# Optimal Patronage*

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**Abstract**

We study the design of promotions in an organization where agents belong to groups that advance their cause. Examples and applications include political groups, ethnicities, agents motivated by the work in the public sector and corruption. In an overlapping generations model, juniors compete for promotions. Seniors have two kinds of discretion: direct discretion which allows an immediate advancement of their cause and promotion discretion ("patronage") which allows a biasing of the promotion decision in favour of the juniors from their group. We consider two possible goals of the principal, maximizing juniors' efforts and affecting the steady-state composition of the senior level towards the preferred group, and show that patronage may be strictly positive in both of them. We also apply the second setting to the case of corruption.

*Keywords: motivated agents, contest, promotion, patronage, bureaucracy, corruption*

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# 1 Introduction

This paper is based on a simple observation that people belong to different groups and they care about the group to which they belong.[1] The group identity can be exogenous as in the case of ethnicities, tribes and castes. It may also be endogenous, as in the case of values, for example, political orientation and religions.

The main question of this paper is the following. What implications arise for the organizational design when agents belong to and care about their groups? In particular, can we rationally explain some seemingly welfare detrimental phenomena such as patronage? By patronage we mean unfair promotions for which the group identity is taken into account rather than only ability or performance. The main result of the paper is that, even if the goals of organization are group-neutral, for example, to maximize the efforts or output of the workers, allowing for some patronage might be optimal. We also study the effectiveness of patronage when one group is preferred to the other in which case the composition of the organization matters.

While patronage occurs in private firms too, we mainly have in mind the design of bureaucracies where agents from different groups inevitably work together and where patronage provokes most public outcry. Indeed, the governments usually formally and explicitly do not allow for discrimination, while in reality this is not the case in many countries, especially developing countries.

We build an overlapping generations model in which agents live for two periods. When young, agents work in the organization at junior level. Some will be promoted to senior level and work there when old. Promotions are based on the contest between junior agents, but this contest may be biased. The organizational designer, who we refer to as the planner, may give the senior agents the ability to bias the contest in favour of the juniors they prefer. When this happens, we say that there is *patronage*.

Agents belong to two different groups, and care about the welfare of their group. Senior agents use their discretionary power to contribute to their group welfare in two ways. First, they have *direct discretion*, that is, they can directly increase their group welfare. For example, they can channel public funds towards regions populated by their tribe or they can make public statements and take some decisions that promote their political values. Thus, senior agents prefer to promote juniors of their group because, when they become seniors, they will promote their group. The second kind of discretion is *promotion discretion*, or patronage as described above. Thus, in our

[1] See Burgess, Jedwab, Miguel, Morjaria, and Padró i Miquel (2013), Franck and Rainer (2012), Hodler and Raschky (2014), Iyer and Mani (2012) (for the most recent econometric evidence) and references there.

model patronage is valued only when there is direct discretion.

We consider two possible goals of the planner. First, the planner is group-neutral and his goal is to maximize the efforts of the junior agents either because their efforts are productive or, in the case of training, because their efforts increase their ability when they become seniors. When juniors from the two groups compete for promotion, the identity of the winner matters because the promoted junior, becoming senior, will benefit his group. This is thus a rent-seeking contest for (group) public goods. The attractiveness of the senior position increases with both the direct discretion and patronage.

The trade-off faced by the planner is the following. A higher patronage makes the senior position more attractive and, therefore, increases the juniors' efforts; we call this the *higher stakes effect*. However, a higher patronage means that the contest for promotion is more biased and, since the juniors are symmetric (except for their group identity), this implies a lower effort; this is the *discouragement effect*.

We find that, when direct discretion is neither too large nor too small, the juniors' efforts are maximized for some strictly positive patronage. In other words, even though the planner can make all the promotions merit-based, he chooses to give senior agents the power to bias them as they please. We also show that in general the direct discretion and patronage are not complements nor substitutes, that is, a higher direct discretion has an ambiguous effect on the optimal patronage. The reason is that both the higher stakes and the discouragement effects increase with the direct discretion.

We then turn to the second possible goal of the planner. The planner might prefer one group to another. For example, the planner is a politician who cares about the preferences of the median voter who is likely to belong to the larger group. Alternatively, the direct discretion may be costly for the planner per se in which case he prefers the group which uses it in a less distortionary way. Suppose that the only goal of the planner is to bias the steady-state composition of the senior level towards his preferred group. There are three effects of patronage on the steady-state composition of the senior level: first, it benefits the larger group because it is more likely to use the patronage; second, it benefits the less motivated group since this group is likely to lose the fair contest, and third, it changes the values of promotion for the two groups because they increase with patronage and this effect can go either way. We present an example in which the sign of the third effect depends on the difference of motivations, as the sign of the second effect. Thus, optimal patronage is determined by the size effect and the combined motivation effect. When the planner's preferred group is larger and less motivated, patronage is beneficial through both effects and is set at the maximum level, that is, seniors have full discretion about whom to promote. In the opposite case, when his preferred group is smaller and more motivated, no patronage is optimal. Otherwise, there is a trade-off. We characterize optimal intermediate patronage. Overall, optimal patronage (weakly) increases with the size of the preferred group and decreases with its relative motivation.

We also present an application of this setting to corruption and investigate if patronage could be useful in combatting it. Some agents are corrupt and the planner tries to limit the spreading of corrupt agents in the bureaucracy. In other words, his goal is to minimize the number of corrupt agents at senior level.[2] Allowing for some patronage may then help since the honest seniors use it in order not to promote the corrupt juniors; however, corrupt seniors "sell" the position to corrupt juniors. Even though corrupt agents have no group motivation, the possibility of selling the position creates inter-generational linkage similar to that of altruistic agents. In particular, the value of the position and, therefore, the bribe that is charged for it increase with the power at that position, that is, with patronage. Thus, formally, the model is very similar to the main case of the two groups of altruistic agents. Corrupt agents are motivated by bribes, honest ones are motivated by the desire not to allow corrupt juniors to be promoted; and the optimal patronage depends on the relative size and motivation of the two groups, as described above.

We then study an extension where the planner prefers one group to the other but also cares about the juniors' efforts; we therefore combine the two planner's goals. The patronage plays a double role: it is used to incentivize junior agents and also to affect the composition of the senior level in the direction of the planner's preferred group.

The rest of the paper is organized as follows. We discuss the related literature in the next subsection. The model is introduced in Section 2. In Section 3 the optimal patronage is characterized when the planner cares about juniors' efforts. In Section 4 the planner cares only about the steady-state composition of the senior level. Section 4.1 analyzes the application to corruption. A few extensions are analyzed in Section 5. In Section 5.1 agents have a mixture of warm-glow motivation and pure altruism. Section 5.2 considers the case of antagonistic groups, that is, when group welfare depends negatively on other group favours. In Section 5.3 the planner prefers one group to the other but still cares about juniors' efforts. In section 5.4 the planner can choose monetary incentives. Section 6 concludes. Appendix A contains the proofs. Appendix B considers two alternative contest models that generate similar results.

[2] The composition of the junior level is exogenous as one cannot observe if a person applying for a governmental job will be corrupt.

## 1.1 Related literature

This paper is related to several strands of literature. In Athey, Avery, and Zemsky (2000), Fryer and Loury (2005) and Morgan, Sisak, and Várdy (2012) biasing the contests for promotion is used by the planner to reach some further goals, such as promoting more able agents in the first case, diversity in the second case and attracting talent to the organization in the last case. In other words, the planner affects the composition of the organization in the direction he prefers as in this paper when the planner cares about the composition of the senior level. When the planner biases the contest to give incentives for the agents to work harder, that is, he solves the moral hazard problem. In those papers it is still the planner who administers the biased contest while in our model the senior agents use the biased contest to promote the juniors they like.

Meyer (1992) studies a two-period contest between identical agents. Introducing a small additive bias in a Lazear-Rosen tournament has only a second-order effect on efforts.[3] If it is introduced in the second period to reward the winner of the first period, it has a first-order effect on first-period incentives and, therefore, it is optimal to introduce some bias in the second period. In our terms, the discouragement effect is of the second order while the higher stakes effect is of the first order. We do not rely on this logic since we introduce patronage as the probability that the senior completely decides on promotion, in which case the discouragement effect is always of the first order. In Appendix B we consider a standard setup of a Tullock contest with a multiplicative bias as in Epstein, Mealem, and Nitzan (2011) and Franke, Kanzow, Leininger, and Schwartz (2013) in which the discouragement effect is of the second order. This fact is useful in showing that optimal patronage is positive even when the costs of public funds are low and, therefore, providing monetary incentives is cheap (see Section 5.4).

In Ghatak, Morelli, and Sjöström (2001) credit market imperfections make current borrowers worse off. However, they increase incentives to work hard and self-finance since the rents to self-financed entrepreneurs also increase. Therefore, reduction in credit market imperfections may reduce welfare. Thus, there is the same very general idea that a certain distortion has some current negative effects but also provides more incentives through higher future rents.

