Herbert Simon and Agent-Based Computational Economics

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Abstract

In this article, we review the development of agent-based computational economics (ACE) in light of Herbert Simon's lifetime contributions to a variety of scientific areas. We take an integrated view of Simon's contributions to economics, computational-theoretic underpinnings of bounded rationality (algorithmic foundations of behavioral economics), artificial intelligence, expert systems, problem-solving and decision-making heuristics and cues, near decomposability (modularity) and complex systems. We address how agent-based economic and financial modeling has developed along each of the aforementioned "Simon dimensions" over the last two decades. At the end, we discuss the possible implications of the legacy of Simon for the future of ACE.

1 Introduction

Herbert Simon's place in the field of economics requires no elaboration. He was a scholar who straddled different fields and donned different hats with such ease that is the defining feature of a polymath. He is a quintessential interdisciplinary scholar who has made pioneering contributions concerning the notion of bounded rationality, has built models based on it, and has also made important advances in understanding complex systems. The former was largely realized in the realm of cognitive science and the latter was rephrased as *modularity* by the later scholars.[1] His importance in the field of artificial intelligence, which is in turn the inspiration of agent-based computational economics (ACE), is discussed in detail in Chen (2005). From Simon's side, the only comment that is more explicitly related to ACE was his positive endorsement of genetic algorithms (Simon, 1996b).

Among all the Nobel Laureates in Economics, there are at least three whose work has been acknowledged by the ACE community. They are Friedrich Hayek (1899-1992), Thomas Schelling, and Elinor Ostrom (1933-2012). The last two directly worked on ACE. Schelling's celebrated work on the segregation model is considered as one of earliest publications on ACE (Schelling, 1969, 1971, 1972). Ostrom had contributed to the development of the empirical agent-based models (Janssen and Ostrom, 2006). Hayek did not work on ACE, but the connection of his work to ACE has been pointed out by Vriend (2002). We believe that there is a strong connection between the development of agent-based computational economics and Herbert Simon and that his influence on ACE is not less, if not more, profound than the previous three. However, to the best of our knowledge, there seems to be no single document that from a holistic perspective addresses this linkage explicitly.[2] We conjecture that the burgeoning of ACE was too late for the time of Simon, who ended his professional life in 2001. However, even so, it still surprises us that so few attempts have been made to connect Simon and ACE, particularly considering that the latter was founded on artificial intelligence and cognitive psychology, the two pillars to which the former has contributed substantially.[3].

In this chapter, we attempt to explore and identify the connections between Simon's contributions and the development of ACE. We concentrate on his influence on the conception of an individual within an economic or social system, his philosophy regarding how the social systems are organized and can be understood, and finally about how the underlying rules that govern social interactions can be unearthed by the investigator, in this case a social scientist. We also suggest ways with which the future developments within ACE can be geared to be more Simonian in character and to be closer to his vision.

The rest of the chapter is structured as follows. Section 2 provides an overview of the setting in which we place our arguments. We then divide our arguments into three main departments: individuals, complex systems and the epistemology of ACE. In Section 3, the modeling of software agents in light of Simon's bounded rationality is discussed. In Section 4, various aspects of complex systems are included here. In Section 5, we elaborate on ACE's potential as an alternative to neoclassical economics. We conclude the paper in the last section.

2 The Setting

Simon's contributions to behavioral economics and artificial intelligence are composed of many different, original ideas, all of which are grounded, either explicitly or implicitly, in the theories and models of computation. On the other hand, ACE routinely studies rule-following agents, computing within an environment that can be best considered as a complex adaptive system. ACE, as a research program, can be considered as being mounted up on, at least, *four pillars*: individuals (decomposition), interaction,

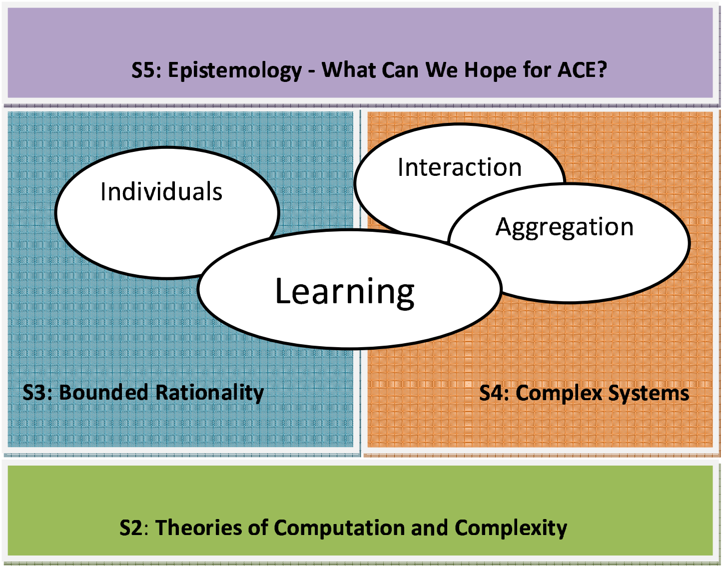

Figure 1: Four Pillars of ACE placed on the foundation of Simon

aggregation and learning (adaptation). Each of these pillars has an important computational component in its characterization. In this background, it is fairly evident that computation, and consequently simulation, can be one possible anchor around which one can attempt to explore the influence of Simon's legacy in the development of ACE.

Since we are exploring the intellectual links between Simon and ACE, it may be instructive to be aware that Simon did not emphasize all of these pillars uniformly. For instance, he placed more emphasis on the characteristics of economic entities (agents, institutions) as being boundedly rational (Simon, 1957, 1976) and adaptive (Simon, 1996b), and on features such as the near decomposability of complex systems (Simon, 1962, 1995). He appears to have focused less on the interaction aspect, except for his celebrated contributions on stochastic models(Simon, 1955b) and the resulting aggregate distributions underpinned by preferential attachment (see Section 5.1). However, despite the varying degrees of emphasis laid by Simon, all of the above ideas seem to have had an important and direct impact on ACE.

Figure 1 illustrates the perspective which we use to organize and develop this chapter. Out of these four pillars, learning seems to be the only pillar that is at the intersection of the ideas from both bounded rationality and complex systems. Learning is an important feature that is relevant for the components within the system (individuals) and also for the system as a whole. Learning or evolution becomes critical when the environment where the agents live and act is ever changing and complex. In such environments, optimal survival rules that are suitable for all agents at all times simply do not exist. Boundedly rational organisms acting in complex environments, has always been Simon's view of economic agents (Simon, 1959).

There has never been any direct conversation between Simon and ACE, at least ACE in the way we understand it today. This is perhaps due to the timing of the development of the latter. However, Simon has commented on many building blocks of ACE, such as various ways of modeling boundedly rational individuals, the form of complexity in social systems, learning and the evolution of complex systems, and the stochastic mechanisms or models that could explain stylized aggregate distributions and so on. In this study, we are also interested in identifying the less direct and the possible unidentified legacies of Simon with regard to ACE. These 'hidden' links include Simon's epistemological point of view towards simulation and his chunking theory (modularity, in its modern form). Besides, Simon's methodological pursuits also provide a hint for carving out a vision for the potential roles of ACE as it evolves, which could include market (more broadly, institutional) design (Marks, 2006), and act as a mode of hypothesis discovery. We argue that these could be important directions that the ACE community could explore in order to broach new and interesting frontiers.

2.1 Computation-Theoretic Underpinning

Given the general frame as depicted in Figure 1, we begin from the bottom. The common thread that connects Simon and ACE is their view on how economic agents and systems should be meaningfully modelled and studied, based on the advantages offered by digital computers. A brief survey on the origins of ACE here can be useful to highlight a common ground, which happens to be the theories of computation and complexity (the bottom level of Figure 1). Four distinct, yet interconnected origins of ACE are identified in a recent survey by Chen (2012). They are market, cellular-automaton, economic-tournament, and experimental economic origins, respectively. Except perhaps for the market origin, we can find Simon's direct or indirect influence on ACE through the other three origins. In particular, we would like dwell a little more on the cellular-automaton origin.

One important and probably the earliest example of the use of cellular automata in social sciences is Schelling's segregation model (Schelling, 1971), which is built upon on a checkerboard topology, also known as a checkerboard model. Albin (1975, 1998) explores the connection between Schelling's checkerboard model and the cellular automata tradition, which, in turn, places ACE on its computational theoretic foundation, underpinned by notions like Turing computability and Wolfram's computational irreducibility. Schelling showed, via many illustrations (Schelling, 1978), how interdependent decisions can lead to unexpected social phenomena, even though the individuals follow simple, or even simplistic, and identical rules. This is an origin that Schelling himself also has acknowledged:

> What I did not know when I did the experiments with my twelve-year-old son using copper and zinc pennies was that I was doing later became known as 'agent-based computational models', or 'agent-based computational economics'. (Schelling (2007), p. xi.)

