$$\dot{K_1} = \begin{cases} \xi_1 K_1 & \text{when} \quad 0 \leq t \leq t_{1,0} \\ \xi_1 K_1 - E_{1,(0,1)}(X_1), & \text{when} \quad t_{1,0} \leq t \leq t_{1,1} \\ \xi_1 K_1 - E_{1,(n,n+1)}(X_1), & \text{when} \quad t_{1,n} \leq t \leq t_{1,n+1}, \text{ for } n \geq 1 \end{cases},$$

$K_1(0)$ is given.

We also have $C_1(t) = \alpha X_1^{\alpha}(t) X_2^{\beta}(t)$.So when $0 \leq t \leq t_{1,0}$, we must have $X_1(t) = L_1$ because labor entails no utility cost for the household, therefore $C_1(t) = \alpha {L_1}^{\alpha} X_2^{\beta}(t)$.The associated discounted-value Hamiltonian with the Lagrangian multipliers is the following

$$H_0 = \frac{C_1(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} + \eta_0 \xi_1 K_1(t) + \zeta_0^0 \left[\alpha {L_1}^{\alpha} X_2^{\beta}(t) - C_1(t)\right].$$

First order condition and K-T condition are

$$
\begin{aligned}
C_1(t)^{-\sigma}e^{-\rho t} &= \zeta_0^0; \\
\zeta_0^0\left[\alpha {L_1}^{\alpha} X_2^{\beta}(t) - C_1(t)\right] &= 0; \\
\alpha {L_1}^{\alpha} X_2^{\beta}(t) - C_1(t) &= 0 \text{ when } \zeta_0^0 > 0.
\end{aligned}
$$

and

$$\eta_0 = -\frac{\partial H_0}{\partial K_1} = -\eta_0 \xi_1.$$

They immediately imply that $C_1^*(t) = \alpha {L_1}^{\alpha} X_2^{\beta}(t)$. No capital is used for production and therefore

$$\dot{K_1}(t) = \xi_1 K_1(t).$$

When capital stock K_1 exceeds $\vartheta_{1,0}$ by an infinitessimal amount, the economy produces both good 0 and good 1. From that point on, the problem is exactly the same as the one we have just solved in the main text. Let $t_{1,0}$ denote the time point when K_1 equals $\vartheta_{1,0}$. Then

$$K_{1,0}e^{\xi_1 t_{1,0}} = \vartheta_{1,0},$$

so $t_{1,0} = \frac{\log \frac{\vartheta_{1,0}}{K_{1,0}}}{\xi_1}$. Therefore

$$C_1^*(t) = \left\{ \begin{array}{ccc} \alpha {L_1}^{\alpha} X_2^{\beta}(t), & \text{when} & t \leq t_{1,0} \\ \alpha {L_1}^{\alpha} X_2^{\beta}(t_{1,0}) e^{\theta_1(t-t_{1,0})}, & \text{when} & t > t_{1,0} \end{array} \right. .$$

Let $t_{1,j}$ denote the time point when only good j is produced, for any $j \geq 1$. Observe that $L_1 e^{h_1(t_{1,j}-t_{1,0})} = X_1(t_{1,j}) = \lambda^j L_1$, so $t_{1,j} = t_{1,0} + \left(\frac{\log\lambda}{h_1}\right) j$, $t_{1,0} = \frac{\log \frac{\vartheta_{1,0}}{K_{1,0}}}{\xi_1}$.

