Marginal Jobs, Heterogeneous Firms, and Unemployment Flows*

Michael W. L. Elsby
University of Edinburgh and NBER

Ryan Michaels
University of Rochester

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Abstract

This paper introduces a notion of firm size into a search and matching model with endogenous job destruction. The outcome is a rich, yet analytically tractable framework that can be used to analyze a broad set of features of both the cross section and the dynamics of the aggregate labor market. In a set of quantitative applications we show that the model can provide a coherent account of a) the salient features of the distributions of employer size, and employment growth across establishments; b) the amplitude and propagation of cyclical fluctuations in flows between employment and unemployment; c) the negative comovement of unemployment and vacancies in the form of the Beveridge curve; and d) the dynamics of the distribution of employer size over the business cycle.

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The study of the macroeconomics of labor markets has been dominated by two influential approaches in recent research: the development of search and matching models (Pissarides, 1985; Mortensen and Pissarides, 1994) and the empirical analysis of establishment dynamics (Davis and Haltiwanger, 1992). This paper provides an analytical framework that unifies these approaches by introducing a notion of firm size into a search and matching model with endogenous job destruction. The outcome is a rich, yet analytically tractable framework that can be used to analyze a broad set of features of both the cross section and the dynamics of the aggregate labor market. In a set of quantitative applications we show that the model can provide a coherent account of a) the salient features of the distributions of employer size, and employment growth across establishments; b) the amplitude and propagation of cyclical fluctuations in flows between employment and unemployment; c) the negative comovement of unemployment and vacancies in the form of the Beveridge curve; and d) the dynamics of the distribution of employer size over the business cycle.

A notion of firm size is introduced by relaxing the common assumption that firms face a linear production technology.[1] Though conceptually simple, incorporating this feature is not a trivial exercise. The existence of a non-linear production technology, and the associated presence of multi-worker firms, complicates wage setting because the surplus generated by each of the employment relationships within a firm is not the same—"the" marginal worker generates less surplus than infra-marginal workers. In section 1, we apply the bargaining solution of Stole and Zwiebel (1996) to derive a very intuitive wage bargaining solution for this environment, something that has been considered challenging in recent research (see Cooper, Haltiwanger and Willis, 2007; and Hobijn and Sahin, 2007). The solution is a very natural generalization of the wage bargaining solution in standard search and matching models. The simplicity of our solution is therefore a useful addition to the literature.[2]

The wage bargaining solution enables us to characterize the properties of the optimal labor demand policy of an individual firm in the presence of idiosyncratic firm heterogeneity. We demonstrate that the labor demand solution is analogous to that of a model of kinked hiring costs in the spirit of Bentolila and Bertola (1990), but where the hiring cost is en-

[1] In its simplest form, this manifests itself in a one firm, one job representation, as in Pissarides (1985) and Mortensen and Pissarides (1994).

[2] Bertola and Caballero (1994) solve a related bargaining problem by taking a linear approximation to the marginal product function and specializing productivity to a two-state Markov process. The present paper relaxes these restrictions. More recent research that models endogenous separations has set worker bargaining power to zero in order to derive wages (Cooper et al., 2007; Hobijn and Sahin, 2007). In the presence of exogenous separations, Acemoglu and Hawkins (2006) characterize wages, but focus instead on a time to hire aspect to job creation, which leads to a more challenging bargaining problem. The wage bargaining solution for models with exogenous job destruction has been characterized by Smith (1999), Cahuc and Wasmer (2001), and Krause and Lubik (2007).

dogenously determined by frictions in the labor market. This yields an analytical solution for the optimal labor demand policy, summarizing microeconomic behavior in the model.

In section 2, we take on the task of aggregating this behavior to the macroeconomic level. This is a challenge because the presence of a non-linear production technology and idiosyncratic heterogeneity imply that a representative firm interpretation of the model doesn't exist. To address this, we develop a method that allows us to solve analytically for the equilibrium distribution of employment across firms (the firm size distribution). In turn, this allows us to determine the level of the aggregate (un)employment stock, which is implied by the mean of that distribution. We also provide a related method that allows us to solve for aggregate unemployment flows (hires and separations) implied by microeconomic behavior. Together, these characterize the aggregate steady state equilibrium of the model economy.

In section 3 we explore the dynamics of the model by introducing aggregate shocks. A difficulty that arises in the model is that, out of steady state, individual firms must forecast future wages, which involves forecasting the future path of the distribution of employment across firms, an infinite-order state variable. A useful feature of our analytical solution for optimal labor demand is that it allows us to simplify part of this problem. In particular, we are able to derive an analytical approximation to a firm's optimal labor demand policy in the presence of aggregate shocks, obviating the need for a numerical solution. Using this, we employ an approach that mirrors the method proposed by Krusell and Smith (1998) to solve for the transition paths for the unemployment stock and flows in the presence of aggregate shocks.

These results form the basis of a series of quantitative applications, which we turn to in section 4. An attractive feature of the model is that, by incorporating both a notion of firm size as well as idiosyncratic heterogeneity, it delivers important cross sectional implications. We show that the model can be used to match key features of the distribution of firm size, and of employment growth across establishments. This is achieved through two aspects of the model. First, due to the existence of kinked hiring costs, optimal labor demand features a region of inaction whereby firms choose neither to hire nor fire workers. This matches a key property of the distribution of employment growth—the existence of a mass point at zero establishment growth—noted at least since the work of Davis and Haltiwanger (1992).[3] Second, informed by the well-known shape of the distribution of firm size, we adopt a Pareto specification for idiosyncratic firm productivity. A surprising outcome of this approach is that the Pareto specification also provides a very accurate description of the tails

[3]Earlier work by Hamermesh (1989), which analyzed data from seven manufacturing plants, also drew attention to the "lumpy" nature of establishment-level employment adjustment.

of the distribution of employment growth, something that cannot be achieved using more conventional lognormal specifications of heterogeneity.

We then use these steady-state features of the model to provide a novel perspective on the cyclical dynamics of worker flows implied by the model. It is well-known that the cyclical amplitude of unemployment, and of the job-finding rate in particular, relies critically on the size of the surplus to employment relationships (Shimer, 2005; Hagedorn and Manovskii, 2007). Intuitively, small reductions in aggregate productivity can easily exhaust a small surplus, and lead firms to cut back substantially on hiring. The presence of large and heterogeneous firms in our model opens up a new approach to calibrating the payoff from unemployment, and thereby the match surplus. Because the model is capable of matching the observed cross-sectional distribution of employment growth, we obtain a sense of the plausible size of idiosyncratic shocks facing firms. Given this, a higher payoff from unemployment implies a smaller surplus, so that jobs will be destroyed more frequently, raising the rate of worker turnover. We discipline the model by choosing the payoff from unemployment that matches the empirical rate at which employed workers flow into unemployment.

Applying this approach to an otherwise standard calibration reveals that our generalized model can replicate both the observed procyclicality of the job finding rate, as well as the countercyclicality of the employment to unemployment transition rate in the U.S.[4] We show that this is a substantial improvement over standard search and matching models. As shown by Shimer (2005), these are unable to generate enough cyclicality in job creation. To overcome this, the standard model must reduce the size of the surplus, which in turn yields excessive employment to unemployment transitions.[5] The generalized model does not face this tension between reproducing the cyclicality of job creation and the rate of worker turnover. Due to the diminishing marginal product of labor, the model generates simultaneously a large average surplus and a small marginal surplus to employment relationships. The former allows the model to match the rate at which workers flow into unemployment, the latter the volatility of the job-finding rate over the cycle.[6]

[4]For evidence on the countercyclicality of employment to unemployment flows, see Perry (1972); Marston (1976); Blanchard and Diamond (1990); Elsby, Michaels, and Solon (2009); Fujita and Ramey (2009); Pissarides (2007); Shimer (2007); and Yashiv (2006).

[5]This formalizes the intuition of recent research that has argued that the average surplus required for the standard model to match the observed cyclicality of the job finding rate is implausibly small (Mortensen and Nagypal, 2007). A small average surplus also jars with widespread evidence for the prevalence of long term employment relationships in the US economy, which researchers have taken to imply substantial rents to ongoing matches (Hall, 1982; Stevens, 2005).

[6]One might imagine that a symmetric logic holds on the supply side of the labor market if there is heterogeneity in workers' valuations of leisure so that "the" marginal worker obtains a low surplus from employment. Interestingly, Mortensen and Nagypal (2007) argue that this is not the case. They show that

A potential concern in models that incorporate countercyclical job destruction, such as the model in this paper, has been that they often cannot generate the observed procyclicality of vacancies (Shimer, 2005; Mortensen and Nagypal, 2008). Importantly, we find that our model makes considerable progress in this regard: Our calibration of the model generates most of the observed comovement between vacancies and output per worker. As a result, it reproduces a key stylized fact of the U.S. labor market: the negative comovement between unemployment and vacancies in the form of the Beveridge curve. The model therefore provides a coherent and quantitatively accurate picture of the joint cyclical properties of both flows of workers in and out of unemployment, as well as the behavior of unemployment and vacancies.

A less well-documented limitation of the standard search and matching model relates to the propagation of the response of the job finding rate to aggregate shocks to labor productivity. The job finding rate is a jump variable in the standard model, responding instantaneously to aggregate shocks, while it exhibits a sluggish response in U.S. data. An appealing feature of the generalized model is that it delivers a natural propagation mechanism: The job finding rate is a function of the distribution of employment across firms, which we show is a slow-moving state variable in our model. Simulations reveal that this aspect of the model can help account for the persistence of the decline in job creation following an adverse shock.

Our efforts to understand the amplification and propagation of labor market flows over the business cycle are related to the important early work of Cooper, Haltiwanger, and Willis (CHW, 2007). They also study a random matching model with decreasing returns. This paper's model differs from theirs in at least two regards. First, CHW suppress worker bargaining power. This simplifies wage bargaining, but at the cost of severing the link between the bargained wage, on the one hand, and market tightness on the other. This paper investigates a more general bargaining rule that allows workers to extract some of the rents. Second, CHW include additional labor market frictions, such as fixed costs to hire and fire. We agree that the interaction of these adjustment costs with matching frictions deserves further attention. However, we find it helpful to abstract from these for a number of reasons. In particular, we are able to make substantial headway in characterizing the analytics of optimal establishment-level labor demand and aggregate job creation. This in turn allows us to highlight the effects of decreasing returns and to contrast our model more

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if firms cannot differentiate workers' types when making hiring decisions, they will base their decision on the average, rather than the marginal, valuation of leisure among the unemployed. The same is unlikely to be true of the model studied here, since firms presumably know their production technology when making hiring decisions.

tightly with that in Mortensen and Pissarides. Thus, we see our paper as complementary to CHW, who study a richer model but rely principally on numerical solution methods.

In the closing sections of the paper, we move beyond characterizing the business cycle properties of the model and turn to evaluating its implications for a number of additional cross-sectional outcomes. First, recent literature has emphasized empirical regularities in the cyclical behavior of the cross-sectional distribution of establishment size: While the share of small establishments with fewer than 20 workers rises during recessions, the shares of larger firms decline (Moscarini and Postel-Vinay, 2009). The model replicates this observation: For each establishment size class considered, it broadly matches the comovement with unemployment over the business cycle observed in U.S. data. Given that these implications of the model are venturing farther afield from the moments it was calibrated to match, we view these results as an important achievement.

In our final quantitative application, we evaluate the model's ability to account for the observation that workers employed in larger firms are often paid higher wages—the employer size-wage effect (Brown and Medoff, 1989). A distinctive attribute of the model is that, by incorporating large firms with heterogeneous productivities, it can speak to this empirical regularity. The magnitude of the size-wage effect implied by the model is mediated by two competing forces, as noted by Bertola and Garibaldi (2001). On the one hand, the existence of diminishing returns in production might lead one to anticipate a negative relation between employer size and wages. On the other, larger firms also tend to be more productive. Quantitatively, the latter dominates, generating one quarter of the empirical size-wage effect.

The remainder of the paper is organized as follows. Section 1 describes the set-up of the model, and characterizes the wage bargaining solution together with the associated optimal labor demand policy of an individual firm. Given this, section 2 develops a method for aggregating this microeconomic behavior up to the macroeconomic level, and uses it to characterize the steady state equilibrium of the model. Section 3 introduces aggregate shocks to the analysis. It presents an approach to computing the out of steady state dynamics of the model through the use of analytical approximations. We then use the model in section 4 to address a wide range of quantitative applications. Finally, section 5 summarizes our results, and draws lessons for future research.

1 The Firm's Problem

In what follows we consider a model in which there is a mass of firms, normalized to one, and a mass of potential workers equal to the labor force, L.[7] In order to hire unemployed workers, firms must post vacancies. However, frictions in the labor market limit the rate at which unemployed workers and hiring firms can meet. As is conventional in the search and matching literature, these frictions are embodied in a matching function, $M = M(U, V)$, that regulates the number of hires, M, that the economy can sustain given that there are V vacancies and U unemployed workers. We assume that $M(U, V)$ exhibits constant returns to scale.[8] Vacancies posted by firms are therefore filled with probability $q = M/V = M(U/V, 1)$ each period. Likewise, unemployed workers find jobs with probability $f = M/U = M(1, V/U)$. Thus, the ratio of aggregate vacancies to aggregate unemployment, $V/U \equiv \theta$, is a sufficient statistic for the job filling (q) and job finding (f) probabilities in the model. Taking these flow probabilities as given, firms choose their optimal level of employment, to which we now turn.

1.1 Labor Demand

We consider a discrete time, infinite horizon model in which firms use labor, n, to produce output according to the production function, $y = pxF(n)$ where $F' > 0$ and $F'' \leq 0$. The latter is a key generalization of the standard search model that we consider: When $F'' < 0$, the marginal product of labor will decline with firm employment, and thereby will generate a downward sloped demand for labor at the firm level. p represents the state of aggregate labor demand, whereas x represents shocks that are idiosyncratic to an individual firm. We assume that the evolution of the latter idiosyncratic shocks is described by the c.d.f. $G(x'|x)$.

A typical firm's decision problem is completely analogous to that in Mortensen and Pissarides (1994), and is as follows. Firms observe the realization of their idiosyncratic shock, x, at the beginning of a period. Given this, they then make their employment decision.

[7] Assuming a fixed number of firms is important for the model to depart from the standard search model. Free entry would yield an economy of infinitesimal firms that converges to the Mortensen and Pissarides (1994) limit. In principle, one could allow for costly firm entry as a middle ground. We abstract from this in part for simplicity. But our choice is also informed by evidence in Davis and Haltiwanger (1992). They find that, in manufacturing, while births and deaths account for around 15 percent of establishment growth, they account for a very small fraction of employment growth. The simple reason is that births and deaths are dominated by the behavior of small establishments that account for a small fraction of total employment. For models that explore the impact of firm entry on job creation, see Garibaldi (2006), Hobijn and Sahin (forthcoming), and Hawkins (2011). The latter, in particular, incorporates this middle ground of costly entry into a framework similar to that studied in this paper.

[8] See Petrongolo and Pissarides (2001) for a summary of empirical evidence that suggests this is reasonable.

Specifically, they may choose to separate from part or all of their workforce, which we assume may be done at zero cost. Any such separated workers then join the unemployment pool in the subsequent period. Alternatively, firms may hire workers by posting vacancies, $v \geq 0$, at a flow cost of c per vacancy. If a firm posts vacancies, the matching process then matches these up with unemployed workers inherited from the previous period. After the matching process is complete, production and wage setting are performed simultaneously.

It follows that we can characterize the expected present discounted value of a firm's profits, $\Pi(n_{-1}, x)$, recursively as:[9]

$$\Pi(n_{-1}, x) = \max_{n,v} \left\{ pxF(n) - w(n, x)\, n - cv + \beta \int \Pi(n, x')\, dG(x'|x) \right\}, \tag{1}$$

where $w(n, x)$ is the bargained wage in a firm of size n and productivity x. A typical firm seeks a level of employment that maximizes its profits subject to a dynamic constraint on the evolution of a firm's employment level. Specifically, firms face frictions that limit the rate at which vacancies may be filled: A vacancy posted in a given period will be filled with probability $q < 1$ prior to production. Thus, the number of hires an individual firm achieves is given by:

$$\Delta n \mathbf{1}^{+} = qv, \tag{2}$$

where Δn is the change in employment, and $\mathbf{1}^{+}$ is an indicator that equals one when the firm is hiring, and zero otherwise. Substituting the constraint, (2), into the firm's value function, we obtain:

$$\Pi(n_{-1}, x) = \max_{n} \left\{ pxF(n) - w(n, x)\, n - \frac{c}{q} \Delta n \mathbf{1}^{+} + \beta \int \Pi(n, x')\, dG(x'|x) \right\}. \tag{3}$$

Note that the value function is not fully differentiable in n: There is a kink in the value function around $n = n_{-1}$. This reflects the (partial) irreversibility of separation decisions in the model. While firms can shed workers costlessly, it is costly to reverse such a decision because hiring (posting vacancies) is costly. In this sense, the labor demand side is formally analogous to the kinked employment adjustment cost model of the form analyzed in Bentolila and Bertola (1990), except that the per-worker hiring cost, $c/q(\theta)$, is endogenously determined.

In order to determine the firm's optimal employment policy, we take the first-order con-

[9] We adopt the convention of denoting lagged values with a subscript, $_{-1}$, and forward values with a prime, $'$.

ditions for hires and separations (i.e. conditional on $\Delta n \neq 0$):

$$pxF'(n) - w(n,x) - w_n(n,x)\,n - \frac{c}{q}\mathbf{1}^{+} + \beta D(n,x) = 0, \text{ if } \Delta n \neq 0, \tag{4}$$

where $D(n,x) \equiv \int \Pi_n(n,x')\,dG(x'|x)$ reflects the marginal effect of current employment decisions on the future value of the firm. Equation (4) is quite intuitive. It states that the marginal product of labor ($pxF'(n)$) net of any hiring costs ($\frac{c}{q}\mathbf{1}^{+}$), plus the discounted expected future marginal benefits from an additional unit of labor ($\beta D(n,x)$) must equal the marginal cost of labor ($w(n,x) + w_n(n,x)\,n$). To provide a full characterization of the firm's optimal employment policy, it remains to characterize the future marginal benefits from current employment decisions, $D(n,x)$, and the wage bargaining solution, $w(n,x)$, to which we now turn.[10]

1.2 Wage Setting

The existence of frictions in the labor market implies that it is costly for firms and workers to find alternative employment relationships. As a result, there exist quasi-rents over which the firm and its workers must bargain. The assumption of constant marginal product in the standard search model has the tractable implication that these rents are the same for all workers within a given firm. It follows that firms can bargain with each of their workers independently, because the rents of each individual employment relationship are independent of the rents of all other employment relationships.

