Privately Optimal Contracts and Sup-Optimal Outcomes in a Model of Agency Costs

Preliminary…comments welcomed.

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Abstract: This paper derives the privately optimal lending contract in the celebrated financial accelerator model of Bernanke, Gertler and Gilchrist (1999). The privately optimal contract includes indexation to the aggregate return on capital and household consumption. Although privately optimal, this contract is not welfare maximizing as it exacerbates fluctuations in real activity. The household's desire to hedge business cycle risk, leads, via the financial contract, to greater business cycle risk. The welfare cost of the privately optimal contract (when compared to the planner outcome) is quite large.

1. Introduction.

The financial accelerator model of Bernanke, Gertler, and Gilchrist (1999), hereafter BGG, is widely used as a convenient mechanism for integrating financial factors into an otherwise standard DSGE model. The BGG model embeds the costly state verification (CSV) model of Townsend (1979) into an environment with risk neutral entrepreneurs, risk averse households, and aggregate risk. Appealing to insurance concerns, BGG assume that the lending contract between the entrepreneur and lender is characterized by a lender return that is invariant to innovations in aggregate variables. Instead, these aggregate innovations feed directly into entrepreneurial net worth. The behavior of net worth is crucial in the BGG model because the agency costs are diminished by increases in net worth. For example, a positive productivity shock shifts wealth to entrepreneurs, lowers agency costs, and thus amplifies the effect of the shock. Hence, BGG's insurance assumption is key to the financial accelerator in their model. This paper revisits this key assumption.

Our principle results include the following. First, the financial contract imposed in the BGG model is not privately optimal. That is, lenders would increase their equity value by offering a loan contract different than the one imposed by BGG. Second, the privately optimal loan contract has the loan repayment varying in response to innovations in the return on capital and innovations in consumption growth. That is, the privately optimal loan contract is indexed to the realization of aggregate shocks. Third, the privately optimal contract is not socially optimal because it leads to large fluctuations in leverage and the risk premium. In fact, the social welfare costs of the privately optimal contract are quite large. In our benchmark calibration the unconditional welfare cost of the privately optimal contract is a one-time payment equal to 47% of steady state household consumption. Further, we demonstrate that the planner outcome represents a Pareto improvement over the competitive equilibrium. In essence, the household's desire to hedge business cycle risk, leads, via the financial contract, to greater business cycle risk. These results suggest the role for a regulatory response, although we do not pursue these issues here.

The paper most closely related to ours is Krishnamurty (2003). Krishnamurty introduces insurance markets into a three period model where borrowing is secured by collateral as in Kiyotaki and Moore (1997). These insurance markets allow for state contingent debt that is indexed to aggregate shocks as in our framework. Krishnamurty shows that such insurance eliminates any feedback from collateral values onto investment and thus reduces the collateral amplification to zero. Although our work is related to Krishnamurty, there are important differences in the analysis. First, we study state contingent debt in a fully calibrated DSGE model. This allows us to examine how debt indexation schemes interact with the endogenous net worth accumulation of borrowers, an effect which is not present in the three-period setup of Krishnamurty. Second, we choose the CSV framework rather than collateral constraints for generating financial frictions. The BGG model is often the preferred model of financial frictions because default occurs in equilibrium and credit spreads arise endogenously. Thus, our contribution is to show how to introduce indexation into this widely used model of credit spreads.

Two other notable precedents for the current paper are Lorenzoni (2008) and Jeanne and Korinek (2010). Although the modeling details differ across the papers, both examine situations in which borrowing is constrained either by limited commitment (Lorenzoni (2008)) or asset value (Jeanne and Korinek(2010)). The common conclusion of the two papers is that the competitive equilibrium is inefficient because of a pecuniary externality. Similar externalities are present in this paper.

The paper proceeds as follows. The next section outlines the competitive equilibrium of the model. Section 3 contrasts this outcome with a constrained social planner's allocation. Section 4 links the contract indexation to BGG. The quantitative analysis is carried out in Section 5. Concluding comments are provided in Section 6.

2. The Model.

<u>Households.</u>

The typical household consumes the final good (C_t) and sells labor input (L_t) to the firm at real wage w_t. Preferences are given by

$$U(C_t, L_t) \equiv \frac{C_t^{1-\sigma}}{1-\sigma} - B\frac{L_t^{1+\eta}}{1+\eta}.$$

The household budget constraint is given by

$$C_t + D_t + Q_t^L S_t \leq w_t L_t + R_{t-1}^D D_{t-1} + (Q_t^L + Div_t) S_{t-1}$$

The household chooses the level of deposits (D_t) which are then used by the lender to fund the entrepreneurs (more details below). The (gross) real rate R_t^d on these deposits is known at time-t. The household owns shares in the final goods firms, capital-producing firms, and in the lender. The former two are standard, so we simply focus on the shares of the lender. This share price is denoted by Q_t^L with Div_t denoting lender dividends, and S_t the number of shares held by the representative household (in equilibrium S_t = 1). The optimization conditions include:

$$-U_L(t)/U_c(t) = w_t \tag{1}$$

$$U_c(t) = E_t \beta U_c(t+1) R_t^d \tag{2}$$

Final goods firms.

Final goods are produced by competitive firms who hire labor and rent capital in competitive factor markets at real wage w_t and rental rate r_t. The production function is Cobb-Douglass where A_t is the random level of total factor productivity:

$$Y_t = \left(K_t^f\right)^\alpha (A_t L_t)^{1-\alpha} \tag{3}$$

The variable K_t^f denotes the amount of capital available for time-t production. This is different than the amount of capital at the end of the previous period as some is lost because of monitoring costs. The optimization conditions include:

$$mpl_t = w_t \tag{4}$$

$$mpk_t = r_t \tag{5}$$

New Capital Producers.

The production of new capital is subject to adjustment costs. In particular, investment firms take $I_t\phi(\frac{I_t}{I_{ss}})$ consumption goods and transform them into I_t investment goods that are sold at price Q_t. Their profits are thus given by $Q_tI_t - I_t\phi(\frac{I_t}{I_{ss}})$, where the function ϕ is convex with $\phi(1) = 1,\ \phi'(1) = 0$ and $\phi"(1) = \psi$. Variations in investment lead to variations in the price of capital.

Lenders.

The representative lender accepts deposits from households (promising sure return R_t^d) and provides loans to the continuum of entrepreneurs. These loans are intertemporal, with the loans made at the end of time t being paid back in time t+1. The gross real return on these loans is denoted by R_{t+1}^L. Each individual loan is subject to idiosyncratic and aggregate risk, but since the lender holds an entire portfolio of loans, only the aggregate risk remains. The lender has no other source of funds, so the level of loans will equal the level of deposits. Hence, dividends are given by $Div_{t+1} = R_{t+1}^L D_t - R_t^d D_t$. The intermediary seeks to maximize its equity value which is given by:

$$Q_t^L = E_t \sum_{j=1}^{\infty} \beta^j \frac{U_c(t+j)}{U_c(t)} Div_{t+j} \tag{6}$$

The FOC of the lender's problem is:

$$E_t \frac{\beta U_c(t+1)}{U_c(t)} \left[R_{t+1}^L - R_t^d\right] = 0 \tag{7}$$

The first-order condition shows that in expectation, the lender makes zero profits, but ex-post profits and losses can occur. We assume that losses are covered by households as negative dividends. This is similar to the standard assumption in the Dynamic New Keynesian (DNK) model, eg., Woodford (2003). That is, the sticky price firms are owned by the household and pay out profits to the household.

These profits are typically always positive (for small shocks) because of the steady state mark-up over marginal cost. Similarly, one could introduce a steady-state wedge (eg., monopolistic competition among lenders) in the lender's problem so that dividends are always positive. But this assumption would have no effect on the model's dynamics so we dispense from it for simplicity.

The expression for the equity value of the bank (6) implies that the household prefers a lender that delivers a dividend stream that covaries negatively with household consumption. The lender is providing loans to the entrepreneurs. Hence, the household prefers a loan contract that requires the entrepreneur to pay back more in periods of low consumption, and vice versa. As we will see below, such a lending contract is privately optimal but socially costly as it exacerbates fluctuations in aggregate activity by making leverage ratios and risk premia countercyclical.

Entrepreneurs and the Loan Contract.

