How capital-based instruments facilitate the transition toward a low-carbon economy

A tradeoff between optimality and acceptability

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Abstract

This paper compares the temporal profile of efforts to curb greenhouse gas emissions induced by two mitigation strategies: a regulation of all emissions with a carbon price and a regulation of emissions embedded in new capital only, using *capital-based* instruments such as investment regulation, differentiation of capital costs, or a carbon tax with temporary subsidies on brown capital. A Ramsey model is built with two types of capital: brown capital that produces a negative externality and green capital that does not. Abatement is obtained through structural change (green capital accumulation) and possibly through under-utilization of brown capital. Capital-based instruments and the carbon price lead to the same long-term balanced growth path, but they differ during the transition phase. The carbon price maximizes social welfare but may cause temporary under-utilization of brown capital, hurting the owners of brown capital and the workers who depend on it. Capital-based instruments cause larger intertemporal welfare loss, but they maintain the full utilization of brown capital, smooth efforts over time, and cause lower immediate utility loss. Green industrial policies including such capital-based instruments may thus be used to increase the political acceptability of a carbon price. More generally, the carbon price informs on the policy effect on intertemporal welfare but is not a good indicator to estimate the impact of the policy on instantaneous output, consumption, and utility.

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Introduction

For the past centuries, the global economy has been on a sub-optimal growth path in the sense that it did not internalize future damages caused by the release

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of greenhouse gases (GHG) into the atmosphere. These emissions are embedded in installed capital — such as fossil-fueled power plants, internal combustion engines in passenger vehicles, heat production — and infrastructure patterns such as transport networks or city density. In order to limit climate change damages, the international community has committed through the UNFCCC to maintain global temperature increase below two degrees compared to the pre-industrial climate, and this requires a drastic reduction of GHG emissions globally (IPCC, 2007).

Doing so in a welfare-maximizing way requires internalizing the externality (damages from greenhouse gases emissions) with a global carbon price, e.g. through a carbon tax or a cap-and-trade system. A carbon price can, in particular, induce a switch from carbon-intensive capital to clean capital such as renewable electricity, electric vehicles, rail transportation and insulated buildings. It can also induce people to reduce the utilization of their polluting capital (Schwerin, 2013).

However, the carbon price does not seem to be society's preferred instrument, and up to now governments have been implementing incentives in favor of green capital[1], such as energy efficiency standards on new capital (e.g. CAFE standards in the US or direct regulation for new buildings and home appliances) or fiscal incentives (see OECD, 2009, for vehicles). This alternative strategy gives firms and households the opportunity to make investments consistent with the turnover of their capital stock, that is to keep using existing capital until it depreciates, while investing in cleaner new capital. Put differently, instead of regulating all GHG gases in the economy — as a carbon price would do — these policies focus on emissions embedded in new capital only.

To give some insights on why such policies seem to be more politically acceptable than a carbon price, we compare analytically a carbon price with "capital-based" instruments, i.e. instruments that focus on capital instead of emissions (e.g. standards or subsidies on investments). Some studies compare the efficiency of such instruments (e.g, Fischer and Newell, 2008; Goulder and Parry, 2008), however, they do not explicitly consider the intertemporal distribution of abatement efforts nor model capital. Vogt-Schilb et al. (2012) model abatement through the deployment of green capital and find that abatement efforts are concentrated over the short term in response to a carbon tax. In this paper, we investigate how the intertemporal distribution of abatement efforts is modified when using alternative mitigation instruments.

We use a simple Ramsey model with two types of capital, as first proposed by Ploeg and Withagen (1991): "brown" capital, which creates a negative externality (greenhouse gases emissions), and "green" capital, which does not. Reducing emissions can be done through two channels. First, through a substitution between brown and green capital, i.e. structural change (as in Acemoglu et al., 2012). This option is slow because it requires capital accumulation in the green sector. Moreover, as investment is supposed irreversible, brown capital can only disappear through depreciation. Second, it is possible to instantly reduce emissions through under-utilization of brown capital, i.e. through a contraction of the output volume. This option allows unlimited short-term abatements.

Starting from a *laissez-faire* equilibrium in which capital is fully utilized and

[1] At the exception of the EU-ETS and a few states that have implemented a carbon tax.

the marginal productivities of brown and green capital are equal, we model a social planner who decides to maintain the concentration of greenhouse gases in the atmosphere below a certain threshold. Two strategies are compared to comply with the ceiling. The first strategy uses a price on carbon emissions, that regulates all GHG emissions. The second strategy regulates emissions from new capital only, using three different capital-based instruments that are equivalent in terms of investments: (i) the carbon tax is completed by a temporary subsidy on brown capital; (ii) the capital cost of brown and green capital are differentiated, e.g. through subsidies for green investment; (iii) brown investments are temporarily regulated.

In the long run the carbon price and the capital-based instruments lead to the same balanced growth path, in which the marginal productivity of brown capital is higher than that of green capital; this compensates for its higher cost, as using brown capital increases GHG atmospheric concentration. The two sets of strategies however induce different trajectories over the short run.

A carbon price yields the first-best optimum pathway, that includes an adjustment of brown capital utilization in the short-run. The partial utilization of brown capital has significant short-term impact on production and possibly consumption. Practically, this impact would primarily affect the owners of brown capital and the workers who depend on them. With capital-based instruments, total discounted welfare is lower than in the optimum, but output is higher over the short-run because brown capital is used at full capacity even during the transition. These capital-based instruments thus smooth out the transition toward a low-carbon economy, and can make more acceptable a carbon price in the longer run.

Our results do not depend on the social cost of carbon, as we model a social planner who imposes the optimal carbon ceiling. Comparing two sets of mitigation strategies, they highlight a trade-off between the optimality of a climate mitigation policy and its short-term impacts, which influence implementation ease. If we compare the instruments in terms of welfare maximization, the carbon tax alone is always the best policy. However, when looking at other criteria such as short-term impacts or ease of implementation of the policy, second-best strategies might appear preferable to many decision-makers. Indeed, strategies that focus on new capital or that subsidize temporarily polluting capital allow reaching the same long-run objective as the optimal policy but delay efforts, with lower short-term impacts on output and higher efforts over the medium-run. Since capital-based instruments postpone mitigation efforts compared with the first-best strategy, they induce higher marginal abatement costs, but they would be preferred by individuals with higher discount rates than the social planner.

Capital-based instruments, since they stimulate the new green sector or temporarily accompany the brown declining sector, are industrial policies. For instance, a temporary subsidy to obsolete brown capital is comparable to Japanese "sunset" industrial policies, that supported declining traditional sectors in the middle of the 20th century (Beason and Weinstein, 1996). On the other hand, a subsidy to green investment is similar to industrial policies that help "sunrise" green industries become competitive. Even though the usual reasons for implementing industrial policies — market failures, increasing returns in the green sector — are not modeled here, our paper brings an additional argument in favor of complementary industrial policies to smooth the transition towards a low-carbon economy.

The remainder of the paper is structured as follows. Section 1 presents the model and section 2 solves for the *laissez-faire* equilibrium. In section 3 we analyze the optimal growth path, that can be obtained with a carbon price, and we compare it with capital-based second-best instruments in section 4. Section 5 concludes.

1. Model

We consider a Ramsey framework with a representative infinitely-lived household, who receives the economy's production from firms y_t, saves by accumulating assets[2], receives income on assets at interest r_t and purchases goods for consumption c_t. At time t, consuming c_t provides consumers with a utility $u(c_t)$. The utility function is increasing with consumption, and strictly concave ($u'(c) > 0$ and $u''(c) < 0$).
The household maximizes their intertemporal discounted utility W, given by:

$$W = \int_0^\infty e^{-\rho t} \cdot u(c_t)\, dt \tag{1}$$

where ρ is the rate of time preference.
Firms produce one final good, using two types of available capital: brown capital k_b and green capital k_g. Green capital encompasses existing green technologies as well as patents, research and development expenses and human capital necessary to develop new green technologies.
Firms may use only a portion q_t of installed capital k_t to produce the flow of output y_t given by:

$$y_t = F(A_t, q_{b,t}, q_{g,t}) \tag{2}$$

$$q_{b,t} \le k_{b,t} \tag{3}$$

$$q_{g,t} \le k_{g,t} \tag{4}$$

F is a classical production function, with decreasing marginal productivities,[3] to which we add the assumption that capital can be under-utilized. A_t is exogenous technical progress, and increases at an exponential rate over time. In the remaining of this paper, q_t will be called utilized capital and k_t installed capital. Although it is never optimal in the *laissez-faire* equilibrium, the under-utilization of installed capital can be optimal when a carbon price is implemented[4]. For instance, coal plants can be operated part-time and low-efficiency cars can be driven less if their utilization is conflicting with the climate objective.

Production is used for consumption (c_t) and investments ($i_{b,t}$ and $i_{g,t}$).

$$y_t = c_t + i_{b,t} + i_{g,t} \tag{5}$$

[2] Assets are capital and loans to other households.

[3] We assume decreasing returns to scale even in the green sector but a further extension of this work will be to assume increasing returns to scale in the short-run.

[4] In this paper, under-utilization of green capital is never optimal so $q_{g,t} = k_{g,t}$.

Investment $i_{b,t}$ and $i_{g,t}$ increase the stock of installed capital, which depreciates exponentially at rate δ:

$$\dot{k}_{b,t} = i_{b,t} - \delta\, k_{b,t} \tag{6}$$

$$\dot{k}_{g,t} = i_{g,t} - \delta\, k_{g,t} \tag{7}$$

The doted variables represent temporal derivatives.

Investment is irreversible (Arrow and Kurz, 1970):

$$i_{b,t} \geq 0 \tag{8}$$

$$i_{g,t} \geq 0 \tag{9}$$

This means that for instance, a coal plant cannot be turned into a wind turbine, and only disappears through depreciation. Arrow and Kurz (1970) show that this constraint can be binding if accumulated capital is higher than the optimal capital level in the steady-state ($k > k^\star$). This constraint is relevant for our problem since in the *laissez-faire* equilibrium brown capital is accumulated without internalizing climate change, so that at some point the amount of brown capital may be higher than the optimal one.

