The Economics of Contract Farming: A Credit and Investment Perspective*

Xiaoxue Du†, Liang Lu†, and David Zilberman‡

†Graduate student, Department of Agricultural and Resource Economics, University of California, Berkeley

‡Professor, Department of Agricultural and Resource Economics, University of California, Berkeley and member of Giannini Foundation

November 9, 2013

Corresponding Author: Liang Lu
Address: 311 Giannini Hall, Department of Agricultural and Resource Economics, UC Berkeley, Berkeley, CA 94720
Phone: 626-673-0363
E-mail: lld2x1515@berkeley.edu

*Research leading to the paper was supported by Energy Biosciences Institute.

The Economics of Contract Farming: A Credit and Investment Perspective

Abstract

Agribusiness firms introduce new products that require agricultural production and then processing. Firms have to decide about processing capacity and assure availability of agricultural feedstock. Some of this is done in-house, and some is secured through contracts. We investigate the allocation of capital between processing capacity and in-house production, while the remainder of agricultural inputs is procured through contracts. Our results show that contract farming will increase with the cost of capital and decline when agribusiness firm has monopsony power over feedstock producers. Moreover, when supply of contracted feedstock is uncertain, expected final output will be less than under certainty and more capital will be allocated to in house production of the feedstock.

Key words: contract farming, vertical integration, uncertainty

JEL classification: Q16, Q42

Introduction

With fast pace of technology innovation and product differentiation, contract farming has become indispensable element of modern agriculture. As reported in MacDonald and Korb (2011), the percentage of contract farming in total U.S. agricultural value product is 11 percent in 1969 and is 39 percent in 2008. Besides the significant growth of contract farming, the other stylized fact is that, as depicted in figure 1, different agricultural sectors demonstrate very dispersed usage of contract farming. Less than 20 percent of corn and cattle production are through contract farming, but over 90 percent of poultry and sugar beets are produced under contracts. Meanwhile, within the sectors that are using contract farming intensively, the forms of contract that different sectors adopt vary drastically. Production contract is the dominating form of contract in broiler sector in the Unite States, whereas marketing contracts is the most influential form of contract in the sector of fresh fruits and vegetables in United States. It is natural for economists to ask what are the features of the industrial organization of contract farming that cause such stylized facts to emerge.

Tremendous literature on contract farming looks at the motivations for adopting this industrial organization and how it benefits either of the parties involved. Early literature such as Cheung (1969) and Glover et al. (1994) focuses on the how contract farming could induce risk-sharing and improve efficiency. The risk-sharing argument states that, under contract farming, both producers and processors receive a predetermined price rather than fluctuating spot market price. Consequently, both parties may find contract farming plausible because of the risk mitigation.

Later literature such as Knoeber and Thurman (1995), Goodhue (2000), and Hueth and Ligon (2002) analyze the contractual relationship in a supply chain. Especially, the question at hand is one party in the contract does not have perfect information about its counterpart (e.g. a manufacturer may not know a producer's productivity or land quality), which is known as the asymmetric information problem. Using contract theory tools, this line of research often looks into the development of contract terms where information rent plays an important role. In terms of agricultural sectors, there are numerous analyses into each sectors, such as Alexander et al. (2000) and Hueth and Ligon (2002) on tomato contracts; Knoeber and Thurman (1995) and Goodhue (2000) on contracts in broiler industry. MacDonald and Korb 2011 mentioned that perishability of certain crops, where number of potential buyers and sellers is small, also may induce contract farming as it could stabilize the supply chain. Du, Lu, and Zilberman (2013) provides a comprehensive survey of literature and summarizes other factors affect contract farming.

The literature, though, ignores the role of contract farming as a vehicle for introduction of new agricultural products or technologies. Boehlje et al. (1998) suggest that modern industrial agriculture is associated with the use of contract farming by agribusiness firms that introduce differentiated products that require both agricultural production and processing. These agribusiness firms develop new technologies and have to allocate their resources and capital between investment in processing and marketing and in-house production, securing the residual feedstock through contract farming. These forms of organization were associated with the introduction of new models of agribusiness like the production and processing of broilers, swine, or biofuel, as well as rubber and palm oil in Africa (Ruf 2009). Companies such as Tyson, Foster Farms, or PERDUE came with an original way to produce or process poultry-and had to decide to what extent to invest in processing and in-house production, and how much to buy from the outside in the form of contracts. In this article, we would like to address this problem. Our analysis shows that contract farming allows agribusiness to overcome credit and other capacity constraints in taking advantage of economies of scale in processing. Credit constraints figured out in traditional landlord-tenant contracts (Jaynes 1982). Likewise, contract farming franchising emerged when the franchiser aims to expand market share but faces capital constraints (Rubin 1978). Bierlen, Parsch, and Dixon (1999) test several contract choice hypotheses using eastern Arkansas data set and provide empirical support for credit constraint hypothesis.

The first objective of this article is to investigate an agribusiness' decision on capital allocation when it has the option of acquiring the feedstock through contracts and self-production. The capital can be allocated on building larger processing facility or on producing the feedstock on its own. If the agribusiness is short of capital, then it is intuitive to see that when the agribusiness is inclined to expand its business by building larger processing capacity, then it will produce less feedstock by itself and rely on contract farming to acquire the remainder of the feedstock. For example, the common practice for broiler processing industry is that processors provide hatched eggs and other necessary inputs for contracted farmers to raise the chicken (Goodhue 2000). Clearly, processors could also raise the chickens by themselves but the broiler houses require huge amount of investment, which may lead to allocating less resource on processing capacity and weaken the processors' core competence. We will investigate the limited capital model under different special cases. Here, we are going to elaborate how processor's monopoly or monopsony power may affect a processor's decision on resource allocation. Product differentiation creates thin markets– there may be only a few sellers that are selling the final product or there may be only a few processors that need the feedstock. Intuitively, either final product or feedstock production may be less than competitive case. We will discuss the implication of such monopoly or monopsony power on agribusiness' capital allocation. Moreover, we will discuss the capital allocation under uncertainty scenario.

