Credit Uncertainty Cycle*

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Abstract

This paper integrates a model of agency cost with time varying uncertainty into an otherwise standard Dynamic New Keynesian model in order to capture a "credit-uncertainty cycle" in the model economy. Deterioration in credit conditions amplifies the aggregate uncertainty of the economy; rising uncertainty further aggravates the information asymmetry between lenders and borrowers, worsens credit conditions and eventually cause more damage than the initial shock. In our model, uncertainty emerges from the volatility in the idiosyncratic productivity of an economic agent (micro uncertainty) as well as in the total factor productivity (macro uncertainty). We describe the time-variant micro and macro uncertainty using stochastic volatility models. In order to obtain independent effects of an uncertainty shock to the economy without interacting with other shocks, we solve our model based on a third-order approximation using perturbation method. We estimate our model using Bayesian Markov Chain Monte Carlo (MCMC) methods to identify the credit, uncertainty as well as the other economic shocks and obtain the steady state parameters of the model economy.

JEL Classification: E32, E44, D8, C32

KEY WORDS: Agency Cost, Uncertainty, Financial Accelerator, Stochastic Volatility, Perturbation Methods.

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1 Introduction

The subprime mortgage crisis together with the collapse of housing prices in the latter part of the 2000s led the US economy into the longest recession since World War II. The outstanding amounts of subprime securitization, however, were by themselves not large enough to have caused the decline in economic activity that were experienced (Gorton (2010)). The unique character of this recession has sparked a renewed interest in the role of two important amplification mechanisms: credit market frictions and uncertainty.

One interpretation of credit frictions stems from the information asymmetry that characterizes credit markets. This gives rise to agency costs that are incorporated in financial contracts that link borrowers and lenders. Recent research develops a credit channel framework which highlights the effect of credit frictions and how they propagate cyclical movements of real economic activity (e.g., Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997), Bernanke, Gertler and Gilchrist (1996, 1999)), and affect monetary policy-making (e.g., Carlstrom, Fuerst and Paustian (2010), Christiano, Motto and Rostagno (2009)). Meanwhile, research which focuses on time-variant second-moment uncertainty has also attracted much attention lately. Aggregate uncertainty, which rises significantly following major economic shocks, delays investment by changing investor sentiments and enhances the "option value of waiting." It also strengthens the precautionary saving motive of economic agents. A shock to time-varying uncertainty has been shown to have strong effects on consumption, output and investment decisions (e.g., Alexopoulos and Cohen (2009), Bloom (2009), Bloom, Floetotto and Jaimovich (2011)).

Even though models of both credit friction and uncertainty have been developed in the literature, the connections between the two are not well established or understood. Exploring those connections is necessary and timely, since the interplay of time-varying uncertainty and imperfect credit markets may work to amplify economic fluctuations. A rise in uncertainty aggravates the information asymmetry between lenders and borrowers and, consequently, worsens credit conditions. This amplifies the initial shock and may raise the degree of uncertainty in the economy. This "credit-uncertainty cycle" suggests that credit frictions, in a model economy with uncertainty tied to them, may yield much larger impact on economic activity than what a conventional model without time-varying uncertainty would predict. It also provides a counter argument to studies that place low quantitative significance on the amplification effect of credit frictions (Kocherlakota (2000), Cordoba and Ripoll (2004)).

The goal of this project is to explore the shock propagation effect of this credit-uncertainty cycle and to identify the separate contributions of credit shocks and uncertainty shocks to economic activity. Discovering the interactions between credit and uncertainty, and differentiating their contribution will be helpful in conducting an effective stabilization policy. Formally, we integrate a model of agency cost with time-varying uncertainty into an otherwise standard Dynamic New Keynesian model. The baseline agency-cost setup is adopted from Carlstrom and Fuerst (1997). An entrepreneur seeks external funds to finance a risky investment project. The riskiness of the project is due to an idiosyncratic technology shock that can only be observed by the entrepreneur costlessly. Hence, lenders must resort to costly monitoring the outcome of the risky projects in order to dissuade the entrepreneurs from misreporting their net revenues. The cost of this monitoring process, the agency cost, is a constant fraction of the value of the project. This agency cost gives rise to the external finance premium required by the lenders and, therefore, raises the costs of borrowing.

To incorporate uncertainty in our context, we first distinguish time-varying uncertainty as "macro uncertainty" and "micro uncertainty" following Bloom *et al.* (2011). The former represents the aggregate uncertainty about the evolution of macroeconomic variables, such as aggregate output and investment; the latter represents the idiosyncratic uncertainty about the evolution of micro economic variables for individual firms or industries, such as firm-level equity returns and sales. Specifically, we let the volatility of total factor productivity (TFP) reflect macro uncertainty as in Bloom (2009) and Bloom *et al.* (2011). The micro uncertainty in our model is modeled as the volatility of the idiosyncratic technology shock of the entrepreneurs. Note that in a model where firms (entrepreneurs) seek for external funds, this idiosyncratic uncertainty about firm-level variables gives rise to the information asymmetry between lenders and borrowers, which is the source of credit frictions (see Christiano, Motto and Rostagno (2003, 2010)).

In order to model time-varying volatility for both types of uncertainty, the literature has proposed three well-developed alternatives: stochastic volatility (SV), GARCH processes, and Markov regime switching models.[1] SV and GARCH reflect a continuously changing process that has innovations in every period. In comparison, Markov regime switching models evolve in a more abrupt, discrete way, with sudden jumps interrupted by periods of calm. In this paper, we use SV to describe the variances of both TFP and idiosyncratic technology shocks. We assume the variances of the aggregate and idiosyncratic technologies evolve over time as an autoregressive process. Thus, TFP has two innovations: one that affects the (log) level of TFP and another that affects the variance of (log) TFP. We also allow aggregate shocks to spillover and effect the distribution of idiosyncratic shocks in that shocks to TFP will also shift the mean of the distribution of idiosyncratic shocks as in Faia and Monacelli (2007).

One usually cannot directly obtain reduced form solutions for a nonlinear Dynamic Stochastic General Equilibrium (DSGE) model such as the one proposed above. The most common approach to handle the nonlinearity in macroeconomics is to approximate the solutions using linear methods, in particular, first-order approximations. However, these techniques are not well suited to properly account for the role of SV. Even with a second-order approximation, time-varying volatilities enter into the solutions in a very restricted way and do not have an independent effect over the dynamics; in turn, it operates through the interaction term of the mean and variance shocks of the same stochastic variable (such as, TFP). To better capture whatever independent effects that time-varying uncertainty have on the decision rules of economic agents, we compute a third-order approximation to our model using a perturbation method as proposed in Fernandez-Villaverde and Rubio-Ramirez (2010) and Fernandez-Villaverde *et al.* (2009).[2]

Using the third order approximation, we are able to examine the effect of a macro-uncertainty shock on our DSGE model. We also consider a micro-uncertainty shock, which has direct impact on the riskiness of the capital production technology. An increase in micro-uncertainty, similarly, lowers entrepreneur consumption. However, it also exacerbates credit frictions and raises external finance premium. This deterioration in credit markets depresses investment and output.

We estimate the parameters that characterize the dynamics of the stochastic processes for TFP and for distribution of entrepreneurial productivity using Bayesian MCMC methods.The response of our model economies to the micro-uncertainty shock simulates what usually follows a typical financial crisis: stagnation of industrial investment and production together with significant job loss. On the contrary, the model economy is able to recover from a macro-uncertainty shock, which has no direct effect on raising the risk in "wall street," much more rapidly.

[1] See Fernandez-Villaverde and Rubio-Ramirez (2010) for a detailed comparison among these three approaches.

[2] See Fernandez-Villaverde and Rubio-Ramirez (2010), Schmitt-Grohe and Uribe (2004), Fernandez-Villaverde, Guerron-Quintana and Rubio-Ramirez (2010), Fernandez-Villaverde *et al.* (2009) for discussion of relative merits of alternative approximations.

The remainder of the paper proceeds as follows: section 2 describes our model with credit market imperfections and micro/macro-uncertainty, while section 3 introduces the perturbation approach that we use to compute a third-order approximation. Section 4 summarizes the parameterization strategy used for the simulations. Section 5 highlights the quantitative findings, and section 6 concludes. General equilibrium conditions, the zero-inflation steady state and all listed tables and figures are provided in the appendix.

2 The Model Economy

The economy is composed of economic agents that lie on a continuum of mass one and is split between two types, households and entrepreneurs. The entrepreneurs own and produce capital goods using final goods. Final goods, in turn, are produced by a perfectly competitive representative firm that is owned by the households. Moreover, the final good is also directly consumed by the entrepreneurs and households, but its production requires a combination of intermediate goods as inputs. There is a continuum of intermediate goods, each produced by a firm with monopolistic power in its own variety. Production of the intermediate goods requires capital (rented from the entrepreneurs and the households) as well as household labor and entrepreneurial labor. In order to partly fund the acquisition of the final goods needed for the production of capital within each period, (impatient) entrepreneurs must tap into the savings of the (patient) households. A representative, competitive financial intermediary channels the savings from the households into loans for the entrepreneurs. Intermediate-goods producers, the final-good firm and the financial intermediary are owned by the households, and profits are distributed back to them.

2.1 Households

The exists a continuum of identical and infinitely-lived households of mass $0 < (1-\eta) \leq 1$ in the economy. The households derive utility from the consumption of final goods, C_t, and disutility from labor, L_t, and maximize their lifetime utility,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, L_t\right), \tag{1}$$

subject to the following sequential budget constraint,

$$P_t C_t + Q_t(K_{t+1}^h - (1-\delta)K_t^h) + B_{t+1} \leq W_t L_t + R_t^k K_t^h + I_t B_t + D_t, \tag{2}$$

where $0 < \beta < 1$ is the intertemporal discount factor and $0 < \delta \leq 1$ is the depreciation rate.

At time t, households supply labor L_t at the prevailing competitive wage rate W_t and rent their capital stock K_t^h at the rental rate R_t^k to the intermediate goods producers. Households then purchase consumption goods C_t at price P_t from the final goods producers, and accumulate new capital produced by the capital goods producers (the entrepreneurs) at a cost Q_t per unit of capital. Since households own the intermediate and final goods firms as well as the financial intermediaries, all profits—and losses—are distributed back to the households every period in the form of dividend payments D_t (including dividends from the final goods firm, the intermediate goods firms, and the financial intermediary). Households also receive a gross nominal risk-free interest rate I_t on their holdings of one-period (uncontingent) bonds B_t maturing at time t, and acquire B_{t+1} bonds maturing at time $t+1$. The bonds are issued by the financial intermediaries and the resources attracted are made available to entrepreneurs in the form of investment loans.

Solving the households' optimization problem we obtain that the Lagrange multiplier on the budget constraint, λ_t^h, must equal to,

$$\lambda_t^h P_t = U_1 \left(C_t, L_t \right), \tag{3}$$

and also the following first-order conditions,

$$\frac{W_t}{P_t} = -\frac{U_2 \left(C_t, L_t \right)}{\lambda_t^h P_t}, \tag{4}$$

$$\frac{Q_t}{P_t} = \beta \mathbb{E}_t \left[\left(\frac{\lambda_{t+1}^h P_{t+1}}{\lambda_t^h P_t} \right) \left(\frac{R_{t+1}^k}{P_{t+1}} + \frac{Q_{t+1}}{P_{t+1}} (1-\delta) \right) \right], \tag{5}$$

$$1 = \beta \mathbb{E}_t \left[\left(\frac{\lambda_{t+1}^h P_{t+1}}{\lambda_t^h P_t} \right) \frac{P_t}{P_{t+1}} I_{t+1} \right], \tag{6}$$

where $U_1 \left(C_t, L_t \right)$ and $U_2 \left(C_t, L_t \right)$ denote the partial derivatives of the utility function with respect to C_t and L_t, respectively. Equation (4) characterizes the households' labor supply, while equations (5) and (6) correspond to the standard intertemporal Euler equations for capital and bonds respectively.

2.2 Final-Good Firm

The representative final-good firm purchases intermediate goods $Y_t(z)$ of each variety $z \in [0,1]$ at a price $P_t(z)$ to produce the final good Y_t according to the following CES technology,

$$Y_t \leq \left[\int_0^1 Y_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{7}$$

where the elasticity of substitution across intermediates is given by $\varepsilon > 1$. The final-good firm maximizes its profits,

$$D_t^f \equiv P_t Y_t - \int_0^1 P_t(z) Y_t(z) \, dz, \tag{8}$$

subject to (7). The optimization problem for the final good firm under flexible prices and perfect competition results in the following demand equation for intermediate goods,

$$Y_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\varepsilon} Y_t, \tag{9}$$

and the following pricing equation,

$$P_t = \left[\int_0^1 P_t(z)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}, \tag{10}$$

where the right-hand side is the nominal marginal cost determined as a function of the prices of the intermediate goods, $P_t(z)$.

Since the representative final-good firm is perfectly competitive and the CES technology exhibits constant returns to scale, the final-good firm makes zero profits and exhausts all its revenue acquiring the intermediate goods. The final-good firm is owned by the households, but it distributes zero dividends to its shareholders (i.e., $D_t^f = 0$). The final good is consumed by the entrepreneurs and the households. Moreover, the entrepreneurs also use the final good as an input in the production of new capital.

2.3 Intermediate-Good Firms

There is a continuum of differentiated intermediate goods that sum to one; they are indexed $z \in [0,1]$ and are each produced by a monopolistically competitive firm. At time t, each intermediate-good firm z acquires labor from households $H_t(z)$, labor from entrepreneurs $H_t^e(z)$, and rents the aggregate capital $K_t(z)$ (including household and entrepreneurial capital) in perfectly competitive factor markets to produce $Y_t(z)$ of its own intermediate variety. The production function for variety z is given by,

$$Y_t(z) \leq A_t F\left(K_t(z), G\left(H_t(z), H_t^e(z)\right)\right). \tag{11}$$

The problem of the intermediate good firm z can be parameterized assuming the following CES production technology

$$F\left(K_t(z), G\left(H_t(z), H_t^e(z)\right)\right) = \left[\alpha \left(K_t(z)\right)^{\varkappa} + (1-\alpha) G\left(H_t(z), H_t^e(z)\right)^{\varkappa}\right]^{\frac{1}{\varkappa}} \tag{12}$$

with capital input, $K_t(z)$, and labor input, which is represented by a CES labor aggregator $G(\cdot)$ which combines household labor and entrepreneurial labor

$$G\left(H_t(z), H_t^e(z)\right) \equiv \left[\theta \left(H_t(z)\right)^{\vartheta} + (1-\theta)\left(H_t^e(z)\right)^{\vartheta}\right]^{\frac{1}{\vartheta}}. \tag{13}$$

This production function can be found in Heckman *et al.* (1998) and Caselli and Coleman (2006) among others. The capital share is given by $0 \leq \alpha \leq 1$, while the labor share $(1-\alpha)$ is split between household labor (a fraction $0 \leq \theta \leq 1$) and entrepreneurial labor (a fraction $(1-\theta)$). The elasticity of substitution between capital $K_t(z)$ and the bundle of labor $G\left(H_t(z), H_t^e(z)\right)$ is given by $\frac{1}{1-\varkappa} \geq 0$. Capital and bundled labor are imperfect substitutes as long as $\varkappa < 1$ (the perfect-substitutability case corresponds to $\varkappa = 1$), the Cobb-Douglas case with unit elasticity of substitution follows when $\varkappa = 0$ and the Leontieff case when $\varkappa \to -\infty$. Similarly, the elasticity of substitution between household labor $H_t(z)$ and entrepreneurial labor $H_t^e(z)$ is given by $\frac{1}{1-\vartheta} \geq 0$. When $\varkappa = \vartheta = 0$, the technology is Cobb-Douglas as in Carlstrom and Fuerst (1997), i.e.,

$$F\left(K_t(z), H_t(z), H_t^e(z)\right) = \left(K_t(z)\right)^{\alpha} \left(\left(H_t(z)\right)^{\theta} \left(H_t^e(z)\right)^{1-\theta}\right)^{1-\alpha}, \tag{14}$$

and this is the specification we adopt as our benchmark. When $\varkappa = 0$, we obtain a model consistent with a constant capital share irrespective of the value of $\vartheta \leq 1$.

A_t denotes the aggregate productivity (or Total Factor Productivity, TFP) at time t. The stochastic process for A_t, which is common to all intermediate good firms z, can be written as,

$$\ln A_t = \rho_a \ln A_{t-1} + \sigma_{a,t} \varepsilon_{a,t}, \tag{15}$$

We introduce stochastic volatility in aggregate productivity as a source of macro-uncertainty by letting $\sigma_{a,t}$ evolve randomly over time. The logarithm of the standard deviation $\sigma_{a,t}$ evolves as an $AR(1)$ process, i.e.,

$$\ln \sigma_{a,t} = (1-\upsilon_{\sigma}) \ln \sigma_a + \upsilon_a \ln \sigma_{a,t-1} + \eta_a u_{a,t}, \tag{16}$$

where $\varepsilon_{a,t}$ and $u_{a,t}$ are i.i.d. $N(0,1)$ and uncorrelated. The shock $\varepsilon_{a,t}$ raises the productivity level (the first moment shock), while $u_{a,t}$ indicates a shock to its volatility (the second moment shock). The parameters $0 < \rho_a < 1$ and $0 < \upsilon_a < 1$ determine the persistence of the productivity level A_t and the persistence of its volatility $\sigma_{a,t}$, respectively. The unconditional expected volatility is given by $\sigma_a > 0$, while $\eta_a \geq 0$ controls the standard deviation of the innovation to the stochastic volatility process. The shock reduces to a conventional homoscedastic set-up when η_a is zero.

2.3.1 Cost-Minimizing Intermediate Firm

The intermediate firm z minimizes real production costs, i.e.,

$$\frac{W_t}{P_t} H_t(z) + \frac{W_t^e}{P_t} H_t^e(z) + \frac{R_t^k}{P_t} K_t(z), \tag{17}$$

by choosing the input vector $\{K_t(z), H_t(z), H_t^e(z)\}$ subject to the production technology in (11).

Solving the cost minimization problem yields the following three input demand equations,

$$\frac{W_t}{P_t} - \lambda_t(z) A_t \frac{\partial F(\cdot)}{\partial H_t(z)} = 0; \tag{18}$$

$$\frac{W_t^e}{P_t} - \lambda_t(z) A_t \frac{\partial F(\cdot)}{\partial H_t^e(z)} = 0; \tag{19}$$

$$\frac{R_t^k}{P_t} - \lambda_t(z) A_t \frac{\partial F(\cdot)}{\partial K_t(z)} = 0; \tag{20}$$

where $\lambda_t(z)$ is the Lagrange multiplier on the technology constraint for intermediate good firm z and, therefore, represents the real marginal cost of production.

