Explaining the Spread of Temporary Jobs and its Impact on Labor Turnover

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Abstract

This paper provides a simple model which explains the choice between permanent and temporary jobs. This model, which incorporates important features of actual employment protection legislations neglected by the economic literature so far, reproduces the main stylized facts about entries into permanent and temporary jobs observed in Continental European countries. We show that the stringency of legal constraints on the termination of permanent jobs has a strong positive impact on the turnover of temporary jobs. We also find that job protection has very small effects on total employment but induces large substitution of temporary jobs for permanent jobs which significantly reduces aggregate production.

Key words: Temporary jobs, Employment protection legislation.

JEL classification: J63, J64, J68.

1 Introduction[1]

It is recurrently argued that the dramatic spread of temporary jobs in Continental European countries is the consequence of the combination of stringent legal constraints on the termination of permanent jobs and of weak constraints on the creation of temporary jobs.[2] Strikingly, however, very little is known about the creation of temporary and permanent jobs, inasmuch as very few contributions have analyzed the choice between these two types of job. There are also very few explanations of the duration of temporary jobs.

Our paper contributes to filling this gap. It provides a model which explains the duration of temporary jobs and the choice between temporary and permanent jobs. This model reproduces important stylized facts that previous models were unable to explain. In particular, for countries with stringent job protection, the model fits the large share of temporary contracts in employment inflows, the huge amount of creation of temporary contracts of very short duration, and the large contribution of inflows into temporary jobs to fluctuations in employment inflows overall. The model sheds new light on the consequences of employment protection. It shows that the stringency of legal constraints on the termination of permanent jobs has very little effect on total employment, but does induce a large-scale substitution of temporary jobs for permanent jobs which significantly reduces aggregate production.

One of the main originalities of our approach is to account for important features of employment protection legislations which have been neglected by the literature so far. In most countries, it is costly to dismiss temporary workers before the date of termination of the contract stipulated when the job starts. More precisely, in the 'French type' regulation, that prevails in Belgium, France, Greece, Italy and Germany, temporary contracts cannot be terminated before their expiration date,[3] whereas in the 'Spanish type' regulation, which covers Spain and

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[2] See Boeri (2011) for a synthesis.

[3] There are obviously exceptions to this general rule, for instance for misbehavior **on the part** of one of the parties. The legislations are described in appendix A. For a given employment spell, it appears that it is generally at least as costly to terminate a temporary contract before its date of termination as to terminate a regular contract.

Portugal, the rule for dismissals before the expiration date of temporary contracts is the same as for permanent contracts.[4] Hence, for a given employment spell, it is generally at least as costly to terminate a temporary contract before its expiration date as it is to terminate a regular contract. In the previous literature, it is generally assumed that it is costly to terminate permanent contracts, whereas temporary contracts can be terminated at no cost at any time. This assumption, made for the sake of technical simplicity, is at odds with many actual regulations. It implies that employers prefer temporary jobs, which can be destroyed at no cost, to permanent jobs, which are costly to destroy, thus making it difficult to explain the choice between permanent and temporary jobs. Our more realistic approach assumes that temporary contracts cannot be terminated before their expiration date.

We consider a job search and matching model where firms hire workers to exploit production opportunities of different expected durations. Some production opportunities are expected to end (i.e. to become unproductive) quickly, others are expected to last longer. This assumption takes into account the heterogeneity of expected durations of production opportunities which is an important feature of modern economies. For instance, firms can get orders for their products for periods of several days, several months or several years, and it is not certain that these orders will be renewed. In the model, jobs can be either permanent or temporary. Permanent employees are protected by dismissal costs. Temporary jobs can be destroyed at zero cost at their expiration date, which is chosen at the instant when workers are hired. But employers have to keep and pay their employees until the date of termination of temporary jobs. These assumptions about employment legislation, which are framed to match the main features of Continental European labor regulations, do not induce Pareto optimal allocations. However, permanent workers protected by firing costs may give moral and political support to such regulations.[5]

When firing costs are sufficiently small, we find that all production opportunities are exploited with permanent jobs. When firing costs are relatively large, permanent jobs are chosen to exploit production opportunities expected to endure for a long time, while temporary jobs are used for production opportunities with short expected durations. In this framework, higher firing costs increase the share of entries into temporary jobs.

[4]Henceforth, we focus on regulation of the French type. We show in Cahuc, Charlot and Malherbet (2012) that the Spanish type yields the same outcome as the French type in the context of our model.

[5]See Saint-Paul (1996, 2002).

We show that our model matches the main stylized facts concerning entries into permanent and temporary jobs in Continental European countries. Moreover, simulation exercises show that the durations of temporary jobs are much shorter than the durations of production opportunities. Therefore, higher firing costs, by increasing the share of temporary jobs, induce a strong excess of labor turnover on production opportunities with relatively short durations. This excess of labor turnover is detrimental to temporary workers whose expected job duration becomes shorter when the employment protection of permanent jobs becomes more stringent. In this context, heightened protection for permanent jobs will have very small negative effects on aggregate employment. However, this small aggregate impact is the net consequence of two large counteracting effects: a strong decrease in the number of permanent jobs and a strong increase in the number of temporary jobs. This large reallocation of jobs, which conforms to empirical evidence,[6] decreases aggregate production, because the production (net of labor turnover costs) of temporary jobs is much smaller than that of permanent jobs. All in all, our model shows that protection of permanent jobs has very small effects on aggregate employment, but induces employment composition effects that significantly reduce aggregate production. Changes in aggregate production are 6.5 to 20 times larger than changes in aggregate employment.

Our paper is related to at least three strands of the literature.

First, we introduce heterogeneity of idiosyncratic productivity shock arrival rates into the job search model. This allows us to explain empirical evidence which indicates that the expected duration of production opportunities is a major motive for using temporary jobs when the destruction of permanent jobs is costly. Indeed, it turns out that the share of temporary contracts is higher in industries with higher labor turnover in countries with stringent job protection (Bassanini and Garnero, 2013). Drager and Marx (2012) find, using a large firm-level data set from 20 countries, that workload fluctuations strongly increase the probability of hiring temporary workers in rigid labor markets, but that no such effect is observed in flexible labor markets. Strikingly, we are not aware of any model that explains such facts. Our model sheds light on the impact of temporary contracts from a perspective different from the one in which temporary contracts are viewed as a way of screening workers before they are promoted into permanent jobs.[7] Actually, in all countries, permanent contracts comprise probationary periods,

[6] See among others Autor (2003), Kahn (2010), Centeno and Novo (2011), Cappellari et al. (2011), Hijzen et al. (2013)

[7] See Bucher (2010), Faccini (2013), Kahn (2010), Nagypal (2002), Portugal and Varejão (2009).

with no firing cost and very short notice, which are used to screen workers into permanent jobs. The maximum mandatory duration of probationary periods is around several months, depending on countries, industries and skills.[8] To the extent that temporary jobs cannot be terminated before their expiration date, it can only be profitable to screen workers by means of temporary contracts if the duration of the probationary period is too short, at least shorter than that of temporary contracts.[9] Accordingly, the view that temporary contracts are used to screen workers can be useful to explain the spread of temporary jobs lasting longer than the probationary period of permanent jobs. But this approach cannot explain the huge amount of creation of temporary contracts of very short spell, much shorter than that of probationary periods.[10] For instance, in France, the average duration of temporary jobs is about one month and a half, while the probationary periods last at least two months and can go to eight months.[11]

Second, we complement the literature on the impact of employment protection legislation[12] by explaining the choice between permanent and temporary jobs. Most of this literature does not explain this choice.[13] Usually, in this literature, temporary jobs, which can be destroyed at zero cost, are preferred to permanent jobs, which are costly to destroy, and it is either assumed that all new jobs are temporary, or that the regulation forces firms to create permanent jobs. As far as we know, four papers explain the choice between temporary and permanent jobs in a dynamic setting.[14] Berton and Garibaldi (2012) propose a matching model with directed search

[8] See: http://www.ilo.org/dyn/eplex/termmain.home?p_lang=uk.

[9] In general, the probationary period of temporary jobs is much shorter than that of permanent jobs. Furthermore, when a temporary job is transformed into a permanent job, the duration of the temporary job has to be subtracted from the duration of the probationary period of the permanent job.

[10] To the extent that workers can be dismissed at zero cost during probationary periods, at first sight it is more profitable to exploit job opportunities expected not to last long with permanent contracts that are terminated at no cost during the probationary periods, rather than with temporary contracts that cannot be terminated before their date of termination even if the job becomes non profitable. However this type of behavior is illegal. An employer who systematically hires workers under permanent contracts and dismisses them during the probationary period instead of using temporary contracts runs the risk of being prosecuted. Our paper does not account for probationary periods, which are left for future research. We merely assume that permanent workers are protected by firing costs from the start of their contract.

[11] In France, the legal maximum duration of the probationary period for permanent contract goes from 2 months for blue collar workers to 4 months for white collar workers. The probationary period can be renewed once if this is stipulated in the labor contract.

[12] See among others, Lazear (1990), Bentolila and Saint-Paul (1992), Saint-Paul (1996), Ljungqvist (2002), l'Haridon and Malherbet (2009).

[13] See, among others: Blanchard and Landier (2002), Cahuc and Postel-Vinay (2002), Boeri and Garibaldi (2007), Sala, Silva and Toledo (2012), Costain, Jimeno and Thomas (2010), Bentolila et al. (2012), Saint-Paul (1996).

[14] Kahn (2010) provides a static two period model where temporary jobs are used to screen workers.

and exogenous wages in which firms are willing to open permanent jobs when their job filling rate is faster than that of temporary jobs. The model features a sorting of firms and workers into permanent and temporary jobs. This model, which provides an endogenous explanation for the coexistence of permanent and temporary contracts, predicts that temporary workers have shorter unemployment durations than permanent workers, which appears to be true in empirical analysis. Caggese and Cunat (2008) consider the optimal dynamic employment policy of a firm that faces capital market imperfections and can hire two types of labor: one that is totally flexible (fixed-term contracts) and one that is subject to firing costs (permanent contracts). They assume that both are perfect substitutes, but that permanent employment is relatively more productive. This implies that a firm without financing constraints would hire permanent workers up to the point where expected firing costs are equal to the productivity gain with respect to temporary workers. Cao, Shao and Silos (2010) provide a matching model where firms find it optimal to offer high-quality matches a permanent contract because temporary workers continue to search on the job while permanent workers do not. Finally, Alonso-Borrego, Galdon-Sanchez and Fernandez-Villaverde (2011) assume that permanent and temporary jobs have different firing costs and hiring costs. In these papers, the duration of temporary jobs is exogenous and it is assumed that firms can dismiss workers before the date of termination of temporary contracts. We use an alternative approach, consistent with actual employment protection legislations of Continental European countries, where the duration of temporary jobs is chosen by employers and workers and where workers cannot be dismissed before the date of termination of temporary contracts or where the rule for dismissals is the same for temporary and permanent contracts .

Third, some papers explain why short-term contracts and long-term contracts may coexist in the absence of employment protection legislation. This issue is particularly relevant to understanding the emergence of temporary contracts in labor markets where there is little difference between the termination costs of temporary and permanent contracts, as in some Anglo-Saxon countries. Smith (2007) has provided a stock-flow matching model where it can be optimal to hire workers of low profitability on a temporary basis in order to be positioned to hire more profitable workers when the stock of job seekers has been sufficiently renewed. This model offers an underlying rationale for why some employment is limited in duration. It also explains the duration of temporary contracts. In our approach, which is complementary, the utilization of temporary contracts does not hinge on a stock-flow matching model but on the heterogeneity of expected production opportunity durations in an environment where there is a legal menu of contracts. Moreover, contrary to Smith, we assume a labor market with free entry. Macho-Stadler, Pérez-Castrillo and Porteiro (2011) provide an alternative explanation where long-term contracts allow the better provision of incentives because firms can credibly transfer payments from earlier to later periods in the life of the workers, and this transfer alleviates the incentive compatibility constraint. In this setup, short-term contracts can emerge in equilibrium because they allow the market to ensure a better matching between agents' abilities and firms' needs.

Our paper is organized as follows. Stylized facts are presented in section 2. A benchmark search and matching model is developed in section 3. Section 4 extends the benchmark model to a more realistic environment and provides simulation exercises that enable us to evaluate the impact of the regulation of job protection on labor turnover, employment and aggregate production. Section 5 states our conclusions.

2 Stylized facts

This section presents three important stylized facts about entries into employment in France and in Spain.[15]

First, most entries into employment are into temporary jobs.[16] Figures 1 and 2 display employment inflows, from unemployment and inactivity, by type of job in France and Spain over the period 2000-2010. These figures show that about 90 percent of entries are into temporary jobs in both countries. These figures do not take into account conversions of temporary jobs into permanent jobs, since they display employment inflows from unemployment and inactivity. In France, about 5.5 percent of temporary jobs are converted into permanent jobs (Le Barbanchon and Malherbet, 2013). This means that about one third of entries into permanent jobs are conversions of temporary jobs, while the other two thirds originate from unemployment and inactivity.[17] In Spain, about 3.5 percent of temporary jobs are transformed into perma-

[15] The choice of France and Spain is motivated by the availability of data (ACOSS and DARES for France, Spanish State Employment Office for Spain). As far as we are aware, other continental European countries have only limited information on entries into employment by type of labor contracts.

[16] In what follows, temporary jobs comprise all fixed-term jobs, including jobs filled through temporary work agencies.

[17] For 100 entries into employment from unemployment and inactivity, there are about 10 entries into perma-

nent jobs,[18] meaning that about one quarter of entries into permanent jobs are conversions of temporary jobs.

The second stylized fact is that the duration of most temporary jobs is very short.

Figure 1 shows that temporary jobs of spells shorter than one month account for two third of entries into employment in France. One month is much shorter than the maximum duration of temporary contracts, which is 24 months. It is also much shorter than the duration of the probationary period of permanent jobs, which is two months for low skilled workers; three months for supervisors and technicians; and four months for managers. The probationary period can be renewed once if expressly provided for under the applicable branch-level collective bargaining agreement. Most collective bargaining agreements provide for probationary periods of between 2 and 3 months for low skilled workers, and between 4 and 6 months for managers, including any renewal. The average probationary period is about 3.75 months (OECD, 2013), while the average duration of temporary jobs is about 1.5 months.

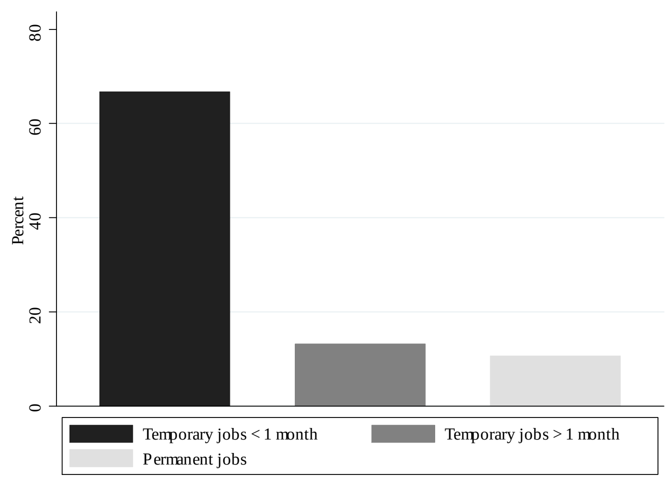

Figure 1: The share of entries into employment according to job type in France over the period 2000-2010. Source: Acoss and DARES, French Ministry of Labor.

In Spain, Figure 2 shows that a large share of entries into temporary jobs are on jobs of very short spell, as in France. The share of entries into contracts of short spell in total employment inflows is large. It amounts to 50 percent for spells below one month and to 10 percent for spells

nent jobs and 90 entries into temporary jobs. 5.5% of these temporary jobs are converted into permanent jobs, which amounts to 4.95 percent of the 100 entries.

[18]Source: Spanish Ministry of Labor and Immigration, Movimiento Laboral registrado, 2012.

between 1 and 2 months. These figures are significantly smaller than in France, suggesting that these entries are less systematically recorded in Spain than in France. One of the reasons might be that data for France come from registers of all new contracts whereas data for Spain come from social security records in which several consecutive contracts in the same firm might be consolidated as a single employment spell. Nevertheless, available information for Spain does confirm that the spell of the vast majority of temporary contracts is far below the upper limit of 24 months. It is also below the average duration of probationary periods. Until 2012, in Spain, the length of probationary periods could not be longer than 6 months for blue collars or two months for other workers (3 months in firms with less than 25 workers). But collective agreements may reduce the length of probationary periods, and in fact they do reduce the probationary period for blue collars. The average length of the probationary period is about 1.5 months for blue collars.[19]

All in all, figures 1 and 2 show clearly that the vast majority of entries into temporary employment are on temporary jobs much shorter than the maximum duration of such jobs and shorter than the probationary period of permanent jobs in France and in Spain. Hence, in most cases, it does not appear that temporary jobs are being created to gain more time to screen workers than the probationary period allows. Most temporary jobs are created because the duration of the production opportunities for which these jobs are created is expected to be short.

As a consequence of the large share of entries into jobs of short duration, the number of entries into employment is very large in both countries, as shown by table 1. In France, the ratio of annual entries into employment over the stock of jobs is equal to 1.88. In Spain, the ratio is about 1.24. As noted above, this ratio might appear smaller than in France due to the fact that not all entries into employment are reported in Spain.

The third stylized fact is that the principal part of fluctuations in employment inflows is due to inflows into temporary jobs. In France changes in total employment inflow are mainly driven by temporary jobs, as shown by figure 3 which displays the deviations of the number of entries into employment with respect to the trend. The average gap between the number of entries and its trend is seven times larger for temporary jobs than for permanent jobs. In particular, at the beginning of the recession that started in 2008, we see a strong drop in entries

[19] Registered collective agreements data set, Ministry of Labor, Spain.

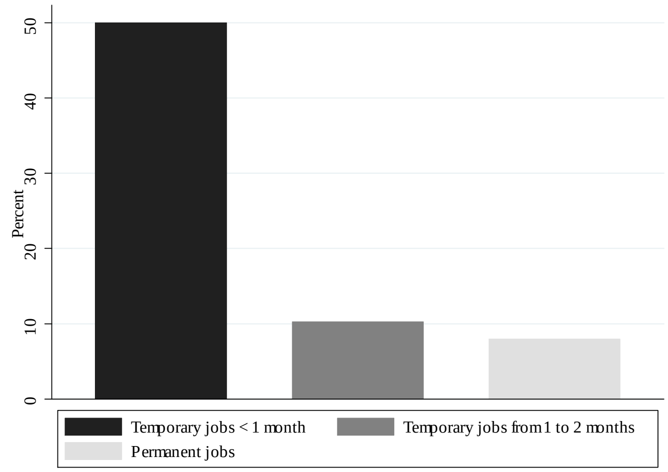

Figure 2: The share of entries into employment according to job type in Spain over the period 2002-2010. Source: Muestra Continua de Vidas Laborales.

	France	Spain
Number of jobs (stock)	15.9	12.9
Annual entries into temporary jobs	26.7	14.4
Annual entries into permanent jobs	3.2	1.6
Number of entries/Number of jobs	1.88	1.24

Table 1: Number of jobs and number of entries (in millions) into employment according to the type of contract. Private non agricultural sector. Period 2000q1 2010q2 for France and 2005q1 2010q2 for Spain. Source: ACOSS and Spanish State Employment Office.

into temporary jobs, much larger than the drop in entries into permanent jobs. Figure 4 shows that employment inflows follow a similar pattern in Spain, where the average gap between the number of entries and its trend is eleven times larger for temporary jobs than for permanent jobs. The collapse of employment inflow in 2008 comes from the drop in entries into temporary jobs. Over the period covered in figure 4, short run fluctuations in employment inflow are mostly driven by temporary jobs.

Let us now provide a model that can explain these three stylized facts.

Figure 3: Number of entries into employment per quarter (in thousands) in France in the private non agricultural sector. Deviations with respect to trends (Hodrick and Prescott filter). Source: ACOSS and DARES.

