Global Gains from Reduction of Trade Costs*

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This draft: September 26, 2013

Abstract

This paper derives a measure of change in global welfare, and then develops a simple equation for computing the global welfare effect of reduction of bilateral trade costs, such as shipping costs or the costs of administrative barriers to trade. The equation is applicable to a broad class of perfect-competition and monopolistic competition models and settings. Balanced trade is not required. We prove that the underlying mechanism driving the result is the envelope theorem. The equation is applied to calculate the global welfare impact of reduction of international shipping costs in the last fifty years, as well as the elasticity of global welfare with respect to trade costs in 2010. We find the estimates to be reasonable and consistent with other estimates in the literature.

Keywords: global welfare, trade cost, gains from trade

JEL Classification codes: F10, F12, F13

*An earlier version of this paper was circulated under the title "Global Gains from Trade Liberalization" (CESifo working paper no. 3775, March 2012). We would like to thank Davin Chor, Gene Grossman, Stephen Yeaple, Tim Kehoe, Jaime Ventura, Jonathan Vogel, Hamid Sabourian and seminar and conference participants in University of Melbourne, University of New South Wales, University of Hong Kong, City University of Hong Kong, HKUST, Shanghai University of Finance and Economics, National University of Singapore, Singapore Management University and Asia-Pacific Trade Seminars for their helpful comments. Naturally, all errors remain ours. The work in this paper has been supported by the Research Grants Council of Hong Kong, China (General Research Funds Project no. 642210).

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1 Introduction

One major theme of international trade economics is the gains from trade. There is a presumption in all trade models that the more integrated is the world, the larger are the gains from trade. Therefore, the lower are trade barriers, the greater global welfare should be. But, how much do trade barriers matter to the world quantitatively? For example, what is global benefit in monetary terms of a one percent reduction in all bilateral trade costs worldwide? How sensitive is the answer to this question to the trade model being used? Answers to these questions would help us evaluate the global benefits of improvement of transportation technology or reduction of administrative barriers to global trade. Understanding the global welfare impact of such changes is important as a larger pie means that there is more room to make every country better off by proper side payments. In fact, international organizations recognize the important of reduction of administrative trade barriers and shipping time to global welfare. OECD, World Bank and World Economic Forum found that trade facilitation could yield enormous economic gains to the world.[1] This paper derives a simple equation for computing the global welfare effect of reduction of bilateral trade costs, such as shipping costs or the costs of administrative barriers to trade, between multiple trading partners. The equation is applicable to a broad class of models and settings. It is then applied to calculate the global welfare impact of reduction of international shipping costs in the last fifty years, as well as the elasticity of global welfare with respect to bilateral trade costs in 2010. We find the estimates to be reasonable and consistent with other estimates in the literature.[2]

Here is a brief description of what we have done. First, we derive a measure of the percentage change in global welfare based on the concept of compensating variation and Kaldor-Hicks' concept of welfare change for a group. We show that the expression $\sum_{i=1}^{n} \frac{E_i}{Y^w} \widehat{U}_i$ (the expenditure-share-weighted average percentage change of country welfare) is a reasonable measure of the percentage change in global welfare, where E_i is the aggregate expenditure of country i, Y^w is the global GDP of the n countries in the world, and $\widehat{U}_i$ is a small percentage change in welfare of country i. As far as we know, we are the first to rigorously justify the use of this expression as a measure of percentage change in global welfare.

Then, we start our analysis with the standard trade model, i.e. a multiple-country, multiple-good, multiple-factor model with constant returns to scale production function, perfect competition and fixed extensive margins of trade (hereinafter abbreviated as PC-FEM model). (Examples include Heckscher-Ohlin model and Armington (1969) model.) We treat this as the benchmark case. We derive a simple equation for computing the total global welfare effect of small reduction in bilateral trade costs for

[1] See for example, World Economic Forum, "*Enabling Trade, Valuing Opportunities,*" 2013, and OECD, Working Party of the Trade Committee, "Quantitative Assessment of the Benefits of Trade Facilitation," TD/TC/WP(2003)31 (19 September 2003).

[2] In 2013, the OECD estimated that reducing global trade costs by 1% would increase worldwide income by more than USD 40 billion, whereas our formula yields USD 144 billion worth of global gains for 1% uniform reduction of all bilateral iceberg trade costs.

multiple pairs of trading partners. We find that the percentage change in global welfare is given by

$$-\sum_{j=1}^{n}\sum_{i=1}^{n}\frac{X_{ij}}{Y^w}\widehat{\tau}_{ij} \ , \tag{1}$$

where X_{ij} is the total value of exports from country i to country j, and $\widehat{\tau}_{ij}$ is a small percentage change in the iceberg cost, τ_{ij}, of exporting from i to j. This turns out to be just the total saving in trade costs divided by global GDP, keeping the values of all bilateral trades unchanged. In other words, the only first order effect is the direct effect of the changes in trade costs. The indirect effects, of changes in allocation of resources to different goods induced by the changes in trade costs, are second order, and therefore do not matter. This is because allocation of resources across outputs and exports were already optimally chosen before the changes in trade costs take place. The equation is independent of changes in terms of trade because of the conservation of physical quantities of goods and it is independent of changes in trade imbalances because they sum to zero. We prove that the envelope theorem is the underlying mechanism of this result. What is remarkable about this equation is that (a) no restriction on preferences is required (except constant returns to scale), and (b) balanced trade is not required for any country.

The use of the envelope theorem to predict market outcome depends on the validity of the first fundamental theorem of welfare economics, which states that when there are no externalities, incomplete markets, incomplete information, or monopoly power, the competitive equilibrium is efficient. This means that the competitive equilibrium yields the same outcome as the solution of a global social planner's global-expenditure maximization problem with the market prices treated as given. We prove that the envelope theorem is applicable in a setting with perfect competition, constant returns to scale, and fixed extensive margins of trade (hereinafter abbreviated as PC-FEM model). The question is, Is the envelope theorem applicable to other models or settings? Thus, our next step is to systematically investigate under what conditions the same equation applies to other trade models and settings.

First, we investigate a perfect-competition model with variable extensive margins of trade (hereinafter abbreviated as PC-VEM model). We find that, under perfect competition, the equation is valid as long as we assume complete specialization (i.e. for each country, each good is supplied by only one country, including the domestic country). Thus, the equation can be applied to a broad class of perfect competition models, including Dornbusch-Fisher-Samuelson (1977) (hereinafter DFS1977) and Dornbusch-Fisher-Samuelson (1980) (hereinafter DFS1980), and Eaton and Kortum (2002) (hereinafter EK2002). Furthermore, we prove that it is applicable to the extensions of these models to multiple sectors, multiple factors, multiple-stage production, existence of tradable intermediate goods, and existence of a non-traded good sector. Again, balanced trade is not required.

Second, we investigate models of imperfect competition. We find that the equation can be applied to models of Krugman (1980) (hereinafter K1980) and Melitz (2003) with Pareto distribution of firm productivity (hereinafter MwP). We further prove that the equation is valid for the extensions of these models to multiple sectors, multiple factors, and existence of a non-traded good sector. Again, balanced trade is not required.

At first blush, there is no reason why the envelope theorem, and therefore expression (1), should be applicable to other models where the extensive margins of trade may not be fixed or firms have monopoly power. Nonetheless, it turns out that expression (1) can be applied to many other settings when certain commonly adopted assumptions are imposed. Basically, these assumptions lead to the cancellation of the effects of adjustments in extensive margins of trade on global welfare. For example, the assumption of complete specialization in the PC-VEM model renders the effects of changes in extensive margins of trade second order in all countries. For the monopolistic competition model of K1980, CES preferences lead to constant markup, which implies that relative prices and quantities are not distorted compared with perfect competition. It also implies that the number of entrants is constant, which in turn implies that the extensive margins of trade are constant, as firms are homogeneous, and they all export to all countries. For the Melitz (2003) model, CES preferences ensure that relative prices and quantities are not distorted as in K1980. If we make the assumption of Pareto firm productivity distribution, then it ensures not only that the number of entrants is constant, but also that the effects of the adjustments in extensive margins of trade induced by changes in cutoff productivities of exporting in different countries cancel each other from the global welfare point of view. As a result, the conclusion of the first fundamental theorem of welfare economics continues to hold despite the existence of monopoly power and heterogeneous firms. Thus, the envelope theorem also predicts the global welfare change of the decentralized markets in these models.

Next, we show that expression (1) continues to hold for the extensions to multi-stage production, multi-sector and multi-factor settings. In some sense, this result is not so surprising. In these extensions, there continues to be no distortions to relative prices and quantities, and the effects of adjustments in extensive margins of trade continues to offset each other from the global welfare point of view. Therefore, the conclusion of the envelope theorem continues to apply. The envelope theorem implies that the global welfare gains from small reduction of trade costs arise entirely from the savings in trade costs when trade flows are assumed to be unchanged. Once this principle is applicable to the one-sector, one-factor model, it is natural that it works for the multi-stage production, multi-sector and multi-factor extensions, as the savings in trade costs keeping bilateral trade flows unchanged would be the same regardless of whether the setting has multi-stage production, multi-sector and multi-factor.

Our work is inspired by Arkolakis, Costinot & Rodriguez-Clare (2012) (hereinafter abbreviated as ACR). They show that for a class of trade models which include Armington (1969), Krugman (1980), Melitz (2003) with Pareto distribution of firm productivity, and Eaton and Kortum (2002), *a country's gains from trade* are always given by the same formula and is determined by only two sufficient statistics: (i) the share of expenditure on domestic goods, and (ii) the trade elasticity. This is no doubt a significant discovery. It helps us to better understand the properties of this class of models. However, there are some limitations to their equation. First, it no longer holds when trade is unbalanced. Second, it no longer holds when extended to multi-sector setting.[3] Third, the elasticity of trade may be hard to estimate.[4] The focus of our analysis is different from that of ACR. In contrast to ACR, our goal is to calculate the *global welfare impact induced by changes in bilateral trade costs*. Our result not only applies to a broader set of models and settings but also does not require the estimation of trade elasticity nor require trade to be balanced.[5]

There are a couple of reasons why our equation applies more broadly than does ACR's.[6] As trade costs are reduced, there are effects that affect the welfare of individual countries that do not affect the welfare of the world as a whole. There are two important effects of this kind. First, the changes in trade balances sum to zero for the world as a whole but the change in trade balance for an individual country can be non-zero. This explains why our equation can allow for trade imbalances whereas ACR cannot. Second, in the multi-sectoral setting, the number of entrants in each sector changes because of the intersectoral resource allocation effect. The welfare impact of the entrants effects on country i originating from different sectors of the same country i do not offset each other. However, the global welfare impact of the entrants effects originating from different sectors of the same country i offset each other. This explains why our equation applies to multi-sectoral setting whereas ACR's does not.

Our work is also partly inspired by Atkeson and Burstein (2010) (hereinafter abbreviated as AB), who prove that the details of firms' responses are of secondary importance to the estimation of the welfare impact of trade costs reduction for an individual country. Under symmetry, they find that though changes in trade costs can have a substantial impact on heterogeneous firms' exit, export, and process innovation decisions, the impact of these changes on a country's welfare largely offset each other. In the end, only the "direct effect" of trade cost reduction matters. They derive their result using a symmetric two-country dynamic model with product and process innovation and tradable intermediate goods. The market structure for intermediate goods in each country is monopolistically competitive. The model is reminiscent of the "lab equipment model" of Romer and Rivera-Batiz (1991). In their appendix, they derive an equation for change in global welfare as a function of trade flows and changes in trade costs for the two-country case with asymmetric countries. It resembles our equation in the two-country case but is not the same. Our paper follows a similar line of thinking as AB, but our focus is very different. Whereas AB focus on proving that the individual and global welfare gains from changes in trade costs depend only on the direct effect based on a particular two-country model, we focus on establishing a simple general formula for the global welfare gains induced by changes in bilateral trade costs that can be applied to as many models and settings as we can find. We carry out

[3] See, for example, Balistreri, Hillberry and Rutherford (2011).

[4] The trade elasticity can be hard to pin down. See, for example, Eaton and Kortum (2002) and Simonovska and Waugh (2013).

[5] Melitz and Redding (2013) recently show that if a K1980 model has the same deep parameters as a Melitz (2003) model, and aggregate firm productivity is the same in the two models, then they cannot simultaneously have the same expenditure share on domestic goods and the same trade elasticity. Nonetheless, our result is not contradictory to theirs.

[6] ACR's equation can be adapted to evaluate the welfare impact of changes in trade costs on an individual country.

rigorous multi-country analysis and careful discussion of how different assumptions play their roles to make the main result hold in different models and settings. We start with the multi-country standard trade model and then progressively extend to other models and settings. Most importantly, we prove that the underlying mechanism for the cancellation of the indirect effects is the envelope theorem. This important insight has not been clearly explained, much less proven, by other papers in the literature, including AB. The standard trade model turns out to be the most important trade model for generating insights in our analysis.

Burstein and Cravino (2012) (hereinafter abbreviated as BC) find that, with balanced trade in each country, changes in world real GDP in response to changes in variable trade costs coincide, up to a first order approximation, with changes in world theoretical consumption. Like us, they associate changes in world real GDP with two set of sufficient statistics, namely, changes in bilateral variable trade costs and export shares of continuing exporting producers. This corroborates our finding, for if our expression (1) is sufficiently general, we should expect that similar result would emerge in many different models and settings. Nonetheless, because the focus of BC is not to establish a general formula for calculating global welfare change induced by changes in bilateral trade costs, they do not attempt to explain the underlying mechanism of the result. For the same reason, though they find that their result can be extended to some other models and to multiple factors, they stop short of carrying out an in-depth and comprehensive generalization to other models and settings. The focus of our paper is very different from that of BC. We start with the standard trade model and establish expression (1). We prove that the underlying mechanism is the envelope theorem. Then, we explain under what conditions expression (1) can be extended to other models and settings. Guided by the knowledge that the underlying mechanism under perfect competition is the envelope theorem, we try to find as many models and settings as possible for which the equation is applicable. Consequently, the set of models and settings for which the equation is found to be applicable is more comprehensive than that in BC.

The following are other key differences between our paper and others in the literature. First, this paper is the first to prove systematically that the conclusion of the envelope theorem can be applied to many different models and settings for calculating the global welfare gains from reduction of trade costs. Second, as far as we know, we are the first in the literature to rigorously justify the use of $\sum_{i=1}^{n} \frac{E_i}{Y^w} \widehat{U}_i$, the expenditure-share-weighted average percentage change of country welfare, as a reasonable measure of the change of global welfare. Others may have used this measure, but they did not justify its use. Third, we do not require balanced trade in any country, while most other papers in the literature, including AB and BC, require balanced trade in all countries. The reason we do not need balanced trade is the same as the reason why envelope theorem is the underlying mechanism driving our main result. According to the first fundamental theorem of welfare economics, the market outcome is the same as the global planner's solution. From the global planner's point of view, global trade balance is equal to zero. Therefore, it does not matter to him. AB consider unbalanced trade, but only under some restrictive conditions, namely, risk sharing between countries. Our finding is important as trade balances can deviate substantially from zero in practice. With this finding, we can be more confident that the equation can be applied to the real world. Fourth, to demonstrate the user-friendliness of our formula, we carry out a few simple empirical applications. For example, we apply it to estimate the global welfare gains from worldwide reduction of international shipping time in the last fifty years, and the estimate is in the range $[2.98\%, 8.81\%]$, which is consistent with estimates by others in the literature, e.g. EK2002.

One class of models that this paper does not include is the imperfect competition models with endogenously variable markups. Such models can capture the pro-competitive gains from trade which models with constant markups cannot. However, there is mixed empirical evidence on the size of the pro-competitive gains from trade. Indeed, pro-competitive effects are sometimes negative so that variable markups reduce the gains from trade (see, for example, Arkolakis, Costinot, Donaldson and Rodriguez-Clare, 2012). In fact, Edmond, Midrigan and Xu (2012) conclude that the size of the gains from trade depends a great deal on the underlying micro-details. Therefore, there is no presumption that our equation will bias the estimate upward or downward by not including this class of model.