The second objective of this article is to analytically show how the interplay between contract farming and uncertainty could affect an agribusiness' capital allocation. As introduced earlier, risk is an important element that may induce contract farming. Katchova and Miranda (2004) find that highly leveraged (more risky) crop producers are more likely to adopt marketing contracts and that marketing contracts were used not only to reduce price risk but also to have an outlet for the harvested crop. Reliance on fixed contracts instead of the spot market has been found to be significantly related to the level or price risk, risk aversion and risk perception among hog producers (Franken, Pennings, and Garcia 2009, Pennings and Wansink 2004). Much of this research has focused on a producer's choice between a contract and selling on the spot market. In this article, we will look at the problem from agribusiness' perspective and compare the scenario of contract farming and vertical integration under uncertainty. From agribusiness' point of view, uncertainty may come from various sources such as price fluctuation in both inputs and outputs; contract defaulting; and in newly developed technologies. Our model will utilize the theoretic framework of Sandmo (1971) and Feder (1977) to analyze how uncertainty could affect a agribusiness' resource allocation.

The rest of the article is arranged as follows: in section 2, we will introduce the limited capital model under certainty case and discuss the implication of monopoly/monopsony on the capital allocation. In section 3 and 4, we will investigate the limited capital model under uncertainty. And in section 5, we will give concluding remarks.

The Certainty Model

Consider a monopoly agribusiness who faces a demand for final output with inverse demand function $p(q)$. We assume that $p' < 0, p'' < 0$. Let $q = f(K_1, x)$ be the production function for final output where K_1 is the capital used for processing facility establishment and x is the feedstock for processing. For the production function, we assume that the production technology is concave. To reflect the complementarity between processing facility and feedstock, we add the assumption that $f_{K_1x} > 0$. The feedstock can be produced through vertical integration with production function $x_1 = g(K_2)$ or contract farming where the unit contract cost function is $w = h(x_2)$. Since we emphasize a case where the processor contract to buy the output of farmers we don't specify whether the contract is a marketing contract or a production contract, as defined in MacDonald and Korb (2011). Then we must have $x = g(K_2) + x_2$. We also require that the feedstock production function $g(K_2)$ is concave. The unit cost of total capital is given by $r = C(K_1 + K_2)$. Then the agribusiness' profit maximization problem is:

$$(1) \quad \max_{K_1,K_2,x_2} p(f(K_1,x_2+g(K_2))f(K_1,x_2+g(K_2)) - C(K_1+K_2)(K_1+K_2) - h(x_2)x_2$$

The agribusiness' decision variables are the amount of inputs to be used. The necessary first order condition gives:

$$(2) \quad p_q f_{K_1} q + p f_{K_1} - C_{K_1}(\cdot) - C(\cdot) = p f_{K_1}(\frac{1}{\varepsilon} + 1) - C'(\cdot) - C(\cdot) = 0,$$

$$(3) \quad p_q f_x g' q + p f_x g' - C_{K_2}(\cdot) - C(\cdot) = p f_x g'(\frac{1}{\varepsilon} + 1) - C'(\cdot) - C(\cdot) = 0,$$

$$(4) \quad p_q f_x q + p f_x - h' x_2 - h(x_2) = p f_x(\frac{1}{\varepsilon} + 1) - h' x_2 - h(x_2) = 0,$$

where ε denotes the demand elasticity. The first order conditions imply that the marginal value product of each input must equal to the marginal cost of the input. Here the marginal value product is calculated by the marginal revenue of final output multiplied by the marginal product of each input. Notice that monopoly firms will only operate in elastic portion of demand curve, then we may divide $1+\frac{1}{\varepsilon}$ on each side of the equalities. Therefore, from equation (3) and (4), we should find:

$$g' = \frac{f_{K_1}}{f_x} = \frac{(K_1+K_2)C'(\cdot)+C(\cdot)}{h'x_2+h(x_2)}. \tag{5}$$

Notice that the first equality implies that the marginal rate of technical substitution between capital and feedstock for final output must equal to the marginal product of self-produced input: $MRTS^1_{K,x} = MP^2_{K_2}$. The intuition is quite straightforward: if $MRTS^1_{K,x} > MP^2_{K_2}$, then the agribusiness could reach the same final output level by allocating one less unit of capital on producing the input and one more unit of capital on processing the final product. By doing so, the production plan requires $MRTS^1_{K,x}$ less units of total input x and producing $MP^2_{K_2}$ less units of inputs x_2. Therefore, the original final output production amount is still feasible but using less units of total input. Meanwhile, if $MRTS^1_{K,x} < MP^2_{K_2}$, by the same logic, it is efficient to allocate more capital on producing the input. Thus, when the production plan is optimized, we must have $MRTS^1_{K,x} = MP^2_{K_2}$.

The other equality in equation (5): $g' = \frac{(K_1+K_2)C'+C(K_1+K_2)}{h'x_2+h(x_2)}$ implies that, at optimal point, the marginal product of K_2 equal to the relative marginal cost of capital and of contract feedstock production. In particular, when marginal cost of capital and contracted feedstock production are constant, we have the following lemma:

Lemma 1 *The capital devoted to self-produced feedstock is fixed and independent of output demand.*

Proof See appendix A1 for the proof.

The illustration of the analysis is given in figure 2 when the unit cost of capital and contracted feedstock are constant. In this case, we know that equation (5) becomes $g'(K_2) = \frac{f_{K_1}}{f_x} = \frac{r}{w}$. Here the equation $g'(K_2) = \frac{r}{w}$ gives a unique solution of K_2^*. At the same time, the equality $MRTS^1_{K,x} = MP^2_{K_2}$ means that, at optimal, the tangent line of the isoquant curve of final output must also equal to $\frac{r}{w}$. Notice that, for final output, the condition of marginal revenue equaling to marginal cost of output determines the optimal final output level. And this condition specifies a unique isoquant curve that $MRTS^1_{K,x}$ should equal to $MP^2_{K_2}$. Then, all of the decision variables are determined.

The lemma provides a basis for the comparative statics analysis. Especially, we would like to see how capital allocation changes with respect to higher unit cost of capital. Moreover, we may explore special cases such as when the agribusiness has monopsony power over contract farming of feedstock production or over capital. The basic conclusions are given in the following corollary.

Corollary 2 *If the agribusiness has monopsony power over contract farming of feedstock production, then more capital will be allocated to self-production of feedstock. If the agribusiness has monopsony power over capital, then less capital will be allocated to self-production of feedstock.*

Proof See Appendix A2 for the proof.

The rationale behind the corollary is that when the agribusiness has monopsony power over contracted feedstock production, he/she may exert the monopsony power by using less contract farming as the unit cost of contracted feedstock is increasing. Then, in order to produce the deficit of feedstock, the agribusiness has to use more capital on producing the input. Similarly, when the agribusiness has monopsony power over capital, less capital will be used. Consequently, the amount of self-produced feedstock will be reduced and more feedstock has to be contracted-out.

