Market Structure, Factor Endowment and Technology Adoption*

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Abstract

A simple model is developed to explore how technology adoption depends on factor endowment when the new technology is more capital-intensive and privately accessible. The endogenous non-competitive market structure of the final good indirectly distorts factor prices in general equilibrium, which results in a non-monotonic impact of capital endowment on both the static allocation efficiency and the dynamic pattern of industrial upgrading. More specifically, the static equilibrium achieves social efficiency when capital endowment is sufficiently large or sufficiently small, irrespective of the resulting market structure. Inefficiency arises only when capital falls onto an intermediate range, in which case the private technology is under-utilized to depress the relative price of capital. Moreover, an increase in the initial capital endowment may delay rather than facilitate the adoption of the capital-intensive technology. Private accessibility to the new technology may also result in premature adoption, over-utilization and multiple equilibria. Welfare-enhancing policies are discussed.

Key Words: Technology Adoption, Capital Accumulation, Market Structure, Industrial Upgrading, Economic Growth

JEL Codes: E23, O14, O33, O41

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1 Introduction

Economic growth is a process of industrial upgrading, along which capital accumulates and technologies advance. At the micro level, we must understand the incentive and behavior of a potential investor (firm) who possesses a new technology and decides whether and when to adopt it in the production. The goal of this paper is to develop a simple dynamic general equilibrium model to theoretically investigate how an existing but privately accessible capital-intensive technology is adopted in a two-factor market economy with endogenous saving and also analyze the allocation efficiency in this industrial upgrading process. In the model, there is one final good, which can be produced by two alternative technologies, new and old. The two technologies differ in two dimensions. First, the new technology is more capital-intensive. Second, the new technology is private to one potential entrant and subject to free imitation one period after adoption, whereas the old one is publicly and freely accessible. Consequently, which technology is better shall depend on the relative factor prices. However, adopting the new technology involves non-competitive market structures for the final good, which may indirectly distort the factor prices in a general-equilibrium fashion even though the factor markets are assumed to be perfect, so the equilibrium factor prices may no longer serve as the accurate signals to guide the socially optimal technology adoption and industry upgrading.

A novel feature of this technology adoption model is that the new technology monopolist (called firm M thereafter) makes adoption and production decisions by taking into account the general equilibrium effect on the intra-temporal and inter-temporal prices of its private accessibility to the new technology. The monopoly rent extracted from this new technology depends on the capital endowment, which accumulates endogenously depending on households' rational expectation on when the new technology is adopted, which in turn affects the market structure and the return to the private technology. Once implemented, the new technology will be fully imitated in the next period and become publicly available afterwards, so the market structure restores perfect competition. Firm M understands that its adoption decision not only affects the dynamics of market structure (due to the lagged information externality) and the current value of profit, but also affects the inter-temporal interest rate (that is, time discounting rate of profit) due to the consumption smoothing motive of the forward-looking households. To make optimal decisions, firm M must take all the aforementioned forces into account when pondering over the following trade-off: Early implementation avoids time discounting of profit, but the instantaneous profit can be larger in the future when the rental-wage ratio becomes lower due to the endogenous capital accumulation.

I first study this problem in a static general equilibrium model, in which I characterize and compare two different cases. One is the first-best benchmark case when both technologies are publicly and freely available, so the market structure is perfect competition. The other is when the capital-intensive technology is privately known only to firm M, so firm M may have monopoly power subject to the limit pricing because the old (labor-intensive) technology is publicly available. In both cases the capital-intensive technology is adopted if and only if the capital endowment is larger than a finite cutoff value. It turns out that this cutoff value is the same for these two different cases. In addition, I show that monopoly still achieves the social (Pareto) efficiency when capital is sufficiently large, which appears to contradict the textbook partial-equilibrium result. The reason is that the monopoly structure has two opposite effects in general equilibrium. One is the conventional negative price effect on demand due to the price markup, but the other is the positive income effect on demand as the monopoly profit now becomes part of the household income. It turns out that these two forces exactly cancel out when capital endowment is sufficiently large.

However, inefficiency arises when and only when capital endowment falls on an intermediate range, in which circumstance both technologies are operating but the total output is depressed compared with the social optimum. Different from the conventional partial-equilibrium rationale, now the new capital-intensive technology is under-utilized mainly because by doing so, firm M can depress the rental price of capital in general equilibrium to maximize its monopoly rent. Despite the lower total output, the old (labor-intensive) technology is overly used from the social efficiency point of view. More precisely, when capital endowment falls to a certain non-empty interval, the labor-intensive technology is fully abandoned if both technologies are publicly available, but the labor-intensive technology continues to operate when the capital-intensive technology is monopolized.

Then I study a simple two-period dynamic model, in which I compare the first-best case and the case with private capital-intensive technology. The dynamic problem is decomposed into two steps. The first step is to decide the inter-temporal capital allocation (optimal saving decision). The second step is to study the intra-temporal allocation across the two technologies given the capital endowment and the technology availability for each period, for which the obtained results for the static model are intensively applied. The first-best case is fully characterized: there are six different dynamic patterns of technology adoption and industrial upgrading, depending on the initial capital endowment and the efficiency in the capital good production. Generally speaking, the capital-intensive technology is adopted earlier and utilized more when the initial capital endowment is larger. However, the analysis becomes much more complicated when the capital-intensive technology is private, because firm M must consider capital allocation, market structures, and prices, both inter-temporally and intra-temporally, when deciding whether and when to adopt the new technology.

One striking result is that the new capital-intensive technology, when privately accessible, may sometimes be adopted even earlier than the case with both technologies publicly available. This socially inefficient premature adoption of the private new technology results from the fact that firm M can earn positive profits only by operating this unique technology, so it tries its best to implement this new technology as much as it could. This departs from the standard result in the existing growth literature of technology adoption, which addresses how the adoption of new technology is inefficiently delayed due to its private accessibility or various diffusion frictions (Parente (1994), Parente and Prescott (1994, 1999), Acemoglu and Robinson (2001), Comin and Hobijn(2010), Wang (2013)). Moreover, in the aforementioned literature the new technology is always strictly better than the old technology, independent of capital endowment, whereas this is not always true in my model because whether the new private technology is socially more efficient than the public technology endogenously depends on the capital endowment.

I also show that, under certain conditions, there exists a non-monotonic relation between the initial capital endowment and the equilibrium timing of adopting the capital-intensive technology. More precisely, when the initial capital endowment is larger than a threshold value, the capital-intensive technology is immediately adopted in the first period; When the initial capital endowment exceeds a higher threshold value, the first adoption of the capital-intensive technology is postponed to the second period; When the initial capital endowment exceeds an even higher threshold value, the capital-intensive technology is again immediately adopted in the first period. This non-monotonicity result stands in contrast with the monotonic relation in the (first-best) case when both technologies are free and public. It occurs mainly because the interest rate (different from the rental price of capital) is endogenously affected by the technology adoption in the Euler equation. Given the one-period lag free imitation, delay in first adoption of the capital-intensive technology yields higher instantaneous profit due to capital accumulation but it also causes inter-temporal discounting by the interest rate. When the initial capital endowment increases from a relatively low level, firm M finds it worthwhile to wait until next period so that capital accumulation can make the second-period instantaneous profit sufficiently larger than the instantaneous profit obtainable in the first period, even after taking into account the endogenous interest rate (discounting factor for profit). However, when the initial capital endowment becomes sufficiently large, the time discounting force becomes dominant, resulting in an immediate adoption of the new technology again.

The idea that the appropriate technology should be consistent with factor endowment can be dated at least back to the Heckscher-Ohlin trade model, where the mechanism is international specialization. Atkinson and Stiglitz (1969) are perhaps the first to formalize the idea that technological change is localized for a small range of capital-labor ratios, so technologies developed in rich countries are not necessarily suitable for developing countries. Basu and Weil (1998) build on that idea and study economic convergence and divergence in a Solow-type growth model with exogenous saving. Lin (2009) explores a wide array of development issues related to the consistency of industries (technologies) with the factor endowment in developing countries. Ju, Lin, and Wang (2013) develop an endogenous growth model with an infinite number of free technologies (industries) which are different in capital intensities, in which all the factor prices are socially efficient signals to guide the first-best technology adoption and industrial upgrading. Optimal adoption of technologies with different capital intensities is also studied in the context of international trade (Wang (2014)) and when there exists Marshallian externality at the industry level (Ju, Lin and Wang (2012)). Whereas these studies assume that all the technologies are publicly available in a perfect competition environment, the model developed here explores technology adoption when the capital-intensive technology is initially only privately accessible and hence the factor price signals may no longer serve as socially efficient signals due to the dynamic general equilibrium effect of the endogenous market structure.

This paper is also closely related to the huge literature of directed technical change, which mainly explores how the relative abundance of different factors affects the direction and the magnitude of endogenous technical change as well as various associated macroeconomic implications such as skill premium across workers and productivity differences across countries (Acemoglu (2002, 2007, 2009), Acemoglu and Zilibotti (2001), Jones (2005)). While it is technically straightforward to modify the current model setting such that the two distinct factors can be reinterpreted as skilled and unskilled labor, the analytical focus of this paper is different from that literature. Instead of exploring how relative factor abundance determines the endogenous technical change rate and favors which production factor dynamically, I mainly study the implications of the private accessibility of a capital-intensive technology for industry upgrading and technology adoption when capital accumulates endogenously.

In the model, firm M extracts monopoly rents from its private technology and the industry upgrades by operating the new technology, which gradually replaces the old technology as the economy grows. These features are shared by the growth literature of vertical innovation or "creative destruction"(Aghion and Howitt (1992, 2009), Grossman and Helpman (1991)). However, there are several key differences. First and the foremost, I focus on how a privately accessible and existing technology is adopted into the economy, whereas the vertical innovation literature explores how a firm optimally acquires the new technology via costly R&D. Second, the driving forces of industry upgrading (technology adoption) are also different. Factor endowment and capital accumulation endogenously determine which existing technology is superior (produces more) and how the new (capital-intensive) technology is adopted in my model, whereas costly R&D is the driving force of industry upgrading and technology adoption in the vertical innovation literature, because the key difference between alternative technologies is in the quality of the good they produce instead of the capital intensity in the production process. Third, the key policy implications are different. The major policy implications of the creative destruction literature concern how a developed economy can achieve the socially optimal level of costly R&D to maintain sustainable technological progress, whereas the major concern in my model is how to ensure an existing privately accessible technology is adopted in a developing economy at the socially optimal stage and scale.[1]

[1] One literature focuses on information externality and studies how the risk and uncertainty affect stragetic investment decisions under asymmetric information such as learning

The paper is structured as follows. Section 2 studies the static model, followed by the analysis of the dynamic model in Section 3. Brief policy suggestions are discussed in Section 4. The last section concludes.

2 Static Model

Consider a static autarky populated by a continuum of identical households with measure equal to unity. Each household is endowed with K units of capital and L units of labor (time). Define the endowment structure as $k \equiv \frac{K}{L}$. There is only one consumption good, which can be produced with two alternative Cobb-Douglas technologies: technology 1 and technology 2. Throughout the paper I also interchangeably call them industry 1 and industry 2, respectively. The corresponding production functions are given by $F^{[1]}(K_1, L_1) = A_1 K_1^{\alpha_1} L_1^{1-\alpha_1}$ and $F^{[2]}(K_2, L_2) = A_2 K_2^{\alpha_2} L_2^{1-\alpha_2}$, where A_i, K_i, L_i and α_i are the total factor productivity, capital, labor, and capital share for technology $i \in \{1, 2\}$. Without loss of generality, assume technology 2 is more capital intensive: $0 < \alpha_1 < \alpha_2 < 1$. Following the pertinent literature, when technology 2 is adopted, it is referred to as industry upgrading. A representative household's utility function is

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}, \text{ where } \sigma \leq 1.$$

I first analyze the competitive equilibrium when both technologies are freely and publicly available, and then I analyze the case in which technology 2 is accessible only to one firm (called firm M thereafter).

2.1 Perfect Competition

The Second Welfare Theorem holds, so the competitive equilibrium can be characterized by solving the following central planner problem:

$$\begin{aligned} G(K, L) \quad \equiv \quad & \max_{\{K_n, L_n\}_{n=1}^{2}} A_1 K_1^{\alpha_1} L_1^{1-\alpha_1} + A_2 K_2^{\alpha_2} L_2^{1-\alpha_2} \\ & s.t. \\ & K_1 + K_2 = K, \\ & L_1 + L_2 = L, \\ & K_n \geq 0, L_n \geq 0, n = 1, 2. \end{aligned}$$

The value function $G(K, L)$ is the endogenous aggregate production friction. The resource allocation problem is a standard nonlinear programming problem with a strictly concave objective function. As a result, there exists a unique

behaviors and strategic delay (Chamley, 2004). Ederington and McCalman (2009) focus on the dynamic tradeoff that an early entry (adoption) implies a larger market share but also an exogenously higher production cost. However, these are partial-equilibrium analyses and the factor endowment plays no role.

solution characterized by the Kuhn-Tucker condition. Let $k_n \equiv \frac{K_n}{L_n}$ denote the capital-labor ratio used for technology n. Define

$$k_1^* \equiv \left[\left(\frac{\alpha_1}{\alpha_2}\right)^{\alpha_2}\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{1-\alpha_2}\left(\frac{A_1}{A_2}\right)\right]^{\frac{1}{\alpha_2-\alpha_1}}, \tag{1}$$

$$k_2^* \equiv \left[\left(\frac{\alpha_1}{\alpha_2}\right)^{\alpha_1}\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{1-\alpha_1}\left(\frac{A_1}{A_2}\right)\right]^{\frac{1}{\alpha_2-\alpha_1}}. \tag{2}$$

Observe $\frac{k_1^*}{k_2^*} = \left(\frac{\alpha_1}{\alpha_2}\right)\left(\frac{1-\alpha_2}{1-\alpha_1}\right) < 1$.

Proposition 1 *In the perfect competitive equilibrium,*
(a) If $\frac{K}{L} \leq k_1^*$, *only technology 1 operates and* $G(K,L) = A_1 K^{\alpha_1} L^{1-\alpha_1}$.
(b) If $k_1^* < \frac{K}{L} < k_2^*$, *both technologies operate with resources allocated as follows:*

$$L_1 = \frac{k_2^* L - K}{k_2^* - k_1^*};\ L_2 = \frac{K - k_1^* L}{k_2^* - k_1^*}, \tag{3}$$

$$K_1 = k_1^* L_1^*;\ K_2 = k_2^* L_2^*. \tag{4}$$

where k_1^* *and* k_2^* *are given by (1) and (2). Moreover,* $G(K,L) = aK + bL$, *where* $a \equiv A_1\alpha_1 (k_1^*)^{\alpha_1 - 1}$ *and* $b \equiv (1-\alpha_1)A_1 (k_1^*)^{\alpha_1}$. *(c) If* $\frac{K}{L} \geq k_2^*$, *only technology 2 operates and* $G(K,L) = A_2 K^{\alpha_2} L^{1-\alpha_2}$.

Proof. If the solution is interior, then it satisfies two first-order conditions that

equate the marginal productivity of labor and capital across the two different technologies, which yields $k_1 = k_1^*$ and $k_2 = k_2^*$ as given by (1) and (2). Using the following factor market clearing conditions

$$k_1^* L_1 + k_2^* L_2 = K,$$
$$L_1 + L_2 = L,$$

together with (1) and (2), I obtain the equilibrium resource allocation across the two technologies (3) and (4). To satisfy the interior physical constraint $L_1 > 0$ and $L_2 > 0$, we must have

$$k_1^* < \frac{K}{L} < k_2^*.$$

Otherwise, the solution is a corner one. ■

From the above proposition, the aggregate production function $G(K,L)$ is constant return to scale, continuously differentiable, concave and strictly increasing in both arguments. Intuitively, the main results in this proposition can be illustrated by Figure 1.

It plots the output per labor y as a function of the capital-labor ratio k. The two different technologies are represented by the two different concave curves, which cross each other at the origin and point Q, where the corresponding capital-labor ratio is denoted by

$$\widetilde{k} \equiv \left(\frac{A_1}{A_2}\right)^{\frac{1}{\alpha_2-\alpha_1}}. \tag{5}$$

Clearly, technology 1 is better than technology 2 if and only if $k \leq \widetilde{k}$. The two curves have one unique cotangent straight line $y = ak + b$ and the x-coordinates of the two tangent points M and N exactly correspond to k_1^* and k_2^* given by (1) and (2). The aggregate production function per labor ($\frac{G(K,L)}{L}$) is the convex envelope of the two technology curves. In particular, when $k_1^* < k < k_2^*$, both technologies are used simultaneously and the aggregate production function per labor is linear (denoted by segment MN), in which case the equilibrium rental price of capital R is just the slope a and the wage rate W is just the intercept b. When $k \leq k_1^*$, only technology 1 is operating so $\frac{G(K,L)}{L} = A_1 k^{\alpha_1}$. When $k \geq k_2^*$, only technology 2 is operating, so $\frac{G(K,L)}{L} = A_2 k^{\alpha_2}$.

