Costly location in Hotelling duopoly*

Jeroen Hinloopen† and Stephen Martin‡

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Abstract

We introduce a cost of location into Hotelling's (1929) spatial duopoly. We derive the general conditions on the cost-of-location function under which a pure strategy price-location Nash equilibrium exists. With linear transportation cost and a suitably specified cost of location that rises toward the center of the Hotelling line, symmetric equilibrium locations are in the outer quartiles of the line, ensuring the existence of pure strategy equilibrium prices. With quadratic transportation cost and a suitably specified cost of location that falls toward the center of the line, symmetric equilibrium locations range from the center to the end of the line.

Key words: Horizontal product differentiation, spatial competition, cost of location

JEL Classification: D21, D43, L13.

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†University of Amsterdam, FEB/ASE, Roetersstraat 11, 1018 WB Amsterdam, The Netherlands; `J.Hinloopen@uva.nl`.

‡Department of Economics, Krannert School of Management, Purdue University, West Lafayette, Indiana 47907-1310, USA; `smartin@purdue.edu`.

1 Introduction

To paraphrase Stigler (1964), no one has the right to invite attention to another extension of Hotelling (1929) without advance indication of the justification for doing so. Our justification is the observation that in a literature where what matters is "location, location, location,"[1] location itself has been treated as a free good. Since economics is sometimes referred to as "the science of scarcity," this seems an odd specification for economists to make, and we introduce a rental cost of location that varies with location on the Hotelling line.[2]

One interpretation of the received approach is that Hotelling implicitly assumed the cost of location to be independent of location, normalized it to 0 for simplicity, and that the literature has followed this approach. But rental costs typically differ by location. In Europe, center-city locations are typically more expensive than those on the periphery. The same was true of the United States through the mid-1950s, and the opposite may be the case for some U.S. cities today.[3]

Hotelling (1929) rebelled against the assumption of homogeneous products because of its implication, in the Bertrand duopoly model, that all demand would switch from one supplier to another in response to an infinitesimal difference in price. Introducing product differentiation and making, as he thought, demands continuous functions of prices, he reached the Principle of Minimum Differentiation, that "Buyers are confronted everywhere with an excessive sameness." In a celebrated comment, d'Aspremont *et al.* (1979) showed that if firms locate "too close" to the center of the Hotelling line, there is no pure-strategy equilibrium in prices, because of precisely the kinds of discontinuities in demand that Hotelling had thought to avoid.[4] Numerical analysis (Osborne and Pitchik, 1987) indicates that equilibrium locations in the two-stage Hotelling game are within the region where there are no pure-strategy equilibria in prices.

d'Aspremont *et al.* (1979) further show that if, keeping all other aspects of Hotelling's specification, transportation cost is made quadratic rather than linear in distance, duopolists will choose maximum rather than minimum differentiation.

We consider a two-stage model, with costly location choice in the first stage, followed by a price-setting stage. Throughout the paper, we emphasize the interpretation of location cost $c(y)$ as a rental cost that varies with distance y from the end of the line. This interpretation naturally suggests the polar opposite cases that cost of location rises moving from the ends to the center and alternatively that cost of location rises moving from the center to the end of the line. One might instead interpret $c(y)$ as an R&D or product-development cost that must be incurred to bring a product of attribute y to market, in which case the Hotelling line represents product characteristic space rather than geographic space. Product development cost may vary with horizontal product characteristics, in a way that is context-specific; the truly black tulip, despite much effort, does not exist. We emphasize the geographic interpretation of the Hotelling line because it suggests natural specifications for $c(y)$.

For the first stage, we derive the general conditions on the cost of location function that must be met for an equilibrium to exist. That is, we find conditions under which location costs ensure the existence of a pure-strategy location-price equilibrium. With linear transportation cost, these conditions require that cost of location rise moving toward the center of the line at an increasing rate. With quadratic transportation cost, the conditions for existence of an equilibrium require that transportation cost do not fall moving toward the ends of the line. We illustrate our general results for specific cost-of-location functions.

In Section 2 we give references to the spatial oligopoly literature. In Section 3 we present a Hotelling Main Street duopoly model with linear transportation cost and location cost rising toward the center of the line. In Section 4 we present the model with quadratic transportation cost and location cost rising toward the ends of the line. Section 5 concludes.

2 Literature

The literature that flows from Hotelling (1929) is vast.[5] Extensions include[6] a finite reservation price (Lerner and Singer (1937), Salop (1979), Economides (1984), Hinloopen and Van Marrewijk (1999)), a circular road (Chamberlin (1953), Vickrey (1964/1999), Samuelson (1967), Salop (1979), Economides (1989)), graphs (Soetevent (2010)), nonlinear transportation costs (d'Aspremont *et al.* (1979), Capozza and Van Order (1982), Economides (1986)), more than two firms (Chamberlin (1933), Lerner and Singer (1937), Shaked (1982)), quantity competition (Hamilton *et al.*, (1994), Gupta *et al.* (1997)), sequential entry with no relocation (Prescott and Visscher (1977), Eaton and Ware (1987)), price-taking firms (Anderson and Engers (1994), Hinloopen (2002)), competition in $n > 1$ dimensions (Irmen and Thisse (1998)), and non-uniform distributions of consumers along the Hotelling line (Anderson, Goeree and Ramer (1997)). If nothing else, this literature establishes that the equilibrium predictions of a spatial oligopoly model are highly sensitive to the details of the specification. Introducing a cost of location is not an exception to this characteristic of the literature.

