

# Dynamic Dependence and Diversification in Corporate Credit\*

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## Abstract

We characterize dependence and tail dependence in corporate credit using a new class of dynamic copula models which can capture dynamic dependence and asymmetry in large samples of firms. We also document important differences between the dependence dynamics for credit spreads and equity returns. Modeling a decade of weekly CDS spreads for 215 firms, we find that copula correlations are highly time-varying and persistent, and that they increase significantly in the financial crisis and have remained high since. Perhaps most importantly, tail dependence of CDS spreads increases even more than copula correlations during the crisis and remains high as well. The most important shocks to credit dependence occur in August of 2007 and in August of 2011, but interestingly these dates are not associated with significant changes to median credit spreads. The decrease in diversification potential caused by the increase in dependence and tail dependence is large. Finally, we find that the CDS volatility, correlation and tail dependence measures that we have constructed using the dynamic copula model are important determinants of credit spreads over time.

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# 1 Introduction

Characterizing the dependence between credit-risky securities is of great interest for portfolio management and risk management, but not necessarily straightforward because multivariate modeling is notoriously difficult for large cross-sections of securities. In existing work, computationally straightforward techniques such as factor models or constant copulas are often used to model correlations for large portfolios of credit-risky securities; alternatively, simple rolling correlations or exponential smoothers are used.

We instead use multivariate econometric models for the purpose of modeling credit correlation and dependence. We use genuinely dynamic copula techniques that can capture univariate and multivariate deviations from normality, including multivariate asymmetries. We demonstrate that by using recently proposed econometric innovations, it is possible to apply copula models on a large scale that is essential for effective credit risk management. We perform our empirical analysis using data on a large cross-section of credit risky securities, namely 5-year Credit Default Swap (CDS) contracts for 215 constituents of the first 18 series of the CDX North American investment grade index. We use a long time series of weekly data for the period January 1, 2001 to August 22, 2012. The 215 firms enter and leave the sample at different time points, but this can easily be accommodated by the estimation methodology we employ. We investigate the dependence between CDS spreads as well as the tail dependence. We also analyze dependence in the underlying equity for comparison. Interestingly, the credit and equity return dynamics differ in important aspects.

We document several important stylized facts, and substantial differences between credit and equity dependence. Copula correlations in CDS spreads vary substantially over our sample, with a significant increase following the financial crisis in 2007. Equity correlations also increase in the financial crisis, but somewhat later, and the increase is less significant and not as persistent. Our estimates indicate fat tails in the univariate credit distributions, but also multivariate non-normalities for CDS spreads. Multivariate asymmetries seem to be less important for credit than for equity returns, confirming the results from threshold correlations. While equity volatility is more persistent than credit volatility, credit copula correlations are more persistent than equity copula correlations. This greatly affects how major events such as the Quant Meltdown, the Lehman bankruptcy, and the U.S. sovereign debt downgrade affect subsequent dependence in credit and equity markets. Tail dependence for credit and equity increases significantly during our sample, more so than copula correlations. Surprisingly, the Lehman bankruptcy affects equity (tail) dependence more strongly than credit (tail) dependence. The US sovereign downgrade in mid 2011 is an important credit event, but this is more apparent when analyzing tail dependence, somewhat less so when analyzing copula correlations.

The increase in cross-sectional dependence is clearly important for the management of portfolio credit risk. We use our estimates to compute time-varying diversification benefits from selling credit protection. We find that the increase in cross-sectional dependence following the financial crisis has substantially reduced diversification benefits, similar to what happened in equity markets. When computing diversification benefits, taking non-normality into account is more important for credit than for equity. Our results also have implications for the management of counterparty risk and the relative pricing of structured products such as CDOs, with tranches that are affected differently by changes in correlation patterns.

Identifying financial and macroeconomic variables that can capture the clustering in defaults and cross-firm default dependence is of great interest for the purpose of modeling portfolio credit risk, but there is no guidance from theory regarding the economic determinants of credit dependence and tail dependence. We use a regression analysis to identify financial and macroeconomic determinants of the time-series variation in credit dependence. Copula correlations increase with the VIX, the overall level of credit spreads, and inflation, and decrease with the level of interest rates and S&P 500 returns. The effect from VIX is robust when including lagged correlations in the regressions. We also perform a regression analysis to investigate if dependence and tail dependence help explain the variation in credit spreads, and we find that this is the case, even after controlling for well-established determinants of credit spreads at the firm level, such as equity volatility, interest rates, and leverage.

We proceed in three steps. The two first steps are univariate. In the first, we remove the short-run dynamics from the raw data by estimating firm-by-firm ARMA models on weekly log-differences. In a second step, we estimate firm-by-firm variance dynamics on the residuals from the first step. We use an asymmetric NGARCH model with an asymmetric standardized  $t$ -distribution following Hansen (1994).<sup>1</sup> Finally, in a third step we provide a multivariate analysis using the copula implied by the skewed  $t$ -distribution in DeMarta and McNeil (2005). Dynamic copula correlations are modeled based on the linear correlation techniques developed by Engle (2002) and Tse and Tsui (2002).<sup>2</sup> Dynamic tail dependence depends on time-varying correlations and degrees-of-freedom, which we capture using a smooth exponential spline function (see Engle and Rangel, 2008). To alleviate the computational burden, we rely on the composite likelihood technique of Engle, Shephard, and Sheppard (2008) and the moment matching from Engle and Mezrich (1996). See Patton (2012) for a recent survey of copula models.

The remainder of the paper is structured as follows. In Section 2 we briefly discuss CDS markets and document stylized facts in our sample. We also discuss existing techniques for

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<sup>1</sup>Engle (1982) and Bollerslev (1986) developed the first ARCH and GARCH models. Bollerslev (1990) first combined the GARCH model with a  $t$ -distribution.

<sup>2</sup>See Engle and Kroner (1995) for an early multivariate GARCH model and Engle and Kelly (2012) for a simplified dynamic correlation model.

modeling credit dependence. Section 3 reports the estimation results from the dynamic models for expected credit spread and volatility that we apply. Section 4 introduces the dynamic copula models and presents the estimation results as well as the key threshold dependence and credit diversification dynamics. Section 5 contains a regression analysis of the determinants of the time-series variation in the copula correlations. Section 6 investigates if the estimated dependence measures help explain time-series variation in credit spreads in our sample. Section 7 concludes.

## 2 CDS Markets, Models and Stylized Facts

We discuss CDS markets and stylized facts characterizing the sample of CDS data we use in our empirical work. We also briefly discuss existing techniques for modeling default dependence.

### 2.1 CDS Markets

A CDS is essentially an insurance contract, where the insurance event is defined as default by an underlying entity such as a corporation or a sovereign country. Which events constitute default is a matter of some debate, but for the purpose of this paper it is not of great importance. The insurance buyer pays the insurance provider a fixed periodical amount, expressed as a “spread” which is converted into dollar payments using the notional principal—the size of the contract.<sup>3</sup> In case of default, the insurance provider compensates the insurance buyer for his loss.

The CDS market exploded in size between 2000 and 2007, standing at over 55 Trillion \$US in notional principal in late 2007, according to the Bank for International Settlements. While the CDS market has subsequently been reduced to approximately 27 Trillion \$US in notional principal as of June 2012, market size seems to have stabilized over the last two years after a sharp drop during the financial crisis. Also, the decline in CDS market size is much less dramatic than the decline for more complex credit derivatives, in particular structured credit products. This suggests that CDS markets have survived the financial crisis, highlighting the importance of a market for single-name default insurance.

Reflecting the growth in market activity, in April 2009 the CDS markets underwent a number of changes. First, the CDS contract has been changed to formalize the auction mechanism for CDS following a credit event. Previously, participants in the CDS market had to sign up for a separate protocol for each auction. Second, committees are now formed to make binding determinations of whether credit and succession events have occurred as well

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<sup>3</sup>Recent changes in the CDS market have made the upfront fee the pricing parameter. However, our data source (Markit) provides us with the spreads.

as the terms of any auction. Third, the effective date for all CDS contract has been changed to current-day less 60 days for credit events and to current-day less 90 days for succession events. Fourth, the North American single-name CDS contracts that we investigate in this paper began trading with a fixed coupon of either 100 basis points or 500 basis points with up-front payments exchanged. Finally, the buyer now has to make a full coupon payment on the first payment date regardless of the date of the trade, and the seller of CDS protection makes an accrual rebate payment to the protection buyer at the time of the trade. See Markit (2009) for the details.

## 2.2 Credit Default Models

Measuring default dependence has always been a problem of interest in the credit risk literature. For instance, a bank that manages a portfolio of loans is interested in how the borrowers' creditworthiness fluctuates with the business cycle. While the change in the probability of default for an individual borrower is of interest, the most important question is how the business cycle affects the value of the overall portfolio, and this depends on default dependence. An investment company or hedge fund that invests in a portfolio of corporate bonds faces a similar problem. Over the last decade, the measurement of default dependence has taken on added significance because of the emergence of new portfolio and structured credit products, and as a result new methods to measure correlation and dependence have been developed.

Different techniques are used to estimate default dependence. The oldest and most obvious way to estimate default correlation is the use of historical default data. In order to reliably estimate default probabilities and correlations, typically a large number of historical observations are needed which are not often available. See for instance deServigny and Renault (2002).

The alternative to historical default data is the combination of a factor model with a model that extracts default intensities or default probabilities. For each of these two tasks, different models have proven especially useful.

For publicly traded corporates, a Merton (1974) type structural model is often used to link equity returns or the prices of credit-risky securities to the underlying asset returns and extract default probabilities.<sup>4</sup> This approach is usually combined with a one-factor model for the underlying equity return to model the default dependence in credit portfolios. Clearly the reliability of the default dependence estimate is determined by the quality of the factor model.

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<sup>4</sup>The structural approach goes back to Merton (1974). See Black and Cox (1976), Leland (1994) and Leland and Toft (1996) for extensions. See Zhou (2001) for a discussion of default correlation in the context of the Merton model.

Alternatively, to model default intensities reduced-form or intensity-based models have become very popular in the academic credit risk literature over the last decade.<sup>5</sup> This approach typically models the default intensity using a jump diffusion, and is also sometimes referred to as the reduced-form approach. Within this class of models, there are different approaches to modeling default dependence. One class of models, referred to as conditionally independent models or doubly stochastic models, assumes that cross-firm default dependence associated with observable factors determining conditional default probabilities is sufficient for characterizing the clustering in defaults. See Duffee (1999) for an example of this approach. Das, Duffie, Kapadia and Saita (2007) provide a test of this approach and find that this assumption is violated. Other intensity-based models consider joint credit events that can cause multiple issuers to default simultaneously, or they model contagion or learning effects, whereby default of one entity affects the defaults of others. See for example Davis and Lo (2001) and Jarrow and Yu (2001). Jorion and Zhang (2007) investigate contagion using CDS data.

This paper instead uses copula methods to model default dependence. See Joe (1997) and Patton (2009a, 2009b, 2012) for excellent overviews of copula modeling. Copulas have been used extensively for modeling default dependence, especially among practitioners and for the purpose of CDO modeling. The advantage of the copula approach is its flexibility, because the parameters characterizing the multivariate default distribution, and hence the correlation between the default probabilities, can be modeled in a second stage, after the univariate distributions have been calibrated. In many cases the copulas are also parsimoniously parameterized and computationally straightforward, which facilitates calibration. Calibration of the correlation structure is mostly performed using CDO data. The simple one-factor Gaussian copula is often used in the literature, but extensions to multiple factors (Hull and White (2010)), stochastic recovery rates (Hull and White (2006)), and non-Gaussian copulas provide a better fit.

In contrast to existing static approaches, in our analysis of default dependence the emphasis is on the modeling of dynamic dependence. Our approach also allows for multivariate asymmetries.<sup>6</sup> Several existing papers use copulas from the Archimedean family to capture dependence asymmetries (see Patton (2004, 2006b) and Xu and Li (2009)), but this approach is difficult to generalize to higher dimensions, and our focus is on the analysis of a large portfolio of underlying credits. To capture time variation in dependence, some existing papers use regime switching models. See Chollete, Heinen, and Valdesogo (2009), Garcia and Tsafack (2011), Hong, Tu, and Zhou (2007), and Okimoto (2008) for examples. We instead follow the autoregressive approach of Christoffersen and Langlois (2013), Christoffersen, Errunza, Ja-

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<sup>5</sup>See Jarrow and Turnbull (1995), Jarrow, Lando and Turnbull (1997), Duffee (1999), and Duffie and Singleton (1999) for early examples of the reduced form approach. See Lando (2004) and Duffie and Singleton (2003) for surveys.

<sup>6</sup>Jondeau and Rockinger (2006) analyze dynamic dependence using symmetric copulas.

cobs, and Langlois (2012), and De Lira Salvatierra and Patton (2013). In independent work, Oh and Patton (2013) also use an autoregressive approach to analyze dynamic dependence for a large portfolio of underlying credits.

## 2.3 CDS Data

One element of the success and resilience of CDS markets has been the creation of market indexes consisting of CDSs, such as the CDX index in North America and the iTraxx index in Europe. Using data from Markit, we consider 5-year CDS contracts on all firms included in the first 18 series of the North American investment grade CDX.NA.IG index. We use the longest possible sample available from Markit for all these firms, starting on January 1, 2001, and ending on August 22, 2012. Many firms do not have CDS quotes available for every day of this sample period. Fortunately, as pointed out by Patton (2006a), the dynamic multivariate modeling approach we employ in our empirical work allows for individual series to begin (and end) at different time points. We make full use of this and include a firm if it has at least one year of consecutive weekly data points. The resulting list of 215 firms is provided in Table 1.

We construct weekly data by using one day each week. We use Wednesdays, which is the weekday that is least likely to be a holiday. We obtain equity data on the sample firms from CRSP. Out of the 215 firms, 12 firms do not have at least a consecutive 52-week history of equity prices, and those are dropped from the sample.<sup>7</sup>

An analysis of dependence can focus on CDS spreads or default intensities. In our empirical work we focus on log-differences in CDS spreads because they are econometrically tractable. For most models, the time series properties of default intensities are very similar. We verified this for our sample by extracting default intensities at each point in time using an assumption of constant default intensity. The conclusions from the dependence analysis on default intensities were very similar to those on spreads, and we therefore do not report the results here.

The solid black line in Panel A of Figure 1 plots the time series of the median CDS spread across firms, and the grey areas represent the interquartile range. Panel B presents the median and interquartile range for the CDS spread volatilities.

Panels C and D of Figure 1 replicate Panels A and B using the equity data. The equity price in Panel C is normalized to one for each firm at the start of the sample.

The vertical lines in Figure 1 denote eight major events during our sample period:

- The WorldCom bankruptcy. July 2002.