Figure 4: Number of entries into employment per quarter (in thousands) in Spain in the private non agricultural sector. Deviation with respect to trends (Hodrick and Prescott filter). Source: Spanish State Employment Office.

3 The model

For the sake of clarity, we start by presenting a simple benchmark model that describes the process of job creation when there is a match between an unemployed worker and a vacant job in a context where production opportunities become unproductive at constant Poisson rates. This setup is extended in the next section to include productivity shocks, as in the search and matching framework of Mortensen and Pissarides (1994) which is more relevant for representing the situation of firms that have fluctuations in the demand for their product. The labor market equilibrium is also determined in the next section when we proceed to quantitative exercises.

3.1 The benchmark setup

There is a continuum of infinitely-lived risk-neutral workers and firms, with a common discount rate $r > 0$. Workers are identical and their measure is normalized to 1. Firms are competitive and create jobs to produce a numéraire output, using labor as sole input. All jobs produce the same quantity of output per unit of time, denoted by $y > 0$, but jobs differ by the rate at which they become unproductive, denoted by $\lambda > 0$. When a job is created, its type λ is randomly selected from within $[\lambda_{\min}, +\infty)$, $\lambda_{\min} > 0$, according to a sampling distribution with cumulative distribution function G and density g. The distribution of λ has positive density over all its support and no mass point. Jobs and workers are brought together pairwise through a sequential, random and time consuming search process. Unemployed workers sample job offers sequentially at a rate that will be determined later in the paper.

There are two types of contract: temporary and permanent. Permanent contracts are the 'regular' type of contract. Permanent contracts stipulate a fixed wage that can be renegotiated by mutual agreement only: renegotiations thus occur only if one party can credibly threaten to leave the match for good if the other refuses to renegotiate. Permanent contracts are open-ended: they do not stipulate any pre-determined duration. Permanent jobs can be terminated at any time at cost F, paid by the employer. F is a red-tape cost, not a transfer from the firm to the worker (such as severance pay). Here we consider only red-tape costs, since it is well known that severance payments only change the timing of the payout – a factor which is basically irrelevant in models with risk-neutral agents. There is a (small) cost to write a contract, either temporary or permanent, which is denoted by $c > 0$.

Temporary contracts stipulate a wage and a fixed duration. Temporary contracts are neither renegotiable nor renewable. The employer must pay the worker the wage stipulated in the contract until the date of termination, even if the job becomes unproductive before this date. At their date of termination, temporary jobs can be either destroyed at zero cost or transformed into permanent jobs. Then, new permanent contracts can be bargained over.[20]

Temporary jobs allow firms to circumvent the legislated protection of permanent jobs. In most countries there are legal constraints on the utilization of temporary jobs. They take different forms: limits on the maximum duration, on the number of renewals, on the circumstances under which temporary jobs can be used (replacement, seasonal work, or temporary increases in company activity). These constraints are difficult to enforce, and indeed are generally very weakly enforced. A good example is France, where temporary jobs account for 90 percent of job creation, although in principle temporary jobs can be created only for replacement, seasonal work or temporary increases in company activity. In light of this, our model which neglects these legal constraints seems to be a reasonable benchmark for analyzing the consequences of job protection in the presence of temporary jobs.[21]

When they meet, workers and employers bargain over a contract that maximizes the surplus of the starting job, which can be either temporary or permanent. A temporary contract is chosen if it yields a higher surplus than a permanent contract. If a temporary contract is selected, the wage profile and the duration of the contract are chosen once for all in the starting contract because it is not permitted to renegotiate the contract.

Let us now define the surplus of permanent and temporary jobs before analyzing the choice between these two types of job.

[20] In the benchmark model, with productivity equal either to y or to zero, there is no wage renegotiation on any type of job since there is no shock that allows any party to have a credible threat with which to trigger renegotiations. In the model with productivity shocks presented in the next section, renegotiations can be triggered by firms only, to the extent that there are no aggregate shocks that increase the outside option of workers. When a productivity shock occurs on a permanent job, the job is destroyed if the surplus becomes negative, and renegotiations can be triggered if the value of the job to the firm becomes negative at the current wage while the surplus of the job is positive. Renegotiations are triggered by the (credible) threat by the firm that is making negative profits at the current wage to destroy the job. When the job is temporary, the firm cannot use this threat because the firm has to pay at least the current wage until the termination date of the contract. Renegotiation can only occur at the date at which the temporary job is transformed into **a** permanent job, provided that this is the case.

[21] The mandatory limit on the duration of temporary contracts and on the number of renewals are analyzed in Cahuc et al. (2012).

3.2 Permanent jobs

The value to a firm of starting permanent jobs with shock arrival rate λ, denoted by $J_p(\lambda)$, can be written as

$$J_p(\lambda) = \int_0^{\infty} \left[\int_0^{\tau} [y - w(\lambda)]\, e^{-rt} \mathrm{d}t - F e^{-r\tau} \right] \lambda e^{-\lambda \tau} \mathrm{d}\tau - c. \tag{1}$$

In this equation, the first term inside brackets, $\int_0^{\tau} [y - w(\lambda)]\, e^{-rt} \mathrm{d}t$, stands for the discounted sum of expected profits, equal to the difference between y, the production, and $w(\lambda)$, the wage, multiplied by the term e^{-rt}, which stands for the discount factor. Profits are expected until some random date τ, at which the job becomes unproductive and is destroyed at cost F. At date τ, since the job has become unproductive, its value is equal to zero. The term $\lambda e^{-\lambda \tau}$ corresponds to the density of the Poisson process governing productivity shocks. The last term, c, is the cost to write the contract.

Similarly, the value to a worker of starting a permanent job with shock arrival rate λ can be written as

$$W_p(\lambda) = \int_0^{\infty} \left[\int_0^{\tau} w(\lambda) e^{-rt} \mathrm{d}t + U e^{-r\tau} \right] \lambda e^{-\lambda \tau} \mathrm{d}\tau. \tag{2}$$

where U denotes the value of unemployment to the worker. The first term, $\int_0^{\tau} w(\lambda) e^{-rt} \mathrm{d}t$, stands for the present value of the wages expected by the worker until date τ, while the second term, $U e^{-r\tau}$, stands for the present value to the worker of searching for a new job in case of separation, an event that occurs at the random date τ.

By definition, the surplus of starting permanent jobs with shock arrival rate λ is

$$S_p(\lambda) = J_p(\lambda) + W_p(\lambda) - U. \tag{3}$$

Using (1) and (2) and rearranging, the surplus $S_p(\lambda)$ can also be written as

$$S_p(\lambda) = \frac{y - rU - \lambda F}{r + \lambda} - c. \tag{4}$$

The properties of the surplus $S_p(\lambda)$ are summarized as follows:

Properties of $S_t(\lambda)$: *function* $S_p(\lambda)$ *is continuous and decreasing in* λ. *It decreases from* $\frac{y}{r} - U - c > 0$ *to* $-c - F < 0$, *so that there exists a unique threshold value*

$$\lambda_p = \frac{y - r(U + c)}{F + c}, \tag{5}$$

such that $S_p(\lambda_p) = 0$ *and* $S_p(\lambda) > 0$ *if and only if* $\lambda < \lambda_p$.

Proof. See appendix C.1. □

3.3 Temporary jobs

The surplus of a temporary job is defined in two stages. We start by defining the expression of the surplus when the duration of the temporary job is given. Then, the expression of the surplus for the optimal duration of the job is derived.

3.3.1 Surplus of temporary jobs when their duration is given

The value to a firm of starting temporary jobs with shock arrival rate λ, and duration Δ, $J_t(\lambda, \Delta)$, can be written as (see appendix B)

$$J_t(\lambda, \Delta) = \int_0^{\Delta} \left[y e^{-\lambda\tau} - w(\lambda, \Delta) \right] e^{-r\tau} \mathrm{d}\tau + \max\left[J_p(\lambda), 0 \right] e^{-(r+\lambda)\Delta} - c. \tag{6}$$

The first term, $\int_0^{\Delta} \left[y e^{-\lambda\tau} - w(\lambda, \Delta) \right] e^{-r\tau} \mathrm{d}\tau$, stands for the discounted sum of expected profits over the duration of the job. In this expression, the level of production y is multiplied by the survival function $e^{-\lambda\tau}$ because the production drops to zero at rate λ. The wage $w(\lambda, \Delta)$ is not multiplied by the survival function because the employer has to keep and pay the employee until the date of termination of the contract. The second term, $\max\left[J_p(\lambda), 0 \right] e^{-(r+\lambda)\Delta}$, is the present value of the option for the firm linked to the possibility of transforming the temporary job into a permanent job at the date of termination of the temporary contract. The present value of this option decreases with the duration of the contract because time is discounted at rate r and because the probability that the job is productive at the date of termination of the contract decreases with the spell of the contract. The last term is the cost of writing the contract.

Similarly, in appendix B, we show that the value to a worker of starting temporary jobs with shock arrival rate λ, and duration Δ, $W_t(\lambda, \Delta)$, can be written as

$$W_t(\lambda, \Delta) = \int_0^{\Delta} \left[w(\lambda, \Delta) - rU \right] e^{-r\tau} \mathrm{d}\tau + \max\left[W_p(\lambda), U \right] e^{-(r+\lambda)\Delta} + U(1 - e^{-(r+\lambda)\Delta}). \tag{7}$$

In this expression, the first term, $\int_0^{\Delta} \left[w(\lambda, \Delta) - rU \right] e^{-r\tau} \mathrm{d}\tau$, stands for the discounted sum of expected gains over the duration of the job's present value. The second term, $\max\left[W_p(\lambda), U \right] e^{-(r+\lambda)\Delta}$, is the present value of the option linked to the possibility of transforming the temporary job into a permanent job at the date of termination of the temporary contract. The last term, $U(1 - e^{-(r+\lambda)\Delta})$, reflects the worker's outside options.

By definition, the surplus of starting temporary jobs with shock arrival rate λ and duration Δ, $S_t(\lambda, \Delta)$, is defined as follows:

$$S_t(\lambda, \Delta) = J_t(\lambda, \Delta) + W_t(\lambda, \Delta) - U, \tag{8}$$

which, using (6) and (7), can be written

$$S_t(\lambda, \Delta) = \int_0^{\Delta} \left(ye^{-\lambda\tau} - rU\right) e^{-r\tau} \mathrm{d}\tau + \max\left[S_p(\lambda), 0\right] e^{-(r+\lambda)\Delta} - c. \tag{9}$$

3.3.2 Optimal duration of temporary jobs

The optimal duration of temporary jobs maximizes the surplus of starting temporary jobs. Therefore, the optimal duration of a temporary job with shock arrival rate λ is defined by the first order condition[22]

$$ye^{-\lambda\Delta} - rU - (r+\lambda) e^{-\lambda\Delta} \max\left[S_p(\lambda), 0\right] = 0. \tag{10}$$

In this expression, the term $ye^{-\lambda\Delta}$ stands for the marginal gain of an increase in the duration of the job. This gain decreases with the duration of the job because the survival probability of production opportunities decreases with the job spell. It goes to zero when the duration goes to infinite. The marginal cost is equal to the sum of the two other terms. The first term, rU, is the flow of value that the employee can get if the job is terminated. The second term is the option value linked to the possibility of transforming the temporary job into a permanent job. The marginal cost decreases with the duration of the job and has a strictly positive lower bound, equal to rU.

The first order condition yields, together with equation (4), the optimal duration as a function of λ, denoted by

$$\Delta(\lambda) = \begin{cases} \frac{1}{\lambda} \ln\left(\frac{rU + \lambda F + (r+\lambda)c}{rU}\right) & \text{if } \lambda \leq \lambda_p \\ \frac{1}{\lambda} \ln\left(\frac{y}{rU}\right) & \text{if } \lambda \geq \lambda_p \end{cases} \tag{11}$$

The properties of the optimal duration $\Delta(\lambda)$ can be summarized as follows:

[22] The second order condition is always fulfilled. When $S_p(\lambda) \leq 0$, the second order condition is $-\lambda ye^{-\lambda\Delta} < 0$. When $S_p(\lambda) > 0$, the derivative of the first order condition with respect to Δ is

$$-\lambda ye^{-\lambda\Delta} + e^{-\lambda\Delta} (r+\lambda) \lambda S_p(\lambda),$$

which is equal to (using the first order condition): $-\lambda rU < 0$.

Properties of optimal duration $\Delta(\lambda)$: *function* $\Delta(\lambda)$ *is continuous, with a kink at* $\lambda = \lambda_p$. *It is monotonically decreasing, and goes from infinite, when the shock arrival goes to zero, to zero when the shock arrival rate goes to infinite.*

Proof. See appendix C.2. □

The optimal duration of temporary contracts is displayed in figure 5. Function $\Delta(\lambda)$ is decreasing with the shock arrival rate λ, and has a kink at $\lambda = \lambda_p$ because temporary jobs are transformed into permanent jobs only if the shock arrival rate is below the reservation value λ_p. Otherwise, the surplus yielded by the creation of permanent jobs is negative, which implies that it is worth neither creating permanent jobs nor transforming temporary jobs into permanent jobs. Note that equation (11) shows that the possibility of transforming temporary jobs into permanent jobs induces firms to shorten the duration of temporary jobs. If it were not possible to transform temporary jobs into permanent jobs, the duration of temporary jobs would be equal to $\ln\left(\frac{y}{rU}\right)/\lambda$ for all λ.[23] Note as well that the expression (11) implies that the actual duration of a temporary job differs from the expected duration of job opportunities, equal to $1/\lambda$.[24]

It turns out that increases in firing costs raise the optimal duration of temporary jobs because they reduce the surplus of permanent jobs and thus the incentive to transform temporary jobs into permanent jobs. Higher firing costs also imply a lower threshold value of λ below which temporary jobs are transformed into permanent jobs. In other words, when firing costs are higher, temporary jobs have longer spells and are less frequently transformed into permanent jobs. The optimal duration of temporary jobs also depends on productivity. Increases in productivity raise the duration of temporary jobs which are not transformed into permanent jobs. Therefore, increases in productivity reduce labor turnover.

Having studied the properties of the optimal duration $\Delta(\lambda)$, we may now examine the properties of the surplus of a temporary contract $S_t(\lambda) = \max_\Delta S_t(\lambda, \Delta)$, which can be summarized as follows

[23] When $\lambda < \lambda_p$, $S_p(\lambda) > 0$ and the expression (4) imply that $y > rU + \lambda F + (r+\lambda)c$, and thus that $\frac{y}{rU} > \frac{rU+\lambda F+(r+\lambda)c}{rU}$.

[24] Of course, we rely on the assumption that the job type λ remains constant over an employment spell. This point is further discussed in section 4.4.3.

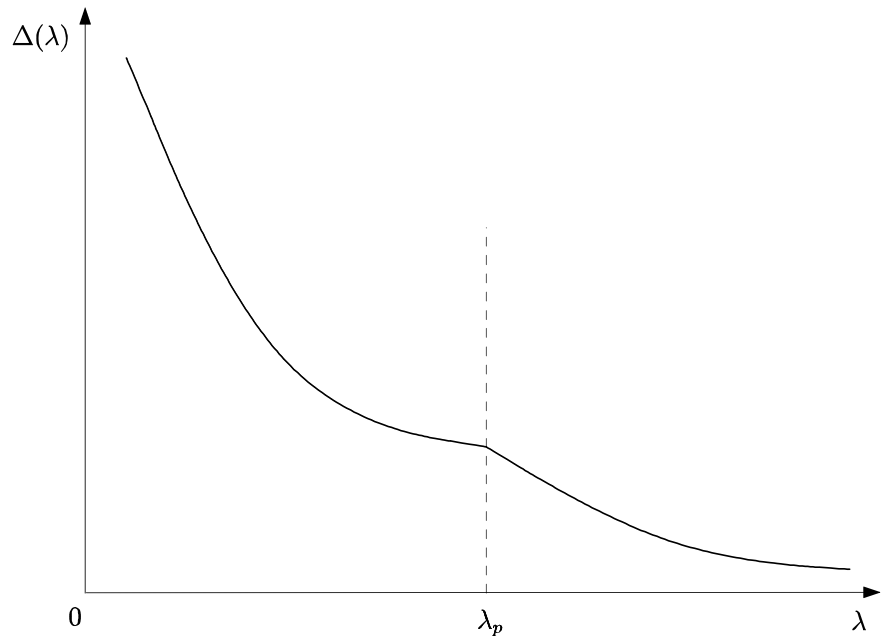

Figure 5: The relation between the shock arrival rate λ and the optimal duration of temporary jobs $\Delta(\lambda)$.

Properties of $S_t(\lambda)$: *function* $S_t(\lambda)$ *is continuous and decreasing in* λ. *It monotonically decreases from* $\frac{y}{r} - U - c > 0$ *to* $-c < 0$, *so that there exists a unique threshold value* λ_t *such that* $S_t(\lambda_t) = 0$, *and* $S_t(\lambda) > 0$ *if and only if* $\lambda < \lambda_t$.

Proof. see Apppendix C.3. □

With these features in mind, it is now possible to study the choice between temporary and permanent contracts.

3.4 Choice between temporary and permanent contracts

When a job is created, firms and workers choose the type of contract that provides the highest surplus. The choice between the two types of contract is described in the following proposition:

Proposition 1. *Choice between temporary and permanent contracts*

Let us define λ_s=$\{\lambda | S_t(\lambda) = S_p(\lambda)\}$, λ_p=$\{\lambda | S_p(\lambda) = 0\}$, $\lambda_t = \{\lambda | S_t(\lambda) = 0\}$. *Then:*

Case 1. *When $c > 0$, $F < U$, and*

$$y\left[\frac{1-e^{-(r+\lambda_p)\Delta(\lambda_p)}}{r+\lambda_p} - e^{-\lambda_p\Delta(\lambda_p)}\frac{1-e^{-r\Delta(\lambda_p)}}{r}\right] > c, \tag{12}$$

there exist unique values $\lambda_t > \lambda_p > \lambda_s > 0$ such that it is optimal to create permanent jobs for $\lambda < \lambda_s$, temporary jobs for $\lambda \in [\lambda_p, \lambda_t]$ and no job for $\lambda \geq \lambda_t$.

Case 2. *When $c > 0$, $F < U$, and condition* (12) *is not fulfilled, there exists a unique value $\lambda_p > 0$ such that it is optimal to create permanent jobs for $\lambda < \lambda_p$ and no job otherwise.*

Case 3. *When $c > 0$ and $F \geq U$, there exists a unique value $\lambda_t > 0$ such that it is optimal to create temporary jobs for $\lambda < \lambda_t$ and no job otherwise.*

Case 4. *When $c = 0$, there exists a unique value $\lambda_t > 0$ such that it is optimal to create temporary jobs for $\lambda < \lambda_t$ and no job otherwise.*

Proof. See appendix C.4. □

There is a trade-off between temporary contracts in which the cost of a productivity shock is an inability to separate for some subsequent period of time, and permanent contracts in which the cost of a negative productivity shock is F. Accordingly, as claimed in proposition 1, depending on the arrival rate of shocks and on the other parameters that determine the value of jobs, it can be optimal to create either temporary or permanent jobs.

The situation that arises in case 1 where $c > 0$, $F < U$ and condition (12) is satisfied, is illustrated by Figure 6, which displays the surplus of permanent jobs and the surplus of temporary jobs for all possible values of the shock arrival rate λ. Condition (12) can be met if c is small. λ_p also has to be small, which corresponds to situations where F is sufficiently large, as shown by equation (5). In this situation, it is optimal to create permanent jobs for values of $\lambda \in [\lambda_{min}, \lambda_s]$ as the arrival rate of productivity shocks is sufficiently small. For larger values of λ, i.e. when λ falls within $[\lambda_s, \lambda_t]$, it becomes optimal to create temporary jobs because the surplus of temporary jobs becomes larger than that of permanent jobs. When $\lambda > \lambda_t$, the arrival rate of productivity shocks is so high that it is never worth creating jobs, either permanent or temporary.