If we trace the connection between cellular automata and economics, one goes all the way up to John von Neumann, whose pioneering study on self-reproducing automata (von Neumann et al., 1966) laid out a general theory, along with his important contributions to general equilibrium theory and the theory of games in economics. von Neumann did not apply cellular automata to study social or economic problems. Despite being the originator of the theory of cellular automata, von Neumann does not seem to dave explored possible, direct economic applications, as seen, for example, in Albin (1998), p. xv. Furthermore, the pioneering work by Thomas Schelling on checkerboard models was not *a priori* motivated by automata theory and hence may be viewed as "serendipitous one-shot play" rather than being a systematic intent to advance a new paradigm. These considerations lead us to place Peter Albin as a pioneer in endowing ACE with a general computation-theoretic underpinning. His two important articles on ACE (Albin, 1982, 1992), reprinted in Albin (1998), along with his first book (Albin, 1975) may qualify him for this position.

Albin (1982), reprinted as Chapter Two of Albin (1998), may be considered as one of the early articles to address computability issues in economics, while a more comprehensive treatment of this issue comes much later in Velupillai (2000, 2010a). Later on in his preface in Albin (1998) and the introductory chapter authored by Duncan Foley, they proposed a *Chomsky-Wolfram synthesis* as a framework to address complexity in economics. In this effort, they were trying to find a thread passing through John von Neumann, Alan Turing, Noam Chomsky, Kurt Gödel, John Conway, and Stephen Wolfram. The thread, called the *automata-theoretic foundation* of economics also nicely connects computer science, linguistics, and dynamical systems.

Albin (1992) applies Wolfram's one-dimensional elementary cellular automata to his proposed spatial (network) prisoner's dilemma game. The class of the spatial prisoner's dilemma game not only provides the simplest explanation of the prevalence of cooperative behavior, but, more importantly, provides the first illustration that ACE models can be richly studied in light of the theory of automata and the associated hierarchy of complexity.

For Simon, his awareness of the automata theory and its possible implications for the social sciences can be dated back to the very early stage of the development. It turns out that von Neumann in fact presented his work *The General Theory of Automata* at a session during the meeting of the Econometric Society held on September 5th, 1950 at Havard University, where Simon was the discussant[4]. Simon made important remarks concerning hierarchies of rationality and their connection with cellular automata. From Simon's discussion, it is evident that social scientists did show interest in drawing an analogy between automata and social organisms at least as early as 1950, much earlier than Schelling's checkerboard models. It is worth noting that Simon's comment dates back to before his proposal of bounded rationality and his theorem proving machine (Newell et al., 1958). Unfortunately, the abstract of the discussion by von Neumann was not made available and the details of the talk by von Neumann remain unknown to us. We can only at best speculate as to whether the concept of automata, underpinned by the theory of computation, was already in Simon's repertoire while he was developing his ideas of procedural rationality. In the light of this background, we will explore more of the content of Simon's ideas that are intertwined with ACE in the rest of this chapter.

3 Agents as Programs - Bounded Rationality

First and foremost, the influence of Herbert Simon on ACE will be apparent once we understand the way agents in an economic system are conceived. Arguably, bounded rationality is one of the most famous terms coined by Simon, and this has been developed in many directions and has managed to acquire many different interpretations over the years. Many of these have forged promising lines of research in their own right. From its definition, as shown in Simon (1957), it states that human minds may often fail to solve the problems to the level that is objectively optimal. Impressed by the human's ability to solve difficult problems, Simon began to painstakingly observe the *processes* of human thinking and devised computer programs to explain the qualitative and quantitative data that he gathered.

This initiative happened during the high wave of *Cognitive Revolution* in the mid-1950s and the 1960s, which is also considered to have marked the birth of artificial intelligence. The results of this research project by Simon gradually developed into the theory of *Human Problem Solving*, in which the information processing systems (IPS) are the models that characterize problem solvers in various domains (Newell and Simon, 1972). This approach lies at the foundation of information processing psychology - an important branch of cognitive science, knowledge engineering and domain expertise in modern computer science. Simon's firm belief seems to be that we have to open the black box of decision making and understand its procedural (algorithmic) aspects. Only then can we begin to appreciate human wisdom and the complexity of the societies that they live in.

3.1 LISP and Genetic Programming

We argue that it might be fruitful to associate bounds of rationality with the complexity of the algorithm (or procedures) that the human agents or software agents can handle and will potentially apply to solve the problems that they encounter. Simon himself has implicitly acknowledged, although with some caveats, this interpretation of bounded rationality through the concept of computational complexity[5]. In any case, the central idea in Simon's conception of an economic agent is that of a *problem solver*; his emphasis was on understanding *how* or, in other words, the procedural aspects. This demand for the transparency of agents is actively answered in ACE in various forms, such as simple programmed agents, entropy-maximizing agents (zero-intelligence agents), human-written programmed agents, and autonomous agents, which constitute a long glossary of artificial agents with transparent behavioral rules (Chen, 2012).[6]

In the context of the problem-solver analogy, it is appropriate to discuss *Logic Theorist* (Newell et al., 1958), which is the very first realization of IPS. Simon viewed prob-

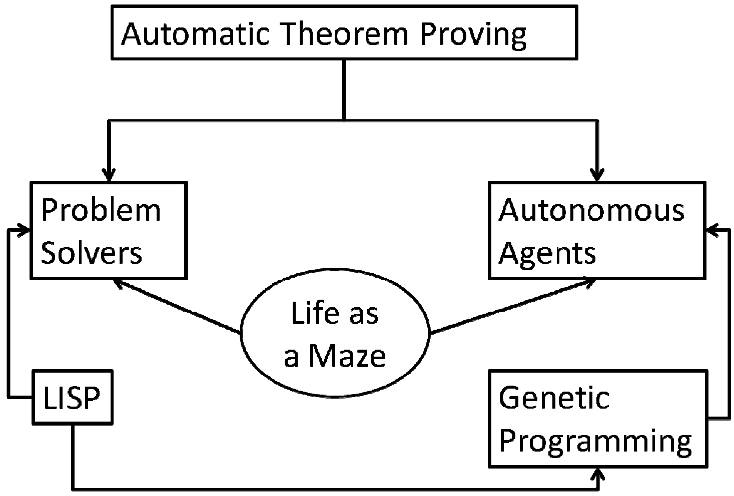

Figure 2: LISP and Genetic Programming

lem solving in general as being analogous to theorem proving, where one starts with a current state and tries to search for the paths to reach the target states with the assumptions or propositions that are available to him/her. *Logic Theorist* was programmed with *Information Processing Language*, a kind of list processing. This language is strongly motivated as a practical implementation of the *lambda* calculus or the recursive function theory that was developed in the 1930s by Alonzo Church (1903-1995), Stephen Klenne (1909-1994) and Alan Turing (1912-1954). It was later formally introduced as LISP by John Marcarthy (1927-2011).

The syntax of LISP has very broad usage to represent problems in many different domains. The universality of LISP is briefly mentioned in the lecture notes of Newell and Simon's Turing Award (Newell and Simon, 1976). One of the successful examples utilizing LISP that Simon developed is the Elementary Perceiver and Memorizer (EPAM) (Feigenbaum and Simon, 1984; Gobet and Simon, 1996). It is an important model in the theories of expert systems that can be used to explain the evolution and the organization of the associative memory of human subjects.

It is worth noting that genetic programming (GP) (Koza, 1992, 1994), which is used in ACE to model the autonomous agents, is essentially inspired by the LISP environment (Figure 2). The connection between GP and Simon, however, goes beyond just the syntax of LISP. Automatic theorem proving, which motivates the *problem solvers* in Simon's idea of rationality and computation, also motivates a notion of autonomous agents in ACE (Figure 2). The latter point has been elaborated in Chen (2012). GP is a tool for autonomous learning or the evolution of programs (can be rules, strategies, or recipes) without any external intervention. It, therefore, equips artificial agents (program solvers) with a *novelty-discovering* or *chance-discovering* capability so that they may constantly exploit the surrounding environment without external intervention, which in turn may also cause the surrounding environment to change or react, and the cycle may continue indefinitely.

