On the Efficiency of Nominal GDP Targeting in a Large Open Economy[1]

Matthew Hoelle[2] M. Udara Peiris [3]

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[2]Department of Economics, Purdue University; mhoelle@purdue.edu

[3]ICEF, NRU Higher School of Economics, Moscow; upeiris@hse.ru

Abstract

Since 2007 there have been increasing calls to abandon a regime of Inflation Rate targeting (IT) in favour of Nominal GDP (NGDP) targeting. One argument in favour of NGDP targeting is that it allows inflation to redistribute resources among bond holders efficiently. This paper formally examines this claim in a large open monetary economy and and provides the necessary (and restrictive) conditions under which NGDP targeting supports the Pareto efficient allocation in a setting with stochastic real uncertainty and incomplete markets.

Key words: monetary policy; open economy; uncertainty; incomplete markets; Pareto efficiency.

JEL classification numbers: D50; E31; E40; E50; F41.

1 Introduction

The abrupt end of the Great Moderation in 2007 has raised calls to abandon a regime of Inflation Rate tageting (IT) and the Taylor Rule[1] in favour of a regime of nominal GDP (NGDP) targeting whereby nominal income, rather than nominal prices, follows a stationary path.[2] We examine the claim that the path of inflation consistent with a NGDP target policy provides additional risk-sharing opportunities to households. We study the role that inflation plays in allocating resources between bondholders across countries in a world with incomplete financial markets and show that, even if agents are unable to insure against risk via existing asset markets (a complete set of asset markets, in an Arrow-Debreu sense, is unavailable), a global policy of NGDP targeting, under specific conditions, provides natural risk sharing opportunities to households so that they can completely hedge risk (achieving the Pareto efficient allocation). However we argue that this result requires restrictive conditions to hold, and which are unlikely to hold in practice.

The most common arguments in favor of the NGDP target are that it (i) stabilizes employment as wage bills depends more on nominal income than on the rate of inflation, (ii) limits asset market instability as asset bubbles tend to form when NGDP growth is higher than average, and (iii) stabilizes demand more effectively as an adverse supply shock is automatically divided between inflation and real GDP, in contrast to the excessive tightening in response to adverse supply shocks by targeting inflation.[3]

[1]IT, while optimal in economies with pricing frictions in the goods market, may be suboptimal in economies with financial frictions such as credit constraints or the possibility of default, since these frictions are persistent and propagate across markets. An example of recent work on monetary models with credit frictions includes Christiano et al. (2010). To fully analyze the welfare implications of IT, information and credit frictions should also be formally modeled. See Faia and Monacelli (2007), Carlstrom et al. (2010) and Fiore and Tristani (2012).

[2]Notable recent proponents include Crook (2011), Krugman (2011), Romer (2011),Frankel (2012), Sumner (2012) and Woodford (2012). See Hall and Mankiw (1994) for an excellent overview of the historical debate on nominal income targeting.

[3]As Koenig (2012*a*) argues:

> Suitably implemented, nominal-GDP targeting is the more-familiar Taylor rule's close cousin. The main difference between the two policy approaches is that nominal-GDP targeting takes a longer-term view of inflation than does the Taylor rule when pointing to a loose or tight policy setting. Consequently, nominal-GDP targeting

The other claim in favour of NGDP targeting, and the one we examine here, stems from the Fisher debt-deflation story whereby inflation redistributes resources among bond holders. The argument, briefly, is that there exists an optimal stochastic rate of inflation, consistent with that obtained under NGDP targeting, that supports an efficient allocation of resources (i.e. optimal risk sharing) in the absence of complete asset markets. To our knowledge, examinations of this claim have required the endowments of households to be perfectly correlated and have focused entirely on closed economies. As a result, they have ignored the role of the current account in distributing resources across countries exposed to *idiosyncratic* fluctuations in real income. Our purpose is to disentangle the effects of inflation in redistributing resources from the specific endowment structure necessary to obtain efficiency. An open economy provides a natural setting to do this.

We study the role that inflation plays in redistributing income across international bond holders who are subject to both *domestic* (*idiosyncratic*) and *world* (*aggregate*) real income risk and show that even in the absence of any insurance markets (only nominal risk-free bonds are available), NGDP targeting by all countries in a large open economy can support the Pareto efficient allocation. If at least one country follows another policy regime in a world economy with the same (real) fundamentals, these risk-sharing opportunities disappear. This is because in a large open economy any regime other than NGDP targeting that could potentially support the Pareto optimal allocation requires a combination of active monetary (interest rate) policy and exchange rate policy. We show that the combination of these polices necessarily violates the implicit debt constraint of the infinite horizon model and is therefore infeasible. Our results are similar in spirit to Koenig (2012*b*) and Sheedy (2012) though we consider a large open economy, with an explicitly modeled monetary structure, and our results do not depend on the transfer of resources between creditors and debtors. Rather, our results describe how inflation affects the

is less forgiving of near-term expected deviations of inflation from long-run target inflation than is the Taylor rule: It requires that any near-term expected inflation shortfall be offset by expected forward inflation overages so that expected average inflation over the medium term holds steady. This unwillingness to let bygones be bygones can be an advantage if the zero bound limits current policy stimulus. Under nominal-GDP targeting, the monetary authority promises future accommodation to offset the current tighter-than-desired policy stance.

efficiency of the transfers of resources between bond holders across countries; the distinction between debtors and creditors is inconsequential in our analysis.

We consider a monetary world economy of large open economies inhabited by homogeneous households. Money has value via a cash-in-advance constraint. The only assets are nominally risk-free bonds denominated in each currency. Monetary policy and exchange rate intervention affect either (i) the money supplies in each country and in turn change the price level and/or (ii) interest rates to affect the real value of domestic output. Under a global policy of nominal GDP targeting, the domestic nominal value of endowments in each country remains constant across states of uncertainty. This implies that the endowments are inversely related to the nominal price level. This setting naturally achieves the desired change in the real value of the nominal bonds as the real payouts span both the individual and the aggregate endowments. There is no role for active exchange rate policy and monetary policy is completely characterized by the monetary growth rate. On the other hand, if an inflation rate is targeted, monetary policy must be contingent on the idiosyncratic endowment realizations. This affects both the real value of the bond payoffs and the real value of endowments via the inflation tax on money balances. Pareto efficiency requires an additional degree of (policy) freedom which is obtained through exchange rate intervention. However such policy violates the implicit debt constraint of the model and hence Pareto efficiency cannot be an equilibrium outcome by targeting inflation.[4]

This paper proceeds as follows. Section 2 presents a stylized economy that captures the essence of the risk sharing argument under different monetary policies. We then present a formal model with an explicit monetary structure in Section 3 and the efficiency of monetary policies in Section 4. Finally, in Section 5 we give some concluding remarks about the robustness of the arguments in favour of NGDP targeting and how extensions and generalizations to the model affect the results.

[4]In a complementary working paper, we show that an optimal combination of monetary and exchange rate policy supports the Pareto efficient allocation under IT in a 2-period model. Such an optimal policy exists because the terminal period provides an additional degree of freedom for monetary authorities.

2 Motivating Example

The debt-deflation story of Irving Fisher requires, in a rational expectations world, the inability to fully insure for inflation to have real effects. Incompleteness of asset markets is a realistic description of the functioning of modern asset markets, and avoids the extreme assumption of the standard Arrow-Debreu model that all agents meet at one moment in time and trade assets that allow for every conceivable contingency in the infinitely long future. The absence of complete contingent markets may arise for several reasons.[5] First, asymmetric information may imply that certain events are unobservable to all market participants. Second, households may not have access to all asset markets, possibly for reasons of moral hazard. Third, transactions costs may prohibit the opening of additional markets.

When there are missing asset or insurance markets (incomplete markets), competitive allocations are generically inefficient[6] and one may envisage a role for active monetary policy and exchange rate management that would otherwise be unnecessary under complete markets. Our purpose in such a setting is to characterize monetary policy that supports the Pareto efficient allocation of resources in a large open economy. Such policy deviates from (current) monetary orthodoxy,[7] but is consistent with the arguments in the theory of incomplete markets that proceeded the seminal works of Arrow and Debreu.

In the open economy context we consider here, the incompleteness of markets may arise quite naturally. We can rationalize the domestic representative agent as the outcome of *domestic markets* being complete, whereas if there are limited risk sharing opportunities internationally, then *global markets* would be incomplete. Under the appropriate conditions, complete markets is observationally equivalent to a representative agent. In this sense, our global market incompleteness captures the inability of households to fully hedge foreign income risk. Though not formally modeled here, reasons that households may be unable to fully hedge foreign risk are incomplete information about foreign markets or even regulations that prohibit

[5] See Geanakoplos (1990) for an excellent overview of the general equilibrium literature on incomplete markets.

[6] See Stiglitz (1982) and Geanakoplos and Polemarchakis (1986).

[7] Nominal income targeting has been formally discussed since the late 1970's in monetary economics. Early contributions to the contemporary literature on nominal income targeting include Meade (1978), von Weizsacker (1978), Tobin (1980) and Bean (1983).

foreign capital flows into certain domestic markets.

Efficiency under incomplete markets requires (following Geanakoplos and Shubik (1990) and LeRoy and Werner (2001)) that the real payouts of the available nominal assets span both the individual and the aggregate endowment. This span condition can be achieved in one of two ways: (i) change the nominal payouts themselves or (ii) change the real value by changing the price level. Any economy with nominal contracts is characterized by its degrees of indeterminacy, nominal and possibly real.[8] This indeterminacy can be removed by formally modeling money and, more precisely, the actions of a monetary authority. If the underlying indeterminacy is real, as in the incomplete markets economy we consider, then these actions determine the equilibrium allocation.[9] In this motivating example we present only the efficiency of prices in equilibrium. In contrast, the formal model that follows in Section 3 will be determinate and will allow us to explicitly characterize the efficiency of monetary policy.