Entrepreneurs are the sole accumulators of physical capital. The time t+1 rental rate and capital price are given by r_{t+1} and Q_{t+1}, respectively, implying that the gross return to holding capital from time-t to time t+1 is given by:

$$R_{t+1}^{k} \equiv \frac{r_{t+1}+(1-\delta)Q_{t+1}}{Q_t}. \tag{8}$$

At the end of period t, the entrepreneurs sell all of their accumulated capital, and then re-purchase it along with any net additions to the capital stock. This purchase is financed with entrepreneurial net worth (NW_t) and external financing from a lender. The external financing is subject to a CSV problem. In particular, one unit of capital purchased at time-t is transformed into ω_{t+1} units of capital in time t+1, where ω_{t+1} is a idiosyncratic random variable with density $\phi(\omega)$ and cumulative distribution $\Phi(\omega)$. The realization of ω_{t+1} is directly observed by the entrepreneur, but the lender can observe the realization only if a monitoring cost is paid. Assuming that the entrepreneur and lender are risk-neutral, Townsend (1979) demonstrates that the optimal contract between entrepreneur and intermediary is risky debt in which monitoring only occurs if the promised payoff is not forthcoming. Payoff does not occur for sufficiently low values of the idiosyncratic shock, $\omega_{t+1} < \varpi_{t+1}$. Let Z_{t+1} denote the promised gross rate-of-return so that Z_{t+1} is defined by

$$Z_{t+1}(Q_t K_{t+1} - NW_t) \equiv \varpi_{t+1} R_{t+1}^k Q_t K_{t+1} \; . \tag{9}$$

We find it convenient to express this in terms of the leverage ratio $\bar{\kappa}_t \equiv \left(\frac{Q_t K_{t+1}}{NW_t}\right)$ so that (9) becomes

$$Z_{t+1} \equiv \varpi_{t+1} R_{t+1}^k \frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1} \tag{10}$$

The CSV problem takes as exogenous the return on capital (R_{t+1}^k) and the opportunity cost of the lender. With $f(\varpi_{t+1})$ and $g(\varpi_{t+1})$ denoting the entrepreneur's share and lender's share of the project outcome, respectively, the lender's ex post realized t+1 return on the loan contract is defined as:

$$R_{t+1}^L \equiv \frac{R_{t+1}^k g(\varpi_{t+1}) Q_t K_{t+1}}{(Q_t K_{t+1} - NW_t)} \equiv R_{t+1}^k g(\varpi_{t+1}) \frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1} \tag{11}$$

where

$$f(\varpi) \equiv \int_{\varpi}^{\infty} \omega \phi(\omega) d\omega - [1 - \Phi(\varpi)]\varpi \tag{12}$$

$$g(\varpi) \equiv [1 - \Phi(\varpi)]\varpi + (1 - \mu) \int_0^{\varpi} \omega \phi(\omega) d\omega \tag{13}$$

Recall that the lender's return is linked to the return on deposits via (7):

$$E_t R_{t+1}^L U_c(t+1) = R_t^d E_t U_c(t+1) \tag{14}$$

The contracting problem takes as given the deposit rate R_t^d and the random variables $U_c(t+1)$ and R_{t+1}^k. The end-of-time-t contracting problem is thus given by:

$$max_{K_{t+1}, \varpi_{t+1}} E_t R_{t+1}^k Q_t K_{t+1} f(\varpi_{t+1}) \tag{15}$$

subject to

$$E_t R_{t+1}^k Q_t K_{t+1} U_c(t+1) g(\varpi_{t+1}) \geq R_t^d E_t U_c(t+1)[Q_t K_{t+1} - NW_t] \tag{16}$$

For a given level of net worth, the choice of K_{t+1} determines the size of the loan, and ϖ_{t+1} determines the state-contingent interest rate on the loan. After some re-arrangement, the optimization conditions include:

$$f'(\varpi_{t+1}) + \Lambda_t U_c(t+1) g'(\varpi_{t+1}) = 0 \qquad (17)$$

$$(\bar{\kappa}_t - 1) E_t R^k_{t+1} f(\varpi_{t+1}) = \Lambda_t E_t U_c(t+1) R^k_{t+1} g(\varpi_{t+1}) \qquad (18)$$

$$E_t U_c(t+1) R^k_{t+1} \frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1} g(\varpi_{t+1}) = R^d_t E_t U_c(t+1) \qquad (19)$$

where Λ_t denotes the multiplier on the constraint (16). Note that ϖ_{t+1} is state-contingent so that (17) holds state-by-state. Expression (19) states that the return to the lender is equal to the certain return R^d_t in expected value. This follows directly from the assumption that the lender maximizes its equity value. In contrast, BGG impose that (19) must hold state-by-state, ie., the lender's return is pre-determined and exactly equal to the deposit rate. Under the POC, the cut-off value for ϖ_{t+1} varies state-by-state and is given implicitly by:

$$\Lambda_t U_c(t+1) = \frac{-f'(\varpi_{t+1})}{g'(\varpi_{t+1})} \equiv F(\varpi_{t+1}) \qquad (20)$$

The **<u>privately optimal contract (POC)</u>** is thus defined by the default cut-off ϖ_{t+1} and leverage ratio $\frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1}$ that satisfy (19)-(20), with Λ_t given by (18). Note that under the POC, ϖ_{t+1} is a function of innovations in household consumption, but does not respond to innovations in R^k_{t+1}. Below we will compare the POC to the contract imposed by BGG.

Entrepreneurs have linear preferences and discount the future at rate β. Given the high return to internal funds, they will postpone consumption indefinitely. To limit net worth accumulation and ensure that there is a need for external finance in the long run, we assume that fraction (1-γ) of the entrepreneurs die each period. These dying entrepreneurs consume their accumulated net worth and exit the economy. Given the exogenous death rate, aggregate net worth accumulation is described by

$$\mathrm{NW_t} = \gamma NW_{t-1}\bar{\kappa}_{t-1}R_t^k f(\varpi_t) \tag{21}$$

The behavior of net worth thus depends upon the response of ϖ_t to innovations in aggregate behavior.

<u>Market Clearing and Equilibrium.</u>

In equilibrium the household holds the shares of the lender, $S_t = 1$, and the lender funds the entrepreneurs' projects, $D_t = Q_t K_{t+1} - NW_t$. Net of monitoring costs, the amount of capital available for production is given by $K_t^f = m(\varpi_t)K_t$. The competitive equilibrium is defined by the variables $\{C_t, L_t, I_t, K_{t+1}, \varpi_t, \Lambda_t, NW_t, \mathrm{C_t^e}, Q_t, R_t^k\}$ that satisfy (8), (17)-(19), (21) and

$$-U_L(t)/U_c(t) = mpl_t \tag{22}$$

$$K_{t+1} \leq (1-\delta)m(\varpi_t)K_t + I_t \tag{23}$$

$$C_t + I_t\phi\left(\frac{I_t}{I_{ss}}\right) + \mathrm{C_t^e} \leq m(\varpi_t)^\alpha K_t^\alpha (A_t L_t)^{1-\alpha} \tag{24}$$

$$\mathrm{C_t^e} \leq (1-\gamma)[Q_t(1-\delta) + mpk_t]f(\varpi_t)K_t \tag{25}$$

$$Q_t = \phi\left(\frac{I_t}{I_{ss}}\right) + \left(\frac{I_t}{I_{ss}}\right)\phi'\left(\frac{I_t}{I_{ss}}\right) \tag{26}$$

where $m(\varpi_t) \equiv f(\varpi_t) + g(\varpi_t) = 1 - \mu\int_0^{\varpi_t} x\phi(x)dx$. Note that $m'(\varpi_t) = -\mu\varpi_t\phi(\varpi_t)$. The marginal products are defined as: $mpl_t \equiv (1-\alpha)Y_t/L_t$, and $mpk_t \equiv aY_t/(m(\varpi_t)K_t)$, where $Y_t \equiv m(\varpi_t)^\alpha K_t^\alpha (A_t L_t)^{1-\alpha}$. We will now contrast the POC competitive equilibrium with the BGG model and the solution to the constrained planner's problem.

3. Comparing the POC to BGG.

In contrast to the POC given by (20), BGG assume that the lender's return is equal to the deposit rate state-by-state, ie., lender profits are zero state-by-state. This is not an implication of the modeling framework, but is instead an assumption. As BGG write, "Since entrepreneurs are risk neutral, we *assume* that they bear all the aggregate risk associated with the contract" (BGG, page 1385, emphasis added). The problem with this assumption is that household risk is linked to consumption, not to the return on capital. The POC provides this consumption insurance; the contract assumed by BGG does not. The behavior of bankruptcy rates in BGG is given implicitly by

$$g(\varpi_{t+1}) = \frac{R_t^d(\bar{\kappa}_t - 1)}{R_{t+1}^k \bar{\kappa}_t} \tag{27}$$

It is useful to compare (20) and (27). In BGG bankruptcy rates depend negatively on the return to capital, but under POC bankruptcy rates do not respond to the return on capital. This necessarily implies that in the POC the promised repayment Z_{t+1} is indexed one-for-one to aggregate behavior. The POC has bankruptcy rates rise when consumption falls, while bankruptcy does not depend on consumption in the BGG model. This response to consumption comes about because under the POC the risk-neutral entrepreneur is willing to offer consumption insurance to the household.

