The Effects of Rare Economic Crises on Credit Spreads and Leverage

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Abstract

Rare disasters have been shown to explain several asset pricing puzzles, such as the equity risk premium and credit spread puzzles. Existing analysis ignores the impact of corporate financing decisions on asset prices by taking them as exogenous. The aim of this paper is to investigate how rare disasters affect endogenous default and capital structure decisions by firms and how, in turn, these corporate financial decisions affect the way in which rare disasters impact credit spreads, leverage and the equity risk premium. We find that the possibility of rare disasters makes firms more conservative in their financial policy, leading to higher interest coverage, but lower leverage ratios, together with larger credit spreads and equity risk premia.

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I Introduction

Research studying the impact of rare and adverse economic events, commonly referred to as 'disasters', 'tail risk' or 'peso problems', shows that when agents believe such rare disasters are possible, many asset pricing puzzles, including the equity risk premium and credit spread puzzles, can be resolved.[1] Asset prices also have important implications for firms' corporate financial policies and vice versa. This can be seen clearly in structural models of corporate finance, such as Leland (1994) and more recently Chen (2010), where explicit first order conditions link default policies and capital structure to asset prices. Despite this, the existing literature on rare disasters focuses almost exclusively on asset prices, taking corporate financial policies as exogenous or ignoring them entirely.

In structural models, default occurs when the present value of bondholders' coupons exceeds the present value of future dividends. With an exogenous capital structure, it is clear that the possibility of rare disasters reduces the present value of future dividend payments, leading to earlier default. However, capital structure is not exogenous: debt is issued so that the present value of tax benefits equals the present value of bankruptcy costs. It not so clear which of the two present values will be reduced more by the possibility of rare disasters. The effect of rare disasters on optimal capital structure and hence default policies is therefore not obvious. Since corporate financial policies affect asset prices, it is no longer obvious how rare disasters impact asset prices. In this paper, we explore how rare disasters impact the joint determination of corporate financial policies and asset prices.

We provide a tractable model of consumption - based asset prices with endogenous corporate financial policies. Aggregate consumption is exogenous as are firms' earnings. Both are subject to rare disasters, which occur once in a hundred years and lead to large contemporaneous drops in consumption and earnings. A firm's earnings are shared between bondholders and equityholders as coupons and dividends, respectively. Valuation is carried out using the state - price density of a representative agent with Epstein - Zin - Weil utility. Optimal default and capital structure decisions are as described above. In addition, firms have an option to restructure their debt if earnings are high, as in Goldstein et al. (2001). The decision to call outstanding debt and issue new debt with a new coupon is made by trading off the benefits of issuing the new debt against the issuance costs

[1] Notable contributions include Rietz (1988), and more recently Barro (2006), Gabaix (2008), and Wachter (2011).

combined with the present value of the associated bankruptcy costs. Consequently, capital structure is dynamic. Within our framework, we study the long - run implications of rare disasters on aggregate, i.e. economy - wide leverage, interest coverage, credit spreads and equity risk premia.

Our paper makes three contributions. First we can explain the term structure of credit spreads, while generating a realistic term structure of default probabilities, and a reasonable equity risk premium with realistic leverage levels.

Second, we derive new testable implications concerning the impact of rare disasters on credit risk. We find that while rare disasters increase the riskiness of corporate debt as measured by credit spreads, interest coverage improves. To see the intuition, observe that rare disasters decrease the present value of tax benefits from debt more than the present value of bankruptcy costs, leading to a lower coupon and hence higher interest coverage. However, the increase in risk premia induced by rare disasters leads to a fall in asset values. This latter effect dominates the fall in the optimal coupon leading to higher spreads.

Third, to the best of our knowledge, ours is the first paper to have both jumps of random size in firms' earnings levels and dynamic capital structure. This enables us to study the long – run implications of rare disasters for the joint determination of corporate financial policies and asset prices. With static capital structure, such a long - run analysis would be impossible.

Related literature

Our paper lies at the focal point of two literatures.

Firstly our paper contributes to the literature on rare disasters. Rietz (1988) is the first paper arguing that allowing for rare disasters may explain the equity risk premium. This idea has been investigated in detail by Barro (2006). In particular, Barro studies a levered equity risk premium with exogenous leverage, in a consumption-based representative-agent framework. Gabaix (2008) investigates the impact of rare disasters on credit spreads when capital structure and default are exogenously specified. Naik and Lee (1990) is the first paper to model jumps in consumption and consider the implications for option pricing. Liu et al. (2005) also model jumps in consumption to investigate option smirks and smiles. The above papers either ignore corporate financing decisions or take them as exogenous. In our paper corporate financial policies are endogenous and co - dependent on asset prices.

Secondly, our paper is part of the literature on structural models of credit risk, for example, Merton (1974), Fischer et al. (1989), and Leland (1994)). More recent contributions include Hackbarth et al. (2006), Chen et al. (2008), Chen and Kou (2009), Bhamra et al. (2010), Chen (2010), Carlson and Lazrak (2010), and He and Xiong (2011). Our paper contributes to this literature by allowing for jumps in earnings levels, random in both timing and size together with a dynamic capital structure.

The paper closest to ours is Gourio (2011), who studies the impact of rare disasters on credit spreads in a general equilibrium model, where firms issue one period debt and can readjust leverage each period without incurring any costs. While our approach is partial equilibrium, firms issue long term debt, and it is costly to adjust leverage.

The paper is organized as follows. Section II describes aggregate consumption, firms' earnings, and the representative agent. Section III describes asset valuation with static capital structure. Section IV describes asset valuation with dynamic capital structure. Section V discusses the quantitative implications of the model. Section VI concludes.

II Model

In this section we introduce a structural-equilibrium model with rare disasters. By using a structural-equilibrium model, as opposed to a pure structural model, such as Hackbarth et al. (2006), credit spreads, optimal default decisions and leverage will depend on the representative agent's preferences and aggregate consumption, as in Bhamra et al. (2010) and Bhamra et al. (2008) . However, our emphasis on rare disasters entails a different modeling approach. In particular, we assume that economic catastrophes can occur, but with very small probability. During a catastrophe, the level of consumption and firms' earnings jump downwards. The exact timing of catastrophes is unknown, i.e. they occur randomly.

II.A The effect of disasters on consumption and firm earnings

In this section, we describe our assumptions about how disasters effect aggregate consumption and firms' earnings cash flows.

The are K firms in the economy. The output of firm k, O_k, is divided between earnings, Y_k, and wages and other human capital income, W_k, paid to workers. Aggregate consumption, C, is equal to aggregate output. Therefore, $C = \sum_{k=1}^{K} O_k = \sum_{k=1}^{K} Y_k + \sum_{k=1}^{K} W_k$. We model aggregate consumption and individual firm earnings directly, and thus aggregate wages are just the difference between aggregate consumption and aggregate earnings.[2]

Aggregate consumption is given by

$$\frac{dC_t}{C_{t-}} = gdt + \sigma_C dB_{C,t} + (e^{z_{C,t}} - 1)dN_t, \tag{1}$$

where g is the constant drift of consumption growth, σ_C is the constant volatility of consumption growth due to small shocks, which we model via a standard Brownian motion, $B_{C,t}$. There is small probability of a crisis occurring in the economy. This crisis occurs with probability per unit time of λ. The aggregate impact of the crisis is characterized by a downward jump in the level of consumption. We model the random nature of the timing of the crisis via a Poisson process, N_t, with intensity equal to the probability per unit time of the crisis occurring. We model the jump in consumption by $e^{z_{C,t}} - 1$, where $z_{C,t}$ is a random variable, independent of $B_{C,t}$ and N_t. The percentage jump size is given by $e^{z_{C,t}} - 1$. We assume that $z_{C,t} < 0$ to ensure that the jump is an undesirable event. In particular, $z_{C,t}$, is exponentially distributed with mean $-1/\epsilon_C < 0$ and variance $(1/\epsilon_C)^2$. To understand the role of ϵ_C, note that if the mean decrease in consumption arising from a rare disaster is J_C, then $\epsilon_C = (1/J_C) - 1$. For example, if the mean decrease in consumption is 10%, then $J_C = 0.1$ and $\epsilon_C = 9$.

The earnings process for firm k is given by $X_{k,t}$, where

$$\frac{dX_{k,t}}{X_{k,t}} = \theta_{k,t}dt + \sigma^s_{Y,k,t} dB^s_{Y,t} + \sigma^{id}_{Y,k,t} dB^{id}_{Y,k,t} + (e^{z_{k,t}} - 1)dN_t \tag{2}$$

where θ_k is the expected earnings growth rate of firm k, and $\sigma^{id}_{Y,k}$ and $\sigma^s_{Y,k}$ are, respectively, the idiosyncratic and systematic volatilities of the firm's earnings growth rate. Total risk from Brownian shocks, $\sigma_{X,k}$, is given by $\sigma_{X,k} = \sqrt{(\sigma^{id}_{X,k})^2 + (\sigma^s_{X,k})^2}$. The standard Brownian motion $B^s_{Y,t}$ is the systematic shock to the firm's earnings growth, which is correlated with aggregate consumption growth:

$$dB^s_{X,t} dB_{C,t} = \rho_{XC} dt, \tag{3}$$

[2] In assuming so, we follow such papers as Kandel and Stambaugh (1991), Cecchetti, Lam, and Mark (1993), Campbell and Cochrane (1999), and Bansal and Yaron (2004).

where ρ_{XC} is the correlation coefficient. The standard Brownian motion $B^{id}_{X,k,t}$ is the idiosyncratic shock to firm k's earnings, which is correlated with neither $B^{s}_{X,t}$, $B_{C,t}$, nor with other firms' idiosyncratic shocks. The earnings level of firm n suffers from a rare disaster at the time as consumption. The percentage change in earnings for firm k is given by $e^{z_{k,t}} - 1$, where $z_{k,t}$ is random variable which is independent across firms and independent from $z_{C,t}$. To ensure that earnings declines during a disaster, we assume $z_{k,t} < 0$. In particular, $z_{k,t}$, is exponentially distributed with mean $-1/\epsilon_k < 0$ and variance $(1/\epsilon_k)^2$. The mean decrease in earnings arising from a rare disaster is J_k, where $\epsilon_k = (1/J_k) - 1$. For ease of notation, we omit the subscript k wherever possible.

II.B Disasters and Risk Prices

We assume the representative agent has the continuous-time analog of Epstein-Zin-Weil preferences, i.e. the representative agent's value function is given by

$$J_t = \max_C E_t \int_t^{\infty} f\left(C_t, J_t\right) dt, \tag{4}$$

where f is the normalized Kreps-Porteus aggregator:

$$f(c,v) = \beta(1-\gamma) v \, u\left(c/h^{-1}(v)\right), \tag{5}$$

for

$$\begin{aligned} u\left(x\right) &= \frac{x^{1-\frac{1}{\psi}} - 1}{1-\frac{1}{\psi}}, \psi > 0, \\ h\left(x\right) &= \begin{cases} \frac{x^{1-\gamma}}{1-\gamma}, & \gamma \geq 0, \gamma \neq 1, \\ \ln x, & \gamma = 1, \end{cases} \end{aligned}$$

where β is the rate of time preference, γ is the coefficient of relative risk aversion (RRA), and ψ is the elasticity of intertemporal substitution under certainty (EIS).[3]

[3]The continuous-time version of the recursive preferences introduced by Epstein and Zin (1989) and Weil (1990) is known as stochastic differential utility (SDU), and is derived in Duffie and Epstein (1992). Schroder and Skiadas (1999) provide a proof of existence and uniqueness. Kraft and Seifried (2008) show the version of SDU we use is well defined under a mixed Brownian-Poisson filtration. We shall need to assume that $\gamma < J_C^{-1} - 1$ to ensure asset prices are well defined.

We start from the fundamental observation that asset prices depend on risk-neutral probabilities and not actual probabilities. Therefore, we begin our analysis with the following proposition that relates the risk-neutral probability of a disaster occuring to its actual probability.

Proposition 1 *The risk-neutral probability per unit time of a disaster occuring is given by*

$$\hat{\lambda} = \lambda\omega, \tag{6}$$

where the risk distortion factor, ω, given by

$$\omega = E_{t-}[e^{-\gamma z_t}] = \frac{1 - J_C}{1 - J_C(1+\gamma)}, \tag{7}$$

is greater than one and increasing in γ, if $\gamma < J_C^{-1} - 1$.

The above proposition is fundamental to understanding our results. It tells us that even though the actual ($\mathbb{P}$ – measure) intensity of a disaster is very small, the risk – neutral ($\mathbb{Q}$ – measure) will be much larger if the risk distortion factor ω is large. For the sake of clarity, suppose that the expected consumption drop from a disaster is 10%, i.e. $J_C = 0.1$ and relative risk aversion is 8. Then the risk distortion factor is 9, implying that a real world disaster intensity of 0.015 (equivalent to a 1.5% probability of a disaster occurring in a given year) becomes a risk – neutral world intensity of 0.135 (equivalent to a 13% probability of a disaster occurring in a given year). We can see that even though disasters may be rare under the physical measure, for plausible values of risk aversion, disasters can occur much more frequently under the risk – neutral measure. Since asset prices are driven by risk – neutral probabilities, it follows that asset prices can be strongly impacted by rare disasters. The novel theme of this paper is the study of how rare disasters impact the interplay between asset prices and corporate financial policies.

Observe also, that a necessary, but not sufficient condition for asset prices to be well defined is that $\gamma < J_C^{-1} - 1$, which ensures that the risk – neutral disaster intensity is strictly positive. Intuitively we can interpret this condition as saying that sufficiently risk averse agent suffers infinite pain from the possibility of a rare disaster – she would still be alive, but her consumption would drop substantially if the disaster occurred.

Proposition 2 *The state-price density of a representative agent with the continuous-time version of Epstein-Zin-Weil preferences is given by*

$$\pi_t = \begin{cases} \left(\beta e^{-\beta t}\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} C_t^{-\gamma} \left(p_C e^{p_C^{-1} t}\right)^{-\frac{\gamma-\frac{1}{\psi}}{1-\frac{1}{\psi}}}, & \psi \neq 1 \\ \beta e^{-\beta \int_0^t [1+(\gamma-1)\ln(V^{-1})]ds} C_t^{-\gamma} V^{-(\gamma-1)}, & \psi = 1 \end{cases}. \tag{8}$$

When $\psi \neq 1$, the price-consumption ratio, p_C, is given by:

$$p_C = \frac{1}{\overline{r} + \gamma\sigma_C^2 - g + \lambda \frac{1-\frac{1}{\psi}}{\gamma-1}\left(\omega \frac{1}{1+\omega\frac{J_C}{1-J_C}} - 1\right)}. \tag{9}$$

where

$$\overline{r} = \beta + \frac{1}{\psi} g - \frac{1}{2}\gamma\left(1 + \frac{1}{\psi}\right)\sigma_C^2. \tag{10}$$

When $\psi = 1$, define V via

$$J = \ln(CV). \tag{11}$$

Then V is given by:

$$\beta \ln V = g - \frac{\gamma}{2}\sigma_C^2 - \lambda \frac{\omega\frac{1}{1+\omega\frac{J_C}{1-J_C}} - 1}{\gamma - 1}. \tag{12}$$

The locally risk-free rate is given by

$$r = \overline{r} + \lambda \frac{\gamma - \frac{1}{\psi}}{\gamma - 1}\left(\frac{\omega}{1 + \omega\frac{J_C}{1-J_C}} - 1\right) - \lambda(\omega - 1). \tag{13}$$

and the risk premium on the claim to aggregate consumption is given by

$$\mu_{R_C,t-} - r_{t-} = \gamma\sigma_C^2 + \lambda E_{t-}[(e^{-\gamma z_t} - 1)(e^{z_t} - 1)]. \tag{14}$$

The price of consumption risk is given by

$$\Theta = \sqrt{\gamma^2\sigma_C^2 + \lambda E_{t-}[(e^{-\gamma z_{C,t}} - 1)^2]}. \tag{15}$$

When substitution (discount rate) effects dominate income (cash flow) effects, increasing the disaster intensity makes the claim to consumption less attractive, leading to a fall in the price - consumption ratio. To this effect, observe that relative to the no disaster case, the price consumption ratio contains one additional term, which is the last term of the denominator of (9), i.e. $\lambda\frac{1-\frac{1}{\psi}}{\gamma-1}\left(\omega\frac{1}{1+\omega\frac{J_C}{1-J_C}}-1\right)$. This term is increasing in λ if $\psi > 1$ & $\gamma > 1$ or if $\psi < 1$ & $\gamma < 1$.