The fundamental comparative statics exercise we are looking for is how would a agribusiness allocate capital when the capital is more expensive. This question is pertinent when a agribusiness faces credit constraint because when a agribusiness faces tighter credit constraint, the shadow cost of capital is larger which is equivalent to higher price of capital. Our first proposition gives an answer to this question under the scenario of constant marginal cost of capital and contracted feedstock.

Proposition 3 *As capital becomes more expensive, the agribusiness will produce less final output; less capital will be used on self-produced feedstock and facility building; more feedstock production will be contracted out.*

Proof See appendix A3 for the proof.

The proof of the proposition is illustrated in figure 3. The original optimal point is K_1^*, K_2^*, x^*. As r increases, we should have the marginal cost of final output being increased. As a consequence, marginal revenue equal to marginal cost condition implies that the final output should be decreased from isoquant Q^* to isoquant Q^{**}. At the same time, as r increases, the tangent line to the $g(K_2)$ function must become steeper. And the new tangent line should also be tangent to the new isoquant curve Q^{**}. As shown in the graph, we can easily see that K_1^*, K_2^*, x^* all decrease as r increases. Also notice that the proportion of x_1 in x is decreased. Therefore, the contracted portion of feedstock must increase.

The other comparative statics question we want to solve is that how agribusiness' input decisions will respond to shifts in output or input market parameter changes. Here, we look at two specific cases: changes in the demand elasticity of final output and in the supply elasticity of feedstock from contractees.

Proposition 4 *If the demand of the final product is less elastic, then less final product will be produced and the share of the in-house production is larger. If the supply of input from contractees is less elastic, then less final output will be produced and the share of in-house production is larger.*

Proof See appendix A4 for the proof.

The intuition behind Proposition 4 is that, when demand of the final product is less elastic, the marginal revenue curve becomes steeper, which makes it intersect the marginal cost curve at a lower production level. And the reduction in output production is achieved by reducing both input uses. But, we know that capital usage on feedstock does not change as demand side variables change, therefore, we must have increased share of in-house feedstock production. When the supply of input from contractees is less elastic, marginal cost curve is steeper. And the other arguments follow the similarly.

Production Uncertainty

Now, we consider the case of uncertainty in final output production. The underlying motivation is that a agribusiness may develop new technology in processing, and the uncertainty in the new technology makes the quantity of final output production uncertain. For instance, a biofuel refinery may consider using second generation feedstock to produce biofuel, but the processing technology, conversion rate in this case, is not mature. Let $q = \theta f(K_1, x)$ denote the final output, where θ is a random variable with mean $\hat{\theta}$ and variance σ^2. One possible interpretation of θ is the processing technology improvement. Let $\pi = p \cdot \theta f(K_1, x_2 + g(K_2) - r(K_1 + K_2) - wx_2$ denote the agribusiness' profit. And, using the framework by Sandmo (1971), the firm's problem is to maximize the utility from profit. i.e.,

$$\max_{K_1, K_2, x_2} EU[p \cdot \theta f(K_1, x_2 + g(K_2) - r(K_1 + K_2) - wx_2]$$

where we assume that $U'(\pi) > 0, U''(\pi) < 0$. We are most interested in how such uncertainty may cause different allocation of resources than the deterministic case. And we have the following proposition:

Proposition 5 *Under production uncertainties, expected final output production is less than certainty case; both capital used on processing and contracted feedstock level are less than certainty amount; the capital on feedstock production remains the same.*

Proof See appendix A5 for the proof.

Uncertainty in Contract Farming

As Acemoglu, Johnson, and Mitton (2009) introduced in their model, there is also uncertainty in contract farming: contractees may default the contract. Consider the case that agribusiness is uncertain about the level of feedstock can be received from farmers. Let θ be the proportion of x_2 that is finally received. The processor can expect to receive a proportion of $\hat{\theta}$. Let $\pi = p \cdot f(K_1, x_2 + g(K_2) - r(K_1 + K_2) - w\theta x_2$ and again, we assume that the agribusiness is maximizing the expected utility of profit.

$$(6) \qquad \max_{K_1, K_2, x_2} EU[p \cdot f(K_1, x_2 + g(K_2) - r(K_1 + K_2) - w\theta x_2],$$

where we again assume that $U'(\pi) > 0, U''(\pi) < 0$.

Proposition 6 *With uncertainty in contracted feedstock, expected final output production is less than certainty case; both capital used on processing and contracted feedstock level are less than certainty amount; the capital on feedstock production is more than certainty amount.*

Proof See appendix A6 for the proof.

The result of this proposition reflects the essence of the Sandmo model: uncertainty will lead to reduction in output production. In this particular case, the decrease in output production comes from the reduction in both of the input levels. Decrease in K_1 means that the agribusiness would build smaller processing facility, which in turn requires less total feedstock. However, the magnitude in the reduction of contracted feedstock is greater than the reduction of total feedstock being required. Consequently, the agribusiness has to use more capital on feedstock production to meet the deficit. One way to look at this proposition is that when a processor agribusiness has better information about contracted farmers, more output will be produced and more contract farming will be carried out. This result may be another possible way to explain the growth of contract farming over time: as the processors know more about the contractees, contract farming is more likely to happen.

To see the interaction between credit constraint and uncertainty, we would like to see the comparative statics of the input levels with respect to cost of capital under contract uncertainties. When the utility function of the agribusiness is decreasing absolute risk aversion, it is easy to see that higher cost of capital will have a direct effect and an indirect effect: the direct effect is that higher cost of capital will lower final output production and self-produced feedstock and increase contracted feedstock, as demonstrated in proposition 3; however, higher cost of capital also indirectly affect the capital allocation in that it decrease the wealth of the agribusiness and consequently makes the agribusiness to be less able to bear risks. As Sandmo (1971) suggests, the indirect effect will further lower the contracted feedstock production and makes the total effect on self-production of feedstock ambiguous.

Concluding Remarks

In this article, we investigate an agribusiness' decision, in both certainty and uncertainty cases, on capital allocation when it has the option of acquiring the feedstock through contracts and self-production. Our analytical model first establishes the fact that, in the certainty case, a monopoly's capital allocated on input production is independent of final output demand. And, at optimal, the marginal rate of technical substitution between input and capital used on output production must equal to the marginal product of capital on input production. Furthermore, our results show that when agribusiness have monopsony power over capital or contractees' supply of input is elastic, there will be more contract farming. When there is uncertainty in output production, expected final output will be less than certainty case and less capital will be allocated to self-production of the input. When there is uncertainty in contracted feedstock, expected final output will be less than certainty case and more capital will be allocated to self-production of the input.