Since the group welfare is essentially a (group) public good, the contest for the promotion is similar to the models of rent-seeking for public goods such as Katz, Nitzan, and Rosenberg (1990) and Linster (1993). Unlike the usual contest where

[3] This is a very general result which holds far beyond the Lazear-Rosen tournament and additive bias, see Drugov and Ryvkin (2013) for details.

each participant cares only about winning the contest, here upon losing each participant also cares about the identity of the winner, that is, whether or not he is from the same group.

The agents in our model are pure altruists, in the sense that they care about their group welfare but not how it is achieved. A few papers, such as Francois (2000), Francois (2007) and Engers and Gans (1998), have considered implications of such agents for the organizational design. However, none of these papers is concerned with the promotion policy. In models where agents have public sector motivation, such as Besley and Ghatak (2005), Delfgaauw and Dur (2008), Macchiavello (2008) and Delfgaauw and Dur (2010) agents have a "warm glow" motive, that is, they value their contribution to the welfare irrespectively of what happens if they do not contribute; we can easily incorporate it into our model (it is equivalent to a higher senior wage). We also consider an intermediate case in which the agents discount their effect on the group welfare depending on how far their action is from the eventual increase in their group welfare. This can be seen as a generalization of impure altruism, see Andreoni (2006) for the definitions and discussion.

Prendergast and Topel (1996) consider an agency model where a supervisor intrinsically cares about his junior being promoted and biases his evaluation report to the principal. The model and the questions there are very different from the ones in this paper, but the same broad lesson emerges. While favouritism creates distortions, completely eliminating it might not be optimal since the agents value exercising it. In Prendergast and Topel (1996) they then agree to a lower wage while in our model they work harder.

As one interpretation of the group welfare is the status of its members, this paper is also related to the small literature on the role of status for incentives, including Auriol and Renault (2001), Auriol and Renault (2008) and Dhillon and Nicolò (2012).

The political economy models of Roberts (1999) and Acemoglu, Egorov, and Sonin (2012) have a similar feature that admitting new members to a club or to a ruling coalition (in our case, promoting) will affect everybody, not only through their direct actions but also via changes in future membership since these new members have voting power.

The application to corruption focuses on the selling of positions which seems completely absent from the corruption literature.[4] Also, very few papers consider organizational design with corrupt agents.

From the modelling perspective, using an overlapping generations model to study

[4]See, for example, the two-volume handbook Rose-Ackerman (2006) and Rose-Ackerman and Søreide (2012).

organizations has been used in the past. For example, it is used in Ghatak, Morelli, and Sjöström (2001) described above. In Meyer (1994) the organization decides how to organize teams in order to learn the best about the workers' abilities. In Carrillo (2000) the focus on fighting corruption with different tools (but not patronage).

# 2 Model

This is an overlapping generations model in which each agent lives for two periods. When young, agents work in the organization, that we call bureaucracy, at the junior level. Some of them will be promoted to the senior level and work there when old. The bureaucracy is organized in departments, each consisting of two junior bureaucrats and one senior bureaucrat. Every period the senior bureaucrat retires and one (and only one) junior of his department is promoted to replace him.[5] The senior bureaucrat gets wage $w$ and some discretionary power that we explain below. The junior who is not promoted gets utility normalized to 0.[6]

## 2.1 Types and utilities of agents

There are two groups, left ($l$) and right ($r$), and each agent belongs to one of them. We call the type of agent the group to which he belongs. The probability that a junior is of type $l$ is $\lambda$. The composition of the departments is random, that is, the types of juniors are independent.[7]

The type of agent matters because agents care about the welfare of their group. That is, the agents' utility has two components: the standard "private" part that depends on their wage and effort costs and an "altruistic" part that depends on the welfare of their group.

## 2.2 Seniors' discretion and group welfare

The discretionary power of the senior bureaucrats takes two forms. First, they can directly benefit their group by amount $d$; we call this *direct discretion*. For example, they administer some funds and can disburse them to the members of their group. Alternatively, they can choose to implement public projects in the places or in the

[5] It does not matter if the promoted junior stays in the same department.

[6] He either leaves the bureaucracy or stays there in some low-level position with a wage equal to zero and no discretionary power.

[7] We discuss the preferences of the planner over the composition of the junior level in Section 3.4.

ways that benefit their group. If the group identity is based on ideology rather than ethnicity, senior bureaucrats can effectively promote their values among the general public since they are highly visible. If the senior position confers status, senior bureaucrats benefit their group by increasing the average status of their group members.

The second form of the seniors' discretionary power is the *promotion discretion* or *patronage*. Senior bureaucrats administer the promotion of the juniors in the department and they can bias it in favour of the junior from their group. The size of the promotion discretion is the focus of this paper. Even if it is possible to eliminate all promotion discretion, that is, to make promotion discretion entirely merit-based, the planner may not find it optimal. We formalize promotion discretion in the simplest way: with probability $p$ a senior bureaucrat has complete discretion about which junior from his department to promote, while with probability $1-p$ the promotion is entirely merit-based.[8]

The welfare of each group is equal to the (discounted) sum of the direct discretions exerted by its seniors, $W_i = d\sum_{t=0}^{+\infty}\delta^t N_i^t$, $i = l, r$, where $\delta$ is the discount factor and $N_i^t$ is the number of seniors of group $i$ in period $t$.[9,10] Note that patronage increases the group welfare only indirectly. A group benefits from its juniors being promoted because they will use their direct discretion when senior (and also promote juniors of the group in the future who will benefit the group when senior, etc.).

## 2.3 Promotion contest

When the promotion is merit-based, the two juniors of the department engage in the contest. If a junior exerts an effort, he generates a high output, exerting no effort results in a low output. The junior with a higher output is promoted; in the case of equal outputs each junior is promoted with probability $\frac{1}{2}$. The cost of effort is $\frac{c}{2}$ and

[8] We discuss different ways of biasing the contest at the end of Section 3.3 and analyse two different contest models in Appendix B. Introducing the bias in this way makes it more difficult to obtain a positive optimal patronage as compared to the standard additive or multiplicative handicaps for one of the players.

[9] In some cases the welfare of each group may decrease with the direct discretion used by the seniors of the other group. For example, agents may care about the relative income or status of their group. Promoting your values is harder when other people promote different (or opposite) values. See Section 5.2 for such an extension.

[10] The group welfare does not include the "private" part of the agents' utilities, that is, their wages and effort costs. This is done so that the different interpretations of group welfare (income, values, status) map into exactly the same model. Also, in the case of income, one can assume that the direct discretion $d$ is much larger than the wage $w$ and omitting $w$ (and effort costs) does not greatly affect the results. Modifying the model to include the "private" part into group welfare is straightforward.

juniors differ in the cost parameter, $c \rightsquigarrow F[\underline{c}, \overline{c}]$, and are privately informed about it.

# 3 Maximizing juniors' efforts

In this section the planner maximizes the (expected) output at the junior level and so he chooses the promotion discretion $p$ to maximize the juniors' efforts. Interpreting the model literally, the senior bureaucrats do not exert any effort since they will be retiring afterwards. Alternatively, their effort may be subject to another (unmodelled) moral hazard problem and is independent of the direct discretion and promotion discretion which are the focus of this paper.

We now solve the model and find the optimal patronage. Set $\delta = 1$. While this makes the welfare of both groups infinite, what matters for the decisions of the agents is the impact they make on the group welfare which is always finite. We consider the case of $\delta < 1$ in Section 3.5.

The first step is to solve the promotion contest and we have to distinguish whether or not the two juniors in a department belong to the same group. We call the first case the "homogeneous department" and the second case the "heterogeneous department".

## 3.1 The contest in a homogeneous department

When both juniors belong to the same group, the welfare of their group does not depend on who gets promoted. The value of the promotion for each of them is only the senior's wage $w$. The senior bureaucrat does not use his promotion discretion as he cannot change the group of the promoted junior.

A junior with cost parameter $c$ exerts an effort if and only if

$$\left(\frac{1}{2}F(\widehat{c}) + 1 - F(\widehat{c})\right) w - \frac{c}{2} \geq \frac{1}{2}(1 - F(\widehat{c}))\, w, \tag{1}$$

where $\widehat{c}$ is the cost threshold of the other junior. Simplifying this inequality gives rise to the following Lemma.

**Lemma 1** *In a homogeneous department a junior exerts an effort if and only if $c \leq w$, that is, with probability $F(w)$.*

Note that $\widehat{c}$ does not matter. By exerting an effort a junior increases his promotion probability by $\frac{1}{2}$ independently of what the other junior is doing. Indeed, if the other junior does not exert an effort, exerting an effort changes the promotion probability from $\frac{1}{2}$ to 1. If he exerts an effort, exerting an effort changes the promotion probability from 0 to $\frac{1}{2}$.

### 3.2 The contest in a heterogeneous department

In a heterogenous department the two juniors belong to different groups. Then, being promoted not only results in the senior wage $w$ but also impacts the group welfare. Indeed, a senior bureaucrat increases the welfare of his group by $d$ directly and by $\Delta W^f$ from possibly biasing the future promotion. The latter occurs in a heterogenous department and with probability $p$ and, when it occurs, the group welfare changes by $d + \Delta W^f$. Solving the equation

$$\Delta W^f = 2\lambda (1 - \lambda) p \left(d + \Delta W^f\right)$$

yields the total impact on the group welfare, $d + \Delta W^f = \frac{d}{1-2\lambda(1-\lambda)p}$.