Brown capital used a time t emits greenhouse gases ($G \times q_{b,t}$) which accumulate in the atmosphere in a stock m_t. GHG atmospheric concentration increases with emissions, and decreases at a dissipation rate[5] ε:

$$\dot{m}_t = G \cdot q_{b,t} - \varepsilon m_t \tag{10}$$

In the following section, we solve for the *laissez-faire* equilibrium. In the last two sections, we adopt a cost-effectiveness approach (Ambrosi et al., 2003) and analyze policies that allow maintaining atmospheric concentration m_t below a given ceiling $\bar{m}$, a proxy for the increase in global temperature (Meinshausen et al., 2009):

$$m_t \leq \bar{m} \tag{11}$$

This threshold can be interpreted as a tipping point beyond which the environment (and output) can be highly damaged. It can also be interpreted as an exogenous policy objective such as the UNFCCC "2C target".

2. *Laissez-faire* equilibrium

The *laissez-faire* equilibrium leads to classical results of a Ramsey model with two types of capital.

Proposition 1. *In the laissez-faire equilibrium, the marginal productivities of green and brown capital are equal. Consumption grows as long as the marginal productivity of capital — net of depreciation — is higher than the rate of time preference.*

[5]The dissipation rate allows maintaining a small stock of brown capital in the balanced growth path.

Proof. Firms rent the services of capital from households, who own it. We denote $R_{b,t}$ and $R_{g,t}$ the rental prices of a unit of brown and green capital respectively. A firm's total cost for capital is $R_{g,t} \cdot k_{g,t} + R_{b,t} \cdot k_{b,t}$. The firm's flow of profit at time t is given by:

$$\Pi_t = F(A_t, q_{b,t}, q_{g,t}) - R_{g,t} \cdot k_{g,t} - R_{b,t} \cdot k_{b,t} \tag{12}$$

A competitive firm, which takes $R_{g,t}$ and $R_{b,t}$ as given, maximizes its profit by using all installed capital and by equalizing at each time t the marginal productivity of brown and green capital to their respective rental prices:

$$\partial_{q_b} F(q_{b,t}, q_{g,t}) = R_{b,t}$$
$$\partial_{q_g} F(q_{b,t}, q_{g,t}) = R_{g,t}$$

Since capital depreciates at the constant rate δ, the net rate of return to the owner of a unit of brown or green capital is respectively $R_{b,t}-\delta$ and $R_{g,t}-\delta$.[6] We model a closed economy, thus the assets owned by the households are installed capital, or loans to other households at rate r_t. At equilibrium, households should be indifferent between investing in brown or green capital, or lending to other households, so that

$$R_{b,t} = R_{g,t} = r_t + \delta \tag{13}$$

When solving for households' utility maximization, we find the Euler equation, that gives the basic condition for choosing consumption over time (see Appendix A):

$$\frac{\dot{c}}{c} = -\frac{u'(c)}{c \cdot u''(c)} \cdot (r_t - \rho) \tag{14}$$

The intertemporal elasticity of substitution is positive ($-\frac{u'(c)}{cu''(c)} > 0$) so consumption grows if the rate of return to saving r_t (i.e. the marginal productivity of capital, net of depreciation) is higher than the rate of time preference. The interest rate r_t is the rate that converts future consumption into a current consumption that is equivalent in terms of social welfare, and equals the rate of return to consumption. If the interest rate equals the rate of time preference, consumption is constant over time. □

As a consequence of Proposition 1, if the output elasticity of brown capital is higher than that of green capital, the ratio of brown capital over green capital is higher than one. In other words, if using brown capital is more productive than using green capital, firms will invest more in brown capital.

In this *laissez-faire* equilibrium, the price of brown and green capital, expressed in units of consumables, does not change over time.[7] In the next section, however, we show that when the climate externality is internalized the price of brown capital decreases, and so does the real rate of returns for the owners of brown capital.

[6] We implicitly assumed that the price of capital in units of consumables is 1, but this will not always be the case when the GHG ceiling is introduced.

[7] The price of brown and green capital, expressed in units of consumables is always 1 in the *laissez-faire* equilibrium (J. Barro and Sala-i Martin, 2004).

3. Discounted welfare maximization: Carbon price

In this section, we solve for the welfare maximization program, in which institutions impose the social cost of emissions on producers and consumers (e.g. an optimal carbon tax, a universal cap-and-trade system) in order to internalize the GHG ceiling constraint. A social planner maximizes intertemporal utility[8] given the economy budget constraint (eq. 5), the capital motion law (eq. 6 and eq. 7), the irreversibility constraint (eq. 8) and the GHG ceiling constraint (eq. 10 and eq. 11). This latter constraint increases the social cost of brown capital, which is the source of the externality. Brown capital may thus be under-utilized in order to instantly reduce GHG emissions. It is however always nonoptimal to under-utilize green capital. Therefore, to keep the model simple, we model these two features (eq. 3 and eq. 8) for brown capital only.

The social planner program is:

$$
\begin{aligned}
&\max_{c,i,k} \int_0^{\infty} e^{-\rho t} \cdot u(c_t)\, dt \\
\text{subject to } & F(q_b, k_g) - c_t - i_{b,t} - i_{g,t} = 0 && (\lambda_t) \\
& \dot{k}_{b,t} = i_{b,t} - \delta k_{b,t} && (\nu_t) \\
& \dot{k}_{g,t} = i_{g,t} - \delta k_{g,t} && (\chi_t) \\
& \dot{m}_t = G\, q_{b,t} - \varepsilon m_t && (\mu_t) \\
& m_t \leq \bar{m} && (\phi_t) \\
& i_{b,t} \geq 0 && (\psi_t) \\
& q_{b,t} \leq k_{b,t} && (\beta_t)
\end{aligned}
$$

The variables in parentheses are the co-state variables and Lagrangian multipliers associated to each constraint. λ_t is the current value of income. ν_t and χ_t are the current values of brown and green capital. μ_t is the current cost of pollution in the atmosphere, expressed in terms of undiscounted utility at time t. The present value Hamiltonian associated to the maximization of social welfare can be found in Appendix B.

We define τ_t as the current price of GHG, expressed in units of consumables.

$$\tau_t = \frac{\mu_t}{\lambda_t} \tag{15}$$

We call t_b the date at which GHG concentration reaches the ceiling[9]: $\forall t \geq t_b$, $m_t = \bar{m}$. We show in Appendix B.1 that the carbon price exponentially grows at the endogenous interest rate plus the dissipation rate of GHG until the ceiling is reached.

$$\dot{\tau}_t = \tau_t[\varepsilon + r_t] \tag{16}$$

When the ceiling is reached, $m_t = \bar{m}$, brown installed capital is constant at $k_{b,t} = \bar{m}\, \varepsilon / G$ and the economy is on the balanced growth path.

[8] The same optimal pathway can be obtained with a lump-sum carbon tax on GHG emissions, as it is shown in Appendix D.

[9] We assume that $m_t = \bar{m}$ on the interval $[t_b, +\infty[$, which is compatible with usual functional forms, like the ones we use here for numerical illustrations.

The main first-order conditions of our problem are (Appendix B):

$$u'(c_t) = \lambda_t = \nu_t + \psi_t = \chi_t \tag{17}$$

$$\partial_{k_g} F = \frac{1}{\lambda}(\delta + \rho)\chi_t - \dot{\chi}_t \tag{18}$$

$$\beta_t = \frac{1}{\lambda}(\delta + \rho)\nu_t - \dot{\nu}_t \tag{19}$$

$$\partial_{q_b} F = \beta_t + \tau_t \cdot G \tag{20}$$

As in Jorgenson (1967), the rental prices of green and brown capital $R_{g,t}$ and $R_{b,t}$ follow:

$$R_{g,t} = \frac{1}{\lambda}[(\delta + \rho)\chi_t - \dot{\chi}_t] \tag{21}$$

$$R_{b,t} = \frac{1}{\lambda}[(\delta + \rho)\nu_t - \dot{\nu}_t] \tag{22}$$

where χ_t and ν_t are respectively the marginal green and brown capital prices. The Lagrangian multiplier associated to the constraint $q_{b,t} \leq k_{b,t}$ (β_t) is therefore equal to the rental price of brown capital $R_{b,t}$ (eq. 19). As a consequence, as long as the rental price of brown capital is positive, brown capital is fully-utilized (complementary slackness condition, see Appendix B). However, if the rental price of brown capital goes down to zero, brown capital may be under-utilized.
We can deduce the following proposition from the first-order conditions.

Proposition 2. *Along the optimal path, the marginal productivity of green capital is equal to the rental price of green capital, which is equal to the interest rate plus the depreciation rate.*

$$\partial_{k_g} F = R_{g,t} = r_t + \delta \tag{23}$$

Along the optimal path, the marginal productivity of brown capital must be equal to the rental price of brown capital plus the carbon price τ_t multiplied by the marginal emissions of production G.

$$\partial_{q_b} F = R_{b,t} + \tau_t\, G \tag{24}$$

Proof. See Appendix B.2. □

In the *laissez-faire* equilibrium, investments were made such that the net marginal productivity of brown capital was equal to the interest rate. This is no longer true when the pollution externality is internalized, as firms have to pay the carbon tax when they use brown capital. Their marginal cost is higher than the interest rate and they must therefore reduce the amount of brown capital used for production, to adjust their marginal benefit.

As previously noted, in the *laissez-faire* equilibrium the price of capital expressed in units of consumables ($\frac{\nu_t}{\lambda_t}$ for brown capital and $\frac{\chi_t}{\lambda_t}$ for green capital) was always equal to 1. Here brown capital can meet the conditions analyzed by Arrow and Kurz (1970), i.e. the amount of installed capital can be higher than the steady-state level. The irreversibility condition can therefore be binding ($\psi_t > 0$) and the price of brown capital can decrease below 1: $\frac{\nu_t}{\lambda_t} = 1 - \frac{\psi_t}{\lambda_t}$ (eq. 17).

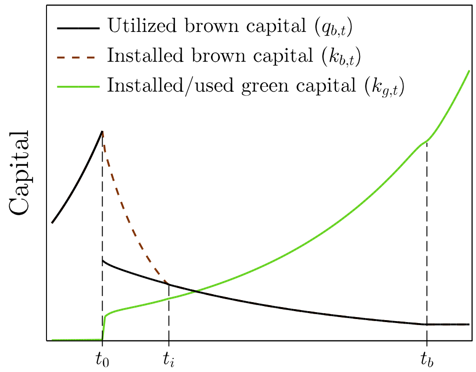

Figure 1: Brown and green installed capital, and utilized brown capital in the first-best optimum. Before t_0, the economy is on the *laissez-faire* equilibrium. At t_0 the carbon price is implemented and brown capital depreciates until t_i ($i_b = 0$). During this period, brown capital may be under-utilized ($q_{b,t} < k_{b,t}$). Brown investments then start again, and the balanced growth path is reached at t_b.

