Firm Sorting and Agglomeration

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Abstract

The distribution of firms in space is far from uniform. Some locations host the most productive large firms, while others barely attract any. In this paper, I study the sorting of heterogeneous firms across locations and analyze policies designed to attract firms to particular regions (place-based policies). I first propose a theory of the distribution of heterogeneous firms in a variety of sectors across cities. Aggregate TFP and welfare depend on the extent of agglomeration externalities produced in cities and on how heterogeneous firms sort across them. The distribution of city sizes and the sorting patterns of firms are uniquely determined in equilibrium. This allows me to structurally estimate the model, using French firm-level data. I find that nearly two thirds of the observed productivity advantage of large cities is due to firm sorting. I use the estimated model to quantify the general equilibrium effects of place-based policies. I find that policies that decrease local congestion lead to a new spatial equilibrium with higher aggregate TFP and welfare. In contrast, policies that subsidize under-developed areas have negative aggregate effects.

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1 Introduction

The distribution of firms in space is far from uniform. Some locations host the most productive large firms, while others barely attract any. Recognizing the key role played by firms in the development of local activity, governments offer financial incentives to attract firms to specific areas. Such interventions aim, in general, to reduce spatial disparities and to increase welfare by fostering local agglomeration externalities. In practice, though, positive local impacts may be counterbalanced by undesirable effects in other regions. In addition, spatial disparities could be reinforced if subsidies attract low-productivity firms while leaving high-productivity ones in booming regions. To understand these effects, I propose a theory of the distribution of heterogeneous firms in a variety of sectors across cities. The model has a unique equilibrium, which is affected by the implementation of these place-based policies. This allows me to conduct a counterfactual policy analysis and to quantify their impact in terms of aggregate productivity and welfare.

Place-based policies are pervasive. Kline and Moretti (2013) report that an estimated 95 billion dollars are spent annually in the United States to attract firms to certain locations. Massive subsidies are targeted to individual firms in a nationwide competition to attract plants.[1] Federal programs provide generous tax breaks in an effort to attract firms to less developed areas, as analyzed most recently by Busso et al. (2013). The European Union regulates these aids and reports a yearly figure of 15 billion euros.[2] In a broader sense, place-based policies also encompass regulations that impact the growth of cities, such as zoning regulations or restrictions on the height of buildings. I analyze the impact of both types of policies in the light of my framework.

To analyze the long-run impact of these policies, one needs to understand how heterogeneous firms react to them and how targeted cities grow or shrink as a result. I develop a model that is uniquely suited to answering these questions. Heterogeneous firms sort across cities. Cities are endogenously impacted by this sorting. They grow when firms choose to locate there and increase the local labor demand. Since firms' location choices are distorted by local policies, cities are, in turn, endogenously affected. Importantly, the model admits a unique equilibrium, which is impacted by local policies. The model captures several forces. Cities are ex ante identical. They are the locus of intangible agglomeration externalities such as thick labor markets or knowledge spillovers (Duranton and Puga (2003)). The sorting of firms into cities of different sizes is driven

[1] Examples abound and are compiled in Story (2012). Recent examples include Apple who obtained a reported 88 million dollars in tax cuts to locate a data center in Reno, Nevada, Caterpillar with a reported 77 million dollars incentives to build a plant in Georgia, or Boeing getting a 120 million dollars subsidy from South Carolina to expand its operations in Charleston.

[2] In contrast to the United States, the European Union regulates the amount and geographical scope of these aids. They are restricted to *less advantaged* regions. Both in Europe and in the US, there exists a wider range of policies that are designed with spatial impact in mind but are not accounted for in these figures, such as the design of local tax reporting for multi-location firms. Bartik (2004) reports a taxonomy of such policies.

by a trade-off between gains in productivity, through local externalities, and higher labor costs. I assume that more efficient firms benefit relatively more from local externalities. This generates positive assortative matching: more efficient firms locate in larger cities, reinforcing their initial edge. A class of city developers compete to attract firms to their city. They act as a coordinating device in the economy, leading to a unique spatial equilibrium.

Using firm-level data, I show that the model is able to reproduce salient features of the French economy. First, in the model, initial differences in productivity between firms induce sorting across city sizes. This, in turn, reinforces firm heterogeneity, as firms in large cities benefit from stronger agglomeration forces. The resulting firm-size distribution is more thick-tailed for sectors that tend to locate in larger cities. Second, across sectors, firm sorting is shaped by the intensity of input use. In labor-intensive sectors, firms locate more in small cities where wages are lower. Third, within sectors, firms have higher revenues in larger cities. They may have lower employment, though, since they face higher labor costs. I show that these outcomes of the model are broadly consistent with stylized facts about French firms.

I structurally estimate the model. This structure is needed to quantify the impact of place-based policies on the spatial equilibrium, absent a real-life counterfactual. The structural estimation recovers a model-based estimate of the shape of agglomeration externalities. It allows me to disentangle the roles played by agglomeration forces on the one hand, and firm sorting on the other hand, in shaping productivity gains associated with city size. I estimate that nearly two thirds of the measured productivity advantage of large cities comes from the sorting of firms based on their efficiency. The magnitude of the agglomeration economies I estimate are in line with existing estimates in the literature as reported by Rosenthal and Strange (2004).

Finally, I simulate the new spatial equilibrium that results from two policies: a tax-relief scheme targeted at firms locating in smaller cities, and the removal of regulations that hamper city growth, such as zoning or building-height regulations, as advocated by Glaeser and Gottlieb (2008). Firms make different location choices as a result, which modifies local labor demand and in equilibrium impacts the city-size distribution. The productive efficiency of the new equilibrium depends in particular on the new city-size distribution: it drives the extent of agglomeration externalities leveraged by firms in the economy. I find that a policy that subsidizes less productive areas has negative aggregate effects on TFP and welfare. In contrast, a policy that favors the growth of cities leads to a new spatial equilibrium that is significantly more productive, by endogenously creating agglomeration externalities and reducing the impact of market failures.

The paper is related to several strands of the literature. The main contribution of the paper is to propose a model of spatial equilibrium that is well suited to the analysis of place-based policies, since it features freely mobile, heterogeneous firms and endogenous city sizes, and has a unique equilibrium. The literature that studies systems of cities, pioneered by Henderson (1974), has traditionally focused on homogenous firms. Recent contributions have introduced richer heterogeneity in the spatial setting.[3] In a seminal contribution, Behrens et al. (2010) study the spatial sorting of entrepreneurs who produce non-tradable intermediates. I study the polar case of producers of perfectly tradable goods. An important difference for policy analysis is the uniqueness of the equilibrium I obtain here. Another closely related strand of the literature (Eeckhout et al. (2010), Davis and Dingel (2012) and Davis and Dingel (2013)) studies the spatial sorting of workers who differ in skill level, to shed light on patterns of wage inequality and on the spatial distribution of skills. My paper uses similar conceptual tools, borrowed from the assignment literature, to focus on how *firms* sort and impact local labor demand, motivated by the fact that firms are directly targeted by place-based policies.[4]

Second, the model shows novel theoretical and empirical evidence of a link between the geographical pattern of economic activity and the sectoral firm-size distribution. It contributes to the literature that aims to explain the determinants of firm-size distribution by exploiting cross-sectoral heterogeneity. Rossi-Hansberg and Wright (2007a) focus on the role of decreasing returns to scale in determining the shape of the establishment-size distribution, while di Giovanni et al. (2011) consider the role played by endogenous selection into exporting. I focus instead on the role played by location choice. The model also contributes to the literature on city-size distribution. City-size distribution is traditionally explained by random growth models, as in Gabaix (1999) and Eeckhout (2004), for example. Here, as in Behrens et al. (2010), the distribution of city sizes is endogenous to the sorting of heterogeneous firms in a static spatial equilibrium, since cities are the collection of the labor force employed by local firms. The city-size distribution is shaped by properties of the firm-size distribution.

As in Desmet and Rossi-Hansberg (2013) and Behrens et al. (2013), I use structural estimation of a model of a system of cities to assess the welfare implications of the spatial equilibrium. The focus of the analysis is different, since they do not explicitly account for sorting by heterogeneous firms. The paper also contributes to the literature that measures agglomeration externalities, as reviewed in Rosenthal and Strange (2004). Most analyses do not take into account the sorting of heterogeneous agents, workers or firms, across locations. A notable exception is Combes et al. (2008), who use detailed data on workers characteristics to control for worker heterogeneity and sorting in a reduced form analysis. I use a structural approach to explicitly account for the

[3] Early studies of heterogeneous firms in the spatial context include Nocke (2006) and Baldwin and Okubo (2006). They predict that more productive firms self-select in larger markets. These results are obtained in a setting where the size of regions is fixed and exogenously given. In contrast, I focus on how local policies endogenously change the spatial equilibrium, including city sizes themselves, as they attain their objective to see targeted cities grow.

[4] The model borrows insights from the assignment model literature, such as Costinot and Vogel (2010), and in particular Eeckhout and Kircher (2012) and Sampson (forthcoming), who focus on heterogeneous firms matching with heterogeneous workers. Here, firms match with heterogeneous city sizes.

sorting of firms. Combes et al. (2012) also emphasize the heterogeneous effect of agglomeration externalities on heterogeneous firms. They show that the most efficient firms are disproportionally more efficient in large cities, indicating potential complementarities between firm productivity and city size. Their analysis is based on a model without firm location choice, in contrast to the approach I take.

Finally, the counterfactual policy analysis offers a complementary approach to research that assesses the impact of specific place-based policies (see, for example, Busso et al. (2013) for the US, Mayer et al. (2012) for France and Criscuolo et al. (2012) for the UK). The literature has traditionally focused on estimating the local effects of these policies. A notable exception is Kline and Moretti (2013), who develop a methodology to estimate their aggregate effects.[5] For the policy they consider, they estimate that positive local effects are offset by losses in other parts of the country. My approach is similar in spirit, with the difference that I account explicitly for firm sorting. I find a negative aggregate effect of policies targeting the smallest cities. Finally, Glaeser and Gottlieb (2008) study theoretically the economic impact of place-based policies. My analysis brings in heterogeneous firms and the general equilibrium effect of place-based policies on the productive efficiency of the country.

The paper is organized as follows. Section 2 presents the model and its predictions. Section 3 details the empirical analysis. I show salient features from French firm-level data that are consistent with the forces at play in the model. I then structurally estimate the model using indirect inference. In section 4, I conduct a counterfactual analysis using the estimated model. Section 5 concludes.

2 A Model of the Location Choice of Heterogeneous Firms

Consider an economy in which production takes place in locations that I call cities. Cities are constrained in land supply, which acts as a congestion force. The economy is composed of a variety of sectors. Within sectors, firms are heterogeneous in productivity. They produce, in cities, using local labor and traded capital. Non-market interactions within cities give rise to positive agglomeration externalities. I assume that they have heterogeneous effects on firms, in the sense that more efficient firms are more able to leverage local externalities. Firms' choice of city results from a trade-off between the strength of local externalities, the local level of input prices and, possibly, the existence of local subsidies. Heterogeneous firms face different incentives, which yields heterogeneity in their choice. I follow Henderson (1974) and postulate the existence

[5] Albouy (2012) focuses on a related question. He argues that federal taxes impose a de facto unequal geographic burden since they do not account for differences in local cost of living, and estimates he the corresponding welfare cost.

of a class of city developers. In each potential city site, a developer represents local landowners and competes against other sites to attract firms. City developers play a coordination role in the creation of cities, which leads to a unique equilibrium of the economy. The model describes a long-run steady state of the economy and abstracts from dynamics.

2.1 Set-up and agents' problem

2.1.1 Cities

Each potential city site has a given stock of land, normalized to 1. Sites are identical ex-ante. Cities emerge endogenously on these sites, potentially with different population levels L. In what follows, I index cities and all the relevant city-level parameters by city size L. City size is sufficient to characterize all the economic forces at play, in the tradition of models of systems of cities pioneered by Henderson (1974). In particular the distance between two cities plays no role as goods produced in the economy are either freely traded between cities within the country, or are, in the case of housing, non tradable.

Land is used to build housing, which is divisible and consumed by workers. Atomistic landowners construct housing h by combining their land γ with local labor ℓ, according to the housing production function

$$h = \gamma^b \big(\frac{\ell}{1-b}\big)^{1-b}. \tag{1}$$

The land-use intensity b governs the amount of housing that can be produced with a given stock of land. Lowering b increases the housing supply in a city where the stock of land is fixed. This parameter summarizes the technological and regulatory constraints on housing supply in a city. In the policy experiments, I analyze how it impacts productivity and welfare in the spatial equilibrium.

Landowners compete in the housing market, taking both the housing price $p_H(L)$ and the local wage $w(L)$ as given. Since there is a fixed total supply of land equal to 1 in the city, the housing supply equation is

$$H(L) = \big(\frac{p_H(L)}{w(L)}\big)^{\frac{1-b}{b}}, \tag{2}$$

where $H(L)$ is the total quantity of housing supplied in a city of size L.

2.1.2 Workers

Set-up There is a mass N_0 of identical workers. Each worker is endowed with one unit of labor. A worker lives in the city of his choosing, consumes a bundle of traded goods and housing, and is paid the local wage $w(L)$. Workers' utility is

$$U = \left(\frac{c}{\eta}\right)^{\eta} \left(\frac{h}{1-\eta}\right)^{1-\eta}, \tag{3}$$

where h denotes housing and c is a Cobb-Douglas bundle of goods across S sectors of the economy defined as

$$c = \prod_{j=1}^{j=S} c_j^{\xi_j} \;, \;\text{ with } \quad \sum_{j=1}^{j=S} \xi_j = 1.$$

Within a sector j, consumers choose varieties according to the CES aggregator

$$c_j = \left[\int c_j(i)^{\frac{\sigma_j - 1}{\sigma_j}} di \right]^{\frac{\sigma_j}{\sigma_j - 1}}.$$

I denote by P the aggregate price index for the composite good c. Since goods are freely tradable, the price index is the same across cities. Workers are perfectly mobile and ex ante identical.

Workers' problem Workers in city L consume $c(L)$ units of the good and $h(L)$ units of housing to maximize their utility (3), under the budget constraint

$$P\, c(L) + p_H(L)h(L) = w(L).$$

As a result, the aggregate local demand for housing is

$$H(L) = \frac{(1-\eta)w(L)L}{p_H(L)}. \tag{4}$$

The local housing market clears. Equations (2) and (4) pin down prices and quantities of housing produced.[6] The quantity of housing consumed by each worker in city L is

$$h(L) = (1-\eta)^{1-b}\, L^{-b}. \tag{5}$$

Housing consumption is lower in more populous cities because cities are constrained in space. This congestion force counterbalances the agglomeration-inducing effects of positive production

[6] Appendix A gives the equilibrium housing prices and labor hired.

externalities in cities and prevents the economy from complete agglomeration into one city.

Since workers are freely mobile, their utility must be equalized in equilibrium across all inhabited locations to a level $\bar{U}$. In equilibrium, wages must increase with city size to compensate workers for congestion costs, according to

$$w(L) = \bar{w}((1-\eta)\,L)^{b\frac{1-\eta}{\eta}}, \tag{6}$$

where $\bar{w} = \bar{U}^{\frac{1}{\eta}}\,P$ is an economy-wide constant to be determined in the general equilibrium.

2.1.3 Firms

Production The economy consists of S sectors that manufacture differentiated tradable products. Sectors are indexed by $j = 1, ..., S$. Firms have the same factor intensities within their sectors but differ in productivity. Firms produce varieties using two factors of production that have the following key characteristics. One has a price that increases with city size; the other has a constant price across cities. For simplicity, I consider only one factor whose price depends on city size: labor. In particular, I do not consider land directly in the firm production function. I call the other factor capital, as a shorthand for freely tradable inputs. Capital is provided competitively by absentee capitalists. The price of capital is fixed exogenously in international markets, and the stock of capital in the country adjusts to the demand of firms.

Firms differ exogenously in efficiency z. A firm of efficiency z in sector j and city of size L produces output according to the following Cobb-Douglas production function

$$y_j(z, L) \;=\; \psi(z, L, s_j)\; k^{\alpha_j}\; \ell^{1-\alpha_j},$$

where ℓ and k denote labor and capital inputs, α_j is the capital intensity of firms in sector j and $\psi(z, L, s_j)$ is a firm-specific Hicks-neutral productivity shifter, as detailed below. It is determined by the firm's 'raw' efficiency, the extent of the local agglomeration externalities and a sector-specific parameter s_j.

Firms engage in monopolistic competition. Varieties produced by firms are freely tradable across space: there is a sectoral price index that is constant through space. Firms take it as given. What matters for location choice is the trade-off between production externalities and costs of production. The relative input price varies with city size. Wages increase with L (equation (6)), whereas capital has a uniform price. Therefore, the factor intensity of a firm shapes, in part, its location decision. A more labor-intensive firm faces, all else equal, a greater incentive to locate in a smaller city where wages are lower.

Productivity and agglomeration The productivity of a firm $\psi(z, L, s_j)$ depends on its own 'raw' efficiency z, on local agglomeration externalities that increase with city size L, and on a sector-specific parameter s_j. I explain the roles of these parameters in turn.

A key assumption of the model is that ψ exhibits a strong complementarity between local externalities and the efficiency of the firm. Firms that are more efficient at producing are also more efficient at leveraging local agglomeration externalities, such as knowledge spillovers or labor-market pooling. The empirical literature offers some suggestive evidence pointing to such complementarity between firm efficiency and agglomeration externalities. Arzaghi and Henderson (2008) present an empirical study of the advertising industry. They measure the benefit agencies derive from interactions in local networks and provide evidence that agencies that were larger in the first place are more willing than smaller ones to pay higher rents in order to have access to a better local network. Combes et al. (2012) study a wide set of French industries and provide suggestive evidence that more efficient firms are disproportionately more productive in larger cities, pointing to such a complementarity as a potential explanation for this fact. Finally, in Section 3, I present a set of stylized facts on French firms' location and production patterns. They are consistent with sorting, a consequence of the assumed complementarity.

Knowledge spillovers can arguably exhibit this type of complementarity. More efficient firms can better leverage the local information they obtain. A similar idea, though for individual agents, is provided by Davis and Dingel (2012). In their model, more able individuals optimally spend less time producing and more time leveraging local knowledge, which increases their productivity, leading to such a complementarity. In Appendix B, I extend this idea to the set-up of my model where workers are hired to produce the blueprint of the firm. The firm can have workers work at the plant or spend time improving the blueprint by leveraging local information, e.g., on production processes or product appeal. It is optimal for more efficient firms to encourage workers to spend more time discovering local ideas. As a result, the productivity of the firm exhibits complementarity between city size and the firm's own efficiency.[7]

In what follows, I remain agnostic on the source of agglomeration externalities and their specific functional form. This allows me to highlight the generic features of an economy with such complementarities. I let the productivity $\psi(z, L, s)$ have the following properties:

Assumption A *$\psi(z, L, s)$ is log-supermodular in city size L, firm raw efficiency z and sectoral characteristic s, and is twice differentiable. In addition, $\psi(z, L, s)$ is strictly log-supermodular in (z, L). That is,*

$$\frac{\partial^2 log\, \psi(z, L, s)}{\partial L \partial z} > 0, \quad \frac{\partial^2 log\, \psi(z, L, s)}{\partial L \partial s} \geq 0, \quad \textit{and} \quad \frac{\partial^2 log\, \psi(z, L, s)}{\partial z \partial s} \geq 0.$$

[7] The productivity function is log-supermodular in firm efficiency and city size.