The structure of this paper is as follow. In section 2, we derive a measure of the percentage change in global welfare. In section 3, based on the standard trade model, we derive a simple equation for the change in global welfare as a consequence of the small reduction of bilateral trade costs. It will be seen that the effect solely arises from the direct savings in trade costs. We then prove that the underlying mechanism of the result is envelope theorem. In section 4, we extend our analysis to the model with perfect competition and variable extensive margins of trade. In section 5, We consider the monopolistic competition models of K1980 and MwP. We prove in both sections 4 and 5 that the change in global welfare is given by the same equation as derived in section 3. In section 6, we prove that the above equation continues to hold in a number of extensions of the models in sections 4 and 5, e.g. multi-sector, multi-factor, multiple stages of production, the existence of intermediate good, and existence of a non-traded good sector. Section 7 presents some empirical applications of the model. The last section concludes.

2 Definition of percentage change in global welfare

In this section, we present a justification for the use of an equation that we shall use as a measure of percentage change in global welfare. Suppose in the world economy there are n countries that are capable of producing m goods, which are all tradable. We want to define a measure of percentage change in global welfare resulting from **small changes** (infinitesimal ones in the formal analysis) in trade costs. We would like to have a concept of change of global welfare such that an increase in global welfare signifies an enlargement of the global pie so that *potentially* every country can be better off by some proper income transfers between countries. Note that income is transferable but utility is not transferable. Therefore, the sum of utility of all countries is not a good measure of global welfare based on this concept.

As the world consists of a number of sovereign countries, each of which only cares about its own welfare, we have to define the improvement in global welfare based on the concept of Pareto improvement. More specifically, the improvement to global welfare resulting from a shock (such as reduction of trade costs) should be measured by how much potential Pareto improvement is afforded to the world after the change. Thus, we define the percentage increase in global welfare (after trade costs reduction) as the maximum potential equiproportional increase in welfare of all countries after some proper lump sum transfers of income between countries. It measures the potential amount of Pareto improvement to the countries of the world as a whole. Note that this amount can be negative. The above concept of the change in global welfare is consistent with that of Kaldor and Hicks (see, for example, Friedman 1998).[7] Consistent with Kaldor-Hicks' concept of efficiency, an outcome is more efficient if those that are made better off could in principle compensate those that are made worse off, so that a Pareto improving outcome can potentially result. This concept of Pareto improvement does not require compensation actually be paid, but merely that the possibility for compensation exists.

Define E_i, P_i and U_i as the expenditure, exact price index and welfare of country i, respectively. The utility function (or welfare function) of country i, given by $U_i(\mathbf{q_i})$ (where $\mathbf{q_i} = \left(q_i^1, q_i^2, ..., q_i^m\right)$ is the vector of quantities of goods consumed) is assumed to be homogeneous of degree one in $\mathbf{q_i}$. Consequently, we can define an exact price index P_i, which stands for the cost a consumer has to pay to obtain one unit of utility. Therefore, the total utility of all consumers in country i (i.e. welfare of country i) is given by $U_i = E_i/P_i$ for all i. After the reduction of trade costs, the vector of price-cum-welfare of the countries changes from $(P_1, ..., P_n; U_1, ..., U_n)$ to $(P_1 + dP_1, ..., P_n + dP_n; U_1 + dU_1, ..., U_n + dU_n)$. Let μ be the potential equiproportional increase in welfare of all countries after trade costs reduction. Then,

$$\sum_{i=1}^{n} (P_i + dP_i)(U_i + dU_i) = \sum_{i=1}^{n} (P_i + dP_i)(\mu + 1) U_i$$

The LHS is the total expenditure before lump-sum transfers while the RHS is the total global expenditure after lump-sum transfers that leads to an equiproportional increase in welfare for all countries equal to μ. Note that this scheme of lump-sum transfers are equivalent to summing up the compensating variations of all countries and then distribute them proportionately across countries.[8] Re-arranging the

[7] Because we are considering small changes, the use of the Kaldor-Hicks compensation criterion does not carry with it some of the shortcomings cited by its critics.

[8] This is because $(P_i + dP_i)(U_i + dU_i) - (P_i + dP_i) U_i$ is the compensating variation of country i. Note also that the equivalent variation of country i, $P_i(U_i + dU_i) - P_i U_i$, is the same as the compensating variation in the current context, because the changes are infinitesimal.

above equation and simplifying, we have

$$\begin{aligned}\mu =& \frac{\sum_{i=1}^{n}\left(P_i+dP_i\right)\left(dU_i\right)}{\sum_{i=1}^{n}\left(P_i+dP_i\right)U_i}\\ =& \frac{\sum_{i=1}^{n}P_i\left(dU_i\right)}{\sum_{i=1}^{n}P_iU_i}+O\left(\epsilon^2\right)\\ =& \sum_i \frac{P_iU_i}{\sum_k P_kU_k}\cdot\frac{dU_i}{U_i}\\ =& \frac{1}{Y^w}\sum_{i=1}^{n}E_i\widehat{U}_i \end{aligned} \tag{2}$$

where $O\left(\epsilon^2\right)$ denotes second order terms, which can be dropped, as we are considering only infinitesimal changes; $\widehat{U}_i \equiv \frac{dU_i}{U_i}$, $E_i = P_iU_i$ and $Y^w \equiv \sum_k P_kU_k$ is the GDP of the world.[9] Hereinafter, $\widehat{x} \equiv \frac{dx}{x}$, which we call the percentage change of x.

Thus, $\sum_{i=1}^{n}\frac{E_i}{Y^w}\widehat{U}_i$ (or the expenditure-share-weighted average percentage change of welfare of all countries) is the percentage change in global welfare. This makes sense as the importance of a country as reflected in its size should be reflected in the calculation of the change in global welfare. Note that we do not have to define what is global welfare. We only need to define what is the percentage change in global welfare. Note also that utility is cardinal, not ordinal, in this model. To get an intuitive sense of what μ means, note that the welfare impact of a percentage change in global welfare of μ is equivalent to having all consumers in the world potentially increasing their consumption of each good by a fraction of μ, if the same sets of goods are produced, traded and consumed by each country before and after the shock.[10] This expression for the percentage change in global welfare has been used in the literature, such as in Hsieh and Ossa (2011), AB and BC. However, as far as we are aware, we are the first to present a justification for its use.[11]

3 Perfect competition with fixed extensive margins of trade

The Standard Trade Model[12]

Consider a many-country, many-good, many-factor trade model under perfect competition, with complete (meaning that a country cannot import the same good from more than one country, including the home country) or incomplete specialization in each sector in each country , and fixed extensive

[9] See the appendix for a more detailed proof.

[10] This is because we assume that the utility function of each country is homogeneous of degree one in quantities of all goods consumed in that country. Therefore, the percentage change in global welfare is homogeneous of degree one in the percentage change of quantities of all goods consumed in the world as a whole.

[11] Note also that though our formal analysis is based on infinitesimal changes, the equation should be a sufficiently good approximation as long as all percentage changes of P_i and U_i are less than, say, ten percent as a rule of thumb. Therefore, the equation should applicable to many yearly changes.

[12] We would like to thank Gene Grossman for suggesting a two-country two-good version of this model to us.

margins of trade. The extensive margin of country i's exports to country j is the set of goods exported from i to j, i.e. Ω_{ij}. We assume that each of Ω_{ij} for all i and j is unchanged in the face of changes in τ_{ij} for all i and j. This kind of model is often termed the standard trade model. Suppose in the world economy there are n countries that are capable of producing m goods, which are all tradable. Let $\Omega_{ij} \subseteq \Omega = \{1, ..., m\}$ denote the set of goods exported from country i to country j. Therefore, the price $p_j(\omega)$ faced by consumers of good ω in country j satisfies

$$p_j(\omega) = p_i(\omega)\,\tau_{ij} \text{ for } \omega \in \Omega_{ij}$$

where τ_{ij} is an iceberg trade cost such that τ_{ij} units are shipped from the source country i for one unit to arrive at the destination country j (assume that $\tau_{ii} = 1$).[13] We can call $p_i(\omega)$ the F.O.B. price and $p_j(\omega)$ the C.I.F. price of good ω.

E_j denotes the expenditure in country j, while $\mathbb{E}_j[p_j(1), ..., p_j(m), U_j]$ stands for the expenditure function (therefore, by definition $E_j = \mathbb{E}_j$ always), $U_j[q_j(1), ..., q_j(m)]$ denotes the welfare of country j (where $q_j(\omega)$ denotes the consumption of good ω in country j), and the GDP function $R_j = [p_j(1), ..., p_j(m); \mathbf{L_j}]$ (where $\mathbf{L_j}$ is the vector of K factor endowments, which are assumed to be fixed) denotes the total sales revenue of firms in country j, which is also equal to total output Y_j, or the GDP of country j. The net export of country j is equal to $X_j^{net} \equiv R_j[p_j(1), ..., p_j(m); \mathbf{L_j}] - \mathbb{E}_j[p_j(1), ..., p_j(m), U_j]$. Totally differentiating the equation and re-arranging terms, we have

$$\frac{\partial \mathbb{E}_j}{\partial U_j} dU_j = \sum_{\omega \in \Omega} \left[\frac{\partial R_j}{\partial p_j(\omega)} - \frac{\partial \mathbb{E}_j}{\partial p_j(\omega)} \right] dp_j(\omega) - dX_j^{net} \tag{3}$$

Consider the expenditure function $\mathbb{E}_j[p_j(1), ..., p_j(m), U_j]$ and the indirect utility function $U_j = V_j[p_j(1), ..., p_j(m), E_j]$. By definition of the two functions, we have the identity

$$\mathbb{E}_j\{p_j(1), ..., p_j(m), V_j[p_j(1), ..., p_j(m), E_j]\} \equiv E_j.$$

Totally differentiating this identity, we get

$$\frac{\partial \mathbb{E}_j[p_j(1), ..., p_j(m), U_j]}{\partial U_j} \cdot \frac{\partial V_j[p_j(1), ..., p_j(m), E_j]}{\partial E_j} = 1$$

$$\Longrightarrow \frac{\partial \mathbb{E}_j}{\partial U_j} = \left(\frac{\partial V_j}{\partial E_j} \right)^{-1}. \tag{4}$$

If utility $U_j[q_j(1), ..., q_j(m)]$ is homogeneous of degree one in $\{q_j(1), ..., q_j(m)\}$, then the indirect utility function can be written as $V_j[p_j(1), ..., p_j(m), E_j] = E_j \cdot \widetilde{V}_j[p_j(1), ..., p_j(m)]$, which implies that

$$\frac{\partial V_j}{\partial E_j} = \widetilde{V}_j = \frac{V_j}{E_j}.$$

[13] The amount $\tau_{ij} - 1$ can be called the "wastage due to shipping" per unit arriving at the destination, but it should also include the ad-valorem trade cost equivalent of any administrative delay or other non-tariff barriers.

Substituting this into (4), we get

$$\frac{\partial \mathbb{E}_j}{\partial U_j} = \frac{E_j}{V_j} = \frac{E_j}{U_j} \qquad \Longrightarrow \qquad E_j \widehat{U}_j = \frac{\partial \mathbb{E}_j}{\partial U_j} dU_j \tag{5}$$

Therefore, (3) gives the expenditure-weighted percentage welfare change of country j. If we sum it up for all j, we shall obtain the percentage change in global welfare, according to (2).

The question is: what are the effects of τ_{ij} on $E_k \widehat{U}_k$ for $i, j, k \in \{1, ..., n\}$? The answer to this question will guide us to calculate $\frac{1}{Y^w} \sum_{k=1}^{n} E_k \widehat{U}_k$ resulting from changes in τ_{ij} for $i, j \in \{1, ..., n\}$.

3.1 Calculating the global gains from reduction of trade costs

Assumptions:

1. There is perfect competition in all goods markets.

2. Utility $U_j [q_j (1), ..., q_j (m)]$ is homogeneous of degree one in $\{q_j (1), ..., q_j (m)\}$ and satisfies the usual assumptions of a well-behaved utility function.

3. Define $y_j (\omega)$ as the total output of good ω in country j. The production function $y_j (\omega) = \mathcal{F}_j [\mathbf{L_j} (\omega)]$ is homogeneous of degree one in $\mathbf{L_j} (\omega)$, which is the vector of factors employed in the production of good ω in country j, and it satisfies the usual assumptions of a well-behaved production function.

4. The extensive margins of trade are fixed, which means that the set Ω_{ij} $\forall$ i, j remain the same before and after the change in trade costs.

5. Factor market clearing: $\mathbf{L_j} = \sum_{\omega \in \Omega} \mathbf{L_j} (\omega)$ for all j.

Steps in deriving $\sum_{i=1}^{n} \frac{E_i}{Y^w} \widehat{U}_i$:

Define $p_{ij}(\omega)$ as the price of variety ω consumed in country j that is imported from country i. Note that $p_j (\omega) = p_{jj} (\omega)$ when country j exports good ω, and $p_j (\omega) = p_{ij} (\omega)$ when country j imports good ω from country i.

1. For $\omega \in \Omega_{ij}$, τ_{ij} affects $p_{ii} (\omega)$ and $p_{ij} (\omega)$, i.e.

$$\widehat{p_{ij}} (\omega) = \widehat{p_{ii}} (\omega) + \widehat{\tau}_{ij}. \tag{6}$$

2. For $\omega \in \Omega_{ji}$, $p_j (\omega)$ affects R_j. According to the first fundamental theorem of welfare economics, the competitive market outcome is the same as the solution of the GDP maximization problem of a national social planner. Therefore, applying envelope theorem to the maximization problem of the national social planner, we have $\frac{\partial R_i}{\partial p_j(\omega)} = y_i (\omega)$, which is the total production of good ω in country i.

3. For $\omega \in \Omega_{ij}$, $p_j(\omega)$ affects $\mathbb{E}_j$. Similar to Step 2 above, by envelope theorem, we have $\frac{\partial \mathbb{E}_j}{\partial p_j(\omega)} = q_j(\omega)$, which is the total consumption of good ω in country j.

4. Conservation of physical quantities of goods. Note that $y_i(\omega) = \sum_{j=1}^n y_{ij}(\omega)$ where $y_{ij}(\omega)$ denotes the quantity of good ω produced in i that is exported to country j. Thus, $y_{ij}(\omega) > 0$ if i is an exporter of good ω to j and $y_{ij}(\omega) = 0$ if i is not an exporter of good ω to j. Consequently, $y_{ij}(\omega) > 0 \Rightarrow y_{ji}(\omega) = 0$.

Note also that $q_j(\omega) = \sum_{i=1}^n q_{ij}(\omega)$ where $q_{ij}(\omega)$ denotes the quantity of good ω consumed in j that is imported from country i. Thus, $q_{ij}(\omega) > 0$ if i is an exporter of good ω to j and $q_{ij}(\omega) = 0$ if i is not an exporter of good ω to j. Consequently, $q_{ij}(\omega) > 0 \Rightarrow q_{ji}(\omega) = 0$.

Therefore, if i exports good ω to j (i.e. $\omega \in \Omega_{ij}$), then $y_{ij}(\omega) > 0$ and $q_{ij}(\omega) > 0$. Moreover, the quantity of ω shipped out of i to j has to be equal to the quantity of ω shipped into j from i plus the wastage due to shipping. That is,

$$\begin{aligned} \tau_{ij} q_{ij}(\omega) &\equiv y_{ij}(\omega) \quad \text{for } \omega \in \Omega_{ij} \\ \Longrightarrow \quad p_{ij}(\omega) q_{ij}(\omega) &\equiv p_{ii}(\omega) y_{ij}(\omega) \quad \text{for } \omega \in \Omega_{ij} \end{aligned} \tag{7}$$

The LHS represents the total value of j's import of good ω from i, while the RHS represents the total value of i's export of good ω to j. They are both denoted by $x_{ij}(\omega)$.

