Stochastic Idiosyncratic Operating Risk and Real Options: Implications for Stock Returns

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Abstract

We show that introducing stochastic idiosyncratic operating risk into an equity valuation model of firms with growth options explains two empirical anomalies related to idiosyncratic volatility: the positive contemporaneous relation between stock returns and changes in idiosyncratic return volatility, and the poor performance of stocks with high idiosyncratic volatility. The model further predicts that (i) returns correlate positively with idiosyncratic volatility during intervals between large changes in idiosyncratic volatility (the switch effect), (ii) and that the anomalies and the switch effect are stronger for firms with more real options and which undergo larger changes in idiosyncratic volatility. Empirical results support these predictions.

Keywords: Idiosyncratic return volatility, cross section of stock returns, asset pricing, real options, growth options, stochastic volatility, regime switching, mixed jump-diffusion processes.

1 Introduction

Modern portfolio theory and the capital asset pricing model suggest that investors diversify idiosyncratic risks and only systematic risk is priced in equilibrium. The empirical evidence on idiosyncratic return volatility ($IVol$) and stock returns is not readily explained by this simple intuition. One strand of the literature (Duffee (1995); Grullon, Lyandres, and Zhdanov (2010)) establishes that changes in monthly realized $IVol$ are contemporaneously positively related with stock returns (positive $IVol$-return relation or anomaly hereafter), while a different strand (Ang, Hodrick, Xing, and Zhang (2006)) establishes that portfolios of high end-of-month $IVol$ stocks significantly under-perform their low $IVol$ counterparts (negative $IVol$-return relation or anomaly hereafter).[1] Yet, a third strand of the literature establishes that the negative $IVol$ anomaly is due to strong return reversals among a subset of small firms (Huang, Liu, Rhee, and Zhang (2009); Fu (2009)). Given the lack of consensus, it is not surprising that progress in delivering a unified explanation for these findings has been difficult.[2]

In this paper, we reconcile these seemingly disparate empirical regularities via an equity valuation model with growth options and stochastic idiosyncratic operating risk.[3] Our model also provides novel empirical predictions, which are borne out in the data, thereby providing evidence that our choice of model is reasonable. We start by showing that if a firm's equity returns possess two key properties, then the positive and negative $IVol$ anomalies can be resolved. This is true even if the CAPM holds. The first property is that a firm's equity returns and its idiosyncratic volatility are driven by a common idiosyncratic risk factor and the second is that a firm's systematic volatility falls when idiosyncratic risk increases. The first property ensures that firm-level abnormal returns and changes in idiosyncratic risk are positively correlated, which is the positive $IVol$ anomaly. The second property ensures firm-level expected returns, which load on systematic volatility are higher when idiosyncratic risk is lower, giving rise to the negative

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[1]Other papers investigating the positive return-volatility relation are Spiegel and Wang (2006), Fu (2009) and Huang, Liu, Rhee, and Zhang (2009). The negative $IVol$ anomaly is also shown to exist in international stock markets by Ang, Hodrick, Xing, and Zhang (2009).

[2]Earlier empirical papers investigating idiosyncratic volatility and returns in the cross section are Lintner (1965), Tinic and West (1986) and Lehmann (1990).

[3]Starting from a partial equilibrium valuation model with real options similar to Carlson, Fisher, and Giammarino (2004) and Cooper (2007), we introduce stochastic idiosyncratic operating risk, for which we assume a 2-regime Markov switching process. Guo, Miao, and Morellec (2005) and Hackbarth, Miao, and Morellec (2006) also develop a 2-regime Markov switching process in state dynamics to investigate investment and capital structure decisions, respectively.

$IVol$ anomaly. In our theoretical model of a cross-section of firms with growth options, where idiosyncratic operating risk is stochastic, both properties arise endogenously – this resolves the positive and negative $IVol$ anomalies. We exploit the analytical solutions of the model to validate our theoretical analysis via simulation. Using simulated data we redo the main analysis in Duffee (1995) and Ang, Hodrick, Xing, and Zhang (2006)) – we thereby verify that our model, in which the CAPM holds, does indeed resolve both the positive and negative $IVol$ anomalies. Finally, to test whether our choice of model is reasonable, we identify its novel empirical predictions and confront them with the data.

We now explain how a cross-sectional model of firms with growth options and stochastic idiosyncratic operating risk gives rise to firm-level equity returns, which possess the two key properties which allow the resolution of the positive and negative $IVol$ anomalies. A firm's currently producing assets – the assets-in-place – have linear valuations in cashflows, which are invariant with respect to idiosyncratic operating risk. In contrast, a firm's growth options are convex with respect to cashflows and therefore depend on idiosyncratic operating risk. Standard option pricing theory tells us options are levered positions on the underlying asset – an increase in the volatility of the underlying asset increases the option value. Therefore returns on a firm with growth options will be exposed to the same underlying idiosyncratic risk factor as the level of idiosyncratic risk itself – this is just the first key property, and so firm-level returns will be positively correlated with changes in $IVol$. Idiosyncratic risk is by definition non-systematic, and so the increase in growth option and hence firm value stemming from a rise in idiosyncratic risk will be accompanied by a decrease in the proportion of the option value exposed to systematic risk. Hence, the second key property is satisfied: systematic risk falls as idiosyncratic risk rises.[4] Expected returns load on systematic risk, so a firm with growth options will have expected returns which fall when idiosyncratic risk rises, resolving the negative $IVol$ anomaly.

To verify the intuition revealed by our theoretical analysis, we simulate the model.[5] The simulations recreate the $IVol$ anomalies that are qualitatively similar to Duffee (1995) and Ang, Hodrick, Xing, and Zhang (2006), with more pronounced results when we specify larger spreads

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[4]This is in contrast with the embodied technology shocks modelled in Garleanu, Panageas, and Yu (2012), which impact the level of output flow.

[5]By using a 2-regime Markov switching process to model idiosyncratic operating risk, we can derive analytical solutions (see Guo, Miao, and Morellec (2005) and Hackbarth, Miao, and Morellec (2006)).

in volatility between regimes. When we specify a single regime – the standard specification in most real option models – we find that the model generates no statistical $IVol$-return relation, validating that our explanation is the driving mechanism behind the results.

Our model also helps understand the findings that the negative $IVol$-return relation is largely explained by the return reversals of high $IVol$ stocks among a subset of small firms (Huang, Liu, Rhee, and Zhang (2009)); Fu (2009)). The model generates strong return reversals through the risk dynamics embedded in the operations of the firms that possess growth opportunities. Therefore, we rely on a rational theory of firms that face uncertain operating environments which allows for observable firm-characteristics to explain dispersions in equity returns. In this sense, we depart from the explanations based on limits to arbitrage (Pontiff (2006)) or investors' cognitive biases and mispricings in financial markets (Daniel, Hirshleifer, and Subrahmanyam (1998)) for the negative $IVol$ anomaly.

The final part of our paper is empirical – is the data supportive of the key economic mechanisms in our theoretical model? To answer this question, we test the model's novel empirical predictions. First, we focus on verifying the model's predictions that the $IVol$-return relations rely on real options and idiosyncratic operating risks. This is crucial for verifying the economics underlying our explanation for how the $IVol$ anomalies can arise in a rational model. For the positive $IVol$-return relation, we revisit Grullon, Lyandres, and Zhdanov (2010) by recreating many of their empirical proxies for growth options and creating some of our own. We employ similar cross-sectional return regressions plus additional specifications, where we include the difference between the 70th and 30th percentile values of $IVol$ for each stock as a proxy for the spread in idiosyncratic volatility between regimes. For the negative $IVol$-return relation, we initially follow Ang, Hodrick, Xing, and Zhang (2006) by creating $IVol$-sorted portfolios, and then go further by sorting stocks via proxies for real option intensity and $IVol$ spread to compute portfolio returns. We find evidence for stronger positive and negative $IVol$-return relations for more real option intensive firms and which experience more extreme changes in $IVol$. These results lend strong support to our model.

In the final part of our empirical analysis, we focus on a second set of novel predictions. Conditioned on a volatility regime, expected equity returns equate to the sum of a continuous drift term and a jump term that captures the expectation of a change in equity value in the event of

a switch in volatility. A volatility regime that corresponds inversely with the jump term implies a positive correspondence with the continuous drift term. If real options and stochastic idiosyncratic risk are incorporated into firm valuations, stock returns should correlate positively with $IVol$ in intervals between large changes in $IVol$. Using an event study approach, we investigate the difference in 5-month average returns around the month in which stocks experience large changes in $IVol$. We find that the difference between post and pre-switch returns is positive for the up-switch sample, and negative for the down-switch sample, and that this 'switch effect' is stronger for firms with more real options or which experience more extreme changes in $IVol$. Here again the results are in strong agreement with the model.

Motivated by anomalies evidenced in the cross-section of stock returns, Berk, Green, and Naik (1999) were among the first to establish a linkage between corporate investments and expected equity returns.[6],[7] Since then, the literature has been extended in many directions (Carlson, Fisher, and Giammarino (2004); Zhang (2005); Carlson, Fisher, and Giammarino (2006); Cooper (2007); Sagi and Seashole (2007), and Carlson, Fisher, and Giammarino (2010)). We add to this literature by expanding the description of the firm's operating environment to reconcile the $IVol$ anomalies. In our model, idiosyncratic volatility serves as an additional state variable that affects only the systematic volatility of a firm's real options, but not assets-in-place.

To the best of our knowledge few inroads have been made to link idiosyncratic risk to asset pricing. The exceptions are Babenko, Boguth, and Tserlukevich (2013) and Kogan and Papanikolaou (2013) who show that firm-specific shocks contain information about future priced risk. Babenko et al. (2013) view firms as portfolios of systematic and idiosyncratic divisions and rely on additive systematic and idiosyncratic cashflow shocks in the valuation of the firms to explain asset pricing anomalies. Kogan and Papanikolaou (2013), on the other hand, show that the investments of firms with high growth opportunities exhibit higher sensitivity to investment-specific-technology shocks earning a lower risk premia. While one can view these models as strongly complementary, our modeling approach explicitly considers idiosyncratic cashflow shocks with time-varying risk

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[6]Fama and French (1992) provide evidence on the ability of size and book-to-market to explain returns. Fama and French (1996) provide a cross-sectional landscape view of how average returns vary across stocks. Anderson and Garcia-Feijoo (2006) offer empirical evidence on the relation between corporate investments and average returns.

[7]Firm-level investment in a real option context was first pioneered by MacDonald and Siegel (1985), MacDonald and Siegel (1986) and Brennan and Schwartz (1985), and later adopted and extended by many others. Dixit and Pindyck (1994) is a standard reference for a detailed analysis of the literature.

together with an optimal timing decision concerning growth option exercise. Hence, the underlying mechanism in our model is distinct from Babenko et al. (2013) and Kogan and Papanikolaou (2013), allowing us to propose a novel channel between the operating environment faced by the firms and equity returns. The distinct features of our model yield novel testable predictions on the correspondence between $IVol$ and stock returns such as the switch effect, which we test empirically in this paper.

The rest of the paper is organized as follows. In Section 2, we describe two key properties of equity returns, which are sufficient to resolve the positive and negative $IVol$ anomalies. In Section 3, we describe the model and provide analytical solutions to show that the two key properties arise endogenously. In Section 4, we redo the main analysis of Duffee (1995) Ang, Hodrick, Xing, and Zhang (2006), but with simulated data from the model, to show that the positive and negative $IVol$ anomalies arise within the model. We take the novel empirical predictions of our model and confront them with the data in Section 5 and in Section 6 we conclude. The Appendix contains all the proofs and other technical details omitted in the main body of the paper.

2 Two Key Properties of Returns: A Thought Experiment

In this section we describe two properties of returns which lead to the positive $IVol$ anomaly, i.e. the positive contemporaneous relation between stock returns and changes in idiosyncratic return volatility, and the negative $IVol$ anomaly, i.e. the poor performance of stocks with high idiosyncratic volatility.

We first assume the existence of a stochastic discount factor (SDF), π, such that

$$\frac{d\pi_t}{\pi_t} = -r_t dt - \Theta_t dB_t^{sys}. \tag{2.1}$$

The riskfree rate r_t may be stochastic, dB_t^{sys} is a composite systematic risk factor, and Θ_t is the composite price of risk, which can be decomposed as follows:

$$dB_t^{sys} = \sum_{n=1}^{N} \frac{\Theta_{n,t}}{\Theta_t} dB_{n,t}^{sys}, \ \Theta_t = \sqrt{\sum_{n=1}^{N} \Theta_{n,t}^2} \ ,$$

where $B_{n,t}^{sys}$, $i \in \{n, \ldots N\}$ are mutually orthogonal standard Brownian motions under the physical probability measure $\mathbb{P}$, each corresponding to some risk factor, where $\Theta_{i,t}$ is the associated price of risk, which can be stochastic. This way we make it clear that (2.1) nests models such as Bansal and Yaron (2004), where the conditional CAPM does not hold in addition to the conditional CAPM.

Equity returns on a cross-section of firms, $k \in \{1, \ldots, K\}$ are exposed to aggregate risk and idiosyncratic risk. The equity return for Firm k is given by

$$dR_{k,t} = \mu_t dt + \sigma_t^{sys} dB_t^{sys} + \sigma_{k,t}^{id} dM_{k,t}^{id},$$

where the conditional expected return, μ_t, may be stochastic, σ_t^{sys} is conditional systematic volatility, which can be stochastic, $dM_{k,t}^{id}$ is the increment in some idiosyncratic risk factor for Firm k and $\sigma_{k,t}^{id}$ is conditional idiosyncratic volatility, which we assume is positive for all firms. The idiosyncratic risk factors are independent across firms and so is conditional idiosyncratic volatility. [8]

Since π is a SDF, the basic asset pricing equation holds, i.e.

$$E_t[dR_{k,t} - r_t dt] = E_t \left[dR_{k,t} \frac{d\pi_t}{\pi_t} \right],$$

which implies

$$\mu_t = r_t + \sigma_t^{sys} \Theta_t.$$

So far everything has been standard – we have not yet made any novel assumptions. We now do so by assuming returns for firms $k \in \{1, \ldots, K\}$ satisfy the following two properties:

Property 1 *The idiosyncratic risk factor for Firm k's returns is also a risk factor for the firm's idiosyncratic volatility, i.e.*

$$d\sigma_{k,t}^{id} = a_t dt + b_t dM_{k,t}^{id},$$

where $b_t > 0$.

Property 2 *Firm k's systematic volatility is a decreasing function of its idiosyncratic volatility.*

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[8] $M_{k,t}^{id}$ is a martingale under the physical probability measure $\mathbb{P}$ and may be continuous, e.g. a standard Brownian motion or discontinuous such as a compensated Poisson process.

We now explore the implications of Properties 1 and 2 for $IVol$ anomalies via a simple thought experiment.

In the first part of our thought experiment, we ask what would happen if we simulated returns and idiosyncratic volatility and ran a regression of monthly abnormal returns against corresponding monthly changes in volatility? We would find a positive relationship between monthly abnormal returns and changes in idiosyncratic volatility, i.e. the positive $IVol$ anomaly would hold. Why is this so? The answer lies in Property 1.

The dynamics of abnormal returns are given by

$$dR_{k,t}^a = \sigma_t^{sys} dB_t^{sys} + \sigma_{k,t}^{id} dM_{k,t}^{id}.$$

There is a single common risk factor driving both abnormal returns and changes in idiosyncratic volatility. Furthermore, the direction of the changes is the same. The only other risk factor driving abnormal returns is the systematic risk factor, which is independent of changes in idiosyncratic volatility. There are no additional risk factors driving changes in idiosyncratic volatility. Hence, abnormal returns and changes in idiosyncratic volatility will be positively correlated.

To see this more formally, observe that a monthly abnormal return is given by

$$R_{k,t+\frac{1}{12}}^a - R_{k,t}^a = \int_t^{t+\frac{1}{12}} dR_{k,u}^a du = \int_t^{t+\frac{1}{12}} \sigma_u^s dB_u^{sys} + \int_t^{t+\frac{1}{12}} \sigma_{k,u}^{id} dM_{k,u}^{id}.$$

The corresponding monthly change in idiosyncratic volatility is given by

$$\sigma_{k,t+\frac{1}{12}}^{id} - \sigma_{k,t}^{id} = \int_t^{t+\frac{1}{12}} d\sigma_{k,u}^{id} du = \int_t^{t+\frac{1}{12}} a_u du + \int_t^{t+\frac{1}{12}} b_u dM_{k,u}^{id} du.$$

The conditional covariance between abnormal returns and the corresponding change in idiosyncratic volatility is clearly positive:

$$Cov_t \left(R_{k,t+\frac{1}{12}}^a - R_{k,t}^a, \sigma_{k,t+\frac{1}{12}}^{id} - \sigma_{k,t}^{id} \right) = E_t \left[\left(\int_t^{t+\frac{1}{12}} \sigma_{k,u}^{id} dM_{k,u}^{id} \right) \left(\int_t^{t+\frac{1}{12}} b_u dM_{k,u}^{id} du \right) \right] > 0.$$

There is therefore a positive contemporaneous relationship between returns and $IVol$ at the firm level. Of course, idiosyncratic volatility cannot be observed directly, but if we could estimate

it sufficiently accurately via observing abnormal returns, then we would still expect to find the positive $IVol$ anomaly.

The second part of our thought experiment relates to the second property of returns. We observe abnormal returns data over a month, measure the variance of their abnormal returns as a proxy for idiosyncratic volatility, sort firms by their estimated idiosyncratic volatility and then observe their abnormal returns over the next month. How would such portfolios perform over the next month, if we measure performance in terms of returns?

To answer this question consider a firm for which idiosyncratic volatility is currently high. The expected return for a firm is given by

$$E_t[dR_{k,t}] = (r_t + \sigma_t^{sys}\Theta_t)dt,$$

where by assumption σ_t^{sys} is a decreasing function of $\sigma_{k,t}^{id}$. Hence, expected returns will be lower for higher $IVol$ portfolios: the negative $IVol$ anomaly arises naturally from Property 2.

An obvious concern comes to mind. Is the example behind our thought experiment pathological or can the two key properties we have used arise naturally in a standard cross-sectional asset pricing model?

In the next section we show the two key properties are not at all pathological. On the contrary, they arise as endogenous outcomes in a model of a cross-section of firms with growth options provided we make a single crucial assumption: idiosyncratic operating risk is stochastic.

3   Model

We construct a growth option model similar in spirit to the models in Garlappi and Yan (2008) and Carlson, Fisher, and Giammarino (2004).[9] This section describes the firms' economic environment.

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[9]With no loss of generality, we rely specifically on growth options to incorporate convexity of firm valuations in the firms' output price. Other forms of real options that incorporate convexities would accommodate similar results.

3.1 The Environment

There are two types of firms: mature and young, and there is a finite number of each. Mature firms produce at full capacity. In contrast, young firms produce at a lower operating scale, but have the option to make an irreversible investment to increase production and also become mature. Firms are all equity financed. Firm k produces a single commodity that can be sold at time-t in the product market at price $P_{k,t}$, with dynamics

$$P_{k,t} = X_{k,t}Z_t,$$

where X_k and Z are respectively the idiosyncratic and systematic components. Their dynamics are given by

$$\frac{dX_{k,t}}{X_{k,t}} = \sigma_{k,t}^{id}dB_{k,t}^{id},$$
$$\frac{dZ_t}{Z_t} = \mu dt + \sigma^{sys}dB_t^{sys}, \tag{3.1}$$

μ denotes the constant growth rate, σ^{sys} constant systematic volatility, $\sigma_{k,t}^{id}$ stochastic idiosyncratic volatility, and $dB_{k,t}^{id}$ and dB_t^{sys} are the increments of two independent Brownian motions. The increments $dB_{k,t}^{id}$ are independent across firms.