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<sup>7</sup>The twelve firms are: AT&T Mobility LLC, Bombardier Capital Inc., Bombardier Inc., Cingular Wireless LLC, Capital One Bank USA National Association, Comcast Cable Communication LLC, General Motors Acceptance Corp., Intelsat Limited, International Lease Finance Corp., National Rural Utilities Coop Financial Corp., Residential Capital Corp., and Verizon Global Funding Corp.

- The Ford and GM downgrades to junk. May 2005.
- The Delphi bankruptcy. October 8 2005.
- The Bear Stearns subprime funds collapse and quant meltdown. July/August 2007. Henceforth referred to simply as the quant meltdown.
- The Bear Stearns bankruptcy. March 2008.
- The Lehman bankruptcy. September 2008.
- The stock market bottom and CDS Big Bang. March/April 2009. Henceforth referred to simply as the stock market bottom.
- The U.S. sovereign debt downgrade. August 2011.

## 2.4 Stylized Facts

Figure 1 illustrates some important stylized facts regarding the trends in credit risk in our sample period.

Panel A of Figure 1 indicates that the time series of the median CDS spread and the interquartile range reach their maximums during the peak of the financial crisis in 2008. Less dramatic turbulence is also evident during the dot-com bust in 2002 and the US sovereign debt downgrade in 2011.

Panel D of Figure 1 indicates that the time series pattern for equity volatility is similar to that for credit spreads. The relationship between credit spreads and equity volatility is of course suggested by structural credit risk models such as Merton (1974). Panels A and D also suggest that CDS spreads and equity volatility are highly persistent over time.

The interquartile ranges in the four panels of Figure 1 also contain valuable insights into credit and equity dependence. The cross-sectional range of spreads is much wider during the financial crisis compared to the pre-crisis years. This effect lingers on to some extent in the post-crisis period. The high post-crisis range in spreads suggests that investors may be able to at least partly diversify credit risk which is a key topic of interest for us. We observe a similar increase in the cross-sectional range of spreads during the financial crisis for equity volatility in Panel D, but in the post-crisis period the widening of the range is less pronounced. For spread volatility in Panel B, the cross-sectional range widens during the financial crisis, but not significantly more than during other crisis periods that barely show up in spread levels. Dynamics in credit spread levels and credit spread volatility thus seem to differ substantially.

Figure 2 plots the median CDS spread in each industry. The 215 firms in our sample are distributed along the following 10 GIC sectors: Energy (12 firms), Materials (14), Industrials



(25), Consumer Discretionary (64), Consumer Staples (16), Health Care (13), Financials (34), Information Technology (15), Telecommunications Services (14), and Utilities (8). For ease of exposition in Figure 2 we combine the energy and utility sectors which each have few firms.

The impact of the financial crisis is obvious in Figure 2, but interestingly the crisis affected different industries quite differently. Some industries, Information Technology and Telecommunication Services in particular, were affected as much or even more by the 2001-2003 upheaval versus the 2007-2009 crisis.

When examining the time series plots of the 215 individual CDS names (not reported), the magnitude of the firm-specific variation across the sample period is quite remarkable. This should bode well for the potential diversification benefits of investors exposed to corporate credit risk.

In Table 2 we report sample averages across firms for CDS spreads and equity prices. Panel A of Table 2 shows the first four sample moments of weekly log-differences in CDS spreads along with the IQR for each moment. We also report the Jarque-Bera tests for normality as well as the first two autocorrelation coefficients. Note the strong evidence of non-normality as well as some evidence of dynamics in the weekly returns. We will model both of these features below.

In Panel B we report the median sample correlations between log-differences in spreads and equity prices. On the diagonal we report the median and IQR across the correlations between each firm and all other firms. On the off-diagonal we report the median and IQR of the correlation between the CDS spreads and equity returns for the same firm. The relatively high and robust negative correlation between weekly equity returns and weekly spreads is expected. Note that the log-difference in spreads can be viewed as the return on *buying* credit protection and thus reducing credit risk. The negative correlation between spreads and equity returns is thus evidence of a positive correlation between the exposure to credit and equity risk.

Below, we will work solely with the weekly log-differences in CDS spreads and stock prices. For simplicity we will refer to them generically as returns and denote them by  $R_t$ .

In order to further explore the dependence across firms we compute threshold correlations, following Ang and Chen (2002) and Patton (2004) for example. We define the threshold correlation  $\bar{\rho}_{ij}(x)$  with respect to deviations of standardized returns  $\bar{R}_i$  and  $\bar{R}_j$  from their means as

$$\bar{\rho}_{ij}(x) = \begin{cases} \text{Corr}(\bar{R}_i, \bar{R}_j \mid \bar{R}_i < x, \bar{R}_j < x) & \text{when } x < 0 \\ \text{Corr}(\bar{R}_i, \bar{R}_j \mid \bar{R}_i \geq x, \bar{R}_j \geq x) & \text{when } x \geq 0, \end{cases}$$

where we use returns that are standardized by their sample mean and standard deviation, and thus measure  $x$  as the number of standard deviations from the mean. The threshold correlation reports the linear correlation between two assets for the subset of observations

lying in the bottom-left or top-right quadrant. In the case of the bivariate normal distribution the threshold correlation approaches zero when the threshold,  $x$ , goes to plus or minus infinity.

Panels A and C of Figure 3 report the median and IQR of the bivariate threshold correlations computed across all possible pairs of firms. Panel A shows that the CDS spread threshold correlations are high and almost symmetric. The equity threshold correlations in Panel C are also high but show some evidence of asymmetry: Large downward moves are more highly correlated than large upward moves. Panels A and C in Figure 3 show strong evidence of multivariate non-normality. This is evidenced by the large deviations of the solid line (empirics) from the dashed lines (normal distribution). Adequately capturing these non-normalities motivates the non-normal copula approach below.

### 3 Dynamic Models of Credit Spreads

Our dynamic model development proceeds in three steps. In the first step, we model the mean dynamics on the univariate time series of each CDS spread and stock return. In the second step, we model the variance dynamics and the distribution of the time-series residual for each firm. In the third step, we develop dynamic copula models for CDS and equity returns using all the firms in our sample. The first two steps are covered in this section and the third in the subsequent section.

#### 3.1 Mean Dynamics

The log-differencing on the raw data is partly done to remove long memory in the data. However, the weekly data we analyze contain short-run dynamics as well. In order to obtain white-noise innovations required for consistent modeling of correlation dynamics, we fit univariate *ARMA-NGARCH* models to the weekly log-differenced time series. We first fit each of the possible *ARMA* specifications with *AR* and *MA* orders up to two. The *ARMA* order for each time series is then chosen using the finite sample corrected Akaike criterion.

To be specific, in a first step, we use Gaussian quasi-maximum likelihood (QMLE) to estimate nine models nested within the *ARMA*(2, 2) model on the weekly log-differences in CDS spreads and equity prices for each firm

$$R_t = \mu + \phi_1 R_{t-1} + \phi_2 R_{t-2} + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \varepsilon_t \quad (3.1)$$

where  $\varepsilon_t$  is assumed to be uncorrelated with  $R_s$  for  $s < t$ . The conditional mean for  $R_t$  constructed at the end of week  $t - 1$  is then simply

$$\mu_t = \mu + \phi_1 R_{t-1} + \phi_2 R_{t-2} + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$$

### 3.2 Variance Dynamics

In a second step we fit the Engle and Ng (1993) *NGARCH*(1, 1) model to the *ARMA* filtered residuals  $\varepsilon_t$

$$\begin{aligned}\varepsilon_t &= \sigma_t z_t \\ \sigma_t^2 &= (1 - \alpha - \beta)\bar{\sigma}^2 + \alpha(\varepsilon_{t-1} - \gamma\sigma_{t-1})^2 + \beta\sigma_{t-1}^2 \\ z_t &\sim i.i.d. ast(z; \lambda, \nu)\end{aligned}\tag{3.2}$$

where we constrain  $\alpha > 0$ ,  $\beta > 0$ , and  $\alpha + \beta < 1$ , and set the unconditional variance,  $\bar{\sigma}^2$ , equal to the sample variance of  $\varepsilon_t$ . The i.i.d. return residuals,  $z_t$ , are assumed to follow the asymmetric standardized  $t$  distribution from Hansen (1994) which we denote  $ast(z; \lambda, \nu)$ . The skewness and kurtosis of the distribution are nonlinear functions of the parameters  $\lambda$  and  $\nu$ . When  $\lambda = 0$  the symmetric standardized  $t$  distribution is obtained. When  $\lambda = 0$  and  $1/\nu = 0$ , we get the normal distribution. The corresponding cumulative return probabilities are now given by

$$\eta_t \equiv \Pr_{t-1}(R < R_t) = \sigma_t^{-1} \int_{-\infty}^{\sigma_t^{-1}(R_t - \mu_t)} ast(z; \lambda, \nu) dz.\tag{3.3}$$

Note that the individual return-residual distributions are constant through time but the individual return distributions do vary through time because the return mean and variance are dynamic.

Using time series observations on  $\varepsilon_t$ , the parameters  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\lambda$  and  $\nu$  are estimated using a likelihood function based on (3.2) and  $ast(z; \lambda, \nu)$ . For each firm we again estimate two sets of parameters, one based on spreads and one based on equity returns.

### 3.3 Estimates of Mean and Variance Dynamics

Panel A of Table 3 reports for credit spreads and equity the percentage of firms for which each of the nine estimated *ARMA*( $p, q$ ) models were favored by the Akaike criterion. The percentages are quite similar across the nine possible models. The *ARMA*(2, 2) is the single-most selected model, suggesting that perhaps higher lags should be considered. Panel A also shows the median and interquartile range across firms of each *ARMA* coefficient estimate. The parameter values vary considerably across firms. The Ljung-Box test on the  $z_t$  residuals show that the test do not reject the null that the residuals are serially uncorrelated 99% of the time for CDS spreads, and 90% of the time for equity returns. This suggests that the *ARMA* models are able to adequately capture conditional mean dynamics across firms and markets.

Panel B in Table 3 shows the median and interquartile range across firms for each of the three *NGARCH* parameters as well as the two parameters in the asymmetric  $t$  distribution.

Weekly volatility persistence, defined by  $(\alpha(1 + \gamma^2) + \beta)$ , is fairly tightly distributed around the median values of 0.950 for CDS spreads and 0.980 for equity returns. Volatility is clearly highly persistent in both spreads and equity returns. The  $\gamma$  parameter captures the asymmetric volatility response to positive and negative return residuals. For equities, the median  $\gamma$  value is 1.230 and the interquartile range is entirely positive. For CDS spreads the  $\gamma$  is negative and smaller in magnitude. Recall that the CDS spreads capture the returns to *buying* credit protection.

The Ljung-Box test of serial correlation in the  $z_t^2$  shows that the NGARCH model is able to adequately capture variance dynamics. Equity returns, which have the highest volatility persistence, have 13% of NGARCH models rejected by Ljung-Box at the 5% level, which is clearly not drastically above the size of the test.

The  $\nu$  parameter has medians of 3.72 (CDS spreads) and 6.56 (equity returns), indicating fat tails in the conditional distribution. The asymmetry parameter,  $\lambda$ , is generally negative for equities and positive for CDS spreads and roughly equal in magnitude for the two. Recall again that the CDS spreads capture the returns to buying credit protection.

As discussed above, panel B of Figure 1 shows the time paths for the median and interquartile range of CDS spread volatilities. The differences between the path of CDS volatility in Panel B and the path of equity volatility in Panel D are interesting. While the time paths of the medians are clearly moving together, the median path for the CDS spread volatility contains many more sharp peaks. This is also the case for the path of the interquartile range.

Note that the relationship between equity volatility and CDS spreads has been extensively studied because of the Merton (1974) model. The relation between equity returns and equity volatility has also been extensively analyzed in the empirical literature. Panel C of Figure 1 confirms that equity returns are negatively related to equity volatilities in Panel D. This stylized fact is usually referred to as the leverage effect, and it leads to negative skewness in the return distribution. However, little is known about the relation between CDS spread volatility and spreads. Visual inspection of Panels A and B suggests a positive relation. However, Panel B indicates that while most of the spikes in CDS spread volatility match the spikes in spread levels in Panel A, this is not always the case. Note for example that one of the highest peaks in CDS spread volatility occur at the time of the quant meltdown in August 2007, which coincides with only a minor uptick in spreads in Figure 1. We analyze this relation in more detail in the empirical work below.

Finally, note that the volatility patterns in spreads in Panel B of Figure 1 are somewhat different from the volatility in equity in Panel D. An obvious example is May 2005, around the time of the Ford and GM downgrade, when equity volatility in Panel D does not spike up, but median CDS spread volatility sharply increases.

Figure 4 plots the median of the weekly NGARCH dynamic in CDS spreads for the nine

industries from Figure 2. Spread volatility clearly does not seem to be a simple deterministic function of the spreads themselves. The variation of spread volatility across industries is quite dramatic. The high level of CDS spread volatility in the financial crisis is apparent.

Panel A of Table 4 contains descriptive statistics of the ARMA-NGARCH model residuals. Skewness and kurtosis are still present after standardizing by the NGARCH model, which motivated the use of the asymmetric standardized  $t$  innovations. As expected, the residual correlations between CDS spreads and equity prices are not materially different from the raw return correlations in Panel C of Table 2.

Finally, Panels B and D of Figure 3 plot the median and IQR threshold correlations on the weekly ARMA-NGARCH residuals. Comparing with the threshold correlations on raw returns in Panels A and C, we see that the median threshold correlations in residuals are often lower, but still higher than the bivariate Gaussian distribution (dashed lines) would suggest. Overall Figure 3 indicates that the ARMA-NGARCH models by removing univariate non-normality from the data are also able to remove some of the multivariate non-normality from the data. Modeling the remaining multivariate non-normality is the task to which we now turn.

## 4 Dynamic Dependence and Diversification

In this section we first introduce the copula functions that we apply to credit spreads and stock returns. We then discuss the dynamic copula correlation estimates, and report on model-based measures of threshold dependence. Finally, we compute measures of conditional diversification benefits for credit and equity portfolios.

### 4.1 Dynamic Copula Functions

From Patton (2006b), who builds on Sklar (1959), we can decompose the conditional multivariate density function of a vector of returns for  $N$  firms,  $f_t(R_t)$ , into a conditional copula density function,  $c_t$ , and the product of the conditional marginal distributions  $f_{i,t}(R_{i,t})$  as follows

$$\begin{aligned} f_t(R_t) &= c_t(F_{1,t}(R_{1,t}), F_{2,t}(R_{2,t}), \dots, F_{N,t}(R_{N,t})) \prod_{i=1}^N f_{i,t}(R_{i,t}) \\ &= c_t(\eta_{1,t}, \eta_{2,t}, \dots, \eta_{N,t}) \prod_{i=1}^N f_{i,t}(R_{i,t}), \end{aligned} \quad (4.1)$$

where  $R_t$  is now a vector of  $N$  returns at time  $t$ ,  $f_{i,t}$  is the density and  $F_{i,t}$  is the cumulative distribution function of  $R_{i,t}$ .