I introduce a sector-specific parameter s_j that allows sectors to vary in the way they benefit from local urbanization externalities. Rosenthal and Strange (2004) note that empirical studies suggest that the force and scope of agglomeration externalities vary across industries. More specifically, Audretsch and Feldman (1996) suggest that the benefits from agglomeration externalities are shaped by an industry's life-cycle and that highly innovative sectors benefit more strongly from local externalities than mature industries. I index industries such that, in high s sectors, firms benefit from stronger agglomeration forces, for a given city size.[8] In the estimation of the model, I allow for parameter values that shut down the heterogeneous effect between agglomeration externalities and firm efficiency. The specification I retain for ψ nests the typical specification considered in the literature, where only agglomeration forces of the form $\psi = zL^s$ are at play.[9]

Finally, I restrict the analysis to productivity functions $\psi(z, L, s)$ for which the firms' problem is well defined and concave, absent any local subsidies, for all firms. In other words, I assume that the positive effects of agglomeration externalities are not too strong compared to the congestion forces. In particular, given that the congestion forces increase with city size with a constant elasticity, agglomeration externalities must have decreasing elasticity to city size to prevent a degenerate outcome with complete agglomeration of firms in the largest city.

Entry and location choice There is an infinite supply of potential entrants who can enter the sector of their choosing. Firms pay a sunk cost f_{Ej} in terms of the final good to enter sector j, then draw a raw efficiency level z from a distribution $F_j(.)$. I assume that this distribution is an open interval (possibly unbounded) on the real line. This assumption is made for tractability; the results carry through without it, although the notation is more cumbersome. Once firms discover their raw efficiency, they choose the size of the city where they want to produce.[10]

Firms' problem A firms' choice of city size is influenced by three factors. First, relative input prices vary by city size. Second, firm productivity increases with city size, through greater agglomeration externalities. Third, local city developers compete to attract firms to their cities by subsidizing profits at rate $T_j(L)$, which varies by city-size and sector.[11]

[8] In addition, because $\frac{\partial^2 log\ \psi(z,L,s)}{\partial z \partial s}$ is non-negative, for any city size L, the elasticity of firm productivity to its initial efficiency is either equal across sectors, or larger in high s sectors.

[9] In that case $\frac{\partial^2 log\ \psi(z,L,s)}{\partial L \partial z} = 0$, $\frac{\partial^2 log\ \psi(z,L,s)}{\partial s \partial z} = 0$ and $\frac{\partial^2 log\ \psi(z,L,s)}{\partial L \partial s} > 0$.

[10] Contrary to the setting in Melitz (2003), the model abstracts from any selection of firms at entry, since there is no fixed cost to produce. I focus instead on *where* firms decide to produce once they discover their efficiency, and how this shapes the spatial equilibrium of the economy. That is, rather than selection on entry, I focus on selection on city size.

[11] The subsidy offered by city developers $T_j(L)$ may in principle differ across cities for cities of the same size. As I show in the next section, this is not the case in equilibrium. Anticipating this, I abstract from introducing cumbersome notation denoting different cities of the same size.

The firm's problem can be solved recursively. For a given city size, the problem of the firm is to hire labor and capital and set prices to maximize profits, taking as given the size of the city (and hence the size of the externality term), input prices, and subsidies. Then, firms choose location to maximize this optimized profit.

Consider a firm of efficiency z producing in sector j and in a city of size L. Firms hire optimally labor and capital, given the relative factor prices $\frac{w(L)}{\rho}$ – where ρ denotes the cost of capital – and their local productivity $\psi(z, L, s_j)$. This choice is not distorted by local subsidies to firms' profits. Firms treat local productivity as exogenous, so that the agglomeration economies take the form of external economies of scale. Given the CES preferences and the monopolistic competition, firms set constant markups over their marginal cost. This yields optimized profits for firm z in sector j as a function of city size L

$$\pi_j(z, L) = \kappa_{1j}\big(1 + T_j(L)\big)\left(\frac{\psi(z, L, s_j)}{w(L)^{1-\alpha_j}}\right)^{\sigma_j - 1} R_j\, P_j^{\sigma_j - 1}, \tag{7}$$

where P_j is the sectoral price index, R_j is the aggregate spending on goods from sector j and κ_{1j} is a sector-specific constant.[12]

Note that firm employment, conditional on being in a city of size L, is given by

$$\ell_j(z, L) = (1 - \alpha_j)(\sigma_j - 1)\frac{\pi_j(z, L)}{w(L)\big(1 + T_j(L)\big)}. \tag{8}$$

The proportionality between profits - net of subsidies - and the wage bill is a direct consequence of constant factor shares, implied by the Cobb-Douglas production function, and of constant markup pricing.
The problem of the firm thus is to choose the city size L to maximize (7).

2.1.4 City developers

Set-up There is one city-developer for each potential city size. City developers fully tax local landowners and compete to attract firms to their city by subsidizing their profits. Absent these developers, there would be a coordination failure as atomistic agents alone - firms, workers or landowners - cannot create a new city. City developers are, in contrast, large players at the city level. Developers have limited information over firms. They observe the industry of the firm as well as the profits made by firms locally, but they do not observe the level of efficiency of every individual firm. As a result, they do not have the full set of instruments that would allow them

[12] The sectoral constant κ_{1j} is $\kappa_{1j} = \frac{((\sigma_j-1)\alpha_j^{\alpha_j}(1-\alpha_j)^{1-\alpha_j}\rho^{-\alpha_j})^{\sigma_j-1}}{\sigma_j^{\sigma_j}}$

to internalize the externalities in this economy, contrary to the case in Henderson (1974). The equilibrium outcome will not be efficient. Nevertheless, as in Henderson (1974), city developers act as a coordinating device that allows a unique equilibrium to emerge in terms of city-size distribution.[13] There is perfect competition and free entry among city developers, which drives their profits to zero in equilibrium.

City developers' problem Each city developer i announces a city size L and a level of subsidy to local firms' profits in sector j, $T_j^i(L)$. Developers are funded by the profits made on the housing market. As the housing market clears in each city, aggregate landowner profits at the city level are:

$$\pi_H(L) = b(1-\eta)Lw(L), \tag{9}$$

as detailed in Appendix A. It will prove useful when solving for the equilibrium to note that a constant share of the local labor force is hired to build housing, namely

$$\ell_H(L) = (1-b)(1-\eta)L. \tag{10}$$

This impacts the local labor-market-clearing conditions used in equilibrium.

A city developer i developing a city of size L faces the following problem:

$$\begin{aligned}
&\max_{L,\{T_j^i(L)\}_{j\in 1,\dots,S}} \Pi_L = b(1-\eta)w(L)L - \sum_{j=1}^{S}\int_z \frac{T_j^i(L)}{1+T_j^i(L)}\pi_j(z,L)\mathbb{1}_j(z,L,i)M_j dF_j(z),\\
&\text{such that}\\
&\qquad \mathbb{1}_j(z,L,i) = 1 \qquad \text{if } L = \arg\max_L \pi_j(z,L) \text{ and firm } z \text{ chooses city } i,\\
&\qquad \mathbb{1}_j(z,L) = 0 \qquad \text{otherwise.}
\end{aligned} \tag{11}$$

In this expression, M_j denotes the mass of firms in sector j, $F_j(.)$ is the distribution of raw efficiencies in sector j and $\pi_j(z,L)$ is the local profit of a firm of efficiency z in sector j, as defined in (7).

2.2 Spatial equilibrium

Having set up the problems of workers, firms, landowners and city developers, I am now ready to solve for the equilibrium of the economy. I show that this equilibrium exists and is unique.

[13] The model is silent on which location is used by city developers. The equilibrium is unique in terms of city-size distribution, not in terms of the actual choice of sites by developers, as all sites are ex ante identical.

2.2.1 Equilibrium definition

Definition 1 *An equilibrium is a set of cities $\mathcal{L}$ characterized by a city-size distribution $f_L(.)$, a wage schedule $w(L)$, a housing-price schedule $p_H(L)$ and for each sector $j = 1, ..., S$ a location function $L_j(z)$, an employment function $\ell_j(z)$, a capital-use function $k_j(z)$, a production function $y_j(z)$, a price index P_j and a mass of firms M_j such that*

(i) workers maximize utility (equation (3)) given $w(L), p_H(L)$ and P_j,

(ii) utility is equalized across all inhabited cities,

(iii) firms maximize profits (equation (7)) given $w(L), \rho$ and P_j,

(iv) landowners maximize profits given $w(L)$ and $p_H(L)$,

(v) city developers choose $T_j(L)$ to maximize profits (equation (11)) given $w(L)$ and the firm problem,

(vi) factors, goods and housing markets clear; in particular, the labor market clears in each city,

(vii) capital is optimally allocated, and

(viii) firms and city developers earn zero profits.

In what follows, I present a constructive proof of the existence of a such an equilibrium. Furthermore, I show that the equilibrium is unique. As is standard in the literature, I allow for the possibility of a non-integer number of cities of any given size (see Abdel-Rahman and Anas (2004) for a review and more recently Rossi-Hansberg and Wright (2007b) or Behrens et al. (2010)).

Proposition 1 *There exists a unique equilibrium of this economy.*

2.2.2 Constructing the spatial equilibrium

The equilibrium is constructed in four steps. First, I solve for the equilibrium subsidy offered by city developers. Second, I show that it pins down how firms match with city sizes, as well as the set of city sizes generated in equilibrium by city developers. Third, general equilibrium quantities are determined by market clearing conditions and free entry conditions in the traded goods sectors, once we know the equilibrium matching function from step 2. Finally, the city-size distribution is determined by these quantities, using labor-market clearing conditions. In each step, the relevant functions and quantities are uniquely determined; hence, the equilibrium is unique.

Step 1: Equilibrium subsidy

Lemma 2 *In equilibrium, city developers offer a constant subsidy to firms' profit $T_j^* = \frac{b(1-\eta)(1-\alpha_j)(\sigma_j-1)}{1-(1-\eta)(1-b)}$ for firms in sector j, irrespective of city size.*

I sketch the proof in the case of an economy with only one traded goods sector. The formal proof with S sectors follows the same logic and is given in Appendix C. City developers face perfect competition, which drives their profits down to zero in equilibrium. Their revenues correspond to the profits made in the housing sector (equation (9)), which are proportional to the aggregate wage bill in the city $w(L)L$. They compete to attract firms by subsidizing their profits. In equilibrium, irrespective of which firms choose to locate in city L, these profits will also be proportional to the sectoral wage bill $w(L)N$, where N is the labor force hired in the traded goods sector locally, as can be seen from equation (8). Finally, the local labor force works either in the housing sector (equation (10)) or the traded goods sector, so that $N = L(1-(1-b)(1-\eta))$. Profits given by (11) simplify to $b(1-\eta)w(L)L - T\frac{(1-(1-b)(1-\eta))}{(\sigma-1)(1-\alpha)}w(L)L$. The choice of city size is irrelevant, and T^* is the only subsidy consistent with zero profits. City developers that offer lower subsidies will not attract any firm, hence will not create cities. City developers that offer higher subsidies attract firms but make negative profits.

The equilibrium subsidy does not depend on city size, perhaps surprisingly. This comes from the fact that city developers only have a limited set of tools to influence firms' choices, since they only subsidize profits and do not observe firm efficiencies. These tools do not allow them to influence firm choices and internalize the local production externalities. In this economy, the existence of a class of city developers helps pin down the set of cities that is opened up in equilibrium - as seen below - but does not influence firm choices. Therefore, the characterization of the equilibrium in terms of firms' outcomes, which I detail later, does not depend on the existence of city developers. They are also valid in an economy where cities are exogenously given.

Step 2: Equilibrium city sizes and the matching function The city developers' problem determines the equilibrium city sizes generated in the economy. Cities are opened up when there is an incentive for city developers to do so, i.e. when there exists a set of firms and workers that would be better off choosing this city size. Workers are indifferent between all locations, but firms are not, since their profits vary with city size. Given the equilibrium subsidy T_j^* offered by city developers, the profit function of firm z in sector j is:

$$\pi_j^*(z, L) = \kappa_{1j}\big(1 + T_j^*\big)\left(\frac{\psi(z, L, s_j)}{w(L)^{1-\alpha_j}}\right)^{\sigma_j - 1} R_j\, P_j^{\sigma_j - 1} \tag{12}$$

There is a unique profit-maximizing city size for a firm of type z in sector j, under the regularity conditions I have assumed. Define the optimal city size as follows

$$L_j^{**}(z) = \arg\max_{L \geq 0} \pi_j^*(z, L). \tag{13}$$

Assume that, for some firm type z and sector j, no city of size $L_j^{**}(z)$ exists. There is then a profitable deviation for a city developer on an unoccupied site to open up this city. It will attract the corresponding firms and workers, and city developers will make a positive profit by subsidizing firms at a rate marginally smaller than T_j^*. The number of such cities adjusts so that each city has the right size in equilibrium. This leads to the following lemma, letting $\mathcal{L}$ denote the set of city sizes in equilibrium:

Lemma 3 *The set of city sizes $\mathcal{L}$ in equilibrium is the optimal set of city sizes for firms.*

Given this set of city sizes, the optimal choice of each firm is fully determined. Define the matching function

$$L_j^*(z) = \arg\max_{L \in \mathcal{L}} \pi_j^*(z, L). \tag{14}$$

It is readily seen that the profit function of the firm (equation (12)) inherits the strict log-supermodularity of the productivity function in z and L. Therefore, the following lemma holds.

Lemma 4 *The matching function $L_j^*(z)$ is increasing in z.*

This result comes from a classic theorem in monotone comparative statics (Topkis (1998)). The benefit to being in larger cities is greater for more productive firms and only they are willing in equilibrium to pay the higher factor prices there. Furthermore, the matching function is fully determined by the firm maximization problem, conditional on the set of city sizes $\mathcal{L}$. As seen from equation (12), this optimal choice does not depend on general equilibrium quantities that enter the profit function proportionally for all city sizes. Finally, under the regularity assumptions made on ψ as well as on the distribution of z, $F_j(.)$, the optimal set of city sizes for firms in a given sector is an interval (possibly unbounded). The sectoral matching function is invertible over this support. For a given sector, I use the notation $z_j^*(L)$ to denote the inverse of $L_j^*(z)$. It is increasing in L. The set of city sizes $\mathcal{L}$ available in equilibrium is the union of the sector-by-sector intervals.

Step 3: General equilibrium quantities The equilibrium has been constructed up to the determination of the following general equilibrium values. The reference level of wages $\bar{w}$ defined in equation (6) is taken as the numeraire. The remaining unknowns are the aggregate revenues in the traded goods sector R, the mass of firms M_j and the sectoral price indexes P_j.

I use the following notation:

$$E_j = \int \frac{\psi(z, L_j^*(z), s_j)^{\sigma_j - 1}}{L_j^*(z)^{\frac{b(1-\eta)(1+(1-\alpha_j)(\sigma_j-1))}{\eta}}} dF_j(z), \text{ and} \tag{15}$$

$$S_j = \int \left(\frac{\psi(z, L_j^*(z), s_j)}{L_j^*(z)^{\frac{b(1-\eta)(1-\alpha_j)}{\eta}}} \right)^{\sigma_j - 1} dF_j(z), \tag{16}$$

where E_j and S_j are sector-level quantities that are fully determined by the matching functions $L_j^*(z)$ for each sector j. They are normalized measures of employment and sales in each sector.[14]

There is free entry of firms in each sector. Therefore, for all $j \in \{1, ..., S\}$,

$$f_{Ej}\, P = (1 + T_j^*) \frac{\kappa_{1j}}{\kappa_{3j}}\, S_j\, \xi_j R\, P_j^{\sigma_j - 1}, \tag{17}$$

where f_{Ej} is the units of final goods used up in the sunk cost of entry.
Aggregate production in each sector is the sum of individual firms' production. Therefore, for all $j \in \{1, ..., S\}$,

$$\xi_j R = \frac{\sigma_j \kappa_{1j}}{\kappa_{3j}}\, M_j\, S_j\, \xi_j R\, P_j^{\sigma_j - 1}. \tag{18}$$

The mass of workers N_o works either in one of the traded goods sectors or in the construction sector. The national labor market clearing condition is therefore

$$N_o = \sum_{j=1}^{S} M_j \frac{\kappa_{2j}}{\kappa_{3j}}\, E_j\, \xi_j R\, P_j^{\sigma_j - 1} + N_o(1-b)(1-\eta), \tag{19}$$

where the last term derives from equation (10).

Inverting this system of $2S+1$ equations gives the $2S+1$ unknowns P_j, M_j for all $j \in \{1, ..., S\}$ and R, the aggregate revenues in the traded goods sector, as detailed in Appendix D.

Step 4: Equilibrium city-size distribution The city developers' problem and the firms' problem jointly characterize (1) the set of city sizes that necessarily exist in equilibrium and (2) the matching function between firm type and city size. Given these, the city-size distribution is pinned down by the labor market clearing conditions. The population living in a city of size

[14]Given the wage equation (6) and the expression for operating profits (7), aggregate operating profits in sector j are $\frac{\kappa_{1j}}{\kappa_{3j}} M_j\, S_j\, R_j\, P_j^{\sigma_j - 1}(1 + T_j^*)$ where $\kappa_{3j} = (1-\eta)^{b(1-\eta)(1-\alpha_j)(\sigma_j - 1)}$. Similarly, aggregate revenues in sector j are $\frac{\sigma_j \kappa_{1j}}{\kappa_{3j}} M_j S_j R_j P_j^{\sigma_j - 1}$ and aggregate employment in sector j is $\frac{\kappa_{2j}}{\kappa_{3j}} M_j\, E_j\, R_j\, P_j^{\sigma_j - 1}$, where $\kappa_{2j} = \kappa_{1j}(1-\alpha_j)(\sigma_j - 1)$.

smaller than any L must equal the number of workers employed by firms that have chosen to locate in these same cities, plus the workers hired to build housing. That is,

$\forall L > L_{min}$,

$$\int_{L_{min}}^{L} u f_L(u)\, du = \sum_{j=1}^{n} M_j \int_{z_j^*(L_{min})}^{z_j^*(L)} \ell_j(z, L_j^*(z))\, dF_j(z) + (1-\eta)(1-b) \int_{L_{min}}^{L} u f_L(u)\, du,$$

where $L_{min} = \inf(\mathcal{L})$ the smallest city size in the equilibrium.[15]

Differentiating this with respect to L and dividing by L on both sides gives the city size density ($f_L(L)$ is not normalized to sum to 1)

$$f_L(L) = \kappa_4 \frac{\sum_{j=1}^{S} M_j \mathbb{1}_j(L)\, \ell_j(z_j^*(L))\, f_j(z_j^*(L))\, \frac{dz_j^*(L)}{dL}}{L}, \tag{20}$$

where $\kappa_4 = \frac{1}{1-(1-\eta)(1-b)}$ and $\mathbb{1}_j(L) = 1$ if sector j has firms in cities L, and 0 otherwise.

The equilibrium distribution of city sizes $f_L(.)$ is uniquely determined by equation (20).

Lemma 5 *$f_L(.)$ is the unique equilibrium of this economy in terms of the distribution of city sizes.*

Several remarks are in order here. First, the city-size distribution is shaped by the distribution of firm efficiency and by the sorting mechanism. This offers a static view of the determination of the city-size distribution, driven by heterogeneity in firm types. In the empirical exercise, I compute the city-size distribution obtained with equation (20), where firm heterogeneity is estimated from French firm-level data but the city-size distribution is not used in the estimation. It exhibits Zipf's law, consistent with the data on cities. I show in Appendix C.5 that under some parametric restrictions for ψ, consistent with the ones used in the empirical section, the city-size distribution follows Zipf's law if the firm-size distributions follow Zipf's law.

