The Impact of Quantitative Easing on the U.S. Term Structure of Interest Rates*

Robert Jarrow† and Hao Li‡

January 23, 2013

Abstract

This paper estimates the impact of the Federal Reserve's 2008 - 2011 quantitative easing (QE) program on the U.S. term structure of interest rates. We estimate an arbitrage-free term structure model that explicitly includes the quantity impact of the Fed's trades on Treasury market prices. As such, we are able to estimate both the magnitude and duration of the QE price effects. We show that the Fed's QE program affected forward rates without introducing arbitrage opportunities into the Treasury security markets. Short- to medium- term forward rates were reduced (less than twelve years), but the QE had little if any impact on long-term forward rates. This is in contrast to the Fed's stated intentions for the QE program. The persistence of the rate impacts increased with maturity up to 6 years then declined, with half-lives lasting approximately 4, 6, 12, 8 and 4 months for the 1, 2, 5, 10 and 12 year forwards, respectively. Since bond yields are averages of forward rates over a bond's maturity, QE affected long-term bond yields. The average impacts on bond yields were 327, 26, 50, 70, and 76 basis points for 1, 2, 5, 10 and 30 years, respectively.

KEYWORDS: Quantitative easing, the term structure of interest rates, arbitrage-free models, large trader, quantity impact on price.

JEL Classification Codes: G12, E43, E44, E52, E58

*Helpful comments from workshops at the Federal Reserve Bank of New York and the Federal Reserve Board in Washington D.C. are gratefully acknowledged.

†Johnson Graduate School of Management, Cornell University, Ithaca, NY, 14853 and Kamakura Corporation. email: raj15@cornell.edu.

‡Johnson Graduate School of Management, Cornell University, Ithaca, NY, 14853. email: hl624@cornell.edu.

The Impact of Quantitative Easing on the U.S. Term Structure of Interest Rates

Abstract

This paper estimates the impact of the Federal Reserve's 2008 - 2011 quantitative easing (QE) program on the U.S. term structure of interest rates. We estimate an arbitrage-free term structure model that explicitly includes the quantity impact of the Fed's trades on Treasury market prices. As such, we are able to estimate both the magnitude and duration of the QE price effects. We show that the Fed's QE program affected forward rates without introducing arbitrage opportunities into the Treasury security markets. Short- to medium- term forward rates were reduced (less than twelve years), but the QE had little if any impact on long-term forward rates. This is in contrast to the Fed's stated intentions for the QE program. The persistence of the rate impacts increased with maturity up to 6 years then declined, with half-lives lasting approximately 4, 6, 12, 8 and 4 months for the 1, 2, 5, 10 and 12 year forwards, respectively. Since bond yields are averages of forward rates over a bond's maturity, QE affected long-term bond yields. The average impacts on bond yields were 327, 26, 50, 70, and 76 basis points for 1, 2, 5, 10 and 30 years, respectively.

1 Introduction

Following the 2007-2009 financial crisis, between November 2008 and March 2010 the Federal Reserve conducted massive asset purchases known as quantitative easing (QE1) to lower long-term interest rates and spur economic growth. In QE1 the Fed purchased approximately \$1.75 trillion of assets consisting of \$1.25 trillion mortgage-backed securities (MBS), \$300 billion Treasury securities, and \$200 billion federal agency debt. Between November 2010 and June 2011, another phase of quantitative easing known as QE2 was implemented, consisting of an additional purchase of \$600 billion long-term Treasuries. Outstanding is the question as to whether quantitative easing was successful given a federal funds rate (Figure 1) that has been almost zero since the end of 2008.[1] And if effective, which Treasury rates were lowered, by how much, and for how long?

The existing empirical literature provides strong support for the proposition that both QE1 and QE2 were effective in lowering long-term Treasury yields, but it provides less evidence with respect to how long the effects of the asset purchases lasted. Studies of the effectiveness of QE include Bernanke, Reinhart, and Sack (2004), D'Amico and King (2012), Gagnon, Raskin, Remache, and Sack (2010), Hamilton and Wu (2012), Krishnamurthy and Vissing-Jorgensen (2011), Li and Wei (2012), Meaning and Zhu (2011), and Wright (2012). Related work also includes Fuster and Willen (2010) on the QE effects of purchasing mortgages, Oda and Ueda (2005) who investigate the Bank of Japan's zero interest rate policy from 1999 to 2003, Swanson (2011) on the 1961 Operation Twist, and Joyce, Lasaosa, Stevens and Tong (2010) on the QE impacts from the Bank of England. Broadly speaking, there have been two approaches used in this literature to study the

[1] See Bernanke and Reinhart (2004) for a discussion of monetary policies around the zero lower bound for short-term interest rates.

effectiveness of QE1 and QE2: performing an event study on the announcement day of a large asset purchase, or estimating a time series equilibrium or arbitrage-free model for the term structure of interest rates.

In an event study, the window after the announcement date is purposely kept small, usually one or two days, in order to minimize the confounding effect of changing macro-economic conditions on the observed change in Treasury rates. The advantage of an event study is that it does not require the specification of a particular equilibrium or arbitrage-free model. Hence, the results are robust to model misspecification. The key disadvantage of an event study is that it only measures the impact of an asset purchase over the event window considered. Cumulating event window changes over longer time horizons is confounded by changes in macro-economic conditions. In addition, for their validity, event studies require two additional assumptions to hold. One, that the announcement was not leaked before the announcement date, and two, that any price impact is instantaneous and not lagged across time. It is an open question as to the validity of these assumptions in Treasury markets.

In a time series model, to measure the impact of QE, the usual approach is to perform a counter factual experiment. Using a model based on no QE policy, one can estimate the expected path of the term structure of interest rates. This is the counter factual control. This no QE policy forecast is then compared with realized rates (or conditional expectations based on the realized rates) generated under the QE policy. The difference between these two rates is due to random noise and QE. The advantage of a time series model is that it captures the changing macro-economic conditions during the estimation period. The disadvantage of this approach is that the difference between the expected and realized rates may include other components, not due to QE, if the time series model is misspecified. Unfortunately, the models implemented are simplified for analytic or econometric reasons, making this a reasonable concern.

The contribution of our paper is three-fold. First, our paper estimates the impact of QE on the term structure of forward rates, and not bond yields. Forward rates correspond to the "marginal" rate for a future time interval, while yields correspond to the "average" rate over a time horizon starting today and ending with the bond's maturity. As such, since yield are averages of forward rates, yields confound changes in short- and long- term forward rates. Our study isolates the effects of QE on the different maturity forward rates.

The second contribution of our paper is to provide a new and alternative methodology for estimating the impact of QE on the term structure of interest rates. Our methodology uses an arbitrage-free term structure evolution that explicitly includes the impact of the Fed's Treasury purchases on the price. As such, our approach is able to estimate both the magnitude and the duration of QE price impacts on the term structure of Treasuries. The idea is that we can decompose the observed forward rate curve into two components: one is a hypothetical market forward rate curve without the Fed's purchases, and the other is the price impact due to QE. The advantage of our approach, as contrasted with the existing time series methodology, is that we do not need to set up a counter factual experiment. Instead, this relation is explicitly built into our parametric model for the evolution of the term structure of interest rates. Our empirical methodology is based on the literature studying the pricing of derivatives in an arbitrage-free economy with a large trader (see Jarrow (1992), Bank and Baum (2004), Jarrow, Protter, Roch (2011)).

Our paper is most closely related to Jarrow, Protter, and Roch (2011) who study the divergence in an asset's price from fundamental value caused by trading activity. Similar to our methodology, Li and Wei (2012) add a supply factor to Treasury yields, measuring the impact of QE without a counter factual experiment. However, both the theory and empirical methodology in Li and Wei differ from that used in our paper.

The third contribution of our paper is to test whether the Fed's QE purchases introduced arbitrage opportunities into the Treasury security markets. It is plausible that given such large scale purchases, the Fed could over pay for particular maturity Treasuries causing their risk premium to be distorted relative to close maturity substitutes. Although the Fed attempted to only purchase undervalued assets (see Gagnon, Raskin, Remache, and Sack (2010, p. 47)), it is an open question whether their purchases were successful in this regard.

Our estimation shows that the term structure of forward rates were affected by QE, and without the introduction of arbitrage opportunities. Short- and medium- term forward rates declined (up to 12 years) with the size of the impact decreasing in maturity. There were no discernible changes in long-term forward rates (greater than 12 years). The persistence of the price impacts increased with maturity up to 6 years then declined, with half-lives lasting approximately 4, 6, 12, 8 and 4 months for the 1, 2, 5, 10 and 12 year forwards, respectively. The Fed's QE activities did not affect long-term forward rates, contrary to the Fed's stated intentions. This is not surprising, however, given that the Fed's purchase activities were concentrated on bonds with maturities of less than 10 years (see Figure 4).

Since bond yields are averages of forward rates over a bond's maturity, QE did affect long-term bond yields. The average impacts on bond yields were 327, 26, 50, 70, and 76 basis points for 1, 2, 5, 10 and 30 years, respectively. These yield impacts are consistent with those estimated in the existing literature, except for the 1-year rate. Our 1-year estimated yield change is significantly greater than that in the existing literature because it includes the impact of the Fed's monetary policy - keeping short-term rates near the zero lower bound. The existing 1-year estimates come from an event study (see Krishnamurthy and Vissing-Jorgensen (2011)), which only includes the impact of QE alone. Secondly, unlike the estimates from an event study, our estimated changes in bond yields are consistent across the entire term structure. In particular, the 1-year rate change is embedded within the 5-year, 10-year, and 30-year yield changes to be consistent with an arbitrage-free term structure evolution.

These results can be best understood using the modified expectations hypothesis that always holds in an arbitrage-free term structure model (see Jarrow (2009)). The expectations hypothesis is modified for risk aversion using adjusted probabilities, instead of the actual probabilities. As in the classical expectations hypothesis, except for this modification, the time t forward rate for date T is the time t "expected" spot rate for date T. These results show that the impact of QE on the future spot rate is "expected" to disappear after 12 years. And in addition, the effect of a purchase on the future spot rate is "expected" to last longer, the longer the term of the rate up to about 6 years. Perhaps because most monetary policy activities occur on the very short-end of the curve, diminishing the lasting power of any quantity impact on the short-term forward rates.

The outline of this paper is as follows. The next section introduces the model. In

Section 3, we present the estimation methodology along with a description of the data, followed by a discussions of the results. Section 4 provides various model specification tests, Section 5 compares our estimates with those of the existing literature, and Section 6 concludes.

2 The Model

This section constructs a Heath, Jarrow, Morton (1992) arbitrage-free term structure of interest rate model augmented to include the price impacts of a large trader, the Federal Reserve, based on the insights of Jarrow, Protter, Roch (2011). Traded are default-free zero-coupon bonds of all maturities and a money market account in a frictionless market. A frictionless market has no transaction costs, no restrictions on trade (e.g. short sale restrictions), and asset prices are perfectly divisible. All traders, except the Fed, act as price takers believing their trades have no impact on the price of the traded Treasuries. In contrast, the Fed's purchases are assumed to have a significant price impact, the details of which will be presented shortly.

2.1 The Term Structure Evolution

We let $P(t,T)$ denote the time t market price of a zero-coupon bond paying a dollar at time T. This price is observed at time t and reflects the presence of the Fed's purchasing activities. The time t forward rate for date T is denoted $F(t,T)$ and it is implicitly defined by

$$P(t,T) = e^{-\int_t^T F(t,s)ds}. \tag{1}$$

The instantaneous spot rate of interest is defined by $R(t) \equiv F(t,t)$. We note for subsequent usage in the empirical section that the spot rate of interest is a hypothetical construct that is unobservable in actual markets.[2]

We let $p(t,T)$ denote the hypothetical unobserved zero-coupon bond price that would exist in the economy if the Fed did not trade. For convenience, we call $p(t,T)$ the "true" price. We denote the true forward and spot rate by $f(t,T)$ and $r(t)$, respectively.

We assume that the true forward rate process evolves according to an N- factor model:

$$df(t,T) = \mu(t,T)dt + \sum_{n=1}^{N} \sigma_n(t,T)dW_n(t) \tag{2}$$

where $W_i(t)$ for $i = 1, ..., N$ are independent standard Wiener processes, $\mu(t,T)$ is the drift of the forward rate process, and $\sigma_n(t,T)$ is the volatility of the n^{th} factor. Of course, we assume the technical conditions necessary for this stochastic differential equation to exist (see Heath, Jarrow, Morton (1992) for these conditions). At this point, this evolution is quite general. Due to the Wiener processes, the only economic restriction being imposed is that the forward rates' sample path is continuous in time.

[2] This is because the spot rate is defined by the limit condition: $R(t) = \lim_{\Delta \to 0} \left(\frac{1-P(t,t+\Delta)}{P(t,t+\Delta)} \cdot \frac{1}{\Delta} \right)$.

The true price process includes the impact of any expected or unexpected changes in the market's supply/demand for Treasuries caused by changes in the business cycle and normal economic activity, for example, an increase in the foreign demand for Treasuries during "flight-to-quality" episodes in the recent credit crisis. This process also reflects changes in the outstanding supply of Treasury securities as determined by the U. S. Treasury's auction activities.

2.2 The Fed's Price Impact

As mentioned previously, we consider the Fed as a large trader, whose purchase/sales affect the prices of Treasuries. In the large trader literature mentioned previously, a large trader's purchases/sales are private information (see Jarrow (1992), Bank and Baum (2004), Jarrow, Protter, Roch (2011)). This implies that the large trader's price impact occurs at the time of trade, and not before. In contrast, the Fed's purchases/sales are public information, announced in advance of their trades. As such, in contrast to a large private trader, there will be an announcement effect on the price of Treasuries prior to their purchases, which needs to be incorporated into the model.

We consider a partial equilibrium model, similar to Jarrow, Protter, Roch (2011). Let $x(t,T)$ denote the time t cumulative changes in the aggregate demand for the T maturity zero-coupon bond caused by the Fed's activities, both the announcements and trades. As such, this consists of two components:

$$x(t,T) = y(t,T) + z(t,T)$$

where $y(t,T) \geq 0$ is the time t cumulative changes in the Fed's holdings of the T maturity zero-coupon bond due to their purchases, and $z(t,T)$ is the market's time t cumulative change in the demand for the T maturity zero-coupon bond due to the Fed's announcements. For example, at the announcement time in hopes of obtaining future profits, traders may purchase Treasuries in anticipation of a price rise at the time of the Fed's purchases, causing prices to react immediately.

