Purchasing Votes without Cash: Implementing Quadratic Voting Outside the Lab*

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Abstract

We compare behavior under Quadratic Voting with respect to the conventional One Man One Vote system in a context of a discrete valuation distribution with non zero mean for a binary collective decision. Unlike, the One Man One Vote system, in which a majority rule of 50+1 could yield socially inefficient outcomes if the intensity of the preferences in the minority is larger than in the majority, Quadratic Voting allows subjects to express the intensity of their preferences by purchasing votes at a quadratic cost and overcome the inefficient outcome. Once the votes are counted and a particular outcome or public good is provided, the proceeds from the allocation of voters to votes gets redistributed back to the subjects inversely proportional to their allocation of endowments to votes. We conduct an experiment with university students in which we analyze their understanding of the mechanism and their voting behavior. We use tickets for a raffle of a valuable prize instead of cash as the currency in the experiment, compelling us to round the rebate unlike the original model of Quadratic Voting (Weyl, 2013). As a consequence, the equilibria under Quadratic Voting in our setting is a subset of the equilibria under the One Man One Vote system. We find that the overspending of voters under Quadratic Voting is proportional to their exogenous valuation of the policies voted to be implemented.

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1 Introduction

Collective decision making using a majority rule of 50+1 is a standard procedure used by many firms and countries to decide whether a policy should be implemented. However, this mechanism is subject to the "tiranny of the majority" and therefore may give rise to situations that deviate the collective decision-making process from optimality (Posner and Weyl, 2014). Consider for instance a minority that care much more intensely about getting a public good (e.g. a road or a anti-discrimination law) than the majority does for the opposite outcome. With the use of a simple majoritarian rule it might be the case that the losses caused to the majority are greater than the benefits provided to the majority, an inefficient outcome from an utilitarian point of view. More generally, take a binary collective decision with a distribution of preferences such that preferring one of the choices is represented by a positive valuation whereas preferring the complementary choice is represented by a negative valuation. A majority voting system will lead to an inefficient outcome when the expected value and the median value are on opposite sides of the valuation space.

The Quadratic Voting mechanism (henceforth QV) proposed by Weyl (2013) is a novel electoral design which can yield more efficient outcomes than a majority voting system in these situations. Under the QV mechanism individuals buy votes at a quadratic unit cost and receive a reimbursement equal to the average of the others' expenditures in votes. Hence, the marginal cost of an additional vote is proportional to the votes already purchased, and the marginal benefit of an additional vote is proportional to the cardinal value of changing the policy. Furthermore, because individuals receive a reimbursement equal to the average of others' payment, the mechanism is budget balanced. In addition, QV tries to resolve other problems of mechanism design such as the collusion problem of the Vickrey-Clarke-Groves (VCG) mechanism, and the information problems of the Expected Externality mechanism, where a social planner needs to know the distribution of the valuations that agents have.

The QV procedure is inspired in the mechanism proposed by Hylland and Zeckhauser (1979), in which subjects are endowed with "influence" points used to vote for an increase or decrease in the supply of different public goods. The particular feature of this mechanism observed in Weyl (2013) is that "influence" points buy a number of votes equal to the root square of the expenditure in favor (or against) the provision of the selected good. The other feature of Hylland and Zeckhauser's mechanism, the possibility to simultaneously decide among the provision of different goods as a way to express the intensity of the preferences, is adapted in the mechanism with storable votes proposed by Casella (2005). In that model, a committee take one collective decision per period during a total of T periods, with committee members being allowed to save their vote from their contemporary decision to be used in future periods.

Goeree and Zhang (2013) independently propose a similar mechanism to QV, with a quadratic bidding cost and a rebate equal to the average of the other's expenditure, that is individually rational when the ex ante distribution of the valuations is symmetric. This is also the first work testing the QV mechanism in a laboratory setting. Subjects received a “moderate” or “extremist” valuation favoring one of the two policies, and interacted for 40 rounds divided in two stages. In the first 20 rounds individuals make choices under the QV mechanism and a majority rule. Then they choose whether the QV or the majority rule should be implemented in the next 20 rounds, using a majority rule, in order to endogenize the choice of institutions. Goeree and Zhang find that individuals prefer the QV mechanism over a majority system rule, and that the QV is more efficient than the majority system in a laboratory experiment.

Experimental methods are advantageous in the exploration of individual behavior and its interplay with economic and political institutions such as markets and collective-choice mechanisms (Morton and Williams, 2010; Palfrey, 2013). In the particular case of Goeree and Zhang’s experiment the controlled environment is ideal to test some properties of their bidding mechanism with quadratic costs as its robustness to irrational behavior and truthful bidding,[1] as well as observing the subjects’ preferences of this mechanism with respect to the standard majoritarian rule. The repeated nature of interactions is a key component of their experiment for two reasons. First, it allows participants to learn about the mechanism and the distribution through their experience; second, by increasing the exposition of participants to both electoral mechanisms their preferences for one electoral rule over the other are subject to less noise.

An additional advantage of the experimental approach is that it is possible to deal with one of the objections against the QV mechanism: the taboo against vote-buying discussed by Posner and Weyl. The proper framing, in addition to the random allocation of exogenous valuations for a hypothetical policy, minimize the considerations against purchasing votes within a subjects’ pool that anyway expects a monetary reward in exchange for their participation.

An alternative solution to the taboo against purchasing votes is to implement the mechanism with an alternative currency, which may have some properties that decrease the concerns regarding Quadratic Voting: a better distribution of the initial endowments among the electorate and a higher cost of extra-system transferability of monetary units across subjects. The former property decreases the asymmetries in influence according to wealth levels, as well as the concerns regarding credit constraints. The latter property makes harder for voters to incur in extra-system vote buying, a problem raised, but only partially solved, in Posner and Weyl (2014).

We also conducted a classroom experiment prior to the experiment reported in this paper whose goal was to analyze voters’ behavior using different currencies. In that experiment the QV mechanism was used to reach a decision on the final exam’s date. There were two alternative dates on which they could bid, the week before and the week after the spring break.[2] The

[1]The individually rational property applies only in the case of Goeree and Zhang’s mechanism, not to Weyl’s more general model.

[2]The spring break matches the religious holiday corresponding to the Catholic Holy Week.

results of that experiment led us to study further the issue of using different currencies for such a controversial proposal as the Quadratic Voting mechanism. Subjects were requested to submit how many units of their initial endowment, set at five units, they were willing to spend on their preferred exam date. They were told that the number of points added to their preferred policy corresponds to the square root of their expenditure. Students took their expenditure decision using four different payment media: cash, the number of hours they were willing to sacrifice to deliver the take-home exam, the amount of lines they were willing to sacrifice to complete an essay question that was part of the exam, and additional points of a bonus in the grade of the exam. For all the payment media the expenditure was rescaled to a discrete scale allowing participants to spend up to 5 units of their endowment in the QV mechanism (full information on the exchange rates is reported in Table A.1 in the Appendix). Subjects were told that one of the four payment media was previously selected, but that it would not will be revealed until all the students submitted their experimental decisions or send a message disclosing that they abstain from participating. The chosen payment media was cash.