We find that three phases can be distinguished once the carbon price has been implemented (eq. B.7 and Fig. 1): a phase during which the irreversibility constraint is binding and brown investment is nil (between t_0 and t_i in Fig. 1), a phase during which brown investment is strictly positive (between t_i and t_b in Fig. 1), and the balanced growth path (after t_b).

The phase when $i_{b,t} = 0$ may be separated into two different phases depending on the price of brown capital ($\frac{\nu_t}{\lambda_t} = 1 - \frac{\psi_t}{\lambda_t}$): if $0 < \psi_t < \lambda_t$ the price of brown capital is positive, so brown capital is fully-utilized even though no brown investment is made. If $\psi_t = \lambda_t$ the rental price of brown capital is nil and it can be under-utilized. Note that in Fig. 1 brown capital is always under-utilized when $i_{b,t} = 0$.

The remarks made above can be summurized in the following proposition.

Proposition 3. *Two phases can be distinguished during the optimal transition to the new balanced growth path with externality:*

(i) *A phase when the value of brown capital is lower than the marginal utility of consumption. During this phase, the rental price of brown capital is lower than that of green capital and brown investment is nil.*

$$R_{b,t} = R_{g,t} - p \qquad (25)$$
$$\text{with } 0 < p \leq R_{g,t}$$

In particular, when the price of capital is nil ($\nu_t = 0$), the rental price of brown capital $R_{b,t}$ decreases to zero (and $p = R_{g,t}$). In this case installed brown capital is optimally under-utilized such that the marginal productivity of utilized brown capital is equal to the carbon price (multiplied by G).

$$\partial_{q_b} F = \tau_t \, G \text{ with } q_{b,t} < k_{b,t} \qquad (26)$$

(ii) *A phase when brown investments are strictly positive, and the price of brown capital is equal to the price of green capital. During this phase (and* *on the balanced growth path), $R_{b,t} = R_{g,t}$ and:*

$$\partial_{q_b} F = \partial_{k_g} F + \tau_t \, G \tag{27}$$

The marginal productivity of brown capital is higher than that of green capital, to adjust to its higher social cost.

PROOF. See Appendix B.3.

We show in the following proposition that the phase when brown investment is nil necessarily happens first when a carbon price is implemented in the *laissez-faire* equilibrium.

Proposition 4. *When a GHG ceiling is enforced in the laissez-faire equilibrium, the irreversibility constraint is binding ($i_{b,t} = 0$).*

PROOF. At t_0, the carbon price is implemented in a *laissez-faire* equilibrium, in which marginal productivities of brown and green capital are equal (Proposition 1). This condition on marginal productivities determines the ratio of installed brown capital over green capital. On the other hand, eq. 27 ($\partial_{q_b} F = \partial_{k_g} F + \tau_t \cdot G$) implies that in the phase with $i_{b,t} > 0$ and on the balanced growth path the ratio of installed brown capital over green capital is lower than in the *laissez-faire* equilibrium (because of decreasing marginal productivities). Since installed capital is necessarily the same at t_0^- and t_0^+ (just before, and just after the implementation of the carbon price), eq. 27 cannot be true at t_0^+ and the irreversibility constraint is necessarily binding in the beginning. A more complete proof can be found in Appendix C. □

During this first phase, the irreversibility constraint prevents the economy from transforming brown capital into either green capital or consumption even though brown capital is overabundant. The price of brown capital therefore decreases, as well as its rental price. This lower rental price can be seen as a transfer from brown capital owners, who "compensate" firms so that they can keep producing during the first phase. Indeed, since firms that use brown capital suddenly have to pay an additional production cost with the carbon tax, they can only keep producing if the rental price of brown capital decreases (otherwise, their marginal cost would be higher than their marginal benefit).

If the carbon price is very high and the rental price of brown capital decreases down to zero, installed brown capital is optimally under-utilized to reduce carbon emissions. Indeed, if brown capital has no value anymore because τ_t is too high, it may be optimal to stop using it (in Fig. 1 installed brown capital is under-utilized until $t_{i,1}$). In other words, since there is no fixed cost in using brown capital ($R_{b,t} = 0$), firms may reduce capital utilization such as to equalize their marginal cost and marginal revenue (eq. 26). When the rental price is nil, the marginal productivity of brown capital is transfered to households through the tax revenue $\tau_t \, G$ only.

A direct consequence of eq. 26 is that during the first phase, installed brown capital is optimally under-utilized if the carbon price (multiplied by G) is higher than the marginal productivity of brown capital when all brown capital are used: $\tau_t G > \partial_{q_b} F|_{q_{b,t}=k_{b,t}}$. In particular, at t_0 installed brown capital is under-utilized if:

$$\tau_{t_0^+} \geq \frac{1}{G} \, \partial_{q_b} F(t_0^-)$$

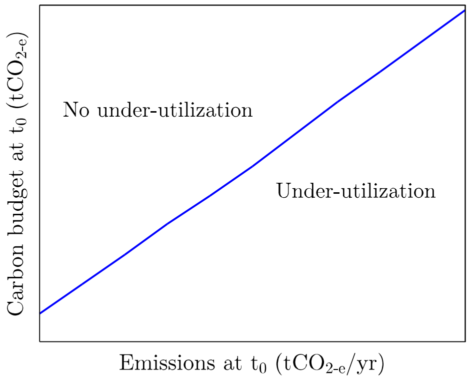

Figure 2: Depending on initial emissions (i.e. initial brown capital $k_{b,0}$) and on the concentration ceiling ($\bar{m}$), brown capital is under-utilized or not in the first-best optimum.

The under-utilization of brown capital depends on the ceiling $\bar{m}$, on initial brown capital $k_b(t_0)$ and on other parameters of the model such as the functional forms of F and u, on the depreciation rate δ and the preference for the present ρ. Put more simply, as it is illustrated in Fig. 2, for a given set of functions and parameters the under-utilization of brown capital only happens if initial brown capital is high (right end of the x-axis) and/or if the ceiling is stringent (lower part of the y-axis).

This can be interpreted in terms of time horizon: for a given level of initial emissions, the lower the ceiling the shorter the time before the ceiling is reached if all brown capital is utilized. If the time is short before the ceiling is reached, it is optimal to under-utilize brown capital in order to reduce emissions faster. Conversely, if the ceiling is to be reached in a long time, it is optimal to use all installed brown capital while it depreciates to a sustainable level.

Because investment is irreversible, the society that we model has to live with past mistakes for a while, once it realizes it has been on a non-optimal growth path. A way to bypass this obstacle is to give up part of installed polluting capital in order to reduce emissions faster (Fig. 1, 2, and Prop. 4). Such a strategy reduces short-term output, but for stringent climate objectives (with regard to past accumulation of polluting capital), it is optimal.

Under-utilization of existing capital may however be politically difficult. First, it appears as a waste of resources and creates unemployment (even though labor is not modeled here). Second, it affects primarily the owners of polluting capital and the workers whose jobs depend on this capital, transforming them into strong opponents to climate policies.

4. Capital-based policies

It is possible to reduce carbon emissions through investment decisions — that is, to redirect investments towards green capital — without creating an incentive to reduce the utilization rate of brown capital, i.e. with no effect on production decisions. In practice, it can be done with capital-based instruments such as energy efficiency standards, fiscal incentives or differentiated interest rates depending on the carbon content of capital (Rozenberg et al., 2013).

In this section, we consider the three following instruments: (i) the carbon tax is completed by a temporary subsidy on brown capital; (ii) the cost of brown and green capital are differentiated, e.g. with a subsidy on green investment; (iii) brown investments are regulated.

All three instruments are equivalent if they are optimally designed to maximize welfare given the ceiling constraint. They allow reaching the same balanced growth path as in the first-best optimum in the long-run, and they induce a full utilization of brown capital in the short-run.

4.1. Carbon tax plus temporary subsidy on brown capital

Proposition 3 implies that when a carbon tax is implemented, if the rental price of brown capital falls down to zero in the short-run (because investment is irreversible) then it is optimal to under-utilize installed brown capital such that the marginal productivity of utilized brown capital is equal to the carbon price (eq. 25 and 26).

As explained before, this might be unacceptable to the owners of brown capital, creating strong opposition against the measure. They may also consider the tax unfair, as they were not aware of the future carbon tax when they bought their capital. Under-utilization can however be prevented by subsidizing unprofitable brown capital in the short-run. Starting from the social optimum, eq. 24 becomes:

$$\partial_{q_b} F = R_{b,t} + \tau_t \, G - s_t \quad (28)$$
$$\text{with } s_t > 0 \text{ if } R_{b,t} = 0 \text{ and } s_t = 0 \text{ otherwise}$$

As explained in Appendix E, when the rental price of brown capital is nil, the subsidy is set as the difference between the carbon tax and the marginal productivity of brown capital when all brown capital is used.[10] Firms thus have no incentive to under-utilize brown capital. This can only happen in the first phase, when brown investment is nil. Whenever $R_{b,t}$ is strictly positive (and in particular in the long-run), brown capital is fully-utilized and the subsidy is equal to zero (so that the subsidy is only a temporary measure to smooth the transition).

Note that since the complementary subsidy is a second-best strategy, the optimal value of the carbon tax is higher in the short-run than the one found in the first-best solution (section 3). However, the total cost borne by producers when they use brown capital is lower than in the first-best case. Here, the temporary subsidy is equivalent to a lower carbon tax in the beginning because we model only one kind of brown capital. If there was a continuum of brown capital with different carbon intensities, the subsidy would only go to carbon-intensive capital that would otherwise be discarded.

In practice, a unique carbon price can be implemented to act as a signal for investments, and it can be completed by temporary subsidies to the most vulnerable firms or households, so that they can keep using their polluting capital. Of course, such policies might create regulatory capture (Laffont and Tirole, 1991), but could be a prerequisite for the implementation of the carbon tax.

[10] Note that the subsidy is always lower than the carbon tax.

Such a subsidy to obsolete brown capital is comparable to Japanese industrial policies, that supported declining traditional sectors during the transition towards higher productivity sectors in the middle of the 20th century (Beason and Weinstein, 1996).

4.2. Differentiation of capital costs

A second solution is to differentiate capital costs, for instance with fiscal incentives such as subsidies on green investment ($\theta_{g,t} < 0$) or taxes on brown investment ($\theta_{b,t} > 0$). Here we model lump-sum taxes on installed capital, as they are easier to model even though they are less realistic. In our model, since investment is irreversible, taxes on capital only have an impact on new capital, i.e. on investment decisions.