Allowing for the possibility of diminishing marginal product of labor $F''(n) < 0$, however, implies that these rents will depend on the number of workers within a firm. Intuitively, the rent that a firm obtains from "the" marginal worker will be lower than the rent obtained on all infra-marginal hires due to diminishing marginal product. An implication of the latter is that the multilateral dimension of the firm's bargain with its many workers becomes important: The rents of each individual employment relationship within a firm are no longer independent.

[10] Because job creation is costly, a firm that hires never simultaneously fires; the first-order condition (4) formalizes this point. As a result, there is no distinction in our model between gross worker flows and net job flows. While this assumption was quite standard at the time of this paper's writing (and remains common), relaxing it would enable one to relate the model to a richer set of cross-sectional facts. For instance, Davis, Faberman, and Haltiwanger (2010) document that the relationship between gross worker flows and net job flows changes over the business cycle. Models that include endogenous job destruction and employer-to-employer flows (e.g., quits) can potentially shed light on this result. For work along these lines within a directed search framework, see Schaal (2010).

To take this into account, we adopt the bargaining solution of Stole and Zwiebel (1996) which generalizes the Nash solution to a setting with diminishing returns.[11] Stole and Zwiebel present a game where the bargained wage is the same as the outcome of simple Nash bargaining over the *marginal* surplus. The game that supports this simple result is one in which a firm negotiates with each of its workers in turn, and where the breakdown of a negotiation with any individual worker leads to the renegotiation of wages with all remaining workers.[12]

In accordance with timing of decisions each period, wages are set after employment has been determined. Thus, hiring costs are sunk at the time of wage setting, and the marginal surplus, which we denote as $J(n,x)$, is equal to the marginal value of labor gross of the costs of hiring:

$$J(n,x) = pxF'(n) - w(n,x) - w_n(n,x)\,n + \beta D(n,x). \tag{5}$$

The surplus from an employment relationship for a worker is the additional utility a worker obtains from working in her current firm over and above the utility she obtains from unemployment. The value of employment in a firm of size n and productivity x, $W(n,x)$, is given by:

$$W(n,x) = w(n,x) + \beta\mathbb{E}\left[sU' + (1-s)\,W(n',x')\,|n,x\right]. \tag{6}$$

While employed, a worker receives a flow payoff equal to the bargained wage, $w(n,x)$. She loses her job with (endogenous) probability s next period, upon which she flows into the unemployment pool and obtains the value of unemployment, U'. With probability $(1-s)$, she retains her job and obtains the expected payoff of continued employment in her current firm, $W(n',x')$. Likewise, the value of unemployment to a worker is given by:

$$U = b + \beta\mathbb{E}\left[(1-f)\,U' + fW(n',x')\right]. \tag{7}$$

Unemployed workers receive flow payoff b, which represents unemployment benefits and/or the value of leisure to a worker. They find a job next period with probability f, upon which they obtain the expected payoff from employment, $W(n',x')$.

Wages are then the outcome of a Nash bargain between a firm and its workers over the

[11] This approach was first used by Cahuc and Wasmer (2001) to generate a wage equation for the exogenous job destruction case.

[12] The intuition for the Stole and Zwiebel result is as follows. If the firm has only one worker, the firm and worker simply strike a Nash bargain. If a second worker is added, the firm and the additional worker know that, if their negotiations break down, the firm will agree to a Nash bargain with the remaining worker. In this sense, the second employee regards herself as being on the margin. By induction, then, the firm approaches negotiations with the nth worker as if that worker were marginal too. Therefore, the wage that solves the bargaining problem is that which maximizes the marginal surplus.

marginal surplus, with worker bargaining power denoted as η:

$$(1-\eta)\left[W(n,x)-U\right]=\eta J(n,x). \tag{8}$$

Given this, we are able to derive a wage bargaining solution with the following simple structure:

Proposition 1 *The bargained wage, $w(n,x)$, solves the differential equation*[13]

$$w(n,x)=\eta\left[pxF'(n)-w_n(n,x)\,n+\beta f\frac{c}{q}\right]+(1-\eta)\,b. \tag{9}$$

The intuition for (9) is quite straightforward. As in the standard search model, wages are increasing in the worker's bargaining power, η, the marginal product of labor, $pxF'(n)$, workers' job finding probability, f, the marginal costs of hiring for a firm, c/q, and workers' flow value of leisure, b. There is an additional term, however, in $w_n(n,x)\,n$. To understand the intuition for this term, consider a firm's negotiations with a given worker. If these negotiations break down, the firm will have to pay its remaining workers a higher wage. The reason is that fewer workers imply that the marginal product of labor will be higher in the firm, which will partially spillover into higher wages ($w_n n<0$). The more powerful this effect is (the more negative is $w_n n$), the more the firm loses from a given breakdown of negotiations with a worker, and the more workers can extract a higher wage from the bargain.

In what follows, we will adopt the simple assumption that the production function is of the Cobb-Douglas form, $F(n)=n^{\alpha}$ with $\alpha\leq 1$. Given this, the differential equation for the wage function, (9), has the following simple solution:

$$w(n,x)=\eta\left[\frac{px\alpha n^{\alpha-1}}{1-\eta(1-\alpha)}+\beta f\frac{c}{q}\right]+(1-\eta)\,b. \tag{10}$$

Setting $\alpha=1$ yields the discrete time analogue to the familiar wage bargaining solution for the Mortensen and Pissarides (1994) model.

[13] An interesting feature of this solution is its similarity to the solution obtained by Cahuc and Wasmer (2001) for the exogenous job destruction model. It is also consistent with Acemoglu and Hawkins' (2010) Lemma 1, except that it holds both in and out of steady state.

1.3 The Firm's Optimal Employment Policy

Now that we have obtained a solution for the bargained wage at a given firm, we can combine this with the firm's first-order condition for employment and thereby characterize the firms optimal employment policy, which specifies the firm's optimal employment as a function of its state, $n(n_{-1}, x)$. Thus, combining (4) and (9) we obtain:

$$(1-\eta)\left[\frac{px\alpha n^{\alpha-1}}{1-\eta(1-\alpha)}-b\right]-\eta\beta f\frac{c}{q}-\frac{c}{q}\mathbf{1}^{+}+\beta D(n,x)=0. \tag{11}$$

Given (11) we are able to characterize the firm's optimal employment policy as follows:

Proposition 2 *The optimal employment policy of a firm is of the form*

$$n(n_{-1},x)=\begin{cases} R_v^{-1}(x) & \text{if} \quad x>R_v(n_{-1}), \\ n_{-1} & \text{if} \quad x\in[R(n_{-1}),R_v(n_{-1})], \\ R^{-1}(x) & \text{if} \quad x<R(n_{-1}), \end{cases} \tag{12}$$

where the functions $R_v(\cdot)$ and $R(\cdot)$ satisfy

$$(1-\eta)\left[\frac{pR_v(n)\,\alpha n^{\alpha-1}}{1-\eta(1-\alpha)}-b\right]-\eta\beta f\frac{c}{q}+\beta D(n,R_v(n))\equiv\frac{c}{q}, \tag{13}$$

$$(1-\eta)\left[\frac{pR(n)\,\alpha n^{\alpha-1}}{1-\eta(1-\alpha)}-b\right]-\eta\beta f\frac{c}{q}+\beta D(n,R(n))\equiv 0. \tag{14}$$

The firm's optimal employment policy will be similar to that depicted in Figure 1. It is characterized by two reservation values for the firm's idiosyncratic shock, $R(n_{-1})$ and $R_v(n_{-1})$. Specifically, for sufficiently bad idiosyncratic shocks ($x<R(n_{-1})$ in the figure), firms will shed workers until the first-order condition in the separation regime, (14), is satisfied. Moreover, for sufficiently good idiosyncratic realizations ($x>R_v(n_{-1})$ in the figure), firms will post vacancies and hire workers until the first-order condition in the hiring regime, (13), is satisfied. Finally, for intermediate values of x, firms freeze employment so that $n=n_{-1}$. This occurs as a result of the kink in the firm's profits at $n=n_{-1}$, which arises because hiring is costly to firms, while separations are costless.

To complete our characterization of the firm's optimal employment policy, it remains to determine the marginal effect of current employment decisions on future profits of the firm, $D(n,x)$. It turns out that we can show that $D(n,x)$ has the following recursive structure:

Proposition 3 *The marginal effect of current employment on future profits, $D(n,x)$, is given by*

$$D(n,x) = d(n,x) + \beta \int_{R(n)}^{R_v(n)} D(n,x')\,dG(x'|x), \tag{15}$$

where

$$d(n,x) \equiv \int_{R(n)}^{R_v(n)} \left\{ (1-\eta)\left[\frac{px'\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} - b\right] - \eta\beta f \frac{c}{q} \right\} dG(x'|x) + \int_{R_v(n)}^{\infty} \frac{c}{q} dG(x'|x). \tag{16}$$

Equation (15) is a contraction mapping in $D(n,\cdot)$, and therefore has a unique fixed point.

The intuition for this result is as follows. Because of the existence of kinked adjustment costs (costly hiring and costless separations) the firm's employment will be frozen next period with positive probability. In the event that the firm freezes employment next period ($x' \in [R(n), R_v(n)]$), the current employment level persists into the next period and so do the marginal effects of the firm's current employment choice. Proposition 3 shows that these marginal effects persist into the future in a *recursive* fashion. Propositions 2 and 3 thus summarize the microeconomic behavior of firms in the model.[14]

To get a sense for how the microeconomic behavior of the model works, we next derive the response of an individual firm's employment policy function to changes in (exogenous) aggregate productivity, p, and the (endogenous) aggregate vacancy-unemployment ratio, θ. To do this, we assume that the evolution of idiosyncratic shocks is described by:

$$x' = \begin{cases} x & \text{with probability} \quad 1-\lambda, \\ \tilde{x} \overset{c.d.f.}{\sim} \tilde{G}(\tilde{x}) & \text{with probability} \quad \lambda. \end{cases} \tag{17}$$

Thus, idiosyncratic shocks display some persistence ($\lambda < 1$) with innovations drawn from the distribution function $\tilde{G}$. Given this, we can establish the following result:

Proposition 4 *If idiosyncratic shocks, x, evolve according to (17), then the effects of the aggregate state variables p and θ on a firm's optimal employment policy are*

$$\frac{\partial R_v}{\partial p} < 0; \frac{\partial R}{\partial p} < 0; \frac{\partial R_v}{\partial \theta} > 0; \text{ and } \frac{\partial R}{\partial \theta} > 0 \iff n \text{ is sufficiently large.} \tag{18}$$

The intuition behind these marginal effects is quite simple. First, note that increases in aggregate productivity, p, shift a firm's employment policy function downwards in Figure

[14] It is straightforward to show that equations (10) to (16) reduce down to the discrete time analogue to the Mortensen and Pissarides (1994) model when $\alpha = 1$.

1. Thus, unsurprisingly, when labor is more productive, a firm of a given idiosyncratic productivity, x, is more likely to hire workers, and less likely to shed workers. Second, increases in the vacancy-unemployment ratio, θ, unambiguously reduce the likelihood that a firm of a given idiosyncratic productivity will hire workers (R_v increases for all n). The reason is that higher θ implies a lower job-filling probability, q, and thereby raises the marginal cost of hiring a worker, c/q. Moreover, higher θ implies a tighter labor market and therefore higher wages (from (9)) so that the marginal cost of labor rises as well. Both of these effects cause firms to cut back on hiring. Finally, increases in the vacancy-unemployment ratio, θ, will reduce the likelihood of shedding workers for small firms, but will raise it for large firms. This occurs because higher θ has countervailing effects on the separation decision of firms. On the one hand, higher θ reduces the job-filling probability, q, rendering separation decisions less reversible (since future hiring becomes more costly), so that firms become less likely to destroy jobs. On the other hand, higher θ implies a tighter labor market, higher wages, and thereby a higher marginal cost of labor, rendering firms more likely to shed workers. The former effect is dominant in small firms because the likelihood of their hiring in the future is high.

2 Aggregation and Steady State Equilibrium

2.1 Aggregation

Since we are ultimately interested in the equilibrium behavior of the aggregate unemployment rate, in this section we take on the task of aggregating up the microeconomic behavior of section 1 to the macroeconomic level. This exercise is non-trivial because each firm's employment is a non-linear function of the firm's lagged employment, n_{-1}, and its idiosyncratic shock realization, x. As a result, there is no representative firm interpretation that will aid aggregation of the model.

To this end, we are able to derive the following result which characterizes the steady state aggregate employment stock and flows in the model:

Proposition 5 *If idiosyncratic shocks, x, evolve according to (17), the steady state c.d.f. of employment across firms is given by*

$$H(n) = \frac{\tilde{G}[R(n)]}{1 - \tilde{G}[R_v(n)] + \tilde{G}[R(n)]}. \tag{19}$$

Thus, the steady state aggregate employment stock is given by

$$N = \int n dH(n), \tag{20}$$

and the steady state aggregate number of separations, S, and hires, M, is equal to

$$S = \lambda \int [1 - H(n)] \tilde{G}[R(n)] dn = \lambda \int H(n) \left(1 - \tilde{G}[R_v(n)]\right) dn = M. \tag{21}$$

Proposition 5 is useful because it provides a tight link between the solution for the microeconomic behavior of an individual firm and the macroeconomic outcomes of that behavior. Specifically, it shows that once we know the optimal employment policy function of an individual firm (that is, the functions $R(n)$ and $R_v(n)$) then we can directly obtain analytical solutions for the distribution of firm size, and the aggregate employment stock and flows.

The three components of Proposition 5 are also quite intuitive. The steady state distribution of employment across firms, (19), is obtained by setting the flows into and out of the mass $H(n)$ equal to each other. The inflow into the mass comes from firms who reduce their employment from above n to below n. There are $[1 - H(n)]$ such firms, and since they are reducing their employment, it follows from (12) that each firm will reduce its employment below n with probability equal to $\Pr[x < R(n)] = \lambda\tilde{G}[R(n)]$. Thus, the inflow into $H(n)$ is equal to $\lambda[1 - H(n)] \tilde{G}[R(n)]$. Similarly, one can show that the outflow from the mass is equal to $\lambda H(n) \left(1 - \tilde{G}[R_v(n)]\right)$. Setting inflows equal to outflows yields the expression for $H(n)$ in (19).[15] Given this, the expression for aggregate employment, (20), follows directly.

The intuition for the final expression for aggregate flows in Proposition 5, (21), is as follows. Recall that the mass of firms whose employment switches from above some number n to below n is equal to $\lambda[1 - H(n)] \tilde{G}[R(n)]$. Equation (21) states that the aggregate number of separations in the economy is equal to the *cumulative* sum of these downward switches in employment over n. To get a sense for this, consider the following simple discrete example. Imagine an economy with two separating firms: one that switches from three employees to one, and another that switches from two employees to one. It follows that two firms have switched from > 2 employees to ≤ 2 employees, and one firm switched from > 1 to ≤ 1 employee. Thus, the cumulative sum of downward employment switches is three, which is also equal to the total number of separations in the economy.

[15] This mirrors the mass-balance approach used in Burdett and Mortensen (1998) to derive the equilibrium wage distribution in a search model with wage posting.

2.2 Steady State Equilibrium

Given (19), (20), and (21), the conditions for aggregate steady state equilibrium can be obtained as follows. First note that each firm's optimal policy function, summarized by the functions $R(n)$ and $R_v(n)$ in Proposition 2, depends on two aggregate variables: The (exogenous) state of aggregate productivity, p; and the (endogenous) ratio of aggregate vacancies to aggregate unemployment, $V/U \equiv \theta$, which uniquely determines the flow probabilities q and f.

In the light of Proposition 5, we can characterize the aggregate steady state of the economy for a given p in terms of two relationships. The first, the *Job Creation condition*, is simply equation (20), which we re-state here in terms of unemployment, making explicit its dependence on the aggregate vacancy-unemployment ratio, θ:

$$U(\theta)_{JC} = L - \int n dH(n;\theta). \tag{22}$$

(22) simply specifies the level of aggregate employment that is consistent with the inflows to (hires) and outflows from (separations) aggregate employment being equal as a function of θ. The second steady state condition is the *Beveridge Curve* relation. This is derived from the difference equation that governs the evolution of unemployment over time:

$$\Delta U' = S(\theta) - f(\theta) U. \tag{23}$$

(23) simply states that the change in the unemployment stock over time, $\Delta U'$, is equal to the inflow into the unemployment pool—the number of separations, S—less the outflow from the unemployment pool—the job finding probability, f, times the stock of unemployed workers, U. In steady state, aggregate unemployment will be stationary, so that we obtain the steady state unemployment relation:

$$U(\theta)_{BC} = \frac{S(\theta)}{f(\theta)}. \tag{24}$$

The steady state value of the vacancy-unemployment ratio, θ, is co-determined by (22) and (24).

3 Introducing Aggregate Shocks

The previous section characterized the determination of steady state equilibrium in the model. However, in what follows, we are interested in the dynamic response of unemployment, vacancies and worker flows to aggregate shocks. To address this, we need to characterize the dynamics of the model out of steady state. The latter is not a trivial exercise in the context of the present model. Out of steady state, firms in the model need to forecast future wages and therefore, from equation (9), future labor market tightness. Inspection of the steady state equilibrium conditions (22) and (24) reveals that, in order to forecast future labor market tightness, firms must predict the evolution of the entire distribution of employment across firms, $H(n)$, an infinite order state variable.

Our approach to this problem mirrors the method proposed by Krusell and Smith (1998). We consider shocks to aggregate labor productivity that evolve according to the simple random walk:

$$p' = \begin{cases} p + \sigma_p & \text{w.p.} \quad 1/2, \\ p - \sigma_p & \text{w.p.} \quad 1/2. \end{cases} \tag{25}$$

Following Krusell and Smith, we conjecture that a forecast of the *mean* of the distribution of employment across firms, $N = \int n dH(n)$, provides an accurate forecast of future labor market tightness. We then exploit the fact that shocks to aggregate labor productivity, denoted by σ_p in equation (25), are small in U.S. data.[16] This allows us to approximate the evolutions of aggregate employment, N, and labor market tightness, θ, around their steady state values N^* and θ^* as follows:

$$\begin{aligned} N' &\approx N^* + \nu_N (N - N^*) + \nu_p (p' - p), \\ \theta' &\approx \theta^* + \theta_N (N' - N^*) + \theta_p (p' - p), \end{aligned} \tag{26}$$

for $\sigma_p \approx 0$. Under these conditions, we can approximate the optimal employment policy of an individual firm out of steady state. To see how this might be done, note from the first order conditions (13) and (14) that to derive optimal employment in the presence of aggregate shocks, one must characterize the marginal effect of current employment decisions on future profits, $D(\cdot)$, out of steady state.