Given the change in aggregate demand, we assume that the Fed's activities (announcements and trades) affect the evolution of the observed forward rates as follows:

$$dF(t,T) = \lambda(t,T)(f(t,T) - F(t,T))dt + df(t,T) - d\Psi(t,T) \tag{3}$$

where $\lambda(t,T)$ corresponds to the rate of mean reversion of the observed to the true forward rate, $g(x(t,T),T)$ corresponds to the marginal impact of the change in aggregate demand $dx(t,T)$ on the observed forward rate, and $d\Psi(t,T) \equiv g(x(t,T),T)dx(t,T)$. As noted, we assume that the forward rate decreases as aggregate demand increases.

Solving this stochastic differential equation, we can alternatively express this assumption as

$$F(t,T) = f(t,T) - \int_0^t e^{-\int_s^t \lambda(u,T)du} d\Psi(s,T). \tag{4}$$

In this form the motivation for this assumption is clear. The observed market forward rate $F(t,T)$ can be decomposed into two components. The first is the true forward rate $f(t,T)$, to which the observed rate mean reverts. The second component is the price impact of the

Fed's activities, which depend on both the rate of mean reversion $\lambda(t,T)$ and the marginal impact of the change in aggregate demand $d\Psi(s,T)$. At this point, $\lambda(t,T)$ and $\Psi(t,T)$ can be very general stochastic processes. The only restrictions are those necessary to make expression (4) well-defined and exist.[3]

For subsequent usage, we note that in this reduced form model, the impact on Treasury prices (T near 0) due to the Fed's short-term interest rate monetary policy are included in this component as well.

Finally, it is important to note that although the Fed's holdings $y(t,T)$ are observable, the accumulated changes in aggregate demand $z(t,T)$ are not. Consequently, $x(t,T)$ is not observable, so that we will not be able to empirically decompose the marginal impact $d\Psi(s,T) = g(x(s,T),T)dx(s,T)$ into its component parts. This explains why we use the simplified notation in expression (4) above.

To facilitate empirical estimation, we impose the following additional structure. First, we assume that the Fed starts its activities with an announcement at time 0, and the purchases end at some known future time τ. Second, we let the Fed's price impact on the $T-$ maturity forward rate be a deterministic function of the rate's maturity, i.e.

$$\Psi(0,T) = 0, \quad d\Psi(t,T) = I_{\{t\leq\tau\}}\psi(T)dt \tag{5}$$

where $\psi(T)$ is the marginal price impact rate, per year, and $I_{\{\cdot\}}$ is an indicator function. This assumption implies that the Fed's purchases do not introduce additional randomness into the forward rate's evolution. Alternatively stated, the forward rate process can be viewed as a controlled process where the Fed chooses the marginal price impact rate.

Last, we let the mean reversion rate also be a deterministic function of the forward rate's maturity, i.e.

$$\lambda(t,T) = \lambda(T). \tag{6}$$

Under these additional assumptions, we can rewrite expression (4) as:

$$F(t,T) = \begin{cases} f(0,T) \\ f(t,T) - \frac{\psi(T)}{\lambda(T)}(1-e^{-\lambda(T)t}), & \text{if } 0 < t \leq \tau \\ f(t,T) - \frac{\psi(T)}{\lambda(T)}(e^{\lambda(T)\tau}-1)e^{-\lambda(T)t}, & \text{if } t > \tau. \end{cases} \tag{7}$$

From expression (7), we can easily derive the evolution of the observed zero-coupon bond price.

For $0 < t \leq \tau$ we have

$$\begin{aligned} P(t,T) &= e^{-\int_t^T F(t,s)ds} \\ &= p(t,T)\exp\{\int_t^T \frac{\psi(s)}{\lambda(s)}(1-e^{-\lambda(s)t})ds\}. \end{aligned} \tag{8}$$

For $t > \tau$, we have

$$P(t,T) = p(t,T)\exp\{\int_t^T \frac{\psi(s)}{\lambda(s)}(e^{\lambda(s)\tau}-1)e^{-\lambda(s)t}ds\} \tag{9}$$

[3]For example, for each T, $\Psi(t,T)$ needs to be a semimartingale.

As indicated, the Fed's purchases increase the true price $p(t,T)$ by the proportionality factor given in expressions (8) and (9). Defining the price impact as $\delta(t,T) \equiv \ln P(t,T) - \ln p(t,T)$, expressions (8) and (9) imply that

$$\delta(t,T) = \begin{cases} \int_t^T \frac{\psi(s)}{\lambda(s)}(1 - e^{-\lambda(s)t})ds, & \text{if } 0 < t \leq \tau \\ \int_t^T \frac{\psi(s)}{\lambda(s)}(e^{\lambda(s)\tau} - 1)e^{-\lambda(s)t}ds, & \text{if } t > \tau. \end{cases} \tag{10}$$

We see that the price impact increases as time t increases, then decreases as the bond approaches its maturity. The price impact is zero at both $t = 0$ and $t = T$. The maximum distortion is achieved at some time $t^* \in (0, \tau]$. Of course, the objective of the empirical section is to estimate the magnitude of this quantity for the different maturity Treasury securities during the Fed's QE.

2.3 The Arbitrage-free Restrictions

We assume that the observed forward rate evolution, before and after the Fed's purchases, is arbitrage-free. This section studies the restrictions that this no arbitrage assumption imposes. To obtain these restrictions, we start with expression (7), rewritten in differential form:

$$dF(t,T) = \begin{cases} df(t,T) - \psi(T)e^{-\lambda(T)t}dt, & \text{if } t \leq \tau \\ df(t,T) + \psi(T)(e^{\lambda(T)\tau} - 1)e^{-\lambda(T)t}dt, & \text{if } t > \tau. \end{cases} \tag{11}$$

Here it is seen that the Fed's buying activity is deterministic and only affects the drift of the observed forward rate's evolution. Otherwise, the evolution of the true forward rate process is unaffected. Thus, one can directly apply the HJM no arbitrage drift conditions (HJM (1992)) to obtain the following theorem. The proof is in the appendix.

Theorem 1 ***No Arbitrage Conditions***

Given $(\sigma_n(t,T), \phi_n(t))$ for all n, and $(\lambda(T), \psi(T))$ for all T, the observed forward rate evolution is arbitrage free if and only if there exist $(\Phi_n(t))$ for all n such that

$$\sum_{n=1}^{N} \sigma_n(t,T)\Phi_n(t) = \sum_{n=1}^{N} \sigma_n(t,T)\phi_n(t) + \begin{cases} \psi(T)e^{-\lambda(T)t}, & \text{if } 0 < t \leq \tau \\ \psi(T)(1 - e^{\lambda(T)\tau})e^{-\lambda(T)t}, & \text{if } t > \tau \end{cases} \tag{12}$$

where $\Phi_n(t)$ ($\phi_n(t)$) are the market prices of risk for factor n with (without) the Fed's price impact.

This theorem shows that in an economy whose term structure evolution is arbitrage-free, the Fed's purchases necessarily change the market prices of risk in the economy (from $\phi_n(t)$ to $\Phi_n(t)$ for all n). This makes intuitive sense because the Fed's purchases, changing aggregate demand, causes a shifting in the economy's equilibrium. To reduce the traders' aggregate demands, to meet the decreased available supply, equilibrium risk premium must adjust and expression (12) shows exactly how.

This shift in risk premium can be better understood by studying a one-factor model. In this case, the HJM no-arbitrage drift restriction is

$$\Phi(t) = \phi(t) + \begin{cases} \frac{\psi(T)e^{-\lambda(T)t}}{\sigma(t,T)}, & \text{if } 0 < t \leq \tau \\ \frac{\psi(T)(1-e^{\lambda(T)\tau})e^{-\lambda(T)t}}{\sigma(t,T)}, & \text{if } t > \tau. \end{cases} \tag{13}$$

Expression (13) shows that when the Fed is buying, risk premium must increase by the positive quantity on the right side of this expression to keep the term structure evolution arbitrage-free.

We will test below to see if expression (12) holds during the QE program. As mentioned in the introduction, although the Fed attempted to only purchase undervalued assets (see Gagnon, Raskin, Remache, and Sack (2010, p. 47)), it is plausible that given such large scale purchases the Fed could have over paid for particular maturity Treasuries causing their risk premium to be distorted relative to close maturity substitutes.

3 Estimation

This section estimates the Fed's QE price impact on the term structure of interest rates using the arbitrage-free term structure model developed in the previous section. Included in this estimation is the Fed's price impact on the spot rate of interest (the difference between $R(t)$ and $r(t)$). Because the spot rate of interest is unobservable and important to the model's formulation, we necessarily estimate the impact parameters using a Kalman filter.[4] We start with estimating a one-factor affine model,[5] and then generalize to more realistic two- and three-factor models.

3.1 The Data

Since QE1 was officially announced on November 25, 2008 and QE2 was completed on June 30, 2011, we choose the sample period spanning from November 24, 2008 to June 30, 2011 to estimate the impact of the Fed's QE activities.

The term structure of interest rate data is the daily instantaneous forward rates time series constructed by Gürkaynak, Sack, and Wright (GSW (2007)) and available on the Federal Reserve website.[6] This data set contains 30 forward rates with maturities ranging from 1 year to 30 years. The GSW data is based on the forward rate smoothing procedure described in Svensson (1994), which assumes a parametric form with six parameters, and it is chosen for easy comparison with the existing literature.[7]

Figure 1 graphs the Federal funds rate before and during the QE1 and QE2 estimation period. As seen, the Fed funds rate drop to near zero corresponds with the start of the QE time period. This implies that our estimates of the Fed's impact on bond prices will also reflect the impact of the Fed's short-term interest rate monetary policy activities as well. As discussed in the introduction to the paper, the Fed funds rate being maintained close to the zero lower bound is an important reason for determining the additional effectiveness of both QE1 and QE2 in lowering long-term interest rates.

Table 1 provides summary statistics for the different maturity forward rates over the sample period. This table provides a benchmark for the level of forward rates and their standard deviations over the estimation period. Figure 2 plots the forward rates' time

[4] Bolder (2001) provides a good technical guide on implementing a Kalman filter.

[5] This is sometimes called a Vasicek (1977) model.

[6] https://www.federalreserve.gov/econresdata/researchdata.htm

[7] We also explored the estimation using forward rates based on a polynomial spline smoothing procedure yielding similar results. For brevity these results are not reported in the subsequent text.

series evolutions. Interestingly, one can see a decline in the observed forward rates between 2008 and 2011, most pronounced for the 1, 2, 3, and 5 year forward rates.

Figure 3 graphs the evolution of the Fed's balance sheet over the sample period.[8] The Fed's purchases mainly focused on mortgage-backed securities (MBS) during QE1 and Treasury securities during QE2. This difference in the types of asset purchased across QE1 and QE2 suggests that there may be differing price impacts. We explore this possibility in our estimation below.

Figure 4 provides a breakdown of the Fed's Treasury holdings by maturity over our sample period. For their Treasury security purchases, the Fed's activities are mostly concentrated on securities with maturities between one and ten years. These holdings will be relevant when discussing the QE's impact on bond price yields in subsequent sections.

Relevant to the Fed's Treasury purchases and their impact on forward rates is the outstanding supply of Treasury securities during the QE period. As mentioned earlier, we do not explicitly adjust our estimates of the Fed's QE impact on forward rates for changes in the outstanding supply of Treasuries. In our methodology, this supply adjustment is implicitly captured through its impact on the estimated true forward rate process (the drift and volatilities) over this time period. A potential concern with our methodology, therefore, is that if the U. S. Treasury purposely increased its auction of Treasuries to take advantage of the Fed's QE activities, then our estimated price impacts would be biased low. To investigate this potential bias, Figure 11 shows the time series of newly auctioned Treasury securities over the QE period.[9] As seen, the newly auctioned Treasuries are quite stable and only slightly increasing across time, the upward trend reflecting an increase in the size of the Federal budget deficit over this same time period. It does not appear that the U.S. Treasury's auction process was directly influenced by the Fed's QE activities, minimizing this potential bias in our estimation methodology.

3.2 A One-Factor Model

This section estimates a one-factor affine model for the evolution of the term structure of interest rates. We start with the one-factor model to both illustrate the methodology and to provide a benchmark for comparing the results for two- and three-factor models.

3.2.1 The Methodology

In the one-factor affine model, the true forward rates evolution given by expression (2) can be written as:

$$f(t,T) = (1 - e^{-k(T-t)})\left(\theta - \frac{\sigma_r^2}{2k^2}\left(1 - e^{-k(T-t)}\right)\right) + e^{-k(T-t)} r_t \tag{14}$$

[8] Data source: http://www.federalreserve.gov/econresdata/

[9] Data source: http://www.treasurydirect.gov/instit/annceresult/press/preanre/preanre.htm

where the state variable r_t is the true instantaneous spot rate. Substitution into expression (7) gives the observed forward rate process, including the Fed's price impact:

$$F(t,T) \tag{15}$$
$$= \begin{cases} (1-e^{-k(T-t)})\left(\theta - \frac{\sigma_r^2}{2k^2}\left(1-e^{-k(T-t)}\right)\right) + e^{-k(T-t)}r_t - \frac{\psi(T)}{\lambda(T)}(1-e^{-\lambda(T)t}), \\ \quad \text{if } 0 < t \leq \tau \\ (1-e^{-k(T-t)})\left(\theta - \frac{\sigma_r^2}{2k^2}\left(1-e^{-k(T-t)}\right)\right) + e^{-k(T-t)}r_t - \frac{\psi(T)}{\lambda(T)}(e^{\lambda(T)\tau}-1)e^{-\lambda(T)t}, \\ \quad \text{if } t > \tau. \end{cases}$$

As mentioned previously, since the spot rate is unobservable[10], to estimate our system we use a Kalman filter. In our Kalman filter, the time-discretized state transition equation for the spot rate is given by

$$r_{t+\Delta t} = \theta(1-e^{-k\Delta t}) + e^{-k\Delta t}r_t + \sigma_r \varepsilon_t \tag{16}$$

where ε_t follows a standard normal distribution.