2.3.2 Price-Setting of Intermediate Firm

Each intermediate good firm z has monopolistic power in setting the price of its own variety. However, it also faces Rotemberg-style quadratic costs to nominal price adjustment given by,

$$s_p(P_t(z), P_{t-1}(z)) = \frac{\varphi_p}{2}\left(\frac{P_t(z)}{P_{t-1}(z)} - 1\right)^2, \ \forall z \in [0,1], \tag{21}$$

where $\varphi_p \geq 0$ measures the degree of the price adjustment cost, whereas $\varphi_p = 0$ indicates that the intermediate good's pricing is perfectly flexible. This adjustment cost depends on the price change considered by each intermediate-good firm, but it is measured in terms of the final good. The Rotemberg adjustment cost function is convex, symmetric and increasing. Every period t, the intermediate-good firm z sells its output of the intermediate good $Y_t(z)$ at price $P_t(z)$, and pays competitive wages W_t and W_t^e to households and entrepreneurs as well as a competitive rental rate R_t^k to both. Capital is perfectly substitutable whether it is rented from households or entrepreneurs, but the same is not true for labor.

Hence, the intermediate firm z chooses a vector $\{K_t(z), H_t(z), H_t^e(z), P_t(z)\}$ to maximize its expected nominal profits, i.e.,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_t^h \left[P_t(z) Y_t(z) - \left(W_t H_t(z) + W_t^e H_t^e(z) + R_t^k K_t(z) \right) - s_p\left(P_t(z), P_{t-1}(z)\right) P_t Y_t \right], \tag{22}$$

where λ_t^h denotes the Lagrange multiplier on the households' budget constraint in (3). Given that households own the intermediate firms, they discount the profit stream generated by these firms accordingly. The intermediate firm's optimization is naturally split into a cost minimization problem (choosing production inputs) and a profit maximization problem (choosing its price).

Given the solution of the real marginal cost $\lambda_t(z)$ from the cost minimization problem, the intermediate firm z maximizes its nominal profits,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_t^h \left[P_t(z) Y_t(z) - P_t \lambda_t(z) Y_t(z) - s_p\left(P_t(z), P_{t-1}(z)\right) P_t Y_t \right],$$

by choosing price $P_t(z)$, subject to the demand function of intermediate good z in (9). We denote λ_t^h the Lagrangian multiplier on the household's budget constraint, implied by equation (3). We set up the following maximization problem by replacing the demand function using equation (15),

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_t^h \left[\left(\frac{P_t(z)}{P_t} \right) \left(\frac{P_t(z)}{P_t} \right)^{-\varepsilon} - \lambda_t(z) \left(\frac{P_t(z)}{P_t} \right)^{-\varepsilon} - s_p\left(P_t(z), P_{t-1}(z)\right) \right] P_t Y_t.$$

Solving the problem yields the following pricing equation:

$$\begin{aligned} & \lambda_t^h Y_t \left[\left(\tilde{p}_t(z)\right)^{-\varepsilon} \left((1-\varepsilon) + \varepsilon \frac{\lambda_t(z)}{\tilde{p}_t(z)} \right) - \varphi_p \left(\frac{\tilde{p}_t(z)}{\tilde{p}_{t-1}(z)} \pi_t - 1 \right) \frac{\pi_t}{\tilde{p}_{t-1}(z)} \right] \\ = \quad & -\beta \varphi_p \mathbb{E}_t \left[\lambda_{t+1}^h Y_{t+1} \left(\frac{\tilde{p}_{t+1}(z)}{\tilde{p}_t(z)} \pi_{t+1} - 1 \right) \tilde{p}_{t+1}(z) \left(\frac{\pi_{t+1}}{\tilde{p}_t(z)} \right)^2 \right], \end{aligned}$$

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$, and $\tilde{p}_t(z) \equiv \frac{P_t(z)}{P_t}$ as the relative price of intermediate good z.

In equilibrium, all intermediate firms z set the same price $P_t(z) = P_t$ (i.e., $\tilde{p}_t(z) = 1$), so we can rewrite the pricing equation as,

$$\lambda_t^h Y_t \left[(1-\varepsilon) + \varepsilon \lambda_t(z) - \varphi_p (\pi_t - 1) \pi_t \right] + \beta \varphi_p \mathbb{E}_t \left[\lambda_{t+1}^h Y_{t+1} (\pi_{t+1} - 1) (\pi_{t+1})^2 \right] = 0. \tag{23}$$

2.4 Entrepreneurs

The entrepreneurs are a fraction of $0 \leq \eta < 1$ of the population. As in Carlstrom and Fuerst (1997), the entrepreneurs only own the capital goods producing technology and are the source of capital formation in the economy.

2.4.1 Micro-Uncertainty

The entrepreneurs have a stochastic technology that contemporaneously transforms X_t units of the final good into $\varpi_t X_t$ units of capital goods. The term ϖ_t describes the entrepreneurs productivity in creating the capital good and is private information that can only be observed if financial intermediaries pay a monitoring cost. The distribution of ϖ_t is time-varying with mean $\varpi_{m,t}$ and a time-varying variance. We denote the density of ϖ_t as $\phi^{\varpi}(\varpi_t; .)$ and the distribution function

$$\Phi^{\varpi}(x; .) = \Pr(\varpi_t \leq x).$$

In the quantitative model below, we assume that ϖ_t is distributed log normal, $\log(\varpi_t)\tilde{}N(\mu_{\varpi,t}, \sigma^2_{\varpi,t})$, where $\sigma_{\varpi,t} = \sigma_{\varpi} e^{\hat{\sigma}_{\varpi,t}}$. What we call micro-uncertainty represents the dispersion of the cross-section distribution of entrepreneur productivity, $\sigma_{\varpi,t}$. Micro uncertainty itself evolves according to

$$\widehat{\sigma}_{\varpi,t} = \upsilon_{\varpi} \widehat{\sigma}_{\varpi,t-1} + \eta_{\varpi} u_{\varpi,t}, \tag{24}$$

where $u_{\varpi,t}$ is normally distributed with zero mean and unit variance.

2.4.2 Optimal financial contract (X_t, $\bar{\varpi}_t$)

We follow Carlstrom and Fuerst (1997) in the development of the optimal financial contract. In time period t, entrepreneurs purchase the input (in final goods) with which capital goods will be created. The nominal value of those purchases is $P_t X_t$. To finance these purchases, entrepreneurs use a combination internal funds or nominal net worth N_t and external funding $(P_t X_t - N_t)$ which is borrowed from financial intermediaries. Given a nominal lending rate R^L_t, the entrepreneur who borrows $(P_t X_t - N_t)$ at the beginning of period agrees to pay back $\left(1 + R^L_t\right)(P_t X_t - N_t)$ at the end of the period. From these resources, the entrepreneur will end up producing $\varpi_t X_t$ units of capital goods priced at Q_t per unit. If the realized revenues $Q_t \varpi_t X_t$ are lower than the cost of repaying the loan after observing the realization of the idiosyncratic shock, then the entrepreneur will default. The range of values ϖ_t over which the entrepreneur will default is given by,

$$\varpi_t < \frac{\left(1 + R^L_t\right)(P_t X_t - N_t)}{Q_t X_t} \equiv \bar{\varpi}_t. \tag{25}$$

The lender receives $\begin{cases} \left(1 + R^L_t\right)(P_t X_t - N_t) \text{ if } \varpi_t \geq \bar{\varpi}_t \\ \varpi_t Q_t X_t \qquad \text{if } \varpi_t < \bar{\varpi}_t \end{cases}$. In the event of default, in order to prevent the entrepreneur from misreporting the true value of ϖ_t, the lender pays a monitoring fee proportional to $Q_t X_t$,, i.e. $\mu \varpi_{m,t} Q_t X_t$, $\mu \in [0, 1]$.

The expected nominal income of the entrepreneurs,

$$\begin{aligned}
& \int_{\bar{\varpi}_t}^{\infty}\left[\varpi_t Q_t X_t-\left(1+R_t^L\right)\left(P_t X_t-N_t\right)\right] d \Phi^{\varpi}\left(\varpi_t ; \varpi_{m, t}, \sigma_{\varpi, t}\right) \\
= & \int_{\bar{\varpi}_t}^{\infty}\left[\varpi_t Q_t X_t-\bar{\varpi}_t Q_t X_t\right] d \Phi^{\varpi}\left(\varpi_t ; \varpi_{m, t}, \sigma_{\varpi, t}\right) \\
= & Q_t X_t \int_{\bar{\varpi}_t}^{\infty}\left(\varpi_t-\bar{\varpi}_t\right) d \Phi^{\varpi}\left(\varpi_t ; \varpi_{m, t}, \sigma_{\varpi, t}\right) \\
= & Q_t X_t\left\{\int_{\bar{\varpi}_t}^{\infty} \varpi_t d \Phi\left(\varpi_t ; A_t, \sigma_{\varpi, t}\right)-\bar{\varpi}_t\left[1-\Phi^{\varpi}\left(\varpi_t ; \varpi_{m, t}, \sigma_{\varpi, t}\right)\right]\right\} \\
\equiv & Q_t X_t f\left(\bar{\varpi}_t, \varpi_{m, t}, \sigma_{\varpi, t}\right)
\end{aligned}$$

where $f\left(\bar{\varpi}_t, \varpi_{m, t}, \sigma_{\varpi, t}\right) \equiv\left\{\int_{\bar{\varpi}_t}^{\infty} \varpi_t d \Phi^{\varpi}\left(\varpi_t ; \varpi_{m, t}, \sigma_{\varpi, t}\right)-\bar{\varpi}_t\left[1-\Phi^{\varpi}\left(\varpi_t ; \varpi_{m, t}, \sigma_{\varpi, t}\right)\right]\right\}$ denotes the fraction of the expected net capital goods output received by the entrepreneurs (borrowers).

Similarly, the expected nominal income of the lenders is given by,

$$\begin{aligned}
& Q_t X_t\left\{\left[\int_0^{\bar{\varpi}_t} \varpi_t d \Phi^{\varpi}\left(\varpi_t ; \varpi_{m, t}, \sigma_{\varpi, t}\right)-\mu \varpi_{m, t} \Phi^{\varpi}\left(\varpi_t ; \varpi_{m, t}, \sigma_{\varpi, t}\right)\right]+\bar{\varpi}_t\left[1-\Phi^{\varpi}\left(\varpi_t ; \varpi_{m, t}, \sigma_{\varpi, t}\right)\right]\right\} \\
\equiv & Q_t X_t g\left(\bar{\varpi}_t, A_t, \sigma_{\varpi, t}\right),
\end{aligned}$$

where $g\left(\bar{\varpi}_t, \varpi_{m, t}, \sigma_{\varpi, t}\right) \equiv\left\{\left[\int_0^{\bar{\varpi}_t} \varpi_t d \Phi^{\varpi}\left(\varpi_t ; \varpi_{m, t}, \sigma_{\varpi, t}\right)-\mu \varpi_{m, t} \Phi^{\varpi}\left(\varpi_t ; \varpi_{m, t}, \sigma_{\varpi, t}\right)\right]+\bar{\varpi}_t\left[1-\Phi^{\varpi}\left(\varpi_t ; \varpi_{m, t}, \sigma_{\varpi, t}\right)\right]\right\}$ denotes the fraction of the expected net capital goods output received by the lenders.

Note that the resource constraint of loanable funds is,

$$f\left(\bar{\varpi}_t, A_t, \sigma_{\varpi, t}\right)+g\left(\bar{\varpi}_t, A_t, \sigma_{\varpi, t}\right)=\varpi_{m, t}(1-\mu \Phi\left(\bar{\varpi}_t ; A_t, \sigma_{\varpi, t}\right)), \tag{26}$$

where $\mu \Phi\left(\bar{\varpi}_t ; A_t, \sigma_{\varpi, t}\right)$ indicates the fraction of capital used to pay the monitoring cost.

Given P_t, Q_t and N_t, the optimal financial contract $(X_t,\ \bar{\varpi}_t)$ maximizes the entrepreneurs' expected income subject to the constraint that the lenders are indifferent between loaning funds and retaining them. Formally, the problem involves solving

$$\underset{X_t, \bar{\varpi}_t}{Max} Q_t X_t f\left(\bar{\varpi}_t, \varpi_{m, t}, \sigma_{\varpi, t}\right) \tag{27}$$

$$\text{s.t. } Q_t X_t g\left(\bar{\varpi}_t, \varpi_{m, t}, \sigma_{\varpi, t}\right) \geq P_t X_t-N_t. \tag{28}$$

Since the optimal contract in this case implies that the constraint in equation (28) will hold with equality. The optimal $\bar{\varpi}_t$ satisfies the following equation

$$f\left(\bar{\varpi}_t, \varpi_{m, t}, \sigma_{\varpi, t}\right) Q_t=\frac{f_{\bar{\varpi}_t}\left(\bar{\varpi}_t, \varpi_{m, t}, \sigma_{\varpi, t}\right)}{g_{\bar{\varpi}_t}\left(\bar{\varpi}_t, \varpi_{m, t}, \sigma_{\varpi, t}\right)}\left[Q_t g\left(\bar{\varpi}_t, \varpi_{m, t}, \sigma_{\varpi, t}\right)-P_t\right], \tag{29}$$

Equation (29) indicates that the returns on a loan depend upon the marginal product that entrepreneurs can obtain from the idiosyncratic capital production technology. Note that a strong assumption imposed here by Carlstrom and Fuerst (1997) as well as all existing models of credit friction (e.g., Bernanke and Gertler (1989), Bernanke, Gertler and Gilchrist (1999), Kiyotaki and Moore (1997)), is that the financial contracts expire within the period.

2.4.3 Entrepreneurs in Equilibrium

Assuming an inelastic labor supply of entrepreneurs, the determinant of net worth is given by,

$$N_t = W_t^e + \left[R_t^k + Q_t^k(1-\delta)\right] K_t^e, \tag{30}$$

where W_t^e is the nominal wage received by the entrepreneurs and K_t^e denotes the capital stock of the entrepreneurs at time period t.

The entrepreneurs choose the input expenditure X_t of capital production, new capital K_{t+1}^e, and consumption expenditure C_t^e to maximize their utility, which is derived from consumption C_t^e,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} (\beta\gamma)^t C_t^e, \ \gamma \in (0,1),$$

subject to the budget constraint,

$$Q_t^k K_{t+1}^e + P_t C_t^e \leq Q_t X_t f\left(\bar{\varpi}_t, \varpi_{m,t}, \sigma_{\varpi,t}\right). \tag{31}$$

The right-hand side of the budget constraint denotes the share of the revenue received by the entrepreneurs solved by the optimal financial contract designed above. As the optimal contract requires $Q_t X_t g\left(\bar{\varpi}_t, \varpi_{m,t}, \sigma_{\varpi,t}\right) = P_t X_t - N_t$, we can write the entrepreneurs' budget constraint as

$$Q_t \frac{W_t^e + \left[R_t^k + Q_t^k(1-\delta)\right] K_t^e}{P_t - Q_t g\left(\bar{\varpi}_t, \varpi_{m,t}, \sigma_{\varpi,t}\right)} f\left(\bar{\varpi}_t, \varpi_{m,t}, \sigma_{\varpi,t}\right) - Q_t^k K_{t+1}^e - P_t C_t^e = 0. \tag{32}$$

Solving the entrepreneurs' problem yields the following first order conditions:

$$1 - \lambda_t^e P_t = 0, \tag{33}$$

$$-\frac{Q_t^k}{P_t} + \beta\gamma \mathbb{E}_t \left[\frac{\frac{Q_{t+1}}{P_{t+1}} f_{t+1}}{1 - \frac{Q_{t+1}}{P_{t+1}} g_{t+1}} \left[\frac{R_{t+1}^k}{P_{t+1}} + \frac{Q_{t+1}^k}{P_{t+1}}(1-\delta) \right] \right] = 0, \tag{34}$$

where λ_t^e denotes the Lagrange multiplier for entrepreneurs, f_t and g_t are simplified notations for functions $f\left(\bar{\varpi}_t; \varpi_{m,t}, \sigma_{\varpi,t}\right)$ and $g\left(\bar{\varpi}_t; \varpi_{m,t}, \sigma_{\varpi,t}\right)$, respectively. The discount factor of the entrepreneurs, $\beta\gamma$, suggests that the entrepreneurs are more impatient than the households and hence become the borrowers.

2.5 Capital Installation

We differ from Carlstrom and Fuerst (1997) by adding adjustment costs to changing the stock of capital. To keep the underlying Carlstrom and Fuerst optimal contracting framework static, we add a capital goods assembler who buys the capital good from entrepreneurs yet faces the adjustment costs of changing the capital stock. Thus, capital goods assembler buys the capital goods, X_t^k, from entrepreneurs. The net addition to the capital stock is given by

$$K_{t+1} - (1-\delta)K_t = s_k\left(X_t^k, X_{t-1}^{agg,k}, K_t^{agg}\right) X_t^k. \tag{35}$$

We assume that adjustment costs $s_k(\cdot)$ depend on lagged aggregate investment and/or aggregate capital and the capital goods assembler takes these as given. Current period profits of the capital goods assembler is:

$$Q_t^k s_k\left(X_t^k, X_{t-1}^{agg,k}, K_t^{agg}\right) X_t^k - Q_t X_t^k. \tag{36}$$

Q_t^k is the price of installed capital while Q_t is the price of uninstalled capital. The optimal choice of X_t^k involves the following condition:

$$Q_t^k\left(\frac{\partial s_{k,t}}{\partial X_t^k} X_t^k + s_{k,t}\right) - Q_t = 0. \tag{37}$$

The adjustment cost function $s_k\left(X_t^k, X_{t-1}^{agg,k}, K_t^{agg}\right)$ can either take a capital adjustment form,

$$s_k\left(X_t^k, X_{t-1}^{agg,k}, K_t^{agg}\right) = s_k\left(\frac{X_t^k}{K_t^{agg}}\right) = 1 - \frac{\varphi_k}{2}\frac{\left(\frac{X_t^k}{K_t^{agg}} - \delta\right)^2}{\frac{X_t^k}{K_t^{agg}}}, \tag{38}$$

or an alternative investment adjustment form,

$$s_k\left(X_t^k, X_{t-1}^{agg,k}, K_t^{agg}\right) = s_k\left(\frac{X_t^k}{X_{t-1}^{agg,k}}\right) = 1 - \frac{\varphi_x}{2}\frac{\left(\frac{X_t^k}{X_{t-1}^{agg,k}} - 1\right)^2}{\frac{X_t^k}{X_{t-1}^{agg,k}}}. \tag{39}$$

Parameters $\varphi_k > 0$ and $\varphi_x > 0$ measure the degree of the adjustment cost in each specification. The circumstance where $\varphi_k = 0$ or $\varphi_x = 0$ implies there are no adjustment costs.