5. Invoking (3) and (5), we have

$$\begin{aligned} E_j \widehat{U}_j &= \sum_{\omega \in \Omega} \left[\frac{\partial R_j}{\partial p_j(\omega)} - \frac{\partial \mathbb{E}_j}{\partial p_j(\omega)} \right] dp_j(\omega) - dX_j^{net} \\ &= \left[\sum_{\omega \in \Omega_{ji}} \sum_{i=1}^n y_{ji}(\omega) \, dp_{jj}(\omega) - \sum_{\omega \in \Omega_{ij}} \sum_{i=1}^n q_{ij}(\omega) \, dp_{ij}(\omega) \right] - dX_j^{net} \quad \text{by steps 2 and 3} \\ &= \sum_{i=1}^n \sum_{\omega \in \Omega_{ji}} p_{jj}(\omega) y_{ji}(\omega) \widehat{p}_{jj}(\omega) - \sum_{i=1}^n \sum_{\omega \in \Omega_{ij}} p_{ij}(\omega) q_{ij}(\omega) \left[\widehat{p}_{ii}(\omega) + \widehat{\tau}_{ij} \right] - dX_j^{net} \text{ by (6)} \\ &= \underbrace{\sum_{i=1}^n \sum_{\omega \in \Omega_{ji}} \widehat{p}_{jj}(\omega) x_{ji}(\omega) - \sum_{i=1}^n \sum_{\omega \in \Omega_{ij}} \widehat{p}_{ii}(\omega) x_{ij}(\omega)}_{\text{terms of trade effect}} - \underbrace{\sum_{i=1}^n X_{ij} \widehat{\tau}_{ij}}_{\text{direct effect}} - \underbrace{dX_j^{net}}_{\text{trade imbalance effect}} \end{aligned} \tag{8}$$

Note that $\widehat{\tau}_{jj} = 0$ for all j by the definition that $\tau_{jj} = 1$ for all j.

The last line follows from $x_{ji}(\omega) = p_{jj}(\omega) y_{ji}(\omega)$, $x_{ij}(\omega) = p_{ij}(\omega) q_{ij}(\omega)$ and $X_{ij} = \sum_{\omega \in \Omega_{ij}} p_{ij}(\omega) q_{ij}(\omega)$, where $x_{ij}(\omega)$ is the exports of good ω from i to j. The various effects are labeled accordingly. **Explanation of the various effects on the last line:** The first term reflects the increase in welfare of j resulting from increases in F.O.B. prices of j the exporter, while the second term is the decrease in welfare of j resulting from increases in F.O.B. prices of j the importer, both keeping the trade costs unchanged. The sum of the first and the second term can be positive or negative. It captures the change in purchasing power induced by the general equilibrium adjustments in relative factor prices (which in turn affects the terms of trade), which we refer to as "terms of trade effect". The third term captures the direct effect of changes in trade costs (saving in trade costs), which we refer to as "direct effect". The last term captures the reduction in expenditure due to trade imbalance, which we refer to as "trade imbalance effect".

Summing $E_j\widehat{U}_j$ over j, we have

$$\begin{aligned}\sum_{j=1}^{n}E_j\widehat{U}_j &= \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{\omega\in\Omega_{ji}}\widehat{p}_{jj}(\omega)\,x_{ji}(\omega)-\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{\omega\in\Omega_{ij}}\widehat{p}_{ii}(\omega)\,x_{ij}(\omega)-\sum_{i=1}^{n}X_{ij}\widehat{\tau}_{ij}-\sum_{j=1}^{n}dX_j^{net}\\ &= -\sum_{j=1}^{n}\sum_{i=1}^{n}X_{ij}\widehat{\tau}_{ij}\end{aligned}$$

The second line follows from interchanging i and j in the first term on the first line, which results in the cancellation of the first and second terms, and from $\sum_{i=1}^{n}dX_i^{net}=0$, which is implied from $\sum_{i=1}^{n}X_i^{net}=0$. The cancellation of the first and second terms on the first line signifies that the effects of changes in terms of trade offset each other from the global welfare point of view. It is the result of the fact that the quantity of ω shipped out of i to j has to be equal to the quantity of ω shipped into j from i plus the wastage due to shipping.[14]

Therefore, percentage change in global welfare is given by

$$\sum_{i=1}^{n}\frac{E_i}{Y^w}\widehat{U}_i=-\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{X_{ij}}{Y^w}\widehat{\tau}_{ij} \tag{9}$$

where $Y^w=\sum_{k=1}^{n}E_k=\sum_{k=1}^{n}Y_k=$ GDP of the world.

Note that the RHS of (9) is equal to the total saving in trade costs, controlling for the volumes of exports, divided by global GDP.[15] This is the first order effect, or direct effect, and, interestingly, the only effect. The second order effects, or indirect effects, such as those due to the changes in terms of trade and in trade balances of different countries, offset each other from the global welfare point of view. As we shall show below, the key to obtaining the simple result in (9) is the envelope theorem. Two important features of this model are that it allows for incomplete specialization and the extensive margins of trade are fixed.

We summarize the main result in the following proposition.

Proposition 1 *Under perfect competition and fixed extensive margins of trade, the change in global welfare associated with small changes in trade costs is given by equation (9).*

[14] Controlling for the trade cost of a good, an autonomous increase in the F.O.B. price (i.e. $p_{ii}(\omega)$ if country i is the exporter of good ω) increases the welfare of the exporter but reduces that of the importer. However, because the F.O.B. value of export of the good by the exporter is equal to the F.O.B. value of import by the importer, the effect on the exporter and that on the importer cancel each other from the global welfare point of view.

[15] The total variable exporting costs for exporting from i to j is equal to $\left[\sum_{\omega\in\Omega_{ij}}p_{ii}(\omega)\,q_{ij}(\omega)\right](\tau_{ij}-1)$. Therefore, the total saving in exporting costs from i to j, controlling for trade flows, is equal to $\left[\sum_{\omega\in\Omega_{ij}}p_{ii}(\omega)\,q_{ij}(\omega)\right]d\tau_{ij}=\left[\frac{\sum_{\omega\in\Omega_{ij}}p_{ij}(\omega)q_{ij}(\omega)}{\tau_{ij}}\right]d\tau_{ij}=X_{ij}\widehat{\tau}_{ij}$.

Examples of models: Heckscher-Ohlin (and its extensions to multi-country, multi-good and multi-factor), Armington, Anderson (1979), Anderson and Van Wincoop (2003).

3.2 Intuition

In this subsection we want to prove that the main result (9) is an outcome of the envelope theorem. It is clear that a global planner should allocate resources so as to maximize global expenditure given the price levels of each country. Suppose he does not, then every country can potentially be made better off by the reallocation of resources that gives rise to higher global expenditure. This means that he does not maximize global welfare if he does not maximize global expenditure.

According to the first fundamental theorem of welfare economics, as there are no externalities, incomplete markets, incomplete information, or monopoly power, the competitive equilibrium is efficient. This means that the competitive equilibrium yields the same allocation of resources to the production of different goods as the solution of a global social planner's global-expenditure maximization problem, given the market prices. Let there be n countries, m goods and K factor inputs. Let $\mathbf{l_{ij}}(\omega)$ denote the vector of K factor inputs for producing output $y_{ij}(\omega)$, and $l_{ij}^k(\omega)$ denote the kth element of $\mathbf{l_{ij}}(\omega)$. As the global planner maximizes expenditure, she solves the following problem, by choosing $l_{ij}^k(\omega), \forall i,j,k,\omega$, taking trade costs τ_{ij} for all i,j, and the market prices $p_{ij}(\omega)$ for all i,j,ω, as given.

$$\begin{aligned}
\max_{\left\{l_{ij}^k(\omega),\forall i,j,k,\omega\right\}} \quad W &\equiv \sum_{j=1}^{n} P_j U_j \\
&= \sum_{j=1}^{n} E_j \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{\omega\in\Omega_{ij}} p_{ij}(\omega)\, q_{ij}(\omega) \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{\omega\in\Omega_{ij}} \frac{p_{ij}(\omega)\, f^{\omega}(\mathbf{l_{ij}}(\omega))}{\tau_{ij}} \text{ s.t. } \sum_{j=1}^{n}\sum_{\omega\in\Omega_{ij}} \mathbf{l_{ij}}(\omega) = \mathbf{L_i} \qquad \text{for all } i,j,\omega
\end{aligned}$$

where $\mathbf{L_i}$ denotes the vector of K factor endowments in country i and $f^{\omega}(\mathbf{l_{ij}}(\omega))$ is the production function for producing $y_{ij}(\omega)$ units of good ω by country i for sales in country j.[16] P_j is a function of the set of prices $\{p_{ij}(\omega)|\ \forall i,j \text{ and } \omega\in\Omega_{ij}\}$, while U_j is a function of the set of quantities $\{q_{ij}(\omega)|\ \forall i,j \text{ and } \omega\in\Omega_{ij}\}$. In the above steps, we have invoked $\tau_{ij}q_{ij}(\omega)\equiv y_{ij}(\omega) = f^{\omega}(\mathbf{l_{ij}}(\omega))$ for all i,j,ω.

[16] Here we assume that the production function of the same good is the same in all countries. We could assume that production functions of the same good are different across countries, but the conclusion will remain the same.

Therefore, the Lagrangean is given by

$$\max_{\{l_{ij}(\omega)\}} W = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{\omega\in\Omega_{ij}} \frac{p_{ij}(\omega) f^{\omega}(\mathbf{l_{ij}}(\omega))}{\tau_{ij}} + \sum_{i=1}^{n}\xi_{\mathbf{i}}\left(\mathbf{L_i} - \sum_{j=1}^{n}\sum_{\omega\in\Omega_{ij}} \mathbf{l_{ij}}(\omega)\right)$$

where $\xi_{\mathbf{i}}$ denotes a vector of K Lagrange multipliers for the K factors in country i.

The optimal value of $l_{ij}^{k}(\omega)$, denoted by $l_{ij}^{k*}(\omega)$, is a function of τ_{ij} and $p_{ij}(\omega)$. Note that the market value of $p_{ij}(\omega)$ is a function of τ_{ij} too. Formally,

$$l_{ij}^{k*}(\omega) = g\left(\tau_{ij}, p_{ij}(\omega)(\tau_{ij})\right)$$

As shown in (2), the percentage change in global welfare is given by

$$\textstyle\sum_{i=1}^{n}\frac{E_i}{Y^w}\widehat{U}_i = \displaystyle\sum_i \frac{P_iU_i}{\sum_k P_kU_k}\cdot\frac{dU_i}{U_i} = \frac{\sum_{i=1}^{n}P_i(U_i+dU_i) - \sum_{i=1}^{n}P_iU_i}{\sum_k P_kU_k} = \frac{\sum_{i=1}^{n}P_i(U_i+dU_i) - \sum_{i=1}^{n}P_iU_i}{Y^w}$$

as $\sum_k P_kU_k$ is equal to global expenditure, which is in turn equal to global GDP Y^w. Therefore, the task for us is to calculate $\sum_{i=1}^{n}P_i(U_i+dU_i) - \sum_{i=1}^{n}P_iU$, which we define as dW, induced by the changes in τ_{ij}. It is in fact the sum of equivalent variations of all countries.

In evaluating the effect of τ_{ij} on W, we have to evaluate how $l_{ij}^{k}(\omega)$ for all i, j, k,and $\omega\in\Omega_{ij}$, are affected, then calculate their effects on U_j for all j. In other words, we have to take into account 1. the direct effect of $\tau_{ij}\longrightarrow W$; 2. plus the indirect effect of $\tau_{ij}\longrightarrow l_{ij}^{k}(\omega)\longrightarrow W$; 3. plus the indirect effect of $\tau_{ij}\longrightarrow p_{ij}(\omega)\longrightarrow l_{ij}^{k}(\omega)\longrightarrow W$. Then, $dW\equiv\sum_{i=1}^{n}P_i(U_i+dU_i)-\sum_{i=1}^{n}P_iU$ is equal to the resulting change in $\sum_{j=1}^{n}E_j$ induced by the above direct and indirect effects of τ_{ij}.

Thus, the total effect of τ_{ij} on W can be written as

$$\begin{aligned}\frac{dW}{d\tau_{ij}} &= \frac{\partial W}{\partial\tau_{ij}} + \sum_{\omega\in\Omega_{ij}}\sum_{k=1}^{K}\frac{\partial W}{\partial l_{ij}^{k}(\omega)}\cdot\frac{\partial l_{ij}^{k*}(\omega)}{\partial\tau_{ij}} + \sum_{\omega\in\Omega_{ij}}\sum_{k=1}^{K}\frac{\partial W}{\partial l_{ij}^{k}(\omega)}\cdot\frac{\partial l_{ij}^{k*}(\omega)}{\partial p_{ij}(\omega)}\cdot\frac{\partial p_{ij}(\omega)}{\partial\tau_{ij}}\\ &= \frac{\partial W}{\partial\tau_{ij}} \text{ by envelope theorem, since } \frac{\partial W}{\partial l_{ij}^{k}(\omega)} = 0 \text{ for all } i,j,k, \text{and } \omega\in\Omega_{ij}\end{aligned}$$

Note that the direct effect of $p_{ij}(\omega)$ on W, $\partial W/\partial p_{ij}(\omega)$, should not be included as we are evaluating $\sum_{i=1}^{n}P_i(U_i+dU_i)-\sum_{i=1}^{n}P_iU_i$, i.e. the change in $\sum_{i=1}^{n}P_iU_i$ after changes in all the U_i have been accounted for but evaluated based on the old set of prices $\{P_i|\ i=1,2,...,n\}$.

Therefore, by envelope theorem, we have

$$
\begin{aligned}
\sum_{i=1}^{n} P_i\left(U_i+dU_i\right)-\sum_{i=1}^{n} P_i U &= \frac{\partial W}{\partial \tau_{ij}} d\tau_{ij} \\
&= -\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{\omega\in\Omega_{ij}} p_{ij}(\omega) f^{\omega}\left(\mathbf{l_{ij}}(\omega)\right)\tau_{ij}^{-2} d\tau_{ij} \\
&= -\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{\omega\in\Omega_{ij}} \frac{p_{ij}(\omega) f^{\omega}\left(\mathbf{l_{ij}}(\omega)\right)}{\tau_{ij}}\cdot\frac{d\tau_{ij}}{\tau_{ij}} \\
&= -\sum_{i=1}^{n}\sum_{j=1}^{n} X_{ij}\widehat{\tau_{ij}} \qquad \text{as} \sum_{\omega\in\Omega_{ij}} \frac{p_{ij}(\omega) f^{\omega}\left(\mathbf{l_{ij}}(\omega)\right)}{\tau_{ij}} = X_{ij} \\
\Longrightarrow \qquad \textstyle\sum_{i=1}^{n} \frac{E_i}{Y^w}\widehat{U}_i &= \frac{\sum_{i=1}^{n} P_i\left(U_i+dU_i\right)-\sum_{i=1}^{n} P_i U_i}{Y^w} \\
&= -\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{X_{ij}}{Y^w}\widehat{\tau_{ij}}
\end{aligned}
$$

3.3 Generalization

There are two characteristics of the standard trade model that make the envelope theorem work. First, the extensive margins of trade are fixed. Second, the market structure is perfect competition, which implies that the market allocation of resources are efficient. We shall relax these two assumptions one by one and see under what conditions the main result (9) continues to hold. We first allow the extensive margins of trade to vary under perfect competition. Then, we allow the market structure to be monopolistic competition. Before we proceed with the generalization, we first make the assumption that there is a continuum of goods instead of discrete number of goods. This will simplify our analysis without much loss of generality.