Second, as all sites used by city developers are identical ex ante, the model has no predictions on which sites are used to build these cities. Finally, for each city size, the share of employment in each sector can be computed using the same method, now sector by sector. For a given city size, the average sectoral composition over all such cities is determined by the model. On the other hand, the model is mute on the sectoral composition of an *individual* city within a given class, which is irrelevant for aggregate outcomes. The model could easily be extended to accommodate *localization* externalities, by assuming that, for a given sector, the agglomeration externality depends on the size of this sector and not the overall city size. This would lift this

[15] I abuse notations here since $z_j^*(L)$ is not defined over the entire set $\mathcal{L}$ but over a convex set strictly included in $\mathcal{L}$.

city-level indeterminacy. Cities would be perfectly specialized in that sector, since the congestion costs depend on the overall city size, but the benefits are sector-specific. This would not change any other characterization of the equilibrium. In particular, the city-size distribution defined in equation (20) and lemma 5 would still hold.

This step completes the full characterization of the unique equilibrium of the economy.

2.3 Characterization

I proceed to derive several properties of the equilibrium, following two objectives. The first is to verify that the assumptions of the model lead to predictions that are broadly consistent with salient features of the data. I derive the theoretical predictions here and, in the empirical Section 3, I present a set of stylized facts from French firm-level data that are broadly consistent with these predictions. My second objective is to propose a model-based decomposition of the impact of place-based policies on welfare and the productive efficiency of the equilibrium, which will guide the counterfactual policy analysis I conduct in Section 4.

2.3.1 Within-sector patterns

Within a given sector j, the revenue, production and employment distributions are all determined by the matching function $L_j^*(z)$. In the sorting equilibrium, for a firm of efficiency z, productivity, revenues and employment are given by

$$\psi_j^*(z) = \psi(z, L^*(z), s_j),$$

$$r_j^*(z) = \sigma_j \kappa_{1j} \left(\frac{\psi(z, L^*(z), s_j)}{w(L_j^*(z))^{1-\alpha_j}} \right)^{\sigma_j - 1} P_j^{\sigma-1} R_j, \tag{21}$$

$$\ell_j^*(z) = \kappa_{2j} \frac{\psi(z, L^*(z), s_j)^{\sigma-1}}{w(L_j^*(z))^{(\sigma-1)(1-\alpha_j)+1}} P_j^{\sigma_j - 1} R_j, \tag{22}$$

where the starred variables denote the outcomes in the sorting equilibrium. Since there is positive assortative matching between a firm's raw efficiency and city size (lemma 4), firm-level observables also exhibit complementarities with city size. Let $\mathcal{L}$ denote the set of city sizes in the economy. In all that follows, the characterizations hold regardless of whether $\mathcal{L}$ is the unique set determined in equilibrium by the city developers problem, or $\mathcal{L}$ is exogenously given.

Proposition 6 *In equilibrium, within each sector, firm revenues, profits and productivity increase with city size, in the following sense. For any $L_H, L_L \in \mathcal{L}$ such that $L_H > L_L$, take z_H such that $L_j^*(z_H) = L_H$ and $L_j^*(z_L) = L_L$. Then, $r_j^*(z_H) > r_j^*(z_L)$, $\pi_j^*(z_H) > \pi_j^*(z_L)$, and $\psi_j^*(z_H) > \psi_j^*(z_L)$.*

These strong predictions on the ranking of the size of firms (in revenues or productivity) vis a vis the city size are a direct consequence of the perfect sorting of firms.[16] In contrast, employment can be either positively or negatively associated with city size through the effect of wages. Within a sector, $\ell^*(z) \propto \frac{r^*(z)}{w(L^*(z))}$, where both revenues and wages increase with city size. Firms may have lower employment in larger cities, even though they are more productive and profitable. More precisely, if $\epsilon_l = \frac{d \log \bar{\ell^*}(L)}{d \log L}$ and $\epsilon_r = \frac{d \log \bar{r^*}(L)}{d \log L}$ are the elasticities of mean employment and mean revenues with respect to city size in equilibrium, then

$$\epsilon_l = \epsilon_r - b\frac{1-\eta}{\eta}, \tag{23}$$

so that ϵ_l is not necessarily positive.

2.3.2 Comparative statics across sectors

I now compare the predicted distribution of firm outcomes across sectors. Sectors differ in their capital intensity α_j and in the strength of their benefit from agglomeration externalities s_j, and both impact the sorting process, leading in turn to differences in observed outcomes. The model delivers predictions on the geographic distribution of sectors as well as on the heterogeneity of the firm-size distribution across sectors, which I use to guide the estimation in Section 3.

Geographic distribution Define the *geographic distribution* of firms in a sector as the probability that a firm from the sector is in a city of size smaller than L. That is, let

$$\tilde{F}(L; \alpha_j, s_j) = P(\text{firm from sector } (\alpha_j, s_j) \text{ is in a city of size smaller that } L).$$

Proposition 7 *In a competitive equilibrium, all else equal, the geographic distribution $\tilde{F}_j$ of a high α_j sector first-order stochastically dominates that of a lower α_k sector. The geographic distribution $\tilde{F}_j$ of a high s_j sector first-order stochastically dominates that of a lower s_k sector.*

The formal proofs are in Appendix C. These results stem from the following observation. As shown before, the matching function $L_j^*(z)$ is always increasing, but its slope and absolute level depend on the capital intensity α_j and the strength of agglomeration externalities s_j in the sector. In labor-intensive sectors, the weight of the wage effect is heavier in the trade-off between the benefits of agglomeration externalities and labor costs. This pushes the matching function down, towards smaller cities. For any city size threshold, there are more firms from a labor-intensive sector that choose to locate in a city smaller than the threshold. In contrast, in sectors with strong

[16] In the data, the sorting is imperfect. I allow for imperfect sorting in the estimation by specifying an error structure around the baseline model.

agglomeration externalities, firms benefit more from a given city size, which pushes the matching function up for all firms. All else equal, they locate more in larger cities.

Note that the model predicts that firms of different sectors coexist in equilibrium within each type of city size.[17] The support of city sizes chosen by firms in each sector is shaped by s_j and α_j; in particular, if the productivity distribution has convex and bounded support, the support of city sizes chosen by firms is summarized by an interval, whose bounds increase with α_j or s_j. The intersection of such supports is not, a priori, empty. Holding city size fixed, a variety of sectors may be present in some types of cities at equilibrium, but each with a different efficiency level, realized productivity, and realized revenues, as the matching function differs by sector.

Firm-size distribution Because firm sorting reinforces initial differences between firms, the intensity of sorting impacts the dispersion of the observed sectoral firm-size distribution. Let $Q_j(p)$ denote the p-th quantile of the firm revenue distribution in sector j.

Proposition 8 *If $(\alpha_2, s_2) \geq (\alpha_1, s_1)$, the observed firm-size distribution in revenues is more spread in Sector 2 than in Sector 1. For any $p_1 < p_2 \in (0,1)$, $\frac{Q_1(p_2)}{Q_1(p_1)} \leq \frac{Q_2(p_2)}{Q_2(p_1)}$.*

In other words, if one normalizes the median of each revenue distribution to the same level, all higher quantiles in the revenue distribution of Sector 2 are strictly higher than in Sector 1, and all lower quantiles are below. The distribution in Sector 1 is more unequal. As a consequence, higher α_j or higher s_j sectors have thicker upper-tails in their firm-size distributions. This leads to a characterization that will prove useful empirically. Firm-size distributions are empirically well approximated by power law distributions, in their right tail. The exponent of this distribution characterizes the thickness of the tail of the distribution. Assume that the revenue distribution of firms in two sectors 1 and 2 can be approximated by a power law distribution in the right tail, with respective exponents ζ_1 and ζ_2. Then the following corollary holds

Corollary 9 *Let $(\alpha_2, s_2) \geq (\alpha_1, s_1)$. The tail of the firm-size distribution in Sector 2 is thicker than the tail of the firm-size distribution in sector 1: $\zeta_2 \leq \zeta_1$.*

2.3.3 Policy implications

Having characterized the equilibrium, I now turn to proposing a model-based decomposition of welfare, which will provide tools to guide the quantitative counterfactual policy analyses I conduct in Section 4.

[17] As noted above, the share of employment in each sector is pinned down on average over all cities of the same size, but is indeterminate for individual cities. If, in contrast to what I assume, externalities were sector-specific (localization externalities), the model would predict the same sectoral shares on average over all cities of a given size. The only difference would be that individual cities would then be fully specialized.

In the quantitative exercise, I analyze two types of policies. First, federal governments subsidize firms to modify their location choice. In general, these policies are designed to attract firms to underdeveloped areas.[18] They are advocated for reasons of equity – policy makers want to smooth out spatial inequalities – and efficiency, since helping cities grow can have multiplying effects on productivity in the presence of agglomeration externalities. In what follows, I discuss how to analyze their aggregate welfare effects in the context of the model. These policies do no affect spatial inequalities in terms of welfare, since utility is equalized across all cities in equilibrium in the model. Nevertheless, they do impact other measures of spatial inequality, such as spatial inequality in real wage or in productivity. I quantify the impact of place-based policies on such measures, together with their aggregate welfare impact, in Section 4.

I model place-based policies as follows. In contrast to the constant subsidies given out by local developers, these policies subsidize firms more when they choose to locate in less productive cities, i.e., smaller cities in the context of the model.[19] This takes the form of subsidies or tax breaks that are linked with the size of the city chosen by firms. This scheme shapes the incentives of firms and distorts the equilibrium matching functions $L_j^*(z)$, leaving the wage schedule unchanged. This policy is funded by raising a lump-sum tax on firms.

Second, I study policies that aim to remove some of the existing constraints on city growth. This class of policies encompasses a broad range of instruments which make the housing supply more elastic, such as lifting existing zoning regulations or building-height regulations. In the model, the housing supply elasticity is driven by the land-use intensity b in the housing production function (1). I model an easing of zoning regulations as a decrease in land-use intensity b. This leads to a direct, mechanical, positive welfare effect that is not the focus here, and for which I control in the quantitative analysis: all else equal, more housing can be built with the same amount of land, which directly increases workers' utility. I focus instead on the indirect effects this policy has on the differentiated goods sectors. Because the utility of workers is equalized across city sizes, the wage elasticity to city size $b\frac{1-\eta}{\eta}$ is also shaped by the land-use intensity. In turn, this elasticity impacts the firm matching function, and, ultimately, the productive efficiency of the differentiated goods sectors. Firms match with larger cities, and cities are larger in equilibrium.

Welfare depends on the productivity of the economy, as well as on the congestion costs borne out by workers. The policies I study impact both. I first examine the channels through which policies impact aggregate productivity in the differentiated goods sectors, before turning to the expression for welfare.

[18] I discuss these policies in more detail in the policy analysis section.

[19] Smaller cities host the least productive firms in the model

Productivity Let Y_j, N_j and K_j denote, respectively, sectoral levels of output, employment and capital. As I show in Appendix D, sectoral productivity $TFP_j \equiv \frac{Y_j}{K_j^{\alpha_j} N_j^{1-\alpha_j}}$ can be expressed

$$TFP_j = M_j^{\frac{1}{\sigma_j - 1}} \frac{S_j^{\frac{\sigma_j}{\sigma_j - 1} - \alpha_j}}{E_j^{1-\alpha_j}}, \tag{24}$$

where E_j and S_j are defined in equations (15) and (16).

Holding fixed the matching function $L_j^*(z)$ and the mass of firms M_j, this expression would be maximized if $w(L)$ did not depend on city size. In that case, TFP_j is simply

$$\left(\int \psi(z, L_j^*(z), s_j)^{\sigma_j - 1} dF_j(z) \right)^{\frac{1}{\sigma_j - 1}}.$$

This is reminiscent of the results in the misallocation literature (Hsieh and Klenow (2009)). Productivity is maximized when all firms face the same input price, whereas in the spatial equilibrium, wages increase with city size. Second, given the wage schedule and the mass of firms M_j, productivity increases if the matching function $L_j(z)$ is pushed up, since firms benefit from more externalities. Finally, TFP_j increases as more firms enter the market, through a standard love of variety effect.[20]

These three channels offer a way to decompose the effects of place-based policies on the productive efficiency of the economy. I quantify each in the quantitative section. Qualitatively, the first type of policies I study affects productivity by pushing down the equilibrium matching function. Firms tend to be pushed down toward samller cities, but on the other hand, in the new spatial equilibrium, city sizes adjust to local labor demand and small cities grow. The aggregate effect on productive efficiency is ambiguous, as is the entry effect. The second type of policies has positive effects both through the misallocation channel, since the wage schedule is flatter, and through a new matching function in which firms sort into larger cities in equilibrium. The entry effect is a priori ambiguous.

Welfare The model lends itself naturally to welfare analysis. Given the choice of numeraire, welfare is given by

$$\bar{U} = \left(\frac{1}{P} \right)^{\eta}$$

[20]The expression for the mass of firms is $M_j \propto \left(\left(\sum_{j=1}^S \xi_j \frac{E_j}{S_j} \right)^{1-\bar{\alpha}} \prod_{j=1}^S S_j^{\frac{\xi_j}{\sigma_j - 1}} \right)^{\frac{\bar{\sigma}-1}{(1-\bar{\alpha})(\bar{\sigma}-1)-1}}$ where $\bar{\alpha} = \sum_{j=1}^S \alpha_j \xi_j$ and $\frac{1}{\bar{\sigma}-1} = \sum_{j=1}^S \frac{\xi_j}{\sigma_j - 1}$.

As shown in Appendix D, welfare depends positively on a measure of aggregate productivity, and negatively on a term that summarizes the aggregate congestion costs in the economy. Comparing two equilibria,

$$\bar{U} \propto \left(\prod_{j=1}^{S} TFP_j^{\xi_j}\right)^{\frac{\eta}{1-\bar{\alpha}}} \left(\prod_{j=1}^{S} (\frac{S_j}{E_j})^{\xi_j(1-\alpha_j)}\right)^{-\frac{\eta}{1-\bar{\alpha}}},$$

where $\bar{\alpha} = \sum_{j=1}^{S} \alpha_j \xi_j$ is an aggregate measure of the capital intensity of the economy.

The term $\prod_{j=1}^{S} TFP_j^{\xi_j}$ is a model-based measure of aggregate productivity. Take the example of a policy that increases TFP by pushing firms to larger cities. It has a direct positive impact on welfare, magnified by the term $\frac{1}{1-\bar{\alpha}}$ that captures the fact that capital flows in response to the increased TFP in the economy, making workers more productive.

This effect is dampened by the second term, which captures the congestion effects that are at play in the economy. Recall that in this economy, wages increase to compensate workers for increased congestion costs in larger cities. Here, $\frac{S_j}{E_j}$ measures the ratio of the average sales of firms to their average employment in a given sector. It is a model-based measure of the representative wage in the economy, since $\frac{r_j^*(z)}{\ell_j^*(z)} \propto w(L^*(z))$ for each firm. Coming back to the example of a policy that tends to push firms into larger cities, such a policy will also tend to increase aggregate congestion in the economy by pushing workers more into larger cities. Individual workers are compensated for this congestion by increased wages, in relative terms across cities, so that all workers are indifferent across city sizes. But the level of congestion borne by the *representative* worker depends on how workers are distributed across city sizes. It increases as the economy is pushed toward larger cities. This negative effect is captured by the second term in the welfare expression that decreases with the representative wage.

3 Estimation of the Model

I now take the model to the data, in order to be able to perform a quantitative policy analysis. Using French firm-level data, I first show that sectors display location patterns and firm-size distribution characteristics that are consistent with the theoretical predictions presented above. I then use these theoretical characterizations of the equilibrium to guide the structural estimation of the model.

3.1 Data

I use a firm-level data set of French firms (BRN). It contains information on the balance sheets of French firms, declared for tax purposes. All firms with revenues over 730,000 euros are included. It reports information on employment, capital, value added, production, and 3-digit industry classification. It is matched with establishment-level data (SIRENE), which indicate the geographical location at the postal code level of each establishment of a given firm-year. As is standard in the literature, the geographical areas I use to measure city size are the 314 French commuting zones, or *"Zones d'emploi"* (employment zones), within metropolitan France. They are defined with respect to the observed commuting patterns of workers and cover all of France.[21] They are designed to capture local labor markets and are better suited than administrative areas, which they abstract from, to capturing the economic forces at play in the model.[22] To measure the size of the city, I use the total local employment of the area, since I need a proxy for externalities such as knowledge spillovers or labor market pooling that depend on the size of the workforce. I use the data for the year 2000 in the estimation procedure.

I retain only tradable sectors in the analysis, consistent with the assumptions of the model. The set of industries is the one considered in Combes et al. (2012), i.e., manufacturing sectors and business services, excluding finance and insurance. Finally, I retain firms with two employees or more. Data for smaller firms tend to be very noisy. I trim the bottom and top 1% of the data. This leaves me with 130,018 firms. Summary statistics are reported in Table 1.

3.2 Descriptive evidence on sorting

Before proceeding to the structural estimation of the model, I present a first look at the raw data. My objective is to check that the comparative statics of the model are broadly consistent with the patterns exhibited in the data. Recall that in the model, the complementarity between firm efficiency and agglomeration forces leads to the sorting of firms across cities of different sizes. This impacts the elasticity of firm-level observables with respect to city size within industries (lemma 6). Furthermore, firm sorting is shaped by two key sector-level parameters, namely, the sectoral strength of agglomeration externalities s_j as well as the sectoral intensity of use of traded inputs α_j. The model shows how these parameters shape (i) the location patterns of firms in a given industry (lemma 7) and (ii) the dispersion of the sectoral firm-size distribution (lemma 8). I turn to examining the raw data in these dimensions.

[21]They are presented as areas where *"most workers live and work, and where establishments can find most of their workforce"*.

[22]The previous definition of these zones was constrained by some administrative borders. A new definition of these zones was published by INSEE in 2011. Updated measures of zone sizes were published for all years after 1997.

To do so, I use the most disaggregated level of industry available in the data. I keep sectors with more than 200 observations, for a total of 148 industries, and present correlations between different sectoral characteristics, guided by the theory. These correlations could be driven by explanations alternative to the ones I propose in the model. To mitigate these concerns, I check that the patterns I find are robust to a set of sectoral controls that I detail below. The broad consistency of the data with the salient features of the model are only suggestive evidence that sorting forces may be at play.

I first investigate how, in each sector, average firm value added and average firm employment change as city size increases.[23] In the model, the elasticity of firm revenues to city size is positive within industries whereas the elasticity of employment to city size is strictly lower and possibly negative. Empirically, I compute the average firm-level value added and employment by industry and city and compute their elasticity with respect to city size.[24] Figure 1 plots the distributions of these elasticities. For value added, it is positive for 95% of the observations, and is significantly negative for only one industry, sawmills. This is intuitive since natural resources drive location in this industry —a dimension the sorting model abstracts from. The elasticity of employment to city size is shifted to the left compared to the elasticity of value added to city size. This is broadly consistent with the intuition of the model.

Second, the model suggests that firm location choices are linked to sectoral characteristics and, in particular, the intensity of input use, which I can measure in the data. The model distinguishes between inputs whose price varies locally and inputs with constant price across space. To proxy for inputs whose price does not systematically increase with city size, I use a measure of "tradable capital" defined as total capital net of real estate assets. I measure tradable capital intensity, α^K, as the Cobb-Douglas share of this tradable capital in value added. I then run the following regression:

$$share_j = \beta_0 + \beta_1 \alpha_j^K + \beta_2 X_j + \epsilon_j,$$

where j indexes sectors, $share_j$ is the share of establishments in sector j located in large cities (i.e., the largest cities that hosts half of the population) and X_j is a set of control variables varying at the industry level. Table 2 reports the coefficient estimates. It shows that industries that use more tradable capital are significantly more likely to be located in larger cities. However, these industries could also be the ones with higher skill intensity, driven to larger cities in search of skilled workers. To control for that, I use an auxiliary data set to measure industry-level skill

[23] Because the model does not feature the use of intermediates, I use value added as the measure of firm output.