2.6 Market Clearing

There are two labor markets, one capital market and one goods market in the economy. The two labor markets are cleared as follows:

$$\int_0^1 H_t(z)\,dz \equiv H_t = (1-\eta) L_t, \tag{40}$$

$$\int_0^1 H_t^e(z)\,dz \equiv H_t^e = \eta. \tag{41}$$

Aggregating over individual capital goods installers, $K_t = K_t^{agg}$ and $X_t^k = X_t^{agg,k}$, equilibrium in the capital goods market implies:

$$X_t^k = \eta\varpi_{m,t}\left[1 - \mu\Phi^{\varpi}\left(\bar{\varpi}_t; A_t, \sigma_{\varpi,t}\right)\right] X_t. \tag{42}$$

The aggregate capital accumulation is given by

$$K_{t+1} - (1-\delta) K_t = s_k\left(X_t^k, X_{t-1}^k, K_t\right) X_t^k, \tag{43}$$

with the right hand side of the equation indicating the output of newly installed capital using units of capital good X_t^k with a nonlinear aggregate capital adjustment cost $s_k\left(X_t^k, X_{t-1}^k, K_t\right)$. Note that a portion $\mu\Phi^{\varpi}\left(\bar{\varpi}_t; \varpi_{m,t}, \sigma_{\varpi,t}\right)$ of the investment output X_t is destroyed by the credit friction (agency cost). The total supply of installed capital equals the capital stock demanded by households and entrepreneurs,

$$K_{t+1} = (1-\eta) K_{t+1}^h + \eta K_{t+1}^e. \tag{44}$$

Finally, in the goods market, final output covers total consumption expenditure, investment expenditure and price adjustment costs,

$$\left[1 - \frac{\varphi_p}{2}(\pi_t - 1)^2\right] Y_t = (1-\eta) C_t + \eta C_t^e + \eta X_t. \tag{45}$$

2.7 Monetary Policy

The Taylor rule is often defined as the trademark of modern monetary policy. In that case the policy instrument of the monetary authority is the short-term rate I_t, while $\bar{I}$ is its corresponding steady state value. We assume that the monetary authorities set short-term nominal interest rates according to Taylor (1993),

$$I_t = (I_{t-1})^{\rho_i}\left[\bar{I}(\Pi_t)^{\psi_\pi}\left(\frac{Y_t}{\bar{Y}_t^*}\right)^{\psi_y}\right]^{1-\rho_i} m_t, \tag{46}$$

where m_t is the monetary policy shock, $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is the (gross) CPI inflation rate, and $\frac{Y_t}{\bar{Y}_t^*}$ is the output level in deviations from its potential $\bar{Y}_t^*$ in a frictionless environment. The frictionless environment is defined as a Real Business Cycle (RBC) model economy without nominal rigidity, any adjustment costs nor credit friction.

Appendix A summarizes the general equilibrium conditions for the model proposed above and the ones for a frictionless model.

3 A Perturbation Approach

The set of equations listed in Appendix A does not have a known analytical solution, so we need to use a numerical method to solve it. The traditional log-linearizing approach is not suitable anymore because second moment volatility does not enter into the decision rules with a first-order approximation of the system. In order to obtain an independent effect of volatility, we apply a perturbation approach following Fernandez-Villaverde and Rubio-Ramirez (2010), Fernandez-Villaverde, Guerron-Quintana and Rubio-Ramirez (2010), Fernandez-Villaverde *et al.* (2009) by solving the system through a third order approximation. Building a higher-order perturbation is an approach that has been shown to be both accurate and fast (Aruoba, Fernández-Villaverde, and Rubio-Ramírez, 2006).

3.1 Rewriting the System

The idea of the perturbation approach is to simply find a Taylor approximation of the decision rules around the steady state of the model. To do so, we must first introduce a perturbation parameter, Υ, and rewrite each stochastic process as,

$$z_{j,t} = \rho_j z_{j,t-1} + \Upsilon \sigma_{j,t} \epsilon_{j,t}, \tag{47}$$

$$\ln \sigma_{j,t} = (1 - \upsilon_j) \ln \sigma_j + \upsilon_j \ln \sigma_{j,t-1} + \Upsilon \eta_j u_{j,t}, \tag{48}$$

where $z_{j,t}$ denotes the jth structural shock (level shock) and $\sigma_{j,t}$ denotes the jth volatility shock. When $\Upsilon = 1$, we get back the original formulation of the problem. On the other hand, if we set $\Upsilon = 0$, we eliminate the sources of uncertainty in the model, and the economy will asymptotically settle at the deterministic steady state.

Next, we rewrite all variables in terms of their deviations with respect to the steady state. That is, $\hat{\mathrm{x}} = \mathrm{x_t} - \mathrm{x}$ for any arbitrary variable $\mathrm{x_t}$ with steady state x, except for $\ln \sigma_{j,t}$ where $\hat{\sigma}_{j,t} = \ln \sigma_{j,t} - \ln \sigma_j$. Accordingly, equations (47) and (48) can be written as,

$$z_{j,t} = \rho_j z_{j,t-1} + \Upsilon \sigma_j e^{\hat{\sigma}_{j,t}} \epsilon_{j,t}, \tag{49}$$

$$\hat{\sigma}_{j,t} = \upsilon_j \hat{\sigma}_{j,t-1} + \Upsilon \eta_j u_{j,t}. \tag{50}$$

As in Fernandez, Guerron and Rubio (2010) and Fernandez *et al.* (2009), each $\epsilon_{j,t}$ and $u_{j,t}$ is assumed to be normally distributed with zero mean and unit variance.

3.2 Structure of the Solution

The set of equilibrium conditions can be written in a compact way as

$$\mathbb{E}_t f\left(\mathbb{Y}_{t+1}, \mathbb{Y}_t, \mathbb{S}_{t+1}, \mathbb{S}_t, \mathbb{Z}_{t+1}, \mathbb{Z}_t\right) = 0, \tag{51}$$

where $\mathbb{E}_t$ denotes the mathematical expectations operator conditional on information available at time t, $\mathbb{Y}_t$ is the vector of control variables, $\mathbb{S}_t$ is a vector of predetermined variables, and vector $\mathbb{Z}_t$ contains all structural shocks, $z_{j,t}$. In our model,

$$\begin{aligned} \mathbb{S}_t &= \begin{bmatrix} \hat{K}_t & \hat{K}_t^e \end{bmatrix}, \\ \mathbb{Z}_t &= \begin{bmatrix} \widehat{A}_t & \hat{\sigma}_{\varpi,t} & \widehat{m}_t \end{bmatrix}. \end{aligned}$$

Note that we assume that all structural shocks follow an SV process of the form represented by equations (49) and the standard deviations of their innovations evolves as in equation (50). We can easily shut down a volatility shock by setting the appropriate entries of υ_j and η_j to zero; that way, the structural shock will return to a homoscedastic shock.

A vector of state variables for the system can be written as

$$s_t \equiv \left(\mathbb{S}_t,\ \mathbb{Z}_{t-1},\ \mathbf{\Sigma}_{t-1},\ \mathbf{E}_t,\ \mathbb{U}_t,\ \Upsilon\right),$$

where vector $\mathbf{\Sigma}_t$ contains all volatility shocks, $\hat{\sigma}_{j,t}$, vector $\mathbf{E}_t$ includes the innovations to the level shocks, $\epsilon_{j,t}$, and vector $\mathbb{U}_t$ includes the innovations to the volatility shocks, $u_{j,t}$. Then, the solution to the system of functional equations given in equation (51) can be expressed in terms of the following two equations:

$$\mathbb{Y}_t = \mathfrak{g}\left(\mathbb{S}_t,\ \mathbb{Z}_{t-1},\ \mathbf{\Sigma}_{t-1},\ \mathbf{E}_t,\ \mathbb{U}_t,\ \Upsilon\right), \tag{52}$$

$$\mathbb{S}_{t+1} = \mathfrak{h}\left(\mathbb{S}_t,\ \mathbb{Z}_{t-1},\ \mathbf{\Sigma}_{t-1},\ \mathbf{E}_t,\ \mathbb{U}_t,\ \Upsilon\right). \tag{53}$$

We are seeking a higher-order approximation to functions $\mathfrak{g}(\cdot)$ and $\mathfrak{h}(\cdot)$ around the steady state where $\mathbb{S}_t = \mathbb{S}$ and $\Upsilon = 0$. As in Fernandez and Rubio (2010), Fernandez, Guerron and Rubio (2010), and Fernandez *et al.* (2011), we find that the first partial derivatives of $\mathfrak{g}(\cdot)$ and $\mathfrak{h}(\cdot)$ with respect to any component of $\mathbf{\Sigma}_{t-1}$ and $\mathbb{U}_t$ as well as to the perturbation parameter Υ evaluated at the steady state equal to zero. This suggests that neither variances nor their evolution enter in the first-order component of the solution of the model.

It is only in the second-order component of the solution that we have terms that depend upon variances. However, even in the second-order form, time-varying volatilities enter into the solution only through the interaction term of the innovations to the structural shocks, $\mathbf{E}_t$, and the innovations to volatility shocks, $\mathbb{U}_t$. That is, we have to hit the model with both a level shock (such as the TFP shock) and its volatility shock (the macro-uncertainty shock) simultaneously to show the effect of stochastic volatility. Under a second-order approximation, a volatility shock does not affect the approximated model economy independently but only plays a role of scaling the effect of a level shock. This has been proved in Schmitt-Grohé and Uribe (2004) and Fernandez, Guerron and Rubio (2010). The second cross-derivatives of $\mathfrak{g}(\cdot)$ and $\mathfrak{h}(\cdot)$ with respect to $\mathbf{\Sigma}_{t-1}$ and $\mathbb{U}_t$ are all zero. Only the cross derivatives of each innovation to the structural shocks with respect to *its own* volatility shock, "$\mathbf{\Sigma}_{t-1}\mathbf{E}_t$," and the cross derivatives of the innovation to the structural shocks with respect to the innovation to *its own* volatility shock, "$\mathbf{E}_t\mathbb{U}_t$."

In order to have time-varying macro-uncertainty enter without interacting with any other variables, we need to compute at least a third-order approximation. We clarify this with Table 1, in which we characterize the third derivatives of $\mathfrak{g}(\cdot)$ and $\mathfrak{h}(\cdot)$ with respect to different variables $(\mathbb{S}_t,\ \mathbb{Z}_{t-1},\ \mathbf{\Sigma}_{t-1},\ \mathbf{E}_t,\ \mathbb{U}_t,\ \Upsilon)$[3]. The way to read the table is as follows: Take an arbitrary entry, for instance, entry (2,3), $\mathbb{S}_t\mathbb{Z}_{t-1}\mathbf{E}_t \neq 0$. It states that the cross-derivatives of $\mathfrak{g}(\cdot)$ and $\mathfrak{h}(\cdot)$ with respect to $\mathbb{S}_t$, $\mathbb{Z}_{t-1}$ and $\mathbf{E}_t$ are different from zero. Based on Table 1, as long as we do not shut down the uncertainty of the model by setting a zero perturbation parameter, a volatility shock enters the system via the nonzero cross-derivative term, $\mathbb{U}_t\Upsilon\Upsilon$, without interacting with any other variables.

Once we obtain the higher-order cross-derivatives of $\mathfrak{g}(\cdot)$ and $\mathfrak{h}(\cdot)$ evaluated at the steady state, we can apply nth-order Taylor expansion of the decision rules

$$\mathfrak{f}\left(\mathrm{x}_{1,\mathrm{t}}, \mathrm{x}_{2,\mathrm{t}}, ..., \mathrm{x}_{\mathrm{n},\mathrm{t}}\right) \approx \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}...\sum_{k_n=0}^{\infty}\left[\frac{\partial^{k_1+k_2+...+k_n}\mathfrak{f}\left(\mathrm{x}_1, \mathrm{x}_2, ..., \mathrm{x}_\mathrm{n}\right)}{\partial \mathrm{x}_1^{k_1}\partial \mathrm{x}_2^{k_2}...\partial \mathrm{x}_\mathrm{n}^{k_n}}\frac{\hat{\mathrm{x}}_{1,\mathrm{t}}^{k_1}}{k_1!}\frac{\hat{\mathrm{x}}_{2,\mathrm{t}}^{k_2}}{k_2!}..\frac{\hat{\mathrm{x}}_{\mathrm{n},\mathrm{t}}^{k_n}}{k_n!}\right]$$

to any arbitrary variable x_t with steady state x. To obtain the third-order Taylor polynomial, we need to sum over $k_1 + k_2 + ... + k_n \leq 3$. We use the pruning approach as in Kim *et al.*'s (2003) to get rid of spurious higher order terms in our model simulations.

[3] The derivation of the results summarized in Table 1 is too long to attach, but is available upon request.

4 Model Parameterization

Table 2 summarizes the model parameters adopted in our model simulations. Since we adopt the set-up of credit friction from Carlstrom and Fuerst (1997), our model parametrization is roughly similar to theirs, except where otherwise noted.

Household preferences are given by a logarithmic utility function, in which the labor supply elasticity is chosen so that a steady state level of household's labor supply L equals 0.3. The intertemporal discount factor, β, equals 0.99, implying a 4 percent annual real interest rate. The elasticity of substitution across varieties in the CES aggregator, ε, is set to be 10, consistent with a price markup of roughly 11 percent.

We assume the consumption good production technology to have a Cobb-Douglas form with constant returns to scale. As in Carlstrom and Fuerst (1997), the technology with a capital share of 0.36 and a household labor share of 0.6399 implies a close-to-zero entrepreneur labor share. Capital depreciation rate is set to be 0.02. The agency cost, μ, is set to 0.25.

Since the parameters associated with the adjustment costs and nominal rigidity cannot be pinned down by the deterministic steady state in which all adjustment costs are zero, we assign conventional values to these parameters following the literature. The Rotemberg price adjustment parameter, φ_p, is chosen such that the coefficient on marginal cost in the Phillips curve is 0.052 as in Carlstrom, Fuerst and Paustian (2010). In an equivalent Calvo price-setting model, such a slope implies that prices are fixed for 5 quarters on average. For the labor adjustment cost parameter, φ_l, we use an estimate of 1.96 following Janko (2007) and Cooper and Willis (2003). Finally, the capital and investment adjustment cost parameters, φ_k and φ_x, are set as 13.25 and 2.12 for a flexible-price model, respectively, and 11.15 and 3.35 for a sticky-price model as in Martinez-Garcia and Søndergaard (2009).

The parameters defines the Taylor rule are chosen as the estimates from Justiniano, Primiceri and Tambalotti (2011). The interest rate inertia parameter, ρ_i, equals 0.858, while the sensitivity of the nominal policy rate to the inflation target, ψ_π, equals 1.709, the sensitivity to the level of output gap, ψ_y, equals 0.051, and the sensitivity to the growth rate of the output gap, ψ_{dy}, equals 0.208.

5 Estimation of Stochastic Volatility Parameters

Unlike most of the other parameters, there is little consensus about what are reasonable parameter values for the stochastic volatility processes. As a result, we estimate these ourselves. Unlike Christiano *et al.* (2009) or Fernandez, Guerron and Rubio (2010) who use the general equilibrium implications of the stochastic volatility shocks to estimate their values, we attempt to get more direct information about these shocks by focusing on a few key observables. For the stochastic volatility process for TFP, we use realized Solow residuals to jointly estimated the autoregressive process and stochastic volatility process for TFP. For micro-uncertainty, we use the model's implications for the external finance premium and the internal rate of return (or alternatively the probability of default) to infer the time series values of $\sigma_{\varpi,t}$, from which we can then estimate SV model for micro-uncertainty.

5.1 Stochastic Volatility Model for TFP (macro-uncertainty)

We use realized Solow residuals from 1954:1 to 2009:4 to jointly estimate the autoregressive process for the TFP and the stochastic volatility model for TFP.[4] We linearly detrended the Solow residuals, allowing for a break in the time trend in 1973:1. The top two panels of Figure 1 display the raw Solow residuals and the detrended Solow residuals.

Recall the model for (log, detrended) TFP is the following:

$$a_t = \rho_a a_{t-1} + \sigma_a e^{\hat{\sigma}_{a,t}} \varepsilon_{a,t}, \tag{54}$$

and

$$\hat{\sigma}_{a,t} = \upsilon_a \hat{\sigma}_{a,t-1} + \eta_a u_{a,t}, \tag{55}$$

where $\varepsilon_{a,t}$ and $u_{a,t}$ are distributed N(0,1). While we observe a_t, we do not observe $\hat{\sigma}_{a,t}$ directly. Our approach is to estimate the parameters ρ_a, σ_a, υ_a, η_a, and $\hat{\sigma}_{a,t}$ $t = 0, 1, ..., T$ by using Bayesian MCMC methods (see appendix for details). The approach is a mixture of Gibbs/Metropolis-Hasting sampling. Given values for a_t, ρ_a, σ_a, υ_a, η_a, and $\hat{\sigma}_{a,\tilde{}t}$ we sample $\hat{\sigma}_{a,t}$ (by Metropolis-Hasting). Similarly, given a_t and $\hat{\sigma}_{a,t}$ t=1,...T, we sample ρ_a and σ_a. Finally, given $\hat{\sigma}_{a,t}$ t=1,...T, we sample υ_a and η_a. We use 100,000 draws to burn in the Markov Chain and 50,000 draws to approximate the posterior distribution. While the literature provides a good sense of reasonable priors for ρ_a and σ_a, we use relatively diffuse priors for the other parameters.

