Consistency of Higher Order Risk Preferences

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Abstract:

Risk aversion (a 2nd order risk preference) is a time-proven concept in economic models of choice under risk. More recently, the higher order risk preferences of prudence (3rd order) and temperance (4th order) also have been shown to be quite important. While a majority of the population seems to exhibit both risk aversion and these higher-order risk preferences, a significant minority does not. We show how both risk-averse and risk-loving behaviors might be generated by a simple type of basic lottery preference for either (1) combining "good" outcomes with "bad" ones, or (2) combining "good with good" and "bad with bad" respectively. We further show that this dichotomy is fairly robust at explaining higher order risk attitudes in the laboratory. In addition to our own experimental evidence, we take a second look at the extant laboratory experiments that measure higher order risk preferences and we find a fair amount of support for this dichotomy. Our own experiment also is the first to look beyond 4th order risk preferences and we examine risk attitudes at even higher orders. The consistency of these results with expected utility theory and with a few non-expected utility theories is also examined.

Keywords: risk apportionment, mixed risk aversion, mixed risk loving, prudence, temperance, edginess, laboratory experiments, moment preference, prospect theory
JEL Codes: C9, D8

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1. INTRODUCTION

The risk attitude of an economic agent has long been expressed as simply being risk averse or risk loving (or neither). How we characterize risk aversion can depend upon model specifics, but typically is consistent with an aversion to mean-preserving spreads. Of course, any measures of the intensity of risk aversion truly are model-specific, such as the widely-used utility-based measures of Arrow (1965) and Pratt (1964). In a similar vein, an individual's (or corporation's) "risk profile" typically is just a metric of how much risk an agent is willing to take, as if "risk" were some kind of homogeneous mass. All of these notions deal with only so-called "second-order effects." But risk comes in many forms.

Recent attention has been given to the fact that one's behavior towards risk depends upon more than just risk aversion. The early expected-utility models of precautionary saving by Leland (1968), Sandmo (1970) and Dréze and Modigliani (1972), which were later re-analyzed by Kimball (1990), all showed how the attribute of "prudence" (a 3rd order effect) can affect such decision making. Even more recently, temperance (a 4th order effect) has become integrated into decision models (see, for example, Gollier (2001)).[1]

Across a wide array of settings a majority of people have been found to exhibit risk aversion; but the minority who are risk loving often only receive passing attention. Except for the occasional attempt to explain risk-loving behavior, an abundance of papers simply include an assumption of risk aversion. Although higher order risk attitudes are less-well understood, researchers have mostly found evidence for prudence and, to a lesser degree, for temperance as well. But again, individuals who do not follow the majority are largely ignored.

Beyond the 4th order, not much at all has been done to examine how these preferences affect decision making. Whether or not theoretical research in this direction is even warranted might depend in part on whether such attitudes are empirically relevant. This paper helps to answer the latter question by examining both 5th and 6th order attitudes in a laboratory setting in addition to reexamining lower order attitudes.

Our motivation in this paper is to examine another framework for viewing risk behavior. In particular, Eeckhoudt, Schlesinger and Tsetlin (2009) propose a method for viewing aversion to higher degree risks as a type of lottery preference for combining relatively good outcomes with relatively bad outcomes; as opposed to the alternative of combining "good with good" and "bad with bad." The particulars of such lottery preference are spelled out below, but Eeckhoudt et al. (2009) pay no real attention to risk lovers.

A recent paper by Crainich, Eeckhoudt and Trannoy (2013) attempts to remedy this situation by examining risk lovers. In particular, they apply the analysis from Eeckhoudt et al. (2009), but

[1] The paper by Noussair et al. (2013) provides a good summary of the many ways prudence, and to a lesser extent temperance, has been applied to many types of economic problems, such as auctions, bargaining, ecological discounting, precautionary saving and rent-seeking contests.

with the assumption that risk lovers prefer to combine “good with good” and “bad with bad.” In particular, Crainich et al. (2013) apply their analysis to 3rd and 4th order risk attitudes to show that risk lovers also *can be* both prudent and intemperate. However, as Ebert (2013) comments, neither prudence nor intemperance *needs* to follow from risk-loving behavior.

Whether or not risk lovers actually do tend to exhibit this type of mixed-risk-loving behavior would seem to be an empirical question. Although Crainich et al. (2013) restrict their theoretical analysis to expected utility, there is no compelling argument to do so, as we explain below. Moreover, although their theoretical analysis only goes up to 4th order risk attitudes, the analysis can easily be extended to risk attitudes of any arbitrary order *n*.

This paper both generalizes the theoretical underpinnings of Crainich et al. (2013) and tests the results using controlled experiments. In the laboratory, respondents stated their preferences for 38 pairs of lotteries. The lotteries were designed to test for risk attitudes (a.k.a. “risk preferences”) of orders 2-6.[2] In this manner we were able to see evidence to support a hypothesis that can be derived from Eeckhoudt et al. (2009) and Crainich et al. (2013). Namely, that lottery preference for either combining “good with bad” or for combining “good with good” more basically describes risk-averse behavior or risk-loving behavior respectively. In particular, we find two distinct patterns of behavior:

> *Risk averters are “mixed risk averse:” they dislike an increase in risk for every degree n*
>
> *Risk lovers are “mixed risk loving:” they like risk increases of even degrees, but dislike increases of odd degrees*

Thus, both risk averters and risk lovers agree on their risk attitudes of odd orders, such as for 3rd order prudence. But risk averters and risk lovers disagree on their risk attitudes of even orders. For example, at the 4th order, risk averters are temperate but risk lovers are intemperate. Mimicking the terminology of Caballé and Pomansky (1995), who use “mixed risk aversion” for the first pattern of behavior within the confines of expected utility, Crainich et al. (2013) use the terminology “mixed risk loving” to characterize the second pattern of behavior. It also should be noted that, although Caballé and Pomansky (1995) characterize mixed risk aversion in terms of utility functions, they never address the question of whether or not this trait is commonly exhibited by risk averse individuals.

Our evidence provides a fair amount of support for the above hypothesis of two distinct patterns of behavior. Since risk lovers seem to follow this consistent pattern (mixed risk loving), it might help to explain their behavior in situations where risk loving alone does not appear to be sufficient. For example, Golec and Tamarkin (1998) find that betting on “long shots” in a horse

[2] We also test preferences being *monotonic* in wealth, which is a first-order risk attitude.

race is consistent with both risk-averse and risk-loving behavior, both of which can be consistent with a preference for less third-degree risk.[3]

In our experiment, risk aversion, prudence and temperance seem to be the more frequent risk attitudes for orders 2-4. This evidence agrees with the handful of experimental evidence to date (Tarazona-Gomez (2004), Ebert and Wiesen (2011), Ebert and Wiesen (2012), Noussair et al. (2013) and Maier and Rüger (2012)). Only one paper to date, Deck and Schlesinger (2010), shows some other trait (intemperance) to be more prevalent, but only modestly so.

To the best of our knowledge, our paper is also the first to make any experimental attempt at determining risk attitudes for orders higher than 4. Our results for 5^{th} order attitudes also support the above-mentioned patterns, although the support is weaker. Likewise, 6^{th} order attitudes are weakly consistent, but behavior at this order is only marginally different from making random choices. Thus, although we can theoretically consider risk preferences for any arbitrary order *n*, restricting any analyses within economic applications to only the first four orders seems a reasonable approximation. We attribute this phenomenon to the ever increasing complexity involved with deciphering higher degrees of risk increases.

The theoretical model that we set up describes preferences over particular 50-50 lotteries pairs. This simple approach stems from the earlier work of Eeckhoudt and Schlesinger (2006), as adapted by Eeckhoudt et al. (2009). Although not constrained to expected-utility theory, we show how our results are consistent with expected utility models: both for risk averters and for the less-examined case of risk lovers.[4] We also show how the evidence can be used to support (or not support) other preference models as well, such as moment preference, rank-dependent expected utility (Quiggin (1982)) and cumulative prospect theory (Tversky and Kahneman (1992)).

We start in the next section by introducing the basic theoretical lottery-preference framework for risk attitudes of orders 2-4: risk aversion, prudence and temperance; and we next extend the analysis to any arbitrary order *n*, paying particular attention to the 5^{th} and 6^{th} orders. Since expected utility is still quite prevalent in much of the literature, especially the literature with applications of higher order risk attitudes, we next explain the theory of how each of the different order risk attitudes works within an expected utility framework. The following two sections present our experimental design and our experimental results, which are shown to add support to the hypothesis of two dichotomous behavior patterns. Finally, we discuss the consistency of our experimental results with both expected utility and with a few non-expected utility models of choice behavior, as well as add a few closing remarks.

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[3] Golec and Tamarkin (1998) use moment-based preferences, which are consistent with our results, as we explain later in the paper.

[4] A look at the seminal papers by Pratt (1964) and Arrow (1965), for example, show typical detailed analyses of risk-averse behaviors, but no regard for how the many theorems and other results might apply to risk lovers. Some extensions are relatively trivial, but others can be quite perplexing.

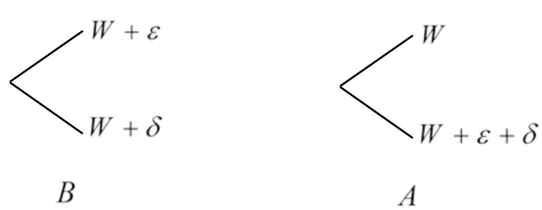
2. RISK AVERSION, PRUDENCE AND TEMPERANCE

Eeckhoudt and Schlesinger (2006) and Eeckhoudt et al. (2009) introduced a canonical method for classifying risk attitudes, based upon a simple set of lottery preferences. Here we present a brief summary of these risk attitudes, starting with the well-known second-order attitude of risk aversion. We assume throughout that all individuals prefer more wealth to less. Since only binary lotteries with equal probabilities are considered in this paper, we will write $[x, y]$ to denote a lottery with a 50-50 chance of receiving either outcome x or outcome y, where it is understood that both x and/or y might themselves be lotteries.