Correspondingly, the capital stock on the equilibrium path is given by

$$K_1(t) = \left\{ \begin{array}{ccc} K_{1,0}e^{\xi_1 t}, & \text{for} & t \in [0, t_{1,0}] \\ \frac{-\frac{aL_1}{\lambda-1}}{h_1-\xi_1} e^{h_1(t-t_{1,0})} + \frac{-aL_1}{\xi_1(\lambda-1)} + \left[\vartheta_{1,0} + \frac{\frac{aL_1}{\lambda-1}}{h_1-\xi_1} + \frac{aL_1}{\xi_1(\lambda-1)}\right] e^{\xi_1(t-t_{1,0})}, & \text{for} & t \in [t_{1,0}, t_{1,1}] \\ F(t), & \text{for} & \begin{array}{c} t \in [t_{1,n}, t_{1,n+1}], \\ \text{any } n \geq 1 \end{array} \end{array} \right. , \tag{69}$$

where

$$
\begin{aligned}
F(t) \equiv & -\frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n}\left[\frac{L_1 e^{h_1 t}}{(h_1-\xi_1)} + \frac{\lambda^n(a-\lambda)L_1}{\xi_1 (a-1)}\right] \\
& + [\lambda^n]^{\frac{-\xi}{h_1}} \left\{K_1(t_{1,n}) + \frac{a^{n+1}-a^n}{\lambda-1} L_1 \left[\frac{1}{(h_1-\xi_1)} + \frac{(a-\lambda)}{\xi_1 (a-1)}\right]\right\} e^{\xi_1(t-t_{1,0})},
\end{aligned}
$$

thus

$$K_1(t_{1,n+1}) = F(t_{1,n+1}) \equiv -\frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n}\left[\frac{L_1 e^{h_1 t_{1,n+1}}}{(h_1-\xi_1)} + \frac{\lambda^n(a-\lambda)L_1}{\xi_1(a-1)}\right]$$
$$+ [\lambda^n]^{\frac{-\xi_1\sigma}{\xi_1-\rho}}\left\{K_1(t_{1,n}) + \frac{a^{n+1}-a^n}{\lambda-1}L_1\left[\frac{1}{(h_1-\xi_1)} + \frac{(a-\lambda)}{\xi_1(a-1)}\right]\right\} e^{\xi_1(t_{1,n+1}-t_{1,0})}$$
$$K(t_{1,n+1}) = \psi_1 \lambda^{\frac{n\xi_1}{h_1}} \frac{a\lambda^{\frac{-\xi_1}{h_1}}\left[1-\left(a\lambda^{\frac{-\xi_1}{h_1}}\right)^n\right]}{1-a\lambda^{\frac{-\xi_1}{h_1}}} + \lambda^{\frac{n\xi_1}{h_1}} K_1(t_{1,1})$$

where

$$\psi_1 \equiv \frac{L_1(a-1)}{\lambda-1}\left\{\left(\lambda^{\frac{\xi_1}{h_1}}-\lambda\right)\frac{1}{(h_1-\xi_1)} + \left(\lambda^{\frac{\xi_1}{h_1}}-1\right)\frac{(a-\lambda)}{\xi_1(a-1)}\right\}.$$

Similar as before, we can derive the transversality condition: $\lim_{t\to\infty} K_1(t)e^{-\xi_1 t} = 0$, which implies

$$\begin{aligned}
\lim_{t\to\infty} F(t)e^{-\xi_1 t} &= 0 \\
&\Rightarrow \lim_{n\to\infty}\left(\lambda^{\frac{-\xi_1}{h_1}} a\right)^n \frac{K_1(t_{1,n})}{a^n} = 0 \\
&\Rightarrow \lim_{n\to\infty}\left(\lambda^{\frac{-\xi_1}{h_1}}\right)^{n+1} \psi_1 \lambda^{\frac{n\xi_1}{h_1}} \frac{a\lambda^{\frac{-\xi_1}{h_1}}\left[1-\left(a\lambda^{\frac{-\xi_1}{h_1}}\right)^n\right]}{1-a\lambda^{\frac{-\xi_1}{h_1}}} + \left(\lambda^{\frac{-\xi_1}{h_1}}\right)^{n+1} \lambda^{\frac{n\xi_1}{h_1}} K_1(t_{1,1}) = 0 \\
&\Rightarrow K_1(t_{1,1}) = -\psi_1 \frac{a\lambda^{\frac{-\xi_1}{h_1}}}{1-a\lambda^{\frac{-\xi_1}{h_1}}}.
\end{aligned}$$

