Optimal timing, cost and sectoral dispatch of emission reductions: abatement cost curves vs. abatement capital accumulation

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Abstract

Greenhouse gas emission reductions may be achieved through a combination of actions. Some bring immediate environmental benefits and little consequences over the long term (e.g., driving less), and are best represented by *abatement cost curves*. Others imply investments and persistent emission reductions (e.g., retrofitting buildings) and are best represented by *abatement capital accumulation*. We investigate the optimal cost, timing and sectoral dispatch of emission reductions in these two models, based on these two formalizations and reach drastically different conclusions. In a model based on abatement cost curves, optimal emission reduction efforts are mostly done in the future, when the carbon price is higher. In a model based on abatement capital accumulation, the carbon price grows over time, but optimal abatement efforts are bell-shaped and concentrated over the short run. The unique carbon price optimally translates in different investment costs (expressed in dollars per abated ton) across sectors. This result reconciles two apparently opposite views on abatement strategies: while a unique carbon price triggers the socially-optimal investment pathway (in the absence of other market failure), short-term efforts should be higher in sectors that require abatement investment and that will take longer to decarbonize, such as transport and urban planning.

Keywords: climate change mitigation; green transition; path dependence

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We thank Alain Ayong Le Kama, Patrice Dumas, Marianne Fay, Michael Hanemann, Frédéric Ghersi, Louis-Gaëtan Giraudet, Christophe de Gouvello, Christian Gollier, Céline Guivarch, Oskar Lecuyer, Baptiste Perissin Fabert, Antonin Pottier, Jean Charles Hourcade, Lionel Ragot, Julie Rozenberg, François Salanié, Ankur Shah, seminar and conference participants at Cired, École Polytechnique, Université Paris Ouest, Paris Sorbonne, Princeton University's Woodrow Wilson School, Toulouse School of Economics, the Joint World Bank – International Monetary Fund Seminar on Environment and Energy Topics, and at the Conference on Pricing Climate Risk held at Center for Environmental Economics and Sustainability Policy of Arizona State University for useful comments and suggestions on various versions of this paper. The remaining errors are the authors' responsibility. We are grateful to Patrice Dumas for technical support. We acknowledge financial support from Institut pour la Mobilité Durable (Renault and ParisTech), The views expressed in this paper are the sole responsibility of the authors. They do not necessarily reflect the views of the World Bank, its executive directors, or the countries they represent.

December 24, 2013

1. Introduction

The international community aims at containing global warming below 2°C above preindustrial levels. This objective can be reached with different emissions pathways, hence with different distribution of the effort over time. There are also many sectors where greenhouse gas (GHG) emission may be reduced, from the use of renewable energy to better building insulation and more efficient cars. Two important questions for public policy are thus to determine when and where (i.e. in which sectors) emission reductions should occur.[2]

Many assessments of the optimal timing of greenhouse gas emissions use abatement cost curves (e.g., Nordhaus, 1992; Goulder and Mathai, 2000). In this framework, mitigation efforts and the carbon price are almost the same thing, and both grow over time along the optimal pathway. We show in this paper that this is because abatement cost curves embed the implicit assumption that the level of emission abatement may be chosen freely at each time step, without any path dependence (section 2). This assumption is valid for some emission reduction actions, such as consumption pattern changes (e.g., driving less kilometer per year), but not for all actions.

Indeed, many authors have re-framed climate mitigation as a transition toward a low-carbon economy, where immediate actions have long-term consequences. Modeling exercises have focused on the dynamics of knowledge accumulation (e.g., Grubb et al., 1995; Gerlagh et al., 2009; Grimaud et al., 2011; Acemoglu et al., 2012). Here, we focus on another accumulation process, namely the replacement of high-emission capital, for instance coal power plants or inefficient cars, by low-carbon capital, such as windmills and electric vehicles.

We build a simple intertemporal optimization model where reducing emission requires to invest in and accumulate abatement capital, a proxy for the replacement of high-emission capital by low-emission capital (section 3). Investments in abatement capital have convex costs: accumulating abatement capital faster is more expensive. Also, there is a maximum abatement potential; this potential is exhausted when all the emitting capital (e.g. energy-intensive dwellings, fossil fuel power plants) is replaced by non-emitting capital (e.g. retrofitted dwellings, renewable power).

In this new framework where abatement is obtained through capital accumulation, the optimal carbon price and the optimal abatement efforts have drastically different dynamics, and are not longer growing alongside. The optimal carbon price still grows exponentially over time. The optimal abatement investment, hovewever, is bell-shaped: it first grows and then decreases over time. For stringent temperature targets, the bulk of emission-reduction efforts (i.e., investments) happens in the short run.

Furthermore, the optimal cost of abatement capital is different from the discounted value of future avoided carbon emissions. Instead, its value is the sum of three terms: (1) the value of avoided emissions; (2) the forgone-opportunity effect, i.e. the fact that each investment in abatement capital reduces future investment opportunities, since the abatement potential is limited; (3) the

[2] A third question, the desirable burden sharing among countries, is not treated in this paper.

replacement-cost effect, that translates the fact that when all avoidable emissions are avoided, the value of the abatement capital is driven by its replacement cost, which is different from the carbon price. As a result, determining the optimal level of abatement investments requires more information than just the shadow price of carbon. It also requires the date when the all emission are abated and the replacement cost of abatement capital in the long term (section 4).

The paper also investigates the consequences on the distribution of emission reduction efforts across sectors. A unique, economy-wide carbon price optimally translates in different investment levels in different economic sectors. On the optimal pathway, short-term efforts, measured in dollars invested today per abated ton of carbon, are higher in the sectors that will take longer to decarbonize, such as urban planing or transportation. In an abatement capital accumulation framework, a unique carbon price is therefore consistent with higher efforts in sectors with long-lived capital (e.g., Jaccard and Rivers, 2007; Lecocq and Shalizi, 2014; Vogt-Schilb and Hallegatte, 2014).

We provide numerical illustrations of these analytical results, using IPCC data (section 5). We show that the optimal dispatch of emission-reductions efforts (over time and across sectors) is both qualitatively and quantitatively different when assessed using abatement cost functions and abatement capital accumulation.

Section 2 provides a brief reminder of the optimal cost and timing of emission reductions in the abatement cost curves framework. In section 3, we introduce a model of abatement capital accumulation and derive some analytical results concerning the optimal timing of carbon abatement in this new framework. In section 4, we turn to the question of the optimal dispatch of emission reductions across sectors. In section 5 we provide numerical illustrations. We conclude in section 6.

2. The abatement cost curves framework

In his seminal contribution, Nordhaus (1992) has framed the question on when and how much to reduce GHG emissions as a cost-benefit analysis, with the Dynamic Integrated model of Climate and the Economy (DICE). In DICE, a social planner chooses at each time step a fraction of GHG emission to abate, and spends a fraction of GDP in emission-reduction activities represented through an abatement cost curve.[3] DICE became a reference in the climate mitigation literature, and has been extended in various directions. For instance, Popp (2004) investigates the role of induced technical change, and Bruin et al. (2009) study how the explicit description of adaptation in the model changes the optimal timing of emission reductions.

Abatement cost curves are also found in more theoretical contributions. For instance, Goulder and Mathai (2000) study how induced technical change changes the optimal timing of carbon abatement in an analytical model, and Pizer (2002) analyses the implications of uncertainty surrounding compliance costs on the optimal climate policy path.

[3] This is denoted TC in equation 12 in Nordhaus (1992).

In this section we show with a simple model that when abatement cost curves are used, the optimal timing and cost of GHG reductions is essentially the same thing as the exponentially-growing carbon price. We also highlight that the abatement-cost-curve framework requires the assumption that the level of emission reductions may be freely decided at each point in time.

2.1. Abatement cost curve

In this framework, the cost of a climate mitigation policy at time t is linked to the abatement a_t through an abatement cost curve γ. The function γ is classically convex, positive and twice differentiable:

$$\begin{aligned} \forall a_t, \quad \gamma''(a_t) &> 0 \\ \gamma'(a_t) &> 0 \\ \gamma(a_t) &> 0 \end{aligned} \tag{1}$$

We make the simplifying assumption that the abatement cost curve γ is constant over time.[4]

The basic idea behind these curves is that some potentials for emission reductions are cheap (e.g. insulating buildings), while other are more expensive (e.g. upgrading power plants with carbon capture and storage). If potentials are exploited in the merit order — from the cheapest to the most expensive — the marginal cost of doing so $\gamma'(a_t)$ is growing in a_t, and $\gamma(a_t)$ is convex.