It is worth noting that capital is not subject to the decreasing return when $k_1^* < k < k_2^*$, even though there is no productivity change in either of the two specific technologies (i.e., A_1 and A_2 are fixed) or any non-convexity such as Marshallian externality. It is the resource reallocation during the technology upgrading that sustains the constant capital return.

2.2 Monopoly in Technology 2

Now suppose technology 2 is accessible to only one potential entrant (called firm M thereafter), which can be interpreted as the effective coalition of all the potential firms that have access to technology 2.[2] Technology 1 is still publicly and freely accessible. The ownership, hence the dividend, of each firm is equally divided by all the households. I explore whether firm M will operate technology 2 given K and L.

Given factor prices, the unit costs under these two technologies are given, respectively, by

$$\mu_1(W,R) = \frac{R^{\alpha_1} W^{1-\alpha_1}}{A_1 (\alpha_1)^{\alpha_1} (1-\alpha_1)^{1-\alpha_1}}$$

and

$$\mu_2(W,R) = \frac{R^{\alpha_2} W^{1-\alpha_2}}{A_2 (\alpha_2)^{\alpha_2} (1-\alpha_2)^{1-\alpha_2}}. \tag{6}$$

Normalize $A_1 = 1$ and let $A_2 = A$. $\mu_2(W,R) < \mu_1(W,R)$ if and only if

$$\frac{R}{W} < \psi \equiv \left[A \frac{\alpha_2^{\alpha_2} (1-\alpha_2)^{(1-\alpha_2)}}{\alpha_1^{\alpha_1} (1-\alpha_1)^{(1-\alpha_1)}}\right]^{\frac{1}{\alpha_2-\alpha_1}}. \tag{7}$$

[2] Firm M could be also imagined as a giant global company that considers making FDI in a host economy or a domestic special interest group that obtains sole permission from a foreign company to operate this new technology.

The factor markets are perfectly competitive. By Shephard Lemma, to produce Q units of output with technology 2 requires the following amount of production factors:

$$L_2^*(Q,W,R) = \frac{Q}{A\left(\frac{\alpha_2}{1-\alpha_2}\right)^{\alpha_2}\left(\frac{W}{R}\right)^{\alpha_2}}; \; K_2^*(Q,W,R) = \frac{Q}{A\left(\frac{\alpha_2}{1-\alpha_2}\right)^{\alpha_2-1}\left(\frac{W}{R}\right)^{\alpha_2-1}}. \tag{8}$$

Similarly, to produce Q units of output with technology 1, it requires

$$L_1^*(Q,W,R) = \frac{Q}{\left(\frac{\alpha_1}{1-\alpha_1}\right)^{\alpha_1}\left(\frac{W}{R}\right)^{\alpha_1}}; \; K_1^*(Q,W,R) = \frac{Q}{\left(\frac{\alpha_1}{1-\alpha_1}\right)^{\alpha_1-1}\left(\frac{W}{R}\right)^{\alpha_1-1}}.$$

Observe that

$$k_1(Q,W,R) \equiv \frac{K_1^*(Q,W,R)}{L_1^*(Q,W,R)} = \frac{\alpha_1}{1-\alpha_1}\frac{W}{R}, \tag{9}$$

$$k_2(Q,W,R) \equiv \frac{K_2^*(Q,W,R)}{L_2^*(Q,W,R)} = \frac{\alpha_2}{1-\alpha_2}\frac{W}{R}. \tag{10}$$

If technology 2 is operated in equilibrium, then $\frac{R}{W} \leq \psi$. The equilibrium output

price P is no larger than $\mu_1(W,R)$ because technology 1 is freely available. When firm M serves the whole economy, it solves the following:

$$\Pi = \max_{P \leq \mu_1(W,R)} \left[P - \mu_2(W,R)\right]\frac{Y}{P},$$

where Y is the total consumption expenditure of the economy. For the moment, suppose firm M takes Y and factor prices W and R as exogenously given, then $P = \mu_1(W,R)$ and

$$\Pi = [1 - \frac{\mu_2(W,R)}{\mu_1(W,R)}]Y. \tag{11}$$

The total expenditure is equal to the total household wealth, sum of profits, labor income and capital rental income:

$$Y = \Pi + WL + RK. \tag{12}$$

Combining (11) and (12) yields

$$Y = \frac{\mu_1(W,R)}{\mu_2(W,R)}\left(WL + RK\right).$$

In this general equilibrium environment, the market clearing conditions imply

$$AK^{\alpha_2}L^{1-\alpha_2} = \frac{WL + RK}{\mu_2(W,R)}. \tag{13}$$

The right-hand side is the total production cost divided by unit cost, so it is equal to the total output given by the left-hand side. (13) and (6) imply

$$\frac{R}{W} = \frac{\alpha_2}{(1-\alpha_2)\,k}, \tag{14}$$

so condition (7) is reduced to

$$k \geq k_2^*, \tag{15}$$

where k_2^* is given by (2). Recall Proposition 1 says that only technology 2 is operated if and only if $k \geq k_2^*$ when both technologies are publicly available. So the cutoff values for the capital labor ratio are identical for perfect competition and monopoly. Moreover, when $k > k_2^*$, the monopoly achieves social optimality as the total output (consumption) is identical to the first-best case, departing from the standard partial-equilibrium result that monopoly is socially inefficient. This is because the negative effect of the price markup on the consumption demand is exactly cancelled out by the positive income effect of the extra profit on the consumption demand through the general equilibrium channel. Nonetheless, the equilibrium output prices and profits are different in these two different market structures and the monopoly profit is

$$\frac{\Pi}{W} = \frac{\left[\left(\frac{k}{k_2^*}\right)^{\alpha_2-\alpha_1} - 1\right]}{(1-\alpha_2)} L \tag{16}$$

So far I assume that firm M serves the whole market by taking W and R as completely exogenous. The total output is pre-determined by the factor market clearing conditions in this general equilibrium. However, it is not always optimal for firm M to grab the whole market. In fact, firm M can also influence the factor prices by choosing its own output level. Now I explore the situation in which firm M is sophisticated enough to take all the general equilibrium effect into account. Factor markets are still perfectly competitive.

When Firm M serves only a fraction of the market, its optimization problem is as follows

$$\Pi = \max_{P \leq \mu_1(W,R);\, Q \geq 0} \left[P - \mu_2(W,R)\right] Q. \tag{17}$$

Combining (8), (9) and factor markets clearing conditions yields

$$\frac{K_1^*}{L_1^*} = \frac{K - \frac{Q}{A\left(\frac{\alpha_2}{1-\alpha_2}\right)^{\alpha_2-1}\left(\frac{W}{R}\right)^{\alpha_2-1}}}{L - \frac{Q}{A\left(\frac{\alpha_2}{1-\alpha_2}\right)^{\alpha_2}\left(\frac{W}{R}\right)^{\alpha_2}}} = \frac{\alpha_1}{1-\alpha_1}\frac{W}{R}, \tag{18}$$

where the second equation can be rewritten as

$$Q = \frac{A\left(\frac{\alpha_2}{1-\alpha_2}\right)^{\alpha_2}(1-\alpha_1)(1-\alpha_2)}{\alpha_2-\alpha_1}\left(\frac{R}{W}\frac{K}{L} - \frac{\alpha_1}{1-\alpha_1}\right)\left(\frac{R}{W}\right)^{-\alpha_2} L. \tag{19}$$

Observe $\left(\frac{R}{W}\right)'(Q) > 0$, because a higher output Q raises the relative demand for capital as technology 2 is more capital intensive, leading to a rise in the relative price of capital $\frac{R}{W}$. Substituting (19) into (17) yields

$$\begin{aligned}\frac{\Pi}{WL} &= \frac{A\left(\frac{\alpha_2}{1-\alpha_2}\right)^{\alpha_2}(1-\alpha_1)(1-\alpha_2)}{\alpha_2-\alpha_1}\left[\frac{R}{W}\frac{K}{L}-\frac{\alpha_1}{1-\alpha_1}\right]\cdot \\ &\left[\frac{\left(\frac{R}{W}\right)^{\alpha_1-\alpha_2}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}}-\frac{1}{A(\alpha_2)^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}}\right].\end{aligned} \tag{20}$$

Profit maximization gives the following first-order condition relative to $\frac{R}{W}$ (via choice of Q):

$$\frac{(1+\alpha_1-\alpha_2)\left(\frac{R}{W}\right)^{\alpha_1-\alpha_2}}{(\alpha_1)^{\alpha_1}}k+\frac{(\alpha_2-\alpha_1)\left(\frac{R}{W}\right)^{\alpha_1-\alpha_2-1}}{(1-\alpha_1)}\alpha_1^{1-\alpha_1}-\frac{(1-\alpha_1)^{1-\alpha_1}}{A(\alpha_2)^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}}k=0. \tag{21}$$

The second-order condition holds, so there exists a unique solution $\frac{R}{W}$ to (21), denoted as $\frac{R}{W}=\Gamma(k)$, where function $\Gamma(k)$ is continuously differentiable and strictly decreasing. Consequently, (20) implies that $\frac{\Pi}{W}$ is strictly increasing in k. Observe that $\Gamma(k_2^*) < \Gamma(k_1^*) = \psi$, so (21) implies that $\frac{R}{W} < \psi \Leftrightarrow k > k_1^* \Leftrightarrow Q > 0$. That is, technology 2 is operating if and only if $k > k_1^*$. This cutoff value is the same as in the perfect competitive equilibrium characterized in Proposition 1. (18) and (19) jointly imply that the aggregate output is

$$\widetilde{G}(K,L) \equiv y(\frac{K}{L})L, \tag{22}$$

where

$$y(k) \equiv \left\{\begin{array}{c}\alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}\left[\frac{\alpha_2}{1-\alpha_2}-\Gamma(k)k\right]\Gamma^{-\alpha_1}(k) \\ +A\left(\frac{\alpha_2}{1-\alpha_2}\right)^{\alpha_2}(1-\alpha_1)\left[\Gamma(k)k-\frac{\alpha_1}{1-\alpha_1}\right]\Gamma^{-\alpha_2}(k)\end{array}\right\}\frac{(1-\alpha_2)}{(\alpha_2-\alpha_1)}. \tag{23}$$

Firm M serves only a fraction of the market, so

$$Q(\frac{R}{W}) < AK^{\alpha_2}L^{1-\alpha_2}. \tag{24}$$

By revoking (10), (24) holds if and only if $\frac{R}{W} < \frac{\alpha_2}{(1-\alpha_2)k_2}$. Combining (21), it implies that

$$k < k^* \equiv \left[\frac{\alpha_2(1-\alpha_1)}{\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2}\right]^{\frac{1}{\alpha_2-\alpha_1}}k_2^*. \tag{25}$$

Let Q_i^m and Q_i^c denote the output produced with technology $i \in \{1,2\}$ when technology 2 is private (monopoly) and public (competitive market), respectively.

Lemma 2 *When $k \in (k_1^*, k^*)$, both technologies are operating. Moreover, $Q_1^m > Q_1^c$, $Q_2^m < Q_2^c$, $Q_1^m + Q_2^m < Q_1^c + Q_2^c$.*

Proof. Please see the appendix. ■

The intuition is that the profit of firm M decreases with the relative capital price, so firm M indirectly depresses this price by under-utilizing the capital-intensive technology. Accordingly, the labor-intensive technology is overly used when compared with the first-best allocation. The market equilibrium is socially inefficient.

When $k = k^*$, only technology 2 is operating and

$$\frac{R}{W} = \left[\frac{\alpha_2^{1-\alpha_2} (1-\alpha_1)^{2-\alpha_1} \alpha_1^{\alpha_1}}{A (1-\alpha_2)^{1-\alpha_2} [\alpha_1\alpha_2 - \alpha_1^2 + \alpha_2 - \alpha_2^2]} \right]^{\frac{1}{\alpha_1 - \alpha_2}}.$$

The profit (20) is given by

$$\Pi = \frac{(\alpha_2 - \alpha_1)^2 WL}{(1-\alpha_2)(\alpha_1\alpha_2 - \alpha_1^2 + \alpha_2 - \alpha_2^2)}. \tag{26}$$

Whenever $k \geq k^*$, only technology 2 is operating and the market structure is monopoly. Since the Inada condition is satisfied, the aggregate output must be $AK^{\alpha_2}L^{1-\alpha_2}$ even though firm M can choose to sell only part of the output. Substituting $k = k^*$ into (16) also yields (26).

Proposition 3 *When $k \leq k_1^*$, only technology 1 is adopted, $\frac{R}{W} = \frac{\alpha_1}{(1-\alpha_1)k}$, and the total output is $\widetilde{G}(K, L) = K^{\alpha_1}L^{1-\alpha_1}$; When $k \in (k_1^*, k^*)$, both technologies are operating, $\frac{R}{W}$ is uniquely determined by (21), the profit is given by (20), and the total output is given by (22); When $k \geq k^*$, only technology 2 is operating with $\frac{R}{W} = \frac{\alpha_2}{(1-\alpha_2)k}$, profit given by (16), and output $\widetilde{G}(K, L) = AK^{\alpha_2}L^{1-\alpha_2}$.*

We can see that the equilibrium is socially efficient when $k \leq k_1^*$ as it is perfectly competitive with only technology 1. The capital-intensive technology is adopted only when the capital-labor ratio exceeds the same threshold value for capital, irrespective of the public accessibility to technology 2, but the total output is smaller than the first best when $k \in (k_1^*, k^*)$, and technology 1 is "overly" used in the sense that industry 1 would have been abandoned if technology 2 were publicly available when $k \in (k_2^*, k^*)$, but it still operates when technology 2 is monopolized. The social efficiency restores when $k \geq k^*$ even though the market structure is monopoly and the prices are different from that in perfect competition (it also differs from the artificial social planner problem). This is because the negative price effect due to monopoly markup exactly cancels out the positive income effect due to the general equilibrium nature that monopoly rent of firm M becomes part of the households' income. To summarize, there exists a non-monotonic relationship between the capital labor ratio and the social efficiency of technology adoption.

3 Dynamic Model

Now I study the dynamic pattern of technology adoption associated with possible changes in the market structure. Again, I first characterize the benchmark case, namely the socially efficient equilibrium, where both technologies are publicly and freely available. Then I explore what happens when the capital-intensive technology is initially privately accessible to only firm M. A comparison of the two cases is discussed afterwards. The key insights can be illustrated with a simple two-period model.[3]

3.1 Perfect Competition

A representative household is endowed with K_0 capital and L labor. E_t and C_t denotes, respectively, the capital used for production and consumption at period $t \in \{1, 2\}$. Capital good is produced with AK technology and cannot be used for consumption. All the capital used for production fully depreciates. Consumption goods are perishable. Therefore, the artificial social planner solves the following problem.

$$\max_{C_1, C_2} \frac{C_1^{1-\sigma} - 1}{1-\sigma} + \beta \frac{C_2^{1-\sigma} - 1}{1-\sigma}, \text{ where } \sigma \in (0, 1]$$

subject to

$$\begin{aligned}
E_2 &= \xi(K_0 - E_1), \\
C_1 &\leq \widetilde{G}(E_1, L), \\
C_2 &\leq \widetilde{G}(E_2, L), \\
E_1 &\geq 0, E_2 \geq 0, C_1 \geq 0, C_2 \geq 0, \\
&\quad K_0 \text{ and } L \text{ are given,}
\end{aligned}$$

where ξ is a parameter capturing the investment-specific technological progress (ISTP thereafter, *a la* Greenwood, Hercowitz and Krusell (1997)) in the capital good sector, and $\widetilde{G}(\cdot, L)$ is given by Proposition 3. Assume $\beta\xi > 1$ so that ISTP is sufficiently quick to rule out industry downgrading. There are six possible patterns of technology adoption summarized below:

Table 1. Dynamic Patterns of Technology Adoption

Patterns	1	2	3	4	5	6
Period 1	1	2	1, 2	1	1	1, 2
Period 2	1	2	1, 2	1, 2	2	2

For example, Pattern 4 refers to that only technology 1 is used in period 1 and both technologies are used in period 2.

[3] For infinite-period models with infinite technologies (industries), please refer to Ju, Lin and Wang (2013) and Wang (2014).