In case 2, which is similar to case 1 except that condition (12) is not met, permanent contracts are always more profitable than temporary contracts. Therefore, there are no temporary jobs. This situation arises when firing costs are small.

In case 3, the surplus of temporary jobs is always larger than that of permanent jobs because firing costs are very large, larger than U. Therefore, there are only temporary jobs.

Eventually, in case 4, where the cost to write contracts is equal to zero, only temporary contracts are created, because it is always preferable to hire workers on temporary jobs, possibly for very short periods of time, and then to transform temporary jobs into permanent jobs rather than directly hiring workers on permanent jobs.[25] This shows that there is no trade-off between permanent jobs and temporary jobs if there are no costs to write contracts. The trade-off would also disappear if it were possible to write a single contract that stipulated a contingent transformation of temporary contract into permanent contract at the instant when the worker is hired. It is likely that such contracts are not observed in the real world because they are too costly to verify.

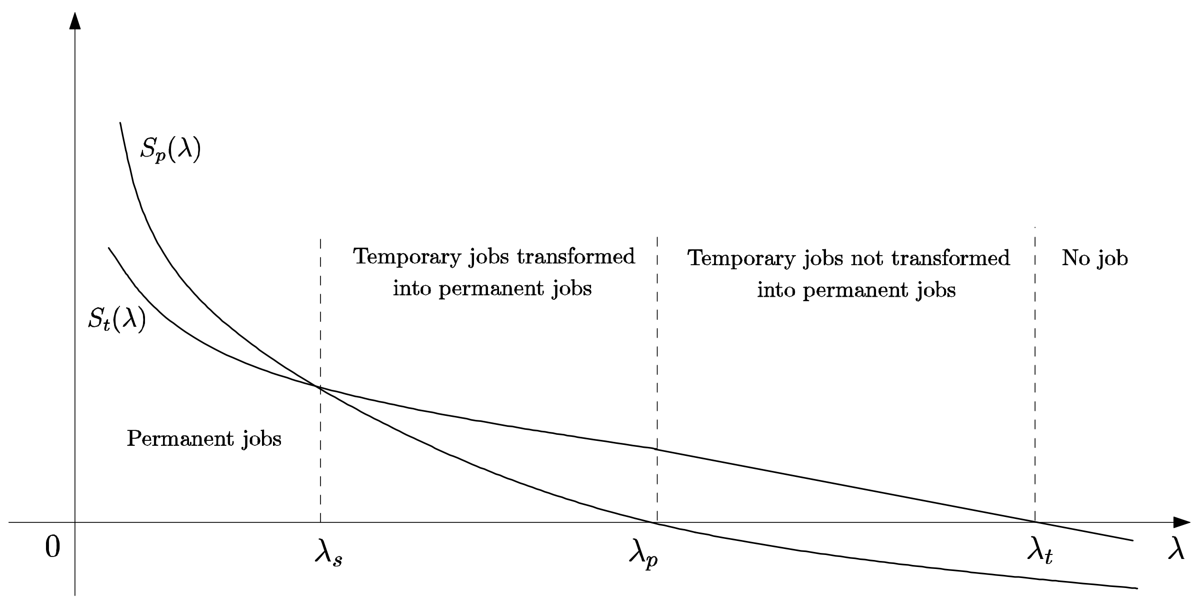

Figure 6: The relation between the shock arrival rate and the type of job creation.

Finally, it is worth noting that our model implies that temporary jobs pay lower wages than permanent jobs even when their productivity is the same. There are two reasons for this property, consistent with empirical evidence.[26] First, the duration of temporary jobs is

[25] Formally, it can be verified that $S_p(\lambda) < S_t(\lambda)$ when $c = 0$, as shown in the appendix. In the simulations, c takes very small values relative to y (about 0.5% of the average monthly production of an employee).

[26] Empirical evidence shows that temporary workers get lower wages than permanent workers controlling for a large cluster of observable characteristics. For instance, Booth et al. (2002) find that temporary workers in

shorter than that of permanent jobs. This induces a lower average surplus for temporary jobs as shown by figure 6. Second, the impossibility of terminating temporary contracts before their date of termination implies that there are situations where employers pay positive wages to unproductive temporary workers. This reduces their entry wage which is not renegotiated.

4 Quantitative evaluation

We now turn to the quantitative evaluation of the model in order to show that it is compatible with the stylized facts highlighted in section 2. Then we study the impact of job protection on the main variables of interest, *i.e.* job inflows, aggregate employment and production. So far we have restricted ourselves to rather simplistic production and destruction processes where output is constant and job destruction is exogenous. Moreover, labor market equilibrium has not yet been defined. To make our numerical exercise more relevant, we now generalize to a richer stochastic environment and consider an extension of the benchmark model where it is assumed that productivity shocks do not strike productivity down to zero once for all, but imply a new value of the productivity drawn from a stationary distribution, as in the model of Mortensen and Pissarides (1994). We proceed to the analysis at market equilibrium in this framework.

For this purpose, let us now assume that the production of an employee is a random variable with distribution $H(y)$ which has upper support y_u and no mass point. The productivity of each employee changes at Poisson rate λ. When productivity changes, there is a draw from the fixed distribution $H(y)$. For the sake of simplicity, it is assumed that the productivity of new matches is equal to the upper support of the distribution, as in Mortensen and Pissarides (1994). In what follows, we show that the model with productivity shocks can be solved in a similar way to the benchmark model. We then turn to the calibration exercise.

4.1 Permanent jobs

Permanent jobs can start either from new matches or from transformations of temporary jobs. In either situation, the value to the firm of a starting permanent job with shock arrival rate λ

Britain earn less than permanent workers (men 8.9 percent and women 6 percent less). Hagen (2002) finds an even larger gap, about 23 percent in Germany, controlling for selection on unobservable characteristics.

and productivity y, denoted by $J_p(y,\lambda)$, satisfies the Bellman equation

$$J_p(y,\lambda)=\int_0^\infty\left[\int_0^\tau [y-w(y,\lambda)]\,e^{-rt}\mathrm{d}t-e^{-r\tau}\int_{-\infty}^{y_u}\max\left[J_c(x,\lambda),-F\right]\mathrm{d}H(x)\right]\lambda e^{-\lambda\tau}\mathrm{d}\tau-c,\tag{13}$$

where $J_c(x,\lambda)$ denotes the value to the firm of a continuing permanent job with shock arrival rate λ and productivity x. The first term inside brackets, $\int_0^\tau [y-w(y,\lambda)]\,e^{-rt}\mathrm{d}t$, stands for the discounted sum of expected profits until date τ at which time a productivity shock hits the job, with $w(y,\lambda)$ denoting the wage. Then, from date τ, the value of the job is $\int_{-\infty}^{y_u}\max\left[J_c(x,\lambda),-F\right]\mathrm{d}H(x)$, as it can either be continued at the new productivity level x, or be destroyed at cost F.

Similarly, the value to the worker of being employed on a starting permanent job with shock arrival rate λ and productivity y, denoted by $W_p(y,\lambda)$, satisfies

$$W_p(y,\lambda)=\int_0^\infty\left[\int_0^\tau w(y,\lambda)e^{-rt}\mathrm{d}t-e^{-r\tau}\int_{-\infty}^{y_u}\max\left[W_c(x,\lambda),U\right]\mathrm{d}H(x)\right]\lambda e^{-\lambda\tau}\mathrm{d}\tau,\tag{14}$$

where $W_c(x,\lambda)$ denotes the expected utility to the worker of a continuing permanent job with shock arrival rate λ and productivity x. The first term inside brackets, $\int_0^\tau w(y,\lambda)e^{-rt}\mathrm{d}t$, stands for the discounted sum of wages paid to the worker until date τ. At some random date τ, the job can be hit by a productivity shock, yielding $\int_{-\infty}^{y_u}\max\left[W_c(x,\lambda),U\right]\mathrm{d}H(x)$, since it can either be continued with the new productivity level x, or can be destroyed, in which case the worker becomes unemployed.

The surplus of a starting permanent job with shock arrival rate λ and productivity y, denoted by $S_p(y,\lambda)$ can be defined as

$$S_p(y,\lambda)=J_p(y,\lambda)+W_p(y,\lambda)-U.\tag{15}$$

Firing costs are paid when a continuing permanent job is destroyed, but not when the employer and the employee destroy a starting job because they cannot achieve an initial agreement. The cost c to sign a contract is paid when the job starts, but not when the job is continued. Accordingly, firing costs and the cost to sign a contract create a difference between the surplus of a starting permanent contract, $S_p(y,\lambda)$, and that of a continuing permanent contract $S_c(y,\lambda)$. The surplus of a continuing permanent contract is equal to

$$S_c(y,\lambda)=S_p(y,\lambda)+F+c.\tag{16}$$

Using this expression and equations, (13), (14) and (15), we get

$$rS_c(y,\lambda) = y - r(U-F) + \lambda\left(\int_{-\infty}^{y_u} \max\left[S_c(x,\lambda),0\right]\mathrm{d}H(x) - S_c(y,\lambda)\right). \tag{17}$$

Continuing permanent jobs are destroyed when their surplus becomes negative. Since $S_c(y,\lambda)$ increases with y, jobs are destroyed if their productivity drops below the reservation value, denoted by $R(\lambda)$, such that $S_c(R,\lambda)=0$. This reservation productivity satisfies

$$R(\lambda) = r(U-F) - \lambda\int_{R(\lambda)}^{y_u} \frac{y-R(\lambda)}{r+\lambda}\mathrm{d}H(y). \tag{18}$$

This equation implies that $R(\lambda)$ is a decreasing function of λ.

The creation of permanent jobs can arise from entries of unemployed workers into employment or from transformations of temporary jobs into permanent jobs. In both cases, permanent jobs are created if their productivity is above the threshold denoted by $T(\lambda)$, such that $S_p(T,\lambda)=0$. This threshold satisfies

$$T(\lambda) = R(\lambda) + (r+\lambda)(F+c). \tag{19}$$

4.2 Temporary jobs

The value to the firm of starting temporary jobs with shock arrival rate λ and duration Δ, $J_t(\lambda,\Delta)$, can be written as (see appendix D)

$$\begin{aligned} J_t(\lambda,\Delta) \;=\; & \int_0^{\Delta}\left[e^{-\lambda\tau}y_u + \left(1-e^{-\lambda\tau}\right)\int_{-\infty}^{y_u} y\mathrm{d}H(y) - w(\lambda,\Delta)\right]e^{-r\tau}\mathrm{d}\tau \\ & + e^{-(r+\lambda)\Delta}\max\left[J_p\left(y_u,\lambda\right),0\right] + e^{-r\Delta}\left(1-e^{-\lambda\Delta}\right)\int_{-\infty}^{y_u}\max\left[J_p\left(y,\lambda\right),0\right]\mathrm{d}H(y) - c. \end{aligned} \tag{20}$$

The integral of the first row stands for the present value of the instantaneous profits obtained over the duration of the temporary contract. The terms of the second row correspond to the present value of the gains expected at the date of termination of the temporary contract minus the cost to write the contract.

In appendix D, we show that the value to the worker of being employed on a starting temporary job with shock arrival rate λ and duration Δ, $W_t(\lambda,\Delta)$, can be written as

$$\begin{aligned} W_t(\lambda,\Delta) \;=\; & \int_0^{\Delta} w(\lambda,\Delta)e^{-r\tau}d\tau + e^{-(r+\lambda)\Delta}\max\left[W_p\left(y_u,\lambda\right),U\right] \\ & + e^{-r\Delta}\left(1-e^{-\lambda\Delta}\right)\int_{-\infty}^{y_u}\max\left[W_p\left(y,\lambda\right),U\right]\mathrm{d}H(y). \end{aligned} \tag{21}$$

The first term corresponds to the present value of wages obtained over the duration of the temporary contract. The second and third terms correspond to the present value of the worker's expected gains at the date of termination of the temporary contract.

The surplus of starting permanent jobs with shock arrival rate λ, can then be written as

$$S_t(\lambda, \Delta) = J_t(\lambda, \Delta) + W_t(\lambda, \Delta) - U, \tag{22}$$

which implies, using (20) and (21), that the surplus of starting temporary jobs with shock arrival rate λ and duration Δ can be written as (see appendix D for more details)

$$\begin{aligned} S_t(\lambda, \Delta) \quad = \quad & \int_0^{\Delta} \left(e^{-\lambda\tau} y_u + \left(1 - e^{-\lambda\tau}\right) \int_{-\infty}^{y_u} y \mathrm{d}H(y) - rU \right) e^{-r\tau} \mathrm{d}\tau + \\ & e^{-(r+\lambda)\Delta} \max\left[S_p(y_u, \lambda), 0\right] + e^{-r\Delta} \left(1 - e^{-\lambda\Delta}\right) \int_{-\infty}^{y_u} \max\left[S_p(y, \lambda), 0\right] \mathrm{d}H(y) - c. \end{aligned} \tag{23}$$

The integral of the first row stands for the present value of the instantaneous surpluses obtained over the duration of the temporary contract. The terms of the second row correspond to the present value of the gains expected at the date of termination of the temporary contract minus the cost to write the contract.

4.2.1 Optimal duration of temporary contracts

Once the value of starting jobs is known, it is possible to determine the optimal duration of temporary contracts and shed light on the choice between temporary and permanent contracts.

The optimal duration of temporary contracts is the value of Δ, denoted by $\Delta(\lambda)$, which maximizes $S_t(\lambda, \Delta)$. We get (see appendix E)

$$\Delta(\lambda) = \begin{cases} \frac{1}{\lambda} \ln \left(\frac{y_u - \bar{y} - (r+\lambda)[S_p(y_u, \lambda) - \chi]}{rU - \bar{y} + r\chi} \right) & \text{if } \lambda \leq \lambda_p \\ \frac{1}{\lambda} \ln \left(\frac{y_u - \bar{y}}{rU - \bar{y}} \right) & \text{if } \lambda \geq \lambda_p \end{cases} \tag{24}$$

where $\bar{y} = \int_{-\infty}^{y_u} y \mathrm{d}H(y)$, $\chi = \int_{T(\lambda)}^{y_u} S_p(y, \lambda) \mathrm{d}H(y)$, and λ_p is defined by the condition $S_p(y_u, \lambda_p) = 0$.

This expression of the optimal duration of temporary contracts is similar to that obtained in the benchmark model (see equation (11)). The optimal duration is continuous, decreases with the shock arrival rate λ and increases with the productivity of starting jobs. It goes to zero when the shock arrival rate λ becomes very large, and has a kink at λ_p.

4.2.2 Choice between temporary and permanent contracts

The choice between the creation of temporary and permanent jobs is determined by the comparison of the values of the surplus of starting jobs. As in the benchmark model, (Proposition 1, case 1), there are values of the parameters such that temporary jobs are preferred to permanent jobs if the shock arrival rate is above a threshold denoted by λ_s, which satisfies $S_p(y_u, \lambda_s) = S_t(\lambda_s)$ (see appendix F). Below this threshold, permanent jobs are created. There also exists an upper finite value of the shock arrival rate, λ_t, such that $S_t(\lambda_t) \equiv \max_{\Delta} S_t(\lambda_t, \Delta) = 0$, above which no job is created. Temporary jobs with shock arrival rate λ falling in the interval (λ_s, λ_t) are transformed into permanent jobs only if their productivity is above the reservation value $T(\lambda)$. Otherwise, they are destroyed.

4.3 Labor market equilibrium

Let us now describe the process of job creation, the matching between workers and jobs, and the bargaining between workers and employers in order to determine the labor market equilibrium.

Firms must invest $\kappa > 0$ to find a production opportunity. κ is a sunk cost. As described above, all production opportunities start with the same level of productivity y_u. Then they are hit by shocks at Poisson rates λ that differ across jobs. Firms draw production opportunities from the distribution $G(\lambda)$ just after the sunk cost κ has been paid. When a production opportunity is found, a job vacancy can be created. The value of a type-λ vacant job (*i.e.* with shock arrival rate λ) is denoted by $V(\lambda)$. Free entry implies that the expected value of vacant jobs is equal to the investment cost

$$\kappa = \int \max\left[V(\lambda), 0\right] \mathrm{d}G(\lambda). \tag{25}$$

Unemployed workers and job vacancies are brought together through a constant returns to scale matching technology which implies that vacant jobs are filled at rate $q(\theta)$, $q'(\theta) < 0$, where $\theta = v/u$ denotes the labor market tightness, equal to the ratio of vacancies, v, over unemployment u. For the sake of simplicity, it is assumed that the instantaneous cost of vacancies equals zero and that firms must re-invest to find new production opportunities when matches are broken. Moreover, bargaining allows workers to get the share $\beta \in (0, 1)$ of the job surplus. Therefore, the value of type-λ vacant jobs satisfies

$$rV(\lambda) = q(\theta)\left[(1-\beta)S(\lambda) - V(\lambda)\right] \tag{26}$$

where $S(\lambda)$ denotes the surplus of type-λ starting filled jobs. Firms create type-λ vacancies only if their expected value is positive. Since it has been shown above that all (temporary and permanent) job surpluses $S(\lambda)$ decrease with λ and become negative when λ goes to infinite, this implies that type-λ vacant jobs are created only if $\lambda < \lambda_{\text{sup}}$ where λ_{sup} equals either λ_t (see figure 6) if the equilibrium comprises temporary and permanent jobs or λ_p if there are permanent jobs only, which occurs when firing costs are sufficiently small.

The matching technology implies that unemployed workers sample job offers at rate $\theta q(\theta)$. Thus, denoting by z the instantaneous income of unemployed workers, the value of unemployment satisfies

$$rU = z + \theta q(\theta)\beta \int_{\lambda_{\min}}^{\lambda_{\text{sup}}} \frac{S(\lambda)}{G(\lambda_{\text{sup}})} \mathrm{d}G(\lambda).$$

Combining the three previous equations, we get

$$rU = z + \frac{\beta\theta\left[r + q(\theta)\right]}{(1-\beta)\, G(\lambda_{\text{sup}})}\kappa. \tag{27}$$

This equation shows that increases in labor market tightness, which increase the arrival rate of job offers, improve the expected gains of unemployed workers.

Now, let us focus on two types of labor market equilibrium: one where there are only permanent jobs and another where there are permanent and temporary jobs.[27]

4.3.1 Equilibrium with permanent jobs only

When firing costs are sufficiently small, all jobs are permanent because the surplus of permanent jobs is always larger than that of temporary jobs. It is possible to find a system of two equations that defines the equilibrium value of (θ, λ_p). From equations (25) and (26), the free entry condition can be written as

$$\kappa = \frac{q(\theta)(1-\beta)}{r + q(\theta)} \int_{\lambda_{\min}}^{\lambda_p} S_p(y_u, \lambda)\mathrm{d}G(\lambda), \tag{28}$$

where $S_p(y_u, \lambda)$ is defined by equations (16) and (17), and by equation (27) which defines U, as U shows up in the expression of $S_p(y_u, \lambda)$. We get another relation between θ and λ_p using the condition that defines the threshold value of shock arrival rates above which no jobs are

[27] As shown in Proposition 1 for the benchmark model, an equilibrium with temporary jobs only can exist in our framework. We rule out this possibility for the sake of realism. We also rule out the trivial equilibrium without entries into employment.

created

$$S_p(y_u, \lambda_p) = 0. \tag{29}$$

Equations (28) and (29) define a unique equilibrium value of (θ^*, λ_p^*) provided that the conditions of existence are satisfied, which is assumed. This leads to the following proposition:

Proposition 2. *Equilibrium with permanent contracts only.*

Provided that an equilibrium with permanent jobs only exists, it is unique and defined by a couple (θ^*, λ_p^*) solving equations (28) and (29).