2.2. Abatement potential

We explicitly model a maximal abatement potential $\bar{a}_t$.[5] It reflects the idea that GHG emissions cannot be reduced beyond a certain point. For instance, if emissions can be reduced to zero but negative emissions are impossible, $\bar{a}_t$ equals current emissions.

$$\forall t, \ a_t \leq \bar{a}_t \tag{2}$$

2.3. Carbon budget

One approach to determine when and how much to abate carbon emissions is to perform a cost-benefit analysis. Due to the various scientific uncertainties surrounding damages from climate change and climate change itself (Manne and Richels, 1992; Ambrosi et al., 2003), it is frequent to use targets expressed in global warming (as the 2C target from UNFCC), or, similarly (Allen et al., 2009), cumulative emissions (Zickfeld et al., 2009).

Here, we constrain cumulative emissions below a given ceiling, a so-called carbon budget B. This keeps the model as simple as possible, and allows us to focus on the dynamics induced by two models of emission reductions (abatement cost curves *vs.* abatement capital accumulation) keeping the dynamics of climate change and climate damages out.[6]

[4] Goulder and Mathai (2000) show that the qualitative shape of the optimal timing and cost of emission reductions is robust to this assumption.

[5] A similar assumption is frequently made implicitly. In particular, in DICE, the abatement cost depends on the fraction of emission abated, and this fraction is caped to 1.

[6] Many contributions based on numerical optimisation (including Nordhaus, 1992; Goulder and Mathai, 2000) factor in some climate change dynamics, without changing the qualitative results exposed in this section.

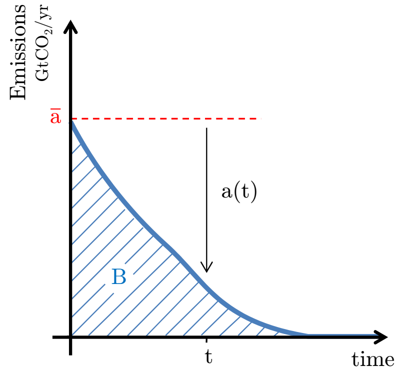

Figure 1: An illustration of the climate constraint.

For simplicity, we assume that emissions would be constant and equal to $\bar{a}$ in absence of abatement.[7] We denote m_t the cumulative atmospheric emissions at date t. The carbon budget reads (dotted variables represent temporal derivatives):

$$\begin{aligned} &m_0 \text{ given} \\ &\dot{m}_t = \bar{a} - a_t \\ &m_t \leq B \end{aligned} \tag{3}$$

B represents the allowable emissions to reach a temperature target (Meinshausen et al., 2009), but can also be interpreted as a tipping point beyond which the environment is catastrophically damaged.

2.4. The social planner's program in the abatement cost curve framework

In the abatement-cost-curve framework, the social planner determines when to abate in order to minimize abatement costs discounted at a given rate r, under the constraints set by the abatement potential and the carbon budget:

$$\min_{a_t} \int_0^\infty e^{-rt}\,\gamma\,(a_t)\,dt \tag{4}$$

$$\begin{aligned} \text{subject to}\quad & a_t \leq \bar{a} && (\lambda_t) \\ & \dot{m}_t = \bar{a} - a_t && (\mu_t) \\ & m_t \leq B && (\phi_t) \end{aligned}$$

We denoted in parentheses the (positive, present-value) Lagrangian multipliers.

2.5. Results in the abatement cost curve framework

With the objective to maintain cumulative emissions below the carbon budget, both the abatement potential $\bar{a}$ and the cumulative emission ceiling B are

[7] Again, Goulder and Mathai (2000), show that the qualitative shape of the optimal cost and timing of emission reductions is robust to this assumption.

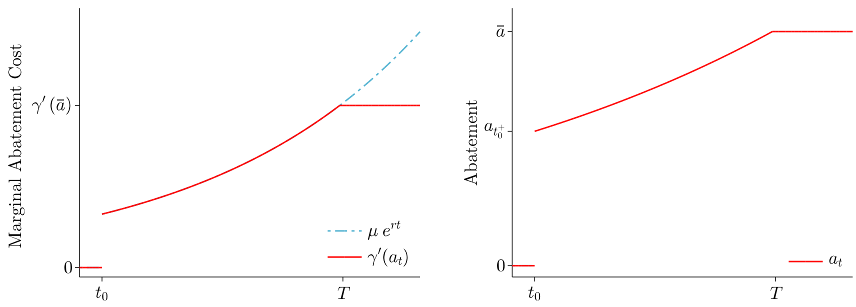

Figure 2: Optimal timing and costs of abatement in the abatement-cost-curve framework. Left: Before the potential is reached, abatement efforts are equal to the carbon price and grow over time. Right: When the social planner imposes a carbon tax at t_0, the level of abatement "jumps".

reached at an endogenous date T.

$$\forall t \geq T, \quad m_t = B$$
$$\implies a_t = \bar{a} \qquad \text{(from eq. 3)}$$

Before this date, the classical result holds: the current carbon price μe^{rt} grows at the discount rate r (Appendix A):

$$\forall t \leq T, \quad \mu_t = \mu \tag{5}$$

This ensures that the present value of the carbon price is constant along the optimal path, such that the social planner is indifferent between one unit of abatement at any two dates.

In the abatement cost curve framework, the optimal abatement cost strategy is to implement abatement options such that the marginal abatement cost is equal to the current carbon price at each point in time, until the potential $\bar{a}$ and the carbon budget are reached (Appendix A):

$$\gamma'(a_t^\star) = \begin{cases} 0 & t \leq t_0 \\ \mu e^{rt} & t_0 < t < T \\ \gamma'(\bar{a}) & t \geq T \end{cases} \tag{6}$$

Where t_0 is the date when the social planner implements the carbon price (Fig. 2).

Proposition 1. *In the abatement cost curve framework, the optimal abatement pathway is such that:*
– Both the abatement efforts $\gamma'(a_t^\star)$ and the abatement level $a_t^\star$ increase over time.
– At each time step t, the optimal amount of abatement may be derived from the current carbon price μe^{rt} and the abatement cost curve γ.
– Abatement jumps when the carbon price is implemented.

PROOF. See (6).[8] □

[8] The result on abatement efforts is general, while the result on the abatement level requires that $\gamma'^{-1}(e^x)$ is a growing function of x. A sufficient condition is that γ is polynomial.

Such jumps are possible because in the abatement-cost-curve framework, the amount of abatement may be decided at each period independently.[9] This simplifying assumption is valid in several cases where abatement action is paid for and delivers emission reduction at the same time, such as driving less or reducing indoor temperatures.

In other cases, such as upgrading to more efficient vehicles or retrofitting buildings, costs are mainly paid when the action is undertaken, while annual emissions are reduced over several decades. These actions are better modelled as accumulation of abatement capital. We show in the next section that in this case, the abatement pathway is continuous (by design, it cannot "jump" anymore); and while the carbon price still grows over time, the cost of the climate policy is bell-shaped. We then discuss important consequences on the optimal dispatch of emission-reduction efforts across economic sectors.

3. Abatement capital accumulation

3.1. A simple model of abatement capital accumulation

In this section, we set up and solve a different model, where all emission reductions require accumulation of abatement capital.[10] Note that this is an extreme assumption, useful to compare the results of this model with those from the abatement-cost-curve framework.

The stock of abatement capital starts at zero (without loss of generality), and at each time step t, the social planner chooses a positive amount of physical investment x_t in abatement capital a_t, which otherwise depreciates at rate δ:

$$a_0 = 0 \tag{7}$$

$$\dot{a}_t = x_t - \delta a_t \tag{8}$$

For simplicity, abatement capital is directly measured in terms of avoided emissions, such that the carbon budget reads as in section 2 :

$$\begin{aligned} &m_0 \text{ given} \\ &\dot{m}_t = \bar{a} - a_t \\ &m_t \leq B \end{aligned} \tag{3}$$

Investment in abatement capital costs $c(x_t)$, where the function c is positive, increasing, differentiable and convex:

$$\begin{aligned} \forall x_t, \quad c''(x_t) &\geq 0 \\ c'(x_t) &\geq 0 \\ c(x_t) &\geq 0 \end{aligned} \tag{9}$$

The cost convexity bears on the investment flow. This captures increasing opportunity costs to use scarce resources (skilled workers and appropriate capital)

[9] Such jumps in the optimal abatement pathway are present in many works, for instance in the optimal pathway to 1.5°C in the original work by Nordhaus (1992) and in the numerical illustrations provided by Goulder and Mathai (2000). Also, Schwoon and Tol (2006) allow explicitly for such jumps, in a model that would otherwise be close to the abatement capital accumulation model presented in section 3.

[10] Similar models have been used by Kolstad (1996) and Fischer et al. (2004).

to build and deploy abatement capital. For instance, x_t will depend on the pace — measured in buildings per year — at which old buildings are being retrofitted at date t (the abatement a_t would then be proportional to the share of retrofitted buildings in the stock). Retrofitting buildings at a given pace requires to pay a given number of scarce skilled workers. If workers are hired in the merit order and paid at the marginal productivity, the marginal price of retrofitting buildings $c'(x_t)$ is a growing function of the pace x_t.

