Understanding Booms and Busts in Housing Markets*

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Abstract

Some booms in housing prices are followed by busts. Others are not. It is generally difficult to find observable fundamentals that are useful for predicting whether a boom will turn into a bust or not. We develop a model consistent with these observations. Agents have heterogeneous expectations about long-run fundamentals but change their views because of "social dynamics." Agents with tighter priors are more likely to convert others to their beliefs. Boom-bust episodes typically occur when skeptical agents happen to be correct. The booms that are not followed by busts typically occur when optimistic agents happen to be correct.

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1 Introduction

Some booms in housing prices are followed by busts. Others are not. It is generally difficult to find observable fundamentals that are useful for predicting whether a boom will turn into a bust or not. We develop a model that is consistent with this observation. Agents have heterogeneous expectations about long-run fundamentals. Some agents are optimistic while others are not. Agents change their views as a result of "social dynamics." They meet randomly and those with tighter priors are more likely to convert other agents to their beliefs. The model generates a "fad" in the sense that the fraction of the population with a particular view rises and then falls. These fads lead to boom-busts or protracted booms in house prices, even if uncertainty about fundamentals is not realized. According to our model, an econometrician would not be able to predict whether a boom will turn into a bust or not. That is because before uncertainty is realized, the data are not informative about which agent is correct.[1]

Our model has three features. First, there is uncertainty about the long-run fundamentals that drive house prices. We assume that in each period, there is a small probability that housing fundamentals will change permanently to a new value. This emphasis on long-run fundamentals is related to the literature on long-run risk (Bansal and Yaron (2004), and Hansen, Heaton, and Li (2008)).

Second, as in Harrison and Kreps (1978), Scheinkman and Xiong (2003), Acemoglu, Chernozhukov, and Yildiz (2007), Piazzesi and Schneider (2009), Dumas, Kurshev, and Uppal (2009), and Geanakoplos (2010), agents in our economy have heterogenous beliefs about fundamentals. Some agents believe that housing fundamentals will improve while others don't. Agents can update their priors in a Bayesian fashion. However, the data do not convey useful information about long-run fundamentals, so agents' priors stay constant over time. In other words, agents agree to disagree and this disagreement persists over time. One group of agents is correct in their views about future fundamentals, but there is no way to know ex-ante which group that is.

The third feature of the model is an element which we refer to as "social dynamics."

[1] Models in which agents have homogeneous expectations can also generate protracted movements in house prices (see e.g. Zeira (1999) and Chu (2012)). But, these models generally imply a close relation between house prices and fundamentals that is difficult to see in the data. Glaeser and Gyourko (2006) argue that it is difficult to explain the observed large changes in house prices with changes in incomes, amenities or interest rates.

Agents meet randomly with each other and some agents change their priors about long-run fundamentals as a result of these meetings. We use the entropy of an agent's probability distribution of future fundamentals to measure the uncertainty of his views. We assume that when agent i meets agent j, the probability that i adopts the prior of j is decreasing (increasing) in the entropy of the prior of i (j). Agents with tighter priors are more likely to convert other agents to their beliefs. Our model generates dynamics in the fraction of agents who hold different views that are similar to those generated by the infectious disease models proposed by Bernoulli (1766) and Kermack and McKendrick (1927).

We consider two cases. In the first case, the agents with the tightest priors are those who expect fundamentals to remain the same. In the second case, the agents with the tightest priors are those who expect fundamentals to improve. Absent realization of uncertainty about long-run fundamentals, the model generates fads. In the first case, there is a rise and fall in the number of people who believe that buying a house is a good investment. Here, the model generates a protracted boom-bust cycle. In the second case, there is a rise and fall in the number of people who believe that housing fundamentals will not change. Here, the model generates a protracted boom in housing prices that is not followed by a bust.

We use the model to compute the price path expected by different agents. These unconditional expected price paths take into account the probability of uncertainty being realized at each point in time. Regardless of which agent happens to be correct, the model has the following implications. Agents who think that fundamentals will improve, expect prices to rise and then level off. Agents who think fundamentals will not change, expect prices to rise and then fall. An econometrician taking repeated samples from data generated by the model would see both boom-busts and booms that are not followed by busts. The boom-bust episodes typically occur in economies where agents who don't expect fundamentals to improve happen to be correct. The episodes in which booms are not followed by busts typically occur in economies where agents who expect fundamentals to improve happen to be correct. Of course, in any given economy an econometrician would not be able to predict ex-ante which type of episode would occur.

We first study the implications of social dynamics in a frictionless asset pricing model of the housing market. While useful for building intuition, the model is too stylized to account for various features of the data. For this reason, we embed our model of social dynamics into a matching model of the housing market of the sort considered by Piazzesi and Schneider

(2009). As these authors show, a small number of optimists can have a large impact on housing prices. However, heterogeneity in beliefs per se is not enough to generate protracted booms or booms and busts of the sort observed in the data. Here, social dynamics play a key role by changing the fraction of agents who hold different views about future fundamentals. In our model these changes introduce non-trivial dynamics into house prices.

We show that a calibrated version of our model can account for some key features of the recent boom-bust episode in the U.S., including the duration and magnitude of the boom. A key question is whether the model relies on agents having unreasonable expectations about movements in house prices. To answer this question, we use data from the Case, Shiller, and Thompson (2012) home buyer survey on the one-year ahead expected increase in home prices. We argue that, if anything, the model understates how much U.S. agents expected house prices to appreciate during the recent episode.

We also present some evidence on three key implications of our model. First, booms (busts) are marked by increases (decreases) in the number of agents who buy homes only because of large expected capital gains. Second, the probability of selling a home is positively correlated with house prices. Third, sales volume is positively correlated with house prices. We find support for all three implications in the data. More generally, we argue that the extensive margin of the number of potential home buyers plays a critical role in house price dynamics.

The remainder of this paper is organized as follows. In Section 2, we study the implications of social dynamics in a frictionless asset-pricing model of the housing market. Section 3 presents a simple matching model of the housing market and describes its transition dynamics. Section 4 incorporates social dynamics into the matching model and generates our main results. We discuss the quantitative properties of the model in Section 5. In Section 6 we present empirical evidence regarding the key mechanisms at work in the model. Section 7 contains concluding remarks.

2 Social dynamics in a frictionless model

In this section we consider a simple frictionless model of the housing market. We use this model to describe the role played by social dynamics and the implied movements in the fraction of agents with different beliefs about long-run fundamentals.

The model economy The economy is populated by a continuum of agents with measure one. All agents have linear utility and discount utility at rate β. Agents are either home owners or renters. To simplify, we assume that each agent can only own one house. One characteristic that distinguishes houses from stocks and other assets, is that houses cannot be sold short. So, we assume that there is no short selling in our model.[2]

For simplicity, we assume that there is a fixed stock of houses, $k < 1$, in the economy. This assumption is motivated by the observation that large booms and busts occur in cities where increases in the supply of houses are limited by zoning laws, land scarcity, or infrastructure constraints.[3] There is a rental market with $1 - k$ houses. These units are produced by competitive firms at a cost of w per period, so the rental rate is constant and equal to w. The momentary utilities of owning and renting a house are ε^h and ε^r, respectively. We assume that the utility of owning a home is higher than the net utility of renting ($\varepsilon^h > \varepsilon^r - w$), so that home prices are positive.

We first consider the equilibrium of a version of the economy with no uncertainty. Agents decide at time t whether they will be renters or home owners at time $t + 1$. The net utility of being a renter at time $t + 1$ is $\varepsilon^r - w$. If an agent buys a house at time t, he pays P_t. At time $t + 1$, he lives in the home and receives an utility flow ε^h. The agent can sell the house at the end of period $t+1$ for a price P_{t+1}. Since all agents are identical, in equilibrium they must be indifferent between buying and renting a house. So, house prices satisfy the following equation:

$$-P_t + \beta\left(P_{t+1} + \varepsilon^h\right) = \beta\left(\varepsilon^r - w\right). \tag{1}$$

The stationary solution to this equation is:[4]

$$P = \frac{\beta}{1-\beta}\varepsilon, \tag{2}$$

where $\varepsilon = \varepsilon^h - (\varepsilon^r - w)$.

We now consider an experiment that captures the effects of infrequent changes in the value of housing fundamentals. Examples include low-frequency changes in the growth rate

[2] We use the conventional meaning of the expression "short sale," which is a transaction in which an invester borrows an asset and sells it with the promise to return it at a later date. In the recent crisis, the term "short sale" has been used with a different meaning. It refers to a situation in which the house's sale price falls short of the mortage value and the bank agrees to accept the proceeds of the sale in lieu of the mortgage balance.

[3] See, e.g. Glaeser, Gyourko, and Saks (2005), Quigley and Raphael (2005), Barlevy and Fisher (2010), and Saiz (2010).

[4] It is well known that there are explosive solutions to equation (1) (see, e.g. Diba and Grossman (1987)). We abstract from these solutions in our analysis.

of productivity which affects agents' wealth and changes in financial regulation or innovations which make it easier for agents to purchase homes. For concreteness, we focus on the utility of owning a home. Suppose that, before time zero, the economy is in a steady state with no uncertainty, so $P_t = P$. At time zero, agents learn that in each period, with small probability ϕ, the value of ε changes permanently to a new level, ε^*. Agents agree about the value of ϕ but disagree about the probability distribution of ε^*. Agents do not receive any information useful to update their priors about the distribution of ε^*.[5] As soon as uncertainty is resolved, agents become homogeneous in their beliefs.

Prior to the resolution of uncertainty, agents fall into three categories depending on their priors about ε^*. We refer to these agents as "optimistic," "skeptical," and "vulnerable." We denote by o_t, s_t, and v_t the percentage of agents who are optimistic, skeptical and vulnerable, respectively. Agent types are indexed by $j = o, s, v$ and are assumed to be publicly observable. Priors are common knowledge, so higher-order beliefs play no role in our model. The laws of social dynamics described below are public information. Agents take into account future changes in the fractions of the population that hold different views.

The new value of the flow utility of owning a home, ε^*, is drawn from the set Φ. For simplicity, we assume that this set contains n elements. An agent of type j attaches the probability distribution function (pdf) $f^j(\varepsilon^*)$ to the elements of Φ.

We assume that, at time zero, there is a very small fraction of skeptical and optimistic agents. Almost all agents are vulnerable, i.e. they have diffuse priors about future fundamentals. Optimistic agents expect an improvement in fundamentals:

$$E^o(\varepsilon^*) > \varepsilon.$$

Skeptical and vulnerable agents do not expect fundamentals to improve:

$$E^s(\varepsilon^*) = E^v(\varepsilon^*) = \varepsilon.$$

For now, we assume that agents do not internalize the possibility of changing their type as a result of social interactions. This assumption rules out actions that are optimal only because agents might change their type in the future. For example, a skeptical agent might buy a home, even though this action is not optimal given his current priors, because there

[5] If agents disagreed about the value of ϕ they would update their priors about ϕ after observing whether a change in fundamentals occurred. We abstract from uncertainty about the value of ϕ to focus our analysis on the importance of social dynamics.

is a chance he might become optimistic in the future. We return to this issue at the end of this section.

We use the entropy of the probability distribution $f^j(\varepsilon^*)$ to measure the uncertainty of an agent's views,

$$e^j = -\sum_{i=1}^{n} f^j(\varepsilon_i^*) \ln\left[f^j(\varepsilon_i^*)\right].$$

The higher is the value of e^j, the greater is agent j's uncertainty about ε^*. This uncertainty is maximal when the pdf is uniform, in which case $e^j = \ln(n)$.

Agents meet randomly at the beginning of the period. When agent l meets agent j, j adopts the prior of l with probability γ^{lj}. The value of γ^{lj} depends on the ratio of the entropies of the agents' pdfs:

$$\gamma^{lj} = \max(1 - e^l/e^j, 0). \tag{3}$$

This equation implies that a low-entropy agent does not adopt the prior of a high-entropy agent. In addition, it implies that the probability that a high-entropy agent adopts the priors of the low-entropy agent is decreasing in the ratio of the two entropies. We use this formulation for two reasons. First, it strikes us as plausible. Second, it is consistent with evidence from the psychology literature that people are more persuaded by those who are confident (e.g. Price and Stone (2004) and Sniezek and Van Swol (2001)).

Throughout, we assume that the entropy of vulnerable agents exceeds the entropy of skeptical and optimistic agents:

$$e^s < e^v, \qquad e^o < e^v.$$

So, the vulnerable are the most likely to change their views. In addition, we make the natural assumption that most agents are vulnerable at time zero and that the initial number of optimistic and skeptical agents is small and identical: $o_0 = s_0$.

The population dynamics generated by our model are similar to those implied by the infectious-disease models of Bernoulli (1766) and Kermack and McKendrick (1927).[6] We consider two cases. In the first, the prior of the skeptical agents has the lowest entropy. In the second, the prior of the optimistic agents has the lowest entropy. In both cases, if uncertainty is not resolved, the entire population converges to the view of the agent with the

[6] Bernoulli (1766) used his model of the spread of smallpox to show that vaccination would result in a significant increase in life expectancy. When vaccination was introduced, insurance companies used Bernoulli's life-expectancy calculations to revise the price of annuity contracts (Dietz and Heesterbeek (2002)).

lowest entropy. The model generates a fad, in the sense that the fraction of the population with the second lowest entropy rises and then falls.

Case 1 In this case, the pdf of the skeptical agents has the lowest entropy:

$$e^s < e^o < e^v.$$

The fractions of optimistic, skeptical and vulnerable agents in the population evolve according to:

$$o_{t+1} = o_t + \gamma^{ov} o_t v_t - \gamma^{so} o_t s_t, \tag{4}$$

$$s_{t+1} = s_t + \gamma^{sv} s_t v_t + \gamma^{so} o_t s_t, \tag{5}$$

$$v_{t+1} = v_t - \gamma^{ov} o_t v_t - \gamma^{sv} s_t v_t. \tag{6}$$

To understand equation (4), note that there are $o_t v_t$ encounters between optimistic and vulnerable agents.[7] As a result of these encounters, $\gamma^{ov} o_t v_t$ vulnerable agents become optimistic. Similarly, there are $s_t o_t$ encounters between skeptical and optimistic agents. As a result of these encounters, $\gamma^{so} s_t o_t$ optimistic agents become skeptical. These two sets of encounters and the value of o_t determine o_{t+1}.

Consider next equation (5). There are $o_t s_t$ encounters between skeptical and optimistic agents, which lead $\gamma^{so} o_t s_t$ optimistic agents to become skeptical. There are also $s_t v_t$ encounters between skeptical and vulnerable agents, which lead $\gamma^{sv} s_t v_t$ vulnerable agents to become skeptical. Finally, equation (6) implies that the fraction of vulnerable agents declines over time because $\gamma^{ov} o_t v_t$ vulnerable agents become optimistic and $\gamma^{sv} s_t v_t$ become skeptical.

Consider a path of the economy along which uncertainty is not realized. In case 1, the model can generate a fad in which the number of optimistic agents rises for a while before declining toward zero. To see how this pattern emerges, suppose that initially a large fraction of the population is vulnerable and that $\gamma^{ov} v_0 > \gamma^{so} s_0$. In conjunction with equation (4), the latter condition implies that the number of optimistic agents initially rises over time as the number of vulnerable agents who become optimistic is larger than the number of optimistic agents who become skeptical. The number of vulnerable agents declines over time as some of these agents become optimistic and others become skeptical (see equation (6)). This decline implies that, eventually, $\gamma^{ov} v_t < \gamma^{so} s_t$. At this point, the fraction of optimistic agents begins to fall. As $t \to \infty$, all optimistic agents become skeptical (see equation (5)).

[7] See Duffie and Sun (2007) for a law of large numbers that applies to pairwise random meetings.

Case 2 In this case, the pdf of the optimistic agents has the lowest entropy:

$$e^o < e^s < e^v.$$

The fractions of optimistic, skeptical and vulnerable agents in the population evolve according to:

$$o_{t+1} = o_t + \gamma^{ov} o_t v_t + \gamma^{os} o_t s_t, \tag{7}$$

$$s_{t+1} = s_t + \gamma^{sv} s_t v_t - \gamma^{os} o_t s_t, \tag{8}$$

$$v_{t+1} = v_t - \gamma^{ov} o_t v_t - \gamma^{sv} s_t v_t. \tag{9}$$

To understand equation (7), note that the $o_t v_t$ encounters between optimistic and vulnerable agents lead $\gamma^{ov} o_t v_t$ vulnerable agents to become optimistic. There are also $o_t s_t$ encounters between optimistic and skeptical, which lead $\gamma^{os} o_t s_t$ skeptical agents to become optimistic. Equation (8) reflects the fact that the $o_t s_t$ encounters between skeptical and optimistic agents result in $\gamma^{os} o_t s_t$ skeptical agents becoming optimistic. The $s_t v_t$ encounters between skeptical and vulnerable agents result in $\gamma^{sv} s_t v_t$ vulnerable agents becoming skeptical. Finally, equation (9) implies that v_t declines, since $\gamma^{ov} o_t v_t$ vulnerable agents become optimistic and $\gamma^{sv} s_t v_t$ become skeptical.

This version of the model also generates a fad but here, it is the number of skeptical agents that rises for a while before declining toward zero. A fad arises when initially a large fraction of the population is vulnerable and $\gamma^{sv} v_t > \gamma^{os} o_t$. The basic difference between case 1 and case 2 is that in the latter case skeptics are converted into optimists, so that eventually all agents become optimistic.

Equilibrium in the frictionless model House prices are determined by the marginal buyer. To identity this buyer, we sort agents in declining order of their house valuations. The marginal buyer is the agent who is at the kth percentile of house valuations. When the fraction of optimistic agents is lower than k for all t, the marginal home buyer is always a non-optimistic agent. Since these agents do not expect changes in the utility of owning a home, the price is constant over time at the value given by equation (2). In order to generate a boom-bust cycle, at least k agents must be optimistic at some point in time.

It is useful to define the time-t fundamental value of a house before the resolution of uncertainty for a given agent, assuming that this agent is the marginal buyer until uncertainty is resolved. We denote these fundamental values for the optimistic, skeptical, and vulnerable agents by P_t^o, P_t^v, and P_t^s, respectively. The value of P_t^o is given by:

$$P_t^o = \beta \left\{ \phi \left[E^o(\varepsilon^*) + \beta \frac{E^o(\varepsilon^*)}{1-\beta} \right] + (1-\phi)(\varepsilon + P_{t+1}^o) \right\}. \tag{10}$$

The logic that underlies this equation is as follows. With probability ϕ uncertainty is resolved. In this case, the expected utility flow and house price at time $t+1$ are $E^o(\varepsilon^*)$ and $\beta E^o(\varepsilon^*)/(1-\beta)$, respectively. With probability $1-\phi$, uncertainty is not resolved. In this case, the agent receives a utility flow, ε, and values the house at P_{t+1}^o. Since we are deriving the fundamental value under the assumption that the optimistic agent is always the marginal home buyer, $P_t^o = P_{t+1}^o = P^o$. Solving equation (10) for P^o, we obtain:

$$P^o = \beta \frac{\phi E^o(\varepsilon^*)/(1-\beta) + (1-\phi)\varepsilon}{1-\beta(1-\phi)}. \tag{11}$$

Vulnerable and skeptical agents expect ε^* to equal ε, so:

$$P^s = P^v = \frac{\beta}{1-\beta}\varepsilon. \tag{12}$$

We begin by characterizing the equilibrium of the economy in case 1. Recall that in this case, the fraction of optimistic agents first rises and then falls. Suppose that the number of optimistic agents is lower than k for $t < t_1$ and exceeds k for $t \in [t_1, t_2]$, where $t_2 < \infty$. For $t > t_2$, the marginal home buyer is a skeptical agent, so the price is given by:

$$P_t = P^s, \quad \text{for } t \geq t_2 + 1. \tag{13}$$

Using P_{t_2+1} as a terminal value we can compute recursively the prices for $t \leq t_2$ that obtain if uncertainty is not realized. Since the marginal home buyer between period t_1 and period t_2 is an optimistic agent, we have:

$$P_t = \beta\{\phi[E^o(\varepsilon^* + P_{t+1}^*)] + (1-\phi)(\varepsilon + P_{t+1})\}, \quad \text{for } t_1 \leq t \leq t_2.$$

Here, P_{t+1} and P_{t+1}^* are the $t+1$ prices when uncertainty is not realized and when uncertainty is realized, respectively.

