Tipping the Climate Dominoes

Derek Lemoine and Christian Traeger

January 2014

Anthropogenic emissions of greenhouse gases activate a number of feedback processes in the climate system. Scientists warn that some feedbacks can lead to abrupt and irreversible changes in climate dynamics, so-called tipping points. The threat of tipping points plays a major role in demands for aggressive emission reductions and for limiting warming to 2°C, as agreed upon in the Copenhagen Accord. We extend a benchmark integrated assessment model of climate change to account for three interacting, irreversible tipping points: (i) a feedback increasing climate sensitivity, (ii) a feedback reducing carbon sink uptake, and (iii) a tipping point directly affecting economic damages. Each tipping point is triggered by an imperfectly known temperature threshold. Optimal mitigation policy has to account for the impact of today's emissions on future carbon stock, temperatures, and damages in different possible futures where tipping points may have been triggered at different temperature thresholds. Optimal mitigation today also has to incorporate how future decision makers will respond to current actions, observations, and possible tipping. We show that the presence of the three tipping points approximately doubles the currently optimal carbon tax and that the presence of multiple tipping points cuts the optimal peak temperature by approximately 1°C. The effects of the individual tipping points are approximately additive, suggesting that interactions among tipping points are not crucial for policy.

1 Introduction

The threat of climate tipping points plays a major role in calling for aggressive emission reductions that might limit warming to 2 degrees Celsius (Hansen et al., 2008; Ramanathan and Feng, 2008; Rockström et al., 2009). Yet integrated assessments of climate change commonly only model slow and reversible climate impacts and quantitative, economic policy analysis of tipping points is sparse. Previous integrated assessments analyzed a single type of tipping point that directly affects economic output (Keller et al., 2004; Lontzek et al., 2012) or a single tipping point that alternatively modifies the carbon cycle or the sensitivity of the climate to emissions (Lemoine and Traeger, 2014). However, scientists are particularly concerned about potential interactions among tipping points (Lenton et al., 2008; Kriegler et al., 2009; Levermann et al., 2012; Kopits et al., 2013; Lenton and Ciscar, 2013). Reducing the effectiveness of carbon sinks or increasing climate sensitivity would amplify future warming, which would in turn make further tipping points more likely. Here, we simultaneously interact tipping points that alter the carbon cycle, strengthen warming feedbacks, and reduce economic output. We solve for the optimal policy in a stochastic integrated assessment model with anticipated Bayesian learning, which recognizes that future policymakers have new information about the location of temperature thresholds and can react to any tipping points that may have already occurred.

2 Model

The first tipping point makes temperature more sensitive to CO_2 emissions. It reflects the possibility that warming mobilizes large methane stores locked in permafrost and in shallow ocean clathrates (Hall and Behl, 2006; Archer, 2007; Schaefer et al., 2011). It also reflects the possibility that land ice sheets begin to retreat on decadal timescales: the resulting loss of reflective ice could double the long-term warming predicted by models that hold land ice sheets fixed (Hansen et al., 2008). Temperature dynamics in DICE depend on the climate sensitivity parameter, which is the equilibrium warming from doubling CO_2. The value of 3°C used in DICE is inferred from climate models that hold land ice sheets and most methane stocks constant. We represent a climate feedback tipping point as increasing climate sensitivity to 5°C (see Supplementary Material for a formal description).

The second tipping point increases the time during which emissions affect the climate. It reflects the possibility that carbon sinks weaken beyond the predictions of coupled climate-carbon cycle models. Warming-induced changes in oceans (Le Quéré et al., 2007), soil carbon dynamics (Eglin et al., 2010), and standing biomass (Huntingford et al., 2008) could affect the uptake of CO_2 from the atmosphere. We represent these weakened sinks by decreasing the decay rate of atmospheric CO_2 by 50%.

The third tipping point directly affects the economic damage function. This damage function encapsulates all impacts from warming, including damages from sea level rise, from habitat loss, and from a weakening thermohaline circulation (Gulf Stream). An unexpected, abrupt change in any of these channels would change this damage function. For example, if the West Antarctic or Greenland ice sheets collapse, sea levels could rise quickly and dramatically (Oppenheimer, 1998; Oppenheimer and Alley, 2004; Vaughan, 2008; Notz, 2009). These higher sea levels would interact with the existing pathways by which warming causes damages. By stressing adaptive capacity, higher sea levels or other loss of productive habitat make damages increase faster with warming. We model such a tipping point as changing the DICE damage function from the conventional assumption of a quadratic in temperature to a cubic in temperature. Whereas others have applied a cubic damage function directly (Stern, 2007; Ackerman et al., 2010), we instead allow the damage function to become cubic only upon triggering this tipping point.