In addition, some authors (Ha-Duong et al., 1997; Schwoon and Tol, 2006) claim that capital stock turnover justifies this cost convexity: if emitting capital is replaced by low-carbon capital faster than the natural turnover rate, the value of the scrapped capital should be added to the cost of building low-carbon capital.[11]

This convexity is of different nature than the convexity of γ in the classical approach presented in section 2, where it arises from heterogeneity in abatement options (e.g., different abatement costs for frequently-driven and occasionally-driven vehicles) while in this model it arises from production decreasing returns (e.g. for car manufacturers) and capital early-scrapping (e.g. replacing a working classical vehicle with an electric one).

The social planner chooses when to invest in abatement capital in order to meet a carbon budget (section 2.3) at the lowest inter-temporal cost, under the constraint set by the maximum abatement potential (section 2.2):

$$\begin{aligned} &\min_{x_t} \int_0^\infty e^{-rt} c(x_t)\, dt && (10) \\ \text{subject to} \quad &\dot{a}_t = x_t - \delta a_t && (\nu_t) \\ &a_t \leq \bar{a} && (\lambda_t) \\ &\dot{m}_t = \bar{a} - a_t && (\mu_t) \\ &m_t \leq B && (\phi_t) \end{aligned}$$

Note that the social planner does not control directly abatement a_t, but investment x_t, that is the speed at which emission are reduced.

The Greek letters in parentheses are the present-value Lagrangian multipliers (chosen such that they are positive): ν_t is the value of abatement capital, μ_t is the cost of carbon emissions, and λ_t is the cost of the maximum abatement potential.

3.2. Optimal cost and timing of abatement investment

Proposition 2. *Along the optimal path, abatement increases until it reaches the maximum potential $\bar{a}$ at an endogenous date denoted T (Fig. 3). Marginal investment costs depend on this date T, the depreciation rate of the abatement*

[11] In this simple model, we do not distinguish between abatement capital that reduces emissions without producing any output, and abatement capital that produces the same output than polluting capital, but without emitting pollution.

The cost $c(x_t)$ may thus be interpreted as the cost of investing in abatement capital (e.g., retrofitting an existing building, upgrading a fossil-fuelled power plant with carbon capture and storage), or the cost of building low-carbon capital instead of, or in replacement for, polluting capital.

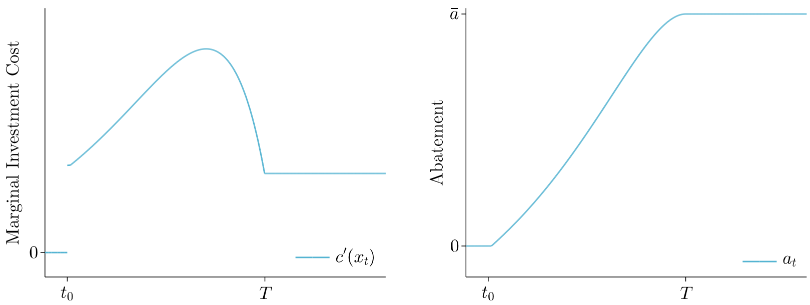

Figure 3: Optimal cost and timing of abatement in the abatement capital accumulation framework. Left: Abatement efforts are bell-shaped. Right: The level of abatement is continuous over time (by design).

capital δ, and the current carbon price μe^{rt}:

$$\forall t \leq T,$$
$$c'(x_t^\star) = \underbrace{\mu e^{rt} \int_t^\infty e^{-\delta(\theta - t)}\, d\theta}_{\mathcal{R}} - \underbrace{\mu e^{rt} \int_T^\infty e^{-\delta(\theta - t)}\, d\theta}_{\mathcal{O}} + \underbrace{e^{-(r+\delta)(T-t)}\, c'(\delta \bar{a})}_{\mathcal{K}} \tag{11}$$

Proof. Appendix B ☐

Equation 11 states that at each time step t, the social planner should invest in abatement capital up to the pace at which marginal investment costs (Left-hand side term) are equal to marginal benefits (RHS term).

The marginal benefits decomposes in three terms. The first term $\mathcal{R}$ relates to emission reductions. As before, the current carbon price μe^{rt} grows at the discount rate. It is multiplied by the quantity of GHG saved by the marginal abatement equipment during its lifetime: $\int_t^\infty e^{-\delta(\theta-t)}\, d\theta$ (a *naive* value of abatement capital). With a higher depreciation rate δ, this benefit is lower, as one investment results in less GHG saved.

The second term $\mathcal{O}$ comes from a forgone-opportunity effect. The limited potential $\bar{a}$ behaves here like a finite resource, an abatement deposit. Each investment in abatement capital brings closer the date T when all emissions are avoided. After T, accumulating more abatement capital does not allow to reduce emissions. The value $\mathcal{O}$ of this forgone opportunity is the value of the GHG that the maximum potential prevents to save after T. The shorter it takes to cap all emissions, that is the lower T, the greater is this effect.[12]

The third term $\mathcal{K}$ is the contribution of the marginal investment to the present value of the final stock of green capital (a classical transversality condition). Note that here, the abatement capital is valued at its replacement cost $c'(\delta\bar{a})$. It is not valued at the carbon price μe^{rt} because after T, the binding

[12] In Appendix C we study these two effects separately from the third one in a variant with infinitely-lived abatement capital ($\delta = 0$). In this case, the date T is a growing function of the potential $\bar{a}$.

constraint is to maintain abatement capital at its maximum potential, not to reduce emissions.

Corollary 1. *The optimal cost of abatement capital is different from the value of avoided carbon emissions over its lifetime.*

Proof. Combining the two first terms ($\mathcal{R} + \mathcal{O}$), equation 11 can be rewritten as:

$$\forall t \leq T,$$
$$c'(x_t^\star) = \int_t^T \mu e^{r\theta} e^{-(\delta+r)(\theta-t)}\, d\theta + \int_T^\infty (r+\delta)\, c'(\delta\bar{a})\, e^{-(\delta+r)(\theta-t)}\, d\theta \quad (12)$$

The output of abatement capital is valued at the current carbon price ($\mu e^{r\theta}$) before T, and at the replacement value of abatement capital $(r+\delta)\, c'(\delta\bar{a})$ after T.[13]

Corollary 2. *Optimal investment costs are bell-shaped.*

Proof. The optimal marginal investment cost can also be written as:

$$c'(x_t^\star) = \mu e^{rt}\, \frac{1 - e^{-\delta(T-t)}}{\delta} + e^{-(\delta+r)(T-t)}\, c'(\delta\bar{a}) \quad (13)$$

The first term reflects a complex trade-off: investing soon allows the planner to benefit from the persistence of abatement efforts over time, and prevents investing too much in the long-term; but it brings closer the date T, removing the option to invest later, when the discount factor is higher. This results in a bell-shaped distribution of mitigation costs over time: in the short term, the effect of discounting may dominate – if either T, r or δ is sufficiently large – and the effort may grow (nearly exponentially); in the long term, the effect of the limited potential dominates and accumulation of new abatement capital decreases to zero (Fig. 3). □

This section has shown that taking into account abatement capital changes drastically the optimal cost and timing of greenhouse gas abatement. In the next section, we discuss how to assess investment made to reduce emission in different economic sectors.

4. The optimal dispatch of emission reductions across sectors

In this section, we extend the model of abatement capital accumulation to the case of several sectors. The economy is partitioned in a set of sectors indexed by i. For simplicity, we assume that abatement in each sector does not interact with the others.[14] Each sector is described by an abatement potential

[13] $(r+\delta)\, c'(\delta\bar{a})$ can be interpreted as both the rental cost and the levelized cost of abatement capital during the steady state (see next section).

[14] This is not entirely realistic, as abatement realized in the power sector may actually reduce the cost to implement abatement in other sectors, using electric-powered capital (Williams et al., 2012).

$\bar{a}_i$, a depreciation rate δ_i, and a cost function c_i. The social planner's program becomes:

$$\begin{aligned} &\min_{x_{i,t}} \int_0^\infty e^{-rt} c_i(x_{i,t})\, dt && (14) \\ \text{subject to} \quad & \dot{a}_{i,t} = x_{i,t} - \delta_i\ a_{i,t} && (\nu_{i,t}) \\ & a_{i,t} \leq \bar{a}_i && (\lambda_{i,t}) \\ & \dot{m}_t = \sum_i (\bar{a}_i - a_{i,t}) && (\mu_t) \\ & m_t \leq B && (\phi_t) \end{aligned}$$

The value of abatement capital $\nu_{i,t}$ and the cost of the sectoral potentials $\lambda_{i,t}$ now depend on the sector i, while the carbon price μ_t is still unique for the whole economy.