Following Christoffersen, Errunza, Jacobs, and Langlois (2012), and Christoffersen and Langlois (2013) we allow for dependence across the return residuals using the copula implied by the skewed  $t$  distribution discussed in Demarta and McNeil (2005). The skewed  $t$  copula cumulative distribution function,  $C_t$ , for  $N$  firms can be written as

$$C_t(\eta_{1,t}, \eta_{2,t}, \dots, \eta_{N,t}; \Psi, \lambda_C, \nu_{C,t}) = t_{\Psi, \lambda_C, \nu_{C,t}}(t_{\lambda_C, \nu_{C,t}}^{-1}(\eta_{1,t}), t_{\lambda_C, \nu_{C,t}}^{-1}(\eta_{2,t}), \dots, t_{\lambda_C, \nu_{C,t}}^{-1}(\eta_{N,t})), \quad (4.2)$$

where  $\lambda_C$  is a copula asymmetry parameter,  $\nu_{C,t}$  is a time-varying copula degree of freedom parameter,  $t_{\Psi, \lambda_C, \nu_{C,t}}$  is the multivariate skewed  $t$  density with correlation matrix  $\Psi$ , and  $t_{\lambda_C, \nu_{C,t}}^{-1}$  is the inverse cumulative distribution function of the corresponding univariate skewed  $t$  distribution.

Note that the copula correlation matrix  $\Psi$  is defined using the correlation of the copula residuals  $z_{i,t}^* \equiv t_{\lambda_C, \nu_{C,t}}^{-1}(\eta_{i,t})$  and not of the return residuals  $z_{i,t}$ . If the marginal distribution in (3.3) is close to the copula  $t_{\lambda_C, \nu_{C,t}}$  distribution, then  $z_{i,t}^*$  will be close to  $z_{i,t}$ .

We now build on the linear correlation techniques developed by Engle (2002) and Tse and Tsui (2002) to model dynamic copula correlations. We use the copula residuals  $z_{i,t}^* \equiv t_{\lambda, \nu}^{-1}(\eta_{i,t})$  as the model's building block instead of the return residuals  $z_{i,t}$ . In the case of non-normal copulas, the fractiles do not have zero mean and unit variance, and we therefore standardize the  $z_{i,t}^*$  before proceeding.

The copula correlation dynamic is driven by

$$\Gamma_t = (1 - \beta_C - \alpha_C)\Omega + \beta_C\Gamma_{t-1} + \alpha_C\bar{z}_{t-1}^*\bar{z}_{t-1}^{*\top} \quad (4.3)$$

where  $\beta_C$  and  $\alpha_C$  are scalars, and  $\bar{z}_t^*$  is an  $N$ -dimensional vector with typical element  $\bar{z}_{i,t}^* = z_{i,t}^*/\sqrt{\Gamma_{ii,t}}$ . The conditional copula correlations are defined via the normalization

$$\Psi_{ij,t} = \Gamma_{ij,t}/\sqrt{\Gamma_{ii,t}\Gamma_{jj,t}}.$$

To allow for general patterns in tail dependence, we allow for slowly-moving trends in the degrees of freedom. Following Engle and Rangel (2008), who model the trend in volatility, we define the degree of freedom at time  $t$ ,  $\nu_{C,t}$ , using an exponential quadratic spline

$$\nu_{C,t} = \underline{\nu}_C + \delta_{C,0} \exp\left(\delta_{C,1}t + \sum_{j=1}^k \delta_{C,j+1} \max(t - t_{j-1}, 0)^2\right) \quad (4.4)$$

where  $\underline{\nu}_C$  is the lower bound for the degrees of freedom, which is equal to four for the skewed  $t$  copula,  $\delta_{C,0}, \dots, \delta_{C,k+1}$  are scalar parameters to be estimated, and  $\{t_0 = 0, t_1, \dots, t_k = T\}$  denotes a partition of the sample in  $k$  segments of equal length. The exponential form ensures

that the degrees of freedom are positive and above their lower bound at all times. The  $k$  different segments allows us to capture periods of positive and negative trends in the degrees of freedom. Note that we model degree-of-freedom dynamics using splines and not lagged returns, because—unlike for variance and correlation—it is not obvious what the functional form of the lagged return should be when updating the degree-of-freedom process.

In the next section we investigate the time-variation in both correlations and tail dependence. Whereas correlation at time  $t$  is driven by the dynamic in Equation (4.3), tail dependence is determined by both the time-varying correlation and the degrees of freedom. Hence, our model allows for changes in tail dependence that are separate from those in correlation.

Below we refer to the model using (4.2) and (4.3) as the Dynamic Asymmetric Copula (DAC) model. The special case where  $\lambda_C = 0$  we denote by the Dynamic Symmetric Copula (DSC). In this case the lower bound for the degree of freedom is  $\underline{\nu}_C = 2$ . When we additionally impose  $1/\nu_C = 0$  we obtain the Dynamic Normal Copula (DNC).

Following Engle, Shephard and Sheppard (2008), we estimate the copula parameters  $\alpha_C$ ,  $\beta_C$ ,  $\lambda_C$ , and  $\nu_C$  using the composite likelihood ( $CL$ ) function defined by

$$CL(\alpha_C, \beta_C, \lambda_C, \nu_C) = \sum_{t=1}^T \sum_{i=1}^N \sum_{j>i} \ln c_t(\eta_{i,t}, \eta_{j,t}; \alpha_C, \beta_C, \lambda_C, \nu_C), \quad (4.5)$$

where  $c_t$  is the copula density from (4.1). Note that the  $CL$  function is built from the bivariate likelihoods so that the inversion of large-scale correlation matrices is avoided. In a sample as large as ours, relying on the composite likelihood approach is imperative. The unconditional correlations are estimated by unconditional moment matching (See Engle and Mezrich, 1996)

$$\widehat{\Omega}_{i,j} = \frac{1}{T} \sum_{t=1}^T \bar{z}_{i,t}^* \bar{z}_{j,t}^* \quad (4.6)$$

which is another crucial element in the feasible estimation of large-scale dynamic models.

As discussed above, the estimation of dynamic dependence models using long time series and large cross-sections is computationally intensive. In our case, estimating the dynamic copula models for 215 firms is possible only because we implement unconditional moment matching and the composite likelihood approach. An additional advantage of the composite likelihood approach is that we can use the longest time span available for each firm-pair when estimating the model parameters, thus making the best possible use of a cross-section of CDS time series of unequal length.

## 4.2 Copula Correlation and Tail Dependence Estimates

Panel B of Table 4 contains the Dynamic Asymmetric Copula (DAC) parameter estimates and composite likelihoods from fitting a single model to the 215 firms in our sample. We again present separate results for models estimated on the weekly residuals in CDS spread and equity log-differences. The copula correlation persistence is higher for CDS spreads (0.98) and considerably lower at 0.94 in the case of equity prices. Comparing with volatility persistence in Table 3, it is interesting to note that equities have relatively higher volatility persistence and lower correlation persistence when compared with credit spreads. This finding demonstrates the importance of modeling separate dynamics for volatility and correlation.

Panel C in Table 4 reports the parameter estimates for the Dynamic Symmetric Copula model where  $\lambda_C = 0$  and Panel D reports on the Dynamic Normal Copula where we also impose  $1/\nu_{C,t} = 0$ . While we do not have asymptotic distribution results available for testing differences in composite likelihoods, the results suggest that the improvements in fit are largest when going from the normal copula in Panel D to the symmetric  $t$  copula in Panel C. When going from the symmetric  $t$  copula in Panel C to the asymmetric  $t$  copula in Panel B, the improvement in fit seems to be largest for equities. This result matches the patterns in the threshold correlations in Figure 3 which show the strongest degree of bivariate asymmetry for equities.

We estimate a model with a simple time trend for the degrees of freedom. The estimates in the third and fourth columns of Panel B indicate that degrees of freedom have been trending down for both CDS and equity. In unreported results, we allow for more complex shapes in degrees of freedom by increasing the number of splines in Equation (4.4), and find that the decreasing time trend is robust.

Figure 5 plots the median and IQR of the DAC copula correlations for CDS spreads and stock returns. The level of the CDS spread correlation is higher than that of equity correlation throughout the sample. Credit correlations in Panel A show a pronounced and persistent uptick in 2007 around the time of the Quant Meltdown, and a pronounced but less persistent uptick in mid 2011 following the US sovereign downgrade. The equity correlations in Panel B show less persistent upticks in late 2008 following the Lehman bankruptcy, and again in mid 2011 following the US sovereign downgrade. The differences in persistence following these major events are of course related to the differences in copula correlation persistence between credit and equity mentioned earlier.

Figure 6 plots the median and IQR of the DAC copula tail dependence for CDS spreads and stock returns. For equity we plot lower tail dependence, because it is economically the most interesting of the two tails. This corresponds to upper tail dependence for CDS spreads. Lower (upper) tail dependence measures the probability that two returns will both be below



(above) a small (high) quantile.<sup>8</sup> Several very interesting conclusions obtain. First, equity and credit tail dependence increase much more over the sample than copula correlations in Figure 5. Second, similar to the pattern in correlations in Figure 5, CDS tail dependence increases earlier than equity tail dependence. Third, in the first part of the sample equity tail dependence is higher than credit tail dependence, but this changes in the second part of the sample. Fourth, the impact of credit events on tail dependence is sometimes more dramatic than their impact on correlations. The most obvious example is the US sovereign downgrade in mid 2011, which shifts credit tail dependence up significantly for the remainder of the sample, whereas in Figure 5 the correlations revert back quicker to the earlier levels. These findings have important implications for portfolio diversification.

In Figure 7 we plot the median DAC correlation of CDS spreads and equity returns for the nine industries in Figure 2. Figure 8 does the same for the tail dependence of credit and equity. While the uptick in credit correlation during 2007 is evident for most industries, the variation across industries is large. The time paths of credit and equity (tail) dependence are clearly different from each other and are fairly similar across industries because they all share the same trend in degrees of freedom.

### 4.3 Conditional Diversification Benefits

Consider an equal-weighted portfolio of the constituents of the on-the-run CDX investment grade index in any given week. We want to assess the diversification benefits of the portfolio using the dynamic, non-normal copula model developed above. As in Christoffersen, Errunza, Jacobs and Langlois (2012), we define the conditional diversification benefit by

$$CDB_t(p) \equiv \frac{\overline{ES}_t(p) - ES_t(p)}{\overline{ES}_t(p) - \underline{ES}_t(p)}, \quad (4.7)$$

where  $ES_t(p)$  denotes the expected shortfall with probability threshold  $p$  of the portfolio at hand,  $\overline{ES}_t(p)$  denotes the average of the  $ES$  across firms, which is an upper bound on the portfolio  $ES$ , and  $\underline{ES}_t(p)$  is the portfolio  $VaR$ , which is a lower bound on the portfolio  $ES$ . The  $CDB_t(p)$  measure takes values on the  $[0, 1]$  interval, and is increasing in the level of diversification benefit. Note that by construction  $CDB$  does not depend on the level of expected returns. Expected shortfall is additive in the conditional mean which thus cancels out in the numerator and denominator in (4.7).

The  $CDB$  measure depends on the threshold probability  $p$ . Below we consider  $p = 5\%$  and  $p = 50\%$ . The  $CDB$  measure is not available in closed form for our dynamic copula model

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<sup>8</sup>Tail dependence is formally defined as the probability limit as the quantile goes to 0 or 1. We obtain an approximation by simulating from our model, and using the quantile 0.001 for lower and 0.999 for upper tail dependence.

and so we compute it using Monte Carlo simulations. We also report on a volatility-based measure which is defined by

$$VolCDB_t = 1 - \frac{\sqrt{\mathbf{1}^\top \Sigma_t \mathbf{1}}}{\mathbf{1}^\top \sigma_t}, \quad (4.8)$$

where  $\mathbf{1}$  denotes a vector of ones, and where  $\Sigma_t$  denotes the usual matrix of linear correlations computed in our case via simulation from the DAC model. One can show that under conditional normality,  $VolCDB_t$  will coincide with  $CDB_t(50\%)$  so that the difference between these two measures indicates the degree of non-normality from a diversification perspective.

Each week  $t$  we form an equally weighted portfolio of the 125 companies currently in the CDX.NA.IG index. We use the longest available history of returns up to week  $t - 1$  to estimate the unconditional correlation matrix for the 125 firms, and then compute the conditional correlations from our DAC model. In order to have sufficient historical data available, we keep only firms with at least two years of data, and start on September 22, 2004, which is the first day of Series 3 of the index.

The solid black line in Figure 9 shows the  $CDB(5\%)$  measure for an equal-weighted portfolio *selling* credit protection as well as for an equal-weighted portfolio of equity returns. First consider Panel A: Diversification benefits for CDS have declined from above 70% at the end of 2003 to below 50% at the end of our sample. The majority of the decline took place during the mid 2007 to mid 2008 period and was relatively gradual. Panel B shows that the decline in diversification benefits in equity markets has been smaller in magnitude, from just over 70% in 2007 to just above 60% at the end of our sample. The majority of the decline in equity market diversification benefits took place from early 2007 to early 2009 and it was relatively gradual as well.

Figure 9 also depicts the average volatilities (in grey, on the right-hand axis) and the average correlations (the dashed line, on the left-hand axis). Intuitively, changes in the diversification measure should be related to changes in correlation, which captures risk that is more systematic in nature, and changes in average volatility, which proxies for whatever idiosyncratic risk is left in the portfolio. For the equity portfolio in Panel B, the time series for average correlation and average volatility are highly correlated, and the drops in diversification benefits in 2008 and 2011 could be due to either measure. For the credit portfolio in Panel A, the conclusions are very different. On the one hand, at certain times the changes in average volatilities and correlations are highly related, for instance in August 2011 at the time of the U.S. sovereign debt downgrade, when correlations and volatilities increase and diversification benefits decrease. On the other hand, there are long periods of time during which the changes in average volatilities are not related to the changes in diversification benefits, for instance between 2008 and mid-2011. Overall diversification benefits seem much more highly related to average correlations.