The presence of uncertainty shocks (see for example Bloom (2009)) in the idiosyncratic component of the volatility of price growth is the novel feature of our model. In other words, idiosyncratic operating risk is stochastic. The economic rationale for our assumption comes from allowing firms to have random and time-varying potential to realize monopolistic rents.[10]

We model shocks to Firm k's idiosyncratic volatility by assuming $\sigma_{k,t}^{id}$ follows a 2-state Markov chain: $\sigma_{k,t}^{id} \in \{\sigma_L^{id}, \sigma_H^{id}\}$, where $0 < \sigma_L^{id} < \sigma_H^{id}$ and the probability of entering state $s_{k,t} \in \{L, H\}$ within the infinitesimally small time dt is $\lambda_{s_{k,t}}dt$.[11] Firms do not have to be in the same idiosyncratic volatility state and switches between states are independent across firms. Essentially,

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[10]Dixit and Pindyck (1994) and Caballero and Pindyck (1996) show that idiosyncratic shocks translate to a firm's ability to retaining monopolistic rents – a firm that experiences a positive idiosyncratic technology shock experiences an advantage that cannot be stolen by its competitors, while a positive aggregate shock is shared with the firm's competitors. Some plausible micro-economic examples for a change in idiosyncratic operating risk are: shifts in consumer needs and wants, persistent changes in production technology, or changes in the general operating environment of the firm or the firm's industry, among others.

[11]Using a Markov chain with more than 2 states is also possible, but does not add to the underlying economics.

each firm has its own idiosyncratic volatility state and Markov chain, which is i.i.d. across firms –
hence the appearance of the subscript k on $\sigma_{k,t}^{id}$ and $s_{k,t}$, but not on σ_L^{id} and σ_H^{id}. Both $P_{k,t}$ and the
volatility regime $s_{k,t}$ are observable for any given firm. We subscript quantities with $s_{k,t} \in \{H, L\}$
throughout to denote their dependence on Firm k's volatility regime.

Investors in the stock market can hedge market risk in the firms' operations by trading on two
securities. Let B_t denote the price of the risk free asset with dynamics

$$\frac{dB_t}{B_t} = rdt,$$

where r is the constant riskless rate and let S be the price of a risky security with dynamics

$$\frac{dS_t}{S_t} = \mu_S dt + \sigma_S dB_t^{sys}.$$

S has a beta equal to one and $\Theta = \frac{\mu_S - r}{\sigma_S}$ is the constant market price of risk. The proportion of S
held in a replicating portfolio determines the beta of the portfolio. This is equivalent to assuming
an exogenous stochastic discount factor π, where

$$\frac{d\pi_t}{\pi_t} = -rdt - \Theta dB_t^{sys}.$$

It is important to note that the risk-free rate and market price of risk are constant – this is
purely for clarity. We could accommodate both a stochastic risk-free rate and market price of
risk. However, we choose not to, because they are not central to the economics underlying this
paper.

We can carry out the valuation of firms under the risk-neutral measure $\mathbb{Q}$. Working under $\mathbb{Q}$
changes the dynamics of the systematic component of the product market price to

$$\frac{dZ_t}{Z_t} = \hat{\mu}dt + \sigma^{sys}d\hat{B}_t^{sys},$$

where the risk-neutral drift, $\hat{\mu} = \mu - \sigma^{sys}\Theta$, is by assumption strictly less than the risk-free rate,
r, and $d\hat{B}_t^{sys} = \Theta dt + dB_t^{sys}$ is the increment in a standard Brownian motion under $\mathbb{Q}$.

3.2 The Value of a Mature Firm

We now derive the value of a mature firm. We shall omit firm subscripts throughout the rest of this section for convenience.

The cash flow of a mature firm stems solely from its assets-in-place, which allow it to create output. Consider assets-in-place which produce a unit of output per unit time. The cost of producing a unit of output is c per unit of time and so the profit per unit time is $P_t - c$. The value of the profit stream stemming from producing a unit of output is

$$A_t = A(P_t) = E_t^{\mathbb{Q}} \left[\int_t^\infty e^{-r(u-t)}(P_u - c)du \right] = \frac{P_t}{r - \hat{\mu}} - \frac{c}{r}. \tag{3.2}$$

The scale of production for a mature firm is ξ_M, and so its profit flow is $\xi_M(P_t - c)$. The value of a mature firm is therefore given by $\xi_M A_t$.

3.3 The Value of a Young Firm

A young firm derives value from the sum of its assets-in-place and a growth option.

The value of assets-in-place for a young firm is smaller than for mature firm, because of a smaller production scale, $\xi_Y < \xi_M$. The value of a young firm's assets-in place is thus given by $\xi_Y A_t$.

A growth option allows a young firm to increase its production scale by $\xi = \xi_M - \xi_Y$. Hence, at the moment of exercise, τ, the value of the young firm's assets-in-place increases by $(\xi_M - \xi_Y)A_\tau = \xi A_\tau$. At the moment of exercise, the young firm pays a one time irreversible investment of I. The expected present-value of the payoff $\xi A_\tau - I$ gives the value of the growth option

$$G_{s_t} = G_{s_t,t} = E_t^{\mathbb{Q}}[e^{-r(\tau-t)}(\xi A_\tau - I)|s_t], \ t \leq \tau,$$

which depends on the volatility regime. This is indicated by $E_t^{\mathbb{Q}}[\cdot|s_t]$, the expectation operator under $\mathbb{Q}$ conditional on date-t, when the current volatility regime is s_t. For simplicity, we assume all equity financing.

Summing together assets-in-place and the growth option gives a young firm's total equity

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value:

$$V_{Y,s_t} = V_{Y,s_t,t} = \xi_Y \, A_t + G_{s_t}. \tag{3.3}$$

After exercise, $t > \tau$, the young firm is mature and so the growth option value is merely the value of the additional assets in place and is no longer dependent on the volatility regime, $G_t = \xi \, A_t$, and so $V_{Y,t} = \xi_Y \, A_t + \xi \, A_t = \xi_M \, A_t$.

The dependence of the growth option value on the idiosyncratic volatility regime comes from our assumption that idiosyncratic volatility is stochastic. It is the convexity of the option value with respect to P_t that ensures the optimal decision to expand and hence young firm value depend on the volatility regime s_t (Guo, Miao, and Morellec (2005)).

The random stopping time, τ, is chosen to maximize firm value and is defined by the optimal investment thresholds P_L^*, P_H^*, which are determined by smooth pasting conditions (see Guo, Miao, and Morellec (2005)). In the low idiosyncratic volatility state, the growth option is worth less, making it optimal to exercise the option earlier: $P_L^* < P_H^*$. When a firm is in the low state exercise occurs as soon as the price process hits the threshold P_L^* from below. In the high state, exercise occurs if the price process hits the threshold P_H^* from below or, if $P_t \geq P_L^*$, as soon as the volatility state switches to low, whichever comes first. As a consequence, investment can occur instantaneously, without a change in the level of the price, purely because of a fall in idiosyncratic volatility.

The following proposition gives the value of the growth option for a young firm.

Proposition 1 *In its low idiosyncratic volatility regime, a young firm's growth option value is given by*

$$G_{L,t} = \begin{cases} \frac{1}{l_1 - l_2} \left[\delta_L(l_2) \left(\frac{P_t}{P_L^*} \right)^{l_1} - \delta_L(l_1) \left(\frac{P_t}{P_L^*} \right)^{l_2} \right], & P_t^{\max} < P_L^* \\ \xi A(P_t) - I, & P_t^{\max} = P_L^* \\ \xi A(P_t), & P_t^{\max} > P_L^* \end{cases} \tag{3.4}$$

where $P_t^{\max} = \sup_{t \geq 0} \{ P_u : u \in [0, t) \}$ is the firm's maximum output price,

$$\delta_L(l) = \frac{\xi P_L^*}{r - \hat{\mu}} - l \, \xi A(P_L^*),$$

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and $l_2 > l_1 > 0$ are the positive roots of the quartic

$$q_L(l)q_H(l) - \lambda_L\lambda_H = 0, \tag{3.5}$$

where

$$q_L(l) = \frac{1}{2}\left[(\sigma_L^{id})^2 + (\sigma^{sys})^2\right]l(l-1) + \hat{\mu}l - (r + \lambda_H),$$

$$q_H(l) = \frac{1}{2}\left[(\sigma_H^{id})^2 + (\sigma^{sys})^2\right]l(l-1) + \hat{\mu}l - (r + \lambda_L).$$

In its high idiosyncratic volatility regime, a young firm's growth option value is given by

$$G_{H,t} = \begin{cases} \frac{1}{l_1-l_2}\left[\epsilon(l_1)\delta_L(l_2)\left(\frac{P_t}{P_L^*}\right)^{l_1} - \epsilon(l_2)\delta_L(l_1)\left(\frac{P_t}{P_L^*}\right)^{l_2}\right], & P_t < P_L^*,\ P_t^{\max} < P_H^* \\ \frac{\lambda_L}{r+\lambda_L}(\xi A(P_t) - I) + \frac{1}{j_1-j_2}\left\{\delta_H(j_2)\left(\frac{P_t}{P_H^*}\right)^{j_1} - \delta_H(j_1)\left(\frac{P_t}{P_H^*}\right)^{j_2}\right\}, & P_t \in [P_L^*, P_H^*),\ P_t^{\max} < P_H^* \\ \xi A(P_t) - I, & P_t^{\max} = P_H^* \\ \xi A(P_t), & P_t^{\max} > P_H^* \end{cases} \tag{3.6}$$

where

$$\epsilon(l) = -\frac{\lambda_L}{q_H(l)},$$

$$\delta_H(j) = \xi P_H^*\left(\frac{1}{r-\hat{\mu}} - \frac{\lambda_L}{r+\lambda_L}\right) - j\frac{r}{r+\lambda_L}\left(\xi A(P_H^*) - I\right),$$

and $j_1 > j_2$ are the roots of the quadratic

$$q_H(j) = 0. \tag{3.7}$$

The optimal investment thresholds P_i^*, $i \in \{L, H\}$ are given by the following system of nonlinear algebraic equations.

$$\frac{1}{l_1-l_2}\left[\epsilon(l_1)\delta_L(l_2) - \epsilon(l_2)\delta_L(l_1)\right] - \frac{1}{j_1-j_2}\left[\delta_H(j_2)\left(\frac{P_L^*}{P_H^*}\right)^{j_1} - \delta_H(j_1)\left(\frac{P_L^*}{P_H^*}\right)^{j_2}\right] - \frac{\lambda_L}{r+\lambda_L}(\xi A(P_L^*) - I) = 0, \tag{3.8}$$

$$\frac{1}{l_1 - l_2} \left[l_1 \epsilon(l_1) \delta_L(l_2) - l_2 \epsilon(l_2) \delta_L(l_1) \right] - \frac{1}{j_1 - j_2} \left[j_1 \delta_H(j_2) \left(\frac{P_L^*}{P_H^*} \right)^{j_1} - j_2 \delta_H(j_1) \left(\frac{P_L^*}{P_H^*} \right)^{j_2} \right] - \frac{\lambda_L}{r + \lambda_L} \frac{\xi P_H^*}{r - \hat{\mu}} = 0.$$

(3.9)

Importantly, the proposition reveals that the value of a young firm's growth option value jumps upward when idiosyncratic volatility shifts up, in stark contrast to assets-in-place, which are independent of idiosyncratic volatility. Therefore, the dependence of a young firm's value on its idiosyncratic volatility regime is attributed entirely to the young firm's growth option.

Insert Figure 1 here

Figure 1 provides a graphical illustration of Proposition 1 for different sets of parameter values for σ_H^{id} and σ_L^{id}. Comparing the graphs across panels reveals that the opportunity to expand has a larger valuation in the high regime than in the low regime, and the difference is increasing in the spread between σ_H^{id} and σ_L^{id}. The last panel reveals that the model results in a single valuation profile if $\sigma_H^{id} = \sigma_L^{id}$, which is the usual specification in standard growth option models.

3.4   Returns

For mature firms, expected returns differ in the cross-section based solely on variation in output prices – idiosyncratic volatility is irrelevant. In contrast, for young firms, variation in idiosyncratic volatility across firms impacts the cross-section of returns via growth options. In particular, returns on the growth option possessed by a young firm satisfy the two key properties described in Section 2, making it possible to resolve the positive and negative $IVol$ anomalies.

We start by analyzing returns for a mature firm. First observe that because the value of a mature firm exhibits constant returns with respect to production scale, the return on a mature firm is independent of the production scale. Consequently, the return on a young firm's assets-in-place is the same as the return on a mature firm, if their output prices are equal. It therefore makes more sense to refer to the return on assets-in-place, which we denote by $dR_{A,t}$ and is given

by

$$dR_{A,t} = [r + (1 + L(P_t))\Theta\sigma^{sys}]dt + (1 + L(P_t))\sigma^{sys}dB_t^{sys} + (1 + L(P_t))\sigma_t^{id}dB_t^{id}, \qquad (3.10)$$

where $L(P_t) = \frac{\frac{c}{r}}{\frac{P_t}{r-\hat{\mu}} - \frac{c}{r}}$.

The return on assets-in-place satisfies neither of the two properties which lead to the resolution of the positive and negative $IVol$ anomalies: unexpected changes in idiosyncratic volatility are driven by a Markov chain which is independent of the idiosyncratic shock, dB_t^{id} and systematic volatility, $(1 + L(P_t))\sigma^{sys}$, is independent of idiosyncratic volatility.

We now look at the returns of a young firm, where stochastic idiosyncratic volatility plays a prominent role. The return on a young firm, dR_{Y,s_t}, is the weighted average of the return on its assets-in-place and growth option, i.e.

$$dR_{Y,s_t} = \left(1 - \frac{G_{s_t}}{V_{Y,s_t}}\right) dR_{A,t} + \frac{G_{s_t}}{V_{Y,s_t}} dG_{s_t}, \qquad (3.11)$$

where dG_{s_t} is the return on the firm's growth option.

We already know that the return on assets-in-place will not help us resolve $IVol$ anomalies, so we focus on the growth option return. Before proceeding, we observe that if a firm's idiosyncratic volatility changes at date-t, we denote the idiosyncratic volatility state just before the change by s_{t-} and just after by s_t. The following proposition gives the return on a young firm's growth option.

Proposition 2 *The growth option return is given by*

$$\frac{dG_{s_{t-}}}{G_{s_{t-}}} = \mu_{G,s_{t-}}dt$$

$$+ \Omega_{s_{t-}}(\sigma^{sys}dB_t^{sys} + \sigma_{s_{t-}}^{id}dB_t^{id}) + \frac{G_{s_t} - G_{s_{t-}}}{G_{s_{t-}}}dM_{s_{t-} s_t,t}^{id}, \qquad (3.12)$$

where

$$\mu_{G,s_{t-}} = \Omega_{s_{t-}}\mu + \frac{G_{s_t} - G_{s_{t-}}}{G_{s_{t-}}}\lambda_{s_t} + \frac{1}{2}\frac{P_t^2}{G_{s_{t-}}}\frac{\partial^2 G_{s_{t-}}}{\partial P_t^2}\left((\sigma^{sys})^2 + (\sigma_{s_{t-}}^{id})^2\right),$$

and

$$\Omega_{s_{t-}} = \frac{P_t}{G_{s_{t-}}} \frac{\partial G_{s_{t-}}}{\partial P_t}, \tag{3.13}$$

is the elasticity of the growth option with respect to the output price,

$$dM^{id}_{s_{t-} s_t,t} = dN^{id}_{s_{t-} s_t,t} - \lambda_{s_t} dt$$

is a compensated Poisson process and hence discontinuous martingale, driven by changes in the young firm's idiosyncratic volatility regime, i.e.

$$dN^{id}_{s_{t-} s_t,t} = \begin{cases} 0 & , s_t = s_{t-} \\ 1 & , s_t \neq s_{t-} \end{cases}. \tag{3.14}$$

In the no action region, $P_t^{\max} < P_L^*$, the growth option's elasticity with respect to the output price is lower when idiosyncratic volatility is high, i.e. $\Omega_L > \Omega_H$, but the growth option's idiosyncratic volatility is higher when idiosyncratic volatility is high, i.e. $\sigma_L^{id}\Omega_L < \sigma_H^{id}\Omega_H$, and the growth option's expected return when there is no change in idiosyncratic volatility is higher when idiosyncratic volatility is high, i.e. $\mu_{G,H} - \frac{G_L - G_H}{G_H}\lambda_L > \mu_{G,L} - \frac{G_H - G_L}{G_L}\lambda_H$.

Since the increment in the idiosyncratic risk factor, $dM^{id}_{s_{t-} s_t,t}$ also drives changes in idiosyncratic volatility, Property 1 holds for the growth option's return. The return on assets-in-place is independent of the firm's idiosyncratic volatility regime, so it follows that Property 1 also holds for a young firm's return.

As idiosyncratic volatility rises, the growth option's elasticity, Ω, falls and so the option's systematic volatility decreases. Hence, Property 2 is satisfied for the growth option's return. However, the young firm's value is the sum of assets-in-place and its growth option, so it does not immediately follow that the young firm's systematic volatility is lower in the high idiosyncratic volatility regime – we need to account for the effect of a change in the weight, $\frac{G_{s_t}}{V_{Y,s_t}}$, in the young firm's return as shown in Equation (3.11). The following proposition establishes that the young firm's systematic volatility is lower in the high idiosyncratic volatility regime.

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Proposition 3 *A young firm's conditional systematic return volatility is given by*

$$\sigma_{R_Y,s_t}^{sys} = \left[\left(1 - \frac{G_{s_t}}{V_{Y,s_t}}\right)(1 + L(P_t)) + \frac{G_{s_t}}{V_{Y,s_t}}\Omega_{s_t}\right]\sigma^{sys}, \tag{3.15}$$

and is lower in the high idiosyncratic volatility regime, i.e.

$$\sigma_{R_Y,H}^{sys} < \sigma_{R_Y,L}^{sys}. \tag{3.16}$$

Propositions 2 and 3 together show that a young firm's return endogenously satisfies Properties 1 and 2.

We now discuss the implications of the properties of growth option returns for $IVol$ anomalies in more detail. From (3.11) and (3.12), we can see that abnormal returns for a young firm are given by

$$\begin{aligned}
dR_{Y,s_{t-}}^a &= dR_{Y,s_{t-}} - E_t[dR_{Y,s_{t-}}|s_{t-}] \\
&= \left[\left(1 - \frac{G_{s_{t-}}}{V_{Y,s_{t-}}}\right)(1 + L(P_t)) + \frac{G_{s_{t-}}}{V_{Y,s_{t-}}}\Omega_{s_{t-}}\right](\sigma^{sys}dB_t^{sys} + \sigma_{s_{t-}}^{id}dB_t^{id}) + \frac{G_{s_t} - G_{s_{t-}}}{V_{Y,s_{t-}}}dM_{s_{t-} s_t,t}^{id}.
\end{aligned}$$

Furthermore, changes in idiosyncratic volatility can be written as

$$d\sigma_{s_{t-}}^{id} = \lambda_{s_t}(\sigma_{s_t}^{id} - \sigma_{s_{t-}}^{id})dt + (\sigma_{s_t}^{id} - \sigma_{s_{t-}}^{id})dM_{s_{t-} s_t,t}^{id}$$

There is one common risk factor driving changes in abnormal returns and idiosyncratic volatility, i.e. $dM_{s_{t-} s_t,t}^{id}$. Since growth option value is higher when idiosyncratic volatility is higher, the change in idiosyncratic volatility, $\sigma_{s_t}^{id} - \sigma_{s_{t-}}^{id}$ and the change in growth option value $G_{s_t} - G_{s_{t-}}$ have the same sign. Property 1 therefore holds and so a young firm's abnormal returns will be positively correlated with changes in its idiosyncratic volatility, contributing to the positive $IVol$-return empirical relation (Duffee (1995)). In our model, the positive $IVol$ anomaly is explained by the jumps in returns of young firms when they experience a switch in their idiosyncratic operating risk.

