Estimating Strategic Complementarity in a State-Dependent Pricing Model*

Marco Bonomo
EPGE, Getulio Vargas Foundation
marco.bonomo@fgv.br

Arnildo da Silva Correa
Central Bank of Brazil
arnildo.correa@bcb.gov.br

Marcelo Cunha Medeiros
Department of Economics, Pontifical Catholic
University of Rio de Janeiro–Puc-Rio
mcm@econ.puc-rio.br

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Abstract

The macroeconomic effects of shocks in models of nominal rigidity depend crucially on the degree of strategic complementarity among price setters. However, the empirical evidence on its magnitude is indirect and ambiguous: the one based on macroeconomic data suggest strong strategic complementarities in price-setting, which seems to be contradicted by some recent studies based on micro data. In this paper we estimate directly the degree of strategic complementarity based on individual price data underlying the CPI-FGV from Brazil for the 1996-2006 period, benefiting from large amount of macroeconomic variation in Brazilian sample during this period. Our identification strategy is to infer the degree of strategic complementarity from the relation between the frictionless optimal price and macroeconomic variables that results from a microfounded model. However, since we observe price changes, and not the change in the frictionless optimal price, in order to estimate the model we assume that firms follow a two-sided asymmetric Ss rule. The resulting econometric model for price-adjustments is a non-standard ordered probit model. By explicitly assuming the Ss pricing rules, our methodology is able to disentangle the effect of strategic complementarity from selection effect. The results, which are based on individual price changes and not on macro effects, indicate a substantial degree of strategic complementarity, contributing to reconcile micro and macro based evidence.

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1 Introduction

In recent years there has been an increasing interest in the study of price rigidity. It became well understood that not only the extent of price rigidity matters for monetary policy effects, but also the type of dynamic price setting policy. In particular, monetary policy shocks tend to have much less effect on models where price setters use state-dependent rules than in time-dependent models[1]. The reason is that in the former models the selection effect leads to higher average price changes in response to shocks than in time-dependent models[2]. Recently, Gertler and Leahy (2008) have shown that when strategic complementarity in prices are large enough, even state-dependent models may generate aggregate price-level stickiness comparable to those in time-dependent model[3]. Thus, we have learned that the macroeconomic effect of shocks depend on the details of the price-rigidity mechanism and the structural interactions among price-setters.

Additionally, the recent access to vast amounts of micro price data enabled the direct measurement of some those micro features, as the frequency and size of price adjustments of price adjustments, while stimulating research strategies to unveil the characteristics that are not directly observable. Price adjustments were found to be more frequent than previously thought (e.g. Bils and Klenow 2004, Klenow and Krivtsov 2008 and Nakamura and Steinson 2008). Although it seems difficult to find a pricing model that fits perfectly the micro data, the evidence seems to be consistent with state-dependency, as frequency of price adjustments seem to move with the macroeconomic environment (e.g. Dias, Marques, and Silva 2007, Klenow and Krivtsov 2008, Midrigan 2008, Gagnon 2009, Barros et al. 2010). Finally, although Klenow and Willis (2006), Burstein and Hellwig (2007), Kryvtsov and Midrigan (2009), and Bils, Klenow and Malin (2009) pursue different strategies in relating aspects of the micro data with the degree of strategic complementarity (real rigidity), they all found indirect evidence against high degree of strategic complementarities in micro data.

As a result, this recent research seems to point out to a large gap between the features consistent with micro data—relatively high frequency of adjustments, state-dependency, and low degree of strategic complementarity—and those necessary to explain macroeconomic effects.

In this paper we revisit the issue of what degree of strategic complementarity is consistent with micro data. However, we use a direct approach, by assuming state-dependency of pricing rules and estimating the degree of strategic complementarity from the dynamics of individual price adjustments. We use individual price data from Brazilian CPI of Getulio Vargas Foundation (CPI-FGV) from 1996 to 2006, benefiting from large amount of macroeconomic variation in Brazilian sample in those eleven years of data. We estimate the degree of strategic complementarity by quasi-maximum likelihood, deriving the likelihood function for price changes from the structural equation for the frictionless optimal price and the assumed state-dependency of pricing policies.

We derive an equation from a structural model that relates changes in the individual frictionless optimal price to changes in the price level, nominal aggregate demand, changes in the exchange rate, idiosyncratic and aggregate shocks. Our identification strategy is then to infer the degree of strategic complementarity from the relation between the frictionless optimal price and macroeconomic variables that result from the microfounded model. However, since we observe price changes, and not the change in the frictionless optimal price, in order to estimate the model we assume that firms follow a two-sided asymmetric Ss rule. The

[1]Examples of state-dependent models include Almeida and Bonomo (2002), Golosov and Lucas (2007), Gertler and Leahy (2008), Midrigan (2009), Nakamura and Steinsson (2009). Time-dependent models consider cases in which prices are fixed between adjustments, as Taylor (1979), Calvo (1983), and Bonomo and Carvalho (2004, 2010), as well as models where prices change continuously and information time is exogenous, as Mankiw and Reis (2004), and Reis (2006).

[2]Extreme cases are Caplin and Spulber (1987) model, for the state-dependent pricing rules and Calvo (1983) for time-dependent pricing rules. In the former case, the selection effect is so strong that it generates money neutrality. In Calvo (1983) there is no selection effect, since the firms that change their prices are randomly drawn.

[3]Since Ball and Romer (1990), it has been known that strategic complementarities amplify the impact of nominal rigidities.

resulting econometric model for price-adjustments is a non-standard ordered probit model, where the emerging autocorrelation and heteroskedasticity in the residuals are treated. The parameter measuring strategic complementarity is obtained from parameter restrictions in the structural model. In addition, by explicitly assuming the Ss pricing rules, our methodology is able to deal with the selection effect and isolate strategic complementarity in a suitable way, providing a more straightforward way to measure real rigidities than those proposed in the literature.

Our approach is based only on the supply side of the economy, and does not depend on the specification of the demand side—including the consumer Euler equation and monetary policy. So, if our assumptions about the supply side are adequate, it should provide results that are not affected by possible mispecifications of the demand side. However, since we do not have a complete model, we cannot evaluate directly the aggregate implications of our estimation.

Our results indicate that the parameter measuring strategic complementarity ranges from 0.03 to 0.11 for the economy as a whole, implying a substantial degree of strategic complementarity. In order to get a sense of the possible macro implications of this range of magnitude, notice that the values for the deep parameters assumed in Gertler and Leahy (2008) entail a degree of strategic complementarity of 0.08, inside the range of our estimations. In their state-dependent model, this magnitude is enough to ensure that monetary shocks have significant real effects.

Barros et al. (2009) and Gouveia (2007) have used this same data set to study the relation between price setting and macroeconomic variables. In particular, Barros et al. (2010) have shown that the frequency of price change is significantly and positively related to inflation, output and exchange rate depreciation. This possibly reflect the relation of the change in the frictionless optimal price with those same variables.

The results differ markedly from those obtained by Bils, Klenon and Malin (2009), who concluded that a state-dependent model without strategic complementarity fits best their micro data. They define theoretical reset price, as the price each firm would like to have in case of adjustment. It turns out that if rules are Ss, as we assume, their theoretical reset price corresponds to the frictionless optimal price (plus a constant). Their methodology does not allow them to recover the theoretical reset price. Then, they construct an auxiliary statistics—empirical reset price inflation—based on empirical reset price. Empirical reset price is the new price set by adjusters, and for non-adjusters is imputed the average empirical reset price of adjusters. In general, it differs from the theoretical reset price because of the selection effect.

In order to investigate whether the difference in methodology or in the data is driving the difference in results, we replicate their methodology with our data. We found a similar pattern of empirical reset inflation with the Brazilian data. If we were to use their methodology to infer the model, we would arrive at the same conclusion as them. However, our frictionless price estimation allow us to recover frictionless price inflation, which corresponds to theoretical reset price inflation, and it is substantially more persistent than the empirical reset price inflation. The impulse response function (IRF) of our frictionless price is similar to their IRF for theoretical reset price simulated with base on a Ss model with strategic complementarities.

We conclude that the difference must come from the methodology. Their exercise is based on complete model simulation, although their focus is on the supply side—price setting and strategic complementarity. As suggested by their own empirical results, the specification of the demand side can make a big difference[4]. By contrast, our methodology focus on estimating the frictionless optimal price equation. The parameters of this equation do not depend on the demand side specification. They are not affected by the selection of price changers either[5].

[4] When they allowed money supply to respond to real aggregate demand, the state-dependent model with strategic complementarity turned out to fit better the impulse response function of reset price inflation than the model without strategic complementarity. However, they reject the model with endogenous money because it generates a too smooth inflation process.

[5] Of course, we could be subject to other problems as, for example, a mispecification of the supply shocks, or in the type of

The remaining parts of the paper ir organized as follows. In the next section we review the various sources of strategic complementarities emphasized in the literature and some papers that use indirect methods to measure their relevance in actual data. In Section 3 we derive the theoretical model and the equation for the frictionless optimal price. We also characterize the Ss pricing rule followed by firms. In Section 4 we derive the econometric methodology and present the identification strategy used to estimate the parameter of interest. Section 5 describes the data set. Section 6 presents the results: strategic complementarity, frictionless price inflation and pricing rules parameters. Section 7 concludes.

2 Real rigidities and their measurement

Real rigidities (strategic complementarities) are mechanisms that dampen the price responses of firms when they have the opportunity of repricing, i.e., reduce the individual firm's willingness to respond to a shock when other firms do not respond right away.

Basically two types of real rigidities have received attention in the literature. The first type is real rigidities at the macro/industry level and emphasizes either input-output linkages across sectors or real factor price rigidities at the aggregate level that cause slow response of real marginal cost to output fluctuations. This slow responsiveness of real marginal cost to shocks is obtained theoretically with variations in the model that include the use of intermediate inputs, as in Basu (1995), segmented input markets as in Woodford (2003) and real wage rigidity as in Blanchard and Galí (2007).

The second type of real rigidities emphasizes the strategic interaction at the micro/firm level. This strategic complementarity arises from losses by an individual firm in having its price deviating from the prices of its competitors. The literature has stressed factors such as upward-sloping marginal cost (decreasing returns to scale) as in Burstein and Hellwig (2007) and Kryvtsov and Midrigan (2009) and non-CES (smoothed kinked) demand curves as in Kimball (1995) and Klenow and Willis (2006) as sources of concavity of the firm's profit function that may lead to strategic complementarities in pricing decisions.

Even though the concept of real rigidity is very appealing, it is hard to identify and measure its presence in the actual data. The difficulty mainly comes from two facts. First, strategic complementarity is linked to the firm's desired price (or frictionless optimal price), which is unobservable. In addition, real rigidities imply a softened response of the firm's prices, conditional on adjustment, to a marginal cost shock, but data on marginal costs is usually unavailable. That is why the literature has basically relied either on calibrations or indirect empirical tests to measure the presence of strategic complementarities in the data. Examples include the use of the reset price inflation concept in Bils, Klenow and Malin (2009) and in Gopinath and Itskhoki (2010), and calibrated models trying to reproduce some features of microdata, such as Klenow and Willis (2006), Burstein and Hellwig (2007), Kryvtsov and Midrigan (2009) among others.

In the present paper we depart from the recent literature by proposing a *direct* way to identify and measure strategic complementarities in pricing decisions. The metric we use to measure real rigidities is in fact very standard in the theoretical literature—it is given by the magnitude of the elasticity of the firm's frictionless optimal price with respect to the aggregate price—and has a direct interpretation—how much the firm's optimal price depends on the prices of its competitors. Although very straightforward, this measure has never been used before in papers trying to estimate real rigidities in the data, possibly because frictionless optimal price is unobserved. Our approach circumvent this fact by treating the frictionless optimal price as a latent variable in an ordered probit model that uses microdata at the firm level. Controlling for the

pricing rule.

aggregate shocks that affect all firms together, we propose a method to match the parameters of the probit to the structural parameters of the state-dependent pricing model, including the one measuring real rigidities.

3 A model of state-dependent pricing

In this section we derive the model on which the econometric estimates are based and define the way we measure strategic complementarities. The model has three main elements. First, households obtain utility from consumption goods and disutility from supplying labor; and firms supply differentiated goods in a monopolistically competitive environment. In the (segmented) labor market, households and firms behave competitively. Second, we assume that firms must pay a fixed adjustment cost to change prices and, therefore, they will follow state-dependent pricing rules. Third, there are aggregate shocks and idiosyncratic productivity shocks.

3.1 Frictionless optimal price[6]

The representative household seeks to maximize the discounted sum of utilities

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[u(C_t;\xi_t) - \int_0^1 v(L_{i,t};\xi_t)di \right] \right\}, \tag{1}$$

where C_t is a consumption index and $L_{i,t}$ is the quantity of labor of type i supplied. $v(L_{i,t};\xi_t)$ represents the disutility of working and ξ_t is a vector of aggregate shocks. The consumption index over which utility is defined is given by

$$C_t = \left[\int_0^1 C_{i,t}^{(\theta-1)/\theta} di \right]^{\theta/\theta-1}. \tag{2}$$

with $\theta > 1$, and $C_{i,t}$ is the consumption of variety i.

