

# Risky, Lumpy Human Capital in Household Portfolios\*

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## Abstract

Illiquid human capital is typically the largest asset in a household's portfolio, for the bulk of working life. Financial assets, which are relatively more liquid, are accumulated throughout life as well, in preparation for both rainy days and predictable life-cycle-related declines in earning ability. Optimal investment in these categories of wealth are necessarily made jointly, however. In this paper we aim to understand the evolution of household wealth and its portfolio composition over the life-cycle, when both human and financial wealth are choices.

We study two environments that differ in their treatment of human capital investment. The first uses the classic Ben-Porath (1967) model. We show that this model is able to capture household financial wealth fairly well over the life-cycle without appeal to transactions costs or other frictions. However, this model cannot be mapped to data that documents systematic differences income, wealth, and portfolio choice among households with differing levels of formal education. We therefore study a second environment that treats investment in higher education in a rich manner. In particular, we allow this investment to reflect three observed features: college is lumpy, risky, and irreversible. We find in this case that while averages can be accounted for, variation across educational groups cannot. Our results thus suggest the strong presence of frictions that drive a wedge between model and data, even when a more abstract model initially suggests success.

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\*The views expressed in this paper are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Richmond or the Federal Reserve System.

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# 1 Introduction

Human capital is arguably the largest asset that households invest in over their lifetime. They also invest in financial assets, which, unlike human capital, are relatively liquid. The levels and stochastic properties of earnings, which are in turn determined by human capital investments, are also likely to influence household financial portfolios. Furthermore, there is a direct trade-off between investment in human capital, which requires forgoing current earnings, and financial investments. Optimal investments in human capital and financial assets are therefore necessarily made jointly and together determine the evolution of household portfolios over the life cycle.

In this paper we aim to understand the evolution of household portfolios, defined broadly enough to include both human and financial wealth positions, over the life-cycle. A key feature of our approach is to include lumpy initial investments (formal education) and subsequent “on-the-job” training à la Ben-Porath (1967), both of which are risky. Specifically, we allow young households to decide whether to attend and complete college, and we allow adult households accumulate human capital while working. To our knowledge we are the first to study human and financial investment decisions in such a setting. An important payoff of our approach is a unified view of household wealth over the life cycle.

Our results show that a rich characterization of human capital investment is important to match facts about household portfolios. We study two environments that differ in their treatment of human capital investment. The first uses the classic Ben-Porath (1967) model. We show that this model is able to capture household financial wealth fairly well over the life-cycle. Quantitatively, a key finding is that our model is able to account for limited stock market participation with no appeal whatsoever to transactions costs. Instead, “corner solutions” to stock purchases emerge naturally from the optimality of front-loading investment in human capital. However, this model cannot be mapped to data that documents systematic differences income, wealth, and portfolio choice among households with differing levels of formal education. We therefore study a second environment that treats investment in higher education in a rich manner. In particular, we allow this investment to reflect three observed features: college is lumpy, risky, and irreversible. We find in this case that while averages can be accounted for, variation across educational groups cannot. Our results thus suggest the strong presence of frictions that drive a wedge between model and data, even when a more abstract model initially suggests success.

Previous literature has examined various aspects of the relationship between human capital investment, returns to human capital and financial portfolios over the life cycle. One strand of the literature focuses on the role of labor-income behavior in determining portfolio allocation. Jagannathan and Kocherlakota (1996) and Cocco et al. (2005) point out that labor income usually

acts as a substitute for holding a riskless asset, and as such, should encourage households to reduce the share of stocks in their portfolio as they age. Note, however, that this is not consistent with observed investment behavior. Others examine the role of labor supply. For example, Gomes et al. (2008) endogenize the labor supply decision, thus allowing households who fare poorly on the stock market to hedge their losses by working more to increase their labor income. They also conclude that the optimal share of stocks in the household's portfolio should decline with age. Yet others allow for shocks to labor income and risky assets returns to be correlated. For instance, Chai et al. (2011) introduce countercyclical risky labor-income dynamics. In contrast to the work cited earlier, these papers find household portfolio allocation patterns to be consistent with empirical evidence. Finally, Roussanov (2010) focuses on the role of human capital investment. Roussanov (2010) endogenizes human capital investment through an indivisible option—agents can either work or study— and matches the life-cycle profile of household portfolios.

One aspect of human capital investment and its impact on household portfolios that has not been explicitly studied in the literature is the decision to invest in college. The investment itself, as well as subsequent labor market outcomes, has many facets that can potentially influence household portfolio choice. Hungerford and Solon (1987) find that returns to education are not a smooth function of the number of years of education; the return to a year of education that culminates in a diploma is higher than the return to a year that does not. In other words, college requires an investment commitment for certain number of years before it pays off. The risk of failure is high; non-completion rates in the U.S. are around 50 percent (Bowen et al., 2009). Moreover, Ozdagli and Trachter (2011) calculate that the probability of dropping out is highest at the two- and three-year mark, implying that costly investments may already have been made for a number of years before the risk of failure is realized. Finally, college is costly, both in terms of explicit costs as well that the opportunity cost of foregone earnings. Paying for college implies reducing assets or taking on non-defaultable debt. All of these factors taken together are likely to put downward pressure on household investment in stocks. On the other hand, earnings are higher (Card, 1999) and unemployment risk is lower (Mincer, 1991) for those who are more highly educated. This may encourage investment in stocks among those who successfully complete college and realize these gains. Our model enables us to assess the resulting quantitative effect of these forces.

Before laying out the model in detail in Section 3, we describe some facts about household portfolios and earnings in the next section. The calibration is laid out in Section 4 and results are provided in Section 5.

To be completed.

## 2 Data

### 2.1 Household Portfolios

We describe salient facts about household financial portfolios and human capital from the Survey of Consumer Finances (SCF). The SCF is a survey of a cross-section of U.S. families conducted every three years by the Federal Reserve Board. It includes information about families' finances as well as their demographic characteristics. While the SCF provides us with rich detail about household finances, it is not a panel so it does not enable us to directly observe the evolution of finances over the life-cycle. To overcome this, we follow a methodology similar to Poterba and Samwick (1997) to create life-cycle profiles.

Our goal is to construct life-cycle profiles of participation in the stock market and stockholdings using cohort-level data. As Deaton (1985) describes, each successive cross-sectional survey of the population will include a random sample of a cohort if the number of observations is sufficiently large. Using summary statistics about the cohort from each cross section, a time series that describes behavior as if for a panel can be generated. In particular, sample cohort means will be consistent estimates of the cohort population mean.

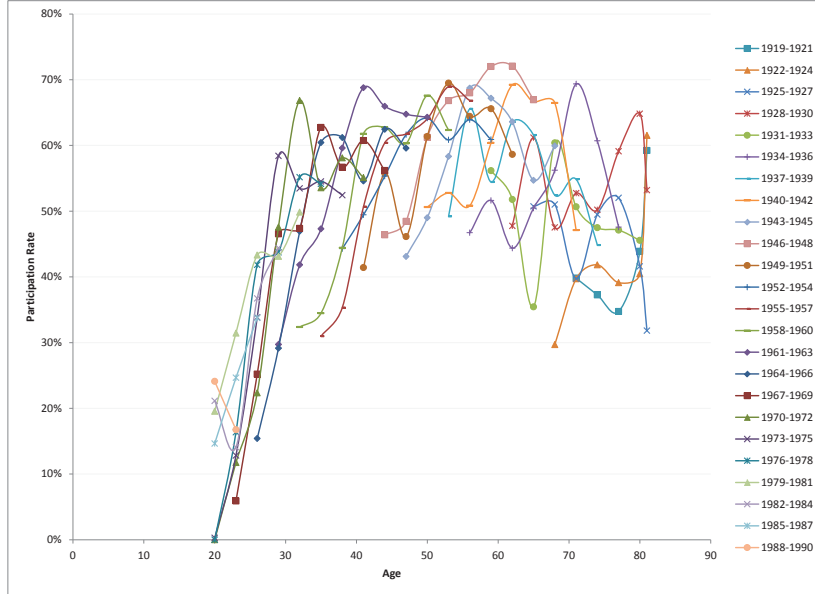
To implement a procedure in this spirit, we begin by pooling households from all eight waves of the 1989-2010 SCF into a single dataset. We assign a household to a cohort if the head of the household is born within the three-year period that defines the cohort. We have 24 cohorts in all, with the oldest consisting of households whose head was born between 1919 and 1921 and the youngest consisting of households with heads born between 1988 and 1990. We include all observations where the household head is between the ages of 18 and 80.

Except for the cohorts that are too young or too old to be represented in all waves of the survey, we have at least a hundred observations of every cohort in each survey year. We use this data to create life-cycle profiles of cohort participation in the stock market. We say that a household participates in the stock market if they have a positive amount of financial assets invested in equity. The variable in the SCF that measures this includes directly held stocks as well as stocks held in mutual funds, IRAs/Keoghs, thrift-type retirement accounts and other managed assets.

In Figure 1, we plot the average participation of a cohort against the age of the cohort (the mid-point of the age range of the cohort) for each of the 24 cohorts. For example, we observe the cohort born in 1949-1951 from the time they are age 38-40 (in the 1989 wave of the SCF) to the time they are age 59-61 (in the 2010 SCF). Figure 1 shows that participation for this cohort increases from roughly 41% to 59%.

The differences in participation rates across cohorts may be the results of three factors: ag-

Figure 1: Household Stock Market Participation Rate by Cohort (SCF)



gregate fluctuations experienced by all cohorts living in a particular year (time effects), lifetime experiences that vary by year of birth (cohort effects) and getting older (age effects). Since we are interested in participation over the life cycle—the changes in a household’s portfolio that result from that household getting older—we need to distinguish age effects from cohort and time effects. The three variables are perfectly collinear (age=year of birth–year of observation), which makes separately identifying the three effects empirically challenging. We follow Poterba and Samwick (1997) in excluding time effects and disentangle age and cohort effects. The decision to invest in stocks can be expressed using a standard probit model

$$S_i^* = \alpha + \sum_{n=2}^{21} \beta_n age_{i,n} + \sum_{m=2}^{24} \gamma_m cohort_{i,m} + \epsilon_i \quad (1)$$

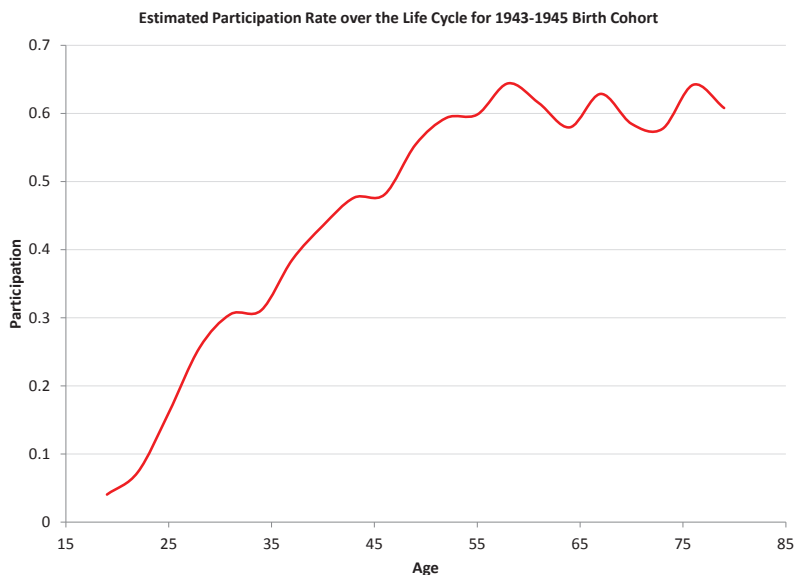
where  $S_i = 1$  if  $S_i^* > 0$  and 0 otherwise.