The key proposition of the model is that contract farming is especially plausible when a agribusiness faces higher cost of capital or credit constraint. Our results can be empirically tested by and applied to many real world applications. For instance, our model could partly answer why Tyson Foods would use contracts to raise chickens– the core competence of Tyson Foods is in processing and marketing its final product and the feedstock requires huge amount of capital. Thus, it will allocate more capital on building processing facility and less on raising the chickens. Similarly, McDonald's may find it more profitable to franchise out its chain stores rather than run a store itself as developing another chain store is costly. Our model could also provide guidance for new industries. For example, a biofuel refinery in Brazil may face the capital allocation problem on refining facility and growing the sugarcane. Our model suggests that the refinery should use contract farming as a tool to deal with its credit constraint and, use more contract farming as the productivity of farmers becomes more certain.

It should be noted that the model may behave differently if we consider the dynamics version of the model, especially when the agribusiness enjoys a learning-by-doing effect when it develops the feedstock. In that case, the value of self-production of the feedstock may grow over time, then there may exist an optimal time to switch from self-production to contract farming, and we leave this as a direction for future research.

References

Acemoglu, D., S. Johnson, and T. Mitton. 2009. "Determinants of Vertical Integration: Financial Development and Contracting Costs." *Journal of Finance* 64:1251–1290.

Alexander, C., R. Goodhue, G. Rausser, et al. 2000. "Do quality incentives matter?" *Working Paper-Department of Agricultural and Resource Economics, University of California, Davis*, pp. .

Bierlen, R., L.D. Parsch, and B.L. Dixon. 1999. "How cropland contract type and term decisions are made: evidence from an Arkansas tenant survey." *The International Food and Agribusiness Management Review* 2:103–121.

Boehlje, M., L.F. Schrader, J. Royer, R. Rogers, et al. 1998. "The industrialization of agriculture: questions of coordination." *The industrialization of agriculture: vertical coordination in the US food system.*, pp. 3–26.

Cheung, S. 1969. *The theory of share tenancy.*. Chicago/London: Univ. Chicago Press.

Du, X., L. Lu, and D. Zilberman. 2013. "Contracting Farming in Biofuel Sector: A Survey." Working Paper, Department of Agricultural and Resource Economics, University of California, Berkeley.

Feder, G. 1977. "The impact of uncertainty in a class of objective functions." *Journal of Economic Theory* 16:504–512.

Franken, J.R., J.M. Pennings, and P. Garcia. 2009. "Do Transaction Costs and Risk Preferences Influence Marketing Arrangements in the Illinois Hog Industry?" *Journal of Agricultural and Resource Economics* 34:297–315.

Glover, D., J.v. Braun, E. Kennedy, et al. 1994. "Contract farming and commercialization of agriculture in developing countries." *Agricultural commercialization, economic development, and nutrition.*, pp. 166–175.

Goodhue, R.E. 2000. "Broiler Production Contracts as a Multi-Agent Problem: Common Risk, Incentives and Heterogeneity." *American Journal of Agricultural Economics* 82:pp.

606–622.

Hueth, B., and E. Ligon. 2002. “Estimation of an efficient tomato contract.” *European Review of Agricultural Economics* 29:237–253.

Jaynes, G.D. 1982. *Economic Theory and Land Tenure, in H. Binswanger, and M. Rosenzweig, eds., Rural Labor Markets in Asia: Contractual Arrangements, Employment and Wages*, New Haven: Yale University Press.

Katchova, A.L., and M.J. Miranda. 2004. “Two-step econometric estimation of farm characteristics affecting marketing contract decisions.” *American Journal of Agricultural Economics* 86:88–102.

Knoeber, C., and W. Thurman. 1995. ““Don’t Count Your Chickens...”: Risk and Risk Shifting in the Broiler Industry.” *American Journal of Agricultural Economics* 77:486–496.

MacDonald, J.M., and P. Korb. 2011. “Agricultural Contracting Update: Contracts in 2008.” Working paper, EIB-72. U.S. Dept. of Agriculture, Econ. Res. Serv.

Pennings, J.M., and B. Wansink. 2004. “Channel contract behavior: The role of risk attitudes, risk perceptions, and channel members’ market structures.” *The journal of business* 77:697–724.

Rubin, P. 1978. “Theory of the Firm and the Structure of the Franchise Contract, The.” *JL & Econ.* 21:223.

Ruf, F. 2009. “Developing Smallholder Rubber Production.” *Ex Post. Agence Française de Développement*, pp. .

Sandmo, A. 1971. “On the theory of the competitive firm under price uncertainty.” *The American Economic Review* 61:65–73.