The firm's flow of profit at time t is given by:

$$\Pi_t = F(q_{b,t}, q_{g,t}) - (R_{g,t} + \theta_{g,t})\, k_{g,t} - (R_{b,t} + \theta_{b,t})\, k_{b,t} \tag{29}$$

The optimal values of $\theta_{g,t}$ and $\theta_{b,t}$ can be obtained with a maximization of social welfare given the ceiling constraint. We solve the firm's maximization problem in Appendix F and find that for all t it is optimal to have:

$$\begin{aligned} q_{b,t} &= k_{b,t} \\ \partial_{q_b} F &= R_{b,t} + \theta_{b,t} \\ \partial_{q_g} F &= R_{g,t} + \theta_{g,t} \end{aligned}$$

Under-utilizing brown capital is never optimal because firms do not pay carbon emissions directly. Instead, they pay a higher fixed price for brown capital than for green one, such that investment in brown capital is not profitable.

Over the short-run, as in the social optimum (Prop. 4) the economy does not invest in new brown capital. As explained in Appendix F, when $\theta_{g,t}$ is implemented, the rental price of green capital increases (as well as the interest rate). Similarly, $\theta_{b,t}$ induces a temporary decrease in the rental price of brown capital. Once brown capital has depreciated to a level compatible with the GHG ceiling, brown investments become profitable and start again.

When brown and green investments are strictly positive, and in particular on the balanced growth path, the marginal productivity of brown capital is equal to that of green capital plus the sum of the tax and the subsidy ($-\theta_{g,t}$ is positive):

$$\partial_{q_b} F(q_{b,t}, q_{g,t}) = \partial_{q_g} F(q_{b,t}, q_{g,t}) + (\theta_{b,t} - \theta_{g,t})$$

To be on the same balanced growth path as in the social optimum, the optimal value of the tax plus the subsidy should be equal to the carbon tax multiplied by the marginal emissions of brown capital:

$$\forall t \geq t_b,\ \theta_{b,t} - \theta_{g,t} = \tau_t \cdot G$$

with t_b the date at which the balanced growth path is reached.

This capital cost differentiation is similar to existing fiscal incentives, feebates programs or concessional loans for high efficiency homes or appliances. With a continuum of brown capital, the differentiation should be proportionate to the carbon content of each new investment. In practice, this tax on brown investment or subsidy to green investment can be done at the firm level but can also be subject to regulatory capture. Capital costs can also be differentiated using financial markets, as in Rozenberg et al. (2013). In this case, the differentiation would be calibrated on the carbon content of investments.

If the cost differentiation is made through a subsidy on green capital only (as in Rozenberg et al. (2013)), the price of brown capital does not decrease in absolute terms when the policy is implemented. Instead, the interest rate increases and becomes higher than the rental price of brown capital. This transition may appear more acceptable to the owners of brown capital.

4.3. Investment regulation

A third possibility to induce a shift from brown to green investment without reducing the utilization rate of brown capital is to regulate brown investment through efficiency standards. In particular, the most polluting brown investments can be forbidden. Here, since we only model one kind of brown capital, we crudely impose brown investments to be nil until brown capital has depreciated to a level allowing to reach the carbon ceiling without using all brown capital.

We come back to the social planner's program (beginning of section 3) and remove the concentration and ceiling constraints (eq. 10 and eq. 11), as well as the irreversibility constraint (eq. 8). Instead, we add a brown investment constraint that forces $i_{b,t}$ to be nil, and we call σ_t its Lagrangian multiplier:

$$\forall t,\ i_{b,t} = 0 \qquad (\sigma_t) \tag{30}$$

The maximization of intertemporal welfare results in the same equations as in the social optimum (Appendix G), except that the rental rate of brown capital is equal to that of green capital plus a positive term n_t which depends on σ_t:

$$\begin{aligned} \partial_{q_b} F &= R_{b,t} \\ R_{b,t} &= R_{g,t} + n_t \end{aligned} \tag{31}$$

Therefore, the rental price of brown capital becomes higher than the interest rate. The brown investment regulation indeed creates a scarcity effect on brown capital, that becomes more expensive than green capital. This outcome should appear acceptable to the owners of brown capital, as they see their wealth increase in the short-run.

Here again, this instrument must be thought of as temporary, since once brown capital has depreciated to a sustainable level, a carbon price can be implemented without inducing under-utilization of brown capital, and thus becomes politically acceptable. Investment regulation can be compared with existing efficiency standards on cars or electric plants, that forbid the construction of the most polluting kinds of brown capital.

4.4. Comparison with the social optimum

All three capital-based instruments, if they are optimally designed given the ceiling constraint, lead to the same emissions and output pathways.

As already noted, if the concentration ceiling is not stringent, these second-best instruments are equivalent to the carbon tax alone, because it is optimal to always use all brown capital in the short-run. On the other hand, if the ceiling is too stringent, such that waiting for brown capital depreciation is not

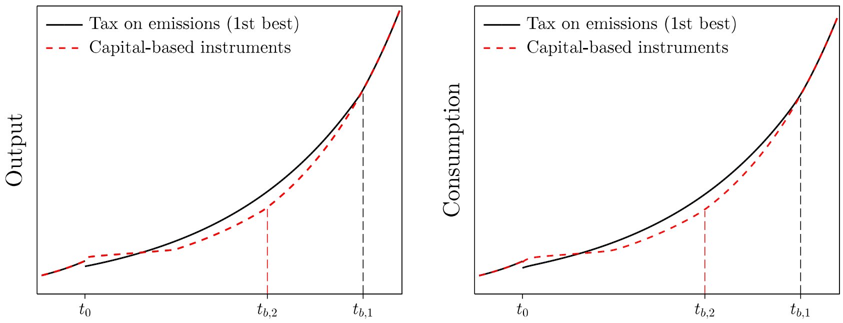

Figure 3: On the left, output y in the two cases. In the short-run output is lower in the first-best case because of the adjustment of brown capital utilization. On the right, consumption c is higher in the second-best case because of a higher output y. t_b is the date at which the balanced growth path is reached, it is reached sooner in the second-best case ($t_{b,2} < t_{b,1}$).

sufficient to remain below the ceiling, these instruments cannot be used to reach the target. This is illustrated in Figure 6 and discussed in section 4.5.

In the middle zone, the one that is interesting for this section, mitigation efforts are increased in the short-run in the first-best optimum with a carbon tax, compared to the second-best alternatives.

Proposition 5. *With capital-based instruments that do not induce brown capital under-utilization, output is higher in the short-run than in the first-best solution with a carbon price.*

Proof. We showed with Proposition 4 that in the first-best optimum, when a carbon price is introduced, the utilization rate of brown capital may be discontinuous. Therefore, in the short-run — that is, when the irreversibility constraint is binding — output is lower in the first-best optimum than in the second-best solution, in which the utilization rate of brown capital is continuous. □

If production is higher over the short-run in the second-best mitigation strategy, consumption can also be higher (we find so in the illustrative simulation of this paper). Analytically, however, the effect on consumption is ambiguous because it involves the offsetting impacts from a substitution effect and an income effect: short-term output is higher, but investments in green capital may also increase since the saving rate is endogenous.

Eventually, all instruments lead to the same balanced growth path, but capital-based policies result in lower discounted welfare than in the social optimum, while they may increase the utility of current generations (Fig. 3). These policies generates higher short-term emissions (Fig. 5) than a carbon price, and because they are sub-optimal, they also generate higher marginal abatement costs (Fig. 4). The marginal abatement cost (MAC) is for instance equal to the carbon price τ_t in the first-best optimum, and to $(\theta_{b,t} - \theta_{g,t})/G$ with differentiated capital costs.

It is interesting to note that in our model, and in particular because investment is irreversible, the carbon tax (or tax plus subsidy) cannot be translated

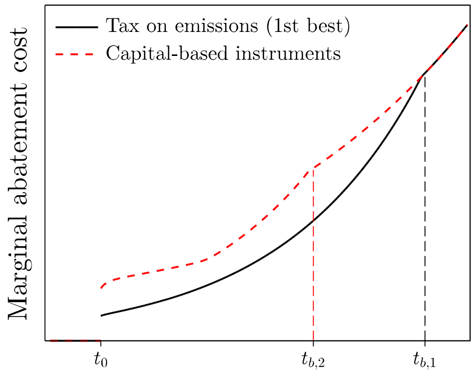

Figure 4: The marginal abatement cost is higher with capital-based instruments than with a carbon price. In the optimal pathway, it is equal to the carbon price (τ).

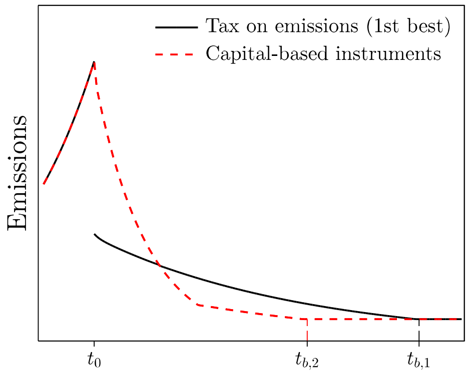

Figure 5: GHG emissions in the two cases. The carbon price induces spare brown capital and thus reduces carbon emissions faster in the short-run.

into consumption losses in a trivial way. At each point in time, the effect of the policy on output and consumption is disconnected from the MAC. Indeed, capital-based instruments are more expensive at each time t in terms of MAC (Fig. 4) while output (and possibly consumption) is higher over the short-run (Fig. 3). Put differently, in our framework the carbon price is not a good indicator to estimate the policy effect on *instantaneous* output and consumption (instead, it gives the impact on intertemporal welfare). On the other hand, the policy design influences the intertemporal distribution of mitigation efforts.

Choosing the best instrument in terms of welfare thus results in choosing the lowest marginal abatement cost but not the highest consumption at each time t. There is however a trade-off between efficiency (intertemporal welfare), intergenerational equity (distribution of efforts over time) and implementation obstacles (political economy). Other criteria than social welfare maximization can be used to decide on the best policy to implement. For instance, Llavador et al. (2011) use the Intergenerational Maximin criterion, which maximizes the minimum utility over the whole trajectory. Using this criterion, capital-based policies would be preferred to the carbon tax alone.