Suppose that juniors know whether the patronage will be used in which case they do not exert any effort.[11] When the patronage is not used, the contest is merit-based and, writing the condition for exerting an effort similar to (1), gives the following Lemma.

**Lemma 2** *In a heterogeneous department when the patronage is not used a junior exerts an effort if and only if* $c \leq w + \frac{d}{1-2\lambda(1-\lambda)p}$, *that is, with probability* $F\left(w + \frac{d}{1-2\lambda(1-\lambda)p}\right)$.

When the contest is merit-based, the juniors exert a higher effort than in a homogenous department and this effort is increasing in the size of patronage $p$.

### 3.3 Characterizing the optimal patronage

Denote $q = 2\lambda (1 - \lambda)$, the probability of having a heterogenous department. Using Lemmas 1 and 2 we can now write the total effort as

$$E = (1 - q) F (w) + q (1 - p) F \left(w + \frac{d}{1 - qp}\right) \tag{2}$$

and the planner maximizes it with respect to $p \in [0, 1]$.

[11] Making the opposite assumption does not change the results qualitatively. See also the discussion at the end of Section 3.3 on the different ways of introducing the bias.

Promotion discretion has two opposite effects on the total effort (2). First, there is a *higher stakes effect*: promotion becomes more valuable since senior bureaucrats have more say in future promotions. Second, there is a *discouragement effect*: there is no effort when the senior promotes the junior of his group for certain.

To understand when the optimal patronage is positive, let us compute the two effects at $p = 0$ (and conditional on being in a heterogeneous department). The value of the promotion is $w + d$. The higher stakes effect is then equal to $f(w + d)\, qd$, that is, the probability of the junior marginal type times the increase in the value of promotion. The discouragement effect is equal to $F(w + d)$ since each junior provides effort with probability $F(w + d)$ in a merit-based contest. The discouragement effect dominates when the direct discretion $d$ is either small or large. When it is small, patronage does not increase the value of promotion by a lot. When it is large, the value of promotion with no patronage, $w + d$, is already large enough to incentivize all or almost all juniors and there is not much to gain from increasing this value further, while the loss due to discouraging effort is large.

When the optimal patronage is positive, it is found from the first-order condition

$$\frac{\partial E}{\partial p} = -F\left(w + \frac{d}{1-qp}\right) + (1-p)\, f\left(w + \frac{d}{1-qp}\right)\frac{qd}{(1-qp)^2} = 0. \tag{3}$$

We proceed with an example in order to have a simple closed-form solution.

**Proposition 1** *Suppose that* $c \rightsquigarrow U[\underline{c}, \underline{c}+1]$. *Optimal patronage* $p^*$ *is* $0$, *if* $d \leq (\underline{c} - w)(1-q)$ *or* $d \geq \frac{\underline{c}-w}{1-q}$, *and otherwise it is*

$$p^* = \frac{1}{q}\left(1 - \sqrt{d\frac{1-q}{\underline{c}-w}}\right). \tag{4}$$

**Proof.** See Appendix A. ■

As we noted above, the patronage is not used if the direct discretion is either too small or too large. Thus, overall, the two kinds of discretion are neither substitutes nor complements. For the case of the uniform distribution considered in Proposition 1, the optimal patronage (4) decreases with $d$. In general, a higher promotion discretion always increases the discouragement effect and increases the higher stakes effect if $f' > 0$. See Figure 1 for an example of where the optimal patronage first increases with $d$ and then decreases while being strictly positive.

The comparative statics of the optimal patronage with respect to other parameters also depends on the relationship between the cost probability density function and its

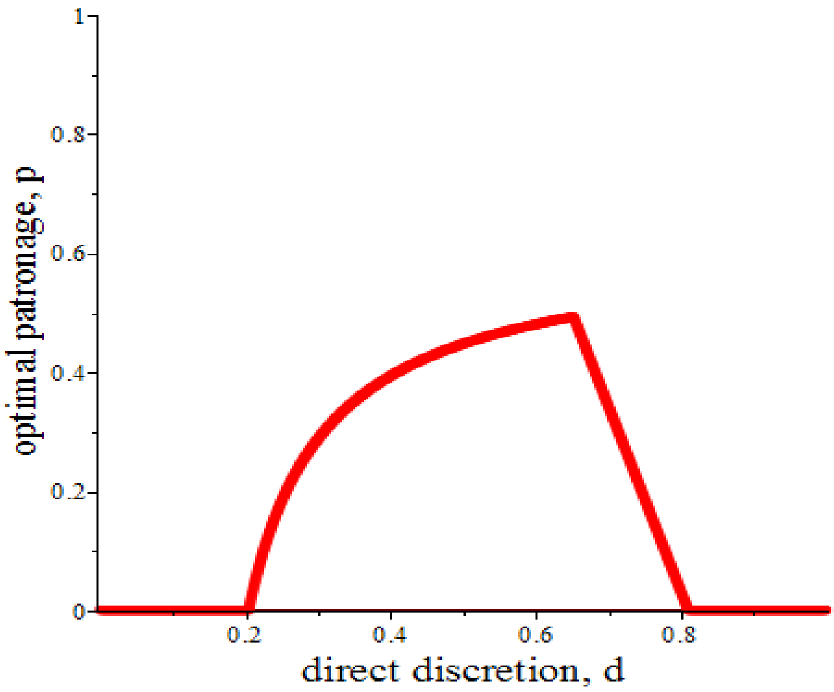


Figure 1: Optimal patronage when the costs are distributed as $Beta\,(5,1)$ $(F\,(c) = c^4)$ and $q = 0.4$, $w = 0.2$.

derivative, that is, $f$ and $f'$. Assuming the uniform distribution drastically simplifies the matters. It is easy to show that the optimal patronage (4) decreases with wage $w$ and increases with the probability of the heterogenous department $q$.

Finally, let us comment on different ways of biasing the contest for promotion and the resulting discouragement effect. Introducing patronage as a probability that the efforts do not matter means that the discouragement effect is always of the first order. This is true for both when the juniors know if patronage will be used, as we assume throughout the paper, and when they do not and, therefore, exert effort that will not matter with some probability. Introducing the bias in a more standard way as is done in the contest literature makes the discouragement effect of the second order at zero bias.[12] Since the higher stakes effect is always of the first order, optimal patronage is then strictly positive for any positive direct discretion. In Appendix B we consider the Tullock contest with the multiplicative bias and show that the optimal patronage $p^* > 0$ for any $d > 0$ (see Proposition 7). To summarize this discussion, introducing patronage as we do in this paper makes it *more* difficult to obtain a strictly positive optimal patronage.

[12]See Meyer (1992) for an early example of an additive bias in a Lazear-Rosen tournament and Franke, Kanzow, Leininger, and Schwartz (2013) for characterization of the multiplicative bias in a general Tullock contest. See Drugov and Ryvkin (2013) for a general condition.

### 3.4 Optimal composition of the departments

Whenever the group identity is observable, which is the case of groups based on ethnicity, caste, religion, etc., two questions arise. Should the planner make departments homogenous or heterogenous? What is the optimal composition of the junior level, i.e., $\lambda$?

**Proposition 2** *The optimal composition of the junior level is balanced, that is, $\lambda = \frac{1}{2}$, and all the departments are heterogenous.*

The efforts are strictly higher in a heterogenous department since the planner can always set the patronage to zero, $p = 0$, in which case the juniors always compete and have higher incentives than in the homogenous department (see Lemmas 1 and 2). Thus, the planner composes heterogenous departments whenever possible, that is, he sets $q = 2\min\{\lambda, 1-\lambda\}$. The optimal composition of the junior level is then to have $\lambda = \frac{1}{2}$.

### 3.5 The effect of the discount factor

When the future periods are discounted with the discount factor $\delta$, in a heterogenous department a promoted junior obtains the utility of $\delta\left(w + d + \Delta W^f\right)$, where $\Delta W^f$ is found from the equation $\Delta W^f = \delta qp\left(d + \Delta W^f\right)$. The total effort (2) becomes

$$E = (1-q)\,F(\delta w) + q(1-p)\,F\left(\delta\left(w + \frac{d}{1-\delta qp}\right)\right).$$

A higher $\delta$ increases both the higher stakes effect, $f(\delta(w+d))\,\delta^2 qd$, and the discouragement effect, $F(\delta(w+d))$ (both computed at $p = 0$). Then, the overall effect is ambiguous. For the case of the uniform distribution considered in Proposition 1, the optimal patronage (4) becomes $\frac{1}{\delta q}\left(1 - \sqrt{d\frac{1-\delta q}{\frac{c}{\delta} - w}}\right)$ and it decreases with $\delta$.