We integrate these three tipping points into the widespread DICE integrated assessment model that is also used by the U.S. government when determining the social cost of carbon for its regulatory analyses (Nordhaus, 1992, 2008; Interagency Working Group on Social Cost of Carbon, 2010; Greenstone et al., 2013). The left schematic in Figure 1 gives a graphical representation of DICE and marks with arrows a, b, and c our tipping point modifications. DICE combines an economic growth model with a simplified climate module. The policymaker decides how to allocate

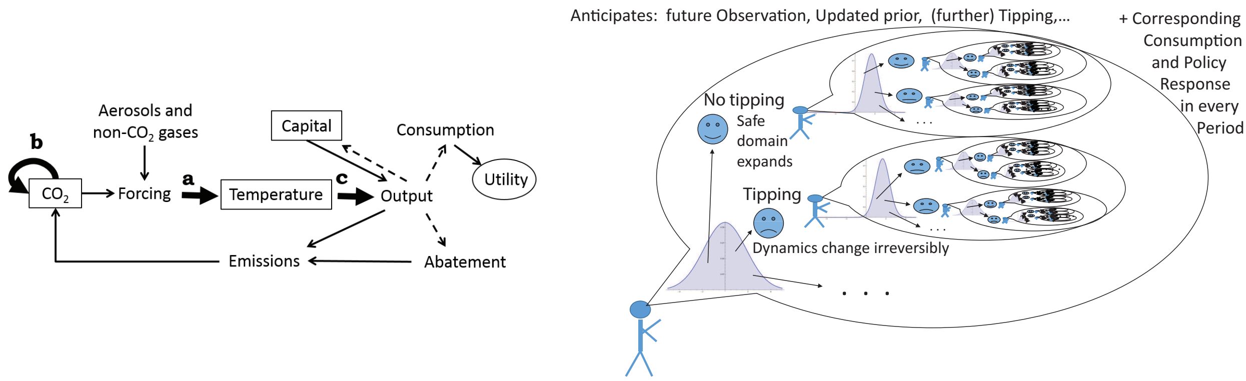

Figure 1: The schematic on the left presents the basic structure of our DICE-based integrated assessment models. Unabated emissions from economic productions accumulate in the atmosphere. Together with other greenhouse gases, the emission stock changes the energy balance of the planet, induces various feedback processes, and increases global surface temperature. The first tipping point (a) increases the feedbacks so that equilibrium warming from a doubling of the CO_2 stock increases from 3°C to 5°C. The second tipping point (b) affects the carbon sinks and reduces the rate of atmospheric carbon removal by 50%. Global surface temperature increase negatively impacts economic production, which is distributed between consumption, investment, and greenhouse gas mitigation expenditure. The third tipping point (c) represents events like sea level rise and habitat loss that further increases temperature stress and changes quadratic in temperature damages to cubic in temperature damages.
The schematic on the right illustrates the decision problem underlying the optimal policy choice. The policy maker anticipates that (i) climate can tip, (ii) future decision makers will adjust policy to the new climate dynamics (iii) the policy maker will learn about the tipping probabilities through observation, including finding that some temperature regions turn out threshold save.

output between consumption, investment in capital, and emission reductions. Unabated emissions accumulate in the atmosphere where they change the radiative balance, induce further warming feedbacks, lead to global (average surface temperature) warming, and economic damages.

An optimal policy balances mitigation, consumption, and capital investment to maximize the welfare of current and future generations.[1] In contrast to the deterministic DICE model, the possibility of tipping makes the policymaker uncertain about the climate dynamics governing the future accumulation of greenhouse gases, temperature, and economic output. We therefore use a recursive, stochastic, dynamic programming implementation of DICE (Kelly and Kolstad, 1999; Leach, 2007; Traeger, 2013), extending Lemoine and Traeger (2014) who introduce a single tipping point into this stochastic dynamic programming assessment model. Solving the model with all three of these tipping points requires solving eight nested dynamic programming problems: when a tipping point affects the earth system, the value of avoiding it depends on the future welfare loss from the altered system, which in turn depends on how future policy optimally responds to these

[1]The DICE model discounts the welfare of future generations at a rate of 1.5% (Nordhaus, 2008). Many have explored the sensitivity of this model's results to alternate discounting assumptions (CITES). We take the DICE model's assumptions as given in order to understand the implications of introducing multiple, interacting climate tipping points into an existing, well-understood benchmark for economic evaluation.

alteration (schematic 1, right). We calibrate identical, uniform Bayesian priors for each tipping point so that a policymaker with year 2005 information expects a first tipping point to occur once the world has warmed by 2.5°C relative to 1900 (see methods section for details).