Proof. See appendix G.1. □

Having determined the equilibrium values of (θ^*, λ_p^*), it is then possible to compute rU^* defined by equation (27), and to substitute it into equation (18) to determine the function $R^*(\lambda)$. Then, the equilibrium unemployment rate, as shown in appendix G.3, results from

$$u^* = \frac{1}{1 + \frac{\theta^* q(\theta^*)}{G(\lambda_p^*)} \int_{\lambda_{\min}}^{\lambda_p^*} \phi^*(\lambda) \mathrm{d}G(\lambda)}, \tag{30}$$

where $\phi^*(\lambda) = 1/\lambda H\left[R^*(\lambda)\right]$ is the expected duration of type-λ permanent jobs. We now turn to the equilibrium with both types of contract.

4.3.2 Equilibrium with permanent and temporary jobs

When firing costs are sufficiently large, starting jobs can be either temporary, with surplus $S_t(\lambda)$, or permanent, with surplus $S_p(y_u, \lambda)$. The free entry condition becomes

$$\kappa = \frac{q(\theta)(1-\beta)}{r + q(\theta)} \left[\int_{\lambda_{\min}}^{\lambda_s} S_p(y_u, \lambda) \mathrm{d}G(\lambda) + \int_{\lambda_s}^{\lambda_t} S_t(\lambda) \mathrm{d}G(\lambda) \right]. \tag{31}$$

This equation defines a relationship between θ and the thresholds. In turn, the conditions

$$S_t(\lambda_t) = 0, \tag{32}$$

$$S_p(y_u, \lambda_s) = S_t(\lambda_s), \tag{33}$$

$$S_p(y_u, \lambda_p) = 0, \tag{34}$$

define the thresholds as a function of θ, once the relation between rU and θ has been taken into account in the expressions of the surpluses S_t and S_p. Then, equations (32), (33), and (34) together with (31) define a unique equilibrium value of the quadruple $(\lambda_s, \lambda_p, \lambda_t, \theta)$, provided that it exists, which is assumed. This leads to the following proposition:

Proposition 3. *Equilibrium with permanent and temporary contracts.*

Provided that an equilibrium with permanent and temporary contracts exists, it is unique and defined by the quadruple $(\lambda_s^*, \lambda_p^*, \lambda_t^*, \theta^*)$ *solving equations (31) to (34).*

Proof. See appendix G.2. □

Once the equilibrium values of $(\lambda_s^*, \lambda_p^*, \lambda_t^*, \theta^*)$ are known, we can determine rU^* defined by (27), and use equation (18) to get $R^*(\lambda)$. Then, $T^*(\lambda)$ is defined by equation (19). It is then possible to compute the duration of temporary contracts in steady-state, $\Delta^*(\lambda)$, defined by (24). The equilibrium unemployment rate, as shown in appendix G.3 is

$$u^* = \frac{1}{1 + \frac{\theta^* q(\theta^*)}{G(\lambda_t^*)} \left[\int_{\lambda_{\min}}^{\lambda_s^*} \phi^*(\lambda) \mathrm{d}G(\lambda) + \int_{\lambda_s^*}^{\lambda_t^*} \Delta^*(\lambda) \mathrm{d}G(\lambda) + \int_{\lambda_s^*}^{\lambda_p^*} \phi^*(\lambda) \left[1 - H(T^*(\lambda)) \left(1 - e^{-\lambda \Delta^*(\lambda)}\right)\right] \mathrm{d}G(\lambda) \right]}, \tag{35}$$

where $\phi^*(\lambda) = 1/\lambda H\left[R^*(\lambda)\right]$ is the expected duration of type-λ permanent jobs.

4.4 Simulation exercises

We now calibrate the model to explore its quantitative properties. In particular, we show that the model is able to reproduce the main stylized facts about entries into employment observed in countries like France and Spain where there is stringent employment protection legislation and a large share of temporary jobs, *i.e.*: *(i)* most entries into employment are into temporary jobs; *(ii)* the duration of most temporary jobs is very short; *(iii)* the main part of fluctuations in employment inflows is due to inflows into temporary jobs. The model is first calibrated to match the labor market of the US economy, where firing costs are close to zero. Then, firing costs are increased to evaluate their impact on entries into permanent and temporary jobs.

4.4.1 Calibration

The parameters and targets used in the calibration refer to the US economy, which represents the benchmark economy without firing costs. Admittedly, this assumption is an approximation, to the extent that we neglect the exceptions to the employment-at-will doctrine which induce firms to use some temporary contracts (see *e.g.* Autor, 2003). However, employment protection legislation remains very weak in the US relative to most other OECD countries, and especially to Continental European countries (Venn, 2009).

The values of the parameters are in the range of those chosen in the literature (see *e.g.* Mortensen and Pissarides, 1999, Shimer, 2005, and Mortensen and Nagypal, 2007). We define the time period to be one month, and consequently set the discount rate r to 0.41%, which corresponds to a 5 percent annual discount rate. The income of unemployed workers (the value of leisure), z, is equal to 0.3, a reasonable value that lies below the upper end of the range of income replacement rates in the United States if interpreted entirely as an unemployment benefit (see *e.g.* Shimer, 2005). As in Mortensen and Pissarides (1994), the distribution of idiosyncratic shocks is assumed to be uniform in the range $[y_{\min}, 1]$. We follow the literature and assume a Cobb-Douglas matching technology of the form $H(v, u) = hv^{\eta}u^{1-\eta}$, where h is a mismatch parameter and η is the elasticity of the matching function with respect to the number of vacancies. We assume η to be equal to 0.5, which falls in the range of the estimates obtained by Petrongolo and Pissarides (2001). Following common practice, we set the bargaining power parameter β to 0.5, a value that internalizes the search externalities in our benchmark specification without firing costs (see *e.g.* Pissarides, 2009). The sampling distribution of type-δ jobs, $\delta = 1/\lambda$, is a truncated log normal distribution. The range of expected durations of production opportunities is comprised between one day (1/22 month, 22 being the average number of days worked per month) and 45 years (540 months).

At this stage, the values of 6 parameters remain to be determined: the parameter of the cumulative distribution function (cdf) of the productivity distribution, $y_{\min}$; the two parameters of the cdf of the sampling distribution of durations of production opportunities, σ and μ; the cost of writing contracts, c; the mismatch parameter, h; and finally the investment cost, κ. The values of the parameters are chosen to match the values of the 6 following variables: the labor market tightness, the minimum job duration, the median and the mean value of the expected durations of production opportunities, the average monthly job finding rate, and the unemployment rate. The calibration strategy then consists in solving a system of 6 equations with 6 unknown parameter values, assuming a flexible economy where $F = 0$. This system is made up of: *(i)* two equations, (28) and (29), that define the equilibrium values of the labor market tightness θ and of the minimum job duration δ_p. We assume that the bottom equilibrium value of $\delta_p = 1/\lambda_p$ is equal to that of the truncated distribution of expected durations (one day) and, as in Shimer (2005), that the average v-u ratio is equal to one; *(ii)* two equations that define the median and the mean value of the expected durations of production opportunities. The median and the mean durations in the cross section of jobs, equal to 4 years (48 months) and 6.67 years (80 months) respectively, are obtained from the CPS, Displaced Workers, Employee Tenure, and Occupational Mobility Supplement, for the private sector in 2008; *(iii)* one equation that targets an average monthly job finding rate of 0.45 (see *e.g.* Shimer, 2005 or Nagypal and Mortensen, 2007); and finally *(iv)* one equation, defined by (30), used to match an unemployment rate equal to 6 percent. This strategy allows us to determine a sextuple $(\kappa, c, \sigma, \mu, h, y_{\min})$ which is a solution to the system described above. Baseline and calibrated parameters are summarized in Table 2.

Table 2: Benchmark parameters values

Baseline parameters			Calibrated parameters		
Parameter	Notation	Value	Parameter	Notation	Value
Bargaining power	β	0.5	Cost of a contract	c	0.0050
Matching elasticity	η	0.5	Mismatch parameter	h	0.45
Discount rate	r	0.41%	*log N* - shape parameter	σ	0.9360
Value of leisure	z	0.3	*log N* - scale parameter	μ	3.0019
Maximum match product	y_u	1	Minimum match product	$y_{\min}$	0.0176
			Investment cost	κ	1.3170

4.4.2 The economy with firing costs and temporary contracts

Let us now look at the consequences of firing costs. We focus on steady states only. This exercise allows us to illustrate the mechanism of the model and to probe whether it can potentially reproduce the three stylized facts presented above in section 2. Obviously, this exercise is illustrative. It is not meant to reproduce the labor market of a specific country, but more generally to illustrate the consequences of the introduction of firing costs in a labor market with frictions and flexible wages. Dealing with a specific country with strong job protection would require taking into account the influence of minimum wage and/or collective bargaining, factors that play an important role in countries with strong job protection.

The first fact is that the share of entries into temporary jobs strongly increases with job protection. Figure 7 shows that the model predicts that firing costs do have a strong impact on the share of entries into temporary jobs. Firms begin to use temporary contracts when firing costs reach about five percent of the average monthly production of an employee. Then, when firing costs increase, the share of entries into temporary jobs rises steadily. It amounts to 85 percent of all entries into employment when firing costs equal about 50 percent of the average monthly production of a job, which is a reasonable order of magnitude given the available estimates.[28] All in all, the model allows us to explain the large share of entries into temporary jobs observed in Continental European countries.

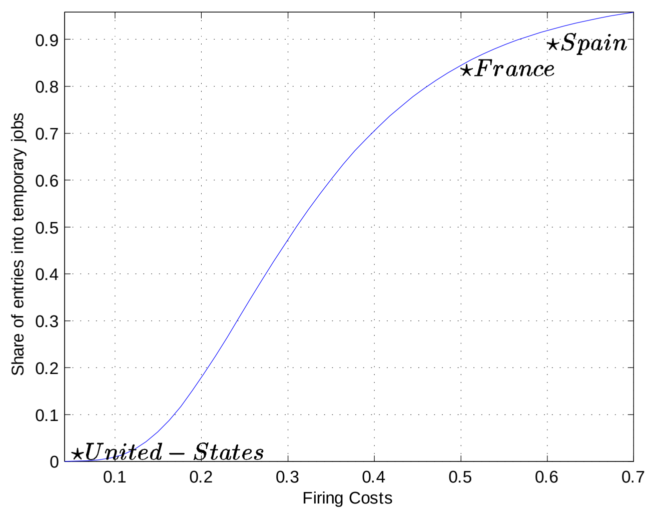

Figure 7: The relation between firing costs (in shares of monthly production of an employee) and the share of entries into temporary jobs in total employment inflows.

The predictions of the model are also in line with the second stylized fact, according to which the average duration of new temporary jobs is very low, about 1.5 months in France. Indeed, the model predicts that the average duration of new temporary jobs is 1.5 months when 90 percent of entries are into temporary jobs.

Figure 8 shows that the model fits the third stylized fact, according to which changes in entries into temporary jobs account for the main share of changes in the total number of entries into employment. This figure represents the relation between changes in the mean productivity of an employee,[29] and changes in the number of entries into temporary and permanent jobs. As

[28] Kramarz and Michaud (2010) estimate that the termination of the contract of a marginal permanent job amounts to 16 percent of the annual wage for an individual layoff and to 50 percent of the annual wage for a collective layoff in France. Since about 4 out of 6 layoffs are individual layoffs, the average cost is 20 percent of the annual wage, which corresponds to 1.5 months of production if the share of wages in production is 2/3. Assuming that red-tape costs amount to about 1/3 of the total layoff costs, we find that red-tape costs represent about 0.5 month of production. For Spain, we assume as do Bentolila et al. (2012), that firing costs are 20 percent higher than in France, so that they amount to 0.6 month of the average production of jobs.

[29] See appendix G.4 for the computation of mean productivity.

can be inferred from the figure, changes in mean productivity induce much larger changes in entries into temporary jobs than into permanent jobs. The rise in entries into temporary jobs following a positive productivity shock is ten times larger than the rise in entries into permanent jobs. This order of magnitude is in line with the facts observed in France and Spain over the period 2000-2010, where the corresponding number lies between 7 and 11, as documented in section 2.

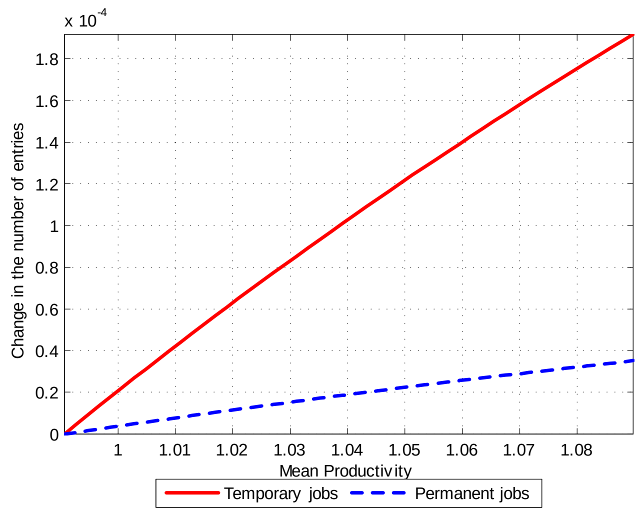

Figure 8: Changes in the number of entries into temporary and permanent employment induced by changes in mean productivity.

4.4.3 Job protection and the excess of job turnover

Our model is particularly useful when it comes to evaluating the impact of job protection on job turnover, employment and production.

Job turnover Our model predicts, in line with empirical evidence, that the average duration of new temporary jobs is short, about 1.5 months, when the share of temporary jobs in entries corresponds to that observed in France or Spain. It is interesting to compare this duration with that of jobs that would be used to exploit the same production opportunities (on the same range of type-λ jobs) in the absence of job protection, where all jobs are permanent according to our model. The result is illustrated in figure 9, which shows that the duration of temporary jobs is much shorter than the duration of the permanent jobs which, absent firing costs, would be utilized to exploit the same production opportunities.

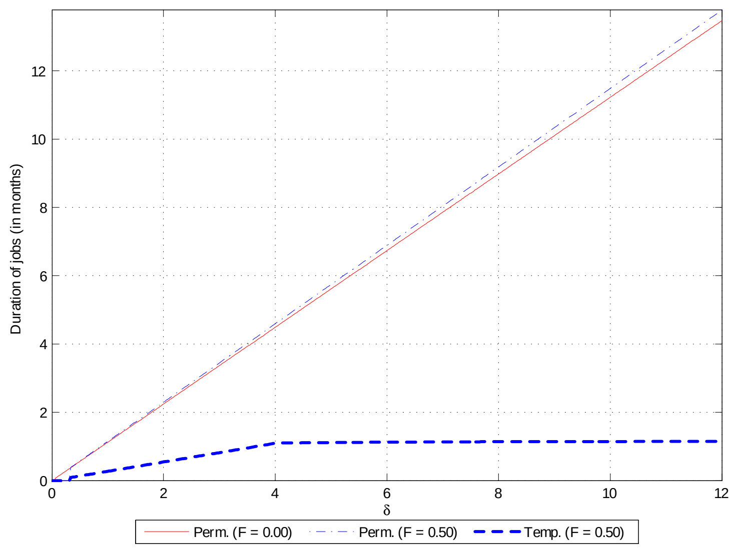

Figure 9: The relation between the average time interval between productivity shocks $\delta = 1/\lambda$ (x-axis) and the expected duration of i) permanent jobs with firing costs (dashed and dotted line), ii) permanent jobs without firing costs (continuous line), iii) temporary jobs (dashed line). Firing costs are expressed in average monthly production of an employee.

For instance, for a value of the average time interval between productivity shocks, δ, equal to six, figure 9 shows that the expected duration of permanent jobs is slightly above six months, while the duration of temporary jobs is about five weeks. This difference is indeed quite large, and it increases when productivity shocks are less frequent. The reason is that firms want to avoid situations where they have to pay unproductive workers. This implies that firing costs can induce, through their impact on the creation of temporary jobs, an important excess of job turnover which in turn can induce large production losses.

Obviously, this result hinges on the assumption that the shock arrival rate follows a Poisson process with constant instantaneous probability. Empirical estimates usually find non-monotonous separation rates that begin to increase with tenure and then decrease toward a level that is lower than that observed at the beginning of the employment spell (see e.g. Booth et al., 1999). The relatively high level of separation rates at the beginning of employment spells suggests that the shock arrival rate is higher at the beginning of job spells. This feature should induce employers to shorten the duration of temporary contracts with respect to a situation where the shock arrival rate is constant. Accordingly, it is likely that the assumption of a constant shock arrival rate leads to an underestimation of the discrepancy between the duration of temporary jobs and that of production opportunities.

Figure 9 also shows that the durations of temporary jobs and permanent jobs react in opposite directions when firing costs increase: when there are higher firing costs, the average expected duration of new temporary jobs is less than the average expected duration of jobs that would have been created to exploit the same production opportunities in the absence of job protection. In other words, firing costs have opposite effects on the duration of jobs created to exploit production opportunities with short duration and on the duration of jobs created to exploit production opportunities with long expected duration. As shown by Figure 10, which represents the density of job durations, higher firing costs increase the dispersion of job durations. When firing costs are higher, there are more jobs with long durations. But there are also more jobs with short durations, because there are more temporary jobs.

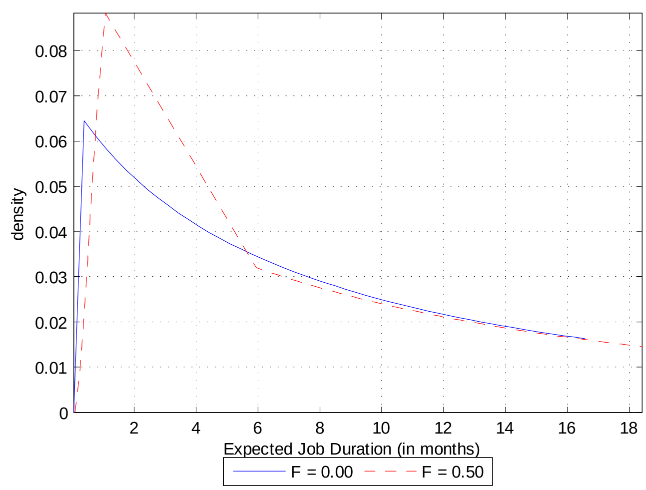

Figure 10: The density of expected job durations (in months) of new jobs for different values of firing costs (in share of monthly production of an employee).

It turns out that these two counteracting effects have a total positive impact on the average job duration in our model. The average job duration can be computed in two different ways. We can compute either the average duration of the stock of existing jobs (*i.e.* the cross-section of jobs) or the average expected duration of new jobs created. As shown by figure 11, increases in firing costs raise the average duration of the stock of jobs (left hand side panel) and of new jobs (right hand side panel). The effects are nevertheless small: increasing dismissal costs from the level observed in the US (equal to zero in the calibration) to that observed in a Continental European country like France (equal to about 50 percent of the average monthly production of jobs), raises the average duration of the stock of jobs by 1.7 percent and the average expected duration of the new jobs by 4.5 percent. This small impact is the net outcome of the two counteracting effects of firing costs on job durations.

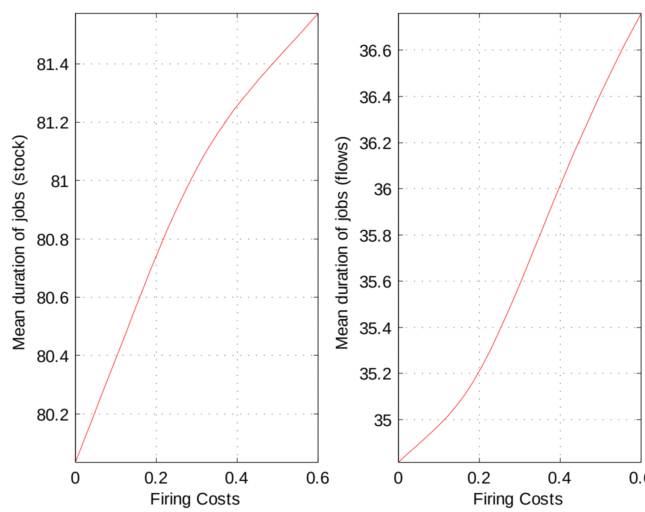

Figure 11: Mean job duration (in months) and firing costs (in share of monthly production of an employee). Left hand side panel: Mean duration of the stock of jobs in cross-section. Right hand side panel: Mean expected duration of new jobs.