From highest to lowest purchased votes we obtained on average: cash (1.51), lines available to complete the essay part of the exam (1.13), hours to deliver the exam (0.71), and bonus points in the exam (0.39) (see Figure A.1 in the Appendix for a comparison of purchased votes across poicies). A majority composed by 65 percent of the electorate, which prefers the later exam date, casted more votes across all payment media. With the exception of the bonus in the exam, the option preferred by the majority accumulated between 73.7 and 76.7 percent of the purchased votes, and the ratio of the average purchased votes by majority members with respect to the minority fell between 1.49 and 1.75. When the bonus in the exam was used as currency the gap between the two choices shrinked: the majority accumulated 61 percent of the votes, and the purchased votes per subject were higher for the minority (the ratio between the average purchased votes from the majority and the minority was 0.84).

With the exception mentioned above, the ratio of the average expenditure from the majority with respect to the minority and the share of total votes in the final outcome were equivalent across currencies.[3] In the light of this positive evidence for the use of other payment methods alternative to cash, we test in this paper the QV mechanism outside the laboratory using tickets for the raffle of a prize. The prize was a Colombia's national soccer team jersey, signed by one of the top players in the squad. It was valued by the participants in the experiment (using a non-incentivized Becker-deGroot-Marschak mechanism) at an average price of $370,000 Colombian

[3]Our explanation for the differences observed when the currency was the bonus exam is that when the payment method is very expensive and subjects are less likely to vote, then the aggregate observed behavior might be driven by a few subjects. If the average low expenditure is anticipated by the most extreme voters in the minority, they may try to increase their influence in the election's outcome, a case reported by Weyl (2013) as an attempt to "buy the election." Indeed, this was the case in our experiment, in which only two out of eight members in the minority purchased votes, but each one of them spent three units of their endowment. The minority's total expenditure was higher than for the majority, but due to the quadratic cost they accumulated fewer votes.

pesos, about 63% the country's monthly minimum wage.

Our goal with this experiment is to explore and discuss the implementability of the QV mechanism in a less controlled environment than the laboratory, but more closely resembling a situation than inexperienced voters will face in case of adopting this mechanism to reach a collective decision. Instead of repeated interactions we only provided our participants with instructions about how the mechanism works: the number of purchased votes as a function of spent tickets and the subsequent rebate. In addition, the use of tickets as payment method imposed two restrictions for us: first, participants' discrete decisions were not based on how many votes do they buy, but rather on how many of the tickets do they forego, as in the original model proposed by Hylland and Zeckhauser (1979). Second, and as a consequence of the first restriction, we rounded the average expenditure of other voters to be reimboursed to each participant. This means that the implement version of the mechanism is not budget balanced.[4]

We implement the QV mechanism and compare it with the conventional One Man One Vote (henceforth 1M1V) system (with rebate) using an electoral size of more than one hundred voters. We conducted our experiment via e-mail. It was open to all the students community at the Universidad de los Andes. We received 510 applications to participate and randomly selected 210 of the interested subjects. We then divided the participants into two groups of 105 subjects with a minority consisting on less than 40% the electorate. Each voting procedure, QV and 1M1V, was randomly assigned to one group.

Subjects bid among two different policies that would define the number of tickets that each participant would receive for the raffle of the announced prize. If their preferred policy was elected subjects received an additional number of tickets equal to their valuation, whereas if the other policy was elected subjects lose that same amount of tickets. The distribution of exogenous valuations was identical across groups. This distribution was asymmetric, endowing subjects in the minority with more intense preferences than those in the majority (*i.e.* their number of tickets is affected more heavily by the elected policy). As a consequence the outcome would be inefficient if the majority's preferred policy is elected. There was one raffle per group with an identical prize for each of them. The only difference between treatments is that subjects in the QV group are allowed to spend any proportion of their endowment to purchase votes, whereas the subjects in the 1M1V can spend at most one of their tickets to purchase votes.

Costly 1M1V has been previously studied by Ledyard (1984). He considers a model in which voters have to choose between two candidates, the distribution of voting costs is independent from the distribution of valuation for the two candidates, and the support of the cost distribution is strictly greater than zero. Under this setting there are some conditions under which 1M1V

[4] An alternative version in which the QV mechanism with rounded rebate remains budget balanced implies that subjects receive a fixed rebate equal to the largest previous integer, plus a chance equal to the decimal fraction of the rebate to receive an additional unit as part of the reimborse. Although it seems theoretically feasible it is necessary to explore its feasibility and if it is properly understood by voters.

with cost replicates QV. In particular, if the density function of the cost random variable is uniform then the average cost is linear in the number votes. Therefore the total cost of voting is quadratic in the number of votes since it is the product between the average cost and the number of votes. However, in our experimental design the 1M1V treatment considers a cost distribution which is just a strictly positive constant, given the expenditure contraint from voters. Therefore we do not expect that 1M1V replicates QV in our setting.

Despite the strategy set being wider under QV, in our particular setting the set of symmetric equilibria in pure strategies under QV is a subset of the equilibria under 1M1V. This is a direct consequence of rounding of the rebate to the closest integer (in absence of the rounding condition we do not have an equilibrium in pure strategies under the QV mechanism) and having a discrete distribution of types. We find that voters in the QV mechanism deviate upwards from the predicted (low) expenditure levels. The number of purchased votes is positively correlated with the intensity of their preferences. This is true for all but the most extreme types in the minority, who are actually characterized by an expenditure level below the most moderate minority members. Under the 1M1V mechanism, the likelihood to vote in the majority is monotonically increasing with the intensity of the preferences, whereas in the minority almost all the subjects with moderate to high intensity of preferences vote.

The rest of the paper is organized as follows. In Section 2 we describe the theoretical model of the QV mechanism. In Section 3, we present the experimental design and a description of the procedure. We report our results in Section 4, followed by a discussion of their implications in Section 5. We conclude in Section 6.

2 The Quadratic Voting model

A voting procedure is set to determine the outcome between two alternatives A and B. Each one of the N voters has a valuation ν_i^A for alternative A to be implemented, and a valuation ν_i^B for alternative B. Following Goeree and Zhang (2013) and Weyl (2013) we will assume that $\nu_i^A = -\nu_i^B$ or simply ν_i, *i.e.*, the implementation of the least preferred outcome generates a negative disutility of the same magnitude than the preferred policy (full polarization).