$S_i$  is the discrete dependent variable that equals one if household  $i$  invests in stocks and zero otherwise.  $S_i$  is determined by the continuous, latent variable  $S_i^*$ , the actual amount invested in stocks.  $S_i^*$ , and thus  $S_i$ , is specified in the above as a function of  $age_{i,n}$  and  $cohort_{i,m}$ . We include 21 dummies for age categories ranging from 18–20 to 78–80 with  $age_{i,n}$  being the dummy variable

that indicates whether the current age of the household head lies in one of the these intervals. We include 24 cohort dummies  $cohort_{i,m}$  to represent cohorts born one of the three-year interval in the range 1919–1921 to 1988–1990. Since our theoretical model starts with high-school completion, we exclude from our sample those households whose head has less than a high school diploma. The SCF oversamples wealthy households and therefore needs to be weighted to obtain estimates that are representative of the U.S. population. As in Poterba and Samwick (1997), we estimate Equation (1) using year-specific sample weights normalized such that the sum of the weights (which equals the population represented) remains constant over time.

The results of the estimation are reported in Table 1.<sup>1</sup> We use the coefficients to construct our estimate of the life-cycle profile of stock-market participation for the cohort born in 1943–1945. Figure 2 shows the results. By our estimation, participation in the stock market increases till age 50, after which it levels off.

Figure 2: Estimated Participation Rate over the Life Cycle for 1943-1945 Birth Cohort (SCF)



We now divide our sample into three groups based on their highest level of educational attainment: those with only a high-school degree, those with some college education but no degree, and

<sup>1</sup>We use all five imputates from the SCF in our estimation. While this provides accurate coefficients, the statistical significance of the results may be inflated. We only need the values of the coefficients to construct life-cycle profiles; therefore we do not report the results of the significance tests.

Table 1: Probit for Stock Market Participation (SCF), N=29,621

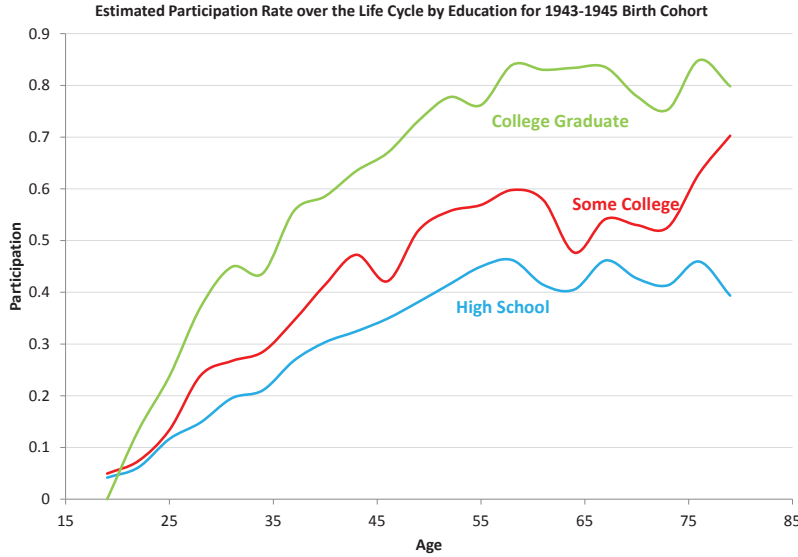
Age	Coefficient	Cohort	Coefficient
18-20	(omitted)	1919-1921	(omitted)
21-23	0.2938	1922-1924	-0.0091
24-26	0.7520	1925-1927	0.2060
27-29	1.0893	1928-1930	0.3663
30-32	1.2352	1931-1933	0.2430
33-35	1.2519	1934-1936	0.3250
36-38	1.4523	1937-1939	0.4309
39-41	1.5821	1940-1942	0.4961
42-44	1.6851	1943-1945	0.5364
45-47	1.6975	1946-1948	0.7041
48-50	1.8814	1949-1951	0.6306
51-53	1.9813	1952-1954	0.7145
54-56	1.9940	1955-1957	0.7423
57-59	2.1145	1958-1960	0.8111
60-62	2.0373	1961-1963	0.9843
63-65	1.9457	1964-1966	0.9545
66-68	2.0733	1967-1969	1.0466
69-71	1.9588	1970-1972	1.1272
72-74	1.9399	1973-1975	1.1686
75-77	2.1090	1976-1978	1.2229
78-80	2.0188	1979-1981	1.2603
		1982-1984	1.1852
		1985-1987	1.2844
Constant	-2.2811	1988-1990	1.1377

those with a college degree. We run regression (1) separately for each group, and repeat the procedure described above to construct life-cycle profiles of participation for each group. The results are reported in Figure 3 for the 1943–1945 birth cohort. Participation rates are quite distinct by educational attainment, and generally increase with the level of education.

We are also interested in portfolio allocation over the life cycle conditional on participation. In other words, we want to know how the fraction of assets invested in stocks evolves over the life cycle. As we will describe later, our model will have one risk-free asset  $b$  and one risky asset  $s$ , so the measure in which we are interested is  $\frac{s}{s+b}$ .

In our model, the risk-free asset  $b$  will be used to save or borrow. To construct the data parallel for  $b$ , we take the value of a household’s financial assets and deduct from it the value of equity held and the value of credit card debt. We treat this as the net value of the household’s risk free assets. As described earlier, the risky asset is the value of equity that the household holds, which

Figure 3: Estimated Participation Rate over the Life Cycle by Education for 1943-1945 Birth Cohort (SCF)



includes directly held stocks and stocks in mutual funds, retirement accounts and other managed assets.

We calculate the fraction  $\frac{s}{s+b}$  only for those who have non-negative net risk free assets and equity. This measure lies between 0 and 1 by construction, so we want our life-cycle estimate of it to lie between 0 and 1 as well. To ensure this, we construct a logistic transformation to obtain the variable  $Y_i = \ln \frac{\frac{s}{s+b}}{1 - \frac{s}{s+b}}$ . We run the following Ordinary Least Squares (OLS) regression on this variable.<sup>2</sup>

$$Y_i = \alpha + \sum_{n=2}^{21} \beta_n age_{i,n} + \sum_{m=2}^{24} \gamma_m cohort_{i,m} + \epsilon_i \quad (2)$$

The results are reported in Table 2. As we did for participation, we use the reported coefficients to estimate the life-cycle profile of portfolio allocation for the cohort born in 1943–1945. Figure 4 shows the results. The estimated share of risky assets conditional on participation increases steadily

<sup>2</sup>Note that, unlike Poterba and Samwick (1997), we do not use Tobit to estimate this equation. By construction, our data is not censored— values below 0 and above 1 are infeasible. Moreover, since our variable of interest the share of risky assets in the household’s portfolio conditional on participation, it will always be strictly positive. It is possible for it to exactly equal 1, but we have very few observations with this value, and in this instance we set it to 0.999999.



after age 25.

Table 2: OLS for Share of Risky Assets in Household Portfolio (SCF), N=17,692

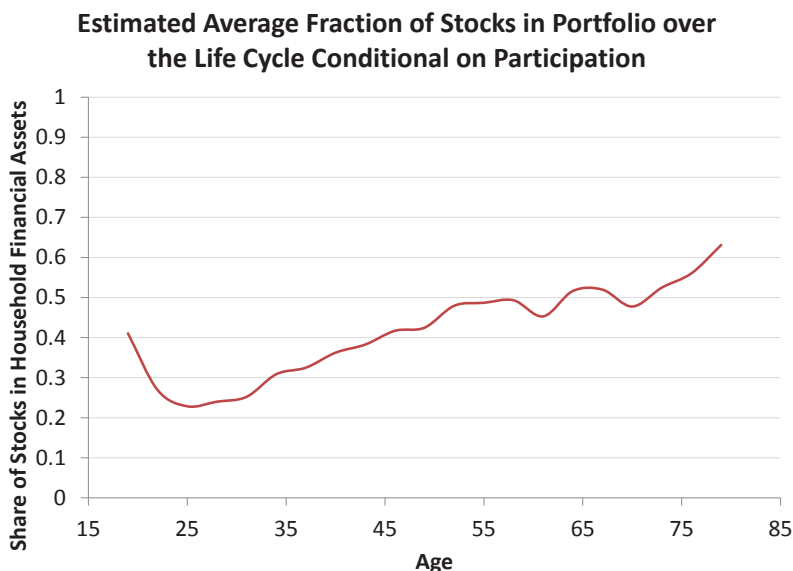
Age	Coefficient	Cohort	Coefficient
18-20	(omitted)	1919-1921	(omitted)
21-23	-0.6370	1922-1924	0.3780
24-26	-0.8546	1925-1927	0.5470
27-29	-0.7905	1928-1930	0.6343
30-32	-0.7250	1931-1933	0.5141
33-35	-0.4445	1934-1936	0.7919
36-38	-0.3688	1937-1939	0.9093
39-41	-0.2029	1940-1942	0.8164
42-44	-0.1164	1943-1945	1.0617
45-47	0.0268	1946-1948	0.9714
48-50	0.0590	1949-1951	1.2370
51-53	0.2798	1952-1954	1.3458
54-56	0.3093	1955-1957	1.2436
57-59	0.3343	1958-1960	1.4706
60-62	0.1733	1961-1963	1.5724
63-65	0.4273	1964-1966	1.8096
66-68	0.4405	1967-1969	1.7859
69-71	0.2720	1970-1972	2.0377
72-74	0.4614	1973-1975	1.7372
75-77	0.6057	1976-1978	2.1898
78-80	0.8993	1979-1981	1.8189
		1982-1984	1.5368
		1985-1987	1.3430
Constant	-1.4237	1988-1990	1.1124

Finally, we run regression (2) separately on three subsamples based on educational attainment and estimate life-cycle profiles of portfolio allocation for each subsample. Figure 5 reports the results for the 1943–1945 birth cohort. In contrast to participation rates, portfolio allocation conditional on participation does not vary greatly by educational attainment. All three series show an increase in the share of risky assets in the household portfolio, from around 20% at age 25 to between 50% and 70% at age 80.