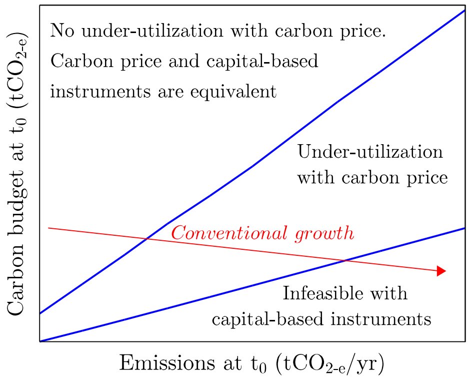

Figure 6: Depending on initial emissions (i.e. initial brown capital $k_{b,0}$) and on the carbon budget ($\bar{m} - m_0$), the carbon tax and capital-based instruments can lead to different or similar outcomes (for a given set of parameters, and in particular ρ and δ). If the carbon budget is too stringent, such that waiting for brown capital depreciation is not sufficient, the capital-based instruments cannot be used. If the carbon budget is not stringent, there is no under-utilization of brown capital in the first-best optimum with the carbon tax and capital-based instruments are equivalent. While the economy is on the laissez-faire growth path (red arrow), brown capital accumulates and the carbon budget is reduced for a given climate objective.

4.5. Lock-in

The carbon price does not always lead to under-utilize brown capital. In fact, this depends on the stringency of the climate target and the level of emissions embedded in installed capital (Fig. 6). As long as climate policies are absent (or very lax), the global economy accumulates brown capital, making GHG emissions grow and reducing the carbon budget for a given climate target (the arrow "conventional growth" in Fig. 6).

At a low development level (left hand side, Fig. 6), a carbon tax does not lead to under-utilization of brown capital and reaching the climate target is possible and optimal without a downward step in income. In this case, the carbon price consistent with the climate target leads to the exact same growth path as capital-based policies. This is a situation of "flexibility" in which a country can chose a brown or a green development path at low cost, using either a carbon price or capital-based instruments.

At one point, brown capital reaches the level when a carbon price induces capital under-utilization and its negative political economy consequences. From there, a carbon price becomes more difficult to implement. But the alternative option of using capital-based instruments is available, leading to higher inter-temporal costs but no immediate drop in income. There is a window of opportunity, during which alternative capital-based instruments may induce a smooth and acceptable transition to a low-carbon economy.

If this occasion is missed (right hand side, Fig. 6), it becomes impossible to reach the climate target without under-utilization of brown capital and capital-based options are not available any more (if the climate objective is not revised). This "capital-based infeasibility zone", i.e. the zone in which brown capital must be under-utilized to remain below the ceiling, depends on the capital depreciation rate δ, the GHG dissipation rate ε, initial GHG concentration m_0

and initial brown capital k_0. It is expressed analytically in Appendix H and if the carbon dissipation rate is small compared to the capital depreciation rate ($\varepsilon \ll \delta$) it can be approximated by:

$$\bar{m} < m_0 + \frac{G\, k_0}{\delta}$$

According to Davis et al. (2010), the level of existing polluting infrastructure in 2010 is still low enough to achieve the 2°C target without under-utilizing brown capital, suggesting that the global economy is not in this last region yet. They show that if existing energy infrastructure was used for its normal life span and no new polluting devices were built, future warming would be less than 0.7 degrees Celsius. Yet, reaching the 2 degrees target might imply to stop investing in polluting capital tomorrow, which depends on our ability to overcome infrastructural inertia and develop clean energy and transport services (Davis et al., 2010; Guivarch and Hallegatte, 2011). Note that Davis et al. (2010) do not discuss whether an optimal climate policy (i.e. a policy that minimizes the discounted cost) would lead to under-utilization, that is whether we are in the top or the middle triangle in Fig. 6.

When we get in the last area (right hand side, Fig. 6), not only the economic cost of reaching the climate target is higher, but the political economy also creates a carbon lock-in: the only option to reach the climate target has to have a significant short-term cost, making it more difficult to implement successfully a climate policy consistent with the target.

5. Conclusion

Current economies have expanded thanks to the installation and use of polluting capital (infrastructure, production processes, energy extraction) and have now come to realize that this accumulation is unsustainable. Reaching ambitious mitigation objectives such as the "2 degrees" target requires decreasing global emissions within the next one or two decades. Doing so with a carbon tax is likely to impose the early retirement of capital that could be operated for several more years (a process sometimes referred to as early-scrapping or mothballing) while progressively accumulating green capital. Such a strategy can be unacceptable if people have an aversion for capital under-utilization, as it creates unemployment and may necessitate compensations for the owners of brown capital. Also, it reduces the income and consumption of current generations for the benefit of future ones, which may appear unattractive to individuals with high discount rates.

We find that when all production capital is used — in our second-best strategy relying on capital-based instruments — the outcome in terms of discounted intertemporal welfare is lower but current generations have a higher income than when a carbon price is implemented alone. Such a transition towards a low-carbon economy can be triggered by green incentives that shift investments towards green capital without penalizing existing brown capital (e.g., efficiency standards on new cars or home appliances). It can also be done by subsidizing brown capital to ensure it is fully used until the end of its lifetime, despite the carbon tax.

In those cases, current generations keep using their inefficient buildings and combustion engines, while redirecting their investment towards green capital.

After some time, the only remaining "brown capital" is the one that does not need to be substituted by green capital, as in the optimal case. A carbon price can then be implemented more easily, since all instruments are then equivalent. Capital-based policies therefore only differ temporarily from the first-best pathway, in a way that smooths the transition costs: they decrease efforts in the short-run, increase them in the medium-run, and leave them unchanged in the long-run.

These results are important for the political economy of climate change that has to deal, in particular, with the issues of sensitivity and preference heterogeneity. Implementing a unique price on carbon emissions penalizes the owners of brown capital that would have to be compensated with interindividual transfers, and such transfers often face technical difficulties (Kanbur, 2010). Because capital-based instruments do not penalize (as much) the owners of brown capital and their employees, they mitigate these difficulties.

Second, time preference heterogeneity (Greene, 2010; Heal and Millner, 2013) makes it unappealing for some people to pay now for remote future benefits. This is even more so because future generations are likely to be richer while being the ones benefiting from reduced climate change damages.

In this analysis, we have modeled a social planner who takes decisions given a set of parameters, and in particular given a discount rate (ρ) for welfare calculation. This discount rate is taken into account in the carbon ceiling, that increases social welfare compared to a baseline scenario with climate change damages. Given these preferences, the social planner sets the GHG ceiling and implements instruments to comply with the ceiling. But some individuals might have different time preferences from the social planner's (Goulder and Williams, 2012). In particular, some individuals might have preferred a less restrictive GHG ceiling, i.e. lower short-term costs and higher long-term costs. Given the time profile of consumption with the two abatement strategies, it is clear that those individuals prefer the second option, because it shifts mitigation efforts to the future, compared to the first-best option. This suggests that the second-best strategy is more robust to preference heterogeneity than the first-best one, and supports the idea that policies focusing on new capital are more politically acceptable than a carbon price.

The capital-based instruments that we modeled in this paper are industrial policies: they subsidize the declining brown sector in order to avoid political opposition to the climate policy, or they trigger investment in the new green sector. As such, they create the same risks from capture and rent-seeking as most industrial policies (Laffont and Tirole, 1991). Applying them is therefore challenging and requires strong institution settings and controls (Rodrik, 2008). However, we do not model here the usual factors that justify the use of industrial policies: learning-by-doing, imperfect appropriability of knowledge spillovers, increasing returns. Instead, we show that industrial policies can facilitate the transition towards a low-carbon economy because they prevent the under-utilization of existing capital and reduce short-term output losses. In a further paper, we will consider increasing returns in the green sector to get a more exhaustive picture of the potential risks and benefits from green industrial policies.

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References

Acemoglu, D., Aghion, P., Bursztyn, L., Hemous, D., Feb. 2012. The environment and directed technical change. American Economic Review 102 (1), 131–166.

Ambrosi, P., Hourcade, J., Hallegatte, S., Lecocq, F., Dumas, P., Ha Duong, M., 2003. Optimal control models and elicitation of attitudes towards climate damages. Environmental Modeling and Assessment 8 (3), 133–147.

Arrow, K. J., Kurz, M., Mar. 1970. Optimal growth with irreversible investment in a ramsey model. Econometrica 38 (2), 331–344, ArticleType: research-article / Full publication date: Mar., 1970 / Copyright © 1970 The Econometric Society.
URL http://www.jstor.org/stable/1913014

Beason, R., Weinstein, D. E., 1996. Growth, economies of scale, and targeting in japan (1955-1990). The Review of Economics and Statistics, 286–295.
URL http://www.jstor.org/stable/10.2307/2109930

Davis, S. J., Caldeira, K., Matthews, H. D., 2010. Future CO2 emissions and climate change from existing energy infrastructure. Science 329 (5997), 1330 –1333.

Fischer, C., Newell, R. G., Mar. 2008. Environmental and technology policies for climate mitigation. Journal of Environmental Economics and Management 55 (2), 142–162.

Goulder, L. H., Mathai, K., Jan. 2000. Optimal CO2 abatement in the presence of induced technological change. Journal of Environmental Economics and Management 39 (1), 1–38.

Goulder, L. H., Parry, I. W. H., Jul. 2008. Instrument choice in environmental policy. Review of Environmental Economics and Policy 2 (2), 152–174.

Goulder, L. H., Williams, R. C., 2012. The choice of discount rate for climate change policy evaluation. Tech. rep., National Bureau of Economic Research.

Greene, D. L., 2010. How consumers value fuel economy: A literature review. Assessment and standards division office of transportation and air quality U.S. environmental protection agency, EPA-420-R-10-008. Washington, DC: US Environmental Protection Agency.
URL http://www.epa.gov/otaq/climate/regulations/420r10008.pdf

Guivarch, C., Hallegatte, S., Oct. 2011. Existing infrastructure and the 2C target. Climatic Change 109 (3-4), 801–805.

Heal, G., Millner, A., Apr. 2013. Discounting under disagreement. Working Paper 18999, National Bureau of Economic Research.

IPCC, 2007. Summary for policymakers. In: Climate change 2007: Mitigation. Contribution of working group III to the fourth assessment report of the intergovernmental panel on climate change, b. metz, O.R. davidson, P.R. bosch, r. dave, L.A. meyer (eds) Edition. Cambridge University Press, Cambridge, UK and New York, USA.