We can log-linearize both models to gain further insight. In log-linear form (lower case), the equations (17)-(19) for the POC are given by:

$$\Psi\varpi_{t+1} = (\lambda_t - \sigma c_{t+1}) \tag{28}$$

$$(\lambda_t - \sigma E_t c_{t+1}) = \frac{\kappa}{\kappa - 1}\kappa_t + (\Theta_f - \Theta_g)E_t\varpi_{t+1} \tag{29}$$

$$E_t(r_{t+1}^k - r_{t+1}^L) = \left(\frac{1}{\kappa - 1}\right)\kappa_t - \Theta_g E_t\varpi_{t+1} \tag{30}$$

where $\Psi \equiv \frac{\varpi_{ss}F'(\varpi_{ss})}{F(\varpi_{ss})}$, with $\Psi > 0$ by the second order condition, $\Theta_g \equiv \frac{\varpi_{ss}g'(\varpi_{ss})}{g(\varpi_{ss})}$, $0 < \Theta_g < 1$, and $\Theta_f \equiv \frac{\varpi_{ss}f'(\varpi_{ss})}{f(\varpi_{ss})} < 0$. Taking expectations in (28) and combining with (29)-(30) we have a convenient expression for the risk spread in terms of leverage:

$$E_t(r_{t+1}^k - r_{t+1}^L) = \left[\frac{(\Psi - \theta_f + \theta_g) - \kappa\theta_g}{(\kappa - 1)(\Psi - \theta_f + \theta_g)}\right]\kappa_t \equiv \nu\kappa_t \tag{31}$$

Note that increases in leverage are associated with increases in the risk premium. Ceteris paribus, deterioration in borrower net worth increases this premium. Similarly, increases in borrowers' net worth decrease how much the economy responds to net worth. Using (30)-(31) to solve for ϖ_{t+1} in (28) we that the POC bankruptcy rate is given by:

$$\varpi_{t+1}^{POC} \equiv \frac{[1-\nu(\kappa-1)]}{\Theta_g(\kappa-1)}\kappa_t - \frac{\sigma}{\Psi}(c_{t+1} - \mathrm{E}_t c_{t+1}) \quad (32)$$

From (9) and (11), the promised payment and lender's return is given by:

$$z_{t+1} = \varpi_{t+1} + r_{t+1}^k - \frac{1}{\kappa-1}\kappa_t \quad (33)$$

$$r_{t+1}^l \equiv -\frac{1}{(\kappa-1)}\kappa_t + \Theta_g \varpi_{t+1} + r_{t+1}^k \quad (34)$$

Substituting (32) into these expression we have that under the POC these are given by

$$z_{t+1}^{POC} = r_t^d + \frac{(1-\Theta_g)[1-\nu(\kappa-1)]}{\Theta_g(\kappa-1)}\kappa_t + \left(r_{t+1}^k - E_t r_{t+1}^k\right) - \frac{\sigma}{\Psi}(c_{t+1} - \mathrm{E}_t c_{t+1}) \quad (35)$$

$$r_{t+1}^{l,POC} = r_t^d + \left(r_{t+1}^k - E_t r_{t+1}^k\right) - \frac{\sigma\Theta_g}{\Psi}(c_{t+1} - \mathrm{E}_t c_{t+1}) \quad (36)$$

The POC is thus defined by (32) and (35)-(36).

The POC contract is quite different than the one imposed by BGG. As mentioned above, BGG assume that (19) holds state-by-state (the lender return equals pre-determined deposit rate state-by-state). The BGG contract is thus given by (28)-(29) and

$$r_{t+1}^k - r_t^d = \left(\frac{1}{\kappa-1}\right)\kappa_t - \Theta_g \varpi_{t+1} \quad (37)$$

Using this expression we have that the BGG contract is given by

$$\varpi_{t+1}^{BGG} = \frac{[1-\nu(\kappa-1)]}{\Theta_g(\kappa-1)}\kappa_t - \frac{1}{\Theta_g}(r_{t+1}^k - E_t r_{t+1}^k) \quad (38)$$

$$z_{t+1}^{BGG} = r_t^d + \frac{(1-\Theta_g)[1-\nu(\kappa-1)]}{\Theta_g(\kappa-1)}\kappa_t + \left(\frac{\Theta_g - 1}{\Theta_g}\right)\left(r_{t+1}^k - E_t r_{t+1}^k\right) \quad (39)$$

$$r_{t+1}^{l,BGG} = r_t^d \quad (40)$$

The key difference between the POC and BGG is the response of bankruptcy rates ϖ_{t+1} and the promised repayment z_{t+1} to innovations in consumption and the return to capital. Under the POC, the household receives consumption insurance from the entrepreneur. For example, when aggregate consumption falls unexpectedly the POC has the entrepreneur increase the promised repayment to the lender. That is, when the marginal utility of consumption is high, the POC has the lender's dividend stream being high. This positive covariance is preferred by households and increases the equity value of the lender. In contrast, a lender that offered the BGG contract would have a lower equity value. This suggests that the BGG contract would not arise in a competitive financial market.

The second difference between the two contracts has nothing to do with risk aversion as it arises even with $\sigma = 0$. Under the POC the promised repayment moves one-for-one with innovations in the return on capital, thus implying that the bankruptcy rate is unaffected by these innovations. This is preferred as it minimizes fluctuations in bankruptcy costs, costs that are convex in ϖ_{t+1}. In contrast, under BGG, bankruptcy costs fluctuate with these observed aggregate shocks. This is suboptimal as it exacerbates bankruptcy costs.

Although the POC is privately optimal, we will see below that it does not maximize aggregate welfare because it exacerbates fluctuations in agency costs by exacerbating fluctuations in net worth and the risk premium (see (31)). In log deviations, the evolution of net worth (21) is given by:

$$nw_{t+1} = nw_t + \kappa_t + r_{t+1}^k + \Theta_f \varpi_{t+1} \tag{41}$$

Using the alternative expressions for the two contracts (POC and BGG) we have

$$nw_{t+1}^{POC} = nw_t^{POC} + r_t^d + \left\{\frac{\Theta_f[1-\nu(\kappa-1)]}{\Theta_g(\kappa-1)} + 1 + \nu\right\}\kappa_t + (r_{t+1}^k - E_t r_{t+1}^k) - \frac{\sigma\Theta_f}{\Psi}(c_{t+1} - \mathrm{E}_t c_{t+1}) \tag{42}$$

$$nw_{t+1}^{BGG} = nw_t^{BGG} + r_t^d + \left\{\frac{\Theta_f[1-\nu(\kappa-1)]}{\Theta_g(\kappa-1)} + 1 + \nu\right\}\kappa_t + \left(1 - \frac{\Theta_f}{\Theta_g}\right)(r_{t+1}^k - E_t r_{t+1}^k) \tag{43}$$

The two lending contracts differ by the response of net worth to innovations in aggregate variables. Under the POC, innovations in the return on capital are shared equally by the lender and the entrepreneur. But under BGG, net worth responds to productivity innovations by twice as much than under the POC (for the calibration used below, $\frac{\Theta_f}{\Theta_g} \approx -0.9$).

The more important difference in net worth behavior comes from consumption innovations. Under the BGG contract, net worth is entirely unresponsive to consumption shocks. But under the POC, net worth responds *sharply* to consumption innovations: for the calibration used below, $-\frac{\sigma\Theta_f}{\Psi} \approx 12.3$ (!). The household privately prefers owning shares in a bank that offers the POC lending contract because it provides a hedge against consumption risk (and the entrepreneur is indifferent as he is risk neutral). Consequently, the lender seeking to maximize its equity value will offer such a contract. However, this privately optimal behavior is socially costly as it results in sharp movements in net worth and the risk premium. For example, suppose that a negative TFP shock leads to a decline in consumption. Under the POC, net worth declines sharply as a result. This decline in net worth is highly persistent and implies a persistent increase in the risk premium.

4. The Planner's Problem.

The constrained planner maximizes the discounted value of utility with weight of ϵ on the entrepreneurs:

$$E_t \sum_{j=0}^{\infty} \beta^j \left[U\left(c_{t+j}, L_{t+j}\right) + \epsilon c_{t+j}^{e}\right] \tag{44}$$

subject to (23)-(26). The planner is constrained by the familiar resource constraints (23)-(24) but also by the consumption allocation implied by the financial contract (25). Physical capital is the only endogenous state variable in the planner's problem. Entrepreneurial net worth is a state variable in the competitive equilibrium because it affects leverage ratios and thus the return to the lender (see (11)). But net worth does not constrain the planner because the planner has access to a wide spectrum of distortionary subsidies and taxes that can be used to conform the household's behavior to the planner's choices. For example, if net worth is low, the competitive equilibrium implies a high risk premium and low level of capital accumulation. But the planner can alter this behavior by subsidizing capital accumulation appropriately. The existence of such subsidies and taxes makes the risk premium and the history of net worth irrelevant to the planner.

Instead, the planner's problem is ultimately a problem of risk sharing between a risk-averse household and a risk-neutral entrepreneur. But this problem is complicated by the fact that risk sharing can only be done via the financial contract. That is, the planner does not have access to lump sum taxes, but can reallocate consumption across agents only by altering entrepreneurial consumption via (25). For example, an increase in ϖ_t will lower entrepreneurial consumption and increase household consumption. But this redistribution is costly because of monitoring costs.