Genetic programming as one of the most powerful models of autonomous agents has been widely acknowledged in the ACE literature (Duffy, 2006; Chen, 2008). Applications of genetic programming to modeling the constant searching for better strategies or products have been illustrated in various ACE models, including double auction markets (Chen and Tai, 2003; Chen and Yu, 2011), artificial stock markets (Kampouridis et al., 2012a,b), and oligopolistic competition with product innovation (Chie and Chen, 2013, 2014).

The modeling principle of leaving artificial agents a larger degree of autonomy arises because the problem introduced by the environment is frequently not well-defined, and may vary with agents' perceptions of the problem. In this case, external intervention is neither necessary nor feasible, and leaving agents to go wild on their own is the proper way of modeling this process as they are placed in a jungle or maze, searching for "truth" or proving a "theorem". This jungle is everywhere in life, but a good theoretic model to exemplify such a complex and perplexing environment is not often seen. In this regard, Simon's story of *Apple* (Simon, 1991) provides an illuminating demonstration, a subject to which we now turn.

3.2 Environment as a Maze

Two of the features of human problem solving (or decision making, in general) that are recurrent in the discussions by Simon are *representations* and *procedures*, that is, *what* and *how*. Representation is the subjective description of the problem or the solution (goal) by the decision making organism (not the observer!); a procedure, on the other hand, provides a sequence of actions which the problem solver can follow to reach the desired solution. One of the common and important characteristics is that the representation and the procedures are not constant over time, but are dynamic (and are shaped by the perceptions of the environment), even if the exact same problem (to be more precise, problem space) is encountered by the problem solver. The following quotation from Simon (1996b) describes the idea concisely:

> The external environments of thought, both the real world and long-term memory, undergo continual change. In memory the change is adaptive. It updates the knowledge about the real world and adds new knowledge. It adds new procedures that contribute to the skills in particular task domains and improve existing procedures. A scientific theory of human thinking must take account of this process of change in the contents of memory. (Ibid, p.100)

Simon's story of Apple serves as the best material for illuminating this idea. In our view, the constantly changing representation and the resultant procedures to be applied are in turn a direct consequence of the bounded rationality, subject to the combinatorial complexity of the environment that surrounds the decision makers. This idea was first presented formally with equations in Simon (1956)[7] and shall be briefly reviewed here since it enables us to better see the connection between Simon and ACE in their articulation of autonomous agents.

This story tells us that the problem space in which we try to search for the answer may often be too enormous for us to have a complete picture of it. With limited time, energy and memory to explore the environment in its totality, our tastes and goals are in turn shaped and altered from time to time during the process of search (either arbitrary or directed). The message of this metaphor is that even if the environment in which one is acting is absolutely static, we can end up having very different knowledge and representation about this environment, thereby creating different subproblems for ourselves and developing different strategies to survive.

Hugo, the ordinary and solitary man in Simon's Apple story, who lives in the "castle" all his life, portrays our everyday decision making in environments whose entire picture is often not known to us. Hugo desires several aspects of daily life, such as preferred food, aesthetic surroundings and comfort, while the limitation of resources he suffers is very rigid - his awake time, the energy he has before he collapses due to hunger, his memory, and a notebook on which he writes down the history of his traverse in detail[8]. A well-trained economist might soon formulate this problem in terms of optimizing multiple objectives subject to constraints. Simon realized very early on that the allocation or search problem based on marginal analysis does not work at all (Simon, 1983). Instead, Hugo is formally characterized as a set of rules that are gradually shaped by Hugo's understanding of the castle.

How is this related to the characterization of agents in the models of ACE? We address this question by teasing out the role of GP in these models. It is worth pointing out that the time-variant procedures used by agents to solve problems can be interpreted as a kind of evolutionary algorithm. If one subscribes to this interpretation, then its relation with GP is quite straightforward. The evolutionary computation as demonstrated by GP allows us to capture the phenomenon of agents' changing (evolving) representations with the change in their employed modules or chunks[9], and accordingly the change in their survival strategies in light of the possible change in their preferences (the fitness criteria). The notion of adaptivity that Simon discussed is well inherited by GP in the context of economic agents. In this vein, the literature on modeling innovation in the light of consumers' changing preferences using GP is very relevant here (Chen and Chih, 2007; Chen and Wang, 2011). However, the rules of evolution in GP can be enriched in the direction of heuristics rather than stochastic forces, as they stand now. We will suggest ways of doing so a little later.

3.3 Selectivity, Satisficing and Aspiration Levels

In his Nobel Laureate speech Simon (1979b), Simon spoke on the processes that people use to make difficult decisions and solve complex problems.

> Selectivity, based on rules of thumb or "heuristics", tends to guide the search into promising regions, so that solutions will generally be found after search

of only a tiny part of the total space. Satisficing criteria terminate search when satisfactory problem solutions have been found. (Ibid, p. 507)

In this section, our discussion is based upon the notion of intelligence (heuristics) - a major theme in Simon's research - in *characterizing different agents*. We do so against the backdrop of bounded rationality that is increasingly gaining acceptance as the appropriate way of characterizing agents even within mainstream economics. In particular, we will focus on the *characterization of intelligence* in terms of *selectivity*, *satisficing* and *aspiration levels*, the key components of bounded rationality as propounded by Simon, and see the development of ACE models in this light.

3.3.1 Selectivity

The idea of selectivity becomes important in situations when boundedly rational agents are acting in complex environments, where the problem space is huge and the agents are cognitively constrained. Humans' cognitive constraints, such as short-term memory or working memory capacity, allow them to process a limited portion of information to which they can get access, and can only effectively deal with limited number of alternatives at a time (Miller, 1956). In both cases - that is, a large problem space and limited cognitive capacity, selectivity helps them to deal with the difficulties of decision making. While these are the reasons why selectivity may be important, we also need to understand how such a selection is executed. One way for human beings to be effective in their selection is to apply familiar chunks[10] to increase their memory span and information-processing and decision-making capability (see also Section 4.2).

In ACE, chunks can be acquired by autonomous agents through information encapsulation and compression, and genetic programming can allow us to model such capability of autonomous agents. The *automatic defined terminals* (ADTs) as proposed in Chen and Chih (2007) in their ACE model of product evolution is an example. By searching for and encapsulating useful chunks, autonomous agents can compress the knowledge that they have acquired incrementally into simple but effective decision rules. Perhaps this avenue of exploration will develop GP in such a way that it can evolve fast and frugal heuristics (Gigerenzer and Selten, 2001; Gigerenzer, 2004). In fact, in some ACE models, genetic programming has already been applied to enable autonomous agents to develop their decision heuristics in the form of *evolutionary decision trees* (Kampouridis et al., 2012a).[11]

It is worth mentioning that partially in light of the recent developments in cognitive experimental economics some ACE models have explicitly considered the cognitive constraints (working-memory constraints) of autonomous agents. For example, the parameter *population size* of genetic programming or genetic algorithms has been chosen as a proxy for the working memory capacity or, simply, intelligence, of agents (Casari, 2004; Chen and Tai, 2010). By setting artificial agents with different population sizes, their working memory capacity, therefore, become heterogeneous. The consequences of heterogeneity in a human's cognitive capability can then be simulated by using these models jointly with human-subject experiments. This is entirely in the spirit and the vision of Simon (2000).[12]

3.3.2 Satisficing and the Aspiration Level

Satisficing is the other intimately-related notion which is of importance to ACE. Satisficing, as opposed to optimization, is the objective of a boundedly rational agent according to Simon (Simon, 1955a). This objective is achieved by looking for good enough solutions, which in turn are judged by *aspiration levels*. Intuitively, satisficing is more general than optimization, because one can aspire to find the 'best'. Satisficing is a natural consequence of a limited computational capacity and is also a common characteristic of various decision making organisms.[13] In the story *Apple* (Section 3.2), Hugo with his set of heuristics (rules) is an example of a satisficing agent, whose sub-problems can be seen as: *what to focus on*, *how to evaluate*, and *when to stop*.

A prototype of a computer agent (a machine with built-in mechanisms) behaving in a satisficing manner, and placed in an uncertain environment, can be found in Newell (1955). Newell's program appears to be a more concrete format of what Simon has proposed in Simon (1955a, 1956), including his story *Apple*. The intention of Newell (1955) is to program the computer to *learn* to play good chess. Chess is one of Simon's favorite theoretical settings, because its problem space is as massive as Hugo's castle and yet bounded; in fact, its possible states can be entirely derived from the rules of the game. In Newell's program, the machine decides which action to take based on answering questions that are arranged in a *goal structure*, and the program is characterized by a set of rules. Learning happens when the set of rules changes over time.