3 Results

Figure 2 presents the optimal tax on a ton of CO_2 in 2015 and 2050 without tipping points, with individual tipping points, with two simultaneous tipping points, and with all three tipping points interacting. In the absence of tipping point concerns, the optimal tax in 2015 is \$5.7 per ton of CO_2; in the presence of three potential tipping points, the optimal tax in 2015 approximately doubles to \$11. The damage tipping point has the strongest individual effect, increasing the optimal emission tax by \$2.4. The feedback tipping point increases the optimal emission tax by \$1.5, and the carbon sink tipping point increases it by \$1.1. Similar findings hold for 2050: the possibility of three interacting tipping points approximately doubles the optimal carbon tax from \$15.3 to \$29.7 per ton of CO_2, and the damage tipping point remains the single most important.

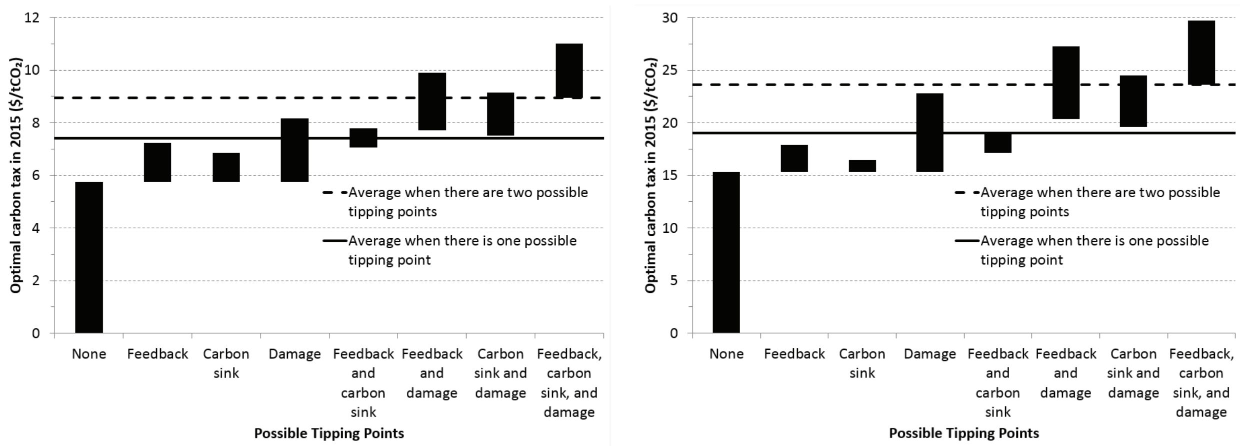

Figure 2: Optimal carbon tax in the years 2015 and 2050, assuming no tipping point has yet occurred. The columns plot scenarios without any possible tipping points; scenarios with a single possible tipping point; scenarios with two possible tipping points; and scenarios with three possible tipping points. The upper end of each bar corresponds to the optimal carbon tax, and the lower end of the bar indicates the average optimal tax when eliminating one of the corresponding tipping points. The horizontal lines show the average carbon tax among the scenarios with only one or two tipping points. The numbers assume that the given year has been reached without having passed a tipping threshold.

We find that interactions between tipping points do raise the optimal tax. Introducing three possible tipping points raises the optimal tax by 5% more in 2015 (and by 30% more in 2050) than would be suggested by adding the effects of each individual tipping point. The primary interaction is between the feedback and damage tipping points: introducing only these two tipping points raises the optimal tax by 6% more in 2015 (and by 19% more in 2050) than would be suggested by summing the effects of these two individual tipping points. If interactions were unimportant, then the optimal tax in a model with three possible tipping points should be less than suggested by summing the optimal tax in each of the three models with a single tipping point because tipping point considerations primarily raise the tax in order to make future tipping points less likely (Lemoine and Traeger, 2014), and a tax increase to avoid one tipping point already helps avoid another.