4 Perfect Competition with variable extensive margins of trade

To deal with variable extensive margins of trade, we note that under incomplete specialization, it is possible that the global welfare impact of the adjustments of extensive margins of trade is first order, not second order. In that case, we have to take into account the adjustments of extensive margins of trade when calculating the global impact of the reduction of trade costs. In contrast, as shown in footnote 17, under complete specialization the impact of the adjustments of extensive margins of trade on global welfare is of second order. For equation (9) to hold, therefore, it is sufficient to assume complete specialization so that the effects of changes in extensive margins of trade is rendered second order, leaving the direct effect as the only first order one.

The following proof is based on the assumption of perfect competition and complete specialization.

The utility function only needs to be constant returns to scale.

Proof:

Step 1: First, $\widehat{U}_j = \widehat{E}_j - \widehat{P}_j \implies E_j\widehat{U}_j = E_j\widehat{E}_j - E_j\widehat{P}_j$. Second, $E_j = Y_j - X_j^{net} \implies E_j\widehat{E_j} = Y_j\widehat{Y}_j - dX_j^{net} \implies$

$$E_j\widehat{E}_j = Y_j\widehat{w}_j - dX_j^{net} \tag{10}$$

Since $Y_j = \sum_{i=1}^{n} X_{ji}$, *the change in welfare of country j is given by:*

$$E_j\widehat{U}_j = \sum_{i=1}^{n} X_{ji}\widehat{w}_j - dX_j^{net} - E_j\widehat{P}_j \tag{11}$$

Step 2: Complete specialization implies that[17]

$$\widehat{P}_j = \sum_{i=1}^{n} \frac{X_{ij}}{E_j}\widehat{p}_{ij} \tag{12}$$

$$\implies \quad E_j\widehat{P}_j = \sum_{i=1}^{n} X_{ij}\widehat{p}_{ij} = \sum_{i=1}^{n} X_{ij}\left(\widehat{w}_i + \widehat{\tau}_{ij}\right) \tag{13}$$

Step 3: From steps 1 and 2, we have

$$E_j\widehat{U}_j = \sum_{i=1}^{n} \underbrace{\left(X_{ji}\widehat{w}_j - X_{ij}\widehat{w}_i\right)}_{\text{terms of trade effect}} - \underbrace{\sum_{i=1}^{n} X_{ij}\widehat{\tau}_{ij}}_{\text{direct effect}} - \underbrace{dX_j^{net}}_{\text{trade imbalance effect}} \tag{14}$$

Therefore,

$$\frac{1}{Y^w}\sum_{j=1}^{n} E_j\widehat{U}_j = \frac{1}{Y^w}\left(\sum_{j=1}^{n}\sum_{i=1}^{n} X_{ji}\widehat{w}_j - \sum_{j=1}^{n}\sum_{i=1}^{n} X_{ij}\widehat{w}_i - \sum_{j=1}^{n}\sum_{i=1}^{n} X_{ij}\widehat{\tau}_{ij} - \sum_{j=1}^{n} dX_j^{net}\right)$$
$$= -\sum_{j=1}^{n}\sum_{i=1}^{n} \frac{X_{ij}}{Y^w}\widehat{\tau}_{ij}$$

where the second line stems from $\sum_{j=1}^{n} dX_j^{net} = 0$, which is implied from $\sum_{j=1}^{n} X_j^{net} \equiv 0$, and from interchanging i and j in the first term on RHS of line one, which results in cancellation of the first and second terms on line one.

From the perspective of global welfare, the terms of trade effects completely offset each other simply because of the conservation of physical quantities of goods — the quantity of ω shipped out of i to j has to be equal to the quantity of ω shipped into j from i plus the wastage due to shipping. The trade imbalance effects completely offset each other as $\sum_{j=1}^{n} X_j^{net} = 0$.

Therefore, we have

[17] The consumer price index P_j can be write as follows:

$$P_j = \min \int_0^{\Omega} p_j(\omega)\, q_j(\omega)\, d\omega \text{ s.t. } \; U_j\left(\{q_j(\omega) | \, \omega \in \Omega\}\right) \leq E_j$$

where $p_j(\omega) = p_{ij}(\omega)$ if variety ω is imported from country i; $q_j(\omega) = q_{ij}(\omega)$ if variety ω is imported from country i. Since the utility function is homogeneous of degree one, envelope theorem implies that $\widehat{P}_j = \int_0^{\Omega} \lambda_j(\omega)\, \widehat{p}_j(\omega)\, d\omega$ where $\lambda_j(\omega)$ denotes the expenditure share on good ω in country j. Complete specialization implies that $\widehat{P}_j = \sum_{i=1}^{n} \lambda_{ij}\widehat{p}_{ij} = \sum_{i=1}^{n} \frac{X_{ij}}{E_j}\left(\widehat{w}_i + \widehat{\tau}_{ij}\right)$ where λ_{ij} denotes the expenditure share of country j on goods from country i.

Proposition 2 *Under perfect competition, variable extensive margins of trade and complete specialization, (9) holds.*

Examples of models to which this proposition applies are DFS1977 and EK2002.

We next analyze under what conditions (9) continues to hold assuming monopolistic competition and variable extensive margins of trade (hereinafter abbreviated as MC-VEM models). We find that in addition to complete specialization, we also need additional assumptions in order for the conclusion of the envelope theorem to hold.

5 Monopolistic Competition with Variable Extensive Margins

For the conclusion of the envelope theorem to be applicable to a model, we want (a) no distortion of the relative prices and relative quantities compared to perfect competition; (b) complete offset of the effects of changes in extensive margins of trade from the global welfare point of view. The first condition is required so that relative prices and relative quantities under the monopolistically competitive market is the same as those under perfect competition, where there is marginal cost pricing. The second condition serves to nullify the effects of the extensive margins of trade, which the envelope theorem does not account for.

It turns out that CES preferences satisfy condition (a) above. For the case of homogeneous firms, condition (b) is automatically satisfied under CES preferences. Thus, (9) will work for K1980. For the case of heterogeneous firms, condition (b) is satisfied by assuming Pareto distribution of firm productivity, as will be shown below. Thus, (9) will work for MwP as well. Axtell (2001) shows that firm revenues in the US follow a Pareto distribution. Since the assumption of Pareto distribution of firm productivity implies Pareto distribution of revenue in our model, the assumption can be justified as being realistic.

We consider a world economy consisting of n countries indexed by $i = 1, ..., n$, with a single factor of input, labor, which is inelastically supplied and immobile across countries. There is a continuum of goods indexed by $\omega \in \Omega = [0, N]$ that can potentially be made available to consumers. We use L_i and w_i to denote the total endowment of labor and the (endogenous) wage level in country i respectively.

5.1 Preferences, Technology and Market Structure

Preferences: We assume CES preferences, which lead to constant markup, which in turn implies that relative prices and relative quantities are not distorted (i.e. they are the same as under perfect competition). In each country j, the representative consumer chooses her consumption bundle to maximize her utility subject to the budget constraint. Her utility is given by the Dixit-Stiglitz function:

$$U_j = \left[\sum_{i=1}^{n}\int_{\omega\in\Omega_{ij}} q_{ij}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}} \tag{15}$$

where $\sigma > 1$ is the elasticity of substitution.[18] Consequently, we have

$$P_j = \left[\sum_{i=1}^{n}\int_{\omega\in\Omega_{ij}} p_{ij}(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}$$

Technology: For every good $\omega \in \Omega$, there is a blueprint that can be acquired by one or many firms depending on the market structure (to be described below). If a firm from country i produces $\mathbf{q} \equiv \{q_{ij}(\omega)\}$ units of good ω to be sold to country j, its cost function is given by

$$C_i(w_i, \mathbf{q}, \omega) = \sum_{j=1}^{n}\left[\tau_{ij} w_i a_i(\omega) q_{ij}(\omega) + \xi_{ij} w_i \cdot \mathbf{1}(q_{ij}(\omega) > 0)\right]$$

and $\mathbf{1}(q_{ij}(\omega) > 0)$ is an indicator function, w_i is the wage in country i, $\tau_{ij} w_i a_i(\omega)$ denotes the constant marginal cost inclusive of trade cost, and $\xi_{ij} w_i$ denotes the fixed cost of exporting from i to j, where $\xi_{ij} \geq 0\ \forall\ i, j$ is exogenous. Note that we treat a country-i firm serving market i as exporting from i to i. We assume that ξ_{ii} can be non-zero. The exogenous unit labor requirement parameter $a_i(\omega)$ reflects the heterogeneity of productivities across blueprints.[19]

Market Structure: We consider monopolistic competition with free entry.[20] There is a large number of firms in each country, and all goods-markets and labor markets clear. A firm from country i needs to hire F_i units of labor to develop a blueprint, which confers it with monopoly power. The measure of number of entrants N_i in country i is endogenously determined by the zero profit condition. In equilibrium, the entry cost, $w_i F_i$, is equal to the expected profit of each firm. In the case of homogeneous firms, all entrants survive and all surviving firms export to all countries. In the case of heterogeneous firms, some entrants do not survive, and not all of those who survive export to all countries, depending on their productivity.

Below, we prove the following proposition:

Proposition 3 *Under the monopolistic competition models of Krugman (1980) and Melitz (2003) with Pareto distribution of firm productivity, (9) holds.*

[18] If $i = j$, then the domestically produced good ω is consumed.

[19] In fact, equation (9) will continue to be valid even if we adopt more general assumptions on the technology such as $C_i(w_i, \mathbf{q}, \omega) = \sum_{j=1}^{n}\left[\tau_{ij} w_i a_{ij}(\omega) t_j^{\frac{1}{1-\sigma}} q_{ij}(\omega) + \xi_{ij} w_i b_{ij}(\omega) m_{ij}(t_j) \mathbf{1}(q_{ij}(\omega) > 0)\right]$, similar to that of ACR. Please refer to ACR (2012) for detail. We make simpler assumptions here so as to highlight our intuition more clearly.

[20] The case with restricted entry is straightforward to analyze, and so is omitted in this paper. The results will be exactly the same as the ones with free entry.

5.2 One-sector K1980

Firms in each country are homogeneous. This is a special case when $\xi_{ij} = 0 \; \forall \; i, j$ and $a_i(\omega) = a_i$ for all ω.

Proof:

Step 1: Same as Step 1 in section 4

Step 2: It can be easily shown that

$$P_j = \left[\sum_{i=1}^{n} N_i \left(\frac{\sigma}{\sigma - 1} w_i \tau_{ij} a_i \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

Totally differentiating the logaritham, we have

$$\begin{aligned} \widehat{P}_j =& \sum_{i=1}^{n} \frac{X_{ij}}{E_j} \left[\widehat{w}_i - \underbrace{\left(\frac{1}{\sigma - 1} \right) \widehat{N}_i}_{\text{entrants effect}} + \widehat{\tau}_{ij} \right] \\ \Longrightarrow \quad E_j \widehat{P}_j =& \sum_{i=1}^{n} X_{ij} \left(\widehat{w}_i + \widehat{\tau}_{ij} \right) \end{aligned}$$

which is the same as (12). The second line follows from the fact that free entry condition and constant markup (due to CES preferences) imply that $\left(\frac{1}{\sigma}\right) N_i w_i F_i = \left(\frac{1}{\sigma}\right) \Pi_i = R_i = w_i L_i \quad \Rightarrow \quad \widehat{N}_i = 0$, which means that the entrants effect is zero.

This is a key step to prove that the Krugman (1980) model yields the same result (9) as the perfect competition model. The rest of the proof is the same as in section 4.

Step 3: Same as Step 3 in section 4.

Thus, (9) is applicable to the Krugman (1980) model.

5.3 One-sector MwP

Firms in each country are heterogeneous. $\xi_{ij} \geq 0 \; \forall \; i, j$ and the labor productivity $\varphi \equiv 1/a_i(\omega)$ of a firm in country i follows a Pareto distribution: $G_i(\varphi) = 1 - \left(\frac{b_i}{\varphi}\right)^{\gamma}$ and $g_i(\varphi) = \frac{\gamma b_i^{\gamma}}{\varphi^{\gamma+1}}$, where $\gamma > \sigma - 1$. Define φ_{ij}^* as the cutoff productivity for a firm in country i to profitably export to country j. As implied from Melitz (2003), the (unconditional) expected average productivity of an entrant is given by $\overline{\varphi}_{ij} \equiv \int_{\varphi_{ij}^*}^{\infty} (\varphi)^{\sigma-1} g_i(\varphi) \, d\varphi$. Thus $\overline{\varphi}_{ij}$ is a function of φ_{ij}^* . The value of $\overline{\varphi}_{ij}$, in turn, directly affects P_j, as shown below.

Two properties of this model will be found useful in the proof below:

P1: In each country, total amount of firm profits gross of fixed entry costs, Π_i, is proportional to total firm revenues R_i.

P2: The elasticity of the expected average productivity of an entrant ($\overline{\varphi}_{ij}$) with respect to the cutoff productivity (φ^*_{ij}) is a constant. In other words, $\widehat{\overline{\varphi}_{ij}}/\widehat{\varphi^*_{ij}}$ is a constant.[21]

P2, together with P1, will eventually help us to nullify the global welfare effects caused by changes in cutoff productivities of countries. In fact, it can be shown that with CES preferences, the only firm productivity distribution for the Melitz (2003) model that satisfies P1 and P2 is Pareto distribution.

Proof:

- The price index is given by:

$$\begin{aligned} P_j &= \left[\sum_{i=1}^n N_i \int_0^\infty \left[p_{ij}(\varphi)\right]^{1-\sigma} g_i(\varphi)\, d\varphi\right]^{\frac{1}{1-\sigma}} \\ &= \left\{\sum_{i=1}^n N_i \int_{\varphi^*_{ij}}^\infty \left[\left(\frac{\sigma}{\sigma-1}\right)\frac{w_i \tau_{ij}}{\varphi}\right]^{1-\sigma} g_i(\varphi)\, d\varphi\right\}^{\frac{1}{1-\sigma}} \\ &= \left\{\sum_{i=1}^n N_i \left[\left(\frac{\sigma}{\sigma-1}\right) w_i \tau_{ij}\right]^{1-\sigma} \overline{\varphi}_{ij}\right\}^{\frac{1}{1-\sigma}} \end{aligned}$$

- Based on the Pareto distribution assumption, the price index becomes:

$$P_j = \left\{\sum_{i=1}^n \left[\frac{\gamma}{\gamma-(\sigma-1)}\right] b_i^\gamma N_i \left(\frac{\sigma}{\sigma-1} w_i \tau_{ij}\right)^{1-\sigma} \left(\varphi^*_{ij}\right)^{-\gamma+\sigma-1}\right\}^{\frac{1}{1-\sigma}}$$

- Totally differentiate the logarithm of the above equation and invoking complete specialization yield:

$$\widehat{P}_j = \sum_{i=1}^n \frac{X_{ij}}{E_j}\left\{\widehat{w}_i - \left(\frac{1}{\sigma-1}\right)\widehat{N}_i + \widehat{\tau}_{ij} + \left[\frac{\gamma-(\sigma-1)}{\sigma-1}\right]\widehat{\varphi^*_{ij}}\right\} \tag{16}$$

- Fixed exporting cost $= \xi_{ij} w_i$ where ξ_{ij} is a constant. The zero cutoff profit condition implies that for the marginal firm:

$$\pi_{ij}(\varphi^*_{ij}) = \frac{E_j}{\sigma}\left[\left(\frac{\sigma-1}{\sigma}\right)\frac{P_j}{\tau_{ij} w_i}\varphi^*_{ij}\right]^{\sigma-1} = \xi_{ij} w_i$$

  which implies that the cutoff productivity satisfies:

$$\widehat{\varphi^*_{ij}} = \widehat{w}_i + \widehat{\tau}_{ij} + \frac{1}{\sigma-1}\left(\widehat{w}_i - \widehat{E}_j\right) - \widehat{P}_j \tag{17}$$

[21] The condition is equivalent to saying that the expression $\left(\varphi^*_{ij}\right)^\sigma g_i\left(\varphi^*_{ij}\right) / \int_{\varphi^*_{ij}}^\infty (\varphi)^{\sigma-1} g_i(\varphi)\, d\varphi$ is constant. The reason we need this assumption is that $\widehat{P}_j = \sum_{i=1}^n \frac{X_{ij}}{E_j}\widehat{P_{ij}}$, where $P_{ij} = N_i \int_{\varphi^*_{ij}}^\infty \left[\left(\frac{\sigma}{\sigma-1}\right)\frac{w_i\tau_{ij}}{\varphi}\right]^{1-\sigma} g_i(\varphi)\, d\varphi = N_i \left(\frac{\sigma}{\sigma-1} w_i \tau_{ij}\right)^{1-\sigma} \overline{\varphi}_{ij}$ is the component of P_j attributed to country i and we want $\widehat{P_{ij}}$ to be a linear combination of $\widehat{w}_i, \widehat{N}_i, \widehat{\tau}_{ij}$ and $\widehat{\varphi^*_{ij}}$ as seen in equation (16).