It is also interesting to relate diversification benefits in Panel A to equity volatilities in Panel B. The average volatilities in Panel B are highly correlated with the VIX and with other indicators of turmoil in equity markets. Clearly the majority of the decline in diversification benefits in credit markets took place well before the peak in equity market volatility. The credit market CDB actually increased a bit during late 2008 and early 2009 when the equity market turmoil was most intense. We conclude that while the data confirm the relationship between the level of credit spreads and equity volatility predicted by Merton-type structural models (see Figure 1), credit diversification benefits are more tightly linked with correlations in credit markets.

In Figure 10 we plot the  $CDB(50\%)$  and  $VolCDB$  measures for CDS spreads in Panel A and for equities in Panel B. Comparing Figures 9 and 10 (note the scales are different) we see that the dynamic patterns are broadly similar, which is not surprising. Panel A suggests that non-normality plays a large role in a well-diversified credit portfolio, and that relying on  $VolCDB$  would exaggerate the benefits from credit diversification. Comparing Panels A and B, the differences between the  $CDB(50\%)$  and  $VolCDB$  are a bit larger for the credit portfolio than for the equity portfolio.

## 5 Economic Determinants of Credit Dependence

Are our new dynamic measures of credit dependence related to traditional economic determinants of credit risk? To answer this question, we now consider regressions of copula correlations on various economic and financial determinants. Another important objective of this exercise is to reflect on variable selection in factor models of credit risk.

As discussed in Section 2.2, an important class of credit default models uses observable macroeconomic factors to characterize the clustering in defaults and cross-firm default dependence. We have obtained estimates of default dependence without relying on such observable factors, and it is useful to investigate how closely our estimates are related to economic variables that are commonly used as factors. We focus on explaining median dependence because our first concern is to verify if the macroeconomic variables can explain the time-variation in the dependence measures. The cross-sectional variation in dependence and the loadings of different firms on the dependence measures are also of interest, but we leave this topic for future work.

There is no acknowledged theory on the selection of economic and financial factors that can capture cross-firm default dependence; perhaps as a result the existing empirical literature is very extensive, and many different economic variables have been used as factors. Duffie, Saita, and Wang (2007) provide an excellent discussion of the existing literature, and choose one-year trailing S&P 500 returns and interest rates as macro variables to capture default dependence

in their own empirical implementation. Duan and Van Laere (2012) also use index stock returns and interest rates, and Collin-Dufresne, Goldstein, and Martin (2001) use S&P 500 returns, interest rates, and the VIX. Campbell, Hilscher, and Szilagyi (2005) use S&P 500 returns to normalize firm returns in their analysis of default and credit risk. Doshi, Ericsson, Jacobs, and Turnbull (2013) use term structure variables and the VIX, and the latent model in Duffee (1999) uses interest rate factors to capture the dependence in credit spreads. See Blume and Keim (1991), Fons (1991), Helwege and Kleiman (1997), Hillegeist, Keating, Cram, and Lundstedt (2004), Jonsson and Fridson (1996), Keenan, Sobehart, and Hamilton (1999), McDonald and Van de Gucht (1999), and Pesaran, Schuermann, Treutler, and Weiner (2006) for examples of other macroeconomic variables that are useful for explaining and forecasting credit spreads and default. See Pospisil, Patel, and Levy (2012) for a list of macroeconomic variables used by Moody’s Analytics for dependence modeling.

In our regressions we limit ourselves to macroeconomic variables that are available at the weekly frequency because we have modeled dependence at the weekly frequency, and we want to capture as much of the time series variation as possible. An analysis of lower frequency macroeconomic variables would be interesting but we keep it for future work. In the absence of explanatory variables suggested by theory, we therefore consider economy-wide measures of risk in equity and default insurance markets, risk-free (government) term structures, and other macro variables that are reasonable additional metrics of the state of the economy, and that have explanatory power for credit spreads documented by the papers cited above. More precisely we use the following regressors:

- The log of the CDX North American investment grade index level is used to proxy for the overall level of risk in credit markets.
- The log of the VIX index represents equity market risk.
- The return on the S&P 500 captures the changes in stock market capitalization. We use the one-week return as well as a trailing one-year return.
- The term structure is captured by a level variable, the 3-month US Constant Maturity Treasury (CMT) index, and a slope variable, the 10 year CMT index minus the 3-month CMT.
- The difference between the interest rate on interbank loans and on short-term government debt, that is the TED spread.<sup>9</sup>

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<sup>9</sup>The TED spread is an indicator of liquidity in fixed income markets. The funding liquidity variable in Fontaine and Garcia (2012) provides an alternative liquidity indicator, but is not available at the weekly frequency.

- The log of crude oil price as measured by the West Texas Intermediate Cushing Crude Oil Spot Price.
- The breakeven inflation level implied by Treasury Inflation Protected Securities. Unlike standard inflation measures, this series is available at the weekly frequency.
- The Aruoba-Diebold-Scotti (ADS) business condition index from the Federal Reserve Bank of Philadelphia.

Table 5 presents the regression results. For the CDX, VIX, S&P 500 returns, and term structure variables, Panel A reports on univariate regressions in columns (i)-(vi). Column (vii) presents the results of a multivariate regression including all variables. In Panel B we include the lagged median correlation as a regressor in all the specifications. All regressors are lagged one week and all results are obtained using OLS with Newey-West standard errors using  $T^{1/4} \approx 5$  lags, where  $T$  is sample size.

In the univariate regressions in Panel A of Table 5, the higher the level of risk in the credit market, as measured by the CDX index, the higher the CDS-based copula correlations. The higher the level of risk in the equity markets, as measured by the VIX, the greater the CDS-based correlations. We present results for two measures of stock market returns: the one-week S&P 500 return as well as the one-year trailing S&P 500 return. Both measures have been used in the literature. Collin-Dufresne, Goldstein, and Martin (2001) use the monthly S&P 500 return in their study of monthly changes in credit spreads. Duffie, Saita, and Wang (2007) and Duan and Van Laere (2012) use the one-year trailing S&P 500 return. The univariate regressions for both measures indicate the a priori expected negative sign, but only the one-year trailing return is statistically significant, and the R-squares clearly indicate that the one-year trailing return has more explanatory power. Interestingly, while the weekly S&P 500 return is weakly negatively correlated with the VIX in our sample, the correlation between the VIX and the one-year trailing return is very strong, at  $-0.60$ . Regression (v) in Panel A indicates that the loading on the interest rate level has the expected negative sign and is statistically significant. Finally, the yield curve slope in (vi) also gives a statistically significant result.

We conclude that the univariate regressions in Panel A all provide intuitively plausible results: when times are bad and the economy experiences negative shocks, the CDX and the VIX are high, stock returns and interest rates are low, and this poor economic environment is associated with higher dependence and fewer diversification opportunities. However, the R-squares indicate that stock market returns explain much less of the time-series variation in copula correlations than term structure variables. Even more strikingly, the VIX explains more of the variation in dependence than the two S&P 500 returns. Finally, the R-square in

regression (i) indicates that the level of credit risk, represented by the CDX index, is a prime candidate for explaining dependence in credit markets. While this may seem self-evident, note that the existing literature mainly relies on equity market indexes for capturing credit dependence. Perhaps the literature has ignored the use of credit indexes because until recently they were not available at high frequencies.

The strong persistence of the median correlation is of course a concern when assessing its relationship with other variables. In Panel B we therefore include the lagged median correlation as a regressor in all the specifications. As expected, the coefficient on lagged median correlation is close to one in the univariate specifications (i)-(vi). Interestingly, while their significance is of course lower now, the sign of all the coefficients remain as in Panel A. Perhaps the most important conclusion from Panel B is that VIX is the most significant driver of credit correlations once lagged correlations are controlled for. Note that the one-year trailing S&P 500 return has lost its significance, presumably because it is picking up the stochastic trend in correlations now captured by the lagged median correlation. The weekly S&P 500 return, however, is now significant in picking up what is effectively weekly changes in credit correlation. The CDX index retains some of its significance from Panel A as well. Finally, the yield curve level and slope are no longer significant, presumably again because they are mainly picking up the stochastic trend in correlations in Panel A.

So far we have separately analyzed each individual coefficient which may thus be influenced by omitted variables bias. In specification (vii) in Panels A and B we therefore include all the variables simultaneously. The results from these multivariate regressions are interesting. Note first that while the lagged median correlation is highly significant in Panel B it is lower than in the univariate specifications which suggests that the economic variables overall have explanatory power. Surprisingly, in Panel A, the stock market returns have a positive sign but this is not the case in Panel B. The interest rate level remains statistically significant in Panel A but not in Panel B. Our most robust finding is that the VIX turns out to be important for explaining credit dependence, in spite of the fact that volatilities do not always co-move with credit correlations, as documented in Figure 9 and Section 4.3. In fact, somewhat surprisingly, in Panel B the estimated coefficient on VIX is significant but the coefficient on CDX is not. The insignificance of CDX is partly due to the smaller point estimate of the coefficient—which is still positive—and partly due to the larger standard deviation on the coefficient when all variables are included.

It may prove interesting to extend these time-series results by investigating the loadings of different firms on these candidate factors. Additionally, one could study how incorporating firm-specific variables helps explain the cross-section of credit correlations. We keep these topics for future work.

## 6 Economic Determinants of Credit Spreads

We now investigate if our new dynamic credit risk measures drive credit spreads when controlling for the usual economic drivers of credit spreads. The determinants of credit spreads have been extensively studied both theoretically and empirically. Most notably, following the analysis of Merton (1974), structural models of credit risk have established volatility, interest rates, and leverage as prime candidates to explain credit spreads.

Partly based on these theories, there is an extensive empirical literature regarding the determinants of credit spreads, both using bond data and CDS data. This literature provides some support for structural models of credit risk, and has also documented other macro-economic and firm-specific determinants of credit risk. See Collin-Dufresne, Goldstein, and Martin (2001), Campbell and Taksler (2003), Cremers, Driessen, Maenhout, and Weinbaum (2008), and Ericsson, Jacobs, and Oviedo (2009) for existing evidence.

The existing literature and the ongoing debate on the determinants of credit risk spreads motivate our empirical approach as well as our selection of regressors. First, existing theory specifies some firm-specific determinants of credit spreads and therefore it is important to specify regressions using firm-specific measures of credit spreads, and not median credit spreads as in Section 5. Second, when testing the ability of new variables to explain credit spreads, it is important to investigate if their explanatory power is robust to the presence of variables suggested by theory in the regression.

We want to investigate if our dependence measures can help explain credit spreads. We therefore present results for univariate regressions of credit spreads on the average copula correlation and tail dependence for each firm with all other firms, but we also present results for multivariate regressions where these dependence measures are added to equity volatility, term structure variables, and leverage, the determinants of credit risk according to the Merton (1974) model. Several studies have specifically questioned the ability of regressors suggested by theory to explain time-series variation in spreads (see Collin-Dufresne, Goldstein, and Martin (2001)), so we focus on time-series regressions. We include lagged spreads as regressors because of the persistence in the spreads. The signs of the estimated coefficients do not change when lagged spreads are not included (not reported).

The results are presented in Table 6. We run firm-by-firm time-series regressions and we report the average point estimates across firms and statistical significance based on the estimated time-series coefficients. A first important conclusion is that overall the results support the theory underlying structural credit risk models. Credit spreads increase with equity volatility. Leverage is mostly estimated with a positive sign, but it is usually not statistically significant.

We are interested in whether the CDS risk measures in our analysis, credit correlation and

tail dependence, help explain credit spreads. We also analyze the impact of CDS volatility on credit spreads. In regressions (ii), (iii), and (iv), we see that CDS volatility, correlation, and tail dependence are positively related to credit spreads. When we include other variables in regressions (vi) and (vii), the relationship between CDS spreads and CDS volatility, CDS correlation and tail dependence is still significantly positive.

We conclude that the CDS volatility, correlation and tail dependence measures that we have constructed using the dynamic copula model are important determinants of the time series dynamics in credit spreads.

## 7 Conclusion

This paper documents cross-sectional dependence in CDS spreads, and compares it with dependence in equity returns. Our results are complementary to existing correlation and dependence estimates, which are typically based on historical default rates or factor models of equity returns, and to existing intensity-based studies, which characterize observable macro variables that induce realistic correlation patterns in default probabilities (see Duffee (1999) and Duffie, Saita and Wang (2007)). Importantly, we use econometric techniques that allow us to estimate a model with multivariate asymmetries and time-varying dependence using a long time series and a large cross-section of CDS spreads.

We document six important stylized facts. First, copula correlations in CDS spreads vary substantially over our sample and increase significantly following the financial crisis in 2007. Equity correlations also increase in the financial crisis, but somewhat later, and the increase is less significant and not as persistent. Second, our estimates indicate fat tails in the univariate distributions, but also multivariate non-normalities. Multivariate asymmetries seem to be less important for credit than they are for equities. Third, credit dependence is more persistent than equity persistence, and this greatly affects how major events such as the Quant Meltdown, the Lehman bankruptcy, and the U.S. sovereign debt downgrade affect subsequent dependence in credit and equity markets. Fourth, tail dependence increases more significantly than do copula correlations. Fifth, VIX is an important driver of credit correlations over time. Sixth, the dependence and tail dependence measures are related to the time-series variation in credit spreads, even after accounting for other well-known firm-level determinants of spreads.

These stylized facts, and the increase in cross-sectional dependence in particular, have important implications for the management of portfolio credit risk. We illustrate these implications by computing the diversification benefits from selling credit protection. The increase in cross-sectional dependence following the financial crisis has reduced diversification benefits, not unlike what happened in equity markets. When computing diversification benefits, taking non-normalities into account is more important for credit than for equity.



Several other important implications of our results deserve further study. First, given the richness and complexity of the equity and credit dependence, it may prove interesting to explore the implications for the pricing of structured products. In particular, following Berd, Engle, and Voronov (2007), it would be interesting to investigate if the CDO pricing model suggested by the estimated dynamics removes the observed correlation smile in CDO tranches. Second, our estimates can be used to manage a portfolio of counterparty risks. Third, our approach can be used to integrate credit and equity dependence dynamics in a single model that allows for diversification across asset classes. Finally, a possible extension is to investigate alternative measures of credit portfolio risk building on Vasicek (1991, 2002).