Proposition 3 shows that a young firm's systematic volatility is higher when idiosyncratic

volatility is low, i.e. Property 2 holds. The intuition follows from standard option pricing results – when volatility increases the value of an option increases, but the option also becomes less sensitive to changes in the underlying, i.e. the option delta falls. In the context of our model, the growth option delta and hence its elasticity with respect to output price is lower when idiosyncratic volatility is higher. This drives down the systematic volatility of the growth option and hence young firm returns in the high idiosyncratic volatility state. Applying the basic asset pricing equation to the returns of a young firm, $dR_{Y,s_{t-}}$, we obtain

$$E_t[dR_{Y,t}|s_{t-}] = \left(r + \sigma^{sys}_{R,s_{t-}} \Theta \right) dt.$$

Therefore, expected returns will be lower when idiosyncratic volatility is high.[12] Observe that in portfolio-based tests, sorting firms based on end-of-month realized $IVol$ is akin to sorting based on the firms' most recent idiosyncratic volatility regime. If real options and stochastic idiosyncratic risk are incorporated into firm valuations, our model implies that return realizations of portfolios sorted on idiosyncratic volatility reflect differences in expected returns, thereby creating a negative correspondence between future equity returns and current idiosyncratic risk (Ang, Hodrick, Xing, and Zhang (2006)). In summary, our model generates reversals in equity returns correlating positively with contemporaneous changes in $IVol$ and inversely with past realized $IVol$, reconciling the positive and the negative $IVol$ anomalies. We explore this feature of the model in the sequel with numerical simulations and empirical tests.

Lastly, our model offers the basis for a novel prediction on the relation between stock returns and idiosyncratic return volatility. Proposition 2 shows that a growth option's expected return contains a continuous component, i.e.

$$\mu^c_{G,s_{t-}} = \Omega_{s_{t-}} \mu + \frac{1}{2} \frac{P_t^2}{G_{s_{t-}}} \frac{\partial^2 G_{s_{t-}}}{\partial P_t^2} \left((\sigma^{sys})^2 + (\sigma^{id}_{s_{t-}})^2 \right),$$

plus a probability weighted jump term

$$\frac{G_{s_t} - G_{s_{t-}}}{G_{s_{t-}}} \lambda_{s_t}.$$

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[12]This is consisent with Johnson (2004), who shows that increasing uncertainty about the value of a firm's assets while holding the risk premium constant lowers the expected returns of levered firms.

Proposition 2 states that the continuous component of the expected return is positively correlated with the idiosyncratic volatility regime. Hence, our model implies that stock returns should correlate positively with $IVol$ during times between large changes in $IVol$. We test this novel empirical prediction in the sequel.

Insert Figure 2 here

Figure 2 provides a graphical illustration of the ideas conveyed in Proposition 2 for different sets of parameter values for σ_H^{id} and σ_L^{id}. Panel (a) shows that there is a negative difference in systematic volatility $(\Omega_H - \Omega_L)\sigma^{sys}$, while Panels (b) and (c) show that there is a positive difference in total volatility from diffusion risk, $\Omega_H \sqrt{(\sigma^{sys})^2 + (\sigma_H^{id})^2} - \Omega_L \sqrt{(\sigma^{sys})^2 + (\sigma_L^{id})^2}$, and the continuous drift terms $\mu_{G,H}^c - \mu_{G,L}^c$ between regimes. Panel (d) of the figure shows that there is a negative difference in jump terms $\frac{G_L - G_H}{G_H} - \frac{G_H - G_L}{G_L}$. All the differences are increasing in the spread between σ_H^{id} and σ_L^{id}, suggesting that the relation between returns and idiosyncratic volatility should be stronger the greater the variation in the firms' operating risk. Lastly, the differences in all quantities are identically zero if the volatility values are the same in both regimes, which is the usual specification in standard growth option models.

4   Simulations

In this section, we verify that our model is able to simultaneously produce the positive and the negative $IVol$-return relations by using data simulated from the model itself to carry out the main analysis in Duffee (1995) and Ang et al. (2006).

Using the analytical solutions of the model, we simulate a large panel of daily firm values by first simulating a single path of B_t^{sys}. Then we simulate 2,500 separate paths of P_t and idiosyncratic volatility values using the stochastic processes defined in (3.1).[13, 14] Each simulated path of P_t corresponds to the output price series for a single firm. The time horizon is 50 years with 20 trading days in each month, corresponding to a total of 12,000 daily observations for each firm. Then, for each day and each firm, we compute firm values using equations (3.2), (3.3), (3.4),

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[13]After applying our filters, the data in our empirical study contains an average of 2,412 firms each month with non-missing sales growth observations.

[14]Hanson (2007) is a good reference for numerical simulations of diffusion and Poisson processes.

and (3.6).

Initial maturities are drawn from a uniform distribution with equal probabilities of young and mature firms, which are updated daily. To ensure mature firms do not dominate the sample over time, mature firms exit the sample upon the arrival of an independent Poisson event with intensity $\lambda_{exit} = 0.01$ per unit time or if the firm value reaches zero due to low realizations of P_t values. Exiting firms are replaced by new young entrants.

We compute daily abnormal returns relative to the $CAPM$ based on $CAPM$ beta expressions for assets-in-place and the growth option. Then for each firm and each month, we compute $IVol$ as the standard deviation of the abnormal returns. The beta of a young firm is computed as a weighted average of the beta of the firm's assets-in-place and the beta of the firm's growth option where the weights are based on the proportion of firm value in the growth option.

We use the simulated returns to carry out the main analysis in Duffee (1995) and Ang et al. (2006) and store the results. Then, we repeat the entire process 99 more times in order to arrive at a set of 100 estimates allowing us to carry out t-tests in order to investigate the statistical significance of the results. To investigate the model's reliance on stochastic idiosyncratic risk, the simulation steps described thus far are repeated using three different sets of values for σ_H^{id} and σ_L^{id}. Table 1 summarizes the set of parameters used to solve the model.

Insert Table 1 here

Using the baseline set of model parameters, Figure 3 shows the month-end values of a single simulated path of P, the corresponding firm values, idiosyncratic volatility regimes, realized idiosyncratic return volatilities $IVol$ and realized returns. Panels (a) and (b) reveals that firm value, V_{s_t}, follows a similar pattern to P, as expected. Panels (c) to (d) show that returns and $IVol$ appear to be regime dependent, consistent with Proposition 2.

Insert Figure 3 here

4.1 The Positive Return-Volatility Relation

Using the simulated data, we fit Fama and MacBeth (1973) monthly cross-sectional regressions of log return r_t on $\Delta IVol_t$ in order to investigate if the model can create the positive $IVol$-return relation. The cross sectional regression model for month t is

$$r_t = \gamma_{0,t}\iota + \gamma_{1,t}\Delta IVol_t + \eta_t$$

where ι is a vector of ones, r_t is a vector of $r_{j,t}$ and $\Delta IVol_t$ is a vector of $\Delta IVol_{j,t}$ of all the firms $j \in J$.

Insert Table 2 here

Table 2 reports the results. The table shows that if $\sigma_H^{id} > \sigma_L^{id}$ the model is able to produce the positive $IVol$-return relation. The table also shows that the positive $IVol$ anomaly is more pronounced for larger spreads between σ_H^{id} and σ_L^{id}, but negligible and insignificant if $\sigma_H^{id} = \sigma_L^{id}$, confirming that the stochastic nature of idiosyncratic risk is crucial to generate the anomaly.

4.2 The Negative Return-Volatility Relation

Using the simulated data, we form portfolios based on $IVol$ in order to investigate if the model can create the negative $IVol$-return relation. At the end of each month, we sort firms based on $IVol$ into five equally sized groups. Then, we compute value-weighted one-month portfolio returns for each of the five groups. The portfolios are rebalanced at the end of each month.

Insert Table 3 here

Table 3 reports the results. $IVol$-sorted portfolios are reported across columns. The zero-cost (high minus low) $IVol$ portfolios are reported in the last column. Figure 4 provides a visual illustration of the average returns reported in the table. The zero-cost $IVol$ portfolio has a highly significant and negative average return if $\sigma_H^{id} > \sigma_L^{id}$, with more amplified results for larger spreads between σ_H^{id} and σ_L^{id}. The model offers negligible and insignificant results if $\sigma_H^{id} = \sigma_L^{id}$, confirming

that the stochastic nature of idiosyncratic risk is crucial to generate the anomaly.

Insert Figure 4 here

We conduct further analysis by fitting Fama and MacBeth (1973) monthly cross-sectional regressions of returns on lagged $IVol$. The regression model for month t is

$$r = \gamma_{0,t}\iota + \gamma_{1,t}IVol_{t-1} + \eta_t$$

Table 2 reports the results. There is a negative and highly statistically significant return-lag $IVol$ relation if $\sigma_H^{id} > \sigma_L^{id}$ with more amplified results for larger spreads between σ_H^{id} and σ_L^{id}, but negligible and insignificant result if $\sigma_H^{id} = \sigma_L^{id}$. These results reaffirm the earlier portfolio results.

Taken together, the simulations confirm the prediction that real options and stochastic idiosyncratic operating risk play a significant role in reconciling the two $IVol$ anomalies.

5   Empirical Analysis

In this section, we test the novel empirical predictions of our model. This is in contrast with the previous section, where we showed that the positive and negative $IVol$ anomalies are present in simulated data from our model.

5.1   Data, Variable Descriptions and Summary Statistics

Daily and monthly stock returns are from CRSP. Daily and monthly factor returns and risk-free rates are from Ken French's website.[15] All accounting variables are from annual COMPUSTAT files. Our sample period is from January, 1971 to December, 2010 for all market-based variables.[16] We consider only ordinary shares traded on the NYSE, AMEX and Nasdaq with primary link to companies on COMPUSTAT with US data source. We eliminate utility (SIC codes between 4900 and 4999) and financial companies (SIC codes between 6000 and 6999), companies with less than one year of accounting data, stock price of zero and negative book equity values. In order to

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[15]http:// mba.tuck.dartmouth.edu/ pages/faculty/ken.french/data library.html

[16]The annual number of firms on COMPUSTAT with non-missing sales and net income observations is relatively low prior to the 70's after applying the reported filters.

remove the effects of delisting, we eliminate return observations within one year of delisting if the delisting code has the first digit different from 1. The final sample size is over 1 million monthly observations with non-missing return and idiosyncratic return volatility values.

5.1.1 Idiosyncratic Volatility

Our empirical study requires a measure for the firms' idiosyncratic operating risk. Stock return volatility is commonly used as a proxy for the volatility of the firms' operations (Leahy and Whited (1996); Bulan (2005); Grullon, Lyandres, and Zhdanov (2010)). Following Ang, Hodrick, Xing, and Zhang (2006), for each firm j and month t, we estimate idiosyncratic return volatility $IVol$ as the standard deviation of the daily stock returns relative to the Fama and French 3 factor model:

$$r_{j,\tau} = \alpha_i + \beta_{j,MKT}MKT_\tau + \beta_{j,SMB}SMB_\tau + \beta_{j,HML}HML_\tau + \varepsilon_{j,\tau}$$

where $IVol_{j,t} = \sqrt{var(log(1 + \varepsilon_{j,\tau}))}$ and $\varepsilon_{j,\tau}$ for $\tau \in (t-1, t]$ are the residuals from fitting regression (5.1.1).[17] Furthermore, we define $\Delta IVol_{j,t}$ as the change in $IVol$ from previous month, i.e., $IVol_{j,t} - IVol_{j,t-1}$.

We also require an empirical proxy to capture the variability in the idiosyncratic risk of the firms. Towards this end, for each firm, we consider the stock's 70th and 30th percentile values of $IVol$ to be the thresholds that define the volatility regimes for the firm, and we denote the spread $\overline{\Delta IVol_j}$ to be to be the difference between the 70th and 30th percentile values.

5.1.2 Firm Characteristics

We require several variables shown in the literature to be determinants of stock returns as controls when conducting cross-sectional return regressions. They are: log market equity; log book-to-market; past stock returns; CAPM beta; and trading volume.[18]

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[17]Following Grullon, Lyandres, and Zhdanov (2010), we use the logarithm of the residuals in order to mitigate the potential mechanical effects of return skewness on the relation between return and volatility (Duffee (1995); Chen, Hong, and Stein (2001); Kapadia (2007)).

[18]Following Fama and French (1993), market value of equity is defined as the share price at the end of June times the number of shares outstanding. Book equity is stockholders' equity minus preferred stock plus balance sheet deferred taxes and investment tax credit if available, minus post-retirement benefit asset if available. If missing, stockholders' equity is defined as common equity plus preferred stock par value. If these variables are missing, we use book assets less liabilities. Preferred stock, in order of availability, is preferred stock liquidating value, or preferred stock redemption value, or preferred stock par value. The denominator of the book-to-market ratio is the

5.1.3   Real Option Proxies

We also require empirical proxies for the extent that firms incorporate real options. We follow Grullon, Lyandres, and Zhdanov (2010) in the selection of our main growth option variables, and additionally, create some of our own.

The most common type of real options come in the form of future growth opportunities (Grullon, Lyandres, and Zhdanov (2010); Brennan and Schwartz (1985); MacDonald and Siegel (1986); Majd and Pindyck (1987); Pindyck (1988)). We consider firm size and firm age as inverse measures of growth opportunities because larger and older firms tend to be more mature and have larger proportions of their values from assets-in-place, while smaller and younger firms tend to derive value from future growth opportunities (Brown and Kapadia (2007); Carlson, Fisher, and Giammarino (2004); Lemmon and Zender (2010)). We define two measures of firm size: the book value of total assets and the market value of equity. Age is defined as the difference between the month of the return observation and the month in which the stock first appeared on CRSP.

Growth opportunities are revealed in growth capitalized in the future in the form of increased sales, profits or investments. Therefore, for our third set of growth variables, we define future sales growth as the sum of the sales growth rates starting 2 years and ending 5 years after the stock return observation. Future profit and future investment growth are defined similarly.[19]

We consider a novel proxy for real option intensity. The equity of a firm is akin to a call option on the firm's assets with the strike price amounting to the total value of the firm's debt (Merton (1974) and Merton (1992)). Since the vega of an option captures the option's sensitivity to the

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December closing stock price times the number of shares outstanding. We match returns from January to June of year t with COMPUSTAT-based variables of year $t-2$, while the returns from July until December are matched with COMPUSTAT variables of year $t-1$. This matching scheme is conservative and ensures that the accounting information-based observables are contained in the information set prior to the realization of the market-based variables. We employ the same matching scheme in all our matches involving accounting related variables and CRSP-based variables. We define past returns as the buy-and-hold gross compound returns minus 1 during the six-month period starting from month $t-7$ and ending in month $t-2$. Following Karpoff (1987), trading volume is trading volume normalized by the number of shares outstanding during month t. Lastly, stock CAPM beta is the estimated coefficient from rolling regressions of monthly stock excess returns on the market factor's excess returns. We use a 60-month window every month requiring at least 24 monthly return observations in a given window, and use the procedure suggested in Dimson (1979) with a lag of one month in order to remove biases from thin trading in the estimations.

[19]One caveat with these growth variables is the possibility of look-ahead bias. Following Grullon, Lyandres, and Zhdanov (2010), we are not concerned with potential issues related to look-ahead bias since the focus of our paper is on investigating the relation between return and volatility, and not on predicting future stock returns. Also, we alleviate concerns of spurious correlation between contemporaneous surprises in growth and monthly returns by merging month t returns with growth variables starting two years following the return observation.

volatility of the underlying asset, the relation between $IVol$ and stock returns should be stronger for firms with higher equity vegas. To test this hypothesis, for each firm j and year n, we utilize the firms' capital structure and the Black and Scholes' formula to define the firms' equity vega as follows:

$$vega_{j,n} = V_{j,n}N'(d_{j,n})\sqrt{5}$$

where $d_{j,n} = \frac{ln\left(\frac{V_{j,n}}{D_{j,n}}\right) + \left(r_{f,n} - \frac{\sigma_{j,n}^2}{2} \times 5\right)}{\sigma_{j,n}\sqrt{5}}$, $N'(x) = \frac{exp(-x^2/2)}{\sqrt{2\pi}}$, $r_{f,n}$ is the annualized risk free rate, $\sigma_{j,n}$ denotes firm j's annualized six-month rolling window idiosyncratic volatility based on the Fama French 3 factor model, $V_{j,n}$ denotes the sum of the firm's market equity value and book value of debt, and $D_{j,n}$ is the firm's book value of debt. For simplicity, we assume that firms have a debt maturity of 5 years. Option vegas are relatively invariant over most of the range of possible values for the underlying asset.[20] Therefore, we also classify firms based on equity vega values in relation to the other firms in the sample. To this end, we categorize high vega firms as firms with vegas in the top tercile based on breakpoint values found among NYSE firms in the sample.

We expand the set of proxies for option intensity described thus far by classifying firms as small, young, high sale growth, high investment growth and high profit growth if the corresponding option intensity proxies have values that fall in top or bottom tercile values based on breakpoint values found among NYSE firms in the sample.

Lastly, it is natural to think that firms in certain industries possess more growth options than others, and real option intensity may be captured by the firms' industry membership. Following Grullon, Lyandres, and Zhdanov (2010), we consider three main classifications of industries based on the 49 industries of Fama and French (1997). We define firms with membership in Fama and French (FF) industries 27 (precious metals), 28 (mining), and 30 (oil and natural gas) as natural resource firms. We classify firms in FF industries 22 (electrical equipment), 32 (telecommunications), 35 (computers), 36 (computer software), 37 (electronic equipment), and 38 (measuring and control equipment) as high-tech firms. Membership in FF industries 12 (medical equipment) and 13 (pharmaceutical products) are defined as biotechnology or pharmaceutical firms. Firms with membership in any one of these three industry classifications are defined as all-growth industry

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[20]A call option's vega is greatest when the option is at the money, and relatively low and invariant over the remainder of possible prices for the underlying stock (see Hull (2011)).

firms.

5.1.4 Summary Statitics

Table 4 reports summary statistics for the main variables in our study. Mean (median) excess return in our sample is 0.9976% (-0.41%) per month or about 11.9712% (-4.92%) per year. Mean (median) daily idiosyncratic stock return volatility $IVol$ is 2.9476% (2.2782%) or about 44.0171% (34.0208%) annually. Our $IVol$ estimates are similar to those reported in Ang, Hodrick, Xing, and Zhang (2006) and Grullon, Lyandres, and Zhdanov (2010). Mean (median) month-to-month change in $IVol$ is -.0023% (-0.011%). The standard deviation is 2.1096% and similar to the value reported in Grullon, Lyandres, and Zhdanov (2010).

Insert Table 4 here

5.2 The Switch Effect

To the extent that real options and stochastic idiosyncratic risk are incorporated into firm values, the model predicts that stock returns should correlate positively with $IVol$ during intervals between large changes in $IVol$ (the switch effect hereafter) reflecting the dependence of the options' returns on the volatility regime. That is, post-switch returns should be greater than pre-switch returns for stocks that experience up switches in $IVol$, and lower for stocks that experience down switches in $IVol$.