The proceeding analysis shows that when markets are incomplete, optimal monetary policy in a large open economy must take the form of nominal income targets *globally*. That is the price level should vary inversely with future output to allow inflation to redistribute resources among bond holders. We illustrate our argument with a stylized 2-period economy which captures the essence of the argument concisely. Importantly, the analysis of this 2-period economy dictates that our results are not specific to the particular monetary structure that we later impose.

2.1 Stylized Large Open Nominal Economy

Consider a 2-period world economy with two large open economies/countries, incomplete markets, and nominal assets with S states of uncertainty in the second period. In the initial period, households trade assets, but do not consume nor receive endowments. In the final period, one of S possible shocks $s \in \mathcal{S} = \{1, ..., S\}$ is realized. In each state, one physical commodity is traded. Two countries exist: home (h) and foreign (f). Each country is populated by homogeneous households of equal measure, with real GDP (endowment) vectors given by $y = (y(s))_{s\in\mathcal{S}}$ and

[8] There is an extensive literature on indeterminacy of monetary equilibria. An important example is Geanakoplos and Mas-Colell (1989).

[9] See Bloise and Polemarchakis (2006) for an overview on formally modeling money in a general equilibrium setting.

$y^* = (y^*(s))_{s\in\mathcal{S}}$. Total world output is denoted $Y = (Y(s))_{s\in\mathcal{S}}$ where $Y = y + y^*$. As the goods are identical, the law-of-one-price prevails and the real exchange rate is trivially one. The assets available to the households are nominal bonds, one bond issued by each country, with risk-free nominal payouts in the issuing country's currency.

Denote the equilibrium commodity price for the home country as $p = (p(s))_{s\in\mathcal{S}}$ and that for the foreign country as $p^* = (p^*(s))_{s\in\mathcal{S}}$. We now introduce three payout matrices, where each matrix contains S rows corresponding to each of the possible states and 2 columns corresponding to the 2 available assets, nominal bonds from each country.

$$R = \begin{bmatrix} \frac{1}{p(1)} & \frac{1}{p^*(1)} \\ \vdots & \vdots \\ \frac{1}{p(S)} & \frac{1}{p^*(S)} \end{bmatrix} \quad R' = \begin{bmatrix} \frac{y(1)}{M(1)} & \frac{y^*(1)}{M^*(1)} \\ \vdots & \vdots \\ \frac{y(S)}{M(S)} & \frac{y^*(S)}{M^*(S)} \end{bmatrix} \quad R'' = \begin{bmatrix} y(1) & y^*(1) \\ \vdots & \vdots \\ y(S) & y^*(S) \end{bmatrix}$$

The first payout matrix R is the real payout matrix associated with the asset structure. Define the span of the payout matrix as $\langle R\rangle$. Mathematically, the span is the 2−dimensional linear subspace of the S−dimensional Euclidean space that can be achieved through a vector of bond holdings (home and foreign): $\langle R\rangle = \left\{x \in \mathbb{R}^S : x = R\theta \text{ for some } \theta \in \mathbb{R}^2\right\}$.

Definition. Span Condition

The endowment vectors (y, y^*) both lie in the span of the payout matrix $R : y \in \langle R\rangle$ and $y^* \in \langle R\rangle$.

The *span condition* implies that the total endowment Y will also lie in the span, $Y \in \langle R\rangle$. Provided that the preferences of the representative agents in each country are identical and homothetic, the Pareto efficient allocation is such that all households consume a constant fraction of the total world output in all states: θ for home households and θ^* for foreign households with $\theta + \theta^* = 1$.

The Pareto efficient allocation can be supported by optimal bond holdings of both households (which satisfy market clearing) iff the excess demand vector $\theta Y - y_h \in \langle R\rangle$. A similar condition holds for the foreign household, but is trivially satisfied given market clearing.

2.2 Policies and the Span Condition

We now turn to what equilibrium prices and monetary policies must prevail to ensure Pareto efficiency. Provided that velocity is state-independent (though not necessarily identical across countries), the quantity theory of money states that the money supply in each country is linearly related to both the commodity price and domestic output. We consider a monetary structure in which the quantity theory of money holds. This implies that $\langle R \rangle = \langle R' \rangle$.

If the money supplies are state-independent, then $\langle R' \rangle = \langle R'' \rangle$. This invariance is how we define nominal GDP targeting. Given that $y \in \langle R'' \rangle$ and $y^* \in \langle R'' \rangle$ hold trivially, then the *span condition* is satisfied under nominal GDP targeting.

Notice that the result permits both aggregate risk and idiosyncratic risk.

With an arbitrary number of states of uncertainty (consider $S > 2$), the span $\langle R \rangle$ is a measure zero subset of the $S-$dimensional Euclidean space. For any policies other than global nominal GDP targeting, the real payout matrix R has span $\langle R \rangle \neq \langle R'' \rangle$. Consequently, theory dictates that the Pareto efficient allocation is typically unable to be supported as an equilibrium allocation. Thus, the deviation of one country from nominal GDP targeting suffices to destroy the risk-sharing opportunities of the financial structure.

The risk-sharing implications of this motivating example are the central message from Koenig (2012*b*) and Sheedy (2012). In fact, the Koenig (2012*b*) result is a special case of the above result for a closed economy with aggregate risk only (the individual household risk is perfectly correlated in Koenig (2012*b*)). However, in a monetary economy, the role of policy and its effect on the determinacy of prices in sustaining Pareto optimality is nontrivial and an issue to which we soon turn.[10]

2.3 Supporting the Pareto Efficient Equilibria

Knowing now that prices exist that support the Pareto efficient allocation, what can be said about the *implementability* of these prices and which ones are supported by monetary policy (i.e. the determinacy of the equilibrium)? In general,

[10]Determinacy is an important result in our chosen monetary structure. In a closed economy with an OLG monetary structure, Sheedy (2012) also verifies the efficiency of nominal GDP targeting, but not the determinacy of such a policy (see Kehoe and Levine (1990) and Feng (2012) for the difficulties of obtaining determinacy in an OLG framework).

the equilibrium above displays both nominal and real indeterminacy: setting either money supplies or interest rates is not enough to determine a locally unique optimal policy.[11] These questions cannot be answered from the above analysis, which only explores the risk-sharing implications of a nominal economy. To answer these questions, we must formally model a monetary structure. Such analysis provides not only an incentive for households to hold money, but also makes prices determinate and a function of policy. Thus, the task for the remainder of the paper is to introduce a proper monetary structure and investigate if the risk-sharing implications of this example continue to hold in a determinate monetary economy where central bank policy has real effects.[12]

3 The Model

Consider an economy with an infinite time horizon. In each discrete period $t \geq 0$, one of S possible shocks $s \in \mathcal{S}$ is realized. Denote the shock occurring in period t as s_t. We represent the resolution of uncertainty by an event tree Σ, with a given date-event $\sigma \in \Sigma$. Each date-event σ is characterized by the history of shocks up to and including the current period $s^t = (s_0, ..., s_t)$. The root of Σ is the date-event σ_0 with realization s_0, where $s_0 \in \mathcal{S}$ is a fixed state of the economy. Each $\sigma \in \Sigma$ has S immediate successors that are randomly drawn from $\mathcal{S}$ according to a Markov process with transition matrix Π. Each $\sigma \in \Sigma$ has a unique predecessor, where the unique predecessor of the date-event s^t is s^{t-1}. Finally, for a variable $x(s^t)$ at date-event s^t, the variables in the set of successor date-events $s^{t+1} \succ s^t$ are $\left(x(s^t, s)\right)_{s \in \mathcal{S}}$.

3.1 Households

There are two countries: home (h) and foreign (f).[13] In each country there is a continuum of homogeneous households of measure 1. At each date-event, households are endowed with an amount of the single, homogeneous good, which is identical in

[11] There is a long standing literature in the general equilibrium with incomplete markets literature on the indeterminacy of equilibria with financial contracts. For an important example in the context of an open economy see Polemarchakis (1988).

[12] The conditions for the determinacy of monetary equilibria with nominal assets and incomplete markets in a closed setting are provided by Dubey and Geanakoplos (2006), while the conditions for a deterministic setting of an open economy are provided by Tsomocos (2008).

[13] We have adopted the notational conventions of Svensson (1985).

both countries. In date-event s^t, the endowment of the home household is $y(s_t) > 0$ and the endowment of the foreign household is $y^*(s_t) > 0$; that is, endowments only depend on the current realization s_t. The economy does not contain any aggregate risk, meaning that $y(s_t) + y^*(s_t) = Y$ for any shock $s_t \in \mathcal{S}$. This assumption is made for expositional convenience only, and our results remain valid without it.

The home households in date-event s^t choose the consumption vector $\left(c_h(s^t), c_f(s^t)\right)$, where $c_h(s^t) \geq 0$ is the consumption of the home commodity and $c_f(s^t) \geq 0$ is the consumption of the foreign commodity. The goods are identical across countries, so the home household preferences are only defined over the sum $c(s^t) = c_h(s^t) + c_f(s^t)$. Similarly, the foreign household consumption vector in date-event s^t is defined by $\left(c_h^*(s^t), c_f^*(s^t)\right)$ with $c^*(s^t) = c_h^*(s^t) + c_f^*(s^t)$.

The probability that the date-event characterized by s^t is realized is defined recursively as

$$\pi(s^t) = \pi(s^{t-1}) \cdot \pi(s_t | s_{t-1}),$$

where the initial probabilities $\pi(s_0)$ are parameters of the model and $\pi(s_t | s_{t-1})$ are elements of the Markov transition matrix Π.

The household preferences are identical and characterized by the utility function

$$\sum_{t,s^t} \beta^t \pi(s^t) u\left(c(s^t)\right), \tag{1}$$

where $\beta \in (0,1)$ is the discount factor. The utility function, $u(c)$, satisfies standard conditions:

Assumption 1. *The utility function, $u : \mathbb{R}_{++} \to \mathbb{R}$, is C^1, differentiably strictly increasing, strictly concave, and satisfies homotheticity. The Inada condition holds:*

$$\lim_{c \to 0} u_1(c) = \infty.$$

3.2 Monetary Structure

We follow the monetary cash-in-advance structure of Nakajima and Polemarchakis (2005) and Peiris and Polemarchakis (2012).[14] Our timing convention is such that

[14] These models are closely related to the open economy models of Lucas (1982) and Geanakoplos and Tsomocos (2002) and the open economy model with incomplete markets of Peiris and Tsomocos (2012).

transactions occur after uncertainty is realized so that there is only a transactions demand for money. We assume a unitary velocity of money.