Employment and production Employment protection creates labor hoarding on permanent jobs and increases the share of temporary jobs. Labor hoarding means that firms retain jobs with low productivity. Since temporary jobs have lower average productivity than permanent jobs, the rise in the share of temporary jobs reduces average productivity. It also induces higher labor turnover costs. We find that all these effects imply that the impact of employment protection on aggregate production is much larger than on aggregate employment. Aggregate production is defined as the sum of home production and the production of filled jobs, minus the cost of vacant jobs and the cost of writing contracts. Firing costs are not included in lost output in the benchmark computation.[30]

According to our simulation exercises, aggregate production is 0.3 percent lower in the economy with firing costs equal to 50 percent of the average monthly production of jobs than it is in the economy without job protection. Employment is 0.04 percent lower. When firing costs are included in lost output, productions drops by 1.6 percent. This shows that changes in production are much larger than changes in employment. This result is the consequence of a large degree of substitution between permanent and temporary jobs. When job protection increases, firms and workers are willing to create more temporary jobs and fewer permanent jobs in order to circumvent the cost of employment protection.

Table 3, which displays the impact of an increase in firing costs from 50 percent to 60 percent of the monthly average production of jobs, corresponding to the French and the Spanish situations respectively, sheds more light on this issue. The three bottom rows show that job protection induces a strong decrease in the number of permanent jobs which is almost compensated by the increase in the number of temporary jobs, so that the net impact of job protection on total employment is very small, equal to 0.015 percent. The variation in total employment is very small compared to the variation in permanent jobs, meaning that job protection entails a strong reallocation of jobs and negligible effects on total employment. This reallocation has important consequences on production. Rows 2 and 3 of table 3 show that job protection decreases the production of permanent jobs and raises the production of temporary jobs. These two counteracting effects also entail a negative effect on total production, equal to 0.095 percent, which is 6.5 times larger than the relative drop in employment. The relative drop in production becomes 20 times larger than the relative drop in employment if firing costs are included in lost output. This large difference between the change in aggregate production and the change in aggregate employment is the consequence of the increase in the share of unstable jobs, which reduces labor productivity and raises labor turnover costs. In other words, the detrimental effects of job protection are mainly due to its impact on the reallocation between permanent and temporary jobs.

Table 3 highlights a key result of our paper. The calibration assumes however that, absent employment protection, the Hosios condition for efficiency is satisfied. Hence, to assess the robustness of our results, we depart from the standard Hosios condition and focus on two

[30] See appendix G.4 for the details on how aggregate production is computed.

alternative cases which fall in the range of values $[0.4, 0.6]$ recommended by Petrongolo and Pissarides (2001) and those considered by Millard and Mortensen (1997) ($\beta = 0.3$) and Shimer (2005) ($\beta = 0.7$).[31] Results are provided in the last two columns of table 3. It is evident that our results are robust to these alternative parametric specifications.

All in all, our results point to a large degree of substitution between permanent and temporary jobs. These results are in line with empirical papers which show that job protection has strong effects on the composition of jobs. Centeno and Novo (2012) find that a reform that increased the employment protection of open-ended contracts in Portugal induced an increase in the share of temporary contracts consistent with a high degree of substitution between open-ended and fixed-term contracts. Cappellari et al. (2011) find similar results for Italy. Furthermore, Hijzen et al. (2013) show that these substitution effects induce significant drops in labor productivity.

		$\beta = \eta = 0.5$	$\beta = 0.7$	$\beta = 0.3$
Variation in aggregate production	ΔY	-0.0871	-0.1005	-0.0807
Variation in temp. jobs production	ΔY_s	0.4571	0.4473	0.4521
Variation in perm. jobs production	ΔY_p	-0.5442	-0.5478	-0.5328
Variation in the number of jobs	$\Delta(1-u)$	-0.0139	-0.0101	-0.0117
Variation in the number of temp. jobs	Δs	0.7518	0.7016	0.7709
Variation in the number of perm. jobs	Δp	-0.7658	-0.7117	-0.7825

Table 3: Decomposition of the impact of an increase in F from 0.50 to 0.60 on production and employment. At F=0.50, employment is equal to 93.96 and production to 91.41.

5 Conclusion

By taking into account the situation in which temporary contracts cannot be destroyed at zero cost before their date of termination we have been able to explain not only the choice between temporary and permanent contracts but also the duration of temporary contracts in a search and matching model of the labor market. This model reproduces some important stylized facts about temporary jobs observed in Continental European countries. Our framework shows that job protection of permanent jobs has a negligible impact on total employment but does entail a strong substitution of temporary jobs for permanent jobs, which decreases total production

[31] See Pissarides (2009) for further discussion on this point.

much more than it decreases employment. All in all, this model is useful for explaining and understanding the consequences of the huge creation of temporary jobs observed in Continental European countries characterized by stringent job protection legislations.

This model could be enriched in different ways. In particular, it neglects on-the-job search, a factor which may contribute to explaining the drop in entries into temporary jobs during downturns, when there are fewer voluntary quits associated with job-to-job shifts. It might also prove useful for analyzing the consequence of risk aversion. Another extension might be to consider a framework where the distribution G of durations of production opportunities is endogenous due, for instance, to match-specific investments undertaken by workers and firms to increase the longevity of jobs.

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APPENDIX

A Termination of temporary contracts

This appendix describes the legal rules for the termination of temporary contracts for 7 OECD countries. Other rules governing the conditions of creation, the maximal duration and the renewal of temporary jobs are described in detail in the ILO Employment protection legislation database[32] and in the OECD indicator of job protection.[33] We describe here the total dismissal costs, including severance payments, but our calibration exercises take into account "red tape" costs only. In general, a reasonable approximation of red-tape costs is that they are identical, within each country, for all dismissals, whether on permanent jobs or on temporary jobs before the date of termination stipulated in the contract.

Belgium: In principle regular dismissals (of the kind available in the case of an open-ended contract) are not possible for temporary jobs. The contract has to expire. The party which breaks the contract before the date of expiration without serious cause has to provide a severance payment the amount of which is equal to the minimum of the payment due until the date of expiration of the contract, and twice the payment due during the advance notice if the contract was permanent.

France: Regular dismissals (of the kind available in the case of an open-ended contract) are not possible for temporary jobs. The contract has to expire. Employers can only dismiss fixed-term workers if there is a credible "valid reason" which makes the continuation of employment unacceptable, e.g. fraudulent behavior by the employee. Conversely, the employee can quit if he finds an open-ended contract.

Germany: Regular dismissals (of the kind available in the case of an open-ended contract) are not possible for temporary jobs. The contract has to expire. Employers can only dismiss fixed-term workers if there is a credible "valid reason" which makes the continuation of employment unacceptable, e.g. fraudulent behavior by the employee.

Greece: Employers can only dismiss fixed-term workers if there is a credible "valid reason" which makes the continuation of employment unacceptable, e.g. fraudulent behavior by the employee. If the contract expires and the worker continues to be employed under the same conditions doing similar or the same work, then the worker is considered as being under an open-ended contract with the corresponding rules applying.

Italy: Regular dismissals (of the kind available in the case of an open ended contract) are not possible. The contract has to expire. Employers can only dismiss fixed-term workers if there is a credible "valid reason" which makes the continuation of employment unacceptable, e.g. fraudulent behavior by the employee. Conversely, the employee can quit if he finds an open-ended contract.

Portugal: the rule for individual dismissal is the same for fixed-term and open-ended contracts. Individual dismissals can be carried out solely for disciplinary reasons, which entails a fairly long disciplinary process. Among OECD countries Portugal is the one with the most stringent legislation for individual dismissals. So, in practice employers avoid this route, either waiting for the end of the fixed term contract (typically a one year contract, renewable for up to three years) or paying the corresponding severance pay (a minimum of three months); or, in the case of open-ended contracts,

[32] See http://www.ilo.org/dyn/terminate/
[33] See www.oecd.org/employment/protection

they negotiate a separation and very often pay out the stipulated amount of severance (one month for each year of tenure).

Spain: If the employer wishes to terminate the contract in advance, he would follow exactly the same procedures as a permanent contract and therefore would pay 20 days for an economic dismissal; but workers can go to court and the employer will normally pay at least the penalty rate of 45 days. So, usually, employers wait for expiration, unless the worker has committed a really serious offence (fraud, etc.).

B Asset Values and the surplus of temporary jobs in the benchmark model

B.1 Firms

Temporary jobs can be in one of the following two states: *(i)* "productive" with productivity $y > 0$; *(ii)* "unproductive" with zero productivity. All jobs start with productivity y. They are hit by shocks which arrive at idiosyncratic Poisson rate λ. When there is a shock, productivity irreversibly goes to zero and the job stays idle until the contract expires, whereupon the job is destroyed. Conversely, if the productivity of the job remains constant throughout the duration of the contract, the job can be converted into a permanent contract.

Let us denote by Δ the duration of the temporary contract, which is decided when the job starts. Let us denote by τ the spell of the job from its date of creation. Once the cost c to sign the contract has been paid, the present discounted value for a firm of a temporary job with shock arrival rate λ, contract duration Δ, spell τ and productivity $x = y, 0$, is denoted by $J_t(\lambda, \Delta, x, \tau)$. The Bellman equations for a firm satisfy:

$$rJ_t(\lambda, \Delta, y, \tau) = y - w(\lambda, \Delta) + \lambda [J_t(\lambda, \Delta, 0, \tau) - J_t(\lambda, \Delta, y, \tau)] + \dot{J}_t(\lambda, \Delta, y, \tau), \tag{B1}$$

$$rJ_t(\lambda, \Delta, 0, \tau) = -w(\lambda, \Delta) + \dot{J}_t(\lambda, \Delta, 0, \tau), \tag{B2}$$

where $\dot{J}_t(\lambda, \Delta, x, \tau) = \partial J_t(\lambda, \Delta, x, \tau)/\partial\tau$.

At the date of termination of the temporary job, there are two possible outcomes. On one hand, if the job has not been hit by a productivity shock, it can be converted into a permanent job. The formal condition reads, when $\tau = \Delta$, as

$$J_t(\lambda, \Delta, y, \Delta) = \max[J_p(\lambda), 0], \tag{B3}$$

where $J_p(\lambda)$ denotes the value of a permanent job for the firm. On the other hand, if a shock did occur, the job is destroyed as soon as the contract reaches its term. The formal condition reads, when $\tau = \Delta$, as

$$J_t(\lambda, \Delta, 0, \Delta) = 0. \tag{B4}$$

Let us find the solution to the system of equations (B1), (B2) with terminal conditions (B3), (B4). A general solution to the first-order linear differential equation (B2) (with constant coefficient and constant term) is given by:

$$J_t(\lambda, \Delta, 0, \tau) = Ae^{r\tau} + B, \tag{B5}$$

where A and B are constants to be determined. Differentiation of (B5) with respect to τ yields $\dot{J}_t(\lambda, \Delta, 0, \tau) = rAe^{r\tau}$. Plugging this expression together with (B5) into (B2), one gets: $B = -w(\lambda, \Delta)/r$. Making use of the terminal condition $J_t(\lambda, \Delta, 0, \Delta) = 0$, we get $Ae^{r\Delta} + B = Ae^{r\Delta} - w(\lambda, \Delta)/r = 0$, and it follows that: $A = \frac{w(\lambda,\Delta)}{r} e^{-r\Delta}$. Finally, using (B5) we get:

$$J_t(\lambda, \Delta, 0, \tau) = -\left(1 - e^{-r(\Delta-\tau)}\right) \frac{w(\lambda, \Delta)}{r}. \tag{B6}$$

Let now rewrite (B1) as:

$$\dot{J}_t(\lambda, \Delta, y, \tau) = (r + \lambda) J_t(\lambda, \Delta, y, \tau) - (y - w(\lambda, \Delta)) - \lambda J_t(\lambda, \Delta, 0, \tau),$$

which is a first-order linear differential equation (with constant coefficient and variable term) of the form

$$\dot{J}_t(\lambda, \Delta, y, \tau) = C J_t(\lambda, \Delta, y, \tau) + D(\tau),$$

where $C = (r + \lambda)$ and $D = -(y - w(\lambda, \Delta)) - \lambda J_t(\lambda, \Delta, 0, \tau)$. A general solution to this equation is given by:

$$\begin{aligned} J_t(\lambda, \Delta, y, \tau) &= e^{C(\tau-\Delta)} \left[J_t(\lambda, \Delta, y, \Delta) + \int_\Delta^\tau e^{-C(\zeta-\Delta)} D(\zeta)\, \mathrm{d}\zeta \right] \\ &= e^{(r+\lambda)(\tau-\Delta)} \left[J_t(\lambda, \Delta, y, \Delta) - \underbrace{\int_\Delta^\tau e^{-(r+\lambda)(\zeta-\Delta)} \left[(y - w(\lambda, \Delta)) + \lambda J_t(\lambda, \Delta, 0, \zeta) \right] \mathrm{d}\zeta}_{\Gamma} \right] \end{aligned} \tag{B7}$$

Using (B6), it is possible to rewrite Γ as:

$$\Gamma = \left[y - (r + \lambda) \frac{w(\lambda, \Delta)}{r} \right] \left(\frac{1 - e^{-(r+\lambda)(\tau-\Delta)}}{r + \lambda} \right) + \frac{w(\lambda, \Delta)}{r} \left(1 - e^{-\lambda(\tau-\Delta)} \right).$$

Multiplying both sides of this expression by $e^{(r+\lambda)(\tau-\Delta)}$, we get:

$$\Gamma e^{(r+\lambda)(\tau-\Delta)} = \left[y - (r + \lambda) \frac{w(\lambda, \Delta)}{r} \right] \left(\frac{e^{(r+\lambda)(\tau-\Delta)} - 1}{r + \lambda} \right) + \frac{w(\lambda, \Delta)}{r} \left(e^{(r+\lambda)(\tau-\Delta)} - e^{r(\tau-\Delta)} \right).$$

Using this expression together with (B7) yields:

$$J_t(\lambda, \Delta, y, \tau) = y \frac{1 - e^{(r+\lambda)(\tau-\Delta)}}{r + \lambda} - \frac{w(\lambda, \Delta)}{r} \left[1 - e^{r(\tau-\Delta)} \right] + e^{(r+\lambda)(\tau-\Delta)} J_t(\lambda, \Delta, y, \Delta).$$

Finally, using the fact that $J_t(\lambda, \Delta, y, \Delta) = \max[J_p(\lambda), 0]$, then setting $\tau = 0$ and rearranging, the starting value of a temporary job writes as:

$$J_t(\lambda, \Delta, y, 0) = y \frac{1 - e^{-(r+\lambda)\Delta}}{r + \lambda} - \frac{w(\lambda, \Delta)}{r} \left(1 - e^{-r\Delta} \right) + \max[J_p(\lambda), 0]\, e^{-(r+\lambda)\Delta}. \tag{B8}$$

Dropping the last two arguments of $J_t(\lambda, \Delta, y, 0)$ in order to alleviate the notations, we denote as $J_t(\lambda, \Delta) = J_t(\lambda, \Delta, y, 0) - c$, the starting value of the discounted expected profit of a temporary job, including the cost of writing contracts. This last expression is similar to equation (6) displayed in the main text:

$$J_t(\lambda, \Delta) = \int_0^\Delta \left(y e^{-\lambda\tau} - w(\lambda, \Delta) \right) e^{-r\tau} \mathrm{d}\tau + \max[J_p(\lambda), 0]\, e^{-(r+\lambda)\Delta} - c. \tag{B9}$$

B.2 Workers

The definition of value functions for workers is obtained with the same method as that for firms. Let us denote by $W_t(\lambda, \Delta, x, \tau)$ the present discounted value for a worker of a temporary job with shock arrival rate λ, contract duration Δ, spell τ and productivity $x = y, 0$. The Bellman equations for a worker satisfy:

$$rW_t(\lambda, \Delta, y, \tau) = w(\lambda, \Delta) + \lambda [W_t(\lambda, \Delta, 0, \tau) - W_t(\lambda, \Delta, y, \tau)] + \dot{W}_t(\lambda, \Delta, y, \tau), \tag{B10}$$

$$rW_t(\lambda, \Delta, 0, \tau) = w(\lambda, \Delta) + \dot{W}_t(\lambda, \Delta, 0, \tau), \tag{B11}$$

where $\dot{W}_t(\lambda, \Delta, y, \tau) = \partial W_t(\lambda, \Delta, y, \tau)/\partial \tau$. The resolution of the Bellman equations for the worker operates symmetrically with the resolution proposed for the firm. At the date of termination of the temporary job, there are again two possible outcomes. On one hand, if the job has not been hit by a productivity shock, it can be converted into a permanent job. The formal condition reads, when $\tau = \Delta$, as

$$W_t(\lambda, \Delta, y, \Delta) = \max [W_p(\lambda), U]. \tag{B12}$$

where $W_p(\lambda)$ and U denote the expected utility of a permanent job for the worker and the expected utility of an unemployed worker respectively. On the other hand, if a shock did occur, the worker loses his job and becomes unemployed as soon as the contract reaches its term. The formal condition reads, when $\tau = \Delta$, as

$$W_t(\lambda, \Delta, 0, \Delta) = U. \tag{B13}$$

Equation (B11) is a first-order linear differential equation with constant coefficients which, using the same steps as for the firm, has the solution:

$$W_t(\lambda, \Delta, 0, \tau) = Ue^{-r(\Delta-\tau)} + \frac{w(\lambda, \Delta)}{r}(1 - e^{-r(\Delta-\tau)}). \tag{B14}$$

Let now rewrite (B10) as

$$\dot{W}_t(\lambda, \Delta, y, \tau) = (r + \lambda) W_t(\lambda, \Delta, y, \tau) - w(\lambda, \Delta) - \lambda W_t(\lambda, \Delta, 0, \tau),$$

which is a first-order linear differential equation with constant coefficient and variable term which admits the general solution

$$W_t(\lambda, \Delta, y, \tau) = e^{\widetilde{C}(\tau-\Delta)} \left[W_t(\lambda, \Delta, y, \Delta) + \underbrace{\int_{\Delta}^{\tau} e^{-\widetilde{C}(\zeta-\Delta)} \widetilde{D}(\zeta) \, \mathrm{d}\zeta}_{\widetilde{\Gamma}} \right],$$

where $\widetilde{C} = (r + \lambda)$ and $\widetilde{D}(\zeta) = -w(\lambda, \Delta) - \lambda W_t(\lambda, \Delta, 0, \zeta)$. Using (B14) and proceeding as for the firm we get that

$$\widetilde{\Gamma} e^{(r+\lambda)(\tau-\Delta)} = \frac{w(\lambda, \Delta)}{r} \left(1 - e^{r(\tau-\Delta)}\right) - U \left(e^{(r+\lambda)(\tau-\Delta)} - e^{r(\tau-\Delta)}\right),$$

so that

$$W_t(\lambda, \Delta, y, \tau) = e^{(r+\lambda)(\tau-\Delta)})W_t(\lambda, \Delta, y, \Delta) + \frac{w(\lambda, \Delta)}{r}\left(1 - e^{r(\tau-\Delta)}\right) - U\left(e^{(r+\lambda)(\tau-\Delta)} - e^{r(\tau-\Delta)}\right).$$

Finally, using (B12), then setting $\tau = 0$ and rearranging, the value of a temporary job for a worker writes as

$$W_t(\lambda, \Delta, y, 0) = e^{-(r+\lambda)\Delta}\max\left[W_p(\lambda), U\right] + \frac{w(\lambda, \Delta)}{r}(1 - e^{-r\Delta})) - U\left(e^{-(r+\lambda)\Delta} - e^{-r\Delta}\right). \tag{B15}$$

Dropping the last two arguments of $W_t(\lambda, \Delta, y, 0)$ in order to alleviate the notations, we denote as $W_t(\lambda, \Delta)$ the starting value of a temporary job for a worker. This last expression is similar to equation (7) displayed in the main text

$$W_t(\lambda, \Delta) = \int_0^{\Delta}(w(\lambda, \Delta) - rU)e^{-r\tau}\mathrm{d}\tau + e^{-(r+\lambda)\Delta}\max\left[W_p(\lambda), U\right] + U\left(1 - e^{-(r+\lambda)\Delta}\right). \tag{B16}$$

B.3 Surplus of a temporary job

Let us define by $S_t(\lambda, \Delta)$ the starting value of the surplus of a temporary job with duration Δ and Poisson rate λ. We have

$$S_t(\lambda, \Delta) = J_t(\lambda, \Delta) + W_t(\lambda, \Delta) - U. \tag{B17}$$

Making use of (B9) and (B16), and reinserting in (B17) we get:

$$\begin{aligned} S_t(\lambda, \Delta) &= \int_0^{\Delta}\left(ye^{-\lambda\tau} - w(\lambda, \Delta)\right)e^{-r\tau}\mathrm{d}\tau + \max\left[J_p(\lambda), 0\right]e^{-(r+\lambda)\Delta} - c + \int_0^{\Delta}(w(\lambda, \Delta) - rU)e^{-r\tau}\mathrm{d}\tau \\ &\quad + e^{-(r+\lambda)\Delta}\max\left[W_p(\lambda) - U, 0\right] \\ &= \int_0^{\Delta}\left(ye^{-\lambda\tau} - rU\right)e^{-r\tau}\mathrm{d}\tau + \max\left[\underbrace{J_p(\lambda) + W_p(\lambda) - U}_{=S_p(\lambda)}, 0\right]e^{-(r+\lambda)\Delta} - c, \end{aligned}$$

where $S_p(\lambda)$ denotes the starting value of the surplus of a permanent job with shock arrival rate λ. This last expression is similar to equation (9).