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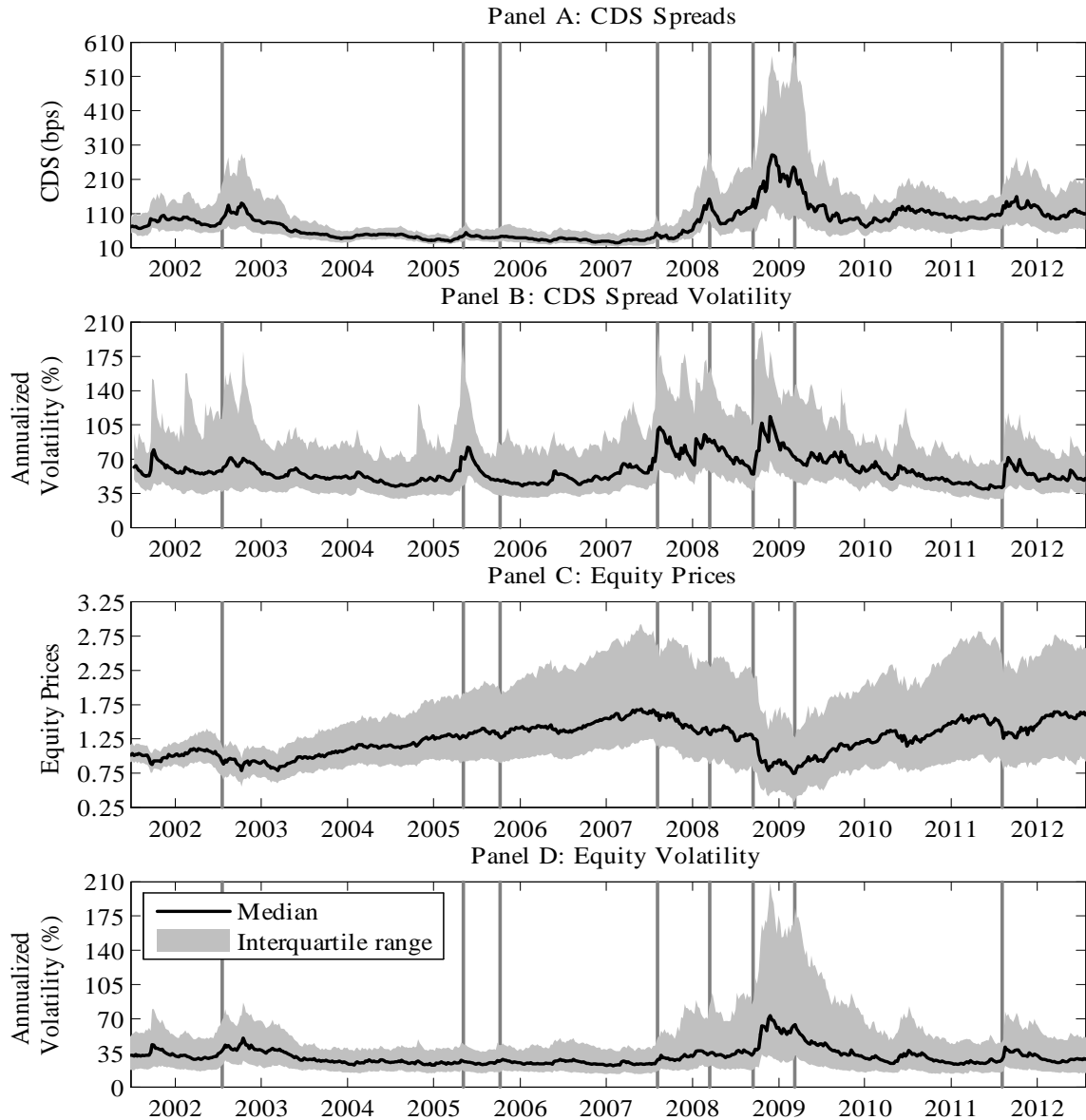
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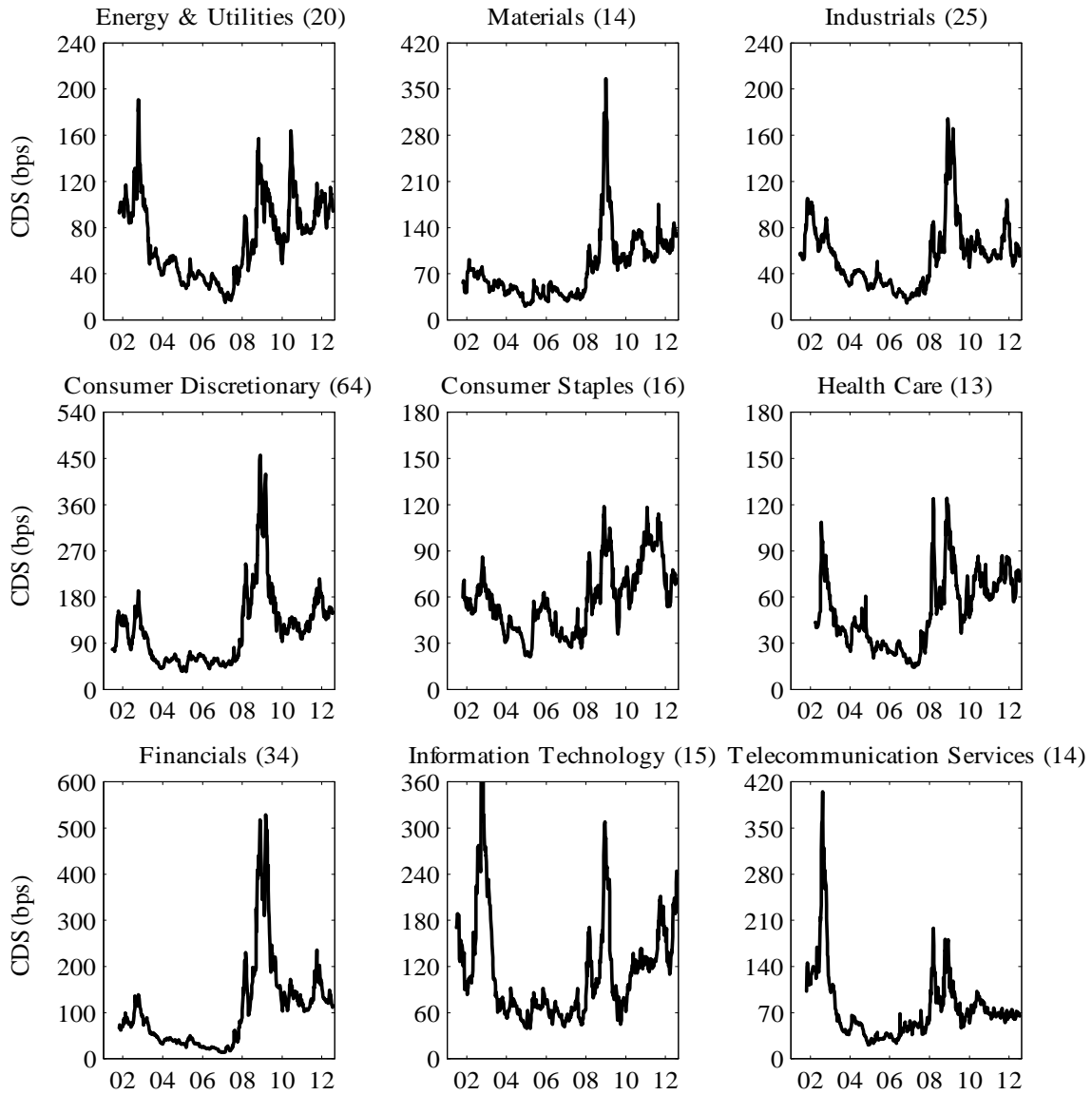
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Figure 1. Quantiles of CDS Spreads, Equity Prices and their Volatilities



Notes to Figure: We plot weekly quantiles across the 215 firms listed in Table 1 for CDS spreads, equity prices, and their volatilities from GARCH models. In each panel, the black line reports the median across firms for each week, and the grey area shows the interquartile range across firms. The vertical lines indicate the major events during the sample period listed in Section 2.3.

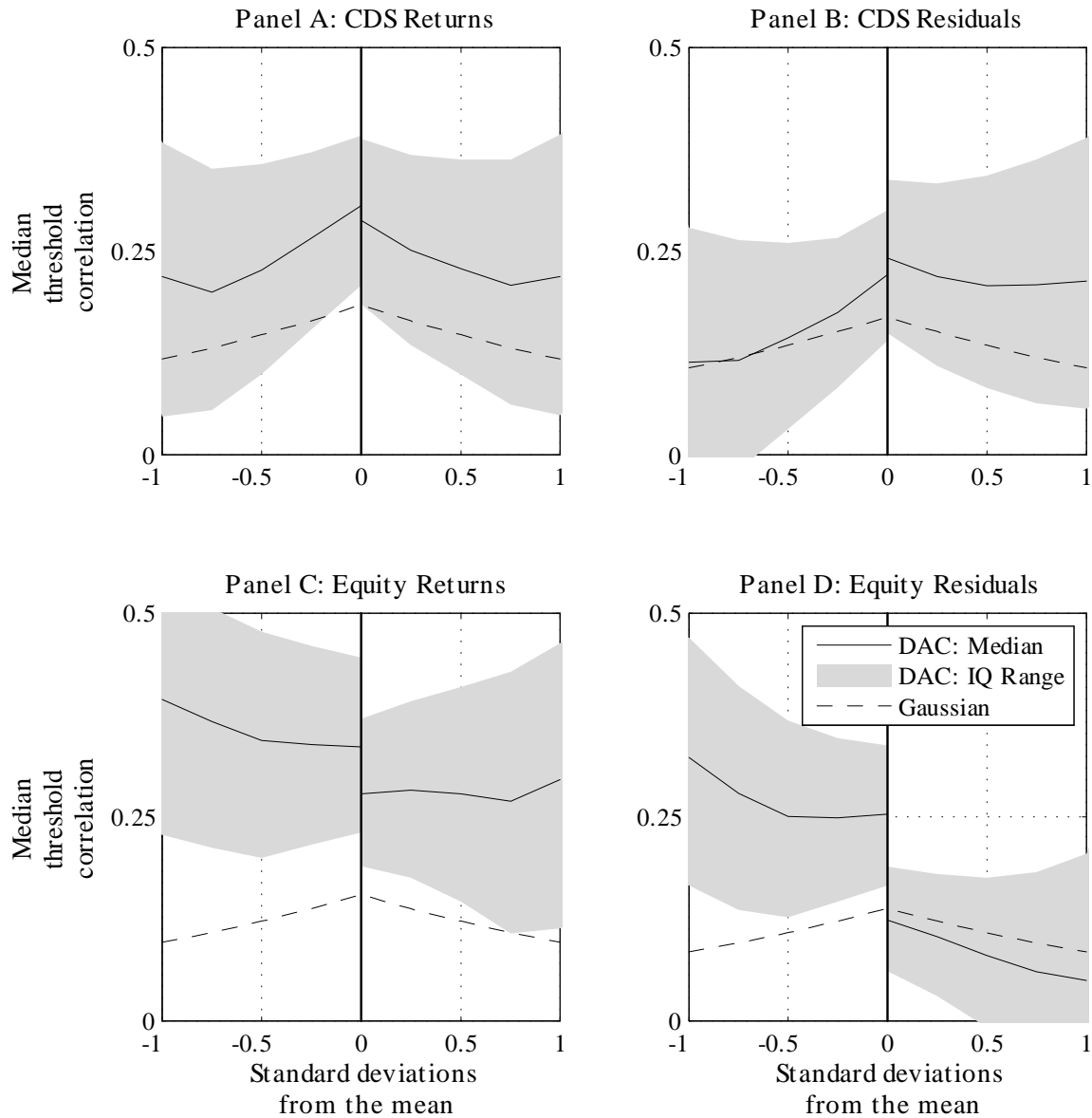
Figure 2. Median CDS Spread by Industry



Notes to Figure: We report the weekly median CDS spread by industry using the GIC sectors in Table 1. We combine the energy and utility sectors. Each panel title indicates the total number of firms available for each industry throughout the sample. Note that the scale differs across industries.

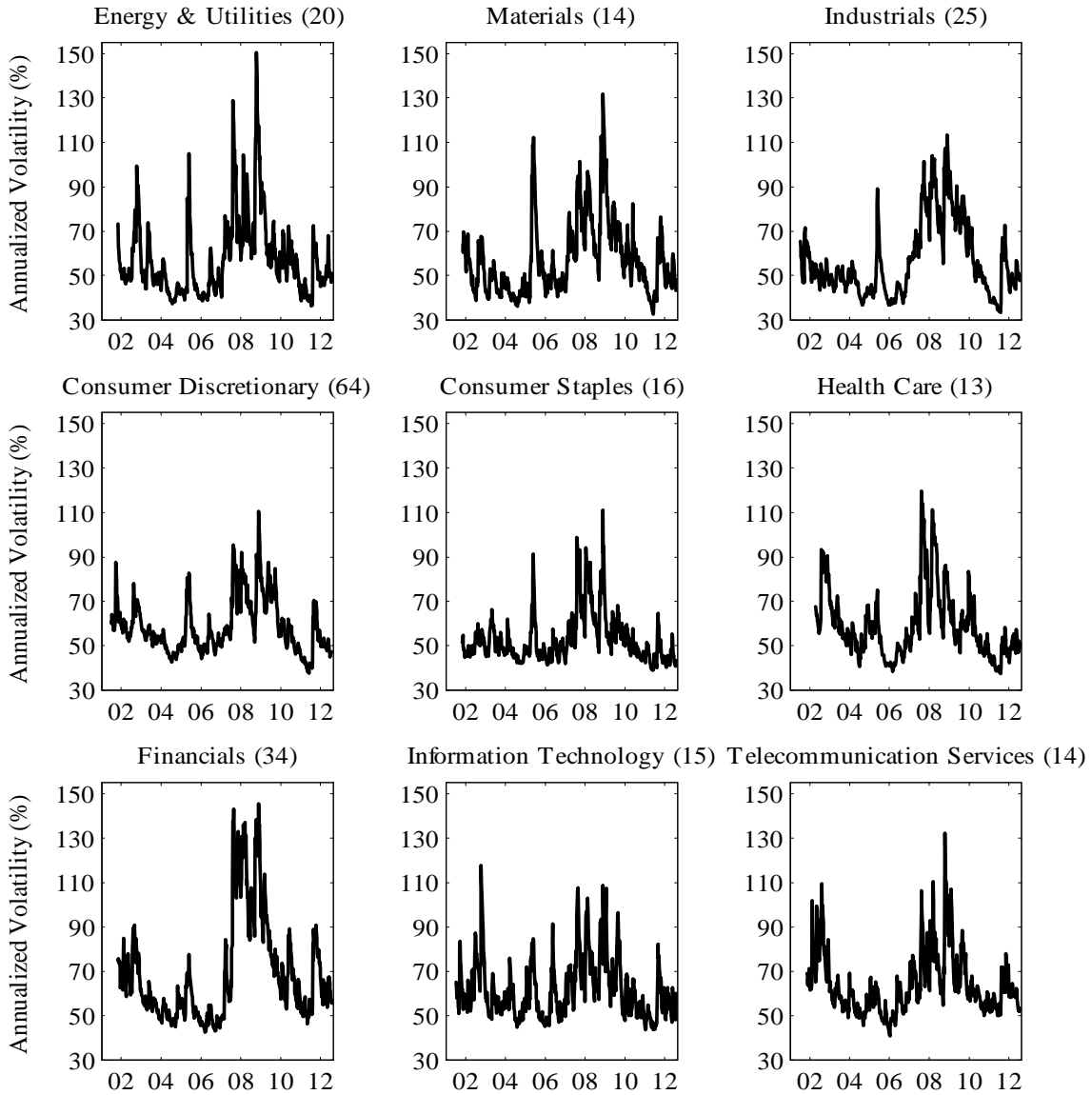


Figure 3. Threshold Correlations for Weekly Log-Differences and ARMA-NGARCH Residuals: CDS Spreads and Equity Returns