- Substituting (17) into (16) and re-arranging yields

$$E_j\widehat{P}_j = \textstyle\sum_{i=1}^{n} X_{ij}\left[\widehat{w}_i + \widehat{\tau}_{ij} + \underbrace{\left(\frac{1}{\sigma-1} - \frac{1}{\gamma}\right)\left(\widehat{w}_i - \widehat{E}_j\right)}_{\text{cutoff productivity effect}} - \underbrace{\frac{1}{\gamma}\widehat{N}_i}_{\text{entrants effect}}\right] \tag{18}$$

- Comparing (18) with the corresponding equation for K1980 in subsection 5.2, we see one extra effect under MwP: cutoff productivity effect, which reflects the effect of changes in the cutoff productivities for exporting on global welfare.

- $\sum_{i=1}^{n} X_{ij}\left(\widehat{w}_i - \widehat{E}_j\right) = \sum_{i=1}^{n} X_{ij}\widehat{w}_i - \left(\sum_{i=1}^{n} X_{ij}\right)\widehat{E}_j = \sum_{i=1}^{n} X_{ij}\widehat{w}_i - E_j\widehat{E}_j = \sum_{i=1}^{n} X_{ij}\widehat{w}_i - \sum_{i=1}^{n} X_{ji}\widehat{w}_j + dX_j^{net}$ where the last equality follows from (10) and the fact that $Y_j = \sum_{i=1}^{n} X_{ji}$.

- Substituting the RHS of the above expression into (18), invoking (11), and then re-arranging terms, we have

$$\begin{aligned} E_j\widehat{U}_j =& \underbrace{\textstyle\sum_{i=1}^{n} X_{ji}\widehat{w}_j - \sum_{i=1}^{n} X_{ij}\widehat{w}_i}_{\text{terms of trade effect}} + \underbrace{\textstyle\sum_{i=1}^{n} X_{ij}\widehat{\tau}_{ij}}_{\text{direct effect}} - \underbrace{dX_j^{net}}_{\text{trade imbalance effect}} - \underbrace{\frac{1}{\gamma}\textstyle\sum_{i=1}^{n} X_{ij}\widehat{N}_i}_{\text{entrants effect}} \\ & - \underbrace{\left(\frac{1}{\sigma-1} - \frac{1}{\gamma}\right)\left(\textstyle\sum_{i=1}^{n} X_{ij}\widehat{w}_i - \sum_{i=1}^{n} X_{ji}\widehat{w}_j + dX_j^{net}\right)}_{\text{cutoff productivity effect}} \end{aligned} \tag{19}$$

- Comparing (14) and (19), we see two extra effects under MwP compared with perfect competition: the cutoff productivity effect and the entrants effect (which correspond to the "Extensive Margin: Selection" and "Extensive Margin: Entry" effects in equation (14) of Costinot and Rodriguez-Clare, 2013) under MwP. Both of these constitute the total effect of changes in the extensive margins of trade on $E_j\widehat{U}_j$.

- Free entry condition implies: $\left(\frac{\sigma\gamma}{\sigma-1}\right) N_i w_i F_i = \left(\frac{\sigma\gamma}{\sigma-1}\right)\Pi_i = R_i = w_i L_i \quad\Rightarrow\quad \widehat{N}_i = 0$. In other words, Property P1 ensures that the "entrants effect" for an individual country is zero, which implies that it does not affect the welfare of an individual country.[22]

- $\sum_{j=1}^{n}\sum_{i=1}^{n} X_{ji}\widehat{w}_j - \sum_{j=1}^{n}\sum_{i=1}^{n} X_{ij}\widehat{w}_i = 0$ conservation of mass.

- $\sum_{j=1}^{n} dX_j^{net} = 0$ trade imbalance effects offset each other.

- From the last two steps, we see that the cutoff productivity effects also offset each other when summing over all countries. This is the consequence of property P2.

- Since the entrants effect and cutoff productivity effect are zero, the extensive margins effect is nullified from the global welfare point of view.

[22] However, the entrants effect for an individual country will not be zero under the multi-sector extension, as we shall see in section 6.1.

- As only the direct effect remains, global welfare change is given by[23]

$$\frac{1}{Y^w}\sum_{j=1}^{n} E_j \widehat{U}_j = -\sum_{j=1}^{n}\sum_{i=1}^{n} \frac{X_{ij}}{Y^w}\widehat{\tau}_{ij}$$

The proofs for the one-sector perfect competition models in section 4 and one-sector monopolistic competition models in section 5 form the basis of the proofs in the extensions discussed in sections 6 below.

5.4 Discussion

Why does the conclusion of the envelope theorem apply to the two MC-VEM models discussed in this section? In section 4, we show that the envelope theorem applies to the PC-VEM model because the extensive margins effect is zero in each country (due to complete specialization). Likewise, in order for the envelope theorem to apply to K1980 and MwP, the extensive margins effects must cancel from the global welfare point of view. What is the underlying mechanism that leads to the cancellation of the extensive margins effects in these two models?

In the homogeneous firm model of K1980, CES preferences implies P1, which implies that the entrants effect is zero in each country, i.e. the number of entrants is constant in each country. As all entrants survive and all goods are sold in all markets, this means that the number of varieties produced and exported by each country is constant. Therefore, the extensive margins effect is zero for each country.

In the MwP model, CES preferences and Pareto distribution of firm productivities implies P1 and P2. P1 ensures that the entrants effect is zero in each country. Although there are changes to the productivity cutoffs for exporting, P2 ensures that the cutoff productivity effects cancel each other when summing over all countries. Therefore, in the end, the extensive margins effects is equal to zero from the global welfare point of view.

6 Extensions

In this section, we consider a number of extensions of the one-sector models in the last two sections to the cases with multiple sectors and multiple factors, existence of a non-traded good sector and the

[23] If we assume that the fixed exporting cost is of the form $Aw_i^{\mu}w_j^{1-\mu}$, global welfare change is equal to $\frac{1}{Y^w}\sum_{j=1}^{n}E_j\widehat{U}_j = \frac{1}{Y^w}\left[\sum_{j=1}^{n}X_j^{net}\left(\frac{1}{\sigma-1}-\frac{1}{\gamma}\right)(1-\mu)\,\widehat{w}_j - \sum_{j=1}^{n}\sum_{i=1}^{n}X_{ij}\widehat{\tau}_{ij}\right]$. In other words, we need balanced trade in order for (9) to hold. However, according to Axtell (2001), the obsered distribution of firm revenue is Pareto with a shape parameter very close to one. Based on MwP, the distribution of firm revenue is also Pareto, with shape parameter equal to $\frac{\gamma}{\sigma-1}$. For this to be close to one means that $\gamma \approx \sigma - 1$, and so the effects of trade balance X_j^{net} is theoretically non-zero, but empirically unimportant. Consequently, our main result (9) continues to hold as a good approximation.

existence of tradable intermediate goods. We conclude that Propositions 2 and 3 continue to hold under these settings. In other words, the structure of the trade model does not affect the property that the change in global welfare only depends on the direct effect of the reduction of trade costs. The proofs in these cases are fundamentally similar to those explained in the last section. Because of the proofs are long, we only outline the intuition below while relegating the detailed proofs to the appendix.

6.1 Multiple Sectors and multiple factors

The virtue of extending to the multiple sector setting is that we can allow for different trade costs in different sectors, which is an important empirical fact. Suppose that the set of goods $\omega \in \Omega$ is separated into groups denoted by Ω^s where $s = 1, ..., S$, and Ω^s is referred to as sector s.

Consumers in country j have their preferences represented by the following utility function:[24]

$$U_j = \prod_{s=1}^{S} [U_j(s)]^{\eta_s}$$

where $0 \leq \eta_s \leq 1$ denotes the (constant) share of expenditure on goods in sector s in any country, $q_{ij}^s(\omega)$ denotes the consumption of variety ω in sector s in country j of goods originating from country i. Moreover, $\sum_s \eta_s = 1$ for the utility function to be constant returns to scale.

PC-VEM

Under perfect competition, the sectoral sub-utility function $U_j(s)$ can be any well-behaved constant returns to scale function of the form $U_j(s) = U_{js}\left(\left\{ q_{ij}^s(\omega) \middle| \ \forall i \text{ and for } \omega \in \Omega_{ij}^s \right\}\right)$. The proof for the perfect competition case is quite straightforward, and is relegated to the appendix.

MC-VEM

Under monopolistic competition, we need to assume the CES utility function:

$$U_j(s) = \left[\sum_{i=1}^{n} \int_{\omega \in \Omega_{ij}^s} q_{ij}^s(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}}$$

We start with multiple sectors and single factor setting in order to gain the basic intuition. Then we generalize the result to multiple sectors and multiple factors, which is rather straightforward.

We prove the following proposition in the appendix.

Proposition 4 *If there are multiple sectors, multiple factor inputs and in each sector there is* (i) *perfect competition, variable extensive margins of trade and complete specialization, or* (ii) *monopolistic competition as modeled by Krugman (1980) or Melitz (2003) with Pareto distribution of firm productivity,*

[24] Assuming CES preferences across sectors instead of Cobb-Douglas preferences does not affect our result stated in Proposition 4.

then the change in global welfare is given by

$$\sum_{j=1}^{n} \frac{E_j}{Y^w} \widehat{U}_j = -\sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{s=1}^{S} \frac{X_{ij}^s}{Y^w} \widehat{\tau_{ij}^s}.$$

Therefore, if we assume that $\tau_{ij}^s = \tau_{ij}$ for any $s = 1, ..., S$, the change in global welfare again reduces to (9).

Multiple Sectors and Single Factor

As before, we assume that there is a single factor input labor. Under monopolistic competition with free entry, a firm in any sector in country i needs to pay an entry cost of $w_i F_i$ to develop a blueprint, which gives it monopoly power.[25]

Here we outline the intuition of Proposition 4 by comparing the multi-sector models with the one-sector models. Under perfect competition, the proof of Proposition 4 concerning each sector is very similar to that of the proof of Proposition 2, and so will not be elaborated here. Under Krugman (1980), we have

$$E_j \widehat{U}_j = \left(\sum_{i=1}^{n} \sum_{s=1}^{S} X_{ji}^s \widehat{w}_j - \sum_{i=1}^{n} \sum_{s=1}^{S} X_{ij}^s \widehat{w}_i - dX_j^{net} \right) - \sum_{i=1}^{n} \sum_{s=1}^{S} X_{ij}^s \left(\widehat{\tau}_{ij}^s - \frac{\widehat{N_i^s}}{\sigma - 1} \right)$$

where the superscript s denotes variables pertaining to sector s.

Under MwP, the change of welfare in country j is given by

$$E_j \widehat{U}_j = \left(\frac{1}{\sigma - 1} - \frac{1}{\gamma} + 1 \right) \left(\sum_{i=1}^{n} \sum_{s=1}^{S} X_{ji}^s \widehat{w}_j - \sum_{i=1}^{n} \sum_{s=1}^{S} X_{ij}^s \widehat{w}_i - dX_j^{net} \right) - \sum_{i=1}^{n} \sum_{s=1}^{S} X_{ij}^s \left(\widehat{\tau}_{ij}^s - \frac{\widehat{N_i^s}}{\gamma} \right) \qquad (20)$$

It is instructive to compare the above two equations with (14) and (19) in the one-sector monopolistic competition model. One important difference between the two sets of equations is that the entrants effect on individual country's welfare captured by $\widehat{N_i^s}$ here is not zero anymore under free entry with a fixed overhead cost for each variety. In other words, $\widehat{N_i^s} \neq 0$ under free entry, as resources reallocate across sectors as trade costs change. For example, Fan, Lai and Qi (2012) describe the "inter-sectoral resource allocation" (IRA) effect in the face of trade liberalization (same as the entrants effect in this paper) in a multi-sectoral model with intrasectoral firm heterogeneity. In general, the change in number of entrants in a sector may have a positive or negative welfare effect on domestic and foreign countries. From the global welfare point of view, the entrants effects from different sectors of the same country completely offset each other, as shown below. As before, the terms of trade effects and trade imbalance effects from different countries offset each other. This leaves the direct price effect as the only relevant effect on global welfare.

The mechanism is as follows. An increase in the number of entrants in a country increases the consumer welfare in other countries as the variety of goods available for the latter increases. Therefore,

[25] Our results continue to hold if we assume the entry costs are different across sectors.

a more positive $\widehat{N_i^s}$ tends to increase $\widehat{U}_j$. We calculate global welfare impact, $\sum_{j=1}^{n}\frac{E_j}{Y^w}\widehat{U}_j$, based on the equation for $\widehat{U}_j$ above. In the appendix, we show that $\sum_{i=1}^{n}\sum_{s=1}^{S}\sum_{j=1}^{n}\left(X_{ij}^s\widehat{N_i^s}\right)=0$ due to property P1 and $w_iL_i=\sum_{s=1}^{S}R_i^s$. This means that the global welfare impact of the entrants effects originating from different sectors of the same country i offset each other.

Note that the multi-sector model cannot collapse into a single-sector model because of the inter-sectoral resource allocation (IRA) effect. The IRA effect explains why ACR's equation works for a single-sector model but not in a multi-sector model, as they need the welfare impact of the entrants effects on country i originating from different sectors of the same country i to offset each other, which is not true. For us, the IRA effect does not invalidate our equation because all we need is for the global welfare impact of the entrants effects originating from different sectors of the same country i to offset each other, which is true.

Examples of models to which this equation applies are: multi-sector extensions of MwP, Hsieh and Ossa (2011), multi-sector extensions of Eaton-Kortum (2002), Chor (2010), Donaldson (2010), Costinot, Donaldson and Komunjer (2012).

Multiple Sectors and Multiple Factors

Now, we assume that the factor endowment in country i is given by L_{ki}, where $k=1,...,K$ denote indexes of factors. The utility function is the same as in the last subsection. The production function of a firm in sector s of country i is given by $f_i^s(l_1,...,l_K;\omega)=A_i(\omega)\prod_{k=1}^{K}(l_k)^{\beta_k(s)}$, where $\beta_k(s)$ is the cost share of factor k in sector s so that $\sum_{k=1}^{K}\beta_k(s)=1$ for all s; l_k is the input of factor k; $A_i(\omega)$ denotes the firm's productivity. To be consistent with the one-sector model, the cost function for producing quantities $\mathbf{q}\equiv\{q_{ij}^s(\omega)\}$ is given by

$$C_i^s(v_i^s,\mathbf{q},\omega)=\sum_{j=1}^{n}\left[\tau_{ij}^s v_i^s a_i(\omega)\,q_{ij}^s(\omega)+\xi_{ij}v_i^s\cdot\mathbf{1}\left(q_{ij}^s(\omega)>0\right)\right]$$

where $v_i^s=\prod_{k=1}^{K}\left(\frac{w_{ki}}{\beta_k(s)}\right)^{\beta_k(s)}$ is called the aggregate factor cost in sector s of country i; and total factor productivity in the production of good ω is $A_i(\omega)\equiv 1/a_i(\omega)$. Compared with the single-factor model, the relevant factor cost is aggregate factor cost v_i^s instead of wage w_i. The environment of the perfect competition model is the same as discussed under the heading PC-VEM in this subsection. Under monopolistic competition with free entry, each firm from country i needs to pay $v_i^sF_i$ to acquire a blueprint. The proof of the multi-sector, multi-factor case is very similar to that of the multi-sector, single factor case above. We simply replace w_i by v_i^s and $\widehat{w}_i$ by $\widehat{v}_i^s$ for all i.