Since the marginal home buyer for $t < t_1$ is a vulnerable/skeptical agent, we have:

$$P_t = \beta\{\phi[E^s(\varepsilon^* + P_{t+1}^*)] + (1-\phi)(\varepsilon + P_{t+1})\}, \quad \text{for } t < t_1.$$

In writing this equation, we use the fact that: $E^s(\varepsilon^*) = E^v(\varepsilon^*)$.

The following proposition characterizes the equilibrium in case 1.

Proposition 1 *The equilibrium price path in case 1 when uncertainty is not realized is given by:*

$$P_t = \begin{cases} P^s + [\beta(1-\phi)]^{t_1-t}\,(P_{t_1} - P^s), & t < t_1, \\ P^o - [\beta(1-\phi)]^{t_2+1-t}\,(P^o - P^s)\,, & t_1 \le t \le t_2, \\ P^s, & t > t_2. \end{cases} \tag{14}$$

The equilibrium price path when uncertainty is realized is given by:

$$P_t = \frac{\beta}{1-\beta}\varepsilon^*. \tag{15}$$

The intuition for this proposition is as follows. Before time t_1 the marginal buyer is a vulnerable agent. If uncertainty is not realized, the marginal buyer at time t_1 is an optimistic agent. The latter agent is willing to buy the house at a value that exceeds P^s because he realizes a capital gain, $P_{t_1} - P^s$, with probability $(1-\phi)^{t_1-t}$. The equilibrium price is equal to P^s plus the expected discounted capital gain, which is: $[\beta(1-\phi)]^{t_1-t}\,(P_{t_1} - P^s)$ (see first line of equation (14)). The price jumps at time zero from P^s to $P^s + [\beta(1-\phi)]^{t_1}\,(P_{t_1} - P^s)$ because of the expected capital gain associated with the change in the marginal buyer at time t_1. As long as uncertainty is not realized, the price rises before time t_1 because the expected discounted capital gain increases at the rate $\beta(1-\phi)$.

Between time t_1 and t_2 the marginal buyer is an optimistic agent. However, if uncertainty is not realized, the marginal buyer at time t_2 is a skeptical agent who is willing to buy the house at a price $P^s < P^o$. So, the equilibrium price is equal to P^o minus the expected discounted capital loss, $[\beta(1-\phi)]^{t_2-t}\,(P^o - P^s)$ (second line of equation (14)). As long as uncertainty is not realized, the price falls before $t_2 + 1$, because the expected discounted capital loss rises at rate $\beta(1-\phi)$. After time t_2+1 there are no more changes in the identity of the marginal buyer. So, unless uncertainty is realized, the price remains constant and equal to the fundamental value of a house to a skeptical agent, P^s. Finally, once uncertainty is realized, agents have homogeneous expectations about fundamentals and the price of a house is given by equation (15).

Proposition 1 implies that the model generates a boom-bust cycle in house prices as long as uncertainty is not realized. Of course, the model can also generate a boom-bust as well as a protracted boom depending upon when uncertainty is realized and the realization of ε^*.

The following proposition characterizes the equilibrium in case 2. Recall that, in this case, the fraction of the population that is optimistic converges monotonically to one. We define t_1 as the first time period in which there are more optimistic agents than homes ($o_t \ge k$).

Proposition 2 *The equilibrium price path in case 2 when uncertainty is not realized is given by:*

$$P_t = \begin{cases} P^s + [\beta(1-\phi)]^{t_1-t}(P^o - P^s), & t < t_1, \\ P^o, & t \geq t_1. \end{cases} \tag{16}$$

The equilibrium price path when uncertainty is realized is given by:

$$P_t = \frac{\beta}{1-\beta}\varepsilon^*. \tag{17}$$

The intuition for this proposition is as follows. From time t_1 until uncertainty is resolved, the marginal home buyer is an optimistic agent. So, absent resolution of uncertainty, the price is equal to P^o for all $t \geq t_1$. Before t_1, the marginal home buyer is a vulnerable/skeptical agent who has a fundamental house value P^s. The equilibrium price is equal to P^s plus the discounted expected value of the capital gain that results from selling the house to an optimistic agent at time t_1, $[\beta(1-\phi)]^{t_1-t}(P^o - P^s)$.

A simple numerical example We now consider a simple numerical example that illustrates the properties of the model summarized in the previous proposition. In case 1, equations (5)–(6) imply that the maximum value of o_t is 0.22. So, the presence of optimistic agents affects prices only if $k < 0.22$. We assume that $k = 0.1$.

We choose the normalization $\varepsilon = E^v(\varepsilon^*) = E^s(\varepsilon^*) = 1$. Given this choice, the equilibrium depends on three additional features of agents' priors about future fundamentals: $E^o(\varepsilon^*)$, e^o/e^v and e^s/e^v. In both case 1 and 2 we assume that:

$$E^o(\varepsilon^*) = 2.55.$$

In case 1 we assume that:

$$e^o/e^v = 0.892,\ e^s/e^v = 0.883.$$

In case 2, we assume that:

$$e^o/e^v = 0.883,\ e^s/e^v = 0.892.$$

We think of each time period as representing one month and choose β so that the implied annual discount rate is six percent. We assume that there is a very small number of optimistic and skeptical natural renters at time zero: $o_0 = s_0 = 2 \times 10^{-6}$. The remainder of the population is vulnerable. We choose ϕ, the probability that uncertainty is realized in each period, to equal 1/120. Absent resolution of uncertainty, this value, together with our other assumptions, implies that a boom-bust pattern emerges over the course of roughly 20 years.

Case 1 Our choices for e^o/e^v and e^s/e^v imply:

$$\gamma^{so} = 0.010,\ \gamma^{ov} = 0.108,\ \text{and}\ \gamma^{sv} = 0.117.$$

Panel (a) of Figure 1 shows the evolution of the fraction of skeptical, optimistic and vulnerable agents absent resolution of uncertainty about ε^*. Consistent with the intuition above, the fraction of optimistic agents in the population initially increases slowly. The infection then gathers momentum until the fraction of optimistic agents peaks at 0.22 percent in year 12. Thereafter, this fraction declines toward zero. Between t_1, the middle of year 10, and t_2, the middle of year 20, the optimistic agents are the marginal buyers since they exceed k percent of the population. The fraction of vulnerable agents falls over time and converges to zero as these agents become either skeptical or optimistic. The fraction of skeptical agents rises monotonically over time until everybody in the economy is skeptical.

Consistent with Proposition 3.1, Figure 2 shows that the price jumps at time zero and then continues to rise slowly until optimistic agents become the marginal home buyers at time t_1. Thereafter, the price drops rapidly, reverting to its initial steady-state value.

Figure 2 also displays the one-period-ahead annualized rate of return that different agents expect at each point, conditional on uncertainty not having been realized at time t:

$$r_t^j = \frac{\phi\left[E^j\left(\varepsilon^*\right) + \beta E^j\left(\varepsilon^*\right)/(1-\beta)\right] + (1-\phi)(\varepsilon + P_{t+1})}{P_t} - 1. \tag{18}$$

The figure also displays the volume of transactions implied by the model computed under the assumption that trade only occurs when at least one of the agents has a motive for transacting.

A key feature of Figure 2 is that agents have heterogeneous beliefs about the expected rate of return to housing. This basic feature of our model is consistent with the findings in Piazzesi and Schneider (2009) who document this heterogeneity using the Michigan Survey of Consumers.[8]

The annualized real rate of return to the marginal home owner is constant and equal to six percent. Before t_1, the skeptical/vulnerable agents are the marginal home owners. Optimistic agents expect very high rates of return which reflect the high value of $E^o\left(\varepsilon^*\right)$. So, all newly optimistic agents ($\gamma^{ov} o_t v_t$) buy homes.[9] During this period, prices rise and all

[8] Vissing-Jørgensen (2003) provides evidence of substantial heterogeneity of beliefs regarding the returns to other assets, such as stocks.

[9] We assume that the vulnerable agents sell since they are indifferent between holding and selling. The

transactions are initiated by agents who buy homes. Prices and transaction volume peak simultaneously at time t_1.

Between time t_1 and t_2, the marginal buyer is an optimistic agent. During this period, the skeptical/vulnerable agents expect negative rates of return because they have a low expected value of ε^*. So, all newly skeptical agents ($\gamma^{so} s_t o_t$) sell their homes to optimistic agents who are indifferent between buying and holding. During this period, prices fall and all transactions are initiated by agents who sell homes. Figure 2 displays the time series of transactions volume. Prices and transactions volume peak simultaneously at time t_1. Transaction volume collapses once prices start to fall because at this point optimistic agents own all the houses. After time t_1, the number of transactions recovers as some optimistic agents become skeptical and sell their homes.

After time t_2, the marginal home owner is, once again, a skeptical/vulnerable agent. Optimistic agents expect very high rates of return that are not reflected in market prices, so all newly optimistic agents ($\gamma^{ov} o_t v_t$) buy homes. But, there are so few vulnerable agents that the number of transactions is close to zero.

From Figure 2 we see that, while the identity of the marginal home owner changes over time, the rate of return to the marginal owner is always six percent. Since pricing is determined by the marginal owner, the expectations of infra-marginal agents are not reflected in home prices.

Finally, Figure 2 displays the price paths expected by optimistic and skeptical/vulnerable agents at time zero. These paths are given by:

$$E_0^j(P_t) = (1-\phi)^{t+1}(\varepsilon + P_{t+1}) + \left[1-(1-\phi)^{t+1}\right]\left[E^j(\varepsilon^*) + \beta E^j(\varepsilon^*)/(1-\beta)\right], \tag{19}$$

for $j = o,\ s,\ v$.

Optimistic agents expect prices to rise very rapidly until time t_1. Thereafter, expected prices continue to rise but at a lower rate, reflecting the fall in actual market prices that occurs if uncertainty is not realized. This fall is outweighed by the high value of ε^* expected by optimistic agents. Finally, expected prices rise at a slightly higher rate after time t_2, because the price remains constant if uncertainty is not realized.

Consider next the price path expected by skeptical and vulnerable agents at time zero.

optimistic agents could induce them to sell by offerering an arbitrarially small premium. Some optimistic agents become skeptical during this time period. However, for $t \leq t_1$ optimistic agents who become skeptical are indifferent between holding and selling, so we assume that they do not transact.

Equation (19) implies that:

$$E_0^o(P_t) - E_0^s(P_t) = \left[1 - (1-\phi)^{t+1}\right] \frac{E^o(\varepsilon^*) - E^s(\varepsilon^*)}{1-\beta}. \quad (20)$$

So, the difference between $E_0^o(P_t)$ and $E_0^s(P_t)$ reflects agents' different expectations about ε^*. This difference implies that skeptical/vulnerable agents always expect a lower price than optimistic agents. The skeptical/vulnerable agents expect prices to rise between time zero and time t_1 because the price appreciation that occurs, as long as uncertainty is not realized, outweighs the fall in price that occurs if uncertainty is realized. Between time t_1 and t_2, the latter effect outweighs the former effect and the skeptical/vulnerable agents expect prices to fall. After time t_2, the market price corresponds to the skeptical/vulnerable agent's fundamental price, so expected prices are constant.

Case 2 Our choices for e^o/e^v and e^s/e^v imply:

$$\gamma^{os} = 0.010,\ \gamma^{ov} = 0.117, \text{ and } \gamma^{sv} = 0.108.$$

Panel (b) of Figure 1 shows the evolution of the fraction of skeptical, optimistic, and vulnerable agents absent resolution of uncertainty about ε^*. The dynamics are the same as in panel (a) of Figure 1, except that the skeptical and optimistic have switched places. Here there is a fad, in the sense that the number of skeptical agents rises for roughly 12 years before falling to zero. The fraction of optimistic agents rises monotonically over time until everyone is optimistic.

Consistent with Proposition 3.2, Figure 3 shows that the price jumps at time zero and then continues to rise until the middle of year 9, when all homes are owned by optimistic agents. From this moment on, the price is constant and equal to the optimistic agent's fundamental value.

Figure 3 also shows the volume of transactions implied by the model. At time zero, all homes are owned by vulnerable agents. Between time zero and time t_1, all newly optimistic agents ($\gamma^{ov} o_t v_t$) buy homes. At time t_1, all the homes are owned by optimistic agents and there are no new transactions because optimistic agents do not become skeptical.

Finally, Figure 3 displays the price path expected by optimistic and skeptical/vulnerable agents at time zero. This path is computed using equation (19). Optimistic agents expect prices to rise very rapidly until t_1. From this point on, the expected price continues to increase because there is a rise over time in the probability that uncertainty is realized and optimistic agents receive a large capital gain. Figure 3 also displays the price path expected by skeptical and vulnerable agents at time zero. As in case 1, the difference between $E_0^o(P_t)$ and $E_0^s(P_t)$ is fully accounted for by different expectations about ε^* (see equation (20)).

Interpreting cross-sectional data on house prices A well-known property of housing markets is the presence of both boom-bust episodes and episodes in which booms are not followed by busts. Piazzesi and Schneider (2009) document this property using post-war data. A longer perspective is provided by Ambrose, Eichholtz, Lindenthal (2010) who document this property using four centuries of housing data for Holland. Eitrheim and Erlandsen (2004) provide analogous evidence using two centuries of housing data for Norway.

It is useful to briefly quantify some stylized facts about the post-war episodes. Our results are based on quarterly OECD data on real house prices for 25 countries for the period 1970 to 2012. An operational definition of a boom or a bust requires that we define turning points where upturns and downturns in house prices begin. To avoid defining high-frequency movements in the data as upturns or downturns, we first smooth the data. Let y_t denote the logarithm of an index of real house prices. Also let x_t denote the centered-moving average of $y_{t,}$ $x_t = \sum_{j=-n}^{n} y_{t+j}$. We define an upturn as an interval of time in which $\Delta x_t > 0$ for all t and a downturn as an interval of time in which $\Delta x_t < 0$. A turning point is the last time period within an upturn or downturn. A boom is an upturn for which $y_T - y_{T-L} > z$, and a bust is a downturn for which $y_T - y_{T-L} < -z$. Here, T is the date at which the boom or bust ended, L is the length of the boom or bust and z is a positive scalar. The results discussed below are generated assuming that $n = 5$, so that x_t is defined as an 11-month centered moving average of y_t. In addition, we assume that $z = 0.15$ so that booms and busts are defined as price moments greater than or equal to 15 percent. Our findings are not sensitive to small changes in the assumed values of n and z.

Three key features emerge from our analysis. First, every country in the sample experienced house price booms and busts.[10] The median sizes of booms and busts are 44.7 and -28.5 percent, respectively. Second, booms and busts occur over protracted periods of time. The median lengths of booms and busts in our sample are 5.5 and 5.38 years, respectively. Third, in many cases booms are followed by protracted busts. We identified 62 booms and 38 busts in our sample. Of the 62 booms, 33 were followed within 6 months by a bust. So,

[10] Australia and Austria only experienced booms, while Germany and Slovenia only experienced busts. The data for Austria and Slovenia are available only since 2000 and 2007, respectively.

roughly half of the booms turned into a bust, while the other half did not.

Our frictionless model is consistent, at least qualitatively, with these three features of the data. Regardless of whether we are in case 1 or case 2, optimistic agents expect prices to rise and eventually level off, while skeptical agents expect prices to rise and then fall. An econometrician taking repeated samples from our data would see both boom-busts and booms that are not followed by busts. The boom-bust episodes would typically occur in economies where the skeptical agents happen to be correct. The booms that are not followed by busts would typically occur in economies where the optimistic agents happen to be right. Of course, in any given economy the econometrician cannot predict which type of episode he would see because, by construction, the data are not informative about which agent is correct.

Robustness In our analysis we assume that agents do not take into account the possibility of changing their type as a result of social interactions. In Appendix A, we assess the quantitative impact of this assumption by calculating equilibrium prices when agents do internalize this possibility. We find that internalizing these changes makes virtually no difference to our results because the probability of switching types is small. In Appendix B, we discuss a second type of robustness. There, we describe a Bayesian environment which generates social dynamics that are similar to those of our model.

3 A matching model

The frictionless model shows the potential of social dynamics to account for observed house-price dynamics. However, this model has three unattractive features. First, to generate a boom-bust cycle in case 1, the fraction of agents that are optimistic must exceed k percent of the population for at least some period of time. According to the Bureau of the Census, on average, during the period 1965-2011, 65 percent of American households owned homes. So, the model requires that a very large fraction of the population become optimistic.[11] Second, the price rise that occurs at time zero is large relative to the peak rise in prices. Third, the model is too stylized to account for the fact that the volume of transactions and time to sell

[11] An alternative strategy for remedying this shortcoming is to allow for heterogeneity in the utility of owning a home. For example, suppose there is a group of agents that would never sell their home because they derive such a high utility from it. The presence of this group is equivalent to a reduction in k, so that it is easier to generate a boom in the price of homes that are potentially for sale.

are highly correlated with average house prices.

To address these shortcomings, we use an extended version of the matching model proposed by Piazzesi and Schneider (2009). In this section, we consider a version of the model in which agents have homogeneous expectations. The basic structure of this model coincides with that of the frictionless model described in Section 2. The economy is populated by a continuum of agents with unit measure. All agents have linear utility and discount utility at the rate β. There is a fixed stock of houses, $k < 1$, and a rental market with $1 - k$ houses. Rental units are produced by competitive firms at a cost w per period, so the rental rate is constant and equal to w.

There are four types of agents in the economy: home owners, unhappy home owners, natural home buyers, and natural renters. We denote the fraction of these agents in the population by h_t, u_t, b_t, and r_t, respectively. Home owners and unhappy home owners own homes at time t. All homes are occupied in equilibrium so that:

$$h_t + u_t = k. \tag{21}$$

Both natural buyers and natural renters rent homes at time t so that:

$$b_t + r_t = 1 - k. \tag{22}$$

We describe the state of the economy by $z_t = (h_t, b_t)'$. We now discuss the problems faced by the different agents in the economy.

Home owners A home owner derives momentary utility ε from his home. The agent's value function, $H(z_t)$, is given by:

$$H(z_t) = \varepsilon + \beta \left[(1 - \eta)H(z_{t+1}) + \eta U(z_{t+1})\right]. \tag{23}$$

With probability η, a home owner's match with his home goes sour and he becomes an unhappy home owner. We denote the value function of an unhappy home owner by $U(z_t)$.

Unhappy home owners We normalize the momentary utility that an unhappy home owner derives from his home to zero. This agent decides whether or not to put up his home for sale. To simplify, we abstract from the transactions costs of selling a home. When a house is up for sale, it sells with probability $q^s(z_t)$. Once the sale occurs, the unhappy home owner becomes a natural renter. The value function of an unhappy home owner is given by:

$$U(z_t) = \max[U^{sell}(z_t), U^{stay}(z_t)]. \tag{24}$$

The value function of an unhappy home owner who puts his house up for sale is:

$$U^{sell}(z_t) = q^s(z_t)\left\{P(z_t) + \beta\left[(1-\lambda)R(z_{t+1}) + \lambda B(z_{t+1})\right]\right\} + [1 - q^s(z_t)]\beta U(z_{t+1}). \qquad (25)$$

Here, $P(z_t)$ denotes the price that an unhappy home owner expects to receive, conditional on a sale, $R(z_t)$ denotes the value function of a natural renter and $B(z_t)$ denotes the value function of a natural home buyer. The utility of a natural home buyer appears in equation (25) because any agent who is a natural renter at the beginning of time $t+1$ is, with probability λ, subject to a preference shock that turns them into a natural buyer.