Relative to the no disaster case, the risk - free rate contains two additional terms. The simplest additional term is $-\lambda(\omega-1)$, the difference between the actual and risk - neutral disaster intensities, which is negative, since the possibility of rare consumption disasters increases demand for precautionary savings. The second additional term is $\lambda\frac{\gamma-\frac{1}{\psi}}{\gamma-1}\left(\frac{\omega}{1+\omega\frac{J_C}{1-J_C}}-1\right)$, whose sign depends on whether the representative agent prefers earlier ($\gamma > 1/\psi$) or later ($\gamma < 1/\psi$) resolution of uncertainty and whether or not her relative risk aversion is greater than unity. Intuitively, an agent who is more risk averse and prefers earlier resolution of uncertainty will avoid saving as the disaster intensity increases, since she would rather consume today while consumption is high instead of postponing consumption only to be faced with a consumption disaster.

The price of consumption risk, Θ, contains two components, a standard component, $\gamma\sigma_C$, stemming from small, but frequent (Brownian) shocks to consumption growth and a disaster risk component, $\sqrt{\lambda E_{t-}[(e^{-\gamma z_{C,t}}-1)^2}$.

III Asset valuation with static capital structure

While our main goal is to explore the behavior of the economy in a dynamic financing equilibrium, to provide clearer intuition, in this section we derive the prices of all assets in the economy assuming that capital structure is static. As we shall see the valuation principles are identical in both cases.

III.A Arrow-Debreu Default Claims

We introduce two Arrow-Debreu default claims. The first Arrow-Debreu default claim, denoted by $q_{D,t}$, is the time-t value of a unit of consumption paid upon default. In other words, if earnings either reaches the boundary X_D from above for the first time or jumps below the boundary X_D from above for the first time, one unit of consumption is paid that instant, i.e.

$$q_{D,t} = E_t^{\mathbb{Q}}[e^{-r(\tau_D-t)}], \tag{16}$$

where

$$\tau_D = \inf_{t>0}\{X_t \leq X_D\}. \tag{17}$$

The second Arrow-Debreu default claim, denoted by $q^X_{D,t}$, is the time-t value of a the random cash flow X, paid at default, i.e.

$$q^X_{D,t} = E^{\mathbb{Q}}_t[e^{-r(\tau_D - t)} X_{\tau_D}]. \tag{18}$$

When there are no rare disasters, X does not jump. Hence, $X_{\tau_D} = X_D$, and $q^X_{D,t} = X_D q_{D,t}$. The possibility of a rare disaster implies that earnings can jump below the default boundary, and so it is possible that $X_{\tau_D} < X_D$. Furthermore, the possibility of a rare disaster increases the probability of default and so the price of the first Arrow-Debreu default claim is higher that in the no-disaster case. Also, since

In the proposition below, we provide exact closed-form expressions for the two Arrow-Debreu default claims.

Proposition 3 *The prices of the Arrow-Debreu default claims are given by*

$$q_{D,t} = \frac{\epsilon - \theta_1}{\epsilon} \frac{\theta_2}{\theta_2 - \theta_1} \left(\frac{X_D}{X_t} \right)^{\theta_1} + \frac{\theta_2 - \epsilon}{\epsilon} \frac{\theta_1}{\theta_2 - \theta_1} \left(\frac{X_D}{X_t} \right)^{\theta_2}, \tag{19}$$

and

$$q^X_{D,t} = X_t \left[\frac{\epsilon - \theta_1}{\theta_2 - \theta_1} \frac{1 + \theta_2}{1 + \epsilon} \left(\frac{X_D}{X_t} \right)^{1+\theta_1} + \frac{\theta_2 - \epsilon}{\theta_2 - \theta_1} \frac{1 + \theta_1}{1 + \epsilon} \left(\frac{X_D}{X_t} \right)^{1+\theta_2} \right], \tag{20}$$

where $\theta_1 < \theta_2$ are the positive roots of the cubic $g(\theta) - r = 0$, where

$$g(\theta) = -\left(\hat{\mu}_X - \frac{1}{2}\sigma_X^2 \right) \theta + \frac{1}{2}\sigma_X^2 \theta^2 + \lambda\omega \left(\frac{\epsilon}{\epsilon - \theta} - 1 \right). \tag{21}$$

III.B Abandonment value

The firms liquidation, or abandonment value, denoted by $A(X_t)$, is the after-tax value of the unlevered firms future earnings:

$$A(X_t) = (1 - \eta) X_t E_t \left[\int_t^\infty \frac{\pi_s X_s}{\pi_t X_t} ds \right]. \tag{22}$$

The liquidation value in (22) is a function of the current earnings level and is time-independent. The next proposition derives the value of $A(X_t)$ in terms of fundamentals of the economy.

Proposition 4 *The liquidation value is given by*

$$A(X_t) = \frac{(1 - \eta) X_t}{r_A}, \tag{23}$$

where

$$r_A = \overline{\mu} - \mu_X + \hat{\lambda} J, \tag{24}$$

and

$$\overline{\mu} = r + \gamma \rho_{XC} \sigma_X^s \sigma_C, \tag{25}$$

is the discount rate in the Gordon growth model.

III.C Credit spreads and the levered equity risk premium

The generic value of debt at time t, denoted by B_t, is given by

$$B_t = E_t \left[\int_t^{\tau_D} \frac{\pi_s}{\pi_t} c ds \right] + E_t \left[\frac{\pi_{\tau_D}}{\pi_t} \alpha A_{\tau_D} \right]. \tag{26}$$

The above expression can be simplified to give

$$B_t = \frac{c}{r} (1 - E_t^{\mathbb{Q}} [e^{-r(\tau_D - t)}]) + \alpha E_t^{\mathbb{Q}} [e^{-r(\tau_D - t)} A(X_{\tau_D})], \tag{27}$$

which can then rewritten in terms of fundamental Arrow-Debreu default claims:

$$B_t = \frac{c}{r} (1 - q_{D,t}) + \alpha (1 - \eta) \frac{1}{r_A} q_{D,t}^X. \tag{28}$$

We can also rewrite the bond price as

$$B_t = \frac{c}{r} (1 - l_D q_{D,t}), \tag{29}$$

where

$$l_D = \frac{\frac{c}{r} - \alpha (1 - \eta) \frac{1}{r_A} \frac{q_{D,t}^X}{q_{D,t}}}{\frac{c}{r}} \tag{30}$$

is the loss ratio at default.

The next proposition gives the corporate bond spread in terms of the risk-free rate, loss ratio, and the Arrow-Debreu default claim, q_D. Note that we define the credit spread as the yield on corporate debt less the yield on an equivalent risk-free security of the same maturity.

Proposition 5 *The credit spread at time t, s_t, is given by*

$$s_t = \frac{c}{B_t} - r = r \frac{l_D q_{D,t}}{1 - l_D q_{D,t}}. \tag{31}$$

Current levered equity value is given by the expected present value of future cash flows less coupon payments up until bankruptcy:

$$S_t = (1-\eta)E_t\left[\int_t^{\tau_D} \frac{\pi_s}{\pi_t}(X_s - c)\right].$$

We can show that the above equation simplifies to give

$$S_t = A(X_t) - (1-\eta)\frac{c}{r} + q_{D,t}(1-\eta)\frac{c}{r} - q_{D,t}^X(1-\eta)\frac{1}{r_A}. \tag{32}$$

In the next proposition we derive the levered equity risk premium and levered stock return volatility of an individual firm.

Proposition 6 *The conditional levered equity risk premium is*

$$\mu_R - r = \gamma\rho_{XC}\sigma_R^s\sigma_C + \Pi, \tag{33}$$

where σ_R^s is the systematic volatility of stock returns given by

$$\sigma_R^s = \frac{\partial \ln S_t}{\partial \ln X_t}\sigma_X^s. \tag{34}$$

and Π is a jump risk premium given in the Appendix. Conditional levered stock return volatility is

$$\sigma_R = \sqrt{\left(\sigma_R^{B,id}\right)^2 + \left(\sigma_R^{B,s}\right)^2 + \lambda\left(\sigma_R^P\right)^2}, \tag{35}$$

where

$$\sigma_R^{B,id} = \frac{\partial \ln S_t}{\partial \ln X_t}\sigma_X^{id}, \tag{36}$$

is the idiosyncratic volatility of stock returns and σ_R^P is from the jump component in stock returns.

III.D Optimal default boundary and optimal static capital structure

Equityholders maximize the value of their default option by choosing when to default and also optimal capital structure. Intuitively, the endogenous default boundary X_D, depends on the extent to which the rare disaster impacts consumption, firm level earnings and of course the probability of the disaster occurring. The default boundary satisfies the following standard smooth-pasting condition:

$$\left.\frac{\partial S}{\partial X}\right|_{X=X_D} = 0, \tag{37}$$

the solution of which leads to the following proposition.

Proposition 7 *For a given coupon level, the optimal default boundary, X_D is given by*

$$X_D = r_A \frac{c}{r} \frac{\theta_1 \theta_2}{(1+\theta_2)(1+\theta_1)} \frac{1+\epsilon}{\epsilon}. \tag{38}$$

Equityholders choose the optimal coupon to maximize firm value at date 0 by balancing marginal tax benefits from debt against marginal expected distress costs. As is standard in the capital structure literature (e.g., see Leland (1994)), by maximizing firm value equityholders internalize debtholders' value at date 0. However, in choosing default times they ignore the considerations of debtholders. This feature creates the usual conflict of interest between equity and debtholders. We assume a proportion ι of the bond's value is lost due to issuance costs.[4] Therefore equityholders choose the coupon to maximize date-0 firm value net of issuance costs, $F_0 = B_0(1-\iota) + S_0$, i.e.

$$c_0 = argmax\, F_0(c).$$

Optimal default boundaries depend on the coupon.

IV Asset valuation with dynamic capital structure

In this section, we extend our model to incorporate dynamic capital structure by allowing equityholders to restructure firm's financial obligations over time. This extension is necessary for two reasons. First, empirical evidence suggests that firms follow a target leverage ratio even though they restructure infrequently. Second, to correctly compare the implications of our model with the data for credit spreads and the risk premium, we must compute the credit spread and equity risk premium as cross-sectional averages over individual firm values. The cross-sectional distribution of firms used to compute these averages should be the one implied by our model.[5] Since this exercise is impossible with static capital (in this case, leverage attenuates in the long-run), we introduce dynamic capital structure.

The pricing of unlevered assets, such as the firm's abandonment value, is the same under static and dynamic capital structure, and so in the following sections we explain how dynamic capital structure is modeled and how debt and equity are priced.

[4] We introduce issuance costs to make the static capital structure results more comparable with the dynamic capital structure results.

[5] A growing literature highlights the importance of doing this, as opposed to simply averaging over equilibrium credit spreads and risk premia, where every firm has the same earnings level (see e.g., Berk et al. (1999) and Strebulaev (2007)).

IV.A Refinancing

The key difference between dynamic and static capital structure lies in the possibility to restructure a firm's debt obligations. In the static model, debt is issued only at time 0. In the dynamic model, firms may restructure at the time of their choice. They prefer to refinance infrequently, since each refinancing is costly (Fischer et al. (1989)). Intuitively, at each refinancing, equityholders choose a new coupon to maximize their value. We now explain how we implement this in an economy where consumption and earning levels can jump downwards.

There are two corporate events in the model: default and refinancing. Since leverage is altered at refinancing dates it is convenient to divide time into periods. A *period* is the time interval between two consecutive refinancing dates. It is convenient to denote the beginnings of such periods by date 0. Within each period, default occurs when a firm's cash flow level reaches a lower boundary, X_D. Restructuring occurs when earnings reach an upper boundary, X_U.

IV.B Homogeneity property

In the dynamic capital structure model specification that we consider below, a homogeneity property holds, as in Fischer et al. (1989) and Goldstein et al. (2001)). Denote ξ to be scaling factor defined as:

$$\xi = \frac{X_U}{X_0}. \tag{39}$$

The homogeneity property holds when ξ is time-invariant and level-invariant.

Using the homogeneity property, we can relate optimal coupons and boundaries between two consecutive periods. In particular,

$$c' = \xi c \tag{40}$$

and

$$X'_D = \xi X_D, \tag{41}$$

where $'$ denotes a variable for a new period.

IV.C Debt and equity valuation

Denote by $B_t(X_t, c_0)$ the value of debt where c_0 is the current coupon. We can write the value of debt in terms of fundamental Arrow-Debreu securities as

$$B_t(X_t, c_0) = \frac{c_0}{r} + \alpha(1-\eta)\frac{q_D^X(X_t)}{r_A} - \frac{c}{r}q_D(X_t) + q_U(X_t)\left(R - \frac{c_0}{r}\right). \tag{42}$$

The Arrow-Debreu restructuring claim $q_U(X_t)$ pays out a unit of consumption at restructuring if the firm has not yet defaulted. The key difference between the default claims, q_D and q_D^X in the static and dynamic cases is that in the dynamic case q_D and q_D^X pay off provided that restructuring has not already occurred in the current period.

In (42), c_0/r is the value of a risk-free consol bond paying coupon c_0. The net two terms represent the recovery value of the firm received by bondholders if default takes place less the value of the coupon payments lost due to default, multiplied by an Arrow-Debreu default claim, q_D. Observe that the recovery value of the firm received by bondholders if default takes place is given by

$$\alpha E_t^{\mathbb{Q}}[e^{-r(\tau_D - t)}A_t(X_{\tau_D})] = \alpha(1-\eta)\frac{E_t^{\mathbb{Q}}[e^{-r(\tau_D - t)}X_{\tau_D}]}{r_A} = \alpha(1-\eta)\frac{q_D^X(X_t)}{r_A}.$$

The final term is the payment made to bondholders at refinancing, denoted by R, less the value of the coupons lost, multiplied by an Arrow-Debreu default restructuring claim, q_U.

Observe that (42) holds for a general refinancing payment, R. The exact form of the refinancing payment depends on the bond indenture provisions such as callability and seniority. For example, if debt is callable at its book value, then R is the original par value of debt. If debt is non-callable, R is the continuation value of debt. For simplicity, we assume that debt is non-callable and issued *pari passu*, i.e. all outstanding debt issues have equal seniority. Dilution is on a per-coupon basis, so that if the coupon at the previous refinancing is c_0, and at restructuring the new coupon is $c_1(c_0)$, then the continuation value of the original debt issued at the previous refinancing date is[6]

$$R_{0,1} = \frac{c_0}{c_1(c_0)}B_t(X_U^0, c_1(c_0)). \tag{43}$$

Based on the above structure of the refinancing payment, we can derive bond prices at refinancing dates and hence at all dates, as shown in the following proposition.

[6] Other definitions of R can be incorporated in the model but with a loss of the homogeneity property.

Proposition 8 *Suppose that the refinancing payment R is given by (43). Then the homogeneity property (see (39)) holds, the date-t debt value is given by*

$$B(X_t, c^0) = \frac{c^0}{r}(1 - l_D q_{D,t} - l_U q_{U,t}), \tag{44}$$

and the credit spread, $s_t(X_t, c^0)$, is given by

$$s_t(X_t, c^0) = r\frac{q_D(X_t, X_D^0)l_{D,t} + q_{U,t}(X_t, X_U^0)l_{U,t}}{1 - q_{D,t}(X_t, X_D^0)l_{D,t} - q_{U,t}(X_t, X_U^0)l_{U,t}}, \tag{45}$$

where loss ratios conditional on default and restructuring are given, respectively, by (30) and where

$$l_U = \frac{\frac{c^0}{r} - B(X_0^0, c^0)}{\frac{c^0}{r}}. \tag{46}$$

and $B(X_0^0, c_0)$ is the bond value at restructuring, given by

$$B(X_0^0, c^0) = \frac{\frac{c^0}{r} + (1-\eta)\frac{\alpha q_D^X(X_0^0)}{r_A} - q_D(X_0^0)\frac{c^0}{r} - q_U(X_0^0)\frac{c^0}{r}}{1 - q_U(X_0^0)}. \tag{47}$$

.

To value equity, we must distinguish between equity value just after refinancing, S_0, and equity value just prior to refinancing, E_0. The value of equity just after refinancing is

$$S_{\nu_t}(X_t, c^0) = Div_t(X_t, c^0) + q_{U,t}(X_t)E_0, \tag{48}$$

where Div_t is the present value of dividends paid to equityholders during the current refinancing period and can be written as

$$\begin{aligned} Div_t(X_t, c^0) &= A_t(X_t) - (1-\eta)\frac{c^0}{r} + (1-\eta)\left[q_{D,t}(X_t)\frac{c^0}{r} - \frac{q_{D,t}^X(X_t)}{r_A}\right] \\ &\quad + q_{U,t}(X_t)\left[(1-\eta)\frac{c^0}{r} - A_t(X_U^0)\right]. \end{aligned} \tag{49}$$

The third term in (49) shows that, if default occurs, equityholders no longer pay coupons but lose the right to future dividends. The third term shows a similar adjustment for the effect of restructuring.