Proposition 6 *If a) aggregate shocks evolve according to (25); b) a forecast of N provides an accurate forecast of future θ; c) aggregate shocks are small ($\sigma_p \approx 0$); and d) idiosyncratic shocks evolve according to (17), then the marginal effect of current employment on future profits is given by*

$$D(n, x; N, p, \sigma_p) \approx D(n, x; N^*, p, 0) + D_N^* (N - N^*), \tag{27}$$

[16] Examples of other studies that have exploited the fact that aggregate shocks are small include Mortensen and Nagypal (2007) and Gertler and Leahy (2008).

where D_N^* is a known function of the parameters of the forecast equation (26) and the steady state employment policy defined in (13) and (14).

Proposition 6 shows that, in the presence of aggregate shocks, the forward looking component to the firm's decision, $D(n, x; N, p, \sigma_p)$, is approximately equal to its value in the absence of aggregate shocks, $D(n, x; N^*, p, 0)$, plus a known function of the deviation of aggregate employment from steady state, $D_N^*(N - N^*)$. Practically, Proposition 6 allows us to derive analytically an approximate solution for the optimal policy function in the presence of aggregate shocks, for given values of the parameters of the forecast equation (26).

To complete our description of the dynamics of the model, we need to aggregate the microeconomic behavior summarized in the employment policies of individual firms. A simple extension of the result of Proposition 5 implies that the aggregate number of separations and hires in the economy at a point in time are respectively given by:

$$\begin{aligned} S(N,p) &= \lambda \int [1 - H_{-1}(n_{-1})] \tilde{G}[R(n_{-1}; N, p)] \, dn_{-1}, \\ M(N,p) &= \lambda \int H_{-1}(n_{-1}) \Big(1 - \tilde{G}[R_v(n_{-1}; N, p)]\Big) \, dn_{-1}, \end{aligned} \tag{28}$$

where $H_{-1}(n_{-1})$ is the distribution of lagged employment across firms. Notice that the timing is emphasized in the out of steady state case.

A number of observations arise from this. First, the aggregate flows depend on the level of aggregate employment, N. Recalling the accumulation equation for N yields:

$$N = N_{-1} + M(N,p) - S(N,p). \tag{29}$$

It follows that, to compute aggregate employment, all one need do is find the fixed point value of N that satisfies equation (29). This allows us to compute equilibrium labor market tightness by noting that

$$f(\theta) = M/(L - N). \tag{30}$$

A second observation from equation (28) is that, in order to compute the path of aggregate unemployment flows, and hence employment, we need to describe the evolution of the distribution of employment across firms, $H(n)$. It turns out that the evolution of $H(n)$ can be inferred by a simple extension of the discussion following Proposition 5. Recall that the change in the mass $H(n)$ over time is simply equal to the inflows less the outflows from that mass. Following the logic of Proposition 5 provides a difference equation for the evolution of $H(n)$:

$$H(n) = H_{-1}(n) + \lambda \tilde{G}[R(n;N,p)][1 - H_{-1}(n)] - \lambda \left(1 - \tilde{G}[R_v(n;N,p)]\right) H_{-1}(n). \quad (31)$$

This allows us to update the aggregate flows $S(N,p)$ and $M(N,p)$ over time, and hence derive the evolution of equilibrium employment.

The previous results allow us to compute the evolution of aggregate employment and labor market tightness for a given configuration of the parameters of the forecast equations (26). This of course does not guarantee that those parameters are consistent with the behavior that they induce. To complete our characterization of equilibrium in the presence of aggregate shocks, we follow Krusell and Smith and iterate numerically over the parameters $\{\nu_N, \nu_p, \theta_N, \theta_p\}$ to find the fixed point. In the simulations of the model that follow, the fixed point of the conjectured forecast equations in (26) provides a very accurate forecast in the sense that the R^2s of regressions based on (26) exceed 0.999.

4 Quantitative Applications

The model of sections 2 and 3 yields a rich set of predictions for both the dynamics and the cross-section of the aggregate labor market. In this section we draw out these implications in a range of quantitative applications, including the cross sectional distributions of establishment size and employment growth, the amplitude and propagation of unemployment fluctuations, the relationship between vacancies and unemployment in the form of the Beveridge curve, the dynamics of the distribution of establishment size, and the employer size-wage effect.

4.1 Calibration

Our calibration strategy proceeds in two stages. The first part is very conventional, and mirrors the approach taken in much of the literature. The time period is taken to be equal to one week, which in practice acts as a good approximation to the continuous time nature of unemployment flows. The dispersion of the innovation to aggregate labor productivity σ_p is set to match the standard deviation of the cyclical component of output per worker in the U.S. economy of 0.02.

We assume that the matching function is of the conventional Cobb-Douglas form, $M = \mu U^{\phi} V^{1-\phi}$, with matching elasticity ϕ set equal to 0.6, based on the estimates reported in

Petrongolo and Pissarides (2001).[17] A weekly job finding rate of $f = 0.1125$ is targeted to be consistent with a monthly rate of 0.45. As in Pissarides (2007), we target a mean value of the vacancy-unemployment ratio of $\theta = 0.72$. Noting from the matching function that $f = \mu\theta^{1-\phi}$, the latter implies that the matching efficiency parameter $\mu = 0.129$ on a weekly basis.

Vacancy costs c are targeted to generate per worker hiring costs c/q equal to 14 percent of quarterly worker compensation. This is in accordance with the results of Silva and Toledo (2007), who cite an estimate of the labor costs of posting vacancies published by the human resources consulting firm, the Saratoga Institute. Hall and Milgrom (2008) also adopt this calibration target. In the context of the model, this implies a value of c approximately equal to 29 percent of the average worker's wage.[18]

Our calibration of worker bargaining power (η) is designed to hold constant the elasticity of the real wage across the linear (Mortensen-Pissarides) and nonlinear (this paper) models. Pissarides (2007) reports an elasticity of the wage with respect to output per worker in the linear model of 0.985. Our calibration of η yields virtually the same result. The idea is to rule out differences in wage dynamics as a source of the differences in the cyclical behavior of the two models. This strategy helps us to isolate the more fundamental properties of a model with decreasing returns and how they generate different quantitative behavior relative to the standard setting.[19]

The production function parameter α is determined by targeting an aggregate labor share based on the estimates reported in Gomme and Rupert (2007). These suggest a labor share for market production of 0.72. Alternatively, to calibrate α, one might consult estimates of plant-level labor demand models. Cooper, Haltiwanger, and Willis (2004) estimate a dynamic labor demand problem on plant-level employment data available from the Longitudinal Research Database. They find that $\alpha = 0.64$, which is similar to the value implied by

[17] An issue that can arise when using a Cobb–Douglas matching function in a discrete time setting is that the flow probabilities f and q are not necessarily bounded above by one. This issue does not arise here due to the short time period of one week.

[18] We want to equate the per worker hiring cost c/q to 14 percent of quarterly wages, $0.14 \cdot [13 \cdot \mathbb{E}(w)]$. (There are 13 weeks per quarter.) Note that the implied weekly job filling probability is given by $q = \mu\theta^{-\phi} = 0.129 \cdot 0.72^{-0.6} = 0.16$. Piecing this together yields $c/\mathbb{E}(w) = 0.16 \cdot 0.14 \cdot 13 = 0.29$.

[19] It is well known that the component of wages that is relevant for the cyclicality of labor market flows is the cyclicality of the new hires' wage, not the average wage among all workers (Shimer, 2004; Hall, 2005; and Hall and Milgrom, 2008). An alternative strategy, then, would be to calibrate η by targeting the elasticity of new hires' real wages with respect to output per worker. Haefke, Sonntag, and van Rens (2008) report an elasticity of 0.77 over the period 1985-2007 for new hires out of unemployment. To the extent we overstate the flexibility of the real wage, we likely understate the model's potential to obtain amplification of labor market flows.

targeting labor share.[20]

To complete the first part of our calibration, we choose the size of the labor force L to match a mean unemployment rate of 6.5 percent.[21] Given the remainder of the calibration that follows, this is equivalent to choosing the labor force to match a weekly job-finding rate of 0.1125.

Idiosyncratic Shocks and the Value of Unemployment A more distinctive feature of our strategy is the calibration of the evolution of idiosyncratic firm productivity and the flow payoff from unemployment to a worker. For the former, we modify slightly the production function in sections 2 and 3 to incorporate time invariant firm specific productivity, denoted by φ, so that $y = p\varphi x F(n)$. Firm specific fixed effects φ are introduced to reflect permanent heterogeneity in firm productivity that is unrelated to the uncertainty that individual firms face over time in the form of the innovation x. Since φ is fixed, its presence does not affect the derivation of the optimal labor demand policy; the rules to hire and fire are simply conditioned on φ.[22]

An important feature of the model of sections 2 and 3 is that it allows a flexible specification of the distribution of shocks. This is useful because conventional parameterizations, such as log-normal shocks, fail to capture the well-known Pareto shape of the cross sectional distribution of firm size. Reacting to this, we set $\varphi \sim Pareto(\varphi_m, k_\varphi)$ and $x \sim Pareto(x_m, k_x)$.[23] The minimum value of the fixed effect φ_m is chosen to yield a minimum establishment employment level of one worker, and its shape coefficient k_φ is chosen to match a mean establishment size of 17.38, based on data from the Small Business Administration.[24] Inno-

[20] In calibrating their search-and-matching model, Cooper, Haltiwanger, and Willis (2007) use a similar estimate, setting $\alpha = 0.65$.

[21] Strictly speaking, since we normalize the mass of production units to one, L should be interpreted as the number of workers per employer.

[22] The fixed effects discipline the model-implied firm-size distribution. Absent φ, we would likely have to select the variance of the time-varying component of idiosyncratic productivity, x, to replicate this distribution. But that would be a mistake. Over the short run, relatively large firms stay large and relatively small firms stay small, suggesting a sort of persistent heterogeneity from which our model abstracts. Thus, it seems advisable to incorporate fixed effects, which in turn allows one to calibrate the variance of x based on moments that are arguably more informative for it.

[23] A Pareto distributed random variable z is parameterized by a minimum value z_m and a "shape" parameter k, and has a density function given by kz_m^k/z^{k+1}.

[24] These data are from the Statistics of U.S. Businesses and can be obtained from http://www.sba.gov/advocacy/849/12162. Complete data on the size of establishments is not available for 1992 and 1993, so mean size is computed for years 1994-2006. We note that mean size is computed among establishments with strictly positive employment; so-called nonemployers, which have no paid employees, are excluded. The Census reports that nonemployers are generally self-employed individuals who operate small unincorporated businesses. See the Census' Nonemployer Statistics program for more, http://www.census.gov/econ/nonemployer/index.html.

vations to idiosyncratic productivity x are normalized so that the mean innovation is equal to one. This implies that $x_m = 1 - k_x^{-1}$. Given this, we solve for firms' optimal employment policy using the results in sections 2 and 3 (see Appendix A for details). This enables us, in turn, to derive the steady-state distribution of weekly employment growth:

Proposition 7 *For a given time-invariant productivity φ, the steady state density of employment growth, $\delta = \Delta \ln n$, across firms is given by:*

$$h_\Delta(\delta|\varphi) = \begin{cases} \lambda \int e^\delta n \tilde{G}' \left[R' \left(e^\delta n \right) \right] dH(n|\varphi) & \text{if} \quad \delta < 0, \\ \lambda \int \left(\tilde{G}\left[R_v(n) \right] - \tilde{G}\left[R(n) \right] \right) dH(n|\varphi) & \text{if} \quad \delta = 0, \\ \lambda \int e^\delta n \tilde{G}' \left[R_v' \left(e^\delta n \right) \right] dH(n|\varphi) & \text{if} \quad \delta > 0, \end{cases} \tag{32}$$

where $H(n|\varphi)$ is the distribution of employment n conditional on fixed firm productivity φ derived in Proposition 5. The unconditional employment growth density is $h_\Delta(\delta) = \int h_\Delta(\delta|\varphi) \, d\Phi(\varphi)$, where Φ is the (known) c.d.f. of φ.

Proposition 7 provides us with a novel approach to calibrating the remaining parameters of the process of idiosyncratic shocks, λ and k_x. There is abundant evidence on the properties of the cross sectional distribution of employment growth $h_\Delta(\delta)$ since the seminal work of Davis and Haltiwanger (1992). Empirically, this distribution is characterized by a dominant spike at zero employment growth, with relatively symmetric tails corresponding to job creation and job destruction (see, for example, Figure 1.A in Davis and Haltiwanger, 1992). Note that this is exactly the form of the employment growth distribution implied by the model in Proposition 7.[25]

We now select λ and k_x to match the share of mass around zero growth and the dispersion of employment growth. Intuitively, the cross sectional distribution of employment is a manifestation of the idiosyncratic shocks x across firms. The more often these shocks arrive (the higher is λ in the model), the more likely a firm is to alter its employment, and the smaller is the implied spike at zero employment growth. Likewise, the greater the dispersion of the innovations x, the larger the implied adjustment that firms will make. Therefore, a higher k_x raises the mass in the tails of the distribution and reduces the mass just to the left and right of zero growth. More specifically, we target the following two moments: the share

[25] The available data on the employment growth distribution reports the annual change in establishment-level employment. Thus, to relate the model to the empirical cross section, we must use the model-implied distribution of annual employment growth. It is possible, but exceptionally cumbersome, to derive this analytically, as we did above for the weekly distribution in Proposition 7. Yet the essential form—a large mass around zero growth, with relatively symmetric tails—will remain in the annual distribution, though of course the magnitudes will differ. In what follows, then, we recover the annual distribution by simulation.

of establishments with growth less than |5%|, which is 41.5%, and the standard deviation of annual employment growth, which is 41.6%.[26] These moments are computed using data on continuing establishments in the Longitudinal Business Database for the years 1992 to 2005.[27]

The latter calibration of the process governing the evolution of idiosyncratic shocks is crucial for our calibration of workers' flow payoff from unemployment b. Since the work of Hagedorn and Manovskii (2007), it has been recognized that the value of b plays a central role in determining the cyclical volatility of aggregate unemployment, and specifically the job-finding rate. Intuitively, higher values of b lead to a smaller surplus to employment relationships. As a consequence, small reductions in aggregate productivity can easily exhaust that surplus, and lead firms to cut back substantially on hiring. Since one of the quantitative applications we consider is the cyclical volatility of worker flows, the parameterization of b is key.

Our model suggests a novel approach to calibrating the payoff from unemployment: For a given level of dispersion in idiosyncratic shocks implied by our calibration of the evolution x, a higher value of b reduces the surplus and implies that jobs will be destroyed more frequently, raising the inflow rate into unemployment s. Thus, we choose b in such a way as to yield employment rents that match the empirical unemployment inflow rate of $s = 0.0078$ on a weekly basis, consistent with estimates reported in Shimer (2007).

The parameter values implied by our calibration are summarized in Table 1. In what follows, we summarize the implications of the calibrated model for a range of cross-sectional and aggregate outcomes.

4.2 Establishment Size and Employment Growth Distributions

An important component of our calibration strategy is to match key properties of the cross-sectional distributions of employment and employment growth across firms. The model's implications for these two outcomes are summarized in Propositions 5 and 7 above. In this section, we compare the steady-state distributions implied by the model with their empirical counterparts.

Figure 2 plots the distribution of establishment size in the calibrated model and recent

[26] In the simulations, annual employment growth is not measured as log changes, as in Proposition 7, but in the same way as in Davis and Haltiwanger (1992): If n_t denotes employment in the current week and n_{t-1} employment in the same week last year, we calculate $\frac{n_t - n_{t-1}}{0.5(n_t + n_{t-1})}$. We do this because the Census Bureau tabulations we use (see above) are based on the Davis-Haltiwanger growth rate.

[27] Thanks to John Haltiwanger, Ron Jarmin, and Javier Miranda for providing us with the tabulations from the LBD that allowed us to make these calculations.

data. Both axes are on a log scale to emphasize the Pareto shape of the distributions. The dots plot the empirical establishment size distribution using pooled data from the Small Business Administration on employment by firm size class for the years 1992 to 2006. The dashed line indicates the analogue implied by the calibrated model. Figure 2 reveals that the model accounts well for the empirical establishment size distribution. While this outcome is not surprising given the Pareto shocks fed through the model, it does highlight the benefit of using a flexible form for the distribution of idiosyncratic productivity in the model of sections 2 and 3.[28]

What is perhaps more surprising is that the model also does a remarkable job of matching the distribution of employment growth across establishments. The dotted line in Figure 3 illustrates the empirical employment growth distribution using data for continuing establishments from the Longitudinal Business Database. As noted above, this displays the classic features of a mass point at zero employment growth, and relatively symmetric tails. The dashed line overlays the employment growth distribution implied by the calibrated model. This bears a very close resemblance to the empirical distribution. This is more noteworthy than it might at first appear: While the use of Pareto shocks was informed by the character of the establishment size distribution in Figure 2, the message of Figure 3 is that it also provides a remarkably good account of the tails of the employment growth distribution, something that has not been emphasized in the literature on establishment dynamics.[29]

4.3 The Cyclicality of Worker Flows

It is now well-known that standard search models of the aggregate labor market cannot generate enough cyclical amplitude in unemployment, and in particular the job finding rate, to match that observed in U.S. data (Shimer, 2005). A natural question is whether the generalized model analyzed here can alleviate this problem. To address this, we feed through a series of shocks to aggregate labor productivity using equation (25), and simulate the implied dynamic response of the model using the results of section 3. Following Mortensen

[28] The Small Business Administration (SBA) provides data on the distribution of firm size rather than establishment size. The County Business Patterns (CBP) provides data on the latter, though the size bins are much coarser. For instance, the CBP reports the number of establishments with $20-49$ workers, whereas the SBA divides this interval into six smaller bins, each with a width of 5 workers. For this reason, we prefer the SBA data. But we have reproduced the plot shown in Figure 2 with CBP data and find virtually the same pattern.