Consider the initial date-event s_0. The home household and the foreign household begin this date-event with nominal assets $w(s_0)$ and $w^*(s_0)$, respectively, where each is valued in terms of the local currency.

The timing proceeds as follows. First, the asset market opens, in which cash and the bonds, one from each country, are traded. Additionally, the currency market opens, in which cash denominated in one currency is traded for cash denominated in another currency. Let $e^*(s_0)$ be the nominal exchange rate for the foreign country (number of units of home currency for each unit of foreign currency) and $e(s_0)$ is the exchange rate for the home country, where $e(s_0) = \frac{1}{e^*(s_0)}$. Let $r(s_0)$ and $r^*(s_0)$ denote the nominal interest rates for the home and foreign country, respectively, implying that $\frac{1}{1+r(s_0)}$ is the price of the nominal bond in the home currency and $\frac{1}{1+r^*(s_0)}$ is the price of the nominal bond in the foreign currency. Accounting for the foreign exchange markets, the budget constraint for the home household in terms of the home currency (and similarly for the foreign household) is given by:

$$\hat{m}_h(s_0) + e^*(s_0)\hat{m}_f(s_0) + \frac{b_h(s_0)}{1+r(s_0)} + e^*(s_0)\frac{b_f(s_0)}{1+r^*(s_0)} \leq w(s_0). \tag{2}$$

The variables $\hat{m}_h(s_0)$ and $\hat{m}_f(s_0)$ are the amounts of the home and foreign currency held, while $b_h(s_0)$ and $b_f(s_0)$ are the home and foreign bond positions (net savings). Cash amounts are nonnegative variables, while the bond holdings can take any values.

The market for goods opens next. Denote $p(s_0)$ and $p^*(s_0)$ as the commodity prices in the home and foreign country, respectively. The purchase of consumption goods is subject to the cash-in-advance constraints:

$$\begin{aligned} p(s_0)c_h(s_0) &\leq \hat{m}_h(s_0), \\ p^*(s_0)c_f(s_0) &\leq \hat{m}_f(s_0). \end{aligned} \tag{3}$$

The home household also receives cash by selling its endowment, $y(s_0)$. Hence, the amount of cash that it carries over to the next period is

$$\begin{aligned} m_h(s_0) &= p(s_0)y(s_0) + \hat{m}_h(s_0) - p(s_0)c_h(s_0). \\ m_f(s_0) &= \hat{m}_f(s_0) - p^*(s_0)c_f(s_0). \end{aligned} \tag{4}$$

Given (4), the cash-in-advance constraints (3) are equivalent to

$$\begin{aligned} m_h(s_0) &\geq p(s_0)y(s_0), \\ m_f(s_0) &\geq 0. \end{aligned} \tag{5}$$

In equilibrium, the Law of One Price must hold, meaning that $p(s_0) = e^*(s_0)p^*(s_0)$. Using this fact and substituting for $m_h(s_0)$ and $m_f(s_0)$ from (4) into (2) yields the budget constraint in date-event s_0 :

$$p(s_0)c(s_0) + \frac{b_h(s_0)}{1+r(s_0)} + e^*(s_0)\frac{b_f(s_0)}{1+r^*(s_0)} \leq w(s_0). \tag{6}$$

The transactions of the home household in all date-events s^t for $t > 0$ are similar. Maintain the notational convention with nominal exchange rates as $e(s^t)$ and $e^*(s^t)$ for home and foreign respectively, nominal interest rates as $r\left(s^t\right)$ and $r^*\left(s^t\right)$ for home and foreign respectively, and commodity prices $p(s^t)$ and $p^*(s^t)$ for home and foreign respectively. The budget constraint that the home household faces in date-event s^t is given by

$$\begin{aligned} &p(s^t)c(s^t) + \frac{b_h(s^t)}{1+r(s^t)} + e^*(s^t)\frac{b_f(s^t)}{1+r^*(s^t)} \\ &\qquad \leq p(s^{t-1})y(s^{t-1}) + b_h(s^{t-1}) + e^*(s^t)b_f(s^{t-1}). \end{aligned} \tag{7}$$

Similar to (5), the cash-in-advance constraints are given by:

$$\begin{aligned} m_h(s^t) &\geq p(s^t)y(s^t), \\ m_f(s^t) &\geq 0. \end{aligned} \tag{8}$$

The debt constraint requires that the real value of the portfolios are uniformly bounded for all sequences of random variables determined by the Markov process:

$$\inf_{t,s^t}\left[\frac{b_h(s^t)}{(1+r(s^t))\,p(s^t)} + \frac{b_f(s^t)}{(1+r^*(s^t))\,p^*(s^t)}\right] > -\infty. \tag{9}$$

These debt constraints suffice to rule out Ponzi schemes and guaranteee an optimal solution to the household problem.[15] We consider the real values of the portfolios as we allow for inflation in the economy resulting from the selected monetary policy.

[15] See Hernandez and Santos (1996) and Beker and Chattopadhyay (2010) for the sufficiency of such a condition.

Define the choice vectors as $c, m_h, m_f \in \ell_+^\infty$ and $b_h, b_f \in \ell^\infty$ for the home household (and $c^*, m_h^*, m_f^* \in \ell_+^\infty$ and $b_h^*, b_f^* \in \ell^\infty$ for the foreign household), where $c = \left(c\left(s^t\right)\right)_{t\geq 0, s^t}$ is the infinite sequence of consumption choices for all date-events, with similar definitions for all other choice vectors. Define the home equilibrium price vectors as $e, r, p \in \ell_+^\infty$ and the foreign equilibrium price vectors as $e^*, r^*, p^* \in \ell_+^\infty$, where $r = \left(r\left(s^t\right)\right)_{t\geq 0, s^t}$ is the infinite sequence of nominal interest rates.[16]

The problem for the home household is given as follows (and symmetrically for the foreign household):

$$
\begin{array}{ll}
\max\limits_{c, m_h, m_f, b_h, b_f} & \sum_{t, s^t} \beta^t \pi(s^t) u\left(c(s^t)\right) \\
\textit{subj. to} & \text{budget and CIA constraints (6) and (5) in } s_0, \\
& \text{budget and CIA constraints (7) and (8) in } s^t, \\
& \text{debt constraint (9).}
\end{array}
\tag{10}
$$

3.2.1 The Monetary Authority

Each country contains a monetary authority whose responsibilities include monetary (interest rate) policy and exchange rate policy.

The parameters $W\left(s_0\right)$ and $W^*\left(s_0\right)$ are the nominal payments owed to the household in the home and foreign country, respectively, where the debt is owed by the monetary authority in each country. In the initial date-event s_0, the monetary authority in the home country chooses the domestic money supply $M\left(s_0\right)$, the domestic debt obligations $B_h\left(s_0\right)$, and the foreign debt obligations $B_f\left(s_0\right)$. The money supplies are nonnegative, while the debt obligations can be either positive (net borrow) or negative (net save). The similar choices for the monetary-fiscal authority in the foreign country are $M^*\left(s_0\right)$, $B_h^*\left(s_0\right)$, and $B_f^*\left(s_0\right)$. The constraint in s_0 for the monetary-fiscal authority in the home country (and similarly for the foreign country) is given by:

$$
M(s_0) + \frac{B_h\left(s_0\right)}{1 + r(s_0)} + e^*\left(s_0\right) \frac{B_f\left(s_0\right)}{1 + r^*(s_0)} = W(s_0). \tag{11}
$$

There are no direct seigniorage transfers made from the monetary authorities to the households. Rather, seigniorage revenue must balance with the future debt

[16] ℓ^∞ is the set of bounded infinite sequences, with respect to the sup norm. It is a metric space and more specifically a Banach space (one can apply the contraction mapping theorem to verify that the value function is well-defined and the Bellman equation is valid). ℓ_+^∞ is the set of nonnegative bounded infinite sequences. Further details on this can be found in Bertsekas (1976).

obligations. As the debt obligations are traded on competitive asset markets (net zero), indirect transfers from monetary authorities to households via changes in interest rates replace the direct transfers considered in stylized models of monetary policy.

The constraint in date-event s^t for any $t > 0$ is given by:

$$M(s^t)+\frac{B_h\left(s^t\right)}{1+r(s^t)}+e^*\left(s^t\right)\frac{B_f\left(s^t\right)}{1+r^*(s^t)} = M(s^{t-1})+B_h(s^{t-1})+e^*\left(s^t\right)B_f(s^{t-1}). \quad (12)$$

Define the choice vectors as $M \in \ell_+^\infty$ and $B_h, B_f \in \ell^\infty$ for the home household (and $M^* \in \ell_+^\infty$ and $B_h^*, B_f^* \in \ell^\infty$ for the foreign household), where $M = \left(M\left(s^t\right)\right)_{t\geq 0, s^t}$ is the infinite sequence of money supplies for all date-events, with similar definitions for all other choice vectors.

3.3 Sequential Competitive Equilibria

The market clearing conditions are such that $w(s_0) = W(s_0)$ and $w^*(s_0) = W^*(s_0)$ hold in the initial date-event and for all date-events s^t :

$$\begin{aligned}
c_h(s^t) + c_h^*(s^t) &= y(s_t), \\
c_f(s^t) + c_f^*(s^t) &= y^*(s_t), \\
m_h(s^t) + m_h^*(s^t) &= M(s^t), \\
m_f(s^t) + m_f^*(s^t) &= M^*(s^t) \\
b_h(s^t) + b_h^*(s^t) &= B_h\left(s^t\right) + B_h^*(s^t), \\
b_f(s^t) + b_f^*(s^t) &= B_f\left(s^t\right) + B_f^*(s^t).
\end{aligned}$$

A sequential competitive equilibrium is defined as follows.