Let Λ_{1t}, Λ_{2t}, and Λ_{3t}, denote the multipliers on (23)-(25), respectively. We find it convenient to treat Q_t parametrically as defined by (26) so that $Q_I(t)$ denotes the derivative of (26) with respect to investment. The following are the FONC to the planner's problem:

$$\Lambda_{1t} + \Lambda_{3t}\tau_t Q_I(t)(1-\delta)m(\varpi_t)K_t = U_c(t)Q_t \tag{45}$$

$$\epsilon = U_c(t) + \Lambda_{3t} \tag{46}$$

$$-U_L(t) = U_c(t)mpl_t + \Lambda_{3t}\alpha mpl_t\tau_t \tag{47}$$

$$\Lambda_{1t} = \beta E_t m(\varpi_{t+1})\left\{\begin{matrix} \Lambda_{1t+1}(1-\delta) + U_c(t+1)mpk_{t+1} \\ +\Lambda_{3t+1}\tau_{t+1}[\alpha mpk_{t+1} + (1-\delta)Q_{t+1}] \end{matrix}\right\} \tag{48}$$

$$\frac{m'(\varpi_t)}{f'(\varpi_t)} = \frac{-\Lambda_{3t}(1-\gamma)[Q_t(1-\delta)+mpk_t]}{[\Lambda_{1t}(1-\delta)+U_c(t)mpk_t-\Lambda_{3t}\tau_t(1-\alpha)mpk_t]} \tag{49}$$

where we define

$$\tau_t \equiv (1-\gamma)\frac{f(\varpi_t)}{m(\varpi_t)}. \tag{50}$$

and we have used $U_c(t) = \Lambda_{2t}$.

It is instructive to compare the planner's behavior (45)-(49) to the competitive equilibrium. The competitive equilibrium includes the marginal conditions

$$-U_L(t) = U_c(t)mpl_t \tag{51}$$

$$E_t U_c(t+1)R^k_{t+1}\frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1}g(\varpi_{t+1}) = U_c(t) \tag{52}$$

The competitive equilibrium has employment (51) satisfying the traditional RBC margin, but the investment decisions (52) is distorted relative to familiar RBC behavior. Comparing (51)-(52) to the complementary (47)-(48) it is quite clear that the planner's allocations will differ sharply from the competitive equilibrium. There are two notable differences. First, leverage ratios and net worth do not constrain the planner for the reasons noted above. Second, the multiplier Λ_{3t} alters both of the planner's conditions (47)-(48) considerably from the competitive equilibrium. From (46), the multiplier Λ_{3t} denotes the difference in the marginal utilities between the entrepreneur and the household. The planner wants to equate these two by transferring consumption units. But (25) constrains the planner: entrepreneurial consumption can be altered only by altering variables in (25). It is this constraint that colors all the planner's choices. Consider first the planner's choice of ϖ_t. Since $f'(\varpi_t)$ and $m'(\varpi_t)$ are both negative, (32) implies that Λ_{3t} is negative. That is, the planner sets $\varpi_t > 0$ and tolerates the associated costs of positive bankruptcy rates only because on the margin he desires to transfer consumption units from the entrepreneur back to the household. This redistribution mechanism illuminates the remaining differences between the planner and the competitive equilibrium. To lower entrepreneurial consumption, the planner prefers a lower price of capital as implied by (45); a lower level of work effort as implied by (47); and a lower level of physical capital as implied by (48).

One can see this distribution motive clearly by considering a special case. Suppose the planner had access to a lump sum transfer that could be used to transfer consumption across agents. In this case (25) would no longer be a constraint and Λ_{3t} would be identically zero. The planner would set ϖ_t identically to zero, and choose labor and investment behavior identical to a two-agent RBC model in which one agent is risk-neutral agent and provides perfect consumption insurance to households.[1] That is, the employment and investment margins would be given by:

$$-U_L(t) = U_c(t)mpl_t \tag{53}$$

[1] Suppose entrepreneurs do not consume, eg., when they die their assets are handed over to households. In this case the planner sets ϖ_t identically to zero, and chooses behavior identical to the RBC model but in which there is not a risk-neutral agent that provides perfect consumption insurance to households.

$$U_c(t) = \beta E_t U_c(t+1) R_{t+1}^k \quad (54)$$

Hence, if the planner was not constrained in his ability to redistribute income, he would choose the traditional RBC behavior. (The consumption insurance provided by entrepreneurs implies that this two-agent RBC economy will respond sharply to TFP shocks compared to a one-agent RBC model.) This RBC behavior could be decentralized in the competitive equilibrium by a time-varying subsidy on capital accumulation so that (52) would coincide with (54). That is, under this special case, there is an obvious government policy that will achieve the planner's allocation in the competitive equilibrium.

5. Quantitative Analysis.

Calibration

Our benchmark calibration will largely follow BGG. The discount factor β is set 0.99. Utility is assumed to be logarithmic in consumption (σ=1), and the elasticity of labor is assumed to be 1/3 (η = 3). The production function parameters include α = 0.35, investment adjustment costs ψ = 0.25, and quarterly deprecation is δ = .025. As for the credit-related parameters, we calibrate the model to be consistent with: (i) a steady state spread between $R^k$and R^d of 200 bp (annualized), (ii) monitoring costs μ = 0.12, and (iii) a leverage ratio of κ = K/NW = 1.954. These values imply a death rate of γ = 0.98, a standard deviation of the idiosyncratic productivity shock of 0.28, and a quarterly bankruptcy rate of .75% (ϖ_{ss} = 0.486). This then implies ν = 0.041. We assume that total factor productivity follows an AR(1) process with ρ^A = 0.95. The financial accelerator is driven by fluctuations in the price of capital. The size of these movements is driven by the autocorrelation in the TFP shocks and the capital adjustment cost ψ. Hence, we perform sensitivity analysis over these two parameters.

We investigate three allocations: (i) the planner, (ii) competitive equilibrium under POC, and (iii) competitive equilibrium under BGG. To reiterate, under a laissez faire assumption only the POC is a competitive equilibrium as it maximizes equity value. The planner and BGG allocations would be supported under a competitive equilibrium only if there are time-varying governmental interventions. For the planner's behavior, we need to assume a welfare weight for entrepreneurial consumption (ϵ). We find it convenient to choose this weight so that the steady state level of capital is identical for the planner and the POC. This is helpful for welfare comparison as we need not adjust for alternative steady state capital stocks.

To develop intuition, Figures 1-2 present impulse response function for the case of iid shocks, ρ^A = 0. The increase in TFP leads to an increase in household consumption. The planner redistributes some of this back to the entrepreneur via a small decline in bankruptcy rate. Note that the planner responds to this iid shock in something of an iid fashion. That is, there is very little persistence in the planner's behavior because net worth is not a state variable, and physical capital has modest effects on persistence. Matters are much different with BGG and POC. Because of the financial accelerator, both BGG and POC over-respond to the TFP shock (in comparison to the planner). This amplification is particularly strong under POC. Under the POC, the increase in consumption leads to a sharp fall in repayment and thus a significant increase in entrepreneurial net worth. This surge in net worth leads to a sharp increase in investment. These effects diminish only slowly as entrepreneurial net worth returns to normal levels.

Figures 3-4 look at the case of an auto-correlated TFP shock. In comparison to the planner, both POC and BGG over-respond to the shock. Again, the planner's response to the shock is less persistent than BGG and POC. Note in particular that bankruptcy rates decline very modestly under the planner so that entrepreneurial consumption rises only modestly. But under both BGG and POC, the financial contract shifts net worth and thus consumption sharply towards the entrepreneur. As before, this persistent movement in net worth leads to a decline in the risk premium and hence a sub-optimal amplification of investment and output.

Table 1 reports the standard deviation of the key variables in the model. This statistic is calculated in a conditional sense by examining the impulse response function to a technology shock. Two key parameters are the degree of investment adjustment costs (ψ), and the autocorrelation in the productivity shock (ρ). The Table provides sensitivity analysis for both of these. For the benchmark calibration of $\psi = 0.25$ and $\rho = 0.95$, we see that the planner results in lower variability of output and consumption compared to BGG and POC. Recall that the private-agent motivation for the POC was to hedge the household's business cycle risk. But the POC contract (and to a lesser degree the BGG contract) actually amplifies consumption volatility. This effect is true throughout Table 1, with notable exceptions being the case with no adjustment costs $\psi = 0$ and high shock autocorrelation $\rho = 0.99$.