Figures

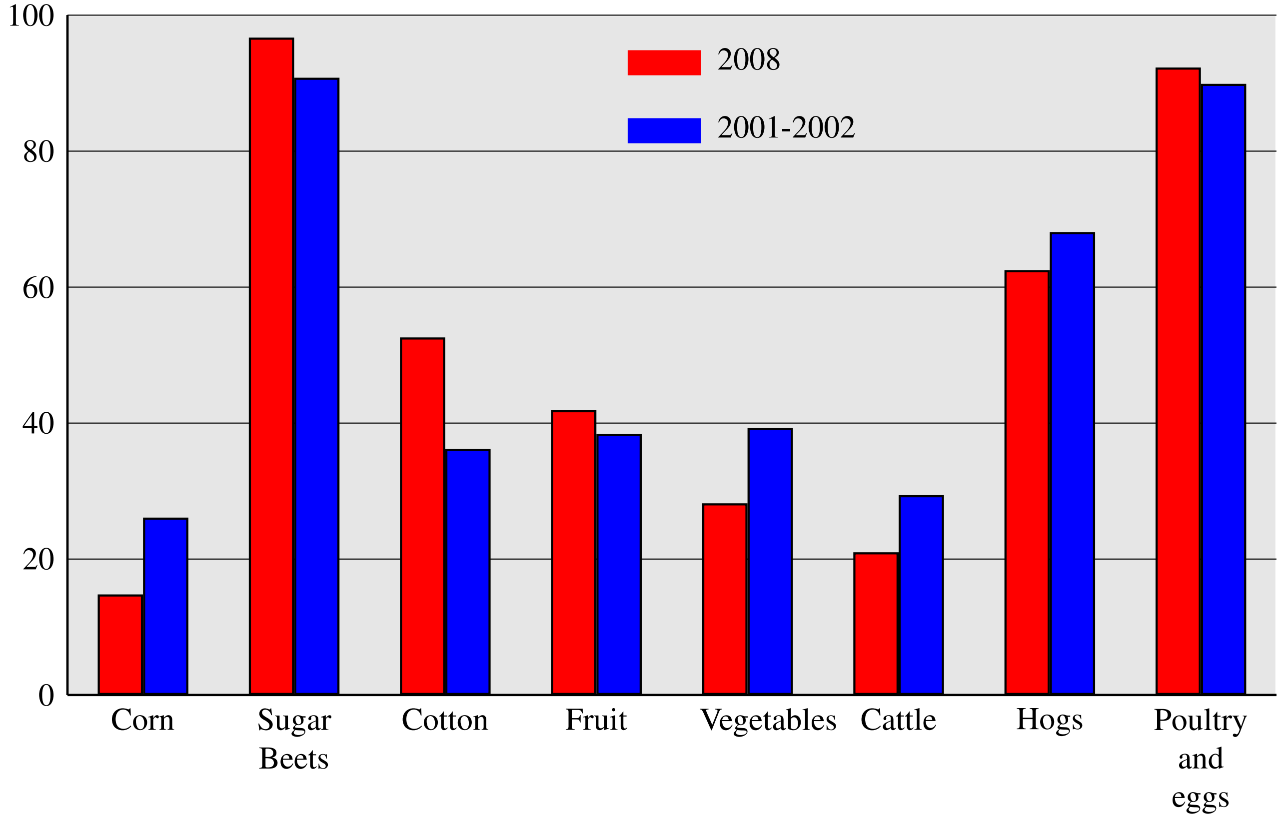

Figure 1. Share of commodity production under contract in 2001 and 2008

Source: MacDonald and Korb (2011)