<u>*Risk aversion* (*2nd order risk apportionment*)</u>

Consider an individual with an initial wealth $W > 0$.[5] Let $k_1 > 0$ and $k_2 > 0$. Consider the two 50-50 lotteries $A_2 \equiv [W, W - k_1 - k_2]$ and $B_2 \equiv [W - k_1, W - k_2]$. To avoid bankruptcy issues, we assume that all variables are defined so as to maintain a strictly positive total wealth. An individual is *risk averse* if and only if lottery B_2 is preferred to lottery A_2 for all possible values of W, k_1 and k_2. The reader can easily verify that the characterization above coincides with a dislike for mean-preserving spreads (see Rothschild and Stiglitz (1970)), as well as with a concave utility function. Eeckhoudt and Schlesinger (2006) describe the preference for B_2 over A_2 as a preference for "disaggregating the harms." The "harms" are the losses of k_1 and k_2. Since they extend this idea to higher-order preferences, they generically label this second-order risk attitude as "risk apportionment of order 2." We note here that a *risk lover* would have exactly the opposite lottery preference. In other words, someone who always prefers the lottery A_2 over lottery B_2 is a risk lover.

This preference is shown graphically as a probability tree in Figure 1, where the branches each have a probability of $p = \frac{1}{2}$. In Figure 1, we let ε and δ represent two generic "harms." Here, each harm is the loss of a fixed amount of money $\varepsilon \equiv -k_1$ or $\delta \equiv -k_2$.



<u>Figure 1</u>: Risk apportionment as lottery preference

[5] We note here that initial wealth can also be random, so long as it is statistically independent of any random additions to wealth. This is in the exact same spirit as Pratt and Zeckhauser (1987). To keep the story somewhat simpler, we assume that initial wealth is an arbitrary, but fixed, constant.

Prudence (3^{rd} order risk apportionment)

To define the third-order risk attitude of *prudence*, Eeckhoudt and Schlesinger (2006) replace one of the "harms" of a sure loss with a zero-mean random variable. Let $\varepsilon \equiv \tilde{\varepsilon}$ now be any zero-mean random variable.[6] In other words, replace the "harm" $\varepsilon \equiv -k_1$ with the "harm" $\varepsilon \equiv \tilde{\varepsilon}$. Define $A_3 \equiv [W, W + \tilde{\varepsilon} - k_2]$ and $B_3 \equiv [W + \tilde{\varepsilon}, W - k_2]$. Prudence is defined as a preference for lottery B_3 over lottery A_3 for every arbitrary W, $k_2 > 0$ and zero-mean $\tilde{\varepsilon}$. This lottery preference is equivalent to a convex marginal utility in expected-utility models, $u''' > 0$. In addition this same lottery preference for B_3 is a preference for decreases in downside risk, as defined by Menezes, Geiss and Tressler (1980), which is itself equivalent to a preference for a decrease in 3^{rd} degree risk, as defined by Ekern (1980). An individual who has the opposite preference, who always prefers A_3 to B_3, is classified as *imprudent*.

Temperance (4^{th} order risk apportionment)

To define the fourth-order risk attitude of *temperance*, now replace the sure loss of $-k_2$ with a second zero-mean risk $\tilde{\delta}$, where the distribution of $\tilde{\delta}$ is assumed to be statistically independent to that of $\tilde{\varepsilon}$.[7] Someone who is temperate will always prefer lottery $B_4 \equiv [W + \tilde{\varepsilon}, W + \tilde{\delta}]$ to lottery $A_4 \equiv [W, W + \tilde{\varepsilon} + \tilde{\delta}]$, whereas someone who is intemperate will always prefer A_4 to B_4. For a risk averter, zero is preferred to either $\tilde{\varepsilon}$ or $\tilde{\delta}$. Thus, Eeckhoudt and Schlesinger (2006) describe temperate behavior as a preference for "disaggregating the harms." Alternatively, suppose that the risk $\tilde{\varepsilon}$ already appears in one state of nature. A temperate individual would prefer to receive an unavoidable second risk $\tilde{\delta}$ in the state of nature where there is no risk, as opposed to the same state of nature with $\tilde{\varepsilon}$. Kimball (1993) refers to the two harms, $\tilde{\varepsilon}$ and $\tilde{\delta}$, in this setting as being "mutually aggravating."

3. HIGHER-ORDER RISK ATTITUDES

Here we present a more general approach that derives from Eeckhoudt et al. (2009). Consider the pair of random variables $\{\tilde{X}_1, \tilde{Y}_1\}$. We assume that the random variable $\tilde{Y}_1$ has more n^{th} degree risk than $\tilde{X}_1$.[8] Also consider a second pair of random variables $\{\tilde{X}_2, \tilde{Y}_2\}$ and assume that

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[6] The terminology "prudence" is due to Kimball (1990), who examined precautionary effects within an expected-utility framework.

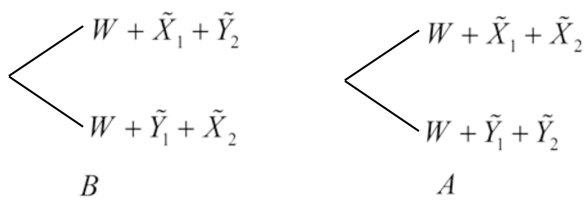
[7] The terminology "temperance," to the best of our knowledge, was first coined by Kimball (1992), and its usefulness in analyzing background risks was examined by Gollier and Pratt (1996) and by Eeckhoudt et al. (1996).

[8] Random variable $\tilde{Y}$ is said to have more first-degree risk than $\tilde{X}$ if $\tilde{X}$ dominates $\tilde{Y}$ via first-order stochastic dominance. $\tilde{Y}$ is said to have more n^{th}-degree risk than $\tilde{X}$, $n > 1$, if $\tilde{X}$ dominates $\tilde{Y}$ via n^{th}-order stochastic dominance and these random variables have the same first n-1 moments. See Ekern's (1980). An increase in 2^{nd}-

the random variable $\tilde{Y}_2$ has more m^{th} degree risk than $\tilde{X}_2$. We also assume that all of the above random variables are statistically independent of one another. The main result in Eeckhoudt et al. (2009) is the following:

Theorem (Eeckhoudt, Schlesinger and Tsetlin, 2009): *Given* $\{\tilde{X}_1, \tilde{Y}_1\}$ *and* $\{\tilde{X}_2, \tilde{Y}_2\}$ *as described above, the 50-50 lottery* $[W+\tilde{X}_1+\tilde{X}_2, W+\tilde{Y}_1+\tilde{Y}_2]$ *has more* $(m{+}n)^{th}$ *degree risk than the lottery* $[W+\tilde{X}_1+\tilde{Y}_2, W+\tilde{Y}_1+\tilde{X}_2]$.

For someone who is risk apportionate of every order, all of the $\tilde{X}_i$ random variables are relatively "good" and all of the $\tilde{Y}_i$ random variables are relatively "bad." Hence, this individual is someone who prefers "combining good with bad." For differentiable expected utility, "risk apportionate of every order" is equivalent to "mixed risk aversion," as defined by Caballé and Pomansky (1995), a point not made explicitly in Eeckhoudt et al. (2009)



<u>Figure 2</u>: Risk apportionment as combining "good" with "bad"

Risk apportionate of order m+n is illustrated by always having a preference for lottery B over lottery A when considering the two lotteries shown in Figure 2, as described in the above Theorem.

Examples:

<u>Risk aversion</u>: Set $n = m = 1$, and define $\tilde{X}_1 = \tilde{X}_2 = 0$, $\tilde{Y}_1 = -k_1$ and $\tilde{Y}_2 = -k_2$.

<u>Prudence</u>: Set $n = 2$, $m = 1$, and define $\tilde{X}_1 = \tilde{X}_2 = 0$, $\tilde{Y}_1 = \tilde{\varepsilon}$ and $\tilde{Y}_2 = -k_2$.

<u>Temperance</u>: Set $n = 2$, $m = 2$, and define $\tilde{X}_1 = \tilde{X}_2 = 0$, $\tilde{Y}_1 = \tilde{\varepsilon}$ and $\tilde{Y}_2 = \tilde{\delta}$.

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degree risk also was analyzed by Rothschild and Stiglitz (1970), who referred to it as a "mean-preserving increase in risk."

The above examples show how our earlier descriptions can all be illustrated as particular applications of the above Theorem. Moreover, the Theorem allows us to provide alternative characterizations. For example, we can set n = 1, m = 3, and define $\tilde{X}_1 = 0$, $\tilde{Y}_1 = -k_1$, $\tilde{X}_2 = \tilde{\theta}$ and $\tilde{Y}_2 = \tilde{\delta}$, where $\tilde{\delta}$ has more 3rd degree risk than $\tilde{\theta}$, which provides an alternative characterization of temperance. [9]

The fifth-order attitude of *edginess*[10] can be characterized by setting $\tilde{X}_1 = 0$, $\tilde{Y}_1 = -k_1$, $\tilde{X}_2 = \tilde{\theta}$ and $\tilde{Y}_2 = \tilde{\delta}$, where $\tilde{\delta}$ has more 4th degree risk than $\tilde{\theta}$; or by setting $\tilde{X}_1 = \tilde{\varepsilon}_1$, $\tilde{Y}_1 = \tilde{\varepsilon}_2$, $\tilde{X}_2 = \tilde{\theta}_1$ and $\tilde{Y}_2 = \tilde{\theta}_2$, where $\tilde{\varepsilon}_2$ has more 2nd degree risk than $\tilde{\varepsilon}_1$ and $\tilde{\theta}_2$ has more 3rd degree risk than $\tilde{\theta}_1$.

To obtain *risk apportionment of order 6*, we can once again apply the Theorem and Figure 2 and choose any positive integers n and m with $n+m$ = 6. This gives us three different ways to construct lotteries characterizing risk apportionment of order 6 (with $n+m$ equaling either 1+5, 2+4 or 3+3). Risk apportionment of order 6 follows from a preference for lottery *B* over *A* in any of the above settings. The opposite preference (for *A* over *B*) will be called *anti-risk apportionment of order 6*. Of course, we need not stop at 6 and the Theorem can be used to define any arbitrarily high order of risk apportionment.