Examples of models to which this equation applies are: Bernard, Redding and Schott (2007), Burstein and Vogel (2010) and Heckscher-Ohlin models with complete specialization such as Dornbusch-Fisher-Samuelson (1980).

6.2 Multi-stage Production, Perfect Competition, VEM

In this extension we allow some goods to be used only as intermediate inputs into the production of other goods in a multi-stage production setting. We assume perfect competition, complete specialization and variable extensive margins of trade. To simplify exposition, we assume that there are two stages of production, though the result is valid for a setting with more than two stages. The final good is produced in two sequential stages. Utility is a constant returns to scale function of the consumption of a continuum of stage-2 outputs indexed by $\omega \in \Omega$. So, each variety of stage-2 output is a final good. The production of final output of variety ω (which shall be called "final good ω") requires the input of stage-1 output for ω (which shall be called "stage-1 ω") and labor. The production of stage-1 ω requires only labor. All stage-1 and final outputs are tradable, and all countries possess the technologies of production for all stages. The market structure for all goods is assumed to be perfect competition. The production function in the first stage in country i is given by

$$y_{1i}(\omega) = A_{1i}(\omega)\, l_{1i}(\omega) \qquad \text{where } \omega \in \Omega$$

where $y_{1i}(\omega)$ is the output of stage-1 ω, $A_{1i}(\omega) \equiv 1/a_{1i}(\omega)$ is country i's labor productivity associated with the production of stage-1 ω, and $l_{1i}(\omega)$ is country i's labor input in the production of stage-1 ω. Stage-1 ω and labor are combined in a nested Cobb-Douglas production function at the second stage to produce final good ω:

$$y_{2i}(\omega) = x_{1i}(\omega)^{\theta} \left[A_{2i}(\omega)\, l_{2i}(\omega)\right]^{1-\theta} \quad \text{where } \omega \in \Omega$$

where $y_{2i}(\omega)$ is the output of final good ω; $A_{2i}(\omega) \equiv 1/a_{2i}(\omega) \Longrightarrow \left[A_{2i}(\omega)\right]^{1-\theta} \equiv \left[a_{2i}(\omega)\right]^{\theta-1}$, which is country i's total factor productivity associated with the production of final good ω; $l_{2i}(\omega)$ is country i's labor input in the production of final good ω; $x_{1i}(\omega)$ is country i's use of stage-1 ω for final good production, and the constant θ represents the cost share of the intermediate input $x_{1i}(\omega)$. The utility function is constant returns to scale, given by

$$U_j = U_j\left(\{x_{2j}(\omega) |\, \omega \in \Omega\}\right),$$

where $x_{2j}(\omega)$ is country j's consumption of final good ω. The consumer price index in country j is therefore given by

$$P_j = P_j\left(\{p_{ij}(\omega) |\, \omega \in \Omega_{ij},\, \forall\, i\}\right)$$

where $p_{ij}(\omega)$ is the price in j of final good ω imported from i.

We prove the following proposition in the appendix.

Proposition 5 *When there is multi-stage production under perfect competition, with variable extensive margins of trade and complete specialization, equation (9) holds.*

A detailed proof is given in the Appendix.

When there are many stages of production, the effect of a small change in trade cost in each stage will accumulate to affect the consumer price index, and this direct effect in each stage is only related to the trade value and trade cost in that stage. Again, the indirect effects offset each other from the global welfare point of view, just like in section 4. As a result, the change in global welfare resulting from small changes in trade costs is only affected by the cumulative trade value over all stages between each and every pair of trading partners and changes in trade costs between each and every pair of trading partners.

Examples of models to which this result applies is the one-factor version of the models in Yi (2003) and Yi (2010).

6.3 Other extensions

In the appendix, we prove the following two propositions. The first one is

Proposition 6 *When there is a tradable intermediate good in the production process, then under perfect competition and complete specialization, equation (9) holds.*

In contrast, ACR (2012) show that their equation does not hold in this setting even under perfect competition. Examples of models to which this equation applies are Eaton and Kortum (2002), Dekle, Eaton and Kortum (2007, 2008), Alvarez and Lucas (2007).

The next one is

Proposition 7 *If there exists a non-traded good sector, and, for the tradable-good sector, there is* (i) *perfect competition, variable extensive margins of trade and complete specialization, or* (ii) *monopolistic competition as modeled by Krugman (1980) or Melitz (2003) with Pareto distribution of firm productivity, then equation (9) holds.*

To summarize, equation (9) can be regarded as a rather general result. It is valid under perfect competition, with fixed or variable extensive margins of trade, multiple sectors, multiple factors, multiple-stage production, existence of a tradable intermediate good and existence of a non-tradable good sector. There is no need to assume balanced trade or any utility function. Therefore, its application to the perfect competition setting is really general.

Even under the monopolistic competition settings of Krugman (1980) and MwP, two of the most commonly used models in its category, (9) is valid when extended to multiple sectors, multiple factors and the existence of a non-tradable good sector.

In other words, the envelope theorem is very powerful in explaining the global welfare impact of reduction of trade costs. In fact, we believe it can be applied also to changes in population, productivity and other exogenous shocks to one or many countries at the same time. This is left for future research.

7 Empirical Applications

Equation (9) can be easily implemented empirically without any complicated computation or econometrics.

First, we apply equation (9) to estimate the change in global welfare due to the reduction of trade costs throughout the world in the last five decades. Hummels (2007) and Hummels and Schaur (2012) find that there were declines in the costs of air freight and overland transport, but rises in the ocean freight rates in the post-WWII period. Consequently, there were structural changes in the composition of the modes of international shipping and declines in the costs of distant transport. They estimate the time costs in shipping and interpret them as trade costs. They show that each day of shipping time is equivalent to an ad-valorem trade cost of 0.6 to 2.3 percent. We adopt their estimate and follow their method by assuming that average shipping time was 40 days for international trade in 1960. At that time, almost all international shipments were through ocean freight. We also assume that there was gradual reduction of ocean shipping time and a gradual substitution towards air freight since 1960. The average international ocean shipping time in 2010 was 20 days and air-shipped trade rose from (approximately) 0% to 50% from 1960 to 2010.[26] Thus, the average shipping days dropped from 40 to $0.5 * 20$ (by sea) + $0.5 * 1$ (by air) = 10.5 days during the period 1960-2010 (assuming that air freight takes just one day). As we assume the cost reduction process to be gradual (i.e. the rate of reduction is constant throughout the fifty years), the number of shipping days dropped by $(40 - 10.5)/50 = 0.59$ per year during the period 1960-2010, which is equivalent to a reduction of ad-valorem trade cost by $[0.354\%, 1.357\%]$ per year. Based on the data of the share of trade in GDP from World Development Indicator (WDI) published by the World Bank, we calculate from equation (9) the cumulative global welfare gains in the last five decades from the saving in shipping time to be $[2.98\%, 8.81\%]$.[27] Note that we obtain these estimates by calculating the annual percentage increase in global welfare each year from 1960 through 2009, and then calculate the cumulative welfare gains from these fifty numbers.[28] The

[26] These numbers are based on U.S. trade statistics. Given that the shipping industry has been very competitive, we think it is reasonable to assume that these numbers also apply to all other countries of the world. A sharp speeding up of ocean transport followed from the introduction of containerization in the late 1960s and 1970s. To simplify the calculation, we assume that the annual rate of reduction of trade cost is constant during this period.

[27] These estimates seem to be quite small. However, they are roughly in line with the finding of, say, Eaton and Kortum (2002). From Table IX there, we can calculate the GDP-share-weighted average of the welfare gains of the nineteen OECD countries from moving from autarky to the trade situation in 1990 to be 1.72%.

[28] Let μ_t be the percentage global welfare gain in year t. The cumulative percentage increase in global welfare is equal to $\left[\prod_{t=1960}^{2009}(1+\mu_t)\right] - 1$.

yearly changes in trade costs were sufficiently small for our equation to be valid, as our equation only applies to small percentage changes.[29],[30]

In another empirical application, we consider a counterfactual experiment of a one-percent reduction in all bilateral trade costs in the year 2010. This reduction can be due to technological improvements or other exogenous changes. Based on our theory, we only need to know the share of total trade value in world GDP in order to calculate the impact on global welfare. The data shows that global trade volume was equal to 22.8% of world GDP in the year 2010. It follows from (9) that the elasticity of global welfare with respect to trade cost is equal to 0.228. In other words, one percent reduction in all bilateral trade costs would increase global welfare by 0.228% of world GDP in 2010, i.e. approximately USD 144 billion. Likewise, shortening international shipping time by one day would have yielded global gains in the range [107, 328] billion USD in the year 2010, based on Hummels and Schaur's estimates above.

8 Conclusion and caveats

Our paper is motivated by the following question: How much does trade facilitation matter to the world? Guided by this question, we derive a simple equation to evaluate the quantitative impact of reduction of bilateral trade costs to world welfare. Although our equation cannot evaluate the distribution of gains for different countries, it informs us of the increase in the size of the global pie. Given that there are many channels for transfers between countries, a larger pie can very well make every country better off. Larger global gains from bilateral trade costs reduction provide stronger support to the advocates of global trade facilitation such as WTO, OECD, and the World Bank.

We have derived a simple equation for computing the global welfare gains from small reduction of bilateral trade costs, such as shipping costs or the costs of administrative barriers to trade. Surprisingly, the equation is applicable to a broad class of models and settings. We then carry out a few empirical applications. We find the estimates to be reasonable and consistent with other estimates in the literature.

Our paper distinguishes from other works in the literature in a few key aspects. First, not only have we proved that only the direct effect matters, but we have also proved that the underlying mechanism driving the result is the envelope theorem. This deep intuition has not been explained clearly in the literature. Second, we rigorously justify the use of the expenditure-share-weighted average percentage change of country welfare as a measure of the change of global welfare, based on the concept of com-

[29] Any change in trade costs less than 10% can be regarded as small enough for our equation to hold with sufficient accuracy.

[30] If we had a panel data of pairwise shipping time for all trading partners, or a panel of detailed data on freight rates for all importers, we could carry out a more accurate calculation of global welfare gains due to savings in shipping costs or other types of trade costs. Unfortunately, these data are not readily available at present. Therefore, we leave it for future research.

pensating variation. Third, we do not require balanced trade in any country, while most other papers in the literature require balanced trade in all countries. The reason we do not need balanced trade is the same as the reason why envelope theorem is the underlying mechanism driving our main result. According to the first fundamental theorem of welfare economics, the market outcome is the same as the global planner's solution. From the global planner's point of view, global trade balance is equal to zero. Therefore, it does not matter to him. Fourth, we are able to demonstrate the user-friendliness of our formula by carrying out a few simple empirical applications.

One class of models that this paper does not include is the imperfect competition models with endogenous and variable markups. However, the empirical evidence about whether the existence of such pro-competitive effect leads to upward or downward bias of the estimate of welfare impact is mixed so far. Another effect our model does not capture is the dynamic gains from trade costs reduction, such as learning by doing or learning by exporting. This is an interesting and potentially important source of gains from trade. It is left for future research.

Our simple equation provides an excellent benchmark for future research. The framework can be extended to analyze more sophisticated settings and to answer more questions. Here are a few directions for future research. First, our framework can also be applied to analyze the worldwide effects of other shocks, such as endowment changes or innovation originating from a set of countries. Second, while the global size of gains from reduction of trade costs is independent of the indirect effects, the global distribution of gains is not. It would be interesting to modify the present analysis to calculate the distribution of global gains from trade cost reduction using different trade models, and compare the results. Third, our present analysis can also be extended to calculate the welfare impacts of tariff reductions when the effects of tariff revenues are properly accounted for.

Appendix

(Note that some of these appendixes can be put online in the interest of space once the paper is accepted for publication)

A Proof for section 2

$$
\begin{aligned}
&\frac{\sum_{i=1}^{n}(P_i+dP_i)(dU_i)}{\sum_{i=1}^{n}(P_i+dP_i)U_i}-\frac{\sum_{i=1}^{n}P_i(dU_i)}{\sum_{i=1}^{n}P_iU_i}\\
&=\frac{\left[\sum_{i=1}^{n}(P_i+dP_i)(dU_i)\right]\left[\sum_{i=1}^{n}P_iU_i\right]-\left[\sum_{i=1}^{n}P_i(dU_i)\right]\left[\sum_{i=1}^{n}(P_i+dP_i)U_i\right]}{\left[\sum_{i=1}^{n}(P_i+dP_i)U_i\right]\left[\sum_{i=1}^{n}P_iU_i\right]}\\
&=\frac{\left[\sum_{i=1}^{n}P_i(dU_i)\right]\left[\sum_{i=1}^{n}P_iU_i\right]-\left[\sum_{i=1}^{n}P_i(dU_i)\right]\left[\sum_{i=1}^{n}P_iU_i\right]}{\left[\sum_{i=1}^{n}(P_i+dP_i)U_i\right]\left[\sum_{i=1}^{n}P_iU_i\right]}+O\left(\epsilon^2\right)\\
&=0
\end{aligned}
$$

where the last line stems from ignoring $O\left(\epsilon^2\right)\equiv\frac{\sum_{i=1}^{n}\sum_{j=1}^{n}dP_i\cdot P_j\cdot(dU_iU_j-U_idU_j)}{\left[\sum_{i=1}^{n}(P_i+dP_i)U_i\right]\left[\sum_{j=1}^{n}P_jU_j\right]}$, which is a residual that contains all terms of order higher than the first.

B Proof of Proposition 4 (Multi-sector and multi-factor)

Multiple Sectors and Single Factor

We proceed with the proof in two steps. The consumer price index is given by

$$P_j=\prod_{s=1}^{S}\left[P_j(s)\right]^{\eta_s} \tag{21}$$

where $P_j(s)$ denotes the aggregate price index for sector s.

Step 1: The *change in global welfare is given by:*

$$\sum_{j=1}^{n}\frac{E_j}{Y^w}\widehat{U}_j=\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{s=1}^{S}\frac{X_{ji}^{s}}{Y^w}\widehat{w}_j+\sum_{j=1}^{n}\frac{dX_j^{net}}{Y^w}-\sum_{j=1}^{n}\sum_{s=1}^{S}\frac{\eta_sE_j}{Y^w}\widehat{P}_j(s). \tag{22}$$

Proof: Totally differentiating the consumer price index (21), we get

$$\widehat{P}_j=\sum_{s=1}^{S}\eta_s\widehat{P}_j(s).$$

Similar to step 1 of the proofs in Propositions 2 and 3, we have $\widehat{U}_j=\frac{Y_j}{E_j}\widehat{w}_j-\frac{dX_j^{net}}{E_j}-\widehat{P}_j$ under both perfect competition and monopolistic competition. Then, combining the fact that $\sum_{s=1}^{S}\eta_s=1$ and the last equation, we have

$$\widehat{U}_j=\frac{Y_j}{E_j}\widehat{w}_j-\frac{dX_j^{net}}{E_j}-\sum_{s=1}^{S}\eta_s\widehat{P}_j(s)$$

Therefore,

$$\textstyle\sum_{j=1}^{n}\frac{E_j}{Y^w}\widehat{U}_j=\sum_{j=1}^{n}\frac{Y_j}{Y^w}\widehat{w}_j-\sum_{j=1}^{n}\frac{dX_j^{net}}{Y^w}-\sum_{j=1}^{n}\sum_{s=1}^{S}\frac{\eta_s E_j}{Y^w}\widehat{P}_j\left(s\right)$$

which implies (22), since $Y_j=\sum_{i=1}^{n}\sum_{s=1}^{S}X_{ji}^{s}$.