[29] It should be noted that the model understates the literal spike at zero growth, and overstates slightly the mass just to the left and right of zero in the interval $|5\%|$. This can be traced to the fact that few establishments make very small employment adjustments in the data. This feature is consistent with the existence of some fixed costs of adjustment, which our model abstracts from. Getting the right allocation of the mass within this area around zero is an important challenge for future work.

and Nagypal (2007), we compute the model-implied elasticities of labor market stocks and flows with respect to output per worker, and compare them with their empirical counterparts.

Model Outcomes Panel A of Table 2 summarizes the results of this exercise. Outcomes in brackets are moments that the model is calibrated to match: the mean levels of the job-finding rate f, the unemployment inflow rate s, and the vacancy-unemployment ratio θ. The aim of the exercise is to draw out the implications of the model for the outcomes that the model is *not* calibrated to match, i.e. the cyclical elasticities of these outcomes with respect to output per worker.

The results in Table 2.A are remarkably encouraging: On all dimensions, the model-implied elasticities lie in a neighborhood close to the cyclicality observed in the data. Specifically, the model implies an elasticity of the job finding rate of 2.55, a little below its empirical analogue of 2.65.[30] In addition, the model-generated cyclical elasticity of the unemployment inflow rate of -1.64 lies only a little below the magnitude observed in the data.

Comparison with Mortensen and Pissarides (1994) These results make substantial progress relative to the standard Mortensen and Pissarides (1994) model. To see this, panels B and C of Table 2 provide two comparison exercises.[31] First, taking as given the process for idiosyncratic shocks implied by the distribution of employment growth derived above, we calibrate the standard model to match the mean levels of the job finding rate f and the unemployment inflow rate s, as well as the elasticity of s with respect to output per worker implied by the generalized model in panel A.[32] This allows the model to speak to the implied elasticity of the job finding rate, and thereby the elasticities of vacancies and labor

[30] The cyclical elasticities reported in Table 2 differ slightly from those implied by Shimer (2005) and Mortensen and Nagypal (2007) as a result of revisions to U.S. GDP data.

[31] In practice, we have to slightly modify the Mortensen and Pissarides (1994) model since we use a distribution of idiosyncratic shocks that has no upper bound. Rather than impose that matches form with initial productivity equal to one, which is the upper bound on productivity in the original model, we endow matches with initial productivity equal to the mean, i.e., $E[x|x > R]$, where R is the reservation threshold.

[32] We use a process for x that induces the observed employment growth distribution in order to discipline the Mortensen and Pissarides (1994) model to be consistent with a realistic amount of firm-specific risk. Alternatively, one may try to calibrate the MP model directly to another, related indicator of firm-specific risk. For instance, although the standard model does not generate a cross section of employment growth, it does generate a cross section of sales growth. Perhaps the most comprehensive source of information on sales is the Census' Longitudinal Business Database. In practice, though, the construction of consistent series for sales in the LBD is a difficult task, and these tabulations have not been made available (Davis, Haltiwanger, Jarmin, and Miranda (2006)). A related measure of firm-level sales volatility has been developed based on data for publicly traded firms. This measure appears to suggest just as much volatility as its employment counterpart (see Davis, Haltiwanger, Jarmin, and Miranda, 2006).

market tightness.[33] The outcomes in panel B confirm what Shimer (2005) demonstrated: that the standard model is unable to generate enough cyclicality in job creation.[34] The model-implied elasticity of the job-finding rate is 0.91, nearly one third of the empirical elasticity. In contrast, the generalized model studied in the present paper can account for all of the observed cyclical comovement between f and output per worker.

The Role of the Payoff from Unemployment Panel C of Table 2 provides a new perspective on the standard model's inability to match the cyclicality of unemployment flows. In this case, we again take as given the process for idiosyncratic shocks implied by the empirical distribution of employment growth. However, instead of targeting the mean level of the inflow rate into unemployment s, we now allow the standard model to match the elasticity of the job finding rate f generated by the model of sections 2 and 3, and then draw out the implications for s.[35] Panel C reveals that the model must dramatically overstate the magnitude of unemployment inflows in order to match the cyclical comovement of f: The implied weekly inflow rate of 0.027 is more than three times that observed in the data.

This result sheds light on a recent debate in the literature. In order to match the cyclical variation in the job finding rate, the standard model requires a small surplus to employment relationships, a point emphasized by Hagedorn and Manovskii (2007) and Mortensen and Nagypal (2007).[36] Mortensen and Nagypal further argue that the required surplus is unrealistically small. The results of Table 2 formalize this intuition: For realistic variation in idiosyncratic shocks to firms, a surplus small enough to match the cyclicality of f implies an employment to unemployment transition rate that is more than three times larger than what is observed empirically. Intuitively, a small surplus implies that small idiosyncratic shocks to employment relationships are enough to exhaust the surplus and lead to destruction of a match. Consequently, realistic dispersion in idiosyncratic shocks generates excessive worker turnover.

Thus, the Mortensen and Pissarides model faces a tension: To match plausible levels of

[33] Fixing the arrival rate, λ, of the idiosyncratic shock and its variance, there are three remaining parameters: b, c, and η. The calibration consistent with the targeted moments is $b = 0.53$, $c = 0.072$, and $\eta = 0.9$.

[34] Shimer's (2005) calibration of the standard model with exogenous job destruction yields an elasticity of f equal to 0.48. Mortensen and Nagypal (2007) favor a different calibration that yields an elasticity of f equal to 1.56 (see their section 3.2). Pissarides' (2007) calibration of the standard model with endogenous job destruction obtains an elasticity of f equal to 1.54.

[35] The calibration consistent with the targeted moments is $b = 0.855$, $c = 0.56$, and $\eta = 0.34$.

[36] A common diagnostic for the size of the flow surplus is the ratio between worker's payoff from unemployment, b, and the average product of labor. For panel A of Table 2, this ratio equals 0.61; for panel C, it equals 0.71. Thus, the Mortensen and Pissarides model demands a smaller surplus to match the volatility of the job finding rate.

unemployment inflows, the model must generate a sufficiently large surplus at the expense of matching the cyclicality of the job finding rate. Conversely, to generate the cyclical variation in the job finding rate, the surplus must be small, which in turn yields excessive employment to unemployment transitions.

Understanding Amplification The results in Table 2 raise the question of why the generalized model yields amplification of the response of job creation to cyclical shocks. The following result provides a sense for where this amplification comes from by approximating the steady-state response of job creation to a change in aggregate labor productivity:

Proposition 8 *For small λ, the shift in the Job Creation condition (22) induced by a change in aggregate productivity p is given approximately by*

$$\left.\frac{d\ln\theta}{d\ln p}\right|_{JC} \approx \frac{(1-\eta)\,\tilde{p}}{\omega\phi\left[(1-\eta)(\tilde{p}-b)-\eta\beta c\theta\right]+\eta\beta c\theta}, \tag{33}$$

where ω is the steady state employment share of hiring firms, and $\tilde{p}\equiv\rho\overline{apl}+(1-\rho)\overline{mpl}$ where $\overline{apl}$ and $\overline{mpl}$ are respectively the average and marginal product of labor of the average-sized firm, and $\rho\equiv\frac{\alpha\eta}{1-\eta(1-\alpha)}$.

Corollary 1 *The elasticity of the vacancy-unemployment ratio to aggregate productivity in the $\alpha=1$ case (Mortensen and Pissarides, 1994) is approximately equal to*

$$\frac{d\ln\theta}{d\ln p} \approx \frac{(1-\eta)\,p}{\phi\left[(1-\eta)(p-b)-\eta\beta c\theta\right]+\eta\beta c\theta}. \tag{34}$$

Equation (34) echoes results presented in Mortensen and Nagypal (2007, 2008): The cyclical response of the vacancy-unemployment ratio θ is amplified when the average flow surplus to employment relationships, $p-b$, is small. Equation (33) generalizes this result to the model studied here. Inspection of (33) and (34) reveals that there are two channels through which the generalized model yields amplification of the cyclicality of labor market tightness. First, the effective surplus that matters for amplification is now given by $\tilde{p}-b$, a weighted average of the average and marginal flow surpluses. This lies below the average flow surplus as a result of the diminishing marginal product of labor in the model.

This provides a sense for why the generalized model is able simultaneously to match the rate at which workers flow into unemployment s, as well as the volatility of the job-finding rate f over the cycle: The former requires a large average surplus; the latter requires a small marginal surplus.[37] The standard Mortensen and Pissarides model cannot achieve this because of its inherent linearity.

Equation (33) also suggests that there is an additional effect at work in the form of the variable ω, the steady state employment share of hiring firms. To understand the significance of this term, note that in the standard Mortensen and Pissarides model where $\alpha = 1$, ω is equal to one: With a linear technology, a firm that reduces its employment will shed all of its workers since, if one worker is unprofitable at a firm, all workers are unprofitable. As a result, all surviving firms at a point in time are hiring firms in the standard model. In contrast, in the generalized model, shedding firms do not reduce their employment to zero because reducing employment replenishes the marginal product of labor. Hence ω will be less than unity, and inspection of (33) and (34) reveals that this will lead to greater amplification relative to the standard model.[38] Intuitively, it is as if the economy has to rely on a smaller mass firms to hire workers from the unemployment pool, which in turn leads to a larger increase in unemployment in times of recession.

4.4 The Beveridge Curve

Until now, we have been concentrating on the cyclicality of worker flows implied by the generalized model. Readers of Shimer (2005), however, will recall that the standard search and matching model also fails to match the observed cyclical volatility in the vacancy rate in the U.S., and especially so if one allows job destruction to move countercyclically.

Table 2 reiterates this message: While vacancies are markedly procyclical, with an empirical elasticity with respect to output per worker of 2.91, the calibration of the Mortensen and Pissarides model in Panel B yields a *counter*cyclical vacancy elasticity.[39] Shimer (2005) has emphasized that this feature of the standard search and matching model in turn leads to a dramatic failure to account for a key stylized fact of the U.S. labor market: the negative relation between vacancies and unemployment, known as the Beveridge curve.

Figure 4 plots the Beveridge curve relation from model-generated data (the hollow cir-

[37] Mortensen and Nagypal (2007) favor an average flow match surplus of $[b/(Y/N)]^{-1} - 1 = \frac{1}{0.73} - 1 = 37$ percent. The corresponding value implied by our calibration is $\frac{1}{0.61} - 1 = 64$ percent. The worker's surplus in our simulation is also substantial: Workers obtain a $(\mathbb{E}[w] - b)/b = 19.5$ percent flow surplus from employment over unemployment.

[38] The reader may worry whether $(1-\eta)(\tilde{p} - b) - \eta\beta c\theta$ is positive or not. To see that it is, note that we can rewrite it as $(1-\eta)\left(\frac{p\alpha x n^{\alpha-1}}{1-\eta(1-\alpha)} - b\right) - \eta\beta c\theta$, and observe from equations (13) and (14) that it is, in fact, the marginal flow surplus of a firm, and therefore must be positive.

[39] This arises because countercyclical job destruction leads to an offsetting increase in hires in times of recession to maintain balance between unemployment inflows and outflows, and thereby stymies the procyclicality of vacancies (Shimer, 2005; Mortensen and Nagypal, 2008).

cles), and compares it with the empirical analogue using vacancy data from the Job Openings and Labor Turnover Survey (the dots). While the model-generated Beveridge curve has a slightly shallower slope, it nonetheless lies very close to the array of observations witnessed in the data. This can be traced to the results in Table 2: The cyclicality of vacancies in the model lies very close to its empirical counterpart, with a cyclical elasticity of 2.47 lying somewhat below the value of 2.91 in the data.[40]

What emerges from Table 2 and Figure 4 is a coherent and quantitatively accurate picture of the joint cyclical properties of both flows of workers in and out of unemployment, as well as the behavior of unemployment and vacancies. In addition to providing a plausible mechanism for the cyclical amplitude of the job-finding rate, the model also presents an environment in which this can be reconciled with the cyclical behavior of job destruction and vacancy creation.

4.5 Propagation

A less well-documented limitation of the standard search and matching model relates to the propagation of the response of equilibrium labor market tightness to aggregate shocks to labor productivity. In the Mortensen and Pissarides model, the vacancy-unemployment ratio is a jump variable and therefore moves contemporaneously with changes in labor productivity. Empirically, however, the vacancy-unemployment ratio displays sluggish behavior, and is much more persistent than aggregate labor productivity, a point emphasized by Shimer (2005) and Fujita and Ramey (2007).

An appealing feature of the model presented in sections 2 and 3 is that it admits a natural channel for the propagation of the response of the vacancy-unemployment ratio to aggregate shocks. The determination of θ over time depends on the evolution of the distribution of employment across establishments $H(n)$. Inspection of equation (31), the law of motion for the distribution of employment across firms, reveals that $H(n)$ is not a jump variable, but is instead a slow moving state variable in the model. In particular, rewriting (31) yields

$$H(n) - H_{-1}(n) = -\vartheta(n)\left[H_{-1}(n) - H^*(n)\right], \tag{35}$$

where $\vartheta(n) \equiv \lambda\left(1 - \tilde{G}[R_v(n)] + \tilde{G}[R(n)]\right)$,[41] and $H^*(n)$ is the steady state distribution that sets $H(n) - H_{-1}(n) = 0$.

[40] A linear regression of de-trended vacancies on de-trended unemployment yields a coefficient of 0.53. When the same regression is run on model-generated data, the coefficient is 0.51.

[41] For notational simplicity, we suppress the dependence of R, R_v, and H on the aggregate state variables N and p.

Equation (35) provides an important source of intuition for what factors are likely to drive propagation in the model. In particular, the rate of convergence to steady state $\vartheta(n)$ is determined by two factors. First, less frequent idiosyncratic shocks, as implied by a lower value of λ, will cause fewer firms to adjust employment, and thereby slow the reallocation of employment across firms. Second, the size of adjustment costs will determine the gap between $R_v(n)$ and $R(n)$ in a firm's optimal employment policy function in Figure 1. Larger adjustment costs will widen this gap, reducing $\vartheta(n)$ in equation (35), and slowing the dynamics of $H(n)$. This suggests that the magnitude of labor adjustment costs has important implications for the propagation of the response of unemployment to aggregate shocks.

Figure 5 plots the dynamic response of unemployment, labor market tightness, the job-finding rate, and the unemployment inflow rate following a permanent one percent decline in aggregate labor productivity using simulated data from the model. This confirms that the generalized model yields some propagation of the response of unemployment and labor market tightness (and thereby the job-finding rate). It takes around 20 months for unemployment to adjust to the shock in the model, and 9 months for the response of θ and f to dissipate.

This is a substantial improvement over the instantaneous response of θ and f implied by the standard search model. However, the magnitude of the propagation implied by the model is not quite enough to account fully for the persistence of the vacancy-unemployment ratio observed in the data. In their detailed analysis of the empirical dynamics of labor market tightness, Fujita and Ramey (2007) show that θ takes around five quarters, or 15 months, to adjust to an impulse to aggregate labor productivity in U.S. data.

An additional message of Figure 5 concerns the dynamics of the unemployment inflow rate s. Panel D reveals that s spikes upward instantaneously following a reduction in aggregate labor productivity, subsides in the immediate aftermath of the shock, and then converges toward a new steady state value over the course of the next two years. In the wake of a recessionary shock a discrete mass of jobs becomes unprofitable and is destroyed immediately, mirroring the implications of the standard Mortensen and Pissarides model (Mortensen, 1994). Following the shock, the inflow rate begins rising again as firms receive productivity shocks at rate λ.

Viewed together, the joint dynamics of the job-finding and inflow rate in the model resemble the qualitative features of the response of f and s over the cycle: Recessions are characterized by a wave of inflows which then recedes and is accompanied by persistent declines in rates of job-finding, just as observed in empirical worker flows in U.S. data (see, for example, Elsby, Michaels and Solon, 2009).

4.6 Extensions

The previous sections have shown that the model derived in sections 2 and 3 can account for many of the cross-sectional and cyclical features of the U.S. labor market. In this section, we push the model harder. We consider two additional outcomes which the model can speak to, but was not designed to account for: the cyclical dynamics of the cross-sectional distribution of establishment size, and the employer size-wage effect.

Cyclical Dynamics of the Employer Size Distribution Until now, we have focused separately on the implications of the model for the cross section of employers and for the aggregate dynamics of labor stocks and flows. Recent literature, however, has sought to understand the joint dynamics of the cross section. Moscarini and Postel-Vinay (2009) in particular emphasize empirical regularities in the cyclical behavior of the cross-sectional distribution of establishment size: The share of small establishments rises during recessions, while the shares of larger firms decline. Figure 6 reiterates this finding. It uses annual data from County Business Patterns for the years 1986 to 2007 on the number of establishments by employer size, and plots the log deviations from trend of the establishment size shares against the unemployment rate.[42] The dots plot the data, and the dot-dashed lines the corresponding least squares regression lines. The share of establishments with 1 to 19 workers rises with unemployment, while the shares of establishments with more than 20 employees decline as unemployment rises.

The blue dashed lines in Figure 6 plot the analogous relationships implied by simulations of the model of sections 2 and 3. The results are encouraging: The model replicates the observation that the share of establishments with fewer than 20 employees increases in times of recession, while the shares of the larger size classes decline. In addition, the model also provides a reasonable account of the magnitude of the cyclicality of the establishment shares. It comes very close to replicating the cyclical sensitivities for the 20 to 99 employee and the 1000+ employee groups, and implies around one half of the cyclical sensitivity of the 1 to 19 and 100 to 999 employer size classes.

The observation that the share of smaller establishments rises in a recession in both the model and the data is not in itself a surprising fact, since aggregate employment, the mean of the distribution of establishment size, falls during recessions. What is noteworthy about the model is that it replicates the position in the distribution—at around 20 employees—at

[42] Here, we use County Business Patterns data rather than data from the Small Business Administration (see Section 4.2) because the former affords us a longer consistent time series for the size classes that we are interested in for this exercise.

which this effect takes hold, as well as the magnitude of the cyclicality in some of the size classes. Given that these implications of the model are venturing even farther afield from the moments it was calibrated to match, we view the results of Figure 6 as an important achievement.

Employer Size-Wage Effect Our final application relates to the observation that workers employed in larger firms are often paid higher wages—the employer size-wage effect noted in the influential study by Brown and Medoff (1989). An attractive feature of the model is that, by incorporating a notion of employer size, and by modeling the wage bargaining process between a firm and its many workers, it can speak to such issues.