As indicated, this evolution allows the spot rate to be negative with positive probability. Although alternative evolutions could be used that preclude negative rates, both economic theory and the empirical evidence are more consistent with evolutions that allow negative (nominal) rates with positive probability. Indeed, from a theoretical perspective, large financial institutions cannot store currency, they can only invest it in either deposits or securities; and consequently, negative rates are possible. Empirically, negative rates on Treasuries were observed in each of November 2009, June 2011, and August 2011;[11] and the Bank of New York Mellon paid negative deposit rates in August 2011.[12]

For the Kalman filter, the measurement equation is given by the evolution of the observed forward rate process:

$$F(t,\Lambda_i) = A_i + B_i r_t + u_t(\Lambda_i) \tag{17}$$

where

$$\begin{aligned} A_i &= \begin{cases} (1-e^{-k\Lambda_i})\left(\theta - \frac{\sigma_r^2}{2k^2}\left(1-e^{-k\Lambda_i}\right)\right) - \frac{\psi_i}{\lambda_i}(1-e^{-\lambda_i t}), & \text{if } 0 < t \leq \tau \\ (1-e^{-k\Lambda_i})\left(\theta - \frac{\sigma_r^2}{2k^2}\left(1-e^{-k\Lambda_i}\right)\right) - \frac{\psi_i}{\lambda_i}(e^{\lambda_i \tau}-1)e^{-\lambda_i t}, & \text{if } t > \tau \end{cases} \\ B_i &= e^{-k\Lambda_i}, \text{ and } \Lambda_i = T_i - t. \end{aligned} \tag{18}$$

[10] Instead, one could obtain estimated spot rates using the intercept of the smoothed GSW forward rate curve with the y-axis. We choose not to use these estimates because the intercept with the y-axis explicitly depends on the functional form of the smoothing function, which in turn, is greatly influenced by the prices of the long-term Treasuries. In reality, short-term Treasury rates (less than one year) are influenced more by the impact of the Fed's short-term interest rate policies than the assumed shape of a smoothing function. Our estimation methodology avoids this potential bias.

[11] See WSJ Blog, Market Beat, November 20, 2009, "Some Treasury Bill Rates Negative Again Friday;" Bloomberg, November 19, 2009, "U.S. 3-month Bills Turn Negative on Concern Risk Rally Overdone;" Bloomberg, June 27, 2011, "Treasury 4-week Bill Rates Negative for First Time since 2010;" WSJ Blog, Market Beat, August 4, 2011, "From One Crisis to Another: One Month T-Bill Yields go Negative Again."

[12] See Bloomberg.com/news, August 5, 2011, "BNY Mellon Makes Clients Pay for Deposits as Investors Seek Safety in Cash;" Online WSJ, August 5, 2011, "New Fee to Bank Cash."

For simplicity, we assume $u_t(\Lambda_i)$ follows an independent normal distribution.

We estimate the parameters using three forward rate series ($\Lambda_i = 1yr,\ 2yr,\ 3yr$). The parameters to be estimated are (k, θ, σ_r, ψ_1, λ_1, ψ_2, λ_2, ψ_3, λ_3).

3.2.2 The Results

The parameter estimates are shown in Table 2 and the evolution of the true spot rate is plotted as the dashed curve in Figure 5.

The spot rate, with the Fed's impact included, is the limit of expression (15) as $T \to t$, i.e.

$$R_t = r_t - \begin{cases} \frac{\psi(0)}{\lambda(0)}(1 - e^{-\lambda(0)t}), & \text{if } 0 < t \leq \tau \\ \frac{\psi(0)}{\lambda(0)}(e^{\lambda(0)\tau} - 1)e^{-\lambda(0)t}, & \text{if } t > \tau. \end{cases}$$

To obtain an estimate of this spot rate, we use the estimates of $(\lambda(1), \psi(1))$ instead of $(\lambda(0), \psi(0))$. The corresponding estimated short rate (denoted by R_t) is plotted as the dotted curve in Figure 5. The difference $(r_t - R_t)$ is the Fed's price impact, which is plotted as the solid curve. A positive and upward trending price impact curve in Figure 5 is consistent with the facts that the Fed's monetary policy was targeting near zero short-term rates, and the Fed had been continuously purchasing Treasury securities over the estimation period. It shows that, under the one-factor model, the Fed's price impact on the short rate has been increasing since QE started, and stayed in the range of 2.3% $\sim$ 2.4% until the end of June 2011.

Table 2 presents the estimated price impact parameters for the one-, two-, and three-year forward rates (λ_i, ψ_i) for $i = 1, 2, 3$. The marginal impact parameter is decreasing in maturity, i.e. $\psi_1 > \psi_2 > \psi_3$. In contrast, the mean reversion parameter is increasing with maturity, i.e. $\lambda_1 > \lambda_2 > \lambda_3$. This implies that the duration of the price impact increases with maturity. Defining the half-life of the price impact as the time $t_0^i = \ln(2)/\lambda_i$ for $i = 1, 2, 3$, then $t_0^1 = 0.32 \approx 3.8$ months and $t_0^2 = 0.54 \approx 6.5$ months. The half life of the price impact of the 3-year forward rate is not defined since λ_3 is insignificantly different from zero.

These results can be best understood using the modified expectations hypothesis that always holds in an arbitrage-free term structure model (see Jarrow (2009)). The expectations hypothesis is modified for risk aversion using adjusted probabilities, instead of the actual probabilities.[13] As in the classical expectations hypothesis, except for this modification, the time t forward rate for date T is the time t "expected" spot rate for date T. These results show that the impact of QE on the future spot rate is "expected" to decline as time progresses. And in addition, the effect of a purchase on the future spot rate is "expected" to last longer, the longer the term of the rate. Perhaps because most monetary policy activities occur on the very short-end of the curve, diminishing the lasting power of any quantity impact on the short-term forward rates.

[13] These adjusted probabilities are called the forward price martingale probability measures, see Jarrow (2009).

3.3 A N-Factor Model

The above estimation procedure can be extended to a N - factor affine model, where the short rate is a sum of N factors

$$r(t) = \sum_{n=1}^{N} z_n(t). \tag{19}$$

Each factor $z_n(t)$ evolves as

$$dz_n(t) = k_n(\theta_n - z_n(t))dt + \sigma_n dW_n(t) \tag{20}$$

where $W_i(t)$ for $i = 1, ..., N$ are independent standard Wiener processes.

Under this framework, one can show that the zero-coupon bond price is[14]

$$P(t,T) = \exp\left\{ C(t,T) - \sum_{n=1}^{N} D_n(t,T) z_n(t) \right\} \tag{21}$$

where

$$\begin{aligned} D_n(t,T) &= \frac{1 - e^{-k_n(T-t)}}{k_n} \\ C(t,T) &= -\sum_{n=1}^{N} \theta_n \left[T - t + \frac{e^{-k_n(T-t)} - 1}{k_n} \right]. \end{aligned}$$

The corresponding forward rates are

$$\begin{aligned} F(t,T) &= -\frac{\partial \ln P(t,T)}{\partial T} \\ &= \sum_{n=1}^{N} \frac{\partial D_n(t,T)}{\partial T} z_n(t) - \frac{\partial C(t,T)}{\partial T} \\ &= \sum_{n=1}^{N} e^{-k_n(T-t)} z_n(t) + \sum_{n=1}^{N} \theta_n \left[1 - e^{-k_n(T-t)} \right]. \end{aligned} \tag{22}$$

Therefore, the time-discretized state transition equation can be written as

$$z_n(t + \Delta t) = \theta_n(1 - e^{-k_n \Delta t}) + e^{-k_n \Delta t} z_n(t) + \varepsilon_n(t) \qquad n = 1, ..., N \tag{23}$$

where $\varepsilon_n(t)$ follow zero-mean normal distributions with the following variance and covariance

$$\begin{aligned} Var\left[\varepsilon_n(t) | \mathcal{F}_{t-\Delta t}\right] &= \frac{\sigma_n^2}{2k_n}\left(1 - e^{-2k_n \Delta t}\right) \\ Cov[\varepsilon_n(t), \varepsilon_m(t) | \mathcal{F}_{t-\Delta t}] &= 0 \qquad n \neq m \end{aligned}$$

where $\mathcal{F}_t$ is the natural filtration generated by the state variables process up to time t.

[14] For the technical details, see Chapter 4 of Brigo and Mercurio (2006), Chapter 2 of Jeanblanc, Yor and Chesney (2009), and Bolder (2001).

Recall that expression (7) describes the relation between the unobserved forward rates without the Fed's impact ($f(t, T)$) and the observed forward rates with the Fed's impact ($F(t, T)$). Combining expressions (7) and (22), we obtain the measurement equation:

$$F(t, \Lambda_i) = A_i + \sum_{n=1}^{N} B_{i,n} z_n(t) + u_t(\Lambda_i) \tag{24}$$

where $u_t(\Lambda_i)$ are assumed to follow independent normal distributions,

$$B_{i,n} = e^{-k_n \Lambda_i}, \text{ and } \ \Lambda_i = T_i - t.$$

For $0 < t \leq \tau$,

$$A_i = \sum_{n=1}^{N} \theta_n \left[1 - e^{-k_n \Lambda_i}\right] - \frac{\psi_i}{\lambda_i}(1 - e^{-\lambda_i t}).$$

For $t > \tau$,

$$A_i = \sum_{n=1}^{N} \theta_n \left[1 - e^{-k_n \Lambda_i}\right] - \frac{\psi_i}{\lambda_i}(e^{\lambda_i \tau} - 1)e^{-\lambda_i t}.$$

3.3.1 The Results

This section estimates both two- and three-factor models. We estimate the parameters using four forward rates ($\Lambda_i = 1,\ 2,\ 3,\ 4$ years). For the two-factor model, the parameters to be estimated are $(k_i,\ \theta_i,\ \sigma_i,\ (\psi_j,\ \lambda_j)_{j=1,2,3,4})_{i=1,2}$ and the results are shown in Table 3. For the three-factor model, the parameters to be estimated are $(k_i,\ \theta_i,\ \sigma_i,\ (\psi_j,\ \lambda_j)_{j=1,2,3,4})_{i=1,2,3}$ and the results are shown in Table 4. For comparison with the existing literature estimating affine models without Fed purchases, Table 4 provides the estimates for this model as well. These estimates without the Fed purchases included are consistent with those found in the existing literature (see Babbs and Nowman (1999)).

The estimates of θ_n $(n = 1, 2, 3)$ have large standard errors because the expression for A_i reveals that the model has poor identification for the individual θ_n. However, λ_i and ψ_i can be estimated with much higher precision.

Consistent with the results from the one-factor model, we find that the magnitude of the impact on the i- year forward rate becomes smaller as i gets larger ($\psi_j > \psi_i$ for $j < i$), while the impact on the i- year forward rate lasts longer for larger i ($1/\lambda_j < 1/\lambda_i$ for $j < i$). The half-lives of the impact for the two-factor model is $t_0^1 = 0.34 \approx 4.1$ months, $t_0^2 = 0.45 \approx 5.4$ months, $t_0^3 = 0.76 \approx 9.1$ months, and the half-lives of the three-factor model is $t_0^1 = 0.34 \approx 4.1$ months, $t_0^2 = 0.46 \approx 5.5$ months, $t_0^3 = 0.61 \approx 7.3$ months, $t_0^4 = 0.95 \approx 11.4$ months. The fact that the two-factor and three-factor models give similar results shows the robustness of the estimation procedure.

To determine the impact of the Fed's QE program on long-term rates, Table 5 presents the Fed's impact parameters estimated using a three-factor model for all maturity forward rates ranging from one to thirty years. This is the key Table of the paper. These parameter estimates are obtained by fitting a three-factor model using the forward rates ($\Lambda = 1,\ 2,\ 3,\ i$ years) for $i = 5,\ 6,\ \ldots, 30$ where the parameters $(\psi_j,\ \lambda_j)_{j=1,2,3}$ are fixed at their values given in Table 4. Hence, only the parameters $(k_i,\ \theta_i,\ \sigma_i)_{i=1,2,3}$ and $(\psi_i,\ \lambda_i)$ are reestimated where i corresponds to the longest term forward rate used in the estimation. This two-step procedure is invoked because there are too many parameters to estimate in the larger system of equations, given the size of our data set.

As seen in Table 5, only the first 12 year forward rates' marginal price impact parameters (ψ, λ) are significantly different from zero. Because the impacts on rates beyond twelve years are all insignificantly different from zero, only the results for maturities less than 14 years are shown. A graphic representation of these estimates is given in Figure 6. The top panel plots the half-life of the impact for each maturity forward rate. These results show that the half-life increases as the maturity increases up to about 6 years, then declines thereafter. The lower panel plots the magnitudes of the marginal price impacts. They decrease monotonically as the maturity increases.

These results show that the Fed's QE program affects only short- and medium- term forward rates, up to about 12 years. After 12 years, the Fed's QE program has no discernible effect on forward rates. This is in contrast to the Fed's stated intention of QE to affect long-term rates. The absence of any impact on long-term forward rates is not surprising given that the Fed concentrated its Treasury purchases on maturities of less than 10 years (see Figure 4). Although there is a spill-over effect on the 11- and 12- year maturity Treasuries, there is little if any spill-over on the 20 and 30 year bonds. If the Fed hopes to affect the long-term forward rates, the evidence suggests that they need to purchase the long-term bonds directly.

This does not mean, however, that the Fed's QE program does not affect long-term bond yields. It does because long-term bond yields are an average of the forward rates over the bond's life, and the Fed's QE program has a large impact on short-term forward rates. The impact of the Fed's QE program on bond yields is presented in a subsequent section.

3.4 Separating QE1 and QE2

As mentioned earlier and shown in Figure 3, the Fed's asset purchases differed across QE1 and QE2. For this reason, it is likely that the price impact on Treasuries differed across these two periods. This section addresses this possibility by estimating the model's parameters separately for each of the two time periods. To capture any information leakage, we choose the estimation periods for both QE events to start one day ahead of the official announcement, i.e., the QE1 estimation period spans from November 24, 2008 to March 31, 2010, and the QE2 estimation period ranges from November 2, 2010 to June 30, 2011.

Due to the small sample size, estimating a three-factor affine model for each sub-period generates too large a set of sample errors and inconclusive results. Therefore, in order to get more reliable estimates, we fit a one-factor model. To justify this simplification, we performed the analysis on only the short- to medium- section of the term structure, up to 8 years. A principal component analysis (PCA) using forward rates with maturities of less than eight years confirms that the first principal component accounts for 93% of the variation, showing that a one-factor model provides a good approximation for this section of the term structure.

The estimation results for QE1 and QE2 are shown in Table 6 (Figure 7 and Figure 8 provide graphic views). Similar to the previous results obtained using the whole sample period (Table 5 and Figure 6), we find that the impacts of both QE's are limited to the one- to four- year forward rates. For maturities longer than four years, the estimates of the mean reversion parameter λ are denoted "Large" because in the numerical convergence procedure use for optimizing the likelihood function, the estimates always reach the pre-set upper bound, even when the upper bound is set at very large values (> 50). Since the half-life is the inverse of λ, a large λ means that the price impact lasts for only a very short period.

Perhaps not surprising given that QE2's purchases concentrated on Treasuries instead of MBS and agencies as in QE1 (see Figure 3), we find that for maturities less than four years, QE2's price impact on Treasury rates is larger than that of QE1. However, the duration of the impact lasts longer in QE1 than in QE2. For instance, QE1's impact on the two-year rate lasts for 5.6 months with magnitude of 4.2% per year, while QE2's impact on the same rate lasts for 3.8 months with a magnitude of 6.5% per year. It is important to note that although the direct purchases of Treasury securities have a larger price impact on Treasury rates than do the purchases of MBS and agencies, the price impact on Treasuries of purchasing these alternative assets is significant. These results confirm the belief that asset substitution is an important effect of Fed purchases in fixed income security markets (see Bernanke and Reinhart (2004)).