Proposition 4 *Suppose $\beta\xi > 1$ and $\sigma \in (0,1]$. When $\beta\xi \geq \left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{\sigma}$, the dynamic equilibrium follows Pattern 1 when $\frac{K_0}{L} \in (0,\theta_1]$, Pattern 4 when $\frac{K_0}{L} \in (\theta_1,\theta_2)$, Pattern 5 when $\frac{K_0}{L} \in [\theta_2,\theta_5]$, Pattern 6 when $\frac{K_0}{L} \in (\theta_5,\theta_6)$, and Pattern 2 when $\frac{K_0}{L} \in [\theta_6,\infty)$. When $\beta\xi < \left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{\sigma}$, the dynamic equilibrium follows Pattern 1 when $\frac{K_0}{L} \in (0,\theta_1]$, Pattern 4 when $\frac{K_0}{L} \in (\theta_1,\theta_3]$, Pattern 3 when $\frac{K_0}{L} \in (\theta_3,\theta_4)$, Pattern 6 when $\frac{K_0}{L} \in [\theta_4,\theta_6)$, and Pattern 2 when $\frac{K_0}{L} \in [\theta_6,\infty)$. The threshold values are given by*

$$
\begin{aligned}
\theta_1 &\equiv \frac{1+\xi^{-1}(\beta\xi)^{\frac{1}{-\alpha_1+1+\alpha_1\sigma}}}{(\beta\xi)^{\frac{1}{-\alpha_1+1+\alpha_1\sigma}}}k_1^* \qquad (27)\\
\theta_2 &\equiv \left[(\beta\xi)^{\frac{1}{-\sigma\alpha_1+\alpha_1-1}}\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{\frac{\sigma}{\sigma\alpha_1-\alpha_1+1}} + \frac{\alpha_2(1-\alpha_1)}{\alpha_1(1-\alpha_2)\xi}\right]k_1^*\\
\theta_3 &\equiv \frac{(\beta\xi)^{\frac{1}{\sigma}}+\xi\alpha_1-(1-\alpha_1)}{\alpha_1\xi}k_1^*\\
\theta_4 &\equiv \left[\frac{1}{\xi}+\frac{(\beta\xi)^{-\frac{1}{\sigma}}-(1-\alpha_2)}{\alpha_2}\right]\frac{\alpha_2(1-\alpha_1)}{\alpha_1(1-\alpha_2)}k_1^*\\
\theta_5 &\equiv \left[\frac{\alpha_2}{\alpha_1}\left[\beta\xi^{(1-\sigma)\alpha_2}\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{(1-\alpha_2)((1-\sigma))}\right]^{\frac{1}{1-\alpha_2+\sigma\alpha_2}}+1\right]k_1^*\\
\theta_6 &\equiv \frac{\alpha_2(1-\alpha_1)}{\alpha_1(1-\alpha_2)}\left[1+\xi^{-1}(\beta\xi)^{\frac{1}{-\alpha_2+1+\alpha_2\sigma}}\right]k_1^*.
\end{aligned}
$$

Proof. Please see the appendix. ■

As shown in the proof, I first explore the equilibrium choice of technologies in each period for a given inter-temporal capital allocation, which is the static problem characterized in Proposition 1. Then I make sure that the inter-temporal output satisfies the Euler equation so that the dynamic saving decision is optimal. The six threshold values in the above proposition are found by ensuring that the domain of capital allocation for each period is consistent with the corresponding technology choice in that period.

This proposition states that the dynamic pattern depends on the initial capital labor ratio and the ISTP parameter ξ. The results can be intuitively illustrated by the following two figures:

To summarize, there is always a unique equilibrium; a larger initial capital-labor ratio implies a quicker adoption of the more capital-intensive technology, or faster industrial upgrading. Moreover, Pattern 3 and Pattern 5 are mutually exclusive and the former (both technologies are used in both periods) occurs only if the ISTP is sufficiently slow ($\beta\xi < \left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{\sigma}$).

3.2 Monopoly with Private Technology 2

Now consider a dynamic environment identical to the previous one except that technology 2 is privately accessible to only firm M, which decides whether and when to implement this new technology. Technology 1 is still public and free. If technology 2 is implemented in period 1, then this technology becomes publicly known in period 2 because people can successfully imitate it after one period operation. If adoption of technology 2 is delayed until period 2, then the monopoly rent is reaped in period 2. The ownership shares of all the firms are equally divided among all the households. Since the second welfare theorem is not applicable, we have to solve the decentralized optimal decisions of households and firms. A representative household solves

$$\max \frac{C_1^{1-\sigma}-1}{1-\sigma}+\beta\frac{C_2^{1-\sigma}-1}{1-\sigma}$$

subject to

$$\begin{aligned} P_1C_1+\frac{P_2C_2}{\widetilde{R}} &\leq (W_1L+R_1E_1+\Pi_1)+\frac{(W_2L+R_2E_2+\Pi_2)}{\widetilde{R}}, \qquad (28)\\ E_2 &= \xi(K_0-E_1),\\ E_1 &\geq 0, E_2\geq 0, C_1\geq 0, C_2\geq 0,\\ &\quad K_0 \text{ is given.} \end{aligned}$$

Household optimization yields

$$\beta\left(\frac{C_2}{C_1}\right)^{-\sigma}=\frac{P_2}{P_1\widetilde{R}}, \qquad (29)$$

which, together with (28), implies

$$\begin{aligned} C_1 &= \frac{(W_1L+R_1E_1+\Pi_1)+\frac{(W_2L+R_2E_2+\Pi_2)}{\widetilde{R}}}{P_1+\beta P_1\left(\frac{P_2}{\beta P_1\widetilde{R}}\right)^{1-\frac{1}{\sigma}}}, \qquad (30)\\ C_2 &= C_1\left(\frac{P_2}{\beta P_1\widetilde{R}}\right)^{-\frac{1}{\sigma}}. \qquad (31) \end{aligned}$$

To ensure positive consumption in both periods, we must have the inter-temporal interest rate

$$\widetilde{R}=\frac{\xi R_2}{R_1}. \qquad (32)$$

Substituting (32) back to (29) yields

$$\beta\xi\left(\frac{C_2}{C_1}\right)^{-\sigma}=\frac{P_2/R_2}{P_1/R_1}. \qquad (33)$$

All the firms maximize its total profit. Those with access to only the public technology take the factor prices as given. However, firm M understands that it can potentially affect the relative factor prices via its output decision through the general equilibrium channel when technology 2 is operated for the first time. Firm M has three options. Option 1 is to start operating technology 2 in period $1(\Pi_1 > 0; \Pi_2 = 0)$; Option 2 is to first start operating technology 2 in period 2 $(\Pi_1 = 0; \Pi_2 > 0)$; Option 3 is never to operate technology 2 $(\Pi_1 = \Pi_2 = 0)$. There are nine possible patterns of technology adoption, as summarized in Table 2.

Table 2. Dynamic Patterns when Technology 2 is Private

Patterns	A1-a	A1-b	A1-c	B1-a	B1-b	B1-c	A2	B2	C
Period 1	1, 2	1, 2	1, 2	2	2	2	1	1	1
Period 2	2	1, 2	1	2	1	1, 2	2	1, 2	1

As before, the dynamic problem is analyzed in two steps. First, dynamic capital allocation (E_1 and E_2) is determined in the optimal saving decision. Second, given E_1 and E_2, the optimal technology adoption decision is made, which affects the market structure in both periods. For Option 1, the second-period problem is identical to the perfect competition case in Subsection 2.1 because both technologies become publicly available at period 2; the first-period problem is identical to that in Subsection 2.2. For Option 2, the second-period problem is identical to that in Subsection 2.2, whereas the first-period problem is to solve a perfect competition model with only technology 1. For Option 3, the optimization in both periods must be consistent with the static analysis in Subsection 2.2 to justify why technology 2 is never adopted.

The necessary and sufficient conditions for the existence of each of the nine different patterns have to be found first, and then we can determine the optimal technology adoption choice of firm M based on the present value of profit from each possible adoption strategy for given initial conditions. Technology 2 is first adopted in period 1 rather than period 2 if and only if $\Pi_1 > \frac{\Pi_2}{\widetilde{R}}$, where $\widetilde{R}$ is endogenously determined in the general equilibrium.

Lemma 5 *Suppose $\beta\xi > 1$. Patterns A1-c, B1-b and B1-c never occur.*

Proof. Refer to the Appendix. ■

The intuition is that the consumption in the second period should be no smaller than the first period, as implied by the Euler equation and $\beta\xi > 1$. Patterns A1-c, B1-b and B1-c all imply the opposite, so they cannot occur in equilibrium. Therefore, we have six different patterns left, same as in Table 1 in Subsection 3.1, but the market structure may not always be perfect competition, depending on when technology 2 is first adopted.

Proposition 6 *Suppose $\sigma < 1$ and $\beta\xi > 1$. There exists a non-empty interval $(\theta_0, \theta_1]$ such that for any $\frac{K_0}{L} \in (\theta_0, \theta_1]$, technology 2 is adopted in period 2 when it is privately accessible (Pattern B2), but technology 2 is never adopted when it is publicly available. Here θ_1 is given by (27) and θ_0 is uniquely determined by the following equation*

$$\beta\xi \left(\frac{y(\xi(\frac{K_0}{L} - \theta_0))}{\theta_0{}^{\alpha_1}} \right)^{-\sigma} = \left(\frac{1}{\theta_0 \Gamma(\xi(\frac{K_0}{L} - \theta_0))} \frac{\alpha_1}{1 - \alpha_1} \right)^{1-\alpha_1}, \tag{34}$$

where $y(\cdot)$ is defined in (23) and $\Gamma(\cdot)$ is defined in (21).

Proof. See the appendix. ■

This proposition indicates that the capital-intensive technology, when privately accessible, may be prematurely implemented from the social efficiency point of view. It occurs for two reasons. First, firm M would earn a positive profit whenever the market can support the adoption but no adoption means zero profit. Therefore, as a monopolist of this capital-intensive technology, the profit-seeking firm M is always incentivized to implement technology 2, unless it is not supportable by the market due to capital scarcity. Second, the initial capital endowment has to be sufficiently large so that capital in the second period can be made high enough to support the adoption of the new technology via sufficient saving in the first period. The increase in saving is feasible in the dynamic equilibrium because consumptions in the two periods are sufficiently substitutable (inter-temporal elasticity of substitution of consumption $\frac{1}{\sigma}$ is larger than unity).[4]

Lemma 7 *Suppose $\sigma = 1$ and $\beta\xi > \frac{1-\alpha_1}{1-\alpha_2}$. Pattern C is realized when $\frac{K_0}{L} \in (0, \widetilde{\theta}_1]$; Pattern B2 is realized with $\frac{K_0}{L} \in (\widetilde{\theta}_1, \widetilde{\theta}_2)$; Pattern A2 is supportable by the market when $\frac{K_0}{L} \in [\widetilde{\theta}_2, \infty)$; Pattern A1-a is supportable by the market when $\frac{K_0}{L} \in (\widetilde{\theta}_0, \widetilde{\theta}_5)$; Pattern B1-a is supportable by the market when $\frac{K_0}{L} \in [\widetilde{\theta}_5, \infty)$, where*

$$\begin{aligned}
\widetilde{\theta}_0 &\equiv \left(\beta \frac{\alpha_2}{\alpha_1} + 1 \right) k_1^* \\
\widetilde{\theta}_1 &\equiv \frac{1+\beta}{\beta\xi} k_1^*; \\
\widetilde{\theta}_2 &\equiv \frac{\alpha_1 (1 - \alpha_1)}{\alpha_1\alpha_2 - \alpha_1^2 + \alpha_2 - \alpha_2^2} \frac{k^*}{\beta\xi} + \frac{k^*}{\xi}; \\
\widetilde{\theta}_5 &\equiv \beta k^* + k^*.
\end{aligned}$$

Proof. Refer to the appendix. ■

[4] In the proof, it is shown that the result in this proposition no longer holds when $\sigma = 1$ and keeping everything else unchanged.

The above lemma can be illustrated by Figure 4.First of all, "a pattern being supportable by the market" just means that the pattern is feasible to implement as a market equilibrium to the extent that firm M has not yet decided whether it is the most profitable choice, so it is a necessary but insufficient condition for the pattern to be a market equilibrium. When there are multiple patterns supportable by the market over certain interval, firm M will choose the most profitable pattern for itself, which is the market equilibrium as well. If there is a unique pattern supportable by the market over some interval, then this pattern must be the market equilibrium for this interval. In addition, multiple equilibria exist if two patterns deliver the same highest total profit.

Notice that Pattern A1-b is not supportable by the market as it is ruled out by condition $\beta\xi > \frac{1-\alpha_1}{1-\alpha_2}$. This figure indicates that there is a unique equilibrium pattern when $\frac{K_0}{L} < \widetilde{\theta}_0$, qualitatively similar to Figure 2 in terms of inter-termporal technology choices and their ordering along the dimension of initial capital labor ratio. Observe that $\widetilde{\theta}_1 = \theta_1$ when $\sigma = 1$. It means that the threshold value of the initial capital labor ratio for the capital-intensive technology to be adopted is independent of whether this technology is publicly or privately accessible in this dynamic environment, preserving the same property in the static economy (recall Proposition 3). When $\frac{K_0}{L} \in [\widetilde{\theta}_0, \widetilde{\theta}_5]$, it is obvious that Pattern A1-a is the equilibrium ($\frac{\Pi_{A2}}{\widetilde{R}} < \Pi_{A1a}$) when β is sufficiently small. When β is large enough, the equilibrium could be Pattern A2, depending on the specific values for different parameters.[5]

Also notice that $\widetilde{\theta}_2 < \theta_2 < \widetilde{\theta}_0$ when $\sigma = 1$ and $\beta\xi > \frac{1-\alpha_1}{1-\alpha_2}$. It means that when $\frac{K_0}{L} \in (\widetilde{\theta}_2, \theta_2)$, the labor-intensive technology is completely abandoned in period 2 if the capital-intensive technology is private, whereas the labor-intensive technology is still operating (together with the capital-intensive technology) in period 2 when the capital-intensive technology is public. In other words, the private accessibility to the capital-intensive technology now leads to a premature abandonment of the old technology and an over-utilization of this new technology from the social efficiency point of view. The reason is that firm M wants to extract as much profit as possible by utilizing this private technology, similar to the case in Proposition 6. These results are in stark contrast with the static model, which predicts that private accessibility could result in an over-utilization of the old technology (Lemma 2 in Subsection 2.1).

Proposition 4 shows that when both technologies are publicly available, the capital-intensive technology is adopted weakly earlier whenever the initial capital endowment becomes larger. Is such a monotonic relationship still valid when the capital-intensive technology is private? The following proposition says no.

[5] For example, $\Pi_{A1a} > \frac{\Pi_{A2}}{\widetilde{R}}$ when $\alpha_1 = \frac{1}{3}$ and $\alpha_2 = \frac{2}{3}$, or when $\alpha_1 = \frac{1}{2}$ and $\alpha_2 = 1$, or when $\alpha_1 = 0$ and $\alpha_2 = \frac{1}{2}$, but the opposite is true when $\alpha_1 = \frac{1}{4}$ and $\alpha_2 = \frac{1}{2}$.

For the convenience of exposition, define

$$x_{1,2} \equiv \frac{1}{2}\left(\frac{\beta\frac{\alpha_2}{\alpha_1}+1}{A^{\frac{1}{1-\alpha_1}}}\right)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}}\left((1+\beta)\mp\sqrt{(1+\beta)^2-4\beta\left[\frac{(1+\beta)}{\beta\xi\left(\beta\frac{\alpha_2}{\alpha_1}+1\right)}\right]^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}}}\right),$$

$$\upsilon_{1,2} \equiv \frac{\left(\beta\frac{\alpha_2}{\alpha_1}+1\right)^{\alpha_2-\alpha_1}\left[\left(\frac{\alpha_1}{\alpha_2}\right)^{\alpha_1}\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{1-\alpha_1}\frac{\alpha_2(1-\alpha_1)}{\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2}\right]^{\frac{1-\alpha_1}{1-\alpha_2}}}{\left[\frac{1}{2}\left((1+\beta)\mp\sqrt{(1+\beta)^2-4\beta\left(\frac{1+\beta}{\beta\xi\left(\beta\frac{\alpha_2}{\alpha_1}+1\right)}\right)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}}}\right)\right]^{\frac{(1-\alpha_1)(\alpha_2-\alpha_1)}{1-\alpha_2}}}.$$

Proposition 8 *Suppose $\sigma = 1$, $\beta\xi \geq 1$ and $\frac{K_0}{L} \in [\left(\beta\frac{\alpha_2}{\alpha_1}+1\right)k^*, \infty)$. The dynamic market equilibrium is the following: [1] When $A > \upsilon_1$, Pattern B1-a is realized if $\frac{K_0}{L} \in [\left(\beta\frac{\alpha_2}{\alpha_1}+1\right)k^*, x_1) \cup (x_2, \infty)$ and Pattern A2 is realized if $\frac{K_0}{L} \in (x_1, x_2)$. Both patterns are equilibria when $\frac{K_0}{L} = x_1$ or x_2. [2] When $A \in (\upsilon_2, \upsilon_1)$, Pattern B1-a is realized if $\frac{K_0}{L} \in (x_2, \infty)$ and Pattern A2 is realized if $\frac{K_0}{L} \in [\left(\beta\frac{\alpha_2}{\alpha_1}+1\right)k^*, x_2)$. Both patterns are equilibria when $\frac{K_0}{L} = x_2$; [3] When $A < \upsilon_2$, Pattern B1-a is realized whenever $\frac{K_0}{L} \in [\left(\beta\frac{\alpha_2}{\alpha_1}+1\right)k^*, \infty)$.*

Proof. See the appendix. ■

Part [1] of this proposition is most interesting. It says that there exists a non-monotonic relationship between the optimal time to adopt the capital-intensive technology and the initial capital-labor ratio $\frac{K_0}{L}$. More specifically, when $\frac{K_0}{L}$ increases across the threshold x_1, surprisingly, the adoption of the capital-intensive technology is delayed from the first period to the second period. To understand why, observe that, on one hand, the current value of first-period profit Π_{B1-a} increases with $\frac{K_0}{L}$ in Pattern B1-a:

$$\Pi_{B1a} = \left[\left(A\left[\frac{K_0/L}{\beta\frac{\alpha_2}{\alpha_1}+1}\right]^{\alpha_2-\alpha_1}\right)^{\frac{1}{1-\alpha_1}} - 1\right]W_1L.$$

On the other hand, the discounted present value of the second-period profit also increases with $\frac{K_0}{L}$ in Pattern A2:

$$\frac{\Pi_{A2}}{\widetilde{R}_{A2}} = \left(1-\left[A\left(\frac{K_0\beta\xi}{L(1+\beta)}\right)^{\alpha_2-\alpha_1}\right]^{-\frac{1}{1-\alpha_1}}\right)\beta LW_1.$$

Mathematically, it turns out that $\frac{\Pi_{A2}}{\widetilde{R}_{A2}}$ exceeds Π_{B1-a} when $\frac{K_0}{L} \in (x_1, x_2)$. However, when $\frac{K_0}{L} > x_2$, $\widetilde{R}_{A2}$ increases with $\frac{K_0}{L}$ sufficiently more than Π_{A2}, so it pays to switch back to Pattern B1-a.