## 4 Affecting the senior level

We now turn to the scenario which is in some ways opposite to the one in Section 3 and in which the planner cares only about the composition of the senior level.[13] For

[13] He then probably cares about the overall composition of the bureaucracy, but the composition of the junior level is exogenous. For example, it might be illegal to hire based on the group identity or the group identity may not be observable at the entry stage, as in the case of groups based on values.

example, the planner is a politician who cares about the preferences of the median voter who is likely to belong to the larger group. Alternatively, the direct discretion may be costly for the planner per se, in which case he prefers the group which uses it in a less distortionary way.

As we will see, the effect of patronage depends on how relatively motivated the two groups are. Thus, we allow for the direct discretion to be different between the two groups, $d_l$ and $d_r$. For example, diverting funds of a given size is more valuable for a poorer group. Alternatively, exerting the direct discretion may be costly for the agents if they need to exert an effort or can be caught, and groups differ in how much the agents are motivated.

Suppose that the planner prefers the left group to the right one and, therefore, maximizes the steady-state share of left seniors, $\lambda^S$. It is found from the equation[14]

$$\lambda^S = \lambda^2 + 2\lambda(1-\lambda)\left[p\lambda^S + (1-p)\frac{1}{2}(1+F_l-F_r)\right], \tag{5}$$

where $F_i = F\left(w + \frac{d_i}{1-2\lambda(1-\lambda)p}\right)$, $i = l, r$. In what follows, we will sometimes refer to $\frac{d_i}{1-2\lambda(1-\lambda)p}$ as the *motivation* of group $i$.

The left seniors come from 1) homogenous departments where both juniors are left, 2) heterogenous departments headed by a left senior who uses promotion discretion, and 3) heterogenous departments where promotion is merit-based and the left junior wins it.

The effect of patronage on $\lambda^S$ can be decomposed into three effects.[15] First, there is the size effect, proportional to $\lambda - \frac{1}{2}$: the promotion discretion benefits the larger group because it is more likely to use it. The second and the third effects arise because patronage changes the likely winner of the fair contest. The second effect is the relative motivation effect proportional to $F_r - F_l$: the patronage benefits the less motivated group because on average this group loses the fair contest. Finally, the third effect is the change in the relative motivation, proportional to $\frac{\partial(F_r-F_l)}{\partial p}$ since the patronage changes the motivations. The sign of this effect depends on the cost distribution $F$ and group motivations.

[14] When the contest is merit-based, the probability that a left junior is promoted in the heterogenous deparment is equal to

$$\frac{1}{2}(F_lF_r + (1-F_l)(1-F_r)) + F_l(1-F_r) = \frac{1}{2}(1+F_l-F_r).$$

[15] See Lemma 3 in the Appendix A for the details.

Expressing $\lambda^S$ from (5) yields

$$\lambda^S = \frac{\lambda}{1 - 2\lambda(1-\lambda)p}\left[\lambda + (1-\lambda)(1-p)(1 + F_l - F_r)\right], \tag{6}$$

The planner maximizes (6) by choosing promotion discretion $p \in [0, 1]$. As before, we will proceed with a particularly well-behaved example when $F$ is linear, that is, when the junior costs are distributed uniformly. In this case the relative motivation is proportional to the difference in direct discretions, $d_r - d_l$. Then, both motivation effects of patronage mentioned above, of relative motivation and of the change in the relative motivation, are proportional to $d_r - d_l$; they can be jointly labelled as the motivation effect. Therefore, there are only two parameters in the planner's problem, $\lambda$ and $d_r - d_l$, which simplifies the characterization of the optimal patronage. See the next proposition and Figure 2.

**Proposition 3** *When the planner maximizes the steady-state share of left seniors and $F$ is linear, the optimal patronage is*

- *Maximum, $p^* = 1$, if $d_r - d_l \geq \max\{\frac{1-2\lambda}{1-2\lambda(1-\lambda)}, 1 - 2\lambda\}$;*
- *Intermediate, $p^* \in (0, 1)$ if $\lambda > \frac{1}{2}$ and $d_r - d_l < 1 - 2\lambda$;*[16]
- *No patronage, $p^* = 0$, otherwise.*

**Proof.** See Appendix A. ■

This Proposition is illustrated in Figure 2. Consider the upper right quadrant. The left group is larger, $\lambda > \frac{1}{2}$, and less motivated, $d_l < d_r$, that is, both the size and motivation effects of a higher patronage are positive. The optimal patronage is then maximum, $p^* = 1$. The lower left quadrant in Figure 2 is the opposite case: the left group is smaller and more motivated. A higher patronage decreases $\lambda^S$ via both effects and it is optimal to set patronage to zero, $p^* = 0$.

The two effects are opposed in the other two quadrants. In the lower right quadrant the left group is larger, $\lambda > \frac{1}{2}$, but also more motivated, $d_l > d_r$. When the motivations are close, the first effect dominates and optimal patronage is at the maximum, $p^* = 0$. As the gap in motivations increase, the second effect becomes more important and the optimal patronage becomes less than maximum and then further

[16]More precisely, it is equal to $\frac{2\lambda-1}{2\lambda(1-\lambda)}\frac{1+(2\lambda-1)(d_r-d_l)}{2\lambda-1-(d_r-d_l)}$ if $d_r - d_l \geq -4\lambda^2 + 6\lambda - 3$ and to $\frac{1+d_r-d_l}{2\lambda(1-\lambda)}$ otherwise.

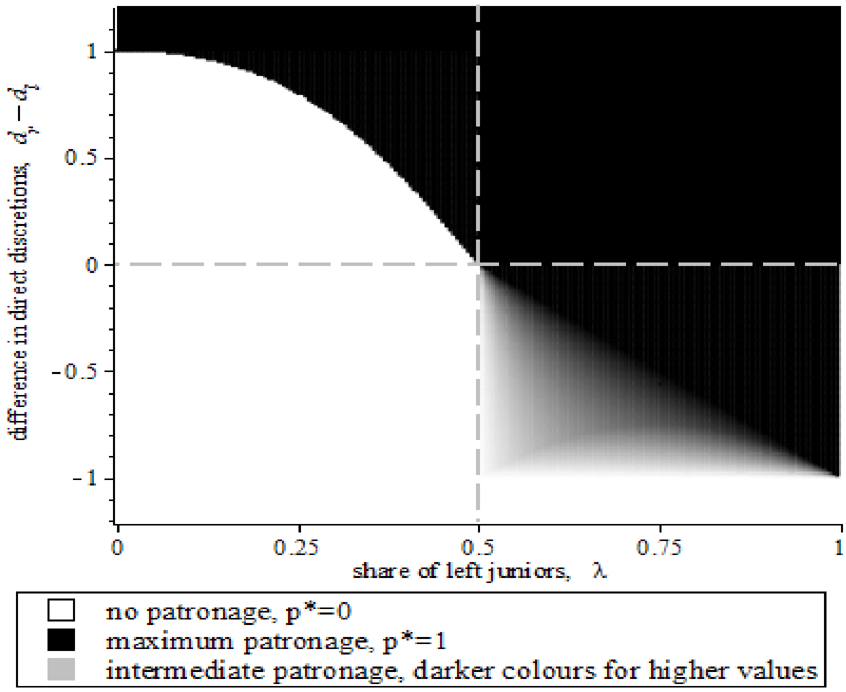


Figure 2: Optimal patronage depending on the share of left juniors, $\lambda$, and the difference in direct discretions, $d_r - d_l$.

decreases. Increasing $\lambda$ makes the larger left group even larger and, therefore, the optimal patronage increases.[17] In the opposite, upper left quadrant the two effects are reversed: now the left group is smaller, $\lambda < \frac{1}{2}$, but also less motivated, $d_r > d_l$. However, in this case $\lambda^S$ is U-shaped in patronage and, therefore, the optimal patronage is either zero or the maximum one.

Finally, when the difference in motivations is very high, the situation changes since one group always wins the fair contest.[18] If $d_r - d_l \geq 1$, the right juniors win it and, therefore, patronage is the only chance for the left ones to be promoted. Thus, it is set at the maximum level. If $d_r - d_l \leq -1$, the left juniors always win the fair contest, in which case no patronage is optimal.

The comparative statics just discussed leads to the following corollary.

**Corollary 1** *Optimal patronage $p^*$ (weakly) increases with the share of left juniors, $\lambda$, and with the difference in direct discretions, $d_r - d_l$.*

[17] There are two cases of the intermediate patronage as mentioned in fn. 16. The second case appears when the left juniors always win the fair contest. Then, further increasing patronage gives more chances to the right juniors. In other words, optimal patronage scales up the difference in motivations just enough so that the left juniors always win the fair contest.

[18] This is the consequence of the fact that the uniform distribution has a bounded support.

## 4.1 Corruption

Let us apply the analysis of the previous section to the case of corruption. One group is "honest" and the other is corrupt; the planner is honest and minimizes corruption by minimizing the number of corrupt agents. Since it is impossible to distinguish the corrupt agents at entry level, the planner minimizes the number of corrupt agents at senior level. We thus abstract from the incentive problem of the juniors considered before. As seniors have much more power, juniors' efforts have only a second-order effect on the social welfare as compared to the number of the corrupt agents at the senior level. Another reason is that the previous literature has extensively studied the agency problem when some agents may engage in a corrupt behaviour (see, for example, Mishra (2006) for a survey), while looking at the spreading of corrupt agents in an organization is new, to the best of our knowledge.