Definition. Given initial nominal obligations $W(s_0)$ and $W^*(s_0)$, a sequential competitive equilibrium consists of an allocation (c, c^*), household money holdings $\left(m_h, m_f, m_h^*, m_f^*\right)$, household portfolios $\left(b_h, b_f, b_h^*, b_f^*\right)$, money supplies (M, M^*), monetary-fiscal authority debt positions $\left(B_h, B_f, B_h^*, B_f^*\right)$, interest rates (r, r^*), commodity prices (p, p^*), and exchange rates (e, e^*) such that:

1. the monetary-fiscal authorities satisfy their constraints (11) and (12).
2. given interest rates (r, r^*), commodity prices (p, p^*), and exchange rates (e, e^*), households solve the problem (10).
3. all markets clear.

3.4 Existence

This paper focuses attention on two policies of monetary-fiscal authorities: nominal GDP (NGDP) targeting and inflation stabilizing (IT), though the results can equivalently be stated in terms of global NGDP and the complementary set of all other policy combinations. The policy of NGDP does not require monetary authorities to hold foreign debt obligations, while the policy of IT does. Thus, rather than specify existence and determinacy results in terms of the target values for the money supplies (for NGDP) and the target values for the interest rates (for IT, since the rate of inflation is directly related to the interest rate via the Fisher equation), the specified policies of the monetary-fiscal authorities will be the debt positions $\left(B_h, B_f, B_h^*, B_f^*\right)$.

An appropriately chosen vector of debt positions $\left(B_h, B_f, B_h^*, B_f^*\right)$ can support NGDP and IT under the constraints (11) and (12). Thus, the global policy choice is characterized by the vector $\left(B_h, B_f, B_h^*, B_f^*\right)$, which can be viewed as parameters to the households in the economy.

The existence of a sequential competitive equilibrium follows standard arguments. The existence of a truncated equilibrium is known from Theorem 3 of Bai and Schwarz (2006) for the case of initial fiscal transfers $W(s_0)$ and $W^*(s_0)$ and zero fiscal transfers in all future period $t > 0$. The existence of the infinite-horizon analogue of the aforementioned truncated equilibrium follows the standard approach of Levine and Zame (1996) or Magill and Quinzii (1994), as the debt constraints (9) suffice to obtain the necessary uniform bounds.

The existence claim is stated as follows, where the proof follows standard arguments referenced above.

Claim 1. *For any given policy vector* $\left(B_h, B_f, B_h^*, B_f^*\right)$, *there exists a sequential competitive equilibrium.*

3.5 Determinacy and Monetary Policy

As we are concerned not only about the efficiency of different monetary policies, but also the *implementability* of these policies, it is important that we verify that the model allows the policy vector $\left(B_h, B_f, B_h^*, B_f^*\right)$ to select a determinate allocation. It is of limited value to claim that efficiency arises under a particular policy vector if such a policy is also consistent with other equilibria that are inefficient.

The Fiscal Theory of the Price Level (see, e.g., Woodford (1994)) argues that the specification of fiscal transfers suffices to pin down the initial price level and the vector of state prices, provided that the fiscal transfers are fixed. Fixed fiscal transfers imply in a 2-period model that the terminal constraints are not satisfied for off-equilibrium vectors of money supplies, interest rates, and monetary-fiscal authority debt positions. This is the exact setting of our infinite horizon model, as our model only contains strictly positive nominal transfers in the initial period, $W(s_0)$ and $W^*(s_0)$, and zero fiscal transfers in all other date-events. Further, the infinite horizon analogue of the terminal constraint is the debt constraint, which need only be satisfied in equilibrium and not for all possible off-equilibrium vectors.

Theorems 3 and 4 from Bai and Schwarz (2006) verify the determinacy of cash-in-advance equilibria in a closed 2-period economy. Their model notably limits monetary authorities to only holding riskless domestic debt positions. Thus, our determinacy result, for the infinite horizon model with the possibility for foreign exchange intervention, subsumes Bai and Schwarz (2006) as special cases. We first describe the special cases and then provide intuition for our determinacy result[17].

Consider the policy vector $\left(B_h, B_f, B_h^*, B_f^*\right)$ that is consistent with *nominal GDP targeting* for both countries. As previously discussed, the monetary authorities can support both the nominal GDP targets and the optimal allocation by only holding domestic debt positions. This is the setting of Bai and Schwarz (2006). Thus, Theorem 4 of Bai and Schwarz (2006) is directly applicable and for every policy vector $\left(B_h, B_f, B_h^*, B_f^*\right)$ consistent with a global policy of *nominal GDP targeting*, the set of sequential competitive equilibria is determinate.

Consider the policy vector $\left(B_h, B_f, B_h^*, B_f^*\right)$ that is consistent with *inflation stabilizing* for both countries. The monetary authorities may attempt to support the inflation targets using only domestic debt positions, but as previously mentioned, we provide them with additional policy tools to improve risk-sharing. Specifically, monetary authorities can inject or withdraw money from their respective domestic economies in a state-contingent manner by holding foreign debt positions. This additional policy tool allows the monetary authorities to target the nominal exchange rate to support a stationary path of inflation. As previously discussed, this additional

[17] As we impose a cash-in-advance constraint in each country we avoid the problems highlighted in Kareken and Wallace (1981). Furthermore, as we pre-specify foreign bond holdings of each monetary authority, as in Daniel (2001), we avoid the problems in Dupor (2000)

policy can achieve the Pareto efficient allocation in a 2-period model in which the terminal condition must bind for all monetary authority choice vectors. Thus, the results of Bai and Schwarz (2006) are not applicable, but intuition dictates that the additional policy tool (international debt positions) and the additional target (nominal exchange rate) will maintain the regularity of the equilibrium system.

The extension of Bai and Schwarz (2006) to an open economy is natural, so we claim that for any combination of policies for the two monetary authorities, $\left(B_h, B_f, B_h^*, B_f^*\right)$, the set of sequential competitive equilibria is determinate. This encompasses a global policy of *nominal GDP targeting*, a global policy of *inflation stabilizing*, and all other policy combinations, including asymmetric ones.

Claim 2. *For any given policy vector* $\left(B_h, B_f, B_h^*, B_f^*\right)$, *the set of sequential competitive equilibria is determinate.*

3.6 Stationary Markov Equilibria

In addition to the set of sequential competitive equilibria, we are also interested in the set of stationary Markov equilibria, a subset of the former.[18] If one exists, a stationary Markov equilibrium can be numerically approximated using standard computational methods. Additionally, such an equilibrium allows the monetary authority policy to depend on the smallest subset of observations, avoiding any (unmodeled) implementation problems involving incomplete information.

Definition. A stationary Markov equilibrium is a sequential competitive equilibrium in which functions $F_b, F_{b^*}, F_B, F_{B^*}, F_r$, and F_{r^*} exist such that for any date-event s^t and all conditional realizations in the following period $\left(s^t, s\right)_{s\in\mathcal{S}}$:

$$\begin{aligned}
\left(b_h\left(s^{t+1}\right), b_f\left(s^{t+1}\right)\right) &= F_b\left(b_h\left(s^t\right), b_f\left(s^t\right), s_{t+1}\right), \\
\left(b_h^*\left(s^{t+1}\right), b_f^*\left(s^{t+1}\right)\right) &= F_{b^*}\left(b_h^*\left(s^t\right), b_f^*\left(s^t\right), s_{t+1}\right), \\
\left(B_h\left(s^{t+1}\right), B_f\left(s^{t+1}\right)\right) &= F_B\left(B_h\left(s^t\right), B_f\left(s^t\right), s_{t+1}\right), \\
\left(B_h^*\left(s^{t+1}\right), B_f^*\left(s^{t+1}\right)\right) &= F_{B^*}\left(B_h^*\left(s^t\right), B_f^*\left(s^t\right), s_{t+1}\right), \\
r\left(s^{t+1}\right) &= F_r\left(s_{t+1}\right), \\
r^*\left(s^{t+1}\right) &= F_{r^*}\left(s_{t+1}\right).
\end{aligned}$$

[18] Stationary Markov equilibria are also referred to as recursive equilibria by Ljungqvist and Sargent (2012).

Given the above financial variables specified by stationary transition functions, the remaining variables can also be defined by stationary transition functions using the budget constraints (7), the monetary authority constraints (12), and the price equations $M\left(s^{t+1}\right) = p\left(s^{t+1}\right) y_h\left(s_{t+1}\right)$ and $e^*\left(s^{t+1}\right) = \frac{p\left(s^{t+1}\right)}{p^*\left(s^{t+1}\right)}$.

4 Efficiency of Policies

This section considers the normative effects of the following policies under incomplete markets: Theorem 1 considers nominal GDP targeting by both countries; Theorem 2 considers inflation stabilizing by both countries; and Theorem 3 considers all global policies in between those just mentioned.

4.1 Nominal GDP Targeting

Definition. Nominal GDP Targeting
If the monetary policy of the home country satisfies $p\left(s^{t+1}\right) y\left(s_{t+1}\right) = p\left(s^t, s\right) y\left(s\right) = p\left(s^t, 1\right) y\left(1\right)$ for all possible realizations $s \in \mathcal{S}$, then the home country is implementing *nominal GDP targeting*.

Define the nominal GDP target growth rates as μ and μ^* for the home and foreign country, respectively.[19] Define the largest expected endowment growth in the home country (similarly for the foreign country) as:

$$G = \max_{s \in \mathcal{S}} \frac{\sum_{s' \in \mathcal{S}} \pi\left(s'|s\right) y\left(s'\right)}{y(s)}.$$

To ensure nonnegative interest rates and a consistent equilibrium definition, we impose the following bounds on the growth rates.