## 2.2 Earnings

Next, we compute statistics of age-earnings profiles for each education group from the CPS for 1969-2002 using a synthetic cohort approach. Specifically, we distinguish between the three education

Figure 4: Estimated Average Fraction of Stocks in Portfolio over the Life Cycle Conditional on Participation for 1943-1945 Birth Cohort (SCF)

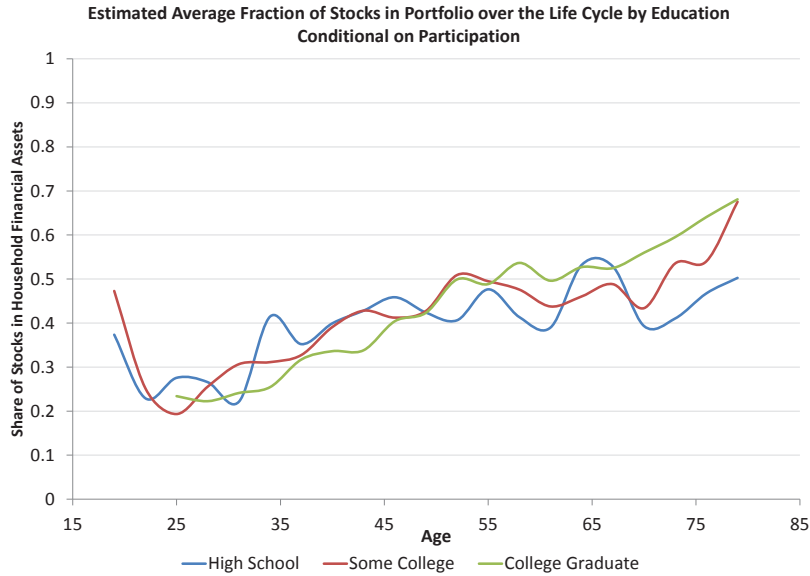


groups in our model, namely, those with 12 years of schooling (high-school), those with at least 12 years but less than 16 years of completed schooling (some college) and those with at least 16 years of completed schooling (college graduates).<sup>3</sup> We compute mean real earnings (real values are calculated using the CPI for 1982-1984 as the base), inverse skewness, and Gini of individuals of type  $(a, k)$  by averaging over the earnings of household heads between the ages of  $a - 2$  and  $a + 2$  in education group  $k$  for the appropriate year.

To be precise, we use the 1969 CPS data to calculate the earnings statistics of 20-year-old high school graduates, of 22-year-olds with some college, and of 24-year-old college graduates; the 1970 CPS data to compute earnings statistics of 21-year-old high school graduates, 23-year-olds with some college, and of 25-year-old college graduates; and so on. Life-cycle profiles for the three education groups for all three statistics are shown in Figure 15 in the Appendix. Our estimates of average real earnings imply that the present value of life-cycle earnings for the high-school group is 0.66 million, for the some college group is 0.74 million and for the college graduates group is 0.99 million. Our estimates imply a college premium of 49.6 percent and a premium for acquiring

<sup>3</sup>Education groups in the model are identified by years of schooling in the CPS data since information on the type of degree obtained is not available.

Figure 5: Estimated Average Fraction of Stocks in Portfolio over the Life Cycle by Education Conditional on Participation for 1943-1945 Birth Cohort (SCF)



a Bachelor degree over some college of 34.9 percent. The implied premia are consistent with micro-studies. Willis (1986) and Card (2001) find that the increase in lifetime earnings from each additional year in college is between 8 and 13 percent, and using CPS 1991 data, Jaeger and Page (1996) find that the marginal effect of acquiring a bachelor’s degree over completing some college is 33 percent. Restuccia and Urrutia (2004) use a 10 percent rate of return, which corresponds to a lifetime college premium of about 1.5.

With these facts in hand, we turn to the description of the model.

### 3 Model

#### 3.1 Overview

Agents start life in the model as youth with a high-school diploma, endowed with a level of human capital,  $h^{HS}$ . At this stage in life, they decide whether or not they will attend college. They also decide how to allocate any wealth they have between a risky asset  $s_t$  and a risk-free asset  $b_t$ . If they choose to attend college, and have wealth, they can use it to finance their education. They

can also borrow using non-defaultable debt  $b_t < 0$  and take out student debt  $d$  to finance their education.

At the end of four years in college, the probability of completion, which depends on the agent’s innate ability and human capital accumulation while in college, is realized. Those who complete college start their adult life with human capital  $h^{CG}$  while those who fail to complete accumulate human capital  $h^{SC}$ , where  $SC$  denotes “some college.” Those who choose not to go to college start with human capital  $h^{HS}$ .

Working adults divide their time between work and the accumulation of human capital, as in the model of Ben-Porath (1967). They consume and, as before, allocate any savings between stocks and bonds. As adults, they also have the option to borrow, that is  $b_t \geq -\underline{b}$ , with  $\underline{b} > 0$ , may be positive or negative. As before, they cannot default on this debt. We believe that having nondefaultable debt is a good abstraction because individuals close to default will likely have not accumulated resources to have interesting portfolios, and therefore the option to default on consumer debt is not central for bond and stock market choices.

We allow for three potential sources of heterogeneity across agents — their immutable learning ability,  $a$ , human capital stock,  $h$ , and initial assets,  $x$ . The set of these characteristics are jointly drawn according to a distribution  $F(a, h, x)$  on  $A \times H \times X$  and we allow for and estimate correlations between returns to stocks, bonds, and human capital.

Time is discrete and indexed by  $t = 1, \dots, T$  where  $t = 1$  represents the first year after high school graduation. Agents work and accumulate human capital until  $t = J - 1$  and start retirement in period  $t = J$  when they face a simple consumption-savings problem.

## 3.2 Preferences

The general problem of an individual is to choose consumption over the life-cycle,  $\{c_t\}_{t=1}^T$  to maximize the expected present value of utility over the life cycle

$$\max_{\{c_t\} \in \Pi(\Psi_0)} E_0 \sum_{t=1}^T \beta^{t-1} u(c_t), \quad (3)$$

where  $u(\cdot)$  is strictly concave and increasing. Also,  $\Pi(\Psi_0)$  denotes the space of all feasible combinations  $\{c_t\}_{t=1}^T$ , given initial state  $\Psi_0$ . Agents have a common discount factor,  $\beta$ . Preferences are represented by a standard time-separable CRRA utility function over consumption. Agents value consumption and do not value leisure.

### 3.3 Financial Markets

There are two financial assets in which the agent can invest, a risk-free asset,  $b_t$ , and a risky asset,  $s_t$ .

#### Risk-free assets

An agent can borrow or save using asset  $b_t$  which can be 0, positive, or negative. Savings will earn the risk-free interest rate,  $R_f$ . We assume that the borrowing rate,  $R_b$ , is higher than the savings rate:  $R_b = R_f + \phi$ . Debt is non-defaultable and comes with a borrowing limit  $\underline{b} > 0$ .

#### Risky assets

We call the risky assets stocks and we denote the agent's holdings of equity between period  $t$  and  $t + 1$  by  $s_{t+1}$ . This amount entitles the owner to its stochastic return in period  $t + 1$ ,  $R_{s,t+1}$ . This represents a gross real return and its excess return is given by:

$$R_{s,t+1} - R_f = \mu + \eta_{t+1}, \quad (4)$$

where  $\eta_{t+1}$ , the period  $t + 1$  innovation to excess returns, is assumed to be independently and identically distributed (i.i.d.) over time and distributed as  $N(0, \sigma_\eta^2)$ . We will allow innovations to excess returns to be correlated with innovations to the aggregate component of permanent labor income.

Given asset investments at age  $t$ ,  $b_{t+1}$  and  $s_{t+1}$ , financial wealth at age  $t + 1$  is given by  $x_{t+1} = R_i b_{t+1} + R_{s,t+1} s_{t+1}$ , with  $R_i = R_f$  if  $b \geq 0$  and  $R_i = R_b$  if  $b < 0$ .

### 3.4 Human Capital

Agents can invest in their human capital in two ways — by investing in a college education when young and by apportioning some of their time to acquiring human capital as adults through on-the-job training. Human capital stock refers to “earning ability” and can be accumulated over the life cycle, while learning ability is fixed at birth and does not change over time. We assume that the technology for human capital accumulation is the same during and after college and that human capital is not productive until graduation.

### 3.4.1 Human capital investment as on-the-job training

College graduates, college dropouts and high school graduates who do not enroll in college optimally allocate time between market work and human capital accumulation as on-the-job training during the adult phase of their life.

Human capital evolves according to the human capital production,  $H(a, h_t, l_t)$ , which depends on the agent's immutable learning ability,  $a$ , human capital,  $h_t$ , and the fraction of available time put into human capital production,  $l_t$ . Human capital depreciates at a rate  $\delta_i$  with  $i \in \{cg, nc\}$  and  $\delta_{cg} > \delta_{nc}$  where  $nc$  stands for both individuals with some college,  $SC$  and for high-school graduates who do not go to college,  $HS$ . The law of motion for human capital is given by

$$h_{t+1} = h_t(1 - \delta_i)H(h_t, l_t, a) \quad (5)$$

Following Ben-Porath (1967), the human capital production function is given by  $H(h, l, a) = a(hl)^\alpha$  with  $\alpha \in (0, 1)$ .

### 3.4.2 College investment

Agents who wish to acquire a college degree optimally divide time between work and human capital while in college; they may invest in both risky and risk-free assets. In addition, they may choose to borrow to finance their college education. They face two types of risks: dropping out of college and uncertainty in their earnings after college. The college dropout risk depends on the human capital stock at the end of college, which in turn is determined by the agent's decision to allocate time to human capital accumulation during college. There are several sources of college financing: family contributions, need-based student loans and non-defaultable debt. Students may also use their labor income and savings (if any) during college to finance their college education.

During college, students may choose to work at the wage rate  $w_{col}$ , but their human capital is not productive until they leave college. Working during college diverts time from human capital accumulation and may increase students' chances of leaving college without acquiring a degree. At the same time, college students have jobs that pay a low wage and do not necessarily value students' human capital stock or contribute to human capital accumulation (Autor et al. (2003)). However, students of high ability may be hired in better paid jobs than students of low ability.<sup>4</sup> Thus, we

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<sup>4</sup>Our modeling is motivated by several important observations. Numerous studies show that working while in school can adversely affect academic performance (Stinebrickner and Stinebrickner, 2003) and increase the likelihood of dropping out (Braxton et al., 2003). Thus, unlike in an environment where the possibility of dropping out from college is not modeled (or is completely exogenous), allowing for the choice to allocate time to work versus human capital accumulation during college is a key ingredient when accounting for the risk of investing in human capital.

model a wage rate per time units worked in college,  $w_{col}(a)$ , instead of per efficiency units; this rate increases with the ability level of the student. This assumption prevents low-ability students from enrolling in college only to enjoy earnings during college that are much higher than the earnings they would have earned had they not enrolled in college. We assume that the growth rate in earnings during college is 0.

Agents are allowed to take out student loans up to  $d(x) = \bar{d} - x$ , which represents the full college cost,  $\bar{d}$ , minus the expected family contribution,  $x$ . They choose the loan amount,  $d$ , at the beginning of college, and they receive equal fractions of the loan each period in college. After college they will repay this loan in equal payments,  $p$  which are determined by the loan size,  $d(x)$ , interest rate on student loans,  $R_g$  and the duration of the loan,  $P$ . Consistent with the data, the interest rate on student loans is  $R_f < R_g < R_b$ .