Corrupt agents take bribes for (not) doing their job and corrupt senior bureaucrats also "sell" the promotions to corrupt juniors. Other agents are honest in that they dislike corruption. They do not take bribes and they try to prevent corruption if they can. We assume that inside the organization or, at least, inside each department people know who is corrupt and who is not but honest agents cannot reveal this to the outside world, either for the lack of hard proof or for the fears for personal safety.[19] Thus, the only way the honest agents can fight corruption is by not promoting corrupt juniors whenever they have such an opportunity. Corrupt seniors use patronage to sell their position, while the honest ones use it to not promote corrupt juniors.

The share of the honest juniors in the bureaucracy is $\lambda$. Honest agents derive utility $g$ when a corrupt junior is not promoted. It may come from their moral satisfaction that an honest rather than a corrupt agent obtains the job. It can also be their valuation of the harm for the society that a corrupt senior will do if they have some prosocial or public sector motivation. If corrupt seniors do not provide much effort, $g$ may be the contribution of the honest senior towards the good of the society. In practice, of course, all three reasons might coexist. Proceeding in the same way as in Section 3.2 yields the value of the promotion for the honest juniors as equal to $w + \frac{g}{1-2\lambda(1-\lambda)p}$.

A corrupt senior bureaucrat takes $b$ in bribes using his direct discretion. For example, he can take kickbacks for placing governmental orders, bribes for granting a licence or for not enforcing some rules. He can also literally sell the position. Whenever he exerts his promotion discretion, he can charge a bribe for promotion to

[19] The movie *Serpico* (based on the true story of Frank Serpico, a New York policeman) is a good illustration of how corruption may be open and visible inside a department, and yet how difficult and dangerous it is to expose it.

a corrupt junior, if there is at least one in his department. This bribe may depend on whether one or both juniors are corrupt in his department; denote it $b_1$ and $b_2$ for the cases of one and two corrupt juniors, respectively.

The total expected bribe income of a corrupt senior bureaucrat is then

$$B = b + \left(2\lambda\left(1-\lambda\right)b_1 + \left(1-\lambda\right)^2 b_2\right)p. \tag{7}$$

Suppose that $b_1$ and $b_2$ are proportional to $B$ with coefficients $k_1$ and $k_2$ which represent the bargaining power of the senior bureaucrat vis-à-vis the junior ones.[20] If $k_1 = k_2 = 1$, the senior bureaucrat has all the bargaining power and extracts all the surplus. However, his bargaining power is likely to be lower if the juniors cannot collect so much in bribes themselves and are credit-constrained. It might also be reasonable to assume that $k_2 > k_1$ since the senior bureaucrat can essentially auction the promotion when both juniors are corrupt. Plugging $b_i = k_i B$, $i = 1, 2$, into (7) we obtain

$$B = \frac{b}{1 - Kp}, \tag{8}$$

where $K = 2\lambda\left(1-\lambda\right)k_1 + \left(1-\lambda\right)^2 k_2$ is the average bargaining power of corrupt seniors.

The steady-state composition of the senior level, $\lambda^S$, is (6) with $F_l = F\left(w + \frac{g}{1-2\lambda(1-\lambda)p}\right)$ and $F_r = F\left(w + \frac{b}{1-Kp}\right)$.

Consider first

$$K = 2\lambda\left(1-\lambda\right), \tag{9}$$

in which case the problem of the planner is exactly as before, that is, to maximize (6) with $d_l = g$ and $d_r = b$. Proposition 3 and Corollary 1 apply. In particular, optimal patronage increases as the honest group becomes larger and relatively less motivated, that is, as $b - g$ increases.

Let us now discuss the effect of the bargaining power of corrupt seniors, $k_1$ and $k_2$. When they increase, the motivation of corrupt juniors $\frac{b}{1-Kp}$ is scaled up more for any value of patronage. This affects optimal patronage via two opposed effects. On the one hand, patronage should decrease to counterbalance the scaling up of the motivation of corrupt juniors. On the other hand, a higher motivation of corrupt juniors means that their chances become higher in a fair contest, which calls for a higher patronage. The total effect is ambiguous and, therefore, a higher bargaining

[20] The bribes might be proportional to $B + w$, in which case the senior bureaucrat can sell the promotion even if he cannot take any direct bribes himself, i.e., when $b = 0$. This does not affect the results qualitatively.

power of corrupt seniors may lead to a higher or lower optimal patronage.

# 5 Extensions

## 5.1 Warm glow and impure altruism

People often value their own contribution to a public good irrespectively of what others do or would do if they do not contribute. This is called "warm glow" (see Andreoni (2006)). Impure altruists combine pure altruism (that is, the total amount of the public good enters the utility function) and warm-glow motivation (that is, their contribution directly enters the utility function). Introducing the warm glow or impure altruism in our model is straightforward: the own direct discretion $d$ has a positive weight in the agents' utility function. Hence, it is equivalent to increasing the senior wage $w$.

Another question arises, however, once an agent is not a pure altruist. How should he care about the actions of the junior he promoted? How should this agent care about the actions of the junior who is promoted by the junior he promoted? What about the junior promoted in his department ten generations later? It seems natural that an agent cares more about the actions of the junior he promoted than of the one a few generations later, even though his decision is necessary for both. One of the reasons is that in the latter case there are other seniors that contribute to the promotion. In other words, the distance between the promotion decision and the eventual increase in the group welfare affects how the agent values this increase. We can then introduce an "altruism" factor to reflect this imperfect altruism. The difference with the time discount factor is that imperfect altruism does not discount the wage but only group welfare gains.

More specifically, suppose that a senior agent assigns an "altruism" factor $\alpha \leq 1$ to the increase in the group welfare brought about by a junior he promoted, $\alpha^2$ to the increase in the group welfare done by a junior promoted by a junior he promoted, etc. In a heterogenous department a promoted junior then obtains the utility of $w + d + \Delta W^f$, where $\Delta W^f$ is found from the equation $\Delta W^f = \alpha qp \left(d + \Delta W^f\right)$. The total effort becomes

$$E = (1-q)\,F(w) + q\,(1-p)\,F\left(w + \frac{d}{1-\alpha qp}\right).$$

The effect of the altruism factor $\alpha$ is then the same as the one of the probability of a heterogenous department $q$. For the case of the uniform distribution considered in Proposition 1, the optimal patronage (4) becomes $\frac{1}{\alpha q}\left(1-\sqrt{d\frac{1-\alpha q}{\underline{c}-w}}\right)$ and the effect of $\alpha$ is ambiguous.

## 5.2 Antagonistic and asymmetric groups

In Section 3 the two groups are symmetric and care only about their own direct discretion. In Section 4 the two groups have different direct discretions, $d_l$ and $d_r$. We also mentioned in the corruption application in Section 4.1 that part of the motivation of the honest agents may come from preventing the harm to the society that corrupt seniors will do. Thus, they are motivated not by the possibility to use their own direct discretion but by the possibility to block the direct discretion of the other group. We call this antogonism and it may be important in a wide range of situations. For example, the effectiveness of the left-wing propaganda decreases when there is more right-wing propaganda. This is also the case when the groups care about their relative income or status. This antagonism can be captured by parameter $\beta_i \geq 0$, $i = l, r$, such that the welfare of group $i$ decreases by factor $\beta_i$ when the senior from the other group exerts direct discretion.

The welfare of the left group becomes

$$W_l = d_l \sum_{t=0}^{+\infty} \delta^t N_l^t - \beta_l d_r \sum_{t=0}^{+\infty} \delta^t N_r^t$$

and analogously for the right group. Then, each time a junior of group $i$ is promoted instead of a junior from group $-i$, the direct impact on the welfare of group $i$ is $d_i + \beta_i d_{-i}$, $i = l, r$.

The groups may also differ in the weight with which the group welfare enters the agents' utility function. We have implicitly assumed throughout the paper that this weight is 1 for both groups. Here the weights are $\gamma_i > 0$, $i = l, r$. A higher $\gamma_i$ corresponds to a group with a higher group altruism. Proceeding in the same way as in Section 3.3, we can find the total output

$$E = (1-q) F(w) + \frac{1}{2} q (1-p) \left[ F\left( w + \gamma_l \frac{d_l + \beta_l d_r}{1-qp} \right) + F\left( w + \gamma_r \frac{d_r + \beta_r d_l}{1-qp} \right) \right] \tag{10}$$

For our example with the uniform distribution of costs, it is the average base motivation which matters; denote

$$\overline{d} = \frac{1}{2}\left[ (d_l + \beta_l d_r)\gamma_l + (d_r + \beta_r d_l)\gamma_r \right].$$

**Proposition 4** *Suppose that* $c \rightsquigarrow U[\underline{c}, \underline{c}+1]$. *Optimal patronage* $p^*$ *is* 0, *if* $\overline{d} \leq (\underline{c}-w)(1-q)$ *or* $\overline{d} \geq \frac{\underline{c}-w}{1-q}$, *and otherwise it is*

$$p^* = \frac{1}{q}\left(1 - \sqrt{\overline{d}\frac{1-q}{\underline{c}-w}}\right). \tag{11}$$

**Proof.** See Appendix A. ■

The optimal patronage (11) is very similar to the one in (4), with the only difference that $d$ is replaced by $\overline{d}$ which is the average one-period increase in the group welfare from the promotion. Since the altruism towards the group matters only in the heterogenous department, where there is one left junior and one right junior by definition, it is the average altruistic motivation that determines the total effort (also because $F$ is linear).