3.5 Test for Arbitrage

Given the parameter estimates from Table 5, we can test for the satisfaction of expression (12) to see whether the Fed's QE Treasury purchases distorted risk premium and introduced arbitrage opportunities into the economy. Since the QE purchases only affected forward rate maturities of less than or equal to 12 years, we only use these rates to test this proposition.

For the three-factor model, from expression (12), we have that

$$\begin{aligned} & \sigma_1(t,\Lambda_j)\Delta\phi_1(t) + \sigma_2(t,\Lambda_j)\Delta\phi_2(t) + \sigma_3(t,\Lambda_j)\Delta\phi_3(t) \\ = \;& \psi(\Lambda_j)e^{-\lambda(\Lambda_j)t} \text{ for } 0 < t \leq \tau \end{aligned} \tag{25}$$

where $\Delta\phi_n(t) = \Phi_n(t) - \phi_n(t)$ and ($\Lambda_j = 1,\ ..., 12$ years).

To understand the intuition underlying our testing procedure, consider solving expression (25) for $(\Delta\phi_1(t), \Delta\phi_2(t), \Delta\phi_3(t))$ using any three-tuple of distinct maturity forward rates. In general, the solution for $(\Delta\phi_1(t), \Delta\phi_2(t), \Delta\phi_3(t))$ will depend upon the particular forward rate maturities selected. Theorem 1 states that the evolution is arbitrage-free if and only if this is not the case, i.e. no matter which three-tuple of forward rates is selected, the same solution for $(\Delta\phi_1(t), \Delta\phi_2(t), \Delta\phi_3(t))$ must occur. We test this observation below.

To formulate our test statistic, fix a time t in the QE time period, and let $y_{jt} = \psi(\Lambda_j)e^{-\lambda(\Lambda_j)t}$, $x_{jt} = (\sigma_1(t,\Lambda_j), \sigma_2(t,\Lambda_j), \sigma_3(t,\Lambda_j))'$, and $\beta_t = (\Delta\phi_1(t), \Delta\phi_2(t), \Delta\phi_3(t))$. Note that in this notation, we are assuming that β_t does not depend on the forward rate's maturity. First, we estimate β_t using a simple linear regression

$$y_{jt} = \beta_t x_{jt} + \varepsilon_{jt} \quad for\ j = 1, ..., 12 \tag{26}$$

where ε_{jt} are assumed to be *i.i.d.* normal distributions with zero mean, representing observational noise in the data.

If expression (25) is true, the null hypothesis, excluding the noise in the data we would expect to see $\varepsilon_{jt} \equiv 0$ for all j. However, given noise in the data, we would expect to see $var(\varepsilon_{jt})$ small relative to $var(y_{jt})$. To test this expectation, we form the test statistic $s_t = \sum_{j=1}^{12}(y_{jt} - x_{jt}\widehat{\beta}_t)^2/var(y_{jt})$ where $\widehat{\beta}_t$ represents the regression estimate from expression (26). The test statistic s_t has a χ^2 distribution with 9 degrees of freedom (12 data points are used to estimate 3 parameters). If s_t is large, we can reject the null hypothesis of no arbitrage.

Estimating expression (26) for each day (t) over the sample period we obtain a time-series of the estimated market prices of risk $\widehat{\beta}_t$, which are plotted in Panel A of Figure 10, and the test statistic, which is graphed in Panel B of Figure 10. As seen, the test statistic is well below the 5% significance threshold. We can not reject the null hypothesis that there is no arbitrage over the QE period. As intended, the Fed's QE program appears to have been successful in not introducing arbitrage opportunities into the economy. A qualification of our results needs to be noted. Since our parameters are estimated with smoothed Treasury price data, the smoothing procedure could itself remove arbitrage opportunitites, providing only a weak test of our hypothesis. A better test would involve using unsmoothed Treasury prices directly.

4 Model Specification Tests

This section provides various model specification tests that support the model's validity.

4.1 A Comparison Pre- QE

To test the model's specification, we estimated the three-factor model for two time periods before the onset of QE1. One is from January 2, 2001 to August 1, 2003, when the Fed lowered interest rates, and the second from January 2, 2004 to August 1, 2006, when the Fed increased interest rates. If the additional structure in our model captures the Fed's QE activities, one would expect to see the mean reversion and marginal impact parameters (λ, ψ) insignificantly different from zero during these time periods. The parameters are estimated using four forward rates ($\Lambda_i = 1,\ 2,\ 3,\ 4$ years) and the results are presented in Tables 7 and 8.

For the period when the Fed was lowering interest rates, all of the impact parameters $(\lambda,\ \psi)$ are insignificantly different from zero, except for the price impacts of the one- and two- year rates. Although the one-year rate impact is significant, its magnitude (0.071) is less than its magnitude (0.081) in the QE period (see Table 4). The same is true for the two-year rate. These impacts on the shortest term forward rates are consistent with the Fed's direct monetary policy activities having a spill over effect on the one- and two- year rates.

For the period when the Fed was increasing interest rates, all of the marginal impact parameters (ψ) are insignificantly different from zero, except for the four-year rate. The mean reversion parameters (λ) are significant for years two through four. The significance for years two and three are irrelevant, since the market impact parameter is not different from zero. The significance of both of the price impact parameters (λ, ψ) for the four-year rate is probably due to noise in the data, but it could be due to the simplicity of the model being estimated. A resolution of these two possibilities awaits the estimation of more complex models in subsequent research.

4.2 Likelihood Ratio Tests, Pre- and Post- QE

This section provides likelihood ratio tests for the model with and without the Fed's impact function for all three sample periods, both pre- and post- QE. These results are given in Tables 4, 7, and 8. The test statistic is $2(ln(L_1)-ln(L_2))$, where L_1 (L_2) is the maximized likelihood value with (without) the price impact term.

At the 5% significance level, the likelihood ratio test rejects the model without the price impact for all three sample periods. This is to be expected since this is an in sample test, and the price impact model has more parameters. More insightful is a comparison of the magnitudes of the changes in the likelihood values over the different sample periods. For the 1/2/2001-8/1/2003 sample (no QE), the log-likelihood increases by 0.6% after adding the price impact term. For the 1/2/2004-8/1/2006 (no QE) sample period, the increase is only 0.4%. In contrast, for the QE period, the log-likelihood increases the most, by 1%, after adding the price impact term. These relative changes in the likelihood ratio tests are consistent with the validity of the model.

4.3 Pricing Errors

Another way to test the model's specification is to study the model's pricing errors in matching the observed forward rates. Table 9 presents the statistical properties of the forward rate errors for our three factor model (whose parameters are given in Table 4). Panel A shows the result for the model with the price impact term (call it the "adjusted model") and Panel B shows the result without the price impact term (call it the "conventional model").

The pricing errors for the adjusted model (Panel A) are quite small, on the order of 2 basis points. Compared to Panel B, one can see that for maturities within five years, the average pricing errors estimated from the conventional model are significantly larger than those from the adjusted model. For maturities longer than five years, the two models generate pricing errors with similar magnitudes. This evidence is consistent with the Fed's price impact on long-term forward rates being small. The pricing errors for both models exhibit autocorrelations, perhaps indicating that a more complex model may provide a better fit.

It is important to note that these results are similar in magnitude to the pricing errors obtained in the 4-factor affine model estimated by Adrian, Crump and Moench (2012, Table 4), where instead of adding the Fed's deterministic price impact component, one adds an additional Brownian motion random shock to the forward rate's evolution. The ability of the deterministic price impact component to match the performance of an additional random factor lends credence to the validity of the model.

5 Comparison to Existing Literature

This section compares our price impact estimates with those in the existing empirical literature. The estimates in the existing empirical literature are summarized in Table 10, Panel A for five studies: D'Amico and King (2011), Gagnon, Raskin, Remache, and Sack (2010), Krishnamurthy and Vissing-Jorgensen (2011), Li and Wei (2012), and Meaning and Zhu (2011). The existing literature studies the price impact on bond yields for maturities ranging between 1 - 30 years. As seen, the estimated price impact is around 40 basis points for short-term rates (less than 5 years), and 75 - 100 basis points for long-term yields (greater than 5 years). To make this comparison, we need to transform our estimated impacts on forward rates from the three-factor model in Table 5 to changes in bond yields.

This transformation is a multi-step process. First, we compute the changes in the true and observed constant maturity zero-coupon bond prices using expression (10). Then, given these true and observed constant maturity zero-coupon bond prices, we compute the true constant maturity par-bond yields for bonds with maturities 2 - 30 years.[15] These true par-bond yields give the coupon payments to use for computing the prices of the observed bonds, using the observed zero-coupon bond prices. Finally, from these observed bond prices, we can compute the observed yields. A comparison of the true par-bond yields with the observed yields generates the desired change in the Treasury yields due to the Fed's QE activities. These yield changes are contained in Table 10, Panel B and graphed in Figure 9.

As seen, the average yield changes are 327, 26, 50, 70, and 76 basis points for the 1, 2, 5, 10, and 30 year bond yields. Except for the 1-year rate, our numbers are similar in magnitude to those in the previous literature. Our estimate for the 1-year rate is significantly larger. As discussed previously, this difference is due to the fact that our estimates include the impact of the Fed's short-term interest rate monetary policy activity during the QE period.

6 Conclusion

This paper provides a new framework for analyzing the price impact of the Fed's trading activities on the Treasury yield curve. To test our theory, we estimated an arbitrage-free affine model that includes the price impact of a large trader, the Fed, over the time period when the Fed conducted its quantitative easing (QE) program: late 2008 to the middle of 2011. Our findings indicate that the QE program generated significant price impacts on short- and medium- term Treasury forward rates of up to 12 years without introducing arbitrage opportunities into the markets. In contrast to the Fed's stated intentions, however, the impact on long-term forward rates appears to have been insignificant. The half-life of the forward rate impacts increased with the maturity of the forward rate up to approximately 6 years, and then declined thereafter. The largest half-life estimated is approximately 1.4 years in duration. Since yields are averages of forward rates, QE did have an impact on long-term bond yields. Our estimates of the magnitude of the QE yield

[15] A par bond yield is that coupon payment that makes a bond's current price equal its face value ($100). We compute the true coupon bond's par-bond yield using the true zero-coupon bond prices.

changes are similar to those that appear in the existing literature.

The model estimated herein was simplified in order to facilitate an analytic representation and the use of maximum likelihood estimation procedures. As such, the model can and should be generalized to explore its empirical validity. Two immediate extensions are to have more a complex large trader impact process and a more complex evolution for the term structure of interest rates. These extensions, however, await subsequent research.

7 Appendix

Proof of Theorem 1

From expression (11), for $t \le \tau$, we have

$$dF(t,T) = (\mu(t,T) - \psi(T)e^{-\lambda(T)t})dt + \sum_{n=1}^{N} \sigma_n(t,T)dW_n(t)$$

The HJM condition on $f(t,T)$ implies that

$$\mu(t,T) = -\sum_{n=1}^{N} \sigma_n(t,T)\left[\phi_n(t) - \int_t^T \sigma_n(t,s)ds\right] \tag{27}$$

The HJM condition on $F(t,T)$ implies that

$$\mu(t,T) - \psi(T)e^{-\lambda(T)t} = -\sum_{n=1}^{N} \sigma_n(t,T)\left[\Phi_n(t) - \int_t^T \sigma_n(t,s)ds\right] \tag{28}$$

where $\Phi_i(t)$ ($\phi_i(t)$) is the price of risk for factor i with (without) the Fed's price impact.

From expression. (27) and (28), we obtain the difference in risk premium:

$$\sum_{n=1}^{N} \sigma_n(t,T)[\Phi_n(t) - \phi_n(t)] = \psi(T)e^{-\lambda(T)t} > 0$$

From expression (11), for $t > \tau$, we have

$$dF(t,T) = [\mu(t,T) + \psi(T)(e^{\lambda(T)\tau} - 1)e^{-\lambda(T)t}]dt + \sum_{n=1}^{N} \sigma_n(t,T)dW_n(t)$$

The HJM condition on $F(t,T)$ implies that

$$\mu(t,T) + \psi(T)(e^{\lambda(T)\tau} - 1)e^{-\lambda(T)t} = -\sum_{n=1}^{N} \sigma_n(t,T)\left[\Phi_n(t) - \int_t^T \sigma_n(t,s)ds\right] \tag{29}$$

From expressions (27) and (29), we obtain the difference in risk premium:

$$\sum_{n=1}^{N} \sigma_n(t,T)[\Phi_n(t) - \phi_n(t)] = \psi(T)(1 - e^{\lambda(T)\tau})e^{-\lambda(T)t} < 0$$

To sum up, the Fed's impact on the risk premium is

$$\sum_{n=1}^{N} \sigma_n(t,T)[\Phi_n(t) - \phi_n(t)] = \begin{cases} \psi(T)e^{-\lambda(T)t}, \text{ if } t \le \tau \\ \psi(T)(1 - e^{\lambda(T)\tau})e^{-\lambda(T)t}, \ \text{ if } t > \tau \end{cases}$$

In the special case of a one-factor model, we have

$$\Phi(t) - \phi(t) = \begin{cases} \frac{\psi(T)e^{-\lambda(T)t}}{\sigma(t,T)}, \text{ if } t \le \tau \\ \frac{\psi(T)(1-e^{\lambda(T)\tau})e^{-\lambda(T)t}}{\sigma(t,T)}, \ \text{ if } t > \tau \end{cases}$$

■

8 References

Adrian, T., R. Crump, and E. Moench, 2012, "Pricing the Term Structure with Linear Regression," working paper, Federal Reserve Bank of New York.

Ang, A., and M. Piazzesi, 2003, “A No-Arbitrage Vector Autoregression of Term Structure Dynamics with Macroeconomics and Latent Variables,” Journal of Monetary Economics, 50, 745-787.

Babbs, S. and K. B. Nowman, 1999, "Kalman Filtering of Generalized Vasicek Term Structure Models," Journal of Financial and Quantitative Analysis, 34(1), 115-130.

Bank, P. and D. Baum, 2004, "Hedging and Portfolio Optimization in Financial Markets with a Large Trader," Mathematical Finance, 14, 1 - 18.

Bernanke, B., and V. Reinhart, 2004, “Conducting Monetary Policy at Very Low Short-Term Interest Rates,” American Economic Review, 94(2), 85-90.

Bernanke, B., V. Reinhart, and B. Sack, 2004, “Monetary Policy Alternatives at the Zero Bound: An Empirical Assessment,” Brookings Papers on Economic Activity, (2), 1-78.

Bolder, D., 2001, "Affine Term-Structure Models: Theory and Implementation," Bank of Canada Working Paper.