Table 2 provides a welfare analysis of the three models. As with Table 1, we provide sensitivity analysis for the degree of investment adjustment costs (ψ), and the autocorrelation in the productivity shock (ρ). The table presents both unconditional welfare, and welfare conditional on the steady state level of capital. To see if there are Pareto improvements, data is also presented for household and entrepreneurial welfare. In all cases the results are reported as numerical differences from the planner's welfare levels. The welfare measures we report are computed based on a second-order approximation to the nonlinear equilibrium conditions of each model. Our welfare measure is the conditional expectation of a weighted average of household and entrepreneurial discounted lifetime utility. The conditional welfare measure is chosen since agents in the model solve an explicitly conditional optimization problem. We choose the weights on entrepreneurs utility, such that the capital stock in the steady state is the same for the planner problem as for the BGG model and the privately optimal contracts model. Our conditional welfare measure requires us to make an assumption about the initial value for the state variables of the model that we condition upon. There is some arbitrariness in this choice. However, the second-order approximation to the conditional welfare function is particularly easy to compute numerically, if we assume that in the initial period, all state variables are at their deterministic steady state. As shown by Schmitt Grohe and Uribe (2005, p. 47), the second order approximation involves only coefficients associated to the perturbation parameter which indexes the volatility of the shocks. In our case, the steady state welfare under the planner solution may also differ across models and we also account for this in our welfare metric.

Let us first focus on comparing the planner to the POC. For the benchmark calibration of $\psi = 0.25$ and $\rho = 0.95$, we see that the planner's allocation is a Pareto improvement. The unconditional welfare gain is large: 47%. Since the calibration is log utility, this is equivalent to a one-off payment equal to 47% of steady state household consumption, or (using a discount rate of 4%), an annual flow of .47% of household consumption. These effects are magnified for higher adjustment costs and higher levels of autocorrelation. With $\psi = 1$ and $\rho = 0.99$, the unconditional welfare loss is 100% of household consumption, or 65% in a conditional sense. Except for iid shocks, the planner is always a Pareto improvement over the POC.

Comparing BGG to the planner, it is curious that for the benchmark calibration, the welfare costs of the BGG contract are quite modest, 3% in conditional welfare, 6% in unconditional welfare. For high adjustment costs and high autocorrelation, these welfare effects become larger, 15% conditional and 22% unconditional. But in all cases the planner allocation is not a Pareto improvement. This is primarily because we set ϵ to match steady state capital stocks. We conjecture that if we could vary ϵ arbitrarily, we could always find a Pareto improvement.

6. Conclusion.

Two basic functions of financial markets are to intermediate between borrowers and lenders, and to provide a mechanism to hedge risk. Both of these motivations are present here. The risky debt contract is an efficient way of mitigating the informational asymmetries arising from the CSV problem. This then allows for funds to flow from the household- lenders to the entrepreneurial-borrowers. Since the entrepreneurs are risk neutral, households prefer contracts that index the loan repayment to innovations in aggregate consumption. This provides the household with a hedge against business cycle risk, and intermediaries that offer such loan contracts have a higher equity value than others. But as in Lorenzoni (2008) and Jeanne and Korinek (2010), in environments with credit constraints, financial markets can go awry. This is the case here. In this model with CSV-inspired credit constraints, the household's desire to hedge consumption risk results, paradoxically, in greater business cycle risk.

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APPENDIX

1. Linearized Model (POC).

$$E_t(r_{t+1}^k - r_{t+1}^l) = \nu\kappa_t \tag{A1}$$

$$nw_t = \kappa\frac{\gamma}{\beta}\left(r_t^k - r_t^l\right) + \frac{\gamma}{\beta}(r_t^l + nw_{t-1}) + \gamma\kappa\frac{rp}{\beta}(k_t + q_t + r_t^k) \tag{A2}$$

$$r_{t+1}^{l,POC} = r_t^d + \left(r_{t+1}^k - E_t r_{t+1}^k\right) - \frac{\sigma\Theta_g}{\Psi}(c_{t+1} - \mathrm{E_t}c_{t+1}) \tag{A3}$$

$$z_{t+1}^{POC} = r_t^d + \frac{(1-\Theta_g)[1-\nu(\kappa-1)]}{\Theta_g(\kappa-1)}\kappa_t + \left(r_{t+1}^k - E_t r_{t+1}^k\right) - \frac{\sigma}{\Psi}(c_{t+1} - \mathrm{E_t}c_{t+1}) \tag{A4}$$

$$\varpi_t^{POC} \equiv \frac{[1-\nu(\kappa-1)]}{\Theta_g(\kappa-1)}\kappa_{t-1} - \frac{\sigma}{\Psi}(c_t - \mathrm{E_{t-1}}c_{t+}) \tag{A5}$$

$$r_t^k = \epsilon q_t + (1-\epsilon)mpk_t - q_{t-1} \tag{A6}$$

$$\kappa_{t-1} = (q_{t-1} + k_t - nw_{t-1}) \tag{A7}$$

$$\sigma c_t + \eta l_t = \alpha k_t + (1-\alpha)a_t - \alpha l_t \tag{A8}$$

$$r_t^d = \sigma(E_t c_{t+1} - c_t) \tag{A9}$$

$$q_t = \psi(i_t - k_t) \tag{A10}$$

$$k_{t+1} = \delta i_t + (1-\delta)k_t \tag{A11}$$

$$\left(1 - \frac{\alpha\beta\delta}{1-\epsilon}\right)c_t + \left(\frac{\alpha\beta\delta}{1-\epsilon}\right)i_t \;\; = \alpha k_t + (1-\alpha)(a_t + l_t) \tag{A12}$$

where $\epsilon \equiv \frac{1-\delta}{mpk_{ss}+(1-\delta)}$. Also we have $\kappa \equiv K_{SS}/NW_{ss}$, Q_{ss}= 1, $R_{ss}^s = 1/\beta$. Finally, we set $\mu\int_0^{\varpi_{ss}}\omega\phi(\omega)d\omega \approx 0$ so that monitoring costs do not appear in (A12).

2. The Derivation of the spread.

$$f'(\varpi_{t+1}) + \Lambda_t U_c(t+1) g'(\varpi_{t+1}) = 0 \tag{A13}$$

$$(\bar{\kappa}_t - 1) E_t R^k_{t+1} f(\varpi_{t+1}) = \Lambda_t E_t U_c(t+1) R^k_{t+1} g(\varpi_{t+1}) \tag{A14}$$

$$E_t U_c(t+1) R^k_{t+1} \frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1} g(\varpi_{t+1}) = R^d_t E_t U_c(t+1) \tag{A15}$$

Linearized we have:

$$\Psi \varpi_{t+1} = (\lambda_t - \sigma c_{t+1}) \tag{A16}$$

$$(\lambda_t - \sigma c_{t+1}) = \frac{\kappa}{\kappa - 1} \kappa_t + (\theta_f - \theta_g) E_t \varpi_{t+1} \tag{A17}$$

$$E_t(r^k_{t+1} - r^L_{t+1}) = \left(\frac{1}{\kappa - 1}\right) \kappa_t - \Theta_g E_t \varpi_{t+1} \tag{A18}$$

Substitute (A16) into (A17):

$$E_t \varpi_{t+1} = \frac{\kappa}{\kappa - 1} \frac{1}{(\Psi - \theta_f + \theta_g)} \kappa_t$$

Then into (A18):

$$E_t(r^k_{t+1} - r^L_{t+1}) = \left[\frac{(\Psi - \theta_f + \theta_g) - \kappa \theta_g}{(\kappa - 1)(\Psi - \theta_f + \theta_g)}\right] \kappa_t \equiv \nu \kappa_t$$

where $\Psi \equiv \frac{\varpi_{ss} F'(\varpi_{ss})}{F(\varpi_{ss})} > 0$, by the second order condition, and $\Theta_g \equiv \frac{\varpi_{ss} g\prime(\varpi_{ss})}{g(\varpi_{ss})}$, where $0 < \Theta_g < 1$, and $\Theta_f \equiv \frac{\varpi_{ss} f\prime(\varpi_{ss})}{f(\varpi_{ss})} < 0$.

3. <u>Going from the household budget constraint to the planner.</u>

More generally, why does the household behave differently than the planner? Evidently there are some effects that the household takes as exogenous but that are internalized by the planner. To gain some insight into this, let us begin with the budget constraint of the household:

$$C_t + D_t + Q^L_t S_t \le w_t N_t + R^D_{t-1} D_{t-1} + (Q^L_t + Div_t) S_{t-1} + P_t \tag{A19}$$

where P_t denotes the profit flow of the capital-producing firm. For future reference, $P_t \equiv Q_t I_t - I_t \phi(\frac{I_t}{\delta K_{ss}})$. Suppose the household internalized all factor prices and dividend flows, that is, the household internalized the following equilibrium conditions:

$Div_t = R^L_t D_{t-1} - R^d_{t-1} D_{t-1}$
$S_t = 1$
$D_t = Q_t K_{t+1} - NW_t$
$w_t = MPN_t$
$r_t = MPK_t$

$$R_t^k \equiv \frac{r_t + (1-\delta)Q_t}{Q_{t-1}}$$

Further let us define the risk premium as

$$R_t^L \equiv R_t^k g(\varpi_t)\frac{\bar{\kappa}_{t-1}}{\bar{\kappa}_{t-1}-1} \equiv R_t^k - rp_t$$

Substituting these expressions into the household's budget constraint (A19) we have

$$C_t + Q_t[K_{t+1} - (1-\delta)K_t] \leq K_t^{\alpha}(A_tL_t)^{1-\alpha} - \{\boldsymbol{rp_t Q_{t-1}}\}K_t + \{\boldsymbol{NW_t - R_t^L NW_{t-1}}\} + P_t \qquad \text{(A20)}$$

Suppose that the household maximized utility subject to (A20), taking as exogenous the two bold terms in braces. That is, suppose the household internalized all factor prices except for the behavior of the risk premium and net worth dynamics. It is straightforward to show that the competitive equilibrium of this framework is identical to the original competitive equilibrium. But if we substitute out for these terms in braces, we are lead to the planner's constraints (23)-(25).