Under QV subjects are offered the possibility to make a bid for their preferred alternative knowing that the option with more aggregate bids will be chosen. Each subject receives an endowment e, and they are allowed to make a bid b_i that cost them $C(b_i) = \alpha b_i^2$. For clarity purposes, and to match the models in Goeree and Zhang (2013) and Weyl (2013), we assume that positive bids (and valuations) correspond to policy A and negative bids (and valuations) correspond to policy B. Each voter receives a rebate R_i equal to the absolute average expenditure of the remaining $N-1$ participants. This rebate guarantees that the mechanism is budget balanced. For a voter i with a valuation ν_i that bids b_i, his expected payoff is given by

$$\pi_i(\nu_i, b_i) = e + G(b_i)\nu_i - (1 - G(b_i))\nu_i - C(b_i) + R_i$$

Where $G(b_i)$ is the expected probability that the aggregate bids are positive, that can be written as $Prob(b_i + \sum_{j\neq i} b_j > 0)$, or simply the probability that alternative A is chosen. The rebate received by player i is independent of his own bid and can be written as $\sum_{j\neq i} \alpha b_j^2/(N-1)$. By substituting these expressions in the payoff function we have:

$$\pi_i(\nu_i, b_i) = e + \nu_i \left(2\, Prob \left(b_i + \sum_{j\neq i} b_j > 0 \right) - 1 \right) - \alpha b_i^2 + \frac{\alpha \sum_{j\neq i} b_j^2}{N-1} \tag{1}$$

In our experiment we have an endowment of 10 tickets ($e = 10$) and the following distribution for ν_i:

$$\nu_i = \begin{cases} \{-8, -6, -4, -2\} & \text{with } p = 2/21 \\ \{0\} & \text{with } p = 1/21 \\ \{+1, +2, +3, +4\} & \text{with } p = 3/21 \end{cases}$$

We have a total of $N = 105$ voters, sixty with a positive ν_i preferring policy A (fifteen for each one of the types), and another forty with a negative ν_i (ten for each of the types) preferring policy B. In this assymetric distribution the mean value of ν_i is equal to -10/21 whereas its median value is 1. Having opposite signs for the mean (reflecting intensity of preferences) and the median (reflecting the expected elected outcome by a majority rule) makes the conventional 1M1V inefficient.

The parameter α in the cost function is set at 1/9. It does not alter the set of equilibria in our game with respect to setting α equal to a unity. By rescaling the mapping between expenditure and purchased votes we make less familiar our 1M1V treatment to the conventional majority rule system, reducing the pre-existent differences in the interpretation of instructions between our two treatments.

There are two substantial differences in our experimental setting with respect to Weyl's model: subjects in our experiment are choosing the expenditure level $C_i(b_i) = b_i^2/9$ rather than their bid b_i, and the average expenditure of the other participants is rounded before they are reimboursed. Taking into account these restrictions equation (1) can be written as:

$$\pi_i(\nu_i, C_i) = e + \nu_i \left(2\, Prob \left((\alpha C_i)^{1/2} + \sum_{j\neq i} (\alpha C_j)^{1/2} > 0 \right) - 1 \right) - C_i + \left|\left| \frac{\sum_{j\neq i} C_i}{N-1} \right|\right| \tag{2}$$

Where $||x||$ represents the nearest integer of x.

We compute numerically the set of equilibria in pure strategies for this game. We find multiple symmetric equilibria for the QV treatment, in which subjects decide the expenditure

$C_i \in \{-10,\ldots,10\}$. These sets of equilibria under QV are a subset of the equilibria for the 1M1V treatment, in which the subjects' expenditure is limited to $C_i \in \{-1,0,1\}$. The equilibria under QV and 1M1V, fully reported in Table 1, are characterized by an expenditure $C_i = -1$ for all members from one of the four types in the minority ($\nu_i \in \{-8,-6,-4,-2\}$), and an expenditure $C_i = 1$ for all members from three of the four types in the majority ($\nu_i \in \{1,2,3,4\}$), whereas all the other types do not spend any ticket from their endowment $C_i = 0$. The different combinations of these strategies give us a total of 16 different equilibria. In addition, the action profile in which all the subjects in the minority play $C_i = 0$ and all the subjects in the majority play $C_i = 1$ also yields an equilibrium. The sets of equilibria under QV and 1M1V are independent of the value of α, but they are affected by the rounding function defining the rebate term. In fact, there is no equilibria in pure strategies under QV when the rebate is not rounded, whereas for the 1M1V we still find a set of equilibria even in this case (see the action profiles at the bottom of Table 1).

Table 1: Set of equilibria for Quadratic Voting (QV) and One Man One Vote (1M1V). Numbers in bold correspond to the exogenous valuation ν_i. Each row corresponds to a strategy set characterized by the expenditures C_i from all the types.

Equilibria under QV and 1M1V with Rounded Rebate									
$\boldsymbol{\nu_i}$	**-8**	**-6**	**-4**	**-2**	**0**	**1**	**2**	**3**	**4**
$C = \prod C_i$	-1	0	0	0	0	0	1	1	1
$C = \prod C_i$	-1	0	0	0	0	1	0	1	1
$C = \prod C_i$	-1	0	0	0	0	1	1	0	1
$C = \prod C_i$	-1	0	0	0	0	1	1	1	0
$C = \prod C_i$	0	-1	0	0	0	0	1	1	1
$C = \prod C_i$	0	-1	0	0	0	1	0	1	1
$C = \prod C_i$	0	-1	0	0	0	1	1	0	1
$C = \prod C_i$	0	-1	0	0	0	1	1	1	0
$C = \prod C_i$	0	0	-1	0	0	0	1	1	1
$C = \prod C_i$	0	0	-1	0	0	1	0	1	1
$C = \prod C_i$	0	0	-1	0	0	1	1	0	1
$C = \prod C_i$	0	0	-1	0	0	1	1	1	0
$C = \prod C_i$	0	0	0	-1	0	0	1	1	1
$C = \prod C_i$	0	0	0	-1	0	1	0	1	1
$C = \prod C_i$	0	0	0	-1	0	1	1	0	1
$C = \prod C_i$	0	0	0	-1	0	1	1	1	0
$C = \prod C_i$	0	0	0	0	0	1	1	1	1
Equilibria under 1M1V with Rounded Rebate									
$\boldsymbol{\nu_i}$	**-8**	**-6**	**-4**	**-2**	**0**	**1**	**2**	**3**	**4**
$C = \prod C_i$	-1	-1	-1	-1	0	0	0	0	1
$C = \prod C_i$	-1	-1	-1	-1	0	0	0	1	0
$C = \prod C_i$	-1	-1	-1	-1	0	0	1	0	0
$C = \prod C_i$	-1	-1	-1	-1	0	1	0	0	0
Equilibria under 1M1V with and without Rounded Rebate									
$\boldsymbol{\nu_i}$	**-8**	**-6**	**-4**	**-2**	**0**	**1**	**2**	**3**	**4**
$C = \prod C_i$	0	0	0	0	0	0	0	0	1
$C = \prod C_i$	0	0	0	0	0	0	0	1	0
$C = \prod C_i$	0	0	0	0	0	0	1	0	0
$C = \prod C_i$	0	0	0	0	0	1	0	0	0

Intuitively, rounding the rebate gives origin to a set of equilibria in which subjects aim to reach an expenditure slightly higher than one half, guaranteeing an additional ticket as rebate for all the electorate and creating at the same time an indifference between spending or not the ticket. For all the strategy sets that are equilibria under QV and 1M1V the selected policy preferred by the majority is always elected. Therefore, the efficiency property of Quadratic Voting is lost if the currency used in the mechanism forces to round the reimbursement to the closest integer. Moreover, there is another subset of equilibria in the 1M1V with rounded rebate that are not equilibria under QV in which the elected outcome is the one preferred by the minority, in which efficiency is reached.