Figure 3 presents the optimal emission and temperature trajectories, conditional on not having crossed a threshold by the corresponding date. In line with the emission tax results in Figure 2, we see that the potential for a carbon sink tipping point only slightly reduces emissions and temperature. Recognizing the potential for a feedback tipping point reduces peak emissions by 1 Gt and peak temperature by approximately 0.3°C. The potential for a damage tipping point reduces optimal peak emissions by over 2Gt and suppresses peak temperature by 0.65°C. Anticipating all three potential tipping points implies a reductions of peak emissions by almost 3Gt reduces the optimal peak temperature from close to 4°C to just below 3°C. The anticipation of the tipping points induces an optimal tranjectory along which both emissions and temperatures start to fall more than half a century earlier than in an optimal policy scenario without tipping points. On the climate side, the sum of the individual tipping point adjustments in separate scenarios is slightly larger then the joint contribution under full interaction of all tipping points. We emphasize that each reduction in emissions and temperature is beneficial to reducing the probability of all tipping points. Moreover, it is significantly more costly to bring temperatures further down if they are already at a lower level. Thus, the fact that the individual tipping point contributions are almost additive shows the relevance of the tipping points' interaction.

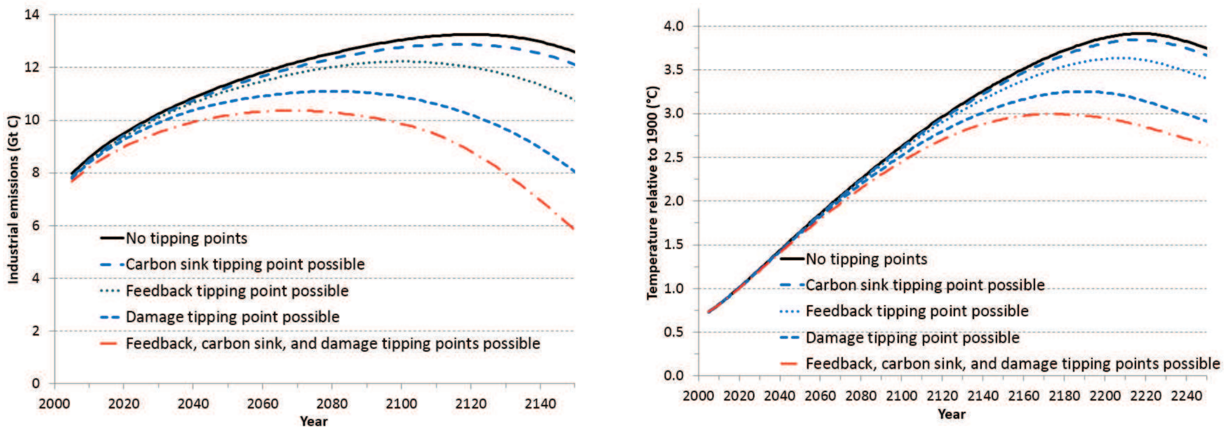

Figure 3: shows how emissions and temperature react optimally to the presence of three different tipping points (blue lines) and their interaction (red lines).

Figure 4 shows the optimal carbon tax in year t if the policy maker delays optimal action until year t and follows the IPCC's RCP 6 scenario of moderate policy in the meanwhile. The RCP number specifies the radiative forcing by the end of the century and the scenarios of the latest Assessment Report IPCC (2013) are 2.8 (or 3), 4.5, 6, and 8.5, where 8.5 is a scenario absent of any climate policy, RCP 6 has a rather moderate climate policy during the current century then catching up strongly to stabilize radiative forcing, RCP 4.5 corresponds to a climate policy close to our optimal policy, and RCP 2.8 is a scenario of extreme mitigation effort turning industrial CO_2

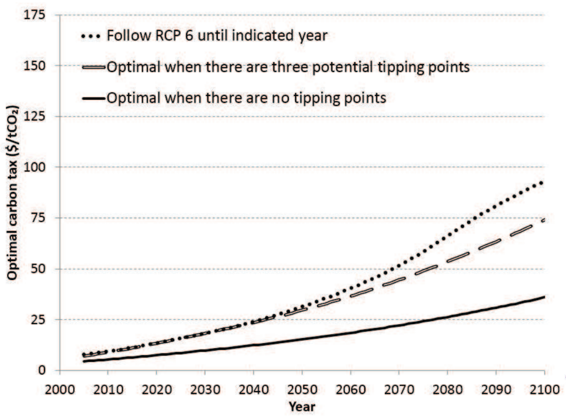

Figure 4: shows the impact of delaying optimal policy for t years. In the meanwhile, CO_2 emissions follow the IPCC's RCP 6 scenario, which assumes moderate climate policy during the current century. The values present the optimal initial carbon tax in the year policy switches to optimal policy – and stays optimal including optimal reactions to possible threshold crossings. The graph assumes that year t has been reached without crossing any tipping threshold – otherwise stronger post threshold policies would be applied.

emissions into net carbon capture before the end of the century Moss et al. (2007).