C The properties of functions $S_p(\lambda)$ and $S_t(\lambda)$

This section proves the properties of $S_p(\lambda)$ and $S_t(\lambda) = \max_{\Delta} S_t(\lambda, \Delta)$ presented in section 3 and provides a proof for proposition 1. We begin by analyzing the properties of $S_p(\lambda)$, then we continue with the properties of $S_t(\lambda)$ and finally we prove proposition 1.

C.1 Analysis of $S_p(\lambda)$

The function

$$S_p(\lambda) = \frac{y - rU - \lambda F}{r + \lambda} - c, \tag{C18}$$

is continuous. It is decreasing as $S_p'(\lambda) = \frac{-y+rU-rF}{(r+\lambda)^2} \leq 0$. It decreases from $S_p(0) = \frac{y}{r} - U - c > 0$ to $\lim_{\lambda\to+\infty} S_p(\lambda) = -c - F < 0$. Thus, there exists a unique threshold value $\lambda_p = \frac{y-rU-rc}{F+c}$ such that $S_p(\lambda_p)\gtreqless 0$ if and only if $\lambda \gtreqless \lambda_p$, as indicated in section 3.

C.2 Properties of the optimal duration $\Delta(\lambda)$

We have:

$$\Delta(\lambda) = \begin{cases} \frac{1}{\lambda}\ln\left(\frac{rU+\lambda F+(r+\lambda)c}{rU}\right) & \text{if } \lambda \leq \lambda_p \\ \frac{1}{\lambda}\ln\left(\frac{y}{rU}\right) & \text{if } \lambda \geq \lambda_p \end{cases} \tag{C19}$$

Function $\Delta(\lambda)$ is continuous and has a kink at λ_p. Let us now show that it is decreasing. This is obvious when $\lambda \geq \lambda_p$. When $\lambda \leq \lambda_p$, we get

$$\begin{aligned} \Delta'(\lambda) &= \frac{1}{\lambda^2}\ln\left(\frac{rU}{rU+\lambda F+(r+\lambda)c}\right) + \frac{1}{\lambda}\left(\frac{F+c}{rU+\lambda F+(r+\lambda)c}\right) \\ &= \frac{1}{\lambda^2}\left[\ln\left(\frac{rU}{rU+\lambda F+(r+\lambda)c}\right) - \left(\frac{rU}{rU+\lambda F+(r+\lambda)c} - 1\right) - \frac{rc}{rU+\lambda F+(r+\lambda)c}\right], \end{aligned}$$

which is negative, because $\ln(x) < x - 1$ for all $x > 0$.

Finally, over the interval $(0,\infty)$, $\Delta(\lambda)$ goes from $\lim_{\lambda\to 0}\Delta(\lambda) = \lim_{\lambda\to 0}\frac{1}{\lambda}\ln\left(\frac{rU+\lambda F+(r+\lambda)c}{rU}\right) = +\infty$ to $\lim_{\lambda\to+\infty}\Delta(\lambda) = \lim_{\lambda\to+\infty}\frac{1}{\lambda}\ln\left(\frac{y}{rU}\right) = 0$.

C.3 Analysis of $S_t(\lambda)$

- Let us show that $S_t(\lambda)$ is continuous and decreasing with $\lim_{\lambda\to 0} S_t(\lambda) = \frac{y}{r} - U - c > 0$, $\lim_{\lambda\to\infty} S_t(\lambda) = -c$.

- When $\lambda \geq \lambda_p$, we get, from equation (9):

$$S_t'(\lambda) = y\frac{e^{-(r+\lambda)\Delta(\lambda)}\left[(r+\lambda)\Delta(\lambda)+1\right]-1}{(r+\lambda)^2},$$

  which is negative because $e^{-x} < 1/(x+1)$ when $x > 0$. Equations (9) and (10) allow us to write

$$S_t(\lambda) = y\frac{1-e^{-(r+\lambda)\Delta(\lambda)}}{r+\lambda} - U\left[1-e^{-r\Delta(\lambda)}\right] - c.$$

  From the definition of $\Delta(\lambda)$ we know that $\lim_{\lambda\to\infty}\Delta(\lambda) = 0$ and $e^{-\lambda\Delta(\lambda)} = rU/y$, so that $\lim_{\lambda\to\infty}\frac{1-e^{-(r+\lambda)\Delta(\lambda)}}{r+\lambda} = 0$ and $\lim_{\lambda\to\infty} 1 - e^{-r\Delta(\lambda)} = 0$. Therefore, $\lim_{\lambda\to\infty} S_t(\lambda) = -c$.

- When $\lambda < \lambda_p$, we get, from equation (9)

$$S_t'(\lambda) = y\frac{e^{-(r+\lambda)\Delta(\lambda)}\left[(r+\lambda)\Delta(\lambda)+1\right]-1}{(r+\lambda)^2} - e^{-(r+\lambda)\Delta(\lambda)}\left[\Delta(\lambda)S_p(\lambda) - S_p'(\lambda)\right],$$

  which is negative, since it has just been shown that the first term $y\frac{e^{-(r+\lambda)\Delta(\lambda)}[(r+\lambda)\Delta(\lambda)+1]-1}{(r+\lambda)^2}$ is negative. Moreover, $S_p(\lambda) > 0$ when $\lambda < \lambda_p$, and $S_p'(\lambda) < 0$. From the definition of the surplus we get

$$\lim_{\lambda\to 0} S_t(\lambda) = \lim_{\lambda\to 0} S_p(\lambda) = \frac{y}{r} - U - c.$$

- We have established that $S_t(\lambda)$ monotonically decreases from $\frac{y}{r} - U - c$ to $-c$. Thus, there exists a unique threshold λ_t such that $S_t(\lambda_t) = 0$. Using the expression of the surplus (9), it follows that λ_t solves the following equation

$$y\frac{1-e^{-(r+\lambda_t)\Delta(\lambda_t)}}{r+\lambda_t} - U(1-e^{-r\Delta(\lambda_t)}) - c = 0. \tag{C20}$$

C.4 Proof of proposition 1

This appendix proves proposition 1. For this purpose, it is convenient to define

$$h(\lambda) = S_t(\lambda) - S_p(\lambda).$$

Making use of the definition of the surpluses (9) and (4) above, and of the FOC (10), we get

$$h(\lambda) = \frac{\lambda F - \lambda U(1-e^{-r\Delta(\lambda)})}{r+\lambda}. \tag{C21}$$

Let us study the properties of the function $h(\lambda)$ defined by (C21) over the interval $[0, +\infty)$ to examine the intercept of $S_t(\lambda)$ and $S_p(\lambda)$. Proposition 1 distinguishes four cases.

- Case 1: $c > 0$ and $F < U$ and condition (12) holds. In this case, we can prove that (i) $\exists \lambda | S_t(\lambda) = S_p(\lambda)$; (ii) $\lambda | S_t(\lambda) = S_p(\lambda)$ is unique; (iii) $0 < \lambda_s < \lambda_p < \lambda$, where $\lambda_p = \{\lambda | S_p(\lambda) = 0\}$, $\lambda_t = \{\lambda | S_t(\lambda) = 0\}$ and $\lambda_s = \{\lambda | S_t(\lambda) = S_p(\lambda)\}$.

(i) h is a continuous function defined over the interval $[0, +\infty)$, with $\lim\limits_{\lambda\to 0} h(\lambda) = 0^-$ and $\lim\limits_{\lambda\to\infty} h(\lambda) = F \geq 0$. Besides,

$$h'(\lambda) = \frac{-\lambda U r \Delta'(\lambda) e^{-r\Delta(\lambda)}}{r+\lambda} + \frac{r\,(F-U)}{(r+\lambda)^2} + \frac{rUe^{-r\Delta(\lambda)}}{(r+\lambda)^2}. \tag{C22}$$

The sign of $h'(\lambda)$ is ambiguous when $F < U$.

However, $\lim\limits_{\lambda\to 0} h'(\lambda) = \frac{F-U}{r} < 0$ when $F < U$, as $\lim\limits_{\lambda\to 0} e^{-r\Delta(\lambda)} = 0$ and $\lim\limits_{\lambda\to 0} \Delta'(\lambda) e^{-r\Delta(\lambda)} = 0$. Therefore, h starts from a negative value close to zero, is first decreasing (negatively valued) and must then be increasing over some range to meet the condition $\lim\limits_{\lambda\to+\infty} h(\lambda) = F \geq 0$. By continuity, there exists at least one value of λ, such that $h(\lambda) = 0$.

(ii) Let us now prove that there exists a unique value of λ, denoted by λ_s such that $h(\lambda_s) = 0$. Using (C21), the definition of λ_s implies $F = U(1 - e^{-r\Delta(\lambda_s)})$. Reinserting in (C22) yields

$$h'(\lambda_s) = \frac{-\lambda_s U r \Delta'(\lambda_s) e^{-r\Delta(\lambda_s)}}{r+\lambda_s} \geq 0,$$

which establishes uniqueness, as multiple thresholds would imply $h' \leq 0$ for at least one of those thresholds. As a result, we have $h(\lambda) \leq 0$ for $\lambda \leq \lambda_s$ while $h(\lambda) > 0$ for $\lambda > \lambda_s$.

(iii) Let us show that condition (12) implies $0 < \lambda_s < \lambda_p < \lambda_t$ We have already shown that $\lambda_p = \{\lambda | S_p(\lambda) = 0\}$, $\lambda_t = \{\lambda | S_t(\lambda) = 0\}$ and $\lambda_s = \{\lambda | S_t(\lambda) = S_p(\lambda)\}$ exist and are unique. We now prove that $\lambda_s < \lambda_p$ and that $\lambda_p < \lambda_t$ when condition (12) holds.

Let us prove that $\lambda_s < \lambda_p$. We have established that $h(\lambda) \leq 0$ for $\lambda \leq \lambda_s$ while $h(\lambda) > 0$ for $\lambda > \lambda_s$. Using equations (9) and (10), we can write

$$S_t(\lambda) = \max\left[S_p(\lambda), 0\right] - c + rU\left(e^{\lambda\Delta(\lambda)}\frac{1 - e^{-(r+\lambda)\Delta(\lambda)}}{r+\lambda} - \frac{1 - e^{-r\Delta(\lambda)}}{r}\right). \tag{C23}$$

Condition (12) together with equation (10) implies that $S_t(\lambda_p) > 0$, and hence, $h(\lambda_p) > 0$. Therefore, we have that $\lambda_p > \lambda_s$.

Let us prove that $\lambda_p < \lambda_t$. We have just shown that $S_t(\lambda_p) > 0$. Since S_t is decreasing in λ, we have $\lambda_t > \lambda_p$.

Therefore, when $c > 0$ and $F < U$ and condition (12) holds, temporary contracts are chosen for $\lambda \in [\lambda_s, \lambda_t]$ while permanent contracts are chosen for $\lambda < \lambda_s$, and there are no jobs above λ_t.

- Case 2: $c > 0$, $F < U$ and condition (12) does not hold. In this case, condition (12) together with equation (10) implies that $S_t(\lambda_p) < 0$. Since $S_p(\lambda)$ is decreasing, this implies that $\lambda_p > \lambda_s$. In this case, temporary contracts cannot be profitably utilized and only permanent contracts are chosen for $\lambda < \lambda_p$ while no contract is profitable for $\lambda > \lambda_p$.

- Case 3: $c > 0$ and $F \geq U$. In this case, $h'(\lambda) \geq 0$ according to (C22). Thus $h(\lambda)$ is continuously increasing from 0 to F, so that $h(\lambda) \geq 0$ for all $\lambda \in [0, +\infty)$. It is thus optimal to choose temporary contracts for $\lambda < \lambda_t$, while no contract is profitable for $\lambda > \lambda_t$.

- Case 4: $c = 0$. In this case, from the definition of the surpluses, it is simple to show that $h(\lambda) \geq 0$ for any $\lambda \in [0, +\infty)$.Namely, using (9) and (4), we get

  $$h(\lambda) = U(1 - e^{-r\Delta(\lambda)}) + \frac{rU(1 - e^{-(r+\lambda)\Delta(\lambda)})}{r+\lambda} + \frac{\lambda F(1 - e^{-(r+\lambda)\Delta(\lambda)})}{r+\lambda} - ce^{-(r+\lambda)\Delta(\lambda)},$$

  which is always positive when $c = 0$. Therefore, it is optimal to choose temporary contracts for $\lambda < \lambda_t$, while no contract is profitable for $\lambda > \lambda_t$.

D Surplus of temporary jobs in the model with productivity shocks

D.1 Firms

Let us denote by $J_t(\lambda, \Delta, y, \tau)$ the value to the firm at date τ of temporary jobs with shock arrival rate λ, duration Δ and productivity y. When there is a shock, there is a draw from the constant distribution H of productivities. $J_t(\lambda, \Delta, y, \tau)$ satisfies the Bellman equation

$$rJ_t(\lambda, \Delta, y, \tau) = y - w\left(\lambda, \Delta\right) + \lambda\left[\int J_t(\lambda, \Delta, x, \tau)\mathrm{d}H(x) - J_t(\lambda, \Delta, y, \tau)\right] + \dot{J}_t(\lambda, \Delta, y, \tau), \tag{D24}$$

with $\dot{J}_t(\lambda, \Delta, y, \tau) = \partial J_t(\lambda, \Delta, y, \tau)/\partial\tau$. At date $\tau = \Delta$

$$J_t(\lambda, \Delta, y, \Delta) = \max\left[J_p\left(y, \lambda\right), 0\right]. \tag{D25}$$

We proceed in two steps. Let us (i) determine$\int J_t(\lambda, \Delta, y, \tau)\mathrm{d}H(y)$, and then (ii) solve for $J_t(\lambda, \Delta, y, \tau)$ in (D24).

D.2 Part (i)

Integrating (D24) over productivity, we get

$$r\int J_t(\lambda,\Delta,y,\tau)\mathrm{d}H(y)=\int\left(y-w\left(\lambda,\Delta\right)\right)\mathrm{d}H(y)+\int\dot{J}_t(\lambda,\Delta,y,\tau)\mathrm{d}H(y). \tag{D26}$$

This equation takes the form $\dot{x}(\tau)=rx(\tau)+b$ with $b=-\int\left(y-w\left(\lambda,\Delta\right)\right)\mathrm{d}H(y)$, and with terminal condition $x(\Delta)=\int\max\left[J_p\left(x,\lambda\right),0\right]\mathrm{d}H(x)$. Its general solution is

$$x(\tau)=Ae^{r\tau}+B,$$

with

$$B=\frac{1}{r}\int\left(y-w\left(\lambda,\Delta\right)\right)\mathrm{d}H(y),$$

and

$$A=\left(\int\max\left[J_p\left(\lambda,y\right),0\right]\mathrm{d}H(y)-\frac{1}{r}\int\left(y-w\left(\lambda,\Delta\right)\right)\mathrm{d}H(y)\right)e^{-r\Delta}.$$

Thus:

$$\int J_t(\lambda,\Delta,y,\tau)\mathrm{d}H(y)=\left(\frac{1-e^{-r(\Delta-\tau)}}{r}\right)\int\left(y-w\left(\lambda,\Delta\right)\right)\mathrm{d}H(y)+e^{-r(\Delta-\tau)}\int\max\left[J_p\left(\lambda,y\right),0\right]\mathrm{d}H(y).$$

D.3 Part (ii)

Equation (D24)

$$\dot{J}_t(\lambda,\Delta,y,\tau)=(r+\lambda)J_t(\lambda,\Delta,y,\tau)-\left(y-w\left(\lambda,\Delta\right)\right)-\lambda\int J_t(\lambda,\Delta,x,\tau)\mathrm{d}H(x),$$

is a first order linear differential equation with constant coefficient $C=(r+\lambda)$ and variable term, $D(\tau)=-\left(y-w\left(\lambda,\Delta\right)\right)-\lambda\int J_t(\lambda,\Delta,x,\tau)\mathrm{d}H(x)$. Its general solution is

$$\begin{aligned}
J_t(\lambda,\Delta,y,\tau) &= e^{C(\tau-\Delta)}\left[J_t(\lambda,\Delta,y,\Delta)+\int_\Delta^\tau e^{-C(\zeta-\Delta)}D\left(\zeta\right)\mathrm{d}\zeta\right]\\
&= e^{(r+\lambda)(\tau-\Delta)}\left[J_t(\lambda,\Delta,y,\Delta)+\underbrace{\int_\Delta^\tau e^{-(r+\lambda)(\zeta-\Delta)}D\left(\zeta\right)\mathrm{d}\zeta}_{\Gamma}\right].
\end{aligned}$$

The term Γ can be rewritten

$$\Gamma=-\left(\frac{1-e^{-(r+\lambda)(\tau-\Delta)}}{(r+\lambda)}\right)\left(y-w\left(\lambda,\Delta\right)\right)-\frac{\lambda}{r}\int\left(x-w\left(\lambda,\Delta\right)\right)\mathrm{d}H(x)\left(\frac{1-e^{-(r+\lambda)(\tau-\Delta)}}{r+\lambda}-\frac{1-e^{-\lambda(\tau-\Delta)}}{\lambda}\right)$$
$$-\int\max\left[J_p\left(x,\lambda\right),0\right]\mathrm{d}H(x)\left(1-e^{-\lambda(\tau-\Delta)}\right).$$

Thus

$$
\begin{aligned}
J_t(\lambda,\Delta,y,\tau) &= e^{(r+\lambda)(\tau-\Delta)}\left[J_t(\lambda,\Delta,y,\Delta)+\int_\Delta^\tau e^{-(r+\lambda)(\zeta-\Delta)}D(\zeta)\,\mathrm{d}\zeta\right] \\
&= e^{(r+\lambda)(\tau-\Delta)}J_t(\lambda,\Delta,y,\Delta) \\
&\quad -(y-w(\lambda,\Delta))\,e^{(r+\lambda)(\tau-\Delta)}\left(\frac{1-e^{-(r+\lambda)(\tau-\Delta)}}{r+\lambda}\right) \\
&\quad -\frac{\lambda}{r}e^{(r+\lambda)(\tau-\Delta)}\int (x-w(\lambda,\Delta))\,\mathrm{d}H(x)\left(\frac{1-e^{-(r+\lambda)(\tau-\Delta)}}{r+\lambda}-\frac{1-e^{-\lambda(\tau-\Delta)}}{\lambda}\right) \\
&\quad -e^{(r+\lambda)(\tau-\Delta)}\int \max\left[J_p(x,\lambda),0\right]\mathrm{d}H(x)\left(1-e^{-\lambda(\tau-\Delta)}\right).
\end{aligned}
$$

Since $J_t(\lambda,\Delta,y,\Delta)=\max\left[J_p(\lambda,y),0\right]$, we get

$$
\begin{aligned}
J_t(\lambda,\Delta,y,0) &= (y-w(\lambda,\Delta))\frac{1-e^{-(r+\lambda)\Delta}}{r+\lambda}+e^{-(r+\lambda)\Delta}\max\left[J_p(y,\lambda),0\right] \\
&\quad -\frac{\lambda}{r}\int (x-w(\lambda,\Delta))\,\mathrm{d}H(x)\left(\frac{e^{-(r+\lambda)\Delta}-1}{r+\lambda}-\frac{e^{-(r+\lambda)\Delta}-e^{-r\Delta}}{\lambda}\right) \\
&\quad -\int \max\left[J_p(x,\lambda),0\right]\mathrm{d}H(x)\left(e^{-(r+\lambda)\Delta}-e^{-r\Delta}\right).
\end{aligned}
$$

Rearranging and taking account of the fact that jobs always start at y_u yields

$$
\begin{aligned}
J_t(\lambda,\Delta) &= y_u\left(\frac{1-e^{-(r+\lambda)\Delta}}{r+\lambda}\right)+\int y\mathrm{d}H(y)\left(\frac{\lambda(1-e^{-r\Delta})+r(e^{-(r+\lambda)\Delta}-e^{-r\Delta})}{r(r+\lambda)}\right) \\
&\quad -\frac{w(\lambda,\Delta)}{r}\left(1-e^{-r\Delta}\right)+e^{-(r+\lambda)\Delta}\max\left[J_p(y_u,\lambda),0\right] \\
&\quad +e^{-r\Delta}\left(1-e^{-\lambda\Delta}\right)\int \max\left[J_p(y,\lambda),0\right]\mathrm{d}H(y)-c,
\end{aligned}
\tag{D27}
$$

which is formally equivalent to the expression given by equation (20).