3 Experimental Design

3.1 Participants

The invitation to participate in the study was sent through students' mailing lists and it was also posted on the official Facebook page of the Economics Department from Universidad de los Andes. This call was open to all the university's students and staff members. We received a total of 511 submissions to participate in the experiment: in 73.4% of the cases they heard about the experiment through mail invitations, 15.5% of them through Facebook, 7.6% were invited to participate by other friends, and the remaining 3.5% reported at least two of the previous sources.

The 210 participants were randomly selected using the following procedure: the candidates were divided into six panels of 85 subjects according to the time of submission of the form. The panels were numbered from 1 to 6. In Panel 1 were located the subjects that complete their registration earlier, whereas in Panel 6 were located the subjects that completed it last. Sixty subjects from Panel 1 were randomly selected to participate, fifty from Panel 2, forty from Panel 3, and so on. Early registrations were given more chances to participate under the assumption that those that registered first may have a larger valuation of the signed jerseys raffled as a reward for participation.

Fifty seven percent of participants were men. Although the call was open to all fields of study the sample was predominantly composed of economists with 72% of the sample. Subjects were asked about their valuation of the prize using the Becker-Degroot-Marschak (BDM) mechanism (Becker et al., 1964) as a hypothetical question. The average valuation of the autographed jersey was 370,000 \$cop[5]. This amount corresponds to 226 percent the value of the jersey in stores (without the autograph).

As the experiment was conducted some days before the beginning of the 2014 Soccer World Cup, subjects were asked about their beliefs of the maximum stage that the National Team

[5] This value was calculated over 492 subjects who answered the question using a numerical value, the others responded that the signed jersey goes beyond a monetary value.

will reach in the tournament.[6] The revealed monetary valuation of the jersey and the expected beliefs about the National Team's performance in the World Cup are signs of the attractiveness of the experimental reward, although we did not find a significant correlation between these two variables.

3.2 Design

Each participant was endowed with 10 tickets to participate in the raffle of a prize commonly known to all subjects, plus a random number $\nu_i \in \{-8, \ldots, 4\}$ representing the exogenous valuation for two policies. Participants had to choose between policy A and policy B. Under policy A we add ν_i to the number of tickets each individual has. Under policy B we subtract ν_i to the number of tickets each individual has. As the implementation of the least preferred policy reduces the payoff in $-\nu_i$, we have in this experiment full polarization. Subjects with $\nu_i > 0$ should prefer Policy A, whereas for subjects with $\nu_i < 0$ the election of Policy B is in their best material interest. The distribution of valuations was exogenous. In each group 60 subjects have $\nu_i > 0$ and 40 subjects have $\nu_i < 0$. A majoritarian election rule will favor the group with $\nu_i > 0$ and Policy A will be implemented. However, given that the mean value of the distribution of ν_i is $-10/21$ the efficient outcome is Policy B. With this experiment we aim to explore if subjects express the intensity of their preferences under QV by purchasing a number of votes positively correlated with their valuation ν_i.

The 210 participants were randomly divided into two groups of 105 subjects each. The prize raffled in each group was identical, as well as the distribution of ν and the fact that voting was costly. We apply a different treatment for each group. In the *Quadratic Voting* treatment subjects are allowed to spend any fraction of their endowment ($C_i(b_i) \in \{-10, \ldots, 10\}$) to be converted in points in favor of their preferred policy at a rate of three times the square root of their expenditure, which is equivalent to purchase votes at a quadratic cost given the function $C(b_i) = b_i^2/9$. In the *One Man One Vote* treatment subjects can spend at most one of the ten units from their endowment ($C_i(b_i) \in \{-1, 0, 1\}$) to add 3 points in favor of their preferred policy.

In order to contribute to the discussion of the implementability of the QV mechanism we restraint our design in two aspects. First, we transform the space of discrete strategies offered to players from purchased votes to expenditure, and we introduce a rounding function for the reimbursement in order to conduct the experiment with a non-monetary currency. Second, we limit the number of decisions to a single round. Under this constraint the voting procedure is closer to a scenario with unexperienced voters with limited capacity to infer their optimal strategy given the distribution of valuations and their realization ν_i, a more challenging but

[6]Forty four percent of the sample believed that Colombia was going to advance to the round of 16, another 44% to the quarter-finals, 5.68 % to the semi-final, 0.78% to the final, and 2.15% believed that Colombia was going to be world champion. Only 3% thought that Colombia was going to be knocked out in the first round.

realistic benchmark to analyze implementability. We also introduce a set of four questions prior to the voters' decision to validate if participants have understood how the mechanism works. The four items, shown in Appendix B, were designed to differ the less possible across the two treatments.

One drawback from this experiment is that we cannot test the efficiency of the QV mechanism as in Goeree and Zhang (2013). Given that we maximize the electorate size and minimize the number of interactions, we only dispose of two observations to infer the outcome's efficiency, one per treatment. Therefore, our conclusions will be limited to the observed individual behavior.

3.3 Procedure

The invitation to participate in the experiment was sent by electronic mail and posted online on June 11th, 2014, the day before the kick-off of the Soccer World Cup. The form was open for 36 hours. We received 511 submissions to participate. The randomly selected 210 participants were informed that they reached the "final stage" of the contest via e-mail on June 14th. The link to the experimental form was embedded into this message and they were told that the deadline to complete the form of the "final stage" was June 17th.

The welcome page of the experimental form included a description of the distribution of ν_i, their initial endowment, and requested some data for identification purposes (name, institutional electronic address, and student ID if available). After signing the informed consent participants received their random number, they were informed on the implications of policies A and B on their final number of tickets, and they were shown a table indicating how many votes will be bought as a function of the spent tickets. Subjects then proceed to a small test with four questions used to evaluate if they understood the voting mechanism. After responding these questions subjects were allowed to reveal their preferred policy and decide on the number of tickets they want to spend on it.

Once the form was closed we computed the selected policy for each treatment. We also calculate the final number of tickets per subject as a function of the winner policy, their expenditure and their rebate. Participants were informed of the outcome of the voting procedure and their number of tickets for the raffle. They were also invited to the draw of the winning tickets, which was publicly done in the Economics Department at Universidad de los Andes.

4 Results

Policy A, the one favored by the majority, was elected in both treatments. In the QV treatment this policy accumulated 189.28 points through the expenditure of 104 tickets, whereas Policy B reached 167.64 points with an aggregate expentidure of 99 tickets. With 51.2% of the expenditure Policy A collected 53.1% of the purchased votes, reaching the majority by a closed margin. In the 1M1V treatment the Policy A accumulated 123 points, 27 more points than Policy B. As participants could spend at most one of their tickets the mapping from purchased votes to expenditure is linear. The 41 tickets in favor of Policy A represent 56.2% of the expenditure and the total votes, a larger margin for the elected policy than under QV. Turnout rates were very high in both treatments, reaching 96.2% and 95.2% under 1M1V and QV, respectively.