4.1. Solving for the optimal MICs

The optimal sectoral investment costs are very similar to the optimal cost found in the previous section with only one sector:

Proposition 3. *The unique carbon price grows at the discount rate until all abatement potentials have been reached (at the respective dates T_i). In each sector i, the optimal marginal investment cost reads:*

$\forall i,\ \forall t \leq T_i,$

$$c_i{}'(x^\star_{i,t}) = \int_t^{T_i} \mu e^{r\theta} e^{-(\delta_i + r)(\theta - t)}\, d\theta + \int_{T_i}^\infty (r + \delta_i)\ c_i'\left(\delta_i \bar{a}_i\right)\ e^{-(\delta_i + r)(\theta - t)}\, d\theta \tag{15}$$

Optimal investment costs are therefore different across sectors.

Proof. See Appendix D

4.2. Equilibrium decentralization and the principle of equimarginality

Take the point of view of the owner of one polluting equipment in a sector i, facing the announced carbon price μe^{rt}. One question for this owner is when should this equipment be retrofitted or replaced with low-carbon capital. The following proposition shows that the answer is anytime before T_i:

Proposition 4. *Along the optimal pathway, individual forward-looking agents in each sector i are indifferent between investing in abatement capital at any time before T_i.*

Proof. Let τ be the date when the agent invests in abatement capital. Before τ, the agent pays the carbon price. At τ, he invest in one unit of abatement capital at the price $c'(x^\star_\tau)$. At each time period t after τ, he has to maintain its abatement capital, which costs $\delta c'(x^\star_t)$. The total discounted cost $V_i(\tau)$ of this strategy reads:

$$V_i(\tau) = \mu\tau + e^{-r\tau} c'(x^\star_\tau) + \int_\tau^\infty e^{-rt} \delta c(x^\star_t) dt \tag{16}$$

The first order condition for the individual agent is:

$$V_i'(\tau) = 0 \iff \lambda_{i,\tau} = 0 \qquad \text{(from eq. 20)} \tag{17}$$

This last condition is satisfied when $\lambda_{i,t}$, the social cost of the sectoral potential $\bar{a}_i$ is null, that is for any $\tau \leq T_i$ (complementary slackness condition D.8). □

In line with the general theory (e.g., Arrow and Debreu, 1954), Prop. 4 means that the optimal investment pathway is a Nash equilibrium: if forward-looking, cost-minimizing agents correctly anticipate the carbon price μe^{rt}, optimal investment trajectories $x_{i,t}$, and the resulting cost of abatement capital $c_i{}'(x^\star_{i,t})$, they have no individual interest to diverge from the social optimum.

While a unique carbon price can decentralize the social optimum, investment costs differ across sectors. This apparent paradox may be resolved using the following metric:

Definition 1. *We call* marginal implicit rental cost of abatement capital (MIRCC) *in sector i at a date t the following value:*

$$p_{i,t} = (r + \delta_i)\; c_i{}'(x_{i,t}) - \frac{dc'(x_t)}{dt} \tag{18}$$

This definition extends the concept of the *implicit rental cost of capital* (Jorgenson, 1967) to the case where investment costs are endogenous functions of the investment pace.[15] It corresponds to the market rental price of abatement capital in a competitive equilibrium, and ensures that there are no profitable trade-offs between: (i) lending at a rate r; and (ii) investing at time t in one unit of capital at cost $c_i{}'(x_{i,t})$, renting this unit during a small time lapse dt, and reselling $1 - \delta dt$ units at the price $c_i'(x_{i,t+dt})$ at the next time period (see also Appendix E).

Proposition 5. *In each sector i, before the date T_i, the optimal marginal implicit rental cost of abatement capital equals the current carbon price:*

$$\forall i,\ \forall t \leq T_i, \quad p^\star_{i,t} = (r + \delta_i)\; c_i{}'(x^\star_{i,t}) - \frac{dc_i{}'(x^\star_{i,t})}{dt} = \mu e^{rt} \tag{19}$$

Proof. Appendix D shows that the first order conditions can be written as:

$$\forall (i,t), \quad (r + \delta_i)\; c_i{}'(x^\star_{i,t}) - \frac{dc_i{}'(x^\star_{i,t})}{dt} = e^{rt}\,(\mu_t - \lambda_{i,t}) \tag{20}$$

Where $\lambda_{i,t}$, the Lagrangian multiplier associated with the sectoral potential $\bar{a}_i$, is null before the potential is exhausted at T_i. □

Equation 19 may be interpreted as a simple cost-benefit rule. The LHS is the cost of renting the marginal unit of abatement capital during one time period. For instance, it is the premium at which an electricity producer would rent a gas

[15] We defined *marginal* rental costs. While market price signals correspond to marginal costs, Jorgenson (1963, p. 143) omits the word "marginal". He uses linear investment costs, for which no distinction needs to be done between average and marginal costs.

power plant during one year, compared to a coal power plant (and expressed in dollars per avoided ton of carbon). The RHS is the benefit of doing so, that this the price of avoided GHG emissions.[16] For instance, it stands for the price of carbon in an emission trading system. In this sense, the MIRCC can be called *marginal abatement cost*, and (19) simply stands for the well-known rule that marginal abatement costs should be equal to the carbon price at each point in time and in every sector.[17]

Prop. 5 also means that the cost-efficiency of investments is more complex to assess when investment costs are endogenous than when they are exogenous. Exposing a case of *exogenous* investment cost, Jorgenson (1967, p. 145) emphasized: "*It is very important to note that the conditions determining the values [of investment in capital] to be chosen by the firm [...] depend only on prices, the rate of interest, and the rate of change of the price of capital goods for the current period.*"[18] In other words, when investment costs are exogenous, current price signals contain all the information that private agents need to take socially-optimal decisions.

In contrast, in the case exposed here — with endogenous investment costs and maximum abatement potentials — the signal given by current prices is incomplete. To determine the optimal amount of investment in a given sector, the carbon price at t must be completed with the correct anticipation of the date T_i when the all emissions in sector i will be capped, and with the long-term replacement cost of abatement capital $c'(\delta\bar{a})$.[19]

4.3. An operational metrics: the levelized abatement cost

A natural metric to measure and compare the cost of abatement investments in different sectors is the ratio of investment (e.g in dollars) to abated GHG (e.g in tCO_2).

Definition 2. *We call Levelized Abatement Cost (LAC) the ratio of marginal investment to discounted abatement.*

Practitioners often use LACs when comparing and assessing abatement investments, such as replacing conventional cars with electric vehicles (EV). Assume the additional cost of an EV built at time t, compared to the cost of a conventional car, is 7 000 \$/EV. This figure may include, in addition to the higher upfront cost of the EV, the lower discounted operation and maintenance costs. If cars are driven 13 000 km per year and electric cars emit 110 gCO_2/km less than a comparable internal combustion engine vehicle, each EV allows to save 1.43 tCO_2/yr. The MIC in this case would be 4 900 \$/(tCO_2/yr). If electric cars depreciate at a constant rate such that their average lifetime is 10 years ($1/\delta_i = 10$ yr) , then $r + \delta_i = 15\%$/yr and the LAC is 734 \$/tCO_2.[20]

[16] Recall that abatement capital is measured in terms of avoided carbon emissions.

[17] The optimal cost of carbon μ itself can be called *marginal abatement cost*, it measures the discounted cost of tightening the carbon budget by one ton of CO_2.

[18] In the present model, these correspond respectively to the current price of carbon μe^{rt}, the discount rate r, and the *endogenous* current change of MIC $d{c_i}'(x_{i,t})/dt$.

[19] Alternatively, one can argue that investment made at t requires to know the full price signal $\{\mu e^{r\theta} - \lambda_{i,\theta}\}_{\theta \in [t,\infty)}$, as in Eq. 20 (instead of the date T_i as in Eq. 19).

[20] The MIC was computed as 7 000 \$/(1.43 tCO_2/yr) = 4 895 \$/(tCO_2/yr); and the LAC as $0.15\,\mathrm{yr}^{-1} \cdot 4\,895$ \$/(tCO_2/yr)= 734 \$/tCO_2.

Proposition 6. *Levelized Abatement Costs, denoted ℓ_t, read:*

$$\ell_{i,t} = (r + \delta_i)\, c_i{}'(x_{i,t}) \tag{21}$$

Proof. See Appendix F. □

Like the rental cost introduced introduced by Jorgenson (see above), this metrics can be computed by myopic agents (e.g., investors or regulators) using only current prices. LACs are homogeneous to a carbon price, but unlike the carbon price (see above), the LACs entirely characterise an investment pathway.[21]

LACs may be interpreted as MICs annualized using $r+\delta$ as the discount rate (taking the carbon price as given, one unit of investment in abatement capital generates a flow of real revenue that decreases at the rate $r + \delta$).