The economic intuition is as follows. The private accessibility to this new technology enables firm M to extract more rents by deliberately postponing the implementation till capital becomes more abundant. An increase in initial capital endowment makes second-period capital larger due to the consumption smoothing motives of consumers, but it also increases the inter-temporal interest rate $\widetilde{R}_{A2}$ to discourage adoption delay.[6] It turns out that the waiting benefit is smaller than the waiting cost when $\frac{K_0}{L} \in [\left(\beta\frac{\alpha_2}{\alpha_1}+1\right)k^*, x_1)$, while the opposite is true when initial capital labor ratio becomes larger: $\frac{K_0}{L} \in (x_1, x_2)$, and then the benefit exceeds the cost again when the capital becomes even more abundant: $\frac{K_0}{L} > x_2$. An immediate policy implication is that a limited amount of foreign aid (by increasing $\frac{K_0}{L}$) may sometimes result in a delay in the adoption of capital-intensive technology, different from the first-best case characterized by Proposition 4.

Another distinctive feature of this dynamic model is that there may exist multiple equilibria. This indeterminancy of technology adoption and industry upgrading results from the indirect price manipulations by firm M through its adoption and quantity decisions. This power is fundamentally due to its exclusive accessibility to the new technology. Whereas firm M feels indifferent between the two possible equilibria, they are typically not equivalent in terms of welfare (efficiency). I will revisit this issue soon.

Proposition 8 also indicates that the non-monotonicity result is possible only when A, productivity of technology 2, is sufficiently large. Technically, A determines the intersection of interval $[\left(\beta\frac{\alpha_2}{\alpha_1}+1\right)k^*, \infty)$ and interval (x_1, x_2). When $A > \upsilon_1$, $(x_1, x_2) \subset [\left(\beta\frac{\alpha_2}{\alpha_1}+1\right)k^*, \infty)$, which makes it possible to have a non-monotonic relationship between the adoption time of the capital-intensive technology and the initial capital labor ratio $\frac{K_0}{L}$ on interval $[\left(\beta\frac{\alpha_2}{\alpha_1}+1\right)k^*, \infty)$, so part [1] of the proposition is obtained; when $A \in (\upsilon_2, \upsilon_1)$, we have $x_1 < \left(\beta\frac{\alpha_2}{\alpha_1}+1\right)k^* < x_2$, so we obtain part [2], in which the non-monotonicity result disappears; when $A < \upsilon_2$, the intersection set $(x_1, x_2) \cap [\left(\beta\frac{\alpha_2}{\alpha_1}+1\right)k^*, \infty)$ is empty so Pattern B1-a always dominates Pattern A2 from firm M's point of view, the result as stated in part [3].

The intuition is that there are two competing effects when A increases. One is the substitution effect, making technology 2 more attractive when the capital-

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[6]

$$
\begin{aligned}
\widetilde{R}_{A2} &= \frac{\xi^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}} A^{\frac{1}{1-\alpha_1}} \left(\frac{K_0}{L}\right)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}} W_2}{(1+\beta)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}} \beta^{\frac{1-\alpha_2}{1-\alpha_1}} W_1}, \\
\Pi_{A2} &= \left(A^{\frac{1}{1-\alpha_1}} \left(\frac{\beta\xi K_0}{(1+\beta)L}\right)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}} - 1\right) LW_2.
\end{aligned}
$$

Both increase with $\frac{K_0}{L}$.

labor ratio is larger, so it tends to encourage capital saving in the first period and delay the adoption of technology 2. The other effect is the income effect: as the total income increases, it tends to discourage capital saving (encourage current production and consumption) by increasing the endogenous interest rate, so it facilitates the immediate adoption of technology 2, as long as the initial capital stock is sufficiently large. An increase in the initial capital-labor ratio would interact with these two effects. It turns out that only when A is sufficiently large is it possible to have the reversal of dominance between the substitution effect and the income effect as the initial capital labor ratio increases.

Recall Lemma 7 shows that Pattern C is the unique equilibrium, which is also socially efficient, when the initial capital labor ratio is sufficiently small ($\frac{K_0}{L} \leq \widetilde{\theta}_1 = \theta_1$). We also know that inefficiency arises when the initial capital labor ratio is in the middle range when compared with the threshold values in Proposition 4. Is the market equilibrium socially efficient again when the initial capital labor ratio becomes sufficiently large? In other words, is the non-monotonic relationship between social efficiency and initial capital labor ratio observed in the static model with private technology (see Lemma 2 and Proposition 3) preserved in the dynamic model?

To address these questions, first note that Proposition 8 indicates that Pattern B1-a is the unique market equilibrium when the initial capital labor ratio is sufficiently large ($\frac{K_0}{L} > \max\{\left(\beta\frac{\alpha_2}{\alpha_1}+1\right)k^*, x_2\}$), qualitatively the same pattern as the first-best case as shown in Proposition 4 (and Figure 2). Now we check whether Pattern B1-a is socially efficient when $\frac{K_0}{L}$ is sufficiently large.

Proposition 9 *Suppose $\sigma = 1$ and $\frac{K_0}{L} \geq (\beta+1)\,k^*$. When both technologies are free and public, only technology 2 is used in both periods (Pattern 2 in Table 1) and the first-best capital allocation is*

$$E_1^{FB} = \frac{K_0}{1+\beta};\ E_2^{FB} = \frac{\beta\xi}{1+\beta}K_0.$$

When technology 2 is privately accessible only to firm M, the capital allocations for Pattern B1-a and Pattern A2 are given, respectively, by the following

$$\begin{aligned} E_1^{B1a} &= \frac{K_0}{\beta\frac{\alpha_2}{\alpha_1}+1}; E_2^{B1a} = \frac{\xi\beta\frac{\alpha_2}{\alpha_1}}{\beta\frac{\alpha_2}{\alpha_1}+1}K_0; \\ E_1^{A2} &= \frac{K_0}{1+\beta}; E_2^{A2} = \frac{\beta\xi}{1+\beta}K_0. \end{aligned}$$

Moreover, Pattern B1-a Pareto dominates Pattern A2 if and only if

$$\frac{K_0}{L} > \widetilde{\theta}_6 \equiv \left[\frac{\left(\beta\frac{\alpha_2}{\alpha_1}+1\right)^{\alpha_2+\beta\alpha_2}}{A\left(\frac{\alpha_2}{\alpha_1}\right)^{\beta\alpha_2}(1+\beta)^{\alpha_1+\beta\alpha_2}}\right]^{\frac{1}{\alpha_2-\alpha_1}}. \qquad (35)$$

Proof. See the appendix. ■

Note that the inter-temporal capital allocations are identical for the first-best case and Pattern A2. However, the first-period output is different for these two cases because different technologies are utilized. The second-period output and technologies are the same in these two cases, even though the market structures are different.

On the other hand, observe that the adopted technology is identical for the first-best pattern and Pattern B1-a in each period when $\sigma = 1$, $\beta\xi \geq 1$ and $\frac{K_0}{L} \geq \max\{\left(\beta\frac{\alpha_2}{\alpha_1} + 1\right) k^*, x_2\}$. However, the inter-temporal capital allocations are different for these two patterns, so the outputs are also different in both periods. We conclude that the dynamic equilibrium Pattern A2 is no longer socially efficient even when $\frac{K_0}{L}$ is arbitrarily large, different from the static case. More precisely, the private accessibility of technology 2 leads to excessive saving and capital over-accumulation ($E_1^{B1a} < E_1^{FB}$). The reason is that positive monopoly rent is only available for the first period as technology 2 becomes public in the second period, so the second period income has no monopoly rent component. Consequently, the inter-temporal consumption smoothing requires more capital saving at period 1 to partly offset the missing monopoly rent in the second period.

This proposition also helps rank the welfare associated with each equilibrium when there exist multiple equilibria, as indicated in Proposition 8. Consider the scenario for the part (1) and part (2) in Proposition 8 (namely, $A > \upsilon_2$, $\sigma = 1$, $\beta\xi \geq 1$ and $\frac{K_0}{L} \in [\left(\beta\frac{\alpha_2}{\alpha_1} + 1\right) k^*, \infty)$). There are two equilibria when $\frac{K_0}{L} = x_2$. By (35), Pattern B1-a Pareto dominates Pattern A2 if and only if $x_2 > \widetilde{\theta}_6$, which is true when A is sufficiently large.[7] The opposite ranking is true for other inverals for A.

4 Discussions

Because of the discrepancies between the first-best allocation and the market equilibrium with private technology, there is potentially room for policy intervention. Notice that the two factor markets are always perfectly competitive and

[7] More precisely, it occurs if and only if

$$A > \max\{\upsilon_2, \frac{\left[\left(\beta\frac{\alpha_2}{\alpha_1}+1\right)^{(\alpha_2+\beta\alpha_2)-\frac{(\alpha_2-\alpha_1)^2}{(1-\alpha_1)}}\left(\frac{\alpha_2}{\alpha_1}\right)^{-\beta\alpha_2}(1+\beta)^{-(\alpha_1+\beta\alpha_2)}\right]^{\frac{(1-\alpha_1)^2}{[1-\alpha_2][1-2\alpha_1+\alpha_2]}}}{\left[\frac{1}{2}\left((1+\beta)+\sqrt{(1+\beta)^2-4\beta\left[\frac{(1+\beta)}{\beta\xi\left(\beta\frac{\alpha_2}{\alpha_1}+1\right)}\right]^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}}}\right)\right]^{\frac{(\alpha_2-\alpha_1)(1-\alpha_1)^2}{[1-\alpha_2][1-2\alpha_1+\alpha_2]}}}\}.$$

well functioning, but the factor price signals are distorted by the non-competitive market structure in the final good market through the general equilibrium channel, so these signals no longer accurately reflect the relative abundance of capital over labor as in the first-best scenario and hence can no longer guide the socially efficient technology adoption and industry upgrading.

The root of market inefficiency is the private accessibility of the new technology. The magnitude of inefficiency, however, endogenously depends on the factor endowment. The private accessibility of the new technology leads to too slow adoption (or insufficient utilization) of it in some cases, whereas it results in premature adoption or over-utilization of it in other cases. Moreover, an increase in the initial capital endowment may sometimes delay, rather than facilitate, the adoption of the capital-intensive technology.

Now I discuss two possible sets of welfare-enhancing policies. One is to directly target the private accessibility of the new technology, and the other is to rectify the relative factor prices.

4.1 Enhancing Accessibility to New Technology

The non-competitive market structure appears to be the culprit for inefficiency, but it is actually only a consequence instead of the cause of market inefficiency. For our purpose of studying technology diffusion to the developing economies, we abstract away questions such as which firm can become firm M and how it acquires this technology advantage, which are important questions intensively studied in the endogenous growth literature of innovation and learning (see Acemoglu (2009), Aghion and Howitt(2009)).[8] Instead of proposing punishment on the monopoly power, which may hurt the incentive of firm M to acquire and implement this new technology, we should directly address the root of the inefficiency: private accessibility to the new technology.

Given the existence of the new technology, one sensible way to improve welfare is for government to enhance the public accessibility to this new technology by helping collect and disseminate the information. For example, the government may support the training programs for workers and entrepreneurs, collect the general market information as public service, reduce the regulations and lower the entry barrier for new firms, *etc.* Of course, information collection is costly and it is impossible for the government to collect all the information for all the existing technologies and make them all publicly available. Given that the government does not necessarily know better than individual firms how to select the most promising technology, it may be desirable for the government to conduct timely surveys from a widely covered group of potential investors

[8] In particular, Aghion et al (2005) find that there exists an inverted-U relationship between market competition and innovation, because too much competition may destroy the *ex ante* incentive of the potential investor who spends resource to aquire this new technology, similar to the issue of optimal patent duration design. Meanwhile, too concentrated market power also hurts innovation due to lack of competition. So the antitrust law does not necessarily help or sometimes makes things even worse from the social efficiency point of view.

asking for opinions about which new industry or what kind of information they are mostly interested in, and then the government could help collect the relevant information and provide these public goods and services that are most commonly demanded.[9]

Another useful way is to encourage foreign direct investment to facilitate the technology diffusion (Harrison and Rodriguez-Clare (2009), Wang (2013)). It not only helps facilitate the technology diffusion and learning, but also increase the number of potential first movers and reduce the duration of technology monopoly.

Such suggestions on industrial policies are not new, but my model indicates that these policies may not only facilitate the timely adoption of appropriate technologies, which is the focus of most existing literature, but also prevent inappropriate new technologies from being prematurely adopted and overly utilized.

What seems also insufficiently emphasized in the literature is the set of policies aiming to rectify the relative factor prices to indirectly facilitate the adoption of the appropriate technology and simultaneously induce the timely abandonment of the obsolete technology, which I now turn to.

4.2 Rectifying Relative Factor Prices

Recall in the model firm M, the monopolist of the new capital-intensive technology, may produce less than the socially optimal amount because it fears that the rising rental price of capital would erode its monopoly profit when it increases its output. Thus the relative factor prices are distorted indirectly in general equilibrium, even though the factor markets themselves are perfect.

To facilitate the timely adoption of the new capital-intensive technology, one way to rectify the factor price is to subsidize the adoption of the new technology to the extent that the monopolist is willing to produce the socially optimal amount at the socially efficient time. The subsidy could be in the form of loan credit or investment credit. Tax holidays could also help capital accumulate faster and hence facilitates the adoption of the new technology. The rationale for these subsidies, however, is fundamentally different from the credit constraint argument.

Another way to speed up timely adoption of the new technology is to impose labor income taxes to facilitate the abandonment of the old labor-intensive technology so that the old technology is operated at the socially optimal level, and the production scale of the new technology expands toward the socially optimal amount as the result of the rectification of the market-clearing relative factor prices.

[9] Canda (2006) provide detailed cross-country case studies to illustrate how government plays an important and welfare-enhancing role to help technology adoption and diffusion in several developing countries. Lin and Monge (2010) provide a very concrete six-step policy precedures for the government in the developing countries to identify the right industrial target and then facilitate industrial upgrading.

To prevent the possibility of premature adoption of the new technology as shown in Lemma 8, factor price rectification should work in the opposite direction. As suggested by Proposition 9, when the initial capital endowment is sufficiently large, measures should be taken to prevent too much saving when the capital-intensive technology is private. Such measures include production subsidies in the first period and/or production tax in the second period.

For small economies, would it help improve economic efficiency by liberalizing the capital account and allowing for international capital flow? Not necessarily. First of all, although firm M could no longer affect the price of the international capital market, yet it could still indirectly affect the wage rate by changing its own output because the labor demand would shift across the two technologies unless there is a sufficiently large labor pool, which is less likely in small economies. Moreover, international capital flow may weaken the resource constraint but not the budget constraint for the households or firms, therefore the general equilibrium effect is not clear.

5 Conclusion

In this paper, I develop a simple dynamic general equilibrium model to explore technology adoption and the associated industry upgrading process when the new technology is more capital-intensive and also privately accessible. Private accessibility makes factor prices no longer accurate signals to guide efficient technology adoption even though the factor markets are perfect. It is shown that the private accessibility may lead to adoption delay in some cases and premature adoption in others. Moreover, an increase in initial capital endowment may sometimes delay instead of facilitating adoption of the new capital-intensive technology and multiple equilibria may arise when it is privately accessible. Welfare-enhancing policies are discussed.

Several directions seem interesting to explore for future research. One is to introduce productivity heterogeneity for the new technology in the spirit of Jones (2005). Another is to introduce multiple players (firms) that are all accessible to the new technology and to embed their strategic interaction into the dynamic general equilibrium framework (Bolton and Farrell (1990), Ederington and McCalman (2009)). A third direction is to consider the political economy aspects of this technology adoption problem.

References

[1] Acemoglu, Daron. 2002. "Directed Technical Change," *Review of Economic Studies* 69: 781-810

[2] ______, 2007. "Equilibrium Bias of Technology", *Econometrica*, September (75): 1371-1410.

[3] ______, 2009. *Introduction to Modern Economic Growth.* New Jersey*:* Princeton University Press

[4] ———-, and Fabrizio Zilibotti. 2001. " Productivity Differences", *Quarterly Journal of Economics* 116: 563-606.