J. Barro, R., Sala-i Martin, X., 2004. Economic Growth.
URL http://localhost:8080/xmlui/handle/123456789/1237

Jorgenson, D., 1967. The theory of investment behavior. In: Determinants of investment behavior. NBER.

Kanbur, R., 2010. Macro crisis and targeting transfers to the poor. In: Globalization and Growth: Implications for a Post-Crisis World. Michael Spence, Danny Leipziger, Commission on Growth and Development, The World Bank, Washington DC, p. 342.

Laffont, J.-J., Tirole, J., Jan. 1991. The politics of government decision-making: A theory of regulatory capture. The Quarterly Journal of Economics 106 (4), 1089–1127.
URL http://qje.oxfordjournals.org/content/106/4/1089

Llavador, H., Roemer, J. E., Silvestre, J., 2011. A dynamic analysis of human welfare in a warming planet. Journal of Public Economics 95 (11), 1607–1620.
URL http://www.sciencedirect.com/science/article/pii/S0047272711000922

Meinshausen, M., Meinshausen, N., Hare, W., Raper, S. C. B., Frieler, K., Knutti, R., Frame, D. J., Allen, M. R., Apr. 2009. Greenhouse-gas emission targets for limiting global warming to 2[thinsp][deg]C. Nature 458 (7242), 1158–1162.
URL http://dx.doi.org/10.1038/nature08017

OECD, Sep. 2009. Incentives for CO2 emission reductions in current motor vehicle taxes. Tech. rep., Organisation for Economic Co-operation and Development, Paris.

Ploeg, F. V. D., Withagen, C., Jun. 1991. Pollution control and the ramsey problem. Environmental and Resource Economics 1 (2), 215–236.

Rodrik, D., 2008. Normalizing industrial policy. Working Paper 3, World Bank.

Rozenberg, J., Hallegatte, S., Perrissin-Fabert, B., Hourcade, J.-C., 2013. Funding low-carbon investments in the absence of a carbon tax. Climate Policy 13 (1), 134–141.

Schwerin, H., 2013. Capacity utilization in pollution control.

Vogt-Schilb, A., Meunier, G., Hallegatte, S., 2012. How inertia and limited potentials affect the timing of sectoral abatements in optimal climate policy. World Bank Policy Research (6154).

Appendix A. Maximization of the household's utility

We consider a Ramsey framework with a representative infinitely-lived household, who receives the economy's production from firms y_t, saves by accumulating assets a_t, receives income on assets at interest r_t and purchases goods for consumption c_t. The assets dynamics are given by:

$$\dot{a}_t = r_t \cdot a_t + y_t - c_t \tag{A.1}$$

At time t, consuming c_t provides consumers with a utility $u(c_t)$. The utility function is increasing with consumption, and strictly concave ($u'(c) > 0$ and $u''(c) < 0$).

The household maximizes their intertemporal utility, given by

$$W = \int_0^\infty e^{-\rho t} \cdot u(c_t)\, dt \tag{A.2}$$

where ρ is the rate of time preference.
The present value Hamiltonian is:

$$H_h = e^{-\rho t} \cdot \{u(c_t) + \lambda_t[r_t \cdot a_t + y_t - c_t]\} \tag{A.3}$$

where λ_t is the shadow price of income at time t.
The first order conditions for a maximum of W are:

$$\forall t, \partial_c H_h = 0 \Rightarrow \lambda_t = u'(c_t) \tag{A.4}$$

$$\forall t, \partial_a H_h = -\frac{\partial(e^{-\rho t}\lambda_t)}{\partial t} \Rightarrow \dot{\lambda}_t = (\rho - r_t)\lambda_t \tag{A.5}$$

The doted variables represent temporal derivatives.
If we differentiate eq. A.4 with respect to time and substitute for λ from this equation and $\dot{\lambda}$ from eq. A.5, we get the Euler equation, which gives the basic condition for choosing consumption over time:

$$\frac{\dot{c}}{c} = -\frac{u'(c)}{c \cdot u''(c)} \cdot (r_t - \rho) \tag{A.6}$$

$-\frac{u'(c)}{cu''(c)} > 0$ so consumption grows if the rate of return to saving is higher than the rate of time preference. If the interest rate equals the rate of time preference, consumption is constant over time.

Appendix B. First order conditions for the social optimum (section 3)

The present value Hamiltonian associated to the maximization of social welfare is:

$$\begin{aligned} H_t = e^{-\rho t} \cdot \{&u(c_t) + \lambda_t[F(q_b, k_g) - c_t - i_{b,t} - i_{g,t}] + \nu_t[i_{b,t} - \delta k_{b,t}] \\ &+\chi_t[i_{g,t} - \delta k_{g,t}] - \mu_t \cdot [G\, q_{b,t} - \varepsilon m_t] + \phi_t \cdot [\bar{m} - m_t] \\ &+ \psi_t \cdot i_{b,t} + \beta_t[k_{b,t} - q_{b,t}]\} \end{aligned} \tag{B.1}$$

λ_t is the current value shadow price of income. ν_t and χ_t are the current shadow values of investments in brown and green capacities. μ_t is the current-value shadow price of pollution in the atmosphere, expressed in terms of undiscounted utility at time t.
First order conditions give:

$$\frac{\partial H_t}{\partial c_t} = 0 \Rightarrow \qquad u'(c_t) = \lambda_t$$
$$\frac{\partial H_t}{\partial i_{b,t}} = 0 \Rightarrow \qquad \lambda_t = \nu_t + \psi_t$$
$$\frac{\partial H_t}{\partial i_{g,t}} = 0 \Rightarrow \qquad \lambda_t = \chi_t$$
$$\frac{\partial H_t}{\partial k_{b,t}} = -\frac{\partial(e^{-\rho t}\nu_t)}{\partial t} \Rightarrow \qquad -\nu_t\delta + \beta_t = -\dot{\nu}_t + \rho\nu_t$$
$$\frac{\partial H_t}{\partial k_{g,t}} = -\frac{\partial(e^{-\rho t}\chi_t)}{\partial t} \Rightarrow \quad \lambda_t\partial_{k_g}F(k_{b,t}, k_{g,t}) - \chi_t\delta = -\dot{\chi}_t + \rho\chi_t$$
$$\frac{\partial H_t}{\partial q_{b,t}} = 0 \Rightarrow \qquad \lambda_t\partial_{q_b}F(q_{b,t}, k_{g,t}) - \mu_t \cdot G = \beta_t$$
$$\frac{\partial H_t}{\partial m_t} = \frac{\partial(e^{-\rho t}\mu_t)}{\partial t} \Rightarrow \qquad -\phi_t + \varepsilon\mu_t = \dot{\mu}_t - \rho\mu_t$$

They can be reduced to the following equations:

$$u'(c_t) = \lambda_t = \nu_t + \psi_t = \chi_t \tag{B.2}$$
$$\lambda_t\partial_{k_g}F = (\delta + \rho)\chi_t - \dot{\chi}_t \tag{B.3}$$
$$\lambda_t\partial_{q_b}F = \beta_t + \mu_t \cdot G \tag{B.4}$$
$$\beta_t = (\delta + \rho)\nu_t - \dot{\nu}_t \tag{B.5}$$
$$\dot{\mu}_t = (\rho + \varepsilon)\mu_t - \phi_t \tag{B.6}$$

A simple interpretation for eq. B.2 is that along the optimal path, the current value of income (λ_t) is the marginal utility of consumption at time t. It is also equal to the value of investments in green capital χ_t (eq. B.2). The implicit rental value of green capital expressed in monetary terms is $\frac{1}{\lambda}[(\delta + \rho)\chi_t - \dot{\chi}_t]$ according to the definition given by Jorgenson (1967), where χ_t is the value of a unit of investment acquired at time t. It is always equal to the marginal productivity of green capital (eq. B.3). The complementary slackness conditions associated to the irreversibility constraint are:

$$\forall t,\ \psi_t \geq 0 \text{ and } \psi_t \cdot i_{b,t} = 0 \tag{B.7}$$

ψ_t is such that $0 \leq \psi_t \leq \lambda$ If $\psi_t > 0$, the value of brown capital is lower than the marginal utility of consumption ($\nu_t < \lambda_t$) and thus there is no investment in brown capital. If $\psi_t = \lambda$ then the value of brown capital is zero. In this case $\beta_t = 0$ (eq. B.5), and according to the complementary slackness condition associated to the "under-utilization" possibility, it means that $q_{b,t} = k_{b,t}$ is not necessarily optimal anymore:

$$\forall t,\ \beta_t \geq 0 \text{ and } \beta_t \cdot (k_{b,t} - q_{b,t}) = 0 \tag{B.8}$$

Still according to the definition given by Jorgenson (1967), β_t is equal to the rental value of brown capacities, defined as $(\delta+\rho)\nu_t - \dot{\nu}_t$ (eq. B.5). The marginal productivity of brown capital is thus equal to the rental value of brown capital plus $\frac{\mu_t}{\lambda_t}G$ (eq. B.4), where $\frac{\mu_t}{\lambda_t}$ is the carbon price expressed in unit of consumables.

Appendix B.1. Carbon price

We call τ_t the current price of GHG, expressed in units of consumables.

$$\tau_t = \frac{\mu_t}{\lambda_t}$$

with μ_t the current-value price of pollution in the atmosphere and λ_t the current value income, both expressed in utility terms.

eq. B.6 gives the evolution of μ_t. Using $\dot{\mu}_t = (\dot{\lambda}_t \tau_t + \lambda_t \dot{\tau}_t)$ and $\frac{\dot{\lambda}_t}{\lambda_t} = (\rho - r_t)$, it can be written as the evolution of τ_t (the carbon price):

$$\dot{\tau}_t = \tau_t[\varepsilon + r_t] - \frac{\phi_t}{\lambda_t}$$

Before m reaches the ceiling, it is not binding and $\phi_t = 0$. In that case the carbon price follows the following motion rule:

$$\dot{\tau}_t = \tau_t[\varepsilon + r_t]$$

Once the constraint is reached, $\forall t\ m_t = \bar{m}$, and $\phi_t > 0$. □

These dynamics may be interpreted as a generalized Hotelling rule applied to clean air: along the optimal pathway, and before the ceiling is reached, the discounted abatement costs are constant over time. The appropriate discount rate is $r + \varepsilon$, to take into account the natural decay of GHG in the atmosphere (see for instance Goulder and Mathai, 2000, footnote 11, p6).