The final term in (48) is present because, if restructuring occurs, equityholders derive value from cash flow payments made after restructuring. In the case of the *pari passu* covenant assumed above for the valuation of debt, equity value just prior to refinancing can be written as

$$E_0(X_U^0) \;=\; \left[(1-\iota)B(X_U^0, c^1(c^0)) - R_{0,1}(X_U^0, c^0, c^1(c^0))\right] + S(X_U^0, c^1(c^0)), \tag{50}$$

where a proportion ι of the newly issued bond's value is lost due to restructuring costs.

Given the above expression for equity value just prior to refinancing, we can derive equity values at refinancing dates and hence at all dates, as shown in the following proposition.

Proposition 9 *Suppose that the bond refinancing payment R is given by (43), so that the homogeneity property (see (39)) holds and the value of equity just before refinancing is given by (50). Then equity values are given by*

$$S(X_t, c^0) = Div_t(X_t, c^0) + q_U(X_t)\{B(X_0^0)[(1-\iota)\xi - 1)] + \xi S(X_0^0, c^0)\}, \tag{51}$$

where $S(X_0, c_0)$ is the equity value at restructuring, given by

$$S(X_0^0, c^0) = \frac{Div_t(X_t, c^0) + q_U(X_t)\{B(X_0^0)[(1-\iota)\xi - 1)]\}}{1 - \xi q_U(X_t)}. \tag{52}$$

Note that expressions for the equity risk premium and return volatility are the same as in the static case (see Proposition 6). The only difference is in the functional form of the elasticity, $\partial \ln S_{\nu_t} / \partial \ln X$.

IV.D Optimal default boundary and optimal capital structure

Relative to the static case, equityholders must now also decide on optimal restructuring boundaries as well as optimal coupons and default boundaries. Equityholders choose 3 variables: X_U^0, X_D^0, c^0.

Given the coupons and restructuring boundaries, the optimal default boundary X_D^0 is determined by the following smooth pasting condition

$$\frac{\partial S_1(X_t, c_0)}{\partial X_t}\Big|_{X_t = X_D^0} = 0. \tag{53}$$

The optimal coupon c^0 and the restructuring boundary X_U^0 are thn chosen to maximize levered firm value at restructuring, i.e.

$$(c^0, X_U^0) = argmax\, F(c^0, X_U^0),$$

where $F = B(X_0, c^0)(1-\iota) + S(X_0, c^0)$.

V Model Implications

In this section we study the quantitative implications of the model.

V.A Parameter Estimation

To estimate parameter values we use aggregate US data at quarterly frequency for the period from 1947Q1 to 2005Q4. Consumption is real non-durables plus service consumption expenditures from the Bureau of Economic Analysis. Earnings data are from S&P and provided on Robert J. Shiller's website. We delete monthly interpolated values and obtain a time-series at quarterly frequency. The personal consumption expenditure chain-type price index is used to deflate the earnings time-series. Unconditional parameter estimates are summarized in Table I. We now discuss the estimation exercise in more detail.

Unsurprisingly, given the difficulty of estimating the frequency and size of rare disasters, values for λ, J_C, and J vary in the literature.[7] For example, Longstaff and Piazzesi (2004) use $\lambda = 1/100$, and take constant jump sizes of $J_C = 0.1$ and $J = 0.9$, assuming that a Great Depression like scenario, where US consumption dropped by 10% and corporate earnings decreased by more than 90% is a once in a hundred years event. Barro (2006) assumes $\lambda = 1.7/100$ and $J_C = 0.36$, whereas Barro and Ursua (2008) use $\lambda = 3.6/100$ and $J_C = 0.4$. Using a cross section of international data, Nakamura et al. (2011) estimate that consumption disasters lead to long-run declines in consumption of about 15%. Given, that our model assumes disasters are instantaneous downward jumps in consumption and that there are no upward jumps, we choose the relatively conservative values for parameters governing the distribution of jump sizes $J_C = 0.1$, $J = 0.25$ and set $\lambda = 1.5/100$. Our estimates of g, θ, σ_C, σ_X^s, and ρ_{XC} are obtained by maximum likelihood. For simplicity, we assume expected earnings growth and volatility parameters are identical across firms. We calibrate idiosyncratic earnings volatility so that the total asset volatility is approximately 25%, the average asset volatility of firms with outstanding rated corporate debt (see Schaefer and Strebulaev (2008)). This yields an idiosyncratic earnings volatility of 22.8%. Andrade and Kaplan (1998) report default costs of about 10–25% of asset value

[7] Weitzman (2007) notes the 'inherent implausibility of being able to meaningfully calibrate rational – expectations – equilibria objective frequency distributions of rare disasters because the rarer the event the more uncertain is our estimate of its probability.'

and we assume $\alpha = 0.2$. We assume that the issuance cost, ι, is 2%. The corporate tax rate, η, is set at 20%.

The annualized rate of time preference, β, is 0.015. For the Disaster Model, relative risk aversion, γ, equals 8 and the EIS, ψ, equals 1/3.5.[8] These values ensure that the price of consumption risk is close as possible to historical levels for the Sharpe ratio, without making the risk – free rate too large, and ensuring that the discount rate, r_A, is positive. The latter constraint is the problematic, since it forces the risk – free rate to be almost 1% higher than its historical mean.

V.B Gauging the impact of rare disasters on credit risk and capital structure

In this section we study the impact of rare disasters on credit risk, capital structure and risk premia, relative to a model without rare disasters, i.e. where the disaster arrival intensity, λ, is zero.

We consider two comparisons. In the first comparison, we set $\beta = .015$, $\gamma = 8$, and $\psi = 1/3.5$ for both disaster and non – disaster models. This somewhat naive approach has the disadvantage that the risk – free rate in the non – disaster model will be unrealistically high. In fact the risk – free rate will be an order of magnitude higher than in the disaster model, because of a much reduced demand for precautionary savings. This decreases asset prices, including corporate debt, leading to much greater credit spreads. To avoid the model comparison being contaminated by the trivial effect of a much higher risk – free rate, we carry out a second comparison, where $\beta = .015$, $\gamma = 15$, and $\psi = 1.66$, in the non – disaster model, and $\beta = .015$, $\gamma = 8$, and $\psi = 1/3.5$, in the disaster model. This has the advantage that the risk – free rate and the price of consumption risk are identical in the non – disaster and disaster models. Consequently any differences in credit risk variables will no longer be driven by changes in asset prices caused by changes in the risk – free rate or price of consumption risk. Instead credit risk variables will change purely because of how the possibility of rare disasters impacts corporate financing decisions.

[8] Some studies estimate the EIS is less than one, (see, e.g. Hall (1988) and Campbell (1999)), while others (see, e.g. Hansen and Singleton (1982), Attanasio and Weber (1989), Vissing-Jorgensen (2002), Bansal and Yaron (2004), and Guvenen (2006)) estimate that the EIS is more than one.

The chief empirical difficulty facing a structural - equilibrium model is to obtain realistic credit spreads, leverage ratios, and equity risk prema, while keeping actual default probabilities close to observed values. Historical credit spreads and actual default rates are summarized in Table II.

V.B.1 Static capital structure

Table III reports results with static capital structure at date 0, when $X_t = X_0$.

First, we compare the Disaster Model with the the No Disaster Model, Benchmark I (Naive). We see that the risk – free rate is lower in the Disaster Model, because of increased demand for precautionary savings, while the price of consumption risk is higher, because of consumption disasters. The credit spread is slightly lower in the Disaster Model, which is a consequence of the much lower risk – free rate. The possibility of rare disasters makes firms more conservative when issuing debt, leading to a lower coupon, a higher interest coverage ratio and lower leverage. Since the possibility of rare disasters leads to a lower default boundary the actual probability of default for horizons beyond one year is lower in the disaster model. At the one year horizon, the possibility of a rare disaster increases the actual probability of default – this impact of downward jumps on short horizon default probabilities is well known (see, e.g. Lando (2004)).

Second, we compare the Disaster Model with the the No Disaster Model, Benchmark I (Naive). By construction, the conditional expectation of percentage changes to the state - price density (risk - free rate) and the conditional standard deviation of changes in the log state - price density (price of consumption risk) are identical across models. Credit spreads are higher in the Disaster Model, while leverage and actual default probabilities for horizons greater than one year are larger. This cannot be caused by a change in the risk - free rate or price of consumption risk. To see the intuition note that under the physical measure $\mathbb{P}$, disasters occur with an intensity of $\lambda = 0.015$, whereas under the risk – neutral measure $\mathbb{Q}$, disasters occur with an intensity of $\hat{\lambda} = \lambda\omega = 0.015 \times 9 = .135$. Hence, the term structure of cumulative jump probabilities is substantially higher under $\mathbb{Q}$ than $\mathbb{P}$, as shown in Figure 1. Since corporate financing decisions depend on asset prices, and assets are valued under $\mathbb{Q}$, financing decisions become much more conservative in the Disaster Model, even though disasters are rare. Hence, the optimal coupon and default boundary are lower. The lower coupon leads to lower leverage. The lower default boundary leads to smaller actual default probabilities for horizons greater than a year. Since disasters under the risk – neutral measure, $\mathbb{Q}$ are not rare (the risk – neutral probability of a disaster occurring within 5 years is 0.5), risk – neutral default probabilities are high relative to actual default probabilities, leading to a higher credit spread, despite the lower coupon and default boundary.

V.B.2 Dynamic capital structure at refinancing

Table IV reports results with dynamic capital structure at the time of refinancing, i.e. $X_t = X_0$. Having seen the limitations of comparing models with widely differing risk - free rates and prices of consumption risk, we compare the Disaster Model with a No Disaster Model which has the same risk - free rate and price of consumption risk. We implement this by setting $\beta = .015$, $\gamma = 15$, and $\psi = 1.66$, in the No Disaster model, and $\beta = .015$, $\gamma = 8$, and $\psi = 1/3.5$, in the Disaster model.

Qualitatively, the results are the same as under static capital structure. Quantitatively, there are differences: spreads are larger and leverage is smaller, both well known effects noted in Goldstein et al. (2001). Equity and debt values are more realistic in the Disaster Model, since risk premia are larger in this model, as a consequence of disaster risk being priced.

V.C Dynamic capital structure: aggregate dynamics and long - run behavior of credit risk variables

We now study the long run, aggregate implications of disaster risk for credit spreads, capital structure and equity risk premia, focusing solely on the Disaster Model. By aggregate implications, we mean that we study averages of variables taken over a cross - section of firms, as opposed to studying variables for an individual firm at the time of refinancing. This approach is analogous to Bertola and Caballero (1994), who study aggregate investment dynamics as opposed to individual firm dynamics. By long - run, we mean that we study the behaviour of aggregate variables not at date 0, but in the limit as time goes to infinity.[9] This allows is to study whether rare disasters have any long run implications for firms in the economy.

Specifically, we simulate the earnings processes for 1000 firms in 100 economies over 100 years. Since the variables we are interested in such as credit spreads, depend on normalized earnings X/X_0,

[9]Since, we cannot compute this limit analytically, we use simulation, as discussed below.

where X_0 is the earnings level at the most recent refinancing, rather than raw earnings X, we confine earnings to the interval $[X_D, X_U]$, defined by default boundary, X_D and the restructuring boundary X_U. When default occurs, a firm vanishes and an identical firm immediate enters the economy, with a cash flow level of X_0. When restructuring occurs, default and restructuring boundaries are scaled up by X_U/X_0, which for credit risk variables such as spreads, is equivalent to starting the earnings process again at X_0.

We find that computing long -run cross - sectional averages gives a term structure of credit spreads and default rates, together with leverage, which is close to that observed for BBB firms in the data. Furthermore, the equity risk premium is realistically large.

Importantly, spreads, leverage, default rates, and the equity risk premium are larger than at refinancing. This is because these variables are convex functions of X/X_D, a measure of the distance to default. The distribution of X/X_D is negatively skewed because of rare disasters. Hence, Jensen's inequality implies that rare disasters increase the long - run cross - sectional averages of credit spreads, default rates, leverage, and the equity risk premium.

The interest coverage ratio is linear in X. Consequently it's long - run cross - sectional mean is very close to it's value at refinancing.

VI Conclusion

We develop a dynamic capital structure model, which jointly prices corporate debt and equity in the presence of rare disasters which drive down consumption and firms' earnings.

Since leverage and default decisions are made optimally, we obtain an endogenous term structure of actual default probabilities and credit spreads. Firms are heterogeneous because their earnings growth rates are subject to idiosyncratic shocks. We find that in the presence of rare disasters, the model - implied term structure of credit spreads, actual default probabilities, together with aggregate leverage and the equity risk premium are close to their empirical counterparts.

Further exploration of our model is warranted. Two dimensions seem important. Firstly, implications for credit spreads and default probabilities at very short maturities should be explored, since this is where rare disasters may have quantitatively different implications for credit spreads than long - run risk. Secondly, we have not fully exploited our model's potential to capture cross - sectional heterogeneity in firms, since we have assumed they all have identical expected growth rates and growth rate volatilities and that their bonds have identical recovery rates.

Some caveats are also in order. We model disasters as instantaneous downward jumps and we ignore the possibility of recoveries, which biases our results on credit spreads upwards. Also, the world can undergo significant structural changes in response to rare disasters. One example of this is the tax code and changes in tax rates could significantly alter our results. Another example is inflation, which we have ignored entirely. Both deflation and inflation have occurred during economic disasters, which is clearly relevant for debt values. Of course, these are just two simple examples, based on historical observation. However, by their very nature, rare economic disasters can lead to other changes, which are inherently unpredictable.

A Appendix: Proofs

Proof of Propositions 1. The risk-neutral probability of a disaster occurring is given by

$$\hat{\lambda} = \lambda E_{t-}\left[\frac{\pi_t}{\pi_{t-}}\right]. \tag{A1}$$

From (8) it then follows that

$$\hat{\lambda} = \lambda\omega, \tag{A2}$$

where

$$\omega = E_{t-}\left[\left(\frac{C_t}{C_{t-}}\right)^{-\gamma}\right] \tag{A3}$$

Since C is given by (1), we obtain

$$\omega = E_{t-}[e^{-\gamma z_{C,t}}]. \tag{A4}$$

Note that if z is exponentially distributed with density function $\epsilon e^{\epsilon y}$ for $y \leq 0$ and 0 for $y > 0$, then

$$\begin{aligned} E[e^{\theta z}] &= \int_{-\infty}^{0} e^{\theta y}\epsilon e^{\epsilon y} dy & \text{(A5)}\\ &= \int_{0}^{\infty} e^{-\theta y}\epsilon e^{-\epsilon y} dy & \text{(A6)}\\ &= \frac{\epsilon}{\epsilon+\theta}, \text{if } \theta+\epsilon > 0. & \text{(A7)} \end{aligned}$$

Hence, if $\gamma < \epsilon$

$$\omega = E_{t-}[e^{-\gamma z_{C,t}}] = \frac{\epsilon_C}{\epsilon_C - \gamma}. \tag{A8}$$

Since $\epsilon_C = J_C^{-1} - 1$, then

$$\omega = \frac{1-J_C}{1-J_C(1+\gamma)}. \tag{A9}$$

Also, if $\gamma < \epsilon + 1$, then

$$E_{t-}[e^{-(\gamma-1)z_{C,t}}] = \frac{\epsilon_C}{\epsilon_C + 1 - \gamma} = \frac{1-J_C}{1-J_C\gamma} \tag{A10}$$

We now express $E_{t-}[e^{-(\gamma 1)z_{C,t}}]$ in terms of J_C and ω.

$$\begin{aligned} E_{t-}[e^{-(\gamma-1)z_{C,t}}] &= \frac{1-J_C}{1-J_C\gamma} & \text{(A11)}\\ &= \frac{1-J_C}{1-J_C(\gamma+1)+J_C} & \text{(A12)}\\ &= \left(\frac{1-J_C(\gamma+1)+J_C}{1-J_C}\right)^{-1} & \text{(A13)}\\ &= \left(\frac{1-J_C(\gamma+1)}{1-J_C} + \frac{J_C}{1-J_C}\right)^{-1} & \text{(A14)}\\ &= \left(\omega^{-1} + \frac{J_C}{1-J_C}\right)^{-1} & \text{(A15)}\\ &= \omega\left(1+\omega\frac{J_C}{1-J_C}\right)^{-1} & \text{(A16)}\\ &= \omega\frac{1}{1+\omega\frac{J_C}{1-J_C}}. & \text{(A17)} \end{aligned}$$

Thus, if we define $\overline{h}$ via

$$\overline{h}(x) = \frac{x^{1-\gamma}-1}{1-\gamma}, \tag{A18}$$

then

$$\begin{aligned} E_{t-}[\overline{h}(e^{z_{C,t}})] &= \frac{E_{t-}e^{(1-\gamma)z_{C,t}}-1}{1-\gamma} && \text{(A19)} \\ &= \frac{1-e^{\frac{1-J_C}{1-J_C\gamma}}}{\gamma-1} && \text{(A20)} \\ &= \frac{1-\omega\frac{1}{1+\omega\frac{J_C}{1-J_C}}}{\gamma-1}. && \text{(A21)} \end{aligned}$$