[5] Acemoglu, Daron, Pol Antras and Elhanan Helpman. 2007. "Contracts and Technology Adoption," *American Economic Review* 97: 916-943.

[6] Aghion, Philippe and Peter Howitt. 1992. "A Model of Growth Through Creative Destruction", *Econometrica* 60(2): 323-351

[7] ______. 2009. *The Economics of Growth.* Cambridge, Mass: MIT Press

[8] Aghion Philippe, Nick Bloom, Richard Blundell, Rachel Griffith, and Peter Howitt. 2005. "Competition and Innovation: An Inverted-U Relationship", *Quarterly Journal of Economics* 120: 701-728

[9] Atkinson, Anthony B. and Joseph E. Stiglitz. 1969. "A New View of Technological Change," The *Economic Journal* 79: 573-578

[10] Basu, Susanto and David Weil. 1998. " Apropriate Technology and Growth", *Quarterly Journal of Economics*, 113 (4): 1025-1054

[11] Bolton, Patrick and Joseph Farrell. 1990. "Decentralization, Duplication, and Delay", *Journal of Political Economy* 98(4): 803-826

[12] Canda, Vandana. 2006. *Technology, Adaptation, and Exports: How Some Developing Countries Got It Right.* World Bank Press

[13] Chamley, Christophe. 2004. *Rational Herds: Economics Models of Social Learning.* Cambridge University Press

[14] Comin, Diego. and B. Hobijn. 2010. "An Exploration of Technology Diffusion", *American Economic Review* 100(5): 2031-59

[15] Ederington and McCalman (2009). "International trade and industrial dynamics", *International Economic Review* 50: 961–989.

[16] Greenwood, Jeremy, Zvi Hercowitz and Per Krusell. 1997. "Long-Run Implications of Investment-Specific Technological Progress", *American Economic Review* 87(3): 342-362

[17] Grossman, Gene and Elhanan Helpman. 1991. "Quality Ladders in the Theory of Growth," *Review of Economic Studies* 58: 43-61

[18] Harrison, Ann and Andres Rodriguez-Clare. 2009. Trade, Foreign Investment, and Industrial Policies for Developing Countries. *Handbook of Development Economics*. edited by Dani Rodrik

[19] Jones, Charles. 2005. "The Shape of Production Functions and the Direction of Technical Change," *Quarterly Journal of Economics* 120(2): 517-549.

[20] Ju, Jiandong, Yifu Lin and Yong Wang. 2012. "Marshallian Externality, Industrial Upgrading and Industrial Policies", HKUST working paper

[21] ——, 2013. " Endowment Structurement, Industrial Dynamics and Economic Growth", HKUST working paper

[22] Lin, Justin Yifu. 2009. *Marshall Lectures: Economic Development and Transition: Thought, Strategy, and Viability.* London: Cambridge University Press

[23] —— and Celestin Monge. 2010. "Growth Identification and Facilitation: the Role of the State in the Dynamics of Structural Change," Policy Research Working Paper #5313, World Bank

[24] Murphy, Kevin M.; Andrei Shleifer and Robert W. Vishny, 1989. "Industrialization and Big Push." *Journal of Political Economy* 97(5): 1003-1026

[25] Parente, Stephen.1994. "Technology Adoption, Learning-by-Doing and Economic Growth," *Journal of Economic Theory* 63: 346-369

[26] ——- and Edward Prescott, 1994. "Barriers to Technology Adoption and Development," *Journal of Political Economy* 102(2): 298-321

[27] ——. 1999. "Monopoly Rights: A Barrier to Riches," *American Economic Review* 89(5): 1216-1233

[28] Wang, Yong. 2013. "Fiscal Decentralization, Endogenous Policies, and Foreign Direct Investment: Theory and Evidence from China and India," *Journal of Development Economics* 103: 107–123

[29] ___. 2014. "*Industry Dynamics, International Trade, and Economic Growth*", working paper

6 Appendix

6.1 Appendix 1: This is to prove Proposition 4.

Pattern 1: Only Technology 1 in both periods Establish the Lagrangian:

$$\mathcal{L} = \frac{\left[E_1^{\alpha_1} L^{1-\alpha_1}\right]^{1-\sigma} - 1}{1-\sigma} + \beta \frac{\left[E_2^{\alpha_1} L^{1-\alpha_1}\right]^{1-\sigma} - 1}{1-\sigma} + \lambda \left[\xi(K_0 - E_1) - E_2\right],$$

which yields the following two first-order conditions relative to E_1 and E_2, respectively:

$$\begin{aligned} \alpha_1 \left[E_1^{\alpha_1} L^{1-\alpha_1}\right]^{-\sigma} E_1^{\alpha_1 - 1} L^{1-\alpha_1} &= \lambda\xi, \\ \beta\alpha_1 \left[E_2^{\alpha_1} L^{1-\alpha_1}\right]^{-\sigma} E_2^{\alpha_1 - 1} L^{1-\alpha_1} &= \lambda. \end{aligned}$$

We obtain

$$E_1 = \frac{K_0}{1 + \xi^{-1} (\beta\xi)^{\frac{1}{-\alpha_1+1+\alpha_1\sigma}}};\ E_2 = K_0 \frac{(\beta\xi)^{\frac{1}{-\alpha_1+1+\alpha_1\sigma}}}{1 + \xi^{-1} (\beta\xi)^{\frac{1}{-\alpha_1+1+\alpha_1\sigma}}}.$$

$$\frac{1}{-\alpha_1 + 1 + \alpha_1\sigma}$$

Since $\beta\xi > 1$and $\sigma \in [0, 1]$, to ensure $E_1 \leq k_1^* L$ and $E_2 \leq k_1^* L$, we must have

$$\frac{K_0}{L} \leq \theta_1 k_1^*, \tag{36}$$

where k_1^* is given by (9) and θ_1 is defined as

$$\theta_1 \equiv \frac{1 + \xi^{-1} (\beta\xi)^{\frac{1}{-\alpha_1+1+\alpha_1\sigma}}}{(\beta\xi)^{\frac{1}{-\alpha_1+1+\alpha_1\sigma}}}.$$

In other words, technology 1 alone is adopted in both periods if and only if (36) holds.

Pattern 2: Technology 2 in both periods Following the same method, we have:

$$E_1 = \frac{K_0}{1 + \xi^{-1} (\beta\xi)^{\frac{1}{-\alpha_2+1+\alpha_2\sigma}}};\ E_2 = K_0 \frac{(\beta\xi)^{\frac{1}{-\alpha_2+1+\alpha_2\sigma}}}{1 + \xi^{-1} (\beta\xi)^{\frac{1}{-\alpha_2+1+\alpha_2\sigma}}},$$

To ensure E_1 and E_2 larger than $k_2^* L$, we require

$$\frac{K_0}{L} \geq \theta_6 k_1^*, \tag{37}$$

where k_2^* is given by (10) and θ_6 is defined as

$$\theta_6 \equiv \frac{\alpha_2 (1-\alpha_1)}{\alpha_1 (1-\alpha_2)} \left(1 + \xi^{-1} (\beta\xi)^{\frac{1}{-\alpha_2+1+\alpha_2\sigma}}\right).$$

In other words, technology 2 alone is adopted in both periods if and only if (37) holds.

Pattern 3: Technologies 1 and 2 in both periods We can derive from the first order conditions that

$$\begin{aligned} E_1 &= \frac{\left[1-(\beta\xi)^{\frac{1}{\sigma}}\right](1-\alpha_1)k_1^* L + \alpha_1 \xi K_0}{[(\beta\xi)^{\frac{1}{\sigma}} + \xi]\alpha_1}, \\ E_2 &= \xi\left(\frac{\alpha_1 (\beta\xi)^{\frac{1}{\sigma}} K_0 - \left[1-(\beta\xi)^{\frac{1}{\sigma}}\right](1-\alpha_1)(k_1^*) L}{[(\beta\xi)^{\frac{1}{\sigma}} + \xi]\alpha_1}\right). \end{aligned}$$

To ensure $k_1^* L < E_1, E_2 < k_2^* L$, we must have

$$\theta_3 k_1^* < \frac{K_0}{L} < \theta_4 k_1^*,$$

where

$$\theta_3 \equiv \frac{(\beta\xi)^{\frac{1}{\sigma}} + \xi\alpha_1 - (1-\alpha_1)}{\alpha_1 \xi}; \theta_4 \equiv \left[\frac{1}{\xi} + \frac{(\beta\xi)^{-\frac{1}{\sigma}} - (1-\alpha_2)}{\alpha_2}\right] \frac{\alpha_2 (1-\alpha_1)}{\alpha_1 (1-\alpha_2)}.$$

The non-emptiness of K_0 further requires $\theta_3 < \theta_4$, or

$$(\beta\xi) < \left[\frac{(1-\alpha_1)}{(1-\alpha_2)}\right]^{\sigma}. \tag{38}$$

That is, ξ has to be sufficiently small.

Pattern 4: Technology 1 in period 1, Technologies 1 and 2 in period 2 The Euler equation is

$$\begin{aligned} &\left(A_1 (k_1^*)^{\alpha_1} - \frac{A_2 (k_2^*)^{\alpha_2} - A_1 (k_1^*)^{\alpha_1}}{k_2^* - k_1^*} k_1^*\right) L + \frac{A_2 (k_2^*)^{\alpha_2} - A_1 (k_1^*)^{\alpha_1}}{k_2^* - k_1^*} \xi (K_0 - E_1) \\ = &\left[\frac{\beta\xi \left[A_2 (k_2^*)^{\alpha_2} - A_1 (k_1^*)^{\alpha_1}\right]}{\alpha_1 L^{(1-\alpha_1)(1-\sigma)} (k_2^* - k_1^*)}\right]^{\frac{1}{\sigma}} E_1^{\frac{\sigma\alpha_1 - \alpha_1 + 1}{\sigma}}. \end{aligned}$$

To ensure $E_1 \le k_1^* L$, we need

$$\frac{K_0}{L} \le \theta_3 k_1^*. \tag{39}$$

On the other hand,

$$\left(A_1\left(k_1^*\right)^{\alpha_1}-\frac{A_2\left(k_2^*\right)^{\alpha_2}-A_1\left(k_1^*\right)^{\alpha_1}}{k_2^*-k_1^*}k_1^*\right)L+\frac{A_2\left(k_2^*\right)^{\alpha_2}-A_1\left(k_1^*\right)^{\alpha_1}}{k_2^*-k_1^*}E_2$$
$$=\left[\frac{\beta\xi\left[A_2\left(k_2^*\right)^{\alpha_2}-A_1\left(k_1^*\right)^{\alpha_1}\right]}{\alpha_1 L^{(1-\alpha_1)(1-\sigma)}\left(k_2^*-k_1^*\right)}\right]^{\frac{1}{\sigma}}\left(K_0-\frac{E_2}{\xi}\right)^{\frac{\sigma\alpha_1-\alpha_1+1}{\sigma}},$$

To ensure $k_1^*L<E_2<k_2^*L$, we get

$$\theta_1 k_1^*<\frac{K_0}{L}<\theta_2 k_1^*, \tag{40}$$

where

$$\theta_2\equiv[\beta\xi]^{\frac{1}{-\sigma\alpha_1+\alpha_1-1}}\left[\frac{1-\alpha_1}{1-\alpha_2}\right]^{\frac{\sigma}{\sigma\alpha_1-\alpha_1+1}}+\frac{\alpha_2\left(1-\alpha_1\right)}{\alpha_1\left(1-\alpha_2\right)\xi}.$$

The non-emptiness for the set of K_0 requires $\theta_1<\theta_2$, or equivalently,

$$[\beta\xi]^{\frac{1}{-\sigma\alpha_1+\alpha_1-1}}<\frac{[\beta\xi]^{\frac{1}{\sigma}}-1}{\alpha_1\xi}+1,$$

which must hold because $\beta\xi>1$. Also we require

$$\theta_1<\theta_3,$$

or equivalently,

$$\alpha_1\xi<(\beta\xi)^{\frac{1}{-\alpha_1+1+\alpha_1\sigma}}\left[(\beta\xi)^{\frac{1}{\sigma}}+\xi\alpha_1-1\right],$$

which automatically holds. In summary, the equilibrium demonstrates this industrial pattern if and only if (39) and (40) hold.

Pattern 5: Technology 1 in period 1, Technology 2 in period 2 The Euler equation is

$$\beta\xi\alpha_2\left[\xi(K_0-E_1)\right]^{\alpha_2-1-\sigma\alpha_2}\left(AL^{1-\alpha_2}\right)^{1-\sigma}=\alpha_1E_1^{\alpha_1-1-\alpha_1\sigma}L^{(1-\alpha_1)(1-\sigma)}.$$

so $E_1\leq k_1^*L$ implies

$$\beta\xi\alpha_2\left[\xi(K_0-k_1^*L)\right]^{\alpha_2-1-\sigma\alpha_2}\left(AL^{1-\alpha_2}\right)^{1-\sigma}\geq\alpha_1\left(k_1^*L\right)^{\alpha_1-1-\alpha_1\sigma}L^{(1-\alpha_1)(1-\sigma)},$$

which is equivalent to

$$\frac{K_0}{L}\leq\theta_5 k_1^*,$$

where

$$\theta_5\equiv\left(\frac{\alpha_1}{\alpha_2}\right)^{-1}\left[\beta\xi^{(1-\sigma)\alpha_2}\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{(1-\alpha_2)((1-\sigma))}\right]^{\frac{1}{1-\alpha_2+\sigma\alpha_2}}+1.$$

On the other hand

$$\beta\xi\alpha_2 E_2^{\alpha_2-1-\sigma\alpha_2}\left(AL^{1-\alpha_2}\right)^{1-\sigma}=\alpha_1\left(K_0-\frac{E_2}{\xi}\right)^{\alpha_1-1-\alpha_1\sigma}L^{(1-\alpha_1)(1-\sigma)}$$

so $E_2 \geq k_2^* L$ implies

$$\beta\xi\alpha_2\left(k_2^* L\right)^{\alpha_2-1-\sigma\alpha_2}\left(AL^{1-\alpha_2}\right)^{1-\sigma}\geq\alpha_1\left(K_0-\frac{Lk_2^*}{\xi}\right)^{\alpha_1-1-\alpha_1\sigma}L^{(1-\alpha_1)(1-\sigma)},$$

which is reduced to

$$\frac{K_0}{L}\geq\theta_2 k_1^*.$$

In summary, we must have

$$\theta_2 k_1^*\leq\frac{K_0}{L}\leq\theta_5 k_1^*,$$

the non-emptiness of which requires $\theta_2 \leq \theta_5$, or equivalently

$$\left[(\beta\xi)^{-1}\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{\sigma}\right]^{\frac{1}{\sigma\alpha_1-\alpha_1+1}}\leq\frac{\alpha_2(1-\alpha_1)}{\alpha_1(1-\alpha_2)\xi}\left\{\left[(\beta\xi)^{-1}\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{\sigma}\right]^{-\frac{1}{1-\alpha_2+\sigma\alpha_2}}-1\right\}+1, \tag{41}$$

which holds if and only if (38) does not hold. In other words, Pattern 3 and Pattern 5 are incompatible with each other.

Pattern 6: Technologies 1 and 2 in period 1, Technology 2 in period 2:

$$\begin{aligned}&\beta\xi\alpha_2\left[AE_2^{\alpha_2}L^{1-\alpha_2}\right]^{-\sigma}AE_2^{\alpha_2-1}L^{1-\alpha_2}\\ =&\left[\begin{array}{c}\left(A_1\left(k_1^*\right)^{\alpha_1}-\frac{A_2\left(k_2^*\right)^{\alpha_2}-A_1\left(k_1^*\right)^{\alpha_1}}{k_2^*-k_1^*}k_1^*\right)L\\+\frac{A_2\left(k_2^*\right)^{\alpha_2}-A_1\left(k_1^*\right)^{\alpha_1}}{k_2^*-k_1^*}E_1\end{array}\right]^{-\sigma}\frac{A_2\left(k_2^*\right)^{\alpha_2}-A_1\left(k_1^*\right)^{\alpha_1}}{k_2^*-k_1^*}.\end{aligned}$$

From $k_1^* L < E_1 < k_2^* L$, we obtain

$$\theta_6 k_1^* > \frac{K_0}{L} > \theta_5 k_1^*. \tag{42}$$

To ensure $\theta_6 > \theta_5$, we require

$$\frac{\alpha_2(1-\alpha_1)}{\alpha_1(1-\alpha_2)}\left(1+\xi^{-1}(\beta\xi)^{\frac{1}{-\alpha_2+1+\alpha_2\sigma}}\right)>\frac{\alpha_2}{\alpha_1}\left[\beta\xi^{(1-\sigma)\alpha_2}\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{(1-\alpha_2)((1-\sigma))}\right]^{\frac{1}{1-\alpha_2+\sigma\alpha_2}}+1,$$

which must be true because $\frac{\alpha_2(1-\alpha_1)}{\alpha_1(1-\alpha_2)} > 1$ and

$$\frac{(1-\alpha_1)}{(1-\alpha_2)}\xi^{-1}(\beta\xi)^{\frac{1}{-\alpha_2+1+\alpha_2\sigma}}>\left[\beta\xi^{(1-\sigma)\alpha_2}\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{(1-\alpha_2)(1-\sigma)}\right]^{\frac{1}{1-\alpha_2+\sigma\alpha_2}}.$$

On the other hand $E_2 \geq k_2^* L$ requires

$$\frac{K_0}{L} \geq \theta_4 k_1^*. \tag{43}$$

To ensure $\theta_6 > \theta_4$, we require

$$(\beta\xi)^{\frac{1}{-\alpha_2+1+\sigma\alpha_2}} > \left[\frac{(\beta\xi)^{-\frac{1}{\sigma}} - 1}{\alpha_2}\right]\xi + 1,$$

which automatically holds because $\beta\xi > 1$. In summary, the equilibrium has this pattern if and only if (42) and (43) hold.