Under the assumption that investment costs are linear, LACs computed this way should be equal to the carbon price (see Appendix G). However, when the dynamics of capital accumulation are taken into account, this result does not hold:

Proposition 7. *In general, optimal LACs are different in different sectors, and different from the carbon price.*

Proof. See (15) and (21).

In theory, the LAC is therefore not a good tool to assess and compare abatement investments in different sectors. In the next section, we compute numerically optimal investment pathways calibrated on IPCC data. We find that along the optimal pathway, different sectors may invest in abatement capital at drastically different LACs.

5. Illustrative examples using IPCC abatement costs

In this section, we use the two models (abatement cost curves as in section 2, or abatement capital accumulation as in section 3) to investigate the optimal sectoral abatements over the 2007-2030 period. We set a policy objective over this period only,[22] and use abatement cost information derived from IPCC (2007, Fig. SPM 6). Because of data limitations, this exercise is not supposed to suggest an optimal climate policy. It aims at illustrating the impact of two contrasting approaches to model emission reductions on the optimal abatement strategy — and in particular on the choice of the sectors where short-term emission-reduction efforts should be directed.

5.1. Specification and calibration

We extend the problem exposed in section 2 to the case of seven sectors, assuming separate potentials and quadratic abatement costs. Quadratic costs

[21] The investment can be calculated from the LAC $x_{i,t} = c_i'^{-1}\left(\frac{\ell_{i,t}}{r+\delta_i}\right)$.

[22] The infinite-horizon models exposed in sections 2 and 3 have to be modified; all the results exposed still apply.

	Abatement potential $\bar{a}_i$ [$GtCO_2$/yr]	MAC parameter γ_i^m $\left[\frac{\$/tCO_2}{GtCO_2/yr}\right]$	Depreciation rate δ_i [%/yr]	MIC parameter c_i^m $\left[\frac{\$/tCO_2}{GtCO_2/yr^3}\right]$
Waste	0.76	34	3.3	34
Industry	4.08	17.6	4	17.6
Forestry	2.75	15.9	0.8	15.9
Agriculture	4.39	11.9	5	11.9
Transport	2.1	11.6	6.7	11.6
Energy	3.68	10.3	2.5	10.3
Buildings	5.99	3.6	1.7	3.6

Table 1: Numerical values used for the numerical simulations.

are a simple specification that grants that the γ_i are convex, and simplifies the resolution as marginal abatement costs are linear:

$$\begin{aligned} \forall i,\ \forall a_{i,t} \in [0, \bar{a}_i] \quad \gamma_i(a_{i,t}) &= \frac{1}{2}\, \gamma_i^m\, a_{i,t}^2 \\ {\gamma_i}'(a_{i,t}) &= \gamma_i^m\, a \end{aligned} \tag{22}$$

where γ_i^m are parameters specific to each sector. We calibrate these using the abatements corresponding to a 20 \$/tCO_2 marginal cost in figure SPM.6 in IPCC (2007). We calibrate the sectoral potentials $\bar{a}_i$ as the potential at 100 \$/tCO_2 provided by the IPCC (this is the higher potential provided for each sector). Numerical values are gathered in Tab. 1.

In the abatement accumulation model, we also assume quadratic costs (and therefore linear marginal costs).

$$\begin{aligned} \forall i,\ \forall x_{i,t} \geq 0, \quad c_i(x_{i,t}) &= \frac{1}{2}\, c_i^m\, x_{i,t}^2 \\ {c_i}'(x_{i,t}) &= c_i^m\, x_{i,t} \end{aligned} \tag{23}$$

To calibrate the c_i^m, we ensure that relative costs (when comparing two sectors) are equal in the two models:

$$\forall (i,j), \quad \frac{c_i^m}{c_j^m} = \frac{\gamma_i^m}{\gamma_j^m} \tag{24}$$

This defines all the c_i^m off by a common multiplicative constant. We calibrate this multiplicative constant such that the discounted costs of reaching the same target are equal in the two models (this methodology was first used by Grubb et al. (1995)). This way, we aim at reducing differences in optimal strategies to the different models of emission reductions (abatement cost cuves vs. abatement capital accumulation).

We call $\bar{T} = 23$ yr the time span from the publication date of IPCC (2007) and the time horizon of IPCC data (2030). We set the discount rate to $r = 4\%$/yr. We constrain the cumulative emissions over the period as:

$$\int_0^{\bar{T}} \sum_i (\bar{a}_i - a_{i,t}) \leq B$$

To compute the carbon budget B, we chose the Representative Concentration Pathway RCP 8.5 (from WRI (20113)) as the emission baseline. An emission scenario consistent with the 2°C target is the RCP3-PD.[23] We use the difference in cumulative emissions from 2007 to 2030 in this two RCPs to calibrate $B = 153\,\mathrm{GtCO_2}$.

Finally, we estimate the depreciation rates of capital as the inverse of typical capital lifetimes in the different sectors of the economy (Philibert, 2007; World Bank, 2012, Tab. 6.1). The results are displayed in Tab. 1.

We solve the two models numerically and in continuous time. Data and source code will be available at the corresponding author's web page. Computation and plots use Scilab (Scilab Consortium, 2011).

5.2. Results

Fig. 4 compares the optimal mitigation strategy by the two models and Fig. 5 compares the aggregated pathways in terms of abatement and financial effort. Again, these are not optimal pathways to mitigate climate change: they only consider a target in the 2007-2030 window and are based on crude available data.

The two models give the same result in the long run: abatement in each sector eventually reaches its maximum potential (Fig. 4). By construction, they also achieve the aggregated abatement target at the same discounted cost. And in both models, the carbon price grows exponentially (Fig. 4, upper panels). Where the strategies from the two models differ radically is in terms of the temporal and sectoral distribution of aggregated abatement and costs.

The optimal abatement pathway in the abatement-cost-curve framework includes significant abatements as soon as the climate policy is implemented (to emphasize this, we plotted a null abatement between 2005 and the start of the climate policy in 2007). In contrast, the abatement pathway according to the abatement accumulation model starts at zero and increases continuously (Fig. 4, lower panels).

In terms of the temporal distribution of abatement costs, the abatement capital accumulation model provides an optimal pathway that starts higher and then decreases over time (Fig. 5, right). Abatement efforts are concentrated on the short term, because once all the emissions in a sector has been avoided using abatement capital, the only cost is that of maintaining the stock of abatement capital.

In the abatement-cost-curve framework, the carbon price gives a straightforward indication on where and when efforts should be concentrated. In contrast, the growing carbon price is a poor indicator of the optimal cost and timing of abatement capital accumulation (Fig. 4, higher panels).

To avoid paying too much at any point in time, investment should be spread over a larger period, and longer-to-decarbonize sectors should abate at a higher cost than the others. In this example with quadratic investment costs — i.e. with the same cost convexity accross sectors —, industry is decarbonized faster (in terms of $\delta_i\, x_{i,t}$) and at a higher cost ($\ell^\star_{i,t}$) than forestry, despite forestry being *a priori* cheaper to decarbonize (in terms of c_i^m or $\delta_i \times c_i^m$, see Tab. 1). This is

[23] Remarkably, the difference in carbon emissions in 2030 between this two RCPs amounts to $24\,\mathrm{GtCO_2/yr}$, which matches $\sum_i \bar{a}_i$ as calibrated from IPCC (Tab. 1).