3 Linear transportation cost

We first consider the case of linear transportation cost. Assuming that the location cost function $c(y)$ is twice continuously differentiable, we show that existence of a subgame perfect price-location Nash equilibrium requires that the first and second derivative of $c(y)$ be positive. We illustrate this general result for a specific functional form of the cost-of-location function.

3.1 Stage 2: price setting

The market consists of a line of length $l = 1$, along which consumers are uniformly distributed. There are two firms, A and B, located at distances a and b respectively from the left and right ends of the line with $a + b \leq l$. The firms supply a homogeneous product that yields gross surplus v. Consumers have unit demand, and a consumer located at x (measured from the left side of the market) has net surplus

$$U(x;a) = v - t\,|x-a| - p_A, \tag{1}$$

if buying from firm A, where $t>0$ is the transportation rate, and p_i is the price charged by firm i, $i = A, B$, and net surplus

$$U(x;b) = v - t\,|1-b-x| - p_B. \tag{2}$$

if the product is bought from firm B. v is large enough that all consumers always buy.[7]

The boundary consumer has identical net surplus from either firm, $U(x;a) = U(x;b)$, and is at distance

$$x^* = \frac{1}{2t}\left[p_B - p_A + t(1+a-b)\right] \tag{3}$$

from the left end of the line.

Fixed and marginal cost of production are constant, and (without loss of generality) normalized to be zero. Let $c(y) > 0$ be the location cost function, where y is the distance from the firm's location to the nearest end of the line ($y = a$ for firm A, $y = b$ for firm B). We assume that c is twice continuously differentiable. In a familiar way (see e.g. Martin (2002)), now allowing for the cost of location, the objective functions of firm A and B (conditional on prices and locations) are

$$\pi_A(p_A, p_B, a, b) =$$

$$\begin{cases} p_A - c\,(a) & p_A < p_B - t(1-a-b) \\ \frac{1}{2t} p_A\left[t(1+a-b) + p_B - p_A\right] - c\,(a) & |p_A - p_B| \leq t(1-a-b) \\ -c\,(a) & p_A > p_B - t(1-a-b) \end{cases} \tag{4}$$

and

$$\pi_B(p_A, p_B, a, b) =$$

$$\begin{cases} p_B - c\,(b) & p_B < p_A - t(1-a-b) \\ \frac{1}{2t} p_B\left[t(1-a+b) + p_A - p_B\right] - c\,(b) & |p_A - p_B| \leq t(1-a-b) \\ -c\,(b) & p_B > p_A - t(1-a-b) \end{cases} \tag{5}$$

respectively.

The existence of location cost rules out back-to-back, zero-price equilibria (which would imply negative payoffs). Conditions for the existence of pure-strategy price equilibria with firms at different locations are due to d'Aspremont *et al.* (1979); we state them in the form given by Martin (2002).

Proposition 1 *(d'Aspremont et al. (1979)) For $a+b<1$, a pure-strategy price equilibrium exists if, and only if*

$$a \leq 3 + b - 6\sqrt{b} \tag{6}$$

$$b \leq 3 + a - 6\sqrt{a}. \tag{7}$$

If (6) and (7) are satisfied, equilibrium prices and payoffs, given locations, are

$$p_A^*(a,b) = t\left(1 + \frac{a-b}{3}\right), \tag{8}$$

$$p_B^*(a,b) = t\left(1 - \frac{a-b}{3}\right), \tag{9}$$

$$\pi_A^* = \frac{1}{2t}\left(p_A^*\right)^2 - c\left(a\right), \tag{10}$$

$$\pi_B^* = \frac{1}{2t}\left(p_B^*\right)^2 - c\left(b\right), \tag{11}$$

where asterisks denote second-stage equilibrium values, taking locations as given.

Proof. d'Aspremont *et al.* (1979). ■

As d'Aspremont *et al.* remark, for symmetric locations, (6) and (7) simplify to

$$a = b \leq \frac{1}{4}. \tag{12}$$

3.2 Stage 1: choice of location

Let $\widehat{\pi}_A\left(a,b\right)$ denote firm A's stage 1 payoff function in the Hotelling model without location cost, so that $\pi_A\left(a,b\right) = \widehat{\pi}_A\left(a,b\right) - c\left(a\right)$.