4. MIXED RISK LOVERS vs. MIXED RISK AVERTERS

Recall that mixed risk aversion can be described as preference for combining good with bad. From our previous analysis and from an inspection of the Theorem, it follows easily that risk apportionment of order n is consistent with this preference for combining good with bad. To the extent that combining good with bad is an inherent trait of risk preferences, the risk averse individual will also be prudent, temperate, edgy and satisfy risk apportionment of order 6.

On the other hand, mixed risk loving behavior shows a preference for combining good with good or alternatively combining bad with bad. Using the Theorem, it follows that someone who is mixed risk loving -- that is, who always prefers combining good with good -- will satisfy risk apportionment of order n for all n that are odd (e.g. prudence and edginess), but will satisfy anti-risk apportionment of order n for all n that are even (e.g. risk loving, intemperance, anti-risk apportionment of order 6).

Eeckhoudt and Schlesinger (2006) show that risk apportionment of order n holds for an individual with EUT preferences if and only if $\operatorname{sgn} u^{(n)}(t) = (-1)^{n+1}$, where the notation $u^{(n)}$

[9] Our terminology derives from Ekern (1980). An increase in 2nd-degree risk also was analyzed much earlier by Rothschild and Stiglitz (1970), who referred to it as a "mean-preserving increase in risk."

[10] The terminology "edginess" is from Lajeri-Chaherli (2004), who uses this property to examine whether or not the trait of prudence is maintained in the presence of an independent background risk.

denotes the n^{th} derivative of the utility function u.[11] If the above condition holds for all n, marginal utility is said to be completely monotone.[12] Note that most commonly used utility functions, such as those exhibiting constant absolute risk aversion (CARA) and those exhibiting constant relative risk aversion (CRRA), satisfy this condition. As one example of a fairly common utility function without this property, consider the quadratic utility function $u(t) = t - \beta t^2$, with the assumption that $t < (2\beta)^{-1}$, so that u is everywhere increasing. This utility exhibits risk apportionment of orders 1 and 2 only. For higher orders, such an individual is indifferent between lotteries B and A

Risk lovers need not be mixed risk lovers, as pointed out by Ebert (2013). Likewise, Caballé and Pomansky (1995) do not address the issue of whether or not risk averters are also mixed risk averse. These are empirical issues that we address in our experiment. But, if a risk loving individual is indeed mixed risk loving, her utility will satisfy the property that $u^{(n)}(t) > 1, \forall n$.

Our characterizations of mixed risk aversion and mixed risk loving can be summarized in Table 1. Recall that we assume that everyone prefers more wealth to less, $u' > 0$.

<u>Table 1</u>: Projected higher order risk attitudes

MIXED RISK AVERSE	**MIXED RISK LOVING**
<u>Prefer combining good with bad</u>	**<u>Prefer combining good with good</u>**
Risk averse $(u'' < 0)$	*Risk loving* $(u'' > 0)$
Prudent $(u''' > 0)$	*Prudent* $(u''' > 0)$
Temperate $(u^{(4)} < 0)$	*Intemperate* $(u^{(4)} > 0)$
Edgy $(u^{(5)} > 0)$	*Edgy* $(u^{(5)} > 0)$
Risk apportionate of order 6 $(u^{(6)} < 0)$	*Anti-risk apportionate of order 6* $(u^{(6)} > 0)$

5. EXPERIMENTAL DESIGN

A total of 150 participants were recruited from the University of Arkansas Behavioral Business Research Laboratory's database of volunteers.[13] Subjects were recruited for a 45 minute session and received a $5 participation payment in addition to their salient earnings, which averaged $20.92 (minimum was $1 and maximum was $92).

Upon entering the laboratory, participants were seated at a computer terminal that was visually isolated from the other participants. Participants proceeded to read computerized directions and

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[11] Of course, since we assume that more wealth is desirable, we also have $u' > 0$. For 2nd and 3rd derivatives, we will also use the more common notations u'' and u'''.

[12] See Pratt and Zeckhauser (1987) for other economic significances of this property.

[13] The majority of the people in the database are undergraduates in the business school, but some are undergraduates in other colleges and others are not undergraduate students. None of the participants recruited for this study had participated in any previous related study.

answer a series of comprehension questions, both of which are included in the Appendix. After any remaining questions were answered, the participant began making the 38 choice tasks that comprised the experiment.

The choice tasks that the participants encountered are shown in Table 2. Each task involved a binary comparison of fixed amounts of money and 50-50 lotteries. The 50-50 lotteries were presented to the subjects as circles divided in half with a vertical line to represent that each outcome was equally likely, as shown in Figure 3 below. The payoffs for each 50-50 lottery were shown in the corresponding half of the circle and were some combination of cash amounts and additional 50-50 lotteries. This technique is intended to facilitate participant understanding. This presentation admittedly also facilitates viewing the problem as "combining good with bad" or "combining good with good," rather than presenting the lotteries in a reduced form, which might obfuscate this interpretation.[14]

Table 2: Choice Tasks

Task	Order	Construction	Option A	Option B
1	1	-	$20	$20 + $10
2	1	-	$2	$2 + $5
3	1	-	[$2 + [$10, $20], $20]	[$25, $27 + [$-1, $1]]
4	2	-	[$5, $10 + $5]	[$5 + $5, $10]
5	2	-	[$2, $4 + $8]	[$2 + $8, $4]
6	2	-	[$10, $15 + $5]	[$10 + $5, $15]
7	2	-	[$2, $4 + $3]	[$2 + $3, $4]
8	2	-	[$20, $40 + $30]	[$20 + $30, $40]
9	2	-	[$4, $10]	$7
10	2	-	[$1 | $19]	$10
11	3	-	[$5 + [$-2, $2], $10]	[$5, $10 + [$-2, $2]]
12*	3	-	[$10 + [$-4, $4], $20]	[$10, $20 + [$-4, $4]]
13	3	-	[$5 + [$-4, $4], $10]	[$5, $10 + [$-4, $4]]
14	3	-	[$2 + [$1, $-1], $4]	[$2, $4 + [$1, $-1]]
15	3	-	[$20 + [$10, $-10], $40]	[$20, $40 + [$10, $-10]]
16	3	-	[$8 + [$2, $-2], $10]	[$8, $10 + [$2, $-2]]
17	3	-	[$12 + [$1, $-1], $14]	[$12, $14 + [$1, $-1]]
18	4	2+2	[[$14, $20]+[$14, $20], [$10, $24]+[$10, $24]]	[[$10, $24]+[$14, $20], [$14, $20]+[$10, $24]]

[14] The paper by Maier and Rüger (2012) does just the opposite and present lotteries for risk attitudes of orders 2-4 only in a reduced form. Their basic results do not differ from the other experiments to date, each of which presents the choices as compound lotteries. Only the "intemperance" result of Deck and Schlesinger (2010) appears to be an outlier, which could potentially (since they do not account for risk aversion) stem from a disproportionate number of risk lovers in their sample.

19	4	2+2	[[$7, $10]+[$7, $10], [$5, $12]+[$5, $12]]	[[$5, $12]+[$7, $10)], [$7, $10]+[$5, $12]]
20	4	2+2	[8B + 7B, 8A + 7A]	[8A + 7B, 8B + 7A]
21	4	2+2	[[$1, $16]+[$1, $16], [$5, $12]+[$5, $12]]	[[$5, $12]+[$1, $16], [$1, $16]+[$5, $12]]
22	4	1+3	[$14 + 12A, $24 + 12B]	[$14 + 12B, $24 + 12A]
23	4	1+3	[$7 + 11A, $12 + 11B]	[$7 + 11 B, $12 + 11A]
24	4	1+3	[$1 + 11A, $18 + 11B]	[$1 + 11B, $18 + 11A]
25	5	2+3	[[$7, $10] + 11B, [$5, $12] + 11A]	[[$7, $10] + 11A, [$5, $12] + 11B]
26	5	2+3	[[$10, $4] + 12B, [$2, $12] + 12A]	[[$10, $4] + 12A, [$2, $12] + 12B]
27	5	2+3	[[$50, $40] + 11B, [$20, $70] + 11A]	[[$50, $40] + 11A, [$20, $70] + 11B]
28	5	2+3	[[$5, $12]+[$5, $10+[$-2, $2], [$1, $16]+ [$5+[$-2, $2], $10]]	[[$5, $12]+ [$5+[$-2, $2], $10], [$1, $16]+ [$5, $10+[$-2, $2]]
29	5	1+4	[$5 + 19A, $7 + 19B]	[$5 + 19B, $7 + 19A]
30	5	1+4	[$1+[[$10, $4]+[$7, $10], [$2, $12]+[$5, $12]], $4+[[$2, $12]+[$7, $10], [$10, $4]+[$5,$12]]]	[$1+[[$2, $12]+[$7, $10], [$10, $4]+[$5,$12]], $4+[[$10, $4]+[$7, $10], [$2, $12]+[$5, $12]]]
31	5	1+4	[$1 + 20A, $20 + 20B]	[$1 + 20B, $20 + 20A]
32	6	3+3	[11A + 11A, 11B + 11B]	[11A + 11B, 11B + 11A]
33	6	3+3	[11A + 12A, 11B + 12B]	[11B + 12A, 11A + 12B]
34	6	3+3	[12A + 14A, 12B + 14B]	[12A + 14B, 12B + 14A]
35	6	3+3	16A + 16A, 16B + 16B]	16A + 16A, 16B + 16B]
36	6	2+4	[[$8, $12] + 19B, [$5, $15] + 19A]	[[$5, $15] + 19B, [$8, $12] + 19A]
37	6	2+4	[[$8, $12] + [[$2, $12] + [$7,$10], [$10,$4] + [$5, $12]], [$5, $15] + [[$10, $4] + [$7,$10], [$2,$12] + [$5, $12]]	[[$5, $15] + [[$2, $12] + [$7,$10], [$10,$4] + [$5, $12]], [$8, $12] + [[$10, $4] + [$7,$10], [$2,$12] + [$5, $12]]
38	6	2+4	[[$2, $4] + 20B, [$5, $1] + 20A]	[[$5, $1] + 20B, [$2, $4] + 20A]