■

Proof of Proposition 2. Using simple algebra we can write the normalized Kreps-Porteus aggregator in the following compact form:

$$f(c,v) = \beta(h^{-1}(v))^{1-\gamma}\, u\left(c/h^{-1}(v)\right), \tag{A22}$$

where

$$\begin{aligned} u\,(x) &= \frac{x^{1-\frac{1}{\psi}}-1}{1-\frac{1}{\psi}}, \psi > 0, \\ h\,(x) &= \begin{cases} \frac{x^{1-\gamma}}{1-\gamma}, & \gamma \geq 0, \gamma \neq 1. \\ \ln x, & \gamma = 1. \end{cases} \end{aligned}$$

The representative agent's value function is given by

$$J_t = E_t \int_t^{\infty} f\left(C_t, J_t\right) dt. \tag{A23}$$

? show that the state-price density for a *general* normalized aggregator f is given by

$$\pi_t = e^{\int_0^t f_v(C_s,J_s)dt} f_c\left(C_t, J_t\right), \tag{A24}$$

where $f_c(\cdot,\cdot)$ and $f_v(\cdot,\cdot)$ are the partial derivatives of f with respect to its first and second arguments, respectively, and J is the value function given in (A23). The Feynman-Kac Theorem implies

$$f(C_{t-}, J_{t-})dt + E_{t-}[dJ_t] = 0.$$

Using Ito's Lemma we rewrite the above equation as

$$0 = f\left(C_{t-}, J_{t-}\right) + C_{t-}J_{t-,C}\,g + \frac{1}{2}C_{t-}^2 J_{t-,CC}\sigma_C^2 + \lambda\left(E_{t-}[J_t] - J_{t-}\right). \tag{A25}$$

We guess and verify that

$$J_t = h(C_t V), \tag{A26}$$

where V is given by

$$0 = \beta u(V^{-1}) + g - \frac{1}{2}\gamma\sigma_C^2 + \lambda E_{t-}[\overline{h}(e^{z_{C,t}})], \tag{A27}$$

and $\overline{h}(x) = (x^{1-\gamma} - 1)/(1-\gamma)$. Hence,

$$\beta V^{-\left(1-\frac{1}{\psi}\right)} = k - \lambda\left(1 - \frac{1}{\psi}\right) E_{t-}[\overline{h}(e^{z_{C,t}})], \tag{A28}$$

where

$$k = \overline{r} + \gamma\sigma_C^2 - g, \tag{A29}$$

$$\overline{r} = \beta + \frac{1}{\psi}g - \frac{1}{2}\gamma\left(1 + \frac{1}{\psi}\right)\sigma_C^2. \tag{A30}$$

From (A20) and (A21) it follows that

$$\beta V^{-\left(1-\frac{1}{\psi}\right)} = k + \lambda\left(1 - \frac{1}{\psi}\right)\frac{e^{\frac{1-J_C}{1-J_C\gamma}} - 1}{\gamma - 1}, \tag{A31}$$

and

$$\beta V^{-\left(1-\frac{1}{\psi}\right)} = k + \lambda\left(1 - \frac{1}{\psi}\right)\frac{\omega\frac{1}{1+\omega\frac{J_C}{1-J_C}} - 1}{\gamma - 1}, \tag{A32}$$

respectively.

Substituting (A22) into (A24) and using (A26) gives

$$\pi_t = \beta e^{-\beta\left[1+\left(\gamma-\frac{1}{\psi}\right)u\left(V^{-1}\right)\right]t} C_t^{-\gamma} V^{-\left(\gamma-\frac{1}{\psi}\right)}. \tag{A33}$$

When $\psi = 1$, the above equation gives the second expression in (8). We rewrite (A27) as

$$\beta\left[1 + \left(\gamma - \frac{1}{\psi}\right)u\left(\frac{1}{V}\right)\right] = \overline{r} - \left(\gamma - \frac{1}{\psi}\right)\lambda\left(E_{t-}[\overline{h}(e^{z_t})]\right) - \left[\gamma g - \frac{1}{2}\gamma\left(1+\gamma\right)\sigma_C^2\right], \tag{A34}$$

where $\overline{r}$ is given in (10). Setting $\psi = 1$ in the above equation gives

$$\beta \ln V = g - \frac{\gamma}{2}\sigma_C^2 + \lambda E_{t-}[\overline{h}(e^{z_t})], \tag{A35}$$

which implies that

$$\beta \ln V = g - \frac{\gamma}{2}\sigma_C^2 - \lambda\frac{e^{\frac{1-J_C}{1-J_C\gamma}} - 1}{\gamma - 1}, \tag{A36}$$

and (12).

To derive the first expression in (8) from (A33) we prove that

$$V = (\beta p_C)^{\frac{1}{1-\frac{1}{\psi}}}, \psi \neq 1. \tag{A37}$$

We proceed by considering the optimization problem for the representative agent. She chooses her optimal consumption, C^*, and risky asset portfolio, φ, to maximize her expected utility

$$J_t^* = \sup_{C^*,\varphi} E_t \int_t^\infty f\left(C_t^*, J_t^*\right) dt.$$

Observe that J^* depends on optimal consumption-portfolio choice, whereas the J defined previously in (11) depends on exogenous aggregate consumption. The optimization is carried out subject to the dynamic budget constraint, which we now describe. If the agent consumes at the rate, C^*, invests a proportion, φ, of her remaining financial wealth in the claim on aggregate consumption (the risky asset), and puts the remainder in the locally risk-free asset, then her financial wealth, W, evolves according to the dynamic budget constraint:

$$\frac{dW_t}{W_{t-}} = \varphi_{t-}\left(dR_{C,t} - r_{t-}dt\right) + r_{t-}dt - \frac{C^*_{t-}}{W_{t-}}dt,$$

where $dR_{C,t}$ is the cumulative return on the claim to aggregate consumption. The compensated version of the Poisson process, N_t, is the Poisson martingale

$$N_t^P = N_t - \lambda t.$$

It follows from applying Ito's Lemma to $P_t = p_C C_t$ that the cumulative return on the claim to aggregate consumption is

$$dR_{C,t} = \frac{dP_t + C_t dt}{P_{t-}} = \frac{dC_t}{C_{t-}} + \frac{1}{p_C}dt = \mu_{R_C,t-}dt + \sigma_C dB_{C,t} + (e^{z_t} - 1)dN_t^P,$$

where

$$\mu_{R_C,t-} \quad = \quad g + \lambda E_{t-}[e^{z_t} - 1] + \frac{1}{p_C}. \tag{A38}$$

The total volatility of the return to holding the consumption claim is given by

$$\sigma_{R_C} = \sqrt{\sigma_C^2 + \lambda E_{t-}\left((e^{z_t} - 1)\right)^2}.$$

Note that C^* is the consumption to be chosen by the agent, i.e. it is a control. The Hamilton-Jacobi-Bellman differential equation for the agent's optimization problem is

$$\sup_{C^*,\varphi} f\left(C^*_{t-}, J^*_{t-}\right)dt + E_{t-}\left[dJ^*_t\right] = 0.$$

Applying Ito's Lemma to $J^*_t = J^*(W_t)$ allows us to write the above equation as

$$\begin{aligned} 0 \quad = \quad & \sup_{C^*,\varphi} f\left(C^*_{t-}, J^*_{t-}\right) + W_{t-}J^*_{W,t-}\left(\varphi_i\left(\mu_{R_C,t-} - \lambda E_{t-}[e^{z_t} - 1] - r_{t-}\right) + r_{t-} - \frac{C^*_{t-}}{W_{t-}}\right) + \frac{1}{2}W_{t-}^2 J^*_{WW,t-}\varphi_{t-}^2\sigma_C^2 + \\ & + \lambda E_{t-}\left(J^*_t - J^*_{t-}\right). \end{aligned}$$

We guess and verify that $J^*_t = h(W_t F)$, where F is given by

$$0 = \sup_{C^*,\varphi} \beta u\left(\frac{C^*_{t-}}{W_{t-}F}\right) + \left(\varphi_i\left(\mu_{R_C,t-} - \lambda E_{t-}[e^{z_t} - 1] - r_{t-}\right) + r_{t-} - \frac{C^*_{t-}}{W_{t-}}\right) - \frac{1}{2}\gamma\varphi_{t-}^2\sigma_C^2 + \lambda E_{t-}[\overline{h}(1 + \varphi(e^{z_t} - 1))].$$

The first order conditions of the above equation are:

$$\begin{aligned} C^*_t \quad &= \quad \beta^\psi F^{-(\psi-1)}W_t, \\ \mu_{R_C,t-} - \lambda E_{t-}[e^{z_t} - 1] - r_{t-} - \gamma\sigma_C^2 \quad &= \quad -\lambda E_{t-}[(1 + \varphi(e^{z_t} - 1))^{-\gamma}(e^{z_t} - 1)]. \end{aligned}$$

The market for the consumption good must clear, so $\varphi = 1$, $W_t = P_t$, $C^*_t = C_t$ (and thus $J_t = J^*_t$). Note that this forces the optimal portfolio proportion to be one. Hence

$$\mu_{R_C,t-} - r_{t-} = \gamma\sigma_C^2 + \lambda E_{t-}[(1 - e^{-\gamma z_t})(e^{z_t} - 1)], \tag{A39}$$

and

$$p_C = \beta^{-\psi} F^{\psi - 1}. \tag{A40}$$

The above equation implies that for $\psi = 1$, $p_C = 1/\beta$. The equality, $J = J^*$, implies that $CV = WF$. Hence, $F = p_C^{-1} V$. Using this equation to eliminate F from (A40) gives (A37). Substituting (A37) into (A33) and (A32) gives the expression in (8) for $\psi \neq 1$ and (9).

From (A38) and (A39) it follows that

$$r_{t-} = \overline{r} - \lambda \left(\gamma - \frac{1}{\psi} \right) E_{t-}[\overline{h}(e^{z_{C,t}})] + \lambda E_{t-}[1 - e^{-\gamma z_t}]. \tag{A41}$$

Since $\hat{\lambda} = \lambda \omega$, the risk-free rate can be rewritten as

$$r_{t-} = \overline{r} - \lambda \left(\gamma - \frac{1}{\psi} \right) E_{t-}[\overline{h}(e^{z_{C,t}})] - \lambda(\omega - 1). \tag{A42}$$

Using (A21), it then follows that

$$r_{t-} = \overline{r} + \lambda \left(\gamma - \frac{1}{\psi} \right) \frac{\omega \frac{1}{1+\omega \frac{J_C}{1-J_C}} - 1}{\gamma - 1} - \lambda(\omega - 1). \tag{A43}$$

$$\begin{aligned} E_{t-}[(1 - e^{-\gamma z_{C,t}})(e^{z_{C,t}} - 1)] &= E_{t-} e^{z_{C,t}} - 1 - E_{t-} e^{-(\gamma - 1) z_{C,t}} + E_{t-} e^{-\gamma z_{C,t}} && \text{(A44)} \\ &= \frac{\epsilon_C}{\epsilon_C + 1} - 1 - \frac{\epsilon_C}{\epsilon_C + 1 - \gamma} + \frac{\epsilon_C}{\epsilon_C - \gamma} && \text{(A45)} \end{aligned}$$

Ito's Lemma implies that the state-price density evolves according to

$$\begin{aligned} \frac{d\pi_t}{\pi_{t-}} &= \frac{1}{\pi_{t-}} \frac{\partial \pi_{t-}}{\partial t} dt + \frac{1}{\pi_{t-}} C_t \frac{\partial \pi_{t-}}{\partial C_t} \frac{dC_t}{C_t} + \frac{1}{2} \frac{1}{\pi_{t-}} C_t^2 \frac{\partial^2 \pi_{t-}}{\partial C_t^2} \left(\frac{dC_t}{C_t} \right)^2 \\ &\quad + \lambda \frac{\Delta \pi_t}{\pi_{t-}} dt + \frac{\Delta \pi_t}{\pi_{t-}} dN_t^P, && \text{(A46)} \end{aligned}$$

where $dN_t^P = dN^P - \lambda dt$ and $\Delta \pi_t = \pi_t - \pi_{t-}$. Observe that

$$\begin{aligned} \frac{\Delta \pi_t}{\pi_{t-}} &= \left(\frac{C_t}{C_{t-}} \right)^{-\gamma} - 1 && \text{(A47)} \\ &= e^{-\gamma z_{C,t}} - 1. && \text{(A48)} \end{aligned}$$

$$\frac{d\pi_t}{\pi_{t-}} = -r_t dt - \Theta^B dB_{C,t} - \Theta^P dN_t^P, \tag{A49}$$

where

$$\begin{aligned} \Theta^B &= \gamma \sigma_C && \text{(A50)} \\ \Theta_P &= e^{-\gamma z_{C,t}} - 1. && \text{(A51)} \end{aligned}$$

The price of consumption risk is given by

$$\Theta = \sqrt{\gamma^2 \sigma_C^2 + \lambda E_{t-}[(e^{-\gamma z_{C,t}} - 1)^2]}. \tag{A52}$$

■

Proof of Proposition 3. First, define $x_t = \ln X_t$. It follows from Ito's Lemma that under $\mathbb{Q}$

$$dx_t = \left(\hat{\mu}_X - \frac{1}{2}\sigma_X^2\right) dt + \sigma_X d\hat{B}_{X,t} + z_t d\hat{N}_t, \tag{A53}$$

where $\hat{N}$ is a Poisson process with intensity $\lambda\omega$ under $\mathbb{Q}$. Note that

$$x_{k,t} = \left(\hat{\mu}_X - \frac{1}{2}\sigma_X^2\right) t + \sigma_X \hat{B}_{X,t} + \sum_{n=1}^{N_t} z_n, \tag{A54}$$

where $(z_n)_{n\in\mathbb{N}}$ is an i.i.d. sequence of random variables, exponentially distributed, with a common density, given by

$$f_z(y) = \epsilon e^{\epsilon y} 1_{\{y<0\}}. \tag{A55}$$

Note that $E^{\mathbb{Q}}[z_n] = -1/\epsilon$ and $Var^{\mathbb{Q}}[z_n] = (1/\epsilon)^2$. The distribution for jump size is identical under $\mathbb{P}$ and $\mathbb{Q}$, because jump size is independent of the Brownian motion and Poisson process, which drive innovations in earnings growth.

To use the results in, define $\tilde{x}_t = -x_t$. Hence,

$$d\tilde{x}_t = \tilde{\mu}_x dt - \sigma_X d\hat{B}_{X,t} + \tilde{z}_t d\hat{N}_t, \tag{A56}$$

and

$$\tilde{x}_t = \tilde{\mu}_x t - \sigma_X \hat{B}_{X,t} + \sum_{n=1}^{N_t} \tilde{z}_n, \tag{A57}$$

where

$$\tilde{\mu}_x = -\left(\hat{\mu}_X - \frac{1}{2}\sigma_X^2\right), \tilde{z}_t = -z_t. \tag{A58}$$

The density for $\tilde{z}$ is

$$\tilde{f}_{\tilde{z}}(y) = \epsilon e^{\epsilon y} 1_{\{y>0\}}. \tag{A59}$$

Note that $E^{\mathbb{Q}}[\tilde{z}_n] = 1/\epsilon$ and $Var^{\mathbb{Q}}[\tilde{z}_n] = (1/\epsilon)^2$. Also,

$$\tau_D = \inf_{t>0} \{\tilde{x}_t \geq \tilde{x}_D\}, \tag{A60}$$

where $\tilde{x}_D = \ln(1/X_D)$.