Proposition 2 *Necessary and sufficient conditions for the existence of a subgame perfect pure-strategy location-price equilibrium are*
(a) that (6) and (7) be satisfied for locations satisfying the location first-order conditions

$$\frac{t}{3}\left(1 + \frac{a^* - b^*}{3}\right) - c'\left(a^*\right) \equiv 0 \tag{13}$$

and

$$\frac{t}{3}\left(1 + \frac{b^* - a^*}{3}\right) - c'\left(b^*\right) \equiv 0, \tag{14}$$

(provided the implied $a \geq 0$, $b \leq 1$),
(b) the location second-order conditions

$$\frac{t}{9} - c''\left(a^*\right) < 0 \tag{15}$$

and

$$\frac{t}{9} - c''\left(b^*\right) < 0; \tag{16}$$

(c) the participation constraints

$$\pi_A(a^*, b^*) = \frac{1}{2}t - c(a^*) \geq 0 \tag{17}$$

$$\pi_B(a^*, b^*) = \frac{1}{2}t - c(b^*) \geq 0; \tag{18}$$

and
(d) $c(a)$ *rises more rapidly than* $\widehat{\pi}_A(a, b^*)$ *over the range* $\frac{1}{4} \leq a \leq \widetilde{a}(b^*)$*, where* $\widetilde{a}(b^*)$ *is A's best-response location to* b^* *in the game without location cost.*

Proof. The presence of cost of location that is sunk in the pricing stage does not alter the region where there is a pure-strategy price equilibrium or the region where there is a mixed-strategy price equilibrium, save that it rules out back-to-back, zero-price equilibria. Substitute (8) and (9) into (10) and (11), respectively, to obtain expressions for the first-stage objective functions. The first- and second-order conditions are immediate. The first-order conditions imply that equilibrium is symmetric. Then

$$p_A^*(a^*, b^*) = t. \tag{19}$$

With (10) and the requirement that equilibrium payoffs be nonnegative, one obtains the participation constraints (17) and (18). This establishes that (a^*, a^*) is a local maximum pair of locations on $0 \leq a \leq \frac{1}{4}$. Turning to condition (d), the location first-order conditions (13), (14) imply that if $c'(a)$ is sufficiently small on $\frac{1}{4} \leq a \leq \widetilde{a}(b^*)$, there is a local maximum pair of locations in the range $\frac{1}{4} < a < 0.27$ (Osborne and Pitchik (1986)), and this is the global maximum; if $c'(a)$ is sufficiently large, there is no local maximum pair of locations in the central quartiles. For intermediate values of $c'(a)$, payoff functions have two local maxima, and the global maximum is in the outer quartile; condition (d) is sufficient for the latter case to hold.[8] ■

The first-order conditions imply $c' > 0$; the second-order conditions imply $c'' > 0$. That is, with linear transportation costs for a pure-strategy price-location equilibrium to exist it is necessary that the cost of location rises toward the center of the Hotelling line at an increasing rate.

Corollary 3 *(a) Location best-response lines have negative slope,*

$$\left.\frac{da}{db}\right|_{brf} = \frac{\frac{\partial^2 \pi_A(a,b)}{\partial a \partial b}}{-\frac{\partial^2 \pi_A(a,b)}{\partial a^2}} = \frac{-\frac{1}{9}t}{-\frac{\partial^2 \pi_A(a,b)}{\partial a^2}} < 0 \tag{20}$$

(the denominator on the right is positive by the second-order condition), and
(b) Increases in t *move the symmetric equilibrium location toward the center of the line,*

$$\frac{da^*}{dt} = \frac{1}{3c''(a^*)} > 0. \tag{21}$$

3.3 Example I

Let the location cost function take the form[9]

$$c\left(y\right)=y^{\beta}, \tag{22}$$

for $y=a$, b (now omitting asterisks where possible without confusion, for notational compactness). We show below that $\beta>1$ is one of the conditions for the existence of a pure-strategy price-location equilibrium. For $\beta>1$, the location cost function (22) is a proper fraction raised to a power greater than 1. Larger values of β then imply smaller location cost (see Figure 1).

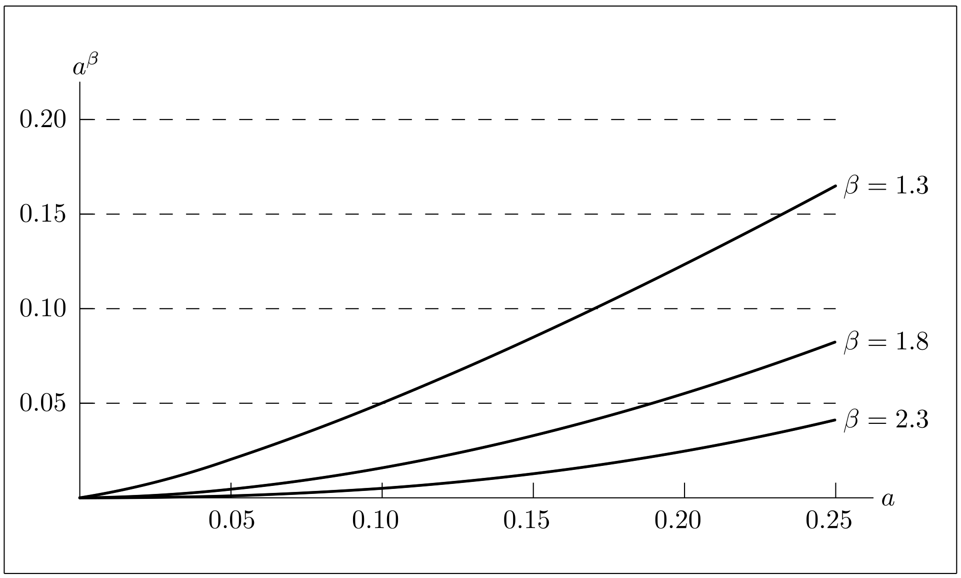

Figure 1: Rental cost function, example I, $\beta=1.2, 1.8, 2.3$.