In an act of deciding what to do, there are a few sub-problems that need to be answered first (an act of divide and conquer); they are problems related to consequences, horizons, evaluations, and alternatives (Newell, 1955). The key to answering these questions lies in a thorough understanding of heuristics, aspiration levels, and sets of rules that the program (computer agent) can use. The architecture of this program coincides with the idea of list processing and genetic programming, which suggests that the satisficing procedures can be more sensibly brought into ACE models.

In fact, the satisficing behavior has been extensively included in many ACE models, in particular, the recent advent of agent-based macroeconomic models. In these models, the behavioral adjustments of households and firms, ranging from consumption, pricing, production, and employment to wages, are not based on the pursuit of optimizing a specific target function, but are based on some satisficing criteria, normally formed as *threshold-based* rules or *routine-based rules*. Take the *buffer stock rule*, one of the frequently used rules in agent-based macroeconomics, as an example. By this rule, the agent will figure out a normal income level, and adjust his consumption based on his current income and this normal income level (Raberto et al., 2008; Cincotti et al., 2012). As a generalized behavioral model of this kind we may assume that agents will basically follow a constant relation between consumption and income, a lifestyle, unless they experience some "unusual" changes characterized by some pre-selected indicators (thresholds). In fact, using what we did yesterday as a default unless some unusual conditions are met may be quite familiar; this habitual heuristic may be considered a kind of fast-and-frugal heuristics.

The adjustment of the aspiration level is one of the key components of satisficing behavior. Simon (1955a) perceives the aspiration level to be tied to the cost of search.

> The aspiration level, which defines a satisfactory alternative, may change from point to point in this sequence of trails. A vague principle would be that as the individual, in his exploration of alternatives, finds it easy to discover satisfactory alternatives, his aspiration level rises; as it finds it difficult to discover satisfactory alternatives, his aspiration level falls. (Ibid., p.111)

This dynamics of the aspiration level has also been incorporated into the ACE literature through its acceptance of prospect theory in general (Mueller and de Haan, 2009; Cincotti et al., 2010) and the *reference points* as an important decision anchor (Hommes, 2011). In addition, the recent developments in the field of ACE indicate that *social preference* can be another determinant of the dynamics of aspiration levels. People may anchor their aspiration levels to the satisfaction levels achieved by their neighbors as defined by their personal networks (Chen and Gostoli, 2012; Zschache, 2012) or anchor their aspiration levels to their social identities (Delli Gatti et al., 2011).

Having reviewed the connections between Simon and ACE at the level of agents (the left block in the middle layer of Figure 1), we now shift our focus to that of the system as whole (the right block).

4 Complex Systems - Modularity

The complex system approach to the social sciences is another aspect of Simon's legacy that has had an influence on ACE. We observe, however, the Simon's impact of complex systems on ACE is weaker than that of bounded rationality. One property that makes Simon's idea of complex system unique is that of *hierarchy*, which is in fact an ubiquitous property of many complex systems. Despite the divergence of views on complex systems between Simon and ACE, we think there is a need to raise the question as to whether Simon's idea of complex systems can have more of an impact on ACE today.

The way in which one can explore and understand a complex system is in itself nontrivial, especially if cognitively constrained agents are engaging in such a task. From an observer's point of view, we try to identify the key forces that govern the behavior of entities in the system. However, from a stakeholder's perspective, his limited capacity may only allow him to act locally with a few relevant entities. It is important to remember that the world may not appear to be so complex for the agent within the system in the same way as it is to the observer who is analyzing the system.

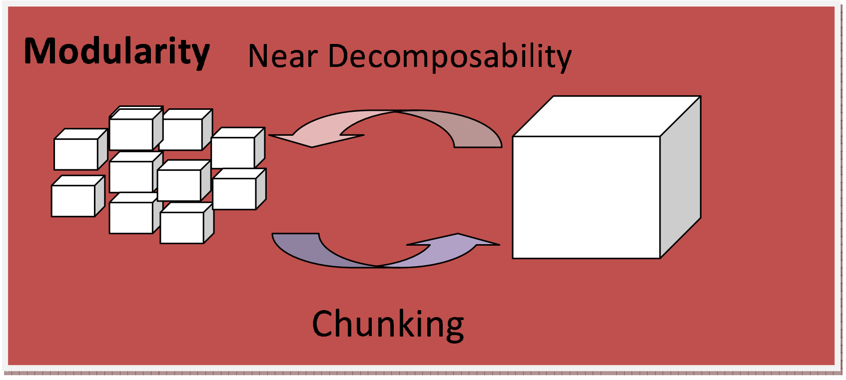

Figure 3: The relationships of modularity, near decomposability and chunking

No matter which perspective we choose to adopt, a recurrent property that aids in understanding and coping with complex systems is that of modularity.

Modularity is also a modern terminology that refers to Simon's idea of hierarchy in complex systems. Callebaut and Diego-Rasskin-Gutman (2005) is a collection of discussions on modularity from different aspects, in which Simon's contributions have been explicitly acknowledged. The word modularity, although not coined or used by Simon, is an adequate and a broad concept that encompasses two distinct ideas promoted by Simon, namely, *near decomposability*[14] and *chunking*. However, there is no clear distinction between the two mentioned in the literature. Therefore, we would like to emphasize here that near decomposability and chunking are two sides of the same coin (modularity): chunking is the bottom up concept of the development of a complex being and near decomposability is a top-down perspective for finding ways to simplify problems. Both of these notions have connections to hierarchy, and consequently, to modularity. See Figure 3 for an illustration. Near decomposability is tied to the perspective of an observer who tries to understand or predict the behavior of a complex system. It is also the unifying hypothesis that helped Simon to understand a variety of complex systems - such as physical systems, symbolic systems, human minds, and social systems (Simon, 1962, 1995, 1996a) as an observer.

Chunking, on the other hand, involves assembling several symbols (pieces of information) into a unit for recognition and operation in problem-solving tasks[15]. A chunk can further be joined with other chunks to form a bigger chunk. The ability to chunk is believed to be an important skill for any effective performance of complex tasks. The role of chunking (interpreted as recognizing and classifying patterns) in complex games such as Chess and Go is well documented (Newell and Simon, 1972; Gobet et al., 2004; Kao, 2013). We consider modularity, in terms of chunking, as an important element for learning and evolution. We will now explore both aspects - chunking and near decomposability - in greater detail with a focus on its relevance to ACE.

4.1 Complex Systems and Near Decomposability

The economic world that we study is often interconnected, complex and not fully decomposable. This in part renders the superiority of ACE, which focuses on the interactions among the actors in a system, and makes it a prominent alternative approach to the conventional economic analysis (Stiglitz and Gallegati, 2011). However, when the linkages among the actors are not equally strong, the weak feedbacks can be ignored in the short run. In that case, the description and the simulation of the model can be simplified. For builders of ACE, near decomposability can help make simulations better structured and a little smarter, following the principles of parsimony in modeling. Koza (Koza, 1994) has also emphasized that fact that decomposition is the regularity that appears in many situations.

Partial equilibrium analysis in economics employs the assumption of *other things being equal*, so that one can analyze the effect of specific changes in a particular market or a sector at a time, when neglecting others. This assumption contravens to the fact that economic sectors do interact or that these interactions can be ignored for the purposes of investigation. When there are changes in one sector and these changes affect the other sectors nontrivially, it may not be admissible to state that the other sectors can remain unchanged. Even in the case of models based on general equilibrium, for example, DSGE models, interactions are very often ignored.

> Built upon the representative agent (RA) framework, [DSGE] rules out most of the key macro-economic interactions by assumption: since most of what is relevant in economics concerns interaction and coordination of heterogeneous agents, the RA framework undermines macroeconomic analysis. ...In order to develop sound micro founded models, requires a methodology which allows for the interactions of economic agents and their links in a networked economy. ...A model with heterogeneous agents (ABM) interacting in a network has to be seen as a first step toward modeling serious microfoundations. (Stiglitz and Gallegati (2011), p.6-7)

When sectors are dependent on each other, the analysis becomes more difficult. This depends on the degree of coupling that exists between sectors, given the specific change that is being investigated. One noticeable observation is that the interaction between two sectors is not necessarily symmetric (Goodwin, 1947). This asymmetric interaction is referred to as 'unilateral coupling' and is related to the idea of 'causal' ordering which is investigated in Simon (1952, 1953) and Simon and Rescher (1966). The notion of a nearly decomposable system plays the role of reducing the difficulty and complication of analyzing coupled systems, a good property of parsimony that science looks for (Simon, 2001).