In this setting, the demand for an individual product has the familiar form:

$$C_{i,t} = C_t \left(\frac{P_{i,t}}{P_t} \right)^{-\theta}, \tag{3}$$

where C_t is the aggregate consumption and the price index is

$$P_t = \left[\int_0^1 P_{i,t}^{1-\theta} di \right]^{1/(1-\theta)}. \tag{4}$$

The optimal quantity of labor of type i supplied is implicitly given by

$$\frac{v_L(L_{i,t};\xi_t)}{u_C(C_t;\xi_t)} = \frac{W_{i,t}}{P_t}, \tag{5}$$

where $W_{i,t}$ is the wage for labor of type i.

[6] See Appendix A for a detailed derivation of the frictionless optimal price from fundamentals.

There is a continuum of monopolistically competitive firms supplying differentiated goods. We assume that each firm has the production function:

$$Y_{i,t} = A_{i,t} L_{i,t}^{\alpha} M_t^{(1-\alpha)}, \tag{6}$$

where $Y_{i,t}$ is the firm's product, $A_{i,t}$ is a productivity factor and M_t is a foreign input used in the production process. The nominal exchange rate $\tilde{E}_t$ is assumed to be exogenous. We also assume that each of the differentiated goods uses a specialized labor input in its production. Therefore, the suppliers do not hire labor from a single homogeneous competitive labor market.

If prices were perfectly flexible, i.e., if producer i could adjust prices continuously, it would choose $P_{i,t}^*$ to maximize profits according to the usual markup rule:

$$\frac{P_{i,t}^*}{P_t} = \mu \psi(Y_{i,t}, Y_t, E_t; \xi_t, A_{i,t}), \tag{7}$$

where $\psi(.)$ is the real marginal cost, Y_t is the aggregate output, E_t is the real exchange rate and $\mu \equiv \theta/(\theta - 1)$ is the firm's desired markup.

A first-order log-linearization of this equation around the steady-state equilibrium in the case of flexible prices leads to the following equation for the frictionless optimal price[7]:

$$\begin{aligned} p_{i,t}^* &= \kappa + \zeta \mathcal{Y}_t + (1-\zeta) p_t + \frac{(1-\alpha)}{1+\alpha\omega\theta} e_t + \tilde{\xi}_t + \tilde{a}_{i,t} \\ &= \kappa + \boldsymbol{x}_t' \boldsymbol{\beta} + \tilde{\xi}_t + \tilde{a}_{i,t}, \end{aligned} \tag{8}$$

where $\mathcal{Y}_t$ is the nominal expenditure, κ is a constant combination of primitive parameters, $\boldsymbol{\beta} = [\zeta, (1-\zeta), (1-\alpha)/(1+\alpha\omega\theta)]'$ and $\boldsymbol{x}_t = (\mathcal{Y}_t, p_t, e_t)'$. The variables $p_{i,t}^*$, $\mathcal{Y}_t$, p_t and e_t are in logarithm. $\tilde{\xi}_t$ is the aggregate shock and $\tilde{a}_{i,t}$ is the idiosyncratic shock.

Observe that when $\zeta < 1$, the optimal price the firm would like to charge depends positively on the prices set by its competitors (in our model represented by p_t), which we refer to as strategic complementarities in price setting. Therefore, the parameter ζ measures real rigidities—the smaller is ζ, the larger is the degree of strategic complementarities in pricing decisions—and a consistent estimation of it can provide an assessment of the intensity of real rigidities in the data.

3.2 The pricing problem

We assume that firms follow a Ss pricing policy parameterized by (s, c, S). The state variable is defined, for item i, as $r_{i,t}^* \equiv p_{i,\tau} - p_{i,t}^*$, where $\tau \equiv \tau_{i,t}$ is the time of the last price adjustment previous to time t. We drop the indexes i and t from τ just to simplify notation. The parametes s and S are the lower and the upper bounds for $r_{i,t}^*$, and c is the target point. Observe that we do not derive the optimal policy here. We just postulate it. But it can be rationalized assuming that firms cannot adjust they prices without paying a lump-sum cost (menu cost) and additional conditions for the $p_{i,t}^*$ process (e.g. a brownian motion). For our purposes, knowing the format of the pricing policy suffices to derive the econometric model to be estimated.

Figure 1 shows an example of path for $r_{i,t}^*$. While variable $r_{i,t}^*$ is inside the range (s, S), the firm maintains its price fixed. When $r_{i,t}^*$ reaches the threshold s, however, $p_{i,t}^*$ is sufficiently above the actual price charged

[7] Lowercase means that the variable is in logarithm.

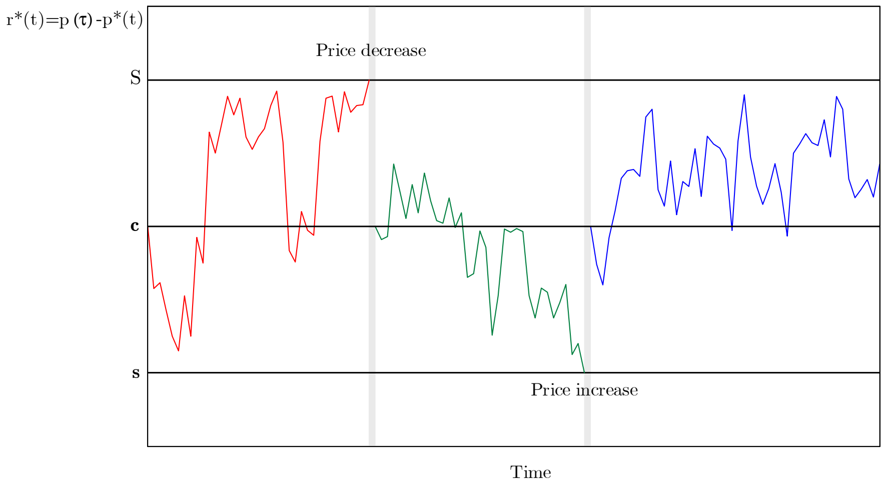

Figure 1: State-dependent pricing rule

and it is optimal for the firm to pay the menu cost and increase the price. On the other hand, when the threshold S is reached, $p^*_{i,t}$ is sufficiently below $p_{i,t}$ and the firm decreases its price. In case of price changes, the variable $r^*_{i,t}$ is set equal to c. The value of c, however, may or may not be equal to zero. As the firm knows that it will maintain the actual price fixed for some time interval, it may be optimal to set $p_{i,t}$ somewhat above $p^*_{i,t}$.

4 Econometric methodology

Using the equation (8) for the frictionless optimal price and the Ss pricing rule, we now derive the econometric methodology to estimate the parameter of strategic complementarity and other parameters of the structural model, like those of the pricing rule.

In the data set we only observe the price of each item charged by the firm over time, $p_{i,t}$, and the dates of price changes. The variable $r^*_{i,t} = p_{i,\tau} - p^*_{i,t}$ is unobservable. Define the observable variable $r_{i,t}$ as

$$r_{i,t} = \begin{cases} 1, & \text{if } \; p_{i,t} > p_{i,t-1} \\ 0, & \text{if } \; p_{i,t} = p_{i,t-1} \\ -1, & \text{if } \; p_{i,t} < p_{i,t-1} \end{cases} . \tag{9}$$

By the above pricing rule, $\lim_{h\to 0} r^*_{i,\tau+h} \equiv \lim_{h\to 0}(p_{i,\tau} - p^*_{i,\tau+h}) = c$. Thus, we will write $p_{i,\tau} = c + p^*_{i,\tau}$ and

$$
\begin{aligned}
r^*_{i,t} = p_{i,\tau} - p^*_{i,t} =& (c + \kappa + \boldsymbol{x}'_\tau \boldsymbol{\beta} + \tilde{\xi}_\tau + \tilde{a}_{i,\tau}) - (\kappa + \boldsymbol{x}'_t \boldsymbol{\beta} + \tilde{\xi}_t + \tilde{a}_{i,t}) \\
=& c + (\boldsymbol{x}_\tau - \boldsymbol{x}_t)'\boldsymbol{\beta} + (\tilde{\xi}_\tau - \tilde{\xi}_t) + (\tilde{a}_{i,\tau} - \tilde{a}_{i,t}) \\
=& c - \boldsymbol{z}'_{i,t}\boldsymbol{\beta} - (\tilde{\xi}_t - \tilde{\xi}_\tau) - (\tilde{a}_{i,t} - \tilde{a}_{i,\tau}),
\end{aligned}
\tag{10}
$$

where $\boldsymbol{z}_{i,t} = \boldsymbol{x}_t - \boldsymbol{x}_\tau$. Note that although only macroeconomic variables are contained in vector $\boldsymbol{x}_t$, $\boldsymbol{z}_{i,t}$ is not the same for all firms as τ depends on both i and t. Once firms change their prices at different points in time, the accumulated variation will be different across firms along the time. For this reason we add the subscript i in $\boldsymbol{z}_{i,t}$. And at the end, what matters for the individual firm's pricing decision is the accumulated change in those variables since the moment of its last price adjustment. It is precisely this difference in the length of each price spell what enables us to estimate the model with only aggregate variables as regressors.

Following the recent literature such as Boivin, Giannoni and Mihov (2009) which provide evidence that the common component of inflation is very persistent, we approximate the aggregate shock by[8]

$$
\tilde{\xi}_t = \tilde{\xi}_{t-1} + v_t, \quad v_t \sim \mathsf{IID}(0, \sigma^2_v). \tag{11}
$$

In addition, to identify the parameters of interest we assume that the idiosyncratic shock is given by $\tilde{a}_{i,t} = a_i + a_{i,t}$, where a_i is an individual fixed effect component and[9]

$$
a_{i,t} = \eta + a_{i,t-1} + \varepsilon_{i,t}, \quad \varepsilon_{i,t} \sim \mathbb{N}(0, \sigma^2). \tag{12}
$$

Therefore, our econometric model also takes into account that there may be individual fixed specificities in the price of each item. But as will be clear ahead, because the idiosyncratic shock appears in difference, these individual effects disappear.

Under these assumptions we can write equation (10) as[10]

$$
\begin{aligned}
r^*_{i,t} \equiv p_{i,\tau} - p^*_{i,t} &= c - \boldsymbol{z}'_{i,t}\boldsymbol{\beta} - (\tilde{\xi}_t - \tilde{\xi}_\tau) - (a_{i,t} - a_{i,\tau}) \\
&= c - \eta\delta_{i,t} - \boldsymbol{z}'_{i,t}\boldsymbol{\beta} - \sum_{j=1}^{T} \gamma_j d_{j,t} - u_{i,t},
\end{aligned}
\tag{13}
$$

where $\delta_{i,t}$ is the time interval between t and τ,

$$
u_{i,t} = \varepsilon_{i,t} + \cdots + \varepsilon_{i,t-\delta_{i,t}+1} \sim \mathbb{N}(0, \delta_{i,t}\sigma^2)
$$

and, for $t = 1, \ldots, T$,

$$
d_{j,t} = \begin{cases} 1, & \text{if } j \in [t - \delta_{i,t} + 1, t] \\ 0, & \text{otherwise} \end{cases}. \tag{14}
$$

[8] Maybe in the long run the process is stationary, but in the short run the process is persistent enough to be approximated by this assumption.

[9] In the main specification we assume that $a_{i,t}$ is a random walk with drift because we would like to take into account that productivity grows over time. But, given the recent evidence that idiosyncratic shock is short-lived, we also estimate the model assuming that $a_{i,t}$ is a white-noise, i.e., $a_{i,t} = \varepsilon_{i,t}$ and $\varepsilon_{i,t} \sim \mathbb{N}(0, \sigma^2)$. See the section with results.

[10] To obtain equation (13) we iterate backward $a_{i,t}$ and $\tilde{\xi}_t$ until the moment of the last price adjustment and use some algebra. Details are given in Appendix B.

Note that the dummy variables assume value 1 in each time between t and τ (the time of the last price adjustment). These terms control for the aggregate shocks that hit the economy since the moment in which the firm has set its price for the last time. In addition, every firm i in our panel data set that repriced on time τ and maintained its price fixed until time t has the same dummy variables with value 1. More generally, every firm that kept its price unchanged during a period overlapping the time interval between τ and t has the respective dummy variables assuming value 1 during the period. By controlling these common shocks in our probit model (in addition to the other regressors), we want to make sure that two firms that change prices at the same time reprice together not because they were affected by the same shock, but because of some kind of strategic interaction between them.

Also observe that the residuals $u_{i,t}$ are naturally correlated. However, given our assumptions, we know the autocorrelation structure—$u_{i,t}$ is a moving average $MA(\delta_{i,t}+1)$ process—and we use this information in the estimation procedure.