Brigo, D., and F. Mercurio, 2006, "Interest Rate Models - Theory and Practice: With Smile, Inflation and Credit," Springer Finance, 2nd Edition.

D'Amico, S., and T. King, 2012, "Flow and Stock Effects of Large-Scale Treasury Purchases: Evidence on the Importance of Local Supply," Finance and Economics Discussion Series 2012-44. Washington: Board of Governors of the Federal Reserve System.

Fuster, A., and P. Willen, 2010, “$1.25 Trillion is Still Real Money: Some Facts About the Effects of the Federal Reserve's Mortgage Market Investments,” Working paper, Federal Reserve Bank of Boston.

Gagnon, J., M. Raskin, J. Remache, and B. Sack, 2010, "Large-Scale Asset Purchases by the Federal Reserve: Did They Work?" Federal Reserve Bank of New York Staff Report no. 441.

Gürkaynak, R., B. Sack, and J. Wright, 2007, "The U.S. Treasury Yield Curve: 1961 to the Present," Journal of Monetary Economics, 54, 2291-2304.

Hamilton, J., and J. Wu, 2012, "The Effectiveness of Alternative Monetary Policy Tools in a Zero Lower Bound Environment," Journal of Money, Credit, and Banking, 44, 3-46.

Heath, D., R. Jarrow, and A. Morton, 1992, "Bond Pricing and the Term Structure of Interest Rates: A New Methodology for Contingent Claims Valuation," Econometrica, 60, 77-105.

Jarrow, R., 1992, "Market Manipulation, Bubbles, Corners, and Short-Squeezes," Journal of Financial and Quantitative Analysis, 27, 311-336.

Jarrow, R., 2009, "The Term Structure of Interest Rates," Annual Review of Financial Economics, 1, 69 - 96.

Jarrow, R., P. Protter, A. Roch, 2011, "A Liquidity Based Model for Asset Price Bubbles," Quantitative Finance, online November.

Jeanblanc, M., M. Yor and M. Chesney, 2009, "Mathematical Methods for Financial Markets," Springer Finance.

Joyce, M., A. Lasaosa, I. Stevens and M. Tong., 2010. "The Financial Market Impact of Quantitative Easing," Bank of England Working Paper 393.

Krishnamurthy, A., and A. Vissing-Jorgensen, 2011, "The Effects of Quantitative Easing on Interest Rates: Channels and Implications for Policy," Working Paper.

Li, C. and M. Wei, 2012, "Term Structure Modelling with Supply Factors and the Federall Reserve's Large Scale Asset Purchase Programs," working paper, Federal Reserve Board of Governors, Division of Moneetary Affairs, Washington, D. C.

Meaning, J. and F. Zhu, 2011, "The Impact of Recent Central Bank Asset Pruchase Programmes," BIS Quarterly Review, December, 73 - 83.

Oda, N., and K. Ueda, 2005, "The Effects of the Bank of Japan's Zero Interest Rate Commitment and Quantitative Monetary Easing on the Yield Curve: A Macro-Finance Approach," Bank of Japan Working Paper No. 05-E-6.

Piazzesi, M., 2005, "Bond Yields and the Federal Reserve," Journal of Political Economy, 113(2), 311-344.

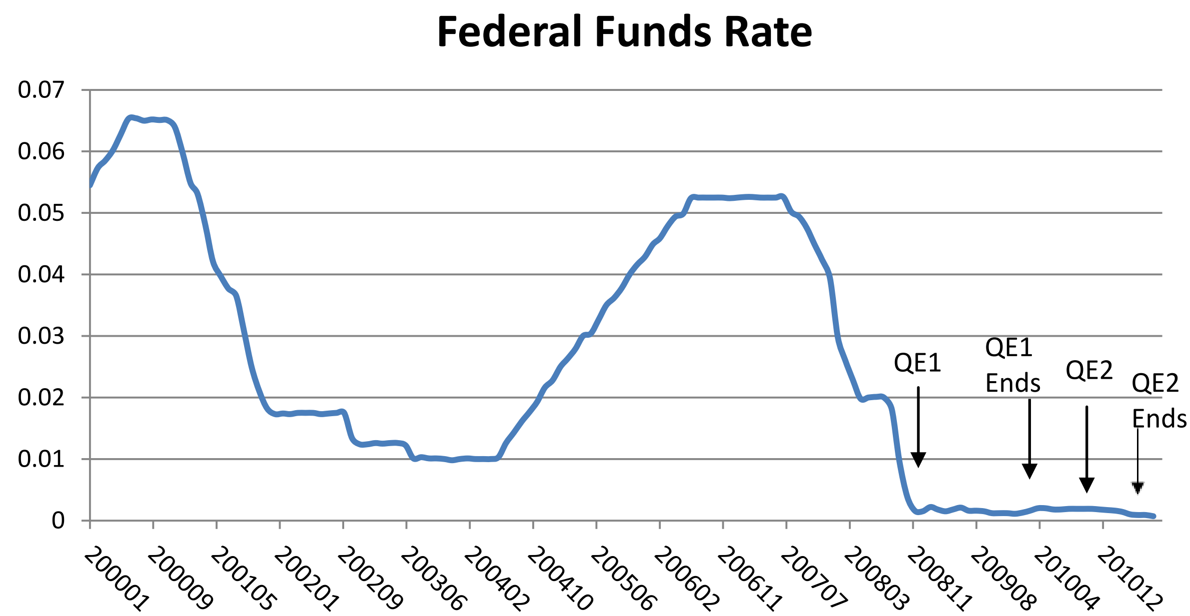
Svensson, L., 1994, "Estimating and Interpreting Forward Rates: Sweden 1992-4," NBER Working Paper.

Swanson, E., 2011, "Let's Twist Again: A High-Frequency Event-Study Analysis of Operation Twist and Its Implications for QE2," Brookings Papers on Economic Activity.

Vasicek, O., 1977, "An Equilibrium Characterization of the Term Structure," Journal of Financial Economics, 5, 177-188.

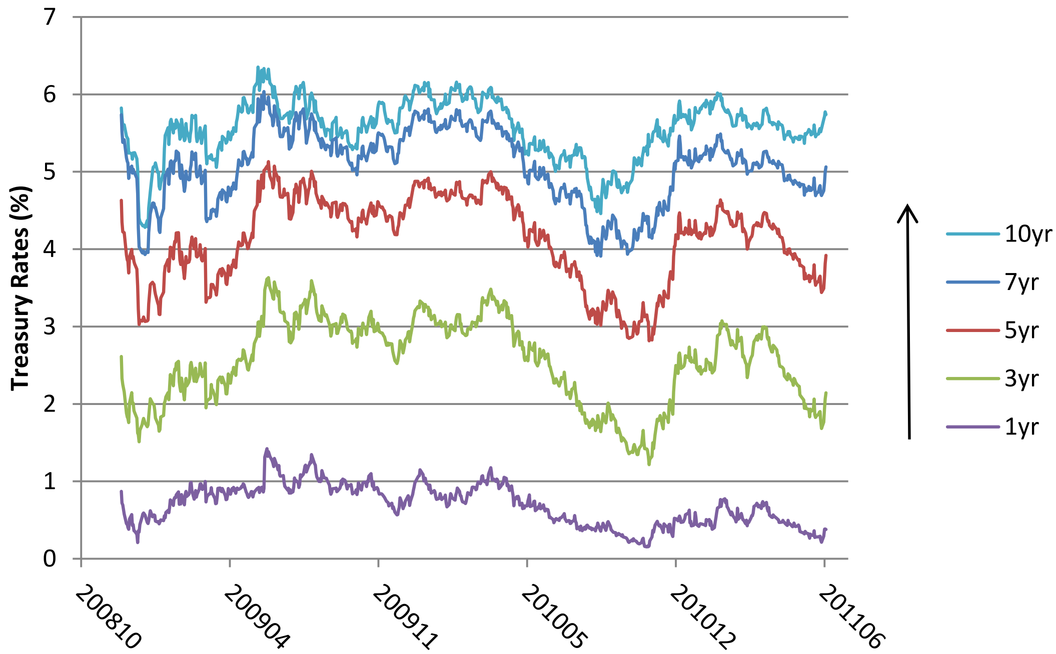
Wright, J., 2012, "What does Mnetary Policy do to Long-term Interest Rates at the Zero Lower Bound?," working paper, Johns Hopkins University.

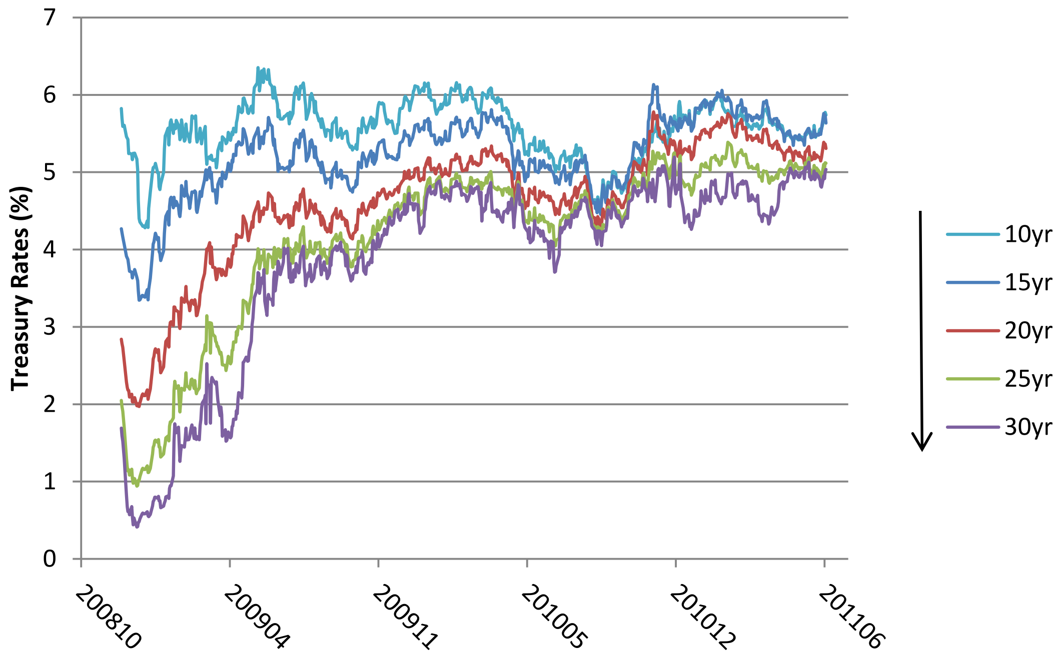
Figure 1. Time-series of Federal Funds Rate



This graph shows the time-evolution of the federal funds rate.

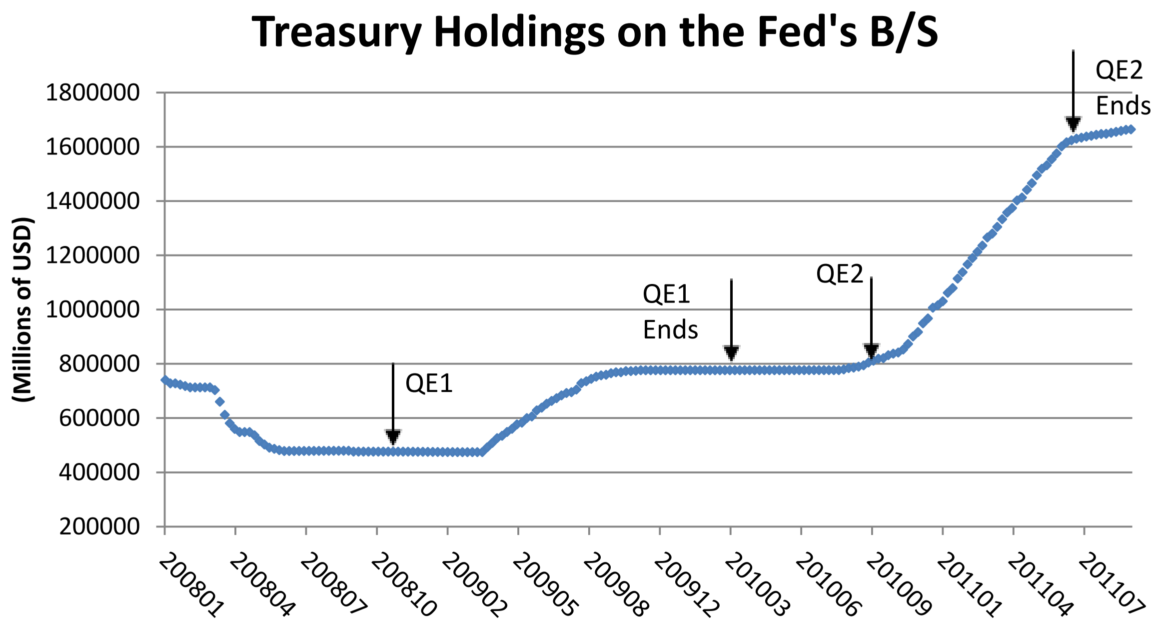
Figure 2. Time-series of GSW Forward Rates

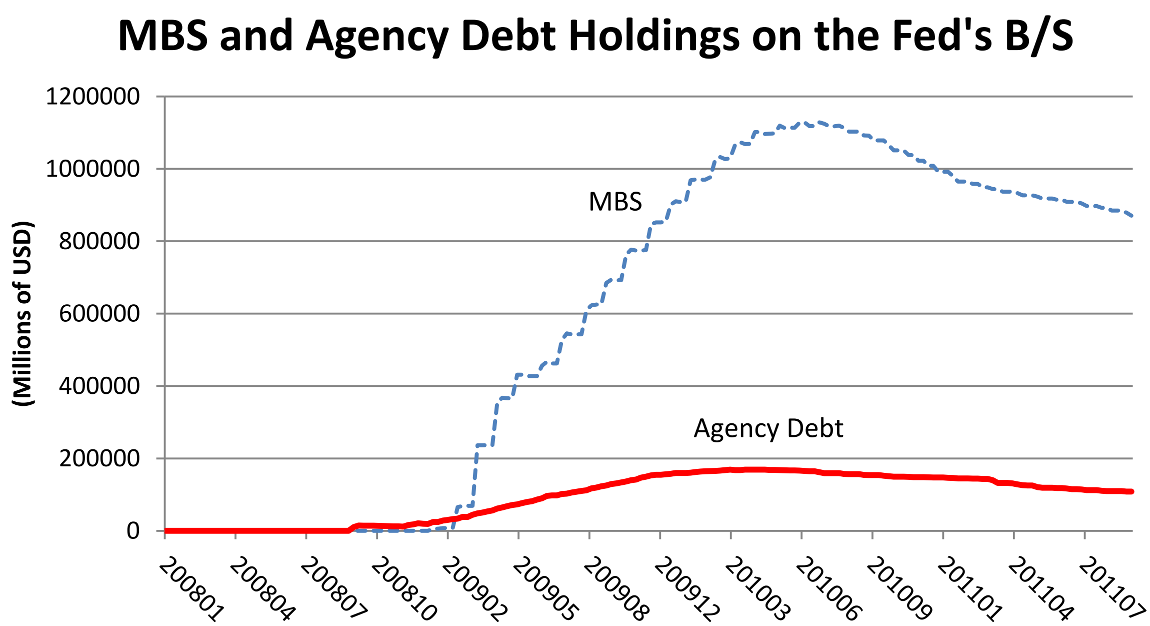




Time-evolution of instantaneous forward rates constructed by Gürkaynak, Sack, and Wright (GSW(2006)). Sample period: November 24, 2008 to June 30, 2011.