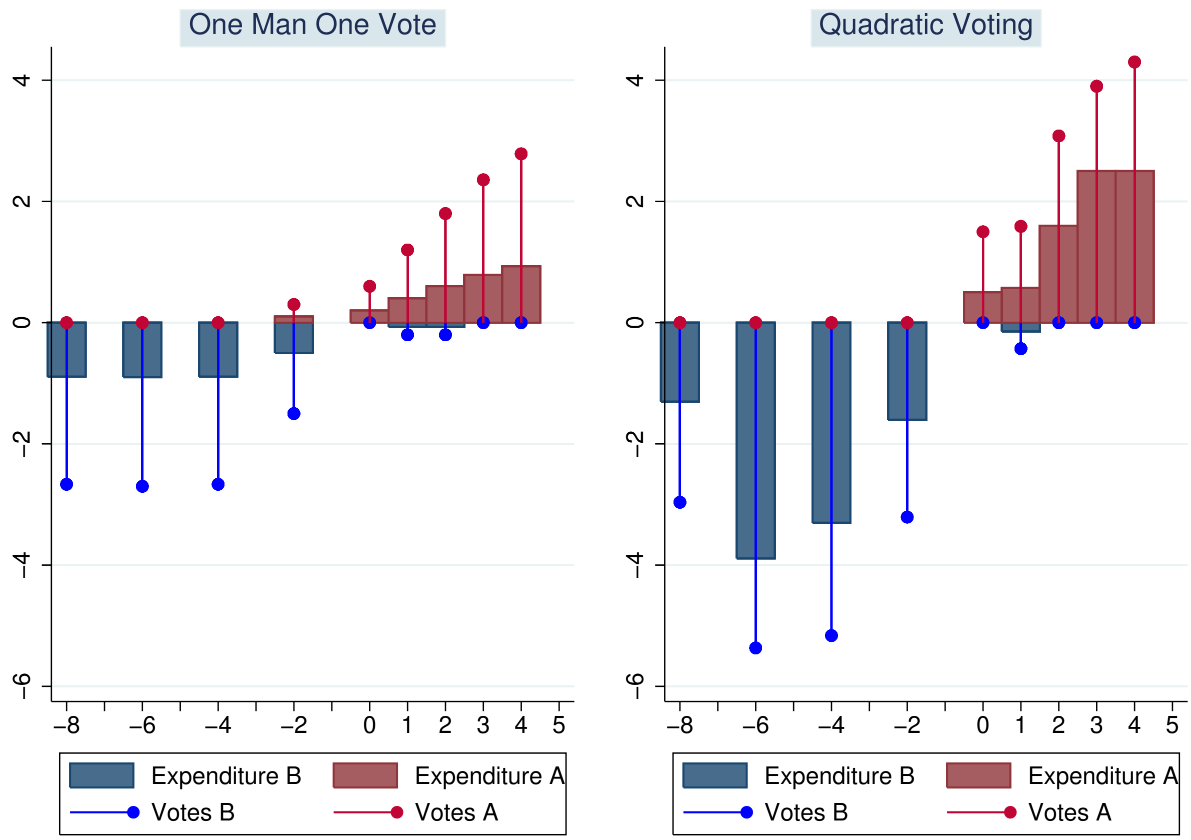
The average purchased votes and expenditure for each one of the nine ν_i types in our setting are shown in Figure 1. On the left panel is shown that in the 1M1V treatment the voters with $\nu_i < -2$ are very likely to spend one of their tickets voting for Policy B. In fact there is only one participant for each type $\nu_i \in \{-8, -6, -4\}$ that completed the online questionnaire and did not spend any of his tickets in the voting mechanism. For minority members with less intense preferences, those with $\nu_i = -2$, only half of them vote for Policy B. In addition, one of these participants voted for Policy A. Within the majoritarian group we observe a monotonic increase in the share of voters with the value of ν_i, ranging from 0.40 when $\nu_i = 1$, to 0.93 (all but one of the voters) when $\nu_i = 4$. We observe a slight support for Policy A from voters with $\nu_i = 0$, which may correspond to a bandwagon effect (Simon, 1954; Fleitas, 1971; Tyran, 2004) or simply noisy behavior.

Behavior in the Quadratic Voting mechanism is shown on the right panel of the same figure. For subjects in the minority we observe that expenditure for Policy B increases monotonically with the intensity of their preferences for $\nu_i \in \{-6, -4, -2\}$, although this is not the case for the most extreme voters ($\nu_i = -8$) who indeed spent on average less than the more moderate minority members ($\nu_i = -2$). For the majority members the expenditure is quasi-monotonic, as participants with $\nu_i = \{3, 4\}$ spend on average 2.5 tickets. Nonetheless, the average purchased votes is monotonic when $\nu_i > 0$. It means that there is more dispersion of expenditures for voters with $\nu_i = 3$, and those with larger expenditures are contributing with less votes per spent ticket than the other subjects with the same valuation. We again observe a tendency of voters with $\nu_i = 0$ to spend tickets in favor of the policy preferred by the majority. In this case, two out of four subjects voted for Policy A.

We also report the results on the validation test took after reading the experimental instructions and right before reaching the voting decision. As is shown in Table 2 the percentage of correct responses to each question was between 83% and 94%, and the average score (assigning 1 point per correct response) was 3.50. Wrong answers were not highly concentrated on the same subjects. Instead, we observe that only 11% of the participants responded two or more questions incorrectly.

The validation test was contextually similar across treatments. The difference was that in the example proposed under QV one of the hypothetical subjects spent more than one ticket, altering the required calculations. Despite this additional complexity we only find a statistically significant difference in the rate of response for question 2, which asks for the rebate received by

Figure 1: Average purchased votes and expenditure for One Man One Vote and Quadratic Voting. Bars correspond to average expenditure for policies A (in red) and B (in blue). Lines with dots correspond to average points contributed to the voting procedure for each valuation. Purchased votes exceed expenditure given the bidding cost $C(b_i) = b_i^2/9$.



one of the participants. We also find that responding incorrectly to these questions is negatively correlated with larger expenditures under QV (although it is not statistically significant). In addition, by exploring the differences in the test score across types we find that subjects in the minority get higher scores than subjects in the majority. This is true separately for QV and 1M1V, but the difference only becomes statistically significant when both treatments are pooled (t test with p-value 0.064).