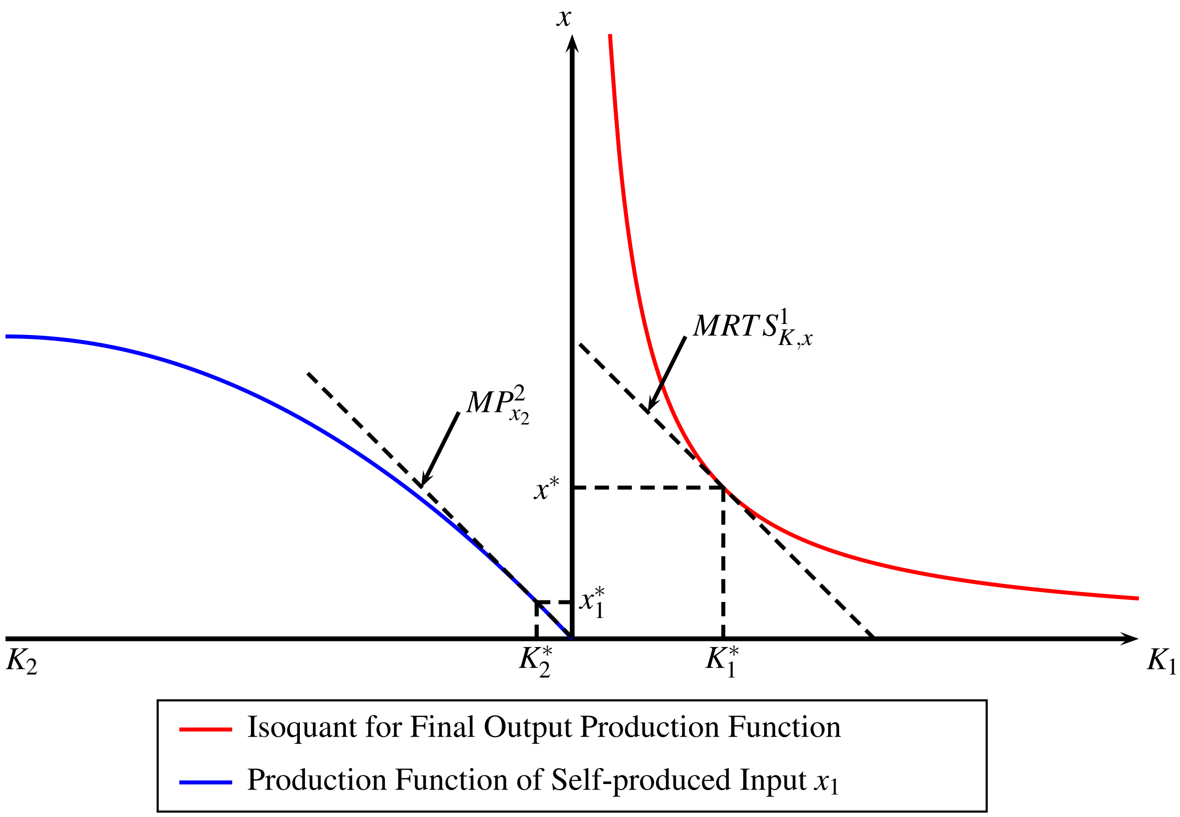

Figure 2. Illustration of capital allocation

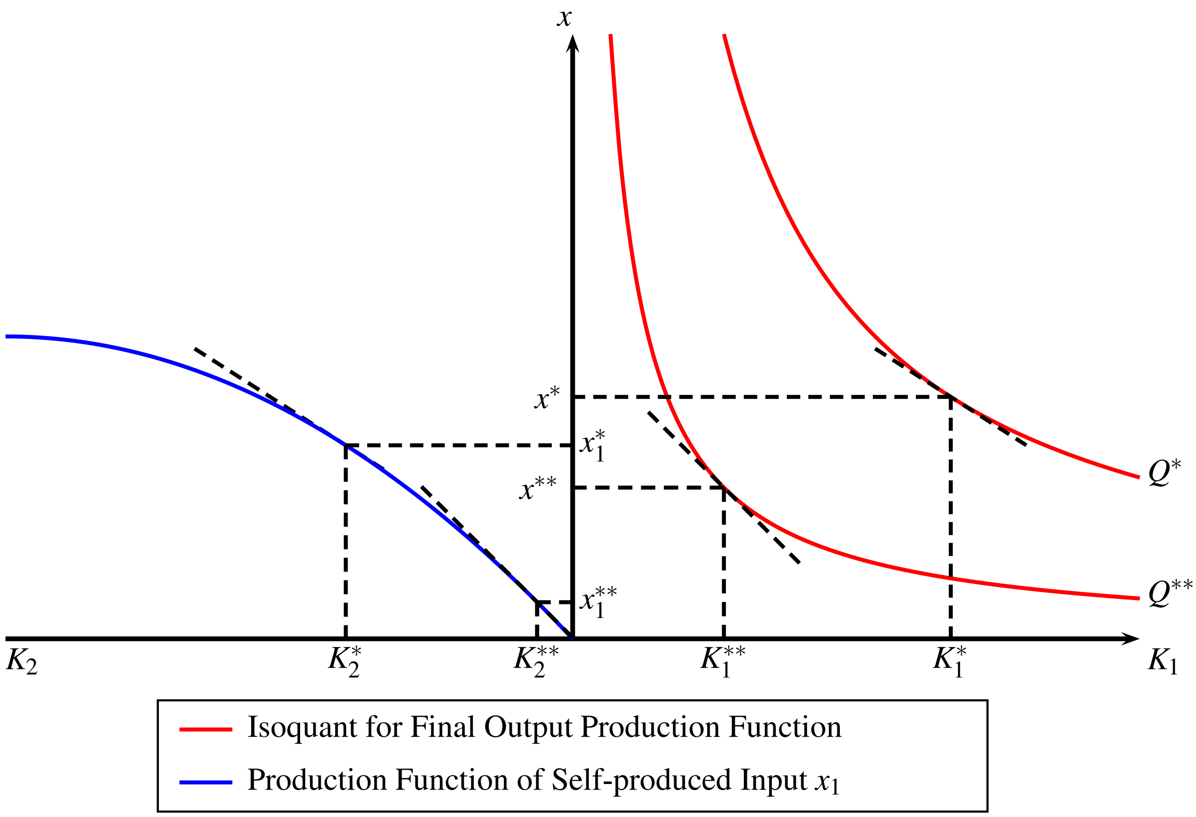

Figure 3. Illustration of capital allocation when capital is more expensive

AJAE Appendix for The Economics of Contract Farming: A Credit and Investment Perspective

A1. Proof for Lemma 1

Proof We have established earlier that

$$(7) \qquad g' = \frac{(K_1+K_2)C'(\cdot)+C(\cdot)}{h'(x_2)x_2+h(x_2)}.$$

Thus, when marginal cost of capital and contracted feedstock production are constant, we have $C' = h' = 0$. And we have $g' = \frac{r}{w}$. Therefore, the optimal K_2 is given by:

$$(8) \qquad K_2^* = (g')^{-1}(\frac{r}{w}).$$

Clearly, demand side variables are not involved in optimal K_2. ∎

A2. Proof for Corollary 2

Proof Suppose the agribusiness has monopsony power over contract farming of feedstock production, then equation (5) implies: $g'(K_2) = \frac{r}{w+h'x_2}$. Let K_2^{**} be the optimal capital under this scenario. And we have:

$$(9) \qquad g'(K_2^{**}) = \frac{r}{h'x_2+w} < \frac{r}{w} = g'(K_2^*).$$

The inequality holds because $h'(x_2) > 0$. Notice that $g'' < 0$ and $g'(K_2^{**}) < g'(K_2^*)$, we must have $K_2^{**} > K_2^*$. The second statement follows from the same proof. ∎

A3. Proof for Proposition 3

Proof We first derive the Hessian for the problem. Let $a = p'q+p$ and $b = a' = p''q+2p'$. Since $a = p(1+\frac{1}{\varepsilon})$, we must have $a > 0$. Also notice that $b < 0$ by the assumption of

$p'' < 0$. Then the Hessian for the problem can be written as:

$$(10) \quad \boldsymbol{H} = \begin{bmatrix} af_{K_1K_1} + bf_{K_1}^2 & af_{K_1x}g' + bf_{K_1}f_xg' & af_{K_1x} + bf_{K_1}f_x \\ af_{K_1x}g' + bf_{K_1}f_xg' & af_{xx}(g')^2 + bf_x^2(g')^2 + af_xg'' & af_{xx}g' + bf_x^2g' \\ af_{K_1x} + bf_{K_1}f_x & af_{xx}g' + bf_x^2g' & af_{xx} + bf_x^2 \end{bmatrix}.$$

Since we have a profit maximization problem, the second order condition requires that $\boldsymbol{H}$ is negative definite. Therefore, we have $|\boldsymbol{H}| < 0$. By total differentiation, we can see that:

$$(11) \quad \boldsymbol{H} \begin{bmatrix} \frac{dK_1^*}{dr} \\ \frac{dK_2^*}{dr} \\ \frac{dx_2^*}{dr} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$$

We use $\boldsymbol{H}_{ij}$ to denote the submatrix of $\boldsymbol{H}$ that eliminates the ith row and jth column of $\boldsymbol{H}$. By Cramer's rule, we find that:

$$(12) \quad \frac{dK_1^*}{dr} = \frac{|\boldsymbol{H}_{11}| - |\boldsymbol{H}_{21}|}{|\boldsymbol{H}|}.$$

Since $\boldsymbol{H}_{11}$ is a principal minor of $\boldsymbol{H}$, and $\boldsymbol{H}$ is a negative definite matrix, we must have $|\boldsymbol{H}_{11}| > 0$. Moreover, it can be easily verified that $|\boldsymbol{H}_{21}| = 0$ (the second column of $\boldsymbol{H}_{21}$ is the first column multiplying g'). Therefore, we have:

$$(13) \quad \frac{dK_1^*}{dr} = \frac{|\boldsymbol{H}_{11}| - |\boldsymbol{H}_{21}|}{|\boldsymbol{H}|} < 0.$$

Again, by Cramer's rule, we get:

$$(14) \quad \frac{dK_2^*}{dr} = \frac{-|\boldsymbol{H}_{12}| + |\boldsymbol{H}_{22}|}{|\boldsymbol{H}|} < 0,$$

which comes from the fact that $|\boldsymbol{H}_{22}| > 0$ as $\boldsymbol{H}_{22}$ is another principal minor of $\boldsymbol{H}$, and the fact that $|\boldsymbol{H}_{12}| = 0$ (the first row of $\boldsymbol{H}_{12}$ is the second row multiplying g').