Analogous to an economic system solving a problem, Simon considers the phenomena of thinking and the evolution of organisms themselves as problem solving (Simon, 1995, 1996a). The key connection according to him is that the mind and organisms have a hierarchical structure[16]. If the structure is nearly decomposable, we can investigate the system with a certain degree of isolation. The speed of convergence - that is, finding a solution- *within* any subsystem is faster than it is *between* subsystems. Therefore, if the system is nearly decomposable, only the output of a certain subsystem will influence other subsystems. Furthermore, a nearly decomposable structure has certain implications for the nature of the evolution of the system as a whole. In Simon (2002), it is concluded that the organisms with a nearly decomposable structure, no matter how complex they are, will evolve faster than the organisms that have an indecomposable structure. Again, if evolution is viewed as a process of finding solutions, organisms with nearly independent components will need less time to find the solution than the ones without nearly independent components. Having discussed the importance of near decomposability in the study of complex systems in general, we now focus on the specific role of near decomposability within ACE.

4.1.1 Why or Why not ACE needs Near Decomposability

Simon had long observed that the complex systems have regularities that range far beyond chaos and unpredictability alone. Near decomposability is a property that allows us to break a system into subsystems, where mutual dependencies are not identically strong. These weak connections can be conveniently assumed to be negligible for the moment and the description of the system can be simplified. However, if at a certain level of decomposition, the model still fails to interpret or predict the real phenomena, we need to break the system further down. We cannot treat the actors in economic systems (agents or organizations) in terms of simple reaction functions. Instead, as each of them constitutes a complex system, their detailed architecture needs to be explored as demanded by the problem at hand. That is also why we see some developments within agent-based modeling that are geared towards moving hand in hand with advances in cognitive psychology and neuroscience.

Taking the point of view of reductionism, all observed phenomena are emergent properties that trace their origin all the way back to quark. However, as a social scientist who tried to understand complex social phenomena, Simon was of the view that decomposition to a neural level was more than enough. In fact, Davis (2013) brings out the idea of decomposability to the emergence of ACE and suggests that the basic unit of simulation is not the individual, but the rules (bits) embedded in the individuals. However, what is perhaps lacking in contemporary agent-based modeling has to do with interactions that take place at an organizational level.

We also find that the ACE literature, as it stands now, lacks a consensus on how to best characterize different institutions within models of economic and social systems. The most prevalent practice seems to be a binary characterization: individuals and the aggregate. Simon's suggestion was to view the relations between each level of the system in a hierarchical fashion, an idea that dates back to Simon (1962). Such relations are recurrent across the board according to Simon, regardless of whether they are between neurons and the mind, mind and behavior, individuals and organizations, or organizations and social systems. He observed that each of these systems, which are potentially complex, are governed by similar modular structures.

However, a comprehensive review of the ACE literature may lead one to find that most agent-based models developed so far are confined to only two levels, such as the individual-market hierarchy and the firm-market hierarchy. The recent development of agent-based macroeconomics does provide a three-level hierarchy, individual-market-aggregate. Nonetheless, even in this case, these levels are given exogenously. For example, firms are not endogenously formed through individuals. Hence, by and large, it may be fair to say that we have not seen agent-based economic models as being able to demonstrate the evolving hierarchical near-decomposable systems as an essential characterization of complex adaptive systems. One may anticipate that the agent-based models of organizations, specifically those focusing on internal organizations, may have some behavioral algorithms to form hierarchies endogenously. However, as Chang and Harrington (2006) have shown in their survey article, such work is rare, and it is still so.

On the other hand, ACE has started to take inspiration from Neuroeconomics (in particular the dual system hypothesis) to build software agents (Chen, 2014). It is not clear how much understanding of the phenomena might be blurred by looking at the relations between neurons and social networks directly and what is the cost of overlooking these important acting middle levels. It is also arguable for the ACE community to question whether the hierarchical fashion in a complex *social* system can be unambiguously defined. Information flow among actors is perhaps the essence of social computations. Due to the breakthroughs in the Internet and platforms of social interaction, the format of information flow has gone beyond the conventional understanding that is underlined by institutions (for example, departments and colleges), and hence has transcended what Simon could have imagined in his time.

Social scientists might argue that in today's world, with the revolution in information technology, everyone is almost fully connected (via the Internet) and thus near decomposability might not be valid any more. So one might argue that the complex social systems that we are capable of studying today, like in ACE, have managed to go beyond the demands of near decomposability in terms of structure, and thereby advancing beyond Simon's vision in some respects. However, it is important to remember that while social media provides new means for people to connect with each other faster and in a less costly way, the question of purposiveness behind the establishment of such a connection or a formation of a social network is yet not fully understood. At the least, we can say that the importance of near decomposability for ACE still remains inconclusive.

4.2 Chunking Theory

Chunking theory has important applications in the organization and characterization of knowledge. The idea of chunks, although not proposed by Simon, was heavily built in Simon's research on expertise, which is an interdisciplinary ground of psychology, cognitive science and computer science. Some of Simon's pupils and colleagues in the area of computer science had elaborated this approach in detail in relation to the field of *expert systems*, with particular reference to human cognition and memory, such as, Feigenbaum and Simon (1984); Gobet and Simon (1996, 2000); Gobet et al. (2004). The central questions of this research program involve understanding how human beings develop expertise in a specific, complex field or a task.

Following the Miller tradition (Miller, 1956), Simon was aware that the size or organization of chunks rather than the number of chunks matters for the excellent performance of agents. However, the natural result that happens to an expert who is in a field for over 10 years is that there are more than 50,000 chunks that are accumulated over time (see, for example, Simon, 1996b, p.89). If the size of short-term memory is really small and similar across human minds, then it is possible to conjecture that experts will further group the isolated chunks into bigger chunks and only retrieve the sub-chunks when necessary. In other words, experts may organize their chunks in a better and more complex fashion, and at the same time evolve better heuristics to access subportions if and when necessary. A piece of evidence can be found in Simon and Schaeffer (1992) which demonstrates the expert-novice differences in the task of memorizing chess board configuration. The experiment shows that the expert can reproduce a configuration from a famous game (around 25 pieces) quite correctly with only 5 seconds of staring time. Unsurprisingly, the novice can at best retrieve 4-5 arbitrary pieces. However, when both of them are presented with a random and 'meaningless' configuration, the expert and the novice perform equally badly. This immediately shows that the expert is not 'smarter' in terms of the ability to memorize, but has the ability to recognize the meaningful chunks on the board in a very short time[17].

One of the immutable laws in the ever changing world is that individuals never cease to learn, and what Simon suggests is that they learn by chunking or modularizing. In fact, for a complex adaptive system to grow or to improve, the ability to chunk can be seen as a necessary property. To handle huge computer programs well, one requires a modular design, and to become an expert in a particular domain, one needs to have a modular thinking and modular memory for keeping an immense amount of symbols and information, despite the severe limitations of memory capacity.

Going back to the Apple story (Section 3.2), Hugo is definitely gaining more understanding about the castle, and the more he knows, the more picky he becomes. We know for sure that he is learning to acquire tastes; however, it is not clear how he gradually prefers one kind of bread over the others. His experience and memory play a heavy role in shaping his tastes. It is quite obvious that his behavior cannot be interpreted well in terms of Bayesian learning (a kind of optimal learning), because the state of affairs that Hugo experiences is neither fixed nor infinite.

It is important to note that GP also uses a similar, modular structure in its encapsulation of knowledge (Roberts et al., 2001). Although chunking theory has been applied to agent-based modeling in other fields, for example, linguistics (Liang and Zhao, 2005), it has not yet gained popularity within the ACE community. We believe that the notion of modularity in genetic programming is perhaps the right direction to explore

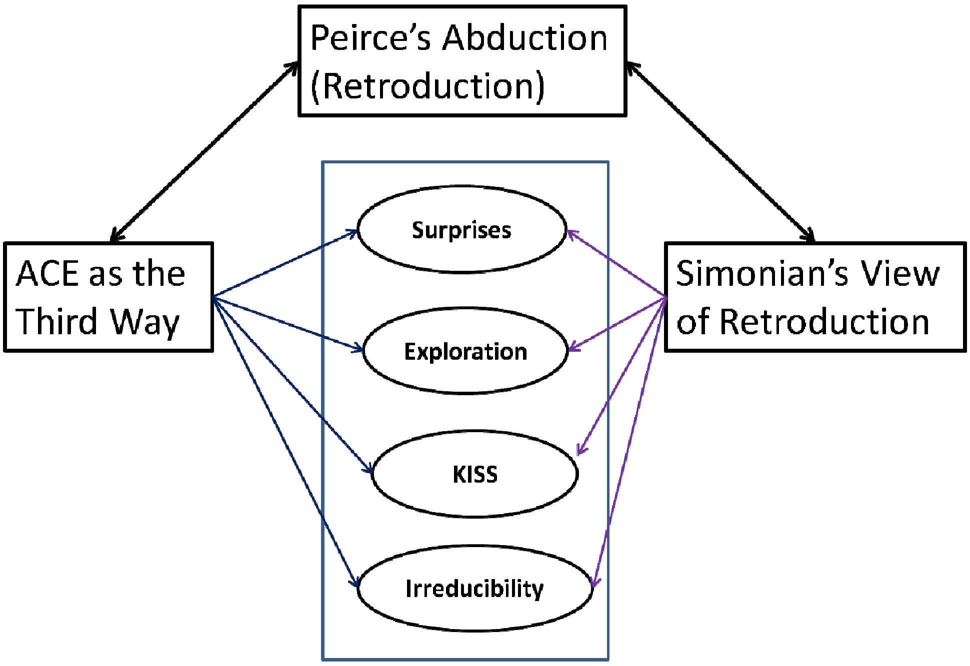

Figure 4: ACE and Simon through Peirce

this property (Chen and Wang, 2011).