<u>Let's dig into the term in braces….show how to get to planner.</u>

$$C_t + Q_t[K_{t+1} - (1-\delta)K_t] \leq K_t^{\alpha}(A_tL_t)^{1-\alpha} - rp_tQ_{t-1}K_t + NW_t - R_t^L NW_{t-1} + P_t \qquad \text{(A20)}$$

Substitute in for rp_t and re-arrange terms.

$$C_t + Q_t[K_{t+1} - (1-\delta)K_t] \leq r_tK_t + w_tL_t + NW_t - R_t^kQ_{t-1}K_t + R_t^L(Q_{t-1}K_t - NW_{t-1}) + P_t$$

Use R_t^k definition:

$$C_t + Q_t[K_{t+1}] \leq w_tL_t + NW_t + R_t^L(Q_{t-1}K_t - NW_{t-1}) + P_t \qquad \text{(A21)}$$

<u>We know:</u>

$$\mathrm{NW_t} = \gamma[Q_t(1-\delta) + mpk_t]f(\varpi_t)K_t = \gamma R_t^k f(\varpi_t)\bar{\kappa}_{t-1}\mathrm{NW_{t-1}}$$

$$\mathrm{C_t^e} = (1-\gamma)[Q_t(1-\delta) + mpk_t]f(\varpi_t)K_t = (1-\gamma)R_t^k f(\varpi_t)\bar{\kappa}_{t-1}\mathrm{NW_{t-1}}$$

$$R_t^L \equiv R_t^k g(\varpi_t)\frac{\bar{\kappa}_{t-1}}{\bar{\kappa}_{t-1}-1}$$

$$R_t^k \equiv \frac{r_t + (1-\delta)Q_t}{Q_{t-1}}$$

$$\bar{\kappa}_t \equiv \left(\frac{Q_tK_{t+1}}{NW_t}\right)$$

Hence, we can write constraint (A21) as:

$$C_t + Q_tK_{t+1} - NW_t \leq w_tN_t + R_t^kQ_{t-1}K_tm(\varpi_t) - R_t^kQ_{t-1}K_tf(\varpi_t) + P_t$$

$$C_t + Q_tK_{t+1} + C_t^e \leq w_tN_t + R_t^kQ_{t-1}K_tm(\varpi_t) + P_t$$

$$C_t + Q_tK_{t+1} + C_t^e \leq w_tN_t + [Q_t(1-\delta) + mpk_t]K_tm(\varpi_t) + P_t$$

Using the definition of P_t, we have the planner constraints:

$$K_{t+1} \leq (1-\delta)m(\varpi_t)K_t + I_t$$

$$C_t + I_t\phi(\frac{I_t}{\delta K_{ss}}) + \mathrm{C_t^e} \leq Y_t$$

$$\mathrm{C_t^e} \leq (1-\gamma)[Q_t(1-\delta) + mpk_t]f(\varpi_t)K_t$$

Table 1: Standard Deviations.

		PLANNER					BGG					PRIVATE			
		ρ = 0	ρ = 0.9	ρ = 0.95	ρ = 0.99		ρ = 0	ρ = 0.9	ρ = 0.95	ρ = 0.99		ρ = 0	ρ = 0.9	ρ = 0.95	ρ = 0.99
ψ = 0	Y	0.0681	0.1612	0.2244	0.4801		0.0656	0.1338	0.1796	0.3866		0.0704	0.1742	0.2351	0.4467
ϵ = .93	L	0.0148	0.0231	0.0246	0.0252		0.0144	0.0163	0.0149	0.0111		0.0158	0.0275	0.0302	0.0316
	C hous	0.0083	0.0641	0.1062	0.2793		0.0085	0.0625	0.1000	0.2558		0.0089	0.0664	0.1095	0.2734
	K	0.2051	1.4016	2.1682	4.9619		0.1934	0.9757	1.4373	3.3182		0.3548	1.6195	2.2826	4.2145
	Spread	0.0001	0.0007	0.0012	0.0031		0.0002	0.0019	0.0022	0.0021		0.0014	0.0040	0.0054	0.0078
	Q	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000	0.0000
	C entr	0.0022	0.0155	0.0241	0.0562		0.0018	0.0071	0.0113	0.0305		0.0057	0.0208	0.0282	0.0475
ψ = .25	Y	0.0419	0.1042	0.1513	0.3567		0.0542	0.1174	0.1587	0.3384		0.1001	0.1817	0.2321	0.4111
ϵ = 1.15	L	0.0030	0.0051	0.0056	0.0057		0.0081	0.0091	0.0084	0.0069		0.0231	0.0275	0.0284	0.0287
	C hous	0.0204	0.0599	0.0926	0.2397		0.0178	0.0583	0.0930	0.2371		0.0319	0.0586	0.0987	0.2536
	K	0.0732	0.5836	0.9845	2.6748		0.3934	0.7847	1.0794	2.3453		1.2253	1.7878	2.1717	3.4046
	Spread	0.0084	0.0018	0.0011	0.0013		0.0043	0.0028	0.0024	0.0021		0.0154	0.0162	0.0168	0.0182
	Q	0.0268	0.0531	0.0671	0.1153		0.0429	0.0688	0.0768	0.1078		0.0862	0.1288	0.1425	0.1704
	C entr	0.0025	0.0093	0.0149	0.0389		0.0121	0.0147	0.0168	0.0298		0.0395	0.0480	0.0526	0.0650
ψ = .50	Y	0.0377	0.0928	0.1345	0.3192		0.0505	0.1089	0.1473	0.3125		0.1055	0.1766	0.2223	0.3858
ϵ = 1.27	L	0.0012	0.0020	0.0021	0.0032		0.0072	0.0065	0.0056	0.0066		0.0253	0.0266	0.0267	0.0264
	C hous	0.0228	0.0605	0.0907	0.2281		0.0208	0.0575	0.0902	0.2275		0.0352	0.0531	0.0913	0.2409
	K	0.0496	0.4080	0.7017	1.9903		0.3787	0.6449	0.8624	1.8186		1.2961	1.6650	1.9391	2.8521
	Spread	0.0082	0.0018	0.0011	0.0010		0.0058	0.0044	0.0039	0.0033		0.0218	0.0222	0.0226	0.0236
	Q	0.0319	0.0655	0.0848	0.1551		0.0608	0.0993	0.1129	0.1634		0.1492	0.2055	0.2256	0.2685
	C entr	0.0028	0.0084	0.0131	0.0339		0.0155	0.0181	0.0198	0.0302		0.0542	0.0614	0.0653	0.0754
ψ=.75	Y	0.0359	0.0874	0.1263	0.2992		0.0482	0.1038	0.1404	0.2966		0.1054	0.1714	0.2144	0.3685
ϵ = 1.35	L	0.0004	0.0008	0.0012	0.0044		0.0068	0.0051	0.0041	0.0073		0.0259	0.0258	0.0253	0.0249
	C hous	0.0239	0.0609	0.0899	0.2220		0.0221	0.0573	0.0888	0.2216		0.0357	0.0494	0.0864	0.2324
	K	0.0388	0.3240	0.5626	1.6303		0.3509	0.5526	0.7247	1.4913		1.2630	1.5389	1.7529	2.4777
	Spread	0.0071	0.0014	0.0008	0.0011		0.0066	0.0052	0.0047	0.0042		0.0255	0.0258	0.0261	0.0269
	Q	0.0347	0.0726	0.0952	0.1793		0.0713	0.1168	0.1341	0.1972		0.1922	0.2552	0.2791	0.3315
	C entr	0.0029	0.0081	0.0123	0.0313		0.0173	0.0200	0.0216	0.0307		0.0629	0.0695	0.0731	0.0821
ψ=1	Y	0.0348	0.0842	0.1213	0.2865		0.0467	0.1005	0.1358	0.2858		0.1041	0.1672	0.2084	0.3562
ϵ = 1.41	L	0.0001	0.0008	0.0016	0.0056		0.0065	0.0042	0.0031	0.0080		0.0261	0.0252	0.0244	0.0238
	C hous	0.0245	0.0611	0.0895	0.2182		0.0229	0.0572	0.0879	0.2177		0.0355	0.0469	0.0830	0.2265
	K	0.0325	0.2734	0.4774	1.4019		0.3265	0.4882	0.6302	1.2685		1.2139	1.4345	1.6103	2.2107
	Spread	0.0055	0.0010	0.0004	0.0013		0.0071	0.0058	0.0053	0.0047		0.0280	0.0282	0.0284	0.0291
	Q	0.0367	0.0778	0.1027	0.1973		0.0784	0.1283	0.1481	0.2201		0.2229	0.2899	0.3163	0.3753
	C entr	0.0030	0.0079	0.0119	0.0298		0.0184	0.0212	0.0228	0.0312		0.0686	0.0750	0.0784	0.0868

Table 2: Welfare Comparisons.