Defining $\boldsymbol{w}_{i,t} = (\delta_{i,t}, \boldsymbol{z}'_{i,t}, \boldsymbol{d}'_t)'$, where $\boldsymbol{d}_t = (d_{1,t}, ..., d_{T,t})'$, and using the pricing rule, we can obtain the probability of observing a price increase:

$$
\begin{aligned}
Pr[r_{i,t} = 1|\boldsymbol{w}_{i,t}] &= Pr[r^*_{i,t} \leq s|\boldsymbol{w}_{i,t}] \\
&= Pr[c - \eta\delta_{i,t} - \boldsymbol{z}'_{i,t}\boldsymbol{\beta} - \sum_{j=1}^{T}\gamma_j d_{j,t} - u_{i,t} \leq s|\boldsymbol{w}_{i,t}] \\
&= Pr\left[\frac{u_{i,t}}{\sqrt{\delta_{i,t}}\sigma} \geq \frac{c - s - \eta\delta_{i,t} - \boldsymbol{z}'_{i,t}\boldsymbol{\beta} - \sum_{j=1}^{T}\gamma_j d_{j,t}}{\sqrt{\delta_{i,t}}\sigma}\right] \\
&= 1 - \Phi\left(\frac{c-s}{\sigma}\frac{1}{\sqrt{\delta_{i,t}}} - \frac{\eta}{\sigma}\sqrt{\delta_{i,t}} - \frac{\boldsymbol{z}'_{i,t}}{\sqrt{\delta_{i,t}}}\frac{\boldsymbol{\beta}}{\sigma} - \sum_{j=1}^{T}\frac{\gamma_j}{\sigma}\frac{d_{j,t}}{\sqrt{\delta_{i,t}}}\right) \\
&= 1 - \Phi\left(\pi_1\ddot{1}_{i,t} - \tilde{\eta}\ddot{\delta}_{i,t} - \ddot{\boldsymbol{z}}'_{i,t}\tilde{\boldsymbol{\beta}} - \sum_{j=1}^{T}\tilde{\gamma}_j\ddot{d}_{j,t}\right)
\end{aligned}
$$

where variables with two dots represent the variables divided by $\sqrt{\delta_{i,t}}$, the parameters with tilde means that the parameters is scaled by σ, $\Phi(.)$ is the cumulative distribution function of a standard normal variable and $\pi_1 = (c-s)/\sigma$. In the third line we have used the fact that $u_{i,t}$ is independent of $\boldsymbol{w}_{i,t}$.

Likewise, we can derive the probability of observing the other two possible outcomes for $r_{i,t}$. This results in the following ordered probit model for price changes:

$$
\begin{aligned}
Pr[r_{i,t} = 1|\boldsymbol{w}_{i,t}] &= 1 - \Phi\left(\frac{c-s}{\sigma}\frac{1}{\sqrt{\delta_{i,t}}} - \frac{\eta}{\sigma}\sqrt{\delta_{i,t}} - \frac{\boldsymbol{z}'_{i,t}}{\sqrt{\delta_{i,t}}}\frac{\boldsymbol{\beta}}{\sigma} - \sum_{j=1}^{T}\frac{\gamma_j}{\sigma}\frac{d_{j,t}}{\sqrt{\delta_{i,t}}}\right) \\
&= 1 - \Phi\left(\pi_1\ddot{1}_{i,t} - \tilde{\eta}\ddot{\delta}_{i,t} - \ddot{\boldsymbol{z}}'_{i,t}\tilde{\boldsymbol{\beta}} - \sum_{j=1}^{T}\tilde{\gamma}_j\ddot{d}_{j,t}\right)
\end{aligned}
$$

$$
Pr[r_{i,t} = 0|\boldsymbol{w}_{i,t}] = \Phi\left(\pi_1\ddot{1}_{i,t} - \tilde{\eta}\ddot{\delta}_{i,t} - \ddot{\boldsymbol{z}}'_{i,t}\tilde{\boldsymbol{\beta}} - \sum_{j=1}^{T}\tilde{\gamma}_j\ddot{d}_{j,t}\right) - \Phi\left(\pi_0\ddot{1}_{i,t} - \tilde{\eta}\ddot{\delta}_{i,t} - \ddot{\boldsymbol{z}}'_{i,t}\tilde{\boldsymbol{\beta}} - \sum_{j=1}^{T}\tilde{\gamma}_j\ddot{d}_{j,t}\right) \quad (15)
$$

$$
\begin{aligned}
Pr[r_{i,t} = -1|\boldsymbol{w}_{i,t}] &= \Phi\left(\frac{c-S}{\sigma}\frac{1}{\sqrt{\delta_{i,t}}} - \frac{\eta}{\sigma}\sqrt{\delta_{i,t}} - \frac{\boldsymbol{z}'_{i,t}}{\sqrt{\delta_{i,t}}}\frac{\boldsymbol{\beta}}{\sigma} - \sum_{j=1}^{T}\frac{\gamma_j}{\sigma}\frac{d_{j,t}}{\sqrt{\delta_{i,t}}}\right) \\
&= \Phi\left(\pi_0 \ddot{1}_{i,t} - \tilde{\eta}\ddot{\delta}_{i,t} - \ddot{\boldsymbol{z}}'_{i,t}\tilde{\boldsymbol{\beta}} - \sum_{j=1}^{T}\tilde{\gamma}_j \ddot{d}_{j,t}\right)
\end{aligned}
$$

for $t = 1, ..., T$ and $i = 1, ..., N$.

The parameters are estimated by quasi-maximum likelihood method, which guarantees consistent estimation. Inference is carried out using a robust variance-covariance matrix for autocorrelation and heteroskedasticity. Since we know the structure of the model in which the probit is based on, given our assumptions we can derive the variance and covariance of $u_{i,t}$ and use this piece of information in the estimation of the parameters' robust variance-covariance matrix. Details are given in Appendix B.

4.1 Estimation of real rigidities and other parameters

The strategy to recover the structural parameter ζ measuring strategic complementarity in the model arises naturally from the restrictions on the frictionless optimal price parameters and the ordered probit model.

Notice that from equation (8) and the probit model, we have:

$$\tilde{\beta}_1 = \frac{\zeta}{\sigma} \tag{16}$$

and

$$\tilde{\beta}_2 = \frac{1-\zeta}{\sigma}. \tag{17}$$

Then, by dividing equation (16) by (17) and isolating ζ we obtain:

$$\zeta = \frac{\tilde{\beta}_1}{\tilde{\beta}_1 + \tilde{\beta}_2}. \tag{18}$$

Similarly, we can estimate the variance of shocks hitting the firms. To obtain σ, substitute equation (18) in (16):

$$\sigma = \frac{1}{\tilde{\beta}_1 + \tilde{\beta}_2}. \tag{19}$$

Since we are able to write ζ and σ as a function of the parameters of the probit model, we can use the Delta method to construct confidence intervals for those parameters. Details are given in Appendix B.

Parameters π_0 and π_1 of the probit model are combinations of the pricing rule's parameters and the variance of shocks. Even though we cannot obtain all parameters of the pricing rules, once we estimate the variance of shocks we can estimate the widths of the top and bottom bands of the pricing rules. They are obtained respectively as

$$c - s = \pi_1\sigma \quad \text{and} \quad c - S = \pi_0\sigma. \tag{20}$$

5 Data

In this section we describe the data set based on which the estimations are made and calculate some statistics of the price changes, such as duration, average price change etc. These statistics will be useful as a guideline to our methodology. In particular, we compare them to some statistics implied by our estimated model and relate them to the estimated pricing rule. This can be viewed as a measure of the goodness of fit of our model.

5.1 Data set description

The data set used here consists of primary information of price quotes of individual products collected and used by the Brazilian Institute of Economics of the Getulio Vargas Foundation (FGV-IBRE) to compute the consumer price index (CPI-FGV). The CPI-FGV has wide coverage and it is one of the oldest Brazilian price indices, being calculated since 1944.

The data set contains information on prices at the firm level collected in the 12 largest Brazilian metropolitan regions, even though the coverage has changed during our sample period[11].

Currently, the CPI-FGV comprises 456 products and services grouped into seven different sectors: Apparel; Education and Recreation; Food; Housing; Medical and Personal Care; Transportation; and Other Goods and Services. The weight of each product or service used in computing the index mirrors the expenditure by households receiving income up to 33 times the minimum monthly wage, obtained from a household consumption survey—Pesquisa de Orcamento Familiar (POF)—also conducted by FGV.

The data are systematically collected by FGV employees. Some data collections are made every ten days, while for other products prices are collected on a monthly basis. We refer to the most disaggregated level of data as an *item*. Each item is identified by a set of very specific characteristics, such as brand, model, packaging, type/variety that is sold in a particular outlet, in a specific city[12]. Each price also includes the exact date of collection. However, while the outlets are identified by codes, for confidential reasons no other feature of the firms, such as size, number of employees, revenues and costs amounts etc is available.

The number of items collected each month is not constant. There is variation due to item exclusions and inclusions over time. However, there is never substitution of an item by a similar one. Therefore, it is possible to follow the same item over time. When the price of a specific item is no longer collected, its price trajectory continues to be registered as missing. Thus, one can be assured that each change recorded is due to an actual price change.

5.2 Data sample and some statistics

In this subsection we describe the sample and calculate some simple statistics of price changes in the data. We have a very representative sample of the overall CPI-FGV (around 85%), containing 243 categories of products and services, whose prices are accompanied by a period of approximately 11 years, from 1996 to 2006. The seven sectors are very well represented.

We start from the original full sample with approximately 7.4 million price quotes and 122 thousand quote lines. The original sample was treated in order to have a set of information more suited to our goals.

[11] Up to the end of 2000, the coverage comprised only the metropolitan regions of Rio de Janeiro and São Paulo. After January 2001, ten other cities were included in the survey: Belo Horizonte, Brasília, Porto Alegre, Recife, Salvador, Belém, Curitiba, Florianópolis, Fortaleza and Goiania. At the beginning of 2005 the last five cities were dropped.

[12] For example, type I black beans of the Combrasil brand, sold in a 1kg package in the outlet number 16,352, in Belém.

First, we would like to have a data set with monthly data. Thus, for products whose prices are collected every 10 days, in the aggregation we choose to keep always the first price quote in each month. Second, the very short price trajectories were excluded. Trajectories with less than 18 observations or with more than 30% of missing observations were dropped. We further refined our sample by treating specifically the remaining gaps. The gaps with only one missing observation were filled using the price collected in the immediately preceding month. Also, gaps with up to three missing observations, when preceded and succeeded by the same price, were filled with the last price available. In the case of four or more missing observations, we maintained the longest uninterrupted part of the price trajectory.

Third, retail prices are characterized by a significant number of temporary price decreases (sales or promotions). Following the recent literature, as Golosov and Lucas (2007), Midrigan (2006) and others, we treated observations identified as sales prices. Because sales are not explicitly classified in our database, we used the following algorithm to identify them. If a price decrease was large enough and reversed in the following months to levels near the one prevailing before the change, we classified that observation as a price sale[13]. Following Midrigan (2006), to deal with large price changes in a V shape, we repeated the algorithm three times. When a "sale" was identified, the price was replaced by the price collected in the immediately preceding period. Outlier were also treated. An observation was classified as an outlier when it was 10 times larger or 10 times smaller than the previous observation.

Finally, to avoid the problem of left-censored spells, for all price trajectories we dropped the observations before the first observed price change.

After this treatment were are left with a final data sample with approximately 3.2 million observations and 63.2 thousand price trajectories, very representative of the seven sectors of the CPI-FGV.

Table 1: Some statistics of the price changes, conditional on adjustment

Statistics	Estimated value
Average price duration (days)	59.3
Average price change (%)	16.0
Standard deviation of the price change distribution	19.4
Kurtosis of the price change distribution	4.2
Average price increase (%)	15.5
Average price decrease (%)	16.7
Percentage of price decrease	44.0
Percentage of small price changes	37.9

Notes: 1) p is defined as the natural logarithm of the item price.
2) All statistics are calculated based on unweighted price changes.
3) Kurtosis is calculated excluding the top and bottom 1% of observations.
4) Small price change is defined as that whose value is lower than 0.5 of the mean of Δp.

Table 1 presents some statistics related to price changes, conditional on adjustment, estimated on the data. The average price duration is estimated in 1.97 months, or approximately 59 days. This value is very similar to the aggregate implied duration of 1.8 months calculated by Barros et al. (2009)[14]. We also

[13]Formally, if $\frac{(p_{t-1}-p_t)}{p_{t-1}} > 25\%$ and $p_{t+1} \geq p_t\left(1+\frac{p_{t-1}-p_t}{p_{t-1}}\right)$. One can always say that 25%is an arbitrary value, and/or that it is high for some sectors and/or low for some others. But we decided to keep a single rule to minimize arbitrariness. Changing this value does not significantly change our main results.

[14]They also find a very different measure by calculating the frequency of price changes for each particular item and then

find that price changes are, on average, large (approximately 16%), even though there is a great fraction of changes that are small. Interestingly, price decreases are estimated to be larger than price increases. While the average price increase is 15.5%, the average price cut is 16.7%.