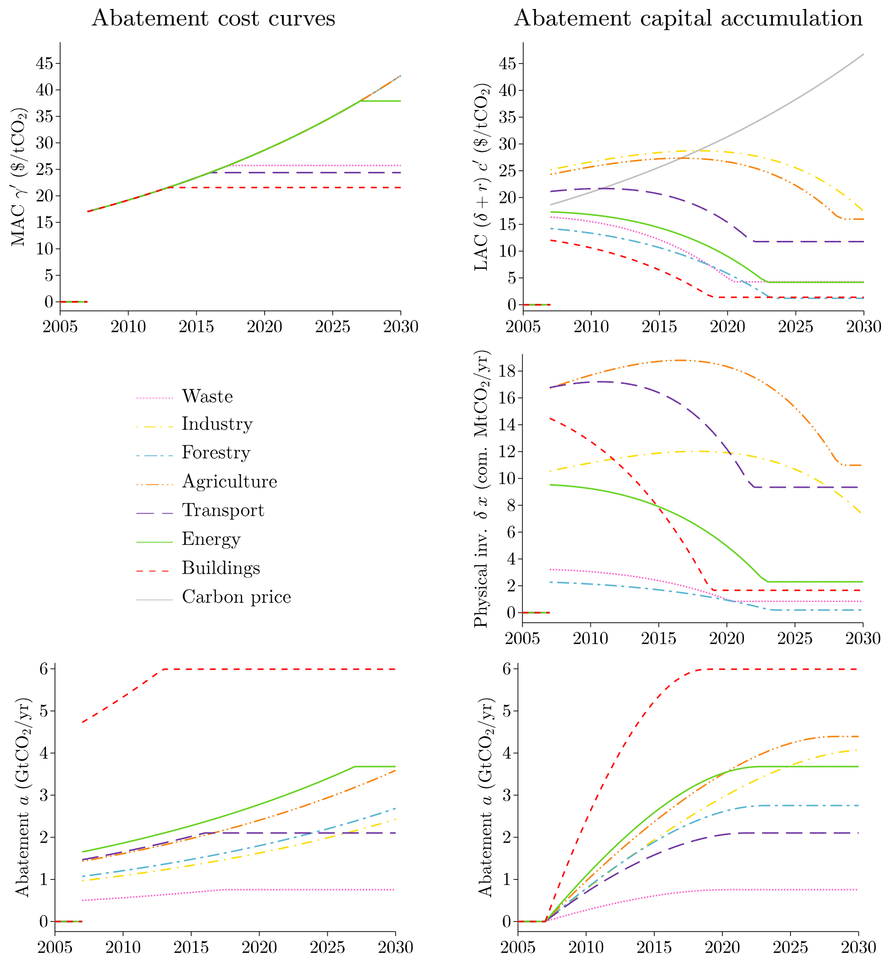

Figure 4: Comparison of optimal abatement strategies to achieve the same amount of abatement, when the costs from IPCC (2007, SPM6) are understood in an abatement-cost-curve framework (left) vs. a abatement capital accumulation framework (right). Note: For clarity, we plotted investment in *committed* $MtCO_2/yr$ ($\delta_i\ x_{i,t}$) instead of crude investment ($x_{i,t}$ in $\mathrm{MtCO_2/yr^2}$). With this metrics, one thousand electric vehicle built in 2010 that will each save 14 $\mathrm{tCO_2}$ during their lifetime count as 1400 $\mathrm{tCO_2/yr}$ in 2010. In the abatement-cost-curve framework, there is no equivalent to the physical abatement investments $x_{i,t}$, as the planner controls directly the abatement level $a_{i,t}$.

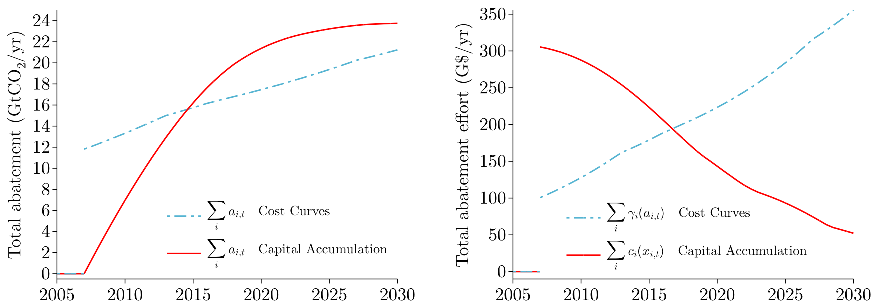

Figure 5: Comparison of the optimal timing and cost of GHG emissions in the two models (abatement cost curves vs abatement capital accumulation). When abatement may be freely chosen on a cost curve at each time step, the abatement can jump from 0 to a large amount instantaneously at the beginning of the period. When abatement requires accumulating capital, abatement has to grow continuously, and the short-term cost are higher than in the abatement-cost-curve framework. We calibrated the two models such that the total discounted cost and the cumulative abatements are equal in the two cases.

because industry takes longer to decarbonize, notably because it has a greater abatement potential (Tab. 1).

6. Conclusion

Emission reductions may be achieved through a combination of actions, some of which are best represented by abatement cost curves and others by abatement capital accumulation. Abatement cost curves are appropriate to represent actions with immediate environmental benefits and little consequences over the long-term, such as changing consumption patterns (e.g., driving less miles per year, using gas power plants more hours per year and coal power less hours per year, or reducing indoor temperatures when heating is needed). On the other hand, when abatement actions imply investments with long-term consequences (e.g., replacing thermal cars with plug-in hybrid or electric vehicles, fossil-fueled power plants with renewable power, or retrofitting buildings), they are best modeled as an accumulation of abatement capital.

We investigated the optimal cost, timing, and sectoral dispatch of greenhouse gas emission reductions in two different models, based either on *abatement cost curves* or *abatement capital accumulation*. While the carbon price grows over time in both models, they offer drastically different recommendations on the optimal abatement strategy.

In a model based on abatement cost curves, emission reductions can be decided independently at each point in time, regardless of previous abatement actions. In this framework, there is almost no difference between the cost of emission reductions and the carbon price. As a result, emission reduction efforts are mostly done in the future, when the carbon price is higher.

In a model based on abatement capital accumulation, the social planner pays for and has control on the rate of emission reductions, rather than on the emission level directly. In this case, the growing carbon price is a more distant indicator of the optimal cost of climate policies, and optimal abatement efforts are bell-shaped and concentrated over the short run.

The abatement capital accumulation model reconciles two apparently opposite views concerning the optimal dispatch of emission reductions across sectors. On the one hand, a unique, economy-wide carbon price would trigger the socially-optimal investment pathway from forward-looking investors (in the absence of any other market failure). On the other hand, higher short-term efforts are justified in sectors in which emission reductions are best represented as an abatement investment and that will take longer to decarbonize, such as transport and urban planning (Jaccard and Rivers, 2007; Lecocq and Shalizi, 2014; Vogt-Schilb and Hallegatte, 2014).

This short-term investment effort and the bell-shaped pathway can be interpreted as a transition to clean capital. In the short term, low-carbon capital has to be built to replace (or retrofit) the existing stock of polluting capital, leading to high investments needs and large transition costs. Over the long term, when the entire stock of emitting capital has been replaced or retrofitted, abatement investments need only to maintain (or grow at the economic growth rate) the stock of clean capital, and the cost is lower than during the transition phase.

Our results should be interpreted cautiously, as the model disregards mechanisms that would affect significantly the cost and timing of climate policies. For instance, we did not take into account knowledge accumulation, knowledge spillovers and economic growth. We also disregarded the effect of uncertainty on climate impacts and future technologies, limited foresight by investors, policy-makers and regulators, and the limited ability of the government to commit.

Finally, we studied abatement cost curve and abatement capital in separate models (for analytical tractability). But as already mentioned, a realistic climate strategy will include some actions that are best represented by the former and some by the latter. Analyzing a realistic strategy would thus require merging the two approaches.[24] Notwithstanding these limitations, this analysis may help clarify public economic issues related to the optimal response to environmental issues.

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Appendix A. Proof of (6) (Abatement cost curves)

The Lagrangian associated with (4) reads:

$$\begin{aligned} L(a_t, m_t, \lambda_t, \mu_t) = &\int_0^\infty e^{-rt} \sum \gamma(a_{i,t})\, dt + \int_0^\infty \lambda_t \left(a_t - \bar{a}\right)\, dt \\ &- \int_0^\infty \dot{\mu}_t m_t \, dt + \int_0^\infty \mu_t \left(\bar{a} - a_t\right)\, dt + \phi_t \left(m_t - B\right) \end{aligned} \tag{A.1}$$

The first order condition is:

$$\begin{aligned} \frac{\partial L}{\partial a_t} = 0 &\iff e^{-rt}\gamma'(a_t) + \lambda_t - \mu_t = 0 \\ &\iff \gamma'(a_t) = e^{rt}(\mu_t - \lambda_t) \qquad \text{(A.2)} \\ \frac{\partial L}{\partial m_t} = 0 &\iff \dot{\mu}_t = \phi_t \qquad \text{(A.3)} \end{aligned}$$

The steady state is reached at a date T when $\dot{m}_t = 0$, that is when the abatement potential $\bar{a}$ is reached, such that:

$$\begin{aligned} &\forall t < T,\ a_t < \bar{a} \text{ and } m_t < B \\ &\forall t \geq T,\ a_t = \bar{a} \text{ and } m_t = B \end{aligned}$$

As the associated Lagrangian multiplier, ϕ_t is null before the carbon budget is reached (complementary slackness condition):

$$\begin{aligned} &\forall t, \quad \phi_t \cdot (m_t - B) = 0 \\ \implies &\forall t < T, \quad \phi_t = 0 \end{aligned} \tag{A.4}$$

This means that the present value of carbon μ_t is constant while the carbon budget has not been reached (A.3):

$$\forall t < T, \quad \mu_t = \mu \tag{A.5}$$

For the same reason, λ_t is null before the sectoral potential becomes binding:

$$\begin{aligned} &\forall t, \quad \lambda_t \cdot (a_t - \bar{a}) = 0 \\ \implies &\forall t < T, \quad \lambda_t = 0 \end{aligned} \tag{A.6}$$