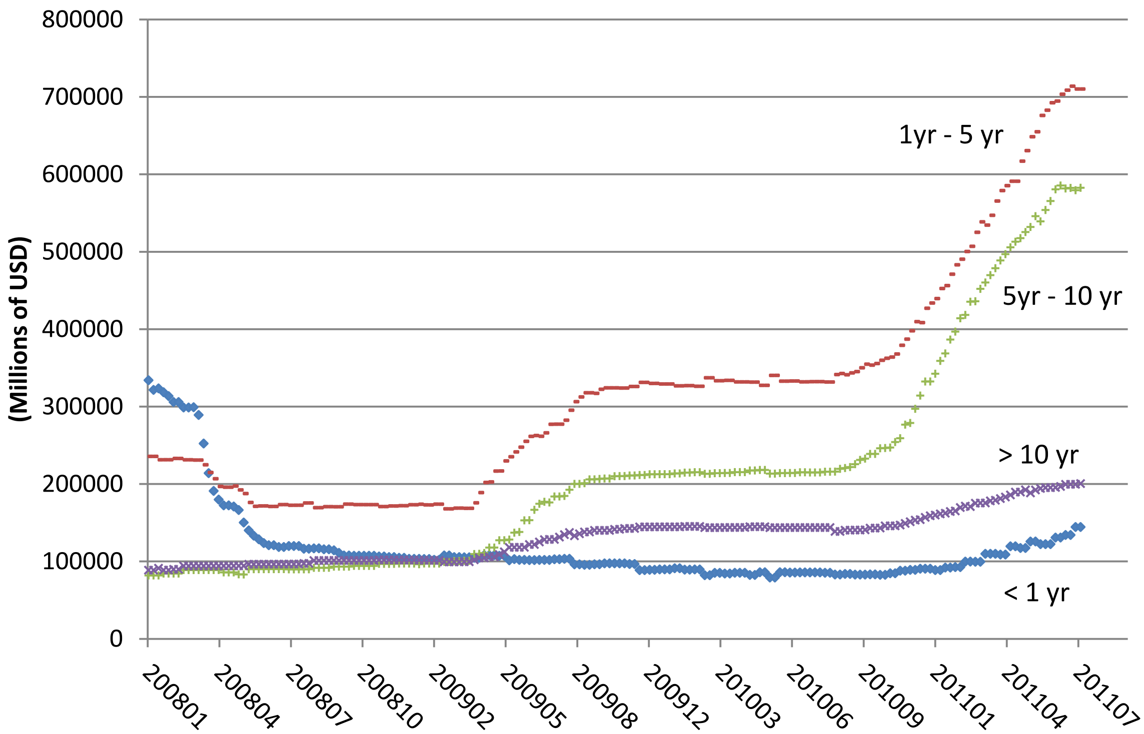
Figure 3. Federal Reserve's Holdings of Treasuries, MBS and Agency Debt





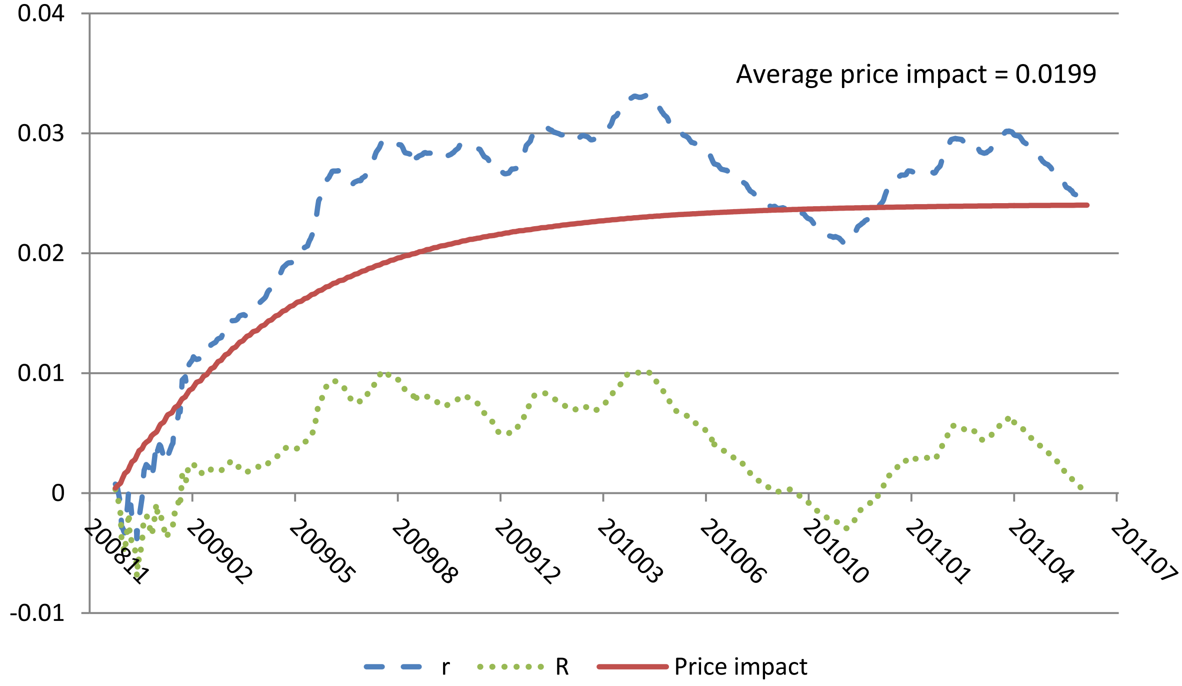
Amount of Treasury securities, mortgage backed securities (MBS) and agency debt held on the Federal Reserve's balance sheet. Data source: http://www.federalreserve.gov/econresdata/

Figure 4. Breakdown of Treasury Holdings by Maturity



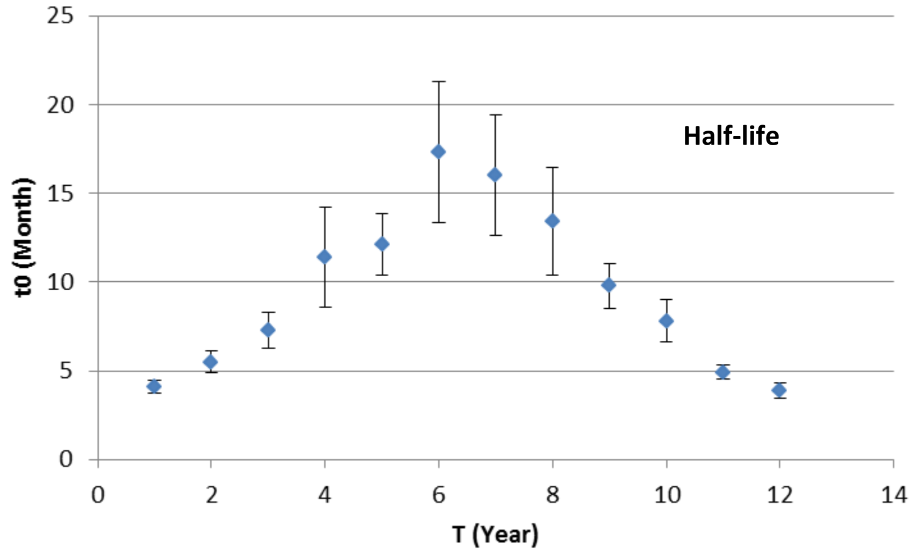
This figure shows the amount of Treasury securities of different maturities held on the Fed's balance sheet. Data source: http://www.federalreserve.gov/econresdata/

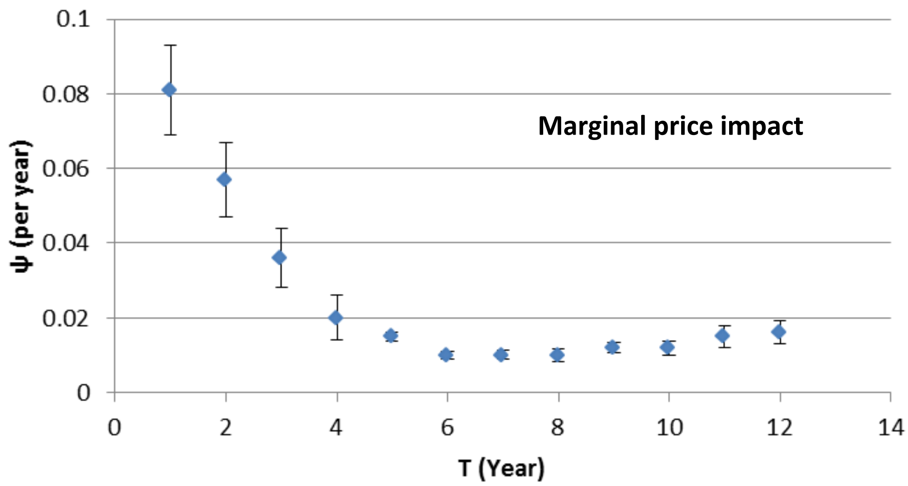
Figure 5. One-factor Estimation of the Instantaneous Short Rate



This figure presents the instantaneous short rate evolution estimated from the one-factor model presented in Section 3. The dashed curve r is the hypothetical short rate estimated by the model. The solid line is the price impact computed with $\lambda = 2.20$ and $\psi = 0.053$ (See Table 2). The dotted curve R is obtained by subtracting the price impact from r. The input data are the 1-year, 2-year, 3-year GSW forward rates spanning from November 24, 2008 to June 30, 2011.

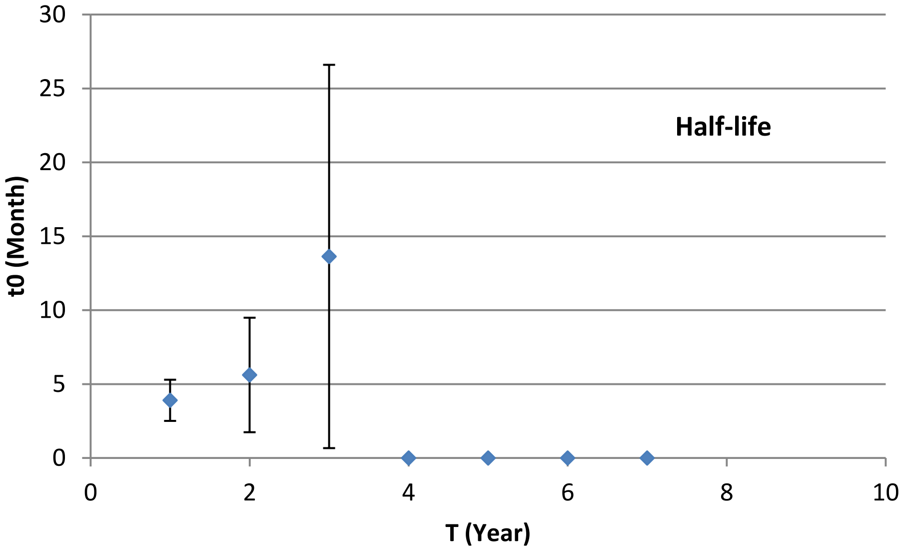
Figure 6. Term Structure of the QE Impact

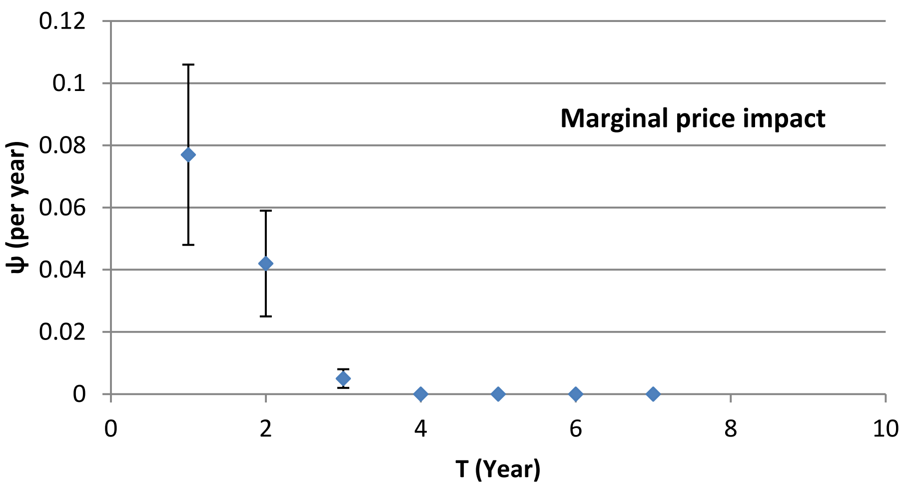




This figure presents the QE impacts on forward rates of different maturities. The top panel shows the duration of the impact and the bottom panel shows the magnitude of the marginal impact. The parameters are estimated by the three-factor affine model. The input data are GSW forward rates spanning from November 24, 2008 to June 30, 2011.