6 Estimation and results

In this section we apply our methodology to the data set of individual prices described in the previous section, combined with aggregate data, to estimate the parameter measuring strategic complementarities in firms' pricing decisions. We also estimate other parameters of the structural model. In particular, we estimate the variance of shocks and some parameters of the pricing policy: the size of positive and negative price adjustments.

6.1 The estimated models

We combine the set of microdata just described with aggregate data to estimate the probit model. The dependent variable is the observed $r_{i,t}$ defined in equation (9). Following the theoretical model, our baseline specification has five explanatory variables and the whole vector of dummies, which are constructed, for each item in our data set, as given in equation (14). The first two variables are, respectively, the square root of the time interval between t and τ, and its inverse. These two piece of information are directly obtained from the pricing history of each item in the data set of microdata. The last three variables are, respectively, the accumulated change in the aggregate price level, $\Delta p_{i,t} = \log(P_t/P_\tau)$, in the nominal expenditure, $\Delta \mathcal{Y}_{i,t} = \log(\mathcal{Y}_t/\mathcal{Y}_\tau)$, and in the exchange rate, $\Delta e_{i,t} = \log(E_t/E_\tau)$, since the last price adjustment in time τ, divided by the square root of the time interval between t and τ, $\sqrt{\delta_{i,t}}$. Therefore, for each item in our database, in every time t, we calculate the changes in these aggregate variables between the moments t and τ. As previously pointed out, it is exactly this difference in the length of price spells what allows us to estimate the model using basically aggregate variables as regressors. Once firms reprice at different points in time, the accumulated changes in those variables since the last adjustment will be different. We must divide the variables by $\sqrt{\delta_{i,t}}$ to account for the heteroskedasticity that appears in the residuals. It is worth noting that two products with the same price adjustment history will have the same regressors.

The baseline specification incorporates the random walk assumption for the idiosyncratic shock and is estimated using the whole sample period. For robustness, we also estimate other specifications considering a white-noise assumption for the idiosyncratic shock, different sample periods and without including the exchange rate variable.

The measure of nominal expenditure we use in our estimations is the monthly nominal GDP series calculated by the Central Bank of Brazil using the quarterly national accounts of the Brazilian Institute of Geography and Statistics – IBGE. Our measure of aggregate price is the Consumer Price Index – IPC-FGV, which is calculated by the FGV from the set of microdata described in the previous section. The exchange rate is the monthly Real/US$ exchange rate (for sale) series provided by the Central Bank of Brazil.

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aggregating the durations implied by these frequencies that were obtained in the more disaggregate level—8.5 months. Gouvea (2007) estimates the aggregate duration between 2.7 and 3.8 months, depending on the calculation method.

6.2 The fit of the models

Before examining the estimation results regarding strategic complementarities, we provide some information about the fit of the model. To measure its performance we compare the implied price duration obtained in the estimated model to price duration calculated directly from the data. Table 2 reports the results. The probability of a price change in the model is obtained by calculating the probability of a price increase, $\Pr[r_{i,t} = 1|\ \overline{\mathbf{w}}]$, and the probability of a price decrease, $\Pr[r_{i,t} = -1|\ \overline{\mathbf{w}}]$, using the average of the explanatory variables in the probit, $\overline{\mathbf{w}}$. The sum of the two probabilities gives the probability of a price change estimated by the model. The implied duration is calculated by the inverse of this estimated probability[15]. The probit model estimate the probability of a price change in 58%, which implies an average price spell duration of approximately 51 days. In turn, the average duration estimated in the data is 59 days. This is a good performance, if we consider that our theoretical model is very simple.

Table 2: Probability of price change and price duration, baseline specification

$\Pr[r_{i,t} = -1 \text{ or } r_{i,t} = 1|\ \overline{\mathbf{w}}]$	Implied duration (in days)	Duration (in days)
0.58	51.4	59.3

6.3 Strategic complementarity

We now proceed to estimate the degree of strategic complementarity in firm's price decisions using the strategy outlined in subsection 4.1. Table 3 reports the estimated parameters of the probit model necessary to compute the parameter of strategic complementarities, using the baseline specification. Detailed results of the probit model are presented in Appendix C.

Before presenting the result regarding strategic complementarities, two points are worth noting. First, the parameters of the probit models are all significant at any of the usual significance levels. They also have the expected signs based on the theoretical model. Therefore, nominal expenditure, inflation and exchange rate affect positively and significantly pricing decisions of firms.

Second, the significance of the probit model parameters means that pricing decisions have a state-dependent component. As already pointed out in the introduction, Barros et al. (2009) have shown using the same set of microdata we use in our estimations that the frequency of price change is significantly and positively related to inflation, output and exchange rate depreciation. In a state-dependent pricing model this connection should result from the relation of the frictionless optimal price with those same variables, and this is what our probit results are capturing.

We infer the parameter of strategic complementarity from this relationship between the frictionless optimal price and the macroeconomic variables derived from the microfounded model. The mapping between the probit model parameters and ζ is made through equation (18). Table 3 reports pontual estimation for ζ, as well as 95% confidence intervals obtained using the Delta method.

The first important piece of evidence that emerges from Table 3 is that the value of $\hat{\zeta}$ is much less than one. This result implies that firms' pricing decisions are strategic complementary rather than substitute. Actually, the upper bound of the confidence interval is far from 1—the lower limit for strategic substitutability.

[15] One can show that, for large samples, the inverse of the estimated probability is a consistent estimator of the durations of price spells. Other papers in the literature use the inverse of the frequency of adjustments. See, for example, Bils and Klenow (2004), Gouvea (2007) and works published by the ECB/IPN (`http://www.ecb.int/home/html/researcher_ipn.en.html`).

Second, our results suggest that not only pricing decisions are complementary, but also that the degree of strategic complementarity is substantial in the data. The estimated value of 0.04 for ζ implies a strong positive interaction between the frictionless optimal price of a given firm and the prices of its competitors.

Table 3: Probit parameters and strategic complementarity, baseline specification

Parameter of $\mathcal{Y}_t$	Parameter of p_t	Parameter of e_t	Strat. complementarity ζ	Conf. interval for ζ
0.44 (0.065)	9.79 (0.044)	0.03 (0.011)	0.04 (0.006)	$0.03 - 0.05$

Notes: The robust standard deviations are in parenthesis. The confidence interval is 95% of confidence. The standard deviation of ζ was obtained by the Delta method. .

We also carried out a number of estimations for robustness check. First, we split the sample in two time periods: from 1997 to 2000 and from 2001 to 2004. The first and second rows of Table 4 show the results. They are consistent with the previous estimations—the estimated ζ is 0.05 in the 1997-2000 period and 0.11 in the 2001-2004 period. We also estimated the model using data of each year separately during the whole sample period. Results are not reported, but they are also consistent with those already presented.

Second, we estimated models without including the exchange rate variable in the frictionless optimal price equation. This formulation can be obtained from our theoretical model if we do not include the foreign input in the production function. In this case the equation for frictionless optimal price is the same as that in Woodford (2002). Results for the aggregate economy are presented in the third row of Table 4. Even though the estimated ζ is larger than those obtained in the previous estimations, it is still well bellow 1 and also supports the evidence of significant real rigidities in the data.

Finally, taking into account the evidence in the recent literature showing that idiosyncratic shocks are typically short-lived, we changed the assumption in equation (12) and estimated the models assuming that the idiosyncratic shock is a white-noise process: $a_{i,t} = \varepsilon_{i,t}$ and $\varepsilon_{i,t} \sim \mathbb{N}(0, \sigma^2)$. The last row of Table 4 presents the results. The estimated value for ζ of only 0.03 shows that our results of strong real rigidities are very robust. Considering all models we have estimated, usually the point estimates of ζ for the aggregate economy is between 0.03 and 0.11.

Table 4: Probit parameters and strategic complementarity (Robustness)

Model	$\mathcal{Y}_t$	p_t	e_t	Strat. comp. ζ	Conf. interval for ζ
Period 1997 - 2000	0.28 (0.154)	5.06 (0.142)	0.62 (0.023)	0.05 (0.028)	$0.00 - 0.11$
Period 2001 - 2004	1.24 (0.087)	10.25 (0.051)	-0.24 (0.014)	0.11 (0.007)	$0.09 - 0.12$
Without Exchange Rate	2.61 (0.053)	3.48 (0.023)	$-$	0.43 (0.006)	$0.42 - 0.44$
Idiosyncratic White Noise	0.44 (0.064)	12.82 (0.064)	0.01 (0.016)	0.03 (0.005)	$0.02 - 0.04$

Notes: The robust standard deviations are in parenthesis. The confidence interval is 95% of confidence. The standard deviation of ζ was obtained by the Delta method. .

6.4 Actual, frictionless and reset price inflation

Our main result indicating substantial degree of strategic complementarities in the data is different from the usual result found in previous studies. In this section we compare our method to those proposed in the literature to estimate real rigidities and provide some other results. In particular we compare our method to that of Bils, Klenow and Malin (2009). In order to investigate whether the difference in results is due to the different data sets, we reproduce their method with our data.

First of all, we emphasize that our procedure is different from those in the literature. Since "desired" prices, marginal costs and markups (variables to which strategic complementarities are related) are not directly observable in practice, or there is no data about them available, generally the studies rely on calibrated models and/or statistical properties of specific observable variables to indirectly infer the presence of real rigidities in the data. For instance, Klenow and Willis (2006), Burstein and Hellwig (2007), Bils, Klenow and Malin (2009), Kryvtsov and Midrigan (2009), Gopinath and Itskhoki (2010) all rely on indirect methods. In contrast, our method, by modeling the frictionless optimal price $p^*_{i,t}$ as a latent variable in an ordered probit model and by deriving the relationship between $p^*_{i,t}$ and p_t from the price-setting model, makes the measurement and interpretation of strategic complementarities more direct and independent of other details of the model. Regarding this point, the comparison with Bils, Klenow and Malin (2009) will be very informative.

Bils, Klenow and Malin (2009) (hereafter BKM) develop a statistical measure called "reset price inflation" to indirectly infer the role for strategic complementarities in the data. They use a general equilibrium price-setting model to simulate the behavior of reset price inflation and compare the simulated results to a measure of this variable constructed using the microdata underpinning the U.S. CPI. They report that a state-dependent model with strategic complementarity is fundamentally at odds with the data—the model displays unrealistically high persistence and low volatility of reset price inflation. Their empirical proxy has low (in fact negative) serial correlation and high standard deviation[16].

In order to compare our method to that developed by BKM we review and simulate their measure of reset price inflation in our data. They define *theoretical reset price* for an individual seller as that price which the seller would choose if he/she implemented a price change in the current period. Observe that this takes into consideration that the changed price is likely to last for several periods. Therefore, it differs from the definition of frictionless optimal price p^*_{it} we use. The *theoretical reset price inflation*, $\pi^*_{i,t}$, is the weighted average change of all reset prices, including those of current price changers and non-changers alike.

Their empirical measure of reset price for an item i at time t, $\hat{p}^*_{i,t}$, is given by:

$$\hat{p}^*_{i,t} = \begin{cases} p_{i,t}, & \text{if } p_{i,t} \neq p_{i,t-1} \\ \hat{p}^*_{i,t-1} + \hat{\pi}^*_t, & \text{if } p_{i,t} = p_{i,t-1} \end{cases}, \tag{21}$$

where $\hat{\pi}^*_t$, the reset price inflation, is given by:

$$\widehat{\pi}^*_t \equiv \frac{\sum_i \omega_{i,t} \left(p_{i,t} - \hat{p}^*_{i,t-1}\right) I_{i,t}}{\sum_i \omega_{i,t} I_{i,t}},$$

where $I_{i,t}$ is equal to 1 if $p_{i,t} \neq p_{i,t-1}$ and zero otherwise.

Notice that since one cannot observe the reset price for those not changing prices, their empirical method updates reset prices for them using the reset inflation $\widehat{\pi}^*_t$. That measure, in turn, is defined by the weighted

[16] They also found that TDP models with or without strategic complementarity generate reset price inflation series that are too persistent.

average of the difference between new prices in t and reset prices defined in $t-1$. The motivation for constructing such measure is similar to ours: to construct a measure that reveals strategic complementarity. As BKM acknowledge, firms that change price may have stronger incentive to do so, which means that there is a selection effect. As a consequence, the theoretical reset price inflation, which is based on desired prices for changers and non-changers, may differ markedly from its empirical counterpart when there is selection effect. In our methodology the selection effect is taken into account by the probit model. Firms whose prices are more out of line are those with larger probabilities of changing prices in the probit.