The bottom panel of Figure 1 displays the median, 5th, and 95th percentiles of the posterior distribution for $\hat{\sigma}_{a,t}$ and $\sigma_a e^{\hat{\sigma}_{a,t}}$. While one observes fluctuations in the values of $\hat{\sigma}_{a,t}$ and $\sigma_a e^{\hat{\sigma}_{a,t}}$ over time, the posterior distribution for individual time periods are often not too precise. Figure 2 displays the actual draws from the MCMC that make up the posterior distribution. From these figures, it appears that the chain has converged. We take the median of the posterior distribution to be the model parameters in our simulation below. For ρ_a the median is 0.87, while for σ_a the median is 0.0099. These are not out-of-line from typical values in the DSGE literature (Heathcote and Perri (2002)). For the stochastic volatility parameters, the median of sample distribution of υ_a is 0.21 while the median for η_a is 0.64. These values are similar to those estimated by Fernandez, Guerron and Rubio (2010).

alternatively the probability of default) to infer the time series values of $\sigma_{\varpi,t}$, from which we can then estimate SV model for micro-uncertainty.

5.2 Stochastic Volatility Model for Entrepreneur Productivity (micro-uncertainty)

The distribution of entrepreneur productivity, ϖ_t is log-normal with a time-varying mean, $\varpi_{m,t}$, and time-varying variance, $\Sigma_{\varpi,t}$. Specifically,

$$\ln(\varpi_t)\tilde{}N(\mu_{\varpi,t}, \sigma_\varpi e^{\hat{\sigma}_{\varpi,t}}) \tag{56}$$

with

$$\hat{\sigma}_{\varpi,t} = \upsilon_\varpi \hat{\sigma}_{\varpi,t-1} + \eta_\varpi u_{\varpi,t} \tag{57}$$

so that $\varpi_{m,t} = \exp(\mu_{\varpi,t} + .5(\sigma_\varpi e^{\hat{\sigma}_{\varpi,t}})^2)$.

[4] For details on the construction of the Solow residuals see Martinez-Garcia (2011).

Given the assumption of log normal distribution, we can write the probability of default as

$$\Phi_t^{default} = \Phi^N \left(\left(\ln(\bar{\varpi}_t) - \mu_{\varpi,t} \right) / (\sigma_{\varpi} e^{\hat{\sigma}_{\varpi,t}}) \right) \tag{58}$$

where $\Phi^N(.)$ is the cdf for a standard normal distribution and $\bar{\varpi}_t$ is the critical default threshold. The external finance premium is given by:

$$R_t^L = \frac{\bar{\varpi}_t}{g_t} - 1 \tag{59}$$

where $g_t = \varpi_{m,t}\Phi^N \left(\left(\ln(\bar{\varpi}_t) - \mu_{\varpi,t} - (\sigma_{\varpi} e^{\hat{\sigma}_{\varpi,t}})^2 \right) / (\sigma_{\varpi} e^{\hat{\sigma}_{\varpi,t}}) \right) + \bar{\varpi}_t(1 - \Phi_t^{default}) - \varpi_{m,t}\mu\Phi_t^{default}$. Note that $\Phi_t^{default}$ and R_t^L depend only on the values of $\bar{\varpi}_t$, $\mu_{\varpi,t}$, and $(\sigma_{\varpi} e^{\hat{\sigma}_{\varpi,t}})$. As in Carlstrom and Fuerst (1997), we normalize $\varpi_{m,t}$ so that in the deterministic steady state, $\varpi_m = 1$ (or $\mu_{\varpi} = -.5{\sigma_{\varpi}}^2$). This allows us to pin down the steady state values for $\bar{\varpi}$ and σ_{ϖ} so that in the steady state $\Phi^{default} = 0.00974$ and the annual external finance premium is 187 basis points .

Similarly, given $\varpi_{m,t}$ (or equivalently, $\mu_{\varpi,t}$), if we had data on quarterly data on expected default probabilities, $\Phi_t^{default}$, and data on external finance premium, R_t^L, we could solve for $\bar{\varpi}_t$ and $\hat{\sigma}_{\varpi,t}$ in each time period. From the times series for $\hat{\sigma}_{\varpi,t}$, we could then estimate the stochastic process for micro-uncertainty. Unfortunately, while there is data reflecting R_t^L, we use the spread between Baa rated bonds and 10 year Treasury notes, we do not have access to time series data reflecting expected default probabilities.

What we do instead is use flow of funds data to provide an additional observation equation with which to back out $\bar{\varpi}_t$ and $\hat{\sigma}_{\varpi,t}$. Rewriting the first order condition of the optimal contract equation (29) yields,

$$\left[q_t \varpi_{m,t}(1 - \mu\Phi_t^{default}) - 1 \right] \frac{x_t}{n_t} = -\frac{f_{\bar{\varpi}}(\bar{\varpi}_t, \mu_{\varpi,t}, \sigma_{\varpi} e^{\hat{\sigma}_{\varpi,t}})}{g_{\bar{\varpi}}(\bar{\varpi}_t, \mu_{\varpi,t}, \sigma_{\varpi} e^{\hat{\sigma}_{\varpi,t}})} - 1 \tag{60}$$

where $q_t = \frac{Q_t}{P_t}$. Given data on the price of investment goods, q_t, and the leverage ratio, $\frac{x_t}{n_t}$, we can use equation (59) along with equation (60) to solve for $\bar{\varpi}_t$ and $\hat{\sigma}_{\varpi,t}$ in each time period.[5] For q_t, we use the NIPA fixed investment deflator divided by the GDP deflator. For the leverage ratio, $\frac{x_t}{n_t}$, we use one plus the ratio of credit instrument liabilities to net worth of nonfarm, nonfinancial corporate business from the Flow of Funds data. Note that the rate of return on internal funds is $R_t^{internalfunds} = \left[q_t \varpi_{m,t}(1 - \mu\Phi_t^{default}) - 1 \right] \frac{x_t}{n_t}$. For the case where $\varpi_{m,t} = 1$ and we do not adjust for the likelihood and cost of default, the unadjusted rate of return on internal funds is just $[q_t - 1] \frac{x_t}{n_t}$.

Figure 3 displays external finance premium and unadjusted rate of return on internal funds, $[q_t - 1] \frac{x_t}{n_t}$. Given the financial deregulation of the early 1980s, our sample runs from 1984:1-2011:2. As there is a substantial downward trend in $[q_t - 1] \frac{x_t}{n_t}$, this suggests that there is likely a substantial upward trend in $\varpi_{m,t}$. Therefore, we add a time trend to our model by letting $\mu_{\varpi,t} = -.5\sigma_{\varpi}^2 + \mu_{\varpi,trend}(t - T/2)$. We then use Bayesian MCMC methods to estimate the values of $\mu_{\varpi,trend}$, υ_{ϖ}, η_{ϖ}, and $\hat{\sigma}_{\varpi,0}$.[6] We use a burn-in period of 10,000 draws and 10,000 draws for the posterior distribution.

The bottom three panels of Figure 3, display the estimated values of $\hat{\sigma}_{\varpi,t}$ and the implied rate of return on internal funds as well as the implied probability of default. The sample means of these implied variables are fairly plausible despite being slightly higher than the implied steady state values of these variables. Given

[5] As these equations are highly nonlinear, it is possible for there to be multiple solutions. In this case, we penalized solutions in which the implied default probabilities and return on internal funds were far from the model's deterministic steady state.

[6] The value of σ_{ϖ} was set using steady state information.

the sharp rise in the external finance premium in 2008, the model implies a sharp rise default probabilities corresponding the financial crisis of 2008. Interestingly, the model suggests an increase in micro-uncertainty in the early 2000s as well as during the 2008 financial crisis. The model implies relatively low rates of return on internal funds during the early 2000s but substantial increases in the return on internal funds during the financial crisis.

Figure 4 displays the actual draws from the MCMC that make up the posterior distribution. From these figures, it appears that the chain has converged. Again, we take the median of the posterior distribution to be the model parameters in our simulations below. For ν_{ϖ} the median is 0.99, suggesting micro-uncertainty variable to be quite persistent. The median of the posterior distribution for the standard deviation of innovations to $\hat{\sigma}_{\varpi,t}$, η_{ϖ} is 0.086. In comparison, using different data and a linear approximated DSGE model, Christiano *et al.* (2009) estimate ν_{ϖ} and η_{ϖ} to be 0.93 and 0.036, respectively.

6 Results

6.1 Summary Statistics Implied by the Models

Table 3 displays some summary statistics for the various models. Allowing SV process on macro-uncertainty results in roughly a 40 percent increase in the volatility of output but does not change much the relative variances or cross-correlations. Allowing SV process on micro-uncertainty results in increase in variance of output of around 20 percent but also changes cross-correlations in a substantial way. In particular, the standard agency-cost model without stochastic volatility results in procyclical movements in external finance premium (and default probabilities). Including stochastic volatility, in particular time varying micro-uncertainty, results in a more plausible countercyclical external finance premium. Adding time varying micro-uncertainty, on the other hand, gives rise to countercyclical price of capital ($log(q_t^k)$), but if one adds adjustment costs to the model the price of capital becomes procyclical. The finding that time-variant micro-uncertainty changes the standard agency-cost model in a qualitatively important way (but not necessarily the macro-uncertainty) is consistent with the financial accelerator literature. Although endogenous credit spreads do not account for much of the business cycles, tying exogenous shocks to the spreads does.

6.2 Impulse Response Analysis

Given the nonlinear nature of the model, we conduct impulse response analysis as in Koop, Pesaran, and Potter (1996) by examining the change in condition expectations given the shock ϵ_t, $E[Y_{t+k}|\epsilon_t, Y_{t-1}] - E[Y_{t+k}|Y_{t-1}]$.[7] As our baseline, we take the initial condition to be the unconditional mean of the distribution for Y_t. Below we also consider alternative initial conditions.

6.2.1 Response to a TFP Shock

In Figure 5, we examine the effect of a TFP shock on the baseline model with credit frictions and stochastic volatility. For comparison, we also display the response for the model with credit frictions and no stochastic

[7] To calculate the $E[Y_{t+k}|.]$, for each period we simulate the model 10,000 draws of shocks, the negative of the same 10,000 shocks, and then average.

volatility and the model with stochastic volatility and no credit frictions. The shock is a positive, one standard deviation shock to log TFP.

One observes that allowing time-varying macro-uncertainty and micro-uncertainty, in general, amplifies the effect of the TFP shock relative to a model that does not include stochastic volatility. A positive technology shock raises investment, consumption, labor and output and, qualitatitvely, the responses of the model economy to the TFP shock are consistent with the findings in Carlstrom and Fuerst (1997). The shock enters through the same channel as in the RBC model and raises the demand for production inputs. A rise in capital demand raises the price of investment (as well as the price of capital with the absence of capital adjustment cost), thus increases the return to internal funds, $\frac{q_t f_t}{1-q_t g_t}$. On impact, the risk-neutral entrepreneurs raise their internal funds holdings (net worth). On the other hand, the rise in investment also indicates an increase in the need for external funds. Since the response of investment is larger than the one of net worth, the reduction of net worth/ investment ratio raises the probability of default as well as the external finance premium, R_t^L, required by the lenders based on equation (25).

Similar as in Carlstrom and Fuerst (1997), the hump shape in investment in the agency-cost models leads to a reverse hump in household consumption after its initial increase. The increase in household labor supply coupled with the increase in labor demand results in a hump-shaped response for hours worked.

6.2.2 A Macro-Uncertainty Shock

Figure 6 presents the impact of a one standard deviation shock to the variance of the stochastic aggregate productivity–a macro-uncertainty shock. We examine the effect of this shock for both models with and without credit frictions. In principle, an increase in TFP volatility raises the expected value future TFP and, hence, has a positive wealth effect (although in this case the persistence of a TFP volatility shock is fairly small) as well as raises uncertainty about future TFP.

Output, consumption, and hours all fall in response to a TFP volatility shock; although, all of these effects are quite small quantitatively. The net effect is that investment and capital goods quantities fall and, as a consequence, prices of investment and capital goods drop. The presence of credit frictions amplifies the effects on investment and capital goods prices but lesses the effect on investment and capital goods quantities relative to the no credit frictions model. Finally, a macro-uncertainty shock increases the external finance premium, the probability of default, and the internal rate of return suggesting that credit frictions are exacerbated when macro-uncertainty rises.

6.2.3 A Micro-Uncertainty Shock

A shock that raises the volatility of the stochastic investment outcome, $\sigma_{\varpi,t}$ increases the dispersion of the cross-section distribution of the entrepreneur productivity. We refer to this shock as a micro-uncertainty shock, although an increase in $\sigma_{\varpi,t}$, given the log-normal distribution, also raises the mean of entrepreneur productivity, $\varpi_{m,t}$.

Although economic agents cannot insure themselves against aggregate shocks (systematic risk), a complete asset market allows for perfect risk sharing and therefore full insurance against idiosyncratic shocks. A frictionless RBC model approximates such a complete asset markets specification and hence, only reflects response to uninsurable aggregate shocks (such as macro-uncertainty), but not idiosyncratic shocks (such as micro-uncertainty). On the other hand, in an agency-cost model, asset markets are incomplete and idiosyncratic shocks cannot be fully insured due to information asymmetry. In our model, we provide the twist which comes from the assumption that the volatility of those idiosyncratic shocks is time-varying and, therefore, the probability and costs of default change with that volatility (which we call micro-uncertainty).

The responses of no credit frictions and agency-cost model economies are reported in Figure 7. Recall that an increase in micro-uncertainty shock raises the mean of the log-normal distribution of entrepreneurial productivity. In a frictionless RBC model, this micro-uncertainty shock is similar to an investment specific technology shock which raises the capital production technology. It lowers the price of capital, increases investment and reduces consumption. The decline in household consumption raises household labor supply and finally causes output to rise.

When credit friction is present, the increase in $\sigma_{\varpi,t}$ has direct impact on the riskiness of the return on investment and shifts the lender's income share $g\left(\bar{\varpi}_t, \varpi_{m,t}, \sigma_{\varpi,t}\right)$. The riskier capital production technology leads to a higher required external finance premium. The high cost of borrowing discourages investment, pushes up the price of capital and internal rate of return, and encourage entrepreneurs to free up more internal funds. In response to the falling investment, households increase consumption and cut down labor input. As a result, output shrinks.

6.3 Sensitivity Analysis

In this section we examine how sensitive the impulse response analysis is extensions in the baseline model. We consider three extensions to the baseline SV-credit frictions model. First, we add capital adjustment costs to the model. This introduces an additional wedge between the price of capital goods and consumption goods. Second, we consider the case where the degree of relative risk aversion is equal to seven (and the intertemporal elasticity of substitution=1/7) rather than one as in the baseline case. Third, we allow entrepreneurial productivity to be correlated with TFP. Specifically, we assume that $\varpi_{m,t} = V\left(A_t\right)$ with $V' > 0$ as in Faia and Monacelli (2007). Note that with equations (25) and (28), we can rewrite the external finance premium as,

$$R_t^L = \frac{\bar{\varpi}}{g\left(\bar{\varpi}_t, \varpi_{m,t}, \sigma_{\varpi,t}\right)} - 1.$$

If the distribution of the risky investment outcome is constant, e.g., $\varpi_m = 1$, the external finance premium is a monotonic increasing function only of the threshold value $\bar{\varpi}$. On the other hand, in the case in which $\varpi_{m,t}$ depends on TFP, the lender's income share $g\left(\cdot\right)$ is also a function of aggregate productivity. Therefore, in the agency-cost model with $\varpi_{m,t} = V\left(A_t\right)$, a TFP shock has a direct impact on the riskiness of the financial contract.

Figure 8 reports the responses of these three extensions together with the benchmark SV-credit frictions model to the TFP shock. The presence of the adjustment cost aggravates the change in the price of capital, q_t^k. Since installing capital using investment goods is costly, investment goods become less desirable. This dampens the increases in investment (due to the positive TFP shock), the price of investment goods, q_t, and accordingly, the increase in internal rate of return, net worth, as well as the external finance premium. Also, due to the capital adjustment cost, households choose to raise more consumption and less labor supply than in the model without adjustment cost. Accordingly, the rise in output is moderated.

Figure 8 also shows that increasing the degree of relative risk aversion or allowing TFP shock to have a direct impact on the distribution (here the mean) of the entrepreneurial technology can exacerbate the effect of the TFP shock on the indicators of credit frictions, including external finance premium and probability of default, as well as investment and output.

Figure 9 displays the comparison of alternative models of agency cost in response to the macro-uncertainty shock. One observes that allowing the entrepreneurial productivity to be correlated to TFP shock (accordingly, to the macro-uncertainty) yields the largest impact as output, hours, investment, capital goods all decline substantially while external finance premium, probability of default, and rate of return on internal funds all rise. For this model increasing uncertainty about future TFP also increases the uncertainty about the future mean of entrepreneur productivity. The increase in external finance premium is large enough to push up the price of investment and capital and, eventually, causes more adverse effects on investment and output.

Figure 10 displays the responses of alternative models to the micro-uncertainty shock. Changing the degree of risk aversion or the mean of entrepreneurial productivity yields similar results as the baseline SV-credit friction model. Allowing capital adjustment cost in the model exacerbates the response of risk premium while dampens the changes of investment, household consumption, hours worked as well as output, compared to the model without adjustment cost.

Given the nonlinear structure of the model, in principle the initial condition (and the size and direction of shock) can affect the responses to shock as well. Figure 11 displays how the responses for the variables to a macro-uncertainty shock can depend on the initial state of the economy. Recall that the baseline case assumes that variables are initially equal to their unconditional means. The figure also reports the responses to when the variables are initially equal to their deterministic steady state values– the unconditional mean and steady state values need not coincide for a nonlinear model. Based on Figure 11, initial conditions matter. Alternative initial conditions lead to different responses of household consumption, hours worked and output. We also consider the case where we average across possible initial conditions[8]. Finally, we compare the results to case where we naively set all future shocks equal to zero, ignoring the implications of nonlinearity for conditional expectations. This naive experiment results in smaller effects of macro-uncertainty than computing generalized impulses.

7 Conclusion and Remarks on Ongoing Work

We examine the effect of macro- and micro-uncertainty in model economies with credit frictions by taking a third-order approximation of an agency-cost model. We model time-varying uncertainty as stochastic volatility process. We estimate the parameters that characterize the dynamics of the stochastic processes for TFP and for distribution of entrepreneurial productivity using Bayesian MCMC methods.

We find that incorporating stochastic volatility amplifies the effects of shocks. Moreover, while adding macro-uncertainty (SV model for TFP) increases the overall volatility of output in the model, allowing time-varying micro-uncertainty rather than macro-uncertainty is more important for changing the qualitative features of the model with credit frictions.