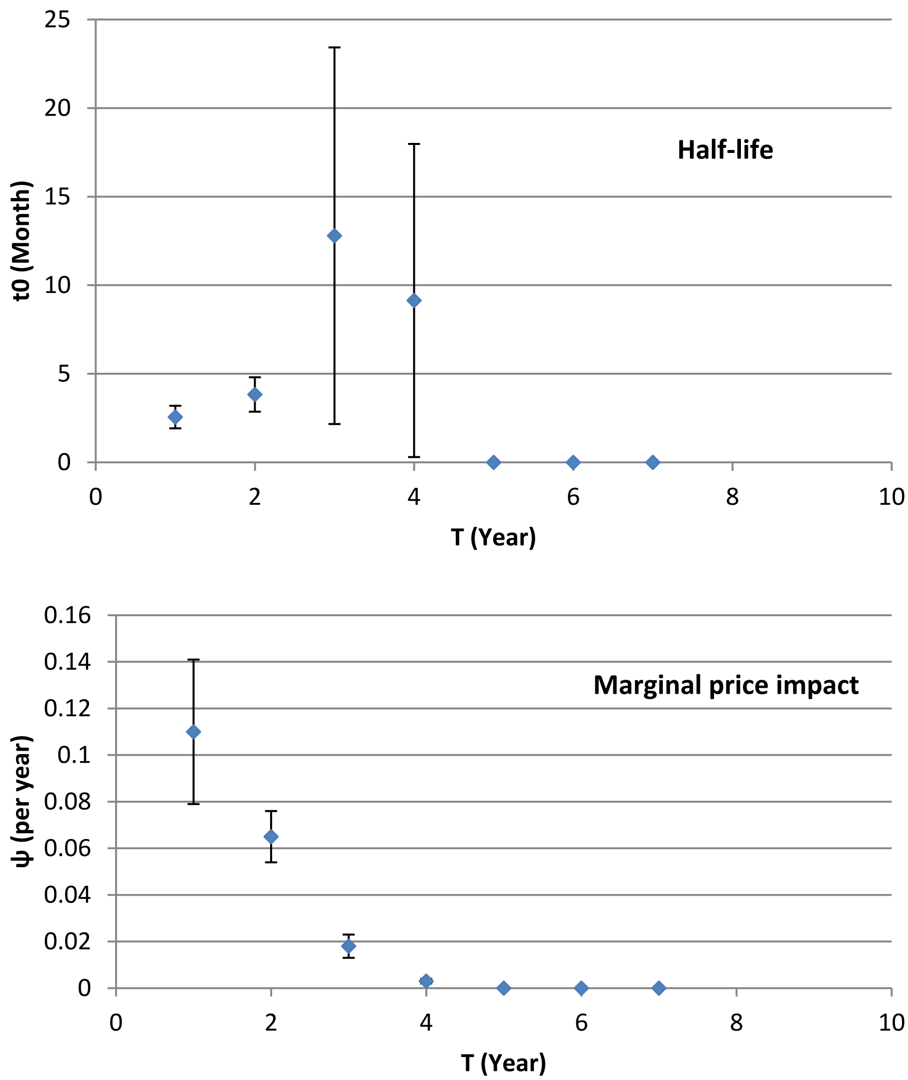
Figure 7. Impact of QE1





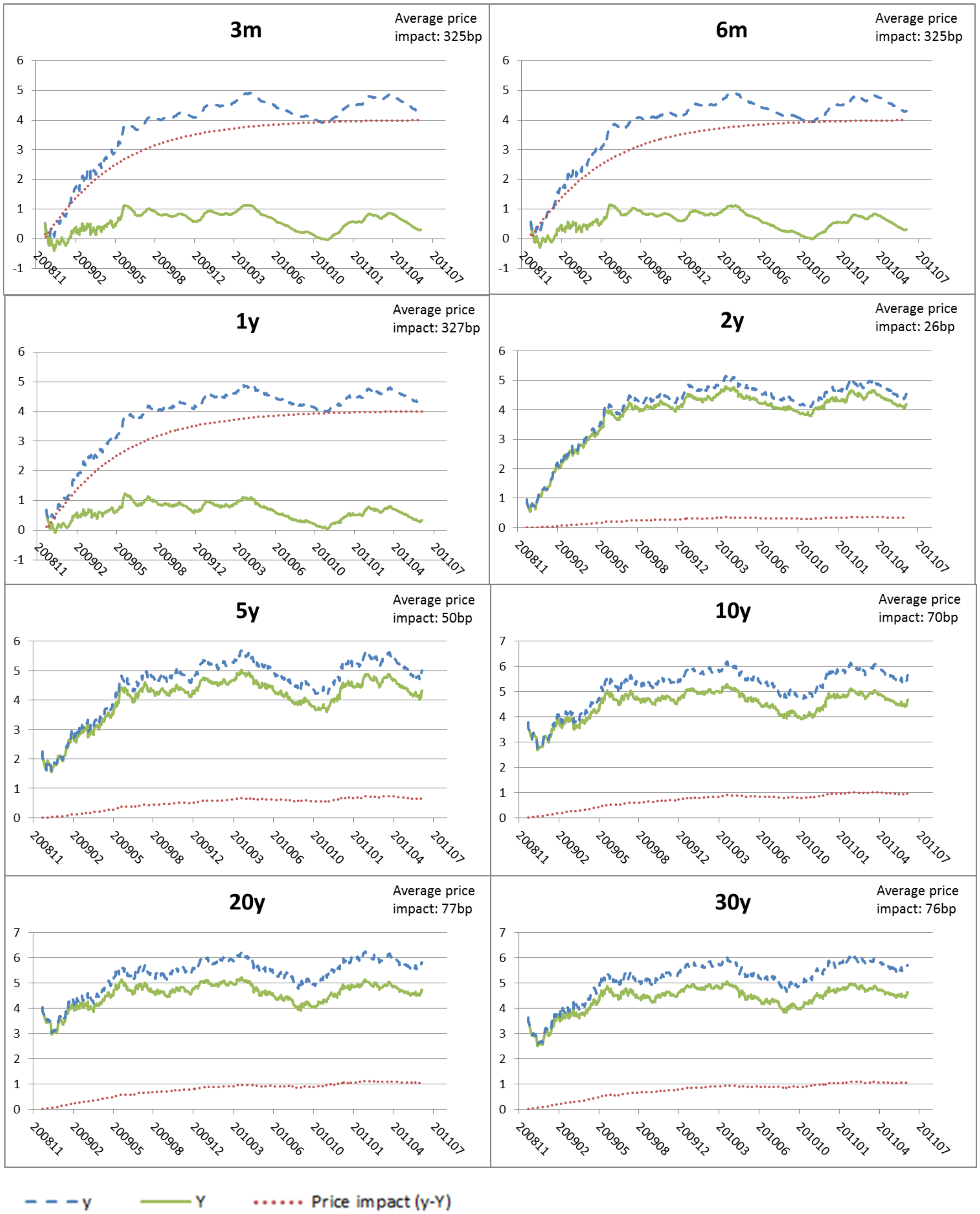
This figure presents the Fed's impacts on forward rates of different maturities. The top panel shows the duration of the impact and the bottom panel shows the magnitude of the marginal impact. The parameters are estimated by the one-factor affine model. The input data are GSW forward rates spanning from November 24, 2008 to March 31, 2010.

Figure 8. Impact of QE2



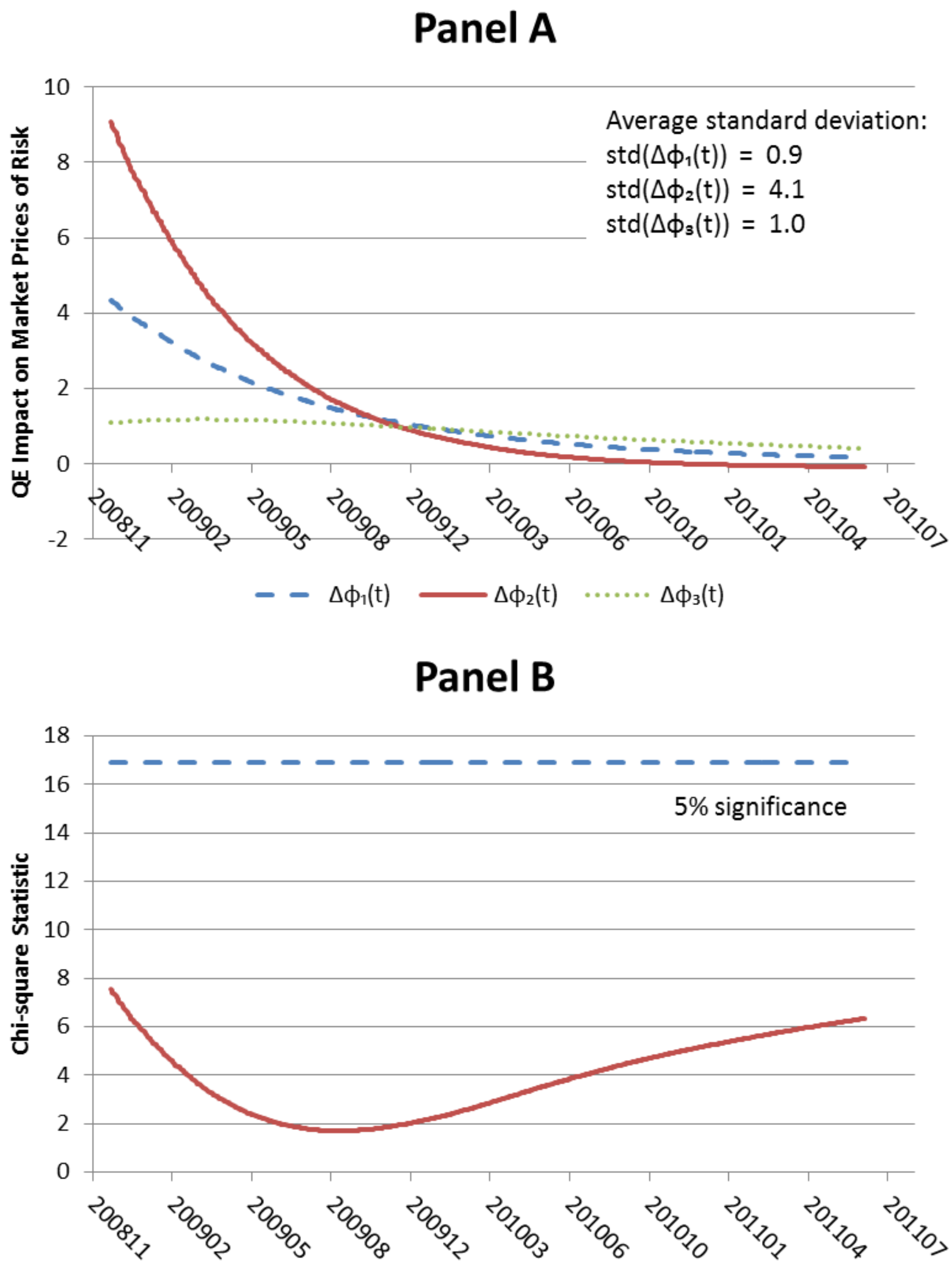
This figure presents the Fed's impacts on forward rates of different maturities. The top panel shows the duration of the impact and the bottom panel shows the magnitude of the marginal impact. The parameters are estimated by the one-factor affine model. The input data are GSW forward rates spanning from November 2, 2010 to June 30, 2011.

Figure 9. QE Impacts on Bond Yields



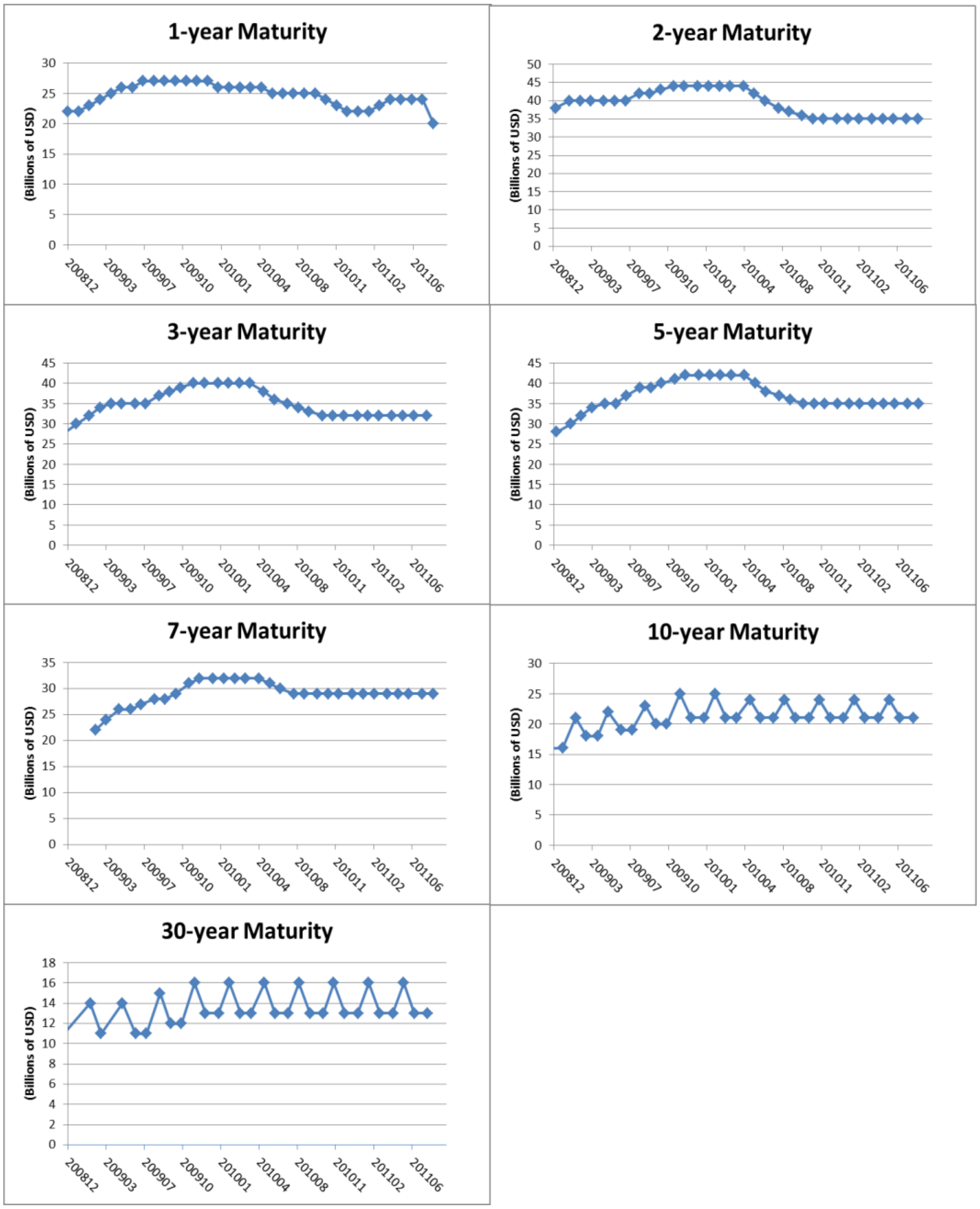
This figure presents the combined impacts of QE1 and QE2 on the yields of 3-month, 6-month, 1-year zero-coupon bonds and par valued coupon bonds with 2-year, 5-year, 10-year, 20-year, 30-year maturities. The unit of vertical axes is percent per year. The average impact (dotted curve) is listed on the top right corner of each panel. The values are estimated by the three-factor affine model.

Figure 10. Test for Arbitrage



Panel A plots time-series evolution of the estimated QE impact on market prices of risk along with the average standard deviations. Panel B shows the related χ^2 statistic and the 5% significance line. The values are estimated by the three-factor affine model.