BKM construct several statistics based on their empirical measure of reset price inflation in order to discriminate among alternative models of price-setting with and without strategic complementarities. They simulate the models, closing them with an exogenous process for the money supply. They compute standard deviation and serial correlation of reset price inflation and inflation. They also show impulse response functions for reset price inflation with base on univariate AR(6) regressions. They conclude that the state-dependent model without strategic complementarity fits better the proposed set of statistics than the competing model. Since the reason for the difference between our findings and their findings could be that we use Brazilian microdata and they use US microdata, we repeat their experiment with our data.[17]

Also notice that in our methodology we can construct a measure of *frictionless optimal price inflation* in the data. The comparison of the statistical properties of the BKM's reset price inflation to the properties of our frictionless optimal inflation can be very informative. In particular, this exercise can verify if the estimated frictionless optimal inflation has the properties that BKM were looking for. After all, strategic complementarities should also make frictionless optimal inflation, like reset price inflation, persistent and stable.

To carry out this exercise we estimate the previous probit model for each of the seven sectors of the CPI-FGV separately, whose parameters are reported in Appendix C, and construct an empirical measure for frictionless optimal price inflation using the following formula:

$$\pi_t^* = \sum_i \omega_{i,t} \pi_{i,t}^*, \tag{22}$$

where $\omega_{i,t}$ is the weight of each item in the CPI-FGV, and the individual frictionless inflation is estimated by

$$\begin{aligned} \pi_{i,t}^* &= p_{i,t}^* - p_{i,t-1}^* = \Delta \boldsymbol{x}_t' \hat{\boldsymbol{\beta}} + (\hat{\tilde{\xi}}_t - \hat{\tilde{\xi}}_{t-1}) + (\hat{\tilde{a}}_{i,t} - \hat{\tilde{a}}_{i,t-1}) \\ &= \hat{\eta} + \Delta \boldsymbol{x}_t' \hat{\boldsymbol{\beta}} + (\hat{\tilde{\xi}}_t - \hat{\tilde{\xi}}_{t-1}) + \hat{\varepsilon}_{i,t}, \end{aligned} \tag{23}$$

where $\hat{\boldsymbol{\beta}}$ and $\hat{\eta}$ here are the parameters of the probit models estimated in the respective sector of the item i. The term $\hat{\tilde{\xi}}_t - \hat{\tilde{\xi}}_{t-1}$ can be constructed using the parameter of the dummy variables controlling for the aggregate shocks in the baseline specification. The only term for which we do not have estimates is $\hat{\varepsilon}_{i,t}$. But we have estimates for the variance of this shock in each sector, estimated using the equation (19). Thus, to construct a measure of frictionless optimal price inflation we carry out a Monte Carlo experiment. For each individual item in our data set we simulate one trajectory for $\{\hat{\varepsilon}_{i,t}\}$ using the distribution $\mathbb{N}(0, \hat{\sigma}_k^2)$, where $\hat{\sigma}_k^2$ is the variance of shocks estimated in sector k, $k = 1..., 7$. We then aggregate the individual frictionless optimal price inflations from (23) using equation (22), to have π_t^*. We repeat this experiment one thousand times and compare the statistical properties of these series to those of the reset price inflation.

[17] We do not have access to disaggregated data on prices collected by the U.S. Bureau of Labor Statistics. Otherwise, we could use these data in our estimations.

Table 5 reports the standard deviation and the serial correlation for reset, frictionless and actual price inflation in our sample. Figure 4 in Appendix C plots the series. For frictionless optimal price inflation the statistics reported correspond to the average obtained across the one thousand simulations. Notice that the statistics for reset price inflation are similar to those reported by BKM for US: the standard deviation is 1.89% and the serial correlation coefficient is -0.31. In fact their simulated state-dependent model without strategic complementarity generates a standard deviation of 1.79% and a serial correlation of -0.31, fitting our data set even better than theirs. It fits also our inflation statistics at least as well as the one in their sample, although not as close. In turn, the estimated frictionless optimal price inflation is much less volatile—with standard deviation equal to 0.67%—, and has dynamics closer to that of actual inflation. Regarding persistence, frictionless optimal price inflation is much more persistent than reset price inflation, with first-order autocorrelation equal to 0.21 These results do not change if we use seasonally adjusted series or not, as the last rows of Table (5) show.

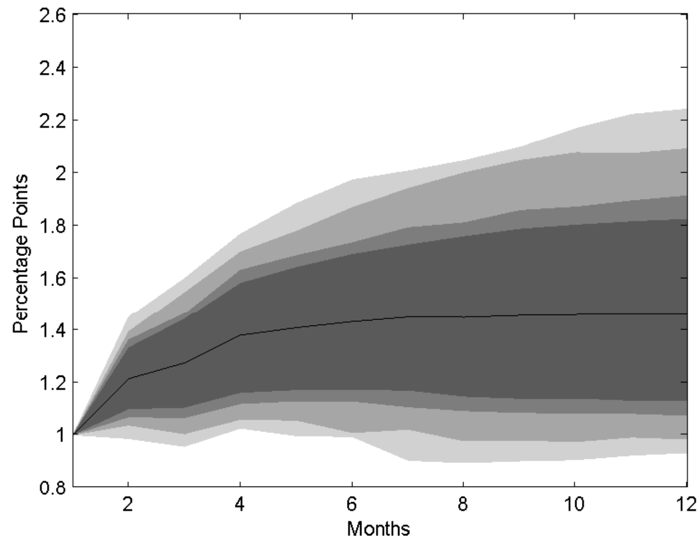
Table 5: Summary statistics for frictionless, reset and actual inflation

Series	Average	Std deviation	Persistence
Actual InflationI PC-FGV	0.57%	0.56%	0.43
Frictionless Inflation	0.57%	0.67%	0.22
Reset Price Inflation	0.58%	1.89%	−0.31
Actual Inflation IPC-FGV, Seas. Adj.	0.57%	0.47%	0.53
Frictionless Inflation, Seas. Adj.	0.57%	0.59%	0.30
Reset Price Inflation, Seas. Adj.	0.58%	1.69%	−0.33

Notes: 1) The statistics are calculated using the period Jun/1996-Aug/2005. 2) Frictionless inflation is obtained by aggregating $\pi^*_{i,t}=\hat{\eta}+\Delta\boldsymbol{x}'_t\hat{\boldsymbol{\beta}}+(\hat{\hat{\xi}}_t-\hat{\hat{\xi}}_{t-1})+\hat{\varepsilon}_{i,t}$, where $\hat{\eta}$ and $\hat{\boldsymbol{\beta}}$ are the parameters estimated in each sector. Aggregation uses the sectoral weight in IPC-FGV. The terms $\hat{\varepsilon}_{i,t}$ come from a Monte Carlo experiment with 1000 simulation. 3) Persistence is measured by the first-order autocorrelation. For frictionless inflation, average, standard deviation and persistence are the averages of the values obtained in the simulation.

We also display in Figures 2 and 3, respectively, the impulse response function (IRF) (in level) for frictionless and for reset price inflation in our data. Figure 2 plots the average response of frictionless price and a fan-chart with various levels of significance obtained in the simulation. It shows that, after a shock, frictionless optimal price has an upward sloping trajectory. The shape is similar to that found by BKM for the impulse response of *theoretical* reset prices generated by their model with strategic complementarities. Moreover, the frictionless optimal price response is consistent with our previous results of strong degree of strategic complementarities in the data: the impact is initially small (as price setters wait for the average price to respond), but accumulates over time as more firms change prices.

On the other hand, visual inspection of the IRF for BKM's simulated model reinforces the conclusion that their state-dependent model without strategic complementarity fits reset price inflation from our data base even more closely than theirs. Figure 3 shows that the response of reset prices is much greater on impact than over time, exactly like those in BKM. Therefore, if we were to use the same methodology as them in our data set, we would arrive at that same conclusion: the state-dependent model without strategic complementarities fits better the data than the state-dependent model with strategic complementarity. If it is not the different data set, what could be the reason for the different results?



Note: Accumulated responses of frictionless optimal price inflation to one unit innovation.

Figure 2: Impulse response of estimated frictionless optimal price, all goods

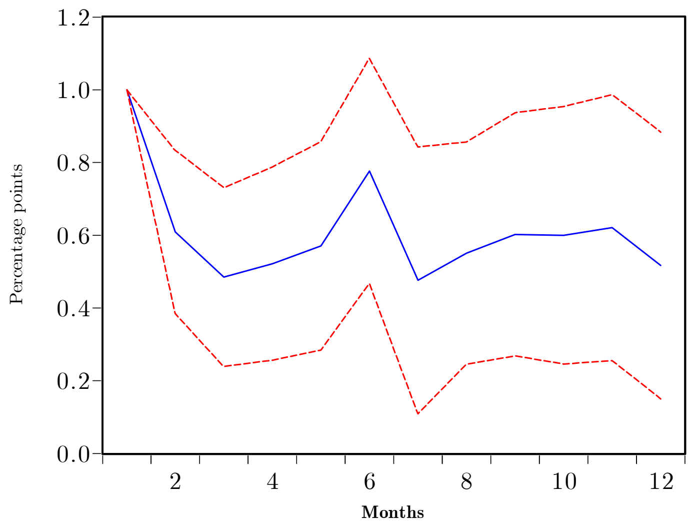
The difference must come from the methodology. Their exercise is based on model simulation. Although their focus in on the supply side—price setting and strategic complementarity,—in order to simulate the model they must close it with a demand side. As their last exercise shows, the specification of the demand side can make a big difference: when they allowed money supply to respond to real aggregate demand the state-dependent model with strategic complementarity turned out to fit better the reset price inflation IRS than the model without strategic complementarity[18]. This result suggests that in simulated models the demand side specification could be as important as the supply side.

By contrast, our methodology focus on estimating the frictionless optimal price equation. The parameters of this equation do not depend on the frequency of price adjustments or on the demand side specification. They are not affect by the selection of price changes either. Of course, we could be subject to other problems as, for example, a misspecification of the supply shocks.

6.5 The estimated pricing rules

Finally, we explore the results of the probit model regarding the pricing rule. Table 6 shows the estimated parameters using the baseline specification. Yet the results of the other specifications estimated for robustness are very close. In fact, in all models we have estimated (splitting the sample in two parts, estimating the model year by year, excluding the exchange rate variable or even using the white noise assumption for idiosyncratic shocks) the parameters of the pricing rules never changed. In this sense, the following results are very robust.

[18] They do not select this model as the best because it generates a inflation rate process that is too smooth.



Note: Accumulated response of reset price inflation to one unit innovation.

Figure 3: Impulse response of estimated reset price, all goods

As can be seen, the intercepts[19] π_1 and π_0 are statistically significant at 1%. Actually, their standard deviations are very small. One fact that may help to explain this large significance is that we take into account the variability in the price spells duration—observe that all variables in the models are divided by the square root of the time interval between t and τ, $\sqrt{\delta_{i,t}}$. Objectively, this means that the intercepts are changing over time, which improves the fit of the model.

Since we are able to estimate the parameter σ, from the intercepts of the probit model we can estimate the pairwise distances between S, s and c using the equations (20)—even though we cannot isolate the own parameters S, s and c, individually. These results are reported in Table 6.

An interesting evidence that emerges from these results is that the estimated Ss band is larger at the top than at the bottom, i.e., the estimated distance between S and c is larger than the distance between c and s—while $c - s$ is 0.06, $S - c$ is equal to 0.09. This means that, if c were zero, on average an individual firm would wait the frictionless optimal price $p^*_{i,t}$ deviate 6% from the actual price before it increasing, and deviate 9% before it reducing its price. Unfortunately, we are not able to estimate the individual value of c. But the estimated adjustments size are smaller than in the sample. One possible explanation is that our estimation does not use adjustment size, but frequency of adjustments. And there are nonlinearities in the relation between size and frequency of price adjustments.

The results in Table 6 also indicate that a typical firm seems to be more tolerant with shocks that decrease its frictionless optimal price than with shocks that increase it. This result is compatible with the evidence raise from the data: that on average price decreases are larger than price increases. Such evidence can be rationalized by the positive average inflation during the period covered by our data. When inflation is

[19] We are calling them intercepts, but observe that our probit is not standard and the variable $1/\sqrt{\delta_{i,t}}$ changes over time.

positive, if a firm is hit by a small negative shock and want to decrease its price, by maintaining its nominal price unchanged the firm will actually have a lower price in real terms. When the distance from its optimal price is sufficiently large, however, the firm reprices and the size of the adjustment will be possibly larger.

Finally, the standard deviation of the idiosyncratic shocks are estimated in 10%, which is equal to that estimated by Klenow and Willis (2006) in the model without strategic complementarity.

Table 6: Estimated parameters of the pricing rule, baseline specification

$\pi_0 = \frac{c-S}{\sigma}$	$\pi_1 = \frac{c-s}{\sigma}$	Std. deviation σ	Top band $S-c$	Bottom band $c-s$	Band width $S-s$
-0.93 (0.01)	0.63 (0.01)	0.10	0.09	0.06	0.15

7 Conclusions

In this paper we have developed an econometric methodology to directly estimate the structural parameter measuring the degree of strategic complementarities in pricing decisions in a state-dependent pricing model. At the micro level, firms face idiosyncratic shocks and fixed costs of adjusting prices. We obtain the firm's frictionless optimal price from fundamentals and, based on the Ss pricing rule, we derive a structural, non-standard ordered probit model. We use a quasi-maximum likelihood method that takes into account the autocorrelation and the heteroskedasticity that emerge from the microfounded model to estimate the probit using the microdata underpinning the CPI-FGV in Brazil for the 1996-2006 period.