This project is currently an ongoing work. Incorporating nominal price frictions will allow us to introduce another source of macro-uncertainty— uncertainty about monetary policy. It would be interesting to examine how this policy uncertainty interacts with micro-uncertainty and credit frictions.

[8] Here we simulate the model 200 periods to determine the initial condition. We than average over 10,000 simulations to get the average response.

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Appendix

A The Sequence of Events in a Given Period

1. The current TFP shock, A_t and macro-uncertainty shock, $\sigma_{a,t}$, are realized.

2. The micro-uncertainty shock, $\sigma_{\varpi,t}$, is realized, thus the distribution of stochastic idiosyncratic technology, ϖ_t, is observed by the households.

3. Intermediate-good firms hire labor and rent capital from households and entrepreneurs. These inputs are used to produce the consumption good, Y_t.

4. Households decide how much of their labor and capital income to consume immediately, and how much to use to purchase the investment good. For each unit of investment that household wishes to purchase, it gives q_t consumption goods to the representative financial intermediary.

5. The financial intermediary use the resources obtained from households to provide loans to the entrepreneurs utilizing the optimal financial contract.

6. Entrepreneurs borrow resources from the financial intermediary and place all of these resources along with their entire net worth into their capital production technology.

7. The stochastic idiosyncratic technology of each entrepreneur, $\varpi_{j,t}$, is realized, where j indexes the infinite number of entrepreneurs. Those entrepreneurs with $\varpi_{j,t} \geq \bar{\varpi}_t$ repay the loan from the financial intermediary. Those with $\varpi_{j,t} < \bar{\varpi}_t$ declare bankruptcy and is monitored by the financial intermediary.

8. Those entrepreneurs who are still solvent make their consumption decision.

B A Summary of General Equilibrium

Lower case variables denote real variables, except that $\hat{\Lambda}_t \equiv \Lambda_t / P_t$. Also, $\tilde{\lambda}_t^h \equiv \lambda_t^h P_t$.

B.1 The Model of Agency Cost

Goods Market $\left(\lambda_t^h, Y_t, C_t, C_t^e, \lambda_t(z), \pi_t\right)$

$$\tilde{\lambda}_t^h - U_1(C_t, L_t) = 0, \tag{61}$$

$$\left[1 - \frac{\varphi_p}{2}(\pi_t - 1)^2\right] Y_t - (1-\eta) C_t - \eta C_t^e - \eta X_t = 0, \tag{62}$$

$$Y_t - A_t F(K_t, G(H_t, H_t^e)) = 0, \tag{63}$$

$$q_t^k K_{t+1}^e + C_t^e - q_t X_t f(\bar{\varpi}_t, \varpi_{m,t}, \sigma_{\varpi,t}) = 0, \tag{64}$$

$$1 - \beta \mathbb{E}_t \left[\left(\frac{\tilde{\lambda}_{t+1}^h}{\tilde{\lambda}_t^h}\right) \frac{I_{t+1}}{\pi_{t+1}}\right] = 0, \tag{65}$$

$$Y_t \left[(1-\varepsilon) + \varepsilon \lambda_t(z) - \varphi_p (\pi_t - 1)\pi_t\right] + \beta \varphi_p \mathbb{E}_t \left[\frac{\tilde{\lambda}_{t+1}^h}{\tilde{\lambda}_t^h} Y_{t+1} (\pi_{t+1} - 1)\pi_{t+1}\right] = 0 \tag{66}$$

Capital Market *($q_t, \tilde{\Lambda}_t, r_t^k, K_{t+1}^h, K_{t+1}^e, K_{t+1}$):*

$$-q_t^k + \beta \mathbb{E}_t \left[\left(\frac{\tilde{\lambda}_{t+1}^h}{\tilde{\lambda}_t^h}\right)\left(r_{t+1}^k + q_{t+1}^k (1-\delta)\right)\right] = 0 \tag{67}$$

$$-q_t^k + \beta \gamma \mathbb{E}_t \left[\frac{q_{t+1} f_{t+1}}{1 - q_{t+1} g_{t+1}}\left(r_{t+1}^k + q_{t+1}^k (1-\delta)\right)\right] = 0 \tag{68}$$

$$q_t^k \left(\frac{\partial s_{k,t}}{\partial X_t^k} X_t^k + s_{k,t}\right) - q_t = 0. \tag{69}$$

$$r_t^k - \lambda_t(z) A_t \frac{\partial F_t}{\partial K_t(z)} = 0, \tag{70}$$

$$K_{t+1} - (1-\delta) K_t - s_{k,t} X_t^k = 0, \tag{71}$$

$$X_i^k - \eta \left[\varpi_{m,t} - \mu \Phi(\bar{\varpi}_t; \varpi_{m,t}, \sigma_{\varpi,t})\right] X_t = 0,$$

$$K_{t+1} - (1-\eta) K_{t+1}^h - \eta K_{t+1}^e = 0, \tag{72}$$

Credit Market *($n_t, X_t, \bar{\varpi}_t$):*

$$n_t - w_t^e - r_t^k K_t^e - q_t^k (1-\delta) K_t^e = 0, \tag{73}$$

$$X_t - \frac{n_t}{1 - q_t g(\bar{\varpi}_t, \varpi_{m,t}, \sigma_{\varpi,t})} = 0, \tag{74}$$

$$q_t f\left(\bar{\varpi}_t, \varpi_{m,t}, \sigma_{\varpi,t}\right) - \frac{f_{\bar{\varpi}_t}\left(\bar{\varpi}_t, \varpi_{m,t}, \sigma_{\varpi,t}\right)}{g_{\bar{\varpi}_t}\left(\bar{\varpi}_t, \varpi_{m,t}, \sigma_{\varpi,t}\right)}\left[q_t g\left(\bar{\varpi}_t, \varpi_{m,t}, \sigma_{\varpi,t}\right) - 1\right] = 0 \tag{75}$$

Labor Market *($w_t, w_t^e, H_t, H_t^e, L_t$)*

$$w_t - \lambda_t(z) A_t \frac{\partial F_t}{\partial H_t(z)} = 0 \tag{76}$$

$$w_t^e - \lambda_t(z) A_t \frac{\partial F_t}{\partial H_t^e(z)} = 0 \tag{77}$$

$$H_t - (1-\eta) L_t = 0 \tag{78}$$

$$H_t^e - \eta = 0 \tag{79}$$

$$w_t + \frac{U_2(C_t, L_t)}{\tilde{\lambda}_t^h} = 0 \tag{80}$$

Monetary policy *(I_t)*

$$I_t = (I_{t-1})^{\rho_i}\left[\bar{I}(\pi_t)^{\psi_\pi}\left(\frac{Y_t}{\bar{Y}_t^*}\right)^{\psi_y}\right]^{1-\rho_i} m_t \tag{81}$$

B.2 Exogenous Shocks

Denoting $\hat{\sigma}_{a,t} \equiv \ln \sigma_{a,t} - \ln \sigma_a$ and $\hat{\sigma}_{\varpi,t} \equiv \ln \sigma_{\varpi,t} - \ln \sigma_\varpi$, we know

$$\sigma_{\varpi,t} = \sigma_\varpi e^{\hat{\sigma}_{\varpi,t}}. \tag{82}$$

TFP shock (SV)

$$\begin{aligned} \ln A_t &= \rho_a \ln A_{t-1} + \sigma_a e^{\hat{\sigma}_{a,t}} \varepsilon_{a,t}, \\ \hat{\sigma}_{a,t} &= \nu_a \hat{\sigma}_{a,t-1} + \eta_a u_{a,t}. \end{aligned}$$

Micro-uncertainty shock

$$\hat{\sigma}_{\varpi,t} = \nu_\varpi \hat{\sigma}_{\varpi,t-1} + \eta_\varpi u_{\varpi,t}.$$

C Estimation of SV-Models

C.1 Estimation of SV-model for Solow Residuals

Define $\widetilde{a}_T = \{a_t, t = 1...T\}$ is the observed data.

For the ith iteration of the Markov Chain:

1. Draw $\hat{\sigma}^{(i)}_{a,t}$ given $\widetilde{a}_T, \rho_a^{(i-1)}, \sigma_a^{(i-1)}, \nu_a^{(i-1)}, \eta_a^{(i-1)}, \hat{\sigma}^{(i)}_{a,t-1}, \hat{\sigma}^{(i-1)}_{a,t+1}$ for t=0,...T.

Draw candidate $\hat{\sigma}^c_{a,t} = \hat{\sigma}^{(i-1)}_{a,t} + s_{\sigma_a}\epsilon$,where $\epsilon\tilde{\ }t(v = 25)$. Calculate acceptance probability:

$$\alpha = \min(1, \frac{\exp(l(\widetilde{a}_T|\hat{\sigma}^c_{a,t}, \rho_a^{(i-1)}, \sigma_a^{(i-1)}, \nu_a^{(i-1)}, \eta_a^{(i-1)}, \hat{\sigma}^{(i)}_{a,t-1}, \hat{\sigma}^{(i-1)}_{a,t+1}))}{\exp(l(\widetilde{a}_T|\hat{\sigma}^{(i-1)}_{a,t}, \rho_a^{(i-1)}, \sigma_a^{(i-1)}, \nu_a^{(i-1)}, \eta_a^{(i-1)}, \hat{\sigma}^{(i)}_{a,t-1}, \hat{\sigma}^{(i-1)}_{a,t+1}))} \frac{g^{\widehat{\sigma}_{a,t}}(\hat{\sigma}^c_{a,t})}{g^{\widehat{\sigma}_{a,t}}(\hat{\sigma}^{(i-1)}_{a,t})} \tag{83}$$

where $l(\widetilde{a}_T|...)$ is the log likelihood given by

$$l(\widetilde{a}_T|\hat{\sigma}^c_{a,t}, ...) = -.5\log(\sigma_a^{(i-1)}\exp(\hat{\sigma}^c_{a,t})) - .5\left(\frac{a_t - \rho_a^{(i-1)}a_{t-1}}{\sigma_a^{(i-1)}\exp(\hat{\sigma}^c_{a,t})}\right)^2 \tag{84}$$

$$-.5\left(\frac{\hat{\sigma}^c_{a,t} - \nu_a^{(i-1)}\hat{\sigma}^{(i)}_{a,t-1}}{\eta_a^{(i-1)}}\right)^2 - .5\left(\frac{\hat{\sigma}^{(i-1)}_{a,t+1} - \nu_a^{(i-1)}\hat{\sigma}^c_{a,t}}{\eta_a^{(i-1)}}\right)^2 + \text{constant} \tag{85}$$

and the prior density $g^{\widehat{\sigma}_{a,t}}(.)$ is N(0,10000). $\hat{\sigma}^{(i)}_{a,t}= \hat{\sigma}^c_{a,t}$ with probability α and $\hat{\sigma}^{(i)}_{a,t}= \hat{\sigma}^{(i-1)}_{a,t}$ with probability $1-\alpha$. We set the scale, s_{σ_a}, so that the acceptance probability is between 25% and 40%.

2. Draw $a_{0,}^{(i)}\rho_a^{(i)}, \sigma_a^{(i)}$ given $\widetilde{a}_T, \hat{\sigma}^{(i)}_{a,t,t=1..T}$.

Draw candidate $\begin{pmatrix} a_0^c \\ \rho_a^c \\ \sigma_a^c \end{pmatrix} = \begin{pmatrix} a_0^{(i-1)} + \epsilon_1 \\ \frac{\exp\left(\log\left(\frac{\rho_a^{(i-1)}}{1-\rho_a^{(i-1)}}\right)+\epsilon_2\right)}{\exp\left(\log\left(\frac{\rho_a^{(i-1)}}{1-\rho_a^{(i-1)}}\right)+\epsilon_2\right)+1} \\ \exp(\log(\sigma_a^{(i-1)}) + \epsilon_3) \end{pmatrix}$,

where $\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{pmatrix} = s_a\Gamma_a \begin{pmatrix} v_1 \\ v_2 \\ \nu_3 \end{pmatrix}$ with $v_i\tilde{\ }t(25)$. Γ_a is the Choleski decomposition of the variance/covariance of the parameter vector $\left(a_0, \log\left(\frac{\rho_a}{1-\rho_a}\right), \log(\sigma_a)\right)$ that maximizes the posterior mode given $\hat{\sigma}^{(i)}_{a,t,t=1..T} =$ 0.The acceptance probability is:

$$\alpha = \min(1, \frac{\exp(l(\widetilde{a}_T|a^c_{0,}\rho^c_a, \sigma^c_a, \hat{\sigma}^{(i)}_{a,t=1,...T}))}{\exp(l(\widetilde{a}_T|a_{0,}^{(i-1)}\rho_a^{(i-1)}, \sigma_a^{(i-1)}, \hat{\sigma}^{(i)}_{a,t=1,...T}))} \frac{g^a(a^c_{0,}\rho^c_a, \sigma^c_a)}{g^a(a_{0,}^{(i-1)}\rho_a^{(i-1)}, \sigma_a^{(i-1)})} \tag{86}$$

where $l(\widetilde{a}_T|...)$ is the log likelihood given by

$$l(\widetilde{a}_T|a^c_{0,}\rho^c_a, \sigma^c_a, \hat{\sigma}^{(i)}_{a,t=1,...T})) = -.5\log(\sigma^c_a\exp(\hat{\sigma}^{(i)}_{a,t}) - .5\left(\frac{a_1 - \rho^c_a a^c_0}{\sigma^c_a\exp(\hat{\sigma}^{(i)}_{a,1})}\right)^2 \tag{87}$$

$$-\sum_{t=2}^{T}\left(.5\log(\sigma^c_a\exp(\hat{\sigma}^{(i)}_{a,t}) + .5\left(\frac{a_t - \rho^c_a a_{t-1}}{\sigma^c_a\exp(\hat{\sigma}^{(i)}_{a,t})}\right)^2\right) \tag{88}$$

and the prior density $g^a(.) = g^{a_0}(a_0|\rho_a,\sigma_a)g^{\rho_a}(\rho_a)g^{\sigma_a}(\sigma_a)$. Here we assume $g^{a_0}(a_0|\rho_a,\sigma_a) = \phi^N(\frac{a_0}{(1-\rho_a^2)^{.5}\sigma_a})$, $g^{\rho_a}(\rho_a) = beta(5.55, 2)$, and $g^{\sigma_a}(\sigma_a) = IG(908, 10)$. We set the scale parameter s_a so that the acceptance probability is in the 25-40% range.

3. Draw $\nu_a^{(i)}, \eta_a^{(i)}$ given $\hat{\sigma}_{a,t=1,...,T}^{(i)}$.

Draw candidate $\begin{pmatrix} \nu_a^c \\ \eta_a^c \end{pmatrix} = \begin{pmatrix} \frac{\exp\left(\log\left(\frac{\nu_a^{(i-1)}}{1-\nu_a^{(i-1)}}\right)+\epsilon_1\right)}{\exp\left(\log\left(\frac{\nu_a^{(i-1)}}{1-\nu_a^{(i-1)}}\right)+\epsilon_1\right)+1} \\ \exp(\log(\eta_a^{(i-1)}) + \epsilon_2) \end{pmatrix}$,

where $\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} = s_{SV} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ with $v_i\tilde{\ }t(25)$. Calculate acceptance probability:

$$\alpha = \min(1, \frac{\exp(l(\hat{\sigma}_{a,t=1,...T}^{(i)}|\nu_a^c,\eta_a^c))}{\exp(l(\hat{\sigma}_{a,t=1,...T}^{(i)}|\nu_a^{(i-1)},\eta_a^{(i-1)}))} \frac{g^{SV_a}(\nu_a^c,\eta_a^c)}{g^{SV_a}(\nu_a^{(i-1)},\eta_a^{(i-1)})} \tag{89}$$

where the log likelihood given by

$$l(\hat{\sigma}_{a,t=1,...T}^{(i)}|\nu_a^c,\eta_a^c) = -.5T\log(\eta_a^c) - \sum_{t=1}^{T} .5\left(\frac{\hat{\sigma}_{a,t}^{(i)} - \nu_a^c\hat{\sigma}_{a,t-1}^{(i)}}{\eta_a^c}\right)^2 \tag{90}$$

and the prior density $g^{SV_a}(.) = g^{\nu_a}(\nu_a)g^{\eta_a}(\eta_a)$. Here we assume $g^{\nu_a}(\nu_a) = beta(1.1, 1.1)$ and $g^{\sigma_a}(\sigma_a) = IG(.02, .51)$. We set the scale vector, s_{SV}, so that the acceptance probability is between 25% and 40%.

C.2 Estimation of SV-model for micro-uncertainty

Recall that given data $Y_t = (R_t^L, q_t, x_t/n_t)$ and the parameters $\varphi = (\sigma_\varpi, \mu_{\varpi,trend})$, we can use equations (59) and (60) to solve for $\overline{\varpi}_t$ and $\hat{\sigma}_{\varpi,t}$. Furthermore, we can solve for the implied default probability, $\Phi_t^{default}$, and the implied rate of return on internal funds, $R_t^{Internal.}$. We denote these as $\hat{\sigma}_\varpi(Y_t,\varphi)$, $\Phi^{def}(Y_t,\varphi)$, $R^{Int}(Y_t,\varphi)$, respectively. We use Bayesian MCMC methods to estimate posterior distributions for $\mu_{\varpi,trend}$, υ_ϖ, and η_ϖ (note the value of σ_ϖ is set to match steady-state values) as well as the posterior distribution for the initial state $\hat{\sigma}_{\varpi,0}$.