We now compute the cumulant generating function of $\tilde{x}_t$, defined by

$$g(\theta) = \frac{1}{t} \ln E^{\mathbb{Q}}[e^{\theta\tilde{x}_t}]. \tag{A61}$$

Note that

$$E^{\mathbb{Q}}[e^{\theta\tilde{x}_t}] = e^{\tilde{\mu}_x t} E^{\mathbb{Q}} e^{-\sigma_X \hat{B}_{X,t} + \sum_{n=1}^{N_t} \tilde{z}_n}, \tag{A62}$$

$$= e^{\tilde{\mu}_x t} E^{\mathbb{Q}}[e^{-\sigma_X \hat{B}_{X,t}}] E^{\mathbb{Q}}[e^{\sum_{n=1}^{N_t} \tilde{z}_n}], \tag{A63}$$

where the previous line is a consequence of independence. Thus,

$$E^{\mathbb{Q}}[e^{\theta x_t}] = e^{\tilde{\mu}_x \theta t + \frac{1}{2}\sigma_X^2 \theta^2 t^2} e^{\lambda\omega t E^{\mathbb{Q}}[e^{\theta z_n} - 1]}, \tag{A64}$$

$$= e^{\tilde{\mu}_x \theta t + \frac{1}{2}\sigma_X^2 \theta^2 t^2 + \lambda\omega t\left(\frac{\epsilon}{\epsilon-\theta} - 1\right)}. \tag{A65}$$

It follows that

$$g(\theta) = \tilde{\mu}_x\theta + \frac{1}{2}\sigma_X^2\theta^2 + \lambda\omega\left(\frac{\epsilon}{\epsilon-\theta} - 1\right) \tag{A66}$$

The equation

$$g(\theta) = r, \tag{A67}$$

reduces to a cubic and has three roots, denoted by are denoted by θ_1, θ_2 and $-\theta_3$, where $0 < \theta_1 < \epsilon < \theta_2$ and $\theta_3 > 0$

From, we know that when $\tilde{x}_0 = 0$, then

$$E[e^{-r\tau_D}] = \frac{\epsilon-\theta_1}{\epsilon}\frac{\theta_2}{\theta_2-\theta_1}e^{-\theta_1\tilde{x}_D} + \frac{\theta_2-\epsilon}{\epsilon}\frac{\theta_1}{\theta_2-\theta_1}e^{-\theta_2\tilde{x}_D}, \tag{A68}$$

which can be rewritten as

$$E[e^{-r\tau_D}] = \frac{\epsilon-\theta_1}{\epsilon}\frac{\theta_2}{\theta_2-\theta_1}X_D^{\theta_1} + \frac{\theta_2-\epsilon}{\epsilon}\frac{\theta_1}{\theta_2-\theta_1}X_D^{\theta_2}. \tag{A69}$$

Therefore,

$$q_{D,t} = E_t[e^{-r(\tau_D-t)}] = \frac{\epsilon-\theta_1}{\epsilon}\frac{\theta_2}{\theta_2-\theta_1}\left(\frac{X_D}{X_t}\right)^{\theta_1} + \frac{\theta_2-\epsilon}{\epsilon}\frac{\theta_1}{\theta_2-\theta_1}\left(\frac{X_D}{X_t}\right)^{\theta_2} \tag{A70}$$

We also know from Chen & Kou (2007) that when $\tilde{x}_0 = 0$

$$E[e^{-r\tau_D + a\tilde{x}_{\tau_D}}] = e^{a\tilde{x}_D}(c_1e^{-\tilde{x}_D\theta_1} + c_2e^{-\tilde{x}_D\theta_2}), \tag{A71}$$

where

$$c_1(a) \quad = \quad \frac{\epsilon-\theta_1}{\theta_2-\theta_1}\frac{\theta_2-a}{\epsilon-a}, \tag{A72}$$

$$c_2(a) \quad = \quad \frac{\theta_2-\epsilon}{\theta_2-\theta_1}\frac{\theta_1-a}{\epsilon-a}. \tag{A73}$$

Rewriting , we obtain

$$E[e^{-r\tau_D + a\tilde{x}_{\tau_D}}] = X_D^{-a}(c_1(a)X_D^{\theta_1} + c_2(a)X_D^{\theta_2}). \tag{A74}$$

Hence,

$$q_{D,t}^X \quad = \quad E_t[e^{-r(\tau_D-t)}X_{\tau_D}] = E_t[e^{-r(\tau_D-t)-\tilde{x}_{\tau_D}}] \tag{A75}$$

$$= \quad X_t\left[\frac{\epsilon-\theta_1}{\theta_2-\theta_1}\frac{1+\theta_2}{1+\epsilon}\left(\frac{X_D}{X_t}\right)^{1+\theta_1} + \frac{\theta_2-\epsilon}{\theta_2-\theta_1}\frac{1+\theta_1}{1+\epsilon}\left(\frac{X_D}{X_t}\right)^{1+\theta_2}\right]. \tag{A76}$$

$$\tau_D \inf_{t>0}\left\{e^{-x_t^-} \le X_D\right\}. \tag{A77}$$

$$E_t[e^{-r(\tau_D-t)}X_{\tau_D}] = d_1e^{-\gamma_1x_t} + d_2e^{-\gamma_2x_t} \tag{A78}$$

■

Proof of Proposition 4. To find the abandonment value,

$$A(X_t) = (1-\eta)E_t\int_t^\infty\left[\frac{\pi_s}{\pi_t}Y_s\right]ds, \tag{A79}$$

we note that

$$A(X_t) = (1-\eta)X_tp^X, \tag{A80}$$

where

$$p^X = E_t \int_t^\infty \left[\frac{\pi_s}{\pi_t} \frac{X_s}{X_t} \right] ds. \tag{A81}$$

Under $\mathbb{Q}$, the basic asset pricing equation implies that

$$E_t^{\mathbb{Q}} \left[\frac{dA + (1-\eta)X}{A} \right] = 0. \tag{A82}$$

Now, under $\mathbb{Q}$

$$\frac{dX_t}{X_{t-}} = \hat{\mu}_X dt + \sigma_X d\hat{B}_X + (e^{z_{k,t}} - 1)(d\hat{N}^P + \hat{\lambda} dt), \tag{A83}$$

where $\hat{B}_X$ is a standard Brownian motion under $\mathbb{Q}$, $\hat{N}^P$ a Poisson martingale under $\mathbb{Q}$,

$$\begin{aligned} \hat{\mu}_X &= \mu_X - \Theta^B \rho_{XC} \sigma_X^s, && \text{(A84)} \\ \Theta_B &= \gamma \sigma_C, && \text{(A85)} \end{aligned}$$

and $\hat{\lambda}$ is the risk-neutral probability of a disaster occurring. Hence,

$$\frac{1}{2}\sigma_X^2 A_{XX}(X_t) + \hat{\mu}_X X A_X(X_t) + \lambda\omega E_{t-}[A(X_t) - A(X_{t-})] + (1-\eta)X_t - rA(X_t). \tag{A86}$$

Thus,

$$\hat{\mu}_X X(1-\eta)p^X + \lambda\omega(1-\eta)p^X X E_{t-}[e^{z_{k,t}} - 1] + (1-\eta)X - rp^X(1-\eta)X = 0. \tag{A87}$$

Hence,

$$p^X = \frac{1}{r - \hat{\mu}_X - \lambda\omega E_{t-}[e^{z_{k,t}} - 1]}. \tag{A88}$$

From (A7) it follows that

$$E_{t-}[e^{z_{k,t}}] = \frac{\epsilon_k}{1+\epsilon_k}. \tag{A89}$$

Hence,

$$\begin{aligned} E_{t-}[e^{z_{k,t}}] &= \frac{J_k^{-1} - 1}{J_k^{-1}} && \text{(A90)} \\ &= 1 - J_k, && \text{(A91)} \end{aligned}$$

and so

$$p^X = \frac{1}{r - \hat{\mu}_X + \hat{\lambda} J_k}. \tag{A92}$$

■

Proof of Proposition 6.

The value of levered equity is given by

$$\begin{aligned} S_t = (1-\eta)E_t^{\mathbb{Q}}\left[\int_t^{\tau_D} e^{-r(s-t)}(X_s - c)\right] &= (1-\eta)E_t^{\mathbb{Q}}\left[\int_t^{\infty} e^{-r(s-t)}(X_s - c)\right] - (1-\eta)E_t^{\mathbb{Q}}\left[\int_{\tau_D}^{\infty} e^{-r(s-t)}(X_s - c)\right] && \text{(A93)} \\ &= (1-\eta)E_t^{\mathbb{Q}}\left[\int_t^{\infty} e^{-r(s-t)}(X_s - c)\right] - (1-\eta)E_t^{\mathbb{Q}}\left[\int_{\tau_D}^{\infty} e^{-r(\tau_D-t)} e^{-r(s-\tau_D)}(X_s - c)\right] && \text{(A94)} \\ &= (1-\eta)E_t^{\mathbb{Q}}\left[\int_t^{\infty} e^{-r(s-t)}(X_s - c)\right] - (1-\eta)E_t^{\mathbb{Q}}\left\{e^{-r(\tau_D-t)} E_{\tau_D}^{\mathbb{Q}}\left[\int_{\tau_D}^{\infty} e^{-r(s-\tau_D)}(X_s - c)\right]\right\} && \text{(A95)} \end{aligned}$$

Now note that

$$E_t^{\mathbb{Q}}\left[\int_t^{\infty} e^{-r(s-t)}(X_s - c)\right] = p^X X_t - \frac{c}{r}. \tag{A96}$$

Hence,

$$\begin{aligned}
S_t &= (1-\eta)\left(p^X X_t - \frac{c}{r} - E_t\left[e^{-r(\tau_D - t)}\left(p^X X_{\tau_D} - \frac{c}{r}\right)\right]\right) && \text{(A97)}\\
&= (1-\eta)\left[p^X X_t - \frac{c}{r} - \left(q_{D,t}^X p^X - q_{D,t}\frac{c}{r}\right)\right] && \text{(A98)}\\
&= (1-\eta)\left[p^X (X_t - q_{D,t}^X) - \frac{c}{r}(1 - q_{D,t})\right] && \text{(A99)}\\
&= A(X_t) - (1-\eta)\frac{c}{r} + q_{D,t}(1-\eta)\frac{c}{r} - q_{D,t}^X(1-\eta)\frac{1}{r_A}. && \text{(A100)}
\end{aligned}$$

Applying Ito's Lemma gives

$$\begin{aligned}
dS_t &= dA(X_t) + dq_{D,t}(1-\eta)\frac{c}{r} - dq_{D,t}^X(1-\eta)\frac{1}{r_A} && \text{(A101)}\\
&= (1-\eta)\frac{1}{r_A}dX_t + \frac{\partial q_{D,t}}{\partial X}dX_t(1-\eta)\frac{c}{r} + \frac{1}{2}\frac{\partial^2 q_{D,t}}{\partial X^2}dX_t.dX_t(1-\eta)\frac{c}{r} - \frac{\partial q_{D,t}^X}{\partial X}dX_t(1-\eta)\frac{1}{r_A} && \text{(A102)}\\
&\quad -\frac{1}{2}\frac{\partial^2 q_{D,t}^X}{\partial X^2}dX_t.dX_t(1-\eta)\frac{1}{r_A} && \text{(A103)}\\
&= \left[(1-\eta)\frac{1}{r_A} + \frac{\partial q_{D,t}}{\partial X}(1-\eta)\frac{c}{r} - \frac{\partial q_{D,t}^X}{\partial X}(1-\eta)\frac{1}{r_A}\right]dX_t + \frac{1}{2}dX_t.dX_t(1-\eta)\left[\frac{\partial^2 q_{D,t}}{\partial X^2}\frac{c}{r} - \frac{\partial^2 q_{D,t}^X}{\partial X^2}\frac{1}{r_A}\right] && \text{(A104)}
\end{aligned}$$

■

Proof of Proposition 7. We use the smooth pasting condition

$$\left.\frac{\partial S}{\partial X}\right|_{X=X_D} = 0 \tag{A105}$$

to derive the optimal default boundary, X_D, for a given coupon c. From (), it follows that

$$\frac{\partial S}{\partial X} = (1-\eta)\frac{1}{r_A} + \frac{\partial q_D}{\partial X}(1-\eta)\frac{c}{r} - \frac{\partial q_D^X}{\partial X}(1-\eta)\frac{1}{r_A}. \tag{A106}$$

$$\begin{aligned}
\left.\frac{\partial q_D}{\partial X}\right|_{X=X_D} &= -\frac{1}{X_D}\left[\frac{\epsilon-\theta_1}{\epsilon}\frac{\theta_2}{\theta_2-\theta_1}\theta_1 + \frac{\theta_2-\epsilon}{\epsilon}\frac{\theta_1}{\theta_2-\theta_1}\theta_2\right] && \text{(A107)}\\
&= -\frac{1}{X_D}\frac{\theta_1\theta_2}{\epsilon(\theta_2-\theta_1)}(\theta_2-\theta_1) && \text{(A108)}\\
&= -\frac{1}{X_D}\frac{\theta_1\theta_2}{\epsilon} && \text{(A109)}
\end{aligned}$$

$$
\begin{aligned}
\left.\frac{\partial q_D^X}{\partial X}\right|_{X=X_D} &= \left[\frac{\epsilon-\theta_1}{\theta_2-\theta_1}\frac{1+\theta_2}{1+\epsilon}+\frac{\theta_2-\epsilon}{\theta_2-\theta_1}\frac{1+\theta_1}{1+\epsilon}\right] && \text{(A110)}\\
&\quad -\left[\frac{\epsilon-\theta_1}{\theta_2-\theta_1}\frac{(1+\theta_2)(1+\theta_1)}{1+\epsilon}+\frac{\theta_2-\epsilon}{\theta_2-\theta_1}\frac{(1+\theta_1)(1+\theta_2)}{1+\epsilon}\right] && \text{(A111)}\\
&= \frac{(\epsilon-\theta_1)(1+\theta_2)+(\theta_2-\epsilon)(1+\theta_1)}{(\theta_2-\theta_1)(1+\epsilon)}-\frac{(1+\theta_2)(1+\theta_1)}{1+\epsilon} && \text{(A112)}\\
&= \frac{1}{1+\epsilon}\left[\frac{(\epsilon-\theta_1)(1+\theta_2)+(\theta_2-\epsilon)(1+\theta_1)}{(\theta_2-\theta_1)}-(1+\theta_2)(1+\theta_1)\right] && \text{(A113)}\\
&= \frac{1}{1+\epsilon}\left[\frac{\epsilon(1+\theta_2)-\theta_1(1+\theta_2)+\theta_2(1+\theta_1)-\epsilon(1+\theta_1)}{(\theta_2-\theta_1)}-(1+\theta_2)(1+\theta_1)\right] && \text{(A114)}\\
&= \frac{1}{1+\epsilon}\left[\frac{\epsilon(\theta_2-\theta_1)+\theta_2-\theta_1}{(\theta_2-\theta_1)}-(1+\theta_2)(1+\theta_1)\right] && \text{(A115)}\\
&= 1-\frac{(1+\theta_2)(1+\theta_1)}{1+\epsilon} && \text{(A116)}
\end{aligned}
$$

Hence,

$$
\begin{aligned}
\left.\frac{\partial S}{\partial X}\right|_{X=X_D} &= (1-\eta)\frac{1}{r_A}+(1-\eta)\frac{c}{r}\left.\frac{\partial q_D}{\partial X}\right|_{X=X_D}-(1-\eta)\frac{1}{r_A}\left.\frac{\partial q_D^X}{\partial X}\right|_{X=X_D} && \text{(A117)}\\
&= (1-\eta)\frac{1}{r_A}-(1-\eta)\frac{c}{r}\frac{1}{X_D}\frac{\theta_1\theta_2}{\epsilon}-(1-\eta)\frac{1}{r_A}\left(1-\frac{(1+\theta_2)(1+\theta_1)}{1+\epsilon}\right) && \text{(A118)}\\
&= -(1-\eta)\frac{c}{r}\frac{1}{X_D}\frac{\theta_1\theta_2}{\epsilon}+(1-\eta)\frac{1}{r_A}\frac{(1+\theta_2)(1+\theta_1)}{1+\epsilon} && \text{(A119)}
\end{aligned}
$$

It then follows from the smooth pasting condition that

$$
\begin{aligned}
-\frac{c}{r}\frac{1}{X_D}\frac{\theta_1\theta_2}{\epsilon}+\frac{1}{r_A}\frac{(1+\theta_2)(1+\theta_1)}{1+\epsilon} &= 0 && \text{(A120)}\\
X_D &= \frac{\frac{c}{r}\frac{\theta_1\theta_2}{\epsilon}}{\frac{1}{r_A}\frac{(1+\theta_2)(1+\theta_1)}{1+\epsilon}} && \text{(A121)}\\
X_D &= r_A\frac{c}{r}\frac{\theta_1\theta_2}{(1+\theta_2)(1+\theta_1)}\frac{1+\epsilon}{\epsilon} && \text{(A122)}\\
X_D = Kc, & && \text{(A123)}
\end{aligned}
$$

where

$$K=\frac{r_A}{r}\frac{\theta_1\theta_2}{(1+\theta_2)(1+\theta_1)}\frac{1+\epsilon}{\epsilon} \tag{A124}$$

■

Proof. The value of the levered firm is given by

$$F=S+B, \tag{A125}$$

i.e.