To determine the sign of $\frac{dx_2^*}{dr}$, first notice that $\frac{dx_2^*}{dr} = \frac{|\boldsymbol{H}_{13}| - |\boldsymbol{H}_{23}|}{|\boldsymbol{H}|}$. Use the fact that $|\boldsymbol{H}_{22}| = |\boldsymbol{H}_{23}|g'$, and $|\boldsymbol{H}_{22}| > 0$, we have $|\boldsymbol{H}_{23}| > 0$. In order to determine the sign of $|\boldsymbol{H}_{13}|$, we explicitly write down the expression of $|\boldsymbol{H}_{13}|$:

$$(15) \quad |\boldsymbol{H}_{13}| = -af_x g''(af_{K_1x} + bf_{K_1}f_x).$$

Once we can show that $af_{K_1x} + bf_{K_1}f_x < 0$, then it follows immediately that $\frac{dx_2^*}{dr} > 0$. To show that $af_{K_1x} + bf_{K_1}f_x < 0$, we first notice that $|\boldsymbol{H}_{11}| > 0$ as it is a principal minor of matrix $|\boldsymbol{H}| > 0$. Here $|\boldsymbol{H}_{11}| = af_x g''(af_{xx} + bf_x^2)$. Then, in order for $|\boldsymbol{H}_{11}|$ to be positive, we must have:

$$(16) \quad af_{xx} + bf_x^2 < 0.$$

Meanwhile, from $|\boldsymbol{H}_{22}| > 0$, we must have $(af_{xx} + bf_x^2)(af_{K_1K_1} + bf_{K_1}^2) > 0$, and thus we have:

$$(17) \quad af_{K_1K_1} + bf_{K_1}^2 < 0.$$

Using equations (16) and (17), we get: $f_x > \sqrt{-\frac{af_{xx}}{b}}$, $f_{K_1} > \sqrt{-\frac{af_{K_1K_1}}{b}}$. Now, we get:

$$\begin{aligned}(18) \quad af_{K_1x} + bf_{K_1}f_x &< af_{K_1x} + b\sqrt{\frac{a^2 f_{K_1K_1} f_{xx}}{b^2}} \\ &< af_{K_1x} - \sqrt{a^2 f_{xx} f_{K_1K_1}} < \sqrt{a^2 f_{xx} f_{K_1K_1}} - \sqrt{a^2 f_{xx} f_{K_1K_1}} = 0\end{aligned}$$

Therefore, as explained earlier, we must have $\frac{dx_2^*}{dr} > 0$. To do the comparative statics for $\frac{dq^*}{dr}$, we first note that

$$(19) \quad \frac{dq^*}{dr} = f_{K_1}\frac{dK_1^*}{dr} + f_x g' \frac{dK_2^*}{dr} + f_x \frac{dx_2^*}{dr},$$

which can be written as:

$$(20) \quad \frac{dq^*}{dr} = \frac{f_{K_1}|\boldsymbol{H}_{11}| + f_x g'|\boldsymbol{H}_{22}| + f_x(|\boldsymbol{H}_{13}| - |\boldsymbol{H}_{23}|)}{|\boldsymbol{H}|}.$$

It can be verified that $|\boldsymbol{H}_{22}| = |\boldsymbol{H}_{23}|g'$, $|\boldsymbol{H}_{11}| = af_x g''(af_{xx} + bf_x^2)$, and $|\boldsymbol{H}_{13}| = -af_x g''(af_{K_1x} + bf_{K_1}f_x)$. Then, we have:

$$(21) \quad \frac{dq^*}{dr} = \frac{a^2 f_x g''(f_{xx} f_{K_1} - f_{K_1x} f_x)}{|\boldsymbol{H}|} < 0,$$

as $f_{xx} < 0$, $f_{K_1 x} > 0$, $g'' < 0$ and $|\boldsymbol{H}| < 0$. ∎

A4. Proof for Proposition 4

Proof In this case, total differentiation with respect to $K_1, K_2, x_2, \varepsilon$, we get the following matrix form equality:

$$(22)\quad \boldsymbol{H}\begin{bmatrix} \frac{dK_1^*}{d\varepsilon} \\ \frac{dK_2^*}{d\varepsilon} \\ \frac{dx_2^*}{d\varepsilon} \end{bmatrix} = \varepsilon^{-2}\begin{bmatrix} pf_{K_1} \\ pf_x g' \\ pf_x \end{bmatrix},$$

where $\boldsymbol{H}$ is the Hessian of the maximization problem as defined in (10). And we use $\boldsymbol{H}_i$ to denote the matrix that replaces the ith column of $\boldsymbol{H}$ by $\varepsilon^{-2}\left[pf_{K_1}\;\; pf_x g'\;\; pf_x\right]^T$. Using Cramer's rule, we have:

$$(23)\quad \frac{dK_1^*}{d\varepsilon} = \frac{|\boldsymbol{H}_1|}{|\boldsymbol{H}|},$$

where $|\boldsymbol{H}_1| = \varepsilon^{-2}\begin{vmatrix} pf_{K_1} & af_{K_1x}g' + bf_{K_1}f_x g' & af_{K_1x} + bf_{K_1}f_x \\ pf_x g' & af_{xx}(g')^2 + bf_x^2(g')^2 + af_x g'' & af_{xx}g' + bf_x^2 g' \\ pf_x & af_{xx}g' + bf_x^2 g' & af_{xx} + bf_x^2 \end{vmatrix}$. Notice that $|\boldsymbol{H}_1| = \varepsilon^{-2}\begin{vmatrix} pf_{K_1} & af_{K_1x}g' + bf_{K_1}f_x g' & af_{K_1x} + bf_{K_1}f_x \\ 0 & af_x g'' & 0 \\ pf_x & af_{xx}g' + bf_x^2 g' & af_{xx} + bf_x^2 \end{vmatrix} = a^2 pf_x g'' \varepsilon^{-2}(f_{K_1}f_{xx} - f_x f_{K_1x}) > 0$, we have $\frac{dK_1^*}{d\varepsilon} = \frac{|\boldsymbol{H}_1|}{|\boldsymbol{H}|} < 0$.

Since K_2^* is not affected by demand side variables, we have $\frac{dK_2^*}{d\varepsilon} = 0$. We can verify this by checking $|\boldsymbol{H}_2| = 0$ (this is true because the second row of $|\boldsymbol{H}_2|$ is a multiple of the third row.