We test the effects of the intensity of preferences and belonging to the majority (or the minority) using the following OLS model:

$$||C_i|| = \beta_0 + \beta_1||\nu_i|| + \beta_2 I(\nu_i > 0) + \beta_3||\nu_i|| \times I(\nu_i > 0) + \boldsymbol{\gamma}\boldsymbol{X_i} + u_i \tag{3}$$

In our specification we use the absolute values of the expenditure C_i and the valuation ν_i in order to capture the effect of the intensity of preferences on voting behavior in a single dimension, and therefore be able to introduce a set of covariates $\boldsymbol{X_i}$ that respond unequivocally

Table 2: Correct responses in the validation test across treatments. The Chi-squared statistical test is used to perform the statistical comparison for all but the variable indicating the average score in the test (adding 1 point per correct response).

	One Man One Vote	Quadratic Voting	p-value
Q1: Points accumulated by each policy	86.1%	92.0%	0.183
Q2: Rebate for one of the participants	94.1%	86.0%	0.056*
Q3: Total payoff for one majority member	87.1%	83.0%	0.411
Q4: Total payoff for one minority member	82.2%	89.0%	0.169
Q1 to Q4 correctly solved	68.3%	63.0%	0.427
Average score (0-4) from Q1 to Q4	3.49	3.50	0.966

*** p<0.01, ** p<0.05, * p<0.1

to the intensity of ν_i regardless if it is positive or negative. The constant term β_0 represents the expenditure for the subjects with the more moderate preferences in the minority, β_1 represents the effect of the intensity of the preferences in the minority; β_2, the parameter corresponding to the indicator function for positive valuations $I(\nu_i > 0)$, represents the expenditure for the subjects with the more moderate preferences in the minority. Finally, β_3, the parameter corresponding to the interaction between $||\nu_i||$ and $I(\nu_i > 0)$ measures the effect of the intensity of preferences in the majority.

The estimation results are shown in Table 3. Columns (1) to (3) correspond to the 1M1V treatment, whereas columns (4) to (6) correspond to the QV treatment. For the first three columns the OLS must be interpreted as a linear probability model, as subjects' expenditure was limited to one ticket in the 1M1V treatment. We find that for the most moderate minority members the likelihood of voting (β_0) is statistically greater than zero at the 1% significance level. Although majority members with moderate preferences are less likely to vote ($\beta_2 < 0$) this difference is not statistically significant. For the minority, the increasing likelihood to vote with the intensity of preferences is not statistically significant. This might be explained by the fact that almost subjects with $\nu_i < -2$ were voting regardless of their valuation. For the majority, on the other hand, the likelihood of voting increases between 10.5 and 13.9 percentage points for each additional unit in ν_i, and it is statistically significant.

In the QV treatment the OLS coefficients can be directly interpreted as an increase in expenditure. According to column (4) subjects in the minority spend on average 2.6 tickets, although we do not observe an effect of the intensity of their preferences. The reason is that the minority members with the most extreme preferences ($\nu_i = -8$) show very low expenditure levels. Excluding these subjects from the sample we find that β_1 becomes positive and significant whereas β_0 loses its significance (see Table A.3 in the Appendix). On the other hand, majority members with moderate preferences spend less than their equivalent type in the minority, and their expenditure increases between 0.65 and 0.72 tickets for each additional unit in their valuation ν_i.

The estimated coefficients reported in columns (2) and (5) show a negative (although not statistically significant) correlation between expenditure and the elicited valuation of the prize,

Table 3: OLS Regression. Dependent variable is the absolute value of expenditure (or cost incurred).

Dependent variable: $\|\|C_i\|\|$	One Man One Vote			Quadratic Voting		
	(1)	(2)	(3)	(4)	(5)	(6)
$\|\|\nu_i\|\|$	0.0455	0.0455	0.0433	-0.0227	-0.0365	-0.0644
	(0.0300)	(0.0295)	(0.0283)	(0.163)	(0.165)	(0.172)
$I(\nu_i > 0)$	-0.256	-0.323	0.226	-2.333**	-2.443**	-1.253
	(0.209)	(0.210)	(0.397)	(1.163)	(1.166)	(3.195)
$\|\|\nu_i\|\| \times I(\nu_i > 0)$	0.105*	0.117**	0.139**	0.648**	0.645**	0.719**
	(0.0570)	(0.0570)	(0.0553)	(0.319)	(0.322)	(0.331)
Prize Valuation (std.)		-0.0376	-0.0897**		-0.141	0.300
		(0.0415)	(0.0438)		(0.245)	(0.406)
$I(\nu_i > 0)\times$ Prize Valuation (std.)			0.271**			-0.762
			(0.107)			(0.514)
Validation score		-0.0623	0.0637		-0.502	-0.298
		(0.0488)	(0.0817)		(0.323)	(0.666)
$I(\nu_i > 0)\times$ Validation Score			-0.167*			-0.411
			(0.100)			(0.766)
Constant	0.591***	0.843***	0.397	2.600***	4.491***	3.903
	(0.163)	(0.243)	(0.341)	(0.891)	(1.538)	(2.787)
Observations	96	93	93	96	95	95
R-squared	0.128	0.167	0.251	0.072	0.099	0.125

Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

and between expenditure and performance in the validation test. An interpretation of these results is that subjects with a higher valuation of the prize or with a better understanding of the mechanism are more likely to engage in strategic voting. We also find that for the 1M1V treatment these two variables have different effects in the majority and the minority. According to column (3), once the prize valuation is interacted with the indicator variable $I(\nu_i > 0)$ the negative effect of this variable becomes significant but only for minority members, whereas for subjects in the majority a higher valuation of the prize leads to a higher likelihood to vote.

5 Discussion

We adapted the original QV mechanism to explore if an alternative framing using a different currency might be useful dealing with the practical difficulties of its implementation. Instead of an action set in which voters bid a value b_i at a cost αb_i^2 for their preferred choice, we framed the decision as an expenditure C_i that adds $(C_i/\alpha)^{1/2}$ points to this choice. Theoretically both problems are equivalent, but in practice deciding in the domain of expenditures rather than in the domain of bids has two potential advantages: subjects might be more aware of their budget constraints and, more importantly, it facilitates the use of alternative currencies within the mechanism. However, a potential cost of choosing C_i is that the predicted linear relationship between ν_i and b_i will be lost. What the right panel in Figure 1 suggests (excluding subjects with $\nu_i = -8$) is that the expenditure is concave in the (absolute) intensity of the preferences.

As a consequence, the indirect bid b_i is at most linearly proportional to ν_i, although the dots in the same panel suggests also a concave relationship. The importance of this result is that the positive decreasing relationship between preference's intensity and expenditure is preserved when subjects are directly asked about C_i. If, for instance, we would have found a linear relationship it would have meant that subjects with higher valuations were not sensitive enough to the quadratic cost, and therefore that they were trying to have more weight in the election than predicted, a potential problem of the mechanism.

One of the advantages aforementioned, the use of alternative currencies, may be accompanied with an additional feature when the chosen payment method is discrete: the need to round the rebate. In our case this condition gives rise to a set of equilibria under QV that, instead of predicting a positive relationship between expenditure and the intensity of preferences, captures different coordination opportunities to produce a rebate of 1 unit through an average expenditure slightly larger than 0.5. If these equilibria are followed in the experiment it might be problematic for the arguments in favor of QV, since it inhibits the expression of the intensity of preferences and leads to inefficient outcomes.