Casual inspection of the wage equation (10) is not heartening in this respect, however: Due to the diminishing marginal product of labor in the model, one might anticipate that the model predicts a negative correlation between wages and employer size, in direct contrast to Brown and Medoff's observation. Further consideration of equation (10), though, reveals that such a conclusion would be premature: While the diminishing marginal product of labor does set in for larger employers, it is also the case that larger employers will be those with higher idiosyncratic productivity x. The implications of the model for the employer size-wage effect depend on which of these forces dominates.[43]

Figure 7 illustrates the relationship between average log wages and log employment implied by the model. It takes simulated data based on the calibration in Table 1, and plots the results of a nonparametric locally weighted (LOWESS) regression of log firm wages on log firm employment. This reveals that the model does in fact predict a positive employer-size wage effect, qualitatively in line with the results of Brown and Medoff. As it turns out, the effect of higher idiosyncratic productivity outweighs the effect of diminishing marginal product.

Figure 7 also provides a sense of the magnitude of the size-wage effect. Brown and Medoff (1989) report that, controlling for observable and unobservable measures of labor quality and for differences in workplace conditions, a worker moving from an establishment with log employment one standard deviation below average to an establishment with log employment one standard deviation above average would receive a wage increase of around

[43] Bertola and Garibaldi (2001) also include a discussion of this point. Their model is somewhat less general than that presented in this paper, however. Like Bertola and Caballero (1994), the authors analyze a linear approximation to the marginal product function. In addition, a more restrictive process for productivity shocks is used: It is assumed that, if a firm receives a negative disturbance, its productivity reverts to the minimum of the distribution's support. Since these transitions are the only means by which the model generates inflows into unemployment, all firms that receive a negative shock must shed workers; inaction is, by assumption, never an optimal response for these employers.

10 percent. As shown in Figure 7, the counterpart implied by the model is a wage premium closer to 2.3 percent.

Thus, while the model yields a positive size-wage effect, it generates only around one quarter of the magnitude of the effect observed in the data. We do not view this necessarily as a problem, however. The mechanism that accounts for the size-wage effect in the model—the interaction of surplus sharing with heterogeneity in employer productivity—is only one of a large number of proposed channels. Oi and Idson (1999) present a summary of these, including efficiency wages, market power, specific human capital, among others. In a model of wage posting, Burdett and Mortensen (1998) demonstrate that on-the-job search in the presence of labor market frictions also can generate a positive employer size-wage effect, as higher-paying firms recruit and retain more workers. The results of Figure 7 suggest that the model presented in this paper leaves room for these additional explanations.

5 Summary and Discussion

In this paper, we have introduced a notion of firm size into a search and matching model with endogenous job destruction. This yields a rich, yet analytically tractable framework. In a series of quantitative applications, we show that the model provides a useful laboratory for analyzing the salient features of both the dynamics and the cross section of the aggregate labor market. Specifically, a calibrated version of the model provides a coherent account of the distributions of establishment size and employment growth; the amplitude and propagation of the cyclical dynamics of worker flows; the Beveridge curve relation between unemployment and vacancies; and the dynamics of the distribution of firm size over the business cycle.

We conclude by placing this paper within the context of more recent research on labor market dynamics and then discuss avenues for future work. Since this paper's first writing, there have been a number of contributions to the search-and-matching literature that study the cross section of employment growth and the cyclical dynamics of labor market flows. The model in Fujita and Nakajima (2009) is closest to our own. They introduce exogenous separations to drive a wedge between gross worker flows and net job changes, which enables them to potentially speak to facts on both of these flows. Hawkins (2011) incorporates a costly entry decision into a framework that is otherwise similar to that introduced in this paper. A few other papers consider decreasing returns within search-and-matching frameworks that depart more noticeably from that studied here. For instance, Kaas and Kircher (2010) study a model of directed search under decreasing returns. The application of directed search enables them to replicate the finding in Davis, Faberman, and Haltiwanger

(2010) that fast-growing firms fill a greater share of their posted vacancies.[44] Schaal (2010) generalizes the on-the-job search model of Menzio and Shi (2009) to include decreasing returns at the firm level, which enables him to engage a rich set of cross-sectional facts (see footnote 10).

Going forward, there are a number of additional avenues for further related work. First, since the model has a well-defined concept of a firm, it lends itself to estimation using establishment level data. As a result, the analytical framework developed here will complement recent research efforts that have sought to solve and estimate search models using numerical methods (e.g. Cooper, Haltiwanger and Willis, 2007).

Second, our interpretation of the standard search and matching model as a model of kinked adjustment costs raises the question of the aggregate implications of other forms of adjustment costs in the labor market. Recent research has emphasized the importance of fixed adjustment costs in explaining the empirical properties of labor demand at the micro level (see for example Caballero, Engel, and Haltiwanger, 1997, and Cooper, Haltiwanger, and Willis, 2004). Incorporating these adjustment costs into the model will provide a unification of the joint insights of the two dominant approaches to the modelling of aggregate labor markets—the search and matching framework, and models of adjustment costs.

A final extension relates to the nature of wage setting. An attractive feature of incorporating firm size into models of the labor market is that an assessment of the multilateral dimension to wage bargaining between a firm and its many workers becomes feasible. This has been of particular interest in recent literature that has emphasized the importance of rigidities in the structure of wages within a firm, as well as of individual wages over time, for determining the volatility of unemployment (Bewley, 1999; Hall, 2005). While the wage bargaining solution derived in the present paper seeks to improve upon approaches in previous work, it is in many ways an idealized environment in which the wages of all workers can be renegotiated costlessly. This idealized setting, however, provides a fruitful benchmark for analyzing the implications of rigidities in renegotiation of wages within a firm, as well as across time.

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7 Appendix

A Solution of the Simulated Model

Here we present technical details of the solution to the model in sections 2 and 3 for the purposes of the quantitative applications in section 4. For simplicity, we present the solution approach for a given fixed firm productivity φ, so we supress this notation in what follows. Aggregation across φ is achieved simply by integrating over the known distribution φ.

Steady State Optimal Employment Policy Idiosyncratic shocks evolve according to (17) with $x \sim Pareto\left(1-k_x^{-1}, k_x\right)$. Denoting the distribution function of x as $\tilde{G}$, we can rewrite the recursion for the function $D\left(n,x\right)$ in Proposition 3 as:

$$\begin{aligned} D\left(n,x\right) &= \left(1-\lambda\right)\chi\left(x\right)+\lambda\int_{R(n)}^{R_v(n)}\chi\left(x'\right)d\tilde{G}\left(x'\right)+\lambda\int_{R_v(n)}^{\infty}\frac{c}{q}d\tilde{G}\left(x'\right) \\ &\quad +\beta\left(1-\lambda\right)D\left(n,x\right)+\beta\lambda\int_{R(n)}^{R_v(n)}D\left(n,x'\right)d\tilde{G}\left(x'\right), \end{aligned} \tag{36}$$

where $\chi(x) \equiv (1-\eta)\left[\frac{px\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} - b\right] - \eta\beta f\frac{c}{q}$. We conjecture that the function $D(n,x)$ is of the form $D(n,x) = d_0 + d_1\chi(x)$. Substituting this into the latter, and equating coefficients, we obtain the following solution for $D(n,x)$:

$$
\begin{aligned}
D(n,x) &= \frac{1-\lambda}{1-\beta(1-\lambda)}\chi(x) \\
&+ \frac{\lambda}{1-\beta(1-\lambda)} \frac{\tilde{G}[R_v(n)] - \tilde{G}[R(n)]}{1-\beta(1-\lambda) - \beta\lambda\left(\tilde{G}[R_v(n)] - \tilde{G}[R(n)]\right)}\mathcal{Q}(n) \\
&+ \frac{1-\tilde{G}[R_v(n)]}{1-\beta(1-\lambda) - \beta\lambda\left(\tilde{G}[R_v(n)] - \tilde{G}[R(n)]\right)}\lambda\frac{c}{q},
\end{aligned} \tag{37}
$$

where $\mathcal{Q}(n) \equiv \mathbb{E}(\chi(x')\,|x' \in [R(n), R_v(n)])$. Substituting into the first order conditions for hires and separations (13) and (14) yields two nonlinear equations in the optimal employment policy $R(n)$ and $R_v(n)$ that are straightforward to solve numerically. The aggregate employment stock and flows are then obtained directly from applying the results of Proposition 5.

Average Product and Average Marginal Product The average product of labor implied by the model is given by $APL = \mathbb{E}[pxn^{\alpha-1}]$. Note that:

$$
\mathbb{E}\left[xn^{\alpha-1}\right] = \int\left[\int xdG(x|n)\right]n^{\alpha-1}dH(n).
$$

Moreover, the optimal employment policy implies that, given n, x must lie in the interval $[R(n), R_v(n)]$, but is otherwise independently distributed. Thus:

$$
\int xdG(x|n) = \frac{\int_{R(n)}^{R_v(n)} xdG(x)}{G[R_v(n)] - G[R(n)]} = \frac{1}{2}[R(n) + R_v(n)], \tag{38}
$$

where the last equality follows from the assumption of uniform idiosyncratic shocks in the simulation. Thus:

$$
APL = \mathbb{E}\left[pxn^{\alpha-1}\right] = p\int \frac{1}{2}[R(n) + R_v(n)]\,n^{\alpha-1}dH(n). \tag{39}
$$

The average marginal product of labor is simply given by $\mathbb{E}[MPL] = \mathbb{E}[px\alpha n^{\alpha-1}] = \alpha APL$.

Average Wages It follows from equation (9) that the average wage across firms is given by:

$$
\bar{w}_f = \frac{\eta}{1-\eta(1-\alpha)}\mathbb{E}[MPL] + \eta\beta f\frac{c}{q} + (1-\eta)b. \tag{40}
$$

To obtain the average wage across workers, which we denote $\bar{w}_w$, note that $\bar{w}_w = \mathbb{E}\left[\frac{n}{\mathbb{E}(n)} w(n,x)\right]$ where $w(n,x)$ is the wage in a given firm defined in (9). That is, it is the employment-weighted average of wages across firms. Thus:

$$\bar{w}_w = \frac{\eta}{1-\eta(1-\alpha)} \frac{1}{\mathbb{E}(n)} \mathbb{E}[px\alpha n^\alpha] + \eta\beta f \frac{c}{q} + (1-\eta) b. \tag{41}$$

This has a very similar structure to the average wage across firms. It follows that:

$$\bar{w}_w = \frac{\eta p \alpha}{1-\eta(1-\alpha)} \frac{1}{\mathbb{E}(n)} \int \frac{1}{2} [R(n) + R_v(n)] n^\alpha dH(n) + \eta\beta f \frac{c}{q} + (1-\eta) b. \tag{42}$$

Finally, the average wage of new hires, which we denote $\bar{w}_m$, is equal to a hiring-weighted average of wages across hiring firms. Noting from (12) that idiosyncratic productivity of hiring firms is given by $x = R_v(n)$, we have that:

$$\bar{w}_m = \mathbb{E}[\mathbb{E}(w(n,x) | n > n_{-1}, n_{-1})] = \int \int_{n_{-1}} w(n, R_v(n)) \frac{dG[R_v(n)]}{1 - G[R_v(n_{-1})]} dH(n_{-1}). \tag{43}$$

B Proofs

Conjecture 1 *The optimal employment policy function is of the form specified in (12).*

We will later verify in the proof of Proposition 2 that the Conjecture is consistent with the solution for the wage equation obtained in Proposition 1.

Proof of Proposition 1. Note first that, under the Conjecture, we can write the marginal surplus to a firm recursively as:

$$J(n,x) = pxF'(n) - w(n,x) - w_n(n,x) n + \beta \int_{R_v(n)}^{\infty} \frac{c}{q} dG(x') + \beta \int_{R(n)}^{R_v(n)} J(n,x') dG(x'). \tag{44}$$

In addition, we can write the value to a worker of unemployment as:

$$U = b + \beta \left\{ (1-f) U' + f \int_0^\infty \int_{R_v(n)}^\infty W\left(R_v^{-1}(x'), x'\right) \frac{dG(x')}{1 - G(R_v(n))} dH(n) \right\}. \tag{45}$$

Upon finding a job, which occurs with probability f, the new job must be in a firm which is posting vacancies. This implies that the idiosyncratic productivity of the firm, $x' > R_v(n)$, and that the level of employment in the hiring firm, $n' = R_v^{-1}(x')$. Moreover, since firms differ in size, there is a distribution of employment levels, $H(n)$, over which an unemployed worker will take expectations when evaluating the expected future benefits of being hired.[45]

[45] The reader may wonder why the integral in (45) is not taken over the joint distribution of n and x'. The reason is that, conditional on $x' > R_v(n)$, n provides no additional information on x'; see the optimal employment policy function (12).

It is useful to rewrite the worker's value of unemployment as:

$$U = b + \beta \left\{ U' + f \int_0^\infty \int_{R_v(n)}^\infty \left[W \left(R_v^{-1}(x'), x' \right) - U' \right] \frac{dG(x')}{1 - G(R_v(n))} dH(n) \right\}. \tag{46}$$

Then note that, due to Nash sharing, the worker's surplus in an expanding firm, $W\left(R_v^{-1}(x'), x'\right) - U' = \frac{\eta}{1-\eta} J\left(R_v^{-1}(x'), x'\right)$, and moreover that, by the first-order condition for a hiring firm (see (4)), $J\left(R_v^{-1}(x'), x\right) = c/q$. Thus, we obtain the simple result:

$$U = b + \beta U' + \beta f \frac{\eta}{1-\eta} \frac{c}{q}. \tag{47}$$

The value of employment to a worker can be written as:

$$\begin{aligned} W(n,x) &= w(n,x) \\ &\quad + \beta \left\{ \int_0^{R(n)} \left[\tilde{s} U' + (1-\tilde{s}) W \left(R^{-1}(x'), x' \right) \right] dG(x') \right. \\ &\quad \left. + \int_{R(n)}^{R_v(n)} W(n,x') \, dG(x') + \int_{R_v(n)}^\infty W \left(R_v^{-1}(x'), x' \right) dG(x') \right\}. \end{aligned} \tag{48}$$

An employed worker's expected future payoff can be split into three regimes. If the firm sheds workers next period ($x' < R(n)$) then the worker may separate from the firm. We denote by $\tilde{s}$ the probability that a worker separates from a firm conditional on the firm shedding workers. If the worker separates, she transitions into unemployment and receives a payoff U'. Otherwise she continues to be employed in a firm of size $n' = R^{-1}(x')$. Note that Nash sharing implies that $W\left(R^{-1}(x'), x'\right) - U' = \frac{\eta}{1-\eta} J\left(R^{-1}(x'), x'\right)$, and that, by the first-order condition, $J\left(R^{-1}(x'), x'\right) = 0$. Thus, $W\left(R^{-1}(x'), x'\right) = U'$. In the event that a firm freezes employment next period ($x' \in [R(n), R_v(n)]$) then Nash sharing implies that $W(n,x') - U' = \frac{\eta}{1-\eta} J(n,x')$. Finally, in the event that the firm hires next period, $W\left(R_v^{-1}(x'), x'\right) - U' = \frac{\eta}{1-\eta} \frac{c}{q}$. Thus, we have that:

$$W(n,x) = w(n,x) + \beta U' + \beta \frac{\eta}{1-\eta} \int_{R_v(n)}^\infty \frac{c}{q} dG(x') + \beta \frac{\eta}{1-\eta} \int_{R(n)}^{R_v(n)} J(n,x') \, dG(x'). \tag{49}$$

Subtracting the value of unemployment to a worker from the latter, we obtain the following description of the worker's surplus:

$$\begin{aligned} W(n,x) - U &= w(n,x) - b + \beta \frac{\eta}{1-\eta} \int_{R_v(n)}^\infty \frac{c}{q} dG(x') \\ &\quad + \beta \frac{\eta}{1-\eta} \int_{R(n)}^{R_v(n)} J(n,x') \, dG(x') - \beta f \frac{\eta}{1-\eta} \frac{c}{q}. \end{aligned} \tag{50}$$

Under Nash, this must be equal to $\frac{\eta}{1-\eta}J(n,x)$, where $J(n,x)$ is as derived in (44) so that we have:

$$w(n,x) = \eta\left[pxF'(n) - w_n(n,x)n + \beta f\frac{c}{q}\right] + (1-\eta)b, \tag{51}$$

as required. ■

Proof of Proposition 2. Given the wage function in (9), it follows that the firm's objective, (3), is continuous in (n_{-1}, x) and concave in n. Thus, it follows from the Theorem of the Maximum that the firm's optimal employment policy function is continuous in (n_{-1}, x). Given this, it follows that the employment policy function must be of the form stated in Proposition 2. This verifies that the Conjecture stated at the beginning of the appendix holds. ■

Proof of Proposition 3. First, note that one can re-write the continuation value conditional on each of the three possible continuation regimes:

$$\Pi(n,x') = \begin{cases} \Pi^{-}(n,x') & \text{if} \quad x' < R(n), \\ \Pi^{0}(n,x') & \text{if} \quad x' \in [R(n), R_v(n)], \\ \Pi^{+}(n,x') & \text{if} \quad x' > R_v(n), \end{cases} \tag{52}$$

where superscripts $^{-/0/+}$ refer to whether their are separations, a hiring freeze, or hires tomorrow. Thus we can write[46]:

$$\int \Pi(n,x')\,dG(x'|x) = \int_0^{R(n)} \Pi^{-}(n,x')\,dG + \int_{R(n)}^{R_v(n)} \Pi^{0}(n,x')\,dG + \int_{R_v(n)}^{\infty} \Pi^{+}(n,x')\,dG. \tag{53}$$

Taking derivatives with respect to n, recalling the definition of $D(\cdot)$, and noting that, since $\Pi(n,x')$ is continuous, it must be that $\Pi^{-}(n,R(n)) = \Pi^{0}(n,R(n))$ and $\Pi^{0}(n,R_v(n)) = \Pi^{+}(n,R_v(n))$, yields:

$$D(n,x) = \int_0^{R(n)} \Pi_n^{-}(n,x')\,dG + \int_{R(n)}^{R_v(n)} \Pi_n^{0}(n,x')\,dG + \int_{R_v(n)}^{\infty} \Pi_n^{+}(n,x')\,dG. \tag{54}$$

Finally, using the Envelope conditions in Lemma 1 below, and substituting into (54) we obtain (15) and (16) in the main text:

$$\begin{aligned} D(n,x) &= \int_{R(n)}^{R_v(n)} \left\{(1-\eta)\left[\frac{px'\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} - b\right] - \eta\beta f\frac{c}{q}\right\} dG(x'|x) \\ &\quad + \int_{R_v(n)}^{\infty} \frac{c}{q}dG(x'|x) + \beta\int_{R(n)}^{R_v(n)} D(n,x')\,dG(x'|x) \\ &\equiv (\mathbf{C}D)(n,x). \end{aligned} \tag{55}$$

[46]Henceforth, "dG" without further elaboration is to be taken as "$dG(x'|x)$".