5 Scientific Discovery and Market Design

After seeing the connections between Simon and ACE at both the individual level and the system level, we now move to the top layer of Figure 1 and address the connection between the two in a simulation from an epistemological viewpoint. The layout of this section is first summarized in Figure 4. We begin by asking: what is the mode of unearthing new knowledge that ACE has to offer? How is it different from the rest of the approaches that already exist, and can we know more? To answer this question, we need to focus on an important aspect of Simon's contributions to the philosophy of science and, in particular, to the logic of scientific discovery (Simon, 1977; Langley et al., 1987).

Social sciences that empirically examine the complex interaction of entities are often categorized as sciences of induction. On the other hand, orthodox economic theorizing that is underpinned by axioms, assumptions and infinite iterations, is in fact best approximated as a branch of applied mathematics, where the possible outcomes of the enquiry are obtained from deduction. While deduction and induction are the two familiar types of reasoning, one has to realize that agent-based computational modeling and simulation constitute neither a method of deduction (theory), nor a method of induction (statistical inference). The distinction from the usual deduction and induction has been well acknowledged by economists and social scientists (Axelrod, 1997; Axelrod and Tesfatsion, 2006; Gallegati and Richiardi, 2011; Halas, 2011).

Axelrod (1997) proposes that agent-based social simulation can be considered as the third approach, that is, in addition to deduction and induction, to science.

> Simulation in general, and ABM in particular, is a third way of doing science in addition to deduction and induction. Scientists use deduction to derive theorems from assumptions, and induction to find patterns in empirical data. Simulation, like deduction, starts with a set of explicit assumptions. But unlike deduction, simulation does not prove theorems with generality. Instead, simulation generates data suitable for analysis by induction. Nevertheless, unlike typical induction, the simulated data come from a rigorously specified set of assumptions regarding an actual or proposed system of interest rather than direct measurements of the real world. Consequently, simulation differs from standard deduction and induction in both its implementation and its goals. Simulation permits increased understanding of systems through controlled computational experiments. (Axelrod and Tesfatsion (2006), p. 1650.)

While Herbert Simon, to the best of our knowledge, did not address this issue directly, he did notice the limitation of normal induction.

> Students are always told that they can't run a successful experiment if they don't have a hypothesis. ...I believe that is a very bad criterion for the design of experiments....If you look down the list of outstanding discoveries in the physical sciences or the biological sciences - look at Nobel awards in those fields - you will note that a considerable number of the prizes are given to people who had the good fortune to *experience a surprise*. (Simon et al. (1992), p. 22; Italics Added)

At this point, agent-based simulations are related to Simon's comment since some emergent phenomena coming out of agent-based simulations bring us novelties and surprises, which inspire us to make hypotheses of these observations (Figure 4). In this sense, some social scientists, such as Gallegati and Richiardi (2011) and Halas (2011), also relate agent-based social simulation to what Charles Peirce (1839-1914) called *abduction* or *retroduction*.[18] Peirce advocated that there is a type of logical reasoning beyond deduction and induction. He called this unique type of reasoning abduction, and suggested that it was the logic of discovery (Peirce, 1997). While, for many philosophers of science, abduction is treated as a part of induction, Peirce forcefully distinguished between the two by indicating that induction is about the test of an established hypothesis using observations, and that abduction is about the formation of the hypothesis.

The connection between Simon and Pierce on abduction is also noticed (Velupillai, 2010a).[19] Simon (1998) explicitly supports the view that the aim of science is to discover, and the process of discovery is neither in the form of pure deduction, nor pure induction. However, Peirce's original notion of abduction is not entirely clear, and its vagueness has invoked many objections. Simon (1973) actually clarifies the notion of Peirce's *retroduction*.

> Peirce coined the term 'retroduction' as a label for the systematic processes leading to discovery....It is the aim of this paper to clarify the nature of retroduction, and to explain in what sense one can speak of a 'logic of discovery' or 'logic of retroduction.' " (Ibid, pp.471-472)

Simon (1973) presents two examples to demonstrate the retroductive process of law discovery: recoding a sequence of letters and concept attainment. He states: "A *law-discovery process*" is a process for recoding, in parsimonious fashion, sets of empirical data. This passage can be understood by reading Simon (2001), where he elaborates that parsimony is the criterion for choosing among possible explanations. This idea has the agreement of what is stated as being the "best explanation" in the following quotation

> Peirce's abduction is now generally identified with more developed and refined version called inference to the best explanation (Harman 1965; Lipton 2004), which seems to solve the problem of both what hypothesis we draw from available data, as well as why we prefer that particular hypothesis. (Halas (2011))

Hence, abductive reasoning is close to hypothesis discovery. When a phenomenon is observed, the first question is to find hypotheses that can explain the phenomenon (what), but there is always more than one explanation, so the next question is to find the best explanation (why). The simplicity principle or the parsimony principle underlying scientific discovery, also known as *Occam's razor*, as suggested by Simon, has a great influence on the ACE practice. In the ACE community, the parsimony principle has received an even more romantic name, the KISS (*keep it simple, stupid*) principle, as originally proposed by Robert Axelrod (Axelrod, 1997).[20]

The cellular automaton models (Section 2), such as the Schelling segregation model (Schelling, 1971), the Albin Spatial Prisoners' dilemma model (Albin, 1992), and the Keenan-O'Brien local oligopolistic competition model (Keenan and O'Brien, 1993), show how complex patterns can be formed using simple agents interacting with each other in a social network by following simple rules. The key message of these models is two-fold. First, complex unpredictable patterns can emerge from very simple homogeneous interacting behavior. Second, a small change in the individual rule may fundamentally change the nature of the system dynamics from a lower hierarchy of complexity to a higher hierarchy of complexity. This also motivates the name, the 'edge' of chaos, that is, a slight change of the rule on this edge will either result in a stable pattern or a chaotic pattern. The unexpected complexity of the behavior of these simple rules leads us to suspect that *complexity in nature may be due to similar simple mechanisms*. [21]

Epstein and Axtell (1996) is probably the first study introducing the agent-based model in the study of big history. In Epstein and Axtell (1996), the fundamental collective behaviors, such as group formation, cultural transmission, combat, and trade, are seen to emerge from the interaction of individual agents following a few simple rules. In agent-based financial models, it is found that the models with simple heterogeneity and simple rules, in particular the variations of the fundamentalist-chartist model, are sufficient to replicate a number of stylized facts. A complex extension of this model may gain additional explanatory power, but so far this power has not been well exploited (Chen et al., 2012). In addition, the simple model makes the later econometric estimation much more feasible.

Maybe, the most prominent example is the simple device, the zero-intelligence agents (Gode and Sunder, 1993). The zero-intelligence device is actually the application of the *maximum entropy approach* to agent-based modeling (Chen, 2012). The capability of this approach to replicate complex financial dynamic systems shows that some aggregate phenomena generated from human-agent systems with complex motives and behavioral rules can be rather well approximated by a system with simple agents characterized by simple motives and simple rules. In a sense, it indicates that adding more complex strategies to the agent-based models may have little macroscopic effect if these complex strategies may interact in such a way that they mutually annihilate the forces of each other. It is this possibility prompting many of us to think about a general physical system which is equipped with the most rudimentary forces but can overarch several seemingly unrelated social phenomena, for example, from pedestrian counterflow to the Schelling segregation model (Vinković and Kirman, 2006), to the El Farol Bar problem (minority games), and then to financial markets.