We observe that a delay up to 50 years does not change the optimal carbon tax (in the policy start year) by much. However, the fact that the optimal carbon tax does not react strongly during the early years does not imply that policy delay is cheap. Assume a policy maker decides to delay enacting the optimal carbon policy until 2050 (and following RCP 6 until then). Moreover, he switches to optimal (post-tipping) policy in case the climate system crosses a threshold already earlier. The cost in our DICE based integrated assessment model of such a delay is \$ 1.9 trillion. In case the policy maker follows RCP 6's moderate first century policy and its strong stabilization policy thereafter unless experiencing a tipping point increases the cost of delay to \$ 2.7 trillion. When delaying optimal policy into the second part of the current century, also the optimal carbon tax increases significantly at the point of switching from RCP 6 to optimal. Finally, note that the optimal policy presented in Figure 4 is conditional on not having crossed any of the climate thresholds in the meanwhile. The welfare costs take the cost of tipping followed by optimal tipping response policy into account. If policy does not react optimally to crossing the first threshold, but only to crossing the second threshold, the welfare costs increases further to \$ 3.9 trillion.

Our findings show that tipping points are of major quantitative importance for climate change policy. The presence of the three potential tipping points in our Bayesian learning model doubles the optimal carbon tax and cuts the optimal peak temperatures by approximately 1°C as compared to the situation without anticipating tipping points. Our direct comparison of individual tipping points in the same model summarizes the differences in policy suggestions of earlier studies. Tipping point studies directly modifying the economic damage function in Keller et al. (2004) and Lontzek et al. (2012) observe a much stronger impact on optimal policy than tipping points that modify the climate dynamics and only endogenously and with delay translate into economic damages as modeled in (Lemoine and Traeger, 2014). We found the strongest interaction between the tipping point triggering feedbacks that increase the climate's sensitivity to greenhouse gas concentrations and the tipping point directly affecting the steepness of the damage function in temperature increase. The potential domino interaction of tipping points manifests itself in the optimal carbon tax: the tipping increment when simultaneously anticipating all three possible tipping points is slightly larger than the sum of the increments when separately analyzing each tipping point. This finding holds despite the fact that any mitigation action to avoid one of the tipping points also reduces the probabilities of the others to happen. Optimizing policy in the face of multiple tipping points with uncertain thresholds is numerically challenging. We find that the optimal carbon tax in the joint tipping scenario is only slightly larger than the sum of the tax increments calculated in separate models with individual tipping points. Thus, our finding also encourages the further, more detailed analysis of individual tipping points and suggests that we obtain a first order approximation to optimal policy by adding the optimal tax adjustments resulting from different tipping point studies.

4 Methods

We reformulate the DICE-2007 model by Nordhaus (2008) into a recursive dynamic programming problem following Kelly and Kolstad (1999) and Traeger (2013). In response to the curse of dimensionality of dynamic programming, we reduce the state space of the climate system's representation. Traeger (2013) shows that this simplification does not impair the model's ability to reproduce climate response as compared with the DICE-2007 or DICE-2013 models, comparing each model to the average of the Atmosphere-Ocean General Circulation Models used in IPCC (2007) and emulated by MAGICC 6.0. We eliminate a damage coefficient increment in DICE that Nordhaus (2008) inserts to compensate for not modeling climate catastrophes and tipping points. This somewhat ad-hoc, smooth, and reversible damage adjustment contributes about half of DICE's damages at a 1°C warming. We introduce tipping points as regime shifts following Lemoine and Traeger (2014).