D.4 Workers

Let us denote by $W_t(\lambda,\Delta,y,\tau)$ the value to the firm at date τ of temporary jobs with shock arrival rate λ, duration Δ and productivity y. When there is a shock, there is a draw from the constant distribution H of productivities. $W_t(\lambda,\Delta,y,\tau)$ satisfies the Bellman equation

$$
rW_t(\lambda,\Delta,y,\tau)=w(\lambda,\Delta)+\lambda\left[\int W_t(\lambda,\Delta,x,\tau)\mathrm{d}H(x)-W_t(\lambda,\Delta,y,\tau)\right]+\dot{W}_t(\lambda,\Delta,y,\tau), \tag{D28}
$$

with $\dot{W}_t(\lambda,\Delta,y,\tau)=\partial W_t(\lambda,\Delta,y,\tau)/\partial\tau$. At date $\tau=\Delta$

$$
W_t(\lambda,\Delta,y,\Delta)=\max\left[W_p(y,\lambda),U\right]. \tag{D29}
$$

We proceed in two steps. Let us (i) determine $\int W_t(\lambda,\Delta,y,\tau)\mathrm{d}H(y)$, and then (ii) solve for $W_t(\lambda,\Delta,y,\tau)$ in (D28).

D.5 Part (i)

Integrating (D28) over productivity, we get

$$r\int W_t(\lambda,\Delta,y,\tau)\mathrm{d}H(y)=\int w\left(\lambda,\Delta\right)\mathrm{d}H(y)+\int \dot{W}_t(\lambda,\Delta,y,\tau)\mathrm{d}H(y). \tag{D30}$$

This equation takes the form $\dot{x}(\tau)=rx(\tau)+b$ with $b=-\int w\left(\lambda,\Delta\right)\mathrm{d}H(y)$, and with terminal condition $x(\Delta)=\int\max\left[W_p\left(x,\lambda\right),U\right]\mathrm{d}H(x)$. Its general solution is

$$x(\tau)=Ae^{r\tau}+B,$$

with

$$B=\frac{1}{r}\int w\left(\lambda,\Delta\right)\mathrm{d}H(y),$$

and

$$A=\left(\int\max\left[W_p\left(\lambda,y\right),U\right]\mathrm{d}H(y)-\frac{1}{r}\int w\left(\lambda,\Delta\right)\mathrm{d}H(y)\right)e^{-r\Delta}.$$

Thus

$$\int W_t(\lambda,\Delta,y,\tau)\mathrm{d}H(y)=\left(\frac{1-e^{-r(\Delta-\tau)}}{r}\right)\int w\left(\lambda,\Delta\right)\mathrm{d}H(y)+e^{-r(\Delta-\tau)}\int\max\left[W_p\left(\lambda,y\right),U\right]\mathrm{d}H(y).$$

D.6 Part (ii)

Equation (D28)

$$\dot{W}_t(\lambda,\Delta,y,\tau)=(r+\lambda)W_t(\lambda,\Delta,y,\tau)-w\left(\lambda,\Delta\right)-\lambda\int W_t(\lambda,\Delta,x,\tau)\mathrm{d}H(x),$$

is a first order linear differential equation with constant coefficient $\widetilde{C}=(r+\lambda)$ and variable term, $\widetilde{D}(\tau)=-w\left(\lambda,\Delta\right)-\lambda\int W_t(\lambda,\Delta,x,\tau)\mathrm{d}H(x)$. Its general solution is

$$\begin{aligned}W_t(\lambda,\Delta,y,\tau) &= e^{\widetilde{C}(\tau-\Delta)}\left[W_t(\lambda,\Delta,y,\Delta)+\int_\Delta^\tau e^{-\widetilde{C}(\zeta-\Delta)}\widetilde{D}\left(\zeta\right)\mathrm{d}\zeta\right]\\ &= e^{(r+\lambda)(\tau-\Delta)}\left[W_t(\lambda,\Delta,y,\Delta)+\underbrace{\int_\Delta^\tau e^{-(r+\lambda)(\zeta-\Delta)}\widetilde{D}\left(\zeta\right)\mathrm{d}\zeta}_{\widetilde{\Gamma}}\right].\end{aligned}$$

The term $\widetilde{\Gamma}$ can be rewritten

$$\begin{aligned}\widetilde{\Gamma}=&-\frac{1-e^{-(r+\lambda)(\tau-\Delta)}}{r+\lambda}w\left(\lambda,\Delta\right)-\frac{\lambda}{r}\int w\left(\lambda,\Delta\right)\mathrm{d}H(x)\left(\frac{1-e^{-(r+\lambda)(\tau-\Delta)}}{r+\lambda}-\frac{1-e^{-\lambda(\tau-\Delta)}}{\lambda}\right)\\ &-\int\max\left[W_p\left(x,\lambda\right),U\right]\mathrm{d}H(x)\left(1-e^{-\lambda(\tau-\Delta)}\right).\end{aligned}$$

Thus

$$
\begin{aligned}
W_t(\lambda,\Delta,y,\tau) &= e^{(r+\lambda)(\tau-\Delta)}\left[W_t(\lambda,\Delta,y,\Delta)+\int_\Delta^\tau e^{-(r+\lambda)(\zeta-\Delta)}D\left(\zeta\right)\mathrm{d}\zeta\right] \\
&= e^{(r+\lambda)(\tau-\Delta)}W_t(\lambda,\Delta,y,\Delta) \\
&\quad -w\left(\lambda,\Delta\right)e^{(r+\lambda)(\tau-\Delta)}\frac{1-e^{-(r+\lambda)(\tau-\Delta)}}{r+\lambda} \\
&\quad -\frac{\lambda}{r}e^{(r+\lambda)(\tau-\Delta)}\int w\left(\lambda,\Delta\right)\mathrm{d}H(x)\left(\frac{1-e^{-(r+\lambda)(\tau-\Delta)}}{r+\lambda}-\frac{1-e^{-\lambda(\tau-\Delta)}}{\lambda}\right) \\
&\quad -e^{(r+\lambda)(\tau-\Delta)}\int \max\left[W_p\left(x,\lambda\right),U\right]\mathrm{d}H(x)\left(1-e^{-\lambda(\tau-\Delta)}\right).
\end{aligned}
$$

Since $W_t(\lambda,\Delta,y,\Delta)=\max\left[W_p\left(\lambda,y\right),U\right]$, we get

$$
\begin{aligned}
W_t(\lambda,\Delta,y,0) &= w\left(\lambda,\Delta\right)\frac{1-e^{-(r+\lambda)\Delta}}{r+\lambda}+e^{-(r+\lambda)\Delta}\max\left[W_p\left(y,\lambda\right),U\right] \\
&\quad -\frac{\lambda}{r}\int w\mathrm{d}H(x)\left(\frac{e^{-(r+\lambda)\Delta}-1}{r+\lambda}-\frac{e^{-(r+\lambda)\Delta}-e^{-r\Delta}}{\lambda}\right) \\
&\quad -\int \max\left[W_p\left(x,\lambda\right),U\right]\mathrm{d}H(x)\left(e^{-(r+\lambda)\Delta}-e^{-r\Delta}\right)
\end{aligned}
$$

Rearranging and taking account of the fact that jobs always start at y_u yields

$$
\begin{aligned}
W_t(\lambda,\Delta) &= \frac{w\left(\lambda,\Delta\right)}{r}\left(1-e^{-r\Delta}\right)+e^{-(r+\lambda)\Delta}\max\left[W_p\left(y_u,\lambda\right),U\right] \\
&\quad +e^{-r\Delta}\left(1-e^{-\lambda\Delta}\right)\int \max\left[W_p\left(y,\lambda\right),U\right]\mathrm{d}H(y),
\end{aligned}
\tag{D31}
$$

which is formally equivalent to the expression given by equation (21).

D.7 Surplus of a temporary job

Let us define by $S_t(\lambda,\Delta)$ the starting value of the surplus of a temporary job with duration Δ and Poisson rate λ. We have

$$
S_t(\lambda,\Delta)=J_t(\lambda,\Delta)+W_t(\lambda,\Delta)-U,
\tag{D32}
$$

making use of (D27) and (D31), and reinserting in (D32) we get

$$
\begin{aligned}
S_t(\lambda,\Delta) &= y_u\frac{1-e^{-(r+\lambda)\Delta}}{r+\lambda}+\int y\mathrm{d}H(y)\left(\frac{\lambda(1-e^{-r\Delta})+r(e^{-(r+\lambda)\Delta}-e^{-r\Delta})}{r(r+\lambda)}\right) \\
&\quad +e^{-(r+\lambda)\Delta}\max\left[J_p\left(y_u,\lambda\right)+W_p\left(y_u,\lambda\right),U\right] \\
&\quad +e^{-r\Delta}\left(1-e^{-\lambda\Delta}\right)\int \max\left[J_p\left(y,\lambda\right)+W_p\left(y,\lambda\right),U\right]\mathrm{d}H(y) \\
&\quad -U-c,
\end{aligned}
$$

then using the definition of $S_p(y,\lambda) = J_p(y,\lambda) + W_p(y,\lambda) - U$, we get

$$\begin{aligned} S_t(\lambda,\Delta) &= y_u \frac{1-e^{-(r+\lambda)\Delta}}{r+\lambda} + \int y\mathrm{d}H(y) \left(\frac{\lambda(1-e^{-r\Delta}) + r(e^{-(r+\lambda)\Delta} - e^{-r\Delta})}{r(r+\lambda)} \right) \\ &\quad + e^{-(r+\lambda)\Delta} \max\left[S_p(y_u,\lambda),0\right] + e^{-r\Delta}\left(1-e^{-\lambda\Delta}\right) \int \max\left[S_p(y,\lambda),0\right] \mathrm{d}H(y) \\ &\quad - U(1-e^{-r\Delta}) - c, \end{aligned}$$

which is formally equivalent to (23) in text.

E Optimal duration of temporary jobs in the model with productivity shocks

The optimal duration of temporary jobs maximizes the surplus of starting temporary jobs. We first consider temporary jobs which are not transformed into permanent jobs because the shock arrival rate is above the threshold value λ_p. Then, the case of temporary jobs that can be transformed into permanent jobs is studied in a second step.

E.1 Case 1: $\lambda \geq \lambda_p$

If $\lambda \geq \lambda_p$, the surplus of a temporary job with shock arrival rate λ and duration Δ is

$$S_t(\lambda,\Delta) = \int_0^{\Delta} \left(e^{-\lambda\tau} y_u + \left(1-e^{-\lambda\tau}\right) \int_{-\infty}^{y_u} y\mathrm{d}H(y) - rU \right) e^{-r\tau}\mathrm{d}\tau - c.$$

The first order condition, $\partial S_t(\lambda,\Delta)/\partial\Delta = 0$, can be written as

$$e^{-\lambda\Delta} y_u + \left(1-e^{-\lambda\Delta}\right) \int_{-\infty}^{y_u} y\mathrm{d}H(y) - rU = 0.$$

The second order condition

$$-\lambda e^{-\lambda\Delta} \int_{-\infty}^{y_u} (y_u - y)\,\mathrm{d}H(y) < 0,$$

is always satisfied. Then, the optimal duration is

$$\Delta(\lambda) = \frac{1}{\lambda} \ln \frac{y_u - \int_{-\infty}^{y_u} y\mathrm{d}H(y)}{rU - \int_{-\infty}^{y_u} y\mathrm{d}H(y)},$$

which corresponds to the expression given by equation (24).

E.2 Case 2. $\lambda \leq \lambda_p$

When $\lambda \leq \lambda_p$, the surplus of a permanent job with shock arrival rate λ and duration Δ is

$$
\begin{aligned}
S_t(\lambda,\Delta) &= \int_0^{\Delta}\left(e^{-\lambda\tau}y_u+\left(1-e^{-\lambda\tau}\right)\int_{-\infty}^{y_u} y\mathrm{d}H(y)-rU\right)e^{-r\tau}\mathrm{d}\tau+\\
&\quad e^{-(r+\lambda)\Delta}S_p(y_u,\lambda)+\left(1-e^{-\lambda\Delta}\right)e^{-r\Delta}\int_{T(\lambda)}^{y_u}S_p(y,\lambda)\mathrm{d}H(y)-c.
\end{aligned}
$$

where $T(\lambda)$ is defined by equation (19).

The first order condition, $\partial S_t(\lambda,\Delta)/\partial\Delta=0$, can be written as

$$
\begin{aligned}
&e^{-\lambda\Delta}y_u+\left(1-e^{-\lambda\Delta}\right)\int_{-\infty}^{y_u}y\mathrm{d}H(y)-rU-r\int_{T(\lambda)}^{y_u}S_p(y,\lambda)\mathrm{d}H(y)\\
&\quad+(r+\lambda)e^{-\lambda\Delta}\left(S_p(y_u,\lambda)-\int_{T(\lambda)}^{y_u}S_p(y,\lambda)\mathrm{d}H(y)\right)=0.
\end{aligned}
$$

The second order condition:

$$
-\lambda e^{-\lambda\Delta}\left[\int_{-\infty}^{y_u}(y_u-y)\,\mathrm{d}H(y)-(r+\lambda)\left(S_p(y_u,\lambda)-\int_{T(\lambda)}^{y_u}S_p(y,\lambda)\mathrm{d}H(y)\right)\right]<0,
$$

is always satisfied. Thus, the optimal duration is

$$
\Delta(\lambda)=\frac{1}{\lambda}\ln\frac{y_u-\int_{-\infty}^{y_u}y\mathrm{d}H(y)-(r+\lambda)\left[S_p\left(y_u,\lambda\right)-\int_{T(\lambda)}^{y_u}S_p\left(y,\lambda\right)\mathrm{d}H(y)\right]}{rU-\int_{y_{\min}}^{y_{\max}}y\mathrm{d}H(y)+r\int_{T(\lambda)}^{y_u}S_p\left(y,\lambda\right)\mathrm{d}H(y),}
$$

which corresponds to the expression given equation (24).

F The properties of functions $S_p(y_u,\lambda)$ and $S_t(\lambda)$ in the model with productivity shocks

F.1 Properties of $S_p(y_u,\lambda)$

The surplus of a starting permanent job with productivity y_u and shock arrival rate λ is

$$
S_p\left(y_u,\lambda\right)=\frac{y_u-rU-\lambda F}{r+\lambda}-c+\frac{\lambda}{r+\lambda}\int_{R(\lambda)}^{y_u}\frac{x-R(\lambda)}{r+\lambda}dH(x) \tag{F33}
$$

From this expression, it is straightforward to prove, assuming that $y_u-rU>rF$, that S_p is continuous in λ and decreases from

$$
\lim_{\lambda\to 0}S_p\left(y_u,\lambda\right)=\frac{y_u}{r}-U-c
$$

to

$$
\lim_{\lambda\to+\infty}S_p\left(y_u,\lambda\right)=-F-c
$$

F.2 Properties of $S_t(\lambda)$

The surplus of a starting temporary job with shock arrival rate λ and optimal duration $\Delta(\lambda) = \max_\Delta S_t(\lambda, \Delta)$ is, using equation (23)

$$\begin{aligned} S_t(\lambda) &= \int_0^{\Delta(\lambda)} \left(e^{-\lambda\tau} y_u + \left(1 - e^{-\lambda\tau}\right) \int_{-\infty}^{y_u} y \mathrm{d}H(y) - rU \right) e^{-r\tau} \mathrm{d}\tau + \\ & e^{-(r+\lambda)\Delta(\lambda)} \max\left[S_p(y_u, \lambda), 0\right] + \left(1 - e^{-\lambda\Delta(\lambda)}\right) e^{-r\Delta(\lambda)} \int_{-\infty}^{y_u} \max\left[S_p(y, \lambda), 0\right] \mathrm{d}H(y) - c. \end{aligned} \tag{F34}$$

From this expression, it is easily checked that $S_t(\lambda)$ is continuous and decreasing from $\lim\limits_{\lambda \to 0} S_t(\lambda) = \frac{y_u}{r} - U - c$ to $\lim\limits_{\lambda \to +\infty} S_t(\lambda) = -c$.

It turns out that $\lim\limits_{\lambda \to +\infty} S_t(\lambda) = -c$ because equation (24) implies that $\lim\limits_{\lambda \to +\infty} \Delta(\lambda) = 0$ and $\max\left[S_p(y_u, \lambda), 0\right] = 0$ when $\lambda \to +\infty$.

Moreover, $\lim\limits_{\lambda \to 0} S_t(\lambda) = \frac{y_u}{r} - U - c$ because equation (24), which implies $\lim\limits_{\lambda \to 0} \Delta(\lambda) = +\infty$, yields, using expression (F34)

$$\lim_{\lambda \to 0} S_t(\lambda) = \frac{y_u}{r} - U - c.$$

F.3 Intercept of S_t and S_p in the model with productivity shocks:

Using the expression (F34) of $S_t(\lambda)$, and keeping in mind that $\lim\limits_{\lambda \to 0} \Delta(\lambda) = +\infty$, we get

$$\lim_{\lambda \to 0} S_t'(\lambda) = \frac{-y_u + \int_{-\infty}^{y_u} y \mathrm{d}H(y)}{r^2}. \tag{F35}$$

Similarly, using the expression (F33) of $S_p\left(y_u, \lambda\right)$, and keeping in mind that $\lim\limits_{\lambda \to 0} R(\lambda) = rU - rF$, we get

$$\lim_{\lambda \to 0} S_p'\left(y_u, \lambda\right) = -\frac{y_u - r\left(U - F\right)}{r^2} + \frac{\int_{r(U-F)}^{y_u} \left[y - r\left(U - F\right)\right] \mathrm{d}H(y)}{r^2}. \tag{F36}$$

Using (F35) and (F36) we get

$$\lim_{\lambda \to 0} S_p'\left(y_u, \lambda\right) > \lim_{\lambda \to 0} S_t'(\lambda) \Leftrightarrow \int_{-\infty}^{r(U-F)} \left[y - r(U - F)\right] \mathrm{d}H(y) < 0,$$

which holds if and only if $U > F$.