[24] For this measure, I restrict the sample to single-establishment firms as the data on value added is only available at the firm level for multi-establishment firms. Single establishment account for 81% of firms and 45% of employment in the sample.

intensity, since the main data set does not have information on the composition of the workforce.[25] Specification (III) in Table 2 shows that controlling for industry-level skill intensity does not affect the results. Specification (IV) runs the same specification, limiting the sample to export-intensive industries. This control aims at mitigating the concern that location choice may be driven by demand-side explanation, whereas the model focuses on supply-side explanations. Again, the results are robust to using this reduced sample. Overall, Table 2 is consistent with the idea that firms location choices are shaped by the intensity of input use in their industry.

Third, the model predicts that firms that locate in large cities benefit disproportionally from agglomeration externalities. As a consequence, the sectoral firm-size distribution is more fat-tailed for industries located in larger cities. Table 3 correlates the thickness of the industry-level firm-size distribution, summarized by its shape parameter ζ_j, with the share of establishments located in large cities in industry j. The shape parameter ζ_j is estimated by running the following regression, following Gabaix and Ibragimov (2011):

$$\log\left(rank_{ij} - \frac{1}{2}\right) = \alpha_j - \zeta_j \log(\text{value added}_{ij}) + \epsilon_{ij}, \tag{25}$$

where j indexes industries, i indexes firms and $rank_{ji}$ is the rank of firm i in industry j in terms of value added. Table 3 shows a negative correlation between ζ_j and the fraction of establishments in industry j located in large cities (defined as in Table 2). In other words, industries that locate more in large cities have thicker-tailed firm-size distributions. This negative correlation is robust to controlling for the number of firms and the average value added in industry i (Specification II), as well as reducing the sample to export-intensive industries (Specification III).

Finally, the model predicts that more efficient firms self-select into larger cities. I investigate this question by focusing on the relocation pattern of movers, i.e. mono-establishment firms that change location from one year to the next. The nature of this question leads me to extend the sample period to 1999-2006. There is no direct way to measure a firm's raw efficiency from the data. However, in the model, within a city-industry pair, firm revenues increase with firm efficiency. I thus compute the following firm-level residual ω_{ijt} and use it to proxy for firm efficiency:

$$\log(\text{value added}^c_{ijt}) = \delta^c + \delta^t + \delta^j + \omega_{ijt},$$

where δ^c, δ^t and δ^j are sets of, respectively, city, year and industry fixed-effects, and i is a firm in industry j located in city c in year t. For all firms relocating from year t to $t+1$, I define $\Delta_t\text{City Size}_i$ as $\log(L_{i,t+1}/L_{i,t})$, where $L_{i,t}$ is the size of the city where firm i is located in year t.

[25]I use the random sample of 1/12 of the French workforce published by INSEE. It contains information on workers' skill level, salary and industry. For each industry, I measure the share of the labor force that is high-skilled. I define a dummy variable that equals one for sectors with above-median skill-intensity.

I then estimate:

$$\Delta_t \text{City Size}_i = \alpha + \beta \omega_{ijt} + X_{it} + \epsilon_{it}$$

where X_{it} is the logarithm of $L_{i,t}$ or a set of initial city fixed effects.[26] Table 4 shows that, conditional on moving, firms that are initially larger tend to move into larger cities. I emphasize that this result is a simple correlation and cannot be interpreted causally in the absence of a valid instrument for the selection into the sample of movers. Table 4 simply shows that, among the set of movers, there exists a positive correlation between initial firm size and the size of the city the firm moves into, a correlation pattern that is consistent with sorting.

3.3 Structural estimation

I now turn to the estimation of the model. The model is estimated industry by industry, on 23 aggregated industries. I use an indirect inference method and minimize the distance between moments of the data and their simulated counterparts to estimate the sectoral parameters that govern the model. Because, in the theory, the strength of agglomeration externalities shapes (i) the location patterns of firms ; (ii) the elasticity of firm-level revenues to city size and (iii) the dispersion of the sectoral firm-size distribution, I use moments that describe these dimensions of the data as targets of the estimation procedure.

3.3.1 Model specification

I first lay out the econometric specification of the model. The literature has traditionally assumed that agglomeration externalities were of the form $\psi(z, L, s_j) = zL^{s_j}$, where s_j measures the strength of externalities. However, in such a framework, externalities enter multiplicatively in the profit function and there is no complementarity between firm productivity and city size. In contrast, the model presented in Section 2 assumes such a complementarity. I thus postulate the following functional form of the productivity function, for each sector $j \in 1 \ldots S$:

Assumption B

$$\log(\psi_j(z, L, s_j)) = a_j \log L + \log(z)(1 + \log \frac{L}{L_o})^{s_j} + \epsilon_{z,L} \quad \textit{for } \log(z) \geq 0 \textit{ and } L \geq L_o \tag{26}$$

$$\log(\psi_j(z, L)) = 0 \quad \textit{for } L < L_o$$

The parameter a_j measures the classic log-linear agglomeration externality. The strength of the complementarity between agglomeration externalities and firm efficiency is captured by s_j. When $s_j = 0$, the model nests the traditional model of agglomeration externalities without complementarity. L_o measures the minimum city size below which a city is too small for a firm to produce in.

[26] These controls absorb the mechanical relationship by which firms in large (resp. small) cities are more likely, conditional on moving, to move to smaller (resp. larger) cities.

It is a normalization parameter in levels that changes proportionally the size of all cities but does not affect the estimation, which relies on *relative* measures. In what follows, I write $\tilde{L} = \frac{L}{L_o}$, and $\mathcal{L}$ the set of normalized city sizes in the simulated economy.[27] I assume that $\log(z)$ is distributed according to a normal distribution with variance ν_Z, truncated at its mean to prevent $\log(z)$ from being negative. This restriction is needed for the productivity of a firm to be increasing in city size in equation (26).

I introduce an error structure by assuming that firms draw idiosyncratic productivity shocks $\epsilon_{z,L}$ for each city size, where $\epsilon_{z,L}$ is i.i.d. across city sizes and firms. It is distributed as a type-I extreme value, with mean zero and variance ν_R. This generates imperfect sorting. This shock captures the fact that an entrepreneur has idiosyncratic motives for choosing a specific location: for example, he could decide to locate in a city where he has a lot of personal connections that make him more efficient at developing his business.[28]

I assume that idiosyncratic shocks are city-size specific, with mean zero and a constant variance across city size bins, and not city-specific as would perhaps be more natural. Still, these shocks can themselves represent the maximum of shocks at a more disaggregated level (e.g., at the city level). The maximum of a finite number of independent draws from a type-I extreme value distribution is also distributed as a type-I EV, with the same variance. Aggregating at the city-size level does not impact the estimation of the variance of the draws. I normalize the mean to be zero. If the model is misspecified and in reality, there is a systematic difference in mean idiosyncratic shocks across different city-size bins, this mean value is not separately identified from the log-linear agglomeration externality term a_j, which will capture both in the estimation.

3.3.2 Estimation procedure

The estimation is conducted in two stages. In the first stage, a set of parameters is calibrated from the data. In the second stage, I use indirect inference to back out the parameters that require a simulation method to be estimated. The indirect inference method is carried through sector by sector. I retain a rather aggregated definition of sectors, corresponding to 23 industries of the French NAF classification, in order to limit the computing requirements of the procedure. I still am able to capture relevant heterogeneity across sectors.

In the first stage, I start by calibrating for each industry its capital intensity α_j and elasticity

[27] L_o changes proportionaly the size of all cities but does not alter the relative matching function and firm-size distributions used in the estimation nor the relative prices. $\tilde{L}$ is the relevant measure for firm choices. L_o is calibrated to match the actual level of city sizes in the data.

[28] I show in Appendix B that under Assumption B, firms in sectors with a lower labor intensity $1-\alpha_j$ or a higher agglomeration parameter s_j locate with a higher probability in larger cities, whereas in the theoretical section this statement was not probabilistic.

of substitution σ_j. The capital intensities are calibrated to the share of capital in sectoral Cobb-Douglas production functions, and the elasticity of substitution is calibrated to match the average revenue to cost margin in each sector.[29] I then calibrate the land-use intensity b and housing share $1-\eta$. b and $1-\eta$ jointly determine (1) the elasticity of wages to city size and (2) the elasticity of the housing supply. I thus calibrate them to match the median elasticity of housing supply measured in the US by Saiz (2010) and the wage elasticity to city size measured in the data.[30] To measure the elasticity of wages to city size, I follow equation (23) and use the difference between the elasticities of average firm revenue to city size and the elasticities of average firm employment to city size across all sectors. Finally, I calibrate the Cobb-Douglas share of each industry ξ_j by simply measuring its share of value-added produced.

In the second stage, I estimate the following parameters for each sector: a_j, which measures the log-linear agglomeration term; s_j, which measures the intensity of complementarity between agglomeration externalities and firm efficiency; ν_{Zj}, which measures the variance of the firms' raw efficiency; and ν_{Rj}, which is the variance of the firm-city size idiosyncratic shock. Firms make a discrete choice of (normalized) city size, according to the following equation

$$\log \tilde{L}_j^*(z) = \underset{\log \tilde{L} \in \mathcal{L}}{\arg\max} \quad \log(z)\big(1+\log\ \tilde{L}\big)^{s_j} + (a_j - b(1-\alpha_j)\frac{1-\eta}{\eta})\log \tilde{L} + \epsilon_{z,L}, \tag{27}$$

which is the empirical counterpart of equation (14). Because the choice equation involves unobservable heterogeneity across firms and is non-linear, I have to use a simulation method to recover the model primitives. I use an indirect inference method (Gouriéroux and Monfort (1997)) to estimate the true parameter $\theta_0^j = (a^j, s^j, \nu_R^j, \nu_Z^j)$ for each sector j. The general approach is close to the one in (Eaton et al. (2011)), except that I also use indirect moments in the inference (namely, regression coefficient estimates). The estimate θ_{II}^j minimizes an indirect inference loss function

$$\|m_j - \hat{m}_j(\theta)\|_{W_j^2} = \big(m_j - \hat{m}_j(\theta)\big)' W_j \left(m_j - \hat{m}_j(\theta)\right), \tag{28}$$

where m_j is a vector stacking a set of moments constructed using French firm data, as detailed below; $\hat{m}_j(\theta)$ is the vector for the corresponding moments constructed from the simulated economy

[29] In each sector, $\hat{\sigma_j}$ and $\hat{\alpha_j}$ are calibrated using $\frac{\hat{\sigma_j}}{\hat{\sigma_j}-1} = mean(\frac{\text{v.a.}}{\text{costs}})$ where costs exclude the cost of intermediate inputs, and $\hat{\alpha_j} = \alpha_j^{CD}\frac{\sigma_j}{\sigma_j - 1}$ where α_j^{CD} is the sectoral revenue-based Cobb-Douglas share of capital.

[30] I use the median measure of housing supply elasticity for the US as there has been no similar study done on French data, to my knowledge.

for parameter value θ; W_j is a matrix of weights.[31] I compute standard errors using a bootstrap technique.[32]

I simulate an economy with 100,000 firms and 200 city sizes. I follow the literature in using a number of draws that is much larger than the actual number of firms in each sector, to minimize the simulation error. I use a grid of 200 normalized city sizes $\tilde{L}$, ranging from 1 to M where M is the ratio of the size of the largest city to the size of the smallest city among the 314 cities observed in the French data. This set of city-sizes $\mathcal{L}$ is taken as exogenously given.[33] In contrast, the corresponding city-size distribution is not given a priori: the number of cities of each size will adjust to firm choices in general equilibrium to satisfy the labor-market clearing conditions.
The algorithm I use to simulate the economy and estimate the parameters for each sector is as follows:

Step 1: I draw, once and for all, a set of 100,000 random seeds and a set of 100,000 $\times$ 200 random seeds from a uniform distribution on $(0, 1)$.

Step 2: For given parameter values of ν_R and ν_z, I transform these seeds into the relevant distribution for firm efficiency and firm-city size shocks.

Step 3: For given parameter values of a and s, I compute the optimal city size choice of firms according to equation (27).

Step 4: I compute the 17 targeted moments described below.

Step 5: I find the parameters that minimize the distance between the simulated moments and the targeted moments from the data (equation (28)) using the simulated annealing algorithm.

[31] The weighting matrix W_j for sector j is a generalized inverse of the estimated variance-covariance matrix Ω_j of the moments calculated from the data m_j. I calculate Ω_j using the following bootstrap procedure. For a sector with N_j firms, (1) I resample with replacement N_j firms from the initial set of firms in this sector (2) for each resampling, I compute m_j^b, the value of the moments for this set of firms (3) I compute

$$\Omega_j = \frac{1}{2000} \sum_{b=1}^{2000} (m_j^b - m_j)(m_j^b - m_j)'.$$

I take its generalized inverse to compute W_j.

[32] I run the estimation procedure 30 times; for each iteration, I take a new set of normalized draws for the firm productivity and the firm-city size idiosyncratic shocks. This accounts for simulation error. For each iteration, I also recompute the targeted moments by resampling firms in the data. This accounts for sampling error. I then estimate the parameter value θ_b^j, which minimizes (28). The standard errors I report for parameters in sector j are the square root of the diagonal elements of

$$V_j = \frac{1}{30} \sum_{b=1}^{30} (\theta_b^j - \theta_{II}^j)(\theta_b^j - \theta_{II}^j)'.$$

[33] As pointed out in the theory section, the characterizations of the economy provided in Section 2 hold if the set of possible city sizes is exogenously given.

The estimation is made in partial equilibrium, given the choice set of normalized city-sizes $\mathcal{L}$. It relies on measures that are independent of general equilibrium quantities, namely the sectoral matching function between firm efficiency and city size, and relative measures of firm size within a sector.[34]

3.3.3 Moments

I use four sets of moments to characterize the economy, guided by the predictions of the model, and provide intuition for how they help identify the parameters of the model.

(i) Sectoral location. The first set of moments summarizes the geographic distribution of economic activity within a sector. I use the share of employment in a given sector that falls into one of 4 bins of city sizes. I order cities in the data by size and create bins using as thresholds cities with less than 25%, 50% and 75% of the overall workforce. I normalize these sizes by the size of the smallest city, and call these thresholds t_i^L. I compute the fraction of employment for each sector in each of the city bins, both in the data and in the simulated sample. The corresponding moment for sector j and bin i is

$$s_i^{L,j} = \frac{\sum\limits_{t_i^L \le L < t_{i+1}^L} \int \ell_j^*(z) \mathbb{1}_{L_j^*(z)=L}\, dF_j(z)}{\int \ell_j^*(z) dF_j(z)},$$

where $\ell_j^*(z)$ is the employment of firm z and $\mathbb{1}_{L_j^*(z)=L}$ is a characteristic function which equals 1 if and only if firm z in sector j chooses to locate in city size L. These moments depend directly on the matching function between firms and city size. This procedure defines 4 moments for each industry.

(ii) Elasticity of revenues to city size. I use the elasticity of average firm revenues to city size as an additional moment. In the model, absent an idiosyncratic productivity shocks, this elasticity would have a closed-form expression that does not depend on the distribution of firm efficiency and would only be a function of the strength of agglomeration externalities. In the presence of an error structure, the elasticity of average firm revenues to city size is also shaped by the presence of idiosyncratic productivity shocks, which dampens the elasticities predicted by perfect sorting.

[34] Specifically, as detailed in the theoretical model, the optimal choice of city size by a firm depends only on its productivity function and on the elasticity of wages with respect to city size. It does not depend on general equilibrium quantities. The sizes of all firms in a given sector depend proportionally on a sector-level constant determined in general equilibrium (see equations (21) and (22)). Normalized by its median value, the distribution of firm sizes within a sector is fully determined by the matching function.

This moment is computed as follows in sector j. Define

$$\bar{r}_j(L) = \frac{\int r_j^*(z)\mathbb{1}_{L_j^*(z)=L} dF_j(z)}{\int \mathbb{1}_{L_j^*(z)=L} dF_j(z)}$$

the average revenues of sector j firms that locate in city L. The elasticity of average firm revenue to city size in sector j is the regression coefficient estimate $\varepsilon_{r,j}$ of the following equation:

$$\log(\bar{r}_j(L_i)) = c_j + \varepsilon_{r,j} \log L_i + \nu_i,$$

where i indexes the city sizes in $\mathcal{L}$. In the data, I compute this moment on the sub-sample of mono-establishment firms, as the information on value-added is only available at the firm level and not at the establishment level.

(iii) Firm-size distribution in revenues. Third, I use moments that characterize the firm-size distribution in revenues. As discussed in Section 2, these are directly impacted by the sorting mechanism. To compute these moments, I first normalize firms value-added within a given sector by their median value. I retrieve from the data the 25, 50, 75 and 90th percentiles of the distribution and denote them $t_i^{r,j}$.[35] These percentiles define 5 bins of normalized revenues. I then count the fraction of firms that fall into each bin

$$s_i^{r,j} = \frac{\int \mathbb{1}_{t_i^{r,j} \leq \tilde{r}_j(z) < t_{i+1}^{r,j}} dF_j(z)}{\int dF_j(z)},$$

where $\tilde{r}_j(z)$ are the normalized revenues of firm z in sector j. I also measure the tail of the distribution, captured by a log rank-log size regression as in equation 25. This procedure define 6 moments for each sector.

If the matching function alone does not allow me to identify separately ν_R from ν_Z, these moments do. In contrast to a classic discrete choice setting, I observe not only the choice of city size made by firms, but also additional outcomes that are impacted by this choice, namely, firm employment and value-added. In particular, these relationships allow me to identify the variance of idiosyncratic shocks separately form the variance of firm's raw efficiency. Intuitively, ν_Z impacts the relative quantiles of the firm-size distribution both indirectly, through the matching function, and directly, through the distribution of raw efficiency z. In contrast, ν_R impacts the relative quantiles of the firm-size distribution only indirectly, through the matching function.

[35]As in Eaton et al. (2011), higher quantiles are emphasized in the procedure, since they capture most of the value added, and the bottom quantiles are noisier.

(iv) Firm-size distribution in employment. Fourth, I construct the same set of moments as in (iii) for employment. In a model with perfect sorting, there would be a perfect correlation between ex-ante firm efficiency and city size and hence between wages and revenues. Thus, with perfect sorting, the information contained in the distribution of revenues and employment would be redundant. However, the presence of idiosyncratic shock leads to imperfect sorting and shapes the difference between the distribution of employment and the distribution of revenues. Having both these distributions as moments contributes to quantifying the amount of imperfect sorting, captured by ν_R.

3.3.4 Model fit

Targeted moments. I first examine the model fit for the set of moments targeted by the estimation procedure. Figure 2 shows the cross-sectoral fit between the estimated moments and their targeted value in the data, for the following moments: (1) the shape parameter of the distribution in revenues (2) the shape parameter of the distribution of employment (3) the elasticity of revenues to city size and (4) the share of employment located in cities below the median city (as defined above). The 45^o line represents the actual moments in the data. Overall, as seen in Figure 2, the model captures well the cross-sectional heterogeneity in these targeted moments. The estimation relies on five other moments of the sectoral firm-size distribution in revenues. To get a sense of the fit of the model fit in this dimension, I show in Figure 3 how the *whole* firm-size distribution compares in the data and in the model. In general, the fit is better for the upper tail than the lower tail, which is intuitive since the estimation focuses on upper-tail quantiles and the initial distribution of z is truncated to the left. The fit for the employment distribution is similar and is not reported. Finally, the estimation relies on the share of sectoral employment in four given city-size bins. I compute more generally for each sector the share of employment by decile of city size and represent the simulated vs. actual shares on Figure 4. The model accurately captures the cross-sectoral heterogeneity in location patterns. The within-sector patterns are noisier, but still follow well the overall trends in the data. Formally, I measure the variance explained by the model as an R^2 of a regression of the simulated share on the actual shares, forcing an origin of 0 and a slope of 1. The R^2 is 0.38 (0.46 when weighting by sector size).