The value function of an unhappy home owner who decides to not put his house up for sale is:

$$U^{stay}(z_t) = \beta U(z_{t+1}) - \psi(z_t). \qquad (26)$$

Here $\psi(z_t)$ denotes a non-pecuniary or regret cost paid by unhappy home owners who don't put their house up for sale. We find that our algorithm for solving the model is much more stable if we scale the non-pecuniary cost by the expected revenue from selling the house $\psi(z_t) = \psi q^s(z_t)P(z_t)$. In addition, this scale factor leads to a particularly simple expression for the reservation price of an unhappy home owner:

$$\bar{P}^u(z_t) = \frac{\beta}{1+\psi}\left\{U(z_{t+1}) - \left[(1-\lambda)R(z_{t+1}) + \lambda B(z_{t+1})\right]\right\}. \qquad (27)$$

This reservation price makes the unhappy home owner indifferent between putting the house on the market and not trying to sell it. As ψ goes to infinity, the reservation price becomes exogenous.

Natural home buyers A natural buyer is a renter at time t. He has to choose between renting at $t+1$ and trying to buy a home. His net flow utility from renting is given by $\bar{\varepsilon} - w$ and his value function is $B(z_t)$. If he decides to continue renting, his value function, $B^{rent}(z_t)$, is given by:

$$B^{rent}(z_t) = \bar{\varepsilon} - w + \beta B(z_{t+1}). \qquad (28)$$

If he tries to buy a house, he succeeds with probability $q^b(z_t)$. In this case, he pays a price $P^b(z_t)$ and his continuation utility is that of a home owner $((1-\eta)H(z_{t+1}) + \eta U(z_{t+1}))$. With probability $1 - q^b(z_t)$, he does not succeed in buying a house and he remains a renter at time $t+1$. So the value function of being a potential buyer, $B^{buy}(z_t)$, is given by:

$$\begin{aligned} B^{buy}(z_t) &= \bar{\varepsilon} - w + q^b(z_t)\left\{-P^b(z_t) + \beta\left[(1-\eta)H(z_{t+1}) + \eta U(z_{t+1})\right]\right\} \\ &\quad + \left[1 - q^b(z_t)\right]\beta B(z_{t+1}). \end{aligned} \qquad (29)$$

The value function of a natural home buyer is given by:

$$B(z_t) = \max[B^{rent}(z_t), B^{buy}(z_t)]. \tag{30}$$

The reservation price, $\bar{P}^b(z_t)$, is the price that makes a natural buyer indifferent between buying and renting:

$$\bar{P}^b(z_t) = \beta\left[(1-\eta)H(z_{t+1}) + \eta U(z_{t+1}) - B(z_{t+1})\right]. \tag{31}$$

Natural renters A natural renter is a renter at time t. His net flow utility from renting is given by $\bar{\varepsilon} - w$ and his value function is $R(z_t)$. The only difference between natural renters and natural buyers is that the former derive lower utility from owning a home. We model this difference by assuming that whenever natural renters buy a house they pay a fixed cost, $\kappa\varepsilon$. This fixed cost represents the expected present value of the difference between their utility from owning a home and the corresponding utility of a natural buyer.[12] We choose the value of κ so that it is not optimal for natural renters to buy a house in the steady state.

In each period, a fraction λ of natural renters receive a preference shock and become natural home buyers. A natural renter can choose whether to continue renting or to try to buy a house. If he continues renting, his value function, $R^{rent}(z_t)$, is given by:

$$R^{rent}(z_t) = \bar{\varepsilon} - w + \beta\left[(1-\lambda)R(z_{t+1}) + \lambda B(z_{t+1})\right]. \tag{32}$$

The continuation utility reflects the fact that a natural renter becomes a natural home buyer with probability λ.

If the natural renter tries to buy a house, he succeeds with probability $q^b(z_t)$. In this case, he pays a price $P^r(z_t)$ and his continuation utility is the same as that of a home owner, $((1-\eta)H(z_{t+1}) + \eta U(z_{t+1}))$, except that he must pay the fixed cost $\kappa\varepsilon$. With probability $1 - q^b(z_t)$ the natural renter does not succeed in buying a house. In this case, he continues to be a renter at time $t+1$. The value function of a natural renter who decides to buy a house, $R^{buy}(z_t)$, is given by:

$$\begin{aligned} R^{buy}(z_t) &= \bar{\varepsilon} - w + q^b(z_t)\left\{-P^r(z_t) - \kappa\varepsilon + \beta\left[(1-\eta)H(z_{t+1}) + \eta U(z_{t+1})\right]\right\} \\ &\quad + \left[1 - q^b(z_t)\right]\beta\left[(1-\lambda)R(z_{t+1}) + \lambda B(z_{t+1})\right]. \end{aligned} \tag{33}$$

[12] Since the fixed cost is paid upfront, all home owners are identical. It does not matter whether they used to be a natural buyer or a natural renter. This property simplifies our analysis by reducing the number of different agents in the economy.

The value function of a natural renter is given by:

$$R(z_t) = \max[R^{rent}(z_t), R^{buy}(z_t)]. \tag{34}$$

The reservation price, $\bar{P}^r(z_t)$, is the price that makes natural renters indifferent between buying and renting:

$$\bar{P}^r(z_t) = \beta\left\{[(1-\eta)H(z_{t+1}) + \eta U(z_{t+1})] - [(1-\lambda)R(z_{t+1}) + \lambda B(z_{t+1})]\right\} - \kappa\varepsilon. \tag{35}$$

Timing The timing of events within a period is as follows. At the beginning of the period, the state variable is z_t. Then, preference shocks occur. With probability η, home owners become unhappy home owners. With probability λ, natural renters become natural buyers. Then agents make decisions.[13] Transactions occur at the end of the period. A fraction $q^s(z_t)$ of unhappy home owners sell their homes while a fraction $q^b(z_t)$ of home buyers buy houses.

The indicator function $J^b(z_t)$ is equal to one if it is optimal for a natural buyer to buy a house when the state of the economy is z_t and and is equal to zero otherwise. The indicator function $J^r(z_t)$ is equal to one if it is optimal for a natural renter to buy a house when the state of the economy is z_t and is equal to zero otherwise. The indicator function $J^u(z_t)$ is equal to one if it is optimal for an unhappy home owner to put up their house for sale when the state of the economy is z_t and is equal to zero otherwise.

The laws of motion for the fraction of home owners, unhappy home owners, natural home buyers and natural renters in the population are given by:

$$h_{t+1} = (1-\eta)h_t + q^b(z_t)\left[(b_t + \lambda r_t)\,J^b(z_t) + r_t(1-\lambda)J^r(z_t)\right], \tag{36}$$

$$u_{t+1} = (u_t + \eta h_t)\left[1 - q^s(z_t)J^u(z_t)\right], \tag{37}$$

$$b_{t+1} = (b_t + \lambda r_t)\left[1 - q^b(z_t)J^b(z_t)\right], \tag{38}$$

$$r_{t+1} = (1-\lambda)r_t[1 - q^b(z_t)J^r(z_t)] + q^s(z_t)J^u(z_t)\,(u_t + \eta h_t). \tag{39}$$

These equations along with equations (21) and (22) define a law of motion for the state vector, z_t, that we denote $z_{t+1} = G(z_t)$.

[13] Given our notation, value functions are evaluated at this point in time.

The matching technology Since agents can only own one home, only natural renters and natural buyers can potentially buy homes. The total number of potential buyers is given by:

$$\text{Buyers}(z_t) = (b_t + \lambda r_t)\, J^b(z_t) + r_t(1-\lambda)J^r(z_t). \tag{40}$$

There is no short selling and home owners only sell when the match with their home goes sour. The total number of potential sellers is given by:

$$\text{Sellers}(z_t) = (u_t + \eta h_t)\, J^u(z_t). \tag{41}$$

There is a technology that governs matches between buyers and sellers. When a match occurs, the transactions price is determined by generalized Nash bargaining. The bargaining power of sellers and buyers is θ and $1-\theta$, respectively. Matches can occur between a seller and a natural buyer or a natural renter. In the first case, the price paid by the natural buyer, $P^b(z_t)$, is:

$$P^b(z_t) = \theta \bar{P}^b(z_t) + (1-\theta)\bar{P}^u(z_t). \tag{42}$$

In the second case, the price paid by the natural renter, $P^r(z_t)$, is:

$$P^r(z_t) = \theta \bar{P}^r(z_t) + (1-\theta)\bar{P}^u(z_t). \tag{43}$$

The average price received by an unhappy home owner, $P(z_t)$, is given by:

$$P(z_t) = \frac{(b_t + \lambda r_t)\, J^b(z_t)P^b(z_t) + r_t(1-\lambda)J^r(z_t)P^r(z_t)}{(b_t + \lambda r_t)\, J^b(z_t) + r_t(1-\lambda)J^r(z_t)}. \tag{44}$$

The number of homes sold, $m(z_t)$, is determined by the matching function:

$$m(z_t) = \mu \text{Sellers}(z_t)^{\alpha} \text{Buyers}(z_t)^{1-\alpha}. \tag{45}$$

The probabilities of selling, $q^s(z_t)$, and buying, $q^b(z_t)$, a house are given by:

$$q^s(z_t) = m(z_t)/\text{Sellers}(z_t), \tag{46}$$

$$q^b(z_t) = m(z_t)/\text{Buyers}(z_t). \tag{47}$$

3.1 Solution Algorithm

In this subsection we discuss our algorithm for solving the model. We begin by describing the steady state and then show how to solve the equilibrium of the model, given arbitrary initial conditions.

Steady State It can be shown that the model economy has a unique steady state in which the fraction of the different types of agents is constant. We now solve for the steady-state values of the probabilities of buying and selling a home (q^b and q^s) and the fraction of the different agents in the population (h, u, b, and r).

We choose a value for η so that the probabilities of buying and selling a house coincide in the steady state:

$$q^s = q^b.$$

Equations (45) and(46) imply that $q^s = q^b = \mu$. This property, together with equations (22) and (38), implies that the steady-state number of natural buyers is given by:

$$b = \frac{(1-\mu)\lambda(1-k)}{\mu + \lambda(1-\mu)}.$$

Given b, we solve for r, u, and h, as functions of μ, λ and k using equations (22), (37) and (21).

The fact that $q^s = q^b$ implies that the number of buyers is equal to the number of sellers (equations (46) and (47)). Since there are $u + \eta h$ unhappy home owners and $b + \lambda r$ buyers in steady state, we set η so that:

$$u + \eta h = b + \lambda r.$$

Given the values of q^s and q^b, we can solve for the steady-state values of the prices, P, P^b, and P^r, the reservation prices of unhappy home owners, natural renters and sellers, $\bar{P}^u$, $\bar{P}^r$, and $\bar{P}^s$, and the value functions of the different agents evaluated in the steady state: H, U, B and R. To do so, we use the steady state versions of equations (23)–(35), (42)–(44) and the fact that, in steady state, $B = B^{buy}$ and $R = R^{rent}$.

Transitional Dynamics We assume that at time $T = 2000$ the system has converged to the steady state. Consequently, we obtain an approximate solution because it takes an infinite number of periods for the model economy to converge to the steady state.

Let the set Z denote all the values of the state variable z_t that occur along the transition path. First, guess that $J^b(z_t) = J^u(z_t) = 1$ and $J^r(z_t) = 0$ for all $z_t \in Z$. Second, using the initial conditions $z_0 = (h_0, b_0)$ and equations (36)-(39), compute the sequence of values of h_t, u_t, b_t, and r_t. Third, use equations (40), (41), (45)–(47) to compute the values of $q^s(z_t)$ and $q^b(z_t)$ for $z_t \in Z$. Fourth, assume that: $H(z_T) = H$, $U(z_T) = U$, $B(z_T) = B$, and $R(z_T) = R$. Then use equations (23) to (35) and (44) to solve backwards for $\{H(z_t), U(z_t),$

$B(z_t),\ R(z_t), P(z_t)\}$ for $t = 1$ to T. Finally, verify whether the guesses for $J^b(z_t)$, $J^r(z_t)$, and $J^u(z_t)$ for $z_t \in Z$ describe the optimal behavior of buyers and sellers along the proposed transition path. If not, revise the guesses for $J^b(z_t)$, $J^r(z_t)$, and $J^u(z_t)$ until a consistent solution is obtained.

3.2 Experiments

We illustrate the properties of the model through a series of experiments.

An expected improvement in fundamentals We first consider the same experiment that we study in the frictionless model assuming that agents have homogeneous beliefs. At time zero, agents suddenly anticipate that, with probability ϕ, the utility of owning a home rises from ε to $\varepsilon^* > \varepsilon$. It is easy to show that there are no transition dynamics and the economy converges immediately to a new steady state with a higher price. So, when beliefs are homogeneous, anticipated future changes in fundamentals are immediately reflected in today's price. Matching frictions per se do not produce interesting price dynamics, at least in the experiment studied here.

Transitional dynamics We now study an experiment that highlights how an exogenous increase in the number of buyers affects home prices. The resulting intuition is useful for understanding the effects of social dynamics that we discuss in the next section. Suppose that the fraction of natural buyers in the population is initially higher than its steady state value, $b_0 > b$. Since $r_0 = 1 - k - b_0$, the fraction of natural renters in the population is initially lower than its steady state value. The time-zero value of the state variable is $z_0 = (b_0, h_0)$, where h_0 is equal to the steady state value of h. Equations (45)–(47) imply that the probability of buying a house at time zero is lower than in the steady state: $q^b(z_0) < q^b(z)$. The time-zero probability of selling is higher than it is in steady state: $q^s(z_0) > q^s(z)$.

We illustrate the transition dynamics of the model using the parameter values summarized in Table 1. These values are discussed in detail in Section 4, where we consider the quantitative properties of our model.

Figure 4 depicts the model's transition dynamics assuming that the time-zero number of natural buyers is 90 percent above its steady-state level. We now discuss the intuition for why the price is initially high and converges to the steady state from above. Along the transition path, only natural buyers want to buy houses, so the transactions price is a weighted average of the reservation prices of the natural buyers and the unhappy home owners:

$$P(z_t) = \theta \bar{P}^b(z_t) + (1-\theta)\bar{P}^u(z_t). \tag{48}$$

Consider first the determinants of $\bar{P}^u(z_t)$. From equation (27), $\bar{P}^u(z_t)$ is increasing in $U(z_{t+1})$ and decreasing in $B(z_{t+1})$ and $R(z_{t+1})$. The utility of an unhappy home owner, $U(z_{t+1})$, converges to the steady state from above, a result that reflects two forces. First, because the number of buyers is high during the transition, the probability of selling is higher than in the steady state. Second, the price received by the seller is higher than in the steady state. Next, consider the determinants of $R(z_{t+1})$. Along the transition path, it is optimal for natural renters to rent. But, with probability λ, they may become natural buyers. Equation (32) implies that $R(z_{t+1})$ is an increasing function of the discounted, expected, present value of $B(z_{t+1})$. Since $B(z_{t+1})$ is low when there is a large number of buyers, so is $R(z_{t+1})$. Given that $U(z_{t+1})$ is high and $B(z_{t+1})$ and $R(z_{t+1})$ are low relative to the steady state, the reservation price of an unhappy home owner, $\bar{P}^u(z_t)$, is high.

Consider second the determinants of $\bar{P}^b(z_t)$. When a new buyer moves into a new home, with probability η he becomes immediately disenchanted and puts the home up for sale. So, the reservation price of a natural buyer is an increasing function of $H(z_{t+1})$ and $U(z_{t+1})$ (see equation (31)). Since the opportunity cost to a natural buyer of being a home owner is $B(z_{t+1})$, his reservation price is a decreasing function of $B(z_{t+1})$. When the number of buyers is high relative to the steady state, unhappy home owners can sell their home more quickly and receive a higher price. This property has three implications. First, the value of being an unhappy of owner, $U(z_{t+1})$, is higher than in the steady state. Second, $H(z_{t+1})$ is also higher than in the steady state. This result reflects that all home owners will eventually become disenchanted and sell their homes. Equation (23) implies that $H(z_{t+1})$ is an increasing function of the discounted expected present value of $U(z_{t+1})$, which is high when the number of buyers is unusually high. Third, given our calibration, the value of being a natural buyer, $B(z_{t+1})$ falls, reflecting the difficulty in buying a house when there are many buyers. All three forces lead to a rise in $\bar{P}^b(z_t)$.

Recall that $P(z_t)$ is a weighted average of the reservation prices of the unhappy home owners and the natural buyers (see equation (48)). Since both of these prices rise, so too does $P(z_t)$.

In summary, in this experiment an increase in the initial number of buyers reduces the probability of buying a house and raises the probability of selling a house. In addition, it lowers the utility of buyers, raises the utility of sellers, and generates prices that are above their steady state values.

These results suggest that a boom-bust episode occurs if, for some reason, there is a persistent increase in the number of buyers followed by a persistent decrease. In the next section we show that social dynamics can generate the required movements in the number of buyers without observable movements in fundamentals.

4 A matching model with social dynamics

In this section we consider an economy that incorporates the social dynamics described in Section 2 into the model with matching frictions described in Section 3. We use this model to study the same basic experiment considered in Section 2. Suppose that before time zero the economy is in a steady state with no uncertainty. At time zero, agents learn that, with a small probability ϕ, the value of ε changes permanently to a new level ε^*. Agents agree about the value of ϕ but disagree about the probability distribution for ε^*. Agents don't receive any information that is useful for updating their priors about the distribution of ε^*. Once uncertainty is resolved agents become homogeneous in their beliefs. At that point, the economy coincides with the one studied in the previous section where the utility of owning a home is ε^*. The economy then converges to a steady state from initial conditions that are determined by social dynamics and the timing of the resolution of uncertainty.

Agents' expectations about ε^* depend on whether they are optimistic, skeptical or vulnerable. In addition, agents can be home owners, unhappy home owners, natural buyers, or natural renters. So, there are twelve different types of agents in the economy. We use the variables h_t^j, u_t^j, b_t^j, and r_t^j to denote the fraction of the population of type j agents who are home owners, unhappy home owners, natural home buyers, and natural renters, respectively. The index j denotes whether the agent is optimistic, skeptical or vulnerable: $j \in \{o, s, v\}$.

As in Section 3, agents are subject to preference shocks which can turn natural renters into natural buyers and home owners into unhappy home owners. The timing of events within a period is as follows. First, uncertainty about ε^* is realized or not. Second, preference shocks occur. With probability η, home owners become unhappy home owners. With probability λ, natural renters become natural buyers. Third, social interactions occur and agents potentially change their views. Fourth, transactions occur.

4.1 Setting up the model

4.1.1 Population dynamics

There are three ways to incorporate social dynamics into the matching model. The first is closest in spirit to Section 3. Here, agents meet in the beginning of the period via random matching at which point social dynamics occur. With this timing, social dynamics impact directly the number of buyers and sellers in the housing market by changing the fraction of agents of different types. The second approach assumes that social interactions occur only when agents transact in the housing market. Under this assumption, transactions drive social dynamics which later on drive further transactions. The third approach assumes that social dynamics occur at the beginning of the period as well as during market transactions.

We choose the first rather than the second approach because it highlights the role of social dynamics in generating transactions. We choose the first rather than the third approach because it is much simpler and it provides a parsimonious way of generating a gradual increase in the number of optimistic agents (see Section 2).