We use an event study methodology to verify this prediction. To this end, for each firm j and month t, we define an up switch in $IVol$ if $IVol_{j,t-1}$ was below the firm's 30th percentile value and if $IVol_{j,t}$ exceeds the firm's 70th percentile value, capturing the notion of an up switch in idiosyncratic volatility. A down switch event is defined similarly. Once all the up and down switch events are identified for each stock and each month in our sample, we compute the 5-month average return ending in the month prior to the month of the event, and the 5-month average returns beginning from the month after the event. Then we investigate how the difference in average returns around switch months relate to option intensity. More specifically, we risk-adjust

monthly returns according to the Fama and French (1993) 3-factor model

$$r_{j,t}^* = r_{j,t} - r_{f,t} - \sum_{k=1}^{3} \widehat{\beta_{j,k}} F_{k,t}$$

where $r_{j,t}$ is the return on stock j in month t, $r_{f,t}$ is the risk-free rate, and $F_{k,t}$, $k \in [1,3]$, denote the three Fama and French factors (market, size, and book-to-market factors).[21] Each month, we estimate the factor loadings $\widehat{\beta_{j,k}}$ for each stock using monthly rolling regressions with a 60-month window requiring at least 24 monthly return observations. The regressions use the Dimson approach with a lag of one month in order to remove biases from thin trading in the estimations (Dimson (1979)). Then, for each firm j and event month t, the difference in 5-month average returns is computed as follows:

$$r_{j,t}^{Diff} = \frac{1}{5} \sum_{\tau=t+1}^{t+6} r_{j,\tau}^* - \frac{1}{5} \sum_{\tau=t-6}^{t-1} r_{j,\tau}^*$$

We run separate Fama MacBeth cross-sectional return regressions for each real option proxy and for each of the up and the down switch samples. The regression model for month t is

$$r_t^{Diff} = \gamma_0 \iota + \gamma_1 RO_{t-1} + \eta_t$$

where r_t^{Diff} is a vector of differences in average returns around the switch month t, ι is a vector of ones, and RO_{t-1} is a vector of real option intensity values. Our model's predictions translate to tests that $\gamma_0 > 0$ and $\gamma_1 > 0$ (or $\gamma_1 < 0$ for inverse RO proxies) for the up switch sample, and $\gamma_0 < 0$ and $\gamma_1 < 0$ (or $\gamma_1 > 0$ for inverse RO proxies) for the down switch sample.

Insert Table 5 here

Table 5 reports the results. The estimates of $\gamma_0 > 0$ are positive for the up switch sample and negative for the down switch sample, and highly statistically significant in all specifications, offering evidence in agreement with the switch effect. The table also shows that the estimates

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[21]The results using unadjusted returns are available from the authors upon request, but they are not materially different from the results using risk-adjusted returns.

of γ_1 on total asset size, market equity value and age are positive for the down switch sample, and negative for the up switch sample, highlighting a positive correspondence between the switch effect and real option intensity.

Using categorical proxies for real option intensity offers consistent results with greater significance for the up switch sample than for the down switch sample. The exception is when the high vega dummy is used as a proxy, whose coefficient estimate is positive and significant for the down switch sample. However, the estimate for the combined small and high vega dummy is significant and consistent with the model's predictions for both the up and down switch samples. The coefficient estimate for the combined young and high vega dummy is also in favor of the model predictions for the up switch sample, while it lacks statistical significance for the down switch sample. Based on these results, we argue that equity vega alone is not a strong measure for real options unless it is combined with other proxies such as size and age. [22]

Using industry dummies as proxies for option intensity offers consistent results as well. While natural resources, high tech or bio tech industries alone do not offer statistically significant estimates, the all-growth option industry dummy offers an estimate consistent with the switch effect for the up switch sample.

Next, we investigate how the switch effect relates to the variability in $IVol$ (spread) and the interaction between option intensity and spread. The regression model for month t is

$$r_t^{Diff} = \gamma_0 \iota + \gamma_1 \overline{\Delta IVol} + \gamma_2 \overline{\Delta IVol} \times RO_{t-1} + \eta_t$$

where r_t^{Diff}, ι and RO_{t-1} are as defined previously, and $\overline{\Delta IVol}$ is a vector of $\overline{\Delta IVol_j}$. Our model's predictions translate to tests that $\gamma_1 > 0$ and $\gamma_2 > 0$ (or $\gamma_2 < 0$ for inverse RO proxies) for the up switch sample, and $\gamma_1 < 0$ and $\gamma_2 < 0$ (or $\gamma_2 > 0$ for inverse RO proxies) for the down switch sample.

Table 6 reports the results. The table shows that the coefficient estimates for $\overline{\Delta IVol}$ is positive for the up switch sample and negative for the down switch sample with statistically significant results in virtually all of the regression specifications. Hence, the switch effect is strongest among

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[22]One way to view these results is that the levered equity of smaller and younger firms experience greater reactions to changes in operating risk than larger and more mature firms.

stocks that experience greater variability in $IVol$, consistent with our model's predictions.

The coefficient estimates for the interaction term between $\overline{\Delta IVol}$ and RO also support the model's predictions. The sign of the estimates are as predicted for age and size if measured as total assets in the up switch sample, while only size is significant in the down switch sample. The dummies for high future profit, sales and investment growth and their combinations with the small dummy all have positive estimates for the up switch sample with varying levels of significance. For the down switch sample, the estimates are not significant. As for the industry dummies, they are not statistically significant. A possible reason for this may be that industry classifications alone are weak proxies for real option intensity since firms within industries may vary widely in real option intensity. We conclude from these results that there is strong evidence for the switch effect which is more pronounced for more real option firms and which have more variable $IVol$, consistent with our model's predictions.

5.3   Positive $IVol$-Return Relation

To the extent that firm valuations reflect real options and are subject to changes in idiosyncratic volatility, our model predicts that the positive $IVol$ anomaly should be stronger for more option intensive firms and firms that experience larger changes in idiosyncratic volatility. In this section, we empirically test this prediction and provide supporting evidence.

We start by revisiting Grullon, Lyandres, and Zhdanov (2010) and estimating monthly return Fama and MacBeth (1973) regressions on changes in idiosyncratic volatility and growth option intensity. The regression model for month t is

$$r_t - r_{f,t} = \gamma_0 \iota + \gamma_1 \Delta IVol_t + \gamma_2 \Delta IVol_t \times RO_{t-1} + \gamma_3 X_{t-1} + \eta_t$$

where r_t, $r_{f,t}$, ι, $\Delta IVol_t$, RO_{t-1} are as defined before, and X_{t-1} is a matrix with columns of vectors of controls for firm size, book-to-market, past returns, trading volume and stock beta. Our model predictions translate to tests that $\gamma_1 > 0$ and $\gamma_2 > 0$ ($\gamma_2 < 0$ for inverse RO proxies)

Insert Table 7 here

Table 7 reports the results. Unsurprisingly, the coefficient estimates for stock beta and log book-to-market are both significantly positive, while the coefficient for log size are significantly negative in all specifications. The coefficient for trading volume is highly significant and positive, consistent with Karpoff (1987) and Grullon, Lyandres, and Zhdanov (2010). The coefficient for the past six month cumulative returns is insignificant and negative in all specifications, and consistent with some specifications reported in Cooper, Huseyin, and Schill (2008) and Grullon, Lyandres, and Zhdanov (2010).[23]

The table also reports a highly significant and positive $IVol$-return relation ($\gamma_0 > 0$) for all specifications. As for the relation with respect to option proxies, firm size (both equity market value and total asset value) offers highly significant and negative estimates of γ_2. While γ_2 has the predicted negative sign for age, it is not statistically significant.

Focusing on the categorical variables for real option intensity, the γ_2 estimates for the high equity vega dummy is positive and highly statistically significant. This result is interesting because equity vega is the only proxy for real option intensity that is not necessarily related to growth. The high investment and high sales growth dummies also offer similar results. While the high profit growth dummy estimate is not significant, the combined high profit growth and small dummy has a highly significant and positive γ_2 estimate. Similar results apply to the combined dummies for high investment growth, high sales growth, and high equity vega dummies when combined with the small size dummy, implying that combining option proxies may capture real option intensity better.

Focusing on the industry dummies, while the γ_2 estimates are positive for natural resources, high technology and bio technology firms, only the natural resources industry dummy offers statistically significant results. However, the all-growth industry dummy has a highly significant and positive estimate in line with the predictions. In sum, the results thus far support a positive $IVol$-return relation that is stronger for more option intensive firms. These results are consistent with the findings in Grullon, Lyandres, and Zhdanov (2010) as well as our model.

Next, our model also predicts that the positive $IVol$-return relation should be stronger for more real option intensive firms and firms with larger spreads in idiosyncratic volatility. We test

[23]Grullon, Lyandres, and Zhdanov show that the coefficient on past returns is sensitive to the set of other independent factors included in Fama Macbeth regressions.

this hypothesis with the following regression:

$$r_t - r_{f,t} = \gamma_0 \iota + \gamma_1 \overline{\Delta IVol} + \gamma_2 \Delta IVol_t + \gamma_3 \overline{\Delta IVol} \times \Delta IVol \times RO + \gamma_3 X_{t-1} + \eta_t$$

where r_t, $r_{f,t}$, ι, $\Delta IVol_t$, RO_{t-1}, X_{t-1} and $\overline{\Delta IVol}$ are as defined previously. Our model prediction translates to tests that $\gamma_3 > 0$ (or $\gamma_3 < 0$ for inverse RO proxies).

Insert Table 8 here

The results are reported in Table 8. The table reports estimates of γ_3 that are highly significant and positive (negative for inverse real option proxies) for virtually all the regression. The only exceptions apply when age and the dummies for young, small and young, and young and high vega are used where the results are statistically insignificant. The remainder of the table reports the results for the industry dummies. While γ_3 estimates are positive for natural resources, high technology and bio technology firms, only natural resources offers significant results. However, the all-growth industry dummy offers highly significant results in line with the predictions.

Collectively, the results are in strong agreement with our model. The positive $IVol$-return relation is more pronounced for more real option firms and firms that experience larger variability in idiosyncratic volatility.

5.4 The Negative $IVol$-Return Relation

Ang, Hodrick, Xing, and Zhang (2006) report that portfolios of high $IVol$ stocks significantly under-perform their low $IVol$ counterparts. Our model predicts that this negative $IVol$-return relation should be more pronounced for more real option intensive firms and firms with larger $IVol$ spreads. We test this prediction and provide empirical support in this section.

At the end of each June, we sort and rank firms into three equally-sized groups based on each one of our real option proxies. For categorical variables, firms are separated into two groups. We merge the rankings with monthly $IVol$ and stock returns, and for each month, we sort and rank stocks into three equally-sized groups based on $IVol$. Then we compute value-weighted portfolio returns for each of the two-way classifications of $IVol$ and option intensity and assess

their performance over the following month. All portfolios are rebalanced monthly. This approach corresponds to the 1/0/1 (formation period/waiting period/holding period) strategy of Ang, Hodrick, Xing, and Zhang (2006) which most of their analysis is concentrated on.

The performance of the portfolios are assessed on a risk-adjusted basis relative to the Fama and French 3 factor model:

$$r_t - r_{f,t} = \gamma_0 + \gamma_1 MKTRF_t + \gamma_2 SMB_t + \gamma_3 HML_t + \epsilon_t$$

where r_t is the portfolio return, $r_{f,t}$ is the riskless rate, $MKTRF$, SMB, and HML are the Fama and French (1993) three factors that proxy for the market risk premium, size and book-to-market factors respectively. In order to investigate the extent to which real option intensity contributes to the negative $IVol$ anomaly, we also estimate the regression for the zero-cost $IVol$ portfolios for each rank of option intensity.[24] A larger intercept γ_0 estimate translates to a greater average risk-adjusted return.

Insert Table 9 here

Tables 9 to 10 report results. Each panel of the tables corresponds to a different real option proxy, with $IVol$ ranks reported across columns. The last column of each panel reports the estimates for the zero-cost $IVol$ portfolios. The real option ranks are listed down the rows. The reported estimates are annualized to facilitate the interpretation of the economic significance. All other reported figures are unadjusted.

Table 9 reveals that the negative $IVol$ anomaly is more pronounced and statistically more significant for the two lowest firm size groups according to total asset value. The negative $IVol$ anomaly for the largest group is not significant. Size according to market equity value and age offer similar patterns, lending strong support for our model predictions. The anomaly is also stronger for high equity vega firms than for low vega firms. This finding is enlightening because equity vega is the only option proxy that is not necessarily related to future growth opportunities.

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[24]For the zero-cost $IVol$ portfolios, we use portfolio returns instead of portfolio excess returns on the left hand side of regression (5.4).

Insert Table 10 here

The evidence for the negative $IVol$ anomaly is even stronger among small and high equity vega firms than for high equity vega firms alone. Hence, evidence for the negative $IVol$ anomaly is even stronger when proxies for real option intensity are combined, lending credence to our option based-explanation for the anomaly. The other panels point to that conclusion as well. While the negative $IVol$ anomaly is not conclusively stronger for high profit, high sale or high investment growth firms, it is stronger for these firms if they are also small in size, and similarly for younger firms and firms that are younger and have high equity vega.

As for the negative $IVol$ anomaly in relation to the firms' industries, the negative $IVol$ anomaly is more pronounced for natural resources and high technology stocks, while bio tech and all growth-industries offer inconclusive evidence. As mentioned earlier, industry membership alone may be a weak proxy for real option intensity because firms within industries can vary widely in their real option intensity. In sum, we find that there is considerable evidence that the $IVol$-return relation relates to real option intensity.

Next we investigate how the negative $IVol$-return relation relates to the spread in idiosyncratic volatility. In addition to the two-way independent sorts based on $IVol$ and each of the real option proxies, we independently sort stocks into three equally-sized groups based on $\overline{\Delta IVol}$. Then, for each of the two-way rank classifications of real option intensity and $\overline{\Delta IVol}$, we assess the value-weighted returns of the zero-cost $IVol$ portfolios relative to the Fama and French 3 factor model.

Insert Table 11 here

Tables 11 and 12 report the results. The negative $IVol$ anomaly is monotonically stronger and more significant for the top $\overline{\Delta IVol}$ group for the size and age proxies. The table also shows that the negative $IVol$ anomaly is stronger among the youngest firms and firms that have the largest $\overline{\Delta IVol}$. These results support our predictions that the negative $IVol$ anomaly should be more pronounced for growth firms that experience more extreme changes in $IVol$. The table also reveals that the negative $IVol$ anomaly seems to be more pronounced for larger firms among

the top $\overline{\Delta IVol}$ stocks. While these results are not in direct support of our model, the negative $IVol$-return remains both statistically and economically significant for small firms.

The main conclusions are similar for high profit, high sale and high investment growth firms. While there is a stronger negative $IVol$-return relation for the high $\overline{\Delta IVol}$ stocks independently of the real option characteristics, the anomaly seems to be weaker for high future growth firms. One reason for these findings may be that the negative $IVol$-return relation could be confounded by the positive returns of high future growth stocks. This is likely to be the case if information on high future growth is reflected in stock returns during the portfolio evaluation period.

Insert Table 12 here

Now focusing on the combined real option proxies, Table 12 shows that the negative $IVol$-return relation is stronger for small and high growth, small and young, and small and high equity vega firms. Hence, the negative $IVol$ anomaly is more evidently related with real options and $\overline{\Delta IVol}$ for the combined real option proxies. In relation to industry membership, the table shows that the negative $IVol$ anomaly is monotonically stronger and statistically more significant for larger $\overline{\Delta IVol}$ independently of industry membership. Natural resources, bio tech and all-growth industry firms within the high $\overline{\Delta IVol}$ have stronger negative $IVol$-return relation, lending support for our model predictions. While high tech stocks exhibit a weaker $IVol$-return relation than low tech stocks within the high $\overline{IVol}$ group, the anomaly still remains significant for the high tech stocks.

Overall, these results demonstrate that the stocks of firms that experience more extreme changes in $IVol$ and incorporate more real options exhibit stronger $IVol$-return relations, lending support for our model.

6 Conclusions

Recent empirical evidence on the correspondence between stock returns and idiosyncratic return volatility at the firm-level have been mixed at best. Some work shows that stock returns and idiosyncratic volatility are contemporaneously positively correlated (Duffee (1995)), the positive $IVol$ anomaly. But other work shows that portfolios of idiosyncratic volatility stocks (Ang, Hodrick, Xing, and Zhang (2006)) underperform their low idiosyncratic volatility counterparts, the negative $IVol$ anomaly.

In this paper, we propose a novel economic explanation for the conflicting findings, which hinges on a simple equity valuation model of a cross-section of firms involving growth options and stochastic idiosyncratic operating risk. More generally, we motivate why empirical work can indicate that idiosyncratic risks are priced in the cross-section of stock returns, even though the CAPM holds.

We introduce a 2-regime Markov switching process (see Guo, Miao, and Morellec (2005) and Hackbarth, Miao, and Morellec (2006)) to incorporate uncertainty in idiosyncratic operating risk. Growth option value is convex in the output price and it does not distinguish between systematic and idiosyncratic risks, a feature that contrasts starkly with the firm's assets-in-place. This gives rise to regime dependency of the firm's equity returns. In particular, we show that two key properties hold, which are sufficient conditions for the positive and negative $IVol$ anomalies to hold. The first property is that returns and idiosyncratic volatility are exposed to the same risk factor, a property which arises immediately in a model of growth options where idiosyncratic operating risk is stochastic and drives the positive $IVol$ anomaly. The second property is that the systematic volatility of returns falls when idiosyncratic risk rises. Idiosyncratic risk is by definition non-systematic. Hence, an increase in a firm's growth option value driven by a rise in idiosyncratic risk leads to a a fall in the proportion of the growth option value exposed to systematic risk. The firm's systematic risk then falls as systematic risk, which is precisely the second key property and leads to the negative $IVol$ anomaly.

In summary, the time-series dynamics of the volatility structure in our model results in an interplay between returns and idiosyncratic return volatility consistent with what has been observed empirically in the cross-section of stocks. We verify our theoretical work with numerical

simulations. To check that the underlying economics of our model is plausible, we empirically test the model's novel implications. Our empirical work shows that the positive and negative $IVol$ anomalies are more pronounced for subsets of firms with more real options or which experience larger shifts in idiosyncratic volatility, which is supportive of our theoretical model.

Previous literature has relied on illiquidity and other market microstructure related explanations for the distributional properties of stock returns related to heteroskedasticity, discontinuities or jumps, and heavy tails. Our model has the capability to parsimoniously generate these features in return distributions from the operating environment that firms face, providing fertile grounds for additional research.[25] Further research in this direction is highly merited.

Finally, our model suggests that jumps in stock returns should coincide with large changes in idiosyncratic return volatility in predictable ways, potentially shedding new insights on the three-way relation between stock returns, idiosyncratic return volatility and expected return skewness (Boyer, Mitton, and Vorkink (2010)). Furthermore, the features of our model may help establish predictability akin to return continuation amenable with the findings of Jegadeesh and Titman (1993), and reversals reported in Jegadeesh (1990). We leave these other interesting extensions for future research.

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[25]The literature has recognized that asset returns must exhibit both stochastic volatility and discontinuous jumps to fit their empirical distributions (Das and Sundaram (1999))

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7 Appendix

This Section provides proofs of the propositions and results stated in the paper.