Moments not targeted. Next, I investigate the model fit for moments not used in the estimation. First, I examine how the firm-size distribution differs for the set of firms located in small cities and the set of firms located in large cities. This is motivated by the analysis in Combes et al. (2012), who show that the productivity distribution of firms is more dilated for the set of firms that locate in larger cities. I measure the ratio of the 75th percentile to the 25th percentile of the firm-size distribution in revenues, both for firms located in cities above the median threshold and in cities below. I define "dilation" as the ratio of this two measures. Figure 5 plots for each sector the log of "dilation" computed on the simulated data against the same variable computed on the actual data. I measure the fit of the model by computing the R^2 of a regression of the simulated dilation on the actual dilation, forcing an origin of 0 and a slope of 1. The R-squared is 0.39 (0.45 when weighting by sector size). The model captures the existence of a dilation effect in firm-size distribution, between small and large cities, as well as the cross-sectoral heterogeneity in dilation.

Finally, another moment not targeted in the estimation is the city-size distribution. The estimation is made on a grid of possible city sizes that have the same maximum to minimum range as in the data. I make, however, no assumption on the number of cities in each size bin, i.e., on the city-size distribution. Armed with sectoral estimations, I can solve for the general equilibrium of the model and in particular compute the city-size distribution that clears labor markets at the estimated parameter values (see Section 2). The estimated city-size distribution exhibits Zipf's law and follows quite well the actual city-size distribution measured here in total local employment of the city, consistent with the data used in estimation. The fit is shown in Figure 6, where the city-size distribution is plotted for the simulated data and the actual data. The fitted lines correspond to a log rank-log size regression run on each of these distributions. Parallel slopes indicate that both distributions have the same tail.[36]

3.3.5 Analysis of the parameter estimates

I turn to the analysis of the parameter estimates of the model. Table 5 reports the estimated parameters industry by industry, with standard errors in parenthesis. The sectoral estimate of s_j, the parameter that governs the strength of the complementarity between firm efficiency and agglomeration externalities, is positive for all but three industries. These three industries correspond to the shoes and leather industry, the manufacture of glass and ceramics (where the coefficient is non significantly different from 0) and metallurgy. All three are relatively mature industries. Three other sectors have positive but insignificant point-estimates for s_j, namely paper products, rubber and plastic products, and office machinery. That more mature industries tend to exhibit different agglomeration forces is reminiscent of the argument in Audretsch and Feldman (1996), who argues that the nature of agglomeration forces depend on the life cycle of industries and show that agglomeration forces tend to decline as industries get more mature and less innovative.

Together, the agglomeration parameters and the variance parameters *jointly* determine the distribution of the realized productivity of firms and, crucially, the productivity gains associated with city size in equilibrium. These gains have been used in the literature as a proxy to measure agglomeration externalities. Here, the productivity gains associated with city size depend not only on the strength of agglomeration externalities, but on the sorting of firms, and on their

[36] The levels are arbitrary and chosen so that the figure is readable.

selection on local idiosyncratic productivity shocks. In what follows, I present direct and counterfactual measures of the elasticity of firm productivity to city size to highlight how these forces interplay and understand how the parameter estimates translate into economic forces. I present average measures across sectors in the main text. Table 6 reports these decompositions industry by industry.

A first raw measure of the observed elasticity of firm productivity to city size (or city density, as all cities have the same amount of land in the model) can be computed by running the following simple OLS regression:

$$\log Prod_{i,j} = \beta_0 + \beta_1 \log\, L_i + \delta_j + \mu_i, \tag{29}$$

where $Prod_{i,j}$ stands for $\psi_{i,j}$, the equilibrium productivity of firm i with efficiency z_i in industry j, $L_i = L_j^*(z_i)$ is the size of the city where firm i has chosen to produce and δ_j is an industry fixed effect. The OLS estimate of β_1, the elasticity of observed firm productivity to city size, is 5.4%. Interestingly, this measure falls within the range of existing measures of agglomeration externalities, as reported in Rosenthal and Strange (2004). They typically range from 3% to 8%. Rosenthal and Strange (2004) note that most studies do not account for sorting or selection effects when estimating he economic gains to density - they are therefore broadly comparable, in scope, to the OLS estimation of β_1 in equation (29).[37] In the estimated model, these observed productivity gains are driven only in part by the existence of agglomeration externalities. Part of these gains come from the sorting of more efficient firms into larger cities, which I examine now.

To measure the contribution of firm sorting in the observed economic gains to density, I conduct the following counterfactual analysis. I simulate the model using the same distribution of firm efficiency and idiosyncratic shocks as in the baseline equilibrium, but constrain firms to choose their city size as if they all had the average efficiency in their sector. I recover from this simulation a new distribution of firm-level observables. In this counterfactual, the difference in firms' location choice is only driven by firm-city size specific iid productivity shocks.[38] This counterfactual thus allows me to compute what would be realized productivities if firms did not sort across cities according to their raw efficiencies. Panel A of Figure 7 compares the relationship between firm-level productivity and city size, in the baseline model and in this counterfactual simulation. This relationship is flatter in the counterfactual simulation. Estimating equation (29) on the counterfactual data leads to an elasticity of firm productivity to city size of 2.1%. By this account, firm sorting accounts for almost two thirds of the productivity gains measured in equilibrium between cities of different

[37] An exception is Combes et al. (2008), who estimate agglomeration externalities using detailed French worker-level data and control for the sorting of workers across locations. They find an estimate of 3.7% of the elasticity of productivity to employment density.

[38] I also keep the set of possible city sizes constant.

sizes.

I now turn to a different exercise and compute a model-based estimate of what would be the impact on firm productivities of an exogenous increase in city size, all else equal. To do so, I first decompose the observed equilibrium firm-level productivities into a "systematic" component and an idiosyncratic component as follows:

$$\log\,\psi_{i,j} = \log\,\tilde{\psi}_{i,j} + \epsilon_{i,L_i}.$$

The "systematic" component in productivity is defined as

$$\log\,\tilde{\psi}_{i,j} = a_j\,\log L_i + \log(z_i)(1 + \log\,\frac{L_i}{L_o})^{s_j}$$

from equation (26), where L_i is the optimal city size chosen by firm z_i in equilibrium. The idiosyncratic shocks are orthogonal to z and L *ex ante*. Because firms select their optimal city size based on their draws, there is a correlation *ex post* between the realized ϵ_{z,L_i} and the firm-level observables. Panel B of Figure 7 plots the observed firm productivity against city size at equilibrium, as well as its systematic and its idiosyncratic components. As Panel B of Figure 7 shows, the magnitude of the realized idiosyncratic shocks decreases with city size. This result can be intuitively explained as follows. Absent idiosyncratic shocks, firms tend to be more profitable in larger cities, on average. To locate in smaller cities, they need to draw a relatively large idiosyncratic shock there. This selection effect tends to dampen the OLS estimate of β_1 in equation (29). Using the structure of the model, I can estimate the effect of an exogenous increase in city size on local firms productivity, by computing the sensitivity of the systematic component of firm productivity $\tilde{\psi}$ to city size L.[39] To do so, I measure $\tilde{\psi}$ in the baseline equilibrium for all firms that share a common efficiency level z but have chosen in equilibrium to locate in different city sizes, driven by the iid shocks ϵ_{zL}. Panel C of Figure 7 plots, for a given level of firm efficiency z, the systematic component of firm productivity as a function of city size. Results are shown for the 25th percentile, the median and the 75% percentile of the raw efficiency distribution z. The corresponding elasticity of firm systematic productivity to city size are estimated, respectively, at 5.8%, 7.2% and 8.3%. These estimates offer a model-based measure of what would be the increase in firm productivity if a city size doubled (and local agglomeration externalities increased accordingly), all else equal. It also illustrates the heterogeneous effects of agglomeration externalities, at the estimated parameter values.

[39] This exercise is by definition partial equilibrium in that I hold constant firms' location and realization of idiosyncratic productivity shocks.

4 The Aggregate Impact of Place-Based Policies

Equipped with the estimates of the model's parameters, I now turn to the evaluation of the general equilibrium impact of a set of place-based policies.

4.1 Local tax incentives

I first study policies that subsidize firms locating in less developed cities. This type of federal program is widespread and has been studied, for example, in Busso et al. (2013) for the US or Mayer et al. (2012) for France. They aim at reducing spatial disparities and are advocated for reason of efficiency. The case for increased efficiency relies on the idea that in the presence of agglomeration externalities, jump-starting a local area by attracting more economic activity can locally create more agglomeration externalities, enhancing local TFP. This argument, however, needs to be refined. As has been pointed out in the literature (Glaeser and Gottlieb (2008), Kline and Moretti (2013)) this effect depends in particular on the overall shape of agglomeration externalities. While smaller cities may in fact benefit from these policies, larger cities marginally lose some resources – and therefore benefit from less agglomeration economies. The net effect on the overall economy is a priori ambiguous. Turning to spatial disparities, since utility is equalized across all workers, there is no welfare inequality in equilibrium in the model. Nevertheless, the economy is characterized by spatial disparities in real wages, and in the productivity of firms located in different cities. Place-based policies impact these spatial inequalities. They tend to benefit the targeted areas, but the extent to which they reduce aggregate measures of inequality depends on the overall reallocation of economic activity in space, which I examine in the quantitative exercise below.

To evaluate these policies in the context of my model, I consider a set of counterfactuals in which firms are subsidized to locate in the smallest cities of the country, which are also the least productive ones. I first calibrate the policy to match the key characteristics of the French "ZFU" program (*Zones Franches Urbaines*), a policy similar to to the Empowerment Zone program in the US. This policy costs 500 to 600 million euros in a typical year, corresponding to extensive tax breaks given to local establishments. It is targeted to geographically delimited zones that cover an overall population of 1.5M, or 2.3% of the French population. I retain this number as the geographical scope of these policies and implement a scheme that subsidizes firms locating in the smallest cities corresponding to 2.3% of the population in the simulated data. I implement a subsidy of 15% of firms' profits in these areas, paid for by a lump-sum tax levied on all firms in the country. The gross cost of this subsidy in the model is 0.03% of GDP, which matches the one reported for France for the ZFU program. A 15% subsidy on profits correspond to a 45% tax break on the French corporate tax, whereas the French ZFU program offers full corporate tax breaks for 5 years as well as other generous labor tax and property tax relief.

To compute the counterfactual equilibrium, I proceed as follows.

Step 1: I start from the equilibrium estimated in the data. I hold fixed the number of workers in the economy, the real price of capital, the set of idiosyncratic productivity shocks for each firm and city-size bin, and the distribution of firms' initial raw efficiencies.

Step 2: I recompute the optimal choice of city-size by firms, taking into account the altered incentives they face in the presence of the subsidy.

Step 3: Because the composition of firms within a given city-size bin changes, total labor demand in a city-size bin is modified. I hold constant the number of cities in each bin and allow the city size to grow (or shrink) so that the labor market clears within each city-size bin. This methodology captures the idea that these policies are intended to"push" or jump-start local areas, which in addition grow through agglomeration effects.[40]

Step 4: As city sizes change, the agglomeration economies and wage schedules are modified, which feeds back into firms' location choice.

Step 5: I iterate this procedure from step 2, using the interim city-size distribution.

The fixed point of this procedure constitutes the new counterfactual equilibrium.

Local effects The model predicts large effects of this policy on the targeted cities. In targeted cities, the number of establishments grows by 21%. The corresponding local increase in population is, however, only 6%. This is because the firms attracted by the policy in these areas are small and have low productivity. These results are roughly consistent with the order of magnitude estimated in Mayer et al. (2012) on the effect of the French ZFU; Mayer et al. (2012) find a 31% increase in the entry rate of establishments in the three years following the policy's implementation and note that these new establishments are small relative to existing establishments. Of course, this is just a "plausibility check" since the two exercises are not directly comparable - the ZFU targets sub-areas smaller than the cities of my model, and the model does not have dynamic effects.

Aggregate effects Beyond evaluating the effects of the policy on the targeted cities, the counterfactual exercise allows me to compute the general equilibrium effect of this type of policy. I compute the aggregate TFP and welfare effects of the policy for different levels of the subsidy, holding constant the targeted areas. Figure 8 reports the TFP and welfare in the counterfactual equilibrium, relative to the reference equilibrium, as a function of the gross cost of the subsidy expressed in percentage of GDP. In the model, the subsidy is paid for by lump-sum transfers and there is no shadow cost of public funds; hence, the welfare estimates are upper bounds on the actual effects of the policy.

[40] I maintain the subsidy to the cities initially targeted as they grow.

The simulation show that these place-based policies have negative long-run effects, both on the productive efficiency of the economy and in terms of welfare. A subsidy to smaller cities that amounts to 1% of GDP leads to a loss of 1.6% in TFP in the aggregate, and a loss of 2.2% in welfare. While such a policy allows to decrease congestion overall (see Section 2), the welfare gain from decreasing congestion is only 0.25%. It is largely dominated by the TFP effect.[41]

I use the counterfactual economy to study the impact of these place-based policies on inequality. I look at two measures of inequality: (1) the Gini coefficient for the distribution of real income across workers (2) a Gini coefficient for the distribution of production outcomes across cities. This second measure summarizes to what extent the fraction of total output produced in each city is distributed (un)equally across cities and therefore captures the inequality in the relative contribution of cities to the aggregate economy. A reason to focus on such a measure is that policy makers may want to smooth out this type of inequality across cities.

Surprisingly, the type of place-based policies I study leads to an increase in spatial inequalities as measured by these two Gini indices (see Panel B of Figure 8). The intuition for this result is as follows. The counterfactual equilibrium is characterized by (1) growth in the size of smaller cities, (2) a decrease in the population of mid-size cities, and (3) an increase in the population of the largest cities. That larger city grows in the counterfactual economy comes from the fact that, as mid-size cities lose population in favor of smaller cities, they offer less agglomeration externalities. As a consequence, these mid-size cities become less attractive than larger cities for a set of firms that were previously indifferent between these mid-size cities and *larger* cities. Small and large cities thus expand at the expense of mid-size cities. Quantitatively, this leads to a rise in the two Gini coefficients.

According to these results, place-based policies may have general equilibrium effects that run counter to their rationale.

4.2 Land-use regulation

Glaeser and Gottlieb (2008) forcefully argue against policies that limit the growth of cities. In this section, I study one such policy, land-use regulation. At a general level, land-use regulation consists in policies that constrain the available housing supply. Zoning regulations or regulations on the type or height of buildings that can be built within a city constitute examples of such land-use regulations.[42] A rationale for these restrictions on land-use development is that they

[41]As shown in section 2, TFP has a magnified impact on welfare as capital flows in and out of the economy in response to the TFP shock.

[42]Empirically, Gyourko et al. (2008) develop an index of differences in the local land-use regulatory climate across US cities based on various measures of the local regulatory environment.

may increase the quality of life for existing residents. On the other hand, by constraining the housing supply and limiting the size of cities, they may dampen the agglomeration effects at play in the economy.

I model the loosening of land-use regulation by decreasing the land-use intensity parameter b in the housing production function (equation (1)). Decreasing this parameter increases the elasticity of housing supply. To quantify the impact of land-use regulation policies, I compare the aggregate TFP and welfare of two counterfactual economies: one where the housing supply elasticity is set at the 25th percentile of the housing supply elasticity distribution, as estimated by Saiz (2010), and one where it is set at the 75th percentile.

Increasing the housing supply elasticity has two separate effects on welfare. First, a direct – mechanical – effect on utility. All else equal, as the housing sector becomes more productive and the housing supply elasticity increases, the housing units available to households increase, which directly raise their utility. This mechanical effect is not the focus here. Beyond this direct effect, an increase in the housing supply elasticity flattens out the wage schedule (see equation (6)), which leads firms in the heterogeneous goods sectors to locate in larger cities. This indirect effect enhances the productive efficiency of these sectors. To focus on this indirect effect, I control for the direct effect on utility of an increase in housing supply as follows. For each value of the housing supply elasticity, I simulate the equilibrium of the economy as described above. To measure welfare per capita, I take into account the spatial reallocation of economic activity, but hold constant b, hence the price of housing, in the utility of workers. Fixing the price of housing mutes the mechanical welfare effect coming from an increase in housing supply.

Figure 9 reports TFP and welfare, relative to the reference equilibrium, for various levels of the housing supply elasticity. An increase in the housing supply elasticity from the 25th to the 75th percentile leads to a 3.1% gain in TFP and a 3.3% indirect gain in welfare.[43]

The 3.1% gain in TFP can be decomposed into an externality effect, an entry effect and a misallocation effect, as described in Section 2. The externality effect stems from the fact that an increase in the housing supply elasticity shifts the spatial equilibrium toward larger cities, that offer larger agglomeration externalities. This increase in productivity drives up firm profits, which in turn attracts more entry, resulting in increased variety for consumers. Combined together, these effects are estimated to lead to a 2.5% increase in aggregate TFP. The remaining effect can be attributed to reduced misallocation in the economy: since the increase in housing supply elasticity flattens out the wage schedule, firms face less distortion in input prices. Factors are allocated to heterogeneous firms in a more efficient way, which leads to an increase in TFP. The effect of

[43] The aggregate welfare gain, including both the direct and the indirect effects of an increase in housing supply, is 20.5%

reduced misallocation is quantitatively smaller than the externality and the entry effect combined (0.6% vs. 2.5%).

This policy experiment illustrates how increasing housing supply in cities can have positive effects beyond directly reducing congestion costs. They allow a more efficient spatial organization of production in the differentiated goods sectors by endogenously creating agglomeration externalities and reducing the extent of misallocation.

5 Conclusion

I offer a new general equilibrium model of heterogeneous firms that are freely mobile within a country and can choose the size of the city where they produce. I show that the way firms sort across cities of different sizes is relevant to understanding aggregate outcomes. This sorting shapes the productivity of each firm and the amount of agglomeration externalities in the economy. It also shapes the allocation of factors across firms: cities are a medium through which the production of heterogeneous firms is organized within a country. The sorting of firms, mediated by the existence of city developers who act as a coordinating device for the creation of cities, lead to a unique spatial equilibrium of this economy. Therefore, the model can be used to conduct policy analysis. It allows the quantification of the complex spatial equilibrium effects of spatial policies. This complements the existing literature, which has traditionally focused on estimating local effects of such policies. Using the structure of the model, I estimate the general equilibrium effects of two types of place-based policies. A policy that explicitly targets firms locating in the least productive cities tends to hamper the productivity of the economy as a whole. For the specific policy I study, spending 1% of GDP on local tax relief leads to an aggregate welfare loss of 2.2% and does not reduce the observed spatial disparities. On the other hand, policies that encourage the growth of all cities - not just the smallest ones - can enhance equilibrium productivity and welfare: moving the housing-supply elasticity from the 25th to the 75th percentile of housing-supply elasticity leads to a 3.3% welfare gain through a spatial reorganization of production.