College investment is risky. If a student with initial human capital  $h_1$  decides to acquire a college degree, the probability with which she succeeds is given by  $\pi(h_5(h_1, a, l_{1,\dots,4}^*))$ . This is a continuous, increasing function of the human capital stock after college years,  $h_5$ , which in turn increases with the initial human capital stock,  $h_1$ , the ability of the individual,  $a$ , and her choice of time devoted to human capital investment during college years,  $l_{1,\dots,4}^*$ . This formulation captures the idea that college preparedness, embodied in  $h_1$ , student's learning capacity, captured in  $a$ , and effort to invest in human capital during college are important determinants of college completion. If the student completes college, she will walk into period 5 as a college graduate, i.e.  $i = CG$ , and if she does not complete college, she will walk into period 5 as a college dropout with some college education, i.e.  $i = SC$ .<sup>5</sup>

### 3.5 Labor Income

During the adult phase,  $t = t_w, \dots, J-1$ , with  $t_w = 1$  for no college and  $t_w = 5$  for college graduates and dropouts, human capital stock is valued each period in the labor market. Earnings are given by product of the stochastic component,  $z_t$ , the rental rate of human capital,  $w_t$ , the agent's human capital,  $h_t$ , and the time spent in market work,  $(1 - l_t)$ .

Therefore, agent's  $i$  earnings in period  $t$  are given by

$$\log(y_t) = G(w_t, h_t, l_t) + z_t \tag{6}$$

with  $G(w_t, h_t, l_t)$  representing the deterministic component as a function of rental rate  $w_t$ , human

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<sup>5</sup>Modeling investment in four-year college and the risk of dropping out at the end of the fourth period in the model are justified by data: according to the BPS 1996/2001, 68.5% of students enroll in four-year colleges. Our findings also show that 89% of college dropouts are enrolled in college at least for three full years.

capital stock at age  $t$ ,  $h_t$  and labor effort,  $1 - l_t$  and  $z_t$  representing the stochastic component. The rental rate of human capital evolves over time according to  $w_t = (1 + g_i)^{t-1}$  with the growth rate,  $g_i$  with  $i \in \{cg, nc\}$ . Depending on whether agents have a college degree or not, they face different growth rates in the rental rate with  $g_{cg} > g_{nc}$ .<sup>6</sup>

The stochastic component,  $z_{it}$  consists of an idiosyncratic temporary shock  $\epsilon_{it} \sim N(0, \sigma_\epsilon^2)$  and a persistent shock  $u_{it} = \rho u_{i,t-1} + \nu_{it}$ , with  $\nu_{it} \sim N(0, \sigma_\nu^2)$  innovation process. The variables  $u_{it}$  and  $\epsilon_{it}$  are realized at each period over the life cycle and are not correlated. The process for  $u_{it}$  is taken to be a random walk, following Gourinchas and Parker (2002) and Hubbard et al. (1994). The latter estimate a general first-order autoregressive process and find the autocorrelation coefficient to be very close to one. We assume that the temporary shock  $\epsilon_{it}$  is uncorrelated across households, but we decompose the permanent shock  $\nu_{it}$  into an aggregate component  $\xi_{it} \sim N(0, \sigma_\xi^2)$  and an idiosyncratic component  $\omega_{it} \sim N(0, \sigma_\omega^2)$ . This decomposition implies that the random component of aggregate labor income follows a random walk, an assumption made in the finance literature (see Fama and Schwert (1977) and Jagannathan and Wang (1996)).

Finally, we allow for the correlations between returns to human capital and financial assets through the aggregate component of the income process,  $\xi_t$ . Let  $\Sigma$  be the covariance matrix for the payoffs on all assets where

$$[R_f, R_s, \xi_t] \sim N(\mu, \Sigma) \quad (7)$$

### 3.6 Means-Tested Transfer and Retirement Income

In addition to labor income, agents receive means-tested transfers from the government,  $\tau_t$ , which depend on age,  $t$ , income,  $y_t$ , and net assets,  $x_t$ . These transfers capture the fact that in the U.S. social insurance is aimed at providing a floor on consumption. Following Hubbard et al. (1994) we specify these transfers by

$$\tau_t(t, y_t, x_t) = \max\{0, \underline{\tau} - (\max(0, x_t) + y_t)\} \quad (8)$$

Total pre-transfer resources are given by  $\max(0, x_t) + y_t$  and the means-testing restriction is represented by the term  $\underline{\tau} - \max((0, x_t) + y_t)$ . These resources are deducted to provide a minimal income level  $\underline{\tau}$ . For example, if  $x_t + y_t > \underline{\tau}$  and  $x_t > 0$ , then the agent gets no public transfer. By contrast, if  $x_t + y_t < \underline{\tau}$  and  $x_t > 0$ , the the agent receives the difference, case in which he has  $\underline{\tau}$

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<sup>6</sup>The growth rates for wages are estimated from data. Evidence shows that wage growth rates for college dropouts and people with no college are similar and are lower than the wage growth rate for college graduates. Also, human capital depreciates faster for college graduates than for college dropouts and individuals who do not enroll in college. See Section 4.1 for details.



units of the consumption good at the beginning of the period. Agents do not receive transfers to cover debts, which requires the term  $\max(0, x_t)$ . Lastly, transfers are required to be nonnegative, which requires the “outer” max.

After period  $t = J$  when agents start retirement, they get a constant fraction  $\phi(y_J)$  of their income in the last period as working adults, which they divide between risky and risk-free investments.<sup>7</sup>

### 3.7 Agent’s Problem

The agent maximizes lifetime utility by choosing asset positions in bonds and stocks, time allocated to market work and to human capital, and borrowing.

We formulate the problem in a dynamic programming framework where any period  $t$  variable  $j_t$  is denoted by  $j$  and its period  $t+1$  value by  $j'$ . The state vector is defined as follows. An individual’s feasible set of consumption and savings is determined by his age,  $t$ , ability,  $a$ , beginning-of-period human capital,  $h$  and net worth,  $x(b, s)$ , current-period realization of the persistent shock to earnings,  $u$ , and current-period transitory shock,  $\nu$ .

We solve the problem backwards starting with the last period of life when agents consume their savings. The value function in the last period of life is set to  $V_T^R(a, h, x) = u(x)$ . For the retirement phase, the value function is given by

$$V^R(t, a, h, b, s) = \sup_{b', s'} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} + \beta V^R(t+1, a, h', b', s') \right\} \quad (9)$$

where

$$c + b' + s' \leq \phi(y_J) + R_i b + R_s s$$

In the above,  $R_i = R_f$  if  $b \geq 0$  and  $R_i = R_b$  if  $b < 0$ .

Retired agents do not accumulate human capital. They face a simple consumption-savings problem but may choose to invest in both risk-free and risky assets.

Given important differences between education groups during the remaining stages of the life cycle, we present the value functions and further details separately for the no-college and college paths.

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<sup>7</sup>We will relax this assumption in the quantitative part of the paper where we introduce an additional uncertainty during retirement (e.g., stochastic medical expenses along the lines of Hubbard et al. (1995)), since this may slow down the pace at which wealth is being depleted.

### 3.7.1 No College

We use  $V_J^{R,i}(t, a, h, b, s)$  from Equation 9 as a terminal node for the adult's problem on the no college path. Note that  $V_R^{HS}(t, a, h, b, s, u, \nu) = V_R^R(a, h', b', s') \forall u, \nu$ .<sup>8</sup> We solve for the set of choices in the working phase, for which the value function is given by

$$V^{HS}(t, a, h, b, s, u, \nu) = \sup_{l, h', b', s'} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} + \beta E_{u'/u} V^{HS}(t+1, a, h', b', s', u', \nu') \right\} \quad (10)$$

where

$$\begin{aligned} c + b' + s' &\leq w(1-l)hz + R_b b + R_s s + \tau(t, y, x) \text{ for } t = 1, \dots, J-1 \\ l &\in [0, 1], h' = h_t(1 - \delta_{nc}) + \pi(a)a(hl)^\alpha \end{aligned}$$

The value function  $V^{HS}(t, a, h, b, s, u, \nu)$  gives the maximum present value of utility at age  $t$  from states  $h$ ,  $b$ , and  $s$ , when learning ability is  $a$  and the realized shocks are  $u$  and  $\nu$ . Solutions to this problem are given by optimal decision rules  $l_j^*(t, a, h, b, s, u, \nu)$ ,  $h^*(t, a, h, b, s, u, \nu)$ ,  $b^*(t, a, h, b, s, u, \nu)$  and  $s^*(t, a, h, b, s, u, \nu)$ , which describe the optimal choice of the fraction of time spent in human capital production, the level of human capital, and risk-free and risky assets carried to the next period as a function of age,  $t$ , human capital,  $h$ , ability,  $a$ , and current assets,  $b$  and  $s$  when the realized state is  $(u, \nu)$ . The value function,  $V^{HS}(1, a, h, x)$ , gives the maximum expected present value of utility if the agent chooses not to go to college from initial state  $h$ , when learning ability is  $a$  and initial assets are  $x$ .

### 3.7.2 College

As before, the problems for college graduates and dropouts are solved backwards, starting with the retirement phase for which the value function is given by Equation 9.

**Working phase,  $j = 5, \dots, J-1$**

We use  $V_J^{R,i}(t, a, h, b, s)$  from the retirement phase as a terminal node and solve for the set of choices in this phase of the life-cycle. We further break down this phase into a post-repayment period and a repayment period. For the post-repayment period,  $t = P+1, \dots, J-1$ , the problem is identical to the one for working adults on the no-college path. Recall that during the repayment period after college,  $t = 5, \dots, P$ , agents have to repay their student loans with a per period payment  $p = \frac{d(x)}{\sum_{t=1}^{P-5} R_g^t}$  with  $d(x_1)$  the size of the loan which depends on initial assets  $x_1$  and  $R_g$  the interest rate on student loans.

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<sup>8</sup>The same idea is used whenever we switch from one phase to another without explicitly stating this.

The value function is given by

$$V^i(t, a, h, b, s, u, \nu) = \sup_{l, h', b', s'} \left\{ \frac{c^t}{1 - \sigma} + \beta E_{u'/u} V^i(t + 1, a, h', b', s', u', \nu') \right\} \quad (11)$$

where

$$\begin{aligned} c + b' + s' &\leq w(1 - l)hz + R_j b + R_s s + \tau(t, y, x) \text{ for } t = P + 1, \dots, J - 1 \\ c + b' + s' &\leq w(1 - l)hz + R_j b + R_s s + \tau(t, y, x) - p(x_1) \text{ for } t = 5, \dots, P \\ l &\in [0, 1], h' = h_t(1 - \delta_i) + \pi(a)a(hl)^\alpha \end{aligned}$$

where  $i = CG, SC$ ;  $R_i = R_f$  if  $b \geq 0$  and  $R_i = R_b$  if  $b < 0$ .