Proof of Proposition 1

Observe that

$$G_{i,t} = E_t^{\mathbb{Q}}\left[e^{-r(\tau-t)}(\xi\,A_\tau - I)|s_t = i\right].$$

The basic asset pricing equation states that

$$E_t^{\mathbb{Q}}[dG_{i,t} - G_{i,t}rdt|s_t = i] = 0, t \leq \tau. \tag{7.1}$$

We shall use $p_t = \ln P_t$ as the state variable. Under $\mathbb{Q}$, we have (using Ito's Lemma and Girsanov's Theorem)

$$dp_t = \hat{\mu}_{p,i}dt + \sigma_i d\hat{B}_t,$$

where

$$\hat{\mu}_{p,i} = \hat{\mu} - \frac{1}{2}\sigma_i^2$$

$$\sigma_i = \sqrt{(\sigma_i^{id})^2 + (\sigma^{sys})^2}$$

$$d\hat{B}_t = \frac{\sigma_i^{id}}{\sigma_i}dB_t^{id} + \frac{\sigma^{sys}}{\sigma_i}d\hat{B}_t^{sys}.$$

Hence, (7.1) implies the following ordinary differential equation system

$$\frac{1}{2}\sigma_i^2\partial_{p_t}^2 G_{i,t} + \hat{\mu}_{p,i}\partial_{p_t}G_{i,t} - rG_{i,t} + \lambda_j(G_{j,t} - G_{i,t}) = 0, \ i,j \in \{L,H\}, \ j \neq i, \ t \leq \tau,$$

where $\partial_{p_t} = \frac{\partial}{\partial p_t}$. We shall use the following notation: I is the 2 by 2 identity matrix, 0_2 is the 2 by 1 vector of zeros, $diag(a_1,a_2)$ is the 2 by 2 diagonal matrix with a_1, a_2 along the diagonal, $\mathcal{D}_p^i = \frac{1}{2}\sigma_i^2\partial_{p_t}^2 + \hat{\mu}_{p,i}\partial_{p_t}$, where $\hat{\mu}_{p,i} = \hat{\mu} - \frac{1}{2}\sigma_i^2$, $D = diag(\mathcal{D}_p^L, \mathcal{D}_p^H)$, Λ is the generator matrix of the 2 state Markov chain, i.e. $\Lambda = \begin{pmatrix} -\lambda_H & \lambda_H \\ \lambda_L & -\lambda_L \end{pmatrix}$. Thus, we can write the ordinary differential equation system in matrix form as

$$(D - rI_2 + \Lambda)\begin{pmatrix} G_L \\ G_H \end{pmatrix} = 0_2. \tag{7.2}$$

When $p < p_L^*$, $t \leq \tau$, and so (7.2) holds. The general solution is given by

$$G_i = \sum_{m=1}^{4} h_{im}e^{l_m p},$$

where $l_1 > l_2 > 0 > l_3 > l_4$ are the roots of the quartic in (3.5).

To ensure that G_i $i \in \{L,H\}$ are finite as $p \to -\infty$, we set $h_{i3} = h_{i4} = 0$, $i \in \{L,H\}$, so we use only

the two positive roots: $l_1 > l_2 > 0$.

$$G_i = \sum_{m=1}^{2} h_{im} e^{l_m p},$$

Substituting the above expression into (7.2) gives

$$(D - rI_2 + \Lambda) \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \begin{pmatrix} e^{l_1 p} \\ e^{l_2 p} \end{pmatrix} = 0_2.$$

Simplifying gives

$$\sum_{m=1}^{2} e^{l_m p} \left[h_{Lm} \left(\frac{1}{2} \sigma_L^2 l_m^2 + \hat{\mu}_{p,L} l_m - (r + \lambda_H) \right) + h_{Hm} \lambda_H \right] = 0$$

$$\sum_{m=1}^{2} e^{l_m p} \left[h_{Hm} \left(\frac{1}{2} \sigma_H^2 l_m^2 + \hat{\mu}_{p,H} l_m - (r + \lambda_L) \right) + h_{Lm} \lambda_L \right] = 0.$$

Consequently

$$\frac{h_{Hm}}{h_{Lm}} = -\frac{\frac{1}{2} \sigma_L^2 l_m^2 + \hat{\mu}_{p,L} l_m - (r + \lambda_H)}{\lambda_H} = -\frac{\lambda_L}{\frac{1}{2} \sigma_H^2 l_m^2 + \hat{\mu}_{p,H} l_m - (r + \lambda_L)}.$$

Thus

$$h_{Hm} = \epsilon(l_m) h_{Lm},$$

where

$$\epsilon(l) = -\frac{q_L(l)}{\lambda_H} = -\frac{\lambda_L}{q_H(l)}.$$

Hence

$$G_L = \sum_{m=1}^{2} h_{Lm} e^{l_m p}$$

$$G_H = \sum_{m=1}^{2} \epsilon(l_m) h_{Lm} e^{l_m p}.$$

Since $e^p = P$, we have

$$G_L = \sum_{m=1}^{2} h_{Lm} P^{l_m}$$

$$G_H = \sum_{m=1}^{2} \epsilon(l_m) h_{Lm} P^{l_m}.$$

In the region $p \in [p_L^*, p_H^*)$, (7.2) still holds, but we also know that investment is immediate if the

volatility state switches from high to low. Hence $G_{L,t} = \xi A_t - I$ just after the switch has occurred, and so

$$\mathcal{D}_p^L G_H - r G_H + \lambda_L \left(\xi \left(\frac{e^p}{r - \hat{\mu}} - \frac{c}{r} \right) - G_L \right) = 0.$$

The general solution of the above ordinary differential equation is given by

$$G_{H,t} = \frac{\lambda_L}{r + \lambda_L} (\xi A_t - I) + \sum_{m=1}^{2} s_m e^{j_m p},$$

where j_m, $m \in \{1, 2\}$ are the roots of the quadratic (3.7).

In the region $p \geq p_H^*$, investment will have occurred, and so

$$G_{L,t} = \xi A_t, \ G_{H,t} = \xi A_t.$$

At the time of investment the above expressions need to be adjusted downward by subtracting the one-off investment cost of I.

Value matching at $p = p_L^*$ for G_L implies that

$$\lim_{p \uparrow p_L^*} G_L = \lim_{p \downarrow p_L^*} G_L.$$

Value matching and smooth pasting at $p = p_L^*$ for G_H implies that

$$\lim_{p \uparrow p_L^*} G_H = \lim_{p \downarrow p_L^*} G_H,$$

$$\lim_{p \uparrow p_L^*} \frac{\partial G_H}{\partial p} = \lim_{p \downarrow p_L^*} \frac{\partial G_H}{\partial p}.$$

Value matching at $p = p_H^*$ for G_H implies that

$$\lim_{p \uparrow p_H^*} G_H = \lim_{p \downarrow p_H^*} G_H.$$

The above four boundary conditions allow us to determine the 4 constants of integration, h_{L1}, h_{L2}, s_1, s_2 in terms of the investment thresholds P_i^*, $i \in \{L, H\}$ and exogenous variables. In addition we have the two smooth pasting conditions, which ensure optimality of the investment thresholds:

$$\sum_{m=1}^{2} h_{Lm} l_m (P_L^*)^{l_m - 1} = \frac{\xi}{r - \hat{\mu}}$$

$$\sum_{m=1}^{2} j_m (P_H^*)^{j_m - 1} + \frac{\lambda_L}{r + \lambda_L} \xi = \frac{\xi}{r - \hat{\mu}}$$

In total, we have 6 equations in 6 unknowns:

$$\sum_{m=1}^{2} h_{Lm} (P_L^*)^{l_m} = \xi \left(\frac{P_L^*}{r - \hat{\mu}} - \frac{c}{r} \right) - I \tag{7.3}$$

$$\sum_{m=1}^{2} \epsilon(l_m) h_{Lm} (P_L^*)^{l_m} - \sum_{m=1}^{2} s_m (P_L^*)^{j_m} = \frac{\lambda_L}{r + \lambda_L} \left[\xi \left(\frac{P_L^*}{r - \hat{\mu}} - \frac{c}{r} \right) - I \right] \tag{7.4}$$

$$\sum_{m=1}^{2} \epsilon(l_m) h_{Lm} l_m (P_L^*)^{l_m} - \sum_{m=1}^{2} s_m j_m (P_L^*)^{j_m} = \frac{\lambda_L}{r + \lambda_L} \xi \frac{P_H^*}{r - \hat{\mu}} \tag{7.5}$$

$$\sum_{m=1}^{2} s_m (P_H^*)^{j_m} = \frac{r}{r + \lambda_L} \left[\xi \left(\frac{P_H^*}{r - \hat{\mu}} - \frac{c}{r} \right) - I \right] \tag{7.6}$$

$$\sum_{m=1}^{2} h_{Lm} l_m (P_L^*)^{l_m} = \xi \frac{P_L^*}{r - \hat{\mu}} \tag{7.7}$$

$$\sum_{m=1}^{2} s_m j_m (P_H^*)^{j_m} = \xi \left(\frac{1}{r - \hat{\mu}} - \frac{\lambda_L}{r + \lambda_L} \right) P_H^*. \tag{7.8}$$

Solving (7.3), (7.5), (7.6), (7.7), (7.8) for h_{L1}, h_{L2}, s_1, s_2 gives:

$$h_{L1} = \frac{(P_L^*)^{-l_1}}{l_1 - l_2} \left\{ \frac{\xi P_L^*}{r - \hat{\mu}} - l_2 \left[\xi \left(\frac{P_L^*}{r - \hat{\mu}} - \frac{c}{r} \right) - I \right] \right\},$$

$$h_{L2} = \frac{(P_L^*)^{-l_2}}{l_1 - l_2} \left\{ l_1 \left[\xi \left(\frac{P_L^*}{r - \hat{\mu}} - \frac{c}{r} \right) - I \right] - \frac{\xi P_L^*}{r - \hat{\mu}} \right\},$$

$$s_1 = \frac{(P_H^*)^{-j_1}}{j_1 - j_2} \left\{ \xi P_H^* \left(\frac{1}{r - \hat{\mu}} - \frac{\lambda_L}{r + \lambda_L} \right) - j_2 \frac{r}{r + \lambda_L} \left[\xi \left(\frac{P_H^*}{r - \hat{\mu}} - \frac{c}{r} \right) - I \right] \right\},$$

$$s_2 = \frac{(P_H^*)^{-j_2}}{j_1 - j_2} \left\{ j_1 \frac{r}{r + \lambda_L} \left[\xi \left(\frac{P_H^*}{r - \mu} - \frac{c}{r} \right) - I \right] + \xi P_H^* \left(\frac{\lambda_L}{r + \lambda_L} - \frac{1}{r - \mu} \right) \right\}.$$

We hence obtain the expressions in (3.4) and (3.6). Having determined the constants of integration we can rewrite (7.4) and (7.5) as (3.8) and (3.9). ∎

Derivation of (3.10)

The return on assets-in-place is given by $dR_{A,t} = \frac{dA_t + (P_t - c)dt}{A_t}$. Applying Ito's Lemma gives (3.10). ∎

Proof of Proposition 2

The return on a young firm's growth option is given by

$$\frac{dG_{s_{t-}}}{G_{s_{t-}}} = \left(\Omega_{s_{t-}} \frac{dP_t}{P_t} + \frac{1}{2} \frac{P_t^2}{G_{s_{t-}}} \frac{\partial^2 G_{s_{t-}}}{\partial P_t^2} \right) + \frac{G_{s_t} - G_{s_{t-}}}{G_{s_{t-}}} dN^{id}_{s_{t-} s_t, t},$$

where $\Omega_{s_{t-}}$ and $dN^{id}_{s_{t-} s_t, t}$ are defined in (3.13) and (3.14), respectively. Now

$$\frac{dG_{s_{t-}}}{G_{s_{t-}}} = \Omega_{s_{t-}} (\mu dt + \sigma^{sys} dB_t^{sys} + \sigma^{id}_{s_{t-}} dB_t^{id}) + \frac{1}{2} \frac{P_t^2}{G_{s_{t-}}} \frac{\partial^2 G_{s_{t-}}}{\partial P_t^2} \left((\sigma^{sys})^2 + (\sigma^{id}_{s_{t-}})^2 \right) dt$$

$$+ \frac{G_{s_t} - G_{s_{t-}}}{G_{s_{t-}}} dN^{id}_{s_{t-} s_t, t}$$

$$= \left(\Omega_{s_{t-}} \mu + \frac{G_{s_t} - G_{s_{t-}}}{G_{s_{t-}}} \lambda_{s_{t-} s_t} + \frac{1}{2} \frac{P_t^2}{G_{s_{t-}}} \frac{\partial^2 G_{s_{t-}}}{\partial P_t^2} \left((\sigma^{sys})^2 + (\sigma^{id}_{s_{t-}})^2 \right) \right) dt$$

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$$+ \Omega_{s_{t-}} (\sigma^{sys} dB_t^{sys} + \sigma_{s_{t-}}^{id} dB_t^{id}) + \frac{G_{s_t} - G_{s_{t-}}}{G_{s_{t-}}} dM_{s_{t-} s_t, t}^{id}.$$

Now observe that for $P_t^{\max} < P_L^*$

$$\Omega_L = w_L(P)l_1 + (1 - w_L(P))l_2,$$

where

$$w_L(P) = \frac{\delta_L(l_2) \left(\frac{P_t}{P_L^*}\right)^{l_1}}{\delta_L(l_2) \left(\frac{P_t}{P_L^*}\right)^{l_1} - \delta_L(l_1) \left(\frac{P_t}{P_L^*}\right)^{l_2}},$$

and

$$\Omega_H = w_H(P)l_1 + (1 - w_H(P))l_2,$$

where

$$w_H(P) = = \frac{\epsilon(l_1)\delta_L(l_2) \left(\frac{P_t}{P_L^*}\right)^{l_1}}{\epsilon(l_1)\delta_L(l_2) \left(\frac{P_t}{P_L^*}\right)^{l_1} - \epsilon(l_2)\delta_L(l_1) \left(\frac{P_t}{P_L^*}\right)^{l_2}}.$$

Note that

$$w_H(P) - w_L(P) = \frac{\delta_L(l_1)\delta_L(l_2) \left(\frac{P}{P_L^*}\right)^{l_1+l_2} (\epsilon(l_2) - \epsilon(l_1))}{\left(\epsilon(l_1)\delta_L(l_2) \left(\frac{P_t}{P_L^*}\right)^{l_1} - \epsilon(l_2)\delta_L(l_1) \left(\frac{P_t}{P_L^*}\right)^{l_2}\right) \left(\delta_L(l_2) \left(\frac{P_t}{P_L^*}\right)^{l_1} - \delta_L(l_1) \left(\frac{P_t}{P_L^*}\right)^{l_2}\right)}$$

$$= \frac{\delta_L(l_1)\delta_L(l_2) \left(\frac{P}{P_L^*}\right)^{l_1+l_2} (q_L(l_1) - q_L(l_2))}{\lambda_H \left(\epsilon(l_1)\delta_L(l_2) \left(\frac{P_t}{P_L^*}\right)^{l_1} - \epsilon(l_2)\delta_L(l_1) \left(\frac{P_t}{P_L^*}\right)^{l_2}\right) \left(\delta_L(l_2) \left(\frac{P_t}{P_L^*}\right)^{l_1} - \delta_L(l_1) \left(\frac{P_t}{P_L^*}\right)^{l_2}\right)}.$$

After some tedious algebra, we can show that $l_1 > l_2 > 0$ implies $q_L(l_1) > 0 > q_L(l_2)$, which in turn implies $w_L(P) < 0 < w_H(P)$, and so $\Omega_L > \Omega_H$ when $P_t^{\max} < P_L$.

Using the same approach, we can show that the fall in the growth option elasticity caused by a rise in idiosyncratic volatility is proportionally smaller than the idiosyncratic volatility increase, and so $\sigma_L \Omega_L < \sigma_L \Omega_H$, and also

$$\Omega_H \mu + \frac{1}{2} \frac{P_t^2}{G_H} \frac{\partial^2 G_H}{\partial P_t^2} \left((\sigma^{sys})^2 + (\sigma_H^{id})^2\right) > \Omega_L \mu + \frac{1}{2} \frac{P_t^2}{G_L} \frac{\partial^2 G_L}{\partial P_t^2} \left((\sigma^{sys})^2 + (\sigma_L^{id})^2\right).$$

∎

Proof of Proposition 3

The derivation of (3.15) is trivial: it merely states that the a young firm's systematic volatility is the weighted average of the systematic volatility of its assets-in-place and growth option.

To prove, (3.16) observe that

$$G_{s_{t-}} \Omega_{s_{t-}} = P_t \frac{\partial G_{s_{t-}}}{\partial P_t}.$$

The same argument we used to prove $\Omega_L > \Omega_H$ when $P_t^{\max} < P_L$ also implies $G_L \Omega_L > G_H \Omega_H$. Since $G_H > G_L$ (the standard option pricing result that volatility increases option value), we also have $V_{Y,H} > V_{Y,L}$. Hence, $\frac{G_L}{V_L} \Omega_L > \frac{G_H}{V_H} \Omega_H$. Since assets-in-place are independent of idiosyncratic volatility and $G_H > G_L$, we have $\frac{G_H}{V_H} > \frac{G_L}{V_L}$. Hence

$$\frac{G_H}{V_H} \Omega_H - \frac{G_L}{V_L} \Omega_L + (1 + L(P)) \left(\frac{G_L}{V_L} - \frac{G_H}{V_H} \right) < 0,$$

which implies that $\sigma_{R_Y,H}^{sys} < \sigma_{R_Y,L}^{sys}$. ∎

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Figure 1: Model's Properties and Solution: Dependence of Growth Option Value on the Idiosyncratic Volatility Regime

The figure shows growth option values for various values of P across idiosyncratic volatility regimes and the exercise thresholds P_l^* and P_h^*. The solid 45 degree line corresponds to the intrinsic value of the growth option. Option values in the high and low volatility states are depicted by dashed and dashed dotted curves, respectively. The exercise thresholds are depicted by the vertical dotted lines where the lower threshold corresponds to the exercise threshold P_l^*, and the higher threshold corresponds to the exercise threshold P_h^*. Panel (a) corresponds to the model solution with parameters $\sigma_H^{id} = 0.5, \sigma_L^{id} = 0.1$, panel (b) corresponds to the model solution with parameters $\sigma_H^{id} = 0.4, \sigma_L^{id} = 0.2$, and panel (c) corresponds to the model solution with parameters $\sigma_H^{id} = 0.3, \sigma_L^{id} = 0.3$.