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Table 1: Summary statistics.

	log value added			log employment			N
	mean	p10	p90	mean	p10	p90	
Manufacture of food products and beverages	7.82	6.27	9.81	2.44	1.10	4.09	12,613
Manufacture of textiles	8.30	6.62	10.10	2.91	1.39	4.56	2,741
Manufacture of wearing apparel	7.83	6.13	9.54	2.63	1.10	4.22	2,858
Manufacture of leather goods and footwear, leather tanning	8.10	6.24	10.06	2.90	1.10	4.87	824
Manufacture and products of wood, except furniture	7.94	6.60	9.40	2.52	1.39	3.83	3,454
Manufacture of pulp, paper and paper products	8.85	6.97	11.01	3.22	1.39	5.25	1,214
Publishing, printing and reproduction of recorded media	7.79	6.28	9.52	2.18	0.69	3.69	8,829
Manufacture of chemicals and chemical products	9.17	6.74	11.98	3.24	1.10	5.57	2,342
Manufacture of rubber and plastic products	8.60	6.92	10.44	3.02	1.39	4.77	3,359
Manufacture of glass, ceramic, brick and cement products	8.23	6.65	10.07	2.61	1.10	4.28	2,848
Manufacture of basic metals	9.24	7.26	11.57	3.58	1.79	5.81	793
Manufacture of fabricated metal products, except machinery	8.17	6.81	9.57	2.59	1.39	3.89	15,269
Manufacture of machinery	8.24	6.64	10.09	2.61	1.10	4.34	7,040
Manufacture of office machinery and computers	8.41	6.49	10.76	2.79	1.10	4.75	263
Manufacture of electrical machinery	8.36	6.65	10.55	2.84	1.10	4.85	2,090
Manufacture of radio, television and communication equipment	8.53	6.75	10.49	2.93	1.10	4.82	1,376
Manufacture of medical, precision and optical instruments	8.02	6.59	9.78	2.37	1.10	4.04	3,704
Manufacture of motor vehicles	8.66	6.76	11.25	3.16	1.39	5.46	1,269
Manufacture of other transport equipment	8.25	6.24	10.72	2.77	1.10	4.98	872
Manufacture of furniture	7.72	6.15	9.47	2.36	0.69	3.94	4,413
Recycling	7.88	6.53	9.39	2.23	1.10	3.58	1,237
Information technology services	8.01	6.30	9.97	2.30	0.69	3.97	7,749
Business services, non I.T.	7.65	6.33	9.17	1.89	0.69	3.26	39,915

Table 2: Share of establishment in larger cities and tradable capital intensity.

Dep. variable	Share of establishments in large cities			
	I	II	III	IV
Sample		*all tradables*		*export intensive*
Tradable capital intensity	0.467**	0.591**	0.566**	0.488**
	(0.146)	(0.142)	(0.162)	(0.178)
High skill intensity			0.057**	0.010
			(0.029)	(0.029)
Nb firms	no	yes	yes	yes
Mean va	no	yes	yes	yes
R-squared	0.066	0.220	0.173	0.128
Observations	148	148	118	89

(*)$p < 0.10$, (**)$p < 0.05$. Tradable capital intensity: share of capital net of real estate assets in a Cobb-Douglas production function with labor, tradable capital and non tradable capital. Large cities: larger cities representing 50% of workers. High skill intensity are sectors above median of skill intensity. Export intensive: industry above median for all sectors in the economy in export intensity, proxied by the ratio of export to domestic sales.

Table 3: Tail of the firm-size distribution (ζ) vs sector location.

	ζ, tail of firm-size distribution		
	I	II	III
Sample		*all tradables*	*export intensive*
Share in large cities	-0.533**	-0.681**	-0.263**
	(0.142)	(0.105)	(0.113)
Nb firms	no	yes	yes
Mean value added	no	yes	yes
R-squared	0.088	0.574	0.487
Observations	148	148	89

(*)$p < 0.10$, (**)$p < 0.05$. Pareto Shape: ζ estimated by $Log(Rank_i - 1/2) = a - \zeta Log(va_i) + \epsilon_i$, on firms above median size, for industries with more than 200 firms. Export intensive: industry above median for all sectors in the economy in export intensity, proxied by the ratio of export to domestic sales.

Table 4: Movers.

	Δ_t City Size			
	I	II	III	IV
Sample	*all tradables*		*export intensive*	
ln(firm size)	0.100**	0.081**	0.092**	0.080**
	(0.027)	(0.024)	(0.030)	(0.029)
Initial City Size	-0.950**		-0.950**	
	(0.066)		(0.059)	
Constant	11.824**	-0.243**	11.761**	-0.290**
	(0.776)	(0.005)	(0.688)	(0.006)
Initial city F.E.		yes		yes
Cluster	city	city	city	city
R-squared	0.504	0.608	0.505	0.611
Observations	6103	6103	3675	3675

(*)$p < 0.10$, (**)$p < 0.05$. Set of mono-establishment firms which move between 2 years, between 1999 and 2005. Δ_t City Size= $ln(\frac{C_{t+1}}{C_t})$, where C_t is the size of the city where the firm locates at time t. Size is measured by the firm value added relative to other firms in the same sector-year-city, as the residual of $ln(VA)_i = DS_i + DT_i + DC_i + \epsilon_i$ where DS is a sector fixed effect, DT a year fixed effect, DC a city fixed effect. Export intensive: industry above median for all sectors in the economy in export intensity, proxied by the ratio of export to domestic sales.

Table 5: Estimated parameters.

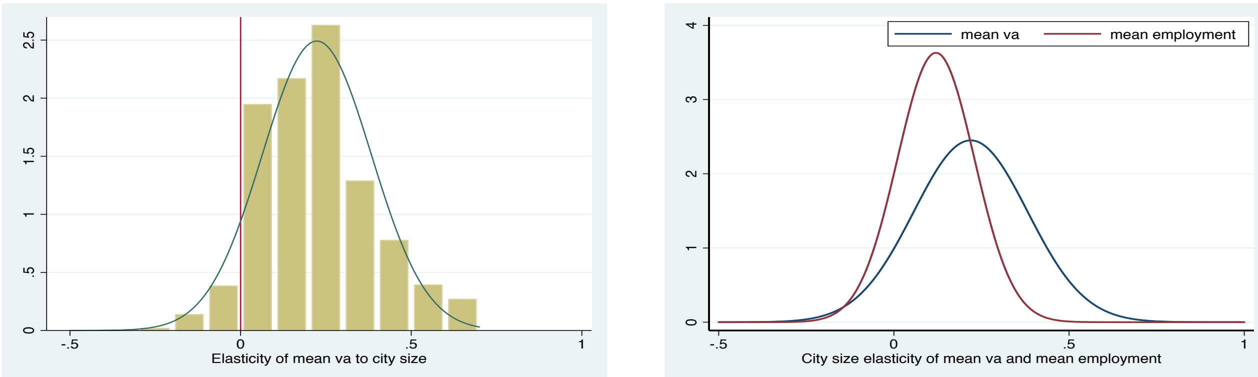
	$\hat{\nu_z}$	$\hat{\nu_R}$	$\hat{a}$	$\hat{s}$
Manufacture of food products and beverages	0.366	0.310	0.025	0.143
	(0.009)	(0.022)	(0.034)	(0.033)
Manufacture of textiles	0.340	0.102	0.014	0.130
	(0.023)	(0.016)	(0.009)	(0.014)
Manufacture of wearing apparel	0.052	0.166	0.015	0.817
	(0.005)	(0.009)	(0.001)	(0.056)
Manufacture of leather goods and footwear, leather tanning	0.241	0.166	0.053	-0.117
	(0.027)	(0.014)	(0.012)	(0.015)
Manufacture and products of wood, except furniture	0.287	0.167	0.006	0.047
	(0.016)	(0.010)	(0.002)	(0.008)
Manufacture of pulp, paper and paper products	0.339	0.230	0.033	0.091
	(0.023)	(0.025)	(0.001)	(0.054)
Publishing, printing and reproduction of recorded media	0.109	0.312	0.012	0.892
	(0.004)	(0.016)	(0.009)	(0.073)
Manufacture of chemicals and chemical products	0.939	0.115	0.000	0.071
	(0.058)	(0.055)	(0.001)	(0.017)
Manufacture of rubber and plastic products	0.371	0.279	0.052	0.053
	(0.052)	(0.024)	(0.028)	(0.042)
Manufacture of glass, ceramic, brick and cement products	0.409	0.348	0.069	-0.179
	(0.006)	(0.035)	(0.034)	(0.113)
Manufacture of basic metals	0.187	0.301	0.049	-0.729
	(0.062)	(0.028)	(0.016)	(0.031)
Manufacture of fabricated metal products, except machinery	0.067	0.180	0.007	0.798
	(0.004)	(0.005)	(0.004)	(0.025)
Manufacture of machinery	0.140	0.208	-0.005	0.512
	(0.013)	(0.011)	(0.024)	(0.135)
Manufacture of office machinery and computers	0.095	0.288	0.236	0.095
	(0.053)	(0.024)	(0.052)	(0.170)
Manufacture of electrical machinery	0.148	0.214	0.063	0.433
	(0.009)	(0.016)	(0.010)	(0.040)
Manufacture of radio, television and communication equipment	0.073	0.254	0.025	0.899
	(0.014)	(0.015)	(0.014)	(0.257)
Manufacture of medical, precision and optical instruments	0.128	0.276	0.100	0.441
	(0.014)	(0.014)	(0.007)	(0.068)
Manufacture of motor vehicles	0.159	0.289	0.099	0.258
	(0.042)	(0.021)	(0.010)	(0.053)
Manufacture of other transport equipment	0.172	0.236	-0.033	0.634
	(0.078)	(0.024)	(0.021)	(0.036)
Manufacture of furniture	0.225	0.240	0.022	0.271
	(0.011)	(0.013)	(0.021)	(0.070)
Recycling	0.271	0.377	0.018	0.505
	(0.019)	(0.025)	(0.012)	(0.078)
Information technology services	0.544	0.096	0.093	0.090
	(0.017)	(0.008)	(0.013)	(0.031)
Business services, non-I.T.	0.130	0.255	0.098	0.673

Table 6: Elasticity of firm productivity to city size: decompositions.

	(I)	(II)	(III)	(IV)	(V)
	With sorting	Without sorting	Systematic Component		
			at 25th perc.	at median	at 75th perc.
Manufacture of food products and beverages	0.04	0.03	0.03	0.04	0.05
Manufacture of textiles	0.05	0.02	0.02	0.03	0.03
Manufacture of wearing apparel	0.04	0.02	0.03	0.05	0.07
Manufacture of leather goods and footwear, leather tanning	0.03	0.03	0.05	0.05	0.04
Manufacture and products of wood, except furniture	0.03	0.03	0.01	0.01	0.01
Manufacture of pulp, paper and paper products	0.03	0.02	0.04	0.04	0.05
Publishing, printing and reproduction of recorded media	0.08	0.01	0.06	0.09	0.14
Manufacture of chemicals and chemical products	0.10	0.01	0.01	0.02	0.03
Manufacture of rubber and plastic products	0.03	0.02	0.05	0.06	0.06
Manufacture of glass, ceramic, brick and cement products	0.01	0.03	0.06	0.05	0.04
Manufacture of basic metals	0.02	0.02	0.09	0.09	0.08
Manufacture of fabricated metal products, except machinery	0.05	0.02	0.03	0.05	0.07
Manufacture of machinery	0.05	0.02	0.01	0.03	0.05
Manufacture of office machinery and computers	0.03	0.03	0.24	0.24	0.24
Manufacture of electrical machinery	0.04	0.02	0.08	0.09	0.10
Manufacture of radio, television and communication equipment	0.06	0.02	0.05	0.08	0.11
Manufacture of medical, precision and optical instruments	0.04	0.03	0.11	0.12	0.14
Manufacture of motor vehicles	0.03	0.02	0.11	0.11	0.12
Manufacture of other transport equipment	0.08	0.02	0.00	0.03	0.07
Manufacture of furniture	0.04	0.02	0.03	0.04	0.06
Recycling	0.07	0.01	0.05	0.08	0.12
Information technology services	0.06	0.02	0.10	0.10	0.11
Business services, non I.T.	0.07	0.02	0.13	0.15	0.18

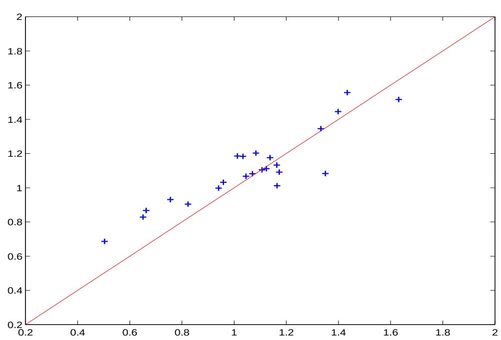
Notes. The elasticity of productivity to city size is computed as the regression coefficient β_j for sector j in the firm-level regression $\log Prod_{i,j} = \beta_0 + \beta_j \log L_i + \nu_i$ run industry by industry, where L_i is the size of the city chosen by firm i and $Prod_{i,j}$ is a measure of the productivity of firm i in sector j evaluated as follows. In Specification (I), $Prod_{i,j}$ is the productivity of firm i in sector j from the baseline model where heterogeneous firms sort across city sizes according to their raw efficiency. In Specification (II), $Prod_{i,j}$ is the productivity of firm i in sector j from a counterfactual exercise where heterogeneous firms do not sort across city sizes according to their raw efficiency. In Specifications (III-V), $Prod_{i,j}$ measures the "systematic" component of firm i productivity, and the regression is run only on firms that share the same raw efficiency z. In Specification

Figure 1: Elasticity of mean value added and employment with city size.

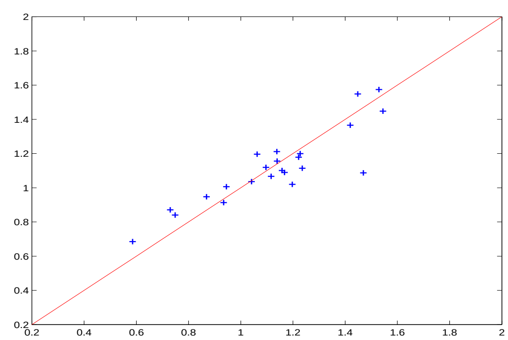


Note: Histogram by firm of β in the regression: $\log \text{mean va}(L_j) = \alpha + \beta \log L_j + \epsilon_j$ (resp. $\log \text{mean empl}(L_j)$), ran sector by sector at the NAF600 level for industries with more than 200 mono-establishment firms.

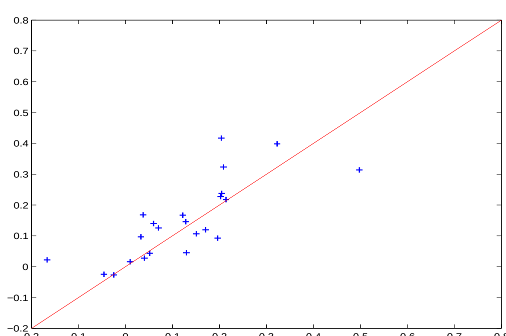
Figure 2: Subset of targeted moments, data and model.



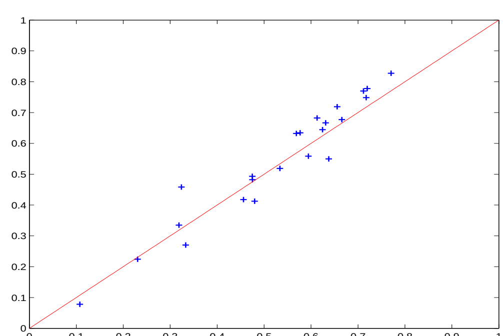
Pareto shapes in revenues



Pareto shapes in employment



Elasticity of revenues to city size



Share of employment below median city

Note: The blue points correspond to sectoral estimates for each moment, plotted agains the sectoral values in the data. The red line is the 45^o line.

Figure 3: Sectoral distribution of firms revenues, model (blue) and data (red).

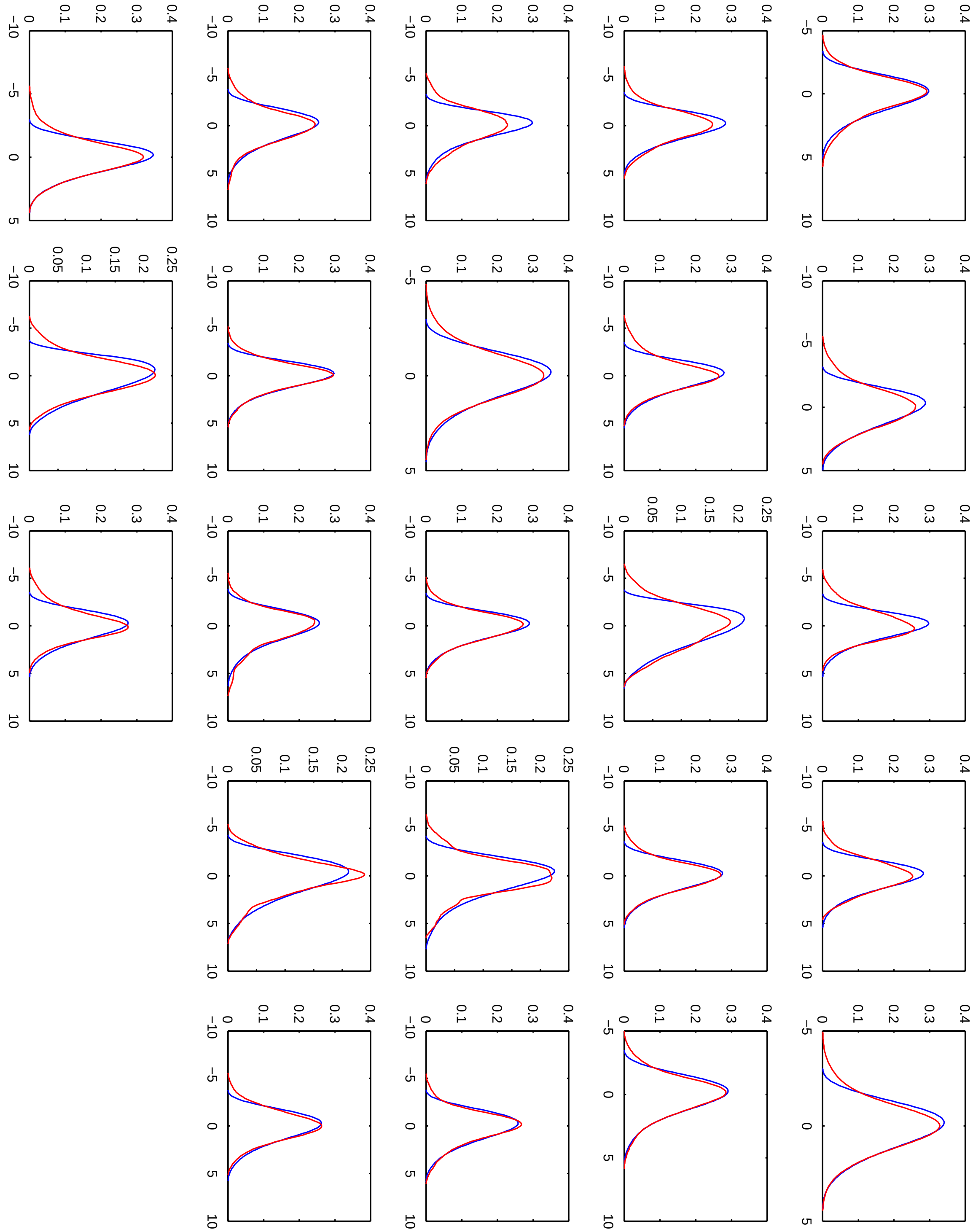


Figure 4: Employment share by decile of city size, model (blue) and data (red).