The predicted equilibria under QV is a subset of the equilibria under 1M1V. This fact is useful to compare the two treatments regarding overexpenditure levels. Although in each treatments the total expenditure should not exceed 55 units, we find that under 1M1V it reached 73 units (1.32 times the expected expenditure) whereas for QV the total expenditure was 203 units (3.69 times the expected expenditure). The aggregate expenditures above the predictions suggest that subjects are not trying to coordinate on these "low spending" equilibria as a way to earn (in probability) an additional unit through the rounded rebate. This lack of coordination is expected given the complexity of the equilibrium, particularly in presence of more than one hundred subjects per group. Unfortunately, we cannot say too much about the behavior in 1M1V with respect to the predictions given the multiplicity of equilibria.

The behavior from two specific player types should be discussed in more depth. First, the low expenditure from the subjects with $\nu_i = -8$ in the QV treatment. On average these subjects spent 1.3 tickets, but the median and the mode was one ticket. A potential explanation for this behavior is that the proximity between their valuation and their endowment lead them to think about their potentially null or negative payoff in case that the undesired outcome were elected. If a disproportional weight was given to this scenario, a manifestation of loss aversion (Kahneman and Tversky, 1984), subjects may vote but in a way that they can still have a positive payoff in the worst case scenario. It suggests that extreme voters are not necessarily maximizing their expected payoff, but rather avoiding very costly outcomes while expressing their preferences the best they can. In practical terms, it would be more likely to be problematic when polarization is at its maximum level (*e.g.* $\nu_i^A = -\nu_i^B$), which tends to be part of the model (and in our case also part of the experiment) but is not necessarily true in most of the collective decisions.

Second, subjects with $\nu_i = 0$, who are not expected to spend none of their endowment in any of the choices as they do not get any material benefit from a particular outcome, are likely to support the majority on both treatments. In the previous section we hypothesize that it would be a signal of a bandwagon effect under 1M1V. If this is true, having found the same pattern in the QV treatment suggests that subjects whose payoff is orthogonal to the elected outcome expect the majority to win using the QV mechanism. Another possibility is that the framing of "adding tickets" is interpreted as positive whereas "subtracting tickets" is interpreted as negative. In any case, more evidence is needed given the small number of subjects of this type in our sample.

6 Conclusion

In this paper we present the results of an alternative implementation of the QV mechanism in which subjects decide on their expenditure for their preferred policy knowing that it will add a number of votes equal to the square root of what they spent. In our effort to test the mechanism with an alternative payment method we we make the participants' payoffs discrete, which implied that the rebate was rounded for all the participants. Given that we use a discrete distribution for the valuation of the policies it created a distortion with respect to the equilibrium described in Weyl (2013): instead of subjects bidding an amount proportional to their valuation (or equivalently spending an amount proportional to the square root of this valuation), the predicted equilibria consisted on low spending levels that granted an additional unit after rounding a rebate slightly higher to one half. We show that subjects do not behave according to these predictions, partly because of the difficulties to coordinate on groups of more than one hundred voters. Instead, voters tend to spend a share of their endowment that is positively correlated with their valuation. The marginally decreasing relationship between valuation an expenditure was unaltered by our indirect bidding procedure, suggesting that voters were sensitive to the quadratic cost. These two patterns of behavior can be interpreted as manifestations of the intensity of voters' preferences, a required condition for the establishment of more efficient voting procedures.

We provide positive evidence for the applicability of the QV mechanism using an alternative currency and an indirect bidding function without losing the expression of preferences' intensity. In terms of practical difficulties for implementation, future work should evaluate the importance of revealing the entire distribution of valuations with respect to more abstract informational sets on bidding behavior.

References

Becker, G. M., DeGroot, M. H., and Marschak, J. (1964). Measuring utility by a single-response sequential method. *Behavioral Science*, 9(3):226–232.

Casella, A. (2005). Storable votes. *Games and Economic Behavior*, 51(2):391–419.

Fleitas, D. W. (1971). Bandwagon and underdog effects in minimal-information elections. *The American Political Science Review*, pages 434–438.

Goeree, J. K. and Zhang, J. (2013). Electoral engineering: One man, one bid. *Working paper.*

Hylland, A. and Zeckhauser, R. (1979). A mechanism for selecting public goods when preferences must be elicited. *Kennedy School of Government Discussion Paper*, 70.

Kahneman, D. and Tversky, A. (1984). Choices, values and frames. *American Psychologist*, 39 (4):341–350.

Ledyard, J. (1984). The pure theory of large two-candidate elections. *Public Choice*, 52 (1):15–33.

Morton, R. and Williams, K. (2010). *Experimental political science and the study of causality: from nature to the lab.* Cambridge University Press.

Palfrey, T. (2013). Experiments in political economy. *Handbook of Experimental Economics Vol. 2.*

Posner, E. and Weyl, G. (2014). Voting squared: Quadratic voting in democratic politics. *Coase-Sandor Institute for Law and Economics Working Paper No. 657.*

Simon, H. A. (1954). Bandwagon and underdog effects and the possibility of election predictions. *Public Opinion Quarterly*, 18(3):245–253.

Tyran, J.-R. (2004). Voting when money and morals conflict: an experimental test of expressive voting. *Journal of Public Economics*, 88(7):1645–1664.

Weyl, G. (2013). Quadratic vote buying. *Working paper.*

Appendix

A Additional Figures and Tables

Table A.1: Exchange rates for each payment media proposed in the classroom experiment discussed in the Introduction.

Treatment	Payment media	Exchange rate
Cash	Money (one subject randomly chosen)	1 bidding point = \$10,000
Time	Hours to deliver take home exam	1 bidding point = 2 hours to be deducted from a total of 50 hours
Space	Lines to write an essay in the exam	1 bidding point = 5 lines to be deducted from a total of 60 lines
Bonus	Points from a bonus in the exam	1 bidding point = 0.1 points to be deducted from a total bonus of 0.5. Maximum score in the exam is 5.0.

Table A.2: Check balance across treatments and types. Standard errors are shown in parentheses. In the last column is reported the p-value for the t tests comparing each variable.

	Balance across treatments				
	One Man One vote		**Quadratic Voting**		p-value
No. of submission	265.6	(13.5)	264.9	(14.0)	0.969
Gender (Men = 1)	0.574	(0.049)	0.560	(0.049)	0.839
Valuation of the prize	369.2	(27.11)	344.2	(18.32)	0.445
Economics' student	0.752	(0.043)	0.68	(0.047)	0.257
Score Verification Test	3.49	(0.087)	3.50	(0.076)	0.966
	Balance across types				
	Minority ($\nu_i < 0$)		**Majority** ($\nu_i > 0$)		p-value
No. of submission	262.0	(15.9)	270.9	(12.7)	0.664
Gender (Men = 1)	0.597	(0.056)	0.539	(0.047)	0.428
Valuation of the prize	360.4	(34.85)	349.1	(15.81)	0.743
Economics' student	0.714	(0.051)	0.696	(0.043)	0.783
Score Verification Test	3.65	(0.078)	3.42	(0.083)	0.064*

*** p<0.01, ** p<0.05, * p<0.1

Figure A.1: Purchased votes for each policy across payment media. Subjects that declare a preference for one of the two exam dates but did not spend part of their endowment are included in the average for the subset of subjects with the same preference.