**College phase,  $t = 1, \dots, 4$**

For this stage of the life-cycle we first take into account the risk of dropping out from college and use  $V^C(5, a, h, b, s, u, \nu) = \pi(h_5)V^{CG}(5, a, h, b, s, u, \nu) + (1 - \pi(h_5))V^{SC}(5, a, h, b, s, u, \nu)$  as the terminal node to solve for the optimal rules. Agents invest in their human capital during college and they may decide to work. Each period in college they pay direct college expenses,  $\hat{d}$ . In the first period, they also choose the loan amount for college education,  $d$ , which will be equally divided in four rounds of loans during college years. The value function is given by

$$\begin{aligned} V^C(t, a, h, b, s) &= \max_{l, h', b', s', d} \left[ \frac{c^{1-\sigma}}{1 - \sigma} + \beta V^C(t, a, h, b, s) \right] \\ c + b' + s' &= w_{col}(1 - l) + R_b b + R_s s + d/4 - \hat{d} \\ l &\in [0, 1], h' = h(1 - \delta_c) + a(hl)^\alpha \\ d &\in D = [0, \bar{d}(x)] \text{ for } t = 1. \end{aligned} \quad (12)$$

Solutions to this problem are given by optimal decision rules  $l_j^*(t, a, h, b, s, u, \nu)$ ,  $h^*(t, a, h, b, s, u, \nu)$ ,  $b^*(t, a, h, b, s, u, \nu)$  and  $s^*(t, a, h, b, s, u, \nu)$ , which describe the optimal choice of the fraction of time spent in human capital production, the level of human capital, and risk-free and risky assets carried to the next period as a function of age,  $t$ , human capital,  $h$ , ability,  $a$ , and current assets,  $b$  and  $s$  when the realized state is  $(u, \nu)$ . The value function,  $V^C(1, a, h, x)$ , gives the maximum expected present value of utility if the agent chooses to go to college from initial state  $h$ , when learning ability is  $a$  and initial assets are  $x$ .

Once the college and no-college paths are fully determined, agents then select between going to college or not by solving  $\max[V^C(1, a, h, x), V^{HS}(1, a, h, x)]$ .

## 4 Mapping the model to the data

There are four sets of parameters that we calibrate: 1) standard parameters, such as the discount factor and the coefficient of risk aversion; 2) parameters for the initial distribution of characteristics; 3) parameters specific to human capital and to the earnings process; and 4) parameters specific to asset markets and default. Our approach includes a combination of setting some parameters to values that are standard in the literature, calibrating some parameters directly to data, and jointly estimating the parameters that we do not observe in the data by matching moments for several observable implications of the model.

Adult age starts at age 20 for households without a college degree, and at age 24 for households with a college degree or with some college education. The per period utility function is CRRA,  $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$ , with the coefficient of risk aversion  $\sigma = 5$ , which is consistent with values chosen in the financial literature. Risk aversion is a key parameter and so we conduct robustness checks on it, in particular we consider higher values up to the upper bound of  $\sigma = 10$  considered reasonable by Mehra and Prescott (1985). We also consider lower values, such as  $\sigma = 3$ . The discount factor ( $\beta = 0.96$ ) chosen is also standard in the literature.

We find the joint distribution of unobserved characteristics by matching statistics of life-cycle earnings in the March Current Population Survey (CPS) for 1969-2002. Agents live 58 model periods, which corresponds to ages 21 to 78.<sup>9</sup>

Education groups are based on years of education completed with exactly 12 years for high school graduates who do not go to college, more than 12 years and less than 16 years of completed schooling for college dropouts, and with 16 and 17 years of completed schooling for college graduates.<sup>10</sup> Life-cycle profiles of earnings are given in Figure 15.

The rental rate on human capital equals  $w_t = (1 + g)^{t-1}$ , and the growth rate is calibrated to match the PSID data on earnings as in Huggett et al. (2006). Given the growth rate in the rental rates,  $g = 0.0014$ , the depreciation rate is set to  $\delta = 0.0114$ , so that the model produces the rate of decrease of average real earnings at the end of the working life cycle. The model implies that at the end of the life cycle negligible time is allocated to producing new human capital and, thus, the gross earnings growth rate approximately equals  $(1 + g)(1 - \delta)$ . We set the elasticity parameter in the human capital production function,  $\alpha$ , to 0.7. Estimates of this parameter are surveyed by

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<sup>9</sup>For each year in the CPS, we use earnings of heads of households age 25 in 1969, age 26 in 1970, and so on until age 58 in 2002. We consider a five-year bin to allow for more observations, i.e., by age 25 at 1969, we mean high school graduates in the sample that are 23 to 27 years old. Real values are calculated using the CPI 1982-1984. For each year the following statistics are computed: mean, inverse skewness and the gini coefficient.

<sup>10</sup>Education groups in the model are identified by years of schooling in the CPS data since information on the type of the degree obtained is not available.

Browning et al. (1999) and range from 0.5 to 0.9.

In the parametrization of the stochastic component of earnings,  $z_{it}$ , we follow Abbott et al. (2013) who use the National Longitudinal Survey of Youth (NLSY) data using CPS-type wage measures to estimate the autoregressive coefficients for the transitory and persistent shocks to wages. For the persistent shock,  $u_{it} = \rho u_{i,t-1} + \nu_{it}$ , with  $\nu_{it} \sim N(0, \sigma_\nu^2)$  innovation process and for idiosyncratic temporary shock  $\epsilon_{it} \sim N(0, \sigma_\epsilon^2)$  they report the following values for high school graduates:  $\rho = 0.951$ ,  $\sigma_\omega^2 = 0.055$ , and  $\sigma_\nu^2 = 0.017$  and college graduates:  $\rho = 0.945$ ,  $\sigma_\omega^2 = 0.052$ , and  $\sigma_\nu^2 = 0.02$ . We use the first set of values for people with no college and some college education, and the second set of values for those who complete four years of college. Recall that the temporary shock  $\epsilon_{it}$  is uncorrelated across households, but the permanent shock  $\nu_{it}$  is divided into an aggregate component  $\xi_{it} \sim N(0, \sigma_\xi^2)$  and an idiosyncratic component  $\omega_{it} \sim N(0, \sigma_\omega^2)$ . We estimate the aggregate component of the income process,  $\sigma_\xi^2$ , together with the returns to financial assets when estimating the covariance matrix for the payoffs,  $\Sigma$  in Equation (7). We set retirement income to be a constant fraction of labor income earned in the last year in the labor market. Following Cocco (2005) we set this fraction to 0.682 for high school graduates and for individuals with some college education and to 0.93 for college graduates.

We turn now to the last set of parameters in the model: those related to financial markets. We consider the mean equity premium  $\mu = 0.06$ . The standard deviation of innovations to the risky asset is set to its historical value,  $\sigma_\eta = 0.157$ . The risk-free rate is set equal to  $R_f = 0.04$ , consistent with values in the literature (McGrattan and Prescott (2000)) and the wedge between the borrowing and risk-free rate is  $\phi = 0.07$  to match the average borrowing rate of  $R_b = 0.11$  (Board of Governors of the Federal Reserve System, 2014). We introduce heterogeneity in the borrowing limit as follows: we group agents in the model by quartiles of initial human capital, compute average earnings over the life cycle for each quartile and set the borrowing limit for all agents within a quartile to be a percentage of the average life-cycle earnings for that quartile. We obtain the relevant percentages from the SCF by dividing the sample into income quartiles and calculating the average credit limit as a percentage of the average income within each quartile. The resulting borrowing limits as a percentage of average earnings by quartiles are: 55%, 48%, 35%, and 27%.<sup>11</sup> The interest rate on student loans is set to  $R_g = 0.068$ . We assume for the time being that the returns to assets are uncorrelated.

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<sup>11</sup>We extrapolate the first percentage from the other three rather than calculating it directly because of the large numbers of zeros in the earnings data for the lowest quartile.

## 4.1 The Distribution of Assets, Ability and Human Capital

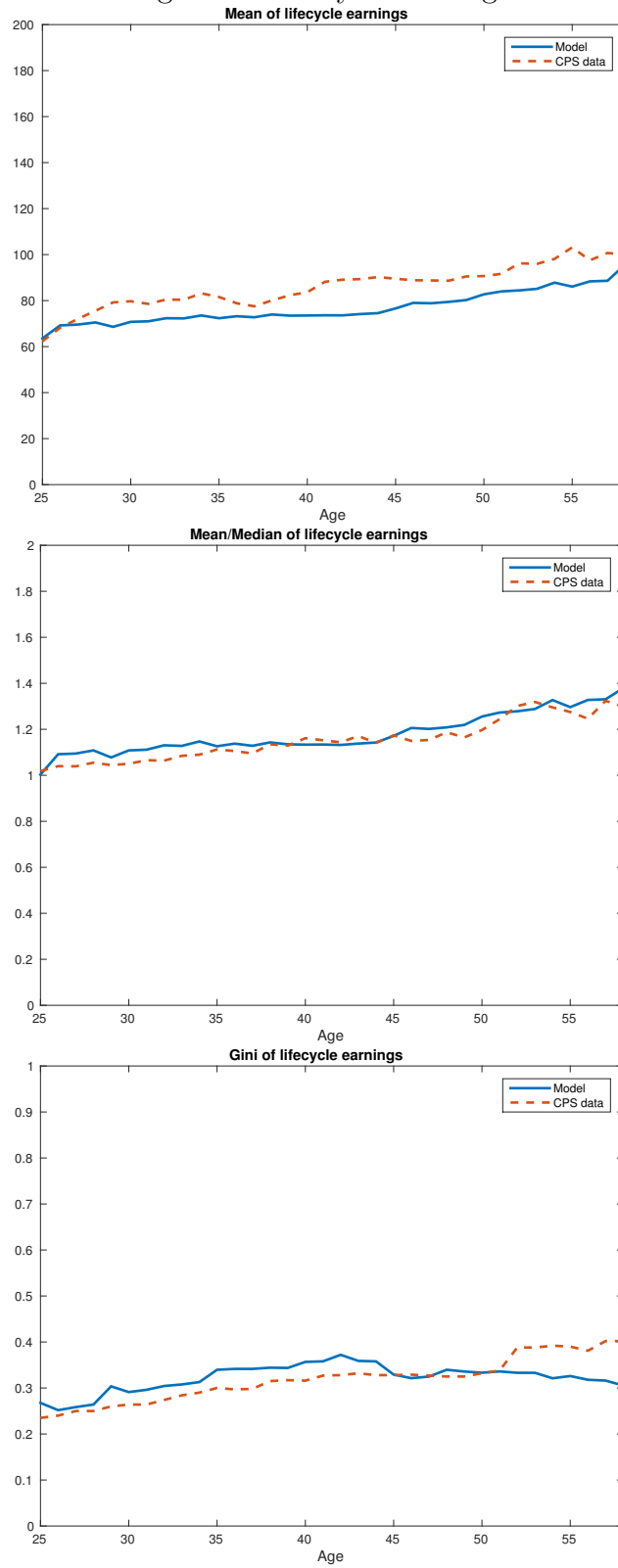
We estimate the joint distribution of initial assets, ability, and human capital by accounting for correlations between all these three characteristics in the following way. First, for the asset distribution, we use the SCF data as described in the Section 2. Second, we calibrate the initial distribution of ability and human capital to match key properties of the life-cycle earnings distribution in the U.S. data. In order to carry out this procedure, we use the CPS 1969-2002 family files for heads of household aged 25 in 1969 and followed until 2002 for life-cycle earnings. Earnings distribution dynamics implied by the model are determined in several steps: i) we compute the optimal decision rules for human capital using the parameters described above for an initial grid of the state variable; ii) we simultaneously compute financial investment decisions and compute the life-cycle earnings for any initial pair of ability and human capital; and iii) we choose the joint initial distribution of ability and human capital to best replicate the properties of U.S. data.