5.1 Preferential Attachment

The search for the universal pattern underlying different disciplinary phenomena is in line with the pursuit of the parsimony principle. In his book *Physics of Social Phenomena: An Essay on Human Development* published in 1835, Adolphe Quetelet (1796-1874) had already attempted to search for some statistical laws underlying a class of social phenomena. Efforts in search of the universal pattern have been further elucidated by Simon (1955b), who tries to identify a class of distributions which are applicable to rather extensive social and natural phenomena. These distributions include two skewed distributions, which are frequently cited in the ACE community, one being the Pareto distribution of income and the other the Zipf distribution of the frequency of the occurrence of words. Simon's pioneering work provides an empirical foundation for one kind of universality which motivates physicists to work on economics or the social sciences.

The skewed distribution studied by Simon has been constantly followed and extended by others in the economic literature and, recently, also pursued by the ACE community. The development of this literature can be roughly characterized by three directions. First, the skewed distributions are found to be applicable to many more economic variables. In addition to income and wealth, they have also been applied to firm size, asset returns, city size, film returns, innovation size, and so on (Gabaix, 2008). The second direction concerns the statistical or econometric techniques chosen to identify the appropriate skewed distribution among many possibilities. In addition to the frequently-cited Pareto and Zipf distributions, there are lognormal and Yule distributions plus many generalizations of them that are often considered by the ACE community. These distributions may look similar by simply eye-browsing. Therefore, the distinction among them requires deliberate statistical analysis (Gallegati et al., 2006).

One important reason for distinguishing different skewed distributions is that they may be associated with different underlying mechanisms. An example shown by Simon is that depending on whether the birth process is involved, one can have either a Yule distribution or lognormal distribution (Simon and Bonini, 1958). Therefore, the third development in this line is to build the theory or offer explanations that underlie these distributions. The mechanism proposed by Simon is a cumulative advantage mechanism, which is based on an early work by a British statistician Udny Yule (1871-1951). Later on, this mechanism, also known as *preferential attachment*, was applied to form the *scale-free network* by Albert-László Barabási and Réka Albert (Barabási and Albert, 1999), and had a great influence on the literature of complex networks in general (Mitzenmacher, 2004; Gabaix, 2008) and on ACE models specifically (Alfarano and Milakovic, 2009; Alam and Geller, 2012; Cederman, 2002; Page, 2012).[22]

5.2 Simulation and Design

> There are two related ways in which simulation can provide new knowledge - one of them obvious, the other perhaps a bit subtle. The obvious point is that, even when we have correct premises, it may be very difficult to discover what they imply. All correct reasoning is a grand system of tautologies, but only God can make direct use of that fact. The rest of us must painstakingly and fallibly tease out the consequences of our assumptions. (Simon (1996b), p.15)

The uniqueness of ACE, as the third way of doing science, is its use of simulation as a primary tool for discovery. In other words, if we don't run a simulation, we simply cannot be assured what may happen. This property is coined as *computational irreducibility* by Stephen Wolfram (Wolfram, 2002). According to Wolfram (2002), if the behavior of a system is not obviously simple, then it will generally be computationally irreducible, which means that the only way to predict its evolution is to run the system itself. There is no simple set of equations that can look into its future. By distinguishing the phenomena known as computationally reducible from those known as computationally irreducible, Wolfram (Wolfram, 2002) argues that the conventional sciences are mainly efforts devoted to computationally reducible phenomena. A new kind of science can then be considered to be a paradigm shift toward the study of computationally irreducible phenomena. Wolfram's proposal is not limited to physics or biology. If one applies this irreducibility characterization to economics or the social sciences, one can equally perceive a new kind of economics or a new kind of social science; for example, see Borrill and Tesfatsion (2011).

> Not only in practice, but now also in theory, we have come to realize that the only option we have to understand the global properties of many *social systems* of interest is to build and run computer models of these systems and observe what happens. (Ibid, p. 230; Italics, added.)

As also mentioned in Vriend (1995), "...we are interested in those regularities that cannot be deduced from the built-in properties of the individual agents or some other microeconomic aspect of the model; at least not by any argument which is substantially shorter than producing that regularity by running the simulation itself. (Ibid, p.212)" As a footnote to this quote, Vriend added "Clearly, the emergent behavior and self-organization *are* a function of the underlying configuration. The relevant point is, however, the following. Given a certain model with a certain parametrization, can one reason, that is, without running a simulation, *which* functions of the parametrization the outcomes are? (Ibid, p. 228)".

Through agent-based modeling and simulation, one can navigate into the territory of computational irreducibility, and explore both "known unknowns" and "unknown unknowns". This endeavor may be useful for market design and policy design. As eloquently described by Mirowski (2007), markets in the history of economic theory have either been taken for granted or have been simplified into just exchange behavior or have been lumped together as abstract entities. The recently arising field of microstructure in finance and market engineering draws our attention to the complexity and variety of markets (Roth, 2002; Mirowski, 2007). Markets are not created by nature, but are artificial; like other man-made artifacts, markets evolve over time. Successes and failures are constantly seen in the course of the evolution. McMillan (2003) has a rich collection of illustrations. Since a market design is not just about a set of rules operating the market, but also involves agents' behavior under these rules, either individually or collectively through their interactions, it will not be hard to be convinced that market processes can generally be computationally irreducible. If so, market design as a science can benefit from the involvement of agent-based modeling and simulation. Currently, ACE have been applied to financial markets, electricity markets, fish markets, housing markets, school admission systems, national lottery design, tax evasion, futures markets, and labor markets. It is a matter of time to see when the state of the art will advance into reality.

For Simon, simulations or viewing economic entities' behavioral rules as computer programs are ways to explore problems and find out possible solutions:

> The use of computer simulations will also enable economics to build realistic theory of firm that will go far beyond the traditional production function and short- and long-run cost curves into characteristics of organization structure and human motivation and their consequences for the decision making process. (Simon (2000), p.36)

ACE can be seen as a continuation of what we quote from Simon. It will provide a realistic theory of markets, hierarchies, and networks by bringing light to deep darkness in the sea of complexity.

6 Concluding Remarks

Based on what we have reviewed in this chapter, it is clear that Simon's connection to ACE is probably more comprehensive than that of Thomas Schelling, Elinor Ostrom, or Friedrich Hayek. Yet, this connection has been much ignored by the ACE community, and, sometimes, has been simplified to just 'bounded rationality'. In this chapter, from a computational-theoretic underpinning, to artificial cognitive and psychological agents, to complex systems, and further to the epistemology of simulation (Figure 1, once again), we give a more thorough systematic treatment, by extending Simon's 'one-dimensional' connection with ACE into a 'multi-dimensional' one.

Of course, by 'connection', we carefully mean that the current state of ACE was developed largely outside of Simon's influence. However, establishing this connection can still be useful. For example, many ACE models tend to ignore their computational theoretical underpinnings, and underestimate the complexity of the ACE models as abstract machines, such as Turing machines. This ignorance and the subsequent ignorance of the undecidability property may lead us to be overconfident for the effectiveness of validation and robustness checks when an effective algorithm to perform these jobs does not even exist (Velupillai and Zambelli, 2011).

In addition, by 'connection', we also mean that ACE may not fully stand on the same side as each argumentation made by Simon. Whether the system has to be near decomposable to be scientifically interesting is not immediately clear for ACE. Presumably, every single unit (decision maker) can depend upon every other decision unit without being guaranteed a fixed sub-structure. The network connecting them may constantly evolve (Davis, 2013), which causes the identification of near decomposable subsystems to be very challenging. However, at this point, ACE cannot formally address this question because as we mentioned earlier most ACE models have only two layers. A truly hierarchical ACE model is yet to be seen.

> "When one finds a vein of gold, was it nature who lost it? If we can find gold that we haven't lost, perhaps we can answer questions that we haven't asked. ...Hence the dictum of Pasteur: "'Accidents happen to the prepared mind."' Simon (1991), p. 369

Simon's scientific odyssey was driven largely by one question: how do human beings solve problems? To his surprise, this question led him to many interesting subproblems that he needed to find answers to in different spheres, and he never had to find a new problem to solve! His wonder was not only restricted to "individual", but also to any economic or behavioral unit. All the economic entities are subject to limited time and computational capacity in solving problems, and yet they are able to cope with the complex environment with their own heuristics. All precisely specifiable heuristics are programmable and hence a paradigm like ACE provides a suitable laboratory for observing the results of interacting economic entities.