By revoking (69), we obtain

$$\frac{-\frac{aL_1}{\lambda-1}}{h_1-\xi_1} e^{h_1(t_{1,1}-t_{1,0})} + \frac{-aL_1}{\xi_1(\lambda-1)} + \left[\vartheta_{1,0} + \frac{\frac{aL_1}{\lambda-1}}{h_1-\xi_1} + \frac{aL_1}{\xi_1(\lambda-1)}\right] e^{\xi_1(t_{1,1}-t_{1,0})} = -\psi_1 \frac{a\lambda^{\frac{-\xi_1}{h_1}}}{1-a\lambda^{\frac{-\xi_1}{h_1}}},$$

which yields

$$\vartheta_{1,0} = \frac{aL_1 h_1\left[\lambda^{1-\frac{\xi_1}{h_1}}-1\right]\left(1-\lambda^{\frac{-\xi_1}{h_1}}\right)}{\left(1-a\lambda^{\frac{-\xi_1}{h_1}}\right)(\lambda-1)(h_1-\xi_1)\xi_1}.$$

It can be verified that this is exactly the same expression as (67) derived before.

Using the similar algorithm, we can fully characterize the case when $K_{1,0} > \vartheta_{1,1}$. Q.E.D.

Appendix 4. Proof of Proposition 7.

The budget constraint for a representative household in country 1 is $P_1 C_{1,1} + P_2(1+\tau_2)C_{1,2} = P_1 X_1 + T_1$. Utility function (2) implies $C_{11} = \alpha(X_1 + \frac{T_1}{P_1})$ and

$C_{12} = \frac{\beta(P_1X_1+T_1)}{P_2(1+\tau_2)}$. Similarly, country 2 household's budget constraint is $P_1(1+\tau_1)C_{2,1}+P_2C_{2,2} = P_2X_2+T_2$.Thus we must have $C_{21} = \alpha\frac{P_2X_2+T_2}{P_1(1+\tau_1)}; C_{22} = \beta(X_2+\frac{T_2}{P_2})$. In the equilibrium, the tariff revenues are $T_1 = \frac{\beta\tau_2P_1X_1}{(1+\alpha\tau_2)}$ and $T_2 = \frac{\alpha\tau_1P_2X_2}{(1+\beta\tau_1)}$. Plugging all these into the market clearing conditions for good 1 and good 2 yields

$$\begin{aligned}
\alpha(X_1 + \frac{P_2\tau_2C_{1,2}}{P_1}) + \alpha\frac{P_2X_2 + P_1\tau_1C_{2,1}}{P_1(1+\tau_1)} &= X_1, \\
\frac{\beta\left(P_1X_1 + P_2\tau_2C_{1,2}\right)}{P_2(1+\tau_2)} + \beta(X_2 + \frac{P_1\tau_1C_{2,1}}{P_2}) &= X_2,
\end{aligned}$$

which imply

$$\begin{aligned}
C_{11} &= \frac{\alpha(1+\tau_2)X_1}{(1+\alpha\tau_2)};\ C_{12} = \frac{\alpha X_2}{(1+\beta\tau_1)}. \\
C_{21} &= \frac{\beta X_1}{(1+\alpha\tau_2)};\ C_{22} = \frac{(\tau_1+1)\,\beta X_2}{1+\beta\tau_1}.
\end{aligned}$$

Then (28) and (29) are obtained naturally. (30) can be derived easily. Observe that the decentralized production decisions in each country remain unaffected by international trade in this static economy, so X_1 and X_2 are exactly given in Table 1. **Q.E.D.**

Appendix 5. **Proof of Proposition 8.**

Proof. By following the same method as in Section 3, we establish the following Hamiltonian equation:

$$H_{n,n+1} = \frac{C_1(t)^{1-\sigma}-1}{1-\sigma}e^{-\rho t} + \eta_{n,n+1}\frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n}\left[\xi_1K(t) - \left[\begin{array}{c}\left[\frac{C_1(t)(1+\alpha\tau_2)^{\alpha}(1+\beta\tau_1)^{\beta}}{\alpha X_2^{\beta}(t)(1+\tau_2)^{\alpha}}\right]^{\frac{1}{\alpha}} \\ -\frac{\lambda^n(a-\lambda)}{a-1}L\end{array}\right]\right].$$