Step 2: The *change in global welfare is given by:*

$$\textstyle\sum_{j=1}^{n}\frac{E_j}{Y^w}\widehat{U}_j=-\sum_{j=1}^{n}\sum_{i=1}^{n}\sum_{s=1}^{S}\frac{X_{ij}^{s}}{Y^w}\widehat{\tau}_{ij}^{s}. \tag{23}$$

Proof: Here, we will show that equation (23) holds under both perfect competition and monopolistic competition.

Under Perfect competition:

Based on the same derivation as in the proof of Proposition 2, equation (12) continues to hold for each of the sectors. In other words, the change in the aggregate price index $P_j\left(s\right)$ in sector s is given by

$$\widehat{P}_j\left(s\right)=\textstyle\sum_{i=1}^{n}\lambda_{ij}^{s}\left(\widehat{w}_i+\widehat{\tau}_{ij}^{s}\right)$$

where $\lambda_{ij}^{s}\equiv X_{ij}^{s}/\left(\eta_s E_j\right)$ is the penetration ratio of country i's exports in sector s of country j. Substituting this expression into equation (22), we obtain

$$\begin{aligned}\textstyle\sum_{j=1}^{n}\frac{E_j}{Y^w}\widehat{U}_j&=\sum_{i,j,s}\frac{X_{ji}^{s}}{Y^w}\widehat{w}_j+\textstyle\sum_{j=1}^{n}\frac{dX_j^{net}}{Y^w}-\sum_{i,j,s}\frac{X_{ij}^{s}}{Y^w}\left(\widehat{w}_i+\widehat{\tau}_{ij}^{s}\right)\\&=-\sum_{i,j,s}\frac{X_{ij}^{s}}{Y^w}\widehat{\tau}_{ij}^{s},\qquad\text{which is (23)}\end{aligned}$$

where the second line stems from $\sum_{j=1}^{n}X_j^{net}\equiv0$ and $\sum_{i,j,s}\frac{X_{ji}^{s}}{Y^w}\widehat{w}_j=\sum_{i,j,s}\frac{X_{ij}^{s}}{Y^w}\widehat{w}_i$, where $\sum_{i,j,s}\equiv\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{s=1}^{S}$

K1980

We have

$$\widehat{P}_j\left(s\right)=\textstyle\sum_{i=1}^{n}\lambda_{ij}^{s}\left[\widehat{w}_i+\widehat{\tau}_{ij}^{s}-\left(\frac{1}{\sigma-1}\right)\widehat{N_i^s}\right]$$

However, free entry condition does not imply that $\widehat{N_i^s}=0$ anymore since L_j^s is not constant due to re-allocation of labor across sectors as τ_{ij} changes. Substituting this expression into (22), we have:

$$\begin{aligned}\textstyle\sum_{j=1}^{n}\frac{E_j}{Y^w}\widehat{U}_j&=\sum_{j=1}^{n}\left(\frac{Y_j}{Y^w}\widehat{w}_j-\frac{dX_j^{net}}{Y^w}\right)-\sum_{i,j,s}\frac{X_{ij}^{s}}{Y^w}\left[\widehat{w}_i+\widehat{\tau}_{ij}^{s}-\frac{\widehat{N_i^s}}{\gamma}\right]\\&=\sum_{j=1}^{n}\left(\sum_{i=1}^{n}\sum_{s=1}^{S}\frac{X_{ji}^{s}}{Y^w}\widehat{w}_j-\sum_{i=1}^{n}\sum_{s=1}^{S}\frac{X_{ij}^{s}}{Y^w}\widehat{w}_i-\frac{dX_j^{net}}{Y^w}\right)-\sum_{j,i,s}\frac{X_{ij}^{s}}{Y^w}\left(\widehat{\tau}_{ij}^{s}-\frac{\widehat{N_i^s}}{\gamma}\right)\\&=-\sum_{j,i,s}\frac{X_{ij}^{s}}{Y^w}\left(\widehat{\tau}_{ij}^{s}-\frac{\widehat{N_i^s}}{\gamma}\right)\end{aligned}$$

MwP

Based on the same derivation as in the proof of Proposition 3, equation (18) continues to hold from the perspective of the sector based on property P2. Hence, the aggregate price index in sector s satisfies:

$$\widehat{P}_j(s) = \sum_{i=1}^{n}\lambda_{ij}^s\left[\left(\frac{1}{\sigma-1}-\frac{1}{\gamma}+1\right)\widehat{w}_i+\widehat{\tau}_{ij}^s-\frac{\widehat{N_i^s}}{\gamma}\right]-\left(\frac{1}{\sigma-1}-\frac{1}{\gamma}\right)\widehat{E}_j$$

where N_i^s is the expected mass of entrants in sector s in country i. Substituting this expression into (22), we have:

$$\begin{aligned}\sum_{j=1}^{n}\frac{E_j}{Y^w}\widehat{U}_j &= \left(\frac{1}{\sigma-1}-\frac{1}{\gamma}+1\right)\sum_{j=1}^{n}\left(\frac{Y_j}{Y^w}\widehat{w}_j-\frac{dX_j^{net}}{Y^w}\right)-\sum_{i,j,s}\frac{X_{ij}^s}{Y^w}\left[\left(\frac{1}{\sigma-1}-\frac{1}{\gamma}+1\right)\widehat{w}_i+\widehat{\tau}_{ij}^s-\frac{\widehat{N_i^s}}{\gamma}\right]\\ &= \left(\frac{1}{\sigma-1}-\frac{1}{\gamma}+1\right)\sum_{j=1}^{n}\left(\sum_{i=1}^{n}\sum_{s=1}^{S}\frac{X_{ji}^s}{Y^w}\widehat{w}_j-\sum_{i=1}^{n}\sum_{s=1}^{S}\frac{X_{ij}^s}{Y^w}\widehat{w}_i-\frac{dX_j^{net}}{Y^w}\right)-\sum_{j,i,s}\frac{X_{ij}^s}{Y^w}\left(\widehat{\tau}_{ij}^s-\frac{\widehat{N_i^s}}{\gamma}\right)\\ &= -\sum_{j,i,s}\frac{X_{ij}^s}{Y^w}\left(\widehat{\tau}_{ij}^s-\frac{\widehat{N_i^s}}{\gamma}\right) \qquad (24)\end{aligned}$$

where the second line stems from $Y_j = \sum_{i=1}^{n}\sum_{s=1}^{S}X_{ji}^s$; the third line follows from $\sum_{j=1}^{n}X_j^{net}\equiv 0$ and $\sum_{i,j,s}\frac{X_{ji}^s}{Y^w}\widehat{w}_j = \sum_{i,j,s}\frac{X_{ij}^s}{Y^w}\widehat{w}_i$.

Both K1980 and MwP

Define ζ as $\frac{1}{\sigma}$ under K1980 and as $\frac{\sigma-1}{\sigma\gamma}$ under MwP. Free entry implies that $\zeta R_j^s = \Pi_j^s = N_j^s w_j F_j$ for all s and j. Therefore, $\widehat{R_j^s} = \widehat{N_j^s}+\widehat{w}_j$. Zero net profits implies that total labor income is equal to total income, $w_j L_j = \sum_{s=1}^{S}R_j^s$, which implies that $\widehat{w}_j = \sum_{s=1}^{S}\frac{R_j^s}{\sum_{s=1}^{S}R_j^s}\widehat{R_j^s} = \sum_{s=1}^{S}\frac{R_j^s}{\sum_{s=1}^{S}R_j^s}\left(\widehat{N_j^s}+\widehat{w}_j\right)$, which in turn implies that $\sum_{s=1}^{S}R_i^s\widehat{N_i^s}=0$. This means that

$$\sum_{i,j,s}\frac{X_{ij}^s}{Y^w}\widehat{N_i^s} = \frac{\sum_{i=1}^{n}\sum_{s=1}^{S}R_i^s\widehat{N_i^s}}{Y^w} = 0$$

Combining this expression with equation (24) under MwP and the corresponding equation under K1980, we have:

$$\sum_{j=1}^{n}\frac{E_j}{Y^w}\widehat{U}_j = -\sum_{i,j,s}\frac{X_{ij}^s}{Y^w}\widehat{\tau}_{ij}^s$$

Multiple Sectors and Multiple Factors

$v_i^s = \prod_{k=1}^{K}\left(\frac{w_{ki}}{\beta_k(s)}\right)^{\beta_k(s)}$ implies $\widehat{v}_i^s = \sum_{k=1}^{K}\beta_k(s)\,\widehat{w}_{ki}$. The consumer price index in country j is given by

$$P_j = \prod_{s=1}^{S}\left[P_j(s)\right]^{\eta_s} \qquad (25)$$

In the following proof, we proceed in two steps.

Step 1: *The change in GDP is given by*

$$\widehat{Y}_j = \frac{\sum_{i=1}^{n}\sum_{s=1}^{S} X_{ji}^s \widehat{v}_j^s}{Y_j} \tag{26}$$

Proof: The production function, the exporting cost function, and the entry cost function in each sector are all Cobb-Douglas. As a result, the total expenditure on factor k in sector s is a fraction $\beta_k(s)$ of the total expenditure on all factors in sector s. Under perfect competition and monopolistic competition with free entry, the total expenditure on all factors in country j in sector s is the revenue of country j in this sector $R_j^s = \sum_{i=1}^{n} X_{ji}^s$. Hence the total expenditure on factor k in sector s in country j is $\beta_k(s)\sum_{i=1}^{n} X_{ji}^s$. Therefore, the total expenditure on factor k in country j is $\sum_{s=1}^{S}\beta_k(s)\sum_{i=1}^{n} X_{ji}^s$, which equals $w_{kj}L_{kj}$ by factor market clearing condition.

Under perfect competition and monopolistic competition with free entry, we have $Y_j = \sum_{k=1}^{K} w_{kj}L_{kj}$, which implies:

$$\begin{aligned}
\widehat{Y}_j &= \sum_{k=1}^{K} \frac{w_{kj}L_{kj}}{\sum_{k=1}^{K} w_{kj}L_{kj}} \widehat{w}_{kj} = \sum_{k=1}^{K} \frac{\sum_{s=1}^{S}\beta_k(s)\sum_{i=1}^{n} X_{ji}^s}{Y_j} \widehat{w}_{kj} \\
&= \frac{\sum_{i=1}^{n}\sum_{s=1}^{S}\sum_{k=1}^{K}\beta_k(s) X_{ji}^s \widehat{w}_{kj}}{Y_j} = \frac{\sum_{i=1}^{n}\sum_{s=1}^{S} X_{ji}^s \left(\sum_{k=1}^{K}\beta_k(s)\widehat{w}_{kj}\right)}{Y_j} \\
&= \frac{\sum_{i=1}^{n}\sum_{s=1}^{S} X_{ji}^s \widehat{v}_j^s}{Y_j}
\end{aligned}$$

where the last line follows from $\widehat{v}_j^s = \sum_{k=1}^{K}\beta_k(s)\widehat{w}_{kj}$.

Step 2: The change in global welfare *is given by (23).*

Proof: This step is the same as Step 1 plus Step 2 of the proof for the multi-sector, single factor case, except that we replace w_i by v_i^s and $\widehat{w}_i$ by $\widehat{v}_i^s$ for all i.

C Proof of Proposition 5 (Multi-stage Production)

We proceed to prove this proposition in two steps.

Step 1: *The change in welfare is given by*

$$\widehat{U}_j = \frac{X_j^{net}}{E_j}\widehat{w}_j - \frac{dX_j^{net}}{E_j} - \sum_{i=1}^{n}\sum_{m=1}^{n}\lambda_{mij}\left[\theta\left(\widehat{w}_m - \widehat{w}_i\right) + \widehat{w}_i - \widehat{w}_j + \theta\widehat{\tau}_{mi} + \widehat{\tau}_{ij}\right] \tag{27}$$

where $\lambda_{mij} \equiv \frac{X_{mij}}{E_j}$ denotes the expenditure share of country j on imported final good from country i, whose production use the stage-1 good from country m.

Proof: Based on similar derivation as in the proof of Proposition 2, we can show that equation (12) continues to hold. However, the marginal cost now is related to where the first-stage production happens. Totally differentiating this expression for P_j, we have

$$\widehat{P}_j = \sum_{i=1}^n \sum_{m=1}^n \lambda_{mij} \widehat{c}_{mij} = \sum_{i=1}^n \sum_{m=1}^n \lambda_{mij} \left[\theta \widehat{w}_m + \theta \widehat{\tau}_{mi} + (1-\theta) \widehat{w}_i + \widehat{\tau}_{ij}\right] \tag{28}$$

where $c_{mij} = \left(\frac{w_m \tau_{mi} a_{1m}}{\theta}\right)^\theta \left(\frac{w_i a_{2i}}{1-\theta}\right)^{1-\theta} \tau_{ij}$ denotes the competitive price of a final good imported into country j from country i, whose production use the stage-1 good from country m. Under perfect competition, $Y_j = w_j L_j$ and $X_j^{net} = Y_j - E_j$, which imply $\widehat{U}_j = \widehat{E}_j - \widehat{P}_j = \frac{X_j^{net}}{E_j} \widehat{w}_j - \frac{dX_j^{net}}{E_j} + \widehat{w}_j - \widehat{P}_j$. Substituting (28) into this equation, and invoking $\sum_{i=1}^n \sum_{m=1}^n \lambda_{mij} = 1$, we get equation (27).

Step 2: *The change in global welfare is given by (9).*

Proof: $X_{ij} \equiv \lambda_{ij} E_j$ now consists of two parts: (i) imports of stage-1 goods from country i to country j for producing final goods; this value equals $\theta \sum_{m=1}^n X_{ijm} \equiv \theta \sum_{m=1}^n \lambda_{ijm} E_m$ since for each dollar of country m's imports of final goods from country j, there are θ dollars of imports of stage-1 goods from country i to country j; (ii) imports of final goods from i to j; this value is equal to $\sum_{m=1}^n X_{mij} \equiv \sum_{m=1}^n \lambda_{mij} E_j$. Define $s_j \equiv E_j / Y^w$. Hence,

$$\begin{aligned} X_{ij} &= \sum_{m=1}^n X_{mij} + \theta \sum_{m=1}^n X_{ijm} \\ \Leftrightarrow \lambda_{ij} E_j &= \sum_{m=1}^n \lambda_{mij} E_j + \theta \sum_{m=1}^n \lambda_{ijm} E_m \\ \Leftrightarrow s_j \lambda_{ij} &= \sum_{m=1}^n s_j \lambda_{mij} + \theta \sum_{m=1}^n s_m \lambda_{ijm} \end{aligned} \tag{29}$$

Thus, from (27), we have

$$\begin{aligned} \sum_{j=1}^n s_j \widehat{U}_j &= \sum_{j=1}^n \frac{X_j^{net}}{Y^w} \widehat{w}_j - \sum_{j=1}^n \frac{dX_j^{net}}{Y^w} - \sum_{j=1}^n \sum_{i=1}^n \sum_{m=1}^n s_j \lambda_{mij} \left[\theta \left(\widehat{w}_m - \widehat{w}_i\right) + \widehat{w}_i - \widehat{w}_j + \theta \widehat{\tau}_{mi} + \widehat{\tau}_{ij}\right] \\ &= \sum_{j=1}^n \frac{X_j^{net}}{Y^w} \widehat{w}_j - \theta \sum_{j=1}^n \sum_{i=1}^n \sum_{m=1}^n s_j \lambda_{mij} \left(\widehat{w}_m - \widehat{w}_i + \widehat{\tau}_{mi}\right) - \sum_{j=1}^n \sum_{i=1}^n \sum_{m=1}^n s_j \lambda_{mij} \left(\widehat{w}_i - \widehat{w}_j + \widehat{\tau}_{ij}\right) \\ &= \sum_{j=1}^n \frac{X_j^{net}}{Y^w} \widehat{w}_j - \sum_{j=1}^n \sum_{i=1}^n \left(\theta \sum_{m=1}^n s_m \lambda_{ijm} + \sum_{m=1}^n s_j \lambda_{mij}\right) \left(\widehat{w}_i - \widehat{w}_j + \widehat{\tau}_{ij}\right) \\ &= \sum_{j=1}^n \frac{X_j^{net}}{Y^w} \widehat{w}_j - \sum_{j=1}^n \sum_{i=1}^n s_j \lambda_{ij} \left(\widehat{w}_i - \widehat{w}_j + \widehat{\tau}_{ij}\right) \\ &= \sum_{j=1}^n \sum_{i=1}^n \frac{X_{ji}}{Y^w} \widehat{w}_j - \sum_{j=1}^n \sum_{i=1}^n \frac{X_{ij}}{Y^w} \widehat{w}_i - \sum_{j=1}^n \sum_{i=1}^n \frac{X_{ij}}{Y^w} \widehat{\tau}_{ij} \\ &= -\sum_{j=1}^n \sum_{i=1}^n \frac{X_{ij}}{Y^w} \widehat{\tau}_{ij} \end{aligned}$$

where the second line comes from $\sum_{j=1}^n X_j^{net} \equiv 0$; the third line arises from switching m to i, i to j, j to m in the second term of line two; the fourth line follows from equation (29); the fifth line is from $s_j = \frac{E_j}{Y^w}$, $\lambda_{ij} = \frac{X_{ij}}{E_j}$, and $\sum_{i=1}^n X_{ij} + X_j^{net} = \sum_{i=1}^n X_{ji}$; the last line comes from interchanging i and j in the first term of the second last line. Hence, equation (9) holds.