Unlike the results in the literature, we find a substantial degree of strategic complementarity in firms' pricing decisions. For the aggregate economy we estimate the parameter ζ measuring strategic complementarities ranging from 0.03 to 0.11. Therefore, we do not find that a state-dependent pricing model with strategic complementarities is fundamentally in contradiction with the data, as suggested by Bills, Klenow and Malin (2009). The difference in the methodology, and not in the data, is responsible for the distinct results. As their methodology depends on the specification of the demand side, a more realistic specification of this part of their model could help to reconcile our results to theirs.

A limitation of our results is that although they seem robust to the specification of the demand side, they are not certainly robust to the type of pricing rule. If the right model is time-dependent, strategic complementarities found in the estimated state-dependent model could be a compensation for the mispecification in the type of pricing rule. Thus, a natural next step should be to develop a similar methodology to estimate the degree of strategic complementarities in a time-dependent pricing model and verify whether the same results are obtained.

The methodology also allows us to estimate some characteristics of the pricing rules. Even though we cannot separately identify the parameters S, s and c, we can easily estimate some characteristics of the pricing rules, like their widths, the length of the top and the bottom bands etc. The variance of shocks is also estimated. All the results are quite consistent with the statistics on price changes calculated directly from the data.

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A Derivation of the frictionless optimal price equation

Here we derive the frictionless optimal price in a general equilibrium framework.

Households. The representative household seeks to maximize the discounted sum of utilities:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(C_t; \xi_t) - \int_0^1 v(L_{i,t}; \xi_t) di \right], \tag{24}$$

subject to its budget constraint. C_t is a consumption index and $L_{i,t}$ is the quantity of labor of type i supplied. The term $v(L_{i,t}; \xi_t)$ represents the disutility of supplying labor and ξ_t is a vector of aggregate shocks.

There is a continuum of differentiated goods $C_{i,t}, i \in [0,1]$. Thus, following Dixit and Stiglitz (1977), C_t is an index

$$C_t = \left[\int_0^1 C_{i,t}^{(\theta-1)/\theta} di \right]^{\theta/\theta-1}, \tag{25}$$

with $\theta > 1$. The expenditure minimization problem of households implies that the optimal demand for an individual product i has the following familiar relation with the aggregated demand:

$$C_{i,t} = C_t \left(\frac{P_{i,t}}{P_t} \right)^{-\theta}, \tag{26}$$

where,

$$P_t = \left[\int_0^1 P_{i,t}^{1-\theta} di \right]^{1/(1-\theta)}. \tag{27}$$

In addition, the household must choose an optimal quantity of each labor type to supply, given the wages that it faces. The optimal quantity $L_{i,t}$ is implicitly given by the first-order condition:

$$\frac{v_L(L_{i,t}; \xi_t)}{u_C(C_t; \xi_t)} = \frac{W_{i,t}}{P_t}, \tag{28}$$

where $W_{i,t}$ is the wage for labor type i at time t.

Firms. There is a continuum of monopolistically competitive firms indexed by $i \in [0,1]$, supplying differentiated goods. We assume that each good i has the following production function:

$$Y_{i,t} = A_{i,t} L_{i,t}^{\alpha} M_t^{(1-\alpha)}, \tag{29}$$

where $A_{i,t}$ is a productivity factor and M_t is a foreign input used in the production process. The nominal exchange rate used to import M_t is given by $\tilde{E}_t$, which is exogenously determined. We may think of capital as being allocated to each firm in a fixed amount, and never depreciating. Moreover, observe the presence of heterogeneity, once $A_{i,t}$ is an idiosyncratic shock affecting only firm i.

Marginal Cost. The optimal quantities of labor and foreign input required to produce the quantity $Y_{i,t}$ of good i is given by

$$L_{i,t} = \left(\frac{\alpha}{(1-\alpha)} \frac{\tilde{E}_t}{W_{it}} \right)^{1-\alpha} \frac{Y_{i,t}}{A_{i,t}}, \tag{30}$$

$$M_t = \left(\frac{(1-\alpha)}{\alpha} \frac{W_{i,t}}{\tilde{E}_t} \right)^{\alpha} \frac{Y_{i,t}}{A_{i,t}}. \tag{31}$$

Then, the nominal cost of supplying the quantity $Y_{i,t}$ is

$$\begin{aligned} W_{i,t} L_{i,t} + \tilde{E}_t m_t &= \alpha^{-\alpha} (1-\alpha)^{(\alpha-1)} \frac{W_{i,t}^{\alpha} \tilde{E}_t^{(1-\alpha)} Y_{i,t}}{A_{i,t}} \\ &= \lambda \frac{W_{i,t}^{\alpha} \tilde{E}_t^{(1-\alpha)} Y_{i,t}}{A_{i,t}}, \end{aligned} \tag{32}$$

where $\lambda = \alpha^{-\alpha}(1-\alpha)^{(\alpha-1)}$. And the nominal marginal cost can be written as

$$\Psi_{i,t} = \lambda \frac{W_{i,t}^{\alpha} \tilde{E}_t^{(1-\alpha)}}{A_{i,t}}. \tag{33}$$

By equation (28), we can see that the quantity of labor is positively related to the firm's product. Therefore, we will rewrite equation (28) as

$$W_{i,t} = \frac{v_L(Y_{i,t}; \xi_t) P_t}{u_C(C_t; \xi_t)} \tag{34}$$

Substituting equation (34) into (33), and considering that $C_t = Y_t$ in equilibrium, leads to the following real marginal cost function:

$$\begin{aligned} \psi(Y_{i,t}, Y_t, E_t; \xi_t, A_{i,t}) \equiv \frac{\Psi_{i,t}}{P_t} &= \frac{\lambda}{A_{i,t}} \left\{ \frac{v_L(Y_{i,t}; \xi_t)}{u_C(Y_t; \xi_t)} \right\}^{\alpha} \left\{ \frac{\tilde{E}_t}{P_t} \right\}^{1-\alpha} \\ &= \frac{\lambda}{A_{i,t}} \left\{ \frac{v_L(Y_{i,t}; \xi_t)}{u_C(Y_t; \xi_t)} \right\}^{\alpha} E_t^{1-\alpha} \end{aligned} \tag{35}$$

where $E_t = \tilde{E}_t / P_t$ is the real exchange rate.

Marginal Revenue. In a model of monopolistic competition, each supplier chooses the price that will maximize its profit, taking into account the demand function given in (3):

$$Y_{i,t} = Y_t \left(\frac{P_{i,t}}{P_t} \right)^{-\theta}. \tag{36}$$

Using this equation, we can write the real revenue as

$$Y_{i,t} \frac{P_{i,t}}{P_t} = Y_{i,t} \left(\frac{Y_{i,t}}{Y_t} \right)^{-1/\theta} = Y_{i,t}^{(\theta-1)/\theta} Y_t^{1/\theta}, \tag{37}$$

which gives us the following equation for the real marginal revenue:

$$\frac{\theta-1}{\theta}\left(\frac{Y_t}{Y_{i,t}}\right)^{1/\theta}. \tag{38}$$

Optimality. Because the good i only contributes with an infinitesimal part to the aggregate output, the supplier chooses its optimal price taking Y_t and P_t as given, i.e., he equals marginal revenue to marginal cost

$$\frac{\theta-1}{\theta}\left(\frac{Y_t}{Y_{i,t}}\right)^{1/\theta} = \psi(Y_{i,t}, Y_t, E_t; \xi_t, A_{i,t}). \tag{39}$$

Once equation (30) shows that $\psi(.)$ is increasing in $Y_{i,t}$, this equation must have a unique solution for $Y_{i,t}$ given Y_t.

Substituting equation (36) into (39), we have the following equation for the frictionless optimal price:

$$\frac{P_{i,t}^*}{P_t} = \mu\psi(Y_{i,t}, Y_t, E_t; \xi_t, A_{i,t}), \tag{40}$$

where $\mu = \theta/(\theta-1)$ is the seller's desired markup.

Log-linearization. When $A_{i,t} = 1, \forall i$, it follows that in equilibrium each good must be supplied in the same quantity, and such common quantity must equal Y_t. We take a first-order log-linearization of the cost function around the steady-state equilibrium in the case of flexible prices and $A_{i,t} = 1$, $\xi_t = 0$. Let $\bar{Y}$ be the output level in this steady state. Then,

$$\begin{aligned}
\log P_{i,t}^* - \log P_t &= \log\left(\frac{P_{i,t}^*}{P_t}\right) - \log(1) \\
&= \widehat{\psi}_{it} \\
&= \alpha\omega\hat{y}_{i,t} + \alpha\vartheta^{-1}\hat{y}_t + (1-\alpha)\hat{e}_t + \frac{\partial log\,\psi_{i,t}}{\partial \xi_t}\xi_t - \log A_{i,t},
\end{aligned}$$

where ω represents the elasticity of real marginal cost with respect to the firm's output; the elasticity of marginal cost with respect to the aggregate demand corresponds to the inverse of the intertemporal elasticity of substitution of private expenditure, $\frac{\partial \log\,\psi}{\partial \log\,Y_t} = -\frac{\partial \log\,u_C}{\partial \log\,Y_t} = -\frac{u_{CC}\bar{Y}}{u_C} = \vartheta^{-1}$ and $\overline{x} = \log\left(\frac{X_t}{\bar{X}}\right)$ for all variables. Therefore,

$$\begin{aligned}
\log P_{i,t}^* - \log P_t = \alpha\omega\left(\log Y_{i,t} - \log\overline{Y}\right) + \alpha\vartheta^{-1}\left(\log Y_t - \log\overline{Y}\right) + \\
+ (1-\alpha)\left(\log E_t - \log\overline{E}\right) + \frac{\partial \log\,\psi_{i,t}}{\partial \xi_t}\xi_t - \log A_{i,t}
\end{aligned}$$

Now, substituting the demand equation (36):

$$\begin{aligned}
\log P_{i,t}^* - \log P_t = \alpha\omega\left(\log\left(\frac{P_{i,t}^*}{P_t}\right)^{-\theta} Y_t - \log\overline{Y}\right) + \alpha\vartheta^{-1}\left(\log Y_t - \log\overline{Y}\right) + \\
+ (1-\alpha)\left(\log E_t - \log\overline{E}\right) + \frac{\partial log\,\psi_{i,t}}{\partial \xi_t}\xi_t - \log A_{i,t}
\end{aligned}$$

And finally, substituting $\mathcal{Y}_t = P_t Y_t$, we have:

$$\log P_{i,t}^* - \log P_t = -\theta\alpha\omega \left(\log P_{i,t}^* - \log P_t\right) + \alpha\left(\omega + \vartheta^{-1}\right) \log\, \mathcal{Y}_t - \\ - \alpha\left(\omega + \vartheta^{-1}\right) \log P_t - \alpha\left(\omega + \vartheta^{-1}\right) \log \overline{Y} + \\ + (1-\alpha)\left(\log E_t - \log \overline{E}\right) + \frac{\partial \log\, \psi_{i,t}}{\partial \xi_t} \xi_t - \log A_{i,t}$$

$$\log P_{i,t}^* = -\frac{\alpha\left(\omega + \vartheta^{-1}\right)}{(1+\theta\omega\alpha)} \log \overline{Y} - \frac{(1-\alpha)}{(1+\theta\omega\alpha)} \log \overline{E} + \\ + \frac{\left(1+\theta\omega\alpha - \alpha\left(\omega + \vartheta^{-1}\right)\right)}{(1+\theta\omega\alpha)} \log P_t + \frac{\alpha\left(\omega + \vartheta^{-1}\right)}{(1+\theta\omega\alpha)} \log\, \mathcal{Y}_t + \frac{(1-\alpha)}{(1+\theta\omega\alpha)} \log E_t + \\ + \frac{1}{(1+\theta\omega\alpha)} \frac{\partial \log\, \psi_{i,t}}{\partial \xi_t} \xi_t - \frac{\log A_{i,t}}{(1+\theta\omega\alpha)}$$

Letting lowercase variables representing variables in logs, we obtain equation (8):

$$p_{i,t}^* = \kappa + (1-\zeta)p_t + \zeta\mathcal{Y}_t + \frac{(1-\alpha)}{(1+\theta\omega\alpha)} e_t + \tilde{\xi}_t + \tilde{a}_{i,t},$$

where $\zeta = \frac{\alpha\left(\omega+\vartheta^{-1}\right)}{(1+\theta\omega\alpha)}$, $\kappa = -\frac{\alpha\left(\omega+\vartheta^{-1}\right)}{(1+\theta\omega\alpha)} \log \overline{Y} - \frac{(1-\alpha)}{(1+\theta\omega\alpha)} \log \overline{E}$, $\tilde{\xi}_t = \frac{1}{(1+\theta\omega\alpha)} \frac{\partial \log\, \psi_{i,t}}{\partial \xi_t} \xi_t$, and

$$\tilde{a}_{i,t} = -\frac{\log A_{i,t}}{(1+\theta\omega\alpha)}$$

B Econometric Estimation

Main Assumptions

Consider the following assumptions.