Using a parametric approach, we search over the vector of parameters that characterize the initial state distribution to minimize the distance between the model and the data statistics. We restrict the initial distribution on the two dimensional grid in the space of human capital and learning ability to be jointly, log-normally distributed. This class of distributions is characterized by 5 parameters. In practice, the grid is defined by 20 points in human capital and ability. We find the vector of parameters  $\gamma = (\mu_a, \sigma_a, \mu_h, \sigma_h, \rho_{ah})$  characterizing the initial distribution by solving the minimization problems  $\min_{\gamma} \left( \sum_{j=5}^J |\log(m_j/m_j(\gamma))|^2 + |\log(d_j/d_j(\gamma))|^2 + |\log(s_j/s_j(\gamma))|^2 \right)$ , where  $m_j, d_j,$  and  $s_j$  are mean, dispersion, and inverse skewness statistics constructed from the CPS data on earnings, and  $m_j(\gamma), d_j(\gamma),$  and  $s_j(\gamma)$  are the corresponding model statistics. Overall, we match 102 moments.<sup>12</sup> Figure 6 illustrates the earnings profiles for individuals in the model versus CPS data when the initial distribution is chosen to best fit the three statistics considered. We obtain a fit of 7.8% (0% would be a perfect fit). The model performs well given riskiness of assets and stochastic earnings in the current paper.<sup>13</sup> The model produces a correlation between ability and human capital of 0.65. We also estimate the correlations between initial assets and ability and human capital to match enrollment and completion rates by initial assets in the data, as delivered by the NCES data. The estimation of the distribution for initial characteristics delivers the following correlations:  $\rho(a, x) = 0.2, \rho(h, x) = 0.16$

<sup>12</sup>For details on the calibration algorithm see Ionescu (2009).

<sup>13</sup>For instance, Huggett et. al. (2008) obtains a fit of 7% (for the same value of the elasticity parameter  $\alpha = 0.7$ ) in a Ben Porath model where the main choice is investment in human capital to maximize lifetime earnings in a framework without investments in financial assets, debt, and earnings uncertainty. As a measure of goodness of fit, we use  $\frac{1}{3J} \sum_{j=5}^J |\log(m_j/m_j(\gamma))| + |\log(d_j/d_j(\gamma))| + |\log(s_j/s_j(\gamma))|$ . This represents the average (percentage) deviation, in absolute terms, between the model-implied statistics and the data.

Figure 6: Life-cycle earnings



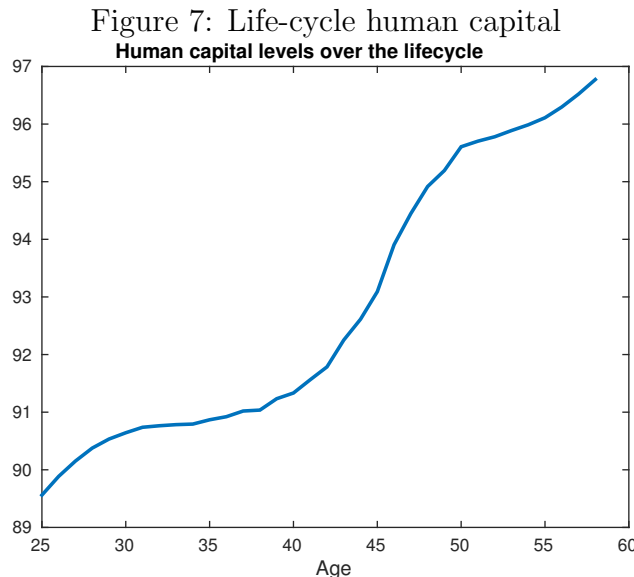
## 5 Results

First we study a version of the model where we restrict attention to human capital investment through on-the-job training as in Ben-Porath (1967). We focus on trade-offs between investments in financial assets and investments in human capital over the life cycle absent any college investment decision and college risk. Results are presented in Section 5.1. Next we study the benchmark model with college enrollment and dropout risk along with student loan debt and study the implications of college investment and financing on life-cycle portfolios. Results are presented in Section 5.2.

### 5.1 On-the-Job Human Capital Investment and Financial Investments

In this section we describe what our model predicts in terms of human capital investment and stock market participation. Note that we did not explicitly “match” these to the data, so they provide an independent metric of how well our model fits the facts.

The first set of figures (Figure 7) shows human capital accumulation over the life cycle. Households invest the greatest share of their time in human capital accumulation when young both because their opportunity cost of doing so is low and their absolute level of human capital is low. Human capital accumulates rapidly till agents are in their mid-30s after which it levels off as the time horizon shrinks and there is less time to collect the returns to human capital investment.

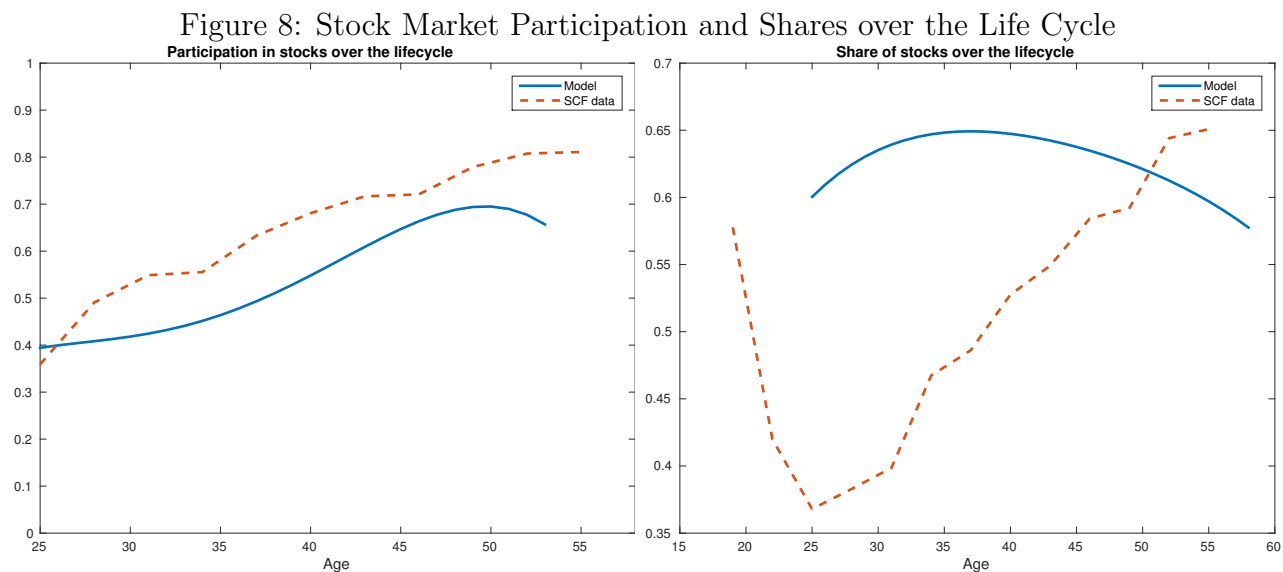


The incentives to invest in human capital over the life cycle dictate investments in risky assets. For young individuals human capital is a relatively high return investment. They would rather



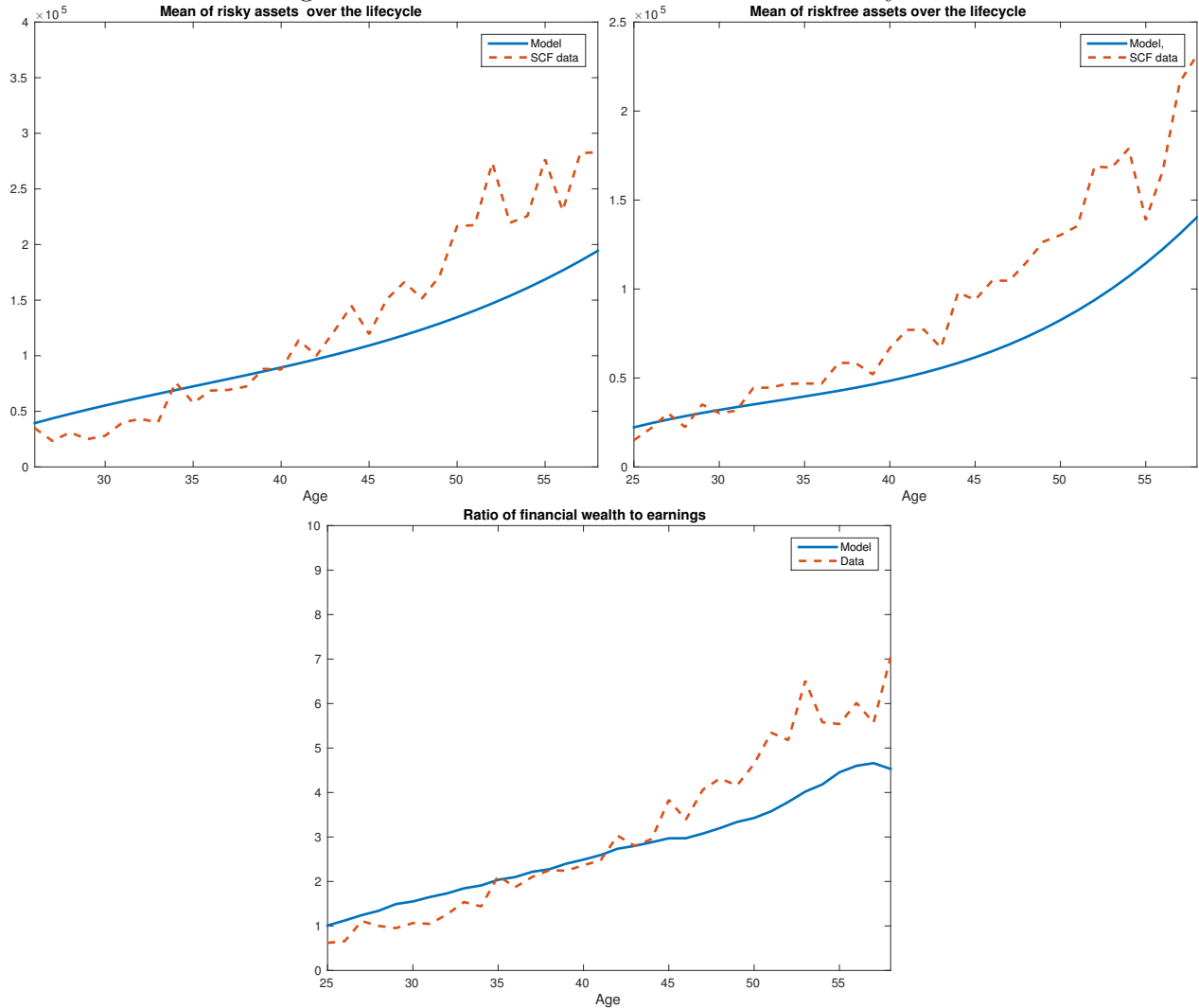
allocate more time to human capital and less to work. Consequently, they have lower participation rates in the stock market and invest a lower amount, on average, in risky assets. Once human capital stock increases, however, human capital investment becomes more costly and delivers lower returns (higher opportunity cost and less time to collect). As a result, older individuals substitute away from human capital investment to financial investments. They choose to devote less time to human capital and instead increase their participation rates and the amount invested in risky assets. This trade-off is key in delivering life cycle financial investments in our model.