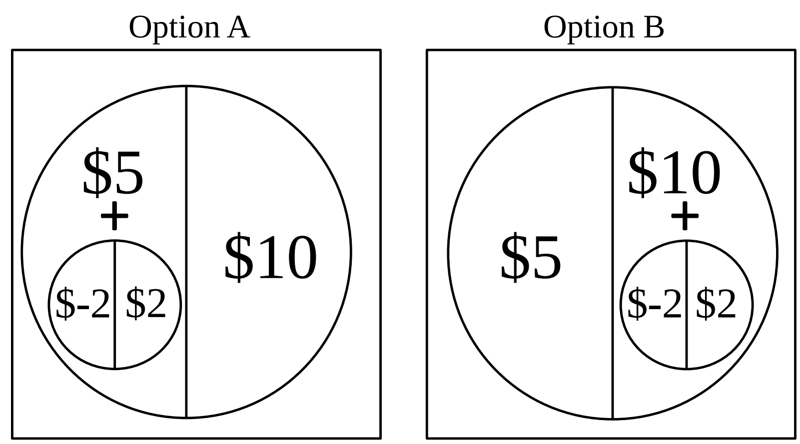
In this table [X,Y] denotes a lottery where there is a 50% chance of receiving X and a 50% chance of receiving Y. “Task” is simply our internal task reference number and table entries of the form #A and #B denote the content of Option A and Option B, respectively for Task #. “Order” refers to the risk-order being tested. “Construction” refers to the *m* and *n* chosen for decomposing $(m+n)^{th}$ order risk, as in section 3 above. Task 12 is marked with an * because the graphic files used in the experiment had an error for Task 12 resulting in the subjects observing two identical choices. Therefore, Task 12 is excluded from all analysis.

The first 3 tasks are designed to verify that participants understand the task under our assumption that they prefer more money to less. For example, Option B of Task 3 is a 50-50 lottery where one would receive either $25 or $27 plus or minus $1 with equal chance. Because the best outcome from Option A is less than the worst outcome from Option B, monotonicity alone is sufficient for one to prefer Option B to Option A. The remaining tasks measure risk apportionment of orders 2 – 6. Figure 3 shows some of the higher order choice tasks as they were presented to the subjects. While all the subjects observed the same tasks in a within subjects design, the order of the tasks was randomized for each person.[15] Which option was listed on the left was also randomized and whatever option was listed on the left was labeled as "Option A" for the participant. In our descriptions here, the label Option B is always used in a manner consistent with the previous part of the paper, i.e. the preferred choice of a mixed risk averter. The preferred choice of a mixed risk lover varies. For example, in Task 11 (see Figure 3), we see that mixed risk lover also prefers Option B, since it mixes the "good" zero-mean lottery [-2,+2] with the good outcome of $10.

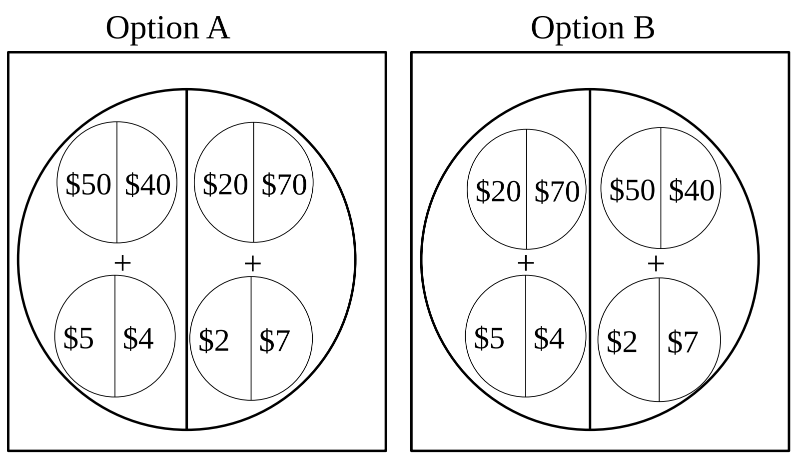
After the participant completed all the choice tasks, one was randomly selected and the participant was paid based upon their choice for that task. This procedure was done to eliminate potential wealth effects that might lead participants to change their behavior over the course of the study if earnings were cumulative.[16] The experimenter approached the participant with a physical spinner to determine the outcome of each lottery. The spinner is a device found in many children's games and available at most educational supply stores. It consists of a metal arrow attached to the center of a square piece of plastic. The arrow is attached in such a way that it will freely move in a circle when pushed. On the plastic was a drawing of a large circle with the diameter shown, similar to the images shown in Figure 3. Note that more complicated tasks required multiple spins. For example, Task 20 requires 4 spins. Participants were allowed to perform the spin themselves so long as the arrow "goes around several times before stopping." Once the payment amount was determined, the experimenter recorded the payoff and the participant's sex, paid the participant, and dismissed him or her from the lab.

[15] In other words, each of the 150 participants had a randomized ordering of the 38 tasks. Task order was not strongly correlated with behavior for any task.

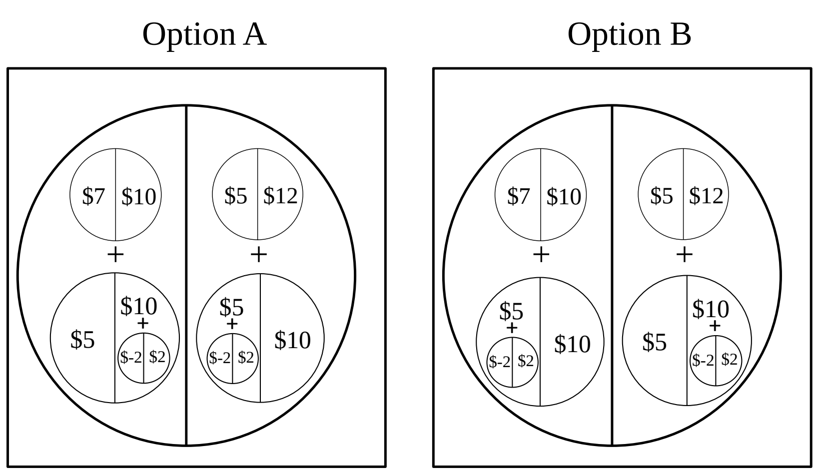
[16] We realize that this method of payment itself, although often used, is still debated as it relies on the independence assumption. See for example Starmer and Sugden (1991) and Laury (2005), as well as the essay by Peter Wakker at http://people.few.eur.nl/wakker/miscella/debates/randomlinc.htm.



Task 11, a 3rd Order Task



Task 20, a 4th Order (2+2) Task



Task 25, a 5th Order (2+3) Task

Figure 3: Sample Tasks as Presented to Participants

6. EXPERIMENTAL RESULTS

The results are presented in two parts. First, we look at aggregate behavior by task order; i.e. aggregate behavior for all of the tasks associated with a specific order of risk preference. Second we look at individual behavior across task orders.

6.1 *Aggregate Behavior*

As all participants faced multiple tasks for each order of risk preference, we can count the number of times a participant selected Option A in each order. Figure 4 shows the distribution of the number of Option A choices participants made for 1st – 6th order tasks. The solid line indicates the frequency with which a given number of A choices would be expected to occur if each participant made a random choice on each task.

Based on the data summarized in Figure 4, we conclude that participants understand the experiment interface and prefer more money to less, with over 92% never selecting the lower payoff Option A on 1st order tasks. The observed distribution is statistically different from what would be observed by chance based on a chi-square (p-value < 0.001). Of those few that did select a lower payoff, only one did so more than once. All of our results in this experiment are qualitatively unchanged, if these participants are excluded.

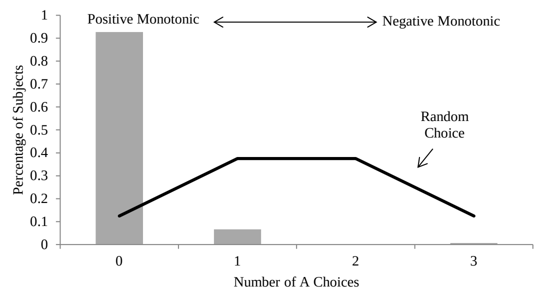
Consistent with the large volume of previous lab experiments, the participants are overwhelmingly risk averse in aggregate as most people made 3 or fewer (out of 7) Option A choices on 2nd order tasks.[17] In fact, a large fraction of the subjects (57%) exhibit fairly strong risk aversion making one or zero Option A choices. Thirty of the 150 participants (20%) were classified as risk loving, i.e. making 4 or more Option B choices. The distribution of 2nd order behavior differs from what would be expected by chance (chi-square test p-value < 0.001).

The aggregate behavior shown in Figure 4 also indicates that the participants were generally prudent, consistent with all of the other previous lab studies to date. The number of prudent choices was more than would occur by chance (chi-square p-value < 0.001). Indeed the strength of prudence as measured by the number of prudent choices seems rather strong here. To the extent that both risk lovers and risk averters would be prudent, this result is to be expected. As discussed in the introduction, most previous research has found respondents to be moderately temperate.[18] Our participants also exhibit modest temperance. The average number of A choices on 4th order tasks was 2.95 out of 7. While a chi square test rejects that 4th order

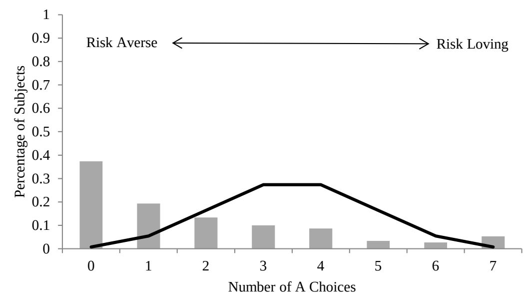
[17] Of course, making even one risk-loving decision might disqualify an individual from being labeled as "risk averse." We will adopt a stochastic type of labeling and refer to someone whose majority of choices is for Option B as being "risk averse." In the next section, we also define a stronger measure, which labels individuals who make either 3 or 4 Option A choices as "risk neutral." See Wilcox (2008) for a good review of these stochastic labels.