In each period, the climate system crosses with a Bayesian prior's probability irreversibly into either of the possible tipping regimes. The individual tipping probabilities are independent in any given state of the world and the hazard of tipping is determined by the global average temperature increase. These assumptions induce a binomial distribution over the number of tipping points triggered between any two periods. When there are n potential tipping points, the probability of crossing k of them between times t and $t+1$ is: $\frac{n!}{k!\,(n-k)!}\,[h(T_t, T_{t+1})]^k\,[1-h(T_t, T_{t+1})]^{n-k}$. The term $h(\cdot)$ is the hazard rate for a single tipping point as a function of current and future temperature. Currently, scientific modeling cannot suggest that one temperature is a more likely threshold than another similar temperature (Kriegler et al., 2009; Smith et al., 2009; Valdes, 2011). Our policymaker therefore believes that each threshold is uniformly distributed between a fixed upper bound $\bar{T}$ and a lower bound subject to learning: $h(T_t, T_{t+1}) = \max\left\{0, \frac{\min\{T_{t+1}, \bar{T}\} - T_t}{\bar{T} - T_t}\right\}$. The hazard rate increases with T_{t+1} because greater warming over the next interval carries a greater risk of tipping over the next interval. The hazard rate's dependence on T_t reflects that if a tipping point is still possible (i.e., it has not been crossed yet), then the policymaker has learned that its threshold must be above the current temperature (or does not exist).

We calibrate the threshold distributions so that in a model with three potential thresholds, the policymaker with year 2005 information expects that 2.5°C of warming relative to 1900 would trigger some tipping point. This requirement implies an upper bound $\bar{T}$ of 7.8°C, which is among the highest values explored in the sensitivity analysis for the single-tipping runs in Lemoine and Traeger (2014). An expected trigger of 2.5°C is consistent with the political 2°C limits for avoiding dangerous anthropogenic interference. Further, in the most recent version of the "burning embers" diagram (Smith et al., 2009), the yellow (medium-risk) shading for the "risk of large-scale discontinuities" begins around 1.6°C of warming relative to 1900, and the red (high-risk) shading begins around 3.1°C of warming relative to 1900.

We solve the tipping scenarios recursively, starting with a world where all tipping thresholds have been crossed. We solve the corresponding Bellman equation by function iteration and approximate the value function using a 10^4 dimensional Chebychev-basis. Each iteration uses the knitro solver to optimize abatement and consumption decisions at the Chebychev nodes. We use the resulting value function in the scenarios where all but one tipping points have been crossed, where it captures the continuation value in the case of further tipping with hazard rate h. We recursively proceed to find all value function until we reach our current world where no tipping threshold has been crossed.

References

Ackerman, Frank, Elizabeth A. Stanton, and Ramón Bueno (2010) "Fat tails, exponents, extreme uncertainty: Simulating catastrophe in DICE," *Ecological Economics*, Vol. 69, No. 8, pp. 1657–1665, DOI: 10.1016/j.ecolecon.2010.03.013.

Archer, D. (2007) "Methane hydrate stability and anthropogenic climate change," *Biogeosciences*, Vol. 4, No. 4, pp. 521–544.

Eglin, T., P. Ciais, S. L. Piao, P. Barre, V. Bellassen, P. Cadule, C. Chenu, T. Gasser, C. Koven, M. Reichstein, and P. Smith (2010) "Historical and future perspectives of global soil carbon response to climate and land-use changes," *Tellus B*, Vol. 62, No. 5, pp. 700–718, DOI: 10.1111/j.1600-0889.2010.00499.x.

Greenstone, Michael, Elizabeth Kopits, and Ann Wolverton (2013) "Developing a social cost of carbon for US regulatory analysis: A methodology and interpretation," *Review of Environmental Economics and Policy*, Vol. 7, No. 1, pp. 23–46, DOI: 10.1093/reep/res015.

Hall, Darwin C. and Richard J. Behl (2006) "Integrating economic analysis and the science of climate instability," *Ecological Economics*, Vol. 57, No. 3, pp. 442–465, DOI: 10.1016/j.ecolecon.2005.05.001.

Hansen, James, Makiko Sato, Pushker Kharecha, David Beerling, Robert Berner, Valerie Masson-Delmotte, Mark Pagani, Maureen Raymo, Dana L. Royer, and James C. Zachos (2008) "Target atmospheric CO2: Where should humanity aim?" *The Open Atmospheric Science Journal*, Vol. 2, pp. 217–231, DOI: 10.2174/1874282300802010217.