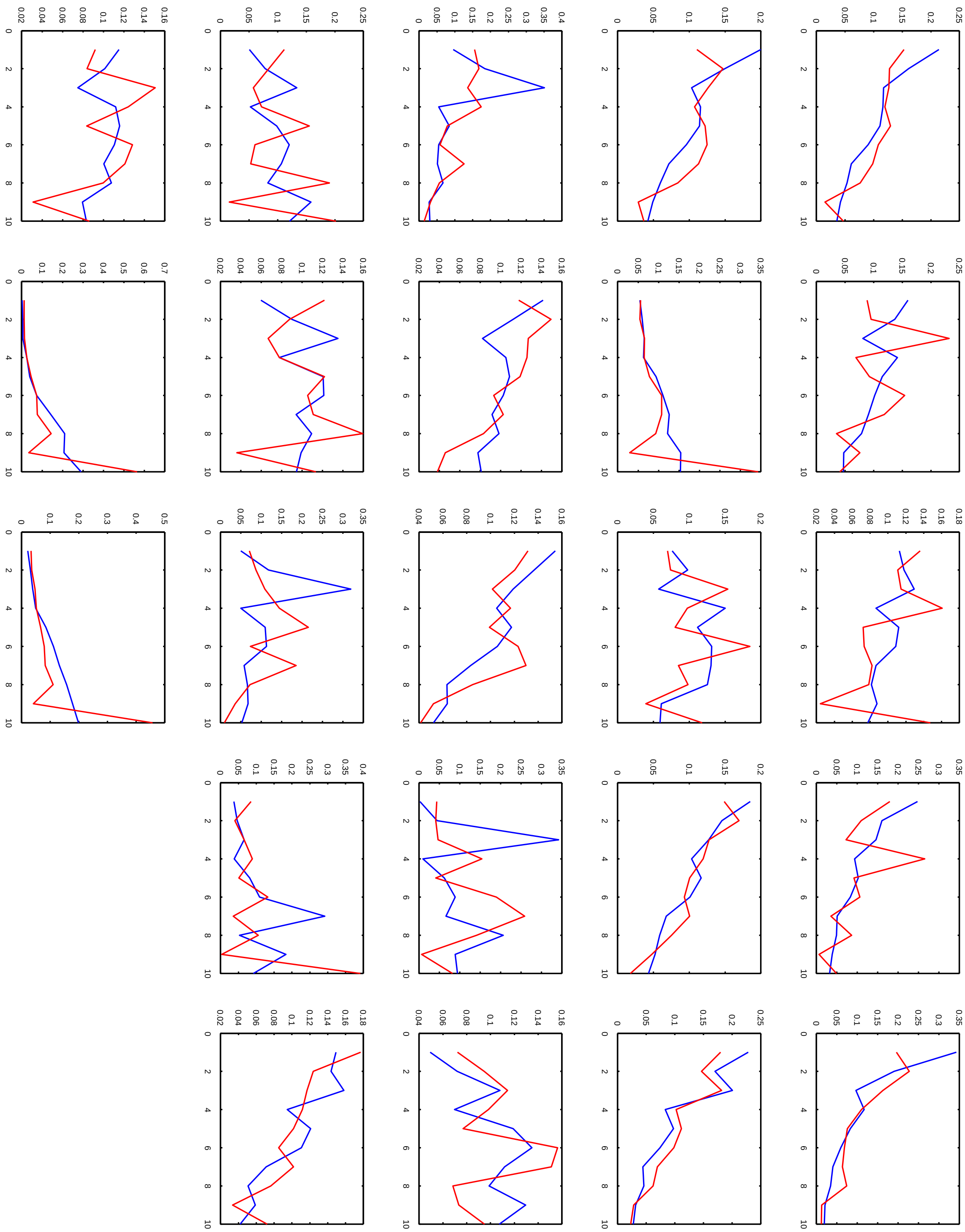
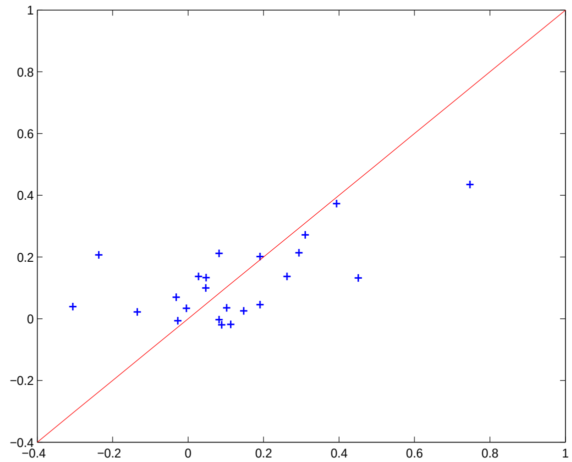
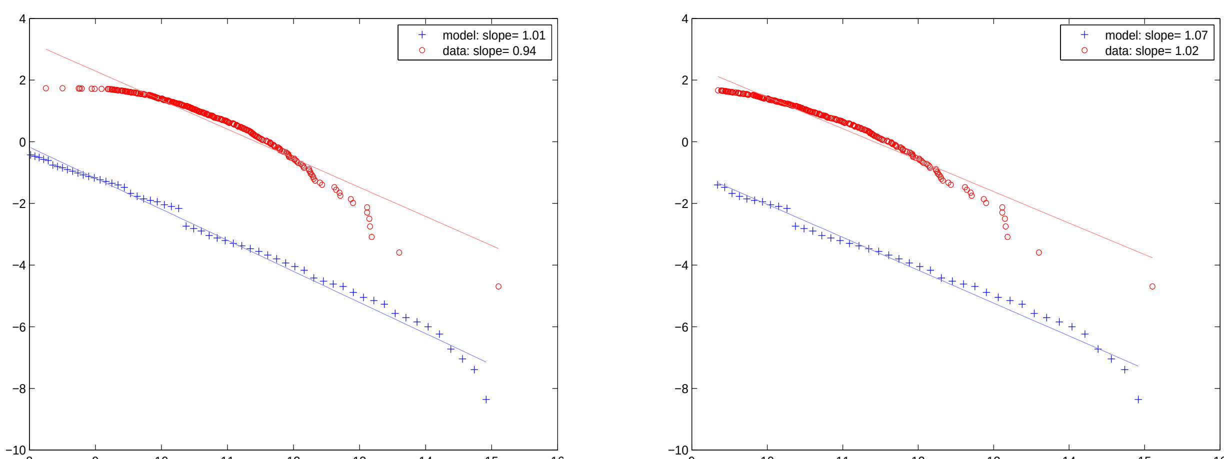


Figure 5: Dilation in firm-size distribution in revenues, large cities vs small cities.



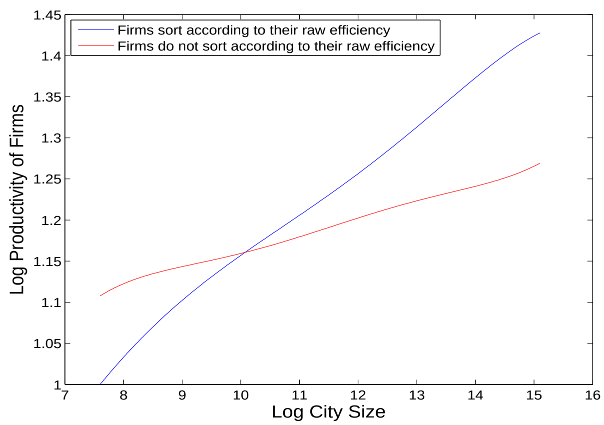
Note: For each sector j, $d_j = \log(\frac{q_{75}}{q_{25}})_{large} - \log(\frac{q_{75}}{q_{25}})_{small}$, where 'large' (resp. 'small') indexes the set of firms that locate above (resp. below) the median threshold of city size. $q(p)$ indicates the pth quantile of revenue distribution. The graph plots this dilation measure $\hat{d}_j$ from the estimated model against d_j in the data.

Figure 6: City size distribution, model and data.

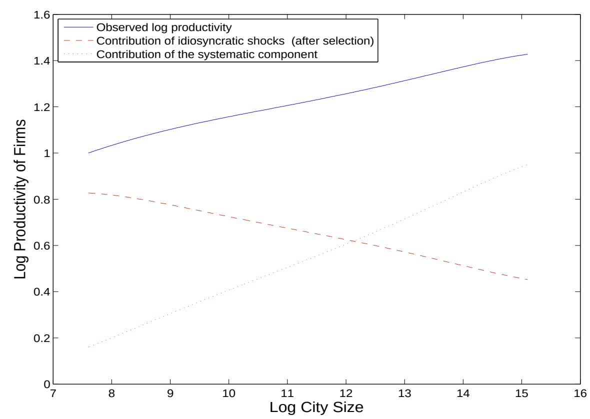


All cities | Right tail

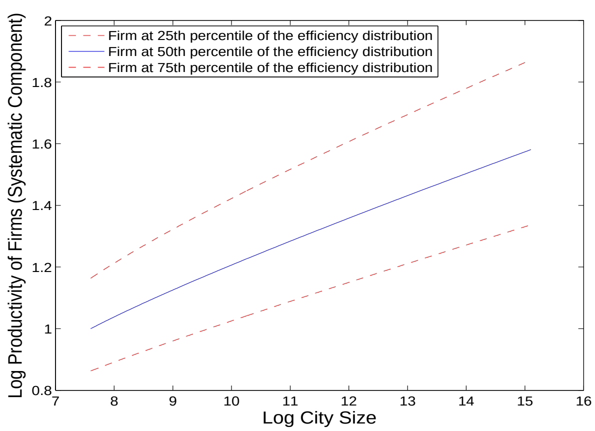
Figure 7: Elasticity of productivity with respect to city size: decompositions.



A. Productivity and city size,
with and without firm sorting



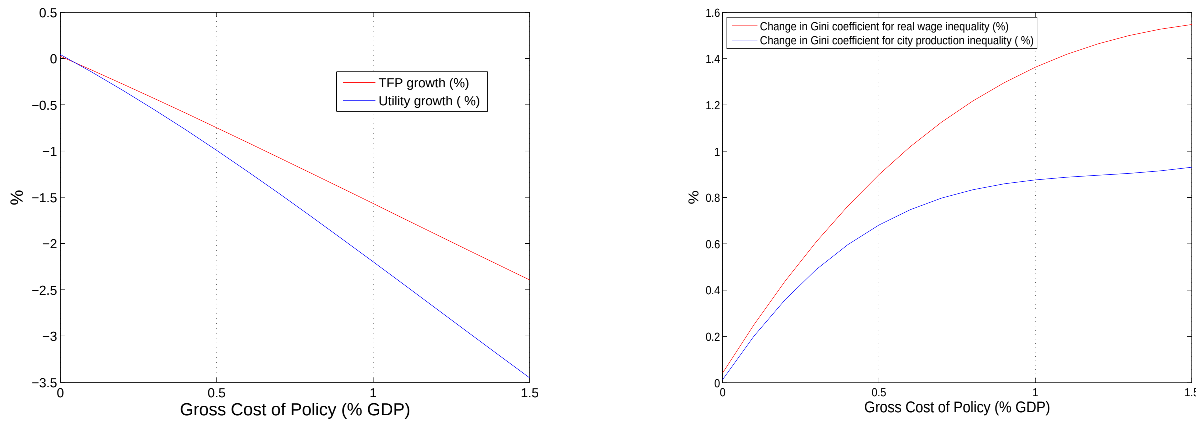
B. Productivity and city size,
systematic and idiosyncratic components



C. Systematic productivity and city size,
for three levels of firm efficiency

Note: these plots represent averages across sectors. Lines are smoothed for readability. The simulated data have a variance around this line as there is not a unique productivity level for each city-size-sector pair.

Figure 8: Aggregate impact of local subsidies, as a function of the cost of the policy.

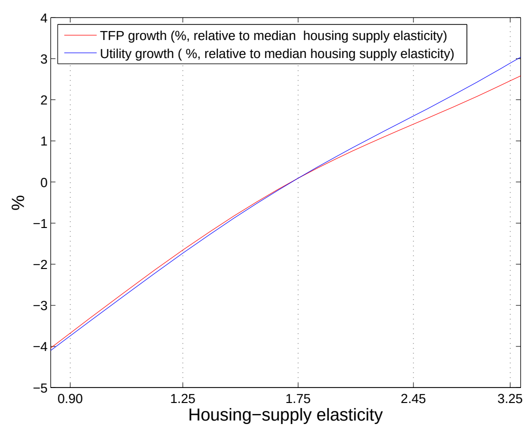


A. TFP and welfare impact, relative to the reference equilibrium

B. Change in the Gini coefficients for real wage inequality and city production inequality

Note: Firms profits are subsidized when they locate in the smallest cities of the reference equilibrium. The targeted area represents 2.3% of the population. The policy is financed by a lump-sum tax on firms.

Figure 9: TFP and indirect welfare effects of increasing housing-supply elasticity.



The horizontal axis measures housing supply elasticity in the economy $\frac{dlogH}{dlogp_H}$. Saiz (2010) reports that median elasticity of housing supply is 1.75, the 25th percentile is at 2.45 and the 75th percentile at 1.25.

Appendix

A Housing market

Housing supply (equation (2)) and demand (equation (4)) equate so that $p_H(L) = (1-\eta)^b\, w(L)\, L^b$. This yields the following labor use and profits in the housing sector:

$$\ell_H(L) = (1-b)(1-\eta)L, \quad \text{and} \tag{30}$$

$$\pi_H(L) = b(1-\eta)Lw(L). \tag{31}$$

The housing supply elasticity is given by $\frac{d\log H(L)}{d\log p_H(L)} = \eta\frac{1-b}{b}$.
Anticipating on the policy discussion, note that a decrease in b increases the housing supply elasticity and also leads to a flatter wage schedule across city sizes, as $\frac{d\log w(L)}{d\log L} = b\frac{1-\eta}{\eta}$.

B Extensions of the model

B.1 Log supermodular productivity function

I adapt the model of Davis and Dingel (2012) who microfound complementarity between entrepreneur skill and city size to a setting where entrepreneurs hire workers to produce output. Workers supply one efficiency unit of labor. They can also spend a share ψ of their time outside of the firm to gather information that will make them better at producing the firm blueprint z, such as information on efficient processes of production, or on the appeal of a product design. The return to this activity is $z\,\phi\, f(L)$ where the return increases in city size through $f(L)$: information gathering is more productive when there is more local information available. As a result of this process, the worker's amount of labor in efficiency units is: $g(z,\phi,L) = (1-\phi)(1+z\,\phi\, f(L))$. The firm chooses the time spent on productive activity $1-\phi$ by maximizing their profit, given their production function $Y = z\,k^\alpha\,(\ell\, g(z,\phi,L))^{1-\alpha}$. As the firm profit function is multiplicatively separable in $g(z,\phi,L)$, they choose optimally ϕ to maximize this term. This yields an optimized $g^*(z,L) = \frac{1}{f(L)z}(f(L)z+1-\alpha)(1+f(L)z)^{1-\alpha}$ at the optimal ϕ^*, which can be shown after some algebra to be log-supermodular in z and L as long as $f(L)$ increases with L ($\frac{\partial^2 log\, g^*(z,L)}{\partial z \partial L} > 0$).

B.2 Model with imperfect sorting

Let $p(z,L_i;j)$ the probability that a firm of type z in sector j chooses city size L_i against other cities L_k for $k=1,...,n$. As the idiosyncratic shocks to productivity follows a type I extreme value distribution,

$$p(z,L;j) = \frac{\pi(z,L_i,j)}{\sum_{k=1}^{n}\pi(z,L_k,j)}.$$

It is readily seen that p inherits the log-supermodularity of π in (z,L,j) (abusing notations, j standing for s_j or α_j). Therefore the probability distribution of establishments in a high j vs a low j sector follows the monotone likelihood ratio property. First order stochastic dominance follows.

C Proofs

C.1 Lemma 2

Proof Consider a given city of size L developed by city developer i. Let

$$N_j(L,i) = \int_z \ell_j(z,L)\mathbb{1}(z,L,i)M_j dF_j(z)$$

denote the number of workers working in sector j in this specific city i. Equation (8) shows that, for a given city size and a given sector, labor hired by local firms is proportionate to the ratio of firms profit to the local wage. The total local profits of firms in sector j is therefore proportional to the sectoral wage bill $w(L)N_j$. The city developers problem (11) then simplifies to

$$\max_{L,\{T_j(L)\}_{j\in 1,\ldots,S}} \Pi_L = b\,(1-\eta)w(L)L - \sum_{j=1}^{S} T_j(L)\,\frac{w(L)N_j}{(1-\alpha_j)(\sigma_j-1)} \tag{32}$$

Free entry pushes the profit of city developers to zero in equilibrium, which drives $T_j(L)$ to T_j^*. The problem is akin to a Bertrand game. Consider a given city size L. First, any deviation from T_j^* downwards leads a city developer to loose all firms from the corresponding sector, as new city developers could offer T_j^* for sector j and 0 for all other sectors, attract all firms for sector j for whom this subsidy is more attractive, and make exactly zero profit. A city L that attracts only firms from sector j attracts $N_j = L - \ell_H(L) = L\big(1-(1-\eta)(1-b)\big)$ workers from sector j, where $\ell_H(L)$ is the labor force hired in the construction sector and the second equality stems from equation (10). It is readily seen from this expression that the subsidy T_j^* for sector j and 0 for all other sectors leads to zero profits. Second, any deviation from T_j^* upwards in any sector j leads to negative profits. To see this, consider all cities of size L, and take the one that offers the highest subsidy city to firms in sector j. Call this city i, and assume that $T_j^i > T_j^*$. From the first step of the proof, we know that in any given city, for all sectors k, either $T_k \geq T_k^*$ and $N_k \geq 0$ or $T_k < T_k^*$ and $N_k = 0$. City developer profits in city i are therefore :

$$\begin{aligned}
\Pi^i &= b\,(1-\eta)w(L)L - \sum_{k=1}^{S} T_k^i\,\frac{w(L)N_k^i}{(1-\alpha_k)(\sigma_k-1)} \\
&< b\,(1-\eta)w(L)L - \sum_{k=1}^{S} T_k^*\,\frac{w(L)N_k^i}{(1-\alpha_k)(\sigma_k-1)} \\
&< b\,(1-\eta)w(L)L - \frac{b(1-\eta)}{1-(1-\eta)(1-b)} w(L)(\sum_{j=1}^{S} N_k) < 0
\end{aligned}$$

where the last inequality comes from $\sum_{j=1}^{S} N_k = L - \ell_H(L)$ and (10). ∎

C.2 Lemma 3

Proof Let L_o denote the suboptimal city size where firms of type (z,j) are located. They get profit $\pi_j^*(z,L_o)$. Denote $\Delta = \frac{\pi_j^*(z_j,L^*(z))}{\pi_j^*(z,L_o)} - 1 > 0$. A city developer can open a city of size $L^*(z)$ by offering a subsidy $\tilde{T_j} = \frac{1+\frac{\Delta}{2}}{1+\Delta}(1+T_j^*) - 1$, which will attract firms as they make a higher profit than at L_o, and allows the city developer to make positive profits. City size distribution adjusts in equilibrium to determine the number of such cities. ∎

C.3 Lemma 4

Proof Fix s. Since $\pi(z, L, s)$ is strictly LSM in (z, L), if follows that for all $z_1 > z_2$ and $L_1 > L_2$, $\frac{\pi(z_1,L_1,s)}{\pi(z_1,L_2,s)} > \frac{\pi(z_2,L_1,s)}{\pi(z_2,L_2,s)}$. So if z_2 has higher profits in L_1 than in L_2, so does z_1. Necessarily, $L^*(z_1) \geq L^*(z_2)$.