To solve the model, we must keep track of the fraction of the different types of agents in the model. Prior to the resolution of uncertainty the number of home owners adds up to k and the number of renters to $1-k$, so we can summarize the state of the economy using a vector of ten state variables:

$$\mathfrak{z}_t = (h_t, b_t, h_t^v, h_t^s, b_t^v, b_t^s, r_t^v, r_t^s, u_t^v, u_t^s)'.$$

To streamline our exposition, we discuss here only the law of motion for the fraction of natural renters who are vulnerable. In the appendix we describe the population dynamics for the other agents. The mechanics of these dynamics are similar to those which we now describe. We denote the fraction of vulnerable natural renters at the beginning of the period, after preference shocks occur, after social interactions occur, and after purchases and sales occur, by r_t^v, $(r_t^v)'$, $(r_t^v)''$, and r_{t+1}^v, respectively. At the beginning of the period, a fraction λ of the natural renters become natural buyers,

$$(r_t^v)' = r_t^v(1-\lambda).$$

Next, social interactions occur. A fraction $\gamma^{sv} s_t$ of the vulnerable natural renters become skeptical and a fraction $\gamma^{ov} o_t$ become optimistic. Consequently, the fraction of vulnerable natural renters after social interactions is given by:

$$(r_t^v)'' = (r_t^v)' - \gamma^{sv} (r_t^v)' s_t - \gamma^{ov} (r_t^v)' o_t.$$

Transactions occur at the end of the period. Let $(u_t^v)''$ denote the fraction of the vulnerable natural sellers that remain after social interactions occur. Let $J^{u,v}(\mathfrak{z}_t)$ denote an indicator function that is equal to one if it is optimal for a vulnerable unhappy home owner to put his house up for sale when the state of the economy at the beginning of the period was $\mathfrak{z}_t$ and zero otherwise. If these agents put their homes up for sale, a fraction $\mathfrak{q}^s(\mathfrak{z}_t)$ succeeds in selling. So, the total number of successful sellers is $\mathfrak{q}^s(\mathfrak{z}_t)J^{u,v}(\mathfrak{z}_t)\,(u_t^v)''$. These sellers become natural renters. Let $J^{r,v}(\mathfrak{z}_t)$ denote an indicator function that is equal to one if it is optimal for a vulnerable natural renter to buy a home when the state of the economy is $\mathfrak{z}_t$ and zero otherwise. The number of vulnerable natural renters who try to purchase a home is equal to $J^{r,v}(\mathfrak{z}_t)\,(r_t^v)''$. A fraction $\mathfrak{q}^b(\mathfrak{z}_t)$ of these agents succeed and become natural home owners. So, the number of vulnerable natural renters at the beginning of time $t+1$ is given by:

$$r_{t+1}^v = (r_t^v)'' - \mathfrak{q}^b(\mathfrak{z}_t)J^{r,v}(\mathfrak{z}_t)\,(r_t^v)'' + \mathfrak{q}^s(\mathfrak{z}_t)J^{u,v}(\mathfrak{z}_t)\,(u_t^v)''\,.$$

Using a similar approach for the other population fractions, we can describe the law of motion of $\mathfrak{z}_t$ as $\mathfrak{z}_{t+1} = \mathcal{G}(\mathfrak{z}_t)$ where $\mathcal{G}$ is a function determined by the deterministic dynamics of the populations of different types.

After uncertainty is realized, of course, agents have homogenous beliefs and population dynamics are described by the same processes given in Section 3.

We now describe the value functions of the different agents in the economy. We begin by displaying the value functions that are relevant after uncertainty about ε^* is realized. We then discuss the value functions that are relevant before ε^* is realized.

4.1.2 Value functions and price functions after realization of uncertainty

The value functions of the different agents and all the price functions are the same as those defined in Section 3 with one difference. The momentary utility, ε, is replaced in value by ε^* when uncertainty is realized. Here, it is useful to explicitly index the value functions H, U, B and R, and the price functions, P, P^b, P^r, $\bar{P}^u$, $\bar{P}^b$ and $\bar{P}^r$ by ε^*. For example, we define the value function of the home owner as:

$$H(z_t, \varepsilon^*) = \varepsilon^* + \beta\left[(1-\eta)H(z_{t+1}, \varepsilon^*) + \eta U(z_{t+1}, \varepsilon^*)\right].$$

Similarly, the equations we used in Section 3 to define U, B and R, and the price functions, P, P^b, P^r, $\bar{P}^u$, $\bar{P}^b$ and $\bar{P}^r$ only need to be modified by replacing ε with ε^*. These functions apply after the resolution of uncertainty because, once uncertainty is resolved, there is no distinction between skeptical, optimistic, and vulnerable agents.

4.1.3 Value functions before the realization of uncertainty

We can write $z_t = \Lambda \mathfrak{z}_t$ where $\Lambda = \begin{pmatrix} I_2 & 0_{2\times 8} \end{pmatrix}$. In order to solve the model, we need to compute value and price functions for the period before uncertainty is realized. Let $\mathcal{H}^j(\mathfrak{z}_t)$, $\mathcal{U}^j(\mathfrak{z}_t)$, $\mathcal{B}^j(\mathfrak{z}_t)$, and $\mathcal{R}^j(\mathfrak{z}_t)$ denote the value functions before uncertainty is realized of a type j home owner, unhappy homeowner, natural buyer and natural renter, respectively. Also define type j's time t expectation of a generic value function at time $t+1$:

$$\mathcal{V}_e^j(\mathfrak{z}_t) \equiv (1-\phi)\mathcal{V}^j[\mathcal{G}(\mathfrak{z}_t)] + \phi \sum_{\varepsilon^* \in \Phi} f^j(\varepsilon^*) V\left[\Lambda \mathcal{G}(\mathfrak{z}_t), \varepsilon^*\right]. \tag{49}$$

Here $\mathcal{V}$ represents $\mathcal{H}$, $\mathcal{U}$, $\mathcal{B}$, or $\mathcal{R}$, V represents, correspondingly, H, U, B or R, $\mathcal{G}(\mathfrak{z}_t)$ is the vector of end-of-period t populations and $\Lambda\mathcal{G}(\mathfrak{z}_t)$ represents the relevant subset of the these variables if uncertainty is resolved at the beginning of period $t+1$.

Given this notation, prior to the resolution of uncertainty the Bellman equation for a homeowner is

$$\mathcal{H}^j(\mathfrak{z}_t) = \varepsilon + \beta\left[(1-\eta)\mathcal{H}_e^j(\mathfrak{z}_t) + \eta\mathcal{U}_e^j(\mathfrak{z}_t)\right], \tag{50}$$

The Bellman equation for a type j unhappy homeowner is

$$\mathcal{U}^j(\mathfrak{z}_t) = \max[\mathcal{U}^{sell,j}(\mathfrak{z}_t), \mathcal{U}^{stay,j}(\mathfrak{z}_t)],$$

with

$$\begin{aligned}
\mathcal{U}^{sell,j}(\mathfrak{z}_t) &= \mathsf{q}^s(\mathfrak{z}_t)\{\mathcal{P}^j(\mathfrak{z}_t) + \beta\left[(1-\lambda)\mathcal{R}_e^j(\mathfrak{z}_t) + \lambda\mathcal{B}_e^j(\mathfrak{z}_t)\right]\} + [1-\mathsf{q}^s(\mathfrak{z}_t)]\beta\mathcal{U}_e^j(\mathfrak{z}_t), \\
\mathcal{U}^{stay,j}(\mathfrak{z}_t) &= \beta\mathcal{U}_e^j(\mathfrak{z}_t) - \psi\mathsf{q}^s(\mathfrak{z}_t)\mathcal{P}^j(\mathfrak{z}_t).
\end{aligned}$$

Here $\mathcal{P}^j(\mathfrak{z}_t)$ denotes the average selling price for type j conditional on a sale, and $\mathsf{q}^s(\mathfrak{z}_t)$ is the probability of a sale. The reservation price of a type j unhappy homeowner is:

$$\bar{\mathcal{P}}^{u,j}(\mathfrak{z}_t) = \frac{\beta}{1+\psi}\left\{\mathcal{U}_e^j(\mathfrak{z}_t) - \left[(1-\lambda)\mathcal{R}_e^j(\mathfrak{z}_t) + \lambda\mathcal{B}_e^j(\mathfrak{z}_t)\right]\right\}.$$

The Bellman equation for a type j natural buyer, $\mathcal{B}^j(\mathfrak{z}_t)$, is given by:

$$\mathcal{B}^j(\mathfrak{z}_t) = \max[\mathcal{B}^{rent,j}(\mathfrak{z}_t), \mathcal{B}^{buy,j}(\mathfrak{z}_t)].$$

with

$$\begin{aligned}
\mathcal{B}^{rent,j}(\mathfrak{z}_t) &= \bar{\varepsilon} - w + \beta\mathcal{B}_e^j(\mathfrak{z}_t), \\
\mathcal{B}^{buy,j}(\mathfrak{z}_t) &= \bar{\varepsilon} - w + \mathsf{q}^b(\mathfrak{z}_t)\{-\mathcal{P}^{b,j}(\mathfrak{z}_t) + \beta[(1-\eta)\mathcal{H}_e^j(\mathfrak{z}_t) + \eta\mathcal{U}_e^j(\mathfrak{z}_t)]\} \\
&\quad + [1-\mathsf{q}^b(\mathfrak{z}_t)]\beta\mathcal{B}_e^j(\mathfrak{z}_t).
\end{aligned} \tag{51}$$

Here $\mathcal{P}^{b,j}(\mathfrak{z}_t)$ denotes the average buying price for a type j natural buyer, and $\mathfrak{q}^b(\mathfrak{z}_t)$ is the probability of a purchase. The reservation price of a type j natural buyer is:

$$\bar{\mathcal{P}}^{b,j}(\mathfrak{z}_t) = \beta[(1-\eta)\mathcal{H}_e^j(\mathfrak{z}_t) + \eta\mathcal{U}_e^j(\mathfrak{z}_t) - \mathcal{B}_e^j(\mathfrak{z}_t)]. \tag{52}$$

The Bellman equation for a type j natural renter, $\mathcal{R}^j(\mathfrak{z}_t)$, is given by:

$$\mathcal{R}^j(\mathfrak{z}_t) = \max[\mathcal{R}^{rent,j}(\mathfrak{z}_t), \mathcal{R}^{buy,j}(\mathfrak{z}_t)].$$

with

$$\begin{aligned} \mathcal{R}^{rent,j}(\mathfrak{z}_t) &= \bar{\varepsilon} - w + \beta\left[(1-\lambda)\mathcal{R}_e^j(\mathfrak{z}_t) + \lambda\mathcal{B}_e^j(\mathfrak{z}_t)\right], \\ \mathcal{R}^{buy,j}(\mathfrak{z}_t) &= \bar{\varepsilon} - w + \mathfrak{q}^b(\mathfrak{z}_t)\{-\mathcal{P}^{r,j}(\mathfrak{z}_t) - \kappa\varepsilon + \beta[(1-\eta)\mathcal{H}_e^j(\mathfrak{z}_t) + \eta\mathcal{U}_e^j(\mathfrak{z}_t)]\} \\ &\quad + [1-\mathfrak{q}^b(z_t)]\beta\left[(1-\lambda)\mathcal{R}_e^j(\mathfrak{z}_t) + \lambda\mathcal{B}_e^j(\mathfrak{z}_t)\right]. \end{aligned} \tag{53}$$

Here $\mathcal{P}^{r,j}(\mathfrak{z}_t)$ denotes the average buying price for a type j natural renter. The reservation price of a type j natural renter is:

$$\bar{\mathcal{P}}^{r,j}(\mathfrak{z}_t) = \beta\left[(1-\eta)\mathcal{H}_e^j(\mathfrak{z}_t) + \eta\mathcal{U}_e^j(\mathfrak{z}_t) - (1-\lambda)\mathcal{R}_e^j(\mathfrak{z}_t) - \lambda\mathcal{B}_e^j(\mathfrak{z}_t)\right] - \kappa\varepsilon. \tag{54}$$

4.1.4 Buyers and sellers

In a slight abuse of notation (since the state variable is now $\mathfrak{z}_t$), the number of buyers and sellers is given by:

$$\text{Buyers}(\mathfrak{z}_t) = \sum_{j=o,s,v} (b_t^j)'' J^{b,j}(\mathfrak{z}_t) + \sum_{j=o,s,v} (r_t^j)'' J^{r,j}(\mathfrak{z}_t), \tag{55}$$

$$\text{Sellers}(\mathfrak{z}_t) = \sum_{j=o,s,v} (u_t^j)'' J^{u,j}(\mathfrak{z}_t). \tag{56}$$

Here $J^{b,j}(\mathfrak{z}_t)$ $[J^{r,j}(\mathfrak{z}_t)]$ is an indicator function that is equal to one if it is optimal for a type j natural buyer [renter] to buy a home when the state of the economy is $\mathfrak{z}_t$ and zero otherwise. Similarly, $J^{u,j}(\mathfrak{z}_t)$ is an indicator function that is equal to one if it is optimal for a type j unhappy home owner to put his house up for sale when the state of the economy is $\mathfrak{z}_t$ and zero otherwise.

The number of homes sold, and the probabilities of buying and selling are given by suitably modified versions of equations (45), (46) and (47).

4.1.5 Transactions prices

There are eighteen different possible transaction prices arising out of potential transactions between three types of sellers, three types of natural buyers, and three types of natural renters. The average price paid by a type j natural buyer is:

$$\mathcal{P}^{b,j}(\mathfrak{z}_t) = \theta \bar{\mathcal{P}}^{b,j}(\mathfrak{z}_t) + (1-\theta) \sum_{\ell=o,s,v} \frac{(u_t^{\ell})'' J^{u,\ell}(\mathfrak{z}_t)}{\text{Sellers}(\mathfrak{z}_t)} \bar{\mathcal{P}}^{u,\ell}(\mathfrak{z}_t). \tag{57}$$

The average price paid by a type j natural renter is:

$$\mathcal{P}^{r,j}(\mathfrak{z}_t) = \theta \bar{\mathcal{P}}^{r,j}(\mathfrak{z}_t) + (1-\theta) \sum_{\ell=o,s,v} \frac{(u_t^{\ell})'' J^{u,\ell}(\mathfrak{z}_t)}{\text{Sellers}(\mathfrak{z}_t)} \bar{\mathcal{P}}^{u,\ell}(\mathfrak{z}_t). \tag{58}$$

The average price received by a type j seller is given by:

$$\mathcal{P}^{j}(\mathfrak{z}_t) = \theta \frac{\sum_{\ell=o,s,v} \left[(b_t^{\ell})'' J^{b,\ell}(\mathfrak{z}_t) \bar{\mathcal{P}}^{b,\ell}(\mathfrak{z}_t) + (r_t^{\ell})'' J^{r,\ell}(\mathfrak{z}_t) \bar{\mathcal{P}}^{r,\ell}(\mathfrak{z}_t) \right]}{\text{Buyers}(\mathfrak{z}_t)} + (1-\theta) \bar{\mathcal{P}}^{u,j}(\mathfrak{z}_t). \tag{59}$$

The average price across all transactions can be obtained by averaging across the seller types:

$$\mathcal{P}(\mathfrak{z}_t) = \sum_{\ell=o,s,v} \frac{(u_t^{\ell})'' J^{u,\ell}(\mathfrak{z}_t)}{\text{Sellers}(\mathfrak{z}_t)} \mathcal{P}^{\ell}(\mathfrak{z}_t). \tag{60}$$

4.2 Solving the model

In this subsection, we describe a solution algorithm to compute the equilibrium of the economy along a path on which uncertainty has not been realized.

We begin by considering case 1. In this case, absent resolution of uncertainty, all agents eventually become skeptical. Since $E^s(\varepsilon^*) = \varepsilon$, if the path under consideration converges then it converges to the initial steady state of the economy.

We use the algorithm described in Section 3 to solve for the steady state associated with all possible realizations of ε^* and for the values of the value functions along the transition to the steady state for any initial condition z_t and realized value of ε^*: $H(z_t, \varepsilon^*)$, $U(z_t, \varepsilon^*)$, $B(z_t, \varepsilon^*)$, and $R(z_t, \varepsilon^*)$. As in Section 3, we denote by Z the set of the values of the state variable z_t that occur along the equilibrium path. We denote by $\mathcal{Z}$ the set of the values of the state variable $\mathfrak{z}_t$ that occur along the equilibrium path.

Our solution algorithm is as follows. First, we specify the initial conditions in the economy: h_0^j, u_0^j, b_0^j, and r_0^j for $j = o, s, v$. We choose these conditions so that the fractions of home owners, unhappy home owners, natural buyers, and natural renters are equal to their initial steady state values. In addition, we assume that all agents are vulnerable except for a small number, ζ, of optimistic and skeptical renters: $h_0^v = h$ and $b_0^v = b$, $u_0^v = u$, $r_0^o = r_0^s = \zeta$, and $r_0^v = 1 - k - b - 2\zeta$.

Second, we guess values of the indicator functions that summarize the optimal decisions of natural buyers, natural renters, and unhappy home owners $J^{b,j}(\mathfrak{z}_t)$, $J^{r,j}(\mathfrak{z}_t)$, and $J^{u,j}(\mathfrak{z}_t)$ for all $\mathfrak{z}_t \in \mathcal{Z}$.

Third, we use equations (62)-(81) in Appendix C to compute the path of $\mathfrak{z}_t$ and the analogs of equations (45)–(47), as well as equations (55) and (56), to compute the values of $\mathfrak{q}^s(\mathfrak{z}_t)$ and $\mathfrak{q}^b(\mathfrak{z}_t)$ for $\mathfrak{z}_t \in \mathcal{Z}$.

Fourth, we compute the limiting values of the value functions of all agents along the path on which uncertainty is not realized. The system of equations that defines these limiting values is given by equations (83)–(95) in Appendix C.

Fifth, we solve backwards for all the value functions using equations (50)–(54) and (57)–(60). As in Section 3, we assume that the economy has reached its steady state at time $T = 2000$.

Sixth, we verify that the initial guesses for the indicator functions $J^{b,j}(\mathfrak{z}_t)$, $J^{r,j}(\mathfrak{z}_t)$, and $J^{u,j}(\mathfrak{z}_t)$ describe the optimal behavior of buyers and sellers along the proposed equilibrium path. If not, we revise the guesses until we obtain a consistent solution. In all the results that we report, the equilibrium values of the indicator functions are: $J^{b,j}(\mathfrak{z}_t) = 1$ and $J^{u,j}(\mathfrak{z}_t) = 1$ for all j, $J^{r,o}(\mathfrak{z}_t) = 1$, $J^{r,s}(\mathfrak{z}_t) = J^{r,v}(\mathfrak{z}_t) = 0$ for all $\mathfrak{z}_t \in \mathcal{Z}$.

We use a similar algorithm to solve for the equilibrium in case 2. An important difference between case 1 and 2 is that, in case 2, absent resolution of uncertainty, all agents become optimistic. Given our conjectures about the optimal decisions of the agents, this means the economy does not converge to a steady state equivalent to an economy in which $\varepsilon^* = E^o(\varepsilon^*)$. This is because in the latter economy natural renters choose to rent, whereas here, in the limit, all natural renters are optimistic and choose to buy. We provide the detailed solution method for case 2 in Appendix C.

4.3 Quantitative properties of the model

In this section we illustrate the properties of the model and assess its ability to account for key features of the U.S. housing market. We interpret the recent boom-bust episode in the U.S. as one in which, *ex post*, the skeptical agents were correct. In what follows, we wish to be eclectic about whether or not the boom ended because uncertainty about future fundamentals was realized. For this reason, we use the quantitative implications of the model in case 1 to guide our calibration. Recall that in this case, a boom-bust pattern occurs both when uncertainty is not realized and when uncertainty is realized and the skeptical agents are correct.[14] In calibrating the model, we think of the U.S. episode as a draw from the skeptical agent's unconditional distribution about future housing fundamentals. So, we focus on properties of this unconditional distribution when comparing the model to the data.

This section is organized as follows. First, we discuss the model calibration. We then discuss the quantitative properties of the model in case 1 and 2.

4.3.1 Model calibration

We first describe a set of parameters chosen to render the steady state of the model consistent with selected first moments of the data. We set the stock of houses, k, to 0.65. Recall that this value is the average fraction of home owners in the U.S. population. We choose μ, the scale parameter in the matching function, so that the average time to sell a house is approximately 6 months. This number is roughly equal to the average time that it takes to sell an existing home, computed using data from the National Association of Realtors for the period 1999 to 2012. It is also consistent with the calibration in Piazzesi and Schneider (2009), which is based on data from the American Housing Survey. We choose η so that the probabilities of buying and selling coincide in the steady state. We choose λ so that, in conjunction with our other assumptions, households sell their homes on average every 15 years. This value is close to the one used in Piazzesi and Schneider (2009). We set both the matching parameter (α) and the bargaining parameter (θ) to 0.5, so as to treat buyers and sellers symmetrically.

It is difficult to obtain direct evidence on the parameter κ. In our benchmark calibration we assume that κ is equal to 62. This value implies that the steady state utility of a natural renter who buys a home is 32 percent lower than that of a natural home buyer. In conjunction with our other assumptions, this value of κ implies that it is not optimal for natural renters to buy homes in the steady state. We find that our results are robust to reasonable perturbations in κ. We choose β to be consistent with an annual discount factor

[14] In case 2, a permanent boom occurs when uncertainty is not realized while a boom-bust pattern occurs only when uncertainty is realized and the skeptical are correct.

of 6 percent. In addition, we normalize ε to one and $\bar{\varepsilon} - w$ to zero.