Again, Using Cramer's rule, we have:

$$(24)\quad \frac{dx_2^*}{d\varepsilon} = \frac{|\boldsymbol{H}_3|}{|\boldsymbol{H}|},$$

where $|\boldsymbol{H}_3| = \varepsilon^{-2} \begin{vmatrix} af_{K_1K_1}+bf_{K_1}^2 & af_{K_1x}g'+bf_{K_1}f_xg' & pf_{K_1} \\ af_{K_1x}g'+bf_{K_1}f_xg' & af_{xx}(g')^2+bf_x^2(g')^2+af_xg'' & pf_xg' \\ af_{K_1x}+bf_{K_1}f_x & af_{xx}g'+bf_x^2g' & pf_x \end{vmatrix} =$ $\varepsilon^{-2} \begin{vmatrix} af_{K_1K_1}+bf_{K_1}^2 & af_{K_1x}g'+bf_{K_1}f_xg' & pf_{K_1} \\ 0 & af_xg'' & 0 \\ af_{K_1x}+bf_{K_1}f_x & af_{xx}g'+bf_x^2g' & pf_x \end{vmatrix}$. Then we can see that

$$(25) \quad |\boldsymbol{H}_3| = a^2pf_xg''\varepsilon^{-2}(f_xf_{K_1K_1}-f_{K_1}f_{K_1x}) > 0.$$

Therefore, we have $\frac{dx_2^*}{d\varepsilon} = \frac{|\boldsymbol{H}_3|}{|\boldsymbol{H}|} < 0$. Since K_2^* does not change, we know in-house feedstock production does not change. But x_2^* have decreased, we know that the share of in-house feedstock production is larger.

Since

$$(26) \quad \frac{dq^*}{d\varepsilon} = f_{K_1}\frac{dK_1^*}{d\varepsilon} + f_xg'\frac{dK_2^*}{d\varepsilon} + f_x\frac{dx_2^*}{d\varepsilon},$$

and $\frac{dK_1^*}{d\varepsilon} < 0, \frac{dK_2^*}{d\varepsilon} = 0, \frac{dx_2^*}{d\varepsilon} < 0$, we must have

$$(27) \quad \frac{dq^*}{d\varepsilon} < 0.$$

When supply of input from contractees is less elastic, we write the equation (4) in the form of: $pf_x(1+\frac{1}{\varepsilon}) = w(1+\frac{1}{\eta})$. And we rewrite equation (5) as $g' = \frac{r}{w(1+\frac{1}{\eta})}$. As η decreases, we should observe that g' also decreases and K_2^* increases. Other comparative static results follow from the same procedure. ∎

A5. Proof for Proposition 5

Proof The first order condition w.r.t K_1 is:

$$E\{u'(\pi)[p\theta f_{K_1} - r]\} = 0$$

Then

$$E\{u'(\pi)[p\theta f_{K_1} - p\hat{\theta} f_{K_1}]\} = E\{u'(\pi)[p\hat{\theta} f_{K_1} - r]\}$$

Notice that

$$\pi = E\pi + (\theta - \hat{\theta})pf(\cdot)$$

Thus, for all $\theta > \hat{\theta}$, we must have $u'(\pi) < u'(E\pi)$. And consequently,

$$u'(\pi)(\theta - \hat{\theta}) > u'(E\pi)(\theta - \hat{\theta}), \forall \theta$$

Therefore,

$$E\{u'(\pi)[p\theta f_{K_1} - p\hat{\theta} f_{K_1}]\} = pf_{K_1} E\{u'(\pi)(\theta - \hat{\theta})\} > pf_{K_1} E\{u'(E\pi)(\theta - \hat{\theta})\} = 0$$

But, this implies

$$p\hat{\theta} f_{K_1} > r.$$

Similarly, we can also show that

$$p\hat{\theta} f_x > w.$$

These two equations imply that, under uncertainty, both f_{K_1} and f_x are higher than the certainty case.

Using the lemmas in Feder (1977), we have the following equality:

$$\frac{f_{K_1}}{f_x} = g'(K_2) = \frac{r}{w}$$

Therefore, the capital used on feedstock production is the same as in certainty case. Now, take total derivative for f_{K_1} and f_x, and realizing that K_2 doesn't change, we get:

$$df_{K_1} = f_{K_1K_1} dK_1 + f_{K_1x} dx_2$$

$$df_x = f_{xK_1} dK_1 + f_{xx} dx_2$$

It can be easily verified that in order for f_{K_1} and f_x to be both increasing, the only possibility is $dK_1, dx_2 < 0$, which means both capital used on processing and contracted feedstock level are less than certainty amount. Also notice that:

$$dq = f_{K_1} dK_1 + f_x dx_2 < 0$$

which means the final output level is less than certainty case as well. $\blacksquare$

A6. Proof for Proposition 6

Proof The first order condition yields:

$$p f_{K_1} = p f_x g' = r$$

Meanwhile,

$$E\{u'(\pi)[p f_x - w\theta]\} = 0$$

Then we get:

$$E\{u'(\pi)[p f_x - w\hat{\theta}]\} = E\{u'(\pi) w(\theta - \hat{\theta})\}$$

Since

$$\pi = E\pi - w(\theta - \hat{\theta}) x_2$$

We have, for all $\theta > \hat{\theta}$, $u'(\pi) > u'(E\pi)$. Thus,

$$u'(\pi)(\theta - \hat{\theta}) > u'(E\pi)(\theta - \hat{\theta})$$

. And consequently, $p f_x > w\hat{\theta}$. In summary, comparing to certainty case, we have larger f_x and f_{K_1} remains unchanged. Therefore, from $\frac{f_{K_1}}{f_x} = g'(K_2)$, we know that $g'(K_2)$ must have been decreased, which means K_2 is larger than the certainty case. Now, we take total derivative for f_x, f_{K_1}:

$$d f_{K_1} = f_{K_1 K_1} dK_1 + f_{K_1 x} dx = 0$$

And

$$df_x = f_{xK_1}dK_1 + f_{xx}dx$$

From the first equation, we get:

$$\frac{dK_1}{dx} = -\frac{f_{K_1x}}{f_{K_1K_1}}$$

Thus,

$$\frac{df_x}{dx} = \frac{f_{xx}f_{K_1K_1} - f_{K_1x}^2}{f_{K_1K_1}} < 0$$

. Then, in order for f_x to be increasing, the total feedstock amount must have been decreased. Also notice that in order for $df_{K_1} = f_{K_1K_1}dK_1 + f_{K_1x}dx = 0$ to hold, we cannot have $dK_1 > 0$. Then, K_1 must have been decreased. Since the total feedstock level and K_1 are both decreasing, from the equation:

$$dq = f_{K_1}dK_1 + f_xdx$$

we know that $dq < 0$, which means the total output is less than the certainty case.