To verify that $\mathbf{C}$ is a contraction mapping, we confirm that Blackwell's sufficient conditions for a contraction hold here (see Stokey and Lucas, 1989, p.54). To verify monotonicity, fix $(n, x) = (\bar{n}, \bar{x})$, and take $\hat{D} \geq D$. Then note that:

$$\begin{aligned}\int_{R(\bar{n})}^{R_v(\bar{n})} \hat{D}(\bar{n}, x')\, dG(x'|\bar{x}) - \int_{R(\bar{n})}^{R_v(\bar{n})} D(\bar{n}, x')\, dG(x'|\bar{x}) \\ = \int_{R(\bar{n})}^{R_v(\bar{n})} \left[\hat{D}(\bar{n}, x') - D(\bar{n}, x')\right] dG(x'|\bar{x}) \geq 0.\end{aligned} \tag{56}$$

Since $(\bar{n}, \bar{x})$ were arbitrary, it thus follows that $\mathbf{C}$ is monotonic in D. To verify discounting, note that:

$$[\mathbf{C}(D + a)](n, x) = (\mathbf{C}D)(n, x) + \beta a [G(R_v(n)|x) - G(R(n)|x)] \leq (\mathbf{C}D)(n, x) + \beta a. \tag{57}$$

Since $\beta < 1$ it follows that $\mathbf{C}$ is a contraction. It therefore follows from the Contraction Mapping Theorem that $\mathbf{C}$ has a unique fixed point. ■

Lemma 1 *The value function defined in (3) has the following properties:*

$$\begin{aligned}\Pi_n^-(n, x') &= 0, \\ \Pi_n^0(n, x') &= (1 - \eta)\left[\frac{px'\alpha n^{\alpha-1}}{1 - \eta(1 - \alpha)} - b\right] - \eta\beta f\frac{c}{q} + \beta D(n, x'), \\ \Pi_n^+(n, x') &= c/q.\end{aligned} \tag{58}$$

Proof of Lemma 1. First, note that standard application of the Envelope Theorem implies that $\Pi_n^-(n, x') = 0$ and $\Pi_n^+(n, x') = c/q$. It is only slightly less obvious what happens when $\Delta n' = 0$, i.e. when the employment is frozen next period. In this case, $n' = n$ and this implies that:

$$\Pi^0(n, x') = px'F(n) - w(n, x')\, n + \beta \int \Pi(n, x'')\, dG(x''|x'). \tag{59}$$

It therefore follows that:

$$\Pi_n^0(n, x') = px'F'(n) - w(n, x') - w_n(n, x')\, n + \beta \int \Pi_n(n, x'')\, dG(x''|x'). \tag{60}$$

Since, by definition $D(n, x') \equiv \int \Pi_n(n, x'')\, dG(x''|x')$, the statement holds as required. ■

Proof of Proposition 4. First note that if x evolves according to (17), then we can rewrite the recursion for $D(n,x)$ as:

$$\begin{aligned} D(n,x) &= \frac{1-\lambda}{1-\beta(1-\lambda)}\chi(x) + \frac{\lambda}{1-\beta(1-\lambda)}\int_{R(n)}^{R_v(n)} \chi(x')\, d\tilde{G}(x') \\ &+ \frac{\lambda}{1-\beta(1-\lambda)}\int_{R_v(n)}^{\infty} \frac{c}{q} d\tilde{G}(x') + \frac{\beta\lambda}{1-\beta(1-\lambda)}\int_{R(n)}^{R_v(n)} D(n,x')\, d\tilde{G}(x'), \end{aligned} \tag{61}$$

where $\chi(x) \equiv (1-\eta)\left[\frac{px\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} - b\right] - \eta\beta c\theta$. It follows that the LHS of the first-order conditions, (13) and (14) are increasing in x, because $\chi(x)$ is increasing in x. Thus, to establish that $\partial R_v/\partial p < 0$ and $\partial R/\partial p < 0$, simply note that the function $D(n,x)$ is also increasing in p and thus the LHS of (13) and (14) are increasing in p.

To ascertain the marginal effects of θ we first need to establish the marginal effect of θ on the function $D(n,x)$. Rewriting $f/q = \theta$ and $q = q(\theta)$ in (61), differentiating with respect to θ, and using the first-order conditions, (13) and (14), to eliminate terms we obtain:

$$D_\theta = -\eta\beta c\frac{1-\lambda(1-p^0)}{1-\beta[1-\lambda(1-p^0)]} - \frac{c}{q}\frac{q'(\theta)}{q}\frac{\lambda p^+}{1-\beta[1-\lambda(1-p^0)]}, \tag{62}$$

where $p^0 \equiv \tilde{G}(R_v(n)) - \tilde{G}(R(n))$, $p^+ \equiv 1-\tilde{G}(R_v(n))$, and $p^- \equiv \tilde{G}[R(n)]$. Note that D_θ is independent of x. Differentiating the first-order condition for a hiring firm, (13), with respect to θ we obtain:

$$-\eta\beta c + \frac{c}{q}\frac{q'(\theta)}{q} + \beta D_\theta = -\frac{\eta\beta c}{1-\beta[1-\lambda(1-p^0)]} + \frac{c}{q}\frac{q'(\theta)}{q}\frac{1-\beta(1-\lambda p^-)}{1-\beta[1-\lambda(1-p^0)]} < 0 \tag{63}$$

since $q'(\theta) < 0$. Thus it follows that $\partial R_v/\partial\theta > 0$. Likewise, differentiating the first-order condition for a shedding firm, (14), with respect to θ we obtain:

$$-\eta\beta c + \beta D_\theta = -\frac{\eta\beta c}{1-\beta[1-\lambda(1-p^0)]} - \beta\frac{c}{q}\frac{q'(\theta)}{q}\frac{\lambda p^+}{1-\beta[1-\lambda(1-p^0)]}. \tag{64}$$

Thus, $\partial R/\partial\theta > 0 \iff n > R_v^{-1}\tilde{G}^{-1}\left(1 + \frac{\eta}{\varepsilon_{q\theta}}\frac{f}{\lambda}\right)$ where $\varepsilon_{q\theta} \equiv \frac{d\ln q}{d\ln\theta}$. ■

Proof of Proposition 5. *Proof of (19) and (20):* See main text.

Proof of (21): First note that a necessary condition for a firm to shed workers is that it receives an idiosyncratic shock, which occurs with probability λ. In this event, the number of separations in a firm that is shedding workers is equal to $n_{-1} - R^{-1}(x)$, since separating firms set employment, $n = R^{-1}(x)$. Now imagine, counterfactually, that all firms shared the same lagged employment level, n_{-1}. Then, the aggregate number of separations in the economy would equal:

$$\Lambda(n_{-1}) = \lambda\int_{x_{\min}}^{R(n_{-1})} \left[n_{-1} - R^{-1}(x)\right] d\tilde{G}(x), \tag{65}$$

where $x_{\min}$ is the lower support of idiosyncratic productivity. Using the change of variables, $x = R(n)$, and integrating by parts:

$$\Lambda(n_{-1}) = \lambda \int_{n_{\min}}^{n_{-1}} (n_{-1} - n) \frac{d\tilde{G}[R(n)]}{dn} dn = \lambda \int_{n_{\min}}^{n_{-1}} \tilde{G}[R(n)]\, dn. \tag{66}$$

Now, of course, the true aggregate number of separations is equal to $S = \int \Lambda(n_{-1})\, dH(n_{-1})$, where $H(\cdot)$ is the c.d.f. of employment. Denoting $n_{\max}$ as the upper support of $H(\cdot)$, further integration by parts reveals that:

$$S = \Lambda(n_{\max}) - \lambda \int \tilde{G}[R(n_{-1})] H(n_{-1})\, dn_{-1} = \lambda \int [1 - H(n)] \tilde{G}[R(n)]\, dn, \tag{67}$$

as required. A similar method reveals that the aggregate number of hires in the economy, $M = \lambda \int H(n) \left(1 - \tilde{G}[R_v(n)]\right) dn$. It follows from the steady state condition for the distribution for employment, (19), that separations, S, are equal to hires, M. ■

Proof of Proposition 6. Given that aggregate shocks evolve according to (25), and denoting the forecast equations for N and θ in (26) as $N'(N,p)$ and $\theta'(N',p)$ respectively, we can write the marginal effect of current employment on future profits as

$$D(n,x,N,p;\sigma_p) = \frac{1}{2} d(n,x,N'(N,p+\sigma_p), p+\sigma_p) + \frac{1}{2} d(n,x,N'(N,p-\sigma_p), p-\sigma_p), \tag{68}$$

where

$$\begin{aligned} d(n,x,N'(N,p'),p') \;=\; & \int_{R(n,N',p')}^{R_v(n,N',p')} \chi(n,x',N',p')\, dG(x'|x) \\ & + \int_{R_v(n,N',p')}^{\infty} \frac{c}{\mu} [\theta'(N',p')]^{\phi}\, dG(x'|x) \\ & + \beta \int_{R(n,N',p')}^{R_v(n,N',p')} D(n,x',N'(N,p'),p')\, dG(x'|x), \end{aligned} \tag{69}$$

and $\chi(n,x,N,p) = (1-\eta)\left[\frac{px\alpha n^{\alpha-1}}{1-\eta(1-\alpha)} - b\right] - \eta\beta c E[\theta'(N',p')|p] \equiv \chi_0 + \chi_1 px + \chi_2 E[\theta'(N',p')|p]$. Note that we are assuming a Cobb-Douglas matching function, which is used in the quantitative analysis described in Section 4. This implies $q(\theta') = \mu[\theta'(N',p')]^{-\phi}$, where μ is a matching efficiency parameter and ϕ is the matching elasticity. Taking a Taylor series approximation to $D(n,x,N,p;\sigma_p)$ around $\sigma_p = 0$ we obtain

$$D(n,x,N,p;\sigma_p) \approx D(n,x,N^*,p;0) + D_{\sigma_p}(n,x,N^*,p;0)\sigma_p + D_N^*(N - N^*), \tag{70}$$

where $D_N^* \equiv D_N(n,x,N^*,p;0)$. It is straightforward to show that $D_{\sigma_p}(n,x,N^*,p;0) = 0$, and that $D_N^* = d_{N'}(n,x,N^*,p)\nu_N$. Under the conjectured forecast equations in (26), we can write[47]

$$
\begin{aligned}
d_{N'}\left(n,x,N'\left(N,p'\right),p'\right) &= \int_{R(n,N',p')}^{R_v(n,N',p')} \chi_{N'}\left(n,x',N',p'\right) dG\left(x'|x\right) \\
&+\int_{R_v(n,N',p')}^{\infty} \frac{c}{\mu}\phi\left[\theta'\left(N',p'\right)\right]^{\phi-1}\theta'_{N'}\left(N',p'\right) dG\left(x'|x\right) \\
&+\beta\int_{R(n,N',p')}^{R_v(n,N',p')} D_N\left(n,x',N'\left(N,p'\right),p'\right) dG\left(x'|x\right). \qquad (71)
\end{aligned}
$$

Evaluating at $N=N^*$ and $p'=p$, and noting that $\chi_{N'}\left(n,x,N',p\right)=\chi_2\theta_N\nu_N$, and $\theta'_{N'}\left(N^*,p\right)=\theta_N$, we obtain

$$
\begin{aligned}
d_{N'}\left(n,x,N^*,p'\right) &= \chi_2\theta_N\nu_N\int_{R(n,N^*,p)}^{R_v(n,N^*,p)} dG\left(x'|x\right)+\frac{c}{\mu}\phi\theta_N{\theta^*}^{\phi-1}\int_{R_v(n,N^*,p)}^{\infty} dG\left(x'|x\right) \\
&+\beta\int_{R(n,N^*,p)}^{R_v(n,N^*,p)} D_N\left(n,x',N^*,p\right) dG\left(x'|x\right). \qquad (72)
\end{aligned}
$$

Recall from above that $D_N^*=d_{N'}\left(n,x,N^*,p\right)\nu_N$. Putting this together yields

$$
\begin{aligned}
D_N^* &= \chi_2\theta_N\nu_N^2\int_{R(n,N^*,p)}^{R_v(n,N^*,p)} dG\left(x'|x\right)+\frac{c}{\mu}\phi\theta_N\nu_N{\theta^*}^{\phi-1}\int_{R_v(n,N^*,p)}^{\infty} dG\left(x'|x\right) \\
&+\beta\nu_N\int_{R(n,N^*,p)}^{R_v(n,N^*,p)} D_{N'}\left(n,x',N^*,p;0\right) dG\left(x'|x\right). \qquad (73)
\end{aligned}
$$

Under the form of idiosyncratic shocks in (idiosync. eq.) we obtain:

$$
\begin{aligned}
D_N^* &= \theta_N\nu_N^2\chi_2\frac{(1-\lambda)+\lambda\left(\tilde{G}\left[R_v^*(n)\right]-\tilde{G}\left[R^*(n)\right]\right)}{1-\beta\nu_N\left[(1-\lambda)+\left(\tilde{G}\left[R_v^*(n)\right]-\tilde{G}\left[R^*(n)\right]\right)\right]} \\
&+\theta_N\nu_N\lambda\frac{c}{\mu}\phi{\theta^*}^{\phi-1}\frac{1-\tilde{G}\left[R_v^*(n)\right]}{1-\beta\nu_N\left[(1-\lambda)+\left(\tilde{G}\left[R_v^*(n)\right]-\tilde{G}\left[R^*(n)\right]\right)\right]}, \qquad (74)
\end{aligned}
$$

where $R_v\left(n,N^*,p\right)\equiv R_v^*(n)$ and $R\left(n,N^*,p\right)\equiv R^*(n)$ summarize the steady state employment policy function. ■

[47] Note that the effects of N' on the limits of integration will cancel by virtue of the first order conditions for optimal hiring and firing.

Proof of Proposition 7. Consider the c.d.f. of employment growth for a given lagged employment level, n_{-1}, and for the case where employment growth is negative:

$$\begin{aligned}\Pr\left(\Delta\ln n<\delta|n_{-1},\delta<0\right) &= \Pr\left(\ln R^{-1}\left(x\right)-\ln n_{-1}<\delta|n_{-1}\right)\\ &= \Pr\left(x<R\left(e^{\delta}n_{-1}\right)|n_{-1}\right)\\ &= \lambda\tilde{G}\left[R\left(e^{\delta}n_{-1}\right)\right]. \end{aligned} \tag{75}$$

It follows that the unconditional c.d.f. of employment growth, given that $\Delta\ln n<0$ is equal to:

$$H_{\Delta}\left(\delta\right)\equiv\Pr\left(\Delta\ln n<\delta\right)=\lambda\int\tilde{G}\left[R\left(e^{\delta}n_{-1}\right)\right]dH\left(n_{-1}\right), \tag{76}$$

It follows that the density of employment growth is given by

$$h_{\Delta}\left(\delta\right)=H_{\Delta}'\left(\delta\right)=\lambda\int\tilde{G}'\left[R'\left(e^{\delta}n_{-1}\right)\right]e^{\delta}n_{-1}dH\left(n_{-1}\right), \tag{77}$$

as stated in the Proposition. A similar method reveals that, in the case where $\Delta\ln n>0$:

$$H_{\Delta}\left(\delta\right)=\lambda\int\tilde{G}\left[R_{v}\left(e^{\delta}n_{-1}\right)\right]dH\left(n_{-1}\right),\text{ and }h_{\Delta}\left(\delta\right)=\lambda\int\tilde{G}'\left[R_{v}'\left(e^{\delta}n_{-1}\right)\right]e^{\delta}n_{-1}dH\left(n_{-1}\right). \tag{78}$$

Finally there is a mass point at zero employment growth. Clearly that is given by:

$$h_{\Delta}\left(0\right)=H_{\Delta}\left(0^{+}\right)-H_{\Delta}\left(0^{-}\right)=\lambda\int\left(\tilde{G}\left[R_{v}\left(n_{-1}\right)\right]-\tilde{G}\left[R\left(n_{-1}\right)\right]\right)dH\left(n_{-1}\right). \tag{79}$$

■

Lemma 2 *If idiosyncratic shocks evolve according to (17), and the matching function is of the form $M\left(U,V\right)=\mu U^{\phi}V^{1-\phi}$, then the marginal firm surplus defined in (44) is given by*

$$\begin{aligned}J &= \frac{\psi p\alpha n^{\alpha-1}}{1-\beta\left(1-\lambda\right)}\left[x+\frac{\beta\lambda p^{0}}{1-\beta\left(1-\lambda\right)-\beta\lambda p^{0}}\mathcal{E}\left(n\right)\right]\\ &\quad -\frac{\left(1-\eta\right)b}{1-\beta\left(1-\lambda\right)-\beta\lambda p^{0}}-\beta\frac{c}{q}\frac{\eta f-\lambda p^{+}}{1-\beta\left(1-\lambda\right)-\beta\lambda p^{0}}, \end{aligned} \tag{80}$$

and the marginal effects of n, p and θ on J are given by

$$\begin{aligned}J_{n} &= -\frac{1-\alpha}{n}\frac{\psi p\alpha n^{\alpha-1}}{1-\beta\left(1-\lambda\right)}\left[x+\frac{\beta\lambda p^{0}}{1-\beta\left(1-\lambda\right)-\beta\lambda p^{0}}\mathcal{E}\left(n\right)\right]\\ J_{p} &= \frac{1}{p}\frac{\psi p\alpha n^{\alpha-1}}{1-\beta\left(1-\lambda\right)}\left[x+\frac{\beta\lambda p^{0}}{1-\beta\left(1-\lambda\right)-\beta\lambda p^{0}}\mathcal{E}\left(n\right)\right]\\ J_{\theta} &= -\beta\frac{c}{q}\frac{1}{\theta}\frac{\eta f-\phi\lambda p^{+}}{1-\beta\left(1-\lambda\right)-\beta\lambda p^{0}}, \end{aligned} \tag{81}$$

where $\psi \equiv \frac{1-\eta}{1-\eta(1-\alpha)}$, $\mathcal{E}(n) \equiv \mathbb{E}(x'|x' \in [R(n), R_v(n)])$, *and* p^0, p^+ *are as defined in the Proof to Proposition 4.*