According to the S&P/Case-Shiller U.S. National Home Price Index, house prices increased from 1994 to 2006 and declined from 2006 to 2012. The value of the inflation-adjusted house price index in 2012 is about the same as in 1994. In light of this observation, and consistent with the frictionless model in Section 2, we choose $E^s(\varepsilon^*) = E^v(\varepsilon^*) = \varepsilon$.

It is more challenging to calibrate the other parameters governing agents' expectations and social dynamics. We choose these parameters so that the model is consistent with key moments of the inflation-adjusted S&P/Case-Shiller Home Price Index for Boston, Los Angeles, and San Francisco (see Figure 1 in the appendix).[15] We focus on prices for these cities because they are included in the Case, Shiller, and Thompson (2012) survey of households' one-year expected increases in home prices. Below, we use this survey to help assess the empirical plausibility of our model. In what follows, we assume that the recent U.S. boom-bust episode began in roughly in 1994 and that prices peaked, in our three sample cities, at the end of 2005.

We choose $E^o(\varepsilon^*)$, ϕ, ψ, e_s/e_v and e_o/e_v so that the model has the following properties.[16] First, house prices do not jump at the beginning of the episode. Second, prices are relatively stable during the first year of the episode. Third, from the perspective of the skeptical agents, the expected time from the beginning of the episode to the peak is roughly 11 years. Fourth, in our sample of cities, the average rise in real house prices from 1994 to the end of is 2005 is 133 percent. So, we require the expected price path from the standpoint of the skeptical agents to be 133 percent from the beginning of the episode to the peak. Fifth, it is useful from a computational standpoint to work with parameter values such that it is optimal for unhappy optimistic home owners to always put their homes up for sale. In fact, there exist parameter configurations for which these agents delay putting their house up for sale until uncertainty is realized in anticipation of a large capital gain. These equilibria are much more difficult to compute because agents follow state-dependent strategies. Interestingly, in these equilibria the increase in home prices is larger because there is a smaller supply of homes in the market. So, in this sense, our benchmark calibration is conservative.

[15] We set the initial number of optimistic and skeptical agents to a very small number, $\zeta = 2 \times 10^{-6}$.

[16] In practice, to compute the equilibrium, we discretize the pdfs over ε^* for different agents subject to the constraint that the implied distributions give rise to our preferred values of $E^j(\varepsilon^*)$, $j = s, o, v$, e_o/e_v, and e_o/e_v. See Appendix D for details.

4.3.2 Case 1

In this case, the pdf of the skeptical agents has the lowest entropy ($e^s < e^o < e^v$). The resulting social dynamics are displayed in Figure 5. Figure 5 describes various features of the model along a path in which uncertainty is not realized. The key features of this path can be summarized as follows. First, average home prices rise and then fall as the infection waxes and wanes. Even though agents have perfect foresight up to the resolution of long-run uncertainty, the initial rise in price is very small. Second, the average transaction price comoves strongly with the number of potential buyers. Third, the number of transactions comoves strongly with the average house price. Fourth, as prices rise, there is a "sellers' market" in the sense that the probability of selling is high and the probability of buying is low.

Consistent with our discussion in Section 3, movements in the number of potential buyers are the key drivers of price dynamics in the model. Over time, the number of potential buyers rises from roughly 2 percent to a peak value of roughly 9 percent of the population and then declines.

In the boom phase of the cycle, the number of potential buyers rises for two reasons. First, in contrast to the model without social dynamics, optimistic natural renters want to buy homes. At the peak of the infection, roughly 11 percent of natural renters are optimistic and account for 35 percent of potential buyers (see Figure 5). Second, as more buyers enter the market, the average amount of time to purchase a house rises from 6 to 25 months, while the average time to sell a house drops from 6 months to 1.5 months. To understand these results, recall that the probabilities of buying and selling a home depend on the ratio of buyers to sellers (see equations (46) and (47)). Other things equal, the inflow of optimistic natural renters into the housing market increases the number of buyers, thereby lowering the probability of buying a house and raising the probability of selling a house. The latter effect reduces the stock of unhappy home owners, thus reinforcing the fall in the probability of buying and the rise in the probability of selling a house. As the infection wanes, the number of buyers falls and the number of sellers rises, so the probabilities of buying and selling a house return to their steady state values.

To understand how changes in the number of buyers and sellers affect prices, we exploit the intuition about transition dynamics discussed in Section 4. The average purchase price is a weighted average of the price paid by four types of agents: optimistic natural renters and optimistic, skeptical and vulnerable natural buyers.

The price paid by each of these agents depends positively on their reservation price (see equations (57) and (58)). Each reservation price is the difference between the value to that agent of being a home owner and a home buyer (see equations (52) and (54)). When the probability of buying is low, the value functions of all potential buyers are low because it is more difficult to realize the utility gains from purchasing a home. When the probability of selling is high, the value functions of unhappy home owners are high because it takes less time to sell a home. The value functions of home owners are also high because, with probability η, they become unhappy home owners.

As more agents become optimistic, the probability of buying falls and the probability of selling rises. As a result, the reservation prices of the different potential buyers rise, leading to a rise in purchase prices.

From Figure 5 we see that the optimistic natural buyers pay the highest price. These agents derive a high utility from owning a home and have a high expectation of ε^*. The next highest price is paid by skeptical natural buyers. These agents also derive a high utility from owning a home but they have a lower expectation of ε^* than optimistic natural buyers. Vulnerable and skeptical natural buyers have the same expectation of ε^* so they pay the same price. Optimistic natural renters pay the lowest price. On the one hand, these agents enjoy the house less than natural buyers. On the other hand, they have a higher expectation of ε^* than skeptical and vulnerable natural buyers. For the case being considered the first effect outweighs the second effect.

The presence of optimistic natural renters has two effects. Taking the prices paid by other agents as given, the presence of optimistic natural renters reduces the average price. However, the presence of optimistic renters increases the number of potential buyers, thereby creating a *congestion effect* that reduces the probability of buying a home. As discussed above, this reduction increases the transactions price paid by the other agents in the system. In our example, the second effect dominates the first effect.

Quantifying the congestion effect One way to quantify the importance of the congestion effect is to redo the experiment but not allow optimistic renters to purchase homes. By construction, in this experiment the probability of buying and selling a home is constant, since the number of potential buyers and sellers is unaffected by the infection. It turns out that the average sale price is hardly affected by social dynamics. The only reason for average prices to go up in this experiment is a rise in the reservation price of optimistic natural buyers. This price is the difference between the value of a being a new home owner who is optimistic, $(1-\eta)\mathcal{H}^o(\mathfrak{z}_{t+1}) + \eta\mathcal{U}^o(\mathfrak{z}_{t+1})$, and the value of being an optimistic natural buyer, $\mathcal{B}^o(\mathfrak{z}_{t+1})$. The value of becoming a home owner increases if a vulnerable agent becomes optimistic. But the value of being an optimistic natural buyer also increases because an optimistic agent has a high expected value of ε^*. In contrast to the situation where the congestion effect is operative, here the probability of buying a home remains constant, so there is no countervailing effect on the optimistic natural buyers' value functions. The net result is a small increase in the reservation price of optimistic buyers.

What happens when uncertainty is resolved? Figure 6 shows the average behavior of the price if uncertainty is realized in year 13. The solid line depicts the actual house price up to the period when uncertainty is realized. The dashed line shows the average price path that vulnerable/skeptical agents expect after uncertainty is realized. Interestingly, these agents do not expect the price to converge immediately to its steady value after uncertainty is realized. The reason is that, when uncertainty is resolved, the number of buyers exceeds its steady state value. The transition to the steady state is governed by the transition dynamics of the homogeneous expectations model. As emphasized in Section 3, when the number of buyers exceeds its steady state value, the price converges to its steady state value gradually from above.

Figure 6 helps us understand why a skeptical or vulnerable natural buyer is willing to buy a house even around the peak in housing prices. At this point, the price is much higher than the steady state price that these agents expect. Even if uncertainty is resolved in the following period, agents expect the fall in the price to be relatively small because the number of potential home buyers is significantly above its steady state value. Even if a home buyer becomes an unhappy home owner, the expected capital loss on the house is expected to be relatively small. As a consequence, the gains from living in the house outweigh the expected capital loss.

Optimistic agents expect a large capital gain when uncertainty is realized. The expected gain is so large that it induces natural renters to try to purchase a home. They are willing to do so because the expected gains from speculation outweigh their disposition to rent rather than buy.

Figure 6 shows that there is a discontinuous jump up or down in house prices when uncertainty is realized. We do not observe these types of jumps in the data. The discontinuity reflects the stark nature of how information is revealed in the model. This feature would be eliminated if information about long-run fundamentals gradually percolates throughout the economy as in Duffie, Giroux and Manso (2010) and Andrei and Cujean (2013).

4.3.3 Case 2

In this case, the pdf of the optimistic agents has the lowest entropy ($e^o < e^s < e^v$). The same economic forces discussed above are at work here. The key difference is that, absent resolution of uncertainty, the entire population becomes optimistic (see panel (b) of Figure 1). As a consequence, the number of optimistic renters rises and remains high until uncertainty is resolved. So, the number of potential buyers remains high and the congestion effect is operative for a much longer period of time. Not surprisingly, in case 2 the probability of buying (selling) is much lower (higher) for a longer period of time than in case 1. Consequently, it takes much longer in case 2 for the volume of transactions to return to its steady state level.

Finally, Figure 7 displays the price path absent resolution of uncertainty. As in case 1, the price rises before year 11, albeit to a higher level, reflecting the larger number of potential buyers in the system. The price stays high until uncertainty is resolved.

4.3.4 Expected price paths

We now discuss the properties of the time-zero expectations of time-t prices for optimistic and skeptical agents. For $t > 0$ these are

$$E_0^j(P_t) = (1-\phi)^t \mathcal{P}[\mathcal{G}^t(\mathfrak{z}_0)] + \sum_{\tau=1}^{t} \phi(1-\phi)^{\tau-1} \sum_{\varepsilon^* \in \Phi} f^j(\varepsilon^*) P\left\{G^{t-\tau}[\Lambda \mathcal{G}^\tau(\mathfrak{z}_0)], \varepsilon^*\right\}. \tag{61}$$

The first term in equation (61) reflects the possibility that uncertainty has not yet been resolved by the end of time t. The probability of this event, $(1-\phi)^t$, is multiplied by $\mathcal{P}[\mathcal{G}^t(\mathfrak{z}_0)]$, the price at time t in that state of the world. Here $\mathcal{G}^t(\cdot)$, is the law of motion of the state variables invoked t times, so it represents $\mathfrak{z}_t$. The second term in equation (61) reflects the possibility that uncertainty is resolved at time $\tau \leq t$, an event that occurs with probability $\phi(1-\phi)^{\tau-1}$. This probability is multiplied by the price that agent j expects to occur at time t if uncertainty is realized at time τ. Here, $G^{t-\tau}[\Lambda \mathcal{G}^\tau(\mathfrak{z}_0)]$ represents z_t since $\mathcal{G}^\tau(\mathfrak{z}_0)$ represents $\mathfrak{z}_\tau$, $\Lambda \mathfrak{z}_\tau$ represents z_τ and $G^{t-\tau}(\cdot)$ represents the law of motion after the resolution of uncertainty invoked $t-\tau$ times.

Panel (a) of Figure 8 depicts, for case 1, the price paths expected by different agents. Optimistic agents expect prices to rise rapidly until year 15 and to remain relatively high. Skeptical agents expect prices to rise up to year 11, although by less than optimistic agents. Thereafter, skeptical agents expect prices to revert gradually to their old steady state levels.

Panel (b) of Figure 8 depicts, for case 2, the price paths expected by different agents. The key property to notice is that, while there are quantitative differences, the patterns are remarkably similar. Skeptical agents expect a boom that is followed by a bust while optimistic agents expect a boom that is not followed by a bust. Qualitatively this is the same result that we obtained with the frictionless model of Section 3. Once again, an econometrician taking repeated samples from our data would see both boom-busts and booms that are not followed by busts. The boom-bust episodes typically occur in economies where the skeptical agents happen to be correct. The booms that are not followed by busts typically occur in economies in which optimistic agents happen to be correct.

4.4 Some evidence on the model's mechanisms

In this subsection, we present some evidence on the mechanisms at work in our model. Our model attributes a key role to buyers with speculative motives. Booms (busts) are marked by increases (decreases) in the number of agents who buy homes primarily because of large expected capital gains (natural renters). It is obviously difficult to measure the importance of such buyers. However, the Michigan Survey of Consumers provides us with some indirect information. The survey includes the question: "is it a good time to buy a home because it is a good investment?" Figure 9 displays the percentage of respondents answering yes to this question from 1996 to 2012, along with the percentage change in the inflation-adjusted Case-Shiller national price index. There is a clear increase in the number of respondents who thought housing was a good investment during the boom period and a sharp decline during the bust phase.

Suppose we assume that a rise in the number of first-time buyers is likely to come from the pool of natural renters. Then, an additional piece of evidence consistent with our model is that housing booms are accompanied by an increase in the number of first-time buyers. Data from the Current Population Survey show that the number of homes owned by individuals 29 years old and younger increased by 34 percent between 1994 and 2005 and decreased by 20 percent between 2005 and 2012. The numbers are even more dramatic for individuals age 25 and younger, with the rise and fall equalling 73 and 24 percent, respectively. In a similar vein, Holmans (1995) documents a large rise and fall in the number of first-time buyers during the U.K. boom-bust episode of the late 1980s and early 1990s. Evidence along the same lines comes from the housing expectations survey data used by Case, Shiller, and Thompson (2012). From 2003 on, first-time buyers had consistently higher expectations of price appreciation than other buyers. The difference between the one-year ahead rate of return expected by first-time buyers and other buyers ranges from 1.1 percent in 2003 to 5.4 percent in 2006. Taken together, the previous evidence is supportive of the important role that our model ascribes to speculative buyers in boom and bust episodes.

A core implication of our model is that a boom (bust) in house prices is associated with an increase (decrease) in the probability of selling a house. To assess this implication, we use data from the National Association of Realtors on inventories and sales of existing homes covering the period 1999 to 2012. We compute the monthly probability of a sale as the ratio of sales to inventories of existing homes for sale. The annual probability of a sale is the average of the monthly probabilities in a given year. Figure 9 shows that there is a striking comovement between this probability and the rate of change in the inflation-adjusted Case-Shiller price index. Both series rise from 1999 until the end of 2005 and then drop sharply until 2008 before recovering.

Our model also implies that booms (busts) are associated with an increase (decrease) in the volume of transactions. Figure 9 provides empirical evidence on the volume of transactions. Transaction volume rose, reaching a peak in 2005, before falling dramatically through 2009. This pattern is consistent with evidence in Stein (1995).

Recall that, according to our model, an unusually large number of potential buyers enter the market during booms. Their desire to purchase a home is primarily driven by speculative motives. The entry of these agents makes it easier to sell existing homes and increases the overall level of activity in the housing market. During the bust phase speculative buyers exit the market driving down the probability of selling a home and the volume of transactions. So, our model implies that the extensive margin plays a critical role in house price dynamics. Taken together, we interpret the evidence in Figure 9 as supportive of this implication.

Agents' expectations about house prices clearly lie at the center of our model. It is also clear that the model can generate large price movements. A key question is whether these movements are driven by quantitatively unreasonable expectations about house prices. To answer this question, we use data from the Case, Shiller, and Thompson (2012) home buyer survey on the one-year ahead expected increase in home prices. This survey was conducted annually in Boston, Los Angeles, and San Francisco between 2003 and 2012.[17] Since the average price in these cities peaked in 2006, we focus on the period from 2003 to 2006. The survey data are quite noisy in part because sample sizes are small (the average number of respondents is about 100 per city per year). So, we focus on the average annual expected real appreciation over the period 2003–06, which is roughly 5.5 percent.[18] To compute the analogue statistic in our model, we proceed as follows. First, we compute the one-year ahead expected appreciation for optimistic, skeptical and vulnerable agents. We then compute the average expectation across agents using their population weights at each point in time. According to our model, the average expected price appreciation in the three-year period before the peak year is 3.1 percent.[19] So, if anything, the model accounts for the large appreciation in housing prices even though it understates agents' one-year ahead expected price appreciation. We conclude that we do not need to invoke quantitatively unreasonable expectations about house prices to generate large price movements.

We conclude by discussing some evidence on the role of social dynamics in asset markets. There is a growing literature that provides evidence on the importance of social dynamics in equity markets.[20] In a recent contribution, Khang (2012) focuses on the role of social dynamics in housing markets. The key question that he addresses is: What is the effect of a 'real estate expert' moving into a community on housing purchases made for investments purposes by other people in the ecommunity? By 'real estate expert,' Khang means someone who has previously purchased housing as an investment property. By community, he means a group of homeowners who reside in the same zip code. The premise of Khang's analysis is that the arrival of an expert is likely to increase the amount of social interactions concerning housing investments.

Khang's analysis is based on comprehensive zip-code-level panel data on housing transactions in 33 states, spanning the period from 2000 to 2011. Where real estate experts choose

[17] The survey also includes results for Milwaukee. We exclude these results because Milwaukee is not included in the S&P/Case-Shiller Home Price Index.

[18] We use the U.S. survey of professional forecasters to convert expected nominal appreciation into expected real appreciation.

[19] The corresponding values for skeptical/vulnerable and optimistic agents are 1.7 percent and 5.1 percent, respectively.

[20] See, e.g. Kelly and Ó. Gráda (2000), Duflo and Saez (2002), Brown, Ivković, Smith, and Weisbenner (2008), Hong, Kubik, and Stein (2005), and Kaustia and Knüpfer (2008).

to reside is clearly endogenous. To deal with this issue, Khang uses a nearest-neighborhood estimator (Abadie el al (2004)) as well as a difference-in-differences approach with zip-code-level fixed effects. His main findings, as they pertain to our model, can be summarized as follows. First, investment property purchases rise by 24 percent in communites where a real estate expert arrives. Second, the effect of the arrival of a real estate expert declines monotonically with the size of the community, with the largest effects occuring in the smallest communities.

Recall that Khang defines a real estate expert as someone with prior housing investment experience. Suppose we make the reasonable assumptions that: (i) a real estate expert has tighter priors about the path of future housing prices than non-real estate experts; and (ii) the percentage of the population that has social interactions with the real estate expert is decreasing in the size of the community.[21] Then, both of Khang's findings are supportive of the basic mechanism at the heart of our model.

5 Conclusion

Boom-bust episodes are pervasive in housing markets. They occur in different countries and in different time periods. These episodes are hard to understand from the perspective of conventional models in which agents have homogeneous expectations. In this paper we propose a model in which agents have different views about long-run fundamentals. Social interactions can generate temporary increases in the fraction of agents who hold a particular view about long-run fundamentals. The resulting dynamics can produce boom-bust cycles as well as booms that are not followed by busts.

Our model abstracts from financial frictions. It is clear to us that the ability of many young buyers to buy a home is influenced by down-payment requirements and credit conditions. An implication of our model is that if young buyers are optimistic but cannot buy a house, say because they are credit constrained, boom-bust cycles in house prices are greatly muted. Indeed, this situation corresponds to the experiment in our model where we lock out optimistic natural renters from the housing market. In this case there are no congestion effects and there are no pronounced boom-bust cycles. But there is no presumption that a policy of requiring high down-payments would be welfare improving because this policy would presumably apply to both natural buyers and natural renters. More generally, poli-

[21] Both of these assumptions hold in our model.

cies aimed at curbing rapid price increases are not obviously welfare improving in our model because, in the end, we do not know who is right about the future: the vulnerable, the skeptical, or the optimistic.

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A Appendix A: Internalizing changes in agent type

In the main text we assume that agents do not take into account the possibility that they may change their type as a result of social interactions. Here we assess the quantitative impact of this assumption by calculating equilibrium prices when agents do internalize the possibility that they may change their type.