## 5.3 The two planner's goals together

We have considered two possible goals of the planner, maximizing juniors' efforts (Section 3) and affecting the composition of the senior level (Section 4), separately. In many cases the planner, however, prefers one group to the other but still cares about the work done by the organization, that is, about juniors' efforts. For example, the planner is the politician for whom the efficiency of the government affects how likely he is to stay in power, but he himself belongs to one of the groups or panders to the bigger group because it contains the median voter. The planner may have group preferences from the efficiency perspective too if he takes into account the cost of the direct discretion and it differs between the two groups. The direct discretion of one group may be less distortionary per se. If one group is richer on average than the other, then a bias in public spending towards this group is more distortionary than an equally sized bias towards the poorer group. Finally, the planner may prefer the group with lower group motivation if agents from this group use the direct discretion less.

The planner may allow for patronage in order to motivate juniors and also to affect the composition of the senior level towards his preferred group. Suppose that the planner (dis)likes the direct discretion $d_i$ with the weight $-h_i$, $i = l, r$. This weight is negative, that is, $h_i > 0$ if the direct discretion means favours, corruption, etc. However, if the direct discretion is used by agents with the intrinsic motivation for public sector, then $h_i < 0$.

Denote

$$F_i = F\left(w + \gamma_i \frac{d_i + \beta_i d_{-i}}{1 - qp}\right), i = l, r,$$

the share of juniors of group $i$ that exert effort. This share is increasing in the group motivation, $\gamma_i (d_i + \beta_i d_{-i})$. The planner's objective function is then to maximize (up to a constant)

$$(1 - q) F(w) + \frac{1}{2} q (1 - p) [F_l + F_r] + H\lambda^S, \tag{12}$$

where $H = h_r d_r - h_l d_l$ is the relative harm of the two groups. If it is positive, the planner prefers the seniors from the left group. The steady-state composition of the senior level, $\lambda^S$, is (6).

For our example with the uniform distribution of costs, the difference in shares $F_r - F_l$ is proportional to the difference in motivations

$$\Delta = \gamma_r (d_r + \beta_r d_l) - \gamma_l (d_l + \beta_l d_r),$$

which we also call the relative motivation of the right group. As we showed in Section 4, the influence of patronage on $\lambda^S$ can be decomposed into the size effect, proportional to $\lambda - \frac{1}{2}$, and the (total) motivation effect, proportional to $\Delta$.[21]

**Proposition 5** *Suppose that* $c \rightsquigarrow U[\underline{c}, \underline{c} + 1]$.

*(i) Optimal patronage* $p^*$ *increases with the relative motivation of the right group,* $\Delta$*, if and only if the planner prefers the left group,* $H > 0$.

*(ii) If* $\Delta = 0$*, optimal patronage* $p^*$ *is* 0*, if* $\overline{d} - \frac{\lambda - \frac{1}{2}}{1-q} H \leq (\underline{c} - w)(1 - q)$ *or* $\overline{d} - \frac{\lambda - \frac{1}{2}}{1-q} H \geq \frac{\underline{c} - w}{1-q}$*, and otherwise it is*

$$p^* = \frac{1}{q}\left(1 - \sqrt{\frac{(1 - q)\overline{d} - \left(\lambda - \frac{1}{2}\right) H}{\underline{c} - w}}\right).$$

**Proof.** See Appendix A. ■

When $\Delta$ increases, patronage favours the left group more since it becomes more likely to lose the fair contest. If $H > 0$, that is, the right group is relatively more harmful for the planner, the planner counteracts higher chances of the right group in a fair contest by allowing for more patronage. If $H < 0$, that is, the planner prefers

[21] As we show there, there are two motivation effects: the relative motivation effect proportional to $F_r - F_l$ and the change in the relative motivation, proportional to $\frac{\partial (F_r - F_l)}{\partial p}$. When $F$ is linear, both are proportional to the difference in motivations.

the right group, he makes the contest more fair if the right juniors are more likely to win it. This is a generalization of Corollary 1 to the case when the planner cares both about the juniors' efforts and composition of the senior level.

For a closed-form solution we need to assume that the two groups do equally well in the fair contest, that is, $\Delta = 0$. Then, there is only the first effect, as we mentioned above, and the patronage unambiguously benefits a larger group. If the planner prefers the left group, $H > 0$, and it is larger, the optimal patronage is higher than in the baseline model (4) as it is used partly to facilitate the promotions of the left juniors.

## 5.4 Monetary incentives

We have so far taken the senior wage as given and abstracted from direct monetary incentives for the juniors. The monetary incentives of course come at the cost of public funds. Taking a standard specification of a biased contest, we can show the following.

**Proposition 6** *Consider the biased Tullock contest from Appendix B2. Optimal patronage is positive for any positive costs of public funds.*

Proposition 7 in Appendix B shows that optimal patronage is strictly positive for any senior wage. Even when providing monetary incentives is very cheap and the senior wage is high, at the margin increasing it still has a first-order cost. In this contest specification as well as in many others patronage has second-order costs at zero but first-order benefits. If juniors can be rewarded not only by the promotion but also by direct monetary incentives for high output, the result still holds for the same reason.[22]

## 5.5 The two groups caring about the same cause

It is possible that the two groups care about the same cause, but to a different extent. For example, public sector workers may all be motivated by (common) social welfare but to a different extent. One group (say, the left) consists of agents that are highly motivated, while the other group (say, the right) consists of workers that are less

[22] There might be then a question whether monetary incentives should be provided directly or as a senior wage but this does not affect the optimality of patronage.

motivated. The social welfare is then

$$W = \sum_{t=0}^{+\infty} d_l \delta^t N_l^t + \sum_{t=0}^{+\infty} d_r \delta^t N_r^t,$$

that is, $\beta_l = \beta_r = -1$. Then, if $d_l > d_r$, both groups want to promote the juniors of the left group since the left seniors contribute more towards the common welfare. In a heterogenous department, the value of promotion for the right juniors is less than wage $w$ since their promotion is worse for the social welfare than the promotion of the left juniors.

# 6 Conclusion

We studied the design of promotions in an organization where agents belong to groups that advance their cause. Examples and applications include political groups, ethnicities, agents motivated by the work in the public sector, and corruption. We showed that the direct discretion of senior agents, that is, the ability to directly benefit their group, leads to the demand for patronage, that is, the discretion in promotions. The optimal patronage then balances the positive effect of the promotion discretion, due to the higher stakes in the juniors' promotion contest, and its negative discouraging effect due to the unfairness of the contest.

We also considered the corruption setting in which some agents are corrupt and others are honest. The corrupt seniors take bribes using their direct discretion and "sell" the promotion to the corrupt juniors. Whenever possible, the honest seniors do not promote corrupt juniors and get a kick in their utility from this action. The planner minimizes the corruption at the senior level (the distribution of junior types is exogenous). Patronage benefits the larger group and the less motivated group. Thus, in some cases the optimal patronage is positive and even becomes maximum, that is, seniors have full discretion in promotions. This is despite the fact that corrupt seniors use it to sell promotions to corrupt juniors.

An interesting avenue for future work is to study the situations when the group identity can be changed or hidden, which is of course most relevant when the groups are based on values. For example, a left-wing person may change his convictions when surrounded by right-wing colleagues. He can also hide that he is from the left if his boss is from the right. In the corruption setting both possibilities are particularly relevant. An honest agent may succumb to temptation of high bribes taken by his colleagues and a corrupt junior may restrain from taking bribes if his senior is honest in order to get the promotion.

# Appendix A. Proofs

**Proof of Proposition 1.** When $c \rightsquigarrow U\left[\underline{c}, \underline{c}+1\right]$, the first-order condition (3) becomes

$$-\left(w+\frac{d}{1-qp}-\underline{c}\right)+\left(1-p\right)\frac{qd}{\left(1-qp\right)^{2}}=0 \tag{13}$$

The second derivative is $-2dq\frac{1-q}{(1-pq)^3}<0$ and the second-order condition is, therefore, satisfied.