The following appendices are not for publication. They are for the perusal of the referee only.

D Proof of Proposition 7 (Existence of a non-traded-good sector)

The consumer price index is given by

$$P_j = w_j^b P_j\left(D\right)^{1-b}$$

where $P_j\left(D\right)$ denotes the price index of differentiated goods. Totally differentiating this expression, we have $\widehat{P}_j = b\widehat{w}_j + (1-b)\,\widehat{P}_j\left(D\right)$. In the following, we present the proofs under perfect competition and monopolistic competition separately.

Under Perfect Competition and K1980

Based on the same derivation as in the proof of Proposition 2, equation (12) continues to hold in the differentiated-good sector, i.e.,

$$\widehat{P}_j\left(D\right) = \textstyle\sum_{i=1}^{n} \frac{\lambda_{ij}}{1-b}\left(\widehat{w}_i + \widehat{\tau}_{ij}\right) \tag{30}$$

where $\lambda_{ij} = \frac{X_{ij}}{E_j}$. Thus, the change in real income is given by:

$$\begin{aligned}\widehat{U}_j &= \widehat{E}_j - b\widehat{w}_j - (1-b)\,\widehat{P}_j\left(D\right)\\ &= \frac{X_j^{net}}{E_j}\widehat{w}_j - \frac{dX_j^{net}}{E_j} - \textstyle\sum_{i=1}^{n}\lambda_{ij}\left(\widehat{w}_i - \widehat{w}_j + \widehat{\tau}_{ij}\right)\end{aligned}$$

where the last line stems from equation (30) and $\sum_{i=1}^{n}\lambda_{ij} = 1-b$. Hence, the change in global welfare is given by:

$$\begin{aligned}\textstyle\sum_{j=1}^{n} s_j\widehat{U}_j &= \textstyle\sum_{j=1}^{n}\frac{X_j^{net}}{Yw}\widehat{w}_j - \sum_{j=1}^{n}\frac{dX_j^{net}}{Yw} - \sum_{j=1}^{n}\sum_{i=1}^{n}\frac{X_{ij}}{Yw}\left(\widehat{w}_i - \widehat{w}_j + \widehat{\tau}_{ij}\right)\\ &= \textstyle\sum_{j=1}^{n}\sum_{i=1}^{n}\frac{X_{ji}}{Yw}\widehat{w}_j - \sum_{j=1}^{n}\sum_{i=1}^{n}\frac{X_{ij}}{Yw}\widehat{w}_i - \sum_{j=1}^{n}\sum_{i=1}^{n}\frac{X_{ij}}{Yw}\widehat{\tau}_{ij}\\ &= -\textstyle\sum_{j=1}^{n}\sum_{i=1}^{n}\frac{X_{ij}}{Yw}\widehat{\tau}_{ij}\end{aligned}$$

where the second line comes from $\sum_{j=1}^{n}X_j^{net} \equiv 0$ and $\sum_{i=1}^{n}X_{ij} + X_j^{net} = \sum_{i=1}^{n}X_{ji}$; the last line comes from interchanging i and j in the first term of the second last line. Hence, equation (9) holds under perfect competition.

Under MwP:

Like in perfect competition,

$$\widehat{U}_j = \widehat{E}_j - b\widehat{w}_j - (1-b)\,\widehat{P}_j\left(D\right) \tag{31}$$

Like in step 1 of the proof in Proposition 3, $E_j = Y_j - X_j^{net}$ and $\widehat{E}_j = \frac{Y_j}{E_j}\widehat{w}_j - \frac{dX_j^{net}}{E_j}$ imply that $\widehat{E}_j = \frac{X_j^{net}}{E_j}\widehat{w}_j + \widehat{w}_j - \frac{dX_j^{net}}{E_j}$. Based on the same derivation as in the proof of Proposition 3, equation (18) continues to hold in the differentiated-good sector, i.e.,

$$\widehat{P}_j(D) = \sum_{i=1}^{n} \frac{\lambda_{ij}}{1-b} \left[\left(\frac{1}{\sigma-1} - \frac{1}{\gamma} + 1 \right) \widehat{w}_i + \widehat{\tau}_{ij} - \frac{\widehat{N}_i}{\gamma} \right] - \left(\frac{1}{\sigma-1} - \frac{1}{\gamma} \right) \widehat{E}_j \tag{32}$$

Under MwP with free entry, $\Pi_j = N_j w_j F_j$. Thus, we have $N_j w_j F_j = \Pi_j = \frac{\sigma-1}{\sigma\gamma} R_j = \frac{\sigma-1}{\sigma\gamma} w_j L_j$. This implies that $\widehat{N}_i = 0$. Equations (31) and (32), together with $\widehat{N}_i = 0$ and $s_j = \frac{E_j}{Y^w}$, imply that:

$$\begin{aligned}
&\frac{1}{Y^w} \sum_{j=1}^{n} E_j \widehat{U}_j \\
&= \sum_{j=1}^{n} \left(\frac{1-b}{\sigma-1} - \frac{1-b}{\gamma} + 1 \right) \frac{E_j}{Y^w} \widehat{E}_j - \sum_{j=1}^{n} \frac{bE_j}{Y^w} \widehat{w}_j - \sum_{j=1}^{n} \sum_{i=1}^{n} \frac{X_{ij}}{Y^w} \left[\left(\frac{1}{\sigma-1} - \frac{1}{\gamma} + 1 \right) \widehat{w}_i + \widehat{\tau}_{ij} \right] \\
&= \sum_{j=1}^{n} \left(\frac{1-b}{\sigma-1} - \frac{1-b}{\gamma} + 1 \right) \left(\frac{X_j^{net}}{Y^w} \widehat{w}_j + \frac{E_j}{Y^w} \widehat{w}_j \right) - \sum_{j=1}^{n} \frac{bE_j}{Y^w} \widehat{w}_j - \sum_{j=1}^{n} \sum_{i=1}^{n} \frac{X_{ij}}{Y^w} \left[\left(\frac{1}{\sigma-1} - \frac{1}{\gamma} + 1 \right) \widehat{w}_i + \widehat{\tau}_{ij} \right] \\
&= \sum_{j=1}^{n} \left(\frac{1-b}{\sigma-1} - \frac{1-b}{\gamma} + 1 \right) \frac{X_j^{net}}{Y^w} \widehat{w}_j - \sum_{j=1}^{n} \sum_{i=1}^{n} \frac{X_{ij}}{Y^w} \left[\left(\frac{1}{\sigma-1} - \frac{1}{\gamma} + 1 \right) (\widehat{w}_i - \widehat{w}_j) + \widehat{\tau}_{ij} \right] \\
&= -b \sum_{j=1}^{n} \left(\frac{1}{\sigma-1} - \frac{1}{\gamma} \right) \frac{X_j^{net}}{Y^w} \widehat{w}_j + \left(\frac{1}{\sigma-1} - \frac{1}{\gamma} + 1 \right) \left(\sum_{j=1}^{n} \sum_{i=1}^{n} \frac{X_{ji}}{Y^w} \widehat{w}_j - \sum_{j=1}^{n} \sum_{i=1}^{n} \frac{X_{ij}}{Y^w} \widehat{w}_i \right) - \sum_{j=1}^{n} \sum_{i=1}^{n} \frac{X_{ij}}{Y^w} \widehat{\tau}_{ij} \\
&= -b \sum_{j=1}^{n} \left(\frac{1}{\sigma-1} - \frac{1}{\gamma} \right) \frac{X_j^{net}}{Y^w} \widehat{w}_j - \sum_{j=1}^{n} \sum_{i=1}^{n} \frac{X_{ij}}{Y^w} \widehat{\tau}_{ij}
\end{aligned}$$

where the second equality follows from $\widehat{E}_j = \frac{X_j^{net}}{E_j} \widehat{w}_j + \widehat{w}_j - \frac{dX_j^{net}}{E_j}$ and $\sum_{j=1}^{n} X_j^{net} \equiv 0$; the third equality follows from $(1-b) E_j = \sum_{i=1}^{n} X_{ij}$; the fourth equality follows from $\sum_{i=1}^{n} X_{ij} + X_j^{net} = \sum_{i=1}^{n} X_{ji}$; the last equality follows from interchanging i and j in the first term inside the third set of parentheses in the fourth equality. When $X_j^{net} = 0$ for all j, equation (9) holds. Even when $X_j^{net} \neq 0$, its effect is empirically unimportant, as $\gamma \approx \sigma - 1$ according to Axtell (2001). Consequently, our main result (9) continues to hold as a good approximation.

From the fourth equality, we can see that the total effect is decomposed into the extensive margin effects, the terms of trade effects and the direct effects. The fifth equality shows that the terms of trade effects offset each other, but the extensive margin effects do not add up to zero. The intuition is that the effect of X_j^{net} on the extensive margin is through the tradable goods sector only, as $(1-b) E_j = \sum_{i=1}^{n} X_{ij}$. As a result, the extensive margin effect is equal to $-b \left(\frac{1}{\sigma-1} - \frac{1}{\gamma} \right) \sum_{j=1}^{n} \frac{X_j^{net}}{Y^w} \widehat{w}_j$ instead of zero.

E Proof of Proposition 6 (Existence of a Tradable Intermediate Good)

The expenditure on final goods is still given by $E_j = Y_j - X_j^{net}$, where $X_j^{net} = \sum_{i=1}^{n} X_{ji} - \sum_{i=1}^{n} X_{ij}$ is net exports and $Y_j = w_j L_j$ is GDP of country j (as GDP is equal to total factor income). To take into account the fact that all goods $\omega \in \Omega$ can either be consumed as final goods or used as input in the production of intermediate good, we can write total expenditure by country j, $\widetilde{E_j}$, as the sum of the expenditures on final and intermediate goods. In other words, $\widetilde{E_j} = E_j + (1-\beta) R_j$ where $R_j = \sum_{i=1}^{n} X_{ji}$ denotes total output of intermediate and final goods. In addition, $Y_j = \beta R_j$ since the expenditure share on labor in the production of intermediate and final goods is β.[31] Hence, $R_j = \left(E_j + X_j^{net}\right)/\beta$. Therefore, total expenditure satisfies:

$$\widetilde{E_j} = E_j + \frac{(1-\beta)}{\beta}\left(E_j + X_j^{net}\right) = \frac{1}{\beta}\left[E_j + (1-\beta) X_j^{net}\right]$$

The marginal cost is now related to the expenditure on the intermediate good.

Based on same derivation as in the proof of Proposition 2, equation (12) continues to hold, i.e.,

$$\widehat{P}_j = \sum_{i=1}^{n} \lambda_{ij}\left[\widehat{\delta}_i + \widehat{\tau}_{ij}\right] \tag{33}$$

where $\widehat{\delta}_i = \beta\widehat{w}_i + (1-\beta)\widehat{P}_i$ replaces $\widehat{w}_i$ in the equation for the one-sector model without intermediate goods and $\lambda_{ij} = X_{ij}/\widetilde{E_j}$. Thus, the change in the welfare of country j is given by:

$$\begin{aligned}
\widehat{U}_j &= \frac{Y_j}{E_j}\widehat{w}_j - \frac{dX_j^{net}}{E_j} - \widehat{P}_j \\
&= \frac{X_j^{net}}{E_j}\widehat{w}_j + \widehat{w}_j - \widehat{P}_j - \frac{dX_j^{net}}{E_j} \\
&= \frac{X_j^{net}}{E_j}\frac{1}{\beta}\left[\widehat{\delta}_j - (1-\beta)\widehat{P}_j\right] + \frac{1}{\beta}\left(\widehat{\delta}_j - \widehat{P}_j\right) - \frac{dX_j^{net}}{E_j} \\
&= \frac{X_j^{net}}{E_j}\widehat{\delta}_j + \frac{E_j + (1-\beta)X_j^{net}}{\beta E_j}\left(\widehat{\delta}_j - \widehat{P}_j\right) - \frac{dX_j^{net}}{E_j}
\end{aligned} \tag{34}$$

(33), (34), $\lambda_{ij} = X_{ij}/\widetilde{E_j}$, $\widetilde{E_j} = \frac{1}{\beta}\left[E_j + (1-\beta) X_j^{net}\right]$ and $\sum_{i=1}^{n}\lambda_{ij} = 1$, together imply:

$$\begin{aligned}
\widehat{U}_j &= \frac{X_j^{net}}{E_j}\widehat{\delta}_j - \sum_{i=1}^{n}\frac{X_{ij}}{E_j}\left[\widehat{\delta}_i - \widehat{\delta}_j + \widehat{\tau}_{ij}\right] - \frac{dX_j^{net}}{E_j} \\
&= \sum_{i=1}^{n}\left[\frac{X_{ji}}{E_j}\widehat{\delta}_j - \frac{X_{ij}}{E_j}\widehat{\delta}_i\right] - \frac{dX_j^{net}}{E_j} - \sum_{i=1}^{n}\frac{X_{ij}}{E_j}\widehat{\tau}_{ij}
\end{aligned} \tag{35}$$

where the second line follows from $\sum_{i=1}^{n} X_{ij} + X_j^{net} = \sum_{i=1}^{n} X_{ji}$. Equation (35), together with $s_j = \frac{E_j}{Y^w}$, imply that the percentage change in global welfare is given by:

$$\begin{aligned}
\frac{1}{Y^w}\sum_{j=1}^{n} E_j\widehat{U}_j &= \sum_{j=1}^{n}\sum_{i=1}^{n}\frac{X_{ji}}{Y^w}\widehat{\delta}_j - \sum_{j=1}^{n}\sum_{i=1}^{n}\frac{X_{ij}}{Y^w}\widehat{\delta}_i - \sum_{j=1}^{n}\frac{dX_j^{net}}{Y^w} - \sum_{j=1}^{n}\sum_{i=1}^{n}\frac{X_{ij}}{Y^w}\widehat{\tau}_{ij} \\
&= -\sum_{j=1}^{n}\sum_{i=1}^{n}\frac{X_{ij}}{Y^w}\widehat{\tau}_{ij}
\end{aligned}$$

where the second line comes from $\sum_{j=1}^{n} X_j^{net} \equiv 0$ and $\sum_{j=1}^{n}\sum_{i=1}^{n}\frac{X_{ji}}{Y^w}\widehat{\delta}_j = \sum_{j=1}^{n}\sum_{i=1}^{n}\frac{X_{ij}}{Y^w}\widehat{\delta}_i$.

[31] Now, the total revenue of country j, R_j, is not equal to its GDP, Y_j.

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