Assumption 1 *The log aggregated price index, p_t, is given by the following weighted sum:*

$$p_t = \sum_{n=1}^{N} \omega_{n,t} p_{n,t}, \tag{41}$$

where $p_{n,t}$ is the individual log price index and $0 < \omega_{n,t} < 1$ is a weight such that

$$\omega_{n,t} \longrightarrow 0 \, as \, N \longrightarrow \infty, \forall \, t. \tag{42}$$

Assumption 2 *The frictionless price follows the model*

$$p^*_{i,t} = \kappa + \zeta \mathcal{Y}_t + (1-\zeta) p_t + \frac{(1-\alpha)}{1+\alpha\omega\theta} e_t + \tilde{\xi}_t + \tilde{a}_{i,t}, \tag{43}$$

where $\mathcal{Y}_t$ is the nominal expenditure, p_t is the aggregated price index, κ and ζ are combinations of deep parameters of the economy, $\tilde{\xi}_t$ is the aggregate shock and $\tilde{a}_{i,t}$ is the idiosyncratic shock.

Assumption 3 *The aggregated shock $\tilde{\xi}_t$ follows a random walk process*

$$\tilde{\xi}_t = \tilde{\xi}_{t-1} + v_t,$$

where $\tilde{\xi}_0 = O_p(1)$,

$$v_t = \sum_{j=0}^{\infty} \varphi_j \epsilon_{t-j},$$

and

$$\sum_{j=0}^{\infty} j |\varphi_j| < \infty.$$

Furthermore, $\{\epsilon_t\}_{t=-\infty}^{\infty}$ is a sequence of independent and identically distributed random variables with zero mean such that $\mathbb{E}\left(\epsilon_t^2\right) < \infty$ and $\mathbb{E}\left(\epsilon_t^{4+\delta}\right) < \infty$, for a $\delta > 0$.

Assumption 4 *For each individual i, the idiosyncratic shocks, $\tilde{a}_{i,t}$, follows one of the following processes:*

1. *$\tilde{a}_{i,t} = a_i + \tilde{a}_{i,t-1} + \varepsilon_{i,t}$, where $\tilde{\xi}_0 = O_p(1)$; or*
2. *$\tilde{a}_{i,t} = a_i + \varepsilon_{i,t}$.*

In both cases a_i is a fixed effect and $\varepsilon_{i,t}$ can be written as

$$\varepsilon_{i,t} = \sum_{j=0}^{\infty} \gamma_{i,j} e_{i,t-j}, \tag{44}$$

$$\sum_{j=0}^{\infty} j |\gamma_{j,t}| < \infty.$$

Furthermore, $\{e_{i,t}\}_{t=-\infty}^{\infty}$ is a sequence of independent and identically distributed random variables with zero mean such that $\mathbb{E}\left(e_{i,t}^2\right) < \infty$ and $\mathbb{E}\left(e_{i,t}^{4+\delta}\right) < \infty$, for a $\delta > 0$. Finally, $\mathbb{E}\left(e_{i,t} e_{j,t}\right) = 0$, $\forall \, t \, and \, j \neq i$.

Derivation of the Econometric Model

Under Assumption 2 we have:

$$r_{i,t}^{*} \equiv p_{i,\tau} - p_{i,t}^{*} = c - \boldsymbol{z}_{i,t}'\boldsymbol{\beta} - (\tilde{\xi}_t - \tilde{\xi}_\tau) - (\tilde{a}_{i,t} - \tilde{a}_{i,\tau}).$$

Under Assumption 3 and iterating $\tilde{\xi}_t$ backward we get

$$\tilde{\xi}_t - \tilde{\xi}_\tau = v_t + \cdots + v_{t-\delta_{i,t}+1}.$$

Let

$$v_t = \sum_{j=1}^{T} \gamma_j \tilde{d}_{j,t}$$

where

$$\tilde{d}_{j,t} = \begin{cases} 1, & \text{if } t = j, \\ 0, & \text{if } t \neq j. \end{cases}$$

Hence,

$$\begin{aligned} \tilde{\xi}_t - \tilde{\xi}_\tau =& v_t + v_{t-1} + \cdots + v_{t-\delta_{i,t}+1} \\ =& \sum_{j=1}^{T} \gamma_j \tilde{d}_{j,t} + \sum_{j=1}^{T} \gamma_j \tilde{d}_{j,t-1} + \cdots + \sum_{j=1}^{T} \gamma_j \tilde{d}_{j,t-\delta_{i,t}+1} \\ =& \sum_{j=1}^{T} \gamma_j d_{j,t}, \end{aligned}$$

where

$$d_{j,t} = \begin{cases} 1, & \text{if } \ j \in [t, t-\delta_{i,t}+1] \\ 0, & \text{if } \ \text{otherwise} \end{cases}$$

Now, under Assumption 4 we have:

$$\begin{aligned} \tilde{a}_{i,t} - \tilde{a}_{i,\tau} =& (\eta + \cdots + \eta) + (\varepsilon_{i,t} + \cdots + \varepsilon_{i,t-\delta_{i,t}+1}) \\ =& \eta\delta_{i,t} + u_{i,t}, \end{aligned}$$

where $\delta_{i,t}$ is the time interval between t and τ, and $u_{i,t} = \varepsilon_{i,t} + \cdots + \varepsilon_{i,t-\delta_{i,t}+1}$ is a moving average MA($\delta_{i,t}-1$) process. Therefore, $u_{i,t} \sim \mathbb{N}(0, \delta_{i,t}\sigma^2)$.

Then, putting all parts together we obtain:

$$r_{i,t}^{*} \equiv p_{i,\tau} - p_{i,t}^{*} = c - \eta\delta_{i,t} - \boldsymbol{z}_{i,t}'\boldsymbol{\beta} - \sum_{j=1}^{T} \gamma_j d_{j,t} - u_{i,t}.$$

Likelihood and Robust Variance-Covariance Matrix

For notational reason, write equations in (15) in the following simpler way:

$$
\begin{aligned}
Pr[r_{i,t}=1|\boldsymbol{w}_{i,t}] &= 1-\Phi\left(\pi_1\ddot{1}_{i,t}-\tilde{\boldsymbol{w}}'_{i,t}\boldsymbol{\theta}\right)\\
Pr[r_{i,t}=0|\boldsymbol{w}_{i,t}] &= \Phi\left(\pi_1\ddot{1}_{i,t}-\tilde{\boldsymbol{w}}'_{i,t}\boldsymbol{\theta}\right)-\Phi\left(\pi_0\ddot{1}_{i,t}-\tilde{\boldsymbol{w}}'_{i,t}\boldsymbol{\theta}\right),\\
Pr[r_{i,t}=-1|\boldsymbol{w}_{i,t}] &= \Phi\left(\pi_0\ddot{1}_{i,t}-\tilde{\boldsymbol{w}}'_{i,t}\boldsymbol{\theta}\right),
\end{aligned}
$$

where $\tilde{\boldsymbol{w}}_{i,t}=(\ddot{\delta}_{i,t},\ddot{\boldsymbol{z}}'_{i,t},\ddot{\boldsymbol{d}}'_t)'$ and $\boldsymbol{\theta}=\left(\tilde{\eta},\tilde{\boldsymbol{\beta}}',\tilde{\gamma}_1,\ldots,\tilde{\gamma}_T\right)'$.

Then, the partial log-likelihood of an observation i at time t is given by:

$$
\begin{aligned}
l_{i,t}(\boldsymbol{\psi}) = \mathbb{I}\{r_{i,t}=1\}\log\left[1-\Phi\left(\pi_1\ddot{1}_{i,t}-\tilde{\boldsymbol{w}}'_{i,t}\boldsymbol{\theta}\right)\right]+\\
+\mathbb{I}\{r_{i,t}=0\}\log\left[\Phi\left(\pi_1\ddot{1}_{i,t}-\tilde{\boldsymbol{w}}'_{i,t}\boldsymbol{\theta}\right)-\Phi\left(\pi_0\ddot{1}_{i,t}-\tilde{\boldsymbol{w}}'_{i,t}\boldsymbol{\theta}\right)\right]+\\
+\mathbb{I}\{r_{i,t}=-1\}\log\left[\Phi\left(\pi_0\ddot{1}_{i,t}-\tilde{\boldsymbol{w}}'_{i,t}\boldsymbol{\theta}\right)\right],
\end{aligned}
$$

where $\boldsymbol{\psi}=(\pi_1,\pi_0,\boldsymbol{\theta}')'$ and $\mathbb{I}\{\cdot\}$ is an indicator function.

Set

$$
\begin{aligned}
\Phi_1(.) &\equiv \Phi\left(\pi_1\ddot{1}_{i,t}-\tilde{\boldsymbol{w}}'_{i,t}\boldsymbol{\theta}\right),\\
\Phi_0(.) &\equiv \Phi\left(\pi_0\ddot{1}_{i,t}-\tilde{\boldsymbol{w}}'_{i,t}\boldsymbol{\theta}\right),\\
\phi_1(.) &\equiv \phi\left(\pi_1\ddot{1}_{i,t}-\tilde{\boldsymbol{w}}'_{i,t}\boldsymbol{\theta}\right),\\
\phi_0(.) &\equiv \phi\left(\pi_0\ddot{1}_{i,t}-\tilde{\boldsymbol{w}}'_{i,t}\boldsymbol{\theta}\right).
\end{aligned}
$$

Then, the score of an observation i at time t is given by:

$$
\boldsymbol{s}_{i,t}(\boldsymbol{\psi}) = \begin{bmatrix} \frac{\partial l_{i,t}}{\partial \pi_1} \\ \frac{\partial l_{i,t}}{\partial \pi_0} \\ \frac{\partial l_{i,t}}{\partial \boldsymbol{\theta}} \end{bmatrix} = \begin{bmatrix} s_{1,i,t} \\ s_{2,i,t} \\ \boldsymbol{s}_{3,i,t} \end{bmatrix},
$$

where,

$$
\begin{aligned}
s_{1,i,t} &= \left\{\frac{\mathbb{I}\{r_{i,t}=0\}}{\Phi_1(.)-\Phi_0(.)}-\frac{\mathbb{I}\{r_{i,t}=1\}}{1-\Phi_1(.)}\right\}\phi_1(.)\ddot{1}_{i,t},\\
s_{2,i,t} &= \left\{\frac{\mathbb{I}\{r_{i,t}=-1\}}{\Phi_0(.)}-\frac{\mathbb{I}\{r_{i,t}=0\}}{\Phi_1(.)-\Phi_0(.)}\right\}\phi_0(.)\ddot{1}_{i,t},\\
\boldsymbol{s}_{3,i,t} &= \left\{\frac{\mathbb{I}\{r_{i,t}=1\}\phi_1(.)}{1-\Phi_1(.)}+\frac{\mathbb{I}\{r_{i,t}=0\}}{\Phi_1(.)-\Phi_0(.)}\left[-\phi_1(.)+\phi_0(.)\right]-\frac{\mathbb{I}\{r_{i,t}=-1\}\phi_0(.)}{\Phi_0(.)}\right\}\tilde{\boldsymbol{w}}_{i,t}.
\end{aligned}
$$

Asymptotic Results

Lemma 1 *Let $\varepsilon_{i,t}$ be defined as in Assumption 4. Under Assumption 1, $\mathbb{E}\left(\varepsilon_{i,t}p_t\right)\longrightarrow 0$ as $N\longrightarrow\infty$, $forall\, t$.*

Proof: Note that

$$\begin{aligned}\mathbb{E}\left(\varepsilon_{i,t}p_t\right) &= \mathbb{E}\left(\varepsilon_{it}\sum_{n=1}^{N}\omega_{n,t}p_{n,t}\right)\\ &= \omega_{i,t}\mathbb{E}(\varepsilon_{i,t}p_{i,t}).\end{aligned}$$

If $-\infty < \mathbb{E}(\varepsilon_{i,t}p_{i,t}) < \infty$, for all i and t, under Assumption 1, $\omega_{i,t}\mathbb{E}(\varepsilon_{i,t}p_{i,t}) \longrightarrow 0$ as $N \longrightarrow \infty$. ■

Define $\boldsymbol{\psi}$ as the vector containing the parameters to be estimated and $\widehat{\boldsymbol{\psi}}$ the quasi maximum likelihood estimator

$$\widehat{\boldsymbol{\psi}} = \underset{\boldsymbol{\psi}\in\boldsymbol{\Psi}}{\mathsf{argmax}}\sum_{i=1}^{N}\sum_{t=1}^{T} l_{i,t}(\boldsymbol{\psi}). \tag{45}$$