3.3.1 Equilibrium locations

For firm A, (13) gives the first-order condition (similarly for firm B)

$$\frac{\partial\pi_A\left(a,b\right)}{\partial a}=\frac{1}{3}t\left(1+\frac{a-b}{3}\right)-\beta a^{\beta-1}\equiv 0, \tag{23}$$

from which the symmetric equilibrium locations are

$$a^*=\left(\frac{t}{3\beta}\right)^{\frac{1}{\beta-1}}. \tag{24}$$

For (24) to be valid, the equilibrium it identifies must satisfy four conditions (i) the second-order condition of the location stage, (ii) the d'Aspremont *et al.* symmetric-equilibrium outer-quartile condition (12), (iii) the firm participation (nonnegative profit) condition, and (iv) the stability condition in location space.

3.3.2 Second-order conditions

Evaluating the second derivative for the equilibrium location (24), the second-order condition is

$$\frac{\partial^2 \pi_A^*}{\partial a^2} = \frac{1}{9}t - \beta(\beta - 1)\left(\frac{t}{3\beta}\right)^{\frac{\beta-2}{\beta-1}} < 0. \tag{25}$$

(25) defines the region in (t, β)-space where the second-order condition is satisfied. $\beta > 1$ is necessary for the second-order condition to be satisfied. For $t = 1$, the second-order condition is satisfied for all $\beta > 1$.

3.3.3 Outer-quartile condition

The outer-quartile condition, which we assume is met, is

$$a^* = \left(\frac{t}{3\beta}\right)^{\frac{1}{\beta-1}} \leq \frac{1}{4}. \tag{26}$$

(26) defines the region in (t, β)-space where the outer-quartile condition is satisfied. If in (26) we normalize $t = 1$, then, numerically,[10] the outer-quartile condition is satisfied for

$$1 < \beta \leq 2.4342. \tag{27}$$

3.3.4 Firm participation constraint

From (8) and (9), symmetric equilibrium prices are

$$p_A^* = p_B^* = t. \tag{28}$$

In equilibrium, each firm supplies half the market, that is,[11]

$$q_A^* = q_B^* = \frac{1}{2}. \tag{29}$$

Then the participation constraint is

$$\pi_A^* = \frac{1}{2}t - \left(\frac{t}{3\beta}\right)^{\frac{\beta}{\beta-1}} \geq 0, \tag{30}$$

which defines the region in (t, β)-space where the firm participation condition is satisfied. For $t \leq 3$, the firm participation constraint is met for all $\beta > 1$.

3.3.5 Stage 1 stability

Necessary conditions for stability are that the trace of the matrix of second-order partial derivatives of payoffs functions be negative, and the determinant positive, when evaluated at equilibrium values. The assumption that the second-order conditions are met means that the trace condition is met.

The stability matrix is

$$\begin{pmatrix} \frac{1}{9}t - \beta(\beta-1)\left(\frac{t}{3\beta}\right)^{\frac{\beta-2}{\beta-1}} & -\frac{1}{9}t \\ -\frac{1}{9}t & \frac{1}{9}t - \beta(\beta-1)\left(\frac{t}{3\beta}\right)^{\frac{\beta-2}{\beta-1}} \end{pmatrix}, \tag{31}$$

with determinant

$$-\beta(\beta-1)\left(\frac{t}{3\beta}\right)^{\frac{\beta-2}{\beta-1}}\left[\frac{2}{9}t - \beta(\beta-1)\left(\frac{t}{3\beta}\right)^{\frac{\beta-2}{\beta-1}}\right]. \tag{32}$$

The determinant of the stability matrix is positive if

$$\frac{2}{9}t - \beta(\beta-1)\left(\frac{t}{3\beta}\right)^{\frac{\beta-2}{\beta-1}} < 0, \tag{33}$$

a stronger condition than (25). If we normalize $t = 1$, condition (33) is met for all $\beta > 1$.