Huntingford, Chris, Rosie A Fisher, Lina Mercado, Ben B.B Booth, Stephen Sitch, Phil P Harris, Peter M Cox, Chris D Jones, Richard A Betts, Yadvinder Malhi, Glen R Harris, Mat Collins, and Paul Moorcroft (2008) "Towards quantifying uncertainty in predictions of Amazon 'dieback'," *Philosophical Transactions of the Royal Society B: Biological Sciences*, Vol. 363, No. 1498, pp. 1857 –1864, DOI: 10.1098/rstb.2007.0028.

Interagency Working Group on Social Cost of Carbon (2010) "Appendix 15a. Social Cost of Carbon for Regulatory Impact Analysis Under Executive Order 12866," United States Government.

IPCC (2007) *Contribution of Working Group I to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change, 2007*, Cambridge, UK: Cambridge University Press.

——— (2013) *Climate Change 2013: The Physical Science Basis*, Geneva: Intergovernmental Panel on Climate Change, Preliminary Draft.

Keller, Klaus, Benjamin M. Bolker, and David F. Bradford (2004) "Uncertain climate thresholds and optimal economic growth," *Journal of Environmental Economics and Management*, Vol. 48, No. 1, pp. 723–741, DOI: 10.1016/j.jeem.2003.10.003.

Kelly, David L. and Charles D. Kolstad (1999) "Bayesian learning, growth, and pollution," *Journal of Economic Dynamics and Control*, Vol. 23, No. 4, pp. 491–518, DOI: 10.1016/S0165-1889(98)00034-7.

Kopits, Elizabeth, Alex L. Marten, and Ann Wolverton (2013) "Moving forward with incorporating "catastrophic" climate change into policy analysis," Working Paper 13-01, National Center for Environmental Economics, U.S. Environmental Protection Agency.

Kriegler, E., J. W. Hall, H. Held, R. Dawson, and H. J. Schellnhuber (2009) "Imprecise probability assessment of tipping points in the climate system," *Proceedings of the National Academy of Sciences*, Vol. 106, No. 13, pp. 5041–5046, DOI: 10.1073/pnas.0809117106.

Le Quéré, Corinne, Christian Rodenbeck, Erik T. Buitenhuis, Thomas J. Conway, Ray Langenfelds, Antony Gomez, Casper Labuschagne, Michel Ramonet, Takakiyo Nakazawa, Nicolas Metzl, Nathan Gillett, and Martin Heimann (2007) "Saturation of the Southern Ocean CO2 sink due to recent climate change," *Science*, Vol. 316, No. 5832, pp. 1735–1738, DOI: 10.1126/science.1136188.

Leach, Andrew J. (2007) "The climate change learning curve," *Journal of Economic Dynamics and Control*, Vol. 31, No. 5, pp. 1728–1752, DOI: 10.1016/j.jedc.2006.06.001.

Lemoine, Derek and Christian Traeger (2014) "Watch your step: Optimal policy in a tipping climate," *American Economic Journal: Economic Policy*, Vol. 6, No. 1, Forthcoming.

Lenton, Timothy M. and Juan-Carlos Ciscar (2013) "Integrating tipping points into climate impact assessments," *Climatic Change*, Vol. 117, No. 3, pp. 585–597, DOI: 10.1007/s10584-012-0572-8.

Lenton, Timothy M., Hermann Held, Elmar Kriegler, Jim W. Hall, Wolfgang Lucht, Stefan Rahmstorf, and Hans Joachim Schellnhuber (2008) "Tipping elements in the Earth's climate system," *Proceedings of the National Academy of Sciences*, Vol. 105, No. 6, pp. 1786–1793, DOI: 10.1073/pnas.0705414105.

Levermann, Anders, Jonathan L. Bamber, Sybren Drijfhout, Andrey Ganopolski, Winfried Haeberli, Neil R. P. Harris, Matthias Huss, Kirstin Krger, Timothy M. Lenton, Ronald W. Lindsay, Dirk Notz, Peter Wadhams, and Susanne Weber (2012) "Potential climatic transitions with profound impact on Europe," *Climatic Change*, Vol. 110, No. 3-4, pp. 845–878, DOI: 10.1007/s10584-011-0126-5.

Lontzek, Thomas S., Yongyang Cai, and Kenneth L. Judd (2012) "Tipping points in a dynamic stochastic IAM," RDCEP Working Paper 12-03, The Center for Robust Decision Making on Climate and Energy Policy.