Assumption 2. *The nominal GDP target growth rates* (μ, μ^*) *are such that* $\beta G \leq 1 + \mu$ *and* $\beta G^* \leq 1 + \mu^*$.

Policies of *nominal GDP targeting* do not require either monetary authorities to hold foreign debt obligations. That is, $B_f\left(s^t\right) = B_h^*\left(s^t\right) = 0$ for all date-events s^t.

[19] The monetary growth rates can vary across time periods. That is, they can be time-invariant, just not state-invariant. As Theorem 1 already guarantees Pareto efficiency without such changes in policy over time, we choose to denote the monetary growth rates as time-invariant for ease of exposition.

Under *nominal GDP targeting,* the monetary authority in the home country chooses a monetary policy consisting only of the initial money supply $M(s_0)$ and the monetary growth rate μ. Given these choices, the infinite sequence of money supplies is trivially determined. Further, the infinite sequence of debt positions B_h is determined to satisfy the monetary authority constraints (12). Thus, the entire policy is automatic given $M(s_0)$ and μ (and similarly for the monetary authority in the foreign country).

Theorem 1. *If both countries adopt* ***nominal GDP targeting,*** *then the stationary Markov equilibrium allocations are Pareto efficient.*

The recursive structure suffices to guarantee the existence of a stationary Markov equilibrium, since Theorem 1 ensures that the welfare theorems are applicable. The proof of Theorem 1 is a constructive argument and ultimately verifies two things: (i) that the Pareto efficient allocation can be supported in the sequential equilibrium budget constraints and (ii) that the transition functions implied from the budget constraints are stationary for all price variables, household choice variables, and monetary-fiscal authority choice variables. Standard arguments from dynamic programming (see Bertsekas (1976)) guarantee that the constructed equilibrium is a stationary Markov equilibrium.

Remark

There is a long standing view that the main cost of inflation lies in the distortion it induces in households' intertemporal savings decisions. The meaningful deviation of our approach from others in the literature is that we do not consider costs of inflation such as inefficient long-term planning (Clarida et al. (1999)) that arises from *unanticipated* inflation. In our framework inflation is perfectly anticipated, in a rational expectations sense, but the incompleteness of markets means that agents cannot insure against this risk. More importantly, under a global policy of NGDP targeting, the fluctuations of inflation are not random but rather correspond exactly to fluctuations in domestic GDP and so inflation does not necessarily inhibit the ability to make long term planning decisions.[20] In the open economy context we consider, the fluctuations in inflation provide a natural hedge against fluctuations in the current account which would occur for reasons outside of the remit of domestic

[20] See Sheedy (2012) for an extended discussion on this point.

monetary-fiscal authorities. In contrast, the next section shows that policies which attempt to stabilize fluctuations in the current account by targeting the nominal exchange rate are not sustainable in the long run.

4.2 Inflation Stabilizing

We next address the implications of global policies of targeting a constant (non-stocastic) rate of inflation: policies of *inflation stabilizing.*

Definition. Inflation Stabilizing

If the monetary policy of the home country satisfies $p\left(s^{t+1}\right) = p\left(s^t, s\right) = p\left(s^t, 1\right)$ for all possible realizations $s \in \mathcal{S}$, then the home country is implementing a policy of *inflation stabilizing.*

Define the expected inflation rates for both the home and foreign country, respectively, as a Markov process that only depends on the current period realization $s \in \mathcal{S} : \{\tau(s)\}_{s\in\mathcal{S}}$ and $\{\tau^*(s)\}_{s\in\mathcal{S}}$. That is, the inflation targets need not be time-invariant, only state-invariant.

Assumption 3. *The inflation rates* $\{\tau(s)\}_{s\in\mathcal{S}}$ *and* $\{\tau^*(s)\}_{s\in\mathcal{S}}$ *are such that* $\beta \leq 1 + \tau(s)$ *and* $\beta \leq 1 + \tau^*(s)$ *for all shocks* $s \in \mathcal{S}$.

Unlike with *nominal GDP targeting*, we permit the monetary authorities to engage in foreign exchange intervention. Formally, this implies $B_f\left(s^t\right) \in \mathbb{R}$ and $B_h^*\left(s^t\right) \in \mathbb{R}$ for all date-events s^t. In a purely nominal economy, such as the one presented in the motivating example, we showed that only a global policy of nominal GDP targeting could support the Pareto efficient allocation. However, the cash-in-advance technology we consider provides an additional mechanism through which targeting inflation may possibly do the same.

The cash-in-advance constraint effectively taxes nominal money balances at the rate of inflation and, in equilibrium, effectively causes agents to defer their nominal incomes by one period. In a present value sense, this is equivalent to discounting *future* income by the nominal interest rate. If real income is subject to stochastic shocks, then nominal interest rates could potentially be chosen in such a way so as to offset the real shock, leaving the present discounted value of current period nominal income state-invariant.

In order to support both the Pareto efficient allocation and a stationary path of prices, the monetary authority needs an additional mechanism to adjust the nominal money supply. This can be done in two ways. The most obvious is for the monetary authority to provide nominal lump sum transfers to agents. Such transfers would, however, be given only to domestic households, which would create wealth effects that offset the stationarity of income obtained through a carefully crafted policy of state contingent nominal interest rates.

The second option for policy makers, and the policy we consider, is that the monetary authority intervenes in the currency markets by purchasing and selling foreign bonds, thereby effectively injecting or withdrawing money from the economy. From the law of one price, this would imply that to maintain stable prices, the exchange rates would be constant over time. That is, the policy we consider is one where the monetary authority uses nominal interest rates to affect domestic aggregate demand and targets a fixed exchange rate to support a stationary path of inflation.

Our result shows that under *inflation stabilizing*, not only is monetary policy insufficient to obtain a Pareto efficient allocation, but so is monetary policy combined with foreign exchange intervention. The equilibrium debt obligations of the monetary authorities that are required to support the Pareto efficient allocation violate the debt constraint under a policy of *inflation stabilizing*.

Under a policy of *inflation stabilizing*, the monetary authority in the home country chooses a monetary policy consisting only of the initial price level $p(s_0)$ and the Markov process for inflation rates $\{\tau(s)\}_{s\in\mathcal{S}}$. The implementation of the sequence of money supplies and debt positions is automatic given that the constraints (12) must be satisfied for all date-events s^t.

Theorem 2. *If both countries adopt **inflation stabilizing**, there do not exist policies that result in a Pareto efficient sequential competitive equilibrium allocation.*

From the statement of the theorem, notice that we do not restrict ourselves to only stationary Markov equilibria. The inefficiency result is much stronger as it holds for the more general equilibrium concept.

The argument nowhere requires the variables to be specified in a stationary Markov manner. The stationary structure is imposed from the Markov endowment process and the Markov process of *inflation stabilizing*. The remaining variables of interest are the bond holdings of households and we do not impose any restrictions on the transition functions for these variables.

4.3 Mixed Global Policies

This section generalizes the result in Theorem 2 to show that for any global policies other than *nominal GDP targeting* by both countries, Pareto efficiency is not supportable under the stationary Markov equilibrium concept. Obviously, both countries adopting *inflation stabilizing* as in Theorem 2 is a special case of global policies other than *nominal GDP targeting* by both countries. While we only show that the Pareto inefficiency holds for the more restrictive definition of equilibrium (stationary Markov), it is this stationary Markov equilibrium concept that is typically considered in the theory and practice of monetary policy.

The definition of a stationary Markov equilibrium does impose minimal structure on the monetary policies that can be implemented. Namely, the definition requires that $\sum_{s\in\mathcal{S}} \pi(s|s_t)\frac{M(s^t)}{M(s^t,s)}$ only depends on the current shock s_t. As special cases are (i) *nominal GDP targeting* in which $\sum_{s\in\mathcal{S}} \pi(s|s_t)\frac{M(s^t)}{M(s^t,s)} = \frac{1}{1+\mu}$ and (ii) *inflation stabilizing* in which $\sum_{s\in\mathcal{S}} \pi(s|s_t)\frac{M(s^t)}{M(s^t,s)} = \frac{y(s_t)}{1+\tau(s_t)}\sum_{s\in\mathcal{S}}\frac{\pi(s|s_t)}{y(s)}$.

From the Fisher equation, the interest rates are in fact stationary functions and defined for the home country (and similarly for the foreign country) as:

$$r\left(s^t\right) = F_r\left(s_t\right) = \frac{y\left(s_t\right)}{\beta\sum_{s\in\mathcal{S}} \pi(s|s_t)y(s)\frac{M(s^t)}{M(s^t,s)}} - 1.$$

The function $F_r\left(s_t\right)$ is stationary iff $\sum_{s\in\mathcal{S}} \pi(s|s_t)\frac{M(s^t)}{M(s^t,s)}$ only depends on the current shock s_t.

This implies that the growth rates for any stationary monetary policy must be of the form:

$$M\left(s^t, s\right) = \left(1 + \mu\left(s_t, s\right)\right) M\left(s^t\right)$$

for all date-events s^t, where the growth rate $\mu\left(s_t, s\right)$ only depends on the two most recent shock realizations.

We must impose assumptions to guarantee that the interests rates are nonnegative.

Assumption 4. *The growth rates* $\{\mu(s_t,s)\}_{s_t,s\in\mathcal{S}}$ *and* $\{\mu^*(s_t,s)\}_{s_t,s\in\mathcal{S}}$ *are such that*
$\beta\sum_{s\in\mathcal{S}}\pi(s|s_t)y(s)\frac{M(s^t)}{M(s^t,s)} \leq y(s_t)$ *and* $\beta\sum_{s\in\mathcal{S}}\pi(s|s_t)y^*(s)\frac{M^*(s^t)}{M^*(s^t,s)} \leq y^*(s_t)$
for all shocks $s_t \in \mathcal{S}$.

The state-contingent definition of *nominal GDP targeting* for the home country specifies that $\mu(s_t,s) = \mu(s_t,1)\ \forall s\in\mathcal{S}$. We allow for this general definition in the following result, rather than the original definition (both time and state invariant) whereby $\mu(s_t,s) = \mu\ \ \forall s_t, s\in\mathcal{S}$.