$$
\begin{aligned}
F &= A(X_t) - (1-\eta)\frac{c}{r} + q_{D,t}(1-\eta)\frac{c}{r} - q^X_{D,t}(1-\eta)\frac{1}{r_A} + \frac{c}{r}(1-q_{D,t}) + \alpha(1-\eta)\frac{1}{r_A}q^X_{D,t} && \text{(A126)}\\
&= A(X_t) - (1-\eta)(1-q_{D,t})\frac{c}{r} - q^X_{D,t}(1-\eta)\frac{1}{r_A} + \frac{c}{r}(1-q_{D,t}) + \alpha(1-\eta)\frac{1}{r_A}q^X_{D,t} && \text{(A127)}\\
&= A(X_t) + \eta(1-q_{D,t})\frac{c}{r} - q^X_{D,t}(1-\eta)\frac{1}{r_A} + \alpha(1-\eta)\frac{1}{r_A}q^X_{D,t} && \text{(A128)}\\
&= A(X_t) + \eta\frac{c}{r} - q_{D,t}\eta\frac{c}{r} - q^X_{D,t}(1-\alpha)(1-\eta)\frac{1}{r_A} && \text{(A129)}
\end{aligned}
$$

$$
\frac{\partial F}{\partial c} = \eta\frac{1}{r} - \frac{\partial q_{D,t}}{\partial X_D}K\eta\frac{c}{r} - q_{D,t}\eta\frac{1}{r} - \frac{\partial q^X_{D,t}}{\partial X_D}K(1-\alpha)(1-\eta)\frac{1}{r_A} \qquad \text{(A130)}
$$

$$
\begin{aligned}
\frac{\partial q_{D,t}}{\partial X_D} &= \frac{1}{X_D}\left[\frac{\epsilon-\theta_1}{\epsilon}\frac{\theta_2\theta_1}{\theta_2-\theta_1}\left(\frac{X_D}{X_t}\right)^{\theta_1} + \frac{\theta_2-\epsilon}{\epsilon}\frac{\theta_1\theta_2}{\theta_2-\theta_1}\left(\frac{X_D}{X_t}\right)^{\theta_2}\right] && \text{(A131)}\\
&= \frac{1}{X_D}\frac{\theta_2\theta_1}{\epsilon(\theta_2-\theta_1)}\left[(\epsilon-\theta_1)\left(\frac{X_D}{X_t}\right)^{\theta_1} + (\theta_2-\epsilon)\left(\frac{X_D}{X_t}\right)^{\theta_2}\right] && \text{(A132)}\\
&= \frac{1}{Kc}\frac{\theta_2\theta_1}{\epsilon(\theta_2-\theta_1)}\left[(\epsilon-\theta_1)\left(\frac{Kc}{X_t}\right)^{\theta_1} + (\theta_2-\epsilon)\left(\frac{Kc}{X_t}\right)^{\theta_2}\right] && \text{(A133)}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial q^X_{D,t}}{\partial X_D} &= \left[\frac{\epsilon-\theta_1}{\theta_2-\theta_1}\frac{(1+\theta_2)(1+\theta_1)}{1+\epsilon}\left(\frac{X_D}{X_t}\right)^{\theta_1} + \frac{\theta_2-\epsilon}{\theta_2-\theta_1}\frac{(1+\theta_1)(1+\theta_2)}{1+\epsilon}\left(\frac{X_D}{X_t}\right)^{\theta_2}\right] && \text{(A134)}\\
&= \frac{(1+\theta_1)(1+\theta_2)}{(\theta_2-\theta_1)(1+\epsilon)}\left[(\epsilon-\theta_1)\left(\frac{X_D}{X_t}\right)^{\theta_1} + (\epsilon-\theta_1)\left(\frac{X_D}{X_t}\right)^{\theta_2}\right] && \text{(A135)}\\
&= \frac{(1+\theta_1)(1+\theta_2)}{(\theta_2-\theta_1)(1+\epsilon)}\left[(\epsilon-\theta_1)\left(\frac{Kc}{X_t}\right)^{\theta_1} + (\epsilon-\theta_1)\left(\frac{Kc}{X_t}\right)^{\theta_2}\right] && \text{(A136)}
\end{aligned}
$$

$$
\begin{aligned}
q_{D,t} &= \frac{\epsilon-\theta_1}{\epsilon}\frac{\theta_2}{\theta_2-\theta_1}\left(\frac{X_D}{X_t}\right)^{\theta_1} + \frac{\theta_2-\epsilon}{\epsilon}\frac{\theta_1}{\theta_2-\theta_1}\left(\frac{X_D}{X_t}\right)^{\theta_2} && \text{(A137)}\\
&= \frac{1}{\epsilon(\theta_2-\theta_1)}\left[(\epsilon-\theta_1)\theta_2\left(\frac{X_D}{X_t}\right)^{\theta_1} + (\epsilon-\theta_2)\theta_1\left(\frac{X_D}{X_t}\right)^{\theta_2}\right] && \text{(A138)}\\
&= \frac{1}{\epsilon(\theta_2-\theta_1)}\left[(\epsilon-\theta_1)\theta_2\left(\frac{Kc}{X_t}\right)^{\theta_1} + (\epsilon-\theta_2)\theta_1\left(\frac{Kc}{X_t}\right)^{\theta_2}\right] && \text{(A139)}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial F}{\partial c} &= \eta\frac{1}{r} - \frac{\partial q_{D,t}}{\partial X_D}K\eta\frac{c}{r} - q_{D,t}\eta\frac{1}{r} - \frac{\partial q^X_{D,t}}{\partial X_D}K(1-\alpha)(1-\eta)\frac{1}{r_A} && \text{(A140)}\\
&= \eta\frac{1}{r} - \frac{1}{Kc}\frac{\theta_2\theta_1}{\epsilon(\theta_2-\theta_1)}\left[(\epsilon-\theta_1)\left(\frac{Kc}{X_t}\right)^{\theta_1} + (\theta_2-\epsilon)\left(\frac{Kc}{X_t}\right)^{\theta_2}\right]K\eta\frac{c}{r} && \text{(A141)}\\
&\quad -\frac{1}{\epsilon(\theta_2-\theta_1)}\left[(\epsilon-\theta_1)\theta_2\left(\frac{Kc}{X_t}\right)^{\theta_1} + (\epsilon-\theta_2)\theta_1\left(\frac{Kc}{X_t}\right)^{\theta_2}\right]\eta\frac{1}{r} && \text{(A142)}\\
&\quad -\frac{(1+\theta_1)(1+\theta_2)}{(\theta_2-\theta_1)(1+\epsilon)}\left[(\epsilon-\theta_1)\left(\frac{Kc}{X_t}\right)^{\theta_1} + (\epsilon-\theta_1)\left(\frac{Kc}{X_t}\right)^{\theta_2}\right]K(1-\alpha)(1-\eta)\frac{1}{r_A} && \text{(A143)}
\end{aligned}
$$

$$0 = \eta\frac{1}{r} - \frac{1}{Kc}\frac{\theta_2\theta_1}{\epsilon(\theta_2-\theta_1)}\left[(\epsilon-\theta_1)\left(\frac{Kc}{X_t}\right)^{\theta_1} + (\theta_2-\epsilon)\left(\frac{Kc}{X_t}\right)^{\theta_2}\right]K\eta\frac{c}{r} \tag{A144}$$

$$-\frac{1}{\epsilon(\theta_2-\theta_1)}\left[(\epsilon-\theta_1)\theta_2\left(\frac{Kc}{X_t}\right)^{\theta_1} + (\epsilon-\theta_2)\theta_1\left(\frac{Kc}{X_t}\right)^{\theta_2}\right]\eta\frac{1}{r} \tag{A145}$$

$$-\frac{(1+\theta_1)(1+\theta_2)}{(\theta_2-\theta_1)(1+\epsilon)}\left[(\epsilon-\theta_1)\left(\frac{Kc}{X_t}\right)^{\theta_1} + (\epsilon-\theta_1)\left(\frac{Kc}{X_t}\right)^{\theta_2}\right]K(1-\alpha)(1-\eta)\frac{1}{r_A} \tag{A146}$$

$$0 = \eta\frac{1}{r} - \frac{\theta_2\theta_1}{\epsilon(\theta_2-\theta_1)}\left[(\epsilon-\theta_1)\left(\frac{Kc}{X_t}\right)^{\theta_1} + (\theta_2-\epsilon)\left(\frac{Kc}{X_t}\right)^{\theta_2}\right]\eta\frac{1}{r} \tag{A147}$$

$$-\frac{1}{\epsilon(\theta_2-\theta_1)}\left[(\epsilon-\theta_1)\theta_2\left(\frac{Kc}{X_t}\right)^{\theta_1} + (\epsilon-\theta_2)\theta_1\left(\frac{Kc}{X_t}\right)^{\theta_2}\right]\eta\frac{1}{r} \tag{A148}$$

$$-\frac{(1+\theta_1)(1+\theta_2)}{(\theta_2-\theta_1)(1+\epsilon)}\left[(\epsilon-\theta_1)\left(\frac{Kc}{X_t}\right)^{\theta_1} + (\epsilon-\theta_1)\left(\frac{Kc}{X_t}\right)^{\theta_2}\right]K(1-\alpha)(1-\eta)\frac{1}{r_A} \tag{A149}$$

$$0 = 1 - \frac{\theta_2\theta_1}{\epsilon(\theta_2-\theta_1)}\left[(\epsilon-\theta_1)\left(\frac{Kc}{X_t}\right)^{\theta_1} + (\theta_2-\epsilon)\left(\frac{Kc}{X_t}\right)^{\theta_2}\right] \tag{A150}$$

$$-\frac{1}{\epsilon(\theta_2-\theta_1)}\left[(\epsilon-\theta_1)\theta_2\left(\frac{Kc}{X_t}\right)^{\theta_1} + (\epsilon-\theta_2)\theta_1\left(\frac{Kc}{X_t}\right)^{\theta_2}\right] \tag{A151}$$

$$-\frac{(1+\theta_1)(1+\theta_2)}{(\theta_2-\theta_1)(1+\epsilon)}\left[(\epsilon-\theta_1)\left(\frac{Kc}{X_t}\right)^{\theta_1} + (\epsilon-\theta_1)\left(\frac{Kc}{X_t}\right)^{\theta_2}\right]K(1-\alpha)\frac{1-\eta}{\eta}\frac{r}{r_A} \tag{A152}$$

$$0 = 1 - \frac{\epsilon-\theta_1}{\theta_2-\theta_1}\left[\frac{\theta_1\theta_2}{\epsilon} + \frac{\theta_2}{\epsilon} + K(1-\alpha)\frac{1-\eta}{\eta}\frac{r}{r_A}\frac{(1+\theta_1)(1+\theta_2)}{1+\epsilon}\right]\left(\frac{Kc}{X_0}\right)^{\theta_1} \tag{A153}$$

$$-\frac{\theta_2-\epsilon}{\theta_2-\theta_1}\left[\frac{\theta_1\theta_2}{\epsilon} + \frac{\theta_1}{\epsilon} + K(1-\alpha)\frac{1-\eta}{\eta}\frac{r}{r_A}\frac{(1+\theta_1)(1+\theta_2)}{1+\epsilon}\right]\left(\frac{Kc}{X_0}\right)^{\theta_2} \tag{A154}$$

Therefore, the optimal coupon satisfies the following nonlinear algebraic equation

$$1 - a_1\left(\frac{Kc}{X_0}\right)^{\theta_1} - a_2\left(\frac{Kc}{X_0}\right)^{\theta_2} = 0, \tag{A155}$$

where

$$a_1 = \frac{\epsilon-\theta_1}{\theta_2-\theta_1}\left[\frac{\theta_1\theta_2}{\epsilon} + \frac{\theta_2}{\epsilon} + K(1-\alpha)\frac{1-\eta}{\eta}\frac{r}{r_A}\frac{(1+\theta_1)(1+\theta_2)}{1+\epsilon}\right] \tag{A156}$$

$$a_2 = \frac{\theta_2-\epsilon}{\theta_2-\theta_1}\left[\frac{\theta_1\theta_2}{\epsilon} + \frac{\theta_1}{\epsilon} + K(1-\alpha)\frac{1-\eta}{\eta}\frac{r}{r_A}\frac{(1+\theta_1)(1+\theta_2)}{1+\epsilon}\right]. \tag{A157}$$

To solve the above equation, define $x = a_1\left(\frac{Kc}{X_0}\right)^{\theta_1}$. Then

$$1 - x = a_2 a_1^{-\frac{\theta_2}{\theta_1}} x^{\frac{\theta_2}{\theta_1}}, \tag{A158}$$

$$q_{D,t} = \frac{\epsilon-\theta_1}{\epsilon}\frac{\theta_2}{\theta_2-\theta_1}\left(\frac{X_D}{X_t}\right)^{\theta_1} + \frac{\theta_2-\epsilon}{\epsilon}\frac{\theta_1}{\theta_2-\theta_1}\left(\frac{X_D}{X_t}\right)^{\theta_2}, \tag{A159}$$

and

$$q^X_{D,t} = X_t \left[\frac{\epsilon - \theta_1}{\theta_2 - \theta_1} \frac{1+\theta_2}{1+\epsilon} \left(\frac{X_D}{X_t} \right)^{1+\theta_1} + \frac{\theta_2 - \epsilon}{\theta_2 - \theta_1} \frac{1+\theta_1}{1+\epsilon} \left(\frac{X_D}{X_t} \right)^{1+\theta_2} \right], \tag{A160}$$

■

Proof. The Arrow-Debreu default claim, which pays off a unit of consumption at default has price, $q_D(X_t)$, given by

$$q_D(X_t) = E_t^{\mathbb{Q}}[e^{-r(\tau_D - t)} I_{\tau_D \leq \tau_U}], \tag{A161}$$

and the Arrow-Debreu refinancing claim, which pays off a unit of consumption at refinancing has price, $q_U(X_t)$, given by

$$q_U(X_t) = E_t^{\mathbb{Q}}[e^{-r(\tau_U - t)} I_{\tau_U \leq \tau_D}]. \tag{A162}$$

From Proposition B1, we know that

$$q_D(X_t) = Z^{(r)}(x_t - x_D) - W^{(r)}(x_t - x_D) \frac{Z^{(r)}(x_U - x_D)}{W^{(r)}(x_U - x_D)}, \tag{A163}$$

and

$$q_U(X_t) = \frac{W^{(r)}(x_t - x_D)}{W^{(r)}(x_U - x_D)}, \tag{A164}$$

where $x_t = \ln X_t$, $x_U = \ln X_U$, $x_D = \ln X_D$, and $g_x(\theta)$ is the cumulant of x, which is given by

$$g_x(\theta) = \mu_x \theta + \frac{1}{2}\sigma_x^2 \theta^2 + \lambda\omega \left(\frac{\epsilon}{\epsilon + \theta} - 1 \right), \tag{A165}$$

where

$$\mu_x = \hat{\mu}_X - \frac{1}{2}\sigma_X^2, \tag{A166}$$

$$\sigma_x = \sigma_X. \tag{A167}$$

From Definition, it follows that

$$\int_0^\infty e^{-\theta y} W^{(r)}(y) dy = \frac{\epsilon + \theta}{\left(\mu_x \theta + \frac{1}{2}\sigma_x^2 \theta^2 - (\lambda\omega + r)\right)(\epsilon + \theta) - \lambda\omega\epsilon}, \theta \geq \Phi(r), \tag{A168}$$

where $\Phi(r)$ is the largest root of $g_x(\theta) - r = 0$. To find $W^{(r)}(y)$ in closed-form we must find the inverse Laplace transform of

$$J(\theta) = \frac{\epsilon + \theta}{\left(\mu_x \theta + \frac{1}{2}\sigma_x^2 \theta^2 - (\lambda\omega + r)\right)(\epsilon + \theta) - \lambda\omega\epsilon}. \tag{A169}$$

We rewrite the denominator of the above expression as

$$\left(\mu_x \theta + \frac{1}{2}\sigma_x^2 \theta^2 - (\lambda\omega + r)\right)(\epsilon + \theta) - \lambda\omega\epsilon = \frac{1}{2}\sigma_x^2 \left\{ \left[\frac{2\mu_x}{\sigma_x^2}\theta + \theta^2 - (\lambda\omega + r)\frac{2}{\sigma_x^2} \right] (\epsilon + \theta) - \frac{2}{\sigma_x^2}\lambda\omega\epsilon \right\}. \tag{A170}$$

Since, $\left[\frac{2\mu_x}{\sigma_x^2}\theta + \theta^2 - (\lambda\omega + r)\frac{2}{\sigma_x^2} \right] (\epsilon + \theta) - \frac{2}{\sigma_x^2}\lambda\omega\epsilon$, is a cubic expression in θ, we can rewrite it as $(\theta - \alpha_1)(\theta - \alpha_2)(\theta - \alpha_3)$, where α_1, α_2, and α_3 are its roots. Thus,

$$J(\theta) = \frac{2}{\sigma_x^2} \frac{\epsilon + \theta}{(\theta - \alpha_1)(\theta - \alpha_2)(\theta - \alpha_3)}. \tag{A171}$$

To find the inverse Laplace transform of $J(\theta)$, we rewrite it using partial fractions to obtain

$$J(\theta) = \frac{2}{\sigma_x^2}\left[\frac{\epsilon+\alpha_1}{(\theta-\alpha_1)(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)} - \frac{\epsilon+\alpha_2}{(\theta-\alpha_2)(\alpha_1-\alpha_2)(\alpha_2-\alpha_3)} + \frac{\epsilon+\alpha_3}{(\theta-\alpha_3)(\alpha_1-\alpha_3)(\alpha_2-\alpha_3)}\right]. \tag{A172}$$