[18] We should note that although Ebert and Wiesen (2011) do not claim to test for temperance per se, they show that a more negatively skewed zero-mean lottery in their 3rd order tasks would lead to more lottery *B* choices; but this is precisely the "good with bad" type of temperance preference that we describe for risk preference of order 4, with a 3+1 construction. Thus, their results also indicate temperate behavior in the aggregate.

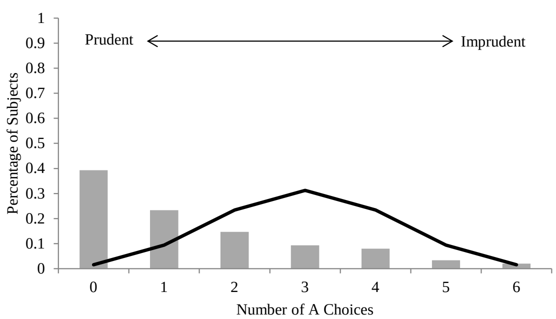
behavior was random (p-value < 0.001), it appears to be the case that too much weight is placed on both tails and too little weight is placed in the center.[19] This is the pattern that would occur, for instance, if some participants were exhibiting clear temperance or intemperance and others were simply randomizing. To the extent that risk lovers (a minority in the population) would be intemperate, we would expect that temperance is exhibited less frequently than prudence. Our result here -- that temperance is less prevalent than prudence -- is consistent with this hypothesis. Noussair, et al. (2013) and Ebert and Wiesen (2012) also show that prudence is exhibited more frequently than temperance.



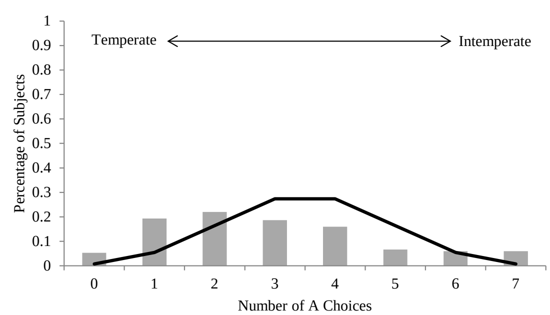
1st Order



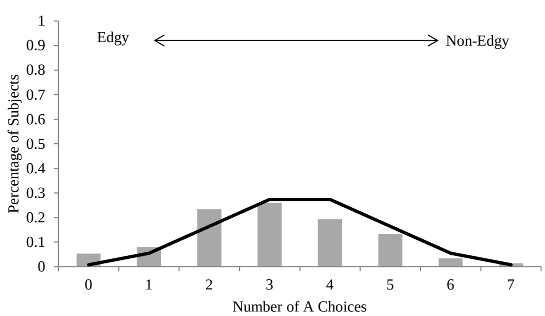
2nd Order



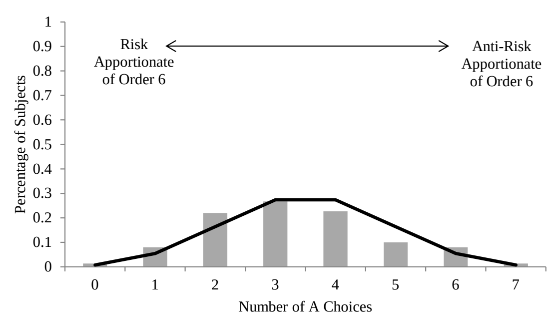
3rd Order



4th Order



5th Order



6th Order

Figure 4: Distributions of Participant Behavior for Each Order

[19] The statistical test can only identify if the distribution differed from the hypothesized form but cannot identify how the distribution differs.

Moving to the 5th order tasks, this pattern of some participants exhibiting clear preferences with others perhaps choosing randomly continues. A chi-square test rejects random behavior (p-value <0.001); but there seems to be no clear preference in the group, only a very slight tendency towards edginess (i.e., towards 5th order risk apportionment.) If risk lovers agree with risk averters about 5th order attitudes, then we would expect most all participants to be edgy. Since 5th order tasks get to be quite a bit more complicated, our results might be interpreted as: many or most subjects choose randomly, but those that have a preference tend toward being edgy. For the 6th order tasks , behavior is only marginally different from random (chi-square p-value = 0.051), although overall a bit more A choices are made. Again, this might be a case where now the complexity is such that most subjects are choosing randomly. If the risk averters who do have a preference are mostly choosing Option B and those who are risk loving mostly choosing Option A, this would again lend some support to Crainich et al. (2013).

In the laboratory, we also recorded the amount of time that subjects took to make each decision. The average time spent on 1st order tasks was 8.3 seconds, clearly sufficient to identify the option with the larger payoff. For orders 2-6, the average number of seconds increased with task order: 9.9, 13.4, 22.9, 27.7 and 30.7 seconds, respectively. While 30.7 seconds may not sound like a long time, it might seem longer if you stop to think about it for, say, a full 30.7 seconds.

That people spent more time on more complicated tasks and behavior still approaches randomness suggests to us that the limit of how deeply people think about uncertainty is limited and that the fifth or sixth order is pushing the upper bound.[20] In fact, Figure 4 shows a gradual evolution from virtual complete agreement to almost a random distribution. Some of the participants asked for and received scratch paper. After the experiment, these subjects indicated that they wanted to calculate the means, at least for the simple problems. While a couple tried to calculate variances, none were found to be calculating higher moments. Of course, they might have also been looking to simplify, if possible, our compound lotteries, which become more complex at higher orders.[21]

Before turning to individual behavior, we briefly report our (lack of) findings regarding gender and behavior. Specifically, we compared male and female behavior for each order. In no case did the behavior differ substantially by sex, although men appear to be nominally more risk

[20] This result could also be interpreted as the compensation was not worth the additional time (i.e. the additional effort). Perhaps even more time would have been used with higher stakes; however the average difference in time between task 7 and task 8, which was for ten times the stakes of task 7, was less than one second. We should also note that people might take more time and effort if 5th and 6th order risk preferences are required real world decisions outside of a laboratory, although we are not aware of any theoretical decisions than depend decisively on such higher-order risk preference, with the possible exception of Lajeri-Chaherli (2004).

[21] Huck and Weizsäcker (1999) examine deviations from maximizing the expected payoff and find that subjects care less about 2nd order risk when the tasks become more complex. This idea seems to also hold for higher orders.

taking than women.[22] Chi-square tests fail to reject that the male and female distributions are the same at the 95% confidence level for each of the six orders. Further, the average percentage of A choices does not differ statistically for males and females on any order at traditional significance levels.

6.2 *Individual Behavior*

As described previously, an individual who is mixed risk averse would be temperate and should pick Option B on 6th order tasks. An individual who is mixed risk loving would be intemperate and pick Option A on 6th order tasks. However, both types of individuals should have monotonic preferences, be prudent (select Option B on 3rd order tasks), and be edgy (select Option B on 5th order tasks). More generally, these two groups should behave similarly on odd numbered tasks and behave differently on even numbered tasks.

To further explore this hypothesis, we examine risk averters and risk lovers separately using two different classification systems. Under the weak classification, a subject making four or more A choices on 2nd order tasks is considered risk loving (30 subjects) while those making three or fewer A choices are considered to be risk averse (120 subjects). Under the strict classification, those making three or four A choices are considered risk neutral (28 subjects) while only those with more extreme behavior are labeled as *strict risk averse* (105 subjects) or *strict risk loving* (17 subjects).

Figures 5 and 6 replicate Figure 4, separating risk-averse participants (shown as white bars) from risk loving participants (shown as black bars) and risk neutral participants (shown as gray bars) for the weak and strict classifications respectively. The patterns revealed in Figures 5 and 6 follow the pattern predicted by Crainich et al (2013) although behavior still appears to become more random as the order increases.

Table 3 reports summarizes statistical tests comparing behavior for each classification with random behavior; behavior of those not classified as risk neutral is highly statistically different from random behavior for orders 3 – 5, but less so for order 6.

Table 4 reports the p-values associated with the chi-square tests that risk averse and risk loving subjects follow the same distribution for each task order versus the alternative that the distributions differ. The results indicate that an alternating pattern is observed. Risk averters and risk lovers are not behaving in the same way on 4th order or 6th order tasks, but are behaving in the same way on 1st, 3rd and 5th order tasks.

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[22] There were 70 females and 80 males in our sample. Males chose the risk-loving Option A 27% of the time on average for the second order tasks whereas women did so 24% of the time. This difference is not significant based upon a two sided t-test (p-value = 0.613).