6.2 Appendix 2: Proof of Lemma 1.

Let $\widetilde{k}_1$ and $\widetilde{k}_2$ denote the equilibrium capital-labor ratios for the two technologies. From the factor market clearing conditions we obtain the total output for each technology:

$$Q_1^m = \Phi(\widetilde{k}_1, \widetilde{k}_2) \equiv \left(\frac{\widetilde{k}_2 L - K}{\widetilde{k}_2 - \widetilde{k}_1}\right)\left(\widetilde{k}_1\right)^{\alpha_1}; \; Q_2^m = \Psi(\widetilde{k}_1, \widetilde{k}_2) \equiv A\left(\frac{K - \widetilde{k}_1 L}{\widetilde{k}_2 - \widetilde{k}_1}\right)\left(\widetilde{k}_2\right)^{\alpha_2},$$

where $\widetilde{k}_1$ and $\widetilde{k}_2$ are given by (9) and (10), respectively and $\frac{R}{W}$ is determined by (21). In the competitive equilibrium, we have

$$Q_1^c = \Phi(k_1^*, k_2^*); \; Q_2^c = \Psi(k_1^*, k_2^*),$$

where k_1^* and k_2^* are given by (1) and (2). Since (9)-(10) always hold, independent of the market structure in the goods market, $\frac{R}{W} < \psi$ implies $\widetilde{k}_1 > k_1^*$ and $\widetilde{k}_2 > k_2^*$. Moreover, $\widetilde{k}_2 > k > \widetilde{k}_1$. Consequently. $Q_1^m > Q_1^c$ and $Q_2^m < Q_2^c$. This is because function $\Phi(\cdot, \cdot)$ strictly increases in both arguments while function $\Psi(\cdot, \cdot)$ strictly decreases in both arguments. Resource allocation is obviously distorted when compared with the competitive equilibrium, so $Q_1^m + Q_2^m < Q_1^c + Q_2^c$.

6.3 Appendix 3: Proof of Lemma 5

Proof. Pattern **A1-c.** $\frac{E_2}{L} \in (0, k_1^*]$ so that only technology 1 is operated in period 2. Note that

$$\frac{P_1}{R_1} = \frac{\left(\Gamma(\frac{E_1}{L})\right)^{\alpha_1 - 1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}}; \; \frac{P_2}{R_2} = \frac{\left(\frac{R_2}{W_2}\right)^{\alpha_1 - 1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}}; \; \frac{W_2}{R_2} = \frac{(1-\alpha_1)E_2}{\alpha_1 L}$$

(33) becomes

$$\beta\xi\left(\frac{\xi^{\alpha_1}(\frac{K_0}{L}-\frac{E_1}{L})^{\alpha_1}}{y_1(\frac{E_1}{L})}\right)^{-\sigma}=\left(\frac{(1-\alpha_1)}{\alpha_1}\xi(\frac{K_0}{L}-\frac{E_1}{L})\Gamma(\frac{E_1}{L})\right)^{1-\alpha_1} \tag{44}$$

which can uniquely pin down $\frac{E_1}{L}$. Clearly $\frac{R_2}{W_2}>\Gamma(\frac{E_1}{L})$, so $\frac{P_1}{R_1}>\frac{P_2}{R_2}$; thus the rhs of (44) is smaller than one, which requires $\frac{\xi^{\alpha_1}(\frac{K_0}{L}-\frac{E_1}{L})^{\alpha_1}}{y_1(\frac{E_1}{L})}>1$ when $\beta\xi>1$ and $\sigma\in(0,1]$. But we must have $y_1(\frac{E_1}{L})>\xi^{\alpha_1}(\frac{K_0}{L}-\frac{E_1}{L})^{\alpha_1}$ because of the adoption pattern. Therefore, this pattern is impossible.

Pattern **B1-b.** Only technology 1 in period 2.

$$\begin{aligned}
P_2/R_2 &= \frac{\left(\frac{W_2}{R_2}\right)^{1-\alpha_1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}}=\frac{\left(\frac{E_2}{L}\frac{1-\alpha_1}{\alpha_1}\right)^{1-\alpha_1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}}\\
\frac{W_2}{R_2} &= \frac{E_2}{L}\frac{1-\alpha_1}{\alpha_1}
\end{aligned}$$

(33) becomes

$$\beta\xi=\left(\frac{[\xi(K_0-E_1)]^{\alpha_1}L^{1-\alpha_1}}{AE_1^{\alpha_2}L^{1-\alpha_2}}\right)^{\sigma}\left(\frac{\xi(K_0-E_1)}{E_1}\frac{1-\alpha_1}{\alpha_1}\frac{\alpha_2}{1-\alpha_2}\right)^{1-\alpha_1}, \tag{45}$$

which uniquely determines E_1. We require $E_1^*\geq k^*L$ and $E_2^*=\xi(K_0-E_1)\leq k_1^*L$, which makes the right hand side of (45) smaller than one, contradicting $\beta\xi>1$.

Pattern **B1-c.** Both technologies in period 2 (that is, $\frac{\xi(K_0-E_1^*)}{L}\in(k_1^*,k_2^*)$)

$$\begin{aligned}
P_1/R_1 &= \frac{1}{\left(\frac{\alpha_2}{1-\alpha_2}\frac{L}{E_1}\right)^{1-\alpha_1}(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}};\\
P_2/R_2 &= \frac{1}{\left(\left[A\frac{\alpha_2^{\alpha_2}(1-\alpha_2)^{(1-\alpha_2)}}{\alpha_1^{\alpha_1}(1-\alpha_1)^{(1-\alpha_1)}}\right]^{\frac{1}{\alpha_2-\alpha_1}}\right)^{1-\alpha_1}(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}};\\
\frac{P_2/R_2}{P_1/R_1} &= \frac{\left(\frac{\alpha_2}{1-\alpha_2}\frac{L}{E_1}\right)^{1-\alpha_1}}{\left(\left[A\frac{\alpha_2^{\alpha_2}(1-\alpha_2)^{(1-\alpha_2)}}{\alpha_1^{\alpha_1}(1-\alpha_1)^{(1-\alpha_1)}}\right]^{\frac{1}{\alpha_2-\alpha_1}}\right)^{1-\alpha_1}}<1.\\
\beta\xi\left(\frac{C_2}{C_1}\right)^{-\sigma} &= \frac{P_2/R_2}{P_1/R_1}.
\end{aligned}$$

Consider the last equation (Euler equation). Whenever $\sigma\in[0,1]$, $LHS>1$ because $\frac{C_2}{C_1}<1$ and $\beta\xi>1$; whereas the $RHS<1$, a contradiction.

6.4 Appendix 4: Proof for Proposition 6

Pattern B2: Only technology 1 in period 1 and both technologies in period 2

$$P_2 = \frac{R_2^{\alpha_1} W_2^{1-\alpha_1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}}; \ \frac{P_2}{R_2} = \frac{\left(\frac{R_2}{W_2}\right)^{\alpha_1-1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}}; \ \frac{R_2}{W_2} = \Gamma(\frac{E_2}{L}).$$

Euler equation (33) becomes

$$\beta\xi\left(\frac{\widetilde{G}(\xi(K_0-E_1),L)}{E_1^{\alpha_1}L^{1-\alpha_1}}\right)^{-\sigma} = \frac{P_2/R_2}{P_1/R_1} = \left(\frac{W_2}{R_2}\frac{R_1}{W_1}\right)^{1-\alpha_1},$$

where function $\widetilde{G}(.,.)$ is defined in (22). This implies (34),which uniquely determines E_1 if it exists, because the LHS increases with E_1 while the RHS decreases with it. Suppose $\sigma = 1$ and $\frac{K_0}{L} = \frac{1+\beta}{\beta\xi}k_1^*$. When $E_1 \to 0$, LHS$\to 0$; RHS$\to\infty$; When $\frac{\xi(K_0-E_1)}{L} \downarrow k_1^*$ (or equivalently,$\frac{E_1}{L} \uparrow \frac{k_1^*}{\beta\xi}$), $LHs \to \beta\xi\left(\frac{(k_1^*)^{\alpha_1}L}{\left(\frac{k_1^*}{\beta\xi}\right)^{\alpha_1}L}\right)^{-1} = (\beta\xi)^{1-\alpha_1}$ while $RHS = \left(\frac{1}{\Gamma(\frac{\xi(K_0-E_1)}{L})}\frac{\alpha_1}{1-\alpha_1}\frac{L}{E_1}\right)^{1-\alpha_1} \to (\beta\xi)^{1-\alpha_1}$. In other words, whenever $\frac{\xi(K_0-E_1)}{L} > k_1^*$, we always have $LHS < RHS$, implying that no solution to (34) exists. Now suppose $\frac{K_0}{L} = \theta_1$ (given by (27)) and $\sigma < 1$. When $E_1 \to 0$, LHS$\to 0$; RHS$\to\infty$. When $\frac{\xi(K_0-E_1)}{L} \downarrow k_1^*$ (or equivalently,$\frac{E_1}{L} \uparrow (\beta\xi)^{-\frac{1}{-\alpha_1+1+\alpha_1\sigma}}k_1^*$), $LHS \to (\beta\xi)^{\frac{\alpha_1+1}{-\alpha_1+1+\alpha_1\sigma}}$ and $RHS \to (\beta\xi)^{\frac{1-\alpha_1}{-\alpha_1+1+\alpha_1\sigma}}$. Thus $LHS > RHS$, so by the Mean Value Theorem, there exists a unique solution $\frac{E_1}{L}$, denoted as θ_0, which falls on $(0,(\beta\xi)^{-\frac{1}{-\alpha_1+1+\alpha_1\sigma}}k_1^*)$ such that (34) is satisfied.

To support Pattern B2, we must require $\frac{\xi(K_0-E_1)}{L} \in (k_1^*, k^*)$, so $\frac{k_1^*}{\xi} < \frac{K_0}{L} < 2\frac{k^*}{\xi}$. In particular, when $\sigma = 1$, (34) becomes

$$\beta\xi y^{-1}(\frac{\xi(K_0-E_1)}{L})\frac{E_1}{L} = \left(\frac{1}{\Gamma(\frac{\xi(K_0-E_1)}{L})}\frac{\alpha_1}{1-\alpha_1}\right)^{1-\alpha_1},$$

so

$$\left[\frac{\alpha_2(1-\alpha_1)}{\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2}\right]^{\frac{1+\alpha_2-\alpha_1}{\alpha_2-\alpha_1}}\frac{(1-\alpha_1)k_1^*}{(1-\alpha_2)\beta\xi} > \frac{E_1}{L} > \frac{k_1^*}{\beta\xi}.$$

On the other hand,

$$\frac{k_1^*}{\xi} + \frac{E_1}{L} < \frac{K_0}{L} < \frac{k^*}{\xi} + \frac{E_1}{L},$$

so we conclude

$$\begin{aligned}
\widetilde{\theta}_1 &\equiv \frac{k_1^*}{\xi} + \frac{k_1^*}{\beta\xi} < \frac{K_0}{L} < \widetilde{\theta}_2 \\
&\equiv \frac{k^*}{\xi} + \frac{\alpha_1 (1-\alpha_1)}{\alpha_1\alpha_2 - \alpha_1^2 + \alpha_2 - \alpha_2^2} \frac{k^*}{\beta\xi}.
\end{aligned}$$

The related present discounted profit is

$$\begin{aligned}
\frac{\Pi_{B2}}{\widetilde{R}} &= \frac{A\left(\frac{\alpha_2}{1-\alpha_2}\right)^{\alpha_2}(1-\alpha_1)(1-\alpha_2)}{\alpha_2-\alpha_1}\left[\Gamma(\frac{E_2}{L})\frac{E_2}{L} - \frac{\alpha_1}{1-\alpha_1}\right] \cdot \\
&\left[\frac{\Gamma(\frac{E_2}{L})^{\alpha_1-\alpha_2}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} - \frac{1}{A(\alpha_2)^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}}\right]\frac{1}{\xi\Gamma(\frac{E_2}{L})}\frac{\alpha_1}{1-\alpha_1}\frac{L}{E_1}LW_1.
\end{aligned} \tag{46}$$

This implies that, for any given K_0, $\frac{\Pi_{B2}}{\widetilde{R}}$ strictly increases when E_2 increases and E_1 decreases, so

$$\begin{aligned}
\frac{\Pi_{B2}}{\widetilde{R}} &< \frac{A\left(\frac{\alpha_2}{1-\alpha_2}\right)^{\alpha_2}(1-\alpha_1)(1-\alpha_2)}{\alpha_2-\alpha_1}\left[\Gamma(k^*)k^* - \frac{\alpha_1}{1-\alpha_1}\right] \cdot \\
&\left[\frac{\Gamma(k^*)^{\alpha_1-\alpha_2}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} - \frac{1}{A(\alpha_2)^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}}\right]\frac{1}{\xi\Gamma(k^*)}\frac{\alpha_1}{1-\alpha_1}\frac{1}{\frac{K_0}{L}-\frac{k^*}{\xi}}LW_1
\end{aligned}$$

In particular, when $\sigma = 1$, (34) becomes

$$\beta\xi\widehat{Y}^{-1}(\xi(K_0-E_1), L, \Gamma(\frac{\xi(K_0-E_1)}{L})) = \frac{1}{E_1}\left(\frac{1}{\Gamma(\frac{\xi(K_0-E_1)}{L})}\frac{\alpha_1}{1-\alpha_1}\right)^{1-\alpha_1}$$

$$\Pi_{B1a} = \left[\left(A\left[\frac{K_0/L}{\beta\frac{\alpha_2}{\alpha_1}+1}\right]^{\alpha_2-\alpha_1}\right)^{\frac{1}{1-\alpha_1}} - 1\right]W_1 L.$$

$$\Gamma(k^*) = \left[\frac{(1-\alpha_1)^{2-\alpha_1}\alpha_1^{\alpha_1}}{A\alpha_2^{\alpha_2-1}(1-\alpha_2)^{(1-\alpha_2)}[\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2]}\right]^{\frac{1}{\alpha_1-\alpha_2}}$$

6.5 Appendix 5: Proof of Lemma 7

6.5.1 Option 1: Immediate Adoption of Technology 2

In period 1, technology 2 can be either operated together with technology 1 (Possibility A) or solely operated (Possibility B). Since the market is perfectly competitive in the second period, we have $\Pi_2 = 0$. By Lemma **, in period 2, either only technology 2 is adopted or both technologies.

Possibility A for Option 1. Both technologies are operated in period 1.

In this case, as suggested by (20), we have

$$\begin{aligned} P_1/R_1 &= \mu_1(W_1,R_1)/R_1 = \frac{\left(\frac{R_1}{W_1}\right)^{\alpha_1-1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} \\ &= \frac{\left(\Gamma(\frac{E_1}{L})\right)^{\alpha_1-1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}}, \end{aligned}$$

where function $\Gamma(\frac{E_1}{L})$ is defined as the implicit solution of $\frac{R_1}{W_1}$ as a function of $\frac{E_1}{L}$ in equation (21) with k replaced by $\frac{E_1}{L}$. Observe $\Gamma'(\frac{E_1}{L}) < 0$ but $\frac{E_1}{L}\Gamma(\frac{E_1}{L})$ is a strictly increasing function of $\frac{E_1}{L}$. To determine $\frac{E_1}{L}$, note that the total output in period 1 is $\widetilde{G}(E_1,L)$, defined in (22). In period 2, both technologies are freely available and the market is perfectly competitive.