By taking the inverse Laplace transform of the above expression, it follows that

$$W^{(r)}(x) = \frac{2}{\sigma_x^2}\left[\frac{\epsilon+\alpha_1}{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}e^{\alpha_1 x} - \frac{\epsilon+\alpha_2}{(\alpha_1-\alpha_2)(\alpha_2-\alpha_3)}e^{\alpha_2 x} + \frac{\epsilon+\alpha_3}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_3)}e^{\alpha_3 x}\right]. \tag{A173}$$

Hence,

$$Z^{(r)}(x) = 1 + \frac{2r}{\sigma_x^2}\left[\frac{\epsilon+\alpha_1}{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}\frac{e^{\alpha_1 x}-1}{\alpha_1} - \frac{\epsilon+\alpha_2}{(\alpha_1-\alpha_2)(\alpha_2-\alpha_3)}\frac{e^{\alpha_2 x}-1}{\alpha_2} + \frac{\epsilon+\alpha_3}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_3)}\frac{e^{\alpha_3 x}-1}{\alpha_3}\right]. \tag{A174}$$

Hence,

$$q_D(X_t) = Z_X^{(r)}(X_t/X_D) - W_X^{(r)}(X_t/X_D)\frac{Z_X^{(r)}(X_U/X_D)}{W_X^{(r)}(X_U/X_D)}, \tag{A175}$$

and

$$q_U(X_t) = \frac{W^{(r)}(X_t/X_D)}{W^{(r)}(X_U/X_D)}, \tag{A176}$$

where

$$W_X^{(r)}(X) = \frac{2}{\sigma_X^2}\left[\frac{\epsilon+\alpha_1}{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}X^{\alpha_1} - \frac{\epsilon+\alpha_2}{(\alpha_1-\alpha_2)(\alpha_2-\alpha_3)}X^{\alpha_2} + \frac{\epsilon+\alpha_3}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_3)}X^{\alpha_3}\right], \tag{A177}$$

and

$$Z_X^{(r)}(X) = 1 + \frac{2r}{\sigma_X^2}\left[\frac{\epsilon+\alpha_1}{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}\frac{X^{\alpha_1}-1}{\alpha_1} - \frac{\epsilon+\alpha_2}{(\alpha_1-\alpha_2)(\alpha_2-\alpha_3)}\frac{X^{\alpha_2}-1}{\alpha_2} + \frac{\epsilon+\alpha_3}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_3)}\frac{X^{\alpha_3}-1}{\alpha_3}\right]. \tag{A178}$$

The second Arrow-Debreu default claim, denoted by $q_D^X(X_t)$, is the time-t value of a the random cash flow X, paid at default, i.e.

$$q_D^X(X_t) = E_t[e^{-r(\tau_D-t)}X_{\tau_D}I_{\tau_D\le\tau_U}]. \tag{A179}$$

Earnings can jump below the default boundary, and so it is possible that $X_{\tau_D} < X_D$. To find a closed-form expression for $q_{D,t}^X = q_D^X(X_t)$, we start by noting that

$$\begin{aligned}
q_{D,t}^X &= E_t[e^{-r(\tau_D-t)+x_{\tau_D}}I_{\{\tau_D\le\tau_U\}}] & \text{(A180)}\\
&= E_t[e^{-r(\tau_D-t)+x_{\tau_D}}I_{\{\tau_D\le\tau_U\ \&\ x_{\tau_D}=x_D\}}] + E_t[e^{-r(\tau_D-t)+x_{\tau_D}}I_{\{\tau_D\le\tau_U\ \&\ x_{\tau_D}<x_D\}}] & \text{(A181)}\\
&= e^{x_D}E_t[e^{-r(\tau_D-t)+(x_{\tau_D}-x_D)}I_{\{\tau_D\le\tau_U\ \&\ x_{\tau_D}=x_D\}}] + e^{x_D}E_t[e^{-r(\tau_D-t)+x_{\tau_D}-x_D}I_{\{\tau_D\le\tau_U\ \&\ x_{\tau_D}<x_D\}}] & \text{(A182)}\\
&= e^{x_D}E_t[e^{-r(\tau_D-t)}I_{\{\tau_D\le\tau_U\}}] + e^{x_D}E_t[e^{-r(\tau_D-t)+x_{\tau_D}-x_D}I_{\{\tau_D\le\tau_U\ \&\ x_{\tau_D}<x_D\}}] & \text{(A183)}\\
&= e^{x_D}q_{D,t} + e^{x_D}E_t[e^{-r(\tau_D-t)+x_{\tau_D}-x_D}I_{\{\tau_D\le\tau_U\ \&\ x_{\tau_D}<x_D\}}]. & \text{(A184)}
\end{aligned}$$

Therefore, it remains to find closed-form expression for $E_t[e^{-r(\tau_D-t)+x_{\tau_D}-x_D}I_{\{\tau_D\le\tau_U\ \&\ x_{\tau_D}<x_D\}}]$. To do this, we exploit the conditional memoryless property (a consequence of assuming that jumps are exponentially distributed), which implies that

$$E_t[e^{-r(\tau_D-t)+x_{\tau_D}-x_D}I_{\{\tau_D\le\tau_U\ \&\ x_{\tau_D}<x_D\}}] = \frac{\epsilon}{\epsilon+1}E_t[e^{-r(\tau_D-t)}I_{\{\tau_D\le\tau_U\ \&\ x_{\tau_D}<x_D\}}]. \tag{A185}$$

Now we define

$$d(x) = E_t[e^{-r(\tau_D - t)} I_{\{\tau_D \le \tau_U \ \& \ x_{\tau_D} < x_D - y\}} | x_t = x] \tag{A186}$$

We know that

$$d(x) = \begin{cases} 0, & x_U \le x \\ 0, & x_D - x \le x \le x_D \\ 1, & x < x_D - y \end{cases}, \tag{A187}$$

and that for $x_D < x < x_U$, we have

$$\frac{1}{2}\sigma_x^2 d'' + \mu_x d' - rd + \hat{\lambda}\int_{-\infty}^{\infty} \left(d(x+w) - d(x)\right) f_z(w)dw = 0, \tag{A188}$$

together with the boundary conditions $d(x_U) = 0$ and $d(x_D) = 0$. We seek a trial solution of the form

$$d(x) = \sum_{k=1}^{3} A_k e^{(x - x_D)\beta_k}, \tag{A189}$$

with constants A_k, $k \in \{1, 2, 3\}$, such that the differential–integral equation in (A188) is satisfied together with its boundary conditions. Note that

$$\begin{aligned}
\int_{-\infty}^{\infty} d(x+w) f_Z(w) &= \int_{-\infty}^{0} d(x+w)\epsilon e^{\epsilon w} dw && \text{(A190)} \\
&= \epsilon \int_{-\infty}^{x_D - x - y} e^{\epsilon w} dw + \epsilon \sum_{k=1}^{3} \int_{x_D - x}^{0} A_k e^{(x+w-x_D)\beta_k} e^{\epsilon w} dw && \text{(A191)} \\
&= e^{\epsilon(x_D - x - y)} + \epsilon \sum_{k=1}^{3} A_k e^{(x-x_D)\beta_k} \int_{x_D - x}^{0} e^{(\beta_k + \epsilon)w} dw && \text{(A192)} \\
&= e^{\epsilon(x_D - x - y)} + \epsilon \sum_{k=1}^{3} \frac{A_k}{\beta_k + \epsilon} e^{(x-x_D)\beta_k} \left(1 - e^{(\beta_k+\epsilon)(x_D - x)}\right) && \text{(A193)} \\
&= e^{\epsilon(x_D - x - y)} + \epsilon \sum_{k=1}^{3} \frac{A_k}{\beta_k + \epsilon} \left(e^{\beta_k(x - x_D)} - e^{\epsilon(x_D - x)}\right). && \text{(A194)}
\end{aligned}$$

Thus, substituting (A189) into (A188) gives

$$\frac{1}{2}\sigma_x^2 \sum_{k=1}^{3} \beta_k^2 A_k e^{(x-x_D)\beta_k} + \mu_x \sum_{k=1}^{3} \beta_k A_k e^{(x-x_D)\beta_k} - (r+\hat{\lambda}) \sum_{k=1}^{3} A_k e^{(x-x_D)\beta_k} + \hat{\lambda}\epsilon \sum_{k=1}^{3} \frac{A_k}{\beta_k + \epsilon} e^{\beta_k(x-x_D)} + \hat{\lambda} e^{\epsilon(x_D - x)} \left(e^{-\epsilon y} - \epsilon \sum_{k=1}^{3} \frac{A_k}{\beta_k + \epsilon}\right). \tag{A195}$$

Thus, β_k, $k \in \{1, 2, 3\}$ are the roots of

$$\frac{1}{2}\sigma_x^2 \beta^2 + \mu_x \beta - (r + \hat{\lambda}) + \hat{\lambda}\frac{\epsilon}{\beta + \epsilon} = 0, \tag{A196}$$

and A_k, $k \in \{1, 2, 3\}$ are determined by

$$e^{-\epsilon y} = \epsilon \sum_{k=1}^{3} \frac{A_k}{\beta_k + \epsilon}, \tag{A197}$$

and two boundary conditions, i.e.

$$\begin{aligned}
A_1 + A_2 + A_3 &= 0 && \text{(A198)} \\
\sum_{k=1}^{3} A_k e^{(x_U - x_D)\beta_k} &= 0. && \text{(A199)}
\end{aligned}$$

Hence,

$$A_1 = \frac{e^{-\epsilon y}}{\epsilon}\frac{(\beta_1+\epsilon)(\beta_2+\epsilon)(\beta_3+\epsilon)\left(e^{\beta_2(x_U-x_D)}-e^{\beta_3(x_U-x_D)}\right)}{(\beta_2-\beta_3)(\beta_1+\epsilon)e^{\beta_1(x_U-x_D)}+(\beta_1-\beta_2)(\beta_3+\epsilon)e^{\beta_3(x_U-x_D)}-(\beta_1-\beta_3)(\beta_2+\epsilon)e^{\beta_2(x_U-x_D)}}, \tag{A200}$$

$$A_2 = -\frac{e^{-\epsilon y}}{\epsilon}\frac{(\beta_1+\epsilon)(\beta_2+\epsilon)(\beta_3+\epsilon)(e^{\beta_1(x_U-x_D)}-e^{\beta_3(x_U-x_D)})}{(\beta_2-\beta_3)(\beta_1+\epsilon)e^{\beta_1(x_U-x_D)}+(\beta_1-\beta_2)(\beta_3+\epsilon)e^{\beta_3(x_U-x_D)}-(\beta_1-\beta_3)(\beta_2+\epsilon)e^{\beta_2(x_U-x_D)}}, \tag{A201}$$

$$A_3 = \frac{e^{-\epsilon y}}{\epsilon}\frac{(\beta_1+\epsilon)(\beta_2+\epsilon)(\beta_3+\epsilon)\left(e^{\beta_1(x_U-x_D)}-e^{\beta_2(x_U-x_D)}\right)}{(\beta_2-\beta_3)(\beta_1+\epsilon)e^{\beta_1(x_U-x_D)}+(\beta_1-\beta_2)(\beta_3+\epsilon)e^{\beta_3(x_U-x_D)}-(\beta_1-\beta_3)(\beta_2+\epsilon)e^{\beta_2(x_U-x_D)}} \tag{A202}$$

■

Proof of Proposition 8.

At restructuring existing debt is diluted on a per coupon basis, so that (43) holds. Note that

$$R = \frac{c^0}{c^1}B(X_U^0, c^1) = \frac{1}{\xi}B(\xi X_0^0, \xi c^0) = \frac{1}{\xi}\xi B(X_0^0, c^0) = B(X_0^0, c^0).$$

Hence

$$B(X_t, c^0) = \frac{c^0}{r} + (1-\eta)\frac{\alpha q_D^X(X_t)}{r_A} - q_D(X_t)\frac{c^0}{r} + q_U(X_t)\left(B(X_0^0, c^0) - \frac{c^0}{r}\right). \tag{A203}$$

Setting $X_t = X_0^0$ in the right-hand side of the above expression and solving for $B(X_0^0, c^0)$ gives

$$B(X_0^0, c^0) = \frac{\frac{c^0}{r} + (1-\eta)\frac{\alpha q_D^X(X_0^0)}{r_A} - q_D(X_0^0)\frac{c^0}{r} - q_U(X_0^0)\frac{c^0}{r}}{1 - q_U(X_0^0)}. \tag{A204}$$

From (A203) it follows that

$$B(X_t, c^0) = \frac{c^0}{r}(1 - l_D q_{D,t} - l_U q_{U,t}), \tag{A205}$$

where

$$l_U = \frac{\frac{c^0}{r} - B(X_0^0, c^0)}{\frac{c^0}{r}}. \tag{A206}$$

The homogeneity property implies that

$$S(X_U^0, c^1(c^0)) = S(\xi X_0^0, \xi c^0) = \xi S(X_0^0, c^0). \tag{A207}$$

Hence

$$E(X_U^0) = [(1-\iota)B(X_U^0, c^1) - R_{0,1}] + S(X_U^0, c^1) \tag{A208}$$

can be rewritten as

$$E(X_U^0) = B(X_0^0, c^0)[(1-\iota)\xi - 1)] + \xi S(X_0^0, c^0), \tag{A209}$$

which implies that

$$S(X_t, c^0) = Div_t(X_t, c^0) + q_U(X_t)\{B(X_0^0)[(1-\iota)\xi - 1)] + \xi S(X_0^0, c^0)\}. \tag{A210}$$

Setting $X_t = X_0^0$ in the above equation and solving for $S(X_0^0, c^0)$ gives

$$S(X_0^0, c^0) = \frac{Div_t(X_t, c^0) + q_U(X_t)\{B(X_0^0)[(1-\iota)\xi - 1)]\}}{1 - \xi q_U(X_t)}. \tag{A211}$$

■

B Supplemental Appendix

In this Supplemental Appendix, we summarize results on the first passage times of spectrally negative Levy processes, i.e. Levy processes with no positive jumps. We use these results to derive the price of Arrow-Debreu default and refinancing claims under dynamic capital structure.

We assume that $Y = \{Y_t, t \geq 0\}$ is a real-valued Levy process defined on a filtered probability space, $\left(\Omega, \mathcal{F}, \mathbb{F} = \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P}\right)$. We assume further that Y may be represented as

$$Y_t = \mu t + \sigma B_t + J_t^-, \tag{B1}$$

where $B = \{B_t, t \geq 0\}$ is a standard Brownian motion and $J^- = \{J_t^-, t \geq 0\}$ is a non-Gaussian spectrally negative Levy process. Both processes are independent. Since the jumps of J^- are all non positive, the moment generating function $E[e^{\theta Y_t}]$ exists for all $\theta \geq 0$. It is a standard result, stemming from the independence and stationarity of their increments that for any Levy process, if the the moment generating function at time t exists, then it satisfies

$$E[e^{\theta Y_t}] = e^{t g_Y(\theta)}, \tag{B2}$$

for some function $g_Y(\theta)$, which is defined for $\theta \in \mathbb{C}$, such that $\Re(\theta) \geq 0$. The function $g(\theta)$ is known variously as the *Levy exponent*, *Laplace exponent* or *cumulant* of Y.

We use the following definitions, taken from Pistorius (2003), the first two of which are modified versions of definitions in Chapter VII of Bertoin (1996).

Definition 1 *Let $q \geq 0$ and the define $\Phi(q)$ as the largest root of*

$$g_Y(\theta) = q. \tag{B3}$$

Definition 2 *For $q \geq$, we define the first scale function, $W^{(q)} : (-\infty, \infty) \to [0, \infty]$, as the unique function whose restriction to $(0, \infty)$ is continuous and has Laplace transform*

$$\int_0^\infty e^{-\theta y} W^{(q)}(y) dy = \frac{1}{g_Y(\theta) - q}, \theta \geq \Phi(q), \tag{B4}$$

and is equal to zero for $y \leq 0$.

Definition 3 *For $q \geq 0$, we define the second scale function, $Z^{(q)} : \mathbb{R} \to [1, \infty]$ by*

$$Z^{(q)}(x) = 1 + q \int_0^y W^{(q)}(z) dz. \tag{B5}$$

Definition 4 *We define the first passage time from above to the barrier a by*

$$\tau_a^- = \inf \{t \geq 0 : Y_t \leq a\}, \tag{B6}$$

and the first passage time from below to the barrier b by

$$\tau_b^+ = \inf \{t \geq 0 : Y_t \geq b\}. \tag{B7}$$

The following proposition gives the Laplace transforms of the two-sided exit time $\min(\tau_a^-, \tau_b^+)$ when Y starts at $y \in (a, b)$.