(a) $\sigma_H^{id} = 0.5$, $\sigma_L^{id} = 0.1$ (b) $\sigma_H^{id} = 0.4$, $\sigma_L^{id} = 0.2$

(c) $\sigma_H^{id} = 0.3$, $\sigma_L^{id} = 0.3$
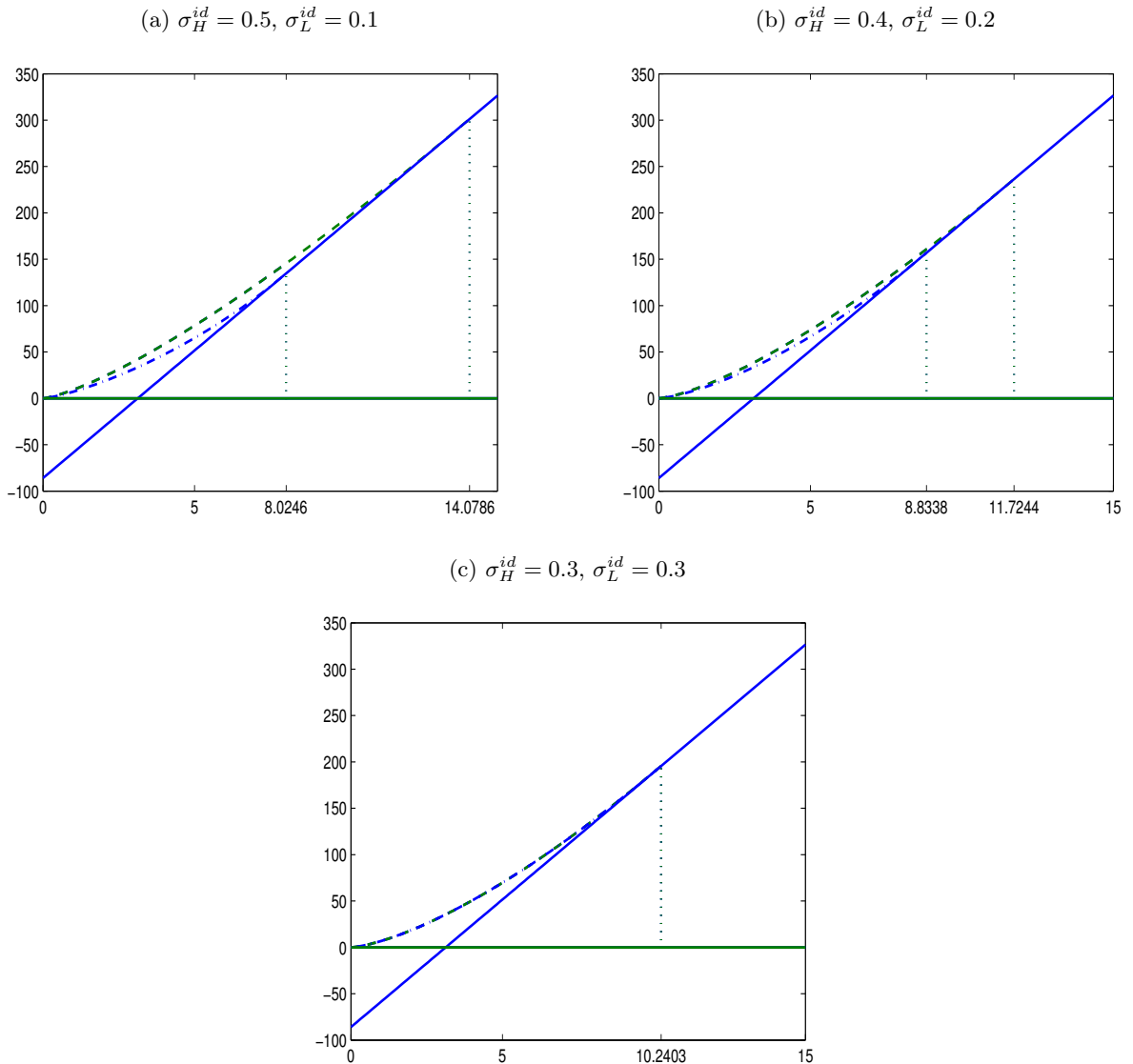
Figure 2: Model's Properties and Solution: Dependence of Return on Idiosyncratic Volatility Regime

The figure shows differences in the growth option's sensitivity to the systematic shock variable, the drift, the jump, and the diffusion terms of the option's value process between the high and low volatility regimes for various values of P based on the model developed in the paper. Panel (a) shows differences in the systematic volatility $(\Omega_H - \Omega_L)\sigma^{sys}$, Panel (b) shows differences in total volatility $\Omega_H\sqrt{(\sigma^{sys})^2 + (\sigma_H^{id})^2} - \Omega_L\sqrt{(\sigma^{sys})^2 + (\sigma_L^{id})^2}$, Panel (c) shows differences in the continuous drift terms $\mu_{G,H}^c - \mu_{G,L}^c$, and Panel (d) shows differences in the jump terms $\frac{G_L - G_H}{G_H} - \frac{G_H - G_L}{G_L}$ between regimes. The figure shows separate results for each set of model parameter values ($\sigma_H^{id} = 0.5, \sigma_{P,L}^{id} = 0.1;\ \sigma_{P,H}^{id} = 0.4, \sigma_{P,L}^{id} = 0.2;\ \sigma_{P,H}^{id} = \sigma_{P,L}^{id} = 0.3$).

(a) Differences in systematic volatility terms

(b) Differences in diffusion terms

(c) Differences in drift terms

(d) Differences in jump terms

Figure 3: Model Properties and Solution: Time-Series Plots

Simulation Results. The figure shows a sample path of simulated variables based on the model developed in the paper for $\sigma_H^{id} = 0.5, \sigma_L^{id} = 0.1$. Panel (a) shows end-of-month values for the output price P, panel (b) shows the corresponding firm values V_{s_t}, panel (c) shows the corresponding end-of-month idiosyncratic volatility regimes where $s_t = 1$ denotes the high volatility regime and $s_t = 0$ denotes the low volatility regime, panel (d) shows the end-of-month realized idiosyncratic return volatility, and panel (e) shows the end-of-month realized returns.

(a) P

(b) V_{s_t}

(c) State s_t: $1 = H, 0 = L$

(d) idiosyncratic return volatility

(e) return

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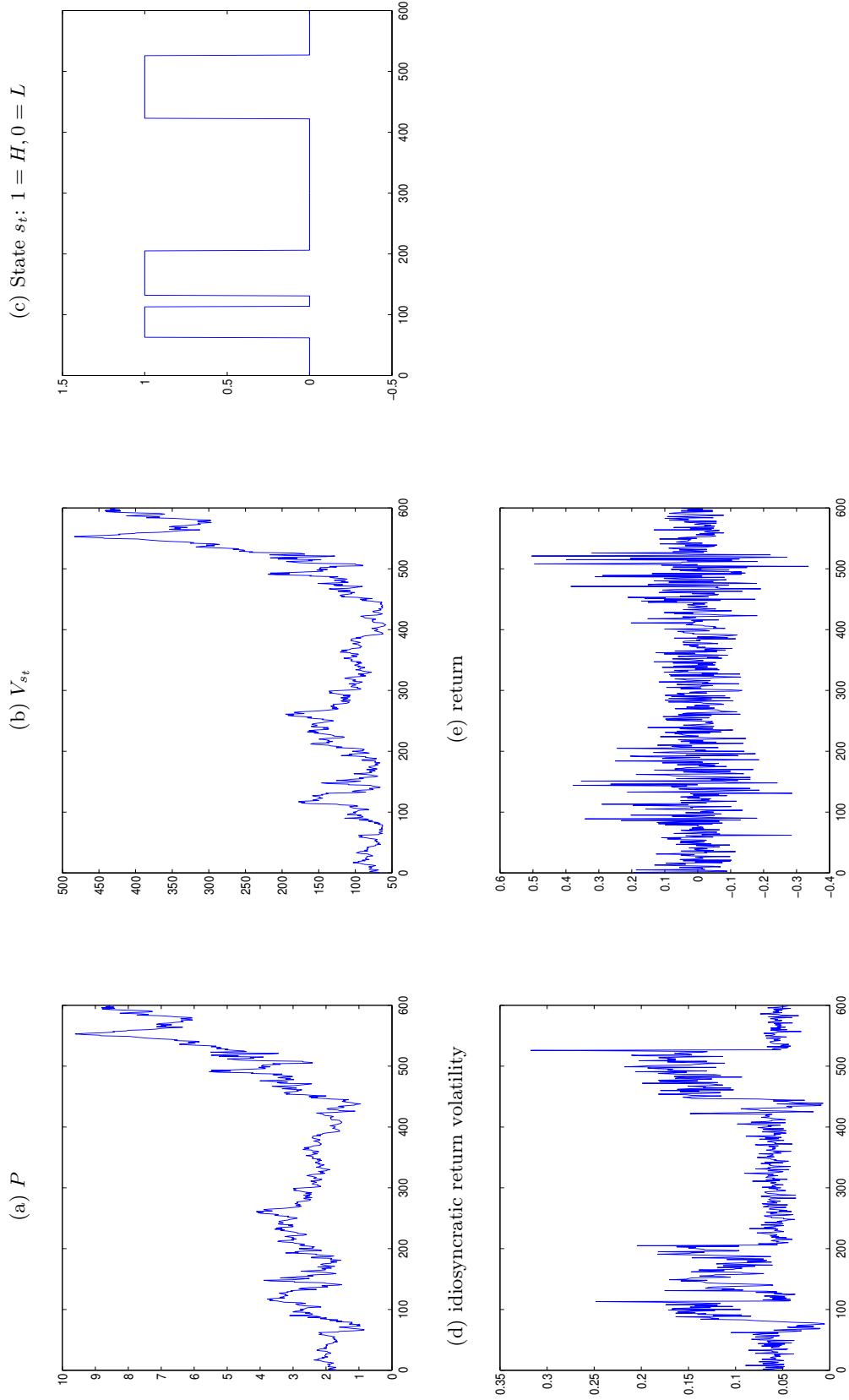
Figure 4: Simulation Results: Idiosyncratic Return Volatility Portfolio Returns

The figure shows the mean value-weighted returns of the portfolios formed after sorting stocks based on the past month return volatility $IVol$ using the simulated data based on the analytical solutions of the model developed in the paper. At the end of each month, stocks are sorted into five equally sized groups based on the past month $IVol$, then value-weighted one-month holding period portfolio returns are computed. The portfolios are rebalanced at the end of each month. The figure shows separate results for each set of model parameter values ($\sigma_H^{id} = 0.5, \sigma_L^{id} = 0.1$; $\sigma_H^{id} = 0.4, \sigma_L^{id} = 0.2$; $\sigma_H^{id} = \sigma_L^{id} = 0.3$).
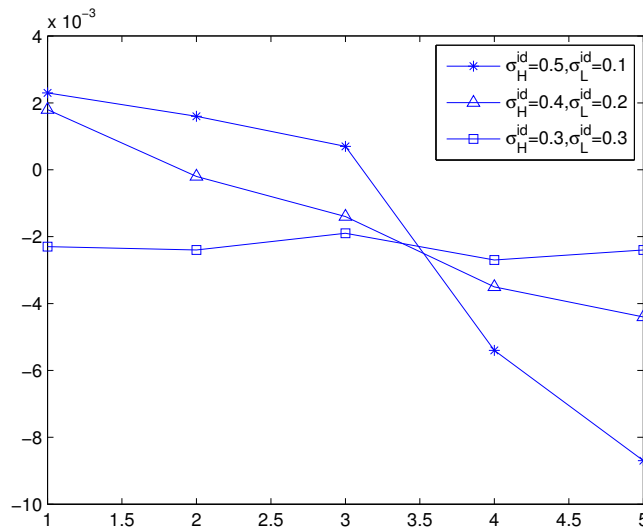
Table 1: Simulation Model Parameters

The table reports the parameter values used to solve and simulate the model developed in the paper. Base case parameter values are distinguished with an asterisk * if more than one value is reported for a variable.

Model Parameters		
Price Dynamics	Variable Description	Values
σ_H^{id}	Output price idiosyncratic volatility in the high regime	0.3,0.4,0.5*
σ_L^{id}	Output price idiosyncratic volatility in the low regime	0.1*,.02,0.3
λ_H	Transition parameter from low to high volatility regime	0.1
λ_L	Transition parameter from high to low volatility regime	0.1
μ	Drift term of the output price process	0.04
σ^{sys}	Systematic volatility of the output price process	0.15
Market	Variable Description	Values
r	Riskless rate	0.05
μ_S	Drift term of tradeable asset (Market)	0.1
σ_S	Diffusion term of tradeable asset (Market)	0.25
Firm's Profit Function	Variable Description	Values
c	Variable cost per unit of output	0.5
ξ_Y	Production scale for young Firms	1
ξ	Difference in production scales between mature and young firms	1.1
I	Investment cost	$\frac{1.5\times(\xi-1)\times20}{r-\mu^*}$
Simulations	Variable Description	Values
N	Number of samples	100
n	Number of firms in each sample	2500
T	Number of years	50
nt	Number of trading days in each month	20
λ_{exit}	Exit parameter for mature firms	0.01

Table 2: Simulation Results: Cross-Sectional Regressions

The table reports coefficient estimates for the regression model $r_t = \gamma_{0,t}\iota + \gamma_{1,t}\Delta IVol_t + \eta_t$ in the first column of each panel, and estimates for the regression model $r_t = \gamma_{0,t}\iota + \gamma_{1,t}IVol_{t-1} + \eta_t$ in the second column of the panels using the simulated data from the analytical solutions from the model developed in the paper. Panels (a), (b) and (c) report separate model estimates corresponding to the simulated samples where $\sigma_H^{id} = 0.5, \sigma_L^{id} = 0.1$, $\sigma_H^{id} = 0.4, \sigma_L^{id} = 0.2$ and $\sigma_H^{id} = \sigma_L^{id} = 0.3$ respectively. T-statistics are reported in square brackets.

	(a) $\sigma_H^{id} = 0.5, \sigma_L^{id} = 0.1$		(b) $\sigma_H^{id} = 0.4, \sigma_L^{id} = 0.2$		(c) $\sigma_H^{id} = 0.3, \sigma_L^{id} = 0.3$	
Intercept	0.0056***	0.0083***	0.0056***	0.0077***	0.0050***	−0.0030***
	[40.1311]	[39.7255]	[44.5101]	[32.1002]	[27.9953]	[-11.7416]
$\Delta IVol_t$	0.1178***		0.0884***		0.0006	
	[16.1998]		[19.1783]		[0.7927]	
$IVol_{t-1}$		−0.1007***		−0.0880***		-0.0001
		[-39.2915]		[-31.0568]		[-0.5524]

Table 3: Simulation Results: $IVol$ Portfolio Returns

The table reports the mean $IVol$ portfolio returns using simulated data based on the analytical solutions of the model developed in the paper. Stocks are sorted into five equally sized groups based on past month $IVol$, then value-weighted one-month holding period portfolio returns are computed. The portfolios are rebalanced at the end of each month. $IVol$ portfolios are reported across columns, and the last column reports the mean return of the zero-cost $IVol$ portfolio. The table reports separate results for each set of model parameter values ($\sigma_H^{id} = 0.5, \sigma_L^{id} = 0.1$; $\sigma_H^{id} = 0.4, \sigma_L^{id} = 0.2$; $\sigma_H^{id} = \sigma_L^{id} = 0.3$). T-statistics are reported in square brackets.
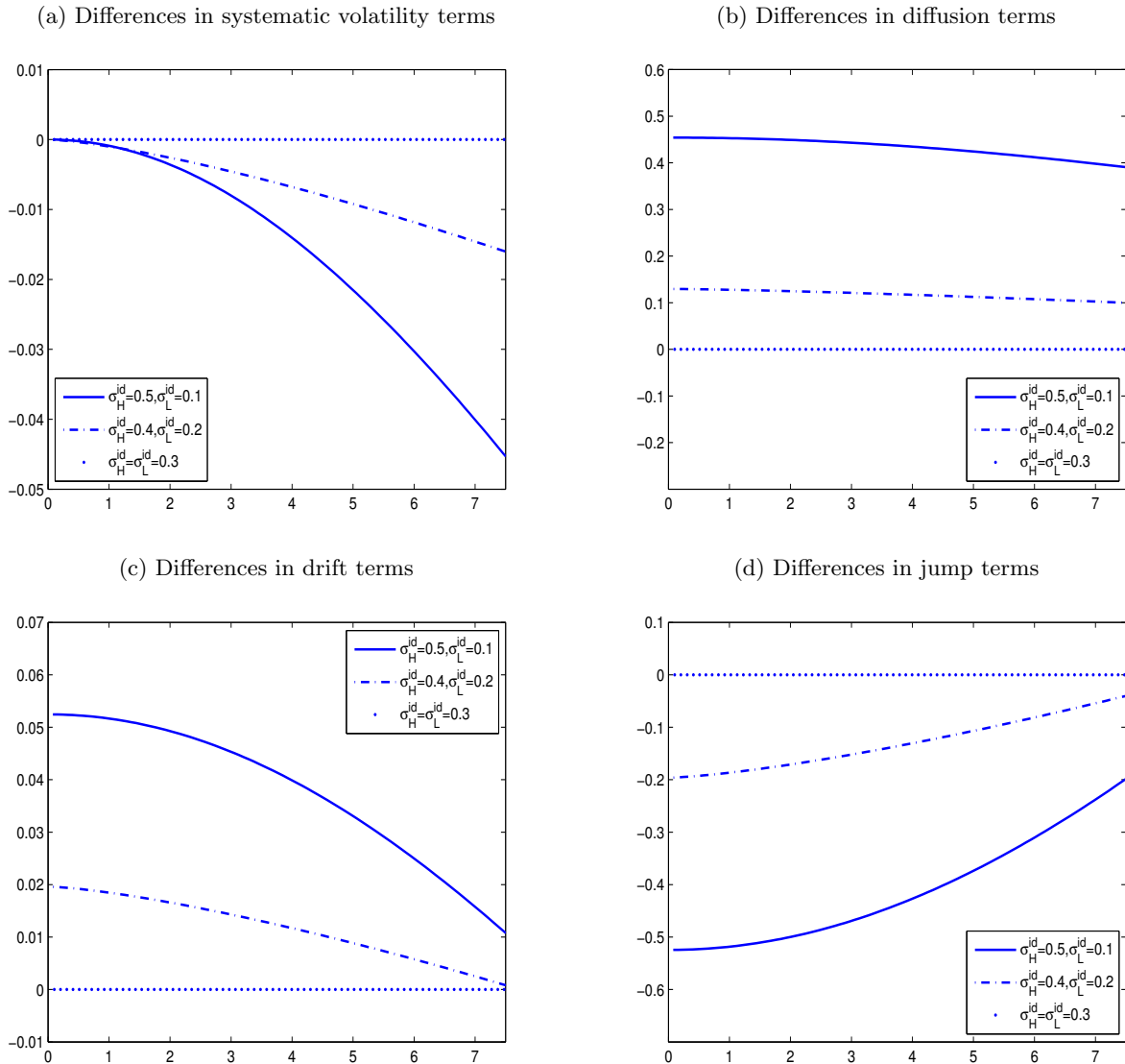
| | IVol Portfolios | | | | | |
	1	2	3	4	5	5-1
$\sigma_H^{id} = 0.5, \sigma_L^{id} = 0.1$	0.0023***	0.0016***	0.0007***	−0.0054***	−0.0087***	−0.011***
	[14.1267]	[8.7286]	[3.0931]	[-15.493]	[-25.1895]	[-33.7894]
$\sigma_H^{id} = 0.4, \sigma_L^{id} = 0.2$	0.0018***	−0.0002	−0.0014***	−0.0035***	−0.0044***	−0.0062***
	[11.1684]	[-0.9962]	[-5.7645]	[-11.3485]	[-14.3503]	[-21.0366]
$\sigma_H^{id} = 0.3, \sigma_L^{id} = 0.3$	−0.0023***	−0.0024***	−0.0019***	−0.0027***	−0.0024	0
	[-7.2206]	[-8.0168]	[-7.8058]	[-9.5375]	[-9.0741]	[-0.0484]

Table 4: Sample Summary Statistics

This table reports sample summary statistics for excess stock returns, idiosyncratic return volatilities $IVol$, month-to-month $IVol$ changes $\Delta IVol$, and the real option intensity proxies. The sample period is from January, 1971 to December, 2010 for all the market-based variables. Excess return is the difference between end-of-month stock return and the risk-free rate. Stock return volatility $IVol$ refers to the end-of-month volatility of the log daily returns risk-adjusted based on the Fama and French 3-factor model. Market equity and total assets are in millions of dollars. Firm age is expressed in months since the firms' first appearance on CRSP. Investment, profit and sale growths are expressed as the sum of the $t+2$ to $t+5$ growth rates where t is the fiscal year of the return observation. $vega$ is computed for each firm according to equation (??).

market variables	Mean	StdDev	P5	Median	P95	N
excess return	0.009976	0.180828	-0.22309	-0.0041	0.272627	1041266
$IVol$	0.029476	0.024979	0.0079	0.022782	0.072884	1038601
$\Delta IVol$	-2.3E-05	0.021096	-0.02552	-0.00011	0.026111	1035935

Real Option variables	Mean	StdDev	P5	Median	P95	N
log(market equity)	4.694734	2.106019	1.5389081	4.521163	8.389149	1040478
log(total assets)	4.804593	2.009753	1.789757	4.62188	8.352702	1041266
log(age)	3.953142	1.540425	0	4.290459	5.746203	1041266
investment growth	0.996235	18.22423	-0.64226	0.225036	2.237907	871778
profit growth	-0.55037	80.99137	-6.71653	0.353252	4.689659	871779
sales growth	1.579677	79.57993	-0.46927	0.29381	1.83045	868519
$vega$	2.84E-69	1.49E-67	9.63E-110	9.89E-81	1.88E-70	1041104

Table 5: Fama MacBeth Regressions: Return Difference Between Post and Pre-Switch Months Following Large Switches in Idiosyncratic Volatility and Its Dependence on Real Options.