(13) can be rewritten as

$$q^2p^2-2qp+1-\frac{1-q}{\underline{c}-w}d=0$$

There are two roots, $\frac{1}{q}\left(1\pm\sqrt{d\frac{1-q}{\underline{c}-w}}\right)$, but the larger is always greater than one. Thus, $p^*=\frac{1}{q}\left(1-\sqrt{d\frac{1-q}{\underline{c}-w}}\right)$. Condition $p^*>0$ gives $d<\frac{\underline{c}-w}{1-q}$ and condition $p^*<1$ gives $d>\left(\underline{c}-w\right)\left(1-q\right)$. ■

**Lemma 3** *Patronage $p$ affects the steady-state composition of the senior level via three effects: 1) by benefiting the larger group; 2) by benefiting the less motivated group and 3) by changing the difference in shares of juniors that exert the effort, $F_l-F_r$.*

**Proof.** Express $\lambda^S$ from (5) as

$$\lambda^S=\frac{\lambda}{1-2\lambda\left(1-\lambda\right)p}\left[\lambda+\left(1-\lambda\right)\left(1-p\right)\left(1+F_l-F_r\right)\right].$$

Its derivative with respect to $p$ is equal to $\frac{\lambda(1-\lambda)}{1-2\lambda(1-\lambda)p}$ multiplied by

$$\frac{2\lambda-1}{1-2\lambda\left(1-\lambda\right)p}+\frac{1-2\lambda\left(1-\lambda\right)}{1-2\lambda\left(1-\lambda\right)p}\left(F_r-F_l\right)+\left(1-p\right)\frac{\partial\left(F_r-F_l\right)}{\partial p}.$$

The first term has the sign of $\lambda-\frac{1}{2}$ and it is thus positive when the left group is larger. The second term has the sign of $F_r-F_l$ and it is positive when the left group is less motivated. The third term has the sign of $\frac{\partial(F_r-F_l)}{\partial p}$ which is ambiguous. Indeed,

$$\frac{\partial\left(F_r-F_l\right)}{\partial p}=\frac{2\lambda\left(1-\lambda\right)}{\left(1-2\lambda\left(1-\lambda\right)p\right)^{2}}\left(d_rf_r-d_lf_l\right),$$

where $f_i=f\left(w+\frac{d_i}{1-2\lambda(1-\lambda)p}\right)$, $i=l,r$. ■

**Proof of Proposition 3.** When $F$ is uniform, the probability that the left junior is promoted in the fair contest, $\frac{1}{2}(1+F_l-F_r)$, is equal to $\frac{1}{2}\left(1-\frac{d_r-d_l}{1-2\lambda(1-\lambda)p}\right)$ provided that both $w+\frac{d_r}{1-2\lambda(1-\lambda)p}$ and $w+\frac{d_l}{1-2\lambda(1-\lambda)p}$ are in the support of $F$. Assume this for now and rewrite (6) as

$$\lambda^S = \frac{\lambda}{1-2\lambda(1-\lambda)p}\left[\lambda+(1-\lambda)(1-p)\left(1-\frac{d_r-d_l}{1-2\lambda(1-\lambda)p}\right)\right] \tag{14}$$

and the first derivative with respect to $p$ is

$$\frac{\partial\lambda^S}{\partial p} = \frac{\lambda(1-\lambda)}{(1-2\lambda(1-\lambda)p)^2}\left((2\lambda-1)+(d_r-d_l)\frac{(1-2\lambda)^2+2\lambda(1-\lambda)p}{1-2\lambda(1-\lambda)p}\right).$$

When $\lambda \geq \frac{1}{2}$ and $d_r \geq d_l$ (the left group is larger and less motivated), both terms in brackets are positive and, therefore, $p^* = 1$.

When $\lambda < \frac{1}{2}$ and $d_r < d_l$ (the left group is smaller and more motivated), both terms in brackets are negative and, therefore, $p^* = 0$.

When the two terms have opposite signs, an interior value of $p$ might be optimal. Solve the first-order condition $\frac{\partial\lambda^S}{\partial p} = 0$ to obtain

$$p^{FOC} = \frac{1}{2\lambda(1-\lambda)}(2\lambda-1)\frac{1+(2\lambda-1)(d_r-d_l)}{2\lambda-1-(d_r-d_l)}.$$

Compute the second derivative of $\lambda^S$ with respect to $p$

$$\frac{\partial^2\lambda^S}{\partial p^2} = \frac{8\lambda^2(1-\lambda)^2}{(1-2\lambda(1-\lambda)p)^3}\left(\lambda-\frac{1}{2}+(d_r-d_l)\frac{1-\lambda(1-\lambda)(3-p)}{1-2\lambda(1-\lambda)p}\right).$$

Plug in $p^{FOC}$ to obtain

$$\frac{\partial^2\lambda^S}{\partial p^2}\mid_{p=p^{FOC}} \propto -\frac{1}{4}\left[2\lambda-1-(d_r-d_l)\right].$$

When $\lambda < \frac{1}{2}$ and $d_r > d_l$ (the left group is smaller and less motivated), $\frac{\partial^2\lambda^S}{\partial p^2}\mid_{p=p^{FOC}} > 0$ and, therefore, the optimal patronage is either 0 or 1. Comparing $\lambda^S\mid_{p=0}$ and $\lambda^S\mid_{p=1}$ we obtain that $p^* = 1$ if $d_r - d_l \geq \frac{1-2\lambda}{1-2\lambda(1-\lambda)}$.

When $\lambda > \frac{1}{2}$ and $d_r < d_l$ (the left group is larger and more motivated), $\frac{\partial^2\lambda^S}{\partial p^2}\mid_{p=p^{FOC}} < 0$ and, therefore, $p^* = p^{FOC}$ provided it is between 0 and 1. Solving $p^{FOC} \leq 1$ we obtain $d_r - d_l \leq 1-2\lambda$. Solving $p^{FOC} \geq 0$ we obtain $d_r - d_l \geq -\frac{1}{2\lambda-1}$. Below, we consider the case $d_r - d_l \geq -1$ separately.

Finally, let us consider what happens when the probability that the left junior is promoted in the fair contest, $\frac{1}{2}(1+F_l-F_r)$, is not equal to $\frac{1}{2}\left(1-\frac{d_r-d_l}{1-2\lambda(1-\lambda)p}\right)$. To bring this possibility to the minimum (as this case does not represent any interest) and to minimize the number of the constraints to consider, assume that the lower bound of the support $\underline{c}=\min\{w+\frac{d_l}{1-2\lambda(1-\lambda)p}, w+\frac{d_r}{1-2\lambda(1-\lambda)p}\}$. Then, we only have to check if

$$\frac{1}{2}\left(1-\frac{d_r-d_l}{1-2\lambda(1-\lambda)p}\right)\in[0,1] \tag{15}$$

and what happens otherwise. A necessary condition for (15) is $d_r-d_l\in[-1,1]$.

When $d_r-d_l\geq 1$, $\frac{1}{2}(1+F_l-F_r)=0$, that is, left juniors are never promoted in a fair contest and, therefore, (6) becomes

$$\lambda^S=\frac{\lambda^2}{1-2\lambda(1-\lambda)p}. \tag{16}$$

It is monotonically increasing in $p$ since patronage is the only way for the left juniors to be promoted. Then, $p^*=1$. When $d_r-d_l\in[0,1)$, the optimal patronage is either 0 or 1 as we found above. When it is zero, (15) is satisfied. When it is one, it might not be satisfied. However, if it is not satisfied, maximizing (16) yields the same result.

When $d_r-d_l\leq -1$, $\frac{1}{2}(1+F_l-F_r)=1$, that is, left juniors are always promoted in a fair contest and, therefore, (6) becomes

$$\lambda^S=\frac{\lambda}{1-2\lambda(1-\lambda)p}\left[\lambda+2(1-\lambda)(1-p)\right]. \tag{17}$$

It is monotonically decreasing in $\lambda$ since patronage is the only chance for the right juniors to be promoted. Then, $p^*=0$. When $d_r-d_l\in(-1,0)$ and $\lambda<\frac{1}{2}$, $p^*=0$ and so (15) is satisfied. If $\lambda\geq\frac{1}{2}$ and $d_r-d_l<1-2\lambda$, $p^*=p^{FOC}$ provided (15) holds. This is the case for $d_r-d_l\geq -4\lambda^2+6\lambda-3$. If not, that is, $\frac{1}{2}\left(1-\frac{d_r-d_l}{1-2\lambda(1-\lambda)p}\right)=1$ at $p=\frac{1+d_r-d_l}{2\lambda(1-\lambda)}<p^{FOC}$, the planner solves

$$\max_{p\in[0,1]}\lambda^S=\left[\begin{array}{l}\lambda^S\text{ in (14) if }p\leq\frac{1+d_r-d_l}{2\lambda(1-\lambda)}\\ \lambda^S\text{ in (17) if }p\geq\frac{1+d_r-d_l}{2\lambda(1-\lambda)}\end{array}\right.$$

Since $\lambda^S$ in (14) is increasing in $p$ for $p<p^{FOC}$ and $\lambda^S$ in (17) is decreasing in $p$, the optimal patronage is then $p^*=\frac{1+d_r-d_l}{2\lambda(1-\lambda)}$.