In case 1, absent resolution of uncertainty, all agents become skeptical as t goes to infinity and the terminal price is equal to the fundamental price of a skeptical agent (P^s in equation (12)). In case 2, absent resolution of uncertainty, all agents become optimistic as t goes to infinity and the terminal price is equal to the fundamental price of an optimistic agent (P^o in equation (11)). Using these terminal prices we can compute the equilibrium price path in a recursive fashion.

When $o_t \geq k$, optimistic agents are the marginal home owners. In this case the equilibrium price is given by:

$$\begin{aligned} P_t &= (1-\gamma^{so}s_t)\beta\left[\phi E^o(\varepsilon^* + P^*_{t+1}) + (1-\phi)(\varepsilon + P_{t+1})\right] + \\ &\quad \gamma^{so}s_t\beta\left[\phi E^s(\varepsilon^* + P^*_{t+1}) + (1-\phi)(\varepsilon + P_{t+1})\right]. \end{aligned}$$

Recall that P_{t+1} and P^*_{t+1} are the $t+1$ prices when uncertainty is not realized and when uncertainty is realized, respectively. Here an optimistic agent takes into into account that with probability $\gamma^{so}s_t$ he becomes skeptical at time $t+1$ and values the house as a skeptical agent. The value of γ^{so} is positive in case 1 but equal to zero in case 2.

When $o_t < k$ and $o_t + v_t \geq k$ vulnerable agents are the marginal home owners even if $s_t \geq k$. Vulnerable agents have higher valuations than skeptical agents because they have a higher probability of becoming optimistic. In this case the equilibrium price is given by:

$$\begin{aligned} P_t &= (1-\gamma^{ov}o_t - \gamma^{sv}s_t)\beta\left[\phi E^v(\varepsilon^* + P^*_{t+1}) + (1-\phi)(\varepsilon + P_{t+1})\right] \\ &\quad +\gamma^{ov}o_t\beta\left[\phi E^o(\varepsilon^* + P^*_{t+1}) + (1-\phi)(\varepsilon + P_{t+1})\right] + \\ &\quad +\gamma^{sv}s_t\beta\left[\phi E^s(\varepsilon^* + P^*_{t+1}) + (1-\phi)(\varepsilon + P_{t+1})\right]. \end{aligned}$$

Here the vulnerable agent takes into account that with probability $\gamma^{ov}o_t$ he becomes optimistic and values the house as an optimistic agent. Also, with probability $\gamma^{sv}s_t$ he becomes skeptical and values the house as a skeptical agent.

Finally, when $o_t < k$ and $o_t + v_t < k$ the marginal home owner is a skeptical agent. In this case the equilibrium price is given by:

$$P_t = \gamma^{os} o_t \beta \left[\phi E^o(\varepsilon^* + P^*_{t+1}) + (1-\phi)(\varepsilon + P_{t+1})\right] + (1-\gamma^{os} o_t)\beta \left[\phi E^s(\varepsilon^* + P^*_{t+1}) + (1-\phi)(\varepsilon + P_{t+1})\right].$$

Here the skeptical agent takes into account that, with probability $\gamma^{os} o_t$ he becomes optimistic and values the house as an optimistic agent. Recall that γ^{os} is zero in case 1 but it is positive in case 2.

We redo the experiment that underlies Figure 2 using the same parameter values. The basic finding is that internalizing changes in agent type makes virtually no difference to our results. The basic reason is that the probability of switching types is small. For instance, in case 1 the maximum value of $\gamma^{ov} o_t$ and $\gamma^{so} s_t$ in our numerical example are three and one-third of one percent, respectively. In the following sections we abstract from this effect to simplify our computations.

B Appendix B: An alternative interpretation of social dynamics

In this appendix we describe an alternative environment which generates social dynamics that are similar to those of our model. In this example agents have heterogeneous priors and receive private signals. Suppose that the agents who are initially optimistic or skeptical have very sharp priors. Agents that are initially vulnerable have very diffuse priors. All agents receive uninformative private signals. Vulnerable agents have sharp priors that the posteriors of optimistic and skeptical agents are the product of initially diffuse priors and very informative signals. So, when a vulnerable agent meets an optimistic (skeptical) agent his posterior becomes arbitrarily close to that of the optimistic (skeptical) agent. We refer to a vulnerable agent who has a posterior that is very close to that of an optimistic (skeptical) agent as optimistic (skeptical).

We reinterpret γ^{lj} as the probability that agents of type l meet agents of type j. We assume that $\gamma^{vs} = \gamma^{vo} = \gamma$ and that $\gamma^{so} = 0$, i.e. skeptical and optimistic agents have no social interactions. Under our assumptions the dynamics of the fraction of population with different views are similar to those generated by our model of social dynamics. Our assumptions about γ^{lj} eliminate the convergence of posteriors that is a generic property of Bayesian environments. As a result, we preserve the property that different agents agree to disagree.[22] To obtain dynamics similar to cases 1 and 2 we need to introduce a slight asymmetry between skeptical and optimistic agents. A simple, albeit mechanical, way to introduce this asymmetry is to suppose that in case 1 (case 2) a small fraction δ of optimistic (skeptical) agents exogenously change their view to those of skeptical (optimistic) agents.

The view of social segmentation embodied in our assumptions about γ^{lj} is consistent with the notion that agents who are strongly committed to a point of view limit their interactions to sources of information and individuals that are likely to confirm their own views. This phenomenon is discussed by Sunstein (2001) and Mullainathan and Shleifer (2005). The latter authors summarize research in psychology, communications and information theory that is consistent with the social-segmentation hypothesis. More recently, Gentzkow and Shapiro (2010) find evidence that people tend to have close social interactions with people who have similar political views. Social segmentation is related to what sociologists call "homophily": contact between similar people occurs at a higher rate than contact among dissimilar people (McPherson, Smith-Lovin, Cook (2001)).

C Appendix C

In this appendix we describe the laws of motion for the fraction of the population accounted for by the twelve types of agents in the model of Section 5. The values of γ^{lj}, which depend on the ratio of the entropies of the pdfs of agents l and j, are defined in equation (3). Recall that $\gamma^{os} = 0$ in case 1 and $\gamma^{so} = 0$ in case two.

Home owners We denote the fraction of home owners of type j $(j = o, s, v)$ in the beginning of the period, after preference shocks occur, after social interactions occur, and after purchases and sales occur by h_t^j, $(h_t^j)'$, $(h_t^j)''$, and h_{t+1}^j, respectively. The laws of motion for these variables are given by:

$$(h_t^j)' = h_t^j(1-\eta), \qquad j = o, s, v, \tag{62}$$

$$(h_t^v)'' = (h_t^v)' - \gamma^{sv}\,(h_t^v)'\,s_t - \gamma^{ov}(h_t^v)'o_t, \tag{63}$$

$$(h_t^s)'' = (h_t^s)' + \gamma^{sv}\,(h_t^v)'\,s_t - \gamma^{os}\,(h_t^s)'\,o_t + \gamma^{so}\,(h_t^o)'\,s_t, \tag{64}$$

$$(h_t^o)'' = (h_t^o)' + \gamma^{ov}\,(h_t^v)'\,o_t - \gamma^{so}\,(h_t^o)'\,s_t + \gamma^{os}\,(h_t^s)'\,o_t, \tag{65}$$

[22] Acemoglu et al. (2007) provide an alternative environment in which agents agree to disagree because they are uncertain about the interpretation of the signals that they receive.

$$h_{t+1}^{j} = (h_t^j)'' + \mathfrak{q}^b(\mathfrak{z}_t)J^{b,j}(\mathfrak{z}_t)(b_t^j)'' + \mathfrak{q}^b(\mathfrak{z}_t)J^{r,j}(\mathfrak{z}_t)(r_t^j)'', \qquad j = o, s, v. \tag{66}$$

Unhappy home owners We denote the fraction of unhappy home owners of type j ($j = o, s, v$) in the beginning of the period, after preference shocks occur, after social interactions occur, and after purchases and sales occur by u_t^j, $(u_t^j)'$, $(u_t^j)''$, and u_{t+1}^j, respectively. The laws of motion for these variables are given by:

$$(u_t^j)' = u_t^j + \eta h_t^j, \qquad j = o, s, v, \tag{67}$$

$$(u_t^v)'' = (u_t^v)' - \gamma^{sv}\,(u_t^v)'\,s_t - \gamma^{ov}(u_t^v)'o_t, \tag{68}$$

$$(u_t^s)'' = (u_t^s)' + \gamma^{sv}\,(u_t^v)'\,s_t - \gamma^{os}\,(u_t^s)'\,o_t + \gamma^{so}\,(u_t^o)'\,s_t, \tag{69}$$

$$(u_t^o)'' = (u_t^o)' + \gamma^{ov}\,(u_t^v)'\,o_t - \gamma^{so}\,(u_t^o)'\,s_t + \gamma^{os}\,(u_t^s)'\,o_t, \tag{70}$$

$$u_{t+1}^j = (u_t^j)'' - \mathfrak{q}^s(\mathfrak{z}_t)J^{u,j}(\mathfrak{z}_t)(u_t^j)'', \qquad j = o, s, v. \tag{71}$$

Natural buyers We denote the fraction of natural buyers of type j ($j = o, s, v$) in the beginning of the period, after preference shocks occur, after social interactions occur, and after purchases and sales occur by b_t^j, $(b_t^j)'$, $(b_t^j)''$, and b_{t+1}^j, respectively. The laws of motion for these variables are given by:

$$(b_t^j)' = b_t^j + \lambda r_t^j, \qquad j = o, s, v, \tag{72}$$

$$(b_t^v)'' = (b_t^v)' - \gamma^{sv}\,(b_t^v)'\,s_t - \gamma^{ov}(b_t^v)'o_t, \tag{73}$$

$$(b_t^s)'' = (b_t^s)' + \gamma^{sv}\,(b_t^v)'\,s_t - \gamma^{os}\,(b_t^s)'\,o_t + \gamma^{so}\,(b_t^o)'\,s_t, \tag{74}$$

$$(b_t^o)'' = (b_t^o)' + \gamma^{ov}\,(b_t^v)'\,o_t - \gamma^{so}(b_t^o)'s_t + \gamma^{os}\,(b_t^s)'\,o_t, \tag{75}$$

$$b_{t+1}^j = (b_t^j)'' - \mathfrak{q}^b(\mathfrak{z}_t)J^{b,j}(\mathfrak{z}_t)(b_t^j)'', \qquad j = o, s, v. \tag{76}$$

Natural renters We denote the fraction of natural renters of type j ($j = o, s, v$) in the beginning of the period, after preference shocks occur, after social interactions occur, and after purchases and sales occur by r_t^j, $(r_t^j)'$, $(r_t^j)''$, and r_{t+1}^j, respectively. The laws of motion for these variables are given by:

$$(r_t^j)' = r_t^j(1 - \lambda), \qquad j = o, s, v, \tag{77}$$

$$(r_t^v)'' = (r_t^v)' - \gamma^{sv}\,(r_t^v)'\,s_t - \gamma^{ov}(r_t^v)'o_t, \tag{78}$$

$$(r_t^s)'' = (r_t^s)' + \gamma^{sv}(r_t^v)' s_t - \gamma^{os}(r_t^s)' o_t + \gamma^{so}(r_t^o)' s_t, \tag{79}$$

$$(r_t^o)'' = (r_t^o)' + \gamma^{ov}(r_t^v)' o_t - \gamma^{so}(r_t^o)' s_t + \gamma^{os}(r_t^s)' o_t, \tag{80}$$

$$r_{t+1}^j = (r_t^j)'' - \mathfrak{q}^b(\mathfrak{z}_t) J^{r,j}(\mathfrak{z}_t)(r_t^j)'' + \mathfrak{q}^s(\mathfrak{z}_t) J^{u,j}(\mathfrak{z}_t)(u_t^j)'', \qquad j = o, s, v. \tag{81}$$

Limiting steady state when uncertainty is not realized, Case 1 We denote by $H(\varepsilon^*)$, $U(\varepsilon^*)$, $B(\varepsilon^*)$ and $R(\varepsilon^*)$ the steady state of the value functions of different agents in the economy when uncertainty is realized and the realized utility of owning a home is ε^*. These values are computed by solving the system of equations (23), (24), (28), (29), (32), (31), and (44), setting $B = B^{buy}$ and $R = R^{rent}$ and replacing ε in equation (23) with the different possible values of ε^*. Then let

$$\bar{V}^j = \sum_{\varepsilon^* \in \Phi} f^j(\varepsilon^*) V(z, \varepsilon^*) \tag{82}$$

where V represents H, U, B or R, and $z = (h, b)'$ with h and b being the steady state values of h and b from Section 3. The limiting value functions of different agents along a path in which uncertainty is not resolved can be obtained by solving the following system of equations for $\mathcal{H}^j$, $\mathcal{U}^j$, $\mathcal{B}^j$, $\mathcal{R}^j$, and $\bar{\mathcal{P}}^{b,s}$ and for $j = o, v, s$, unless indicated otherwise:

$$\mathcal{H}^j = \varepsilon + \beta\{(1-\phi)\left[(1-\eta)\mathcal{H}^j + \eta\mathcal{U}^j\right]) + \phi[(1-\eta)\bar{H}^j + \eta\bar{U}^j]\}, \tag{83}$$

$$\begin{aligned} \mathcal{U}^j &= q^s\mathcal{P}^j + q^s\beta\left\{(1-\phi)\left[(1-\lambda)\mathcal{R}^j + \lambda\mathcal{B}^j\right] + \phi\left[(1-\lambda)\bar{R}^j + \lambda\bar{B}^j\right]\right\} + \\ &\quad (1-q^s)\beta[(1-\phi)\mathcal{U}^j + \phi\bar{U}^j] \end{aligned} \tag{84}$$

$$\bar{\mathcal{P}}^{u,j} = \frac{\beta}{1+\psi}\left\{(1-\phi)\left[\mathcal{U}^j - (1-\lambda)\mathcal{R}^j - \lambda\mathcal{B}^j\right] + \phi\left[\bar{U}^j - (1-\lambda)\bar{R}^j - \lambda\bar{B}^j\right]\right\}. \tag{85}$$

$$\begin{aligned} \mathcal{B}^j &= \bar{\varepsilon} - w - q^b\mathcal{P}^{b,j} + q^b\beta\left\{(1-\phi)\left[(1-\eta)\mathcal{H}^j + \eta\mathcal{U}^j\right] + \phi\left[(1-\eta)\bar{H}^j + \eta\bar{U}^j\right]\right\} + \\ &\quad (1-q^b)\beta[(1-\phi)\mathcal{B}^j + \phi\bar{B}^j] \end{aligned} \tag{86}$$

$$\bar{\mathcal{P}}^{b,j} = \beta\left\{(1-\phi)[(1-\eta)\mathcal{H}^j + \eta\mathcal{U}^j - \mathcal{B}^j] + \phi\left[(1-\eta)\bar{H}^j + \eta\bar{U}^j - \bar{B}^j\right]\right\}. \tag{87}$$

$$\mathcal{R}^j = \bar{\varepsilon} - \omega + \beta\left\{(1-\phi)\left[(1-\lambda)\mathcal{R}^j + \lambda\mathcal{B}^j\right] + \phi\left[(1-\lambda)\bar{R}^j + \lambda\bar{B}^j\right]\right\}, \; j = s, v \tag{88}$$

$$\begin{aligned} \mathcal{R}^o &= \bar{\varepsilon} - w - q^b\mathcal{P}^{r,o} + q^b\beta\left\{(1-\phi)\left[(1-\eta)\mathcal{H}^o + \eta\mathcal{U}^o\right] + \phi\left[(1-\eta)\bar{H}^o + \eta\bar{U}^o\right]\right\} + \\ &\quad (1-q^b)\beta\left\{(1-\phi)\left[(1-\lambda)\mathcal{R}^o + \lambda\mathcal{B}^o\right] + \phi\left[(1-\lambda)\bar{R}^o + \lambda\bar{B}^o\right]\right\} \end{aligned} \tag{89}$$

$$\bar{\mathcal{P}}^{r,j} = \beta(1-\phi)\left[(1-\eta)\mathcal{H}^j + \eta\mathcal{U}^j - (1-\lambda)\mathcal{R}^j - \lambda\mathcal{B}^j\right] + \beta\phi\left[(1-\eta)\bar{H}^j + \eta\bar{U}^j - (1-\lambda)\bar{R}^j - \lambda\bar{B}^j\right] - \kappa\varepsilon. \tag{90}$$

Recall that in the limit all agents are skeptical, and we have conjectured that skeptical natural renters do not choose to buy, thus simplifying expressions for average prices:

$$\mathcal{P}^{b,j} = \theta\bar{\mathcal{P}}^{b,j} + (1-\theta)\bar{\mathcal{P}}^{u,s} \tag{91}$$

$$\mathcal{P}^{r,j} = \theta\bar{\mathcal{P}}^{r,j}(\mathfrak{z}_t) + (1-\theta)\bar{\mathcal{P}}^{u,s} \tag{92}$$

$$\mathcal{P}^{j} = \theta\bar{\mathcal{P}}^{b,s}(\mathfrak{z}_t) + (1-\theta)\bar{\mathcal{P}}^{u,j}(\mathfrak{z}_t). \tag{93}$$

$$\mathcal{P} = \mathcal{P}^s \tag{94}$$

The probabilities of buying and selling are given by:

$$q^s = q^b = \mu. \tag{95}$$

Limiting steady state when uncertainty is not realized, Case 2 In case 2 all agents become optimistic in the limit when uncertainty is not realized. Given our conjectured optimal decision rules, optimistic natural buyers *and* natural renters choose to buy homes prior to the resolution of uncertainty. This means that the limiting populations, h_t, u_t, b_t and r_t do not correspond to their initial steady state values. This is because the initial steady state is a constant solution to equations (36)–(41) and (45)–(47) with $J^r = 0$, whereas here the limiting case is a constant solution to equations (36)–(41) and (45)–(47) with $J^r = 1$. We denote the values of h_t, b_t, q_t^s and q_t^b in this limiting case as $\tilde{h}$, $\tilde{b}$, $\tilde{q}^s$ and $\tilde{q}^b$ and we let $\tilde{z} = (\tilde{h}, \tilde{b})'$.[23]

Next we denote by $H(\tilde{z}, \varepsilon^*)$, $U(\tilde{z}, \varepsilon^*)$, $B(\tilde{z}, \varepsilon^*)$ and $R(\tilde{z}, \varepsilon^*)$ the value functions of different agents in the economy when uncertainty is realized and the realized utility of owning a home is ε^*. These values are computed using the same methods we used to compute utilities for the transitions dynamics case in Section 3. Then we define

$$\bar{V}^j = \sum_{\varepsilon^* \in \Phi} f^j(\varepsilon^*) V(\tilde{z}, \varepsilon^*) \tag{96}$$

[23] In solving the model for Case 2 we have to make a minor modification to the matching technology, and write it as

$$m_t = \min\{\mu \text{Sellers}_t^{\alpha} \text{Buyers}_t^{1-\alpha}, \text{Sellers}_t, \text{Buyers}_t\}.$$

This ensures that the probabilities of buying and selling are bounded between zero and one. In solving Case 1 we found that these constraints never bind. In Case 2, in the limiting case, $\tilde{q}^s = 1$.

where V represents H, U, B or R. With this different definition of $\bar{V}^j$ in place, and using $\tilde{q}^s$ and $\tilde{q}^b$ in place of q^s and q^b, we can solve for the limiting value functions and reservation prices of different agents along a path in which uncertainty is not resolved using equations (83)–(90). Since, in the limit, all agents are optimistic, the expressions for average prices are:

$$\mathcal{P}^{b,j} = \theta\bar{\mathcal{P}}^{b,j} + (1-\theta)\bar{\mathcal{P}}^{u,o} \tag{97}$$

$$\mathcal{P}^{r,j} = \theta\bar{\mathcal{P}}^{r,j}(\mathfrak{z}_t) + (1-\theta)\bar{\mathcal{P}}^{u,o} \tag{98}$$

$$\mathcal{P}^{j} = \theta\frac{[\tilde{b} + \lambda(1-k-\tilde{b})]\bar{\mathcal{P}}^{b,o} + (1-\lambda)(1-k-\tilde{b})\bar{\mathcal{P}}^{r,o}}{1-k} + (1-\theta)\bar{\mathcal{P}}^{u,j}. \tag{99}$$

$$\mathcal{P} = \mathcal{P}^{o} \tag{100}$$

D Appendix D

To simplify our computations we work with a discrete approximation to the beta distributions defined on a symmetric grid with six points. To compute the probability of each of these points we divide the support of the distribution into six intervals of equal size and compute the integral of the beta distribution over these intervals. The support of the discretized distributions for ε^* and the associated pdfs of the different agents in Case 1 are given by:

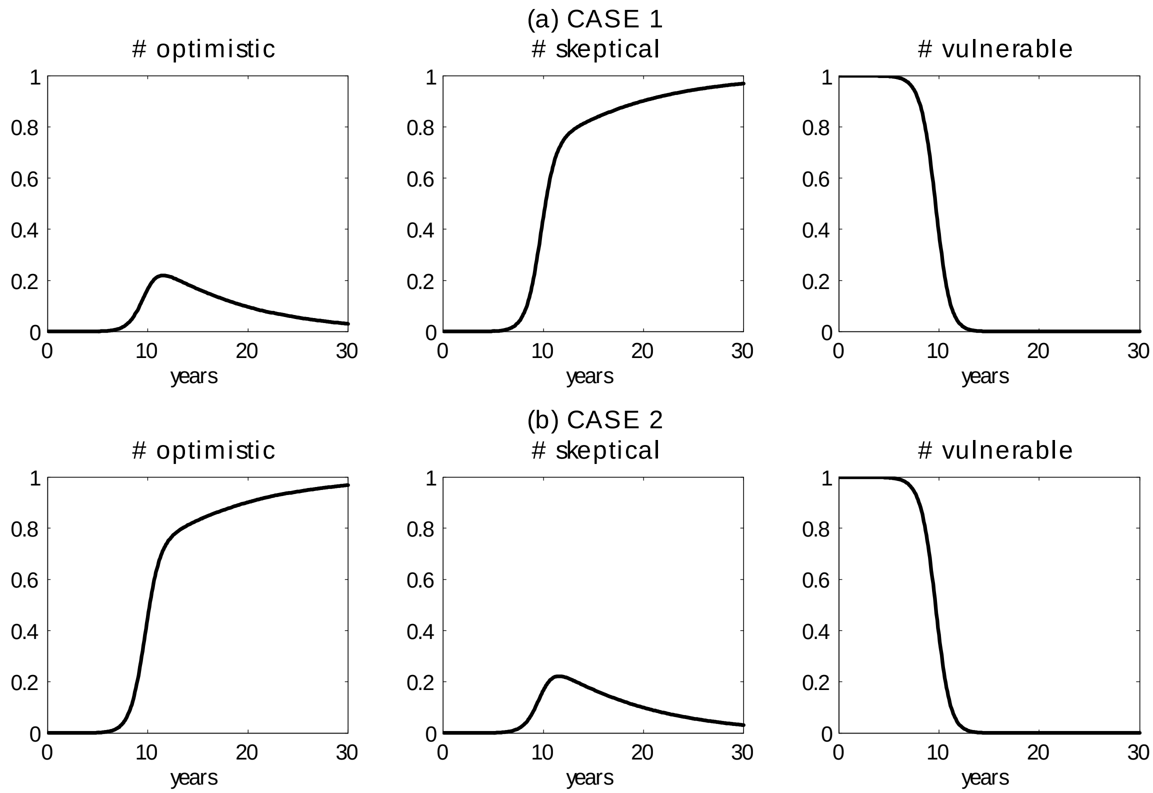
$$\varepsilon^* \in \{0.408, 0.876, 1.344, 1.811, 2.279, 2.747\}$$

$$f^v = [0.069,\ 0.615,\ 0.297,\ 0.0188,\ 0.0000962,\ 0.0000000621]$$

$$f^s = [0.0434, 0.656, 0.292, 0.00841, 0.00000840, 0.00000420]$$

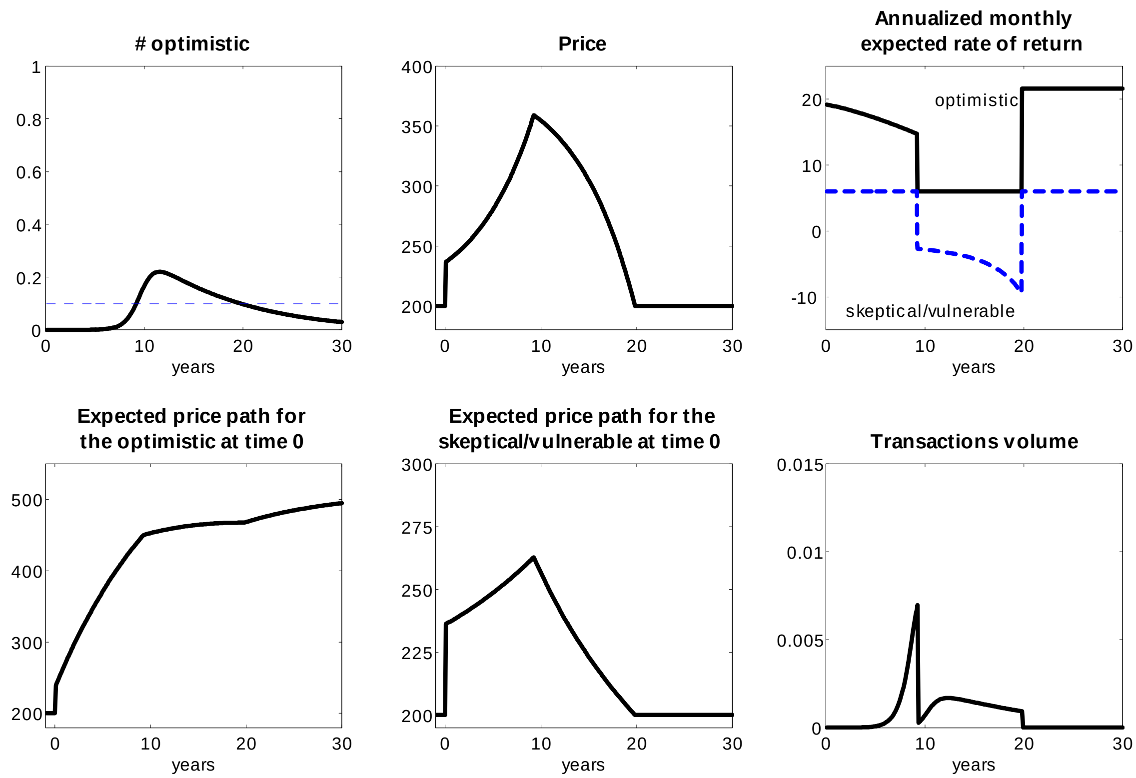
$$f^o = [0.00475, 0.0164, 0.0329, 0.0595, 0.114, 0.772]$$

FIGURE 1: Social Dynamics



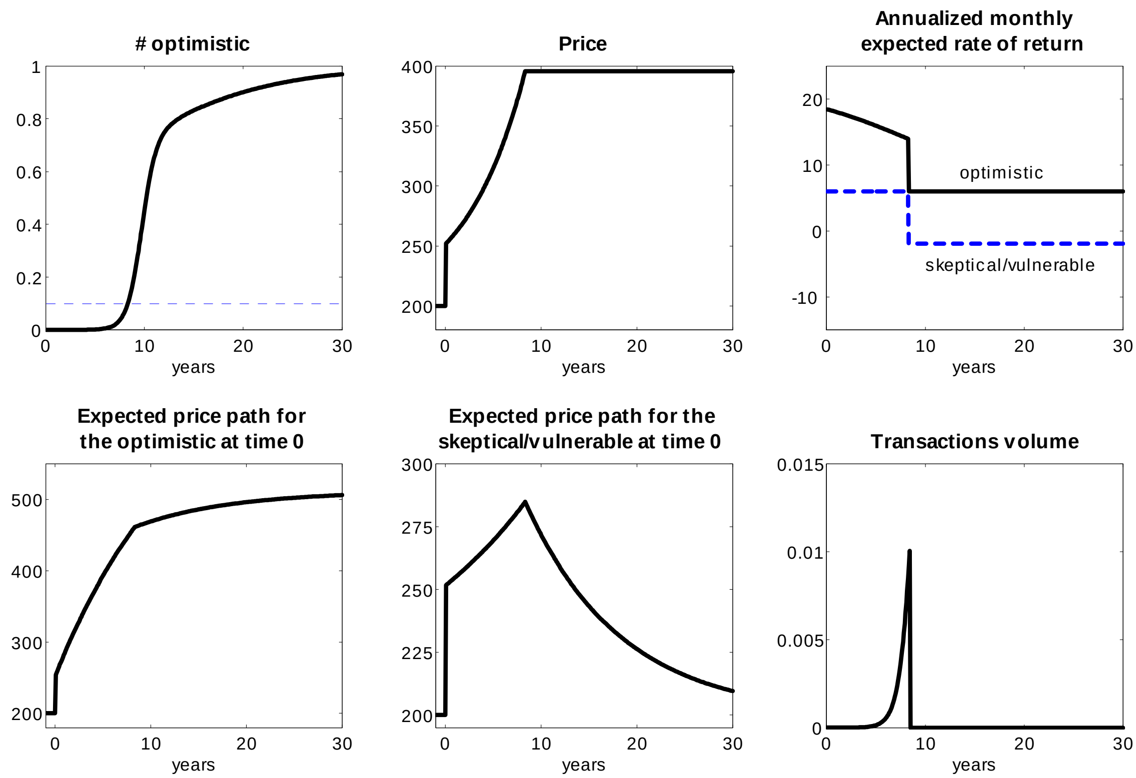
Note: The graphs show the evolution of the populations of each type of agent due to social dynamics. In Case 1, the priors of skeptical agents have the lowest entropy, and the priors of the vulnerable agents have the highest entropy. In Case 2, the priors of optimistic agents have the lowest entropy, and the priors of the vulnerable agents have the highest entropy.

FIGURE 2: Equilibrium of Frictionless Model, Case 1



Note: The graphs show a variety of paths for the frictionless model with social dynamics in Case 1, in which the priors of skeptical agents have the lowest entropy, and the priors of the vulnerable agents have the highest entropy. The number of optimistic agents, the price of a house, the monthly expected rate of return, and transactions volume are all computed under the assumption that uncertainty is not realized. The expected price paths are the expected values of the house price at each date, as of time 0, given the priors of the different agents.

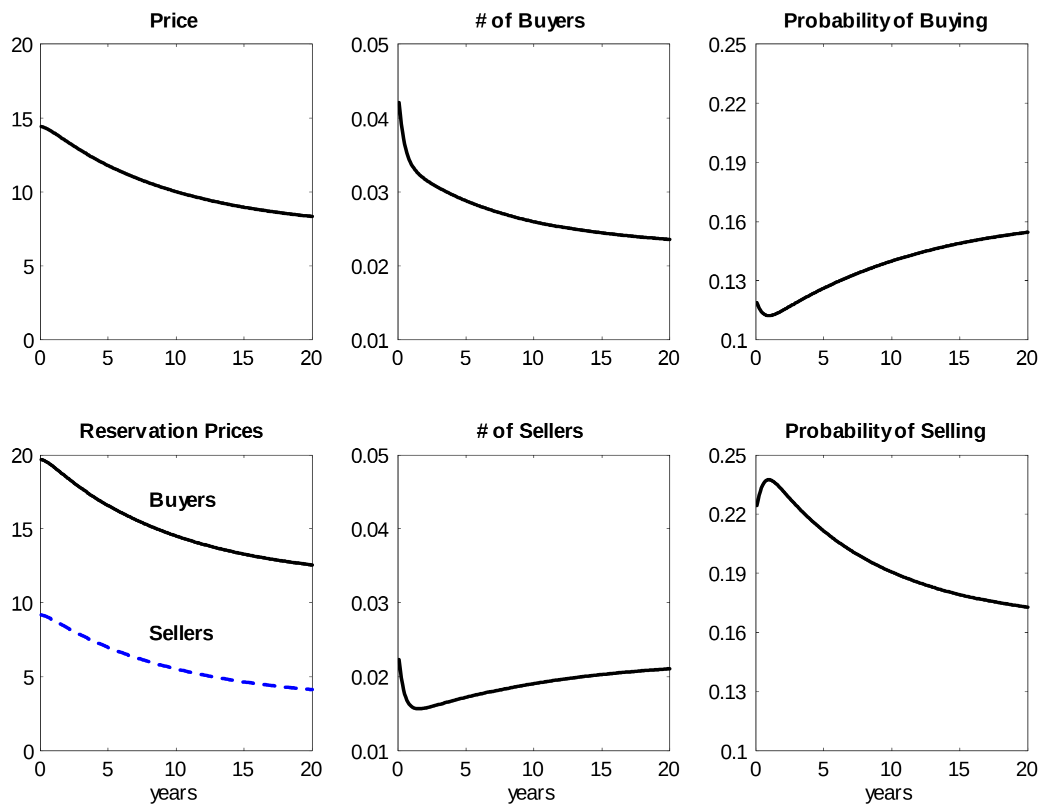
FIGURE 3: Equilibrium of Frictionless Model, Case 2



Note: The graphs show a variety of paths for the frictionless model with social dynamics in Case 2, in which the priors of optimistic agents have the lowest entropy, and the priors of the vulnerable agents have the highest entropy. The number of optimistic agents, the price of a house, the monthly expected rate of return, and transactions volume are all computed under the assumption that uncertainty is not realized. The expected price paths are the expected values of the house price at each date, as of time 0, given the priors of the different agents.

FIGURE 4: TRANSITIONAL DYNAMICS IN A MATCHING MODEL

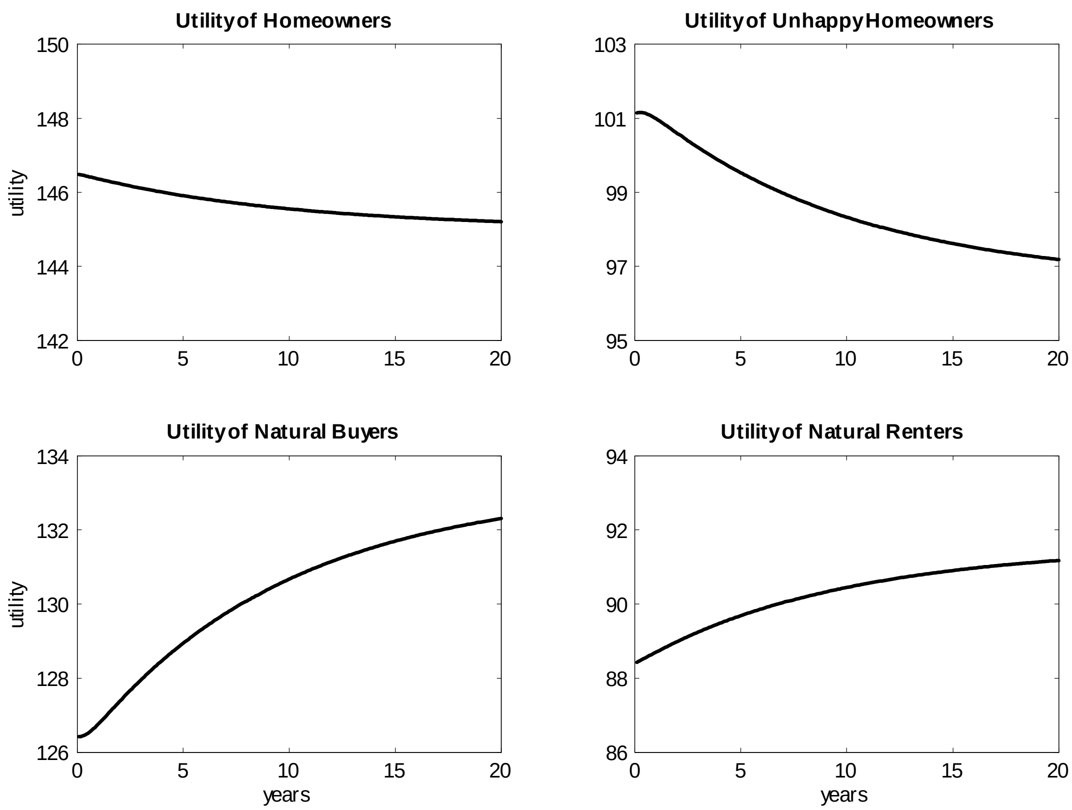
(a) Prices, Buyers, Sellers and Transaction Probabilities



Note: The figures illustrate the transition dynamics associated with the matching model, when there is an initial increase in the number of natural home buyers. Buyers indicates the number of agents who try to buy a home, while sellers indicates the number of agents who try to sell a home. Price is the average price at which homes are sold. B's reservation price is the reservation price of a natural home buyer. Figure 4 continues on the next page.

FIGURE 4: Transitional Dynamics in a Matching Model

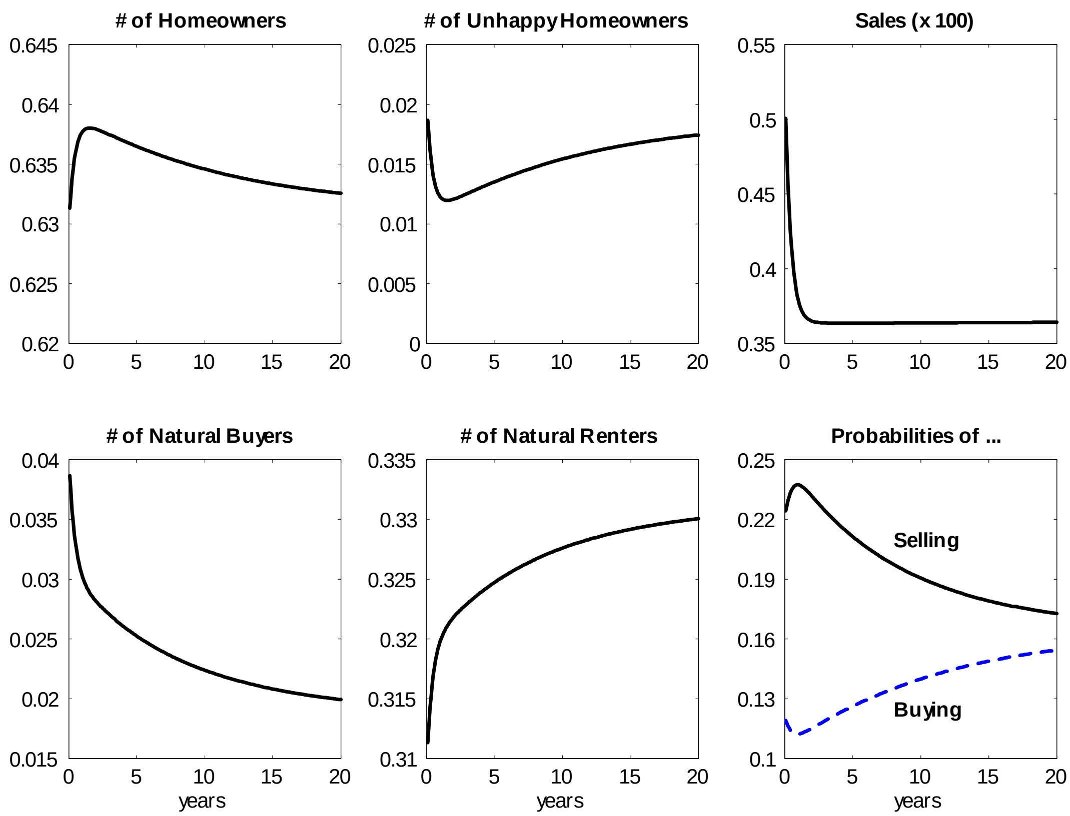
(b) Utility Levels of the Different Agents



Note: The figures illustrate the transition dynamics associated with the matching model, when there is an initial increase in the number of natural home buyers. The utility levels of the four types of agents are indicated. Figure 4 continues on the next page.