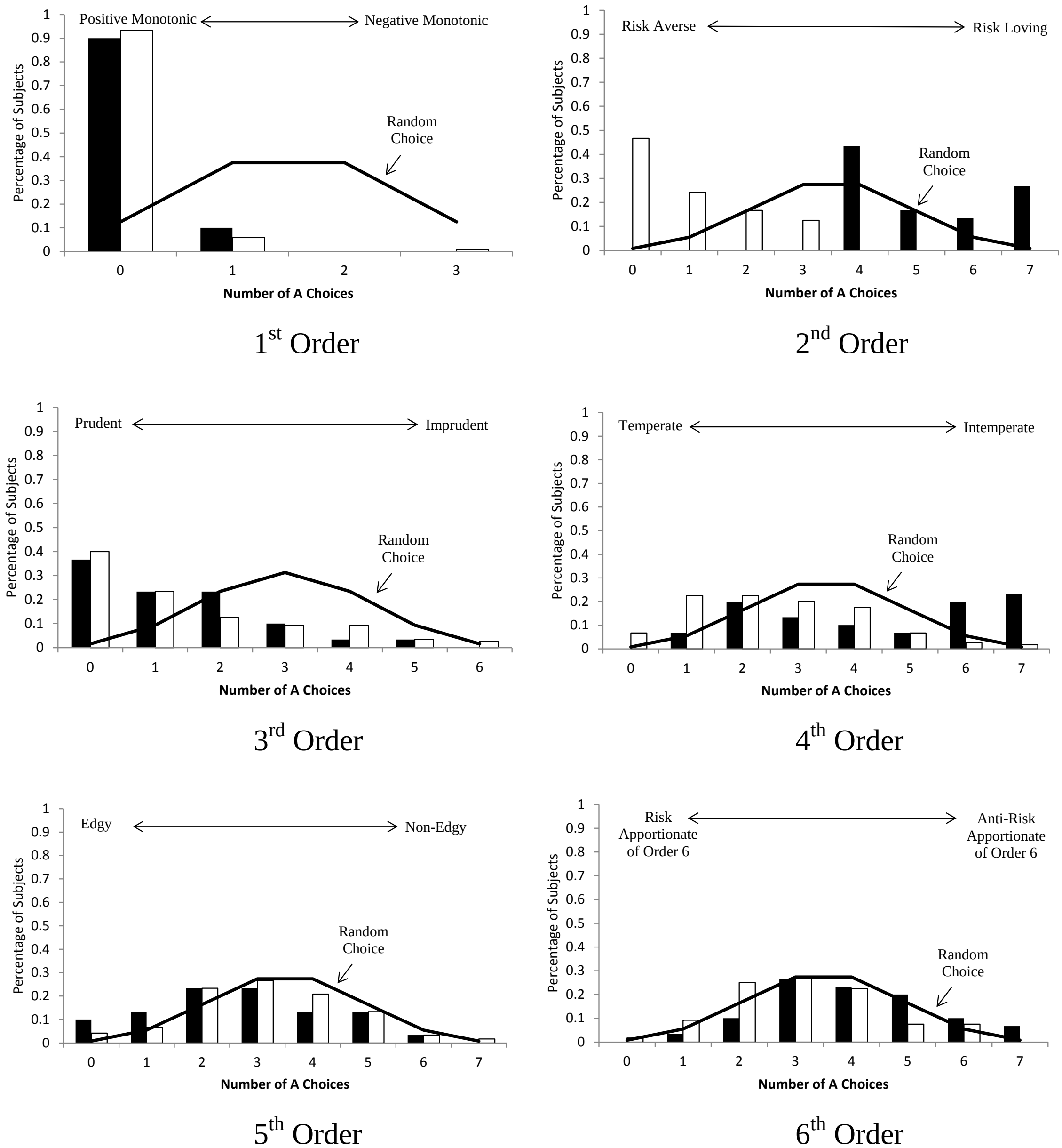
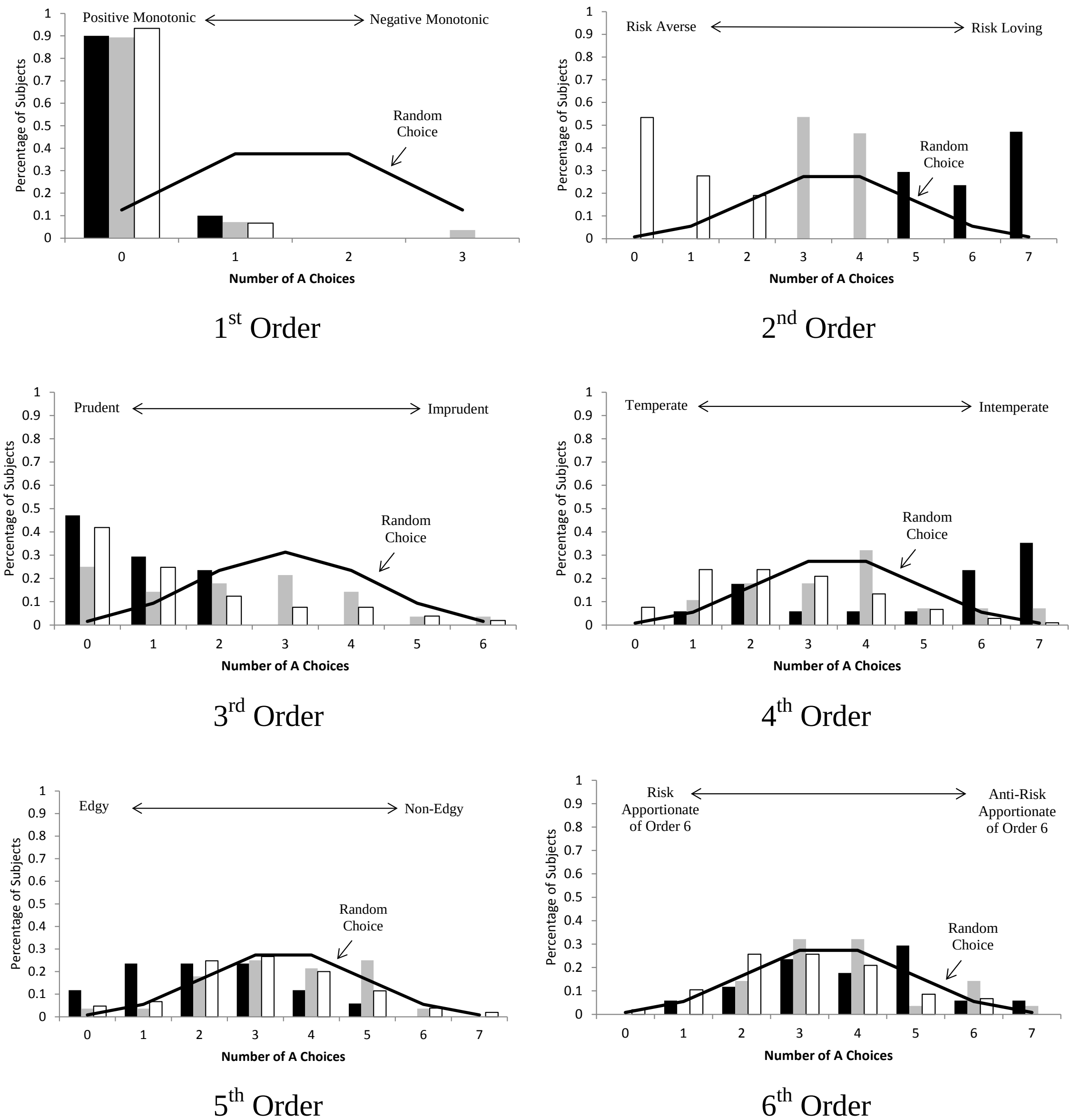


Figure 5: Distributions of Participant Behavior for Each Order by Weak Type [Weak Risk Averse (white) and Weak Risk Loving (black)]



<u>Figure 6</u>: Distributions of Participant Behavior for Each Order by Strict Type [Strict Risk Averse (white), Risk Neutral (gray), and Strict Risk Loving (black)]

Table 3: p-values for Chi-Square Test that Behavior is Random

Order	1st	2nd	3rd	4th	5th	6th
Weak Risk Averse Subjects	<0.001	-	<0.001	<0.001	<0.001	0.011
Weak Risk Loving Subjects	<0.001	-	<0.001	<0.001	<0.001	0.024
Strict Risk Averse Subjects	<0.001	-	<0.001	<0.001	<0.001	0.009
Strict Risk Neutral Subjects	<0.001	-	<0.001	0.008	0.649	0.105
Strict Risk Loving Subjects	<0.001	-	<0.001	<0.001	<0.001	0.294

Table 4: p-values for Chi-Square Test that Risk Loving and Risk Averse Subjects Behave Similarly on Odd Orders and Differently on Even Orders

Order	1st	2nd	3rd	4th	5th	6th
Prediction	Same	-	Same	Different	Same	Different
Weak Classification	0.419	-	0.707	0.003	0.761	0.026
Strict Classification	0.904	-	0.533	<0.001	0.346	0.057

Further evidence for this relationship is found in Table 5, which reports the correlation in individual behavior between tasks of different orders. Specifically, Table 5 gives the correlation between the percentages of times a participant chose Option A on two different selected orders. Given the underlying connection between tasks of different orders, we expect that an individual's choices will be positively correlated between even orders and between odd orders, but uncorrelated between even and odd orders. This is in fact the pattern that is observed.

<u>Table 5</u>: Correlation of Individual Behavior Between Tasks of Different Orders

	% A Choices for 3rd Order	% A Choices for 4th Order	% A Choices for 5th Order	% A Choices for 6th Order
% A Choices for 2nd Order	-0.048	**0.485****	-0.137	**0.259****
% A Choices for 3rd Order	-	0.067	**0.261****	0.131
% A Choices for 4th Order	-	-	0.001	**0.377****
% A Choices for 5th Order	-	-	-	0.049

* and ** indicate significance at the 5% and 1% significance levels, respectively. First order choices are omitted due to the limited variability in behavior.

As a final point, we look at the consistency of individual behavior on higher order tasks with multiple constructions. Fourth order tasks were constructed both as a combination of two 2nd order tasks and a combination of a 1st and 3rd order tasks. Individual behavior was largely consistent between these two constructions (correlation = 0.311, p-value < 0.001). Behavior was also generally consistent between constructions for the more complicated 5th order tasks and 6th order tasks. Fifth order tasks were constructed as a combination of either 1st and 4th orders or of 2nd and 3rd orders (correlation = 0.161 and p-value = 0.024). Sixth order tasks were constructed as a combination of two 3rd order tasks or a 2nd and a 4th order task (correlation = 0.185 and p-value = 0.012). To the extent that such behavior would be inconsistent, it would bring into question the reduction of compound lotteries, but we do not see any strong evidence against reduction of compound lotteries.

7. CONSISTENCY WITH EXPECTED- AND NON-EXPECTED UTILITY BEHAVIORS

<u>Mixing "good with bad" or "good with good" under EUT</u>

Most commonly used utility functions, such as those exhibiting either constant absolute risk aversion or constant relative risk aversion, have derivatives that alternate in sign, which as we have seen is equivalent to having risk apportionment of the various orders (risk aversion, prudence, temperance, etc.).[23] Deck and Schlesinger (2010) point out that we do not often see utility with risk aversion and intemperance (their experimental result). But if we think that Crainich et al. (2013) are correct and that the fundamental behavior of risk lovers is driven by

[23] One interesting exception is quadratic utility. We typically restrict the domain to coincide with an increasing function that is concave. But this utility yields neutrality for all risk orders higher than two. In other words, this individual would be indifferent to our *A* vs. *B* lottery choices for all orders three and higher.

this preference for "combining good with good" -- so that they are also prudent, intemperate, edgy, etc. -- it is relatively easy to find utility functions that have all their derivatives positive. Our experimental evidence seems to support their hypothesis of combining "good with bad" or "good with good."