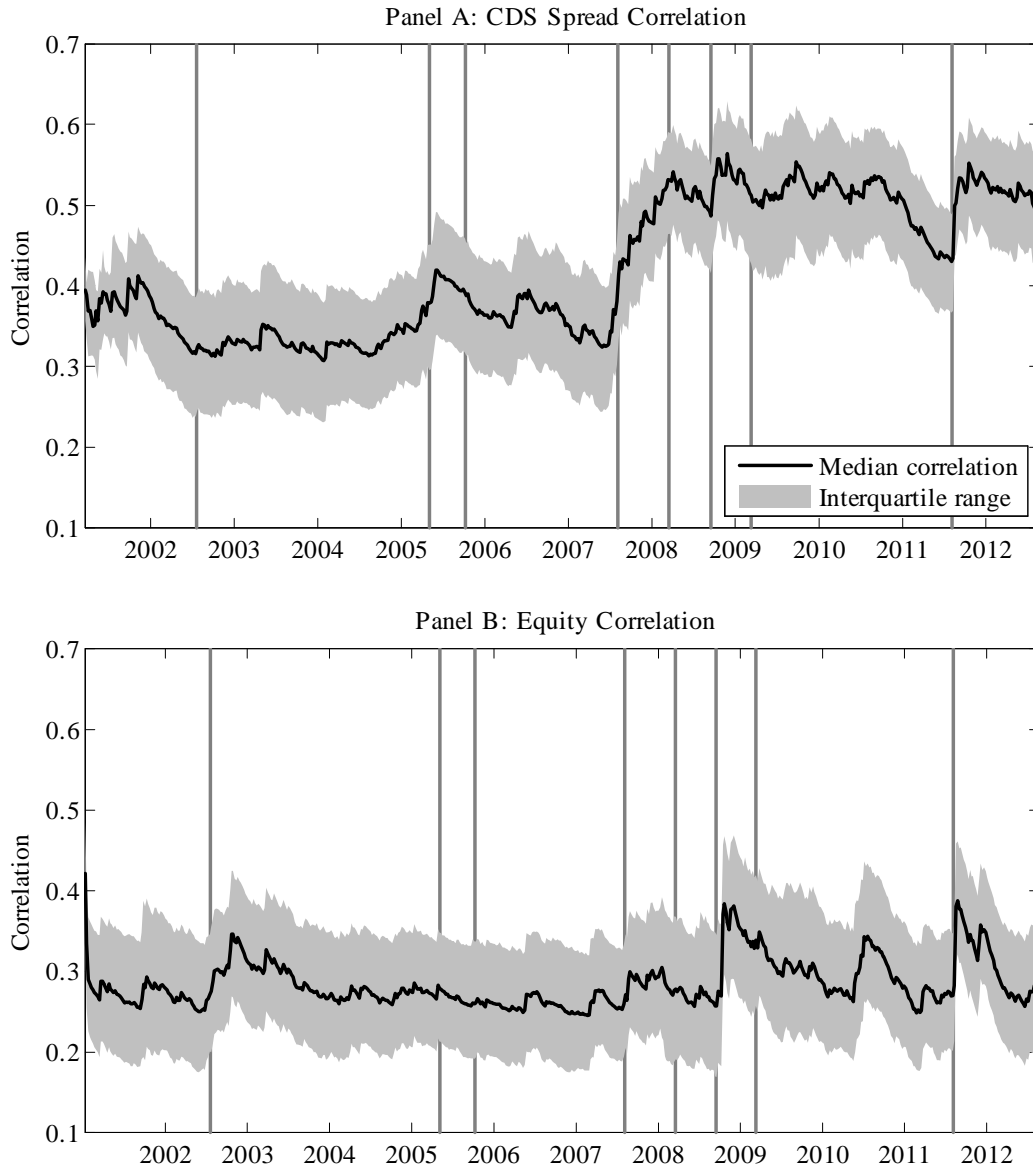
Notes to Figure: For each pair of firms we compute threshold correlations on a grid of thresholds defined using the standard deviation from the mean for each firm (horizontal axis). The solid lines show the median threshold correlations across firm pairs, the gray areas mark the interquartile (IQ) ranges and the dashed lines show the threshold correlations from a bivariate Gaussian distribution with correlation equal to the average for all pairs of firms.

Figure 4. Median Conditional Volatility of CDS Spreads by Industry



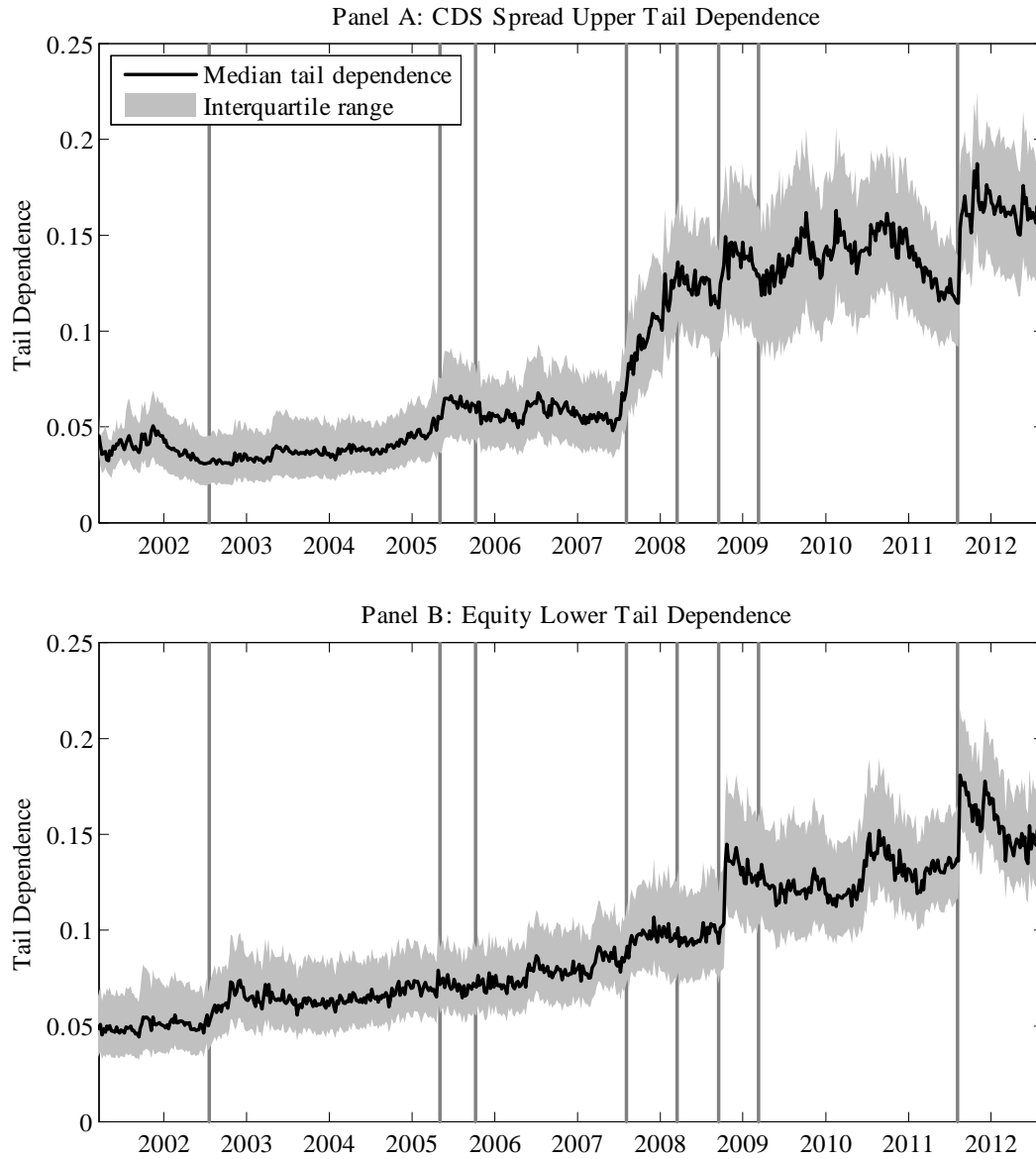
Notes to Figure: We report the weekly median conditional volatility of CDS spreads by industry using the GIC sectors in Table 1. Conditional volatility is estimated using an ARMA-GARCH model for each firm. We combine the energy and utility sectors. Each panel title indicates the total number of firms available for each industry throughout the sample.

Figure 5. Quantiles of Copula Correlations:  
CDS Spreads and Equity Returns



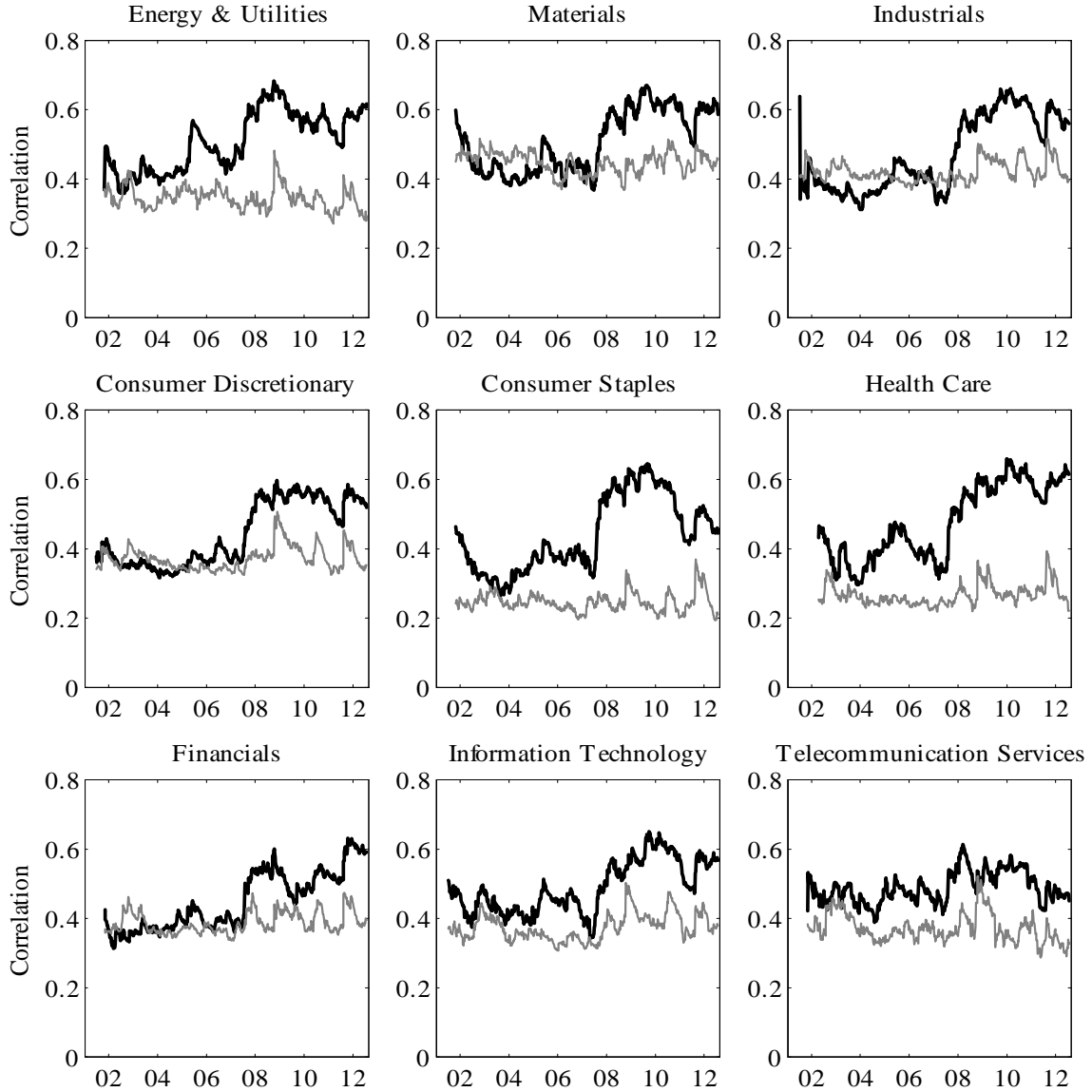
Notes to Figure: Using all available pairs of firms we report the weekly median (black line), and interquartile range (grey area) of the dynamic correlations from the dynamic asymmetric (DAC) copula model.