3.3.6 Location best-responses

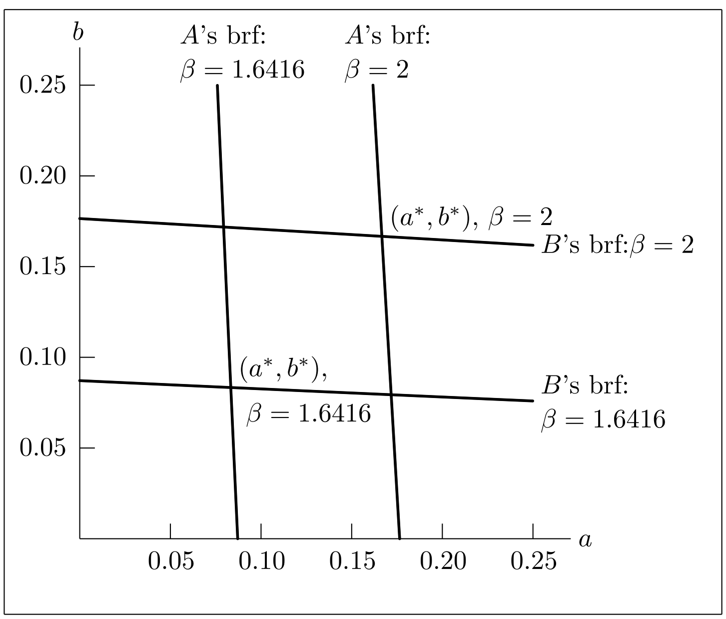

Figure 2: Location best response curves with linear transportation cost and cost of location $c(y) = y^{\beta}$.

Firm A's first-order condition (23) can be solved for b,

$$b = a + 3 - 9\frac{\beta}{t}a^{\beta-1}, \tag{34}$$

to obtain an equation for firm A's location best-response equation, written in inverse form.

Figure 2 shows bests response curves and equilibrium locations for two values of β, $\beta = 1.6416$ and $\beta = 2$.[12] Best-response lines slope downward: location choices are strategic substitutes. A larger value of β means smaller location cost, all else equal, and (as one expects), the equilibrium location shifts toward the center of the line as β increases.

In sum, for transportation cost $t = 1$ and for β satisfying condition (d) of proposition 2, the price-location pair $(p^*, a^*) = \left(1, (3\beta)^{\frac{1}{1-\beta}}\right)$ is a pure strategy subgame perfect Nash equilibrium for the Hotelling model with linear transportation cost and cost of location (24) for $\beta \in (1, 2.4342]$. Figure 3 shows the equilibrium locations over the admissible range of β.

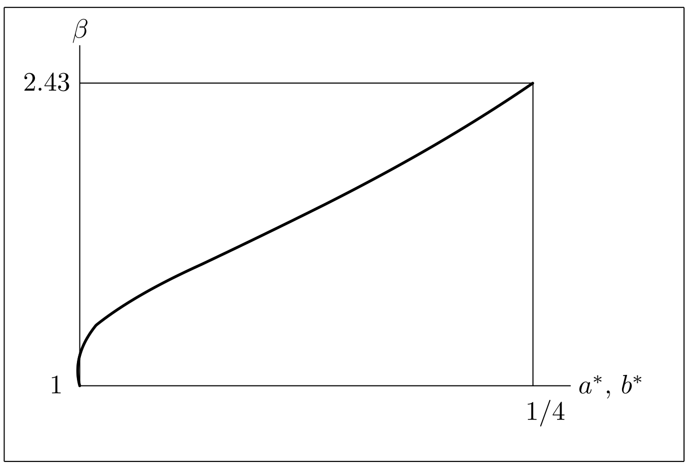

Figure 3: Equilibrium locations in the Hotelling model as a function of β, cost of location $c(x) = x^\beta$ $(t = 1)$.

4 Quadratic transportation cost

4.1 Stage 2: price setting

Following d'Aspremont *et al.* (1979), assume now that a consumer located at x has net utility

$$U(x; a) = v - t\,|x - a|^2 - p_A \tag{35}$$

if buying from firm A,

$$U(x; b) = v - t\,|1 - b - x|^2 - p_B \tag{36}$$

if buying from firm B.

The location of the marginal consumer is

$$x^* = a + \frac{p_B - p_A}{2t(1-a-b)} + \frac{1-a-b}{2}, \tag{37}$$

from which the profits of both firms conditional on price and location follow[13]

$$\pi_A(p_A, p_B, a, b) =$$
$$\begin{cases} p_A - c(a) & a + \frac{p_B - p_A}{2t(1-a-b)} + \frac{1-a-b}{2} > 1; \\ p_A \left[a + \frac{p_B - p_A}{2t(1-a-b)} + \frac{1-a-b}{2}\right] - c(a) & 0 \leq a + \frac{p_B - p_A}{2t(1-a-b)} + \frac{1-a-b}{2} \leq 1; \\ -c(a) & a + \frac{p_B - p_A}{2t(1-a-b)} + \frac{1-a-b}{2} < 0, \end{cases} \tag{38}$$

and

$$\pi_B(p_A, p_B, a, b) =$$
$$\begin{cases} p_B - c(b) & b + \frac{p_A - p_B}{2t(1-a-b)} + \frac{1-a-b}{2} > 1; \\ p_B \left[b + \frac{p_A - p_B}{2t(1-a-b)} + \frac{1-a-b}{2}\right] - c(b) & 0 \leq b + \frac{p_A - p_B}{2t(1-a-b)} + \frac{1-a-b}{2} \leq 1; \\ -c(b) & b + \frac{p_A - p_B}{2t(1-a-b)} + \frac{1-a-b}{2} < 0. \end{cases} \tag{39}$$