Although risk lovers might be in a minority, it is perhaps surprising that more attention has not been given to their potential behavior. Indeed, they do seem to be consistent in their higher order risk preferences, at least for the first several orders. Other papers to date have not explicitly tested for this consistency, although both Noussair et al. (2013) and Ebert and Wiesen (2012) find that prudence is more prevalent than temperance. To the extent that risk lovers are in the minority and they also exhibit prudence and intemperance, these results support our hypothesis. If the theory held perfectly, everyone would be prudent and the proportion that is temperate would equal the proportion that is risk averse.

Although Noussair et al. (2013) claim a statistically significant positive correlation between risk aversion and prudence, a careful look at their evidence shows that this result is driven by their large on-line set of responders. Looking at their subsample of subjects who participated in the laboratory and who were later compensated, they actually find a strong positive correlation only between risk aversion and temperance as we would expect. There is no significant correlation between risk aversion and prudence; and although their correlation between prudence and temperance is positive, it is quite low (0.18) and significant only at a 10% level. Tarazona-Gomez (2004) also tests for the correlation between risk aversion and prudence and concludes that it is not statistically different from zero.

The paper by Maier and Rüger (2012) provides some additional supporting evidence for combining either "good with good" or "bad with bad." In particular, they linearly regress their percent of *Y* choices on their percent of *X* choices, where *X* are particular n^{th} order task choices, $n = 2,3,4$ and *Y* are m^{th} order task choices, with $m < n$. For example, they regress the percent of risk averse choices ($m = 2$) made on the percent of prudent choices ($n = 3$) made for their participants. Although all of their slope coefficients are positive, their best fit ($R^2 = 0.5391$) is when *Y* is risk aversion and *X* is temperance, with a slope coefficient of 0.9062.[24]

<u>Moment preference</u>

For some reason, the paper by Crainich et al. (2013) limits itself to a description within the confines of expected utility theory. As we mentioned earlier, their hypothesis does not need to be so confined. A careful look at each of our n^{th} order tasks, $n \geq 2$, reveals that the first n-1 moments are equal for both Option A and Option B. Moreover, option A within our tasks

[24] For *Y* risk aversion and *X* prudence, their slope is 0.5144 ($R^2 = 0.193$) and for *Y* prudence and *X* temperance, their slope is 0.5978 ($R^2 = 0.3216$).

always has a higher n^{th} moment than Option B. For example, in Tasks 4 – 10 for risk aversion, both Option A and Option B have equal means, but option A has a higher variance. Although we know, for example from Rothschild and Stiglitz (1970) that risk aversion is not "variance aversion," a higher variance is a necessary condition for higher 2^{nd} degree risk. As another illustration, all of our 4^{th} order tasks, Tasks 18-24, have the same first three moments, but option A has a higher kurtosis (more leptokurtic) than option B. Our experimental design was not developed to distinguish between a preference against (or for) n^{th} degree risk and preference against (or for) higher moments.

Oftentimes economic models use a function of the moments to compare distributions of wealth. For example, a function of only the first two moments is still used in some traditional models of portfolio preference. Such a person would always be indifferent between Option A and Option B in our experiment for orders 3-6, since the means and the variances of both options are always identical. Moments higher than the second moment are irrelevant to such an individual.

If we define a moment preference that is consistent with n^{th} degree risk, then someone who is risk apportionate, and thus always dislikes additional n^{th} degree risk, will always prefer higher odd order moments and smaller even order moments. This is the type of person who prefers to combine good with bad. The person who prefers combining good with good will have a preference for a higher n^{th} moment for every *n*.

Our experimental results support these types of moment preferences, at least for smaller orders. Once we get to orders 5 and 6, this preference is very weak. Perhaps it is not a coincidence that most economists, at least anecdotally, are familiar with the names for the first four moments of a probability distribution, but not the fifth or sixth moment.

Cumulative Prospect Theory

The paper by Ebert and Wiesen (2012) purports to use their experimental data to show that cumulative prospect theory (Tversky and Kahneman (1992)) has the best "fit" for explaining observed prudence and temperance. In particular, they use a type of compensating variation (how much cash can be added to Option A before one prefers it to Option B) to measure the intensities of both of these risk attitudes.

Cumulative prospect theory (CPT) generally consists of first defining a so-called reference point, from which we can frame "gains" and "losses," and then defining both an S-shaped value function of wealth and a weighting of the cumulative distribution function. As proposed by Tversky and Kahneman (1992), the value function is concave over the domain of gains but convex over the domain of losses. At the same time, the probability distortion puts more weight into both tails than the objective probability distribution. If we use the expected final payoff as our reference point for 3^{rd} and 4^{th} order risk preferences, an admittedly heroic assumption, the value function and the probability weighting work in opposite directions, as first pointed out by Deck and Schlesinger (2010).[25]

Of course, the issue of defining a reference point illustrates the importance of "framing" in such decision making processes. Particularly noteworthy might be whether decisions are framed as "gains" or "losses" or some mixture of both. While we do not address this framing issue here, the experiment by Maier and Rüger (2012), with CPT in mind, explicitly looks at the framing of gains and losses for both prudence and for temperance and does not find much of an effect, if any. Support for prudence and weak support for temperance seems to prevail in each type of scenario.

Our basic point is that CPT does not seem to be inconsistent with our observations in the laboratory. Since CPT plays a fairly prominent role in decision modeling, much still needs to be developed in this direction.

Rank-Dependent Expected Utility

Rank Dependent Expected Utility was first introduced by Quiggin (1982), although under a different name. Here, we can maintain a value function (i.e. the utility function) over wealth, without regards to a reference point. However, we simultaneously use a probability distortion similar to that used under Prospect Theory.

As one example, we can set the utility function to be the identity function, $u(t) = t$, so that we have only a probability distortion. This is the case examined extensively by Yaari (1987), which he labels the "dual theory." In this model, the same distortion as used in CPT (overweighting both high outcomes and low outcomes) can be potentially interpreted as in the previous section. If we overweight both the best and the worst lottery outcomes, the individual would behave in ways that exhibit both temperance and prudence. Interestingly, such an individual need not exhibit risk aversion.

If the utility function is concave, behavior is partly determined by the details of the shape of utility, exactly as it would be under EUT. However, the probability distortions in isolation would lead to prudent and temperate behavior, the same as they do under CPT. So the extent to which these two effects either reinforce each other or counteract each other can play a role.

There has not been much done to date in the way of analyzing these higher order effects in Rank Dependent Expected Utility Models.[26] Our experimental results here are not inconsistent with

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[25] Deck and Schlesinger (2010) use Tversky and Kahneman (1992) to calibrate their CPT model. Of course, this analysis is sensitive to the choice of a reference point. For example, Maier and Rüger (2011) consider a model with a stochastic "reference point." To the best of our knowledge, other CPT set-ups, as described in Stott (2006), have not been examined with regards to higher order risk preferences.

Rank Dependent Expected Utility, but we are not sure that they say anything convincing in way of support either. Of course, it might be the case that higher order effects and/or mixed risk loving types of behavior induce a completely different shape to the probability weighting for some individuals.

House money effects

Finally, we consider how our experiments might be viewed by many subjects as simply a game show, whose objective is to take the opportunity to win as much money as possible. This "house money" effect is examined in more detail by Thaler and Johnson (1990). But it is well known that many contestants in games of chance are willing to take on more risk if they are "playing" with someone else's money.[27] As a result, observed risk-loving behavior in experiments could be attributed to this house money effect. But is there also a house-money effect for higher orders?

If we look at the tasks of all orders greater than two, it is easy to notice that the choice that combines good with good (Option A for even order tasks and Option B for odd order tasks) is always the choice with highest maximum payoff. For risk lovers: more risk in our Option A binary lotteries implies a higher maximum payoff. For prudence, the higher right skew in our Option B lotteries implies a higher maximum payoff. For temperance, the fatter tails in our Option A lotteries implies a maximum payoff. And this pattern continues for the 5th and 6th orders as well. Thus, a house money effect leading subjects to seek the maximum payoff leads to the same behavior as being a mixed risk lover. Although we cannot rule out such an effect in our experiment, determining the choice that leads to the maximum payoff is relatively easy in this experiment yet no one always follows a strategy of maximizing the maximum possible payoff. In fact only two subjects behave in a manner consistent with such a strategy over 90% of the time, which we take to indicate that house money is not driving behavior.[28]

8. CONCLUDING REMARKS

Empirically investigating higher order risk preferences is an important area of research that is still in its infancy. In this paper we generalized Eeckhoudt et al (2009) and Crainich et al. (2013) into a hypothesis about two distinct ways in which individuals view risk taking, which can be expressed as a basic type of lottery preference:

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[26] Bleichrodt and Eeckhoudt (2006) take one step in this direction by trying capture the effects of probability distortions inside the utility function, albeit for "small" risks.

[27] See, for example, Post et al. (2008) and the references contained therein.

[28] One subject does follow a strategy of maximizing the minimum possible payoff every time and another does so in all but one choice. This behavior is indistinguishable from a preference for combining good with bad.

> *Risk averters are mixed risk averse*: *they dislike an increase in risk for every degree n*
>
> *Risk lovers are mixed risk loving*: *they like risk increases of even degrees, but dislike increases of odd degrees*

Since most studies find a majority of the population is risk averse, the second category above has not been studied much relative to the first category. Indeed, limited to experimental studies of higher orders of risk preference, only the first category above has been directly examined, except by perhaps noting correlations of higher attitudes with risk aversion.

The results of this paper add support to the nascent set of experimental results for the first category above. Most individuals do appear to be not only risk averse, but also prudent, temperate, edgy and, more generally, risk apportionate of order n for any n, as defined by Eeckhoudt and Schlesinger (2006). But those who are not do seem to fit nicely into our second category above.