Pattern A1-a: both technologies in period 1 and only technology 2 in period 2

$$\begin{aligned} P_2/R_2 &= \mu_2(W_2,R_2)/R_2 = \frac{\left(\frac{W_2}{R_2}\right)^{1-\alpha_2}}{A\,(\alpha_2)^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}} \\ \frac{W_2}{R_2} &= \frac{(1-\alpha_2)\,E_2}{\alpha_2 L} = \frac{(1-\alpha_2)\,\xi(K_0-E_1)}{\alpha_2 L}. \end{aligned}$$

In equilibrium, (33) implies

$$\beta\xi A^{1-\sigma}\alpha_2\Gamma^{\alpha_1-1}(\frac{E_1}{L})y(\frac{E_1}{L})^{\sigma} = (\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}\left[\frac{\xi(K_0-E_1)}{L}\right]^{1-\alpha_2+\alpha_2\sigma}, \tag{47}$$

where $y(\cdot)$ is defined by (23). It uniquely determines $\frac{E_1}{L}$ because the two sides of (47) are strictly monotonic in $\frac{E_1}{L}$ but in opposite directions. To support such an equilibrium, we learn from Section 2 that the following is required:

$$\frac{E_1}{L} \in (k_1^*, k^*) \text{ and } \frac{\xi(K_0-E_1)}{L} \geq k_2^*.$$

Or equivalently,

$$\begin{aligned} \beta\xi A^{1-\sigma}\alpha_2\left(\Gamma(k^*)\right)^{\alpha_1-1}\left(y(k^*)\right)^{\sigma} &> \beta\xi A^{1-\sigma}\alpha_2\left(\Gamma(\frac{E_1}{L})\right)^{\alpha_1-1}\left(y(\frac{E_1}{L})\right)^{\sigma} \\ &> \beta\xi A^{1-\sigma}\alpha_2\left(\Gamma(k_1^*)\right)^{\alpha_1-1}\left(y(k_1^*)\right)^{\sigma} \\ (\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}\left(\frac{\xi(K_0-E_1)}{L}\right)^{1-\alpha_2+\alpha_2\sigma} &\geq (\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}k_2^{*1-\alpha_2+\alpha_2\sigma} \end{aligned}$$

Observe that

$$\begin{aligned}\beta\xi A^{1-\sigma}\alpha_2\left(\Gamma(k_1^*)\right)^{\alpha_1-1}\left(y(k_1^*)\right)^{\sigma} &> \alpha_1^{\alpha_1}\left(1-\alpha_1\right)^{1-\alpha_1}k_2^{*1-\alpha_2+\alpha_2\sigma}\\ &\Leftrightarrow \beta\xi>\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{\sigma}\end{aligned}$$

in which case,

$$\begin{aligned}(\alpha_1)^{\alpha_1}\left(1-\alpha_1\right)^{1-\alpha_1}\left(\frac{\xi(K_0-E_1)}{L}\right)^{1-\alpha_2+\alpha_2\sigma} &> \beta\xi A^{1-\sigma}\alpha_2\left(\Gamma(k_1^*)\right)^{\alpha_1-1}\left(y_1(k_1^*)\right)^{\sigma}\\ &\Leftrightarrow\\ \left(\frac{\alpha_1}{\alpha_2}\right)^{-1}\left[\beta\xi\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{(1-\alpha_2)(1-\sigma)}\right]^{\frac{1}{1-\alpha_2+\alpha_2\sigma}}\frac{k_1^*}{\xi}+\frac{E_1}{L} &< \frac{K_0}{L}.\end{aligned}$$

It can be shown that

$$(\beta\xi)^{\frac{1}{1-\alpha_2+\sigma\alpha_2}}\frac{\left[\frac{\alpha_2(1-\alpha_1)}{\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2}\right]^{\frac{1-\alpha_1+\sigma\alpha_2}{(\alpha_2-\alpha_1)(1-\alpha_2+\sigma\alpha_2)}}}{\xi\left(\frac{\alpha_1}{\alpha_2}\right)\left(\frac{1-\alpha_2}{1-\alpha_1}\right)}k_1^*+\frac{E_1}{L}>\frac{K_0}{L},$$

so

$$\begin{aligned}(\beta\xi)^{\frac{1}{1-\alpha_2+\sigma\alpha_2}}\frac{\left[\frac{\alpha_2(1-\alpha_1)}{[\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2]}\right]^{\frac{1-\alpha_1+\sigma\alpha_2}{(\alpha_2-\alpha_1)(1-\alpha_2+\sigma\alpha_2)}}}{\xi\left(\frac{\alpha_1}{\alpha_2}\right)\left(\frac{1-\alpha_2}{1-\alpha_1}\right)}k_1^*+k^* &> \frac{K_0}{L},\\ \left(\frac{\alpha_1}{\alpha_2}\right)^{-1}\left[\beta\xi\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{(1-\alpha_2)(1-\sigma)}\right]^{\frac{1}{1-\alpha_2+\alpha_2\sigma}}\frac{k_1^*}{\xi}+k_1^* &< \frac{K_0}{L}.\end{aligned}$$

In particular, when $\sigma=1$, we have

$$\widetilde{\theta}_4\equiv\beta\frac{\alpha_2\left(1-\alpha_1\right)}{[\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2]}k^*+k^*>\frac{K_0}{L}>\widetilde{\theta}_0\equiv\frac{\alpha_2}{\alpha_1}\beta k_1^*+k_1^*.$$

The profit is given by

$$\frac{\Pi_{A1a}}{W_1L}=\frac{(1-\alpha_1)}{(\alpha_2-\alpha_1)}\left[\Gamma(\frac{E_1}{L})\frac{E_1}{L}-\frac{\alpha_1}{1-\alpha_1}\right]\left[\left(\frac{\Gamma(\frac{E_1}{L})}{\psi}\right)^{\alpha_1-\alpha_2}-1\right],$$

where ψ is given by (7). The first-order condition is given by (21), replacing k with $\frac{E_1}{L}$. It can be rewritten as

$$\left(\frac{\Gamma(\frac{E_1}{L})}{\psi}\right)^{\alpha_1-\alpha_2}=\frac{1}{(1+\alpha_1-\alpha_2)+\frac{(\alpha_2-\alpha_1)\alpha_1}{(1-\alpha_1)\Gamma(\frac{E_1}{L})\frac{E_1}{L}}},$$

which determines $\frac{E_1}{L}$.

Now compare Π_{A1a} with $\frac{\Pi_{A2}}{\widetilde{R}}$ (when $\sigma = 1$), which is given by

$$\frac{\Pi_{A2}}{\widetilde{R}} = \beta\left(1 - \left[A\left(\frac{K_0\beta\xi}{L(1+\beta)}\right)^{\alpha_2-\alpha_1}\right]^{-\frac{1}{1-\alpha_1}}\right)LW_1$$

when

$$\widetilde{\theta}_5 \equiv \beta k^* + k^* \geq \frac{K_0}{L} \geq \widetilde{\theta}_0 \equiv \left(\beta\frac{\alpha_2}{\alpha_1}+1\right)k_1^*.$$

In particular, when $\frac{K_0}{L} = \widetilde{\theta}_5$,

$$\begin{aligned}
\frac{\Pi_{A2}}{\widetilde{R}} &= \beta\left(1 - \left[A\left(k^*\beta\xi\right)^{\alpha_2-\alpha_1}\right]^{-\frac{1}{1-\alpha_1}}\right)LW_1 \\
&= \beta\left(1 - \left[\frac{(\beta\xi)^{\alpha_2-\alpha_1}\alpha_1^{\alpha_1}(1-\alpha_1)}{\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2}\left(\frac{\alpha_2(1-\alpha_1)}{1-\alpha_2}\right)^{1-\alpha_1}\right]^{-\frac{1}{1-\alpha_1}}\right)LW_1 \\
&\geq \beta\left(1 - \left[\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{1+\alpha_2-2\alpha_1}\frac{\alpha_1^{\alpha_1}\alpha_2^{1-\alpha_1}(1-\alpha_1)}{\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2}\right]^{-\frac{1}{1-\alpha_1}}\right)LW_1
\end{aligned}$$

When $\frac{K_0}{L} = \widetilde{\theta}_5$, we know that $\frac{E_1}{L} = k^*$ for pattern A1-a, (26) implies that

$$\Pi_{A1a} = \frac{(\alpha_2-\alpha_1)^2}{[1-\alpha_2]\left[\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2\right]}W_1L.$$

Obviously, $\frac{\Pi_{A2}}{\widetilde{R}} \leq \Pi_{A1a}$ when β is sufficiently small. When $\alpha_2 - \alpha_1 \to 0$, it turns out that $\frac{\Pi_{A2}}{\widetilde{R}} \to 0$ and $\Pi_{A1a} \to 0$. Let us see several numerical examples.

Suppose $\alpha_1 = \frac{1}{3}$ and $\alpha_2 = \frac{2}{3}$, then

$$\Pi_{A1a} = W_1L > \frac{\Pi_{A2}}{\widetilde{R}}.$$

Suppose $\alpha_1 = \frac{1}{2}$ and $\alpha_2 = 1$, then

$$\begin{aligned}
\Pi_{A1a} &= \frac{(\alpha_2-\alpha_1)^2}{[1-\alpha_2]\left[\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2\right]}W_1L \to \infty \\
\frac{\Pi_{A2}}{\widetilde{R}} &= \beta LW_1.
\end{aligned}$$

Suppose $\alpha_1 = 0$ and $\alpha_2 = \frac{1}{2}$, then

$$\begin{aligned}
\Pi_{A1a} &= 2W_1L \\
\frac{\Pi_{A2}}{\widetilde{R}} &= \beta LW_1.
\end{aligned}$$

Suppose $\alpha_1 = \frac{1}{4}$ and $\alpha_2 = \frac{1}{2}$, then

$$\begin{aligned}\Pi_{A1a} &= \frac{2}{5}W_1 L \\ \frac{\Pi_{A2}}{\widetilde{R}} &= \beta\left(1-\frac{5}{9}\left[(\beta\xi)\left(\frac{3}{5}\right)\right]^{-\frac{1}{3}}\right)LW_1 \\ &> \beta\left(1-\frac{5}{9}\left[\frac{9}{10}\right]^{-\frac{1}{3}}\right)LW_1\end{aligned}$$

Notice $\beta\xi > \frac{1-\alpha_1}{1-\alpha_2} = \frac{3}{2}$. So $\frac{\Pi_{A2}}{\widetilde{R}} > \Pi_{A1a}$ when $\beta > \frac{2}{5\left(1-\frac{5}{9}\left(\frac{9}{10}\right)^{-\frac{1}{3}}\right)} > \frac{9}{10}$.

Also, whenever both Pattern A1-a and Pattern B1-a are feasible, the latter gives a strictly larger profit for firm M.

Pattern A1-b: both technologies in both periods

The analysis in Subsection 2.1 suggests that we must require $\frac{E_2}{L} \in (k_1^*, k_2^*)$. Applying Proposition 1, we have

$$C_2 = \alpha_1 k_1^{*\alpha_1 - 1} E_2 + (1-\alpha_1) k_1^{*\alpha_1} L.$$

Observe that

$$\frac{P_1}{R_1} = \frac{\left(\Gamma(\frac{E_1}{L})\right)^{\alpha_1 - 1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}}; \frac{P_2}{R_2} = \frac{1}{\alpha_1 (k_1^*)^{\alpha_1 - 1}}.$$

(33) becomes

$$\beta\xi\left(\frac{a\xi(\frac{K_0}{L}-\frac{E_1}{L})+b}{y_1(\frac{E_1}{L})}\right)^{-\sigma} = \left(\frac{1-\alpha_1}{\alpha_1}k_1^*\Gamma(\frac{E_1}{L})\right)^{1-\alpha_1}, \tag{48}$$

which uniquely determines $\frac{E_1}{L}$. The profit is $\pi_1((\frac{E_1}{L})^*)W_1 L$. To support this equilibrium, we require

$$\xi(\frac{K_0}{L}-\frac{E_1}{L}) \in (k_1^*, k_2^*) \text{ and } \frac{E_1}{L} \in (k_1^*, k^*),$$

which jointly imply

$$\left(\frac{(1-\alpha_1)}{1-\alpha_2}\right)^{-\sigma}\beta\xi k_1^{*-\sigma\alpha_1} < \left(\frac{1-\alpha_1}{\alpha_1}k_1^*\Gamma(\frac{E_1}{L})\right)^{1-\alpha_1} y_1(\frac{E_1}{L})^{-\sigma} < k_1^{*-\sigma\alpha_1}$$

which requires $\beta\xi < \left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{\sigma}$. In that case $k_1^* < \frac{E_1}{L} < \widetilde{k}_1^*(< k^*)$, where $\widetilde{k}_1^*$ is uniquely determined by

$$\left(\frac{1-\alpha_1}{1-\alpha_2}\right)^{-\sigma}\left(\frac{1-\alpha_1}{\alpha_1}\right)^{-(1-\alpha_1)}\beta\xi k_1^{*-\sigma\alpha_1-(1-\alpha_1)} = \Gamma(\widetilde{k}_1^*)^{1-\alpha_1} y_1(\widetilde{k}_1^*)^{-\sigma}.$$

On the other hand

$$\begin{aligned}
\beta\xi\left(a\xi(\frac{K_0}{L}-\frac{E_1}{L})+b\right)^{-\sigma} &< k_1^{*-\sigma\alpha_1} \\
\frac{(\beta\xi)^{\frac{1}{\sigma}}-(1-\alpha_1)}{\alpha_1}\frac{k_1^*}{\xi} &< \frac{K_0}{L}-\frac{E_1}{L}<\frac{k_2^*}{\xi}
\end{aligned}$$

In summary, we must have

$$\frac{(\beta\xi)^{\frac{1}{\sigma}}-(1-\alpha_1)}{\alpha_1}\frac{k_1^*}{\xi}+k_1^*<\frac{K_0}{L}<\widetilde{k}_1^*+\frac{k_2^*}{\xi}.$$

In particular, when $\sigma=1$, it requires

$$\beta\xi<\frac{1-\alpha_1}{1-\alpha_2}.$$

and

$$\frac{\beta\xi-(1-\alpha_1)}{\alpha_1\xi}k_1^*+k_1^*<\frac{K_0}{L}<\frac{\alpha_2}{1-\alpha_2}\frac{(1-\alpha_1)\,k_1^*}{\alpha_1\xi}+\widetilde{k}_1^*$$

Possibility B for Option 1 : only technology 2 in period 1

$$P_1/R_1=\frac{\left(\frac{W_1}{R_1}\right)^{1-\alpha_1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}}=\frac{1}{\left(\frac{\alpha_2}{1-\alpha_2}\frac{L}{E_1}\right)^{1-\alpha_1}(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}}$$

because

$$\frac{R_1}{W_1}=\frac{\alpha_2}{1-\alpha_2}\frac{L}{E_1}$$

The profit is

$$\Pi_{B1}=\frac{\left[\left(\frac{1}{k_2^*}\frac{E_1}{L}\right)^{\alpha_2-\alpha_1}-1\right]}{(1-\alpha_2)}W_1L$$

Pattern B1-a: Only technology 2 in both periods

$$P_2/R_2=\frac{\left(\frac{W_2}{R_2}\right)^{1-\alpha_2}}{A\,(\alpha_2)^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}}=\frac{\left[\frac{\xi(K_0-E_1)}{L}\right]^{1-\alpha_2}}{A\alpha_2}$$

Then (33) yields:

$$\beta\xi=\left[\frac{\xi(K_0-E_1)}{E_1}\right]^{1-\alpha_2+\sigma\alpha_2}\left(\frac{E_1}{L}\right)^{\alpha_1-\alpha_2}\frac{\alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}}{A\alpha_2^{\alpha_1}(1-\alpha_2)^{1-\alpha_1}}. \tag{49}$$

To justify Pattern B1-a, we must have $E_1^*\geq k^*L$ and $E_2^*=\xi(K_0-E_1)\geq k_2^*L$, or equivalently, $K_0 - \frac{k_2^* L}{\xi} \geq E_1 \geq k^* L$,thus $K_0 \geq \frac{k_2^* L}{\xi} + k^* L$which, by (49), are reduced to

$$k_2^{*1+\sigma\alpha_2-\alpha_1}\left(K_0 - \frac{k_2^* L}{\xi}\right)^{\alpha_1-(1+\sigma\alpha_2)} L^{1+\sigma\alpha_2-\alpha_1} \leq \beta\xi \leq \left[\frac{\xi(K_0 - k^* L)}{k^* L}\right]^{1-\alpha_2+\sigma\alpha_2}$$

So ultimately we must require

$$\frac{(\beta\xi)^{\frac{1}{1+\sigma\alpha_2-\alpha_2}}\ k^* L}{\xi} + k^* L \leq K_0. \tag{50}$$

In particular, when $\sigma = 1$, (49) becomes

$$\left(\frac{E_1}{L k_2^*}\right)^{\alpha_2-\alpha_1} = \frac{(K_0 - E_1)}{\beta E_1}, \tag{51}$$

which uniquely determines E_1. Moreover, $\Pi_{B1a} = \frac{\left[\left(\frac{1}{k_2^*}\frac{E_1}{L}\right)^{\alpha_2-\alpha_1} - 1\right]}{(1-\alpha_2)} W_1 L = \frac{\left[\frac{(K_0-E_1)}{\beta E_1} - 1\right]}{(1-\alpha_2)} W_1 L$. Whenever both Pattern A1-a and Pattern B1-a are feasible, the latter gives a strictly larger profit for firm M, so we can refine the previous Lemma can be further refined by adding that Pattern A1-a is an equilibrium only if $\frac{K_0}{L} \in (\widetilde{\theta}_0, \widetilde{\theta}_5)$.

6.5.2 Option 2: Technology 2 is first adopted in Period 2

The market structure is perfectly competitive in period 1 with only technology 1. Thus

$$\begin{aligned}
\frac{P_1}{R_1} &= \frac{\left(\frac{R_1}{W_1}\right)^{\alpha_1-1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} \\
\frac{R_1}{W_1} &= \frac{\alpha_1}{1-\alpha_1}\frac{L}{E_1} \\
P_1 &= \frac{R_1^{\alpha_1} W_1^{1-\alpha_1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} \\
C_1 &= E_1^{\alpha_1} L^{1-\alpha_1}.
\end{aligned}$$

In period 2, either only technology 2 is operated or both technologies are operated.