Moss, Richard, Mustafa Babiker, Sander Brinkman, Eduardo Calvo, Tim Carter, Jae Edmonds, Ismail Elgizouli, Seita Emori, Lin Erda, Kathy Hibbard, Roger Jones, Mikiko Kainuma, Jessica Kelleher, Jean Francois Lamarque, Martin Manning, Ben Matthews, Jerry Meehl, Leo Meyer, John Mitchell, Nebojsa Nakicenovic, Brian O'Neill, Ramon Pichs, Keywan Riahi, Steven Rose, Paul Runci, Ron Stouffer, Detlef van Vuuren, John Weyant, Tom Wilbanks, Jean Pascal van Ypersele, and Monika Zurek (2007) *Towards New Scenarios for Analysis of Emissions, Climate Change, Impacts, and Response Strategies.*, Geneva: Intergovernmental Panel on Climate Change.

Nordhaus, William D. (1992) "An optimal transition path for controlling greenhouse gases," *Science*, Vol. 258, No. 5086, pp. 1315–1319.

——— (2008) *A Question of Balance: Weighing the Options on Global Warming Policies*, New Haven: Yale University Press.

Notz, Dirk (2009) "The future of ice sheets and sea ice: Between reversible retreat and unstoppable loss," *Proceedings of the National Academy of Sciences*, Vol. 106, No. 49, pp. 20590 –20595, DOI: 10.1073/pnas.0902356106.

Oppenheimer, M. and R. B. Alley (2004) "The West Antarctic Ice Sheet and long term climate policy," *Climatic Change*, Vol. 64, No. 1-2, pp. 1–10, DOI: 10.1023/B:CLIM.0000024792.06802.31.

Oppenheimer, Michael (1998) "Global warming and the stability of the West Antarctic Ice Sheet," *Nature*, Vol. 393, No. 6683, pp. 325–332, DOI: 10.1038/30661.

Ramanathan, V. and Y. Feng (2008) "On avoiding dangerous anthropogenic interference with the climate system: Formidable challenges ahead," *Proceedings of the National Academy of Sciences*, Vol. 105, No. 38, pp. 14245 –14250, DOI: 10.1073/pnas.0803838105.

Rockström, Johan, Will Steffen, Kevin Noone, Åsa Persson, F. Stuart Chapin, Eric F. Lambin, Timothy M. Lenton, Marten Scheffer, Carl Folke, Hans Joachim Schellnhuber, Björn Nykvist, Cynthia A. de Wit, Terry Hughes, Sander van der Leeuw, Henning Rodhe, Sverker Sörlin, Peter K. Snyder, Robert Costanza, Uno Svedin, Malin Falkenmark, Louise Karlberg, Robert W. Corell, Victoria J. Fabry, James Hansen, Brian Walker, Diana Liverman, Katherine Richardson, Paul Crutzen, and Jonathan A. Foley (2009) "A safe operating space for humanity," *Nature*, Vol. 461, No. 7263, pp. 472–475, DOI: 10.1038/461472a.

Schaefer, Kevin, Tingjun Zhang, Lori Bruhwiler, and Andrew P. Barrett (2011) "Amount and timing of permafrost carbon release in response to climate warming," *Tellus B*, Vol. 63, No. 2, pp. 165–180, DOI: 10.1111/j.1600-0889.2011.00527.x.

Smith, J. B., S. H. Schneider, M. Oppenheimer, G. W. Yohe, W. Hare, M. D. Mastrandrea, A. Patwardhan, I. Burton, J. Corfee-Morlot, C. H. D. Magadza, H.-M. Fussel, A. B. Pittock, A. Rahman, A. Suarez, and J.-P. van Ypersele (2009) "Assessing dangerous climate change through an update of the Intergovernmental Panel on Climate Change (IPCC) "reasons for concern"," *Proceedings of the National Academy of Sciences*, Vol. 106, No. 11, pp. 4133–4137, DOI: 10.1073/pnas.0812355106.

Stern, Nicholas (2007) *The Economics of Climate Change: The Stern Review*, Cambridge: Cambridge University Press.

Traeger, Christian (2013) "A 4-stated DICE: quantitatively addressing uncertainty effects in climate change," CUDARE Working Paper 1130, University of California, Berkeley.

Valdes, Paul (2011) "Built for stability," *Nature Geoscience*, Vol. 4, No. 7, pp. 414–416, DOI: 10.1038/ngeo1200.

Vaughan, David. G. (2008) "West Antarctic Ice Sheet collapse—the fall and rise of a paradigm," *Climatic Change*, Vol. 91, No. 1-2, pp. 65–79, DOI: 10.1007/s10584-008-9448-3.