Theorem 3. *If one of the countries adopts a policy other than* ***nominal GDP targeting****, there do not exist policies that result in a Pareto efficient stationary Markov equilibrium allocation.*

As previously discussed, this result nests Theorem 2 when we agree to only focus on the policy-relevant equilibrium concept of a stationary Markov equilibrium. In this setting with incomplete markets, our result does not claim that a stationary Markov equilibrium exists, as such a claim is not true in general. Rather, it states that if a stationary Markov equilibrium exists for the specified policies, then the allocation is Pareto inefficient.

5 Concluding Remarks

This paper analyzed the conditions under which a global policy of NGDP targeting permits optimal risk-sharing by households. The key points in this analysis are: (i) while the risk-sharing implications hold for any nominal economy (recall the motivating example), the formal modeling of an appropriate monetary structure is crucial and (ii) the efficiency of NGDP targeting requires all trade partners to pusure an identical policy.

Regarding the delicate nature of the efficiency result under NGDP targeting, we consider extensions and generalizations of the key assumptions of our model. Each result is either found in the classical literature in general equilibrium or follow from them.

1. If multiple commodities are traded and consumed in each date-event, the assumption that preferences are identical and homothetic suffices to guarantee

that a global policy of NGDP supports the Pareto efficient allocation. In contrast, if preferences are no longer homothetic, then the presence of multiple commodities suffices to destroy the efficiency properties.

(a) With all policies supporting allocations inside the Pareto frontier, we require a welfare measure for different monetary policies. No longer maintaining the assumption of homothetic preferences, we can use mean-variance preferences as in Neumeyer (1998) to justify the geometric distance between the complete markets allocation and the marketed subspace (the asset span) as a well-defined welfare measure.

(b) When preferences are no longer identical nor homothetic, we can use the standard theory of constrained suboptimality to claim that for a generic subset of economies, a Pareto-improving monetary policy exists. The specific characterization of such a policy must be computed independently for each economy.

2. If domestic households are heterogeneous with endowment vectors that are not perfectly correlated with the domestic or aggregate output,[21] then in order for a global policy of NGDP targeting to remain efficient, domestic households must be able to completely share risk domestically through either an equity market or a complete set of assets in zero domestic net supply. Without such complete markets, the presence of within-country heterogeneity (i.e. individual real incomes that are not perfectly correlated) destroys the efficiency property.

3. Velocity must be state-independent for a global policy of NGDP targeting support the Pareto efficient allocation. If velocity depends on domestic real output, for example, then this condition is violated. However, NGDP targeting will continue to be efficient if velocity is modeled along the lines of Cagan (1956), because in that heuristic approach velocity depends on the future path of money supplies and these paths are stationary under NGDP.

4. Our results are robust to a model with any finite number of countries. Provided that all countries adopt nominal GDP targeting, then the Pareto efficient allocation can be supported. However, if just one country deviates, then efficiency is broken as the shocks to the current account can no longer be hedged.

[21] This lies in contrast to Koenig (2012*b*) and Sheedy (2012).

Finally it is worth noting that as we have focused on the risk-sharing implications of policies, we cannot make a claim about which policy or combination of policies (and indeed the best response policy) is preferrable in a second-best world, and leave it to future work.

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6 Appendix

Proof of Theorem 1

Proof. Specify the following growth rates for household bonds: $\gamma_h\left(s^t\right)$, $\gamma_f\left(s^t\right)$, $\gamma_h^*\left(s^t\right)$, and $\gamma_f^*\left(s^t\right)$. Specify the following growth rates for the monetary-fiscal authority debt obligations: $\Gamma_h\left(s^t\right)$ and $\Gamma_f^*\left(s^t\right)$. The growth rate definitions are such that

$$b_h\left(s^t\right) = \left(1+\gamma_h\left(s^t\right)\right) b_h\left(s^{t-1}\right), \tag{13}$$

and similarly for each of the 5 other variables. Define the scaled financial variables as

$$\hat{b}_h\left(s^t\right) = \frac{b_h\left(s^t\right)}{\left(1+\mu\right)^t}, \tag{14}$$

and similarly for each of the other 5 variables. The exposition is simplified by expressing all budget constraints and debt constraints in terms of these scaled variables. Consequently, the transition equations for the scaled variables are given by

$$\hat{b}_h\left(s^t\right) = \frac{1+\gamma_h\left(s^t\right)}{1+\mu}\hat{b}_h\left(s^{t-1}\right), \tag{15}$$

and similarly for each of the other 5 variables.

Part (i): Market clearing

Observe that for any date-event s^t, market clearing in scaled variables, $\hat{b}_h\left(s^t\right) + \hat{b}_h^*\left(s^t\right) = \hat{B}_h\left(s^t\right)$, is satisfied iff market clearing in the original variables, $b_h\left(s^t\right) + b_h^*\left(s^t\right) = B_h\left(s^t\right)$, is satisfied.

To support a Pareto efficient equilibrium allocation, the budget constraint for the home household in date-event s^t (and similarly for the foreign household) is:

$$\begin{aligned}
&\theta\left(y\left(s_t\right)+y^*\left(s_t\right)\right)+\frac{b_h(s^t)}{1+r(s_t)}\frac{y\left(s_t\right)}{M(s^t)}+\frac{b_f(s^t)}{1+r^*(s_t)}\frac{y^*\left(s_t\right)}{M^*(s^t)} \\
= \quad & \frac{b_h(s^{t-1})+M(s^{t-1})}{M(s^t)}y\left(s_t\right)+\frac{b_f(s^{t-1})}{M^*(s^t)}y^*\left(s_t\right)
\end{aligned}$$

for some $\theta\in(0,1)$, where θ is the Pareto weight for the home household. Given (15), the budget constraint can be re-written in terms of scaled variables:

$$\begin{aligned}
\theta\left(y\left(s_t\right)+y^*\left(s_t\right)\right) \quad = \quad & \frac{1}{1+\mu}\left(1+\frac{\hat{b}_h\left(s^{t-1}\right)}{M\left(s_0\right)}\frac{r(s_t)-\gamma_h\left(s^t\right)}{1+r(s_t)}\right)y\left(s_t\right) \\
& +\frac{1}{1+\mu^*}\left(\frac{\hat{b}_f\left(s^{t-1}\right)}{M^*\left(s_0\right)}\frac{r^*(s_t)-\gamma_f\left(s^t\right)}{1+r^*(s_t)}\right)y^*\left(s_t\right).
\end{aligned} \tag{16}$$

Sufficient conditions are:

$$\begin{array}{l}
\theta=\frac{1}{1+\mu}\left(1+\frac{\hat{b}_h\left(s^{t-1}\right)}{M(s_0)}\frac{r(s_t)-\gamma_h\left(s^t\right)}{1+r(s_t)}\right) \text{ and} \\
\theta=\frac{1}{1+\mu^*}\left(\frac{\hat{b}_f\left(s^{t-1}\right)}{M^*(s_0)}\frac{r^*(s_t)-\gamma_f\left(s^t\right)}{1+r^*(s_t)}\right)
\end{array}. \tag{17}$$

In exactly the same manner, we obtain the sufficient conditions for the foreign household:

$$\begin{array}{l}
1-\theta=\frac{1}{1+\mu}\left(\frac{\hat{b}^*_h\left(s^{t-1}\right)}{M(s_0)}\frac{r(s_t)-\gamma^*_h\left(s^t\right)}{1+r(s_t)}\right) \text{ and} \\
1-\theta=\frac{1}{1+\mu^*}\left(1+\frac{\hat{b}^*_f\left(s^{t-1}\right)}{M^*(s_0)}\frac{r^*(s_t)-\gamma^*_f\left(s^t\right)}{1+r^*(s_t)}\right)
\end{array}. \tag{18}$$

The sufficient conditions (17) and (18) are satisfied provided that for all date-events s^t, there exists constant variables $\left(\chi_h,\chi_f,\chi^*_h,\chi^*_f\right)\in\mathbb{R}^4$ such that

$$\begin{array}{ll}
\chi_h=\hat{b}_h\left(s^{t-1}\right)\frac{r(s_t)-\gamma_h\left(s^t\right)}{1+r(s_t)} & \chi_f=\hat{b}^*_h\left(s^{t-1}\right)\frac{r(s_t)-\gamma^*_h\left(s^t\right)}{1+r(s_t)} \\
\chi^*_h=\hat{b}^*_h\left(s^{t-1}\right)\frac{r(s_t)-\gamma^*_h\left(s^t\right)}{1+r(s_t)} & \chi^*_f=\hat{b}^*_f\left(s^{t-1}\right)\frac{r^*(s_t)-\gamma^*_f\left(s^t\right)}{1+r^*(s_t)}
\end{array}. \tag{19}$$

For any initial period transfers $w(s_0)$ and $w^*(s_0)$, there exists a range of Pareto weights θ such that the corresponding Pareto optimal allocation can be supported in a stationary Markov equilibrium. For any growth rates μ and μ^*, initial money supplies $M(s_0)$ and $M^*(s_0)$, and any θ in the range discussed above, the constant values $\left(\chi_h,\chi_f,\chi^*_h,\chi^*_f\right)\in\mathbb{R}^4$ are defined as

$$\begin{array}{cc} \chi_h = \theta M(s_0)\left(1+\mu\right) - M(s_0) & \chi_f = \theta M^*(s_0)\left(1+\mu^*\right) \\ \chi_h^* = (1-\theta)M(s_0)\left(1+\mu\right) & \chi_f^* = (1-\theta)M^*(s_0)\left(1+\mu^*\right) - M^*(s_0) \end{array}. \tag{20}$$

The realization s_t determines the nominal interest rate $r(s^t) = F\left(s_t\right)$, so we simply write the interest rate as $r(s_t)$. From (15), (19), and (20), the transition functions for the bond holdings are stationary:

$$\begin{aligned} \hat{b}_h\left(s^t\right) &= \frac{1+r(s_t)}{1+\mu}\left(\hat{b}_h\left(s^{t-1}\right) - \theta M(s_0)\left(1+\mu\right) + M(s_0)\right), \\ \hat{b}_f\left(s^t\right) &= \frac{1+r^*(s_t)}{1+\mu^*}\left(\hat{b}_f\left(s^{t-1}\right) - \theta M^*(s_0)\left(1+\mu^*\right)\right), \end{aligned} \tag{21}$$

and similarly for the foreign household. The transition function is of the form required for a stationary Markov equilibrium:

$$\left(\hat{b}_h\left(s^t\right), \hat{b}_f\left(s^t\right)\right) = F_b\left(\hat{b}_h\left(s^{t-1}\right), \hat{b}_f\left(s^{t-1}\right), s_t\right).$$

Given (15), the monetary-fiscal authority constraint in the home country for any history s^t can be written in terms of scaled variables:

$$\mu M(s_0) = \hat{B}_h\left(s^{t-1}\right)\frac{r(s_t) - \Gamma_h\left(s^t\right)}{1+r(s_t)} \tag{22}$$

and similarly for the foreign country.