Appendix B.2. Marginal productivities

As in Jorgenson (1967), the rental prices of green and brown capacities are defined as follows:

$$R_{g,t} = \frac{1}{\lambda}[(\delta + \rho)\chi_t - \dot{\chi}_t] \tag{B.9}$$

$$R_{b,t} = \frac{1}{\lambda}[(\delta + \rho)\nu_t - \dot{\nu}_t] \tag{B.10}$$

If we differentiate eq. B.2 with respect to time and substitute λ_t and $\dot{\lambda}_t$, we can write:

$$\frac{c_t \cdot u''(c_t)}{u'(c_t)} \cdot \frac{\dot{c}_t}{c_t} = (\rho + \delta - R_{g,t}) \tag{B.11}$$

From eq. A.6 we can thus write:

$$R_{g,t} = r_t + \delta \tag{B.12}$$

Combining eq. B.5 and eq. B.4 we find:

$$\partial_{q_b} F = R_{b,t} + \tau_t \cdot G'(q_{b,t})$$

□

Appendix B.3. Phases in brown capital

Two cases can be distinguished:

- If the irreversibility constraint is not binding, $\psi_t = 0$ (eq. B.7) and eq. B.2 gives $\nu_t = \chi_t$. Combining eq. 24 and eq. B.3 we find

$$\partial_{q_b} F = \partial_{k_g} F + \tau_t \cdot G'(q_{b,t})$$

- When the irreversibility constraint is binding ($\psi_t > 0$), we can differentiate eq. B.2 ($\nu_t = \chi_t - \psi_t$) and substitute ν_t and $\dot{\nu_t}$ in eq. B.5 to obtain

$$\frac{\beta_t}{\lambda_t} = \frac{1}{\lambda_t}\Big((\delta+\rho)\chi_t - \dot{\chi}_t + \dot{\psi}_t - (\rho+\delta)\psi_t\Big)$$

and using eq. B.3 we get

$$\frac{\beta_t}{\lambda_t} = \partial_{k_g} F - \frac{1}{\lambda_t}\Big(-\dot{\psi}_t + (\rho+\delta)\psi_t\Big)$$

We call $p = \frac{1}{\lambda_t}\Big((\rho+\delta)\psi_t - \dot{\psi}_t\Big)$ and get

$$\partial_{q_b} F = \partial_{k_g} F - p + \tau_t \cdot G'(q_{b,t})$$

$0 < \psi_t \leq \lambda_t \Rightarrow 0 \leq \frac{1}{\lambda_t}\Big((\rho+\delta)\psi_t - \dot{\psi}_t\Big) \leq \Big(\rho + \delta - \frac{\dot{\lambda}_t}{\lambda_t}\Big)$,
so that $0 \leq p \leq \partial_{k_g} F$.
When $p = \partial_{k_g} F$, the rental rate of brown capacities is nil and brown capacities may be under-utilized (slackness condition, eq. B.8) in order to adjust the marginal productivity of brown capital to the carbon price:

$$\partial_{q_b} F = \tau_t \cdot G'(q_{b,t})$$

□

Appendix C. Irreversibility constraint: proof of proposition 4

The GHG ceiling is imposed at $t = t_0$. Before that, the economy is in the competitive equilibrium so green and brown capacities have the same marginal productivity and capacities are fully used (Proposition 1). At t_0^-, i.e. just before the ceiling is internalized ($t < t_0$), we thus have the following limits for $q_{b,t}$ and $\partial_{q_b} F$:

$$\lim_{t \to t_0^-} q_{b,t} = k_{b,t} \tag{C.1}$$

$$\lim_{t \to t_0^-} \partial_{q_b} F(q_{b,t}, q_{g,t}) = \partial_{k_g} F(q_{b,t}, q_{g,t}) \tag{C.2}$$

We use a proof by contradiction to show that at t_0^+ (when the constraint is internalized) the irreversibility condition is necessarily binding. Suppose that when $t > t_0$, the irreversibility condition is not binding, i.e. $\psi_t = 0$ (eq. B.7). According to proposition 3, it leads to:

$$\lim_{t \to t_0^+} q_{b,t} = k_{b,t} \tag{C.3}$$

$$\lim_{t \to t_0^+} \partial_{q_b} F(q_{b,t}, q_{g,t}) = \partial_{k_g} F(q_{b,t}, q_{g,t}) + \tau_t \cdot G \tag{C.4}$$

So from eq. C.2 and eq. C.4:

$$\lim_{t\to t_0^+} \partial_{q_b} F \neq \lim_{t\to t_0^+} \partial_{q_b} F \tag{C.5}$$

$\partial_{q_b} F$ is a continuous function of $q_{b,t}$ so eq. C.5 implies that $\lim_{t\to t_0^+} q_{b,t} \neq \lim_{t\to t_0^+} q_{b,t}$, and that is incompatible with eq. C.1 and eq. C.3.
Therefore, the irreversibility condition is necessarily binding at $t = t_0^+$, i.e. $\psi_t > 0$.
Two cases then need to be distinguished, whether brown capacities are fully used or not. If brown capacities are under-utilized ($q_{b,t} < k_{b,t}$ and $\beta_t = 0$), there is a discontinuity in output. □

Appendix D. Decentralized equilibrium with a tax on emissions

In a decentralized economy, it is possible to trigger the same outcome as in the social optimum with a lump-sum tax applied to carbon emissions. In this case, the firm's flow of profit at time t is given by:

$$\Pi_t = F(q_{b,t}, k_{g,t}) - R_{g,t} \cdot k_{g,t} - R_{b,t} \cdot k_{b,t} - \tau_t \, G \, q_{b,t} \tag{D.1}$$

With $R_{b,t}$ and $R_{g,t}$ the rental prices of brown and green capacities respectively, and τ_t the carbon tax. The tax is redistributed through the assets equation:

$$\dot{a}_t = r_t \cdot a_t + y_t - c_t + \tau_t \, G \, q_{b,t} \tag{D.2}$$

As in the centralized equilibrium, the marginal revenue of brown capital is equal to $\partial_{q_b} F = R_{b,t} + \tau_t G$ while the marginal revenue of green capital is $\partial_{k_g} F = R_{g,t}$. The Lagrangian corresponding to the firm's maximization program is:

$$L(t) = \Pi_t + \beta_t (k_{b,t} - q_{b,t}) + \gamma_t (k_{g,t} - q_{g,t}) \tag{D.3}$$

First order conditions are:

$$\partial_{q_g} L = 0 \Rightarrow \qquad \partial_{q_g} F(q_{b,t}, q_{g,t}) = \gamma_t \tag{D.4}$$
$$\partial_{q_b} L = 0 \Rightarrow \qquad \partial_{q_b} F(q_{b,t}, q_{g,t}) = \beta_t + \tau_t \cdot G \tag{D.5}$$
$$\partial_{k_g} L = 0 \Rightarrow \qquad \gamma_t = R_{g,t} \tag{D.6}$$
$$\partial_{k_b} L = 0 \Rightarrow \qquad \beta_t = R_{b,t} \tag{D.7}$$

For all t,

$$\gamma_t \geq 0 \text{ and } \gamma_t \cdot (k_{g,t} - q_{g,t}) = 0$$
$$\beta_t \geq 0 \text{ and } \beta_t \cdot (k_{b,t} - q_{b,t}) = 0$$

(complementary slackness conditions).
With eq. D.4 we have $\gamma_t = \partial_{q_g} F(q_{b,t}, q_{g,t}) > 0$, so $q_{g,t} = k_{g,t}$ for all t.
The combination of eq. D.4 and eq. D.6 gives

$$\partial_{k_g} F(q_{b,t}, k_{g,t}) = R_{g,t}$$

Combining eq. D.5 and eq. D.7, we find

$$\partial_{q_b} F(q_{b,t}, k_{g,t}) = R_{b,t} + \tau_t \cdot G \tag{D.8}$$

In the equilibrium, the rental price of green capacities is equal to the interest rate (plus delta): $R_{g,t} = r_t + \delta$, because green capacities and loans are perfect substitutes as assets for households. When the irreversibility constraint is not binding (see eq. 8), and in particular on the balanced growth path, the rental rate of brown capacities is equal to the interest rate as well and $R_{b,t} = R_{g,t} = r_t + \delta$.

However, when the carbon price in implemented at t_0, the irreversibility constraint is binding (Appendix C). In this case, since the use of brown capacities suddenly becomes too expensive, the rental rate of brown capacities is endogenously reduced. As a consequence of a lower rate of return for owners of brown capital, households stop investing in brown capacities. If the carbon tax is very high, the rental rate of brown capacities can even become nil and brown capacities may be under-utilized. It is possible to determine the rental value of brown capital at t_0^+ thanks of the continuity of capacities between t_0^- and t_0^+: at t_0^+, the marginal productivity of brown capital is equal to $\partial_{q_b} F(t_0^+) = R_b(t_0^+) + \tau_{t_0^+} G$ (eq. D.8).

- If brown capacities are fully-utilized at t_0^+ (that is, $q_{b,t_0^+} = k_{b,t_0}$), the marginal productivity of brown capacities is necessarily equal to that of green capacities ($\partial_{q_b} F(t_0^+) = \partial_{k_g} F(t_0^+)$) because capacities are continuous and $\partial_{q_b} F(t_0^-) = \partial_{k_g} F(t_0^-)$.
  In this case $R_b(t_0^+) = R_b(t_0^-) - \tau_{t_0^+} G$.

- If brown capacities are under-utilized at t_0^+, $q_{b,t_0^+} < q_{b,t_0^-}$, $R_b(t_0^+) = 0$ (eq. D.7) and $\partial_{k_b} F(k_{b,t}, k_{g,t}) = \tau_t \cdot G$.

Brown capacities are thus under-utilized at t_0 if $\tau_{t_0^+} G \geq R_{b,t_0^-}$. In other words, if the carbon tax (multiplied by the carbon intensity of capital G) is higher than the rental value of brown capacities in the laissez-faire balanced growth path, brown capacities are under-utilized in the short-run.

Appendix E. Social optimum with a carbon tax and a temporary subsidy

We start from the first order conditions found in the first-best optimum (section 3) and we modify 24 as follows:

$$\partial_{q_b} F = R_{b,t} + \tau_t G - s_t \tag{E.1}$$

with s_t calculated such that it is equal to$\tau_t G - \partial_{q_b} F|_{q_{b,t}=k_{b,t}} + R_{b,t}$

When the rental price of brown capacities is nil, the subsidy is set as the difference between the carbon tax and the marginal productivity of brown capital when all brown capacities are used. In this case, firms have no incentive to under-utilize brown capacities. Whenever $R_{b,t}$ is strictly positive (and in particular in the long-run), brown capacities are fully-utilized and the subsidy is equal to zero.