Pattern A2: only technology 1 in period 1 and only technology 2 in period 2

$$
\begin{aligned}
P_2 &= \frac{R_2^{\alpha_1} W_2^{1-\alpha_1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} \\
\frac{P_2}{R_2} &= \frac{\left(\frac{R_2}{W_2}\right)^{\alpha_1-1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} \\
\frac{R_2}{W_2} &= \frac{\alpha_2}{(1-\alpha_2)\left(\frac{E_2}{L}\right)} \\
C_2 &= A\left[\xi(K_0-E_1)\right]^{\alpha_2} L^{1-\alpha_2}
\end{aligned}
$$

(33) implies

$$
\beta\xi\left(A\left(\frac{E_1}{L}\right)^{\alpha_2-\alpha_1}\right)^{-\sigma} = \left(\frac{\xi(K_0-E_1)}{E_1}\right)^{-\alpha_1+1+\sigma\alpha_2}\left(\frac{\alpha_2(1-\alpha_1)}{(1-\alpha_2)\alpha_1}\right)^{\alpha_1-1}
$$

which uniquely determines E_1^*. Since $\xi\frac{K_0-E_1}{L} \geq k^*$,

$$
\frac{E_1}{L} \geq \left[\frac{k^{*-\alpha_1+1+\sigma\alpha_2}}{\beta\xi A^{-\sigma}}\left(\frac{\alpha_2(1-\alpha_1)}{(1-\alpha_2)\alpha_1}\right)^{\alpha_1-1}\right]^{\frac{1}{-\alpha_1+1+\sigma\alpha_1}}.
$$

No conditions need to be imposed for period 1 because technology 2 is privately accessible. To justify that firm M serves the whole market in period 2 with technology 2, we require $E_2^* \geq k^* L$, which means $K_0 - \frac{k^*}{\xi}L \geq E_1$, therefore

$$
\frac{E_1}{L} \leq \left(\frac{1}{A}\right)^{\frac{1}{(\alpha_2-\alpha_1)}}\left[\frac{\left(\frac{\alpha_2(1-\alpha_1)}{(1-\alpha_2)\alpha_1}\right)^{\alpha_1-1}}{\beta\xi(k^*)^{\alpha_1-1-\sigma\alpha_2}}\right]^{-\frac{1}{\sigma(\alpha_2-\alpha_1)}}
$$

$$
K_0 \geq \left[\frac{A^\sigma k^{*1+\sigma\alpha_2-\alpha_1}}{\beta\xi}\left(\frac{\alpha_2(1-\alpha_1)}{(1-\alpha_2)\alpha_1}\right)^{\alpha_1-1}\right]^{\frac{1}{\sigma\alpha_1-\alpha_1+1}} L + \frac{k^*}{\xi}L \tag{52}
$$

$$
\begin{aligned}
\widetilde{R} &= \frac{\xi R_2}{R_1} = \frac{\xi\alpha_2(1-\alpha_1)}{\alpha_1\frac{E_2}{E_1}(1-\alpha_2)}\frac{W_2}{W_1} \\
\Pi_{A2} &= \frac{\left[\left(\frac{1}{k_2^*}\frac{E_2}{L}\right)^{\alpha_2-\alpha_1}-1\right]}{(1-\alpha_2)}W_2 L \\
\frac{\Pi_{A2}}{\widetilde{R}} &= \frac{\alpha_1\frac{E_2}{E_1}\left[\left(\frac{1}{k_2^*}\frac{E_2}{L}\right)^{\alpha_2-\alpha_1}-1\right]W_1 L}{\xi\alpha_2(1-\alpha_1)}
\end{aligned}
$$

In addition, we have

$$\frac{\partial E_1^*}{\partial \beta} < 0; \frac{\partial E_1^*}{\partial A} \geq 0; \frac{\partial E_1^*}{\partial L} \leq 0; \frac{\partial E_1^*}{\partial \xi} \geq 0; \frac{\partial E_1^*}{\partial K_0} > 0,$$

where "=" holds if and only if $\sigma = 1$, in which case the Euler equation becomes

$$\beta\xi\left(\xi(K_0 - E_1)\right)^{\alpha_1 - 1 - \alpha_2} = AE_1^{-1}\left(\frac{1}{L}\right)^{\alpha_2 - \alpha_1}\left(\frac{\alpha_2\left(1-\alpha_1\right)}{\left(1-\alpha_2\right)\alpha_1}\right)^{\alpha_1 - 1},$$

or equivalently,

$$E_1 = E_2^{-\alpha_1 + \alpha_2 + 1}\frac{A}{\beta\xi}\left(\frac{1}{L}\right)^{\alpha_2 - \alpha_1}\left(\frac{\alpha_2\left(1-\alpha_1\right)}{\left(1-\alpha_2\right)\alpha_1}\right)^{\alpha_1 - 1}.$$

Thus

$$\begin{aligned}
\frac{\Pi_{A2}}{\widetilde{R}} &= \frac{\alpha_1\frac{E_2}{E_1}\left[\left(\frac{1}{k_2^*}\frac{E_2}{L}\right)^{\alpha_2-\alpha_1} - 1\right]W_1 L}{\xi\alpha_2\left(1-\alpha_1\right)} \\
&= \frac{\beta\left(\frac{\alpha_2(1-\alpha_1)}{(1-\alpha_2)\alpha_1}\right)^{-\alpha_1}\left[\left(\frac{1}{k_2^*}\frac{E_2}{L}\right)^{\alpha_2-\alpha_1} - 1\right]W_1 L}{A\left(1-\alpha_2\right)\left(\frac{E_2}{L}\right)^{\alpha_2-\alpha_1}}
\end{aligned}$$

Recall under B1-a: $K_0 \geq \frac{(\beta\xi)^{\frac{1}{1+\sigma\alpha_2-\alpha_2}}k^*L}{\xi} + k^*L$. Under A2,

$$\begin{aligned}
K_0 &\geq \left[\frac{\left[\frac{(1-\alpha_1)\alpha_2}{\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2}\left(\frac{1-\alpha_1}{1-\alpha_2}\right)\right]^{\sigma}}{\beta\xi}\right]^{\frac{1}{\sigma\alpha_1-\alpha_1+1}}\frac{\alpha_1\left(1-\alpha_2\right)}{\alpha_2\left(1-\alpha_1\right)}k^*L + \frac{k^*}{\xi}L \\
\text{when } \sigma &= 1,\ \frac{K_0}{L} \geq \widetilde{\theta}_2 \equiv \frac{\alpha_1\left(1-\alpha_1\right)}{\alpha_1\alpha_2-\alpha_1^2+\alpha_2-\alpha_2^2}\frac{k^*}{\beta\xi} + \frac{k^*}{\xi}
\end{aligned}$$

$$\frac{\Pi_{A2}}{\widetilde{R}} = \frac{\alpha_1\frac{E_2}{E_1}\left[\left(\frac{1}{k_2^*}\frac{E_2}{L}\right)^{\alpha_2-\alpha_1} - 1\right]W_1 L}{\xi\alpha_2\left(1-\alpha_1\right)}.$$

Now we can compare $\frac{\Pi_{A2}}{\widetilde{R}}$ with Π_{B1}. In particular, when $\sigma = 1$, we have $\frac{\Pi_{A2}}{\widetilde{R}} > \Pi_{B1a} \Leftrightarrow$

$$\frac{\beta\left(\frac{\alpha_2(1-\alpha_1)}{(1-\alpha_2)\alpha_1}\right)^{-\alpha_1}\left[\left(\frac{1}{k_2^*}\right)^{\alpha_2-\alpha_1} - \left(\frac{E_2}{L}\right)^{-\alpha_2+\alpha_1}\right]}{A} > \frac{(K_0 - E_1)}{\beta E_1} - 1.$$

Note that LHS is smaller than $\frac{\beta\left(\frac{\alpha_2(1-\alpha_1)}{(1-\alpha_2)\alpha_1}\right)^{-\alpha_1}\left(\frac{1}{k_2^*}\right)^{\alpha_2-\alpha_1}}{A}$, but for RHS (B1a),

$$\frac{(K_0-E_1)}{\beta E_1}-1=\frac{\beta\left(\frac{\alpha_2(1-\alpha_1)}{(1-\alpha_2)\alpha_1}\right)^{-\alpha_1}\left(\frac{1}{k_2^*}\right)^{\alpha_2-\alpha_1}}{A}$$

if and only if

$$\begin{aligned} E_1 &= \left[1+\frac{(1-\alpha_2)\,\beta}{1-\alpha_1}\right]^{\frac{1}{\alpha_2-\alpha_1}} k_2^* L, \\ K_0 &= \left[\left(1+\frac{(1-\alpha_2)\,\beta}{1-\alpha_1}\right)\beta+1\right]\left[1+\frac{(1-\alpha_2)\,\beta}{1-\alpha_1}\right]^{\frac{1}{\alpha_2-\alpha_1}} k_2^* L \end{aligned}$$

In other words, we know that $\frac{\Pi_{A2}}{\widehat{R}}<\Pi_{B1a}$ when

$$K_0 \geq \left[\left(1+\frac{(1-\alpha_2)\,\beta}{1-\alpha_1}\right)\beta+1\right]\left[1+\frac{(1-\alpha_2)\,\beta}{1-\alpha_1}\right]^{\frac{1}{\alpha_2-\alpha_1}} k_2^* L,$$

6.5.3 Option 3 (C): Technology 2 is never adopted

This occurs only when $\frac{K_0}{L}$ is sufficiently small so that the market cannot support technology 2 in either period. The threshold value is derived when analyzing Pattern B2.

6.6 Appendix 6: Proof of Propositions 8 and 9

Option 2: Technology 2 is first adopted in Period 2.

The market structure is perfectly competitive in period 1 with only technology 1 in operation. Thus

$$\begin{aligned} \frac{P_1}{R_1} &= \frac{\left(\frac{R_1}{W_1}\right)^{\alpha_1-1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} \\ \frac{R_1}{W_1} &= \frac{\alpha_1}{1-\alpha_1}\frac{L}{E_1} \\ P_1 &= \frac{(R_1)^{\alpha_1} W_1^{1-\alpha_1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} \\ C_1 &= E_1^{\alpha_1} L^{1-\alpha_1}. \end{aligned}$$

In period 2, there are two possibilities: Possibility A is that only technology 2 is operated in period 2. Possibility B is that both technologies are operated in period 2.

Pattern A2: only technology 1 in period 1 and only technology 2 in period 2

$$
\begin{aligned}
P_2 &= \frac{(R_2)^{\alpha_1} W_2^{1-\alpha_1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} \\
\frac{P_2}{R_2} &= \frac{\left(\frac{R_2}{W_2}\right)^{\alpha_1-1}}{(\alpha_1)^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}} \\
\frac{R_2}{W_2} &= \frac{\alpha_1}{1-\alpha_1} A^{\frac{1}{1-\alpha_1}} \left(\frac{E_2}{L}\right)^{\frac{1-\alpha_2}{\alpha_1-1}} \\
C_2 &= A\left[\xi(K_0-E_1)\right]^{\alpha_2} L^{1-\alpha_2}
\end{aligned}
$$

(33) implies

$$
\beta\xi A^{1-\sigma}\left(\frac{E_1}{L}\right)^{1-\alpha_1+\sigma\alpha_1} = (\xi\frac{K_0-E_1}{L})^{1-\alpha_2+\sigma\alpha_2},
$$

which uniquely determines E_1^* . No conditions need to be imposed for technology 1 to be implemented in period 1 when technology 2 is privately accessible. To justify that firm M can serve the whole market in period 2 with technology 2, we require $E_2^* \geq k^*L$, which means

$$
\xi\frac{K_0-E_1}{L} \geq k^*; \frac{E_1}{L} \geq \left[\frac{k^{*1-\alpha_2+\sigma\alpha_2}}{\beta\xi A^{1-\sigma}}\right]^{\frac{1}{1-\alpha_1+\sigma\alpha_1}}
$$

therefore

$$
\frac{K_0}{L} \geq \frac{k^*}{\xi} + \left[\frac{k^{*1-\alpha_2+\sigma\alpha_2}}{\beta\xi A^{1-\sigma}}\right]^{\frac{1}{1-\alpha_1+\sigma\alpha_1}} \tag{53}
$$

In addition, we have

$$
\frac{\partial E_1^*}{\partial\beta} < 0; \frac{\partial E_1^*}{\partial A} \leq 0; \frac{\partial E_1^*}{\partial L} \geq 0; \frac{\partial E_1^*}{\partial\xi} \leq 0; \frac{\partial E_1^*}{\partial K_0} > 0,
$$

where "=" holds if and only if $\sigma = 1$, in which case

$$\begin{aligned}
E_1^* &= \frac{K_0}{1+\beta}; E_2^* = \frac{\beta\xi K_0}{1+\beta}; \\
\frac{R_1}{W_1} &= \frac{\alpha_1 (1+\beta)}{(1-\alpha_1)} \frac{L}{K_0}; \frac{R_2}{W_2} = \frac{\alpha_1}{1-\alpha_1} A^{\frac{1}{1-\alpha_1}} \left(\frac{\beta\xi K_0}{L(1+\beta)} \right)^{\frac{1-\alpha_2}{\alpha_1 - 1}}; \\
\Pi_{A2} &= \left(A^{\frac{1}{1-\alpha_1}} \left(\frac{\beta\xi K_0}{(1+\beta) L} \right)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}} - 1 \right) LW_2 \\
\widetilde{R} &= \frac{\xi^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}} A^{\frac{1}{1-\alpha_1}} \left(\frac{K_0}{L}\right)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}} W_2}{(1+\beta)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}} \beta^{\frac{1-\alpha_2}{1-\alpha_1}} W_1} \\
\frac{\Pi_{A2}}{\widetilde{R}} &= \beta \left(1 - \left[A \left(\frac{K_0 \beta\xi}{L(1+\beta)} \right)^{\alpha_2-\alpha_1} \right]^{-\frac{1}{1-\alpha_1}} \right) LW_1
\end{aligned}$$

and (53) becomes

$$\frac{K_0}{L} \geq \frac{k^*}{\xi} + \frac{k^*}{\beta\xi}.$$

For more general $\sigma \in [0,1)$, we have

$$\frac{\Pi_{A2}}{\widetilde{R}} = \left[\frac{L^{\frac{\alpha_1-\alpha_2}{\alpha_1-1}}}{\xi^{\frac{\alpha_1-\alpha_2}{\alpha_1-1}} E_1^* \left[(K_0 - E_1^*)\right]^{\frac{1-\alpha_2}{\alpha_1-1}}} \left(\left(\frac{\beta\xi K_0}{(1+\beta) L} \right)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}} - A^{-\frac{1}{1-\alpha_1}} \right) \right] LW_1.$$

Now we can compare $\frac{\Pi_{A2}}{\widetilde{R}}$ with the profit in possibility B for option1, Π_{B1}. In particular, when $\sigma = 1$, we have

$$\begin{aligned}
\frac{\Pi_{A2}}{\widetilde{R}} &> \Pi_{B1a} \Leftrightarrow \\
& \beta + 1 - \left(A \left[\frac{1}{\beta \frac{\alpha_2}{\alpha_1} + 1} \right]^{\alpha_2-\alpha_1} \right)^{\frac{1}{1-\alpha_1}} (K_0/L)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}} \\
&> \beta \left[A \left(\frac{\beta\xi}{(1+\beta)} \right)^{\alpha_2-\alpha_1} \right]^{-\frac{1}{1-\alpha_1}} (K_0/L)^{-\frac{\alpha_2-\alpha_1}{1-\alpha_1}}
\end{aligned}$$

which holds if and only if

$$x_1 < (K_0/L)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}} < x_2 \tag{54}$$

where

$$
\begin{aligned}
x_{1,2} &\equiv \frac{(1+\beta) \mp \sqrt{(1+\beta)^2 - 4\beta \left(\frac{(1+\beta)}{\beta\xi\left[\beta\frac{\alpha_2}{\alpha_1}+1\right]} \right)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}}}}{2\left(A \left[\frac{1}{\beta\frac{\alpha_2}{\alpha_1}+1} \right]^{\alpha_2-\alpha_1} \right)^{\frac{1}{1-\alpha_1}}} \\
\Delta &\equiv (1+\beta)^2 - 4\beta \left(\frac{(1+\beta)}{\beta\xi\left[\beta\frac{\alpha_2}{\alpha_1}+1\right]} \right)^{\frac{\alpha_2-\alpha_1}{1-\alpha_1}} > 0
\end{aligned}
$$

Therefore, when (54) holds, $\frac{\frac{\Pi_{A2}}{R}}{\Pi_{B1a}} > 1$; otherwise $\frac{\frac{\Pi_{A2}}{R}}{\Pi_{B1a}} \leq 1$. Pattern A2 requires $\frac{K_0}{L} \geq \widetilde{\theta}_1 \equiv \frac{(1+\beta)}{\beta\xi} k^*$.