Moreover, under the technical assumptions made here, $L_j^*(z)$ is a strictly increasing function. Since the set of z is convex, and $\psi(z, L, s)$ is such that the profit maximization problem is concave for all firms, the optimal set of city sizes is itself convex. It follow that $L_j^*(z)$ is invertible. It is locally differentiable (using in addition that $\psi(z, L, s)$ is differentiable), as the implicit function theorem applies and $\frac{dL_j^*(z)}{dz} = -\frac{\frac{\partial(\frac{\psi_2 L}{\psi})}{\partial z}(z,L_j^*(z),s)}{\frac{\partial(\frac{\psi_2 L}{\psi})}{\partial L}(z,L_j^*(z),s)}$. ∎

C.4 Lemma 5

The proof is in the main text.

C.5 City size distribution

I establish here the following characterization of the city-size distribution.

Characterization 10 *If the firm size distribution in revenues follow Zipf's law, a sufficient condition for the city size distribution to follow Zipf's law is that revenues have constant elasticity with respect to city size in equilibrium, under realistic parameter values.*

When the productivity function is $\log(\psi_j(z, L, s_j)) = a_j \log L + \log(z)(1 + \log \frac{L}{L_o})^{s_j}$, equilibrium firm-level revenues have constant elasticity with respect to city size in equilibrium.

Proof There is a bijection between z and firm level observable at equilibrium r^*, ℓ^*, L^*. By an abuse of notation, this functional relationship will be denoted $z(r), z(L), \ell(z)$. All of these relationship pertain to the sorting equilibrium, but I omit the star to keep the notations light.
Step 1: Write $g_j(r)$ the distribution of revenues in sector j. The firm-size distribution in revenues is readily computed through a change of variable, starting from the raw efficiency distribution:

$$g_j(r) = f_j(z(r))\frac{dz}{dr}(r) \tag{33}$$

Step 2: Assume that in all sectors - or at least for the ones that will locate in the largest cities in equilibrium -, firm-size distribution in revenues is well approximated by a Pareto distribution with Pareto shape ζ_j, close to 1. Write $g_j(r)$ the distribution of revenues in sector j:

$$\exists r_{j,o} \ : \ \forall \ r > r_{j,o}, \ g_j(r) \propto r^{-\zeta_j - 1} \tag{34}$$

Step 3: The city size distribution $f_L(.)$ as defined by equation 20 can be written as:

$$f_L(L) = \sum_{j=1}^{S} M_j \tilde{f^j}(L)$$

where $\tilde{f^j}(L) = \frac{\ell_j(z_j^*(L))\, f_j(z_j^*(L))\, \frac{dz_j^*(L)}{dL}}{L}$. Then,

$$\begin{aligned}\tilde{f^j}(L) &\propto\ g_j(r^*(L))\frac{dr_j^*}{dL}\frac{r_j(L)}{L^{b\frac{\eta}{1-\eta}+1}}\\ &\propto \frac{r_J(L)^{-\zeta_j+1}}{L^{b\frac{\eta}{1-\eta}+2}}\end{aligned}$$

The first line uses (33), $\ell_j \propto \frac{r_j(L)}{w(L)}$ and $w(L) \propto L^{b\frac{\eta}{1-\eta}}$. The second line uses the assumption that r^* has constant elasticity with respect to L at the sorting equilibrium, and (34). As $\zeta \sim 1$,

$$\tilde{f^j}(L) \propto \frac{1}{L^{b\frac{\eta}{1-\eta}+2}} \text{ and } f_L(L) \propto \frac{1}{L^{b\frac{\eta}{1-\eta}+2}}.$$

Finally, $b\frac{\eta}{1-\eta} << 1$, as it measures the elasticity of wages with city size, on the order of magnitude of 5%. The tail of the city size distribution is well approximated by a Pareto of shape close to 1. City size distribution follows Zipf's law. ∎

C.6 Proposition 6

Proof Fix j. For productivity, the results comes from the facts that (1) $L_j^*(z)$ is non decreasing in z and (2) that $\psi(z, L, s_j)$ is increasing in L. Revenues are proportional to profits ($r_j * (z) = \frac{\sigma_j}{1+T_j^*}\pi_j^*(z)$). The proof for profits is as follows. $\psi(z_H, L_L, s_j) > \psi(z_L, L_L, s_j)$ as ψ is increasing in z, which leads to $\pi(z_H, L_L) > \pi(z_L, L_L)$, as firms face the same wage in the same city. Finally, $\pi(z_H, L_H, s_j) \geq \pi(z_H, L_L, s_j)$ as L_H is the profit maximizing choice for z_H. Therefore, $\pi(z_H, L_H, s_j) > \pi(z_L, L_L, s_j)$. ∎

In addition, $\epsilon_l = \epsilon_r - (1-\alpha)\frac{1-\eta}{\eta}$.

Proof For a given city size L and a given sector j, $\bar{r_j^*}(L) = \sum_{z\ in\ L} r_j^*(z) \propto \sum_{z\ in\ L} w(L)\ell_j^*(z) \propto w(L)\bar{\ell_j^*}(L)$, where their proportion is constant across city sizes. Therefore $\frac{d\log\ \bar{\ell_j^*}(L)}{d\log\ L} = \frac{d\log\ \bar{r_j^*}(L)}{d\log\ L} - \epsilon_w$, where the elasticity of wages with respect to city sizes is $\epsilon_w = b\frac{\eta}{1-\eta}$ ∎

C.7 Proposition 7

Proof The proof here covers both the case of the main assumptions of the model (continuity and convexity of the support of z and L), and the case where the set of city sizes is exogenously given, and in particular discrete. Let $\mathcal{Z} : \mathcal{L}$ x A x $E \to Z$ be the correspondence that assigns to any $L \in \mathcal{L}$ and $\alpha \in A$ a set of z that chooses L at equilibrium. (It is a function under the assumptions made in the main text (see proof of Lemma 4).) Define $\bar{z}(L, \alpha, s) = \max_z\{z \in \mathcal{Z}(L, \alpha, s)\}$ as the maximum efficiency level of a firm that chooses city size L in a sector characterized by the parameters (α, s). I will use the following lemmas:

Lemma 11 *$log\,\pi$ is supermodular with respect to the triple (z, L, α)*

It is readily seen that: $\frac{\partial^2 log\,\pi(z,L,\alpha,s)}{\partial z \partial L} > 0$, $\frac{\partial^2 log\,\pi(z,L,\alpha,s)}{\partial z \partial \alpha} = 0$ and $\frac{\partial^2 log\,\pi(z,L,\alpha,s)}{\partial L \partial \alpha} = \frac{(\sigma-1)b(1-\eta)}{\eta L} > 0$. This result does not rely on an assumption on the convexity of $\mathcal{L}$. Checking the cross partials are sufficient to prove the supermodularity even if L is taken from a discrete set, as π can be extended straightforwardly to a convex domain, the convex hull of $\mathcal{L}$.

Lemma 12 *$\bar{z}(L, \alpha, s)$ is non decreasing in α, s.*

The lemma is a direct consequence of the supermodularity of $log\,\pi$ with respect to the quadruple (z, L, α, s). Using a classical theorem in monotone comparative statics, if $log\,\pi(z, L, \alpha, s)$ is supermodular in (z, L, α, s), and $L^*(z, \alpha, s) = \max_L log\,\pi(z, L, \alpha, s)$ then $(z_H, \alpha_H, s_H) \geq (z_L, \alpha_L, s_L) \Rightarrow L^*(z_H, \alpha_H, s_H) \geq L^*(z_L, \alpha_L, s_L)$. Note that everywhere, the $\geq$ sign denotes the lattice order on $\mathcal{R}^3$ (all elements are greater or equal than).

Coming back to the proof of the main proposition, we can now write:

$$\begin{aligned}\tilde{F}(L; \alpha, s) &= P(\text{firm from sector}(\alpha, s) \text{ is in a city of size smaller that } L)\\ &= F(\bar{z}(L, \alpha, s))\end{aligned}$$

where $F(.)$ the the raw efficiency distribution of the firms in the industry. Let $\alpha_H > \alpha_L$.
For any $z \in Z$, the previous lemma ensures that $L^*(z, \alpha_H, s) \geq L^*(z, \alpha_L, s)$. In particular, fix a given L and s and write using shorter notation: $\bar{z}_{\alpha_L} = \bar{z}(L, \alpha_L, s)$. Then $L^*(\bar{z}_{\alpha_L}, \alpha_H, s) \geq L^*(\bar{z}_{\alpha_L}, \alpha_L, s) = L$. Because $L^*(z, \alpha_H, s)$ is increasing in z, it follows that:

$$z \in \mathcal{Z}(L, \alpha_H, s) \Rightarrow z \leq \bar{z}_{\alpha_L}$$

and therefore $\bar{z}_{\alpha_H} \leq \bar{z}_{\alpha_L}$ or using the long notation: $\bar{z}(L, \alpha_H, s) \leq \bar{z}(L, \alpha_L, s)$
It follows that $F(\bar{z}(L, \alpha_H)) \leq F(\bar{z}(L, \alpha_L))$ and that $F(L; \alpha, s)$ is decreasing in α. This completes the proof of the first order stochastic dominance of the geographic distribution of a high α sector vs that of a lower α.

The proof is exactly the same for the comparative static in s, we just have to verify that $\pi(z, L, s)$ is log supermodular in (z, L, s). Since $\pi(z, L, s_j) = \kappa\left(\frac{\psi(z,L,s_j)}{w(L)^{1-\alpha}}\right)^{\sigma-1}\frac{R_j}{P_j^{1-\sigma}}$ and $w(L)$ doesn't depend on s, $\pi(z, L, s)$ directly inherits the log supermodularity of $\psi(z, L, s)$ in its parameters. ∎

C.8 Proposition 8

Proof The proof here covers both the case of the main assumptions of the model (continuity and convexity of the support of z and L), and the case where the set of city sizes is exogenously given, and in particular discrete. Within sectors, the revenue function $r_j^*(z)$ at the sorting equilibrium is an increasing function for any j. Let $p_1 < p_2 \in (0, 1)$. Under the assumption, maintained throughout the comparative static exercise, that sectors draw z from the same distribution, there $\exists\ z_1 < z_2$ such that $Q_{j_1}(p_1) = r_{j_1}^*(z_1)$ and $Q_{j_2}(p_1) = r_{j_2}^*(z_1)$ (same thing for z_2 and p_2), ie. the quantiles of the r_{j1}^* and r_{j2}^* distributions correspond to the same quantile of the z distribution. This yields $\frac{Q_{j_1}(p_2)}{Q_{j_1}(p_1)} = \frac{r_{j1}^*(z_2)}{r_{j1}^*(z_1)}$, and $\frac{Q_{j_2}(p_2)}{Q_{j_2}(p_1)} = \frac{r_{j2}^*(z_2)}{r_{j2}^*(z_1)}$.
Finally, it is a classic result in monotone comparative statics (Topkis (1998)) that if $\pi(z, L, \alpha)$ is log-supermodular in (z, L, α), then $\pi^*(z, \alpha) = \max_L \pi(z, L, \alpha)$ is log supermodular in (z, α), or $\frac{\pi_{j2}^*(z_2)}{\pi_{j2}^*(z_1)} \geq \frac{\pi_{j1}^*(z_2)}{\pi_{j1}^*(z_1)}$. Revenues are proportional to profits within sectors, which completes the proof. The same proof applies for s. ∎

C.9 Corollary 9

Proof Let $p_j \in (0, 1)$ be a threshold above which the distribution is well approximated by a Pareto distribution in sector j, and r_j the corresponding quantile of the distribution. The distribution of r conditional on being larger than r_j is:

$$\forall r > r_j,\ H_j(r \,|\, r \geq r_j) \approx 1 - (\frac{r}{r_j})^{-\zeta_j},$$

where ζ_j is the shape parameter of the Pareto distribution for sector j. Thus, if $F_j(r) = p$, one can write:

$$\forall p > p_j, \;\; p \approx F_j(r_j) + H_j(r) \approx p_j + 1 - (\frac{r}{r_j})^{-\zeta_j}$$
$$\frac{r}{r_j} \approx (1 + p_j - p)^{-\frac{1}{\zeta_j}}$$

Letting $p_0 = \max(p_1, p_2)$ and writing $r_j = Q_j(p_0)$ for $j = 1, 2$, and using proposition (8) gives:

$$\begin{aligned} \frac{Q_{j1}(p)}{Q_{j1}(p_0)} &\leq \frac{Q_{j2}(p)}{Q_{j2}(p_0)} \\ (1 + p_0 - p)^{-\frac{1}{\zeta_1}} &\leq (1 + p_0 - p)^{-\frac{1}{\zeta_2}} \qquad \text{for all } p > p_0 \text{ and } p < 1 \\ \zeta_1 &\geq \zeta_2, \end{aligned}$$

where the last inequality comes from $1 + p_0 - p \in (0, 1)$. ∎

D General Equilibrium, TFP and welfare

D.1 General equilibrium quantities

Total manufacturing revenues The national labor market clearing condition (19) together with equation (18) leads to the aggregate revenues in manufacturing,

$$R = N_o \frac{(1 - (1 - b)(1 - \eta))}{\sum_{j=1}^{S} \xi_j \frac{E_j}{S_j}}. \tag{35}$$

Price index and mass of firms Combining equations (18) and (17) leads to the sectoral mass of firms[44]

$$M_j = \frac{\sigma_j \xi_j}{f_{Ej}(1 + T_j^*)} \frac{R}{P}. \tag{36}$$

This equation is still valid when a federal government subsidies firm's profits depending on their location choice, as the policy is financed for by a lump sum tax on all firms profits.
Using equations (36) and (17)

$$P_j^{\sigma_j - 1} = \frac{1}{\sigma_j \frac{\kappa_{1j}}{\kappa_{3j}} M_j S_j} \propto \frac{P^{1+\alpha_j(\sigma_j - 1)}}{R S_j}$$

so that

$$P^{-1} = \prod_{j=1}^{S} (\frac{P_j}{\xi_j})^{-\xi_j} \propto \left(\left(\sum_{j=1}^{S} \xi_j \frac{E_j}{S_j} \right)^{-\frac{1}{\bar{\sigma}-1}} \prod_{j=1}^{S} S_j^{\frac{\xi_j}{\sigma_j - 1}} \right)^{\frac{\bar{\sigma}-1}{(1-\bar{\alpha})(\bar{\sigma}-1)-1}}$$

where $\bar{\alpha} = \sum_{j=1}^{S} \alpha_j \xi_j$ and $\frac{1}{\bar{\sigma}-1} = \sum_{j=1}^{S} \frac{\xi_j}{\sigma_j - 1}$. Hence, $M_j \propto \left(\left(\sum_{j=1}^{S} \xi_j \frac{E_j}{S_j} \right)^{1-\bar{\alpha}} \prod_{j=1}^{S} S_j^{\frac{\xi_j}{\sigma_j - 1}} \right)^{\frac{\bar{\sigma}-1}{(1-\bar{\alpha})(\bar{\sigma}-1)-1}}$.

[44] The price of capital ρ is exogenously given on international market. As I do not choose the price index P as the numeraire, ρ is not fixed in absolute value but relative to P.

D.2 TFP and welfare

TFP Let Y_j, N_j and K_j denote the sectoral output, employment and capital. Sectoral productivity $TFP_j \equiv \frac{Y_j}{K_j^{\alpha_j} N_j^{1-\alpha_j}}$ can be expressed as follows.

$$\begin{aligned} Y_j &= M_j^{\frac{1}{\sigma_j - 1}} \left(\int \ \left(\psi(u, L_j^*(u), s_j) \ k_j^*(u)^{\alpha_j} \ \ell_j^*(u)^{1-\alpha_j} \right)^{\frac{\sigma_j - 1}{\sigma_j}} dF_j(u) \right)^{\frac{\sigma_j}{\sigma_j - 1}} \\ &= K_j^{\alpha_j} N_j^{1-\alpha_j} M_j^{\frac{1}{\sigma_j - 1}} \left(\int \ \left(\psi(u, L_j^*(u), s_j) \ o_j^k(u)^{\alpha_j} \ o_j^\ell(u)^{1-\alpha_j} \right)^{\frac{\sigma_j - 1}{\sigma_j}} dF_j(u) \right)^{\frac{\sigma_j}{\sigma_j - 1}}, \end{aligned} \tag{37}$$

where

$$\begin{aligned} o_j^\ell(z) &= \frac{\ell_j^*(z)}{N_j} = \frac{\frac{\psi(z, L_j^*(z), s_j)^{\sigma_j - 1}}{w(L_j^*(z))^{(1-\alpha_j)(\sigma_j - 1) + 1}}}{\int \left(\frac{\psi(u, L_j^*(u), s_j)^{\sigma_j - 1}}{w(L_j^*(u))^{(1-\alpha_j)(\sigma_j - 1) + 1}} \right) dF_j(u)}, \text{ and} \\ o_k^\ell(z) &= \frac{k_j^*(z)}{K_j} = \frac{\frac{\psi(z, L_j^*(z), s_j)^{\sigma_j - 1}}{w(L_j^*(z))^{(1-\alpha_j)(\sigma_j - 1)}}}{\int \left(\frac{\psi(u, L_j^*(u), s_j)^{\sigma_j - 1}}{w(L_j^*(u))^{(1-\alpha_j)(\sigma_j - 1)}} \right) dF_j(u)}. \end{aligned} \tag{38}$$

After some algebra, this leads to

$$\begin{aligned} TFP_j &= M_j^{\frac{1}{\sigma_j - 1}} \frac{\left(\int \ \frac{\psi(u, L_j^*(u), s_j)^{\sigma_j - 1}}{w(L_j^*(u))^{(1-\alpha_j)(\sigma_j - 1) + 1}} dF_j(u) \right)^{\frac{\sigma_j}{\sigma_j - 1} - \alpha_j}}{\left(\int \ \frac{\psi(u, L_j^*(u), s_j)^{\sigma_j - 1}}{w(L_j^*(u))^{(1-\alpha_j)(\sigma_j - 1) + 1}} dF_j(u) \right)^{1-\alpha_j}} \\ TFP_j &= M_j^{\frac{1}{\sigma_j - 1}} \frac{S_j^{\frac{\sigma_j}{\sigma_j - 1} - \alpha_j}}{E_j^{1-\alpha_j}}, \end{aligned} \tag{39}$$

where S_j and E_j are defined in the main text. Sectoral TFP only depends on the matching function $L_j^*(z)$ and the wage schedule elasticity with respect to city size. To see this, consider the wage schedule defined by equation 6. The wage level $\bar{w}(1-\eta)^{b\frac{1-\eta}{\eta}}$ does not enter expressions (38). The absolute wage level cancels out in the expression of the ratios $o_j^\ell(z)$ and $o_j^k(z)$. It follows that the absolute wage level does not impact equations (37) or (39).

Decomposing Welfare

$$TFP_j = M_j^{\frac{1}{\sigma_j - 1}} (\frac{S_j}{E_j})^{1-\alpha_j} S_j^{\frac{1}{\sigma_j - 1}}$$

Given equation (36) and the expression for the price index,

$$\begin{aligned} P_j^{-1} &= TFP_j (\frac{E_j}{S_j}^{1-\alpha_j}) \frac{1}{P^{\alpha_j}} \\ P^{-1} &\propto \left(\prod_{j=1}^{S} TFP_j^{\xi_j} (\frac{E_j}{S_j})^{\xi_j (1-\alpha_j)} \right)^{\frac{1}{1-\bar{\alpha}}} \end{aligned}$$