We have identified many possible channels through which the legacy of Simon is carried on in today's ACE. We have also suggested ways in which one can carry ACE a little bit further towards Simon's views on rationality and complex systems. For example, first, we are concerned with whether the evolution of heuristics (strategies) can rake place in more human-like ways; second, we feel that adding one more level to the agent-based modeling - as institutions in between individuals and aggregate phenomena - can enhance our understanding and discovery of the complex systems as perceived by Simon.

Simon described himself as a scientist of problem solving and he believed that scientific knowledge is piled up by a series of actions of problem solving. By standing on Simon's shoulders, we aspire that ACE, when it reaches maturity, can provide useful knowledge for extracting gems from complexity.

Acknowledgements

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Notes

[1]Simon may never have used the term modularity, while his notion of near decomposability was frequently renamed as modularity inthe literature (Egidi and Marengo, 2004), and Simon did not seem to object to this different name (Callebaut and Diego-Rasskin-Gutman, 2005). Of course, one has to pay particular attention to the fine difference between the two. In particular, modules, as manifested in many concrete applications, may be viewed as only parts of near-decomposable systems. They serve as the elementary units, which are fully decomposable and fully encapsulated and upon which near decomposable (weakly-interacting) systems can be built. More discussion can be found in Section 4.

[2]John Davis (Davis, 2013) has recently made an attempt to place agent-based models in the context of Simon (1962). Using what is called the *Simon Hierarchy*, his work prompts us to reflect upon agent-based models as complex adaptive systems. The essence of Davis (2013) will be reviewed later in this chapter.

[3]For example, we have had a brief overview of the collected papers of Simon and about Simon, such as Simon (1957, 1977, 1979a, 1982a,b, 1989); Klahr (1989); Simon et al. (1992); Simon (1997); Augier and March (2004), and Herbert A. Simon (3 volume set) (Wood and Wood, 2007), and we have found almost no such connections.

[4]Simon's discussion along with all other abstracts of papers presented at the meeting were reported in *Report of the Harvard Meeting, August 31-September 5, 1950*, Econometrica, Volume 19, Issue 1, pp. 55-72. The one-page discussion was later collected by Simon in Simon (1977), chapter 4.1.

[5]See Velupillai and Kao (2014) for the details.

[6] Simon was quite pleased to see a demonstration of zero-intelligence agents (Gode and Sunder, 1993):

> A parsimonious economic theory, and an empirically verifiable one, shows how human begins, using very simple procedures, reach decisions that lie far beyond their capacity for finding exact solutions by the usual maximizing criteria. A recent example that I like is the work of Shyam Sunder ... and his colleagues on the equilibrium of markets with 'stupid' traders, and the near indistinguishability of such markets from those with optimizing traders. When we have remade economic theory on that model, we will be able to write honest textbooks. (In Simon's letter to Kumaraswamy Velupillai, on 25 May, 2000, reprinted in Velupillai (2010a), p. 409-410.)

The reader can also refer to p. 30 in Simon (2000) for more discussion on zero-intelligence agents by Simon.

[7]Simon (1956) describes a circumstance where an organism is searching in an environment where the aim to satisfy its goals. The problem is designed in such a way that the desired objects are scattered randomly in the environment, which has the form of a *tree structure* (nodes branching into other different nodes). A few parameters quantify the environment, and other parameters quantify the physical constraints of the organism. With the aid of the parameters, the probability that the organism can survive when faced with starvation can be calculated. The situation where the organism cares about multiple goals can also be formulated using this model. This idea can be better understood if the paper is read together with Simon (1955a). A more detailed comment from the point of view of computational complexity can be found in Velupillai and Kao (2014).

[8]Hugo's notebook is an analogy of the long-term memory, in our view. After he collected enough data in the notebook, he began to identify patterns from history. For example, he even started to infer the correlation between the color of the wall and the food presented on the table. In order to make his choice slightly more effective, he needs to organize his database for good inductions. Simon seemed to imply that inference is a very natural action that human beings acquire in the course of decision making.

[9]In genetic programming, these primitives are known as terminals or functions. Hence, technically speaking, the agents' set of terminals and functions may change over time, which helps them gain a different representation of the problem surrounding them, even though the environment remains unchanged.

[10]A chunk is generally defined as an organized or grouped unit that is familiar and can be recognized by the subject. An relevant example is in English language. Each letter of the alphabet can be a familiar chunk, and in turn, the vocabularies composed by permutations of letters of the alphabet are also chunks. Likewise, familiar phrases or sentences composed of vocabularies are chunks as well. By enlarging the organization of a chunk, one can essentially remember more bits of information without being subject to the limitation on the number of chunks.

[11]There are a large number of ACE models built upon evolutionary computation algorithms, including genetic algorithms (GAs) and genetic programming (Chen, 2002). These algorithms can be considered biased search in an immense space, which is close in spirit to Simon's selectivity. In GAs, chunks are known as *building blocks*. The implicit parallelism applied to evaluate a large number of building blocks allows us to identify the promising search area, instead of blindly random search.

[12]See Simon (2000), pp. 34-36.

[13]For the formal discussion of this idea, the interested reader is referred to Velupillai (2010b).

[14]When the data are organized in a matrix, then decomposability can be understood with a rigorous mathematical definition. A square matrix A is said to be decomposable if there exists a permutation matrix P such that

$$PAP^T = \begin{bmatrix} B & 0 \\ C & D \end{bmatrix} \tag{1}$$

Otherwise A is indecomposable. If 0 is replaced by a small amount ϵ, then A is nearly decomposable.

[15]See Simon's famous watchmaker example in Simon (1962)

[16]Such a structure also appears in programs. Koza made it very clear in the introduction of Koza (1994) that the *automatically defined function* can successfully solve many complex problems, especially when the three-step hierarchical problem solving (divide and conquer) is activated. The three steps

are decomposing, solving the subproblems and solving the original problem. This is squarely within Simon's approach to problem solving.

[17]According to other related experiments, the number of chunks that the expert can hold in the working memory is still governed by Miller's magic 7.

[18]Peirce used the terms abduction and retroduction interchangeably in his early writings (Chiasson, 2005).

[19]See Velupillai (2010a), Appendix 2 of Part

[20]Having said that, we must also point out the opposition to this principle. The equally well-known alternative is the *KIDS* (*Keep it descriptive, stupid*) principle, proposed by Bruce Edmonds and Scott Moss (Edmonds and Moss, 2005). It was argued that social simulation models are different from analytical mathematical models, hence the pursuit of simplicity should also change accordingly. The contrast between KISS and KIDS is an on-going research issue in the methodology of social simulation. The interested reader is referred to the special issue on "The Methodology of Simulation Models" of the *Journal of Artificial Societies and Social Simulation* (Vol 12, No. 4, 2009).

Furthermore, even though the simplicity principle is generally known as the *minimum description length* (MDL) *principle* and can be regarded as a *generalized maximum likelihood principle* (Rissanen, 1989), one should be ready to accept any "surprise" that the ACE model may offer, and one such kind of surprise is the inconsistency between the micro motives and macro behavior (Schelling, 1978). In fact, given the observed aggregate phenomenon, by the simplicity principle, the most compelling hypothesis is the one that is consistent between the micro and the macro level. Nothing can be more simple than linear scaling-up. However, if we do so, we are back to the mainstream representative-agent approach to economics, and are no longer doing ACE. Hence, what makes the Schelling model intriguing is that the observed segregation phenomenon can actually emerge from a group of people who can each be tolerant of different kinds (ethnicities) of people.

[21]Wolfram (2002) gives a quite lengthy discussion of the behavior generated by his cellular-automation rule 30 (Ibid., p.27-30), and rule 110 (Ibid., p. 32-38), showing how extremely simple rules can generate highly complex, random, unpredictable patterns. In addition, a one-bit change from the binary string rule #126 to #110, may fundamentally change the nature of the system dynamics from a lower hierarchy of complexity, namely, Class III (linear bounded machines) to a higher hierarchy of complexity, namely, Class IV (Turing machines).

[22]The preferential attachment rule is an intuitive behavioral rule for the new nodes (newcomers, immigrants) for forming their personal networks with the existing nodes (local residents). Basically, the newcomers will consider who are the most important persons in the town and attach higher probabilities to connect with them. In the Barabási-Albert scale-free model, the importance is measured by the number of connections. Hence, the nodes that have been already connected extensively will attract more newcomers than those who are less connected. This idea of *preferential attachment* is similar to the classical "*rich get richer*" model proposed by Simon (1955b). In fact, the Barabási-Albert model which leads to the power-law degree distributions is an independent rediscovery of earlier work by Simon (1955b) on systems with skewed distributions. It can be interpreted as an application of Simon's growth model in the context of networks, readily explaining the emergent scaling in the degree distribution.

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