Figure 6. Quantiles of Tail Dependence:  
CDS Spreads and Equity Returns



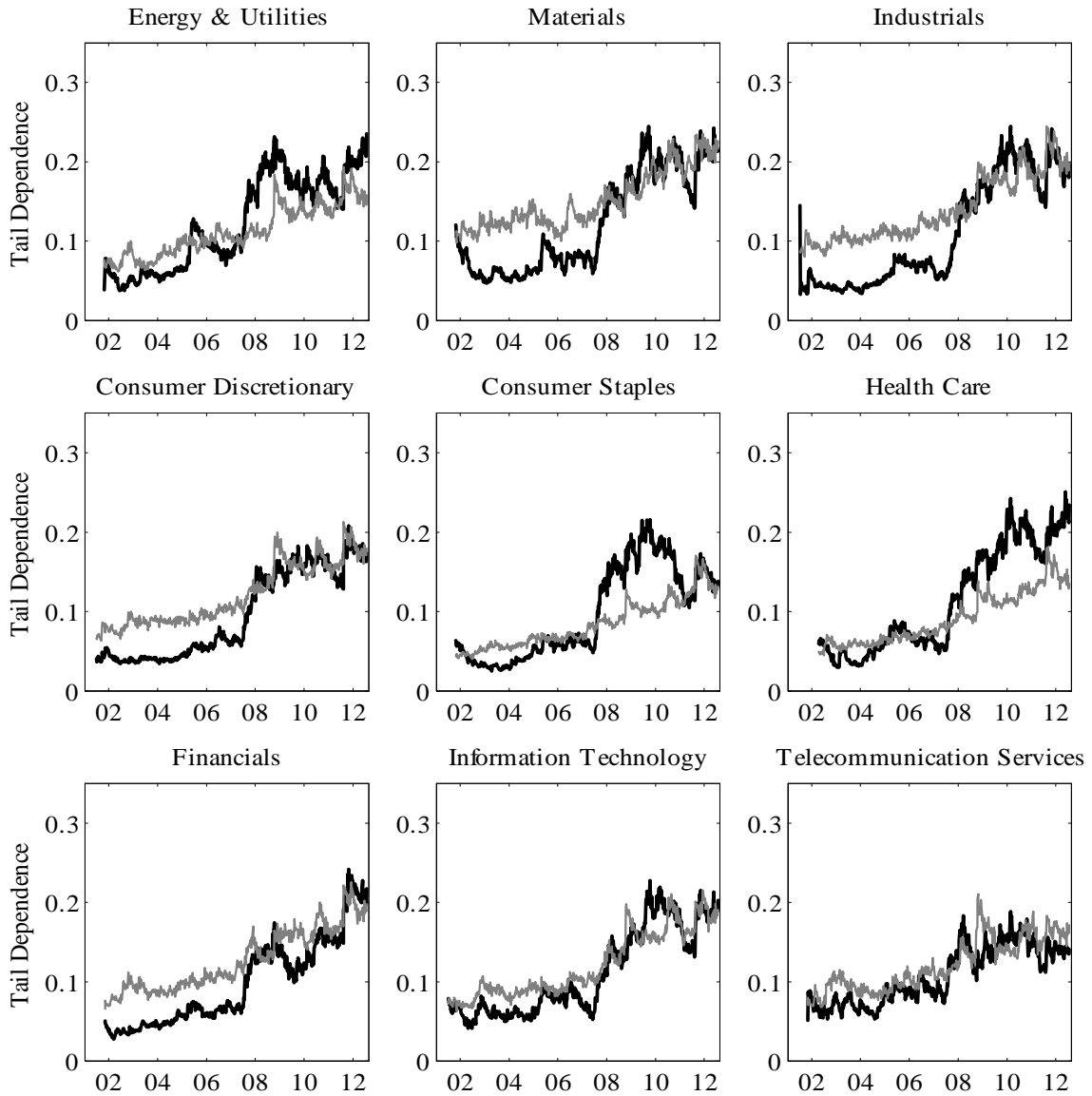
Notes to Figure: Using all available pairs of firms we report the median (black line), and interquartile range (grey area), of the dynamic tail dependence from the dynamic asymmetric copula (DAC) model.

Figure 7. Median Copula Correlations within Nine Industries:  
CDS Spreads and Equity Returns



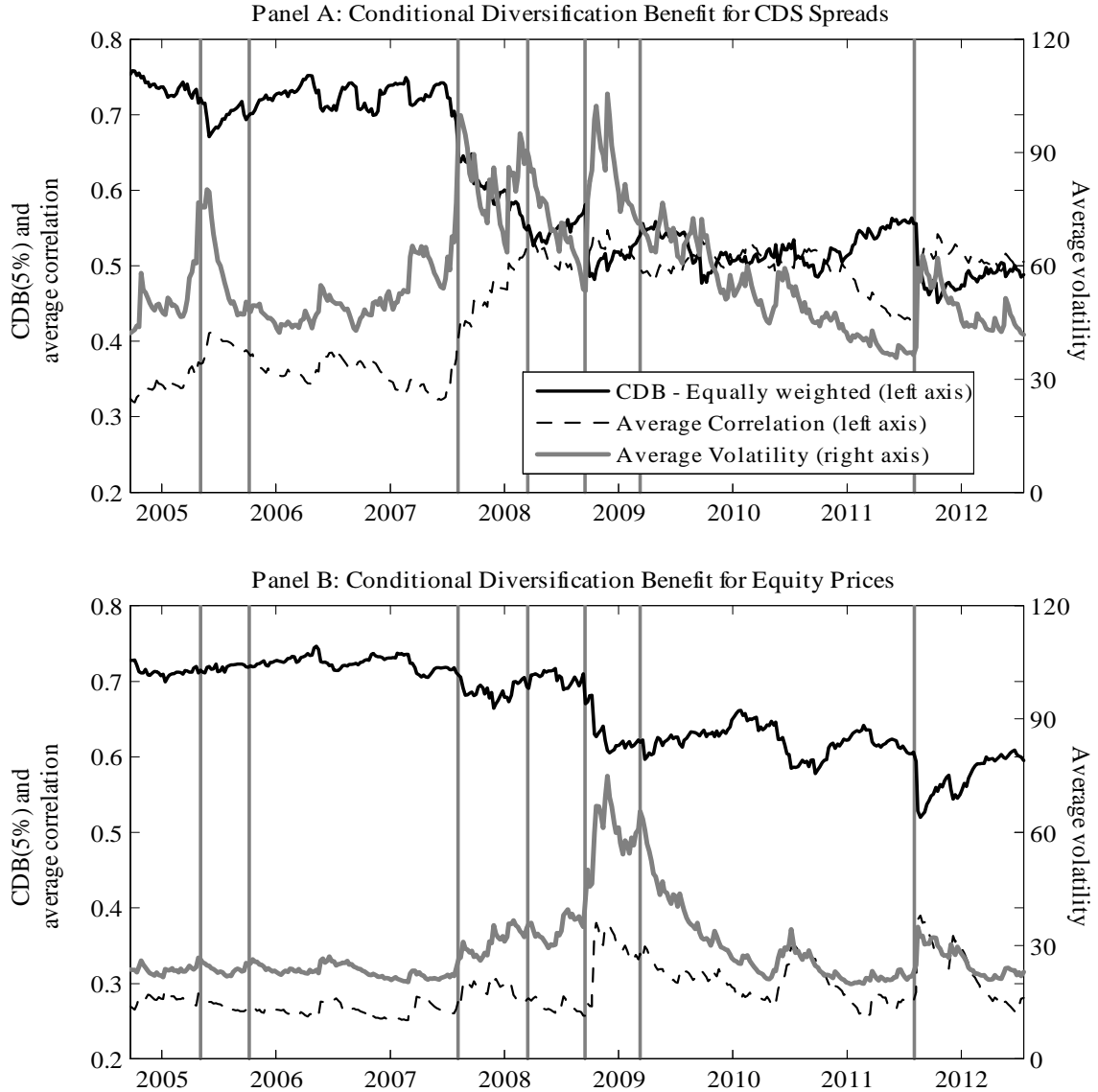
Notes to Figure: We report the median dynamic copula correlation by industry using the GIC sectors in Table 1. The black line shows CDS and the gray line equity correlations. We combine the energy and utility sectors.

Figure 8. Median Tail Dependence within Nine Industries:  
CDS Spreads and Equity Returns



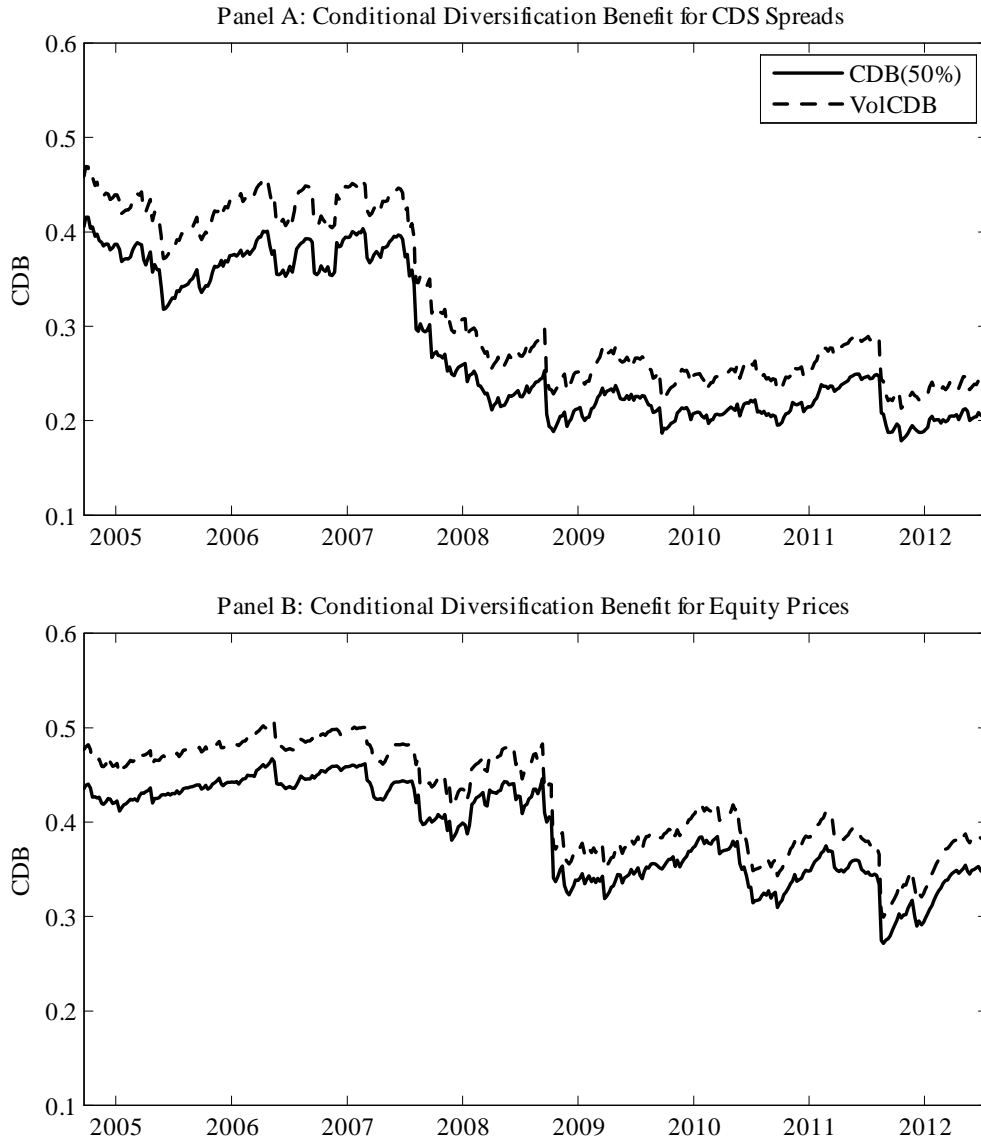
Notes to Figure: We report the median tail dependence by industry using the GIC sectors in Table 1. The black line shows CDS and the gray line equity correlations. We combine the energy and utility sectors.

Figure 9. Conditional Diversification Benefits.  
Credit and Equity Portfolios. 5% Tail.



Notes to Figure: Using equally weighted portfolios of available on-the-run CDX firms in a given week, we compute the 5% conditional diversification benefit (CDB) using the DAC model for CDS spreads (Panel A) and equity returns (Panel B). The credit portfolio sells credit protection by shorting CDS contracts. The dashed line shows the average correlation (on the left-hand axis) and the gray line shows the average volatility (on the right-hand axis). We use the first-two years of the sample to estimate the unconditional correlation matrix, and thus plot CDB starting in 2004.

Figure 10. Conditional Diversification Benefits.  
Credit and Equity Portfolios. 50% CDB and Volatility CDB Measures



Notes to Figure: Using equally weighted portfolios of available on-the-run CDX firms in a given week, we compute the 50% conditional diversification benefit (CDB) using the DAC model for CDS spreads and equity returns. We also show the volatility-based VolCDB measure, which only takes volatilities and linear correlations into account. The credit portfolio sells credit protection by shorting CDS contracts.



**Table 1: Company Names and Industry Classifications**

<b>Energy</b>	<b>Consumer Discretionary</b>	<b>Consumer Discretionary (Cont.)</b>	<b>Financials (Cont.)</b>
Amerada Hess Corp.	American Axle & Manufacturing	Viacom Inc.	Loews Corp.
Anadarko Petroleum Corp.	Autozone Inc.	Visteon Corp.	MBIA Insurance Corp.
Canadian Natural Resources Ltd	Belo Corp.	Walt Disney Co	MBNA Corp.
ConocoPhillips	Black & Decker Corp.	Wendy's International Inc.	Marsh & McLennan Co Inc.
Devon Energy Corp.	Brunswick Corp.	Whirlpool Corp.	MetLife Inc.
Halliburton Co	CBS Corp.	YUM! Brands Inc.	National Rural Utilities Coop
KerrMcGee Corp.	CENTEX Corp.		Radian Group Inc.
Kinder Morgan Energy LP	Carnival Corp.	<b>Consumer Staples</b>	Residential Capital Corp.
Nabors Industries Inc.	Clear Channel Comms Inc.	Albertsons Inc.	SLM Corp.
Transocean Inc.	Comcast Cable Comm. LLC	Altria Group Inc.	Simon Property Group Inc.
Valero Energy Corp.	Comcast Corp.	Beam Inc.	Vornado Realty LP
XTO Energy Inc.	Cox Communications Inc.	CVS Caremark Corp.	Washington Mutual Inc.
	DIRECTV Holdings LLC	Campbell Soup Co	Wells Fargo & Co
<b>Materials</b>	Darden Restaurants Inc.	ConAgra Foods Inc.	Weyerhaeuser Co
Alcan Inc.	Delphi Corp.	General Mills Inc.	XLIT Limited
Alcoa Inc.	Eastman Kodak Co	H. J. Heinz Co	iStar Financial Inc.
Barrick Gold Corp.	Expedia Inc.	Kraft Foods Inc.	
Dow Chemical Co	Ford Motor Credit Co	Kroger Co	<b>Information Technology</b>
E. I. du Pont de Nemours & Co	GAP Inc.	Reynolds American Inc.	1st Data Corp.
Eastman Chemical Co	Gannett Co Inc.	Safeway Inc.	Arrow Electronics Inc.
Freeport McMoran Inc.	Harrah's Operating Co Inc.	Sara Lee Corp.	Avnet Inc.
International Paper Co	Hilton Hotels Corp.	Supervalu Inc.	CA Inc.
MeadWestvaco Corp.	Home Depot Inc.	Tyson Foods Inc.	Cisco Systems Inc.
Olin Corp.	J. C. Penney Co Inc.	Wal Mart Stores Inc.	Computer Sciences Corp.
Rio Tinto Alcan Inc.	Johnson Controls Inc.		Dell Inc.
Rohm & Haas Co	Jones Apparel Group Inc.	<b>Health Care</b>	Electronic Data System Corp.
Sherwin Williams Co	Knight-Ridder Inc.	Aetna Inc.	Hewlett Packard Co
Temple-Inland Inc.	Kohls Corp.	Amgen Inc.	IAC InterActive Corp.
	Lear Corp.	Baxter International Inc.	IBM Corp.
<b>Industrials</b>	Lennar Corp.	Boston Scientific Corp.	Motorola Inc.
Boeing Capital Corp.	Liberty Media Corp.	Bristol Myers Squibb Co	Sabre Holdings Corp.
Bombardier Capital Inc.	Limited Brands Inc.	Cardinal Health Inc.	Sun Microsystems Inc.
Bombardier Inc.	Liz Claiborne Inc.	Cigna Corp.	Xerox Corp.
Burlington Northern Santa Fe	Lowe's Companies Inc.	McKesson Corp.	
CSX Corp.	M.D.C. Holdings Inc.	Pfizer Inc.	<b>Telecommunication Services</b>
Caterpillar Inc.	Macy's Inc.	Quest Diagnostics Inc.	ALLTEL Corp.
Cendant Corp.	Marriott International Inc.	UnitedHealth Group Inc.	AT&T Corp.
Deere & Co	May Department Stores Co	Universal Health Services Inc.	AT&T Inc.
GATX Corp.	Maytag Corp.	Wyeth	AT&T Mobility LLC
General Electric Capital Corp.	McDonald's Corp.		AT&T Wireless Services Inc.
Goodrich Corp.	Mohawk Industries Inc.	<b>Financials</b>	BellSouth Corp.
Honeywell International Inc.	NY Times Co	ACE Limited	CenturyLink Inc.
Ingersoll Rand Co	Newell Rubbermaid Inc.	Allstate Corp.	Cingular Wireless LLC
Lockheed Martin Corp.	News America Inc.	American Express Co	Citizens Communication Co
Masco Corp.	Nordstrom Inc.	American International Group Inc.	Embarq Corp.
Norfolk Southern Corp.	Omnicom Group Inc.	Berkshire Hathaway Inc.	Intelsat Limited
Northrop Grumman Corp.	Pulte Homes Inc.	Boston Properties LP	Sprint Corp.
Pitney Bowes Inc.	RadioShack Corp.	CIT Group Inc.	Verizon Communications Inc.
R. R. Donnelley & Sons Co	Sears Roebuck Acceptance Corp.	Capital One Bank	Verizon Global Funding Corp.
Raytheon Co	Staples Inc.	Capital One Financial Corp.	
Ryder System Inc.	Starwood Hotels & Resorts Inc.	Chubb Corp.	<b>Utilities</b>
Southwest Airlines Co	TJX Companies Inc.	Countrywide Home Loans Inc.	American Electric Power Co Inc.
Textron Financial Corp.	Target Corp.	EOP Operating LP	Constellation Energy Group Inc.
Union Pacific Corp.	Time Warner Cable Inc.	ERP Operating LP	Dominion Resources Inc.
United Parcel Services Inc.	Time Warner Inc.	Freddie Mac	Duke Energy Carolinas LLC
	Toll Brothers Inc.	Fannie Mae	Exelon Corp.
	Toys "R" Us Inc.	General Motors Acceptance Corp.	FirstEnergy Corp.
	Tribune Co	Hartford Financial	Progress Energy Inc.
		International Lease Finance Corp.	Sempra Energy