This table reports regression results of differences in 5-month average returns between post and pre-switch months on real option proxies for stocks that experience switches in idiosyncratic volatility. The regression equation is $r_t^{Diff} = \gamma_{0t} + \gamma_1 RO_{t-1} + \eta_t$. The construction of the real option proxies is described in the paper. Each column corresponds to results for a separate proxy for real options. Separate regression results are reported for the up and the down switch samples. The reported estimates are time-series averages of the monthly coefficient estimates. Newey and West (1987) robust t-statistics are reported in square brackets. RSQR refers to the average of monthly R-squareds.

switch	Coeff.	size (total asset)	size (mkt equity)	age	high vega	high profit	high sale	young	high inv	small high vega	small growth
down	Intercept	-0.0201***	-0.0175***	-0.0111**	-0.0062***	-0.0068***	-0.0079***	-0.0079***	-0.0080***	-0.0076***	-0.0074***
		[-6.3508]	[-5.4812]	[-2.0936]	[-6.8905]	[-6.6695]	[-7.8954]	[-7.7187]	[-8.7388]	[-8.6914]	[-8.1784]
	RO	0.0024***	0.0018***	0.0005	0.0063***	-0.0016	0.0021	-0.0007	0.0025	-0.0116**	-0.0071**
		[4.4752]	[3.1156]	[0.3552]	[-3.1565]	[-1.0178]	[1.2240]	[-0.3579]	[1.5724]	[-2.5072]	[-2.1897]
	RSQR	0.0326***	0.0329***	0.0265***	0.0268***	0.0266***	0.0252***	0.0288***	0.0244***	0.0482***	0.0376***
up	intercept	0.0158***	0.0182***	0.0150***	0.0052***	0.0040***	0.0036***	0.0044***	0.0030**	0.0048***	0.0047***
		[5.3102]	[6.9155]	[3.2173]	[5.2992]	[3.0968]	[3.0411]	[3.9123]	[2.4548]	[4.9601]	[4.7785]
	RO	-0.0022***	-0.0027***	-0.0024**	0.0023	0.0039***	0.0059***	0.0053***	0.0076***	0.0082***	0.0095***
		[-4.4951]	[-6.2128]	[-2.0830]	[1.2730]	[2.7997]	[3.8766]	[3.1845]	[5.2559]	[2.2245]	[3.4609]
	RSQR	0.0283***	0.0279***	0.0285***	0.0252***	0.0251***	0.0269***	0.0255***	0.0261***	0.0408***	0.0349***

switch	Coeff.	small high profit	small high sale	small young	small high inv	young high vega	Natural Resources	High Tech	Bio Tech	All Growth Industry
down	Intercept	-0.0062***	-0.0068***	-0.0069***	-0.0069***	-0.0077***	-0.0083***	-0.0085***	-0.0079***	-0.0082***
		[-7.1963]	[-7.6766]	[-7.6615]	[-8.1033]	[-8.3749]	[-9.2471]	[-8.5055]	[-8.8771]	[-8.5319]
	RO	-0.0088***	-0.0045	-0.0108***	-0.0036	0.001	-0.0015	0.0013	-0.001	0.0002
		[-3.1267]	[-1.4208]	[-3.5391]	[-0.8044]	[0.2391]	[-0.3588]	[0.5947]	[-0.3122]	[0.1036]
	RSQR	0.0394***	0.0388***	0.0384***	0.0439***	0.0357***	0.0333***	0.0273***	0.0288***	0.0296***
up	Intercept	0.0045***	0.0043***	0.0051***	0.0036***	0.0054***	0.0056***	0.0057***	0.0056***	0.0041***
		[3.9771]	[4.0485]	[5.1847]	[3.2879]	[5.2951]	[5.1394]	[5.5452]	[5.5774]	[3.8737]
	RO	0.0088***	0.0107***	0.0077***	0.0164***	0.0079***	0.002	0.0025	0.0028	0.0043**
		[3.0261]	[3.4846]	[2.6551]	[4.7291]	[2.7409]	[0.5846]	[1.3052]	[0.8773]	[2.1847]
	RSQR	0.0365***	0.0368***	0.0324***	0.0397***	0.0325***	0.0299***	0.0237***	0.0265***	0.0290***

Table 6: Fama MacBeth Regressions: Return Difference Between Post and Pre-Switch Months Following Large Switches in Idiosyncratic Volatility and Its Dependence on Real Options and Size of the Switch.

This table reports regression results of differences in 5-month average returns between post and pre-switch months on real option proxies and the 70th and 30th percentile spread in $IVol$ for stocks that experience switches in idiosyncratic volatility. The regression equation is $r_t^{Diff} = \gamma_0 \iota + \gamma_1 \overline{\Delta IVol} + \gamma_2 \overline{\Delta IVol \times RO}_{t-1} + \eta_t$. The construction of the real option proxies is described in the paper. Each column corresponds to results for a separate proxy for real options. Separate regression results are reported for the up and the down switch samples. The reported estimates are time-series averages of the monthly coefficient estimates. Newey and West (1987) robust t-statistics are reported in square brackets. RSQR refers to the average of monthly R-squareds.

switch	Coeff.	size (total asset)	size (mkt equity)	age	high vega	high profit	high sale	young	high inv	small high vega	small growth
down	Intercept	-0.0018 [-0.6873]	0.0011 [0.4085]	0.0015 [0.7341]	0.0002 [0.1085]	-0.0003 [-0.1137]	0 [0.0180]	0.0023 [1.0268]	0.0004 [0.1644]	0.0014 [0.6783]	0.0011 [0.5057]
	$\overline{\Delta IVol}$	-1.0054*** [-3.8117]	-0.8526*** [-3.4519]	-0.2454 [-0.4000]	-0.6433*** [-2.8142]	-0.5962*** [-2.5609]	-0.7080*** [-3.1400]	-1.0003*** [-4.6543]	-0.7249*** [-3.2998]	-0.8262*** [-3.9831]	-0.8361*** [-3.8818]
	$\overline{\Delta IVol} \times RO$	0.1072* [1.7705]	-0.0046 [-0.0742]	-0.1626 [-1.0051]	-0.1921 [-1.2213]	0.0088 [0.0611]	0.1432 [0.9226]	0.2439 [1.4789]	0.1334 [0.8459]	-0.2349 [-1.0029]	0.0228 [0.1089]
	RSQR	0.0882*** [19.6250]	0.0912*** [16.1655]	0.0877*** [17.3481]	0.0871*** [17.1345]	0.0967*** [18.9544]	0.0964*** [19.4351]	0.0898*** [17.9502]	0.0958*** [17.5652]	0.0959*** [17.1672]	0.0962*** [19.9459]
up	Intercept	-0.0003 [-0.1080]	0.0037 [1.4158]	-0.0035 [-1.6219]	-0.0043* [-1.8809]	-0.0066*** [-2.8511]	-0.0056** [-2.4964]	-0.003 [-1.2945]	-0.0059** [-2.4858]	-0.0038* [-1.7371]	-0.0024 [-1.1340]
	$\overline{\Delta IVol}$	0.9565*** [3.9990]	0.9889*** [4.4507]	1.3251** [2.1528]	0.8610*** [4.0885]	0.9418*** [4.4276]	0.8203*** [4.2999]	0.6773*** [3.1748]	0.7773*** [3.9858]	0.7808*** [3.9437]	0.6350*** [3.2559]
	$\overline{\Delta IVol} \times RO$	-0.1229** [-2.2675]	-0.2472*** [-3.9057]	-0.2528 [-1.0236]	-0.0527 [-0.3575]	0.2297* [1.7174]	0.4044*** [2.7888]	0.1471 [1.2512]	0.6317*** [4.7608]	0.0379 [0.1882]	0.2846 [1.5988]
	RSQR	0.0721***	0.0775***	0.0740***	0.0736***	0.0821***	0.0810***	0.0689***	0.0810***	0.0809***	0.0784***

switch	Coeff.	small high profit	small high sale	small young	small high inv	young high vega	Natural Resources	High Tech	Bio Tech	All Growth Industries
down	Intercept	-0.0007 [-0.2803]	0.0001 [0.0229]	0 [0.0216]	-0.0002 [-0.0799]	0.0015 [0.6820]	0.0009 [0.4145]	0.0003 [0.1555]	0.0006 [0.2746]	0.0005 [0.2131]
	$\overline{\Delta IVol}$	-0.5243*** [-2.2498]	-0.6459*** [-2.8497]	-0.7449*** [-3.5372]	-0.6130*** [-2.8459]	-0.8933*** [-4.1053]	-0.7969*** [-3.6453]	-0.7817*** [-3.6182]	-0.7619*** [-3.5135]	-0.8022*** [-3.6652]
	$\overline{\Delta IVol} \times RO$	-0.209 [-1.0722]	-0.0493 [-0.2414]	-0.1334 [-0.7354]	-0.0914 [-0.3688]	0.3063 [1.3479]	-0.0335 [-0.0943]	0.0778 [0.4351]	0.0886 [0.3732]	0.1131 [0.7114]
	RSQR	0.1005*** [20.2486]	0.1007*** [20.3574]	0.0930*** [18.3941]	0.1045*** [18.4894]	0.0941*** [16.2996]	0.0937*** [17.5190]	0.0913*** [19.7699]	0.0888*** [19.0352]	0.0934*** [17.9798]
up	Intercept	-0.0059** [-2.3840]	-0.0054** [-2.1412]	-0.0032 [-1.4104]	-0.0048** [-2.0238]	-0.0031 [-1.4008]	-0.0037* [-1.6704]	-0.0042* [-1.6530]	-0.0037 [-1.4657]	-0.0032 [-1.4908]
	$\overline{\Delta IVol}$	0.9227*** [4.2713]	0.8880*** [4.2814]	0.7471*** [3.7131]	0.7969*** [4.1374]	0.7031*** [3.4753]	0.7761*** [4.0162]	0.8545*** [3.7309]	0.7441*** [3.6893]	0.6572*** [3.5573]
	$\overline{\Delta IVol} \times RO$	0.1712 [0.9805]	0.3161* [1.7296]	0.0841 [0.5751]	0.6159*** [3.2721]	0.0601 [0.3100]	0.339 [1.0305]	0.0486 [0.3155]	0.0432 [0.1791]	0.1858 [1.1833]
	RSQR	0.0847***	0.0841***	0.0701***	0.0842***	0.0777***	0.0753***	0.0723***	0.0711***	0.0773***

Table 7: Fama MacBeth Regressions: Dependence of the Return-$\Delta IVol$ relation on Growth Options.

This table reports the coefficient estimates of Fama and MacBeth (1973) cross-sectional regressions of firm level excess returns on the loading on the market factor (β^{CAPM}), log book-to-market ($Log(BM)$), log market equity ($Log(ME)$), six-month lagged return ($Lag(r)$), monthly trading volume normalized by the number of shares outstanding ($trade$), month-to-month change in firm level idiosyncratic volatility ($\Delta IVol$), and the interaction between real option proxy and $\Delta IVol$. The construction of the real option proxies are described in the paper. The regression model is $r_t - r_{f,t} = \gamma_{0t} + \gamma_1 \Delta IVol_t + \gamma_2 \Delta IVol_t \times RO_{t-1} + \gamma_3 X_{t-1} + \eta_t$. The real option proxies are reported across columns. The reported estimates are the time series average of the monthly coefficient estimates. Newey-West robust t-statistics are reported in square brackets. RSQR refers to the average of monthly R squared.

Coeff.	size (mkt equity)	size (total assets)	age	high vega	high profit	high sale	young	high inv	small high vega	small growth
Intercept	0.0402***	0.0404***	0.0405***	0.0404***	0.0417***	0.0414***	0.0404***	0.0417***	0.0403***	0.0403***
	[6.7509]	[6.8148]	[6.8501]	[6.8507]	[7.0746]	[7.0291]	[6.8406]	[7.0813]	[6.8330]	[6.8257]
$Log(BM)$	0.0057***	0.0059***	0.0057***	0.0057***	0.0052***	0.0052***	0.0057***	0.0052***	0.0057***	0.0058***
	[7.7458]	[8.0586]	[7.7807]	[7.6285]	[6.8186]	[6.8100]	[7.7920]	[6.7996]	[7.7387]	[7.7815]
$Log(ME)$	-0.0034***	-0.0035***	-0.0035***	-0.0035***	-0.0037***	-0.0037***	-0.0035***	-0.0037***	-0.0035***	-0.0035***
	[-5.5311]	[-5.6697]	[-5.7620]	[-5.7513]	[-6.1040]	[-6.0733]	[-5.7558]	[-6.1205]	[-5.7169]	[-5.7682]
β^{CAPM}	0.0015**	0.0015**	0.0016**	0.0016**	0.0014*	0.0015*	0.0015**	0.0014*	0.0016**	0.0015**
	[2.2340]	[2.1974]	[2.0700]	[2.2294]	[1.8310]	[1.9049]	[2.1691]	[1.8073]	[2.2730]	[2.1722]
$trade$	0.0157***	0.0156***	0.0155***	0.0155***	0.0154***	0.0153***	0.0155***	0.0154***	0.0155***	0.0155***
	[14.4667]	[14.3834]	[14.2095]	[14.3831]	[13.4373]	[13.4022]	[14.2209]	[13.4620]	[14.3394]	[14.2345]
$Lag(r)$	-0.0031	-0.0031	-0.0031	-0.0031	-0.0031	-0.003	-0.0031	-0.0032	-0.0032	-0.003
	[-1.1424]	[-1.1612]	[-1.1747]	[-1.1768]	[-1.0903]	[-1.0371]	[-1.1534]	[-1.1049]	[-1.1820]	[-1.1219]
$\Delta IVol$	2.3623***	2.2474***	1.1341***	1.0903***	1.2728***	1.1797***	1.1865***	1.1680***	1.1066***	1.2181***
	[17.5356]	[15.8777]	[6.9660]	[9.8669]	[12.1527]	[11.3973]	[10.1845]	[10.9610]	[10.5451]	[12.3035]
$RO \times \Delta IVol$	-0.3125***	-0.2647***	-0.0094	0.3748***	0.0217	0.2497***	0.0321	0.3273***	0.5297***	0.3139***
	[-10.790]	[-9.8134]	[-0.1847]	[5.0322]	[0.3632]	[3.7902]	[0.3178]	[5.0967]	[5.2866]	[3.4861]
RSQR	0.1150***	0.1128***	0.1099***	0.1107***	0.1104***	0.1111***	0.1099***	0.1110***	0.1127***	0.1107***

Coeff.	small high profit	small highs sale	small young	small high inv	young high vega	Natural Resources	High Tech	Bio Tech	All Growth Option Industries
Intercept	0.0416***	0.0413***	0.0405***	0.0416***	0.0403***	0.0403***	0.0403***	0.0403***	0.0404***
	[7.0506]	[7.0154]	[6.8674]	[7.0715]	[6.8187]	[6.8169]	[6.8440]	[6.8730]	[6.8550]
$Log(BM)$	0.0053***	0.0052***	0.0058***	0.0052***	0.0058***	0.0057***	0.0057***	0.0058***	0.0057***
	[6.8579]	[6.8384]	[7.8430]	[6.7970]	[7.7727]	[7.7483]	[7.7597]	[7.8780]	[7.7787]
$Log(ME)$	-0.0037***	-0.0036***	-0.0035***	-0.0037***	-0.0035***	-0.0035***	-0.0035***	-0.0035***	-0.0035***
	[-6.0680]	[-6.0359]	[-5.7614]	[-6.0951]	[-5.7286]	[-5.7625]	[-5.7303]	[-5.7558]	[-5.7524]
β^{CAPM}	0.0015*	0.0015**	0.0016**	0.0014*	0.0015**	0.0016**	0.0015**	0.0015**	0.0015**
	[1.8461]	[2.0167]	[2.2027]	[1.8043]	[2.1244]	[2.2400]	[2.1177]	[2.0997]	[2.1287]
$trade$	0.0154***	0.0153***	0.0155***	0.0154***	0.0155***	0.0154***	0.0155***	0.0155***	0.0154***
	[13.4108]	[13.4492]	[14.2725]	[13.4390]	[14.1888]	[14.1937]	[14.2384]	[14.3645]	[14.2776]
$Lag(r)$	-0.003	-0.0029	-0.003	-0.003	-0.003	-0.0032	-0.0032	-0.0031	-0.0031
	[-1.0544]	[-1.0095]	[-1.1569]	[-1.0560]	[-1.1339]	[-1.1729]	[-1.1851]	[-1.1741]	[-1.1711]
$\Delta IVol$	1.1829***	1.1282***	1.1573***	1.1504***	1.2322***	1.2437***	1.2796***	1.2852***	1.1634***
	[11.4545]	[10.8841]	[11.6522]	[10.8892]	[12.0615]	[12.0660]	[12.8921]	[12.8305]	[11.8727]
$RO \times \Delta IVol$	0.4097***	0.6017***	0.3011***	0.5906***	0.1640**	0.3629***	0.0536	0.0483	0.2804***
	[4.6175]	[7.4742]	[3.5690]	[7.7597]	[2.0284]	[3.5133]	[0.6618]	[0.4384]	[4.2373]
RSQR	0.1116***	0.1122***	0.1111***	0.1122***	0.1108***	0.1106***	0.1102***	0.1101***	0.1102***

Table 8: Fama MacBeth Regressions: Dependence of the $+ IVol$-Return relation on Real Options and Switch Size.

This table reports the coefficient estimates of Fama and MacBeth (1973) cross-sectional regressions of firm level excess returns on the loading on the market factor (β^{CAPM}), log book-to-market ($Log(BM)$), log market equity ($Log(ME)$), six-month lagged return ($Lag(r)$), monthly trading volume normalized by the number of shares outstanding ($trade$), month-to-month change in firm level idiosyncratic volatility ($\Delta IVol$), the 70th and 30th percentile spread in $IVol$ ($\overline{\Delta IVol}$), real option proxy, and the interaction between real option proxy and $\overline{\Delta IVol}$. The construction of the real option proxies are described in the paper. The regression model is $r_t - r_{f,t} = \gamma_{0t} + \gamma_1 \overline{\Delta IVol} + \gamma_2 \Delta IVol_t + \gamma_3 \overline{\Delta IVol} \times \Delta IVol \times RO + \gamma_3 X_{t-1} + \eta_t$. The reported estimates are the time series average of the monthly coefficient estimates. Newey-West robust t-statistics are reported in square brackets. RSQR refers to the average of monthly R squared.