Our evidence shows that risk lovers, just as risk averters, show a fair degree of consistency when it comes to higher order risk preferences. Moreover, we reexamine results from previous experiments to see if we can glean any support this type of dichotomous behavior, and indeed we can. To the extent that risk lovers are indeed mixed lovers, we might be able to better explain certain types "risk loving behavior."

Both expected utility theory and non-expected utility theory have been modeled around the prototypical risk-averse decision maker; with perhaps some attention given to 3rd and 4th order attitudes (prudence and temperance). In addition to theories mentioned in the previous section of this paper, extensions of higher-order risk attitudes to other types of non-expected utility models are beginning to appear. For example, Kimball and Weil (2009) show that defining prudence in the temporal setting of Kreps and Porteus (1978) can be a bit tricky. A recent experiment by Bostian and Heinzel (2012) shows that subjects do tend to display this type of Kimball-Weil temporal prudence. In another extension, Baillon (2012) shows how one can apply the concept of risk apportionment to models of ambiguity.

Of course risk loving behavior can be described in all of the various theories, but none seems to go beyond the superficial in terms of a deeper understanding of such preferences. What is a counterpart to, say, decreasing absolute risk aversion for risk lovers? Or, what can be said about individuals who favor ambiguity, as opposed to being ambiguity averse?[29] To what extent might higher-order behavior be consistent with basic ambiguity-averse or ambiguity-loving behavior?

[29] One example that is often mentioned is an individual who refuses a genetic test, since she prefers "not to know."

In this paper, our focus was: to what extent might higher-order behavior be characterized by a propensity for combining good with good? Examinations into higher order risk preferences to date, in addition to focusing on risk averters, have only gone as far as the 4th order. Very few results within expected-utility applications can be shown to also consider 5th order risk attitudes[30], but until now, no one has tested for these higher orders. In this paper, we extend experimental tests to also consider both 5th order risk attitudes ("edginess") and 6th order risk attitudes. Although the patterns predicted in our dichotomy above seem to still hold, their significance is rather weak and behavior, at least with respect to our lottery choice, seems to become more and more random with higher orders.

The recent model by Crainich et al. (2013) leads to some very interesting empirical question and we hope that our paper is a good start in answering some of them. Of particular interest is the fact that risk lovers seem to be quite consistent in their higher order preferences. Although Ebert (2012) is correct in stating that risk lovers need not be mixed risk lovers, our experimental evidence shows that they typically are indeed mixed risk lovers. In this regard, confining most economic analyses -- especially theoretical analyses -- to a universe of (mixed) risk averters might be obstructing our view of the forest.

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[30] See Lajeri-Chaherli (2004) as one example.

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APPENDIX: Experiment Directions and Comprehension Quiz

The directions were computerized and self-paced as was the comprehension quiz. Italicized headings were not observed by the participants.

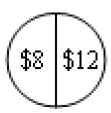
Page 1 of the Directions:

You are participating in a research study on decision making under uncertainty. At the end of the study you will be paid your earnings in cash and it is important that you understand how your decisions affect your payoff. If you have questions at any point, please let a researcher know and someone will assist you. Otherwise, please do not talk during this study and please turn off all cell phones.

Page 2 of the Directions:

In this study there is a series of 38 tasks. Each task involves choosing between Option A and Option B. Once you have completed these tasks, <u>one</u> of the thirty-eight tasks will be randomly selected to determine your payoff.

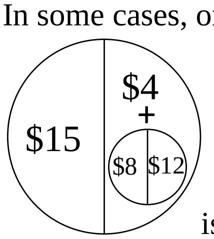
Page 3 of the Directions:

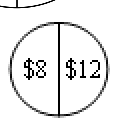
Each option will involve amounts of money and possibly one or more 50-50 lotteries represented as a circle with a line through the middle. A 50-50 lottery means there is a 50% chance of receiving the item to left of the line and a 50% chance of receiving the item to the right of the line. For example, 

is a 50-50 lottery in which you would receive either $8 or $12, each with an equal chance. To determine the outcome of any 50-50 lottery, we will use a spinner. You are welcome to inspect the spinner at any point.

Page 4 of the Directions:

In some cases, one of the items in a 50-50 lottery may be another lottery. For example,



is a 50-50 lottery where you receive either $15 or you receive $4 plus the 50-50 lottery 

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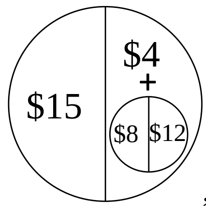
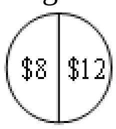
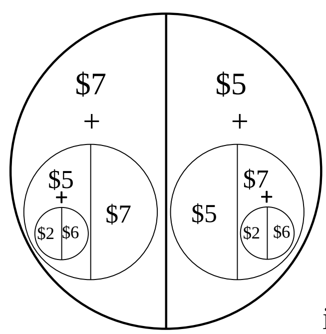
Page 5 of the Directions:

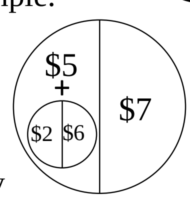
Continuing with the example, 

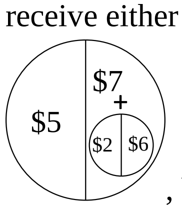
, there is a 50% chance that you would receive $15 in the big 50-50 lottery and that would be it. There is also a 50% chance that you would receive $4 + 

in the big 50-50 lottery. Conditional on this outcome for the big 50-50 lottery, you would then have a 50% chance of receiving an extra $8 and a 50% chance of receiving an extra $12 in addition to the $4. Therefore, the chance that you would end up with $4+$8 = $12 is 0.5 × 0.5 = 0.25 = 25%. The chance that you would end up with $4+$12 = $16 is 0.5 × 0.5 = 0.25 = 25%.

Page 6 of the Directions:

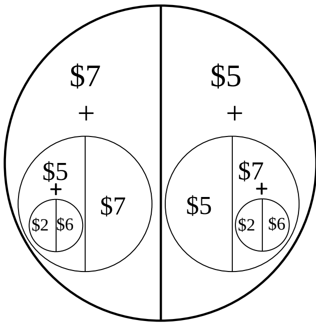
Let's look at a more complicated example. 

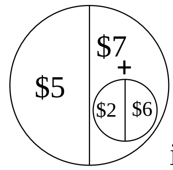
is 50-50 lottery where you receive either $7 plus the 50-50 lottery 

or you receive $5 plus the 50-50 lottery 

, both of which include an additional 50-50 lottery.

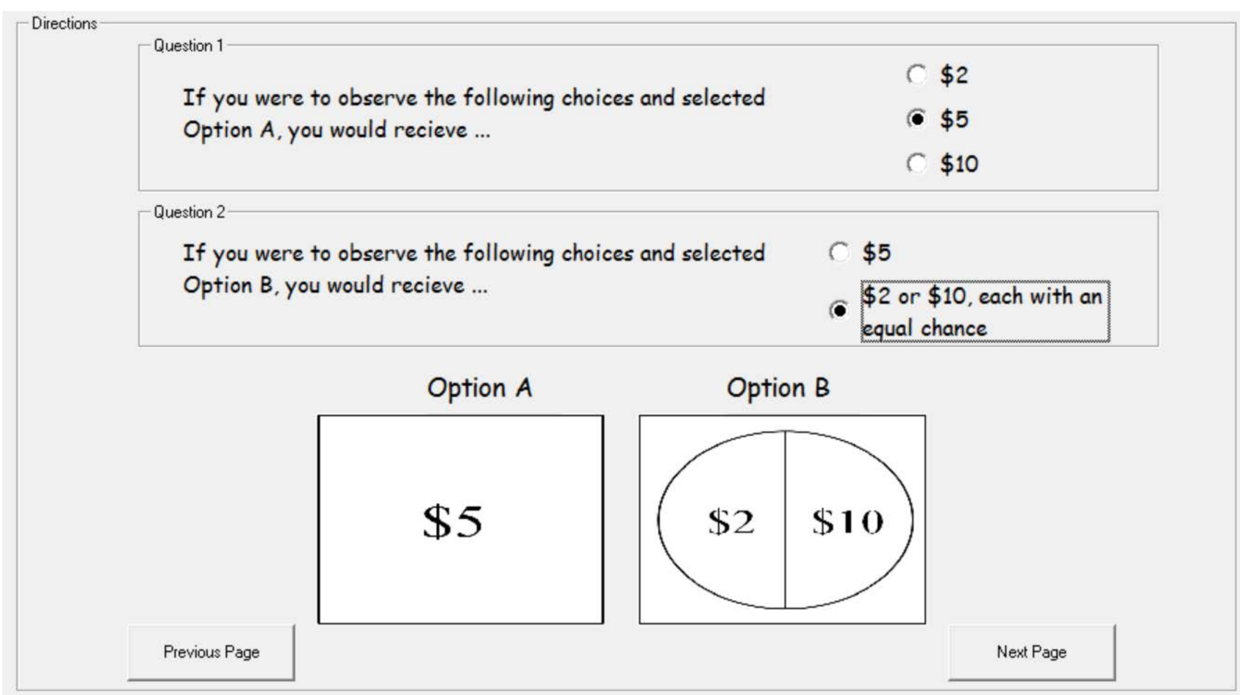
Page 7 of the Directions:

In 

you could earn $10 if you get $5 + 

in the big lottery and then earn $5 in the second lottery. This occurs with a 0.5 x 0.5 = 25% chance. Alternatively, you could earn $14 with a 37.5% chance. Notice that you could earn $14 by 1) earning $7 (in the big lottery) + $5 (in the middle lottery) +$2 (little lottery) which happens with a 0.5 x 0.5 x 0.5 = 12.5% chance or 2) earning $7 (in the big lottery) + $7 (in the middle lottery) which happens with a 0.5 x 0.5 = 25% chance, or 3) earning $5 (in the big lottery) + $7 (in the middle lottery) +$2 (little lottery) which happens with a 0.5 x 0.5 x 0.5 = 12.5% chance. Finally there are two ways that you could earn $18 which occurs with a 0.5 x 0.5 x 0.5 + 0.5 x 0.5 x 0.5 = 25% chance.

Comprehension Quiz Screen 1 (with correct answers added):



Comprehension Quiz Screen 2 (with correct answers added):