For the ith iteration of the Markov Chain:

Draw candidate $\theta^c = \begin{pmatrix} \mu_{\varpi,trend}^c \\ \hat{\sigma}_{\varpi,0}^c \\ \nu_\varpi^c \\ \eta_\varpi^c \end{pmatrix} = \begin{pmatrix} \mu_{\varpi,trend}^{(i-1)} + \epsilon_1 \\ \hat{\sigma}_{\varpi,0}^{(i-1)} + \epsilon_2 \\ \frac{\exp\left(\log\left(\frac{\nu_\varpi^{(i-1)}}{1-\nu_\varpi^{(i-1)}}\right)+\epsilon_3\right)}{\exp\left(\log\left(\frac{\nu_\varpi^{(i-1)}}{1-\nu_\varpi^{(i-1)}}\right)+\epsilon_3\right)+1} \\ \exp(\log(\eta_\varpi^{(i-1)}) + \epsilon_4) \end{pmatrix}$

where $\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \end{pmatrix} = s_\varpi\Gamma_\varpi \begin{pmatrix} v_1 \\ v_2 \\ \nu_3 \\ \nu_4 \end{pmatrix}$ with $v_i\tilde{\ }t(25)$. Γ_ϖ is the Choleski decomposition of the variance/covariance of the parameter vector $\left(\mu_{\varpi,trend}, \hat{\sigma}_{\varpi,0}, \log\left(\frac{\nu_\varpi}{1-\nu_\varpi}\right), \log(\eta_\varpi)\right)$ that maximizes the posterior mode. Note $\varphi^c = (\sigma_\varpi, \mu_{\varpi,trend}^c)$ and $\varphi^{(i-1)} = (\sigma_\varpi, \mu_{\varpi,trend}^{(i-1)})$. Calculate acceptance probability:

$$\alpha = \min(1, \frac{\prod_{t=1}^{T} \exp(l^{\sigma_\varpi}(\hat{\sigma}_\varpi(Y_t, \varphi^c)|\theta^c)) \exp(l^{\Phi^{def}}(\Phi^{def}(Y_t, \varphi^c)) \exp(l^{R^{int}}(R^{int}(Y_t, \varphi^c))}{\prod_{t=1}^{T} \exp(l^{\sigma_\varpi}(\hat{\sigma}_\varpi(Y_t, \varphi^{(i-1)})|\theta^{(i-1)}) \exp(l^{\Phi^{def}}(\Phi^{def}(Y_t, \varphi^{(i-1)})) \exp(l^{R^{int}}(R^{int}(Y_t, \varphi^{(i-1)}))} \frac{g^\theta(\theta^c)}{g^\theta(\theta^{(i-1)})} \tag{91}$$

where

$$l^{\sigma_\varpi}(\hat{\sigma}_\varpi(Y_t, \varphi^c)|\theta^c) = -.5\log(\eta_\varpi^c) - .5\left(\frac{\hat{\sigma}_\varpi(Y_t, \varphi^c) - \nu_\varpi^c \hat{\sigma}_\varpi(Y_{t-1}, \varphi^c)}{\eta_\varpi^c}\right)^2 + \text{constant} \tag{92}$$

$$l^{\Phi^{def}}(\Phi^{def}(Y_t, \varphi^c)) = -.5\left(\frac{\Phi^{def}(Y_t, \varphi^c) - \Phi^{def}_{steadystate}}{sd(\Phi^{def})}\right)^2 + \text{constant} \tag{93}$$

$$l^{R^{int}}(R^{int}(Y_t, \varphi^c)) = -.5\left(\frac{R^{int}(Y_t, \varphi^c) - R^{int}_{steadystate}}{sd(R^{int})}\right)^2 + \text{constant} \tag{94}$$

with the prior density $g^\theta(.) = g^{\mu_{\varpi,trend}}(\mu_{\varpi,trend}) g^{\sigma_{\varpi,0}}(\hat{\sigma}_{\varpi,0}|\nu_\varpi, \eta_\varpi) g^{\nu_\varpi}(\nu_\varpi) g^{\eta_\varpi}(\eta_\varpi)$. The loss functions, $l^{\Phi^{def}}(\Phi^{def}(Y_t, \varphi^c))$ and $l^{R^{int}}(R^{int}(Y_t, \varphi^c))$, act to punish deviations of implied default probability and rate of return on internal funds from the steady state values assumed by the model. It is equivalent to assuming that default probabilities and internal rates of return are iid normal with means equal to their steady states. Here $\Phi^{def}_{steadystate} = 0.0097$ and $sd(\Phi^{def}) = 0.05$ while $R^{int}_{steadystate} = 0.056$ and $sd(R^{int}) = 0.08$. For prior densities, we assume $g^{\mu_{\varpi,trend}}(\mu_{\varpi,trend}) = N(0, 10)$, $g^{\sigma_{\varpi,0}}(\hat{\sigma}_{\varpi,0}|\nu_\varpi, \eta_\varpi) = N(0, \frac{\eta_\varpi^2}{(1-\nu_\varpi^2)^{.5}})$, $g^{\nu_\varpi}(\nu_\varpi) = beta(1.1, 1.1)$, and $g^{\eta_\varpi}(\eta_\varpi) = IG(.02, .51)$. We set the scale, s_ϖ, so that the acceptance probability is between 25% and 40%.

Table 1. Third Derivatives

$\mathbb{S}_t\mathbb{S}_t\mathbb{S}_t \neq 0$	$\mathbb{S}_t\mathbb{S}_t\mathbb{Z}_{t-1} \neq 0$	$\mathbb{S}_t\mathbb{S}_t\mathbf{\Sigma}_{t-1} = 0$	$\mathbb{S}_t\mathbb{S}_t\mathbf{E}_t \neq 0$	$\mathbb{S}_t\mathbb{S}_t\mathbb{U}_t = 0$	$\mathbb{S}_t\mathbb{S}_t\Upsilon = 0$
$\mathbb{S}_t\mathbb{Z}_{t-1}\mathbb{Z}_{t-1} \neq 0$	$\mathbb{S}_t\mathbb{Z}_{t-1}\mathbf{\Sigma}_{t-1} = 0$	$\mathbb{S}_t\mathbb{Z}_{t-1}\mathbf{E}_t \neq 0$	$\mathbb{S}_t\mathbb{Z}_{t-1}\mathbb{U}_t = 0$	$\mathbb{S}_t\mathbb{Z}_{t-1}\Upsilon = 0$	
$\mathbb{S}_t\mathbf{\Sigma}_{t-1}\mathbf{\Sigma}_{t-1} = 0$	$\mathbb{S}_t\mathbf{\Sigma}_{t-1}\mathbf{E}_t \neq 0$	$\mathbb{S}_t\mathbf{\Sigma}_{t-1}\mathbb{U}_t = 0$	$\mathbb{S}_t\mathbf{\Sigma}_{t-1}\Upsilon = 0$		
$\mathbb{S}_t\mathbf{E}_t\mathbf{E}_t \neq 0$	$\mathbb{S}_t\mathbf{E}_t\mathbb{U}_t \neq 0$	$\mathbb{S}_t\mathbf{E}_t\Upsilon = 0$			
$\mathbb{S}_t\mathbb{U}_t\mathbb{U}_t = 0$	$\mathbb{S}_t\mathbb{U}_t\Upsilon = 0$				
$\mathbb{S}_t\Upsilon\Upsilon \neq 0$					
$\mathbb{Z}_{t-1}\mathbb{Z}_{t-1}\mathbb{Z}_{t-1} \neq 0$	$\mathbb{Z}_{t-1}\mathbb{Z}_{t-1}\mathbf{\Sigma}_{t-1} = 0$	$\mathbb{Z}_{t-1}\mathbb{Z}_{t-1}\mathbf{E}_t \neq 0$	$\mathbb{Z}_{t-1}\mathbb{Z}_{t-1}\mathbb{U}_t = 0$	$\mathbb{Z}_{t-1}\mathbb{Z}_{t-1}\Upsilon = 0$	
$\mathbb{Z}_{t-1}\mathbf{\Sigma}_{t-1}\mathbf{\Sigma}_{t-1} = 0$	$\mathbb{Z}_{t-1}\mathbf{\Sigma}_{t-1}\mathbf{E}_t \neq 0$	$\mathbb{Z}_{t-1}\mathbf{\Sigma}_{t-1}\mathbb{U}_t = 0$	$\mathbb{Z}_{t-1}\mathbf{\Sigma}_{t-1}\Upsilon = 0$		
$\mathbb{Z}_{t-1}\mathbf{E}_t\mathbf{E}_t \neq 0$	$\mathbb{Z}_{t-1}\mathbf{E}_t\mathbb{U}_t \neq 0$	$\mathbb{Z}_{t-1}\mathbf{E}_t\Upsilon = 0$			
$\mathbb{Z}_{t-1}\mathbb{U}_t\mathbb{U}_t = 0$	$\mathbb{Z}_{t-1}\mathbb{U}_t\Upsilon = 0$				
$\mathbb{Z}_{t-1}\Upsilon\Upsilon \neq 0$					
$\mathbf{\Sigma}_{t-1}\mathbf{\Sigma}_{t-1}\mathbf{\Sigma}_{t-1} = 0$	$\mathbf{\Sigma}_{t-1}\mathbf{\Sigma}_{t-1}\mathbf{E}_t \neq 0$	$\mathbf{\Sigma}_{t-1}\mathbf{\Sigma}_{t-1}\mathbb{U}_t = 0$	$\mathbf{\Sigma}_{t-1}\mathbf{\Sigma}_{t-1}\Upsilon = 0$		
$\mathbf{\Sigma}_{t-1}\mathbf{E}_t\mathbf{E}_t \neq 0$	$\mathbf{\Sigma}_{t-1}\mathbf{E}_t\mathbb{U}_t \neq 0$	$\mathbf{\Sigma}_{t-1}\mathbf{E}_t\Upsilon = 0$			
$\mathbf{\Sigma}_{t-1}\mathbb{U}_t\mathbb{U}_t = 0$	$\mathbf{\Sigma}_{t-1}\mathbb{U}_t\Upsilon = 0$				
$\mathbf{\Sigma}_{t-1}\Upsilon\Upsilon \neq 0$					
$\mathbf{E}_t\mathbf{E}_t\mathbf{E}_t \neq 0$	$\mathbf{E}_t\mathbf{E}_t\mathbb{U}_t \neq 0$	$\mathbf{E}_t\mathbf{E}_t\Upsilon = 0$			
$\mathbf{E}_t\mathbb{U}_t\mathbb{U}_t \neq 0$	$\mathbf{E}_t\mathbb{U}_t\Upsilon = 0$				
$\mathbf{E}_t\Upsilon\Upsilon \neq 0$					
$\mathbb{U}_t\mathbb{U}_t\mathbb{U}_t = 0$	$\mathbb{U}_t\mathbb{U}_t\Upsilon = 0$				
$\mathbb{U}_t\Upsilon\Upsilon \neq 0$					
$\Upsilon\Upsilon\Upsilon \neq 0$[9]					

[9] The third derivative with respect to the perturbation parameter, $\Upsilon\Upsilon\Upsilon$, depends on the skewness of the distribution of the innovations to structural shocks, $\mathbf{E}_t$. If the innovations are assumed to be normally distributed as in Fernandez and Rubio (2010), Fernandez, Guerron and Rubio (2010), and Fernandez *et al.* (2009), this third derivative equals to zero.

Table 2. Parameters Used in the Model Simulations

Structural Parameters		
Intertemporal Discount Factor	$0 < \beta < 1$	0.99
"Preference Parameter"	$\xi \geq 0$ $(\xi \neq 1)$	1
"Preferences Parameter"	$\kappa \geq 0$	0
"Preferences Parameter"	$\psi \geq 0$	chosen so that $L = 0.3$
"Preferences Parameter"	$\varsigma \geq 0$ $(\varsigma \neq 1)$	0
"Preferences Parameter"	$\varrho \geq 0$	0
Capital Share	$0 \leq \alpha \leq 1$	0.36
Fraction of Labor Share from Household Labor	$0 \leq \theta \leq 1$	0.6399
Elasticity of Subst. between Capital and Labor	$\frac{1}{1-\varkappa} \geq 0$	1
Elasticity of Subst. between Household and Entrepreneur Labor	$\frac{1}{1-\vartheta} \geq 0$	1
Depreciation Rate	$0 < \delta \leq 1$	0.02
Elasticity of substitution across varieties	$\varepsilon > 1$	10
Agency Cost Parameters		
Monitoring Cost	$0 < \mu < 1$	0.25
Std. Dev. of Stochastic Idiosyncratic Technology	$\sigma_{\varpi} > 1$	0.203
Additional Rate of Discounting of Entrepreneur	$0 < \gamma < 1$	0.947
The Bankruptcy Rate	$0 < \Phi(\bar{\varpi}) < 1$	0.00974
The external finance premium	$0 < R^L < 1$	0.0187
Adjustment Cost Parameters		
Labor Adjustment Cost	$\varphi_l \geq 0$	1.96
Rotemberg Price Adjustment Cost	$\varphi_p \geq 0$	173.0769
Capital Adjustment Cost	$\varphi_k \geq 0$	13.25 or 11.15
Investment Adjustment Cost	$\varphi_x \geq 0$	2.12 or 3.35
Taylor Rule Parameters		
Interest Rate Inertia	ρ_i	0.858
Sensitivity to Inflation Target	ψ_π	1.709
Sensitivity to Level of Output Gap	ψ_y	0.051
Sensitivity to Growth Rate of Output Gap	ψ_{dy}	0.208
Exogenous Shock Parameters		
TFP Shock Persistence	$0 < \rho_a < 1$	0.87
TFP Shock Unconditional Volatility	$\sigma_a \geq 0$	0.0099
Persistence of the Stochastic Volatility of TFP	$0 < \upsilon_a < 1$	0.21
Std. Dev. of the Stochastic Volatility of TFP	$\eta_a \geq 0$	0.64
Micro Uncertainty Shock Persistence	$0 < \upsilon_{\varpi} < 1$	0.99
Micro Uncertainty Shock Unconditional Volatility	$\eta_{\varpi} \geq 0$	0.086
Monetary Shock Volatility	$\sigma_m \geq 0$	0.210

Table 3. Summary statistics mean of variables model

SD(X)/SD(log(Y))

Variables	SV-Credit	NoSV-Credit	NoUa-Credit	NoUo-Credit	SV-NoCredit	SV-Credit-Adj. Costs
$log(Y)^*$	0.0637	0.0411	0.0484	0.0581	0.0677	0.0380
$log(C)$	0.4999	0.4683	0.5231	0.4687	0.5076	0.7340
$log(L)$	0.7966	0.7252	0.8381	0.7249	0.8059	0.3853
$log(X)$	3.4307	3.2695	3.5138	3.2547	3.5383	1.9140
$log(X^k)$	3.5739	3.2678	3.7515	3.2531	3.7179	2.1836
$log(q^k)$	0.3879	0.1231	0.5004	0.1232	0.4818	0.6406
$log(q)$	0.3879	0.1231	0.5004	0.1232	0.4818	0.6076
$log(n)$	9.5407	2.8955	12.3224	2.8848	0.9508	15.0032
R^L	1.6677	0.0375	2.1929	0.0375	N/A	2.7172
$\Phi^{def}(\cdot)$	0.7791	0.0140	1.0252	0.0141	N/A	1.2673
R^{int}	0.5097	0.3882	0.5890	0.3891	N/A	0.6500

*This entry is $SD(log(Y))$

Crosscorrelation with Output

Variables	SV-Credit	NoSV-Credit	NoUa-Credit	NoUo-Credit	SV-No Credit	SV-Credit-Adj. Costs
$log(Y)$	1.00	1.00	1.00	1.00	1.00	1.00
$log(C)$	0.62	0.74	0.55	0.74	0.60	0.95
$log(L)$	0.87	0.90	0.85	0.90	0.86	0.79
$log(X)$	0.92	0.95	0.90	0.95	0.92	0.95
$log(X^k)$	0.92	0.95	0.91	0.95	0.93	0.89
$log(q^k)$	-0.25	0.43	-0.39	0.42	-0.37	0.51
$log(q)$	-0.25	0.43	-0.39	0.42	-0.37	-0.15
$log(n)$	0.32	0.89	0.28	0.89	1.00	0.15
R^L	-0.08	0.42	-0.10	0.42	N/A	-0.02
$\Phi^{def}(\cdot)$	-0.07	0.42	-0.09	0.42	N/A	-0.01
R^{int}	0.17	0.42	0.06	0.42	N/A	0.05

Autocorrelation

Variables	SV-Credit	NoSV-Credit	NoUa-Credit	NoUo-Credit	SV-No Credit	SV-Credit-Adj. Costs
$log(Y)$	0.89	0.92	0.88	0.92	0.88	0.87
$log(C)$	0.92	0.91	0.92	0.91	0.98	0.88
$log(L)$	0.84	0.93	0.79	0.93	0.84	0.63
$log(X)$	0.84	0.90	0.82	0.90	0.84	0.85
$log(X^k)$	0.85	0.90	0.83	0.90	0.85	0.88
$log(q^k)$	0.90	0.22	0.93	0.22	0.97	0.88
$log(q)$	0.90	0.22	0.93	0.22	0.97	0.94
$log(n)$	0.97	0.90	0.97	0.90	0.87	0.97
R^L	0.95	0.22	0.95	0.21	N/A	0.97
$\Phi^{def}(\cdot)$	0.95	0.21	0.95	0.21	N/A	0.97
R^{int}	0.16	0.22	0.15	0.22	N/A	0.14