Coeff.	size (mkt equity)	size (total assets)	age	high vega	high profit	high sale	young	high inv	small high vega	small growth
Intercept	0.0620***	0.0627***	0.0635***	0.0636***	0.0628***	0.0622***	0.0633***	0.0626***	0.0635***	0.0633***
	[12.6667]	[12.9225]	[13.0910]	[13.1015]	[13.0393]	[12.8871]	[12.9775]	[12.9719]	[13.0683]	[12.9677]
$Log(BM)$	0.0043***	0.0044***	0.0044***	0.0043***	0.0039***	0.0040***	0.0043***	0.0039***	0.0043***	0.0044***
	[5.9892]	[6.0253]	[6.0236]	[5.8810]	[5.2759]	[5.3703]	[6.0379]	[5.3164]	[5.9482]	[6.1117]
$Log(ME)$	-0.0058***	-0.0059***	-0.0059***	-0.0059***	-0.0059***	-0.0058***	-0.0059***	-0.0059***	-0.0059***	-0.0059***
	[-11.668]	[-11.886]	[-11.888]	[-11.951]	[-12.321]	[-12.203]	[-11.845]	[-12.268]	[-11.906]	[-11.880]
β^{CAPM}	0.0016**	0.0017**	0.0017**	0.0017***	0.0016**	0.0017**	0.0017**	0.0017***	0.0017***	0.0017***
	[2.3965]	[2.4694]	[2.5253]	[2.5956]	[2.1575]	[2.2686]	[2.4378]	[2.1917]	[2.5962]	[2.5885]
$Lag(r)$	-0.0042	-0.0041	-0.0041	-0.0041	-0.0039	-0.0038	-0.0041	-0.004	-0.0041	-0.0041
	[-1.6123]	[-1.5606]	[-1.5916]	[-1.5743]	[-1.4138]	[-1.3623]	[-1.6018]	[-1.4368]	[-1.5860]	[-1.5573]
$trade$	0.0158***	0.0158***	0.0159***	0.0159***	0.0157***	0.0157***	0.0158***	0.0157***	0.0159***	0.0158***
	[14.8382]	[14.7683]	[14.8815]	[14.9759]	[14.1370]	[14.0990]	[14.8001]	[14.1068]	[14.9459]	[14.8546]
$\overline{\Delta IVol}$	-0.9410***	-0.9714***	-1.0111***	-1.0136***	-0.9507***	-0.9369***	-0.9915***	-0.9412***	-1.0108***	-1.0003***
	[-9.4354]	[-9.6958]	[-10.129]	[-10.100]	[-8.9019]	[-8.7162]	[-9.9101]	[-8.8688]	[-10.092]	[-9.9407]
$\Delta IVol$	1.8416***	1.6986***	1.0350***	1.1713***	1.2416***	1.1677***	1.2704***	1.1767***	1.1665***	1.2368***
	[16.2397]	[12.7028]	[8.1893]	[10.8873]	[12.1765]	[11.3403]	[12.5016]	[11.2056]	[11.3075]	[12.3132]
$\overline{\Delta IVol} \times \Delta IVol \times RO$	-10.149***	-7.0635***	-2.5813	10.4454***	5.7354*	15.9548***	-1.0437	18.5515***	13.4981***	11.4536***
	[-6.4869]	[-5.3356]	[-0.5099]	[2.6592]	[1.6515]	[4.5518]	[-0.2828]	[5.1675]	[3.3356]	[2.6497]
RSQR	0.1176***	0.1164***	0.1166***	0.1180***	0.1170***	0.1175***	0.1169***	0.1173***	0.1191***	0.1173***

Coeff.	small high profit	small highs sale	small young	small high inv	young high vega	Natural Resources	Bio Tech	High Tech	All Growth Option Industries
Intercept	0.0629***	0.0623***	0.0636***	0.0628***	0.0634***	0.0627***	0.0627***	0.0627***	0.0631***
	[13.0392]	[12.9103]	[13.0859]	[12.9952]	[13.0304]	[12.9122]	[13.0103]	[12.8832]	[13.0997]
$Log(BM)$	0.0040***	0.0040***	0.0044***	0.0039***	0.0044***	0.0044***	0.0044***	0.0044***	0.0044***
	[5.2952]	[5.3880]	[6.0411]	[5.3075]	[5.9657]	[6.0308]	[6.1201]	[6.0725]	[6.0726]
$Log(ME)$	-0.0059***	-0.0058***	-0.0059***	-0.0059***	-0.0059***	-0.0059***	-0.0058***	-0.0059***	-0.0059***
	[-12.312]	[-12.215]	[-11.882]	[-12.270]	[-11.959]	[-11.903]	[-11.871]	[-11.811]	[-11.952]
β^{CAPM}	0.0017**	0.0017***	0.0017***	0.0017***	0.0018***	0.0018***	0.0016***	0.0018***	0.0017**
	[2.1808]	[2.3133]	[2.4464]	[2.1928]	[2.5693]	[2.6381]	[2.3988]	[2.5412]	[2.4645]
$Lag(r)$	-0.0038	-0.0037	-0.0041	-0.0039	-0.0041	-0.0042	-0.0041	-0.0042	-0.0041
	[-1.3929]	[-1.3437]	[-1.5979]	[-1.4052]	[-1.5618]	[-1.6174]	[-1.6019]	[-1.5995]	[-1.5898]
$trade$	0.0157***	0.0157***	0.0158***	0.0157***	0.0158***	0.0158***	0.0158***	0.0157***	0.0158***
	[14.1190]	[14.1104]	[14.8476]	[14.1089]	[14.8011]	[14.8076]	[14.9660]	[14.8051]	[14.8848]
$\overline{\Delta IVol}$	-0.9580***	-0.9432***	-1.0027***	-0.9531***	-1.0019***	-0.9704***	-0.9709***	-0.9810***	-0.9873***
	[-8.9675]	[-8.8742]	[-9.9681]	[-8.9661]	[-9.9511]	[-9.6355]	[-9.5971]	[-9.7776]	[-9.6676]
$\Delta IVol$	1.2115***	1.1590***	1.2214***	1.1849***	1.2578***	1.2651***	1.2776***	1.2737***	1.1733***
	[11.9304]	[11.2169]	[12.0872]	[11.3227]	[12.4545]	[12.3600]	[12.8883]	[12.7952]	[11.9514]
$\overline{\Delta IVol} IVol \times RO$	14.6836***	24.4176***	5.9097	22.5754***	3.2036	19.3544***	8.6091	5.77	13.0368***
	[3.3967]	[6.0754]	[1.3540]	[5.6070]	[0.7889]	[3.3040]	[1.2756]	[1.4192]	[4.2008]
RSQR	0.1178***	0.1180***	0.1176***	0.1179***	0.1173***	0.1158***	0.1160***	0.1168***	0.1167***

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Table 9: – *IVol*-Return Relation and Real Options

The table reports Fama and French (1993) portfolio alphas along with robust Newey and West (1987) t-statistics in square brackets for the portfolios of stocks sorted by idiosyncratic return volatility *IVol* and the proxies for real option intensity. *IVol* portfolios are reported across columns. Real option portfolios are reported down the rows. The columns labeled '3-1' correspond to the zero-cost *IVol* portfolios within each classification of real option intensity. Idiosyncratic return volatility is computed relative to FF-3. All portfolios are value-weighted and rebalanced monthly. Portfolio alphas are annualized.

IVol Portfolios

Real Option Portfolios	age 1	age 2	age 3	age 3-1	size (market equity) 1	size (market equity) 2	size (market equity) 3	size (market equity) 3-1	size (total asset) 1	size (total asset) 2	size (total asset) 3	size (total asset) 3-1
1	0.7141 [0.5620]	1.4974 [0.8703]	-5.4860** [-2.0730]	-6.2001** [-2.2095]	3.4891** [2.1164]	7.1679*** [4.2677]	-1.4118 [-0.6068]	-4.9009** [-2.0704]	4.0975** [2.4911]	4.0300** [1.9908]	-6.5365*** [-2.8647]	-10.6340*** [-4.3472]
2	0.0977 [0.0690]	2.2573 [1.1694]	-6.8932*** [-2.9674]	-7.0886*** [-3.0333]	2.0518** [2.2206]	2.4464*** [2.6878]	-5.7961*** [-3.0515]	-7.8479*** [-3.3100]	4.1199*** [4.0848]	2.5739* [1.8638]	-5.4489*** [-2.7191]	-9.5688*** [-4.1640]
3	0.2338 [0.1729]	1.0879 [0.7286]	-1.3004 [-0.4450]	-0.999 [-0.3091]	1.1703** [2.0816]	1.1891 [0.8592]	-4.3537 [-1.5122]	-5.5241* [-1.8321]	1.1176** [1.9880]	1.0809 [0.7594]	-3.3977 [-1.0931]	-4.5153 [-1.4095]

Real Option Portfolios	high vega 1	high vega 2	high vega 3	high vega 3-1	high profit 1	high profit 2	high profit 3	high profit 3-1	high sale 1	high sale 2	high sale 3	high sale 3-1
0	1.4045** [2.2822]	2.0348* [1.7913]	-2.361 [-1.1747]	-3.7655* [-1.6816]	0.9203 [1.3045]	-0.3679 [-0.3151]	-3.9303 [-1.6076]	-4.8505* [-1.8132]	0.5376 [0.7579]	-2.7208** [-2.2648]	-8.0188*** [-3.3133]	-8.5564*** [-3.3929]
1	-1.1113 [-0.7644]	-2.3174 [-1.2174]	-10.4032*** [-3.6044]	-9.2920*** [-2.9722]	2.1091* [1.8580]	2.2445 [1.1791]	-2.0649 [-0.7984]	-4.1739 [-1.5930]	4.8059*** [3.9921]	6.1847*** [3.2661]	2.9237 [1.0957]	-1.8821 [-0.6379]

Real Option Portfolios	high investment 1	high investment 2	high investment 3	high investment 3-1	young 1	young 2	young 3	young 3-1	small high vega 1	small high vega 2	small high vega 3	small high vega 3-1
0	-1.6825** [-2.3719]	-5.2831*** [-3.9863]	-11.3864*** [-4.4539]	-9.7039*** [-3.5168]	1.3132 [1.2116]	1.1363 [0.6678]	-3.6190* [-1.7697]	-4.9322** [-2.1477]	1.3118** [2.2125]	1.443 [1.2015]	-4.1226** [-2.0369]	-5.4345** [-2.4447]
1	8.6144*** [9.5657]	10.8817*** [6.3204]	8.8514*** [3.5488]	0.237 [0.0895]	0.4351 [0.3662]	-0.1468 [-0.0634]	-7.3830*** [-2.7331]	-7.8181*** [-2.9091]	7.2256*** [3.1348]	-1.3516 [-0.5017]	-9.4742*** [-2.7450]	-16.6999*** [-5.1945]

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Table 10: − *IVol*-Return Relation and Real Options

The table reports Fama and French (1993) portfolio alphas along with robust Newey and West (1987) t-statistics in square brackets for the portfolios of stocks sorted by idiosyncratic return volatility *IVol* and the proxies for real option intensity. *IVol* portfolios are reported across columns. Real option portfolios are reported down the rows. The columns labeled '3-1' correspond to the zero-cost *IVol* portfolios within each classification of real option intensity. Idiosyncratic return volatility is computed relative to FF-3. All portfolios are value-weighted and rebalanced monthly. Portfolio alphas are annualized.

IVol Portfolios

Real Option Portfolios	small and high profit				small and high sale				small and young			
	1	2	3	3-1	1	2	3	3-1	1	2	3	3-1
0	1.1321* [1.9005]	0.9205 [0.7740]	-4.8299** [-2.2867]	-5.9620*** [-2.6013]	1.1484* [1.8996]	0.8917 [0.7600]	-4.9013** [-2.2452]	-6.0497** [-2.5556]	1.3810** [2.3321]	1.2084 [1.0050]	-3.6206* [-1.7557]	-5.0016** [-2.2157]
1	6.6400*** [3.2080]	5.1844 [1.5296]	-5.9994 [-1.4277]	-12.6394*** [-3.1488]	6.9615*** [3.5611]	7.2097*** [2.6765]	-4.2678 [-1.1800]	-11.2293*** [-3.1720]	3.2628 [1.4685]	-0.4925 [-0.2145]	-12.5558*** [-3.7289]	-15.8186*** [-5.5878]

Real Option Portfolios	small and high investment				young and high vega				natural resources			
	1	2	3	3-1	1	2	3	3-1	1	2	3	3-1
0	1.0777* [1.7810]	0.8581 [0.7173]	-5.3113** [-2.4564]	-6.3890*** [-2.7210]	1.2383** [2.1660]	1.8331 [1.4618]	-3.2394 [-1.5166]	-4.4778* [-1.9169]	0.7477 [0.9722]	0.4518 [0.3995]	-3.7675* [-1.7082]	-4.5152* [-1.8697]
1	13.4407*** [6.5670]	15.6392*** [4.2039]	-0.1405 [-0.0450]	-13.5812*** [-4.3761]	-2.2457 [-1.0604]	-1.7817 [-0.6608]	-11.4735*** [-3.4854]	-9.2278*** [-2.6007]	3.2293 [1.5656]	2.3808 [0.6126]	-4.5156 [-0.9362]	-7.7450** [-2.0684]

Real Option Portfolios	high tech				bio tech				all growth			
	1	2	3	3-1	1	2	3	3-1	1	2	3	3-1
0	1.4748* [1.9199]	0.7158 [0.5762]	-2.7805 [-1.3525]	-4.2553* [-1.8781]	0.6634 [1.0983]	1.2607 [0.8937]	-5.4760** [-2.4337]	-6.1394*** [-2.6350]	0.0523 [0.0518]	-0.6041 [-0.5033]	-5.3537*** [-3.2074]	-5.4061*** [-2.9169]
1	1.4316 [0.8085]	1.2535 [0.4088]	-6.0880* [-1.7998]	-7.5196** [-2.4055]	5.4876*** [3.1616]	5.0094 [1.5461]	0.8018 [0.2237]	-4.6858 [-1.3247]	2.8762*** [2.9638]	3.0485 [1.2202]	-2.9505 [-0.9131]	-5.8266* [-1.8956]

Table 11: Dependence of the − *IVol*-Return Relation on Switch Size and Real Options

The table reports Fama and French (1993) portfolio alphas, along with robust Newey and West (1987) t-statistics in square brackets, for the zero-cost *IVol* portfolios for each of the two-way classifications of real option proxies and $\overline{\Delta IVol}$. The columns labeled 'mean' correspond to the portfolio equally-weighted on the zero-cost *IVol* portfolios for each real option intensity group within each classification of $\Delta IVol$. Idiosyncratic volatility is computed relative to FF-3. All portfolios are value-weighted and rebalanced monthly. Portfolio alphas are annualized.

$\overline{\Delta IVol}$	Real Option Portfolios											
Portfolios	age				size (market equity)				size (total asset)			
	1	2	3	mean	1	2	3	mean	1	2	3	mean
1	1.1583	-2.4428	-3.479	-0.2096	-0.1546	0.8774	-0.7373	-0.0048	1.2953	1.9408	-1.7919	0.4814
	[0.4656]	[-0.8672]	[-1.6132]	[-0.1219]	[-0.1097]	[0.6937]	[-0.3732]	[-0.0040]	[0.7783]	[0.9964]	[-0.8756]	[0.3398]
2	0.5376	-9.0395***	-3.8205	-3.8516*	0.6286	0.2541	-6.2849**	-1.8008	-1.1459	-5.6068**	-5.9637**	-4.2388**
	[0.1828]	[-3.0131]	[-1.1623]	[-1.9194]	[0.2874]	[0.1028]	[-2.2975]	[-1.0100]	[-0.3936]	[-2.1743]	[-2.0455]	[-2.0982]
3	-13.2842***	-9.7927**	-10.4336**	-11.6222***	-6.3840**	-12.7608***	-19.9126***	-13.0192***	-9.1933**	-16.1224***	-14.8518***	-13.3892***
	[-3.2330]	[-2.5055]	[-2.3812]	[-3.8341]	[-2.0049]	[-4.3610]	[-5.4313]	[-5.0374]	[-2.5293]	[-5.0924]	[-3.6702]	[-4.9292]

Table 12: Dependence of the $-IVol$-Return Relation on Switch Size and Real Options

The table reports Fama and French (1993) portfolio alphas, along with robust Newey and West (1987) t-statistics in square brackets, for the zero-cost $IVol$ portfolios for each of the two-way classifications of real option proxies and $\overline{\Delta IVol}$. Idiosyncratic volatility is computed relative to FF-3. All portfolios are value-weighted and rebalanced monthly. Portfolio alphas are annualized.

Real Option Portfolios

$\overline{\Delta IVol}$

Portfolios	high vega 0	high vega 1	high profit 0	high profit 1	high investment 0	high investment 1	young 0	young 1
1	0.147 [0.0618]	-8.1421 [-1.3157]	-0.943 [-0.3316]	-7.1614 [-1.5875]	-6.8576** [-2.0994]	2.6811 [0.6673]	-2.6092 [-0.9174]	1.1289 [0.2094]
2	-3.505 [-1.4618]	-5.6724 [-1.4065]	-1.9006 [-0.6468]	1.2129 [0.3879]	-4.5404 [-1.5994]	0.7809 [0.2560]	-5.2328** [-1.9887]	-1.3772 [-0.4292]
3	-6.8298* [-1.9367]	-5.0545 [-1.2537]	-12.9659*** [-3.1592]	1.8703 [0.4470]	-9.8621** [-2.3589]	-4.5532 [-1.1472]	-8.4579** [-2.3154]	-8.0120** [-2.2161]

Portfolios	small high vega 0	small high vega 1	small growth 0	small growth 1	small high sale 0	small high sale 1	small and young 0	small and young 1	small high investment 0	small high investment 1
1	-2.4541 [-1.0159]	-27.9701* [-1.8613]	-1.2082 [-0.4787]	4.5421 [0.5815]	-4.1531 [-1.5812]	-5.0725 [-0.6850]	-1.8499 [-0.7736]	-0.1049 [-0.0137]	-3.6428 [-1.3841]	-0.432 [-0.0552]
2	-2.187 [-0.9645]	-25.0583*** [-3.7224]	-3.1672 [-1.2881]	-11.2681*** [-2.6043]	-2.8776 [-1.1796]	-5.5105 [-1.0362]	-2.7867 [-1.1933]	-8.3822* [-1.7600]	-2.9884 [-1.1203]	-8.5901* [-1.7891]
3	-9.4226*** [-2.7477]	-12.0750*** [-3.3645]	-7.5165** [-1.9739]	-16.2439*** [-5.2359]	-7.0473* [-1.8367]	-7.3758* [-1.8315]	-8.4557** [-2.2478]	-10.3204*** [-2.8822]	-7.0524* [-1.8104]	-9.6920** [-2.5248]

Portfolios	young and high vega 0	young and high vega 1	natural resources 0	natural resources 1	high tech 0	high tech 1	bio tech 0	bio tech 1	all growth 0	all growth 1
1	-0.8212 [-0.3315]	-12.0280* [-1.7938]	-1.6231 [-0.5745]	6.8209 [1.3028]	0.3569 [0.1429]	0.7912 [0.1257]	-0.8241 [-0.2945]	7.5278 [1.2412]	-0.8772 [-0.4017]	6.3261 [1.0666]
2	-3.0975 [-1.2775]	6.1775 [1.1195]	-1.7178 [-0.6962]	-2.7294 [-0.6421]	-0.8473 [-0.3146]	-8.2843** [-2.2199]	-3.0634 [-1.2751]	-4.2983 [-1.0584]	-0.7051 [-0.3291]	-6.5709** [-2.1681]
3	-8.8177*** [-2.5898]	0.4351 [0.0944]	-7.0048** [-2.1141]	-11.2290** [-2.5374]	-10.9527*** [-3.1950]	-8.6281* [-1.6827]	-7.7035** [-2.1873]	-12.9701** [-2.2989]	-12.3964*** [-4.1275]	-13.3725*** [-3.3579]