Theorem 1 *First, assume that the true parameter vector $\boldsymbol{\psi}$ is in the interior of $\boldsymbol{\Psi}$, a compact parameter space. Under Assumptions 1–4, $\widehat{\boldsymbol{\psi}} \xrightarrow{p} \boldsymbol{\psi}$ and*

$$\sqrt{N}\left(\widehat{\boldsymbol{\psi}} - \boldsymbol{\psi}\right) \xrightarrow{d} \mathsf{N}\left(\mathbf{0}, \boldsymbol{A}^{-1}\boldsymbol{B}\boldsymbol{A}^{-1}\right).$$

Furthermore, $\boldsymbol{A}$ and $\boldsymbol{B}$ can be consistently estimated by

$$\widehat{\boldsymbol{A}} = \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\boldsymbol{s}_{i,t}(\hat{\boldsymbol{\psi}})\boldsymbol{s}_{i,t}(\hat{\boldsymbol{\psi}})'$$

and

$$\widehat{\boldsymbol{B}} = \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\boldsymbol{s}_{i,t}(\hat{\boldsymbol{\psi}})\boldsymbol{s}_{i,t}(\hat{\boldsymbol{\psi}})' + \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{r\neq t}\boldsymbol{s}_{i,r}(\hat{\boldsymbol{\psi}})\boldsymbol{s}_{i,t}(\hat{\boldsymbol{\psi}})',$$

Confidence interval for $\hat{\zeta}$ and for $\hat{\sigma}$

Theorem (Delta Method): Suppose that $\hat{\boldsymbol{\theta}}$ is an estimator of the Px1 vector $\boldsymbol{\theta} \in \boldsymbol{\Theta}$ and that

$$\sqrt{N}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) \xrightarrow{d} \mathbb{N}(\mathbf{0}, \boldsymbol{V}),$$

where $\boldsymbol{V}$ is a PxP positive definite matrix. Let $\boldsymbol{c} : \boldsymbol{\Theta} \to \mathbb{R}^Q$ be a continuous differentiable function on the parameter space $\boldsymbol{\Theta} \subset \mathbb{R}^P$, where $Q \leq P$, and assume that $\boldsymbol{\theta}$ is in the interior of the parameter space. Define $\boldsymbol{C}(\boldsymbol{\theta}) \equiv \nabla_\theta \boldsymbol{c}(\boldsymbol{\theta})$ and the QxP Jacobian of $\boldsymbol{c}$. Then

$$\sqrt{N}[\boldsymbol{c}(\hat{\boldsymbol{\theta}}) - \boldsymbol{c}(\boldsymbol{\theta})] \xrightarrow{d} \mathbb{N}[\mathbf{0}, \boldsymbol{C}(\boldsymbol{\theta})\boldsymbol{V}\boldsymbol{C}(\boldsymbol{\theta})']. \tag{46}$$

Define $\hat{\boldsymbol{C}} \equiv \boldsymbol{C}(\hat{\boldsymbol{\theta}})$. Then plim $\hat{\boldsymbol{C}} = \boldsymbol{C}(\boldsymbol{\theta})$. If plim $\hat{\boldsymbol{V}} = \boldsymbol{V}$, then

$$N[\boldsymbol{c}(\hat{\boldsymbol{\theta}}) - \boldsymbol{c}(\boldsymbol{\theta})]'[\hat{\boldsymbol{C}}\hat{\boldsymbol{V}}\hat{\boldsymbol{C}}']^{-1}[\boldsymbol{c}(\hat{\boldsymbol{\theta}}) - \boldsymbol{c}(\boldsymbol{\theta})] \xrightarrow{d} \chi^2_Q. \tag{47}$$

Proof: See Wooldridge (2002), pp. 44-45 ■

Once ζ and σ are given respectively by $\zeta = \frac{\tilde{\beta}_1}{\tilde{\beta}_1+\tilde{\beta}_2}$ and $\sigma = \frac{1}{\tilde{\beta}_1+\tilde{\beta}_2}$, we can use the theorem above to obtain

$$\sqrt{N}[\hat{\zeta} - \zeta] \xrightarrow{d} \mathbb{N}[0, \boldsymbol{C}_\zeta \boldsymbol{V}_1 \boldsymbol{C}'_\zeta] \tag{48}$$

and

$$\sqrt{N}[\hat{\sigma} - \sigma] \xrightarrow{d} \mathbb{N}[0, \boldsymbol{C}_\sigma \boldsymbol{V}_1 \boldsymbol{C}'_\sigma], \tag{49}$$

where $\boldsymbol{C}_\zeta = [C^1_\zeta \ \ C^2_\zeta]'$, $\boldsymbol{C}_\sigma = [C^1_\sigma \ \ C^2_\sigma]'$ and $C^1_\zeta = \frac{1}{\tilde{\beta}_1 + \tilde{\beta}_2} - \frac{\tilde{\beta}_1}{\tilde{\beta}_1 + \tilde{\beta}_2}$, $C^2_\zeta = -\frac{\tilde{\beta}_1}{(\tilde{\beta}_1 + \tilde{\beta}_2)^2}$, $C^1_\sigma = -\frac{1}{\tilde{\beta}_1 + \tilde{\beta}_2}$, $C^2_\sigma = -\frac{1}{\tilde{\beta}_1 + \tilde{\beta}_2}$. $\boldsymbol{V}_1$ is the respective variance-covariance matrix of $\tilde{\beta}_1$ and $\tilde{\beta}_2$.

Now the confidence interval for the desired significance level can be constructed as usual.

C Additional estimation results

Table 7: Detailed results of the probit model, baseline specification

Parameter	Estimates	Std. error	t-stat	95% conf.	interval
Aggregate Economy					
Log-likelihoood: -12290523863826.87				Number of obs:	2851318
$(c-s)/\sigma$	0.63	0.01	76.81	0.61	0.64
$(c-S)/\sigma$	-0.93	0.01	-113.99	-0.94	-0.91
$\sqrt{\delta_{i,t}}$	-0.35	0.06	-5.90	-0.47	-0.23
$\mathcal{Y}_t$	0.44	0.07	6.79	0.31	0.57
p_t	9.79	0.04	224.11	9.70	9.88
e_t	0.03	0.01	3.07	0.01	0.05
ζ	0.04	0.01	-	0.03	0.05
σ	0.10	0.01	-	0.09	0.11

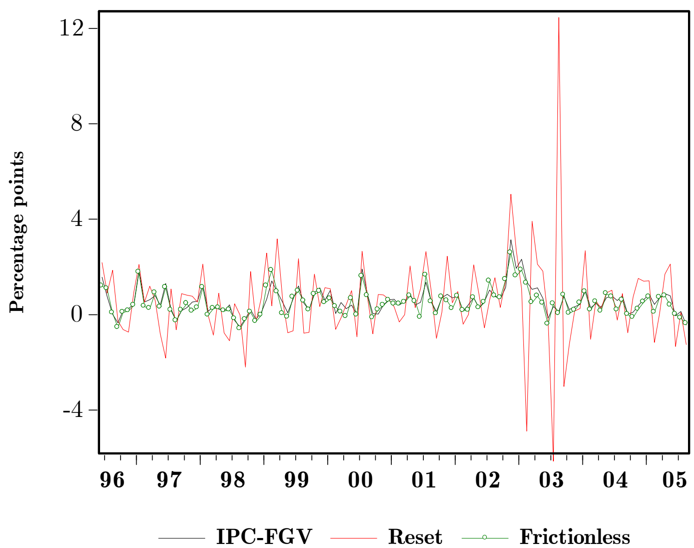

Figure 4: Estimated frictionless, estimated reset and actual in‡ation

Table 8: Detailed results of probit models, sectoral estimation

Sector: Food					
Log-likelihoood: -1639878.65				Number of obs:	1229052
Parameter	Estimates	Std. error	t-stat	95% conf.	interval
$(c-s)/\sigma$	0.48	0.01	38.33	0.46	0.51
$(c-S)/\sigma$	-0.71	0.01	-56.37	-0.73	-0.68
$\sqrt{\delta_{i,t}}$	-0.61	1.02	-0.60	-2.60	1.39
$\mathcal{Y}_t$	-0.33	0.12	-2.84	-0.55	-0.10
p_t	8.36	0.09	91.27	8.18	8.54
e_t	0.26	0.02	12.53	0.22	0.30
ζ	-0.04	0.02	-	-0.07	-0.01
σ	0.12	0.02	-	0.10	0.15

Sector: Other Goods and Services					
Log-likelihoood: -175303.29				Number of obs:	131933
Parameter	Estimates	Std. error	t-stat	95% conf.	interval
$(c-s)/\sigma$	0.72	0.03	29.16	0.67	0.77
$(c-S)/\sigma$	-1.05	0.03	-42.96	-1.10	-1.00
$\sqrt{\delta_{i,t}}$	0.63	1.04	0.60	-1.41	2.67
$\mathcal{Y}_t$	2.44	0.31	7.94	1.84	3.05
p_t	2.69	0.23	56.54	12.25	13.14
e_t	-0.66	0.05	-12.92	-0.76	-0.56
ζ	0.16	0.02	-	0.13	0.20
σ	0.07	0.02	-	0.03	0.11

Sector: Sector: Education and Recreation					
Log-likelihoood: -211577.62				Number of obs:	196576
Parameter	Estimates	Std. error	t-stat	95% conf.	interval
$(c-s)/\sigma$	1.48	0.02	80.32	1.45	1.52
$(c-S)/\sigma$	-2.11	0.02	-115.67	-2.15	-2.07
$\sqrt{\delta_{i,t}}$	-1.08	1.67	-0.65	-4.36	2.19
$\mathcal{Y}_t$	5.28	0.19	28.62	4.92	5.64
p_t	13.68	0.09	156.73	13.51	13.85
e_t	0.58	0.03	21.80	0.53	0.63
ζ	0.28	0.01	-	0.26	0.29
σ	0.05	0.01	-	0.04	0.07

Sector: Housing					
Log-likelihoood: -479116.74				Number of obs:	364110
Parameter	Estimates	Std. error	t-stat	95% conf.	interval
$(c-s)/\sigma$	0.60	0.02	31.55	0.57	0.64
$(c-S)/\sigma$	-1.05	0.02	-55.49	-1.09	-1.01
$\sqrt{\delta_{i,t}}$	-2.08	0.11	-18.89	-2.30	-1.87
$\mathcal{Y}_t$	1.40	0.15	9.15	1.10	1.69
p_t	11.09	0.13	85.32	10.84	11.35
e_t	0.67	0.03	20.03	0.60	0.73
ζ	0.11	0.02	-	0.08	0.14
σ	0.08	0.01	-	0.06	0.10

Table 9: Detailed results of probit models, sectoral estimation (continuation)

Sector:Medical and Personal Care					
Log-likelihoood: -674949.22				Number of obs:	515849
Parameter	Estimates	Std. error	t-stat	95% conf.	interval
$(c-s)/\sigma$	0.83	0.01	79.58	0.80	0.85
$(c-S)/\sigma$	-1.29	0.01	-126.76	-1.31	-1.27
$\sqrt{\delta_{i,t}}$	1.48	0.28	5.32	0.94	2.03
$\mathcal{Y}_t$	1.36	0.14	9.76	1.08	1.63
p_t	11.93	0.08	154.31	11.78	12.09
e_t	-0.92	0.02	-46.45	-0.96	-0.88
ζ	0.10	0.01	-	0.08	0.12
σ	0.08	0.01	-	0.06	0.09

Sector: Transportation					
Log-likelihoood: -123726.40				Number of obs:	95908
Parameter	Estimates	Std. error	t-stat	95% conf.	interval
$(c-s)/\sigma$	0.63	0.01	48.68	0.61	0.66
$(c-S)/\sigma$	-1.25	0.01	-100.01	-1.28	-1.23
$\sqrt{\delta_{i,t}}$	-1.85	0.60	-3.08	-3.03	-0.68
$\mathcal{Y}_t$	2.32	0.36	6.45	1.61	3.02
p_t	11.20	0.24	47.49	10.74	11.66
e_t	0.92	0.06	15.58	0.81	1.04
ζ	0.17	0.02	-	0.13	0.22
σ	0.07	0.02	-	0.03	0.12

Sector: Apparel					
Log-likelihoood: -479116.74				Number of obs:	317890
Parameter	Estimates	Std. error	t-stat	95% conf.	interval
$(c-s)/\sigma$	0.62	0.05	13.53	0.53	0.72
$(c-S)/\sigma$	-0.72	0.05	-15.50	-0.81	-0.63
$\sqrt{\delta_{i,t}}$	0.10	2.22	0.05	-4.24	4.44
$\mathcal{Y}_t$	1.16	0.22	5.24	0.73	1.59
p_t	6.22	0.14	43.18	5.93	6.50
e_t	-0.17	0.04	-4.68	-0.24	-0.10
ζ	0.16	0.03	-	0.11	0.21
σ	0.14	0.03	-	0.08	0.19