Using the first order conditions, we obtain

$$C_1(t)^{-\sigma}e^{-\rho t} = \eta_{n,n+1}\frac{a^{n+1}-a^n}{\lambda^{n+1}-\lambda^n}\frac{1}{\alpha}C_1(t)^{\frac{1}{\alpha}-1}\left[\frac{(1+\alpha\tau_2)^{\alpha}(1+\beta\tau_1)^{\beta}}{\alpha X_2^{\beta}(t)(1+\tau_2)^{\alpha}}\right]^{\frac{1}{\alpha}},$$

which yields

$$(\frac{1}{\alpha}+\sigma-1)\frac{\dot{C_1}(t)}{C_1(t)} = \xi_1 - \rho + \frac{\beta}{\alpha}\frac{\dot{X_2}(t)}{X_2(t)} - \frac{\alpha\dot{\tau}_2}{1+\alpha\tau_2} - \frac{\beta}{\alpha}\frac{\beta\dot{\tau}_1}{1+\beta\tau_1} + \frac{\dot{\tau}_2}{\tau_2+1}.$$

Similarly, for country 2, we have

$$H_{m,m+1} = \frac{C_2(t)^{1-\sigma}-1}{1-\sigma}e^{-\rho t} + \eta_{m,m+1}\left[\xi_2K(t) - \frac{a^{m+1}-a^m}{\lambda^{m+1}-\lambda^m}\left[\begin{array}{c}\left[\frac{C_1(t)(1+\alpha\tau_2)^{\alpha}(1+\beta\tau_1)^{\beta}}{\beta X_1^{\alpha}(t)(1+\tau_1)^{\beta}}\right]^{\frac{1}{\beta}} \\ -\frac{\lambda^m(a-\lambda)}{a-1}L\end{array}\right]\right],$$

which gives

$$(\frac{1}{\beta}+\sigma-1)\frac{\dot{C_2}(t)}{C_2(t)} = \xi_2 - \rho + \frac{\alpha}{\beta}\frac{\dot{X_1}(t)}{X_1(t)} - \frac{\alpha}{\beta}\frac{\alpha\dot{\tau}_2}{1+\alpha\tau_2} - \frac{\beta\dot{\tau}_1}{1+\beta\tau_1} + \frac{\dot{\tau}_1}{\tau_1+1}.$$

Moreover, (31) implies

$$\frac{\dot{C_1}(t)}{C_1(t)} - \frac{\dot{C_2}(t)}{C_2(t)} = \frac{\alpha\dot{\tau}_2}{1+\tau_2} - \frac{\beta\dot{\tau}_1}{1+\tau_1}.$$

(28) implies

$$\frac{\dot{C_1}(t)}{C_1(t)} = \frac{\alpha\dot{\tau}_2}{1+\tau_2} - \alpha\frac{\alpha\dot{\tau}_2}{1+\alpha\tau_2} - \beta\frac{\beta\dot{\tau}_1}{1+\beta\tau_1} + \alpha\frac{\dot{X_1}(t)}{X_1(t)} + \beta\frac{\dot{X_2}(t)}{X_2(t)},$$

and (29) implies

$$\frac{\dot{C_2}(t)}{C_2(t)} = \frac{\beta\dot{\tau}_1}{1+\tau_1} - \alpha\frac{\alpha\dot{\tau}_2}{1+\alpha\tau_2} - \beta\frac{\beta\dot{\tau}_1}{1+\beta\tau_1} + \alpha\frac{\dot{X_1}(t)}{X_1(t)} + \beta\frac{\dot{X_2}(t)}{X_2(t)}.$$

Solving these equations gives (33)-(36). **Q.E.D**.