Figure 1. Solow residuals and estimated stochastic volatility for TFP

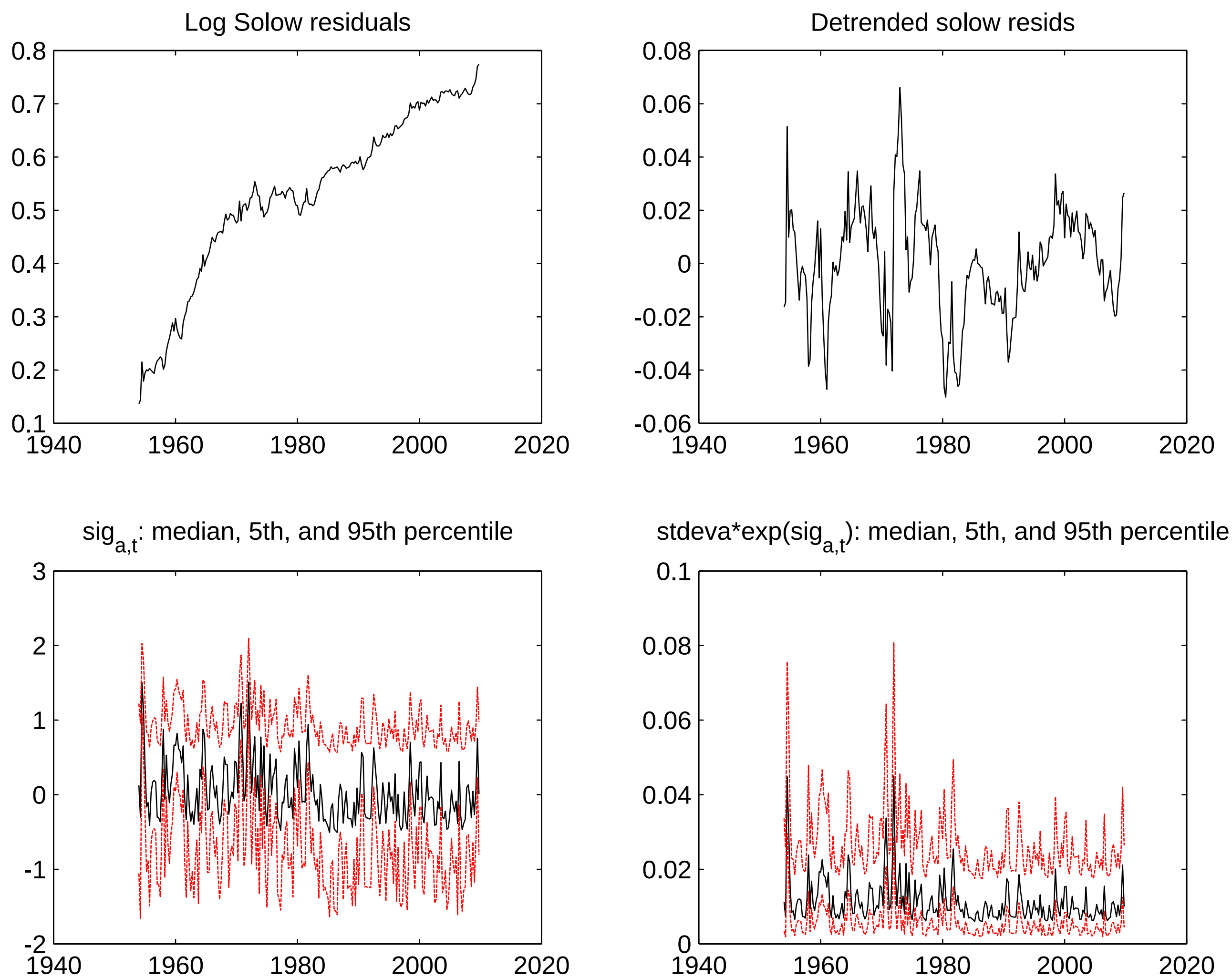

Figure 2. Draws from MCMC for the parameters for stochastic volatility model for Solow Residuals (50,000 draws)

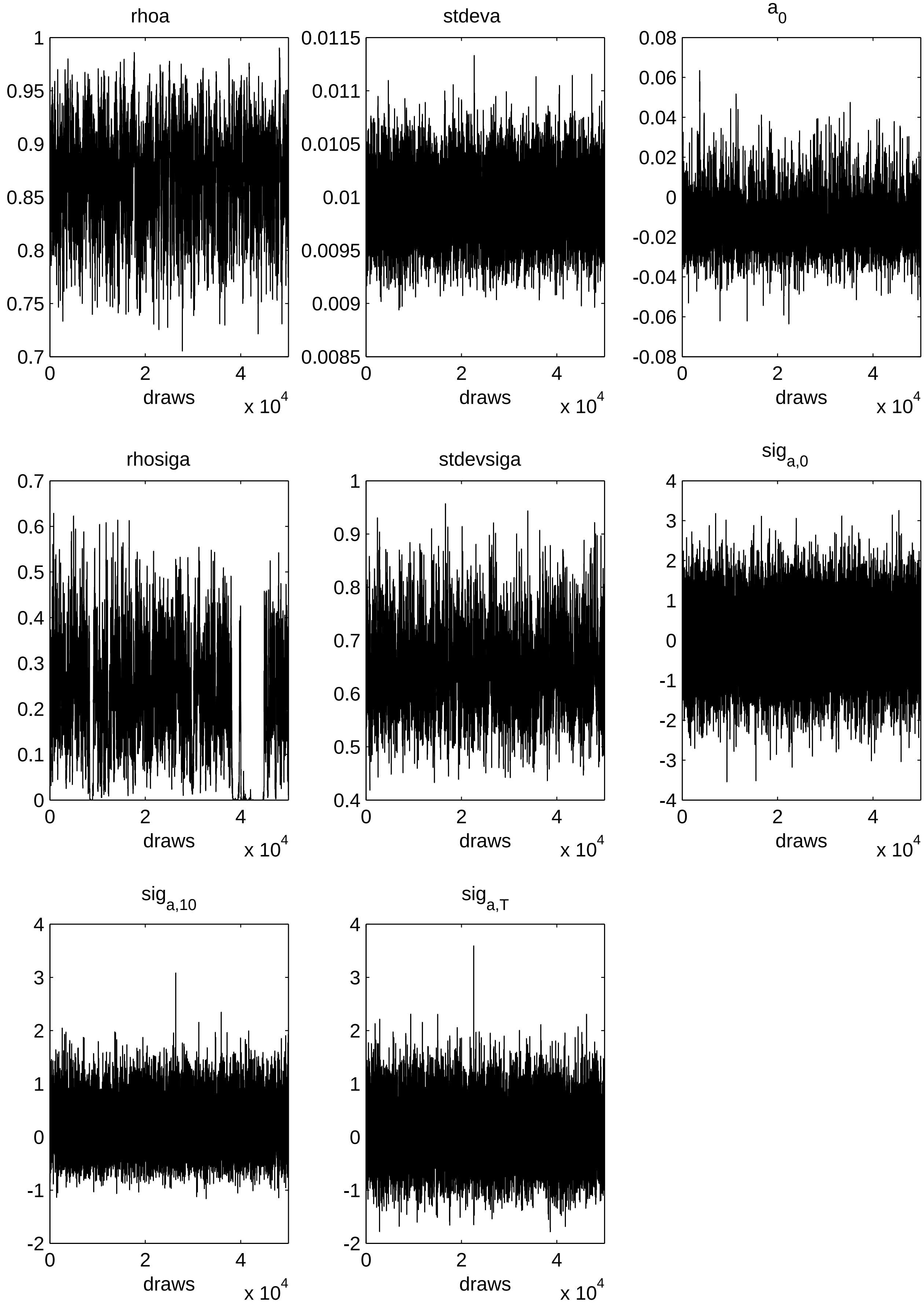

Figure 3. External finance premium, unadjusted internal rate of return, estimated micro-uncertainty process, implied internal rate of return and probability of default.

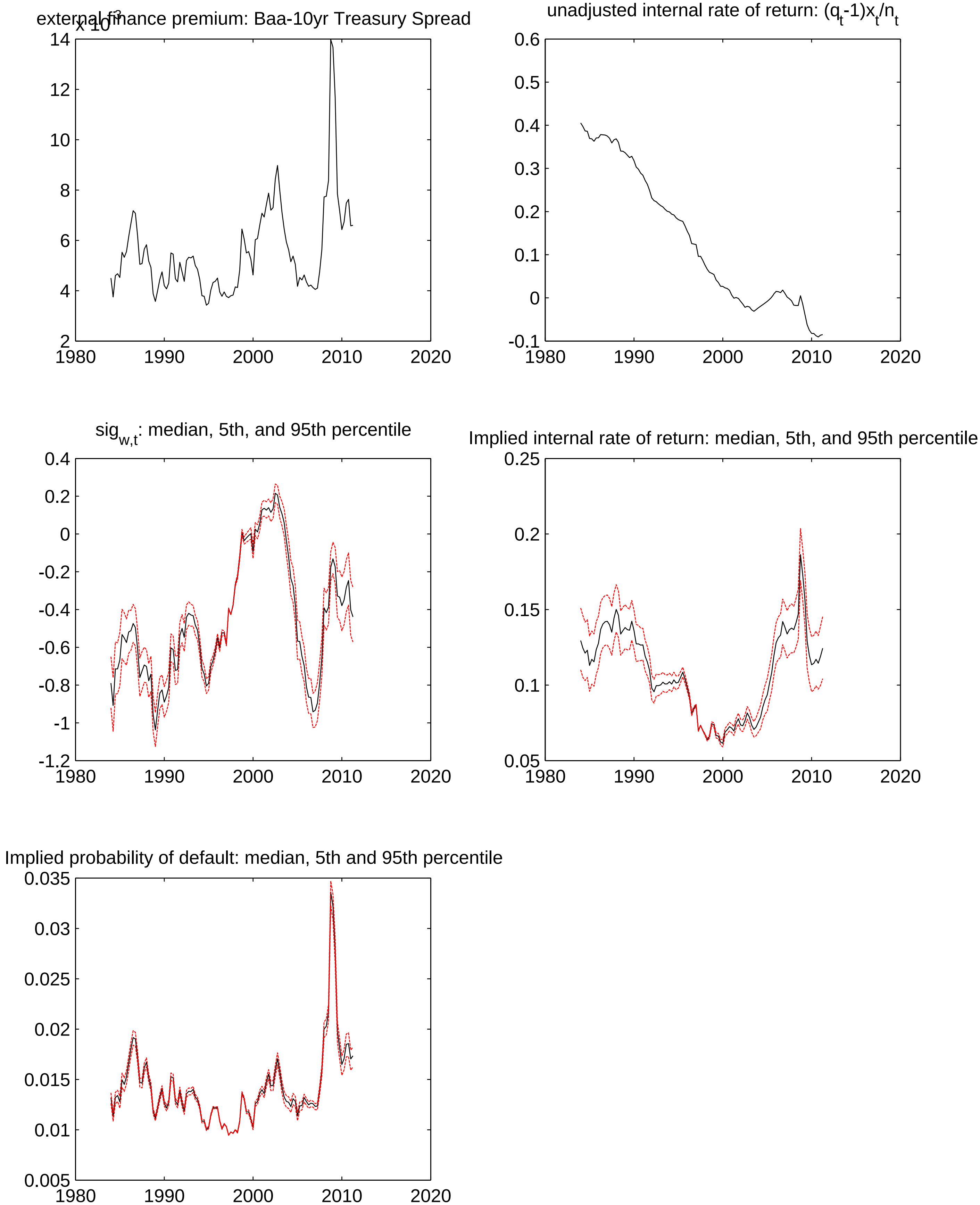

Figure 4. Draws from MCMC for parameters of micro-uncertainty process

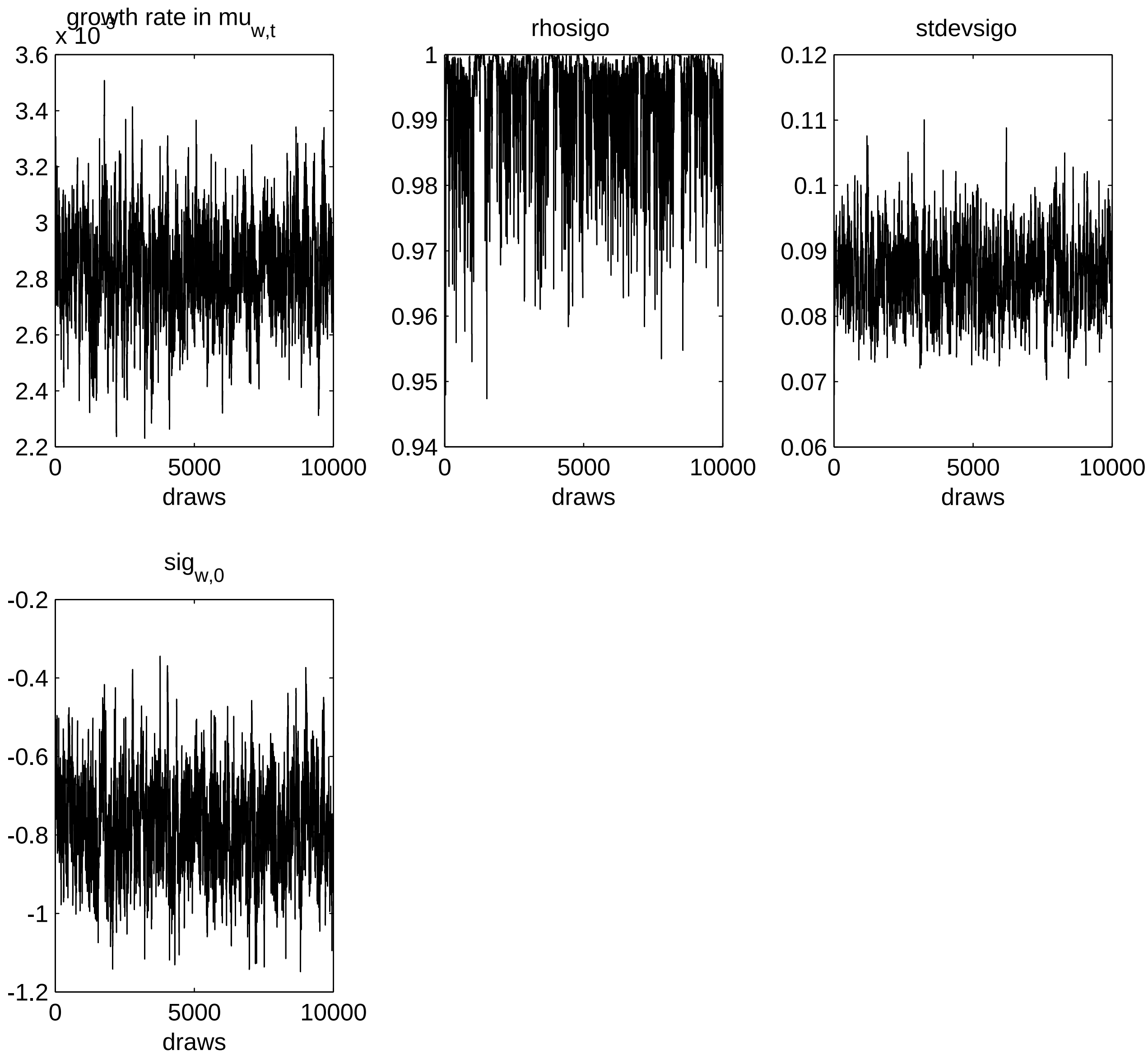

Figure 5. Response to TFP shock (e_{at})

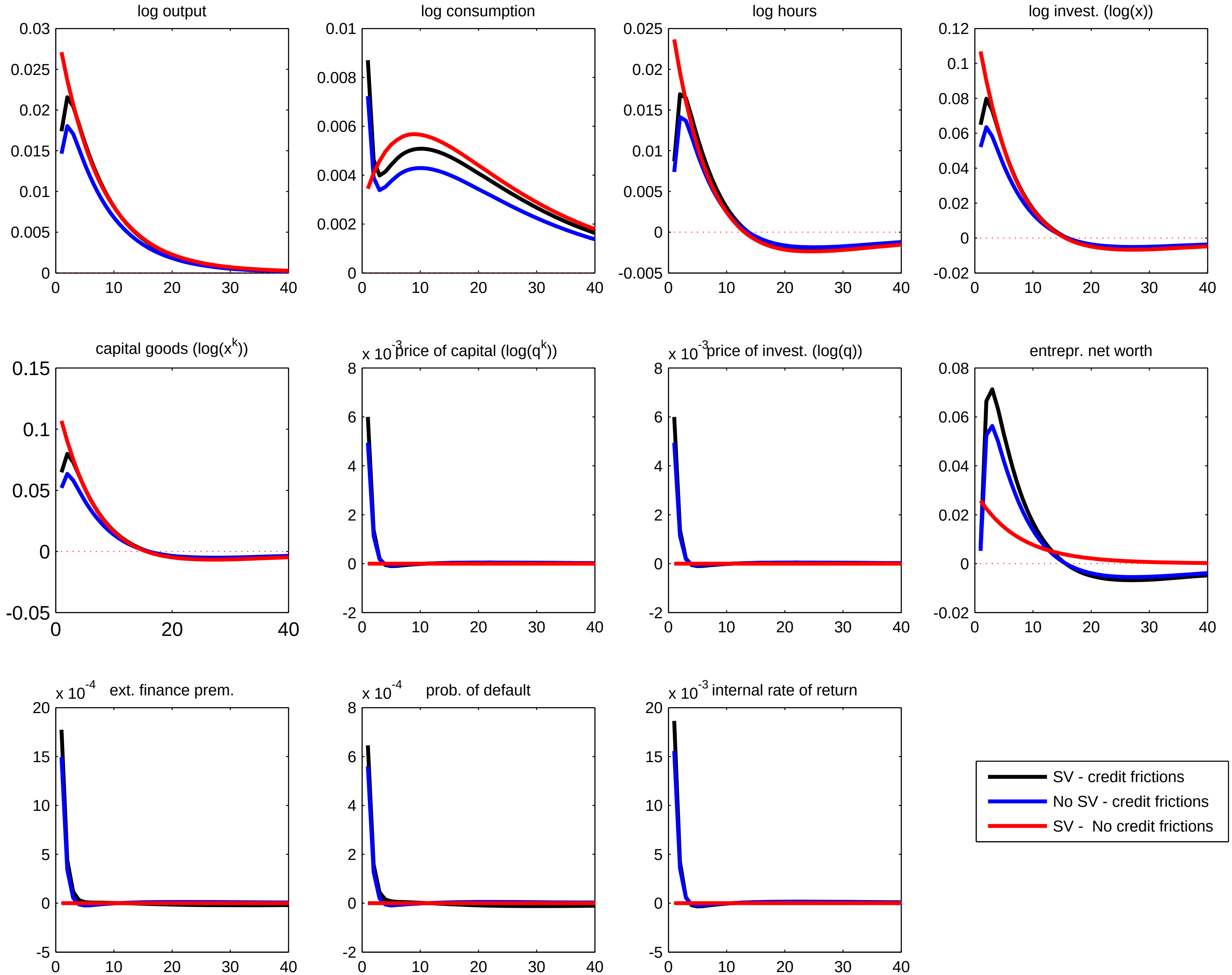

Figure 6. Response to macro-uncertainty shock (u_{at})