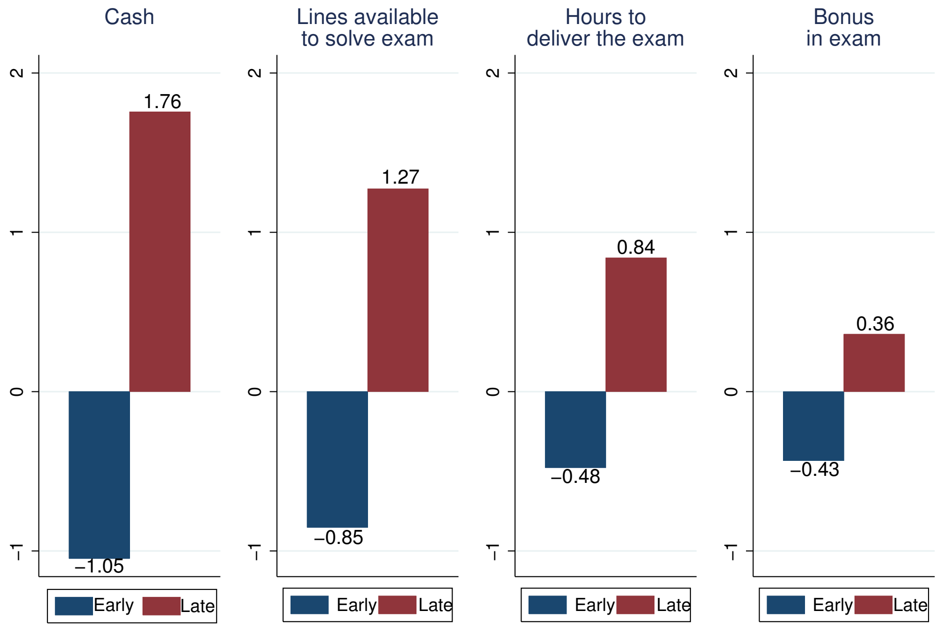


Table A.3: OLS Regression for QV excluding subjects with $\nu_i = -8$.

Dependent variable: $\|\|C_i\|\|$	Quadratic Voting		
	(1)	(2)	(3)
$\|\|\nu_i\|\|$	0.577**	0.537**	0.557**
	(0.264)	(0.268)	(0.280)
$I(\nu_i > 0)$	-0.360	-0.555	2.342
	(1.347)	(1.361)	(3.635)
$\|\|\nu_i\|\| \times I(\nu_i > 0)$	0.0479	0.0773	0.0972
	(0.380)	(0.384)	(0.397)
Prize Valuation (std.)		-0.138	0.432
		(0.261)	(0.464)
$I(\nu_i > 0)\times$ Prize Valuation (std.)			-0.896
			(0.562)
Validation score		-0.447	0.117
		(0.335)	(0.745)
$I(\nu_i > 0)\times$ Validation Score			-0.827
			(0.836)
Constant	0.627	2.406	0.321
	(1.120)	(1.770)	(3.285)
Observations	86	85	85
R-squared	0.148	0.168	0.205

Standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

B Validation Questions

B.1 Quadratic Voting

Suppose there is a group with 5 participants M, N, O, P and Q. All of them initially received 10 tickets for the raffle. M, N, and O received as SECRET NUMBER: +2. P and Q received as SECRET NUMBER: -4.

The expenditure of each participant was:

- M spent 1 of his tickets to buy points for RULE A
- N spent 1 of his tickets to buy points for RULE A
- O spent 1 of his tickets to buy points for RULE A
- P spent 3 of his tickets to buy points for RULE B
- Q did not spend any of his tickets to buy points for RULE B

Q1: How many points will accumulate each rule?

Remember to look in the table how many points are summed according to the spent tickets

- 9.0 points for RULE A and 5.2 points for RULE B
- 9.0 points for RULE A and 9.0 points for RULE B
- 3.0 points for RULE A and 3.0 points for RULE B

Q2: How many tickets will receive participant Q from the expenditure of the other participants?

Remember that each player receives the average of the tickets spend by the other participants, rounded to the closest integer (X.5 is rounded to the next integer).

- He will receive 2 tickets, after the approximation of 1.5 average tickets
- He will receive 1.5 tickets
- He will receive 4 tickets, after the approximation of 3.5 average tickets

Q3: How many tickets will have participant M for the raffle?

Remember that if RULE A is elected the number of tickets is (10 - Spent Tickets + SECRET NUMBER + Average spent tickets of the others)

- 10-1+2+1 = 12
- 10-3+2+1 = 10
- 10-1+2 = 11

Q4: How many tickets will have participant P for the raffle?

Remember that if RULE A is elected the number of tickets is (10 - Spent Tickets + SECRET NUMBER + Average spent tickets of the others)

- 10-3-4+1 = 4
- 10-3+4+1 = 12
- 10-3+2+1 = 10

B.2 One Man One Vote

Suppose there is a group with 5 participants M, N, O, P and Q. All of them initially received 10 tickets for the raffle. M, N, and O received as SECRET NUMBER: +2. P and Q received as SECRET NUMBER: -4.

The expenditure of each participant was:

- M spent 1 of his tickets to buy points for RULE A
- N spent 1 of his tickets to buy points for RULE A
- O did not spend any of his tickets to buy points for RULE A
- P spent 1 of his tickets to buy points for RULE B
- Q did not spend any of his tickets to buy points for RULE B

Q1: How many points will accumulate each rule?

Remember to look in the table how many points are summed according to the spent tickets

- 9.0 points for RULE A and 6.0 points for RULE B
- 6.0 points for RULE A and 3.0 points for RULE B
- 2.0 points for RULE A and 1.0 points for RULE B

Q2: How many tickets will receive participant Q from the expenditure of the other participants?

Remember that each player receives the average of the tickets spend by the other participants, rounded to the closest integer (X.5 is rounded to the next integer).

- He will receive 1 ticket, after the approximation of 0.75 average tickets
- He will receive 0.75 tickets
- He will receive 0 tickets, after the approximation of 0.75 average tickets

Q3: How many tickets will have participant M for the raffle?

Remember that if RULE A is elected the number of tickets is (10 - Spent Tickets + SECRET NUMBER + Average spent tickets of the others)

- 10-1+2+1 = 12
- 10-3+2+1 = 10
- 10-1+2 = 11

Q4: How many tickets will have participant P for the raffle?

Remember that if RULE A is elected the number of tickets is (10 - Spent Tickets + SECRET NUMBER + Average spent tickets of the others)

- 10-1-4+1 = 6
- 10-1+4+1 = 14
- 10-1-4 = 13