	Welfare		Planner-POC				Planner-BGG		
		ρ = 0	ρ = 0.9	ρ = 0.95	ρ = 0.99	ρ = 0	ρ = 0.9	ρ = 0.95	ρ = 0.99
ψ = 0	**conditional**	0.0023	0.0149	0.0253	0.0527	0.0003	0.0065	0.0115	0.0223
ϵ = .93	**unconditional**	0.0042	0.0274	0.0471	0.1216	0.0011	0.0166	0.0315	0.0848
	HH_conditional	0.0189	0.0367	0.0504	0.0738	0.0158	0.0167	0.0150	-0.0022
	HH_unconditional	0.0206	0.0496	0.0727	0.1249	0.0167	0.0263	0.0324	0.0322
	E_conditional	-0.0178	-0.0233	-0.0270	-0.0226	-0.0167	-0.0110	-0.0038	0.0263
	E_unconditional	-0.0177	-0.0238	-0.0275	-0.0036	-0.0167	-0.0103	-0.0009	0.0564
	steady state welfare	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
ψ = .25	**conditional**	0.2400	0.2525	**0.2608**	0.2837	0.0396	0.0279	**0.0255**	0.0286
ϵ = 1.15	**unconditional**	0.3770	0.4389	**0.4659**	0.5347	0.0631	0.0566	**0.0552**	0.0785
	HH_conditional	0.0833	0.1136	**0.1347**	0.1676	-0.1903	-0.2052	**-0.2114**	-0.2360
	HH_unconditional	0.2310	0.3084	**0.3489**	0.4127	-0.1678	-0.1803	**-0.1866**	-0.2162
	E_conditional	0.1368	0.1211	**0.1101**	0.1013	0.2006	0.2035	**0.2067**	0.2309
	E_unconditional	0.1275	0.1138	**0.1021**	0.1065	0.2015	0.2067	**0.2110**	0.2572
	steady state welfare	0.0217	0.0217	**0.0217**	0.0217	0.0217	0.0217	**0.0217**	0.0217
ψ = .50	**conditional**	0.4465	0.4403	0.4404	0.4510	0.0891	0.0730	0.0684	0.0693
ϵ = 1.27	**unconditional**	0.6623	0.6946	0.7107	0.7706	0.1183	0.1059	0.1033	0.1326
	HH_conditional	0.1843	0.2022	0.2167	0.2300	-0.3358	-0.3560	-0.3663	-0.4078
	HH_unconditional	0.4335	0.4878	0.5193	0.5564	-0.3057	-0.3256	-0.3359	-0.3875
	E_conditional	0.2065	0.1876	0.1762	0.1741	0.3346	0.3378	0.3423	0.3757
	E_unconditional	0.1802	0.1628	0.1507	0.1687	0.3339	0.3398	0.3458	0.4096
	steady state welfare	0.0595	0.0595	0.0595	0.0595	0.0595	0.0595	0.0595	0.0595
ψ=.75	**conditional**	0.5906	0.5714	0.5655	0.5677	0.1329	0.1149	0.1093	0.1101
ϵ = 1.35	**unconditional**	0.8448	0.8521	0.8580	0.9078	0.1630	0.1467	0.1432	0.1771
	HH_conditional	0.2539	0.2628	0.2717	0.2646	-0.4428	-0.4659	-0.4793	-0.5376
	HH_unconditional	0.5658	0.6003	0.6222	0.6253	-0.4097	-0.4345	-0.4487	-0.5265
	E_conditional	0.2491	0.2283	0.2174	0.2242	0.4259	0.4297	0.4355	0.4792
	E_unconditional	0.2064	0.1863	0.1744	0.2090	0.4237	0.4301	0.4379	0.5206
	steady state welfare	0.0964	0.0964	0.0964	0.0964	0.0964	0.0964	0.0964	0.0964
ψ=1	**conditional**	0.6952	0.6670	0.6570	0.6538	0.1699	0.1510	0.1450	0.1468
ϵ = 1.41	**unconditional**	0.9703	0.9601	0.9586	1.0014	0.1999	0.1813	0.1772	0.2155
	HH_conditional	0.3037	0.3062	0.3104	0.2837	-0.5225	-0.5477	-0.5640	-0.6388
	HH_unconditional	0.6585	0.6790	0.6931	0.6622	-0.4878	-0.5163	-0.5339	-0.6391
	E_conditional	0.2778	0.2559	0.2459	0.2626	0.4912	0.4957	0.5030	0.5573
	E_unconditional	0.2212	0.1994	0.1884	0.2406	0.4878	0.4948	0.5045	0.6063
	steady state welfare	0.1291	0.1291	0.1291	0.1291	0.1291	0.1291	0.1291	0.1291
	Benchmark calibration is in bold.								

Figure 1

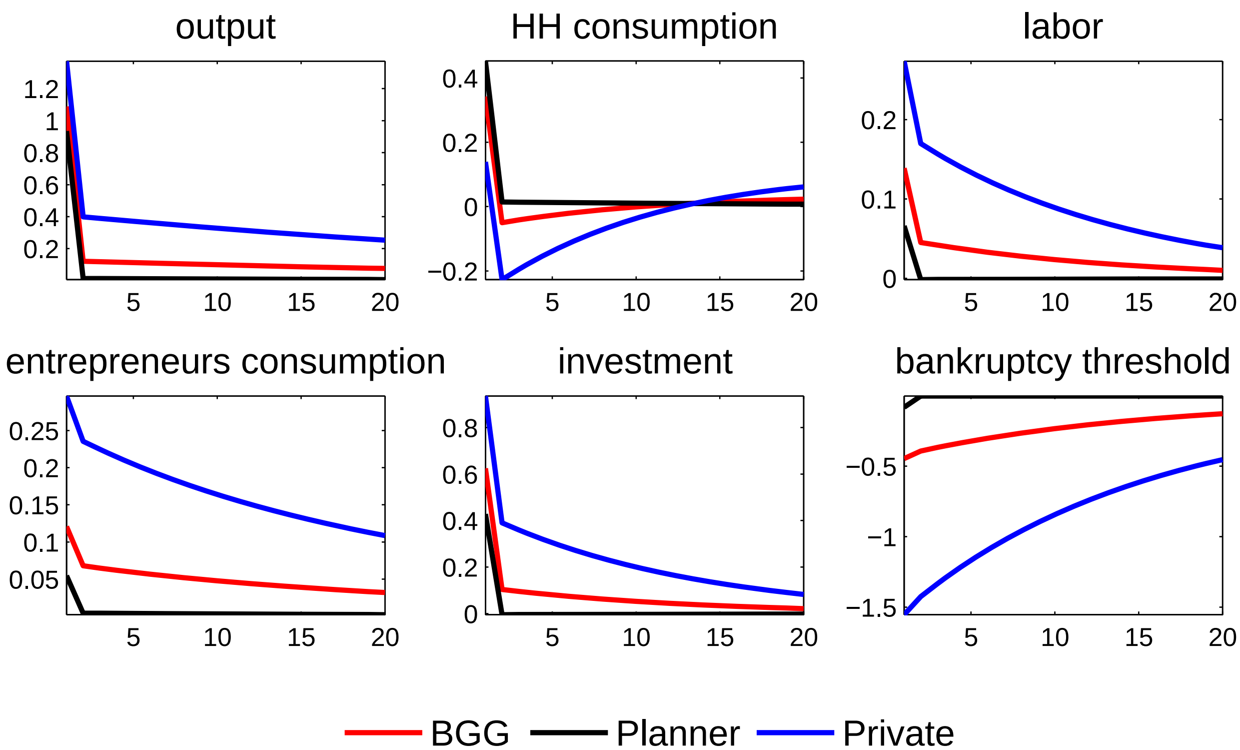

Figure 1: Impulse response to a unit i.i.d. technology shock.