Our second result is the model’s prediction of stock market participation over the life cycle. This is illustrated in Figure 8, in which the results are compared with our life-cycle estimate from the SCF<sup>14</sup>. The profile of participation predicted by the model is qualitatively consistent with the data as well as quantitatively close. Participation increases steadily with age and then starts to flatten out around age 50.



<sup>14</sup>The SCF estimates in Figure 8 are the same as in Figure 1

Figure 9: Wealth Accumulation over the Life Cycle



To summarize, our model does well overall in predicting patterns in life cycle household human capital and financial portfolios that are consistent with the data, with the exception of the share of the portfolio invested in stocks.

The trade-offs between human capital and financial investments vary across individuals with different levels of initial human capital and ability.

### 5.1.1 The importance of initial human capital

The model's predictions of life cycle earnings by quartile of initial human capital show that lifetime earnings are an increasing function of initial human capital levels and the differences across quartiles are quantitatively important. The incentives to invest in human capital are not monotone. While

all groups invest more time in human capital at the beginning of their lives, individuals in the top quartile of initial human capital invest relatively less while individuals in the bottom quartiles devote relatively more time to human capital investment. For the bottom quartile, human capital investment has a low opportunity cost.

As a result, the participation in the stock market varies substantially across individuals with different levels of initial human capital. The participation of households in the top quartile of the distribution of initial human capital steadily increases with age and is consistent with the data on college graduate. The remaining households follow a different pattern of participation. Around 30% of households age 25 participate in the stock market. This figure reduces to around 20% by age 30. Participation increases steadily till age 50 after which it flattens out, in contrast to the pattern seen among agents 50 and older in the top quartile.

### **5.1.2 The Role of Borrowing**

Giving agents in our model the ability to borrow is central to our results. To illustrate this, we look at the model's results when borrowing is shut down. Our findings show that time invested in human capital is lower when we do not allow agents in our model to borrow. The model without borrowing also predicts stock market participation rates that are much lower than in the data in the early part of the life cycle. In the absence of borrowing, agents save in order to invest in risky human capital early in life, and most do not invest in risky financial assets at the same time. Later in life, they invest their savings in stocks, because of which participation in the stock market is much higher than in the case with borrowing (and much higher than in the data). The model without borrowing predicts counterfactually high stock market participation rates late in life for all quartiles of the distribution of initial human capital.

To be completed.

## **5.2 College Investment: Implications for Life-Cycle Portfolios**

In this section we study the interaction between college investment, on-the-job investment in human capital and financial assets over the life-cycle. We focus on the role of college investment and risk for life-cycle financial portfolios.

The model delivers that 49% of high-school graduates enroll in college( and the data counterpart is 47%). College completion in the model is 57% (data is 49%). The average student loan debt is \$13,364 (in 2014 dollars) which is in line with the data as well. The model delivers a college premium of 1.58 and a college degree premium of 1.45, both of which are consistent with the estimates in the literature. Table 5.2 show the model's predictions for college enrollment and

Table1: College investment by initial characteristics

Characteristic	College Enrollment	College Completion
Initial Assets	31	47
	37	54
	51	58
	78	64
Ability	16.2	43.1
	48.1	45.2
	50.2	55.1
	84.8	67.8
Human Capital	36	42.2
	45.8	50.2
	48.2	66.6
	55.5	68.5

Note: Data for enrollment by initial assets is: 34%(Q1),47%(Q2+Q3),62%(Q4) and for completion: 37%,45%,60%.

completion by quartiles of initial characteristics. Recall that the rates by quartiles of initial assets were targeted in the calibration procedure.

Next we show the model's predictions regarding investment in human capital and financial assets over the life-cycle.

Figure 10: Life-cycle human capital  
Human capital levels over the lifecycle

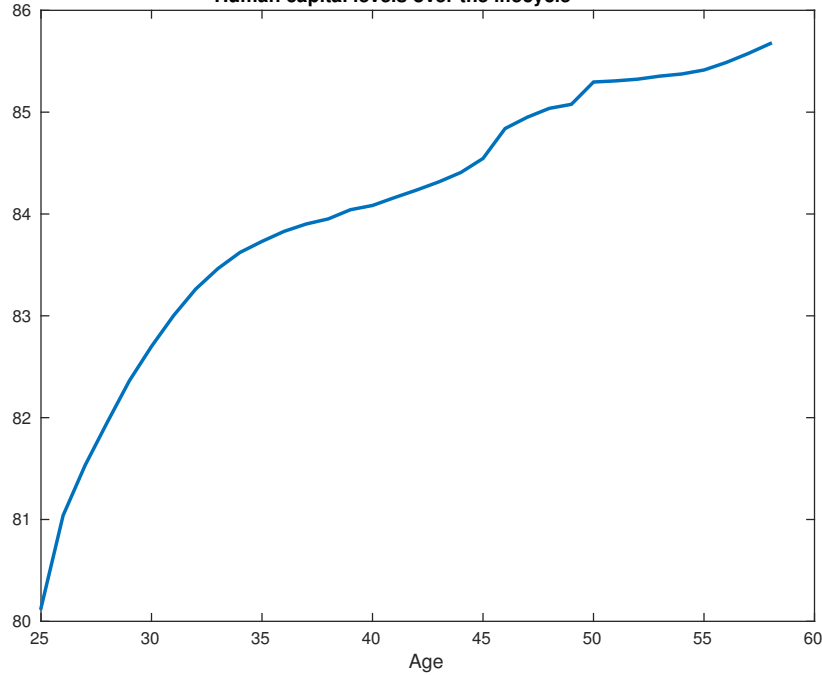


Figure 11: Stock Market Participation and Shares over the Life Cycle

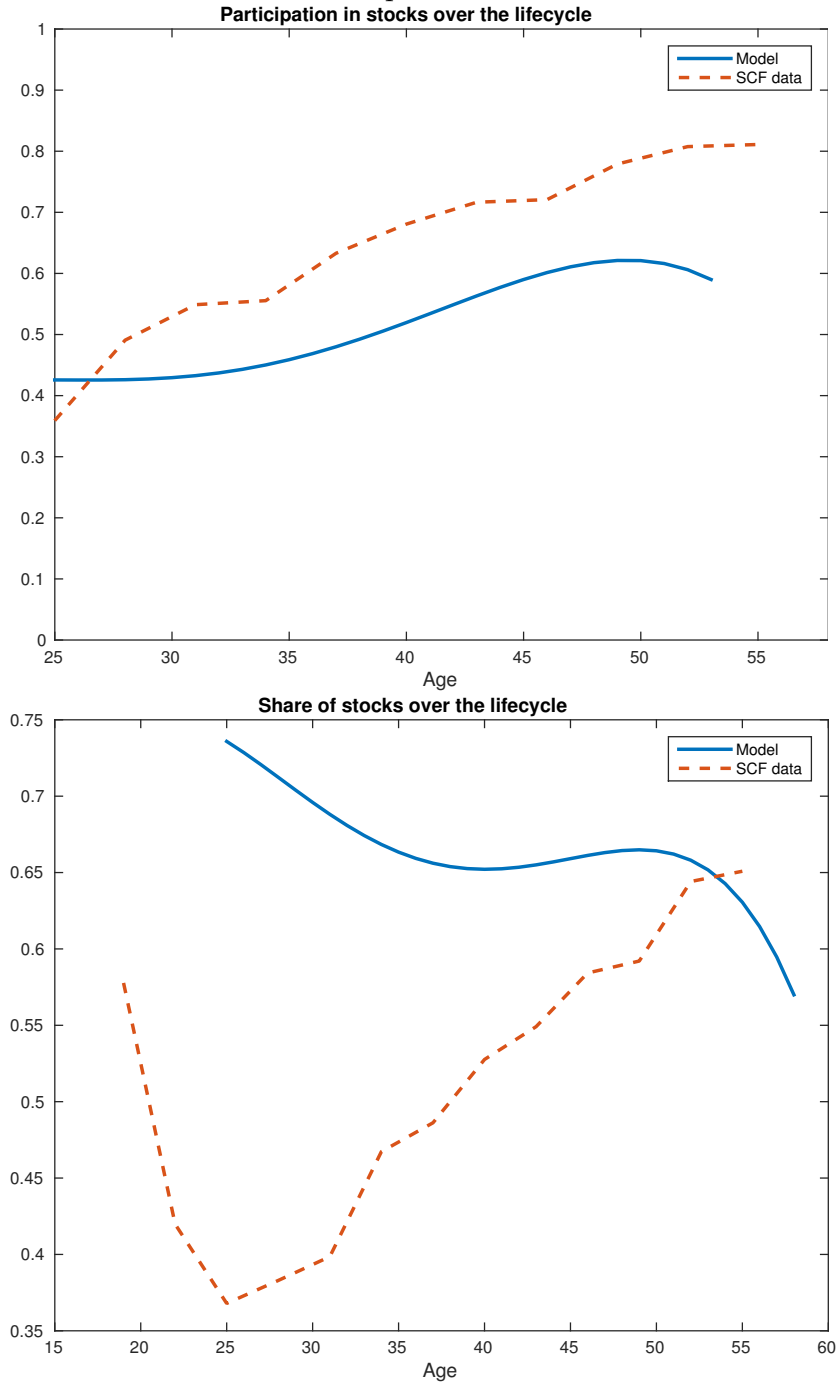
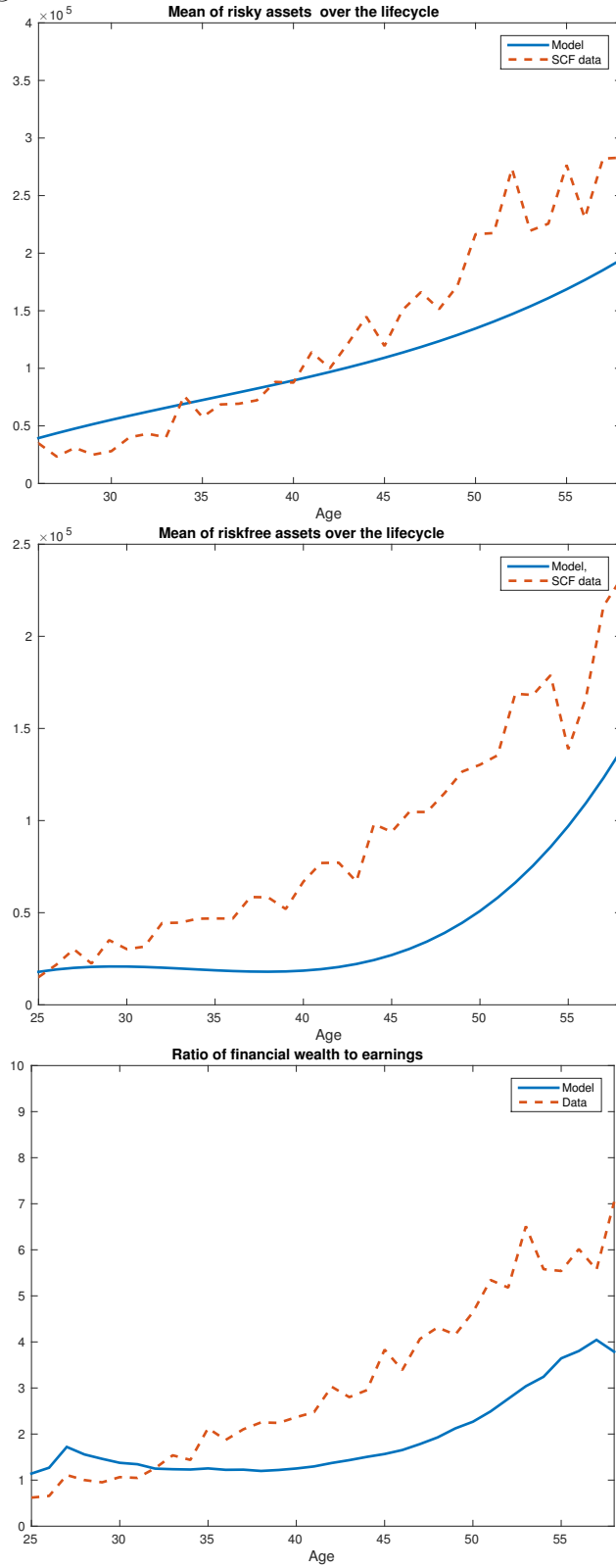