Since $\lim\limits_{\lambda \to 0} S_t(\lambda) = \lim\limits_{\lambda \to 0} S_p\left(y_u, \lambda\right)$, the fact that $\lim\limits_{\lambda \to 0} S_p'\left(y_u, \lambda\right) > \lim\limits_{\lambda \to 0} S_t'(\lambda)$ if and only if $U > F$ implies that there exists a value of $\lambda > 0$ such that $S_p(y_u, \lambda) > S_t(\lambda)$ in the neighborhood of $\lambda = 0$, if and only if $U > F$. Let us assume that this is the case. Then S_p and S_t have at least one positive intercept for positive values of $S_t(\lambda)$ if $S_p(y_u, \lambda_p) = 0 < S_t(\lambda_p)$. This yields a condition similar to (12) in the benchmark. We checked that this intercept is unique in the calibration exercises.

G Labor market equilibrium

G.1 Equilibrium with permanent jobs only

The free entry condition and the condition that defines the threshold value of the shock arrival rate above which no jobs are created are respectively:

$$\kappa = \frac{q(\theta)(1-\beta)}{r+q(\theta)} \int_{\lambda_{\min}}^{\lambda_p} S_p(y_u,\lambda)\mathrm{d}G(\lambda), \tag{G37}$$

$$S_p(y_u,\lambda_p) = 0, \tag{G38}$$

with

$$(r+\lambda)S_p(y_u,\lambda) = y_u - rU - \lambda F + \lambda \int_{R(\lambda)}^{y_u} \frac{y-R(\lambda)}{r+\lambda}\mathrm{d}H(y) - (r+\lambda)\,c, \tag{G39}$$

$$R(\lambda) = r(U-F) - \lambda \int_{R(\lambda)}^{y_u} \frac{y-R(\lambda)}{r+\lambda}\mathrm{d}H(y), \tag{G40}$$

$$rU = z + \frac{\beta\theta\,[r+q(\theta)]}{(1-\beta)\,G(\lambda_p)}\kappa. \tag{G41}$$

Substituting (G41) in (F33) and (G40) implies that the threshold value of shock arrival rates λ_p above which no jobs are created $S_p(y_u,\lambda_p)=0$, can be restated as $\lambda_p \equiv \lambda_p(\theta)$. Then, the free entry condition, which defines the equilibrium value of θ, can be written as follows

$$\Gamma(\theta) \equiv \kappa - \frac{q(\theta)(1-\beta)}{r+q(\theta)} \int_{\lambda_{\min}}^{\lambda_p(\theta)} S_p(y_u,\lambda)\mathrm{d}G(\lambda) = 0. \tag{G42}$$

Differentiating (G42) with respect to θ, keeping in mind that $S_p(y_u,\lambda_p)=0$, yields

$$\Gamma'(\theta) = -(1-\beta)\frac{q'(\theta)r}{[r+q(\theta)]^2} \int_{\lambda_{\min}}^{\lambda_p(\theta)} S_p(y_u,\lambda)\mathrm{d}G(\lambda) - \frac{q(\theta)(1-\beta)}{r+q(\theta)} \int_{\lambda_{\min}}^{\lambda_p(\theta)} \frac{\mathrm{d}S_p(y_u,\lambda)}{\mathrm{d}\theta}\mathrm{d}G(\lambda),$$

where $\frac{\mathrm{d}S_p(y_u,\lambda)}{\mathrm{d}\theta} < 0$ (from equations (F33) and (G41)) so that $\Gamma'(\theta) > 0$. This implies that (G42) defines a unique value of θ provided that the conditions of existence of θ are satisfied.

G.2 Equilibrium with permanent and temporary jobs

The free entry condition and the conditions that define the threshold value of the shock arrival rate above which no temporary jobs are created λ_t, the threshold value of the shock arrival rate above which no permanent job can be profitably created λ_p, and the segmentation threshold between permanent and temporary contracts, λ_s, are respectively:

$$\kappa = \frac{q(\theta)(1-\beta)}{r+q(\theta)} \left[\int_{\lambda_{\min}}^{\lambda_s} S_p(y_u,\lambda)\mathrm{d}G(\lambda) + \int_{\lambda_s}^{\lambda_t} S_t(\lambda)\mathrm{d}G(\lambda)\right], \tag{G43}$$

$$S_p(y_u,\lambda_p) = 0, \tag{G44}$$

$$S_t(\lambda_s) = S_p(y_u,\lambda_s), \tag{G45}$$

$$S_t(\lambda_t) = 0, \tag{G46}$$

where the surplus of a starting permanent contract with productivity y and shock arrival rate λ writes

$$(r+\lambda)S_p(y,\lambda) = y - rU - \lambda F + \lambda \int_{R(\lambda)}^{y_u} \frac{y - R(\lambda)}{r+\lambda} \mathrm{d}H(y) - (r+\lambda)\, c, \tag{G47}$$

where

$$R(\lambda) = r(U-F) - \lambda \int_{R(\lambda)}^{y_u} \frac{y-R(\lambda)}{r+\lambda} \mathrm{d}H(y), \tag{G48}$$

and

$$rU = z + \frac{\beta\theta\,[r+q(\theta)]}{(1-\beta)\,G(\lambda_t)}\kappa, \tag{G49}$$

while the surplus of a temporary contract with shock arrival rate λ writes

$$\begin{aligned} S_t(\lambda) \quad = \quad & \frac{1-e^{-(r+\lambda)\Delta(\lambda)}}{r+\lambda} y_u + \frac{\lambda(1-e^{-r\Delta(\lambda)}) + r(e^{-(r+\lambda)\Delta(\lambda)} - e^{-r\Delta(\lambda)})}{r(r+\lambda)} \int y \mathrm{d}H(y) \\ & + e^{-(r+\lambda)\Delta(\lambda)} \max\left[S_p\left(y_u,\lambda\right),0\right] + e^{-r\Delta(\lambda)}\left(1-e^{-\lambda\Delta(\lambda)}\right) \int \max\left[S_p\left(y,\lambda\right),0\right] \mathrm{d}H(y) \\ & - U(1-e^{-r\Delta(\lambda)}) - c, \end{aligned} \tag{G50}$$

where $\Delta(\lambda)$ is defined by equation (24).

Substituting (G49) in equations (G47), (G48) and (G50), and making use of (G44), (G45) and (G46) imply that we can restate the thresholds as $\lambda_s \equiv \lambda_s(\theta), \lambda_p \equiv \lambda_p(\theta), \lambda_t \equiv \lambda_t(\theta)$, so that the free entry condition, which defines the equilibrium value of θ, can be written

$$\Gamma(\theta) \equiv \kappa - \frac{q(\theta)(1-\beta)}{r+q(\theta)} \left[\int_{\lambda_{\min}}^{\lambda_s(\theta)} S_p(y_u,\lambda)\mathrm{d}G(\lambda) + \int_{\lambda_s(\theta)}^{\lambda_p(\theta)} S_t(\lambda)\mathrm{d}G(\lambda) + \int_{\lambda_p(\theta)}^{\lambda_t(\theta)} S_t(\lambda)\mathrm{d}G(\lambda) \right] = 0. \tag{G51}$$

Differentiating Γ with respect to θ, and keeping in mind that $S_p(y_u,\lambda_s(\theta)) = S_t(\lambda_s(\theta))$ and that $S_t(\lambda_t(\theta)) = 0$, yields

$$\begin{aligned} \Gamma'(\theta) \quad = \quad & -(1-\beta)\frac{q'(\theta)r}{[r+q(\theta)]^2} \left[\int_{\lambda_{\min}}^{\lambda_s(\theta)} S_p(y_u,\lambda)\mathrm{d}G(\lambda) - \int_{\lambda_s(\theta)}^{\lambda_t(\theta)} S_t(\lambda)\mathrm{d}G(\lambda) \right] \\ & - \frac{q(\theta)(1-\beta)}{r+q(\theta)} \int_{\lambda_{\min}}^{\lambda_s(\theta)} \frac{\mathrm{d}S_p(y_u,\lambda)}{\mathrm{d}\theta}\mathrm{d}G(\lambda) - \int_{\lambda_s(\theta)}^{\lambda_t(\theta)} \frac{\mathrm{d}S_t(\lambda)}{\mathrm{d}\theta}\mathrm{d}G(\lambda), \end{aligned}$$

where $\frac{\mathrm{d}S_p(y_u,\lambda)}{\mathrm{d}\theta} < 0$ (from equations (F33) and (G41)) and $\frac{\mathrm{d}S_t(\lambda)}{\mathrm{d}\theta} \leq 0$ (from equations (F34) and (G41)), so that $\Gamma'(\theta) > 0$. Again, the unicity of the equilibrium value of θ follows.

G.3 Unemployment and labor market flows

Once the equilibrium value of the labor market tightness and of the thresholds λ_s, λ_p and λ_t are known, it is possible to define unemployment, and the mass of temporary and permanent jobs at equilibrium (for the sake of simplicity, we only focus on steady state).

Let us begin by defining the steady state unemployment rate in the equilibrium where there are permanent jobs only. The mass of permanent jobs with shock arrival rate λ is denoted by $\ell(\lambda)$. By definition, the unemployment rate is

$$u = 1 - \int_{\lambda_{\min}}^{\lambda_p} \ell(\lambda)\mathrm{d}\lambda. \tag{G52}$$

In steady state, the equality between entries and exits in type-λ jobs is

$$u\alpha_p g(\lambda) = \ell(\lambda)/\phi(\lambda), \tag{G53}$$

where $\phi(\lambda) = 1/\lambda H\left[R(\lambda)\right]$ is the expected duration of type-λ jobs and $\alpha_p = \alpha/G(\lambda_p) = \theta q(\theta)/G(\lambda_p)$.

Equations (G52) and (G53) imply

$$u = \frac{1}{1 + \alpha_p \int_{\lambda_{\min}}^{\lambda_p} \phi(\lambda)\mathrm{d}G(\lambda)}. \tag{G54}$$

This equation shows that the unemployment rate decreases with $\theta q(\theta)$, the arrival rate of job offers, and with the duration of jobs.

Let us now analyze the equilibrium with temporary and permanent jobs. $s_t(\lambda)$ denotes the mass of type-λ temporary jobs which are transformed into permanent jobs. $s_n(\lambda)$ denotes the mass of type-λ temporary jobs which are not transformed into permanent jobs and u denotes the unemployment rate. We can write

$$u = 1 - \int_{\lambda_{\min}}^{\lambda_p} \ell(\lambda)\mathrm{d}\lambda - \int_{\lambda_s}^{\lambda_p} s_t(\lambda)\mathrm{d}\lambda - \int_{\lambda_p}^{\lambda_t} s_n(\lambda)\mathrm{d}\lambda. \tag{G55}$$

There are permanent jobs over the interval $[\lambda_{\min}, \lambda_p]$. The equality between entries into and exits out of permanent jobs with expected duration $\phi(\lambda)$ can be written as

$$\left\{ \begin{array}{ll} \frac{s_t(\lambda)\left[1-H(T(\lambda))\left(1-e^{-\lambda\Delta(\lambda)}\right)\right]}{\Delta(\lambda)} = \frac{\ell(\lambda)}{\phi(\lambda)} & \text{if } \lambda \in [\lambda_s, \lambda_p] \\ u\alpha_t g(\lambda) = \frac{\ell(\lambda)}{\phi(\lambda)} & \text{if } \lambda \in [\lambda_{\min}, \lambda_s] \end{array} \right., \tag{G56}$$

where $\alpha_t = \alpha/G(\lambda_t) = \theta q(\theta)/G(\lambda_t)$. The first row of equation (G56) accounts for the transformations of temporary jobs into permanent jobs. The second row accounts for the entries of unemployed workers into permanent jobs. The equality between entries into and exits out of temporary jobs with expected duration $\Delta(\lambda)$ can be written as

$$u\alpha_t g(\lambda) = \frac{s_t(\lambda)}{\Delta(\lambda)} \text{ if } \lambda \in [\lambda_s, \lambda_p] \tag{G57}$$

$$u\alpha_t g(\lambda) = \frac{s_n(\lambda)}{\Delta(\lambda)} \text{ if } \lambda \in [\lambda_p, \lambda_t] \tag{G58}$$

Equations (G55) to (G58) imply:

$$u = \frac{1}{1 + \alpha_t \left[\int_{\lambda_s}^{\lambda_t} \Delta(\lambda)\mathrm{d}G(\lambda) + \int_{\lambda_{\min}}^{\lambda_s} \phi(\lambda)\mathrm{d}G(\lambda) + \int_{\lambda_s}^{\lambda_p} \phi(\lambda)\left[1 - H(T(\lambda))\left(1 - e^{-\lambda\Delta(\lambda)}\right)\right]\mathrm{d}G(\lambda)\right]}.$$

This equation shows that the unemployment rate decreases with the arrival rate of job offers and with the duration of jobs.

G.4 Production

This appendix presents the computation of aggregate production, equal to home production plus the production of filled jobs minus the cost of job vacancies and the contracting costs.

- Let us first take the case where there are permanent jobs only. Let $l^{\overline{y}}(\lambda)$ and $l^{y_u}(\lambda)$ be the mass of type-λ permanent jobs that have been hit by a productivity shock and the mass of type-λ permanent jobs that are still at the upper bound of the productivity distribution respectively. The production of type-λ filled jobs net of the contracting costs is equal to:

$$Y_p(\lambda) = l^{y_u}(\lambda)y_u + l^{\overline{y}}(\lambda)\frac{\int_{R(\lambda)}^{y_u} y\mathrm{d}H(y)}{1-H(R(\lambda))} - u\alpha_p g(\lambda)c, \tag{G59}$$

with $\alpha_p = \theta q(\theta)/G(\lambda_p)$ and where $l^{y_u}(\lambda)$ and $l^{\overline{y}}(\lambda)$ are defined by the following steady state equations governing the flows between entries into and exits out of type-λ jobs:

$$\begin{aligned} u\alpha_p g(\lambda) &= \lambda l^{y_u}(\lambda) \\ \lambda[1-H(R(\lambda))]l^{y_u}(\lambda) &= \lambda H(R(\lambda))l^{\overline{y}}(\lambda) \end{aligned}$$

To evaluate aggregate production, it is necessary to deduct the entry costs. Let N denote the mass of new jobs in the economy. In steady state, $N = q(\theta)v = \theta q(\theta)u$. Remarking that all firms pay the entry costs but that only a share advertises vacancies, total entry costs amount to $\kappa\frac{N}{G(\lambda_p)} = \kappa\frac{\theta q(\theta)u}{G(\lambda_p)}$. It follows that in an equilibrium where there are permanent jobs only, aggregate production net of contracting and entry costs verifies:

$$Y = uz + \int_{\lambda_{\min}}^{\lambda_p} Y_p(\lambda)\,\mathrm{d}\lambda - \kappa\frac{\theta q(\theta)u}{G(\lambda_p)}.$$

It follows that average productivity (on filled jobs) is then equal to $\frac{Y-uz}{1-u}$.

- Let us now analyze the case with temporary and permanent jobs. We proceed as in the case where there are permanent jobs only, but we now distinguish between different segments over the range $[\lambda_{\min,}\lambda_t]$.

  - In the range $[\lambda_{\min,}\lambda_s]$, there are permanent jobs only and the production of type-λ jobs net of the contracting costs is again defined by equation (G59).
  - In the range $[\lambda_s, \lambda_p]$, there are both permanent and temporary jobs. All type-λ jobs start as temporary and are eventually converted into permanent jobs. The equality between entries into and exits out of temporary jobs with expected duration $\Delta(\lambda)$ is then given by (G57). Let $s_t^{\overline{y}}(\lambda) = s_t(\lambda)\left(1-e^{-\lambda\Delta(\lambda)}\right)$ and $s_t^{y_u}(\lambda) = s_t(\lambda)e^{-\lambda\Delta(\lambda)}$ be the mass of type-λ temporary jobs that have been hit by a productivity shock and the mass of type-λ temporary jobs that are still at the upper bound of the productivity distribution respectively. The production of type-λ jobs net of the contracting costs for temporary and permanent jobs is, respectively:

$$Y_t(\lambda) = s_t^{y_u}(\lambda)y_u + s_t^{\overline{y}}(\lambda)\int_{-\infty}^{y_u} y\mathrm{d}H(y) - u\alpha_t g(\lambda)c, \tag{G60}$$

with $\alpha_t = \theta q(\theta)/G(\lambda_t)$,

$$Y_p(\lambda) = l^{y_u}(\lambda) y_u + l^{\overline{y}}(\lambda) \frac{\int_{R(\lambda)}^{y_u} y \mathrm{d}H(y)}{1 - H(R(\lambda))} - \frac{s_t(\lambda)}{\Delta(\lambda)} \left[\left(1 - e^{-\lambda\Delta(\lambda)}\right) [1 - H(T(\lambda))] + e^{-\lambda\Delta(\lambda)} \right] c, \tag{G61}$$

where the last term in (G61) differs from (G59) due to the fact that only a fraction of the temporary jobs are converted into permanent jobs, and where $l^{y_u}(\lambda)$ and $l^{\overline{y}}(\lambda)$ are defined by the following steady state equations governing the flows between entries into and exits out of type-λ jobs:

$$s_t(\lambda) \frac{1}{\Delta(\lambda)} e^{-\lambda\Delta(\lambda)} = \lambda H(R(\lambda)) l^{y_u}(\lambda) + \lambda[1 - H(R(\lambda))] l^{y_u}(\lambda) \equiv \lambda l^{y_u}(\lambda)$$
$$s_t(\lambda) \frac{1}{\Delta(\lambda)} \left(1 - e^{-\lambda\Delta(\lambda)}\right) [1 - H(T(\lambda))] + \lambda[1 - H(R(\lambda))] l^{y_u}(\lambda) = \lambda H(R(\lambda)) l^{\overline{y}}(\lambda)$$

– In the range $[\lambda_p, \lambda_t]$, there are temporary jobs only. All type-λ jobs start as temporary and are never transformed into permanent jobs. The equality between entries into and exits out of temporary jobs with expected duration $\Delta(\lambda)$ is then given by (G58). Let $s_n^{\overline{y}}(\lambda) = s_n(\lambda)\left(1 - e^{-\lambda\Delta(\lambda)}\right)$ and $s_n^{y_u}(\lambda) = s_n(\lambda) e^{-\lambda\Delta(\lambda)}$ be the mass of type-λ temporary jobs that have been hit by a productivity shock and the mass of type-λ temporary jobs that are still at the upper bound of the productivity distribution respectively. The production of type-λ jobs net of the contracting costs for temporary and permanent jobs is, respectively:

$$Y_t(\lambda) = s_n^{y_u}(\lambda) y_u + s_n^{\overline{y}}(\lambda) \int_{-\infty}^{y_u} y \mathrm{d}H(y) - u\alpha_t g(\lambda) c, \tag{G62}$$

with $\alpha_t = \theta q(\theta)/G(\lambda_t)$.

Finally, using (G59), (G60), (G61) and (G62), aggregate production net of contracting and entry costs verifies:

$$Y = uz + \int_{\lambda_{\min}}^{\lambda_s} Y_p(\lambda) \mathrm{d}\lambda + \int_{\lambda_s}^{\lambda_p} Y_p(\lambda) \mathrm{d}\lambda + \int_{\lambda_s}^{\lambda_p} Y_t(\lambda) \mathrm{d}\lambda + \int_{\lambda_p}^{\lambda_t} Y_t(\lambda) \mathrm{d}\lambda - \kappa \frac{\theta q(\theta) u}{G(\lambda_t)}.$$

It follows that the average productivity (of filled jobs) is then equal to $\frac{Y - uz}{1-u}$.