Figure 11. Amount of Newly Auctioned Treasury Securities



This figure shows the time-evolution of the newly auctioned Treasury securities for various maturities.
Data source: http://www.treasurydirect.gov/instit/annceresult/press/preanre/preanre.htm

Table 1. Summary Statistics

Maturity (year)	**Mean (%)**	**Std**	**Skewness**	**Kurtosis**
1	0.6925	0.2651	0.1080	2.2363
2	1.5845	0.4574	-0.1564	2.2357
3	2.5465	0.5495	-0.3282	2.1799
4	3.4148	0.5733	-0.4328	2.2141
5	4.1280	0.5566	-0.4907	2.3009
6	4.6753	0.5206	-0.5112	2.4173
7	5.0688	0.4783	-0.5192	2.5881
8	5.3303	0.4379	-0.5528	2.8596
9	5.4838	0.4046	-0.6407	3.2392
10	5.5523	0.3831	-0.7762	3.6518
11	5.5559	0.3775	-0.9166	3.9903
12	5.5119	0.3902	-1.0248	4.2518
13	5.4342	0.4203	-1.1017	4.5078
14	5.3340	0.4645	-1.1650	4.7551
15	5.2200	0.5187	-1.2232	4.9368
20	4.6211	0.8293	-1.4136	4.7804
25	4.1592	1.0822	-1.4696	4.2606
30	3.8604	1.2457	-1.4651	3.9361

This table presents the summary statistics of the daily GSW forward rates spanning from November 24, 2008 to June 30, 2011.

Table 2. One-Factor Parameter Estimates

Parameter	Estimate	Std
θ	**0.037**	0.003
k	**0.234**	0.017
σ_r	**0.013**	0.001
λ_1	**2.20**	0.29
λ_2	**1.28**	0.21
λ_3	0.0001	0.13
ψ_1	**0.053**	0.008
ψ_2	**0.023**	0.004
ψ_3	**0.0037**	0.001

This table lists the parameter estimates of the one-factor affine model presented in Section 3. The input data are the 1-year, 2-year and 3-year GSW forward rates spanning from November 24, 2008 to June 30, 2011. Bold indicates significance at the 5% level.

Table 3. Two-Factor Parameter Estimates

Parameter	Estimate	Std
θ_1	0.002	1.647
θ_2	0.058	1.646
k_1	**0.145**	0.009
k_2	0.479	0.046
σ_1	**0.0121**	0.0005
σ_2	0.0001	0.001
λ_1	**2.03**	0.22
λ_2	**1.53**	0.21
λ_3	**0.91**	0.18
λ_4	0.16	0.21
ψ_1	**0.061**	0.009
ψ_2	**0.039**	0.007
ψ_3	**0.018**	0.004
ψ_4	**0.006**	0.002

This table lists the parameter estimates of the two-factor affine model presented in Section 3. The input data are the 1-year, 2-year, 3-year and 4-year GSW forward rates spanning from November 24, 2008 to June 30, 2011. Bold indicates significance at the 5% level.

Table 4. Three-factor Parameter Estimates

	With Impact		**Without Impact**	
Parameter	**Estimate**	**Std**	**Estimate**	**Std**
θ_1	0.003	1.30	0.025	0.803
θ_2	0.039	1.44	0.046	0.818
θ_3	0.031	0.95	0.018	0.797
k_1	**0.104**	0.008	**0.134**	0.006
k_2	**0.446**	0.072	0.134	0.108
k_3	0.104	0.672	**0.588**	0.071
σ_1	**0.0106**	0.0005	**0.0107**	0.0003
σ_2	**0.0039**	0.0007	0.0008	0.0009
σ_3	0.0002	0.0012	0.0001	0.0011
λ_1	**2.01**	0.18		
λ_2	**1.51**	0.17		
λ_3	**1.14**	0.16		
λ_4	**0.73**	0.18		
ψ_1	**0.081**	0.012		
ψ_2	**0.057**	0.010		
ψ_3	**0.036**	0.008		
ψ_4	**0.020**	0.006		
lnL	11171		11065	

This table lists the parameter estimates of the three-factor affine model presented in Section 3. The input data are the 1-year, 2-year, 3-year and 4-year GSW forward rates spanning from November 24, 2008 to June 30, 2011. Bold indicates significance at the 5% level. The bottom row shows the maximized log-likelihood value.

Table 5. Term Structure of the Fed's Impact

Parameter	Estimate	Std	Half-life (months)	Parameter	Estimate	Std
λ_1	**2.01**	0.18	4.1	ψ_1	**0.081**	0.012
λ_2	**1.51**	0.17	5.5	ψ_2	**0.057**	0.010
λ_3	**1.14**	0.16	7.3	ψ_3	**0.036**	0.008
λ_4	**0.73**	0.18	11.4	ψ_4	**0.020**	0.006
λ_5	**0.69**	0.10	12.1	ψ_5	**0.015**	0.0013
λ_6	**0.48**	0.11	17.3	ψ_6	**0.010**	0.0011
λ_7	**0.52**	0.11	16.0	ψ_7	**0.010**	0.0012
λ_8	**0.62**	0.14	13.4	ψ_8	**0.010**	0.0017
λ_9	**0.85**	0.11	9.8	ψ_9	**0.012**	0.0015
λ_{10}	**1.06**	0.16	7.8	ψ_{10}	**0.012**	0.0019
λ_{11}	**1.70**	0.14	4.9	ψ_{11}	**0.015**	0.0030
λ_{12}	**2.14**	0.23	3.9	ψ_{12}	**0.016**	0.0031
λ_{13}	2.59	2.91	-	ψ_{13}	0.007	0.009
λ_{14}	3.16	3.87	-	ψ_{14}	0.008	0.017

This table lists the Fed's impacts on forward rates of different maturities. The left panel shows the duration of the impact and the right panel shows the magnitude of the marginal impact. Half-life is defined as 12*ln(2)/λ. The parameters are estimated by the three-factor affine model. The input data are GSW forward rates spanning from November 24, 2008 to June 30, 2011. Bold indicates significance at the 5% level.

Table 6. Term Structure of the Fed's Impact

Panel A: QE1 Period

Parameter	Estimate	Std	Half-life (months)	Parameter	Estimate	Std
λ_1	**2.13**	0.76	3.9	ψ_1	**0.077**	0.029
λ_2	1.48	1.02	5.6	ψ_2	**0.042**	0.017
λ_3	0.61	0.58	13.6	ψ_3	0.005	0.003
λ_4	Large	-	0	ψ_4	0	-
λ_5	Large	-	0	ψ_5	0	-
λ_6	Large	-	0	ψ_6	0	-
λ_7	Large	-	0	ψ_7	0	-

Panel B: QE2 Period

Parameter	Estimate	Std	Half-life (months)	Parameter	Estimate	Std
λ_1	**3.25**	0.81	2.6	ψ_1	**0.11**	0.031
λ_2	**2.17**	0.55	3.8	ψ_2	**0.065**	0.011
λ_3	0.65	0.54	12.8	ψ_3	**0.018**	0.005
λ_4	0.91	0.88	9.5	ψ_4	**0.003**	0.001
λ_5	Large	-	0	ψ_5	0	-
λ_6	Large	-	0	ψ_6	0	-
λ_7	Large	-	0	ψ_7	0	-

This table lists the Fed's impacts on forward rates of different maturities. The left panel shows the duration of the impact and the right panel shows the magnitude of the marginal impact. Half-life is defined as $12*\ln(2)/\lambda$. The parameters are estimated by the one-factor affine model. The input data for Panel A are GSW forward rates spanning from November 24, 2008 to March 31, 2010 (QE1 period). The input data for Panel B range from November 2, 2010 to June 30, 2011 (QE2 period). Bold indicates significance at the 5% level.

Table 7. Robustness Check

Jan. 2, 2001 – Aug. 1, 2003

	With Impact		**Without Impact**	
Parameter	**Estimate**	**Std**	**Estimate**	**Std**
θ_1	0.021	0.82	0.023	0.82
θ_2	0.020	0.82	0.029	0.82
θ_3	0.022	0.82	0.024	0.82
k_1	0.59	0.31	0.26	0.15
k_2	**0.28**	0.06	**0.25**	0.02
k_3	**0.28**	0.12	**0.25**	0.03
σ_1	0.0001	0.015	0.005	0.009
σ_2	**0.032**	0.009	**0.026**	0.005
σ_3	**0.016**	0.007	**0.021**	0.005
λ_1	**1.35**	0.62		
λ_2	0.75	0.64		
λ_3	0.35	0.71		
λ_4	0.00	0.90		
ψ_1	**0.071**	0.023		
ψ_2	**0.035**	0.015		
ψ_3	0.017	0.011		
ψ_4	0.008	0.008		
lnL	9563		9508	

This table lists the parameter estimates of the three-factor affine model presented in Section 3. The input data are the 1-year, 2-year, 3-year and 4-year GSW forward rates spanning from January 2, 2001 to August 1, 2003. Bold indicates significance at the 5% level. The bottom row shows the maximized log-likelihood value.

Table 8. Robustness Check

Jan. 2, 2004 – Aug. 1, 2006

	With Impact		**Without Impact**	
Parameter	**Estimate**	**Std**	**Estimate**	**Std**
θ_1	0.006	0.94	0.020	0.82
θ_2	0.071	1.25	0.022	0.82
θ_3	0.009	0.95	0.018	0.82
k_1	**0.45**	0.08	**0.98**	0.06
k_2	**0.21**	0.02	**0.20**	0.03
k_3	**0.21**	0.04	**0.20**	0.03
σ_1	0.0001	0.005	**0.069**	0.007
σ_2	**0.019**	0.003	**0.015**	0.002
σ_3	**0.012**	0.005	**0.013**	0.003
λ_1	0.87	0.38		
λ_2	**1.59**	0.31		
λ_3	**1.11**	0.21		
λ_4	**1.20**	0.16		
ψ_1	-0.010	0.007		
ψ_2	0.0006	0.007		
ψ_3	0.010	0.005		
ψ_4	**0.018**	0.004		
lnL	10738		10698	

This table lists the parameter estimates of the three-factor affine model presented in Section 3. The input data are the 1-year, 2-year, 3-year and 4-year GSW forward rates spanning from January 2, 2004 to August 1, 2006. Bold indicates significance at the 5% level. The bottom row shows the maximized log-likelihood value.

Table 9. Summary Statistics of Pricing Errors

Panel A (With the price impact term)

Maturity (year)	Mean (bp)	SE	t	Skew	Kurt	ρ(1)	ρ(10)	ρ(20)
1	-2.10	0.75	-2.80	0.14	2.32	0.93	0.75	0.60
2	-3.97	0.73	-5.45	0.34	2.62	0.88	0.52	0.25
3	-2.30	1.07	-2.16	-0.24	2.61	0.94	0.71	0.50
4	-1.86	1.29	-1.44	-0.53	2.90	0.95	0.76	0.56
5	-1.59	1.44	-1.10	-0.55	2.88	0.97	0.76	0.56
6	0.49	1.47	0.34	-0.51	3.01	0.97	0.74	0.55
7	1.43	1.37	1.04	-0.32	3.19	0.96	0.71	0.50
8	1.29	1.31	0.98	-0.37	3.11	0.96	0.68	0.47
9	3.40	1.24	2.74	-0.47	3.02	0.96	0.67	0.45
10	1.01	1.23	0.82	-0.58	3.13	0.96	0.68	0.46
11	0.37	1.14	0.32	-0.57	2.92	0.96	0.66	0.43
12	-1.08	1.24	-0.87	-0.56	3.12	0.96	0.71	0.49
13	0.64	1.25	0.51	-0.51	2.71	0.96	0.71	0.51
14	-1.36	1.27	-1.07	-0.31	2.45	0.97	0.73	0.54

Panel B (Without the price impact term)

Maturity (year)	Mean (bp)	SE	t	Skew	Kurt	ρ(1)	ρ(10)	ρ(20)
1	-3.84	0.71	-5.43	0.30	2.29	0.93	0.69	0.50
2	-15.08	0.83	-18.16	0.38	3.07	0.91	0.56	0.28
3	-6.14	1.28	-4.79	-0.13	2.31	0.96	0.76	0.56
4	4.84	1.50	3.24	-0.35	2.28	0.97	0.80	0.63
5	2.58	1.52	1.70	-0.43	2.41	0.97	0.79	0.62
6	-0.36	1.47	-0.24	-0.45	2.59	0.97	0.76	0.59
7	-1.96	1.40	-1.40	-0.45	2.81	0.97	0.73	0.55
8	-3.92	1.35	-2.91	-0.50	3.09	0.97	0.72	0.52
9	-3.00	1.32	-2.28	-0.60	3.40	0.97	0.71	0.51
10	-1.41	1.33	-1.06	-0.73	3.66	0.97	0.72	0.51
11	0.60	1.26	0.47	-0.66	3.24	0.96	0.73	0.53
12	-1.62	1.47	-1.10	-0.94	4.10	0.97	0.78	0.59
13	-0.89	1.61	-0.55	-1.03	4.37	0.98	0.82	0.66
14	-1.06	1.76	-0.60	-1.11	4.62	0.98	0.85	0.71

Panel A presents the statistical properties of the pricing errors implied by the affine model including the Fed's impact term. The input data is the daily GSW forward rates spanning from November 24, 2008 to June 30, 2011. The columns from left to right refer to the maturity of the forward rates, sample mean, standard error of the mean, t-ratio, skewness, and kurtosis of the pricing errors; ρ(1), ρ(10), ρ(20) denote the autocorrelation coefficients of order 1, 10, and 20. Panel B provides results estimated from the conventional model without the Fed's impact term.

Table 10. Comparison with Other Papers' Results

Panel A: Other Papers' Results

Paper	**Event**	**Methodology**	**Treasury Yield Changes (bp)**					
			1yr	**2yr**	**3yr**	**5yr**	**10yr**	**30yr**
Gagnon, Raskin, Remache, and Sack (2010)	QE1	Event study (Cumulative response)		-34			-91	
		Time-series regression					-52	
Krishnamurthy and Vissing-Jorgensen (2011)	QE1	Event study (Cumulative response)	-25		-39	-74	-107	-73
	QE2	Event study (Cumulative response)	-2		-8	-20	-30	-21
D'Amico and King (2012)	QE1	Stock effect					-30	
		Flow effect	-3.5bp on the sector purchased					
Li and Wei (2012)	QE1&2	Time-series estimation					-100	
Meaning and Zhu (2011)	QE2	Panel regression	-21bp on the whole yield curve on average					

Panel B: Our Results

Event	**Treasury Yield Changes (bp)**				
	1yr	**2yr**	**5yr**	**10yr**	**30yr**
QE1 and QE2	-327	-26	-50	-70	-76

This table compares our results with other papers that examine the effects of QE on Treasury yields. The numbers in Panel B are also shown in Figure 9.