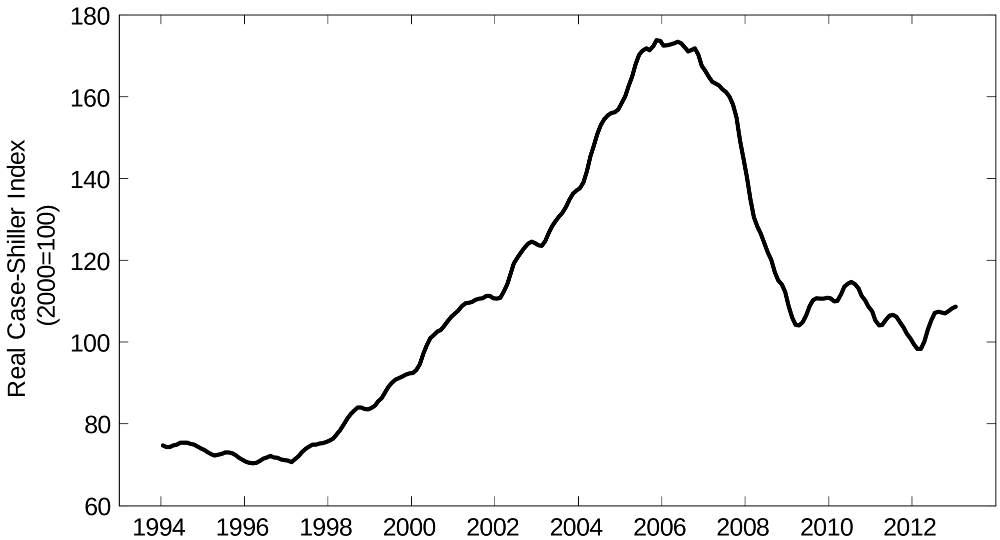
FIGURE 4: Transitional Dynamics in a Matching Model

(c) Agent Populations, Transactions Volume and Transaction Probabilities



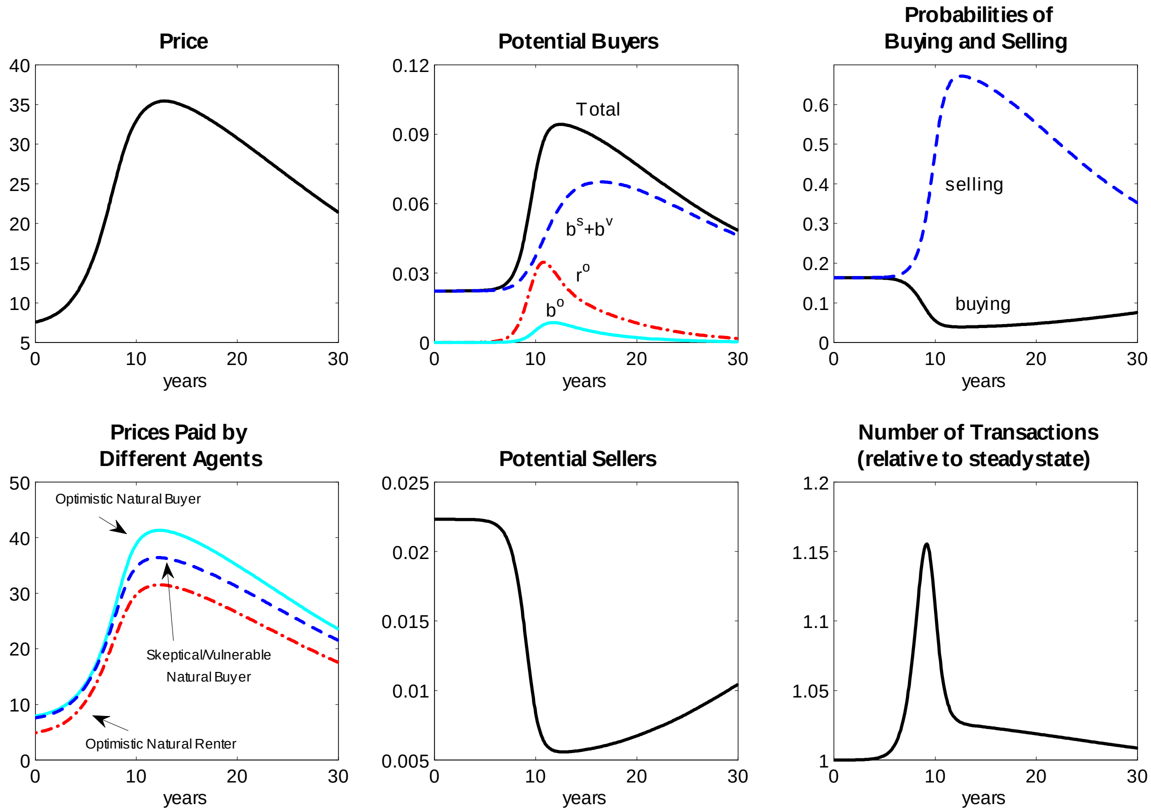
Note: The figures illustrate the transition dynamics associated with the matching model, when there is an initial increase in the number of natural home buyers. The four plots on the left show the number of agents of each type across the transition path. Sales indicates the number of transactions.

FIGURE 5: S&P/Case-Shiller Home Price Index for Three Cities



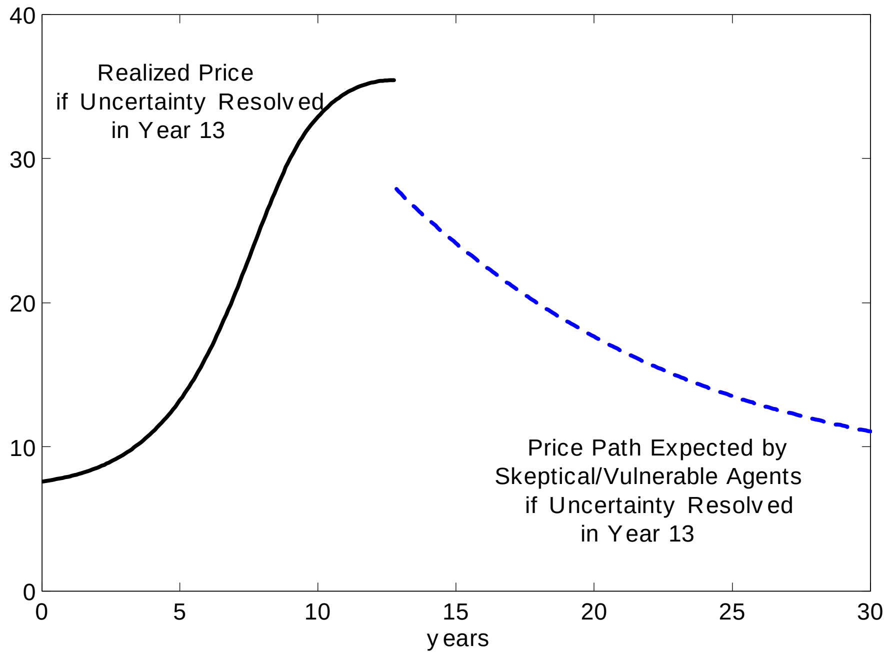
Note: The figure shows the level of the inflation-adjusted Standard & Poors/Case-Shiller Home Price Index for Boston, Los Angeles, and San Francisco.

FIGURE 6: EQUILIBRIUM OF MATCHING MODEL WITH SOCIAL DYNAMICS, CASE 1



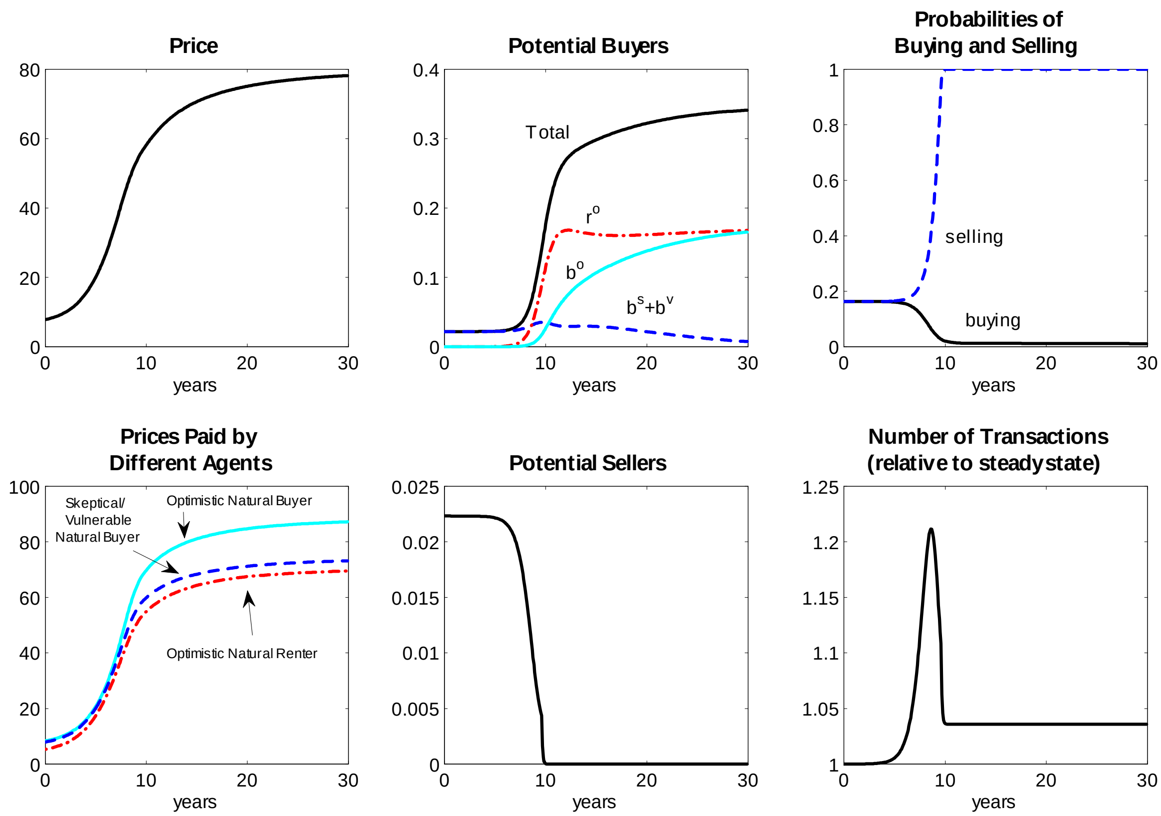
Note: The figures illustrate equilibrium paths when there is no resolution of uncertainty for the matching model with social dynamics in Case 1, in which the priors of skeptical agents have the lowest entropy, and the priors of the vulnerable agents have the highest entropy. The number of potential buyers is calculated after preferences shocks have been realized. Here, b^s, b^v and b^o represent the populations of skeptical, vulnerable and optimistic natural buyers, while r^o is the population of optimistic natural renters.

FIGURE 7: Expected Prices after the Resolution of Uncertainty, Case 1



Note: The figure illustrates the equilibrium price (blue line) if uncertainty is not realized until the end of year 13. The blue dashed line indicates the prices the skeptical and vulnerable agents would expect to observe after year 13, if uncertainly were resolved at that date.

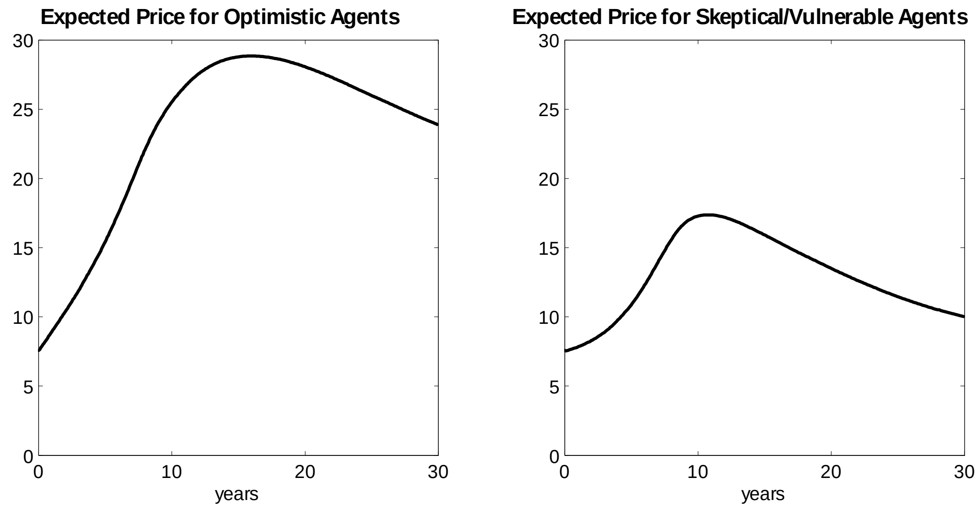
FIGURE 8: Equilibrium of Matching Model with Social Dynamics, Case 2



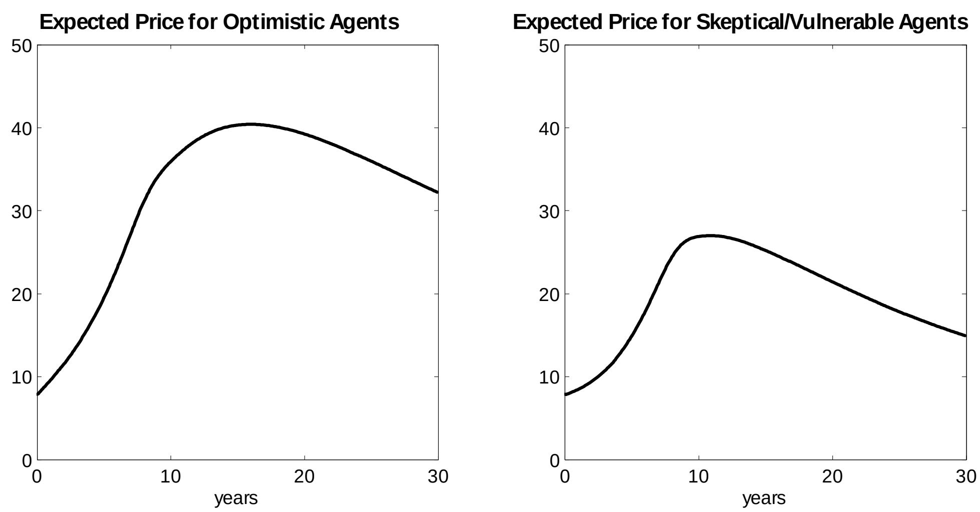
Note: The figures illustrate equilibrium paths when there is no resolution of uncertainty for the matching model with social dynamics in Case 2, in which the priors of optimistic agents have the lowest entropy, and the priors of the vulnerable agents have the highest entropy. The number of potential buyers is calculated after preferences shocks have been realized. Here, b^s, b^v and b^o represent the populations of skeptical, vulnerable and optimistic natural buyers, while r^o is the population of optimistic natural renters.

FIGURE 9: Expected Price Paths, Matching Model with Social Dynamics

(a) Case 1

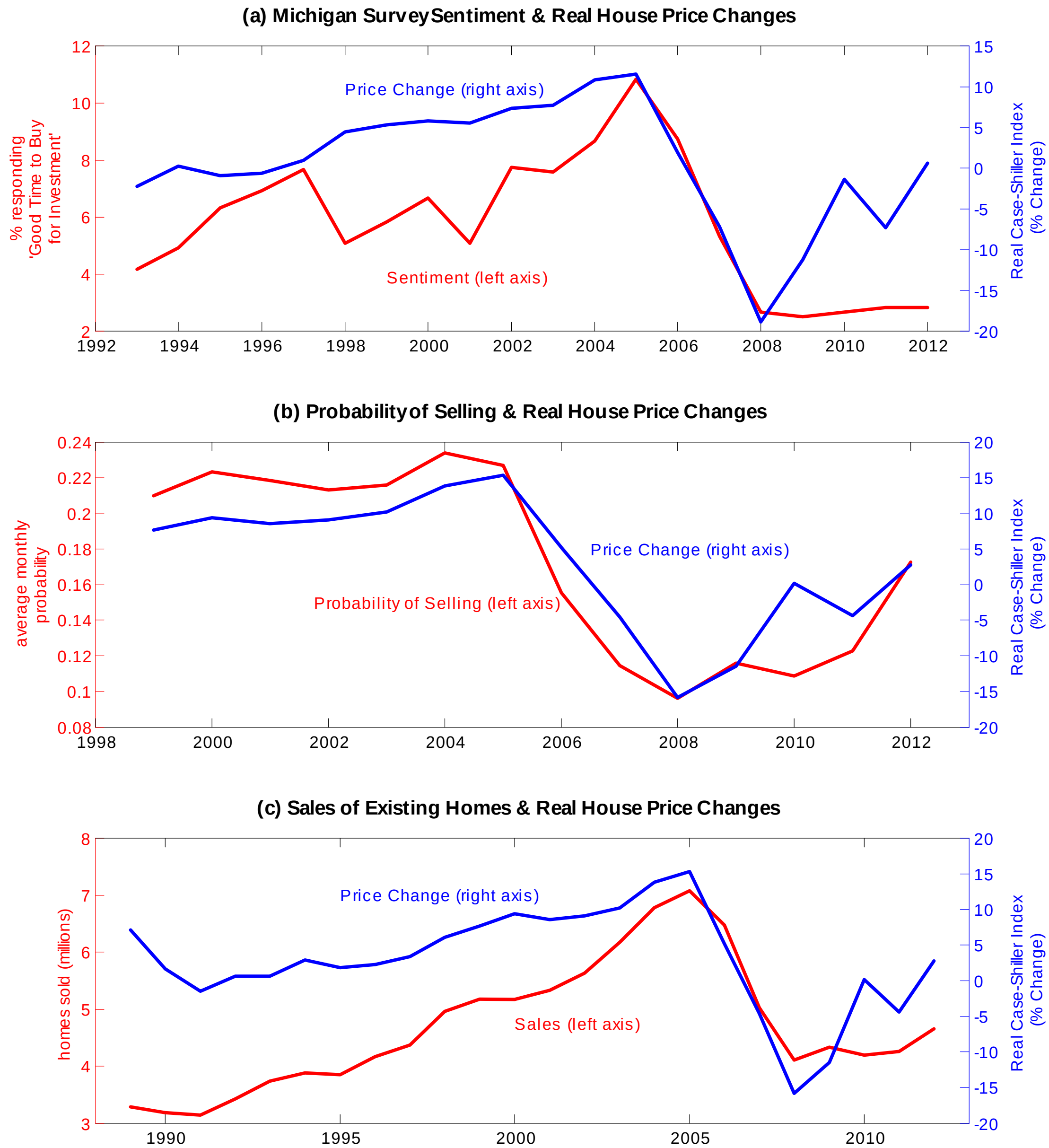


(b) Case 2



Note: The graphs show the price paths expected by different types of agents at time 0. In Case 1, the priors of skeptical agents have the lowest entropy, and the priors of the vulnerable agents have the highest entropy. In Case 2, the priors of optimistic agents have the lowest entropy, and the priors of the vulnerable agents have the highest entropy.

FIGURE 10: Sentiment, Housing Transactions and House Price Changes



Note: In all panels the blue line is the percentage change in the inflation-adjusted Case-Shiller national house price index. Panel (a) shows survey data from the Michigan Survey of Consumers. The red line shows the percentage of respondents answering "yes" to the question: "is it a good time to buy a home because it is a good investment?" Panels (b) and (c) show data from the National Association of Realtors. Panel (b) shows the probability of selling (the ratio of sales to inventories for existing homes) as the red line. Panel (c) shows sales of existing homes as the red line.

TABLE 1: Parameter Values, Matching Model

Parameter	Value	Description
k	0.65	Fraction of home owners in population
β	0.995	Discount factor
ε	1	Utility of owning a home
$\bar{\varepsilon}$	1	Utility of renting, natural buyer and renter
w	1	Rental rate
ϕ	0.0083	Monthly probability that uncertainty is realized
α	0.50	Parameter of matching function
μ	0.163	Parameter of matching function
λ	0.011	Preference shock, natural renters
η	0.0058	Preference shock, home owners
κ	62	Fixed cost of buying, natural renters
θ	0.50	Bargaining power of home buyer
ψ	1/3	Opportunity cost, unhappy home owner