Figure 2

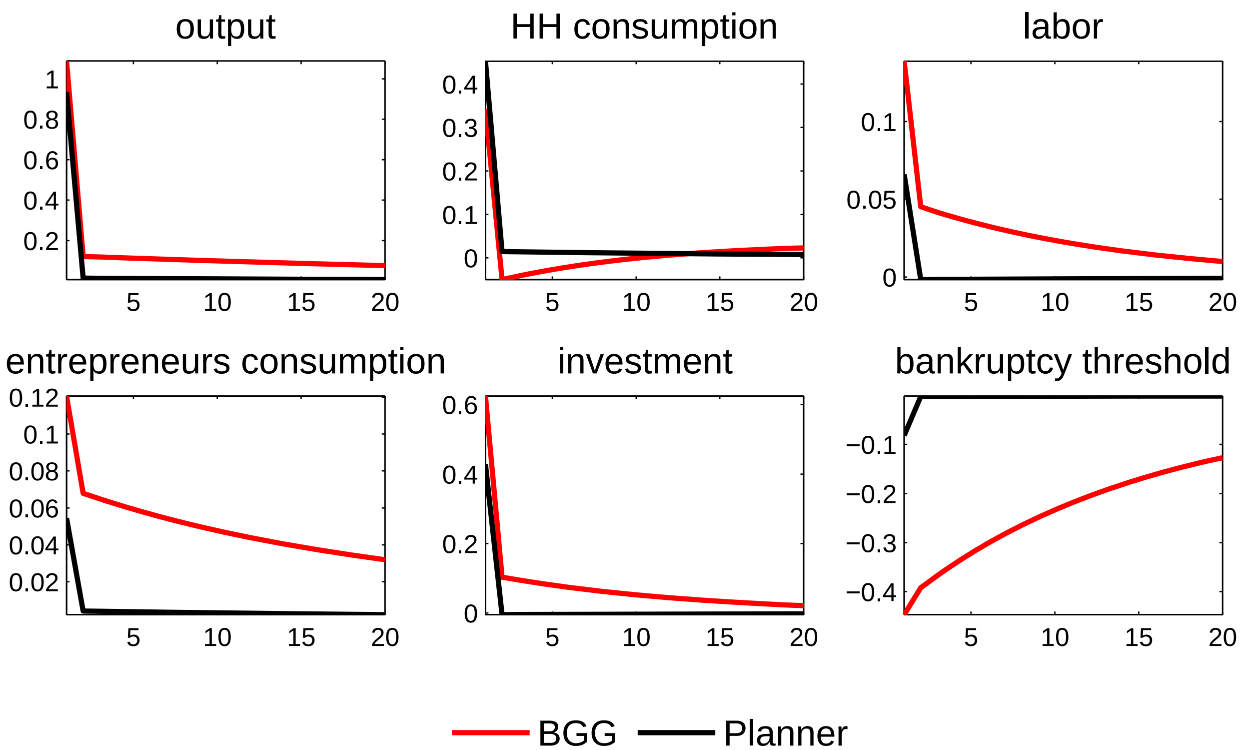

Figure 2: Impulse response to a unit i.i.d. technology shock. BGG vs. Planner allocation

Figure 3

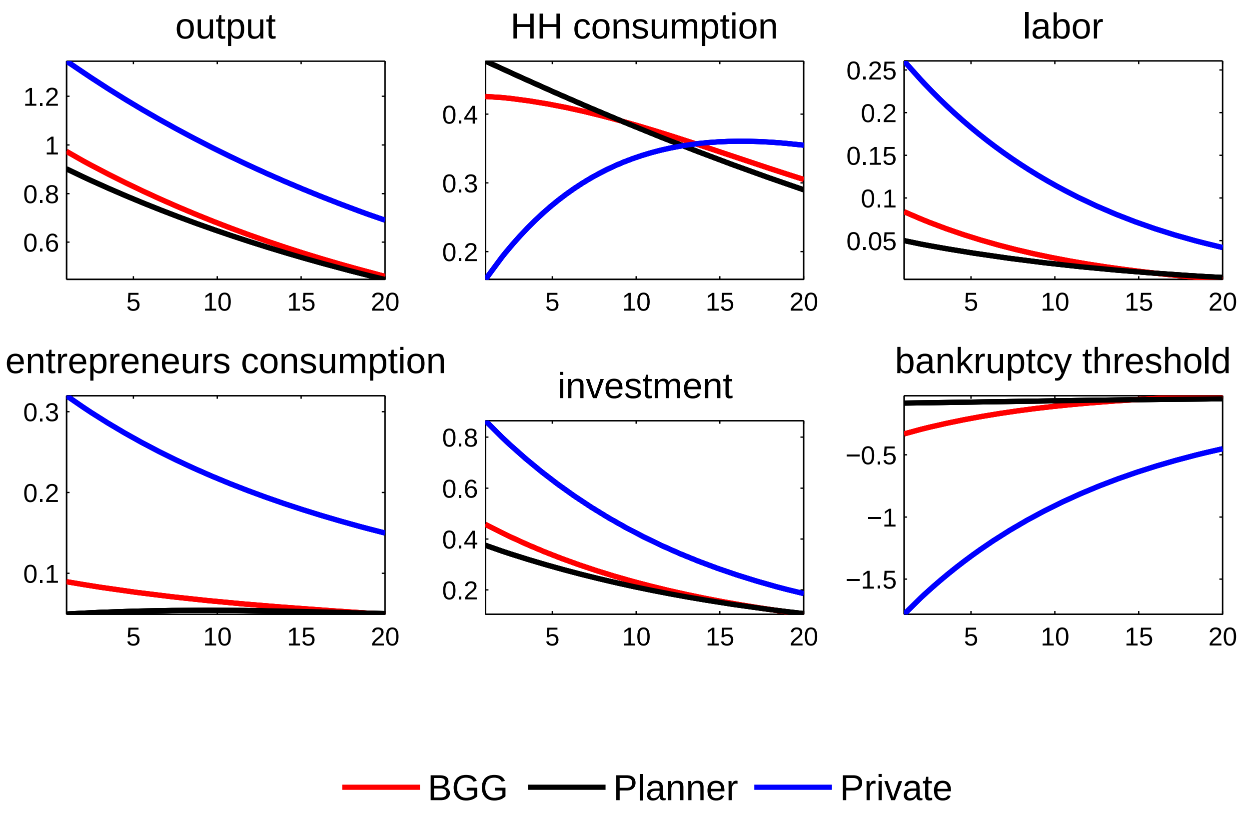

Figure 3: Impulse response to a unit technology serially correlated shock.

Figure 4

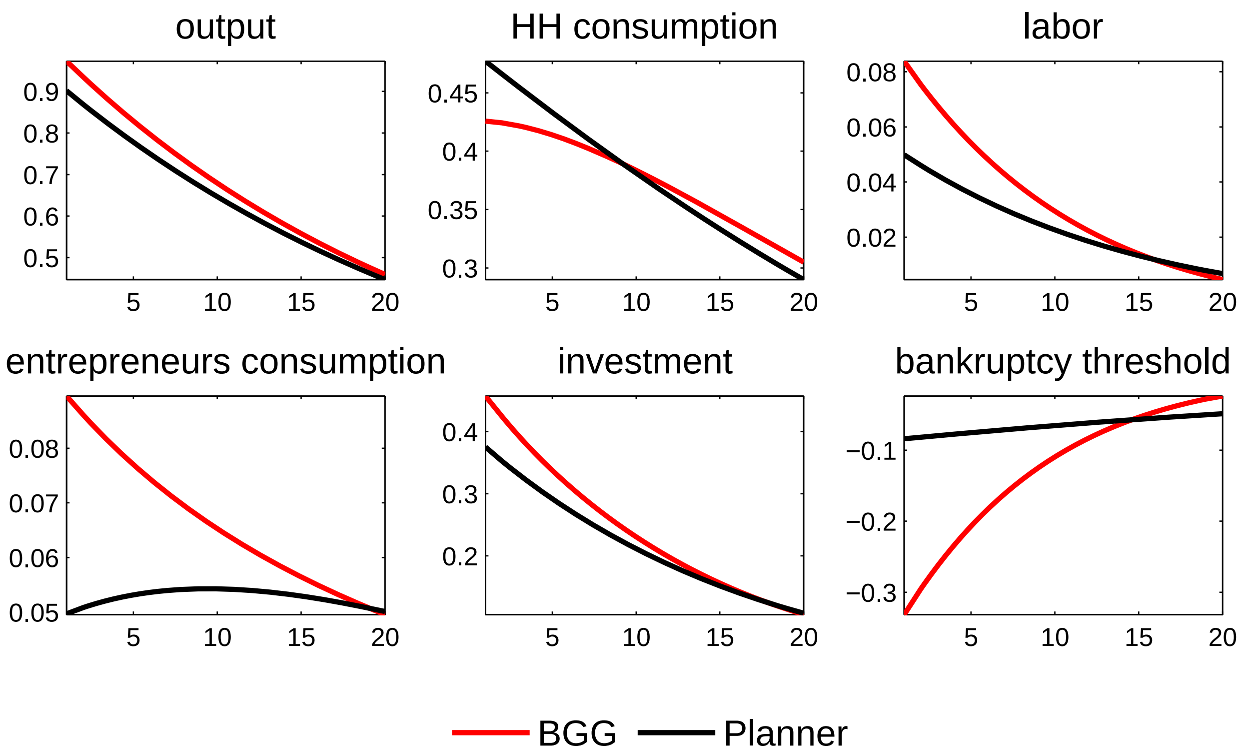

Figure 4: Impulse response to a unit technology serially correlated shock. BGG vs. Planner allocation