Absent cost of location, d'Aspremont *et al.* (1979) show that for this situation, a unique price equilibrium exists for any locations a and b, and that it is given by

$$p_A^*(a,b) = t(1-a-b)\left(1 + \frac{a-b}{3}\right), \tag{40}$$

$$p_B^*(a,b) = t(1-a-b)\left(1 - \frac{a-b}{3}\right). \tag{41}$$

4.2 Stage 1: choice of location

Equilibrium prices (40), (41) fall as a firm approaches the rival's location. d'Aspremont *et al.* (1979) show that in the two-stage game with quadratic transportation cost and without cost of location, firms locate at the ends of the line, each supplying half the market at the maximum noncooperative equilibrium price. Results with location cost are given below, and for a simple parameterization, equilibrium locations can be arbitrarily close to the center of the line.

Proposition 4 *Necessary and sufficient conditions for the existence of a subgame pure-strategy location-price equilibrium are the location first-order conditions*

$$-\frac{t}{18}(3 + a^* - b^*)(1 + 3a^* + b^*) - c'(a^*) \equiv 0, \tag{42}$$

and

$$-\frac{t}{18}(3+b^*-a^*)(1+3b^*+a^*)-c'(b^*)\equiv 0, \tag{43}$$

(provided the implied $a\geq 0$*,* $b\leq 1$*), the location second-order conditions*

$$-\frac{t}{9}(5+3a^*-b^*)-c''(a^*)<0, \tag{44}$$

and

$$-\frac{t}{9}(5+3b^*-a^*)-c''(b^*)<0, \tag{45}$$

and the firm participation constraints

$$\pi_A\left(a^*,b^*\right)=\frac{t}{2}(1-a^*-b^*)-c\left(a^*\right)\geq 0, \tag{46}$$

and

$$\pi_B\left(a^*,b^*\right)=\frac{t}{2}(1-a^*-b^*)-c\left(b^*\right)\geq 0. \tag{47}$$

Proof. Substitute (40) and (41) into (38) and (39), respectively, to obtain expressions for the first-stage objective functions. The first- and second-order conditions are immediate. The first-order conditions imply that equilibrium is symmetric. Then boundary consumers are located at the center of the line, $x^*=1/2$, and equilibrium prices are

$$p_A^*(a^*,b^*)=p_B^*(a^*,b^*)=t(1-a^*-b^*), \tag{48}$$

from which the participation constraints (46) and (47) follow. ■

The first-order conditions imply that $c'<0$. The second-order condition may be satisfied for c'' positive or negative, provided it is not below a lower bound. That is, with quadratic transportation costs for a pure-strategy price-location equilibrium to exist it is necessary that the cost of location does not rise toward the center of the Hotelling line.

Proposition 4 implies

Corollary 5 *(a) Location best-response lines have negative slope,*

$$\left.\frac{da}{db}\right|_{brf}=\frac{\frac{\partial^2\pi_A(a,b)}{\partial a\partial b}}{-\frac{\partial^2\pi_A(a,b)}{\partial a^2}}=\frac{-\frac{1}{9}t\left(1-a-b\right)}{-\frac{\partial^2\pi_A(a,b)}{\partial a^2}}<0 \tag{49}$$

(the denominator on the right is positive by the second-order condition), and (b) Increases in t *move the symmetric equilibrium location toward the end of the line,*

$$\frac{\partial a^*}{\partial t}=-\frac{1}{6}\frac{1+4a}{t+c''\left(a\right)}<0. \tag{50}$$

($c''\left(a\right)\geq -t/9(5+3a^*-b^*$ *by the second-order condition, so the denominator on the right is positive).*

4.3 Example II

Let the location cost function take the form[14]

$$c\,(a) = \rho \left(\frac{1}{2} - a \right)^2 , \tag{51}$$

where $\rho > 0$ is a location cost scale parameter. For this specification, location cost takes its maximum value at the ends of the line, and falls to 0 at the center of line (Figure 4).