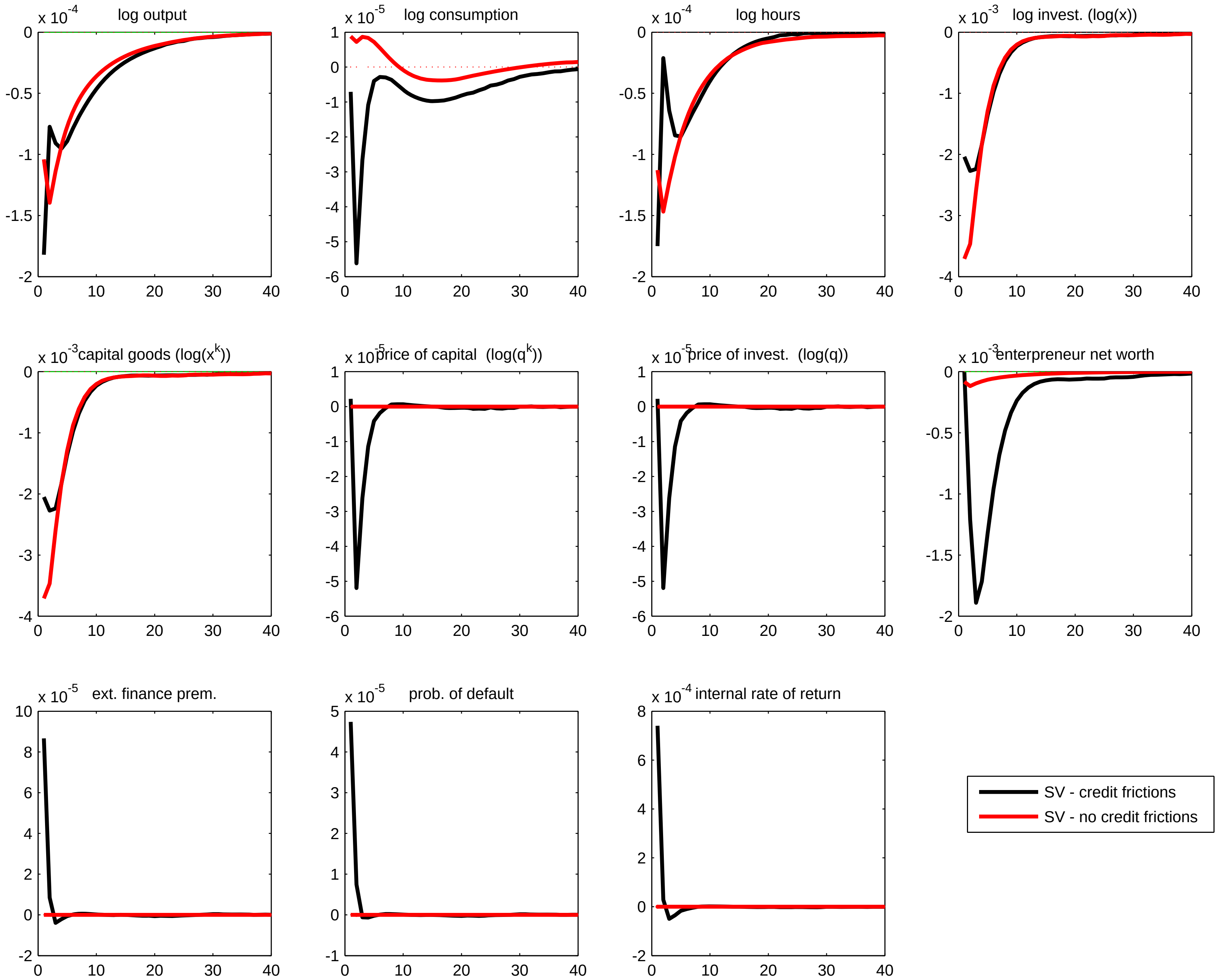

Figure 7. Response to Micro-uncertainty shock (u_{wt})

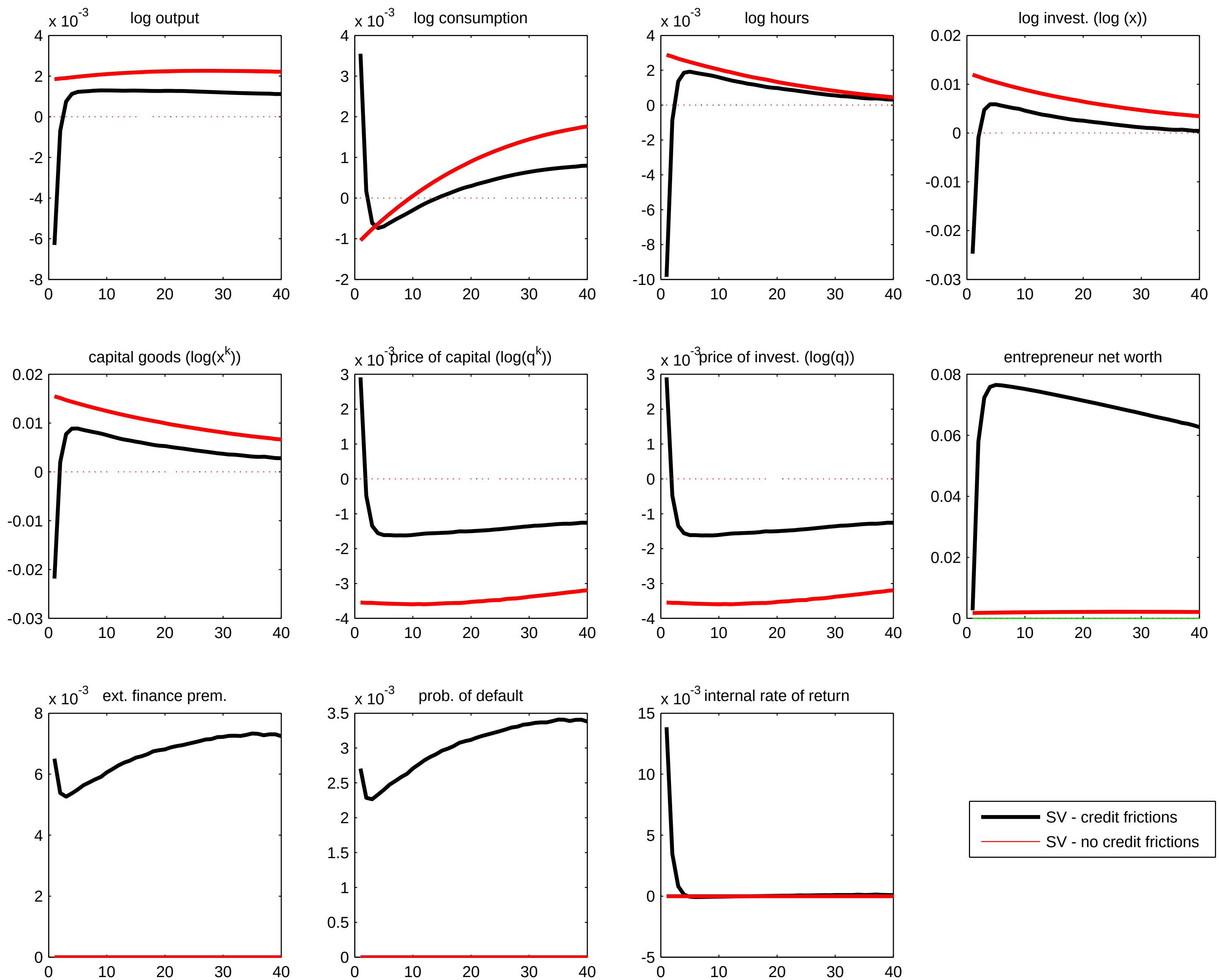

Figure 8. Comparison of alternative models: TFP shock (e_{at})

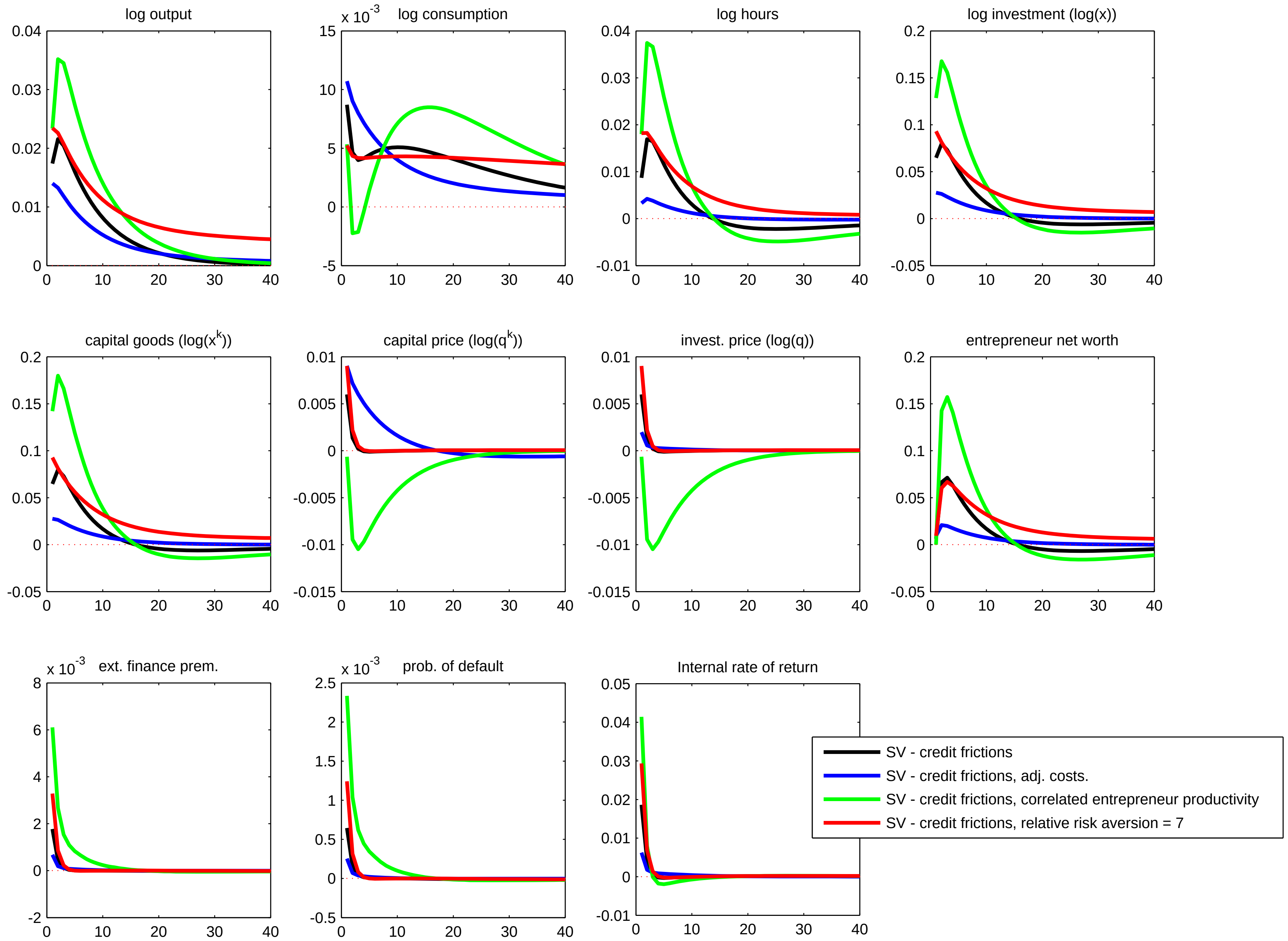

Figure 9. Comparison of alternative models: Macro-uncertainty shock (u_{at})

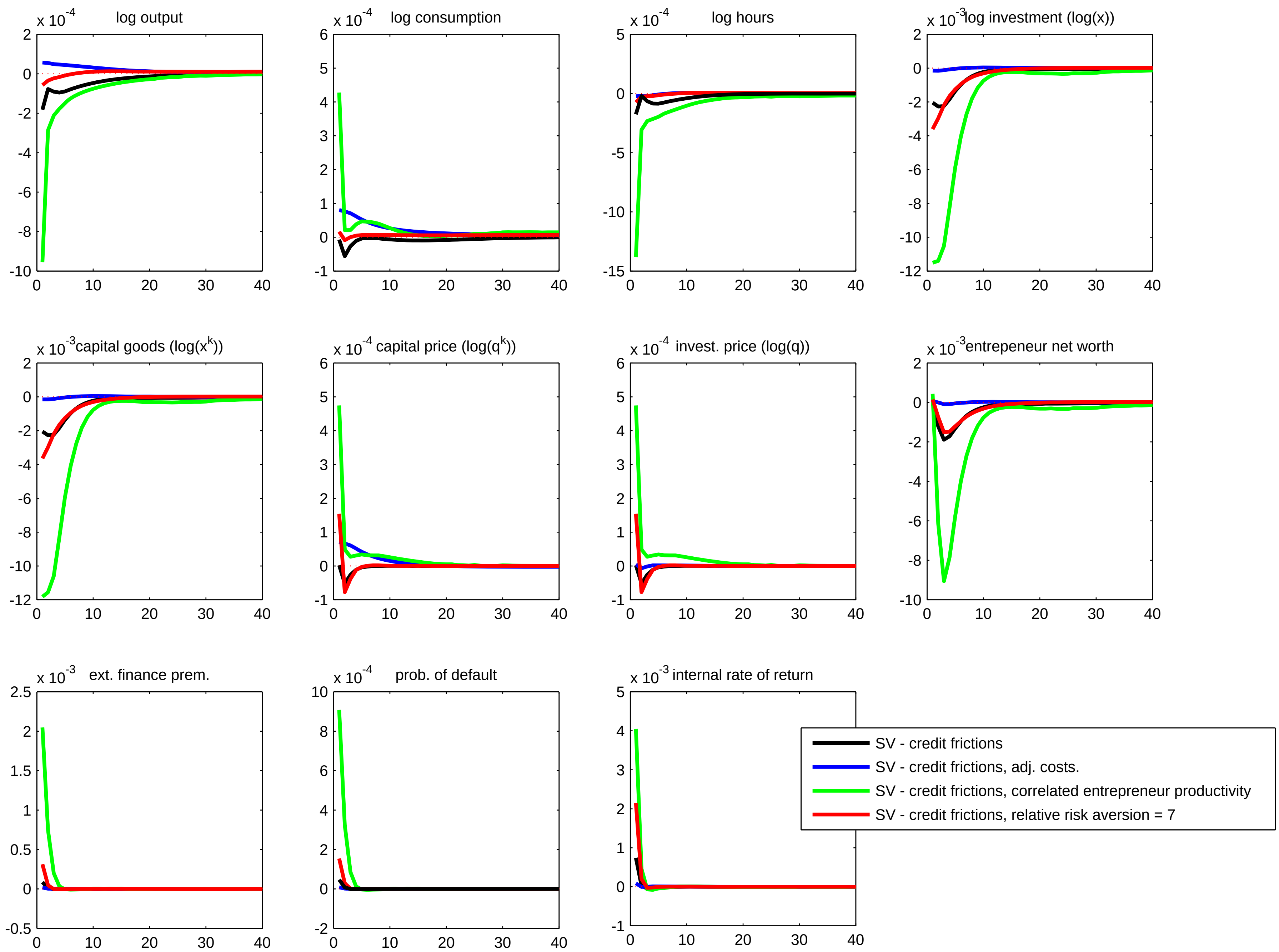

Figure 10. Comparison of alternative models: Micro-uncertainty shock (u_{wt})

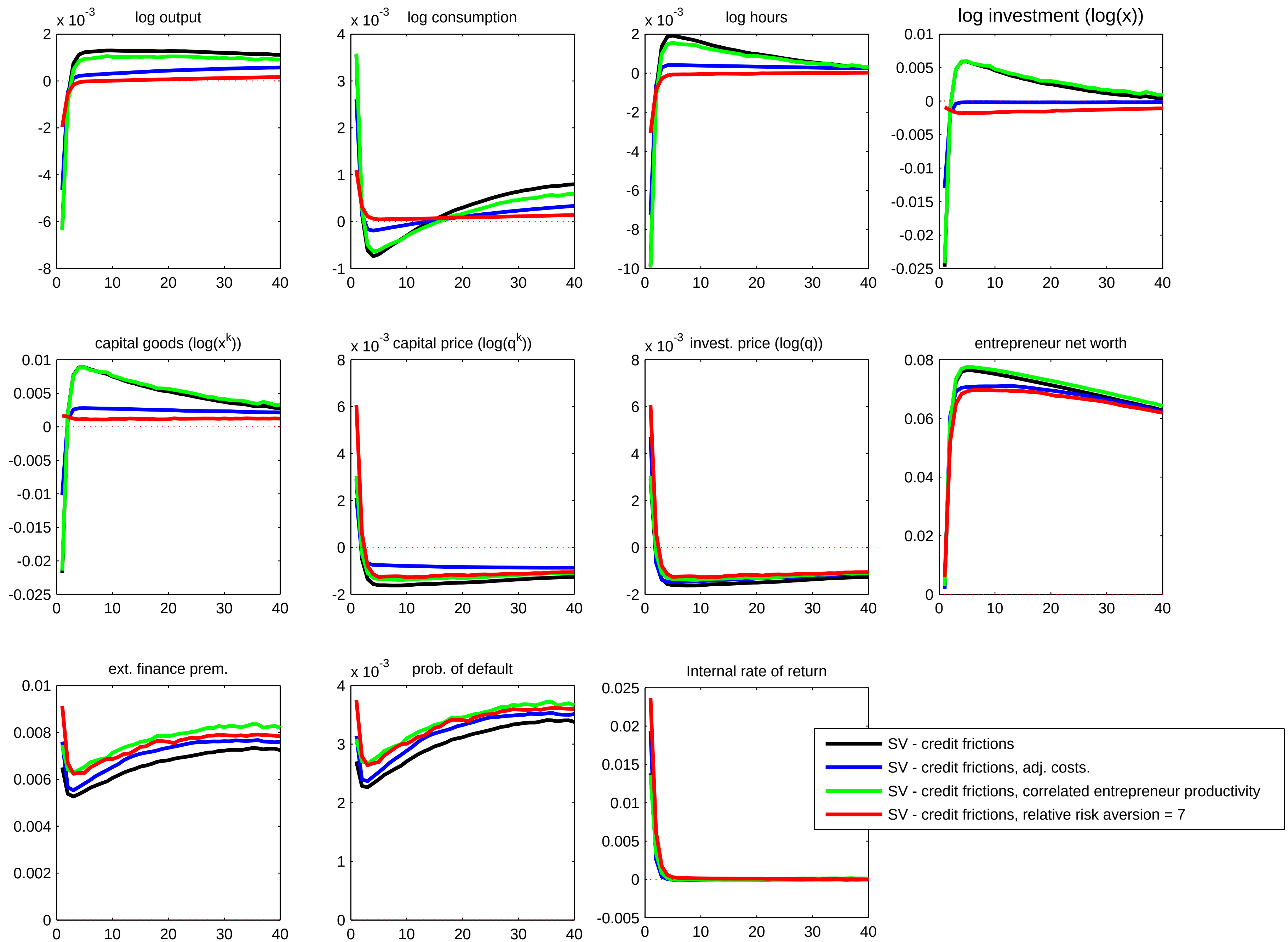

Figure 11. Comparison of alternative GIRFs: Macro-uncertainty shock (u_{wt})