Proof. Since firms only receive an idiosyncratic shock with probability λ each period, we can use the recursion for $J(n,x)$, (44), to write:

$$\begin{aligned} J(n,x) &= \frac{1}{1-\beta(1-\lambda)}\left[\psi p x \alpha n^{\alpha-1} - (1-\eta) b - \eta\beta c\theta\right] \\ &\quad + \frac{\beta\lambda}{1-\beta(1-\lambda)}\frac{c}{q}\int_{R_v(n)} d\tilde{G} + \frac{\beta\lambda}{1-\beta(1-\lambda)}\int_{R(n)}^{R_v(n)} J(n,x')\, d\tilde{G}. \end{aligned} \tag{82}$$

We then conjecture that $J(n,x)$ is of the form $j_0 + j_1 x$. Substituting this assumption into the latter, and equating coefficients yields:

$$\begin{aligned} j_0 &= -\frac{(1-\eta) b}{1-\beta(1-\lambda)} - \beta\frac{c}{q}\frac{\eta f - \lambda p^+}{1-\beta(1-\lambda)} + \frac{\beta\lambda p^0}{1-\beta(1-\lambda)}\left[j_0 + j_1\mathcal{E}(n)\right], \\ j_1 &= \frac{\psi p \alpha n^{\alpha-1}}{1-\beta(1-\lambda)}. \end{aligned} \tag{83}$$

Solving for j_0 we obtain the required solution for $J(n,x)$. Likewise, we can obtain recursions for the marginal effects of n and θ:

$$\begin{aligned} J_n(n,x) &= -\frac{1}{1-\beta(1-\lambda)}\frac{1-\alpha}{n}\psi p x \alpha n^{\alpha-1} + \frac{\beta\lambda}{1-\beta(1-\lambda)}\int_{R(n)}^{R_v(n)} J_n(n,x')\, dG, \\ J_p(n,x) &= \frac{1}{1-\beta(1-\lambda)}\psi x \alpha n^{\alpha-1} + \frac{\beta\lambda}{1-\beta(1-\lambda)}\int_{R(n)}^{R_v(n)} J_p(n,x')\, d\tilde{G}, \\ J_\theta(n,x) &= -\frac{\eta\beta c + \beta\lambda\frac{c}{q^2}q'(\theta)\int_{R_v(n)} dG}{1-\beta(1-\lambda)} + \frac{\beta\lambda}{1-\beta(1-\lambda)}\int_{R(n)}^{R_v(n)} J_\theta(n,x')\, dG. \end{aligned} \tag{84}$$

Again using the method of undetermined coefficients, and noting that the Cobb Douglas matching function implies $q = \mu\theta^{-\phi} \implies \frac{c}{q^2}q'(\theta) = -\frac{c}{q}\frac{\phi}{\theta}$, yields the required solutions for J_n, J_p and J_θ. ■

Proof of Proposition 8. Total differentiation of the Job Creation condition, $U(\theta) = L - N(\theta)$, yields $d\theta/dp = -(\partial N/\partial p)/(\partial N/\partial\theta)$. Indexing firms by i, we can write aggregate employment as $N \equiv \mathbb{E}(n) = \int n(n_{-1}(i), x(i); \xi)\, di$, where $n(n_{-1}, x; \xi)$ is the employment policy function that is common to all firms, which in turn depends on some parameters ξ (which includes p and θ). Differentiating yields:

$$\frac{\partial N}{\partial \xi} = \int \left[\frac{\partial n}{\partial \xi} + \frac{\partial n}{\partial n_{-1}}\frac{\partial n_{-1}}{\partial \xi}\right] di. \tag{85}$$

Note from the form of the employment policy function in (12) that $\partial n / \partial \xi = 0$ if $\Delta n\,(i) = 0$, and $\partial n / \partial n_{-1} = 1$ iff $\Delta n\,(i) = 0$. Substitution and separation of integrals yields

$$\begin{aligned}
\frac{\partial N}{\partial \xi} &= \int_{i:\Delta n_i > 0} \left.\frac{\partial n}{\partial \xi}\right|_{\Delta n_i > 0} di + \int_{i:\Delta n_i = 0} \frac{\partial n_{-1}}{\partial \xi} di + \int_{i:\Delta n_i < 0} \left.\frac{\partial n}{\partial \xi}\right|_{\Delta n_i < 0} di \\
&= \mathbf{p}^{+}\mathbb{E}\left(\left.\frac{\partial n}{\partial \xi}\right| \Delta n > 0\right) + \mathbf{p}^{0}\mathbb{E}\left(\frac{\partial n_{-1}}{\partial \xi}\right) + \mathbf{p}^{-}\mathbb{E}\left(\left.\frac{\partial n}{\partial \xi}\right| \Delta n < 0\right), \qquad (86)
\end{aligned}$$

where $\mathbf{p}^{+}, \mathbf{p}^{0}$, and $\mathbf{p}^{-}$ respectively denote the steady-state probabilities of raising, freezing, and cutting employment. Note further that in steady state $\mathbb{E}\left(\partial n_{-1}/\partial \xi\right) = \mathbb{E}\left(\partial n/\partial \xi\right) = \partial N/\partial \xi$, so that we obtain the result that:

$$\frac{\partial N}{\partial \xi} = \pi\mathbb{E}\left(\left.\frac{\partial n}{\partial \xi}\right| \Delta n > 0\right) + (1-\pi)\,\mathbb{E}\left(\left.\frac{\partial n}{\partial \xi}\right| \Delta n < 0\right), \qquad (87)$$

where $\pi \equiv \mathbf{p}^{+}/\left(1 - \mathbf{p}^{0}\right)$. Thus, we can rewrite the marginal effect of a change in p on θ as:

$$\frac{d\theta}{dp} = -\frac{\pi\mathbb{E}\left(\left.\frac{\partial n}{\partial p}\right| \Delta n > 0\right) + (1-\pi)\,\mathbb{E}\left(\left.\frac{\partial n}{\partial p}\right| \Delta n < 0\right)}{\pi\mathbb{E}\left(\left.\frac{\partial n}{\partial \theta}\right| \Delta n > 0\right) + (1-\pi)\,\mathbb{E}\left(\left.\frac{\partial n}{\partial \theta}\right| \Delta n < 0\right)}. \qquad (88)$$

Then note that the first-order conditions for optimal labor demand set the marginal firm surplus, $J\,(n, x)$ as follows:

$$J\,(n,x) = \begin{cases} c/q\,(\theta) & \text{if} \quad \Delta n > 0, \\ 0 & \text{if} \quad \Delta n < 0. \end{cases} \qquad (89)$$

It is immediate from Lemma 2 that $\partial n/\partial p = -J_p/J_n = \frac{1}{1-\alpha}\,(n/p)$ regardless of whether $\Delta n > 0$ or $\Delta n < 0$. It remains to derive $\partial n/\partial \theta$ in each case. Log-linearizing the function J around n, p, x, and θ, we obtain:

$$\log J \approx \varepsilon_{Jn} \log n + \varepsilon_{Jp}\,(\log p + \log x) + \varepsilon_{J\theta} \log \theta + const. \qquad (90)$$

Using this and totally differentiating the first-order conditions for optimal labor demand with respect to n and θ, we obtain:

$$\varepsilon_{Jn} d\log n + \varepsilon_{J\theta} d\log \theta \approx \begin{cases} -d\log q\,(\theta) & \text{if} \quad \Delta n > 0, \\ 0 & \text{if} \quad \Delta n < 0. \end{cases} \qquad (91)$$

Given the Cobb Douglas matching function assumption, $q\,(\theta) = \mu\theta^{-\phi}$, and it follows that $d\log q\,(\theta) = -\phi d\log \theta$. Thus:

$$\frac{\partial n}{\partial \theta} = \frac{\partial \log n}{\partial \log \theta}\frac{n}{\theta} \approx \begin{cases} \frac{\phi - \varepsilon_{J\theta}}{\varepsilon_{Jn}}\frac{n}{\theta} & \text{if} \quad \Delta n > 0, \\ -\frac{\varepsilon_{J\theta}}{\varepsilon_{Jn}}\frac{n}{\theta} & \text{if} \quad \Delta n < 0. \end{cases} \qquad (92)$$

Substituting this into (88), we obtain:

$$\left.\frac{d\log\theta}{d\log p}\right|_{JC} \approx -\frac{1}{1-\alpha}\frac{\varepsilon_{Jn}}{\omega\phi-\varepsilon_{J\theta}}, \tag{93}$$

where $\omega \equiv \pi\mathbb{E}\left(n|\Delta n>0\right)/\mathbb{E}\left(n\right)$ is the steady state share of employment in hiring firms. In what follows, we evaluate the approximation (90) to the marginal surplus around mean employment, $N\equiv\mathbb{E}\left(n\right)$, and mean productivity conditional on mean employment, $x=\mathcal{E}\left(N\right)\equiv\mathbb{E}\left(x'|x'\in\left[R\left(N\right),R_v\left(N\right)\right]\right)$. Thus, using the results of Lemma 2 it follows that we can write:

$$J_n = -\frac{1}{N}\frac{\left(1-\alpha\right)\psi p\alpha N^{\alpha-1}}{1-\beta\left(1-\lambda\right)-\beta\lambda p^0}\mathcal{E}\left(N\right),$$

and:

$$J = \frac{\psi p\mathcal{E}\left(N\right)\alpha N^{\alpha-1}-\left(1-\eta\right)b-\beta\frac{c}{q}\left[\eta f-\lambda p^+\right]}{1-\beta\left(1-\lambda\right)-\beta\lambda p^0}, \tag{94}$$

where $\psi\equiv\left(1-\eta\right)/\left[1-\eta\left(1-\alpha\right)\right]$. Substituting back into the aggregate elasticity of θ with respect to p, we obtain:

$$\left.\frac{d\log\theta}{d\log p}\right|_{JC} \approx \frac{\psi p\mathcal{E}\left(N\right)\alpha N^{\alpha-1}}{\omega\phi\left[\psi p\mathcal{E}\left(N\right)\alpha N^{\alpha-1}-\left(1-\eta\right)b-\eta\beta c\theta\right]+\eta\beta c\theta-\left(1-\omega\right)\phi\beta\frac{c}{q}\lambda p^+}. \tag{95}$$

Noting that the marginal product of labor in the average-sized firm is equal to $p\mathcal{E}\left(N\right)\alpha N^{\alpha-1}$, and assuming λ is sufficiently small, we obtain:

$$\left.\frac{d\log\theta}{d\log p}\right|_{JC} \approx \frac{\left(1-\eta\right)\tilde{p}}{\omega\phi\left[\left(1-\eta\right)\left(\tilde{p}-b\right)-\eta\beta c\theta\right]+\eta\beta c\theta}, \tag{96}$$

where $\tilde{p}\equiv\rho p\mathcal{E}\left(N\right)N^{\alpha-1}+\left(1-\rho\right)p\mathcal{E}\left(N\right)\alpha N^{\alpha-1}$ and $\rho\equiv\alpha\eta/\left[1-\eta\left(1-\alpha\right)\right]$, as required. ■

Figure 1. Optimal Employment Policy of a Firm

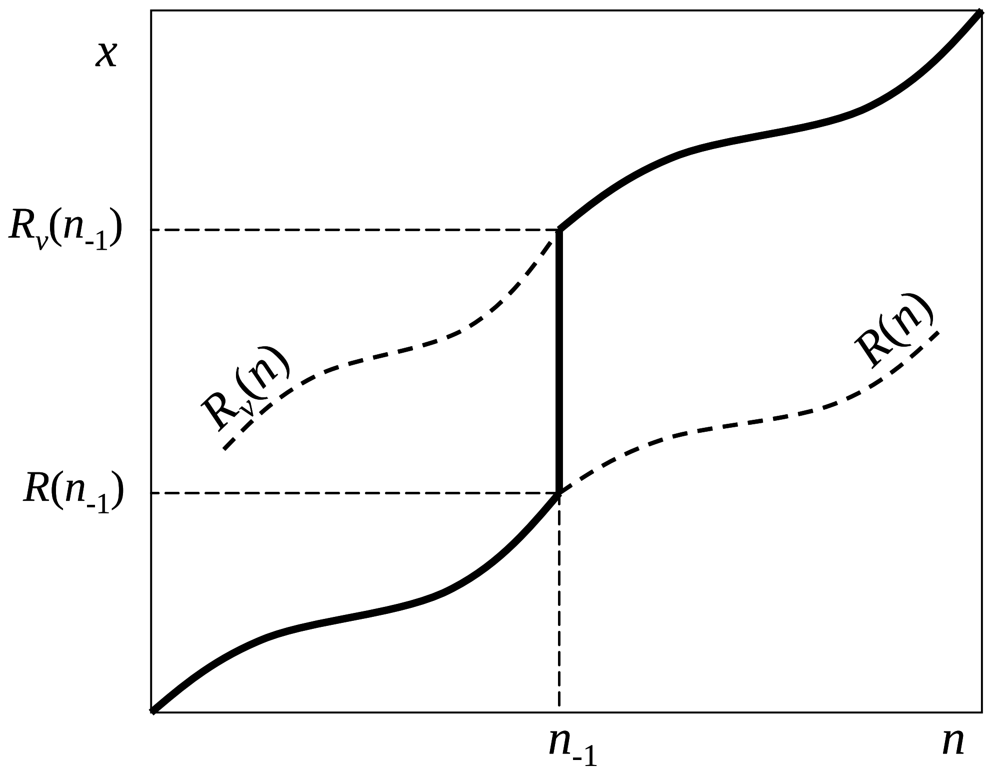
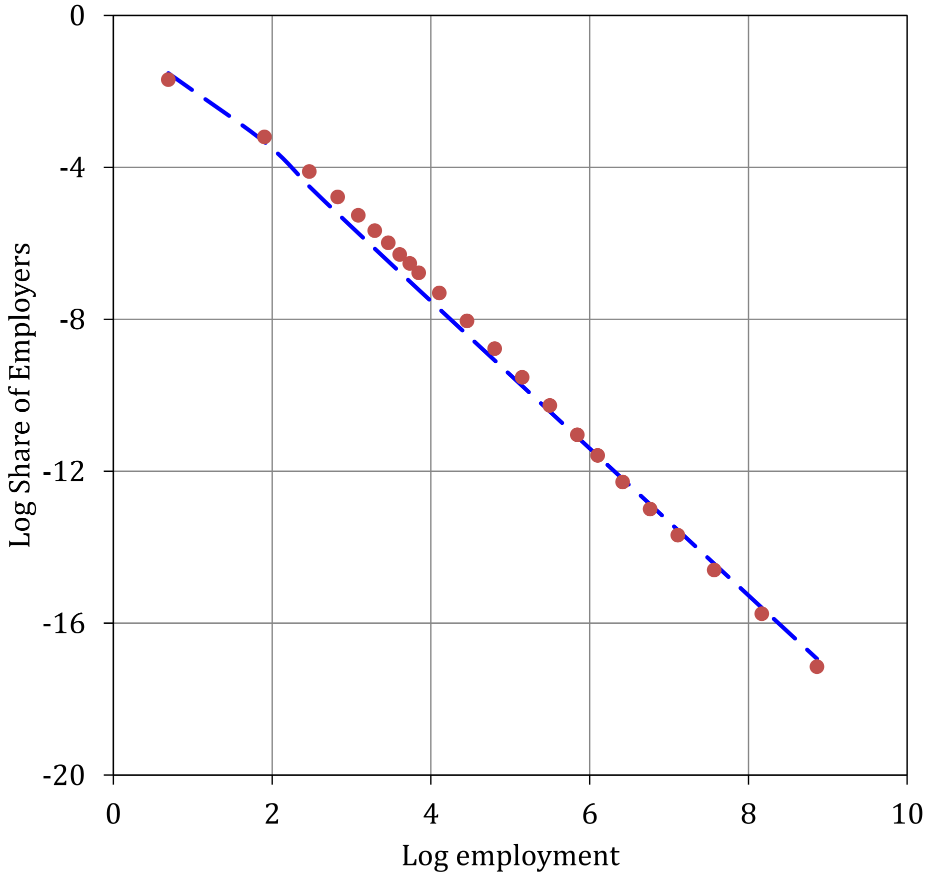


Table 1. Calibrated Model Parameters

Parameter	Meaning	Value	Reason
ϕ	Matching elasticity	0.600	Petrongolo and Pissarides (2001)
μ	Matching efficiency	0.129	Pissarides (2007)
α	$F(n) = n^{\alpha}$	0.601	Labor share = 0.72
β	Discount factor	0.999	Quarterly interest rate = 0.012
b	Value of leisure	0.387	Mean inflow rate = 0.0078
c	Flow vacancy cost	0.133	Hiring cost = 14% quarterly wage
η	Worker bargaining power	0.443	Cyclicality of wage in baseline MP
L	Labor force	18.65	Mean job-finding rate = 0.1125
λ	Arrival rate of x	0.043	LBD data: $\Pr(\Delta \ln n < \lvert 0.05 \rvert)$ = 0.415
$\bar{x}$	Mean of x	1.000	Normalization
σ_x	Std. dev. of x	0.250	LBD data: $\sigma(\Delta \ln n)$ = 0.416
$\bar{\varphi}$	Mean of φ	1.218	Mean employment = 17.38
σ_{φ}	Std. dev. of φ	1.009	Minimum employment = 1

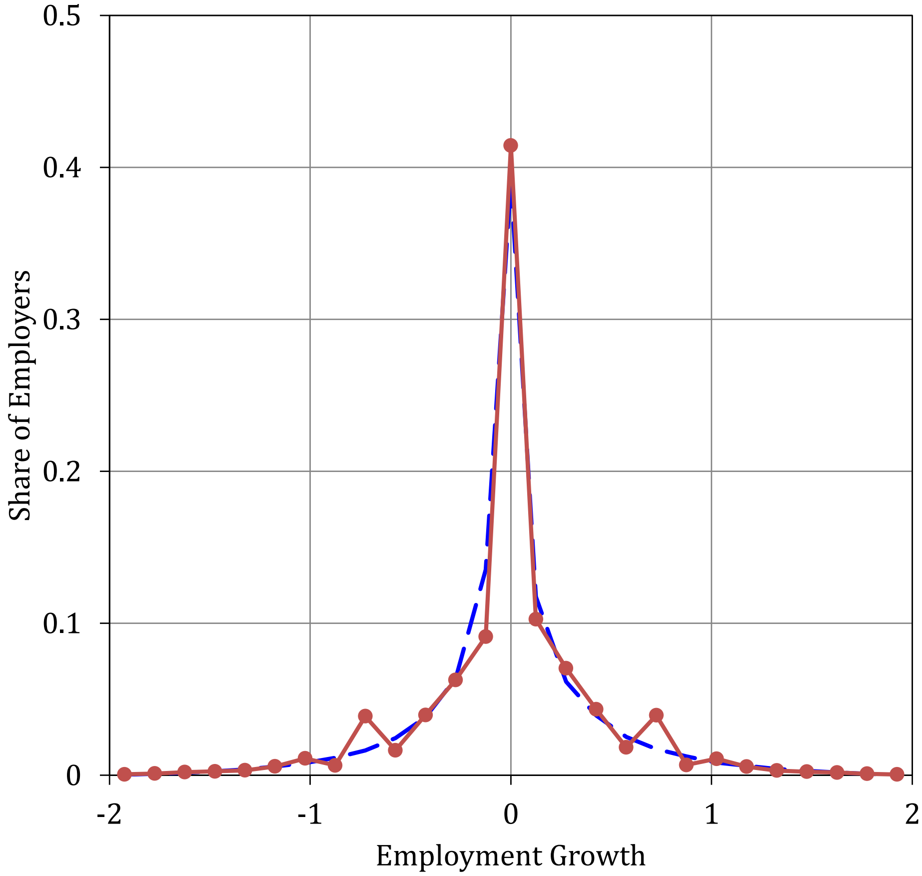
Note: Consistent with the timing of the model, flow parameters are reported at a weekly frequency. First and second moments of fixed firm productivity φ and the innovation to firm productivity x are reported (rather than the parameters of the respective Pareto distributions) for ease of interpretation.