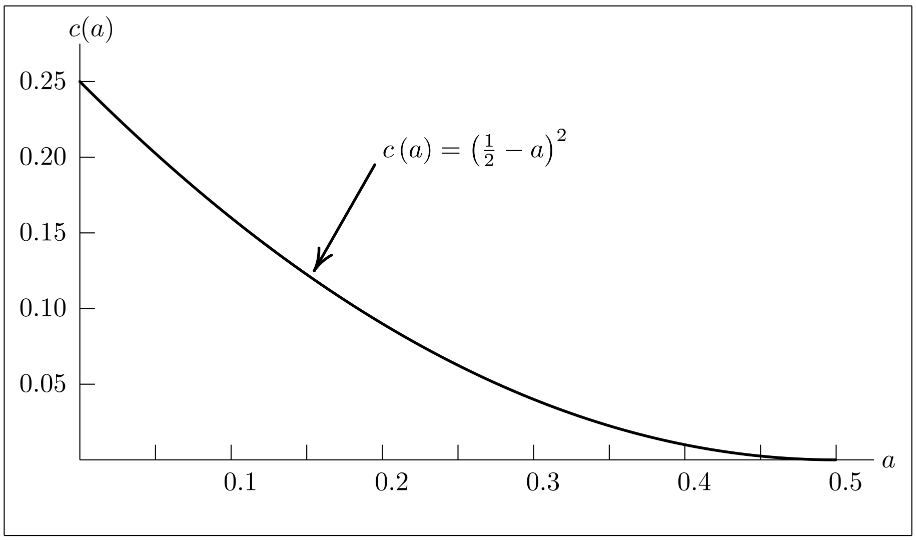

Figure 4: Rental cost function, example II, $\rho = 1$.

It is immediate that the location second-order conditions (44) and (45) are satisfied for (51).

4.3.1 Equilibrium locations

For the location cost function (51), firm A's stage 1 objective function is

$$\pi_A(a, b) = \frac{1}{2} t(1 - a - b) \left(1 + \frac{a - b}{3}\right)^2 - \rho \left(\frac{1}{2} - a\right)^2 . \tag{52}$$

Firm A's location first-order condition is

$$\frac{\partial \pi_A(a, b)}{\partial a} = -\frac{t}{6} \left(1 + \frac{a - b}{3}\right) (1 + 3a + b) + 2\rho \left(\frac{1}{2} - a\right) \equiv 0. \tag{53}$$

In symmetric equilibrium,

$$a^* = \frac{1}{2} \frac{\rho - \frac{t}{6}}{\rho + \frac{1}{3} t} = \frac{1}{2} \frac{\frac{\rho}{t} - \frac{1}{6}}{\frac{\rho}{t} + \frac{1}{3}}. \tag{54}$$

$a^* \geq 0$ requires

$$\rho \geq \rho_{\min} \equiv \frac{t}{6}, \tag{55}$$

and we assume this condition is met.

From (54), a^* increases as $\frac{\rho}{t}$ increases. That is, a^* increases as ρ increases and falls as t increases, ceteris paribus.

(54) implies that a^* is never at the center of the line, although it approaches the center of the line asymptotically as $\rho \to \infty$, or $t \to 0$ for a given ρ (Figure 5).

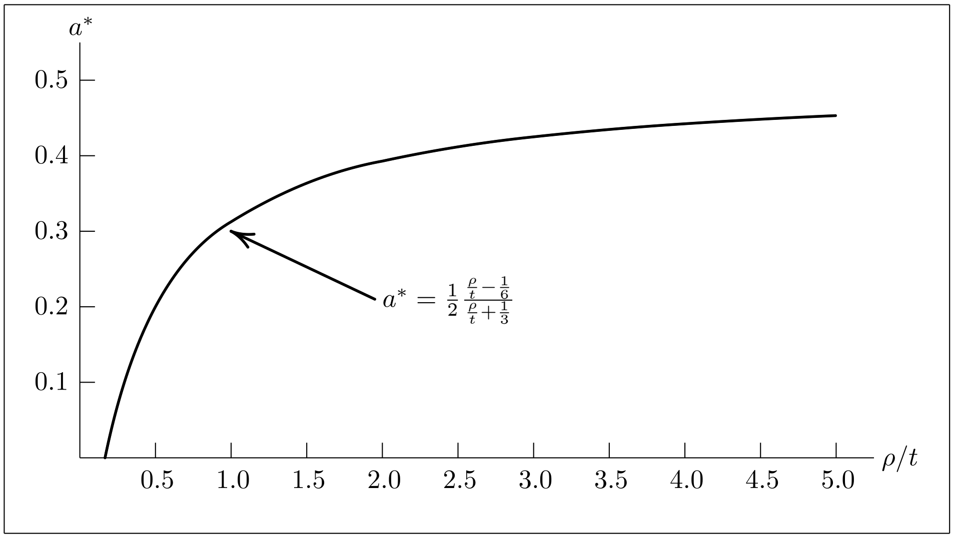

Figure 5: Equilibrium location as a function of ρ/t.

4.3.2 Firm participation constraint

The symmetric equilibrium payoff is

$$\pi_A^* = \frac{3}{16}(9\rho + 4t)\left(\frac{t}{3\rho + t}\right)^2 > 0. \tag{56}$$

The firm participation constraint is always met.

4.3.3 Stage 1 stability

The matrix of equilibrium second derivatives of payoff functions is

$$\begin{pmatrix} -\frac{1}{2}\frac{(6\rho+t)(2\rho+t)}{3\rho+t} & -\frac{1}{6}\frac{t^2}{3\rho+t} \\ -\frac{1}{6}\frac{t^2}{3\rho+t} & -\frac{1}{2}\frac{(6\rho+t)(2\rho+t)}{3\rho+t} \end{pmatrix}. \tag{57}$$

The trace is negative. The determinant of the stability matrix is

$$\frac{2}{9}\left(18\rho^2 + 12t\rho + t^2\right) > 0; \tag{58}$$

the stability condition is satisfied.