Exactly as above for household bond holdings, the transition function for the monetary authority in the home country is

$$\hat{B}_h\left(s^t\right) = \frac{1+r(s_t)}{1+\mu}\left(\hat{B}_h\left(s^{t-1}\right) - \mu M(s_0)\right), \tag{23}$$

with stationary transition function defined as $\hat{B}_h\left(s^t\right) = F_B\left(\hat{B}_h\left(s^{t-1}\right), s_t\right)$ (and similarly for the foreign country).

Market clearing is verified using an inductive argument. The market clearing conditions are $w\left(s_0\right) = W\left(s_0\right)$ and $w^*\left(s_0\right) = W^*\left(s_0\right)$ and in the date-event s_0 the scaled bond holdings are identical to the unscaled holdings. The bond holdings $\left\{\hat{b}_h(s_0), \hat{b}_f(s_0), \hat{b}_h^*(s_0), \hat{b}_f^*(s_0)\right\}$ are set such that the following excess demand condition is satisfied:

$$y\left(s_0\right) - c\left(s_0\right) = \frac{\hat{b}_f\left(s_0\right)}{p^*(s_0)\left(1+r^*(s_0)\right)} - \frac{\hat{b}_h^*\left(s_0\right)}{p(s_0)\left(1+r(s_0)\right)}.$$

As a result, the debt positions $\hat{B}_h(s_0)$ and $\hat{B}^*_f(s_0)$ then trivially satisfy the market clearing conditions $\hat{b}_h(s_0) + \hat{b}^*_h(s_0) = \hat{B}_h(s_0)$ and $\hat{b}_f(s_0) + \hat{b}^*_f(s_0) = \hat{B}^*_f(s_0)$ as

$$\frac{\hat{B}_h(s_0)}{p(s_0)(1+r(s_0))} = \frac{W(s_0)}{p(s_0)} - y(s_0)$$

from the monetary authority constraint of the home country (11) and

$$\frac{\hat{b}_h(s_0)}{p(s_0)(1+r(s_0))} + \frac{\hat{b}_f(s_0)}{p^*(s_0)(1+r^*(s_0))} = \frac{w(s_0)}{p(s_0)} - c(s_0)$$

from the budget constraint for the home household (and similarly for the foreign household). Thus the initialization step of the argument is confirmed: $\hat{b}_h(s_0) + \hat{b}^*_h(s_0) = \hat{B}_h(s_0)$ and $\hat{b}_f(s_0) + \hat{b}^*_f(s_0) = \hat{B}^*_f(s_0)$.

Given these date-event s_0 variables and the monetary growth rates μ and μ^*, we show that specified paths for the bond holdings and debt positions support the Pareto efficient allocation and satisfy market clearing.

For the induction step, fix the date-event $s^{t-1}$and assume that $\hat{b}_h(s^{t-1}) + \hat{b}^*_h(s^{t-1}) = \hat{B}_h(s^{t-1})$ and $\hat{b}_f(s^{t-1}) + \hat{b}^*_f(s^{t-1}) = \hat{B}^*_f(s^{t-1})$. For the home country in any date-event s^t, (21) implies

$$\hat{b}_h(s^t) + \hat{b}^*_h(s^t) = \frac{1+r(s_t)}{1+\mu}\left(\hat{b}_h(s^{t-1}) + \hat{b}^*_h(s^{t-1}) - \mu M(s_0)\right)$$

for any $s^t = (s^{t-1}, s)_{s\in\mathcal{S}}$. Using the market clearing from the inductive step, $\hat{b}_h(s^{t-1}) + \hat{b}^*_h(s^{t-1}) = \hat{B}_h(s^{t-1})$, we obtain:

$$\hat{b}_h(s^t) + \hat{b}^*_h(s^t) = \frac{1+r(s_t)}{1+\mu}\left(\hat{B}_h(s^{t-1}) - \mu M(s_0)\right).$$

Given (23), markets clear in all date-events $s^t = (s^{t-1}, s)_{s\in\mathcal{S}}$:

$$\hat{b}_h(s^t) + \hat{b}^*_h(s^t) = \hat{B}_h(s^t).$$

Part (ii): Debt constraint

The scaled bond holdings $\left\{\hat{b}_h(s^t), \hat{b}_f(s^t), \hat{b}^*_h(s^t), \hat{b}^*_f(s^t)\right\}$ are uniformly bounded random variables. Suppose otherwise, that is, $\lim_{t\to\infty}\sup_{s^t}\left|\hat{b}_h(s^t)\right| = \infty$. From the sufficient condition (19), it must be that

$$\liminf_{t\to\infty} \inf_{s^t}\left|\frac{r(s_{t+1}) - \gamma_h(s^{t+1})}{1+r(s_{t+1})}\right| = 0. \quad (24)$$

With $r(s_{t+1}) \geq 0$ and only dependent on the period $t+1$ realization s_{t+1}, the limit condition (24) is only satisfied if $\lim_{t\to\infty} \inf_{s^t} \left[\gamma_h\left(s^{t+1}\right) - r(s_{t+1})\right] = 0$. This requires that $\gamma_h\left(s^{t+1}\right)$ only depends on the period $t+1$ realization $s_{t+1} : \gamma_h\left(s^{t+1}\right) = \gamma_h\left(s_{t+1}\right)$. If $\gamma_h\left(s_{t+1}\right)$ is independent of the shock s_{t+1}, then (19) cannot be satisfied as $r(s_{t+1})$ varies with the current endowment $y\left(s_{t+1}\right)$ according to the Fisher equation (**??**). If $\gamma_h\left(s_{t+1}\right)$ varies with the shock s_{t+1}, then $\hat{b}_h\left(s^{t+1}\right) = \frac{1+\gamma_h(s_{t+1})}{1+\mu}\hat{b}_h\left(s^t\right)$ varies across the realization s_{t+1}. Taken together, $\hat{b}_h\left(s^{t+1}\right)$ varies with s_{t+1}, while $\frac{r(s_{t+2})-\gamma_h\left(s^{t+2}\right)}{1+r(s_{t+2})}$ varies (only) with s_{t+2}. Thus, (19) cannot be satisfied for all possible histories s^{t+2}. This shows that $\hat{b}_h(s^t)$ is uniformly bounded. The same argument holds for the other 3 financial variables.

In terms of the scaled bond holdings, the debt constraint for the home household is written as:

$$\inf_{t,s^t}\left[\frac{\hat{b}_h(s^t)}{(1+r(s^t))\,p(s_0)} + \frac{\hat{b}_f(s^t)}{(1+r^*\left(s^t\right))\,p^*(s_0)}\right] > -\infty.$$

Since $\left\{\hat{b}_h(s^t), \hat{b}_f(s^t), \hat{b}_h^*(s^t), \hat{b}_f^*(s^t)\right\}$ are uniformly bounded random variables, then the debt constraints for both households are satisfied.

The nominal GDP targets impose the Markov structure on equilibrium prices as discussed leading up to the statement of the theorem. Given that a Pareto efficient allocation can be supported as a stationary Markov equilibrium, then any stationary Markov equilibrium allocation must be Pareto efficient (or else households would not be optimizing). Existence is trivial since the Pareto efficient allocation is stationary in this setting. $\square$

Proof of Theorem 2

Proof. Define the scaled financial variables as

$$\hat{b}_h\left(s^t\right) = \frac{b_h\left(s^t\right)}{\prod_{s'\in s^{t-1}}\left(1+\tau\left(s'\right)\right)}, \tag{25}$$

and similarly for each of the 7 other variables.