Figure 2. Employer Size Distribution: Model vs. Data



Notes: The red dots plot data on the shares of firms in successive employment categories for the years 2002 to 2006 based on data on employment by firm size class from the Small Business Administration. The blue dashed line plots the steady state distribution of employment across firms implied by the generalized model using the parameters reported in Table 1.

Figure 3. Employment Growth Distribution: Model vs. Data



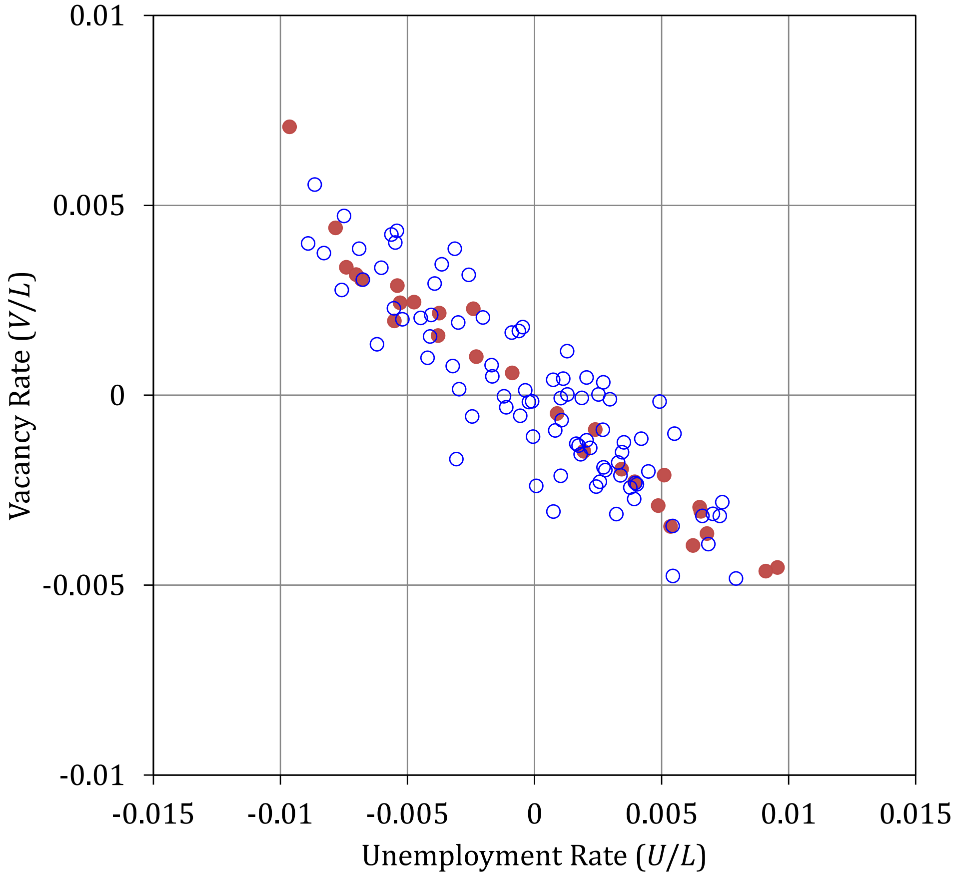
Notes: The red dotted line plots the cross sectional distribution of employment growth based on data for continuing establishments from the Longitudinal Business Database pooled over the years 1992 to 2005. The blue dashed line plots the steady state distribution of employment growth in the model using the parameters reported in Table 1.

Table 2. Cyclicality of Worker Flows: Model vs. Data

Model / Outcome	**Mean Level**		**Elasticity w.r.t. output per worker**	
	Data	*Model*	*Data*	*Model*
A. Generalized				
Job Finding Rate, f	[0.1125]	[0.1125]	2.65	2.55
Inflow Rate, s	[0.0078]	[0.0078]	-1.89	-1.64
Vacancies, V	--	--	2.91	2.47
Tightness, $\theta = V/U$	[0.72]	[0.72]	6.44	6.37
B. MP (*i*)				
Job Finding Rate, f	[0.1125]	[0.1125]	2.65	0.91
Inflow Rate, s	[0.0078]	[0.0078]	-1.89	[-1.64]
Vacancies, V	--	--	2.91	-0.32
Tightness, $\theta = V/U$	[0.72]	[0.72]	6.44	2.28
C. MP (*ii*)				
Job Finding Rate, f	[0.1125]	[0.1125]	2.65	[2.55]
Inflow Rate, s	[0.0078]	0.027	-1.89	[-1.64]
Vacancies, V	--	--	2.91	3
Tightness, $\theta = V/U$	[0.72]	[0.72]	6.44	6.37

Notes: Outcomes reported in brackets are calibrated. Non-bracketed outcomes are implied by the respective model. Flow outcomes are reported on a weekly basis. Empirical elasticities for f and s are computed using quarterly averages of the job-finding rate and the unemployment inflow rate from 1948Q1 to 2007Q1 derived in Shimer (2007). Following Shimer (2005), series are detrended using a Hodrick-Prescott filter with smoothing parameter 10^5. Following Mortensen and Nagypal (2007a), elasticities with respect to output per worker are obtained by regressing the log deviation from trend of f and s on the log deviation from trend of non-farm business output per worker obtained from the Bureau of Labor Statistics.

Figure 4. Beveridge Curve: Model vs. Data



Notes: The red dots plot job openings as a fraction of the labor force against the unemployment rate using quarterly averaged data from the Job Openings and Labor Turnover Survey and the Bureau of Labor Statistics from 2001Q1 to 2007Q4. The blue hollow circles plot the analogous series using simulated data from the model. Series are plotted as deviations from their temporal means.

Figure 5. Model Impulse Responses to a Permanent 1% Decline in Aggregate Labor Productivity, p

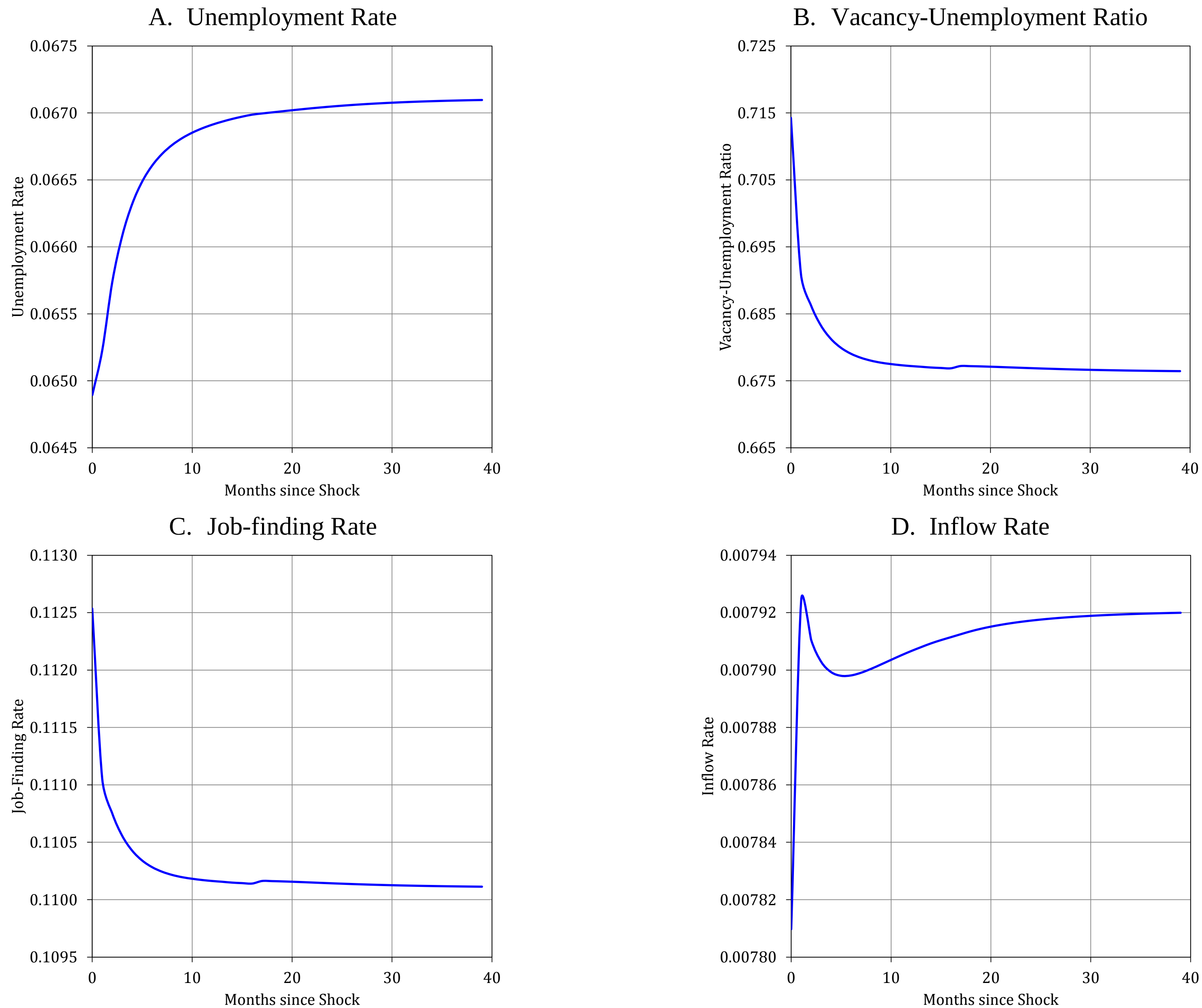
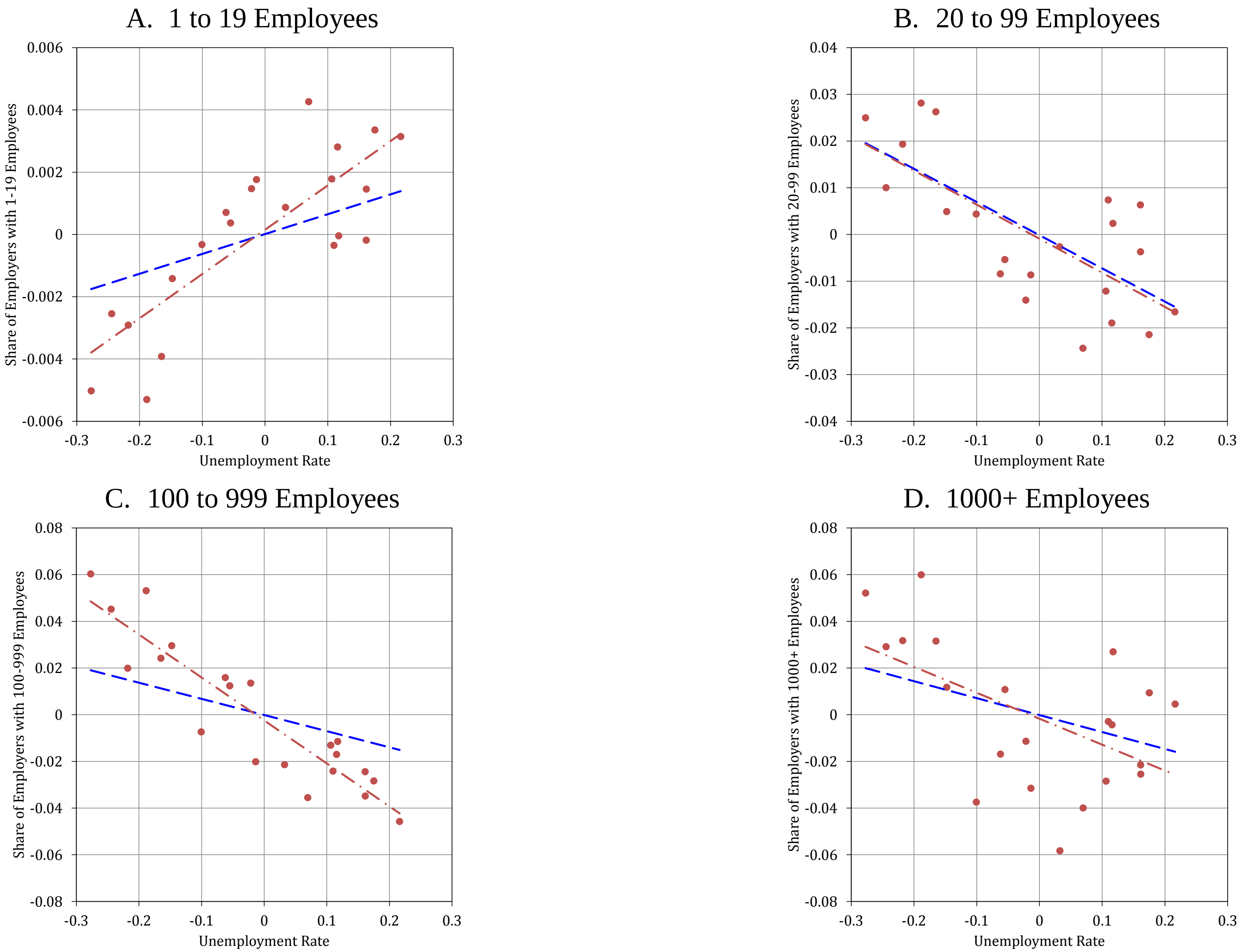
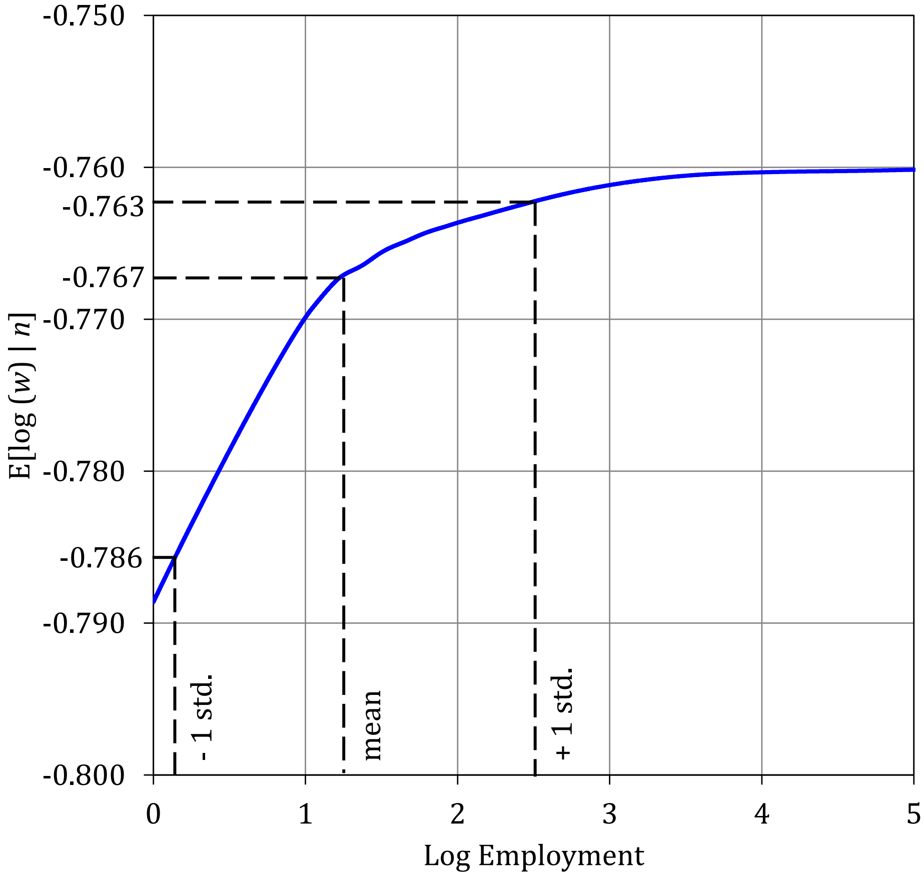


Figure 6. Cyclical Dynamics of the Employer Size Distribution: Model vs. Data



Notes: Red dots plot the log deviation from trend of each employer size class share against the log deviation from trend of the unemployment rate. The red dot-dash lines are fitted least squares regression lines based on the data. The blue dashed line is the analogous relationship implied by the calibrated model. Employer size data are taken from County Business Patterns for the years 1986 to 2007. Annual unemployment rate data are taken from the Bureau of Labor Statistics. Given the short time series, simple linear time trends are used.

Figure 7. Employer Size-Wage Effect implied by the Model



Notes: The blue line plots the mean log wage conditional on employer size as a function of log employer size derived from simulations of the steady state of the model using the parameters reported in Table 1. Simulated data were generated for 250,000 plants over 572 weeks (11 years); the plotted series is based on the cross section of log wages in the final year. The mean log wage conditional on employer size is computed nonparametrically from these simulated data using local weighted (LOWESS) regressions of log wages on log employment.