Combining (A.6) and (A.2), one gets:

$$\gamma'(a_t) = \begin{cases} \mu e^{rt} & t < T \\ \gamma'(\bar{a}) & t \geq T \end{cases} \tag{A.7}$$

Appendix B. Optimal accumulation of abatement capital (Proof of (11))

Appendix B.1. Lagrangian

The Lagrangian associated with (10) reads:

$$\begin{aligned} L(x_t, a_t, m_t, \lambda_t, \nu_t, \mu_t) = &\int_0^\infty e^{-rt} c(x_t)\, dt + \int_0^\infty \lambda_t \left(a_t - \bar{a}\right)\, dt \\ &- \int_0^\infty \dot{\mu}_t m_t\, dt + \int_0^\infty \mu_t \left(\bar{a} - a_t\right)\, dt + \phi_t \left(m_t - B\right) \\ &- \int_0^\infty \dot{\nu}_t a_t\, dt + \int_0^\infty \nu_t \left(\delta a_t - x_t\right)\, dt \end{aligned} \tag{B.1}$$

Appendix B.2. First order conditions

The first order conditions read:

$$c'(x_t) = e^{rt} \nu_t \qquad (\partial x_t) \tag{B.2}$$

$$\dot{\nu}_t - \delta \nu_t = \lambda_t - \mu_t \qquad (\partial a_t) \tag{B.3}$$

$$\dot{\mu}_t = \phi_t \qquad (\partial m_t) \tag{B.4}$$

Where ν_t is the present value of investment in low carbon capital, μ is the present cost of carbon, and λ_t is the social cost of the maximum abatement potential. The first order conditions can be rearranged:

$$\frac{dc'(x_t)}{dt} = e^{rt} \left(\dot{\nu}_t + r\, \nu_t\right) \qquad \text{(from B.2)} \tag{B.5}$$

$$= e^{rt} \left(\delta \nu_t + \lambda_t - \mu_t + r\, \nu_t\right) \qquad \text{(from B.3 and B.11)} \tag{B.6}$$

$$e^{rt} \left(\mu_t - \lambda_t\right) = (r + \delta)\, c'(x_t) - \frac{dc'(x_t)}{dt} \qquad \text{(from D.2)} \tag{B.7}$$

Appendix B.3. Steady state

The steady state is reached at a date T when the carbon budget is reached, emission become null, and investment is used to counterbalance depreciation:

$$\forall t \geq T, \quad \dot{m}_t = 0 \implies a_t = \bar{a} \implies x_t = \delta \bar{a} \tag{B.8}$$

Appendix B.4. Slackness conditions

The complementary slackness conditions mean that the carbon price is constant before the steady state:

$$\forall t < T, \quad m_t < B \ \ \& \ \ \phi_t = 0 \tag{B.9}$$

$$\implies \mu_t = \mu \tag{B.10}$$

and the social cost of the maximum potential is null before the steady state:

$$\begin{aligned} &\forall t < T, \quad a_t < \bar{a} \ \ \& \ \ \lambda_t = 0 \\ &\forall t \geq T, \quad a_t = \bar{a} \ \ \& \ \ \lambda_t \geq 0 \end{aligned} \tag{B.11}$$

Appendix B.5. Marginal Implicit Rental Cost of Capital

Before T, (B.7) simplifies:

$$\forall t \leq T, \quad (r+\delta)\, c'(x_t) - \frac{dc'(x_t)}{dt} = \mu e^{rt} \tag{B.12}$$

The textbook solution of this first order linear differential equation is:

$$\forall t \leq T, \quad c'(x_t) = e^{(r+\delta)t} \int_t^T e^{-(r+\delta)\theta} \mu e^{r\theta} \, d\theta + e^{(r+\delta)t}\, C \tag{B.13}$$

Where C is a constant. Any C chosen such that $c'(x_t)$ remains positive defines an investment pathway that satisfies the first order conditions. The optimal investment pathways also satisfies the following boundary conditions.

Boundary conditions

After T, a_t is constant and the investment x_t is used to counterbalance the depreciation of abatement capital.

$$c'(x_T) = c'(\delta \bar{a}) \qquad \text{(from eq. D.5)} \tag{B.14}$$

Optimal marginal investment costs (MICs)

Injecting B.14 in B.13 and re-arranging, one gets:

$$c'(x_t^\star) = \mu e^{rt} \int_t^T e^{-\delta(\theta - t)} \, d\theta + e^{-(\delta+r)(T-t)}\, c'(\delta \bar{a}) \tag{B.15}$$

Appendix C. Optimal accumulation of infinitely-lived abatement capital

Here, we solve the model (10) in the particular case of infinitely-lived abatement capital (that is $\delta = 0$). In this case, B.13 can be rewritten as:

$$c'(x_t^\star) = e^{rt} \int_t^\infty (\mu - \lambda_\theta)\, d\theta \tag{C.1}$$

Eq. C.1 means that optimal investment cost in abatement capital equals the value of the carbon (μ) that will be saved during the capital lifetime ($\int_t^\infty$) minus the social cost of the maximum potential λ_θ. Again, investment in abatement capital remove both emissions and future investment opportunities.

Along the steady state ($t \geq T$), abatement is constant ($a_t = \bar{a}$), thus $x_t = \dot{a}_t = 0$ hence $c'(x_t) = 0$ and:

$$\forall t \geq T, \quad \lambda_t = \mu \tag{C.2}$$

This means that once the potential $\bar{a}$ becomes binding, the associated shadow cost λ_t equals the value of the carbon that it prevents to abate.

Combining the to equations, we can express the optimal investment:

$$c'(x_t^\star) = \begin{cases} \mu e^{rt}(T - t) & \text{if } t < T \\ 0 & \text{if } t \geq T \end{cases} \tag{C.3}$$

The optimal cost of infinitely-lived abatement capital equals the total social cost of the carbon — expressed in current value (μe^{rt}) — that will be saved thanks to the abatement *before the sectoral potential is reached* — that is during the time span $(T - t)$.

Corollary 3. *When the abatement capital is infinitely-lived ($\delta = 0$) and the carbon price is given, the decarbonizing date T is an increasing function of the abatement potential $\bar{a}$*

Proof. As c' is by assumption (9) strictly growing, it is invertible. Let χ be the inverse of c'; applying χ to (C.3) gives:

$$x_t = \begin{cases} \chi\left(e^{rt}(T - t)\mu\right) & \text{if } t < T \\ 0 & \text{if } t \geq T \end{cases} \tag{C.4}$$

The relation between the sectoral potential ($\bar{a}$), the MICs (through χ), the cost of carbon (μ) and the time it takes to achieve the sectoral potential T reads:

$$\begin{aligned} \bar{a} &= a(T) \\ &= \int_0^T \chi\left(e^{rt}(T - t)\mu\right) dt \end{aligned}$$

Let us define f_χ such that:

$$\begin{aligned} f_\chi(t) &= \int_0^t \chi\left(e^{r\theta}(t - \theta)\mu\right)\, d\theta \\ \implies \frac{df_\chi}{dt}(t) &= \int_0^t e^{r\theta} \chi'\left(e^{r\theta}(t - \theta)\mu\right)\, d\theta \end{aligned}$$

Let us show that f_χ is invertible: $\chi' > 0$ as the inverse of $c' > 0$, thus $\frac{df_\chi}{dt} > 0$ and therefore f_χ is strictly growing. Finally:

$$\bar{a} \mapsto T = {f_\chi}^{-1}(\bar{a}) \quad \text{is an increasing function}$$

For a given marginal cost function,[25] T can always be found from $\bar{a}$. The larger the potential, the longer it takes for the optimal strategy to achieve it.

Appendix D. Optimal investment in multiple sectors

Appendix D.1. Lagrangian

The Lagrangian associated with (14) reads:

$$\begin{aligned} L(x_{i,t}, a_{i,t}, m_t, \lambda_{i,t}, \nu_{i,t}, \mu_t) = & \int_0^\infty e^{-rt} c(x_{i,t})\, dt + \int_0^\infty \lambda_{i,t} \left(a_{i,t} - \bar{a}_i\right)\, dt \\ & - \int_0^\infty \dot{\mu}_t m_t \, dt + \int_0^\infty \mu_t \sum_i \left(\bar{a}_i - a_{i,t}\right) dt + \phi_t \left(m_t - B\right) \\ & - \int_0^\infty \dot{\nu}_{i,t} a_{i,t}\, dt + \int_0^\infty \nu_{i,t} \left(\delta_i a_{i,t} - x_{i,t}\right)\, dt \end{aligned} \tag{D.1}$$

Appendix D.2. First order conditions

The first order conditions read:

$$c_i{}'(x_{i,t}) = e^{rt} \nu_{i,t} \qquad (\partial x_{i,t}) \tag{D.2}$$
$$\dot{\nu}_{i,t} - \delta_i \nu_{i,t} = \lambda_{i,t} - \mu_t \qquad (\partial a_{i,t}) \tag{D.3}$$
$$\dot{\mu}_t = \phi_t \qquad (\partial m_t) \tag{D.4}$$

Where $\nu_{i,t}$ is the present value of investment in low carbon capital, μ is the present cost of carbon, and $\lambda_{i,t}$ is the social cost of the sectoral potential.