Proposition B1 *For $q \geq 0$, we have*

$$E[e^{-q\tau_b^+} I_{\{\tau_b^+ < \tau_a^-\}}] = \frac{W^{(q)}(y-a)}{W^{(q)}(b-a)}, \tag{B8}$$

$$E[e^{-q\tau_b^+} I_{\{\tau_b^+ \geq \tau_a^-\}}] = Z^{(q)}(y-a) - W^{(q)}(x-a) \frac{Z^{(q)}(b-a)}{W^{(q)}(b-a)}. \tag{B9}$$

The proof of (B8) is in Bertoin (1996) and (B9) follows by combining (B8) with Corollary 1 of Bertoin (1996).

References

Andrade, G. and S. N. Kaplan (1998): "How Costly Is Financial (Not Economic) Distress? Evidence from Highly Leveraged Transactions That Became Distressed," *Journal of Finance*, 53, 1443–93.

Attanasio, O. P. and G. Weber (1989): "Intertemporal Substitution, Risk Aversion and the Euler Equation for Consumption," *Economic Journal*, 99, 59–73.

Bansal, R. and A. Yaron (2004): "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles," *Journal of Finance*, 59, 1481–1509.

Barro, R. J. (2006): "Rare Disasters and Asset Markets in the Twentieth Century," *Quarterly Journal of Economics*, 121, 823–66.

Barro, R. J. and J. F. Ursua (2008): "Macroeconomic crises since 1870," Unpublished working paper.

Berk, J. B., R. C. Green, and V. Naik (1999): "Optimal Investment, Growth Options, and Security Returns," *Journal of Finance*, 54, 1553–1607.

Bertoin, J. (1996): *Levy Processes*, Cambridge University Press.

Bertola, G. and R. J. Caballero (1994): "Irreversibility and Aggregate Investment," *Review of Economic Studies*, 61, 223–46.

Bhamra, H. S., L.-A. Kuehn, and I. A. Strebulaev (2008): "The Aggregate Dynamics of Capital Structure and Macroeconomic Risk," Unpublished working paper.

——— (2010): "The Levered Equity Risk Premium and Credits Spreads: A Unified Approach," *Review of Financial Studies*, 23, 645–703.

Campbell, J. Y. (1999): "Asset prices, consumption, and the business cycle," *Handbook of Macroeconomics*, 1, 1231–1303.

Campbell, J. Y. and J. H. Cochrane (1999): "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior," *Journal of Political Economy*, 107, 205–251.

Cantor, R., F. Emery, Kenneth Kim, S. Ou, and J. Tennant (2008): "Default and Recovery Rates of Corporate Bond Issuers," Moody's Investors Service, Global Credit Research.

Carlson, M. and A. Lazrak (2010): "Leverage Choice and Credit Spreads when Managers Risk Shift," *Journal of Finance*, 65, 2323–236.

Cecchetti, S. G., P.-S. Lam, and N. C. Mark (1993): "The Equity Premium and the Risk-Free Rate: Matching the Moments," *Journal of Monetary Economics*, 31, 21–45.

Chen, H. (2010): "Macroeconomic Conditions and the Puzzles of Credit Spreads and Capital Structure," *Journal of Finance*, 65, 2171–2212, unpublished working paper.

Chen, L., P. Collin-Dufresne, and R. S. Goldstein (2008): "On the Relation Between Credit Spread Puzzles and the Equity Premium Puzzle," *Review of Financial Studies, forthcoming.*

Chen, N. and S. G. Kou (2009): "Credit Spreads Optimal Capital Structure and Implied Volatility with Endogenous Default and Jump Risk," *Mathematical Finance*, 19, 343–378.

David, A. (2008): "Inflation Uncertainty, Asset Valuations, and Credit Risk," *Review of Financial Studies*, 21, 2487–2534.

Davydenko, S. and I. A. Strebulaev (2007): "Strategic Actions and Credit Spreads: An Empirical Investigation," *Journal of Finance*, 62, 2633 – 2671.

Duffee, G. R. (1998): "The Relation between Treasury Yields and Corporate Bond Yield Spreads," *Journal of Finance*, 53, 2225–41.

Duffie, D. and L. G. Epstein (1992): "Stochastic Differential Utility," *Econometrica*, 60, 353–94.

Epstein, L. G. and S. E. Zin (1989): "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," *Econometrica*, 57, 937–69.

Fischer, E. O., R. Heinkel, and J. Zechner (1989): "Dynamic Capital Structure Choice: Theory and Tests," *Journal of Finance*, 44, 19–40.

Gabaix, X. (2008): "Variable Rare Disasters: A Tractable Theory of Ten Puzzles in Macro-Finance," *American Economic Review*, 98, 64–67.

Goldstein, R., N. Ju, and H. Leland (2001): "An EBIT-Based Model of Dynamic Capital Structure," *Journal of Business*, 74, 483–512.

Gourio, F. (2011): "Credit risk and disaster risk," Unpublished working paper.

Guvenen, F. (2006): "Reconciling Conflicting Evidence on the Elasticity of Intertemporal Substitution: A Macroeconomic Perspective," *Journal of Monetary Economics*, 53, 1451–72.

Hackbarth, D., J. Miao, and E. Morellec (2006): "Capital Structure, Credit Risk and Macroeconomic Conditions," *Journal of Financial Economics*, 82, 519–550.

Hall, R. E. (1988): "Intertemporal Substitution in Consumption," *Journal of Political Economy*, 96, 339–57.

Hansen, L. P. and K. J. Singleton (1982): "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models," *Econometrica*, 50, 1269–86.

He, Z. and W. Xiong (2011): "Rollover Risk and Credit Risk," *Journal of Finance, forthcoming.*

Huang, J. and M. Huang (2003): "How much of the corporate-treasury yield spread is due to credit risk? A new calibration approach," Unpublished working paper.

Kandel, S. and R. F. Stambaugh (1991): "Asset Returns and Intertemporal Preferences," *Journal of Monetary Economics*, 27, 39–71.

Kraft, H. and F. T. Seifried (2008): "Foundations of Continuous Time Recursive Utility: Differentiability and Normalization of Certainty Equivalents," Unpublished working paper.

Lando, D. (2004): *Credit Risk Modelling, Theory and Applications*, Princeton University Press.

Leland, H. E. (1994): "Corporate Debt Value, Bond Covenants, and Optimal Capital Structure," *Journal of Finance*, 49, 1213–52.

Liu, J., J. Pan, and T. Wang (2005): "An Equilibrium Model of Rare-Event Premia and Its Implication for Option Smirks," *Review of Financial Studies*, 18, 131–164.

Longstaff, F. A. and M. Piazzesi (2004): "Corporate earnings and the equity premium," *Journal of Financial Economics*, 74, 401–421.

Merton, R. C. (1974): "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *Journal of Finance*, 29, 449–70.

Naik, V. and M. Lee (1990): "General Equilibrium Pricing of Options on the Market Portfolio with Discontinuous Returns," *Review of Financial Studies*, 3, 493–521.

Nakamura, E., J. Steinsson, R. J. Barro, and J. F. Ursua (2011): "Crises and Recoveries in an Empirical Model of Consumption Disasters," Unpublished working paper.

Pistorius, M. (2003): "Exit problems of Lévy processes with applications in finance," Ph.D. thesis, Utrecht University.

Rietz, T. A. (1988): "The Equity Risk Premium: A Solution," *Journal of Monetary Economics*, 22, 117–31.

Schaefer, S. M. and I. A. Strebulaev (2008): "Structural Models of Credit Risk are Useful: Evidence from Hedge Ratios on Corporate Bonds," *Journal of Financial Economics*, 90, 1–19.

Schroder, M. and C. Skiadas (1999): "Optimal Consumption and Portfolio Selection with Stochastic Differential Utility," *Journal of Economic Theory*, 89, 68–126.

Strebulaev, I. (2007): "Do Tests of Capital Structure Theory Mean What They Say?" *Journal of Finance*, 62, 1747–1787.

Vissing-Jorgensen, A. (2002): "Limited Asset Market Participation and the Elasticity of Intertemporal Substitution," *Journal of Political Economy*, 110, 825–53.

Wachter, J. (2011): "Can time-varying risk of rare disasters explain aggregate stock market volatility?" Unpublished working paper.

Weil, P. (1990): "Nonexpected Utility in Macroeconomics," *Quarterly Journal of Economics*, 105, 29–42.

Weitzman, M. L. (2007): "Subjective expectations and asset-return puzzles," *American Economic Review*, 97, 1102–1130.

Table I : Parameter estimates

This table reports model parameters. To calibrate the model to the aggregate US economy, quarterly real non-durable plus service consumption expenditure from the Bureau of Economic Analysis and quarterly earnings data from Standard and Poor's, provided by Robert J. Shiller, are used. The personal consumption expenditure chain-type price index is used to deflate nominal earnings. The estimates of consumption growth rate and volatility, earnings growth rate and volatility, and correlation between earnings and consumption growth are based on quarterly log growth rates for the period from 1947 to 2005. All variables are given per annum and in per cent (0.01 means 1% p.a.)

Parameter	Symbol	
Consumption growth rate	g	0.03
Consumption growth volatility	σ_C	0.01
Earnings growth rate	μ_X	0.03
Earnings growth volatility	σ_X^s	0.15
Idiosyncratic earnings growth volatility	σ_X^{id}	0.2258
Correlation	ρ_{XC}	0.1998
Probability per unit time of a rare disaster	λ	0.015
Mean consumption jump size	J_C	0.1
Mean earnings jump size	J	0.25
Annual discount rate	β	0.015
Tax rate	η	0.15
Bankruptcy costs	$1-\alpha$	0.20
Debt issuance cost	ι	0.02

Table II : Empirical default rates and credit spreads

Panel A reports average cumulative issuer-weighted annualized default rates for BBB debt over 5, 10, and 15 year horizons for US firms as reported by Cantor et al. (2008). The first row shows mean historical default rates for the period 1920–2007 and the second row for 1970–2007. Panel B reports the difference between average spreads for BBB and AAA corporate debt, sorted by maturity. Data from Duffee (1998) are for bonds with no option-like features, taken from the Fixed Income Dataset, University of Houston, for the period Jan 1973 to March 1995, where maturities from 2 to 7 years are short, 7 to 15 are medium, and 15 to 30 are long. For Huang and Huang (2003), short denotes a maturity of 4 years and medium of 10 years. The data used in David (2008) are taken from Moody's and medium denotes a maturity of 10 years. For Davydenko and Strebulaev (2007), the data are taken from the National Association of Insurance Companies; short denotes a maturity from 1 to 7 years, medium – 7 to 15 years, and long – 15 to 30 years.

Panel A: Historical BBB Default Probabilities				
Rating	Units	Year 5	Year 10	Year 15
1920 – 2007	%	3.142	7.061	10.444
1970 – 2007	%	1.835	4.353	7.601

Panel B: BBB/AAA Spreads				
Rating	Units	Short	Medium	Long
Duffee (1998)	b.p.	75	70	105
Huang and Huang (2003)	b.p.	103	131	–
David (2008)	b.p	–	96	–
Davydenko and Strebulaev (2007)	b.p	77	72	82

Figure 1: Cumulative disaster probabilities

The dashed (solid) line shows the cumulative disaster probability under $\mathbb{P}$, ($\mathbb{Q}$) as a function of time, i.e. $1 - e^{-\lambda t}$, $(1 - e^{-\hat{\lambda} t})$, where $\lambda = 0.015$ and $\hat{\lambda} = .135$.

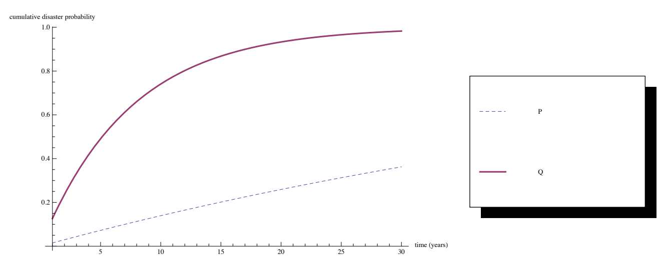

Table III : Credit risk under static capital structure

This table reports the credit risk implications of the static capital structure model for an individual firm at date 0, i.e. when $X/X_0 = 1$. For the No Disaster Model, Benchmark I (Naive), and the Disaster Model, $\beta = .015$, $\gamma = 8$, and $\psi = 1/3.5$, but the risk-free rate and price of consumption risk are differ across models. For No Disaster Model, Benchmark II, $\beta = .015$, $\gamma = 15$, and $\psi = 1.66$, yielding the same risk - free rate and price of consumption risk as in the Disaster Model. Credit spreads are given in basis points, interest coverage is a pure ratio, debt and equity are price values, and all other variables in per cent.

	No Disaster Model		Disaster Model
	Benchmark I (Naive)	Benchmark II	
Risk - free rate, r, %	11.82	3.19	3.19
Price of consumption risk, Θ, %	8.00	15.44	15.44
Credit spread, s, b.p.	60.43	22.01	53.85
Leverage, $B/(B+S)$, %	54.09	47.74	31.45
Interest coverage ratio, X/c	1.41	0.11	3.27
Normalized default boundary, X_D/X_0	0.39	0.32	0.17
1 yr Actual default probability, $p_{D,1}$, %	0.03	0.00	0.00
5 yr Actual default probability $p_{D,5}$, %	10.78	5.13	0.28
10 yr Actual default probability $p_{D,10}$, %	25.56	16.83	3.44
1 yr Risk - neutral default probability, $\hat{p}_{D,1}$, %	0.03	0.00	0.00
5 yr Risk - neutral default probability $\hat{p}_{D,5}$, %	11.10	5.48	0.29
10 yr Risk - neutral default probability $\hat{p}_{D,10}$, %	26.31	17.97	3.63
1 yr Risk - neutral default probability, $\hat{p}_{D,1}$, %	0.03	0.00	0.05
5 yr Risk - neutral default probability $\hat{p}_{D,5}$, %	11.10	5.48	11.29
10 yr Risk - neutral default probability $\hat{p}_{D,10}$, %	26.31	17.97	26.63
Arrow-Debreu default claim, q_D	5.58	7.33	15.74
Debt	5.71	273.28	8.19
Equity	4.85	299.14	17.86
Equity risk premium, $\mu_R - r$, %	0.00	0.00	0.00

Table IV : Credit risk implications at refinancing

This table reports the credit risk implications of the dynamic capital structure model for an individual firm at refinancing, i.e. when $X/X_0 = 1$, for No Disaster and Disaster models with identical risk - free rates and prices of consumption risk. Credit spreads are given in basis points and computed for 'shadow' finite maturity debt. Interest coverage is a pure ratio, debt and equity are price values, and all other variables in per cent.

	No Disasters	Disasters
Normalized earnings level, X/X_0	1.00	1.00
Risk - free rate, r, %	0.03	0.03
Price of consumption risk, Θ, %	0.15	0.15
Credit spread (10 yr), s, b.p.	70.16	95.97
Credit spread (5 yr), s, b.p.	59.64	83.18
Leverage, $B/(B+S)$, %	28.44	26.11
Interest coverage ratio, X/c	0.04	3.82
Normalized default boundary, X_D/X_0	0.15	0.17
1 yr Actual default probability, $p_{D,1}$, %	0.00	0.01
5 yr Actual default probability $p_{D,5}$, %	0.12	0.30
10 yr Actual default probability $p_{D,10}$, %	2.22	3.81
1 yr Risk - neutral default probability, $\hat{p}_{D,1}$, %	0.00	0.00
5 yr Risk - neutral default probability $\hat{p}_{D,5}$, %	0.14	0.76
10 yr Risk - neutral default probability $\hat{p}_{D,10}$, %	2.48	9.07
Arrow-Debreu default claim, q_D	14.84	36.76
Normalized restructuring boundary, X_U/X_0	2.55	2.63
Arrow-Debreu restructuring claim, q_U	37.20	22.12
Debt	482.09	5.06
Equity	1212.88	14.32
Equity risk premium, $\mu_R - r$, %	0.08	6.13

Table V : Long - run aggregate credit risk implications

This table reports long - run cross – sectional averages of credit risk variables in the dynamic capital structure model with disasters, obtained by simulating 100 economies with 1000 firms each for 100 years. Credit spreads are given in basis points, interest coverage is a pure ratio, debt and equity are price values, and all other variables in per cent.

Disasters	
Risk - free rate, r, %	0.03
Price of consumption risk, Θ, %	0.15
Credit spread (10 yr), s, b.p.	101.67
Credit spread (5 yr), s, b.p.	90.11
Leverage, $B/(B+S)$, %	31.11
Interest coverage ratio, X/c	3.72
1 yr Actual default probability, $p_{D,1}$ (mean) , %	0.02
5 yr Actual default probability $p_{D,5}$ (mean) , %	1.30
10 yr Actual default probability $p_{D,10}$ (mean), %	6.81
Equity risk premium, $\mu_R - r$, %	7.01