Part (i): Market clearing

With a Pareto efficient equilibrium allocation, the budget constraint for the home household in date-event s^t (and similarly for the foreign household) is given by:

$$\theta\left(y\left(s_t\right)+y^*\left(s_t\right)\right)+\frac{b_h(s^t)}{p(s^t)\left(1+r\left(s_t\right)\right)}+\frac{b_f(s^t)}{p^*(s^t)\left(1+r^*\left(s_t\right)\right)} \quad (26)$$
$$=\frac{b_h(s^{t-1})}{p(s^t)}+\frac{b_f(s^{t-1})}{p^*(s^t)}+\frac{y\left(s_{t-1}\right)}{1+\tau\left(s_{t-1}\right)}.$$

The real bond payouts are collinear. With dependent assets, a continuum of possible optimal household bond holdings exists, meaning that it is innocuous to assume $b_f(s^t)=b_f(s^{t-1})=0$ and $b_h^*(s^t)=b_h^*(s^{t-1})=0$. In words, households are indifferent and only trade domestic bonds. Given (25),

$$\frac{b_h(s^{t-1})}{p(s^t)}=\frac{\hat{b}_h\left(s^{t-1}\right)}{p(s_0)\left(1+\tau\left(s_{t-1}\right)\right)}.$$

From the Fisher equation (**??**):

$$\frac{b_h(s^t)}{\left(1+r\left(s_t\right)\right)p(s^t)}=\frac{\beta\hat{b}_h\left(s^t\right)}{p(s_0)\left(1+\tau\left(s_t\right)\right)}.$$

Consequently, the budget constraints from (26) can be updated:

$$\theta\left(y\left(s_t\right)+y^*\left(s_t\right)\right)=\frac{y\left(s_{t-1}\right)}{1+\tau\left(s_{t-1}\right)}+\frac{1}{p(s_0)}\left(\frac{\hat{b}_h\left(s^{t-1}\right)}{1+\tau\left(s_{t-1}\right)}-\frac{\beta\hat{b}_h\left(s^t\right)}{1+\tau\left(s_t\right)}\right). \quad (27)$$

The transition function for the home household is then an implicit function of the equation:

$$p(s_0)\left(\theta\left(y\left(s_t\right)+y^*\left(s_t\right)\right)-\frac{y\left(s_{t-1}\right)}{1+\tau\left(s_{t-1}\right)}\right)=\frac{\hat{b}_h\left(s^{t-1}\right)}{1+\tau\left(s_{t-1}\right)}-\frac{\beta\hat{b}_h\left(s^t\right)}{1+\tau\left(s_t\right)}. \quad (28)$$

Mathematically, the difference between the transition function under *inflation stabilizing* and under *nominal GDP targeting* is that the transition function under *inflation stabilizing* must account for the endowment variation $y\left(s_{t-1}\right)$. Not only will this prevent a stationary transition function from being defined, but more importantly will lead to a sequence of scaled bond holdings that violate the debt constraint (9).

An implicit transition function for the monetary authorities can be similarly specified.

Part (ii): Debt constraint

Consider any fixed time period $\tau>0$. It cannot be that both

$$\begin{aligned}\theta\left(y\left(s_\tau\right)+y^*\left(s_\tau\right)\right)-\frac{y\left(s_\tau\right)}{1+\tau\left(s_\tau\right)} &= 0\\ (1-\theta)\left(y\left(s_\tau\right)+y^*\left(s_\tau\right)\right)-\frac{y^*\left(s_\tau\right)}{1+\tau^*\left(s_\tau\right)} &= 0\end{aligned}$$

for all $s_\tau \in \mathcal{S}$. If so, then $1+\tau(s) \geq 1$ $\forall s \in \mathcal{S}$ iff $y(s) \geq \theta(y(s)+y^*(s))$ $\forall s \in \mathcal{S}$ (and similarly, $1+\tau^*(s) \geq 1$ $\forall s \in \mathcal{S}$ iff $y^*(s) \geq (1-\theta)(y(s)+y^*(s))$ $\forall s \in \mathcal{S}$). Taken together, this implies that $y(s) = \theta(y(s)+y^*(s))$ $\forall s \in \mathcal{S}$ (and $y^*(s) = (1-\theta)(y(s)+y^*(s))$ $\forall s \in \mathcal{S}$) meaning that the endowments are perfectly correlated. Such an economy is ruled out by assumption. We then specify that for some $s_\tau \in \mathcal{S}$, $\left(\theta(y(s_\tau)+y^*(s_\tau)) - \frac{y(s_\tau)}{1+\tau(s_\tau)}\right)$ is strictly positive, without loss of generality.

Let the history of shocks s^τ be such that $\hat{b}_h(s^\tau) < 0$ and let the realization s_τ be such that $s_{\tau+k} = s_\tau$ for $k \in \mathbb{N}$ and

$$\theta(y(s_\tau)+y^*(s_\tau)) - \frac{y(s_\tau)}{1+\tau(s_\tau)} > 0. \tag{29}$$

Transition function (28) then implies

$$\frac{\hat{b}_h(s^\tau)}{1+\tau(s_\tau)} - \frac{\beta^k \hat{b}_h(s^{\tau+k})}{1+\tau(s_{\tau+k})} > 0$$

for $k \in \mathbb{N}$. Given that $s_{\tau+k} = s_\tau$ $\forall k \in \mathbb{N}:$

$$\beta^k \hat{b}_h\left(s^{\tau+k}\right) < \hat{b}_h(s^\tau) < 0 \tag{30}$$

for $k \in \mathbb{N}$.

The debt constraint (9) for the lone independent asset in the portfolio and specified in terms of the scaled variable is given by:

$$\inf_{k, s^{\tau+k}} \left[\frac{\hat{b}_h(s^{\tau+k})}{(1+r(s_{\tau+k}))\, p(s_0)}\right] > -\infty. \tag{31}$$

The interest rate $1+r(s_{\tau+k})$ is only a function of the current realization $s_{\tau+k}$ and is uniformly bounded. From the strict inequality (30), for any $k \in \mathbb{N}:$

$$\beta^{\tau+k+1}\hat{b}_h(s^{\tau+k+1}) < \beta^{\tau+k}\hat{b}_h\left(s^{\tau+k}\right) < 0. \tag{32}$$

Given that $\beta < 1$, then the debt constraint (9) is violated for the home household.

On the other hand, if the history of shocks s^τ is such that $\hat{b}_h(s^\tau) < 0$, then let the realization s_τ be such that $s_{\tau+k} = s_\tau$ for $k \in \mathbb{N}$ and

$$\theta(y(s_\tau)+y^*(s_\tau)) - \frac{y(s_\tau)}{1+\tau(s_\tau)} < 0.$$

Using the same logic as before, we obtain an expression similar to (32):

$$\beta^{\tau+k+1}\hat{b}_h(s^{\tau+k+1}) > \beta^{\tau+k}\hat{b}_h\left(s^{\tau+k}\right) > 0.$$

In this second case, market clearing dictates that the debt constraint (31) is violated for the foreign household.

This contradiction completes the argument.

□

Proof of Theorem 3

Proof. Suppose, without loss of generality, that the home country adopts a policy other than *nominal GDP targeting*. Suppose, in order to obtain a contradiction, that the stationary Markov equilibrium allocation is Pareto efficient. The budget constraint for the home household in any given date-event s^t is:

$$\begin{aligned}&\theta\left(y\left(s_t\right)+y^*\left(s_t\right)\right)+\tfrac{1}{p(s^t)}\left(-p\left(s^{t-1}\right)y\left(s_{t-1}\right)+\tfrac{b_h(s^t)}{1+r(s_t)}-b_h(s^{t-1})\right)\\&+\tfrac{1}{p^*(s^t)}\left(\tfrac{b_f(s^t)}{1+r^*(s_t)}-b_f(s^{t-1})\right)=0.\end{aligned}$$

As the home country monetary policy is something other than nominal GDP targeting, then $M\left(s^{t-2},s\right)\neq M\left(s^{t-2},\sigma\right)$ for some $s,\sigma\in\mathcal{S}$, implying that $p\left(s^{t-2},s\right)y_h\left(s\right)\neq p\left(s^{t-2},\sigma\right)y_h\left(\sigma\right)$ for the different shocks $s_{t-1}=s$ and $s_{t-1}=\sigma$.

By the definition of a stationary Markov equilibrium, the bond positions are only functions of three variables:

$$\left(b_h(s^t),b_f(s^t)\right)=F_b\left(b_h(s^{t-1}),b_f(s^{t-1}),s_t\right).$$

Without aggregate risk, $\theta\left(y\left(s_t\right)+y^*\left(s_t\right)\right)$ is constant. This implies that the net portfolio gain

$$\frac{1}{p\left(s^t\right)}\left(\frac{b_h(s^t)}{1+r\left(s_t\right)}-b_h(s^{t-1})\right)+\frac{1}{p^*\left(s^t\right)}\left(\frac{b_f(s^t)}{1+r^*\left(s_t\right)}-b_f(s^{t-1})\right)$$

must be a function of the shock s_{t-1} and must vary $1:1$ with the variation in $\left(p\left(s^{t-2},s\right)y\left(s\right)\right)_{s\in\mathcal{S}}$. Household optimization implies that the net portfolio gain defined above is a nonincreasing function of the portfolio $\frac{b_h(s^{t-1})}{p(s^t)}+\frac{b_f(s^{t-1})}{p^*(s^t)}$. Given this fact and the definition of the stationary transition function

$$\left(b_h(s^t),b_f(s^t)\right)=F_b\left(b_h(s^{t-1}),b_f(s^{t-1}),s_t\right),$$

the requisite variation in the net portfolio gain is possible iff $\frac{b_h(s^{t-1})}{p(s^t)} + \frac{b_f(s^{t-1})}{p^*(s^t)}$ are functions only of the shock s_{t-1}, and independent of the previous period portfolio $\left(b_h(s^{t-2}), b_f(s^{t-2})\right)$.

Now apply the same analysis to the budget constraint in the date-event s^{t-1}. If the portfolio gain in s^{t-1},

$$\frac{1}{p\left(s^{t-1}\right)}\left(\frac{b_h(s^{t-1})}{1+r\left(s_{t-1}\right)} - b_h(s^{t-2})\right) + \frac{1}{p^*\left(s^{t-1}\right)}\left(\frac{b_f(s^{t-1})}{1+r^*\left(s_{t-1}\right)} - b_f(s^{t-2})\right),$$

is a constant function of $\frac{b_h(s^{t-2})}{p(s^{t-1})} + \frac{b_f(s^{t-2})}{p^*(s^{t-1})}$, then it is unable to match the variation in $\left(p\left(s^{t-3}, s\right) y\left(s\right)\right)_{s\in\mathcal{S}}$. The only remaining option is that the portfolio gain in s^{t-1} is a strictly decreasing function of $\frac{b_h(s^{t-2})}{p(s^{t-1})} + \frac{b_f(s^{t-2})}{p^*(s^{t-1})}$. Thus, it cannot be the case that $\frac{b_h(s^{t-1})}{p(s^t)} + \frac{b_f(s^{t-1})}{p^*(s^t)}$ are functions only of the shock s_{t-1}, and independent of the previous period portfolio $\left(b_h(s^{t-2}), b_f(s^{t-2})\right)$. This contradiction finishes the argument.

$\square$