Appendix D.3. Steady state

The steady state is reached at a date T_m when the carbon budget is reached. After this date, emission are null in every sector and investment is used to counterbalance depreciation:

$$\forall t \geq T_m, \quad \dot{m}_t = 0 \implies \forall i, a_{i,t} = \bar{a}_i \implies x_{i,t} = \delta_i \bar{a}_i \tag{D.5}$$

Denoting T_i the date when the all emissions in sector i are capped ($a_{i,t} = \bar{a}_i$), it is immediate that $T_m = \max_i(T_i)$.

[25]When c' is given, χ and therefore f_χ are also given.

Appendix D.4. Slackness conditions

The complementary slackness conditions mean that the carbon price is constant before the steady state:

$$\forall t < T_m, \quad m_t < B \quad \& \quad \phi_t = 0 \tag{D.6}$$

$$\implies \mu_t = \mu \tag{D.7}$$

and the social costs of the sectoral potentials are null before the respective dates T_i:

$$\forall t < T_i, \quad a_{i,t} < \bar{a}_i \quad \& \quad \lambda_{i,t} = 0 \tag{D.8}$$

The rest of the demonstration is similar to the case of one single sector (from Appendix B.5).

Appendix D.5. Optimal marginal investment cost differ across sectors

Proof. We use a proof by contradiction. Let two sectors be such that they exhibit the same investment cost function, the same depreciation rate, but different abatement potentials:

$$\forall x > 0, \; c_1'(x) = c_2'(x), \; \delta_1 = \delta_2, \; \bar{a}_1 \neq \bar{a}_2$$

Suppose that the two sectors take the same time to decarbonize (i.e. $T_1 = T_2$). Optimal MICs would then be equal in both sectors (11). This would lead to equal investments, hence equal abatement, in both sectors at any time (7,8), and in particular to $a_1(T_1) = a_2(T_2)$. By assumption, this last equality is not possible, as:

$$a_1(T_1) = \bar{a}_1 \neq \bar{a}_2 = a_2(T_2)$$

Therefore, different potentials $\bar{a}_i$ have to lead to different optimal decarbonizing dates T_i, and therefore to different optimal LACs $\ell_t^\star$.

A similar reasoning can be done concerning two sectors with the same investment cost functions, same potentials, but different depreciation rates; or two sectors that differ only by their investment cost functions. □

Appendix E. An intuitive approach to the MIRCC

Consider an investment strategy that increases abatement in a sector at one date while keeping the rest of the abatement trajectory unchanged (Fig. E.6). Such a strategy is similar to renting one unit of capital to reduce emissions during one time period. It reduces one unit of GHG, but leaves unchanged any opportunity to invest later in the same sector.

From an existing investment pathway $(x_{i,t})$ leading to an abatement pathway $(a_{i,t})$, the social planner may increase investment by one unit at time θ and immediately reduce investment by $1 - \delta \, d\theta$ at the next period $\theta + d\theta$.

The resulting investment schedule $(\tilde{x}_t)$ leads to an abatement pathway $(\tilde{a}_t)$ that abates one supplementary unit of GHG between θ and $\theta + d\theta$ (Fig. E.6). Moving from $(x_{i,t})$ to $(\tilde{x}_t)$ costs:

$$\mathcal{P} = \frac{1}{d\theta} \left[{c_i}'(x_\theta) - \frac{(1 - \delta_i \, d\theta)}{(1 + r \, d\theta)} \, c_i'(x_{i,\theta + d\theta}) \right] \tag{E.1}$$

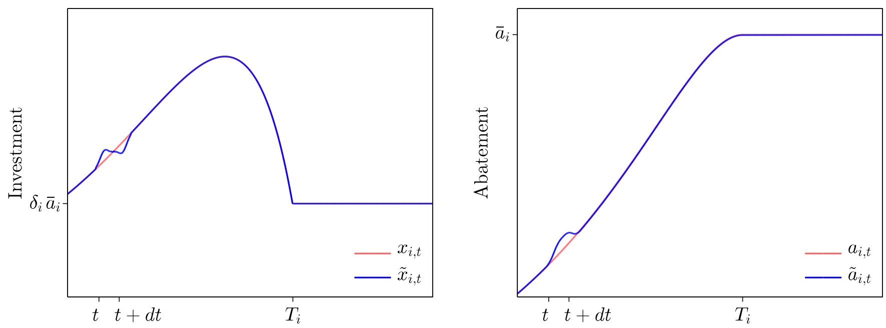

Figure E.6: From a given investment pathway $(x_{i,t})$ leading to the abatement pathway $(a_{i,t})$, saving one more unit of GHG at a date θ without changing the rest of the abatement pathway, as in $(\tilde{a}_t)$, requires to invest one more unit at θ and $(1-\delta_i\,d\theta)$ less at $\theta+d\theta$, as $(\tilde{x}_t)$ does.

For marginal time lapses, this tends to :

$$\mathcal{P} \xrightarrow[d\theta\to 0]{} (r+\delta_i)\; c_i{}'(x_\theta) - \frac{dc_i{}'(x_\theta)}{d\theta} \tag{E.2}$$

$\mathcal{P}$ tends to the cost of renting one unit of abatement capital at t.

Appendix F. Proof of the expression of ℓ_t in Prop. 6

Let h be a marginal physical investment in abatement capital made at time t in sector i (expressed in tCO_2/yr per year). It generates an infinitesimal abatement flux that starts at h at time t and decreases exponentially at rate δ_i, leading to the total discounted abatement ΔA (expressed in tCO_2):

$$\Delta A = \int_{\theta=t}^{\infty} e^{r(\theta-t)} h\, e^{-\delta_i(\theta-t)}\, d\theta \tag{F.1}$$

$$= \frac{h}{r+\delta_i} \tag{F.2}$$

This additional investment h brings current investment from $x_{i,t}$ to $(x_{i,t}+h)$. The additional cost ΔC (expressed in \$) that it brings reads:

$$\Delta C = c_i(x_{i,t}+h) - c_i(x_{i,t}) \underset{h\to 0}{=} h\, c_i{}'(x_{i,t}) \tag{F.3}$$

The LAC ℓ_t is the division of the additional cost by the additional abatement it allows:

$$\ell_t = \frac{\Delta C}{\Delta A} \tag{F.4}$$

$$\ell_t = (r+\delta_i)\, c_i{}'(x_{i,t}) \tag{F.5}$$

Appendix G. Levelized costs and implicit rental cost when investment costs are exogenous and linear

Let I_t be the amount of investments made at exogenous unitary cost Q_t to accumulate capital K_t that depreciates at rate δ:

$$\dot{K}_t = I_t - \delta\, K_t \tag{G.1}$$

Let $F(K_t)$ be a classical production function, and the price of output be the numeraire. Jorgenson (1967) defines current receipts R_t as the actual cash flow:

$$R_t = F(K_t) - Q_t\, I_t \tag{G.2}$$

he finds that the solution of the maximization program

$$\max_{I_t} \int_0^\infty e^{-rt} R_t\, dt \tag{G.3}$$

does not equalize the marginal productivity of capital to the investment costs Q_t:

$$F_K(K_t^\star) = (r+\delta)\, Q_t - \dot{Q}_t \tag{G.4}$$

He defines the implicit rental cost of capital C_t, as the accounting value:

$$C_t = (r+\delta)\, Q_t - \dot{Q}_t \tag{G.5}$$

such that the solution of the maximization program is to equalize the marginal productivity of capital and the rental cost of capital:

$$F_K(K_t^\star) = C_t \tag{G.6}$$

He shows that this is consistent with maximizing discounted economic profits, where the current profit is given by the accounting rule:

$$\Pi_t = F(K_t) - C_t\, K_t \tag{G.7}$$

that is, profits equal revenue less the total cost of renting the capital.

In this case, the (unitary) levelized cost of capital L_t is given by:

$$L_t = (r+\delta)\, Q_t \tag{G.8}$$

And the levelized cost of capital matches the optimal rental cost of capital if and only if investment costs are constant:

$$\dot{Q}_t = 0 \iff F_K(K_t^\star) = L_t \tag{G.9}$$