■

**Proof of Proposition 4.** When $c \rightsquigarrow U\left[\underline{c}, \underline{c}+1\right]$,

$$\frac{1}{2}\left(F\left(w+\gamma_l \frac{d_l+\beta_l d_r}{1-qp}\right)+F\left(w+\gamma_r \frac{d_r+\beta_r d_l}{1-qp}\right)\right)=w+\frac{\overline{d}}{1-qp}-\underline{c}$$

and so the planner's problem to maximize (10) is equivalent to maximizing (2) and Proposition 1 applies with $d=\overline{d}$. ■

**Proof of Proposition 5. Part i)** Using (6) write

$$\lambda^S=\frac{\lambda}{1-2\lambda(1-\lambda)p}\left(1-p+p\lambda-\frac{(1-\lambda)(1-p)}{1-2\lambda(1-\lambda)p}\Delta\right).$$

The second cross-derivative of the planner's problem (12) with respect to $p$ and $\Delta$ is equal to

$$\frac{\partial^2\left(H\lambda^S\right)}{\partial p \partial \Delta}=H\lambda(1-\lambda)\frac{(1-2\lambda)^2+2\lambda(1-\lambda)p}{(1-2\lambda(1-\lambda)p)^3}.$$

Thus, $\operatorname{sgn}\left(\frac{\partial p^*}{\partial \Delta}\right)=\operatorname{sgn}\left(\frac{\partial^2\left(H\lambda^S\right)}{\partial p \partial \Delta}\right)=\operatorname{sgn}(H)$.

**Part ii)** When $\Delta=0$, the first-order condition of problem (12) is

$$q\left(-\left(w+\frac{\overline{d}}{1-qp}-\underline{c}\right)+(1-p)\frac{q\overline{d}}{(1-qp)^2}\right)+\left(\lambda-\frac{1}{2}\right)\frac{q}{(1-qp)^2}H=0. \qquad (18)$$

The second-order condition is $-\frac{2q}{(1-pq)^3}\left(\overline{d}(1-q)-\left(\lambda-\frac{1}{2}\right)H\right)<0$. If it is not satisfied, the problem is convex and $p^*=0$ since at $p=1$ the total effort is zero. Assuming the second-order condition is satisfied, rewrite (18) as a quadratic equation in $p$

$$q^2p^2-2qp+1-\frac{(1-q)\overline{d}-\left(\lambda-\frac{1}{2}\right)H}{\underline{c}-w}=0.$$

There are two real roots, $\frac{1}{q}\left(1\pm\sqrt{\frac{(1-q)\overline{d}-\left(\lambda-\frac{1}{2}\right)H}{\underline{c}-w}}\right)$, but the larger is always greater than one. Thus, $p^*=\frac{1}{q}\left(1-\sqrt{\frac{(1-q)\overline{d}-\left(\lambda-\frac{1}{2}\right)H}{\underline{c}-w}}\right)$ provided it is between 0 and 1. Note that $p^*<1$ implies the second-order condition. ■

# Appendix B. Alternative contest models

Here we present two alternative contest models that generate results very close to the ones obtained before.

## B1. Tournament

As before, patronage $p$ means that with probability $p$ the senior bureaucrat has full discretion in deciding whom to promote, while with probability $1-p$ there is a fair contest. Here, the contest is the standard Lazear-Rosen tournament.

Junior $i$, $i = l, r$, exerts effort $e_i$ at the cost $C(e_i) = \frac{1}{2}e_i^2$ and is promoted if $e_i - e_{-i} + u \geq 0$, where $u \rightsquigarrow U\left[-\frac{1}{2}, \frac{1}{2}\right]$. This is the simplest specification of the Lazear-Rosen tournament and has been used in Meyer (1991), Konrad (2009), Ederer (2010) and Brown and Minor (2012), among others. The probability that junior $i$ is promoted is equal to (assuming interior solution)

$$\Pr\{u \geq e_{-i} - e_i\} = \frac{1}{2} + e_i - e_{-i}.$$

Then, each junior solves

$$\max_{e_i} \left(\frac{1}{2} + e_i - e_{-i}\right) v_i - \frac{1}{2}e_i^2,$$

where $v_i$ is the value of the promotion for junior $i$. It is equal to $w$ in a homogenous department and to $w + \gamma_i \frac{d_i + \beta_i d_{-i}}{1-qp}$ in a heterogenous department. Then, optimal effort is $e_i^* = v_i$ and the aggregate effort in a heterogenous department is

$$(1-p)(e_l^* + e_r^*) = (1-p)\left(w + \gamma_l \frac{d_l + \beta_l d_r}{1-qp} + w + \gamma_r \frac{d_r + \beta_r d_l}{1-qp}\right),$$

which is a particular example of the same object in (10).

## B2. Tullock contest

The contest for the promotion is now a biased Tullock one in which patronage $p \in [0,1]$ is the bias that takes the following form. If junior $i$ is favoured by the senior, he wins the contest with probability

$$\Pr\{i \text{ is promoted}\} = \frac{(1+p)\, e_i^r}{(1+p)\, e_i^r + (1-p)\, e_{-i}^r}, \; r \geq 1. \tag{19}$$

This is a standard specification of a biased Tullock contest as, for example, in Epstein, Mealem, and Nitzan (2011) and Franke, Kanzow, Leininger, and Schwartz (2013). The effort cost is $C(e_i) = \frac{1}{\alpha} e_i^{\alpha}$, $\alpha \geq 1$. As in Section 3, the group motivation is the same in the two groups and is equal to $d$.

**Lemma 4** *Bias $p$ in the contest success function (19) results in the difference in promotion probabilities of the favoured and non-favoured juniors equal to $p$. The equilibrium efforts are $e_i^* = e_{-i}^* = \left(r\frac{1-p^2}{4}\left(w + \frac{d}{1-qp}\right)\right)^{\frac{1}{\alpha}}$.*

**Proof.** Denote the value of the promotion in a heterogenous department as $v$. The favoured junior $i$ maximizes

$$\max_{e_i} \frac{(1+p)\, e_i^r}{(1+p)\, e_i^r + (1-p)\, e_{-i}^r} v - \frac{1}{\alpha} e_i^{\alpha},$$

while the other junior maximizes

$$\max_{e_{-i}} \frac{(1-p)\, e_{-i}^r}{(1+p)\, e_i^r + (1-p)\, e_{-i}^r} v - \frac{1}{\alpha} e_{-i}^{\alpha}.$$

The two first-order conditions are

$$\frac{r e_i^{r-1} e_{-i}^r (1-p^2)}{\left(e_i^r (1+p) + e_{-i}^r (1-p)\right)^2} v = e_i^{\alpha-1}, \qquad \frac{r e_{-i}^{r-1} e_i^r (1-p^2)}{\left(e_i^r (1+p) + e_{-i}^r (1-p)\right)^2} v = e_{-i}^{\alpha-1}.$$

Solving this system yields the equilibrium efforts

$$e_i^* = e_{-i}^* = \left(r\frac{1-p^2}{4} v\right)^{\frac{1}{\alpha}}. \tag{20}$$

Plugging (20) into (19), compute the difference in winning probabilities between the favoured junior $i$ and the non-favoured one $-i$

$$\Pr\{i \text{ is promoted}\} - \Pr\{-i \text{ is promoted}\} = \frac{1+p}{2} - \frac{1-p}{2} = p.$$

The value of the promotion $v$ is then $w + \frac{d}{1-qp}$. ■

The planner maximizes the total effort (up to a monotonic transformation) in a heterogenous department

$$\max_{p\in[0,1]} E^T = \frac{1-p^2}{2}\left(w + \frac{d}{1-qp}\right). \tag{21}$$

**Proposition 7** *Optimal patronage $p^*$ is intermediate, that is, $p^* \in (0,1)$ if and only if direct discretion is strictly positive, $d > 0$. It is increasing in direct discretion $d$ and decreasing in senior wage $w$. When $w = 0$, $p^* = \frac{1-\sqrt{1-q^2}}{q}$.*

**Proof.** The first derivative of (21) with respect to $p$ is

$$\frac{\partial E^T}{\partial p} = \frac{1}{2} d \frac{qp^2 - 2p + q}{(1-pq)^2} - pw$$

and the second is

$$\frac{\partial^2 E^T}{\partial p^2} = -d(1+q)\frac{1-q}{(1-qp)^3} - w < 0.$$

A solution to the first-order condition $\frac{\partial E^T}{\partial p} = 0$ then gives the optimal patronage $p^*$. Since $\frac{\partial E^T}{\partial p} |_{p=0} = \frac{1}{2} d \frac{q}{(1-pq)^2} > 0$, $p^* > 0$. At $p = 1$, $E^T = 0$ while $E^T > 0$ at $p < 1$; thus, $p^* < 1$. To get the comparative statics of $p^*$, note that

$$\frac{\partial^2 E^T}{\partial p \partial w} < 0, \qquad \frac{\partial^2 E^T}{\partial p \partial d} > 0.$$

Finally, take $w = 0$. Then, $\frac{\partial E^T}{\partial p} |_{w=0} \propto qp^2 - 2p + q$ and so

$$p^* |_{w=0} = \frac{1-\sqrt{1-q^2}}{q}.$$

■

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