4.3.4 Location best-responses

The first-order condition (53) implicitly defines firm A's location best-response function. Best response curves slope downward and, as ρ/t rises, equilibrium locations move toward the center of the line (Figure 6).

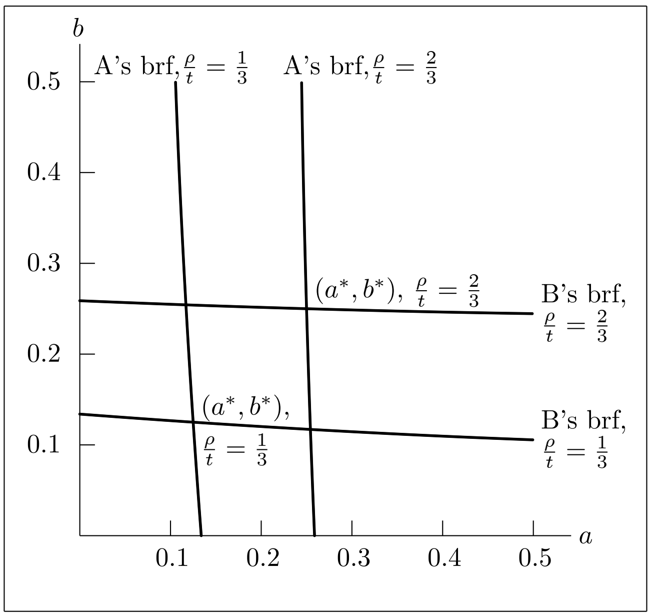

Figure 6: Location best-response curves with quadratic transportation cost and cost of location $c(y) = \rho(1/2 - y)^2$, $\frac{\rho}{t} = \frac{1}{3}$ and $\frac{\rho}{t} = \frac{2}{3}$.

In sum, adding a cost of location that declines moving toward the center of the line to the Hotelling model with quadratic transportation costs can reverse the conclusion of d'Aspremont *et al* (1979). In particular, it can yield a situation approaching endogenous minimum spatial differentiation that does not suffer from the problems in Hotelling (1929).

5 Conclusion

We have motivated our specification with the observation that rent varies with location. In the Hotelling Main Street model with linear transportation cost, if locations toward the center of the line command sufficiently higher rent, rent acts as a centrifugal force that induces firms to locate outside the region where equilibrium prices are in mixed strategies. In the Hotelling Main Street model with quadratic transportation cost, if locations toward the ends of the line command sufficiently higher rent, rent acts as a centripetal force that induces firms to locate toward the center of the line.

We envisage extending the present work to derive the equilibrium rent-location relationship from the distribution of the population. This will permit us to examine conditions leading to the hollowing-out of center cities that is observed in the United States from the latter part of the twentieth century.

Notes

[1] Safire (2009) attributes the phrase to the late Lord Harold Samuel, a British real estate tycoon.

[2] "Cost of location" is something other than "relocation costs." The latter starts from a particular situation, and then tells a dynamic story (although often within a static framework). Our design corresponds to the case of a firm that must incur a location-dependent rental cost to set-up before it can set price.

[3] See Karmon (2012) for a comparison of center-city and suburban rental costs for selected U.S. cities, and Bernard (2010) for a city-suburbs comparison of the overall cost-of-living.

[4] See their footnote 1. See also Vickrey (1964, 1999).

[5] For surveys, see Archibald *et al.* (1986), Morris (1997).

[6] These references do not include the closely-related literature that models basing-point and other spatial pricing policies.

[7] That is, the market is covered. The analysis produces conditions on v for the market to be covered.

[8] If the payoff function is differentiable, the condition is $c'(a) \geq \frac{\partial \widehat{\pi}_A(a,\widetilde{a})}{\partial a}$ for $0.25 \leq a \precsim 0.27$. The condition can be expressed in terms of discrete changes.

[9] Qualitatively similar results are obtained for the quadratic location cost function $c(y) = ry^2$, where $r > 0$.

[10] The upper limit solves $(3\beta)^{-\frac{1}{\beta-1}} = 1/4$.

[11] Since firms locate in the outer quartiles, a firm's most distant customers are located at the center of the line. The net utility of such a consumer is

$v - t\left[3/2 - (t/3\beta)^{\frac{1}{\beta-1}}\right]$, and the market coverage condition is that this be non-negative. The market coverage condition on v, which we assume is met, can be written $v \geq t\left(3/2 - (t/3\beta)^{1/(\beta-1)}\right)$.

[12] The corresponding equilinrium locations are $1/12$ and $1/6$, respectively.

[13] The market coverage condition on v is $v \geq 5t/4$.

[14] Qualitatively similar results are obtained for the linear location cost function $c(a) = \gamma\,(\gamma - a)$, with $\gamma > 0$.

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