Notes to Table: Using data from Markit, we consider all firms included in the first 18 series of the CDX North American investment grade index dating from January 1, 2001 to August 22, 2012. Our sample consists of 215 firms. Firms are ordered alphabetically within each GIC sector.

**Table 2: Descriptive Statistics on CDS Spreads, Default Intensities and Equity Prices****Panel A: Sample Moments on Weekly Log-Differences**

	Annualized Average (%)	Annualized Standard Deviation (%)	Skewness	Kurtosis	Jarque-Bera p-value	AR(1) Coefficient	AR(2) Coefficient
<u>CDS Spreads</u>							
Median	4.08	66.71	0.86	9.75	0.00	0.09	0.05
Interquartile Range	[-4.48, 13.19]	[60.98, 72.98]	[0.48, 1.38]	[7.51, 14.72]	[0.00, 0.00]	[0.06, 0.14]	[0.02, 0.09]
<u>Equity Prices</u>							
Median	4.17	34.83	-0.48	7.91	0.00	-0.03	-0.02
Interquartile Range	[-1.72, 8.60]	[28.38, 43.46]	[-0.86, -0.17]	[6.27, 12.05]	[0.00, 0.00]	[-0.06, 0.01]	[-0.06, 0.02]

**Panel B: Correlations of Weekly Log-Differences**

	CDS Spreads	Equity Prices
<u>CDS Spreads</u>		
Median	0.39	-0.33
Interquartile Range	[0.30, 0.47]	[-0.42, -0.24]
<u>Equity Prices</u>		
Median		0.32
Interquartile Range		[0.25, 0.41]

Notes to Table: We report sample moments on weekly CDS spreads and equity prices across available firms. Panel A reports sample moments computed on the weekly log-differences of spreads and equity prices. Panel B reports average sample correlations across firms using weekly log-differences. On the diagonal we report the median and IQR across the correlations between each firm and all other firms. On the off-diagonal we report the median and IQR of the correlation between spreads and equity prices for the same firm.

**Table 3: Summary of ARMA-NGARCH Estimation on Weekly Log-Differences**

**Panel A: Conditional Mean Dynamics**

<u>Model Chosen by AICC Criterion</u>		CDS Spreads	Equity Prices
ARMA(0,0)		8%	12%
ARMA(0,1)		12%	7%
ARMA(0,2)		10%	9%
ARMA(1,0)		15%	5%
ARMA(1,1)		13%	6%
ARMA(1,2)		5%	6%
ARMA(2,0)		10%	8%
ARMA(2,1)		4%	10%
ARMA(2,2)		23%	36%
<u>Parameter Estimates</u>			
$\mu$	Median	0.0000	0.0000
	Interquartile Range	[-0.00, 0.00]	[-0.00, 0.00]
AR(1)	Median	0.100	-0.040
	Interquartile Range	[-0.27, 0.59]	[-0.84, 0.48]
AR(2)	Median	-0.280	-0.620
	Interquartile Range	[-0.80, 0.10]	[-0.84, -0.10]
MA(1)	Median	0.080	-0.030
	Interquartile Range	[-0.52, 0.38]	[-0.48, 0.79]
MA(2)	Median	0.290	0.630
	Interquartile Range	[0.07, 0.85]	[0.07, 0.87]
L-B(4) p-value > 5%	Proportion	99%	90%

**Panel B: Conditional Volatility Dynamics and Return Distributions**

<u>Parameter Estimates</u>		CDS Spreads	Equity Prices
$\beta$	Median	0.780	0.800
	Interquartile Range	[0.68, 0.85]	[0.71, 0.86]
$\alpha$	Median	0.130	0.050
	Interquartile Range	[0.09, 0.19]	[0.03, 0.09]
$\gamma$	Median	-0.200	1.230
	Interquartile Range	[-0.45, 0.02]	[0.79, 2.05]
Volatility Persistence	Median	0.950	0.980
	Interquartile Range	[0.89, 0.98]	[0.96, 0.99]
$\nu$	Median	3.720	6.560
	Interquartile Range	[3.30, 4.31]	[5.02, 8.13]
$\lambda$	Median	0.100	-0.110
	Interquartile Range	[0.07, 0.14]	[-0.16, -0.07]
L-B(4) p-value $z^2 > 5\%$	Proportion	98%	87%

Notes to Table: For each firm we estimate an ARMA(p,q)-NGARCH(1,1) model where the p and q are chosen by the AICC criterion. The residual distribution is asymmetric  $t$  with parameters  $\nu$  and  $\lambda$ . L-B(4) denotes a Ljung-Box test that the residuals (Panel A) or squared residuals (Panel B) are serially uncorrelated.

**Table 4: ARMA-NGARCH Residual Statistics and Dynamic Copula Parameter Estimation****Panel A: Residual Sample Moments**

		Skewness	Kurtosis	Cross-firm Correlation	Cross-Instrument Correlation
CDS Spreads	Median	0.91	9.29	0.39	-0.33
	Interquartile Range	[0.44, 1.65]	[6.44, 16.13]	[0.30, 0.47]	[-0.42, -0.24]
Equity Prices	Median	-0.45	5.40	0.32	
	Interquartile Range	[-0.75, -0.24]	[4.33, 7.26]	[0.25, 0.41]	

**Panel B: Dynamic Asymmetric Copula Estimation**

	Model I: $v_c(t) = 4 + v_{c,0}$		Model II: $v_c(t) = 4 + v_{c,0} \exp(v_{c,1} t)$	
	CDS Spreads	Equity Prices	CDS Spreads	Equity Prices
$\beta_C$	0.96	0.92	0.96	0.92
$\alpha_C$	0.02	0.02	0.02	0.02
Correlation Persistence	0.98	0.94	0.98	0.94
$v_{c,0}$	6.12	7.52	37.19	18.79
$v_{c,1}$			0.00	0.00
$\lambda_C$	0.08	-0.23	0.00	-0.33
Composite Log-likelihood	1,142,150	696,454	1,144,583	697,187

**Panel C: Dynamic Symmetric Copula Estimation**

	Model I: $v_c(t) = 2 + v_{c,0}$		Model II: $v_c(t) = 2 + v_{c,0} \exp(v_{c,1} t)$	
	CDS Spreads	Equity Prices	CDS Spreads	Equity Prices
$\beta_C$	0.96	0.92	0.96	0.92
$\alpha_C$	0.02	0.02	0.02	0.02
Correlation Persistence	0.98	0.94	0.98	0.94
$v_{c,0}$	10.06	10.81	37.19	18.78
$v_{c,1}$			0.00	0.00
Composite Log-likelihood	1,138,394	686,403	1,144,583	688,493

**Panel D: Dynamic Normal Copula Estimation**

	CDS Spreads	Equity Prices
$\beta_C$	0.959	0.913
$\alpha_C$	0.020	0.019
Correlation Persistence	0.979	0.933
Composite Log-likelihood	1,087,840	651,622

Notes to Table: We report sample statistics on ARMA-NGARCH residuals and estimation results for different copula models. Using the ARMA-NGARCH residuals,  $z$ , we compute in Panel A the median and interquartile range of the skewness, kurtosis, correlations for each pair of firms, and correlations between CDS and equity for each firm. We estimate the dynamic asymmetric copula (DAC) and the dynamic symmetric copula (DSC) with and without at time trend for the degree-of-freedom, and the dynamic normal copula (DNC) models on the 215 firms in our sample. Each of the models is estimated on ARMA-NGARCH residuals from weekly log-differences on CDS spreads and equity prices.

**Table 5: Regressions for Median CDS Correlation****Panel A: Regressions for Median CDS Correlation Without Lagged CDS Correlation**

	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)
Constant	-0.918 **	0.431	4.556 **	4.607 **	5.154 **	3.792 **	-2.702 **
CDX	1.253 **						0.870 **
VIX		1.390 **					0.375 **
S&P 500 return			-1.159				1.571 *
S&P 500 one-year return				-1.320 **			0.393
Interest rate level					-0.316 **		-0.113 *
Yield curve slope						0.426 **	-0.117
TED spread							0.000
Crude oil price							0.494 **
Business conditions index							0.024
Breakeven inflation							0.238
Adjusted R <sup>2</sup>	0.801	0.610	-0.001	0.102	0.683	0.533	0.880

**Panel B: Regressions for Median CDS Correlation With Lagged CDS Correlation**

	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)
Constant	-0.050	-0.037	0.047	0.052 *	0.089	0.058 *	-0.355 *
First lag	0.951 **	0.957 **	0.991 **	0.990 **	0.983 **	0.987 **	0.923 **
CDX	0.064 *						0.055
VIX		0.080 **					0.091 **
S&P 500 return			-0.647 *				-0.325
S&P 500 one-year return				-0.019			0.054
Interest rate level					-0.004		-0.003
Yield curve slope						0.004	-0.009
TED spread							0.000
Crude oil price							0.038
Business conditions index							0.011
Breakeven inflation							0.024
Adjusted R <sup>2</sup>	0.988	0.989	0.988	0.988	0.988	0.988	0.989

Notes to Table: In Panel A, we regress the weekly DAC median CDS correlation on the CDX North American investment grade index, the CBOE implied volatility index, the weekly return and the one-year trailing return on the S&P 500, the 3-month constant maturity U.S. Treasury rate, the difference between the 10-year and the 3-month constant maturity U.S. Treasury rates, the TED spread, the West Texas Intermediate cushing spot crude oil price, the Aruoba-Diebold-Scotti business conditions index, the U.S. breakeven inflation rate. All regressors are lagged, and the first lag of the regressand is included in Panel B. We compute Newey-West standard errors, and significance for regression coefficients at 5% and 1% are denoted by \* and \*\*. All regression estimates are multiplied by 10 for ease of exposition, except for the first lag of the regressand.

**Table 6: Average Coefficients from Predictive Regressions for CDS Spreads**

	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)
Constant	-0.138 **	-0.034 **	-0.113 **	-0.094 **	-0.161 **	-0.198 **	-0.168 **
Lagged CDS	0.960 **	0.986 **	0.983 **	0.983 **	0.955 **	0.953 **	0.951 **
Leverage	0.030				0.059 *	0.010	-0.013
S&P 500 return	-0.421 **				-0.430 **	-0.434 **	-0.439 **
S&P 500 1yr return	0.018 **				0.018 **	0.012 *	0.009
Interest rate level	0.002				0.001	0.004	0.006 *
Yield curve slope	0.007 **				0.007 **	0.010 **	0.013 **
TED spread	0.009 **				0.008 *	0.005	0.005
Equity volatility	0.033 **				0.030 **	0.037 **	0.038 **
CDS volatility		0.014 **			0.012 **		
CDS correlation			0.070 **			0.101 **	
CDS tail dependence				0.098 **			0.236 **
Average Adj. R <sup>2</sup>	0.975	0.977	0.976	0.976	0.976	0.975	0.975

Notes to Table: We report average coefficients and adjusted R<sup>2</sup> from firm-specific time series regressions. The left hand side variable is the weekly CDS spreads for each firm. Right hand side variables include the firm's leverage ratio, the weekly and one-year trailing return on the S&P 500 index, the 3-month constant maturity U.S. Treasury rate, the difference between the 10-year and the 3-month constant maturity U.S. Treasury rates, the TED spread, the firm-level equity GARCH volatility, the firm-level CDS GARCH volatility, the average CDS correlation with all other firms, and the average CDS tail dependence with all other firms. All regressors are lagged, and we also include the first lag of the regressand. We run time-series regressions and then average coefficients across firms. We compute Newey-West standard errors, and significance for regression coefficients at 5% and 1% are denoted by \* and \*\*. Estimates for the TED spreads are multiplied by 100 for ease of exposition.