In a decentralized equilibrium, the subsidy would appear in the profit equation as:

$$\Pi_t = F(q_{b,t}, q_{g,t}) - R_{g,t}\, k_{g,t} - R_{b,t}\, k_{b,t} - (\tau_t - s_t)\, G\, q_{b,t} \tag{E.2}$$

And it would be deducted from the households' budget equation:

$$\dot{a}_t = r_t \cdot a_t + y_t - c_t + \tau_t \, G \, q_{b,t} - s_t \, q_{b,t} \tag{E.3}$$

Note that with the subsidy, in the short-run the optimal value of the carbon tax is different from the one found in the first-best solution (section 3). In the long-run, however, both instruments can lead to the same balanced growth path and the optimal carbon tax is the same.

Appendix F. Firms' maximization problem with differentiation of capital costs

Capital costs can be differentiated with fiscal incentives, e.g. subsidies on new green capacities ($\theta_{g,t} < 0$) or taxes on new brown capacities ($\theta_{b,t} > 0$). Here we model lump-sum taxes on all capacities, but they only have an impact on new investment decisions and are thus equivalent to taxes on new capacities. The optimal values of $\theta_{g,t}$ and $\theta_{b,t}$ can be obtained with a maximization of social welfare given the ceiling constraint. The firm's flow of profit at time t is given by:

$$\Pi_t = F(q_{b,t}, q_{g,t}) - (R_{g,t} + \theta_{g,t}) \, k_{g,t} - (R_{b,t} + \theta_{b,t}) \, k_{b,t} \tag{F.1}$$

The Lagrangian corresponding to the firm's maximization program is:

$$L(t) = \Pi_t + \beta_t (k_{b,t} - q_{b,t}) + \gamma_t (k_{g,t} - q_{g,t}) \tag{F.2}$$

First order conditions are:

$$\partial_{q_g} L = 0 \Rightarrow \qquad \partial_{q_g} F(q_{b,t}, q_{g,t}) = \gamma_t \tag{F.3}$$
$$\partial_{q_b} L = 0 \Rightarrow \qquad \partial_{q_b} F(q_{b,t}, q_{g,t}) = \beta_t \tag{F.4}$$
$$\partial_{k_g} L = 0 \Rightarrow \qquad \gamma_t = R_{g,t} - \theta_{g,t} \tag{F.5}$$
$$\partial_{k_b} L = 0 \Rightarrow \qquad \beta_t = R_{b,t} - \theta_{b,t} \tag{F.6}$$

For all t, the complementary slackness conditions are

$$\gamma_t \geq 0 \text{ and } \gamma_t \cdot (k_{g,t} - q_{g,t}) = 0$$

$$\beta_t \geq 0 \text{ and } \beta_t \cdot (k_{b,t} - q_{b,t}) = 0$$

Note that with the carbon tax, β_t was equal to the rental value of brown capacities while here it is equal to the marginal revenue of brown capital (eq. F.6).
With eq. F.3 we have $\gamma_t = \partial_{q_b} F(q_{b,t}, q_{g,t}) > 0$, so $q_{g,t} = k_{g,t}$ for all t.
Similarly, $\beta_t = \partial_{q_b} F(q_{b,t}, q_{g,t}) > 0$, so $q_{b,t} = k_{b,t}$ for all t.
The combination of eq. F.3 and eq. F.5, and eq. F.4 and eq. F.6

$$\partial_{q_b} F(q_{b,t}, q_{g,t}) = R_{b,t} + \theta_{b,t}$$
$$\partial_{q_g} F(q_{b,t}, q_{g,t}) = R_{g,t} + \theta_{g,t}$$

The irreversibility constraint is never binding for green investments, so as in the laissez-faire equilibrium, green capacities and loans are perfect substitutes as assets, and $R_{g,t} = r_t + \delta$, with r_t the interest rate. Note that when the policies are implemented, the continuity of capacities imposes that $R_g(t_0^+) +$

$\theta_g(t_0^+) = R_g(t_0^-)$. In other words, the rental price of green capacities suddenly increases when the subsidy is implemented. At the same time, the irreversibility constraint is binding for brown capacities and their rental price decreases below that of green capacities. For the same reason as for the green rental price, we have $R_b(t_0^+) = R_b(t_0^-) - \theta_b(t_0^+)$. Therefore, the economy does not invest in new brown capacities during this phase. As with the carbon price, these rental prices variations are transfers between households and firms that compensate for the tax or subsidy when investment is nil in brown capacities. Note that, contrary to the carbon tax, this policy may lead to a negative rental price for brown capacities when brown investments are nil, if $\theta_{b,t}$ is higher than the marginal productivity of brown capital. This negative rental price is equivalent to the subsidy we modelled in section 4.1. It is equal to $R_{b,t} - s_t$ when $s_t > R_{b,t}$.

On the balanced growth path, brown and green investments are positive so the irreversibility constraint is not binding and $R_{b,t} = R_{g,t}$. In this case the marginal productivity of brown capital is equal to that of green capital plus the sum of the tax and the subsidy (note that $(-\theta_{g,t})$ is positive):

$$\partial_{q_b} F(q_{b,t}, q_{g,t}) = \partial_{q_g} F(q_{b,t}, q_{g,t}) + (\theta_{b,t} - \theta_{g,t})$$

To be on the same balanced growth path as in the social optimum, the optimal value of the tax plus the subsidy should be equal to the carbon tax multiplied by the marginal emissions of brown capital:

$$\forall t \geq t_b,\ \theta_{b,t} - \theta_{g,t} = \tau_t \cdot G$$

with t_b the date at which the balanced growth path is reached.

Appendix G. Maximization of social welfare with standards on brown investments

We come back to the social planner's program (beginning of section 3) and remove the concentration and ceiling constraints (eq. 10 and eq. 11), as well as the irreversibility constraint (eq. 8). Instead, we add a brown investment constraint that forces $i_{b,t}$ to be equal to a standard at each point in time, and we call σ_t its Lagrangian multiplier:

$$\forall t,\ i_{b,t} = sd_t \qquad (\sigma_t) \tag{G.1}$$

The standard sd_t can be optimally set to equal brown investments found in sections 4.1 and 4.2. Basically, $sd_t = 0$ until brown capacities have depreciated to a level compatible with the ceiling, and then a carbon price can be implemented. The present value Hamiltonian associated to the maximization of social welfare is:

$$\begin{aligned} H_t = e^{-\rho t} \cdot \{u(c_t) + \lambda_t[F(q_b, k_g) - c_t - i_{b,t} - i_{g,t}] + \nu_t[i_{b,t} - \delta k_{b,t}] \\ + \chi_t[i_{g,t} - \delta k_{g,t}] + \sigma_t \cdot (sd_t - i_{b,t}) + \beta_t[k_{b,t} - q_{b,t}]\} \end{aligned} \tag{G.2}$$

λ_t is the current value shadow price of income. ν_t and χ_t are the current shadow values of investments in brown and green capacities.

First order conditions can be reduced to the following equations:

$$\begin{aligned} u'(c_t) &= \lambda_t = \nu_t - \sigma_t = \chi_t && \text{(G.3)}\\ \lambda_t \partial_{k_g} F &= (\delta+\rho)\chi_t - \dot{\chi}_t && \text{(G.4)}\\ \lambda_t \partial_{q_b} F &= \beta_t && \text{(G.5)}\\ \beta_t &= (\delta+\rho)\nu_t - \dot{\nu}_t && \text{(G.6)} \end{aligned}$$

The maximization of intertemporal welfare results in the same equations as in the social optimum, except for the marginal productivity of brown capital and the rental price of brown capacities:

$$\begin{aligned} &\partial_{q_b} F = R_{b,t} && \text{(G.7)}\\ &R_{b,t} = R_{g,t} + n_t && \text{(G.8)}\\ &\text{with } n_t = \frac{1}{\lambda_t}\left((\rho+\delta)\sigma_t - \dot{\sigma_t}\right) \end{aligned}$$

n_t is positive if the standard imposes lower brown investment than in the *laissez-faire*equilibrium, and it is negative if it imposes higher investments. Here, since we want to force brown investments to be below that of the *laissez-faire* equilibrium, n_t is always positive, which means that the rental price of brown capacities is higher than the interest rate. Indeed, the brown investment standard creates a scarcity effect on brown capital, that becomes more expensive than green capital.

Appendix H. Second-best infeasibility zone

This zone defines the cases when the ceiling is reached before brown capacities have depreciated to a sustainable level. If no investment is made in brown capacities, we have:

$$k_{b,t} = k_0\, e^{-\delta t}$$

Therefore, the stock of pollution follows this dynamic:

$$\dot{m} = k_0\, e^{-\delta t} - \varepsilon\, m$$

The solution to this differential equation is:

$$m_t = -\frac{G\,k_0}{\delta-\varepsilon}\, e^{-\delta t} + \left(m_0 + \frac{G\,k_0}{\delta-\varepsilon}\right)\, e^{-\varepsilon t}$$

This function first increases to a maximum $m_{max} = \frac{G\,k_0}{\delta} e^{-\delta t}$ and then decreases. The maximum date is

$$t_{max} = -\frac{1}{\delta}\, \ln(\frac{m_{max}\,\varepsilon}{G\,k_0})$$

The expression of m at the maximum date gives the limit of the infeasibility zone if $m_{max} = \bar{m}$:

$$\bar{m} = -\frac{G\,k_0}{\delta-\varepsilon}\, e^{\ln(\frac{\bar{m}\,\varepsilon}{G\,k_0})} + \left(m_0 + \frac{G\,k_0}{\delta-\varepsilon}\right)\, e^{\frac{\varepsilon}{\delta}\ln(\frac{\bar{m}\,\varepsilon}{G\,k_0})}$$

This can be rewritten:

$$\bar{m} = \left[\left(m_0 + \frac{G\,k_0}{\delta - \varepsilon} \right) \left(\frac{\varepsilon}{G\,k_0} \right)^{\frac{\varepsilon}{\delta}} \left(\frac{\delta - \varepsilon}{\delta} \right) \right]^{\frac{\delta}{\delta - \epsilon}}$$

The "green incentives infeasibility zone" depends on the capital depreciation rate, the GHG dissipation rate, initial GHG concentration and initial brown capacities.