Figure 12: Wealth Accumulation over the Life Cycle



### 5.2.1 Behavior by education groups

Figure 13: Stock Market Participation and Shares over the Life Cycle

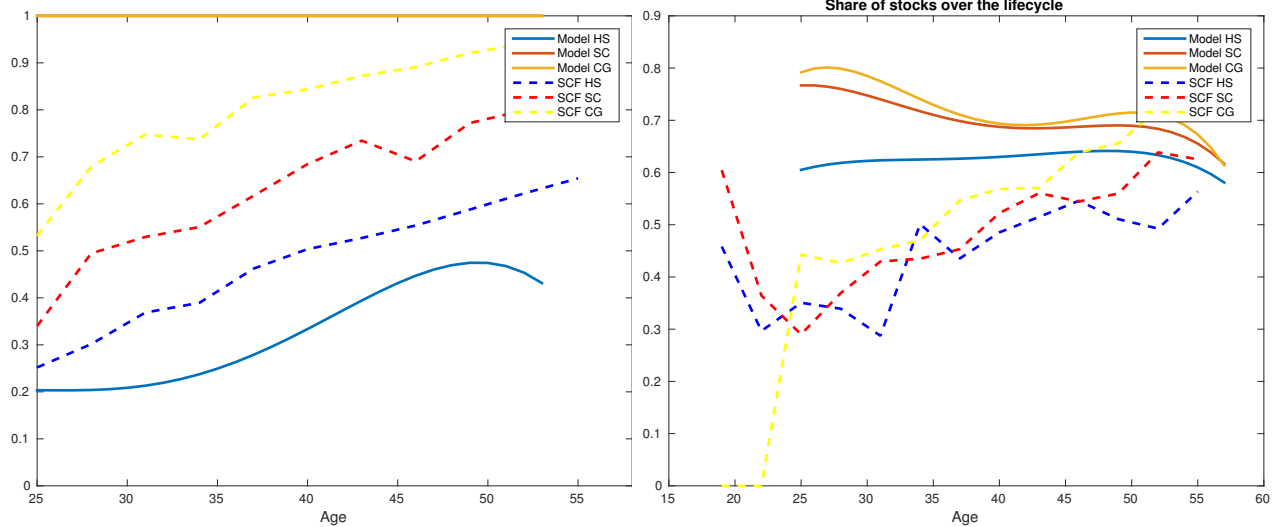
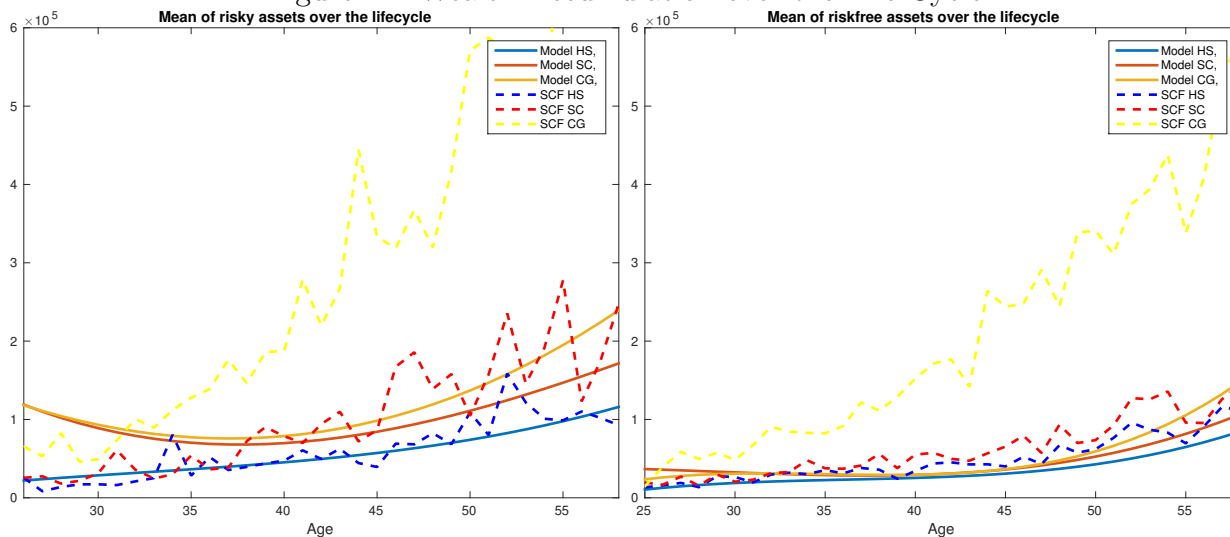


Figure 14: Wealth Accumulation over the Life Cycle



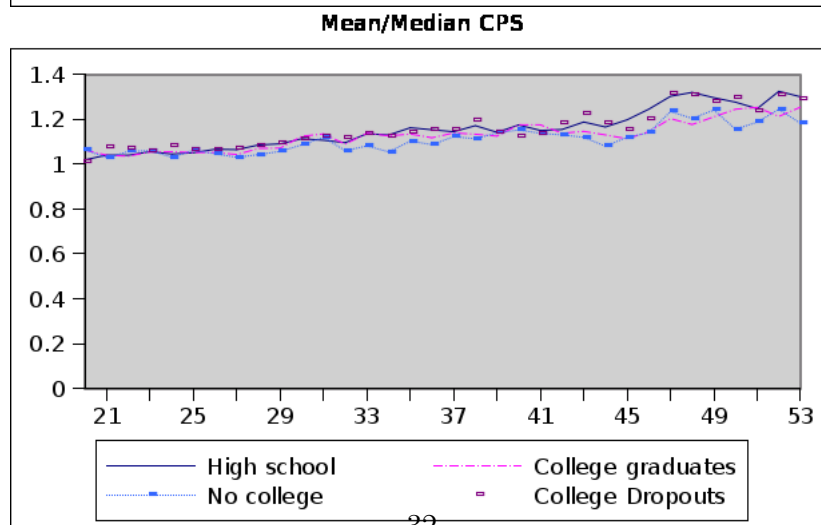
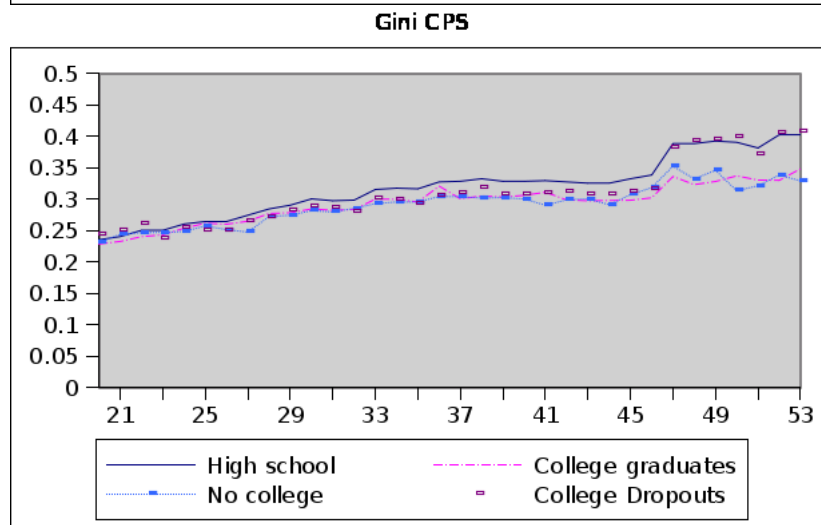
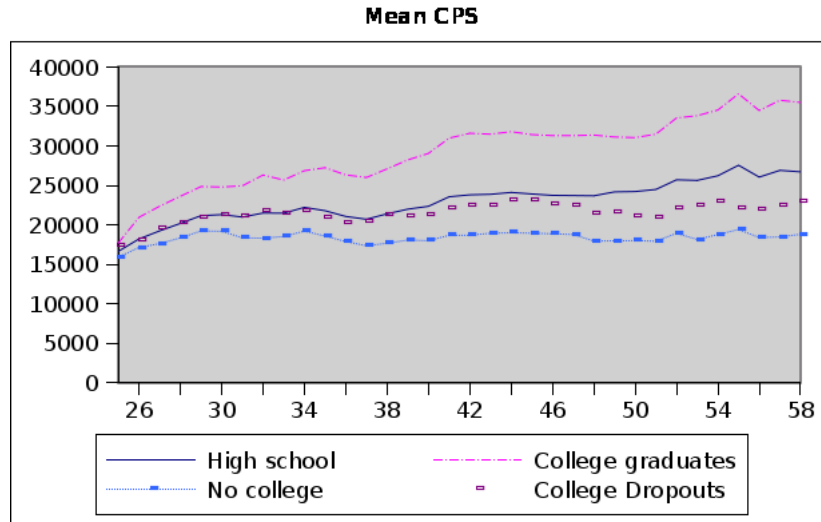
To be completed.

## 6 Conclusion

To be completed.

# A Figures

Figure 15